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A note on evaluating model tidal currents against observations

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Abstract

The root-mean-square magnitude of the vector difference between modeled and observed tidal ellipses is a comprehensive metric to evaluate the representation of tidal currents in ocean models. A practical expression for this difference is given in terms of the harmonic constants that are routinely used to specify current ellipses for a given tidal constituent. The resulting metric is sensitive to differences in all four current ellipse parameters, including phase.

1. Introduction

One of the first steps usually taken to evaluate the performance of an ocean circulation model involves comparing modeled tidal motions with observations. Frequently, this involves the application of harmonic analysis to the available measurements of sea level displacement to extract the harmonic constants of the major tidal constituents. Comparisons with similarly analyzed model sea surface elevations can be used to assess the adequacy of the model, and to help tune model parameters. Similar comparisons are now often made for tidal currents due, in part, to the improved resolution of ocean models and the increasing availability of current measurements from ADCP moorings and HF radar (e.g., Mau et al., 2007). Such comparisons can be helpful to ascertain the effects of physical processes such as baroclinicity or coastally trapped waves on tidal motions, and to assess the influence of data assimilation procedures (e.g., Dunphy et al., 2005; Stammer et al., 2014).

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A straightforward and useful metric to evaluate model tidal elevations is the root-mean-square (rms) difference between the observed displacement of the sea surface, $\eta_o(t)$, and the modeled displacement, $\eta_m(t)$. This is given in terms of harmonic constants by

$$D_\eta = \left\langle \left(\eta_o - \eta_m\right)^2 \right\rangle^{1/2} = \left[\frac{1}{2}\left(h_o^2 + h_m^2\right) - h_o h_m \cos\left(\phi_o - \phi_m\right)\right]^{1/2}, \tag{1}$$

where the angle brackets denote an average over a tidal period, h_o and ϕ_o are the observed (subscript *o*) amplitude and Greenwich phase lag for a given tidal constituent, while h_m and ϕ_m are the modeled (subscript *m*) amplitude and phase (Cummins and Oey, 1997). An equivalent metric that is also widely used is the distance in the complex plane (e.g., Foreman et al., 1993 and 1995; Andersen et al., 1995; Dupont et al., 2005), which is equal to $\sqrt{2}\,D_\eta$.

To our knowledge, a simple formula comparable to (1) with which to assess model tidal currents has not been given. Instead, assessments of models with respect to tidal currents typically have relied on comparisons of the individual harmonic constants used to construct tidal ellipses (e.g., Luyten and Stommel, 1991; Foreman et al, 1995; Davies et al., 2001; Stanton et al., 2001; Blanton et al., 2004; Carbajal and Pohlmann, 2004; Fofonova et al., 2004; Dunphy et al., 2005; Chavanne et al., 2007; Mau et al., 2007; Hannah et al., 2009; Timko et al., 2012 and 2013; Maraldi et al., 2013). As well, graphical comparisons are frequently presented between tidal ellipses calculated from model solutions and observations (e.g., Foreman et al., 1993 and 1995; Cummins and Oey, 1997; Cummins et al., 2000; Mau et al., 2007). Although these kinds of comparisons can be informative, they may be inconclusive as a means of determining the best overall fit to observations from a set of numerical simulations.

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In a few studies, modeled and observed tidal currents for a given constituent have been compared on the basis of the rms magnitude of the vector difference calculated from summing in quadrature the rms difference of the zonal and meridional components of velocity (Foreman et al., 2000; Dupont et al., 2005; Barth et al., 2010; Stammer et al., 2014). This approach takes into account amplitude and phase information to provide an overall quantitative assessment of modeled tidal currents relative to observations. The purpose of the present note is to provide an analytical expression to calculate this same quantity based on ellipse parameters. The resulting expression is analogous to (1) and allows the rms magnitude of the vector difference to be computed directly from the harmonic constants for tidal ellipses that are routinely generated by tidal analysis programs (e.g., Pawlowicz et al., 2002; Codiga, 2011).

2. RMS error for modeled tidal current ellipses

Tidal current ellipses may be written in the complex form,

$$Z(t) = e^{i\theta}\left[A\cos(\omega t - g) + iB\sin(\omega t - g)\right], \tag{2}$$

where $i = \sqrt{-1}$ and ω is the angular frequency of the harmonic (Foreman, 1978). There are four parameters that specify the ellipse for a given tidal harmonic: A and B, the semi-major and semi-minor axes, respectively, the ellipse inclination angle, θ, which is measured in radians counterclockwise from east, and g, the Greenwich phase lag in radians. The latter is defined by the lag in time between alignment of the instantaneous velocity vector along the major axis and the peak amplitude of the equilibrium tide. The current vector rotates in a counterclockwise sense with $B > 0$ and in a clockwise sense with $B < 0$; for rectilinear motion, $B = 0$. Denoting

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the modeled tidal ellipse as Z_m and the observed tidal ellipse as Z_o, we consider the rms magnitude of the vector difference,

$$D_{\mathbf{u}} = \left\langle | Z_m - Z_o |^2 \right\rangle^{1/2}, \tag{3}$$

where the angle brackets represent a time average over the period $T = 2\pi/\omega$ of the tidal harmonic. This metric is analogous to (1) and provides a measure of the overall error of a modeled tidal current for a given harmonic.

To obtain a practical expression, we write $Z_o(t) = Z_o^r + iZ_o^i$ and $Z_m(t) = Z_m^r + iZ_m^i$. Expansion of (2) in terms of the ellipse parameters yields,

$$Z_o^r = \left[C_o^x \cos(\omega t - g_o) + S_o^x \sin(\omega t - g_o) \right] \quad \text{and} \tag{4a}$$

$$Z_o^i = \left[C_o^y \cos(\omega t - g_o) + S_o^y \sin(\omega t - g_o) \right], \tag{4b}$$

where $C_o^x = A_o \cos\theta_o$, $S_o^x = -B_o \sin\theta_o$, $C_o^y = A_o \sin\theta_o$, and $S_o^y = B_o \cos\theta_o$. Likewise,

$$Z_m^r = \left[C_m^x \cos(\omega t - g_m) + S_m^x \sin(\omega t - g_m) \right] \quad \text{and} \tag{5a}$$

$$Z_m^i = \left[C_m^y \cos(\omega t - g_m) + S_m^y \sin(\omega t - g_m) \right], \tag{5b}$$

with $C_m^x = A_m \cos\theta_m$, $S_m^x = -B_m \sin\theta_m$, $C_m^y = A_m \sin\theta_m$, and $S_m^y = B_m \cos\theta_m$. Expanding the right hand side of (3) we have,

$$\left\langle | Z_m - Z_o |^2 \right\rangle^{1/2} = \left[\left\langle Z_m^{r\,2} \right\rangle + \left\langle Z_o^{r\,2} \right\rangle - 2\left\langle Z_m^r Z_o^r \right\rangle + \left\langle Z_m^{i\,2} \right\rangle + \left\langle Z_o^{i\,2} \right\rangle - 2\left\langle Z_m^i Z_o^i \right\rangle \right]^{1/2}. \tag{6}$$

Substituting (4a,b) and (5a,b) into (6) and averaging in time yields,

$$\begin{aligned} D_{\mathbf{u}} = [1/2 & \left(S_m^{x2} + C_m^{x2} + S_o^{x2} + C_o^{x2} + S_m^{y2} + C_m^{y2} + S_o^{y2} + C_o^{y2} \right) \\ & - \cos(g_o - g_m)\left(C_m^x C_o^x + C_m^y C_o^y + S_m^x S_o^x + S_m^y S_o^y \right) \\ & - \sin(g_o - g_m)\left(C_o^x S_m^x + C_o^y S_m^y - C_m^y S_o^y - C_m^x S_o^x \right)]^{1/2}. \end{aligned} \tag{7}$$

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This can be further simplified to give the desired result,

$$D_{\mathbf{u}} = [\frac{1}{2}\left(A_o^2 + B_o^2 + A_m^2 + B_m^2\right) - \cos(g_0 - g_m)\cos(\theta_o - \theta_m)\left(A_o A_m + B_o B_m\right) \\ - \sin(g_0 - g_m)\sin(\theta_o - \theta_m)\left(A_o B_m + A_m B_o\right)]^{1/2}. \quad (8)$$

Equation (8) provides an easily applied and comprehensive metric with which to evaluate model tidal current ellipses relative to observations based on standard harmonic constants. This relation may be regarded as a generalization of the scalar difference (1) to the vector difference in two dimensions. A noteworthy aspect of (8) is that differences in ellipse inclination angle, $\Delta\theta = \theta_o - \theta_m$, and differences in phase lag, $\Delta g = g_o - g_m$, appear as interchangeable variables. As a result, the sensitivity of $D_{\mathbf{u}}$ to $\Delta\theta$ is identical to that due to Δg.

Tidal currents may also be expressed as the sum of two counter-rotating vectors, $Z(t) = Z^{+}(t) + Z^{-}(t)$, where

$$Z^{+}(t) = a^{+}\exp\left(i(\omega t + \varepsilon^{+})\right) \quad (9a)$$

is the counter-clockwise (CCW) rotating component, and

$$Z^{-}(t) = a^{-}\exp\left(-i(\omega t - \varepsilon^{-})\right) \quad (9b)$$

is the clockwise (CW) rotating component (Foreman, 1978). The amplitudes and phases of these components are related to the ellipse parameters as,

$$a^{+} = \frac{1}{2}(A+B), \quad a^{-} = \frac{1}{2}(A-B), \quad \varepsilon^{+} = (\theta - g), \quad \varepsilon^{-} = (\theta + g). \quad (10)$$

Rms differences between modeled and observed CCW and CW components are defined as $D_{ccw} = \left\langle |Z_m^{+} - Z_o^{+}|^2 \right\rangle^{1/2}$ and $D_{cw} = \left\langle |Z_m^{-} - Z_o^{-}|^2 \right\rangle^{1/2}$, respectively. Making use of (9a,b) we have,

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$$D_{ccw} = \left[\left(a_o^{+2} + a_m^{+2} \right) - 2a_o^+ a_m^+ \cos(\varepsilon_o^+ - \varepsilon_m^+) \right]^{1/2}, \tag{11}$$

along with a similar result for D_{cw}. These relations have the same form as the rms difference for sea level displacement (1), differing only by a factor of $\sqrt{2}$. Substituting the relations given in (10) into (11) and into the similar expression for D_{cw}, it may be verified that

$$D_{\mathbf{u}} = \left(D_{ccw}^2 + D_{cw}^2 \right)^{1/2}. \tag{12}$$

Accordingly, the overall model error may be partitioned into errors associated with each of the two counter-rotating components.

Also of interest is the relative rms difference which is given by,

$$D_{\mathbf{u}}^{rel} = D_{\mathbf{u}} \Big/ \left\langle |Z_o|^2 \right\rangle^{1/2} = D_{\mathbf{u}} \Big/ \left((A_o^2 + B_o^2)/2 \right)^{1/2}. \tag{13}$$

Defining the amplitude ratio $R = A_m / A_o$ and introducing the ellipse aspect ratios, $r_o = B_o / A_o$ and $r_m = B_m / A_m$, (13) can be written as,

$$D_{\mathbf{u}}^{rel} = \sqrt{R^2 \left(\frac{1+r_m^2}{1+r_o^2} \right) - 2R \left(\cos\Delta g \cos\Delta\theta \frac{1+r_o r_m}{1+r_o^2} + \sin\Delta g \sin\Delta\theta \frac{r_o + r_m}{1+r_o^2} \right) + 1}. \tag{14}$$

Thus the relative rms difference depends on five quantities: the three non-dimensional parameters, R, r_o, r_m, and the angle differences, Δg and $\Delta\theta$.

A sample of this large parameter space is illustrated in Figure 1 which shows contours of the relative rms difference as a function of R and Δg for two specialized cases in which $r_m = r_o = 1/3$ and $\Delta\theta = 0^{\circ}$ (top panel), or $\Delta\theta = 20^{\circ}$ (lower panel). Due to the identical dependence of the rms difference with respect to Δg and $\Delta\theta$, the figures also represent the

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relative difference as a function of R and $\Delta\theta$ with fixed phase lag differences, $\Delta g = 0^{\circ}$ and $\Delta g = 20^{\circ}$. A striking aspect of Fig. 1a is the rapid increase in $D_{\mathbf{u}}^{rel}$ with differences in phase lag or orientation angle in the vicinity of the center point $\left(R, \Delta g\,(\Delta\theta)\right) = \left(1, 0^{\circ}\right)$ where $D_{\mathbf{u}}^{rel} = 0$. In the case where there is a modest offset of 20° in the orientation angle or phase lag, the minimum relative difference (about 27%) is displaced away from the center point. The sensitivity indicated in these results points to the necessity of representing all four harmonic constants accurately to achieve relatively small model errors.

3. Conclusions

Equation (8) for the rms magnitude of the vector difference provides an easily applied and comprehensive means of evaluating model tidal currents against observations in terms of routinely computed harmonic constants. As such, this result complements equation (1), or equivalently the distance in the complex plane, that are widely used metrics for evaluating sea level differences. As an overall measure of the representation of tidal currents, (8) may be useful in tuning model parameters. Equally, it may be applied to quantify differences between observations of tidal currents from different instruments such as, for example, from HF radar and ADCP moorings (e.g., Kaplan et al., 2005).

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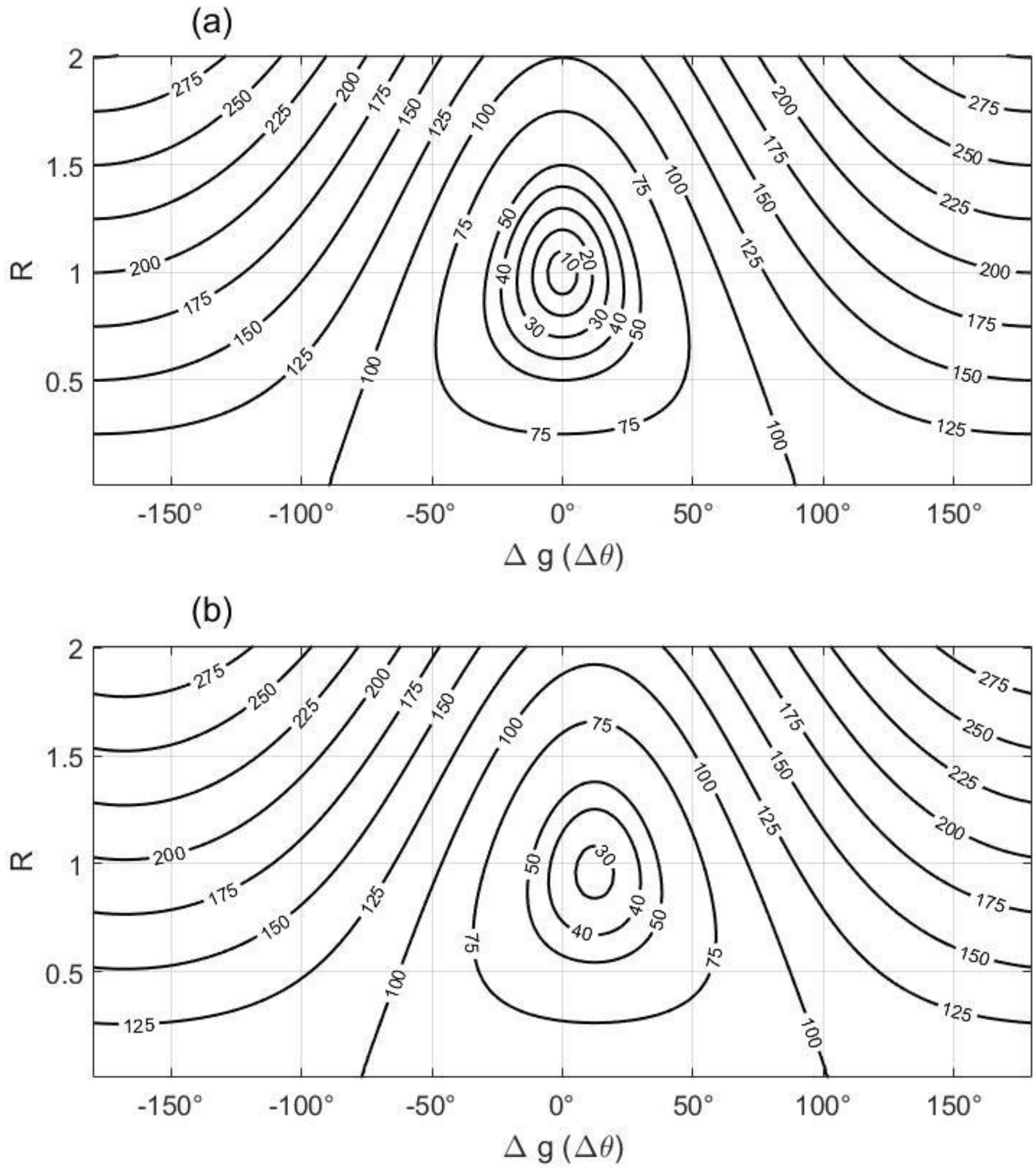

Figure 1. Contours of the percent relative rms difference $\left(100D_{\mathbf{u}}^{rel}\right)$ as a function of the semi-major axis ratio, $R = A_m / A_o$, and the phase difference, $\Delta g = g_o - g_m$, in degrees. The ellipse aspect ratios are fixed at $r_m = r_o = 1/3$, while the difference in ellipse orientation angle, $\Delta\theta = \theta_o - \theta_m$, is fixed at $\Delta\theta = 0^{\text{o}}$ for panel (a), and at $\Delta\theta = 20^{\text{o}}$ for panel (b). The figures also

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represent the relative difference as a function of R and $\Delta\theta$ for cases with (a) $\Delta g = 0^{\mathrm{o}}$ and (b) $\Delta g = 20^{\mathrm{o}}$.

Highlights

- Root-mean-square vector difference between modeled and observed tidal currents
- Practical expression given in terms of harmonic constants for current ellipses

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