Hw 3

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Q1

Suppose X and Y are random variables with nonzero means μ_X and μ_Y, respectively. Let $g(\mu_X, \mu_Y) = \mu_X/\mu_Y$. Show that: $E\left(\frac{X}{Y}\right) \approx \frac{\mu_X}{\mu_Y}$ and
$Var\left(\frac{X}{Y}\right) \approx \left(\frac{\mu_X}{\mu_Y}\right)^2 \left(\frac{VarX}{\mu_X^2} + \frac{VarY}{\mu_Y^2} - 2\frac{Cov(X,Y)}{\mu_X \mu_Y}\right)$

$$T = \begin{pmatrix} X \\ Y \end{pmatrix} \qquad \theta = \begin{pmatrix} \mu_X \\ \mu_Y \end{pmatrix} \qquad g(T) = \frac{X}{Y}$$

$$\begin{aligned} E\left(\frac{X}{Y}\right) &\approx g(\theta) = g(\mu_X, \mu_Y) \\ &\approx \frac{\mu_X}{\mu_Y} \end{aligned}$$

$$\begin{aligned} Var(X/Y) &\approx \left[\frac{\partial g(\theta)}{\partial X}\right]^2 VarX + \left[\frac{\partial g(\theta)}{\partial Y}\right]^2 VarY + 2\frac{\partial g(\theta)}{\partial X}\frac{\partial g(\theta)}{\partial Y} Cov(X,Y) \\ &\approx \left[\frac{1}{\mu_Y}\right]^2 VarX + \left[-\frac{\mu_X}{\mu_Y^2}\right]^2 VarY - 2\frac{1}{\mu_Y}\frac{\mu_X}{\mu_Y^2} Cov(X,Y) \\ &\approx \frac{1}{\mu_Y^2} VarX + \frac{\mu_X^2}{\mu_Y^4} VarY - 2\frac{\mu_X}{\mu_Y^3} Cov(X,Y) \\ &\approx \left(\frac{\mu_X}{\mu_Y}\right)^2 \left(\frac{VarX}{\mu_X^2} + \frac{VarY}{\mu_Y^2} - 2\frac{Cov(X,Y)}{\mu_X \mu_Y}\right) \end{aligned}$$