MARKOV CHAINS

Transition matrix for S&P 500 historical prices

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Assuming that the stock price of S&P 500 obeys Markov property, the **goal** of this research is to construct the transition matrix for this Markov Chain.

For this research I have taken **data** of <u>S&P 500 historical prices</u> for 10 years since 2012 (even though data of historical prices is available since 1928) on a daily basis assuming that trends for the past decade may vary. The time series we get consists of over 2500 observations.

The main perimeters have used are Date (daily), Close price, Return (rate of stock price increase compared to the previous day), Statement (where B= big, S=small, D=decrease, I=increase), which reflects in what direction prices have changed compared to the previous-day prices, Status, which reflects the transition to current statement (BDSI = from statement SI to BD), Dmean (which is the mean of all negative returns), and Imean (mean of all positive returns).

| C2 | → fx | =(B2-B3)/B3 | | | | | |
|----|--------------|-------------|-------------|-------------|----------|-------------|--------------------|
| | А | В | С | D | Е | F | G |
| 1 | Date = | Close* = | Return = | Statement = | Status = | Dmean = | Imean = |
| 2 | Nov 09, 2022 | 3,748.57 | -0.02077788 | BD | BDSI | -0.00725716 | 0.00695301 |
| 3 | Nov 08, 2022 | 3,828.11 | 0.00559788 | SI | SIBI | | |
| 4 | Nov 07, 2022 | 3,806.80 | 0.00961398 | BI | BIBI | | |
| 5 | Nov 04, 2022 | 3,770.55 | 0.01361868 | BI | BIBD | | |
| 6 | Nov 03, 2022 | 3,719.89 | -0.01058598 | BD | BDBD | | |
| 7 | Nov 02, 2022 | 3,759.69 | -0.02500194 | BD | BDSD | | |
| 8 | Nov 01, 2022 | 3,856.10 | -0.00410126 | SD | SDBD | | |
| 9 | Oct 31, 2022 | 3,871.98 | -0.00745438 | BD | BDBI | | |
| 10 | Oct 28, 2022 | 3,901.06 | 0.02462638 | BI | BISD | | |
| 11 | Oct 27, 2022 | 3,807.30 | -0.00608260 | SD | SDBD | | |
| 12 | Oct 26, 2022 | 3,830.60 | -0.00738771 | BD | BDBI | | |
| 13 | Oct 25, 2022 | 3,859.11 | 0.01626665 | BI | BIBI | | |
| 14 | Oct 24, 2022 | 3,797.34 | 0.01188195 | BI | BIBI | | |
| 15 | Oct 21, 2022 | 3,752.75 | 0.02372483 | BI | BIBD | | |
| 16 | Oct 20, 2022 | 3,665.78 | -0.00795094 | BD | BDSD | | |
| 17 | Oct 19, 2022 | 3,695.16 | -0.00667208 | SD | SDBI | | |

Returns that are not positive are considered as a Decrease, elsewhere an Increase. If the value of return is lower than the Dmean, it is denoted as BD, if it is higher (but still negative), it is denoted as SD. On the other hand if the value of return is positive & smaller than Imean it is denoted as SI, and BI if higher than the Imean.

In this research I have taken the means instead of the median quartile values in order to define the statements by high/low-increase/decrease values.

Operations. In order to get a transition matrix we should take every t-1 statement, and count the probability of each transition from t-1 to t states. For example when we take "BI" as t-1 statement, we have 508 observations on that position, and should count the probability for every status containing "BI" as t-1 statement (BDBI, SDBI,

SIBI, BIBI). So we take the number of observations for any status and divide that into the respective statement (BDBI/BI). I have used RStudio to count the number of observations.

```
> table(data$statement)

BD BI SD SI
375 508 774 858
> table(data$states)

BDBD BDBI BDSD BDSI BIBD BIBI BISD BISI SDBD SDBI SDSD SDSI SIBD SIBI SISD SISI
89 96 101 89 118 105 154 131 78 152 224 320 89 155 296 318
```

As a result we get the probabilities for each status, when we already have a certain t-1 statement.

| | BI | SI | SD | BD | SUM |
|---------|--------|--------|--------|--------|------|
| BI(t-1) | 0.2067 | 0.3051 | 0.2992 | 0.1890 | 1.00 |
| SI(t-1) | 0.1527 | 0.3706 | 0.3730 | 0.1037 | 1.00 |
| SD(t-1) | 0.1990 | 0.3824 | 0.2894 | 0.1305 | 1.00 |
| BD(t-1) | 0.3147 | 0.2373 | 0.2080 | 0.2373 | 1.00 |

Conclusion. As we can see the transition matrix we get by this research is not uniform as the values of probabilities in various states visually differ from each other. We have got the highest probability of transition from the state "Small Decrease" to "Small Increase" with the value of 0.38, and the lowest probability of transition from "Small Increase" to "Big Decrease" equal to 0.10.