Decision Analytics for Business and Policy

Homework 3: Integer Optimization

Due Date: Sep 27, 11:59 am ET **Submission:** Canvas

In this homework, you will formulate and implement a linear optimization model. Please pay attention to the following guidelines:

- When asked to "formulate" an optimization model: define decision variables, the objective function, and the constraints of your problem mathematically.
- When asked to "implement" the model: implement the model in code and solve it computationally.
- Please state clearly any modeling or implementation assumptions you made in the write-up.

Problem Description: Emergency Response.

You are planning the deployment of emergency shelters in response to a natural disaster. Given available resources, you have capacity to build only 10 shelters across the city. You have identified 40 potential sites, each with a fixed capacity (i.e., a maximum number of residents that it can assist). All residents will need to access one of the shelters you will construct. You aim to determine which shelters to build to optimize the relief services provided to the population. Given the capacity constraints, you also need to anticipate the assignment of residents to shelters. In this problem, you will model and implement two formulations, which correspond to two different underlying evacuation objectives.

The city under consideration can be modeled as a 20 by 10 miles rectangle, with 200 major residential areas. We index the residential areas by $i = 1, \ldots, 200$ and the shelter sites by $j = 1, \ldots, 40$. Throughout the problem, we consider the "grid distance", or "Manhattan distance" in the city; in other words, if area i is located in the point of coordinates (u_i, v_i) and shelter j is located in the point of coordinates (u_j, v_j), then the distance from i to j is given by $d_{ij} = |u_i - u_j| + |v_i - v_j|$ (hint: calculate d_{ij} from the given data first, and then use d_{ij} values as model parameters).

You have access to the following data files:

- `Pb2_areas.csv`: A matrix of size 200 by 3 that indicates, for each of the 200 residential areas, (i) its x-coordinate, in miles (0 to 20), (ii) its y-coordinate, in miles (0 to 10) and (iii) its number of residents.
- `Pb2_shelters.csv`: A matrix of size 40 by 3 that indicates, for each of the 40 potential shelter locations, (i) its x-coordinate, in miles (0 to 20), (ii) its y-coordinate, in miles (0 to 10) and (iii) its capacity. [Hint: you will need to pre-process some of the data provided. It can be done in Excel or Python.]

For simplicity, we assume that all the residents from the same residential area are assigned to the same shelter. You consider the following decision variables: $x_j = 1$ if shelter j is built, and 0 otherwise, $y_{ij} = 1$ if residential area i is assigned to j.

Questions:

First, you aim to minimize the total distance across all residents required to access their assigned shelter (parts 1-2). Second, you aim to minimize the largest distance required by any resident to access a shelter (parts 3-5).

1. Formulate an integer programming model that optimizes the selection of shelters to minimize total distance. Please specify your objective function and constraints carefully. [20%]

2. Implement your model computationally. Please show your work. What is the total distance across all residents? [20%]

3. Formulate an integer programming model that optimizes the selection of shelters to minimize the maximum access distance. Please specify your objective function and constraints carefully. (Hint: define an additional variable to represent the maximum access distance.) [20%]

4. Implement your model computationally. Please show your work. What is the largest distance between residential areas and shelters? [20%]

5. Plot the histogram of the distance to the assigned shelters across all residents of the city, under each of the two solutions obtained in this problem. Comment on the outputs in view of the two objectives considered. [20%]