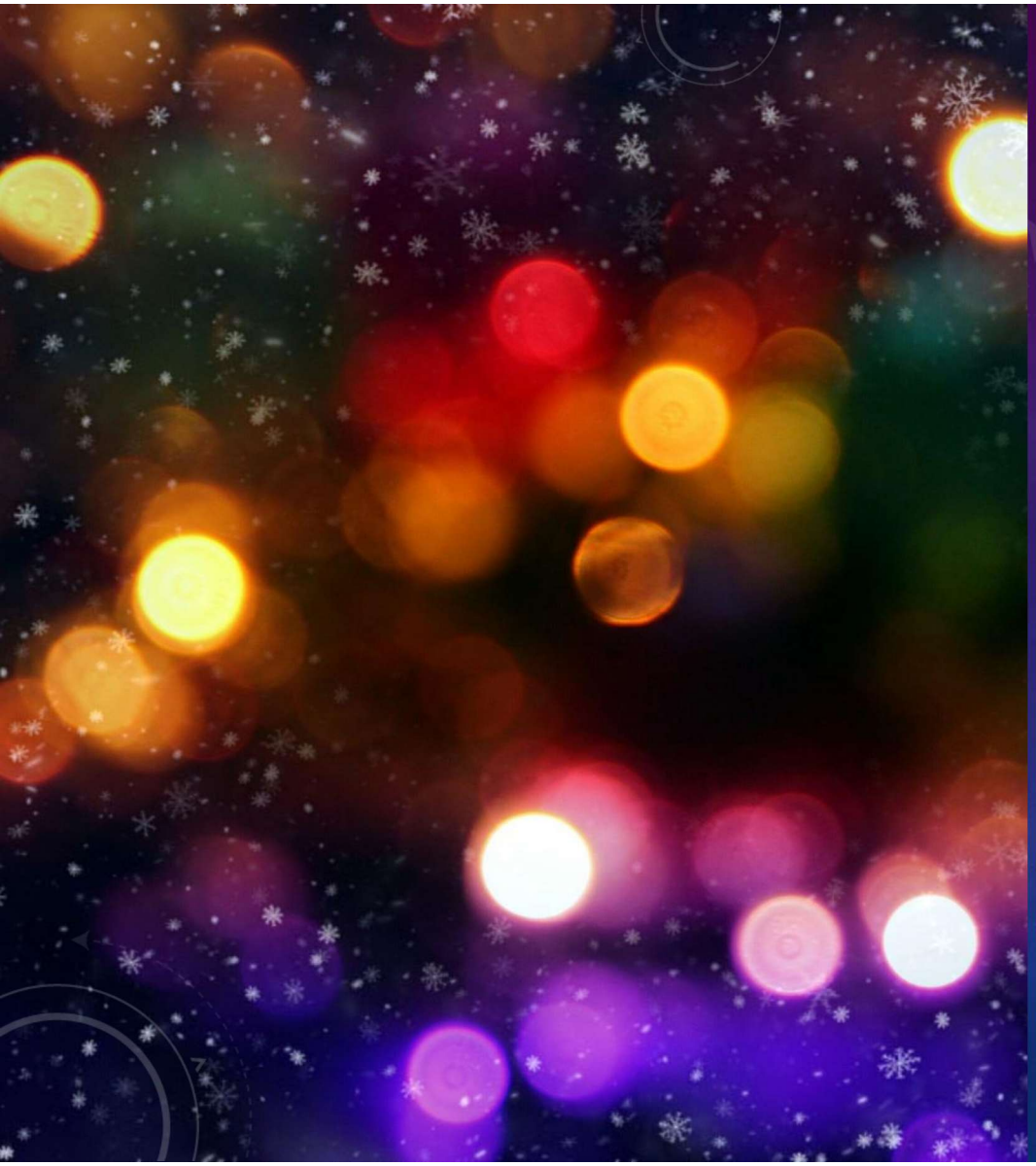

CS210 - DATA STRUCTURES AND ALGORITHMS 1

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QUICKSORT

QUICKSORT

Quicksort is a very useful sorting algorithm

In the majority of situations it operates in O(n*logn) time (a quicker O(n*logn) than mergesort)

It also doesn't need additional memory to run (mergesort needed an extra O(n) amount to store the workspace array)

Like mergesort, quicksort is a recursive sorting algorithm, splitting up an array and calling itself on each half

OVERVIEW

Partition	Partition array (or subarray) into two halves, putting low values into one half and high values into the other half
Call	Call quicksort on the left half
Call	Call quicksort on the right half

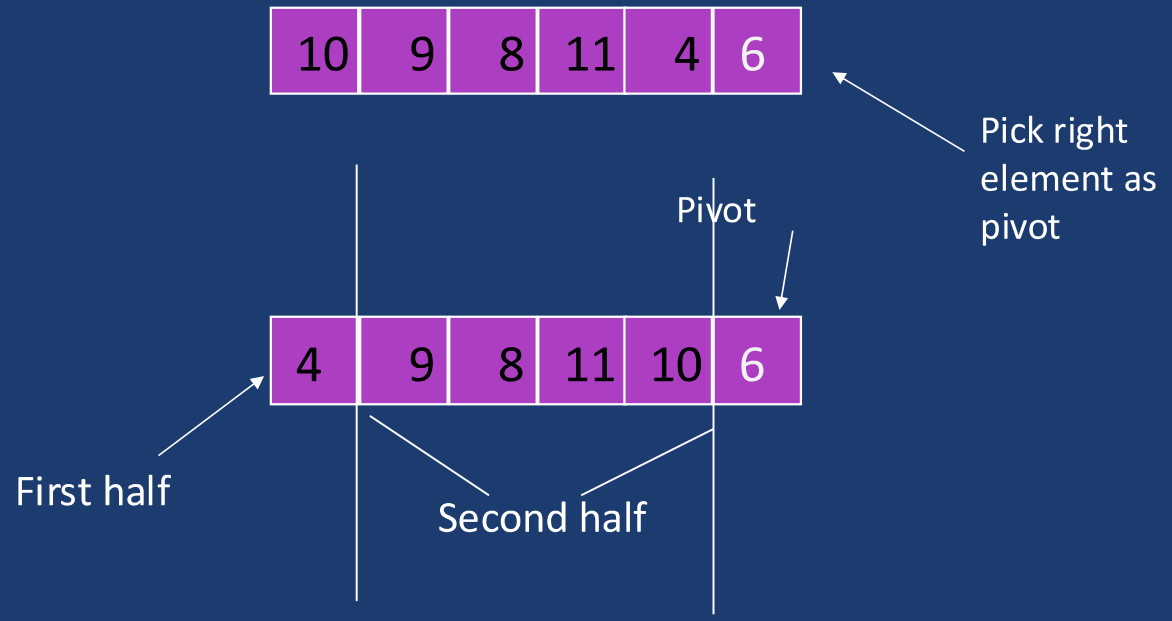
CHOOSING A PIVOT

The idea is that all the elements will be split into two separate lists depending on whether they are bigger or smaller than some **pivot** value

Lets start off by just picking a random pivot – the **rightmost** element in the array to be sorted

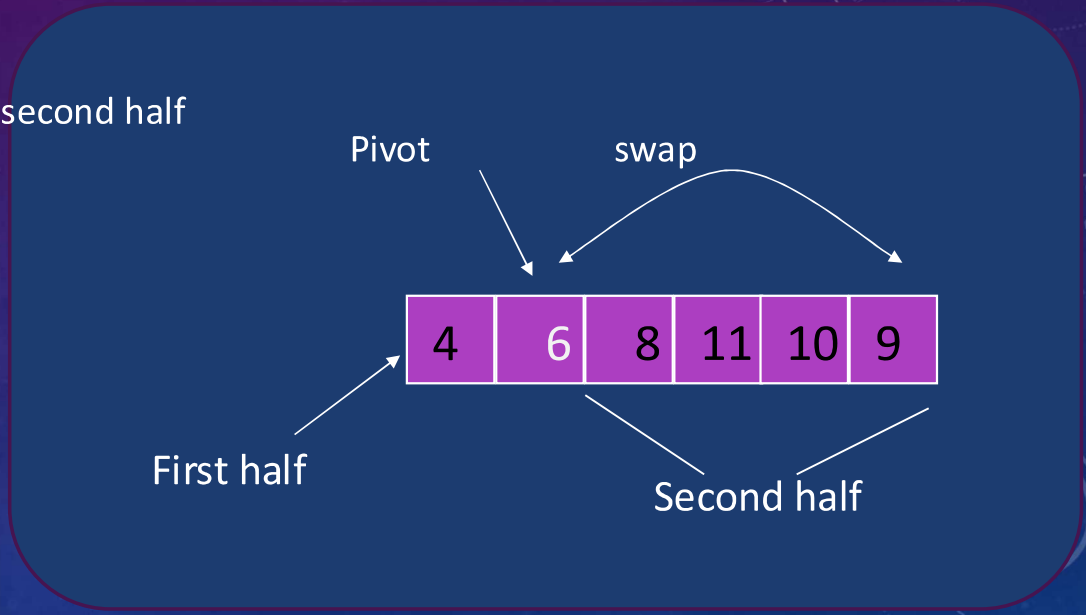
Now split the numbers apart depending on whether they are bigger or smaller than this element

SPLIT INTO TWO HALVES



PUT THE PIVOT IN PLACE

- Now swap the pivot with the first element in the second half
- The pivot is now in its final resting position!



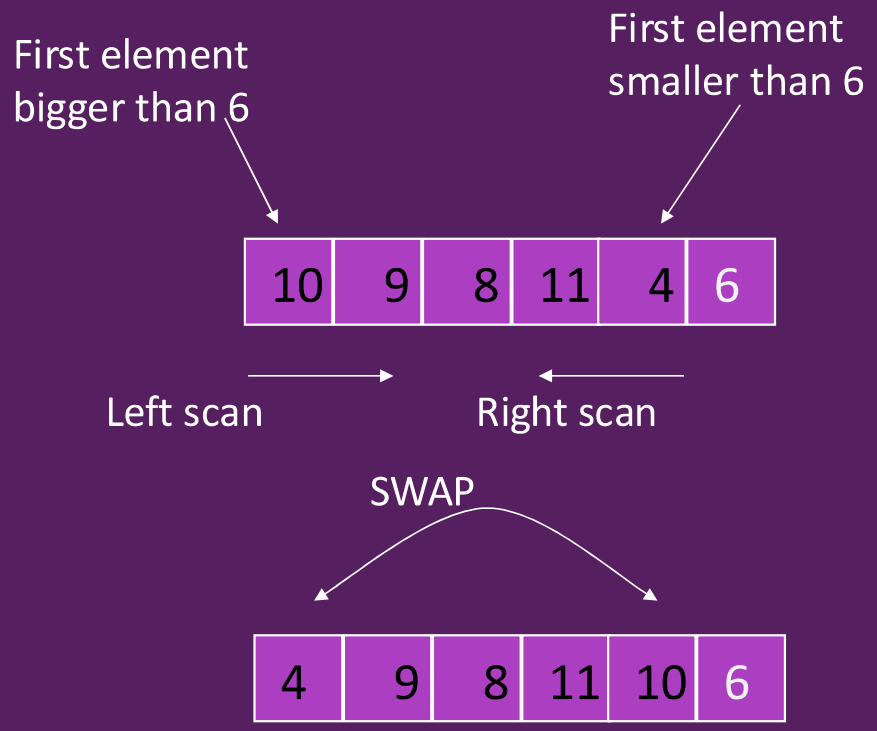
THE CODE FOR THAT…

```
public void recQuickSort(int left, int right) {

      if(right-left <= 0)                 // if size <= 1,
          return;        <- Base Case     //    already sorted
      else{                               // size is 2 or larger

         long pivot = theArray[right];      // rightmost item
                                            // partition range
         int partition = partitionIt(left, right, pivot);
         recQuickSort(left, partition-1);   // sort left side
         recQuickSort(partition+1, right);  // sort right side
      }
}
```

SORTING INTO TWO HALVES

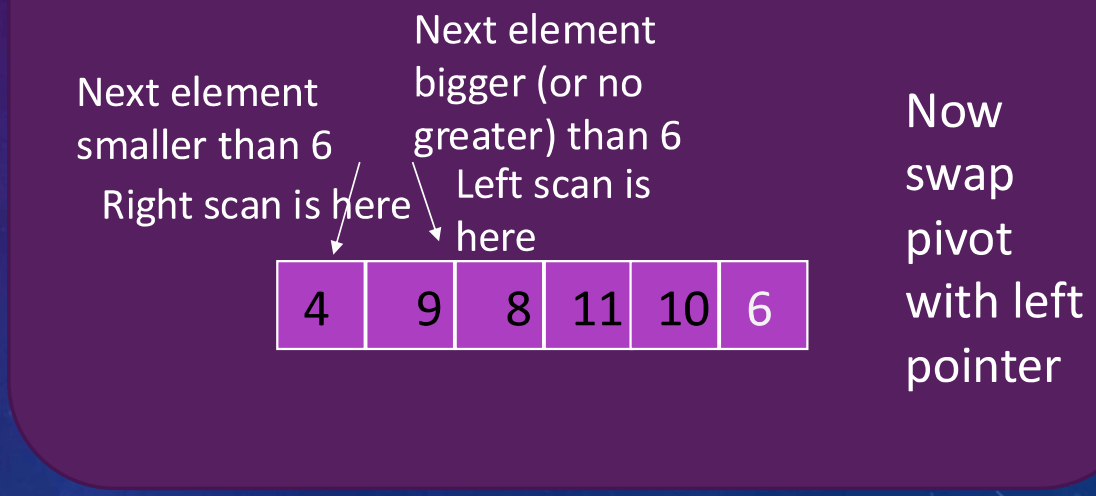
- The sorting works using two '**scans**' of the array
  - One from left to right →.....
  - One from right to left .....←
- The left scan starts at the beginning and searches for the first element that is bigger than the pivot
- The right scan starts at the end and searches for the first element that is smaller than the pivot
- These elements are then swapped

SCAN & SWAP


First element bigger than 6
First element smaller than 6
10 9 8 11 4 6
Left scan
Right scan
SWAP
4 9 8 11 10 6

KEEPING SWAPPING

- The scans keep moving onto the next element and swapping each other's values
- As soon as one scan has gone past the other the swapping method stops
- Now all the values have been separated into two groups

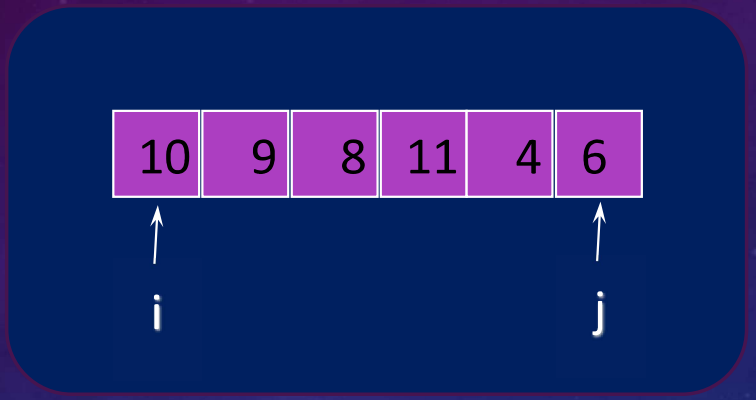


QUICKSORT, NOTATION

- QS(A,Left array position ,Right array position)

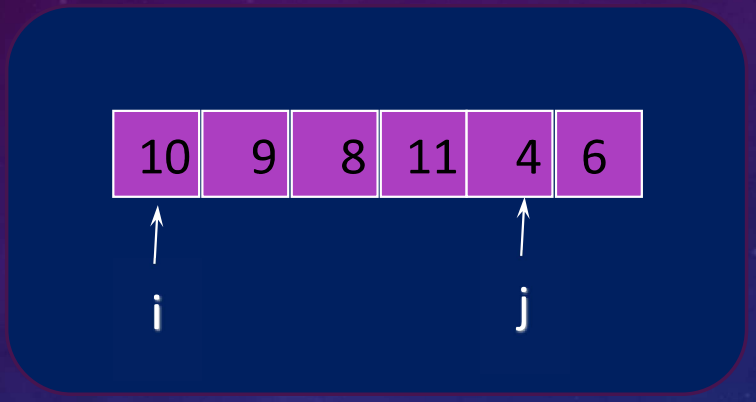
- L: Leftmost position
  R: Rightmost position
- i: L-R scan positions
- j: R-L scan positions
- pivot: We will pick the rightmost element of the array as pivot

QUICKSORT




```
QS(A,0,5)

L:      0
R:      5
i:      0
j:      5
pivot:  5
```

QUICKSORT

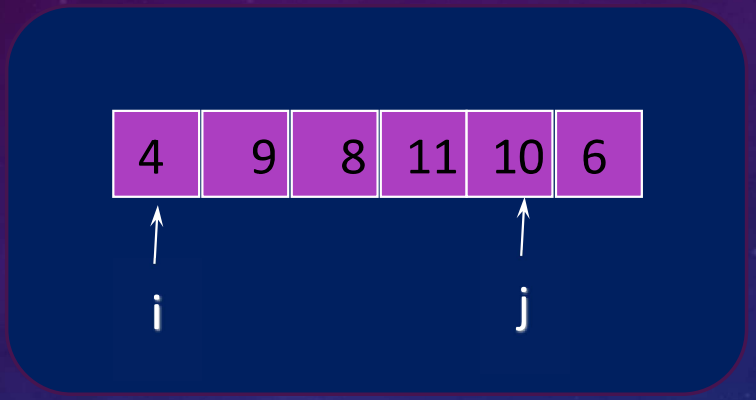


What is the element at position i?
What is the element at position j?
What is the pivot element?

```
QS(A,0,5)

L:      0
R:      5
i:      0
j:      4
pivot:  5
```

Here the element at position i is > pivot while the element at position j is < pivot so swap the elements in i and j

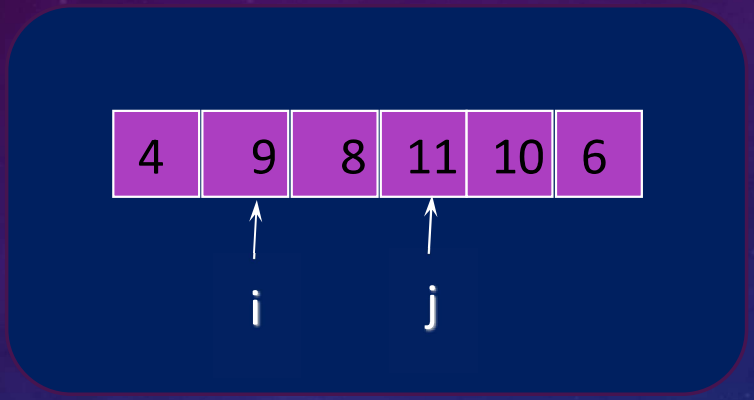
QUICKSORT




```
QS(A,0,5)

L:        0
R:        5
i:        0
j:        5 4
pivot:    5
```

Elements are swapped

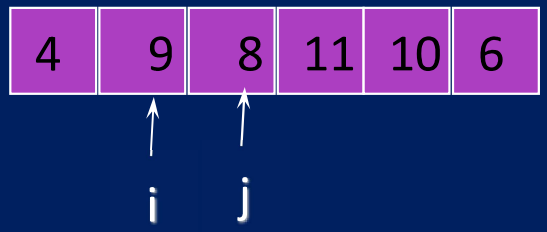
QUICKSORT




```
QS(A,0,5)

L:        0
R:        5
i:        1
j:        5 4 3
pivot:    5
```

Scan from the left has found a value larger than pivot
Scan from the right has NOT found a value smaller than pivot
So, we cannot swap yet

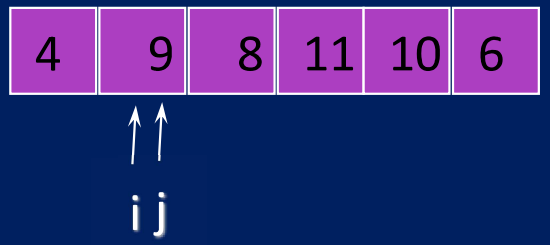
QUICKSORT




```
QS(A,0,5)

L:        0
R:        5
i:        1
j:        5 4 3 2
pivot:    5
```

Move j, and we still have not found a value lower than pivot
So, we still can not swap

QUICKSORT




```
QS(A,0,5)

L:        0
R:        5
i:        1
j:        5 4 3 2 1
pivot:    5
```

i and j are now in the same slot
So, we've found where our pivot should be! Swap the value in i/j with the pivot

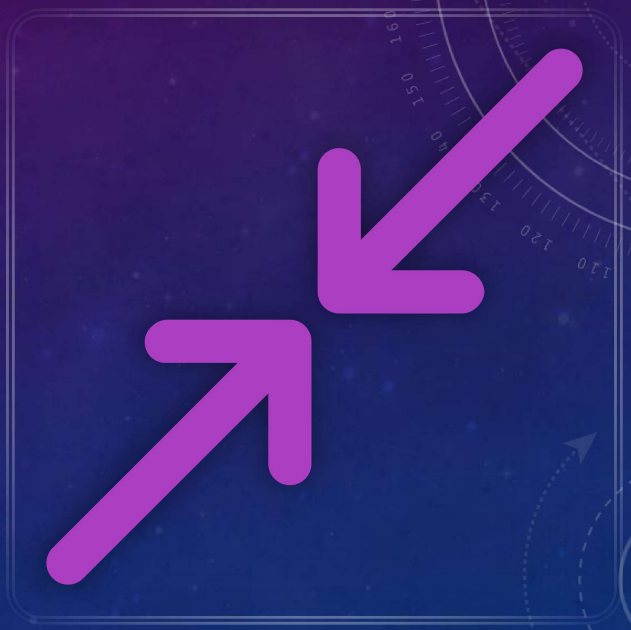
QUICKSORT


```
QS(A,0,5)

L:       0
R:       5
i:       0 1
j:       5 4 3 2
pivot: 4
```

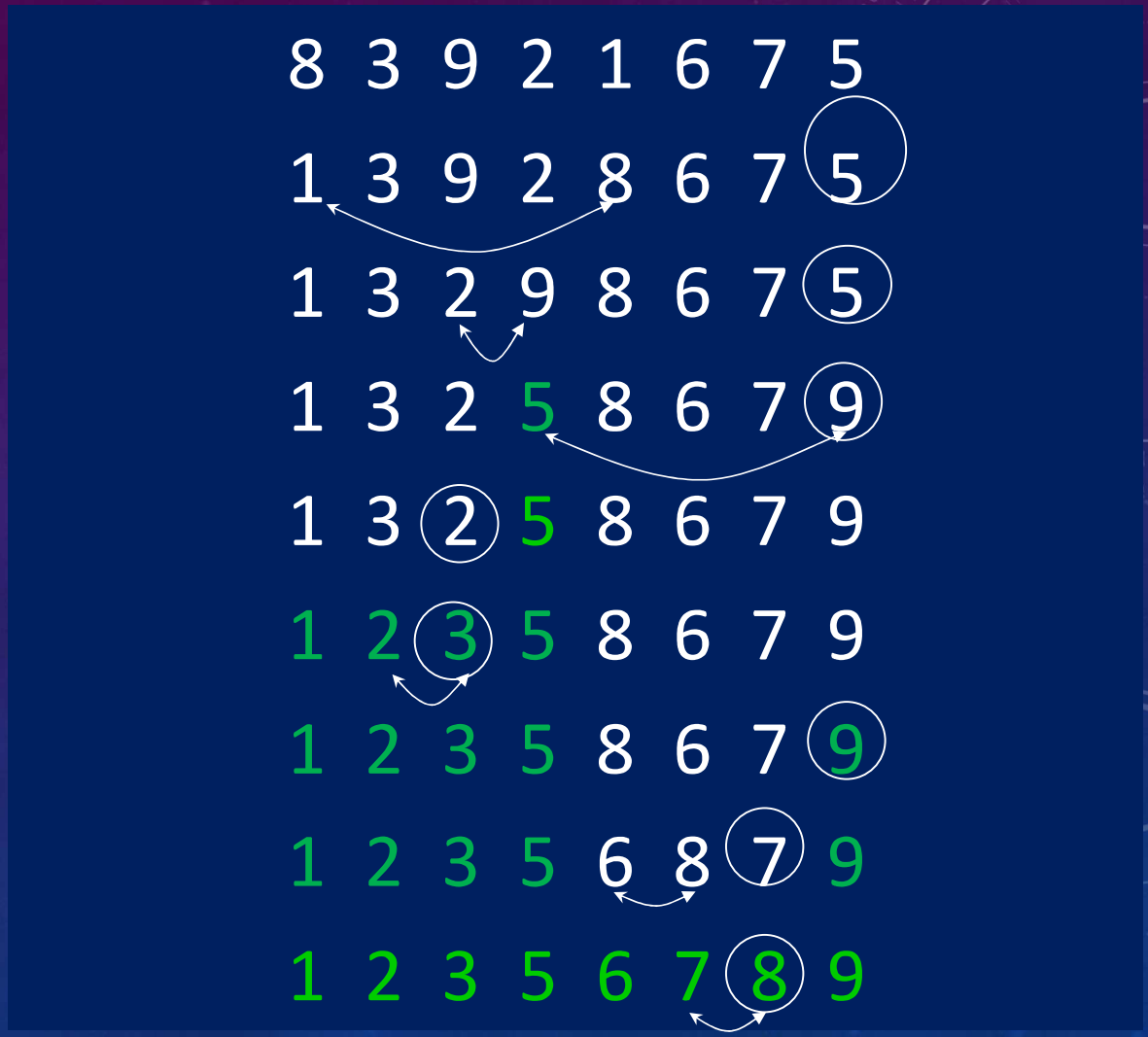
Now recursively Quicksort the sublist to the left of 6 (4) and the sublist to the right of 6 (8, 11, 10, 9)

THINGS TO NOTICE

- The left scan will always stop on the pivot (because the pivot is not smaller than the pivot)
- At this point, the two scans are guaranteed to have crossed paths
- However, the right scan **might go below 0** off the edge of the range so we need to introduce a check to make sure we don't go too far
- This slows down the performance – we'd like to get rid of this
- Leftscan starts at -1 and rightscan at the pivot because they are incremented /decremented before they're used for the first time



- Sort these numbers showing pivots and swaps:
- Remember:
- LeftScan: First number larger than the pivot from the left
- RightScan: First number smaller than the pivot from the right



```
public int partitionIt(int left, int right, long pivot){

      int leftPtr = left-1;           // left    (after ++)
      int rightPtr = right;           // right-1 (after --)
      while(true)
         {
         while( theArray[++leftPtr] < pivot ){}    // scan to the right
         while(rightPtr > 0 && theArray[--rightPtr] > pivot){}
                     // scan to the left

         if(leftPtr >= rightPtr)      // if pointers cross,
            break;                    //    partition done
         else                         // not crossed, so
            swap(leftPtr, rightPtr);  //    swap elements
         }
      swap(leftPtr, right);           // swap pivot into correct place
      return leftPtr;                 // return pivot location
}
```

SWAPS AND COMPARISONS

- **Comparisons**
  - For each partition there will be at most **n+1** or n+2 comparisons
  - Every item will be encountered and compared by one or other of the scans, leading to n comparisons
  - The scans will overshoot each other before they realise it, leading to some additional comparisons

SWAPS AND COMPARISONS

- **Swaps**
  - The number of swaps depends on how the data is arranged
  - If the data is inversely arranged then every pair of values must be swapped, a total of about **n/2** (actually **(n-1)/2 +1**)
  - For random data, half of the elements will already be in the right half position meaning only **n/4** swaps will be required

OVERALL COMPLEXITY

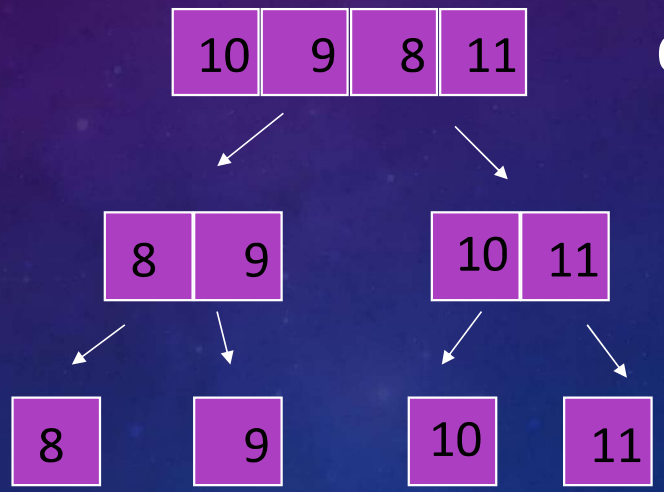
- The order of the algorithm is decided by whichever of swaps or comparisons is greater
- Both are O(n) for one partition
- If each partition halves the array (or subarrays) then the total number of partitionings required is the number of times that n can be halved → **$\log_2 n$**
- O(n) * $\log_2 n$ gives us **O(n*logn)**

COMPLEXITY

$O(\log_2 n)$ partitioning levels

n-1 levels = 3

$\log_2 n = 2$



O(n) Steps

O(n) Steps

Base Case

QUICKSORT PERFORMANCE

- The performance of divide-and-conquer algorithms comes from splitting the problem each time
- If the problem is halved by each split the algorithm will be **O(splitting code) * O(logn)** since the problem can only be split $\log_2 n$ times before resulting in single units
- However, if the problem isn't being split in half then the efficiency is going to be lower
- The split in quicksort depends on the pivot

CHOICE OF PIVOT

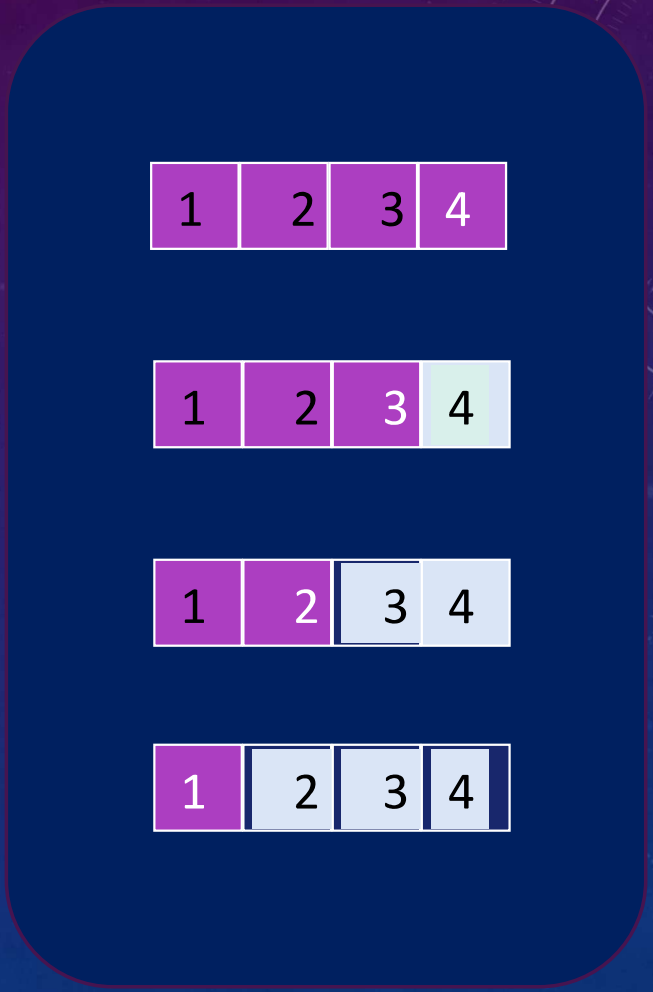
- If the pivot is the middle value in the array, then the array will be split in half perfectly
- However, if the pivot is the highest or lowest element then the split will be completely lop-sided
- In the worst case scenario n number of splits will be required meaning that the performance of quicksort degenerates to $O(n^2)$

Must be split 4 times using rightmost element as a pivot

WORSE PIVOT CHOICE

4 Steps for 4 items = O(n)

What would be the best number to be the pivot?



CHOICE OF PIVOT

- Ideally we should pick a mid-range value for the pivot (as opposed to just a random number)
- Why don’t we calculate the mid-range value?

CHOICE OF PIVOT

- Ideally we should pick a mid-range value for the pivot (as opposed to just a random number)
- If you were to examine all the numbers and calculate the mid-value this would take longer than the sort itself
- A compromise involves **median of three partitioning**
- Pick the first, middle and last elements of the array (or subarray) and take the middle of those three elements

MEDIAN OF THREE

- Select middle value as the pivot
- Sort the three values into the correct positions
- Swap the pivot to the right

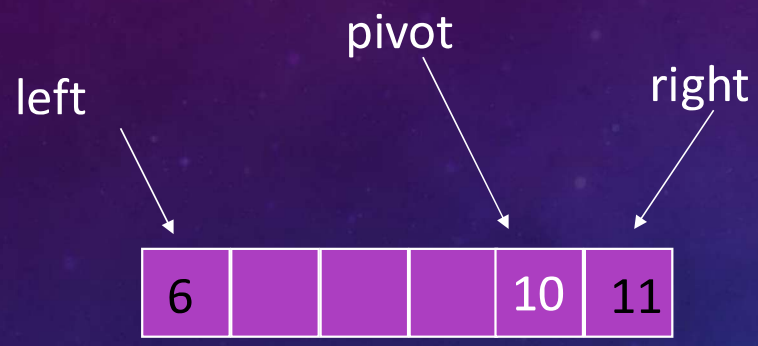
MEDIAN OF THREE

- **<u>Select middle value as the pivot</u>**
- Sort the three values into the correct positions
- Swap the pivot to the right

MEDIAN OF THREE

- Select middle value as the pivot
- **Sort the three values into the correct positions**
- Swap the pivot to the right

MEDIAN OF THREE



- Select middle value as the pivot
- Sort the three values into the correct positions
- **Swap the pivot to the right**

MEDIAN OF THREE

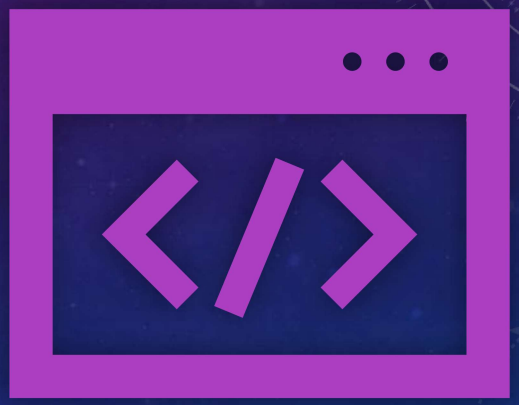
BONUS

- We can start the partition algorithm at left+1 and right-1 because we have already sorted the left and right elements
- We know they are smaller and bigger than the pivot and are therefore in the right place (though not necessarily their final position)
- These extreme values act as buffers which stop the left scan from scanning an element below 0
- The right scan will always stop at the leftmost element because it is guaranteed to be less than the pivot

INCREASES EFFICIENCY

- Small increase in code efficiency – no check required for scanning left

```
while( theArray[++leftPtr] < pivot ) {}; //
  scan right
while( theArray[--rightPtr] > pivot ){}; //
  scan left
```



TODO!

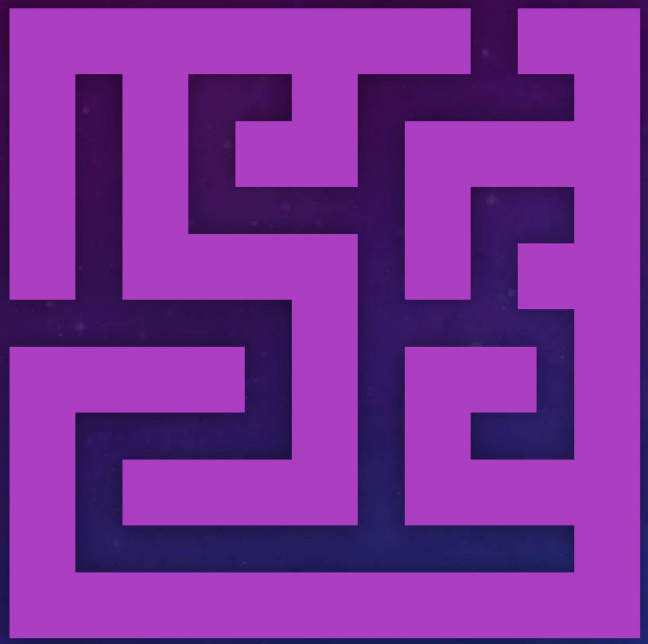
8 3 9 1 6 7 5

……?

- Sort these numbers using the rightmost pivot method AND the median of three method

MEDIAN OF THREE

```
public long medianOf3(int left, int right) {

     int center = (left+right)/2;
                                        // order left & center
      if( theArray[left] > theArray[center] )
         swap(left, center);
                                        // order left & right
      if( theArray[left] > theArray[right] )
         swap(left, right);
                                       // order center & right
      if( theArray[center] > theArray[right] )
         swap(center, right);

      swap(center, right-1);            // put pivot on right
      return theArray[right-1];         // return median value
}
```



NEW PROBLEM

- Can anyone think of a potential problem with our median of three partitioning?

NEW PROBLEM

If you use median-of-three partitioning then the quicksort algorithm can't work for partitions of three or fewer items

You could implement a "manual sort" method to sort three elements

Another option is to use **insertion sort** when the subarray to be sorted becomes suitably small (insertion sort works really well for nearly sorted data)

Rather than a cutoff of 3, you could employ the insertion sort method as soon as the size to be sorted falls below 10 or 20

SWITCHING BETWEEN SORTING ALGORITHMS

```
public void recQuickSort(int left, int right) {

      int size = right-left+1;
       if(size < 10)                  // insertion sort if small
          insertionSort(left, right);
       else{                          // quicksort if large

          long median = medianOf3(left, right);
          int partition = partitionIt(left, right, median);
          recQuickSort(left, partition-1);
          recQuickSort(partition+1, right);
       }
}
```

MAXIMIZING EFFICIENCY

- Knuth recommends a cut-off of 9 for maximal efficiency
- The optimum number depends on the computer, operating system, compiler and of course the data you're sorting
- Another idea is to sort the whole array without bothering to sort small partitions smaller than the cutoff
- Finally, the area is nearly sorted and you can use insertion sort to tidy the whole thing up