Advanced Heuristic

1 Advanced Heuristic Calculation

The heuristic h is defined as:

$$\begin{aligned} h = & \text{ Manhattan distance of the goal piece to the goal place} \\ + & \quad \text{number of pieces that overlap the 2x2 goal place, other than the 2x2 goal piece} \end{aligned} \tag{1}$$

To calculate this heuristic for any puzzle state:

1. Compute the Manhattan distance (d) between the top left corner of the 2x2 goal piece and the top left corner of the goal place $(1, 3)$:

$$d((x, y)) = |x - 1| + |y - 3|$$

2. Count the number of pieces overlapping the 2x2 goal place ($3 \leq i \leq 4, 1 \leq j \leq 2$), excluding the 2x2 goal piece. Use a for-loop to check each piece's info such as coordinates and type. Now, use this info to see if the piece is overlapping the 2x2 goal place.

3. Sum these values to obtain h.

2 Admissibility of the Heuristic

The heuristic h is admissible because:

1. The Manhattan distance d is non-negative and provides a lower bound on the number of moves required to bring the goal piece to the goal place. The number of overlapping pieces at any state is also non-negative.

2. Any piece overlapping the goal place, other than the goal piece, must be moved at least once, ensuring the heuristic does not overestimate the true cost.

Formally, for any state S, $0 \leq h(S) \leq h^*(S)$, where $h^*(S)$ is the actual cost to reach the goal state from S. The non-negativity and necessary movements ensure this inequality, proving admissibility.

3 Dominance Over Manhattan Heuristic

The advanced heuristic h dominates the Manhattan heuristic because it includes the Manhattan distance d plus an additional term accounting for overlapping pieces. Since the number of overlapping pieces is always non-negative, for any state S, we have:

$$h(S) = d(S) + \text{number of overlapping pieces} \geq d(S)$$

We analyze two cases:

- **Case 1:** There are pieces overlapping the goal place G that are not the goal piece g. In this case, g needs to move at least a Manhattan distance d to reach G, and each overlapping piece must move at least once, making $h(S) > d(S)$. The classic initial configuration, as depicted in the assignment handout, is an example of this case.

- **Case 2:** There are no pieces overlapping G other than g. Here, $h(S) = d(S)$.

Thus, $h(S) \geq d(S)$ for all states S. In scenarios with overlapping pieces, $h(S)$ is strictly greater than $d(S)$, providing a tighter bound and making the A* search more efficient, expanding fewer nodes than using d alone.