Homework 2

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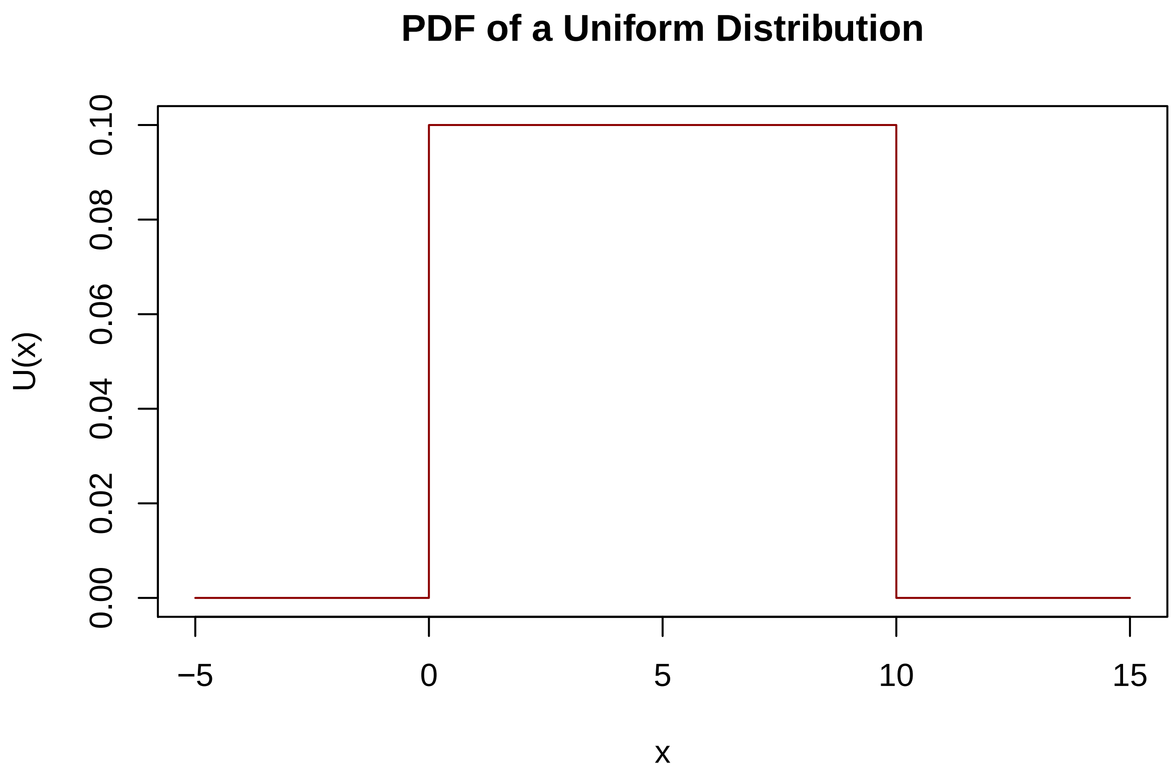
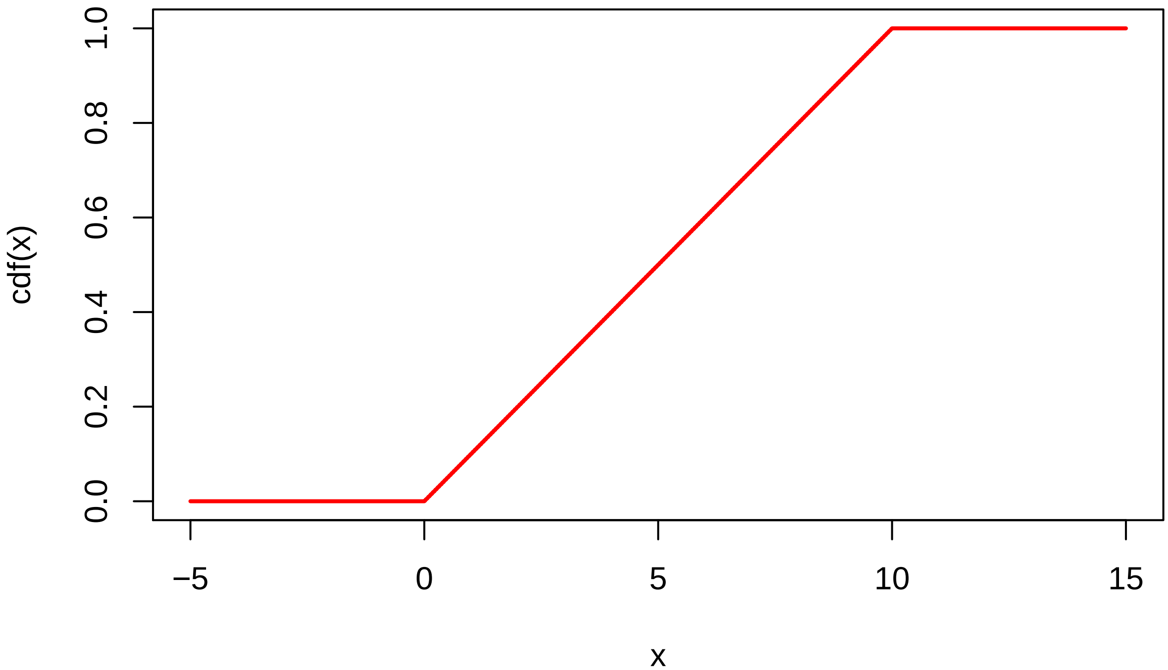
Q. 1b(i)

Sketch the pdf of U(0,10).



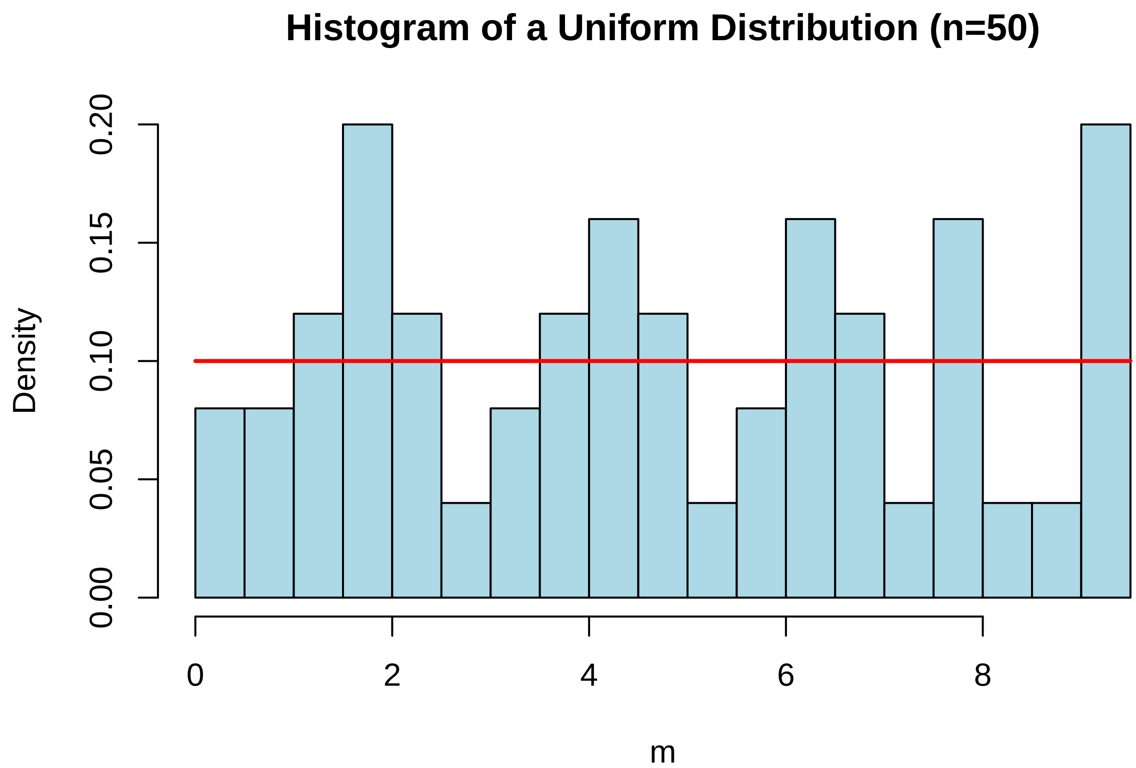
Q.1b(ii)

Sketch the cdf of U(0,10).



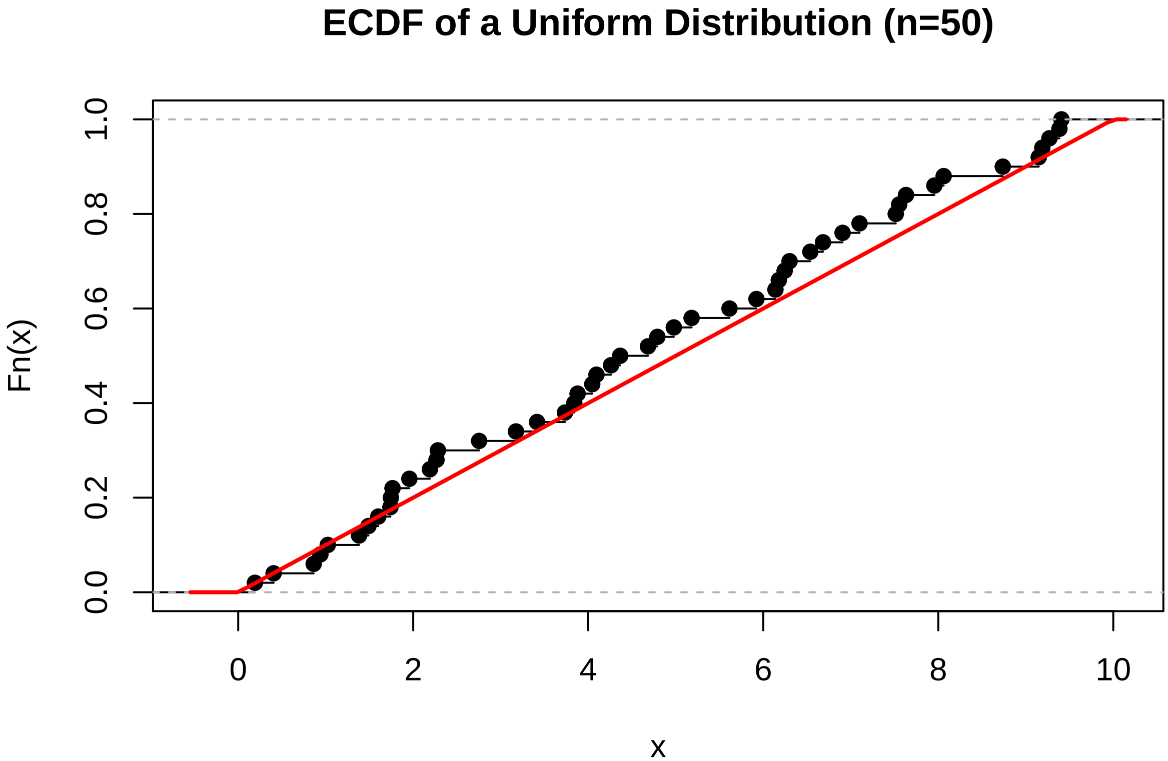
Q. 1c(i)

Use R to generate a random sample of size 50 from Uniform(0, 10) and plot the histogram (overlaid with the true pdf).



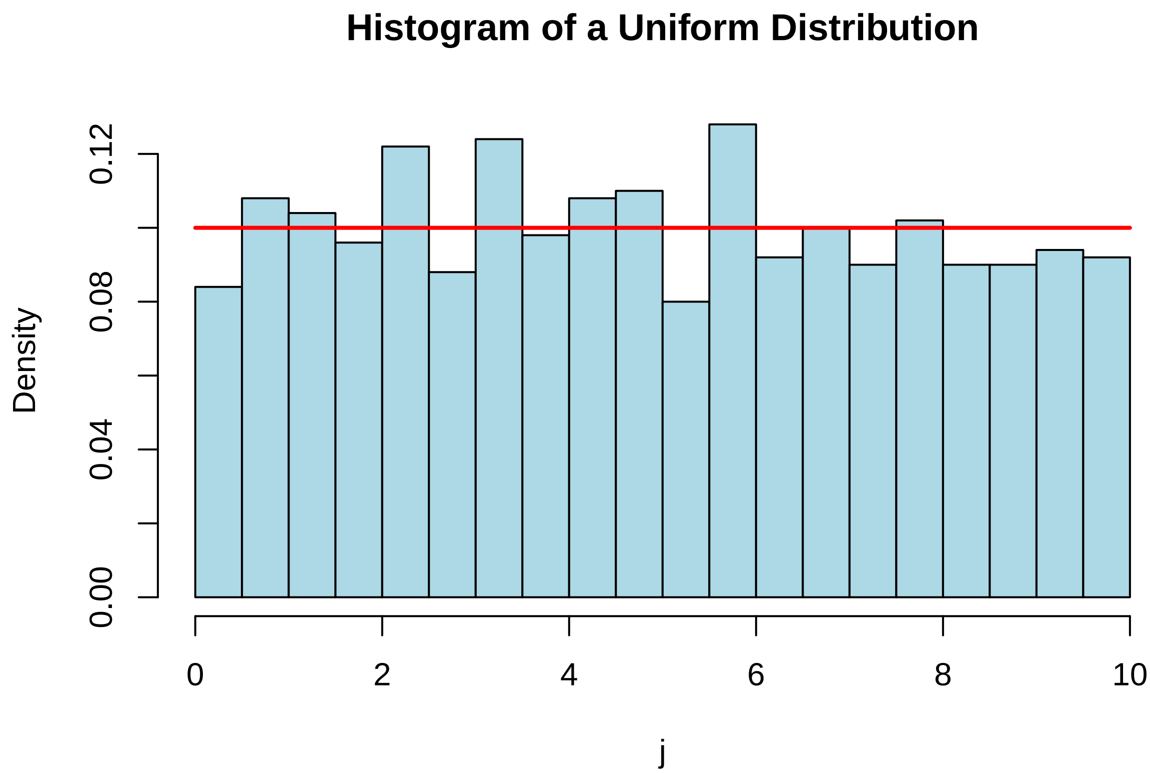
Q. 1c(ii)

plot the emperical cdf overlaid with the theoretical cdf.



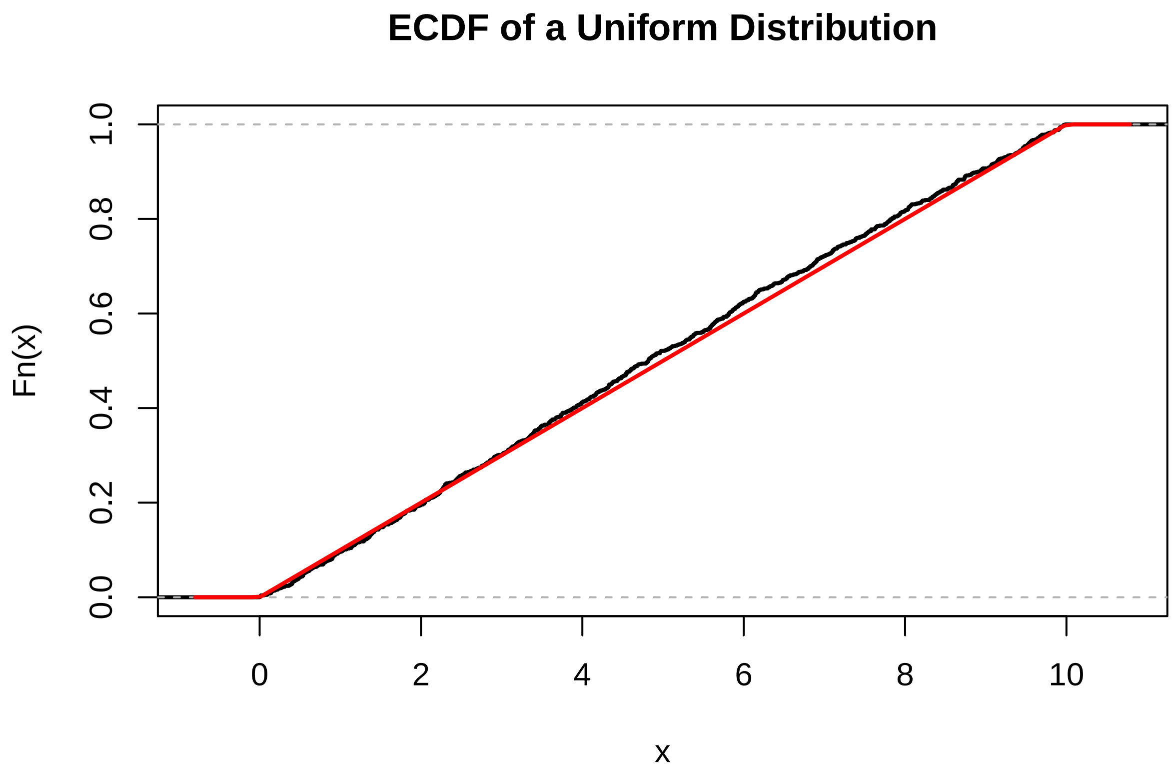
Q. 1d(i)

Use R to generate a random sample of size 1000 from Uniform(0, 10) and plot the histogram (overlaid with the true pdf).



Q. 1d(ii)

Plot the emperical cdf overlaid with the theoretical cdf.

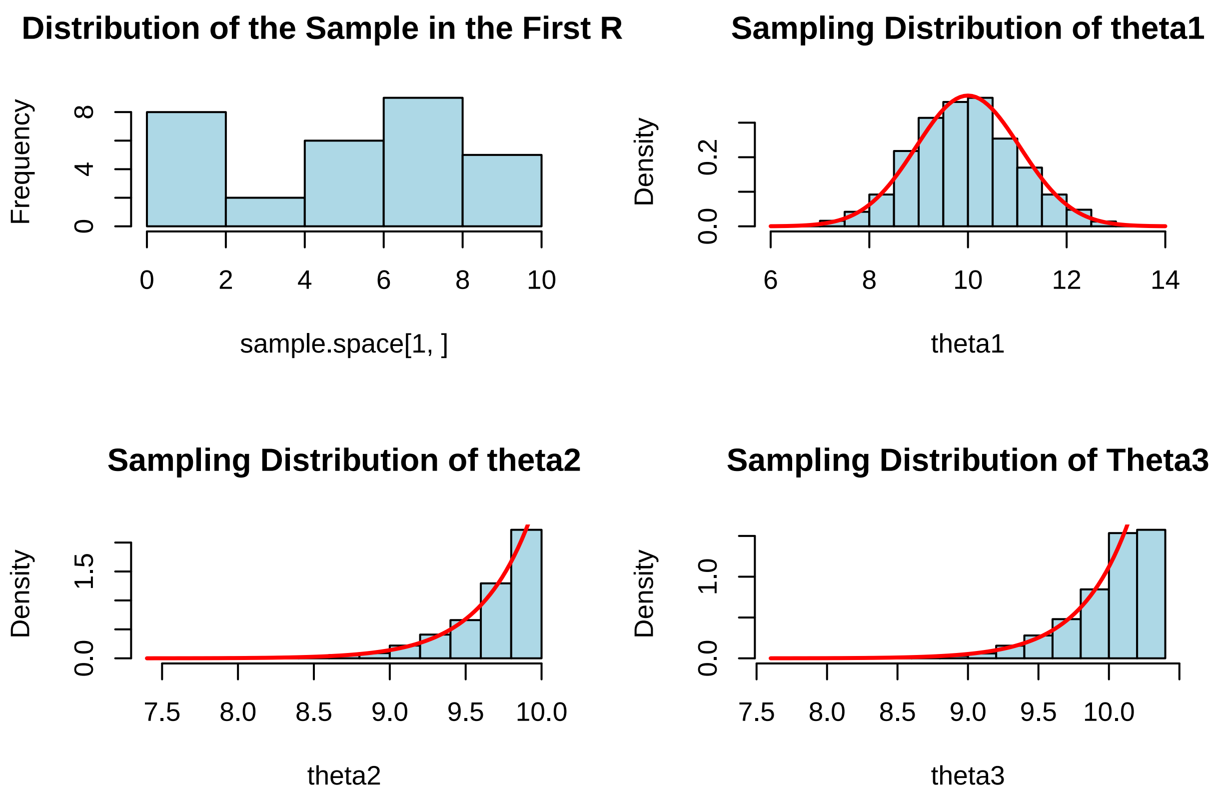


Q. 2d

Use R to generate 1000 independent random samples of size 30 from Uniform(0, 10). Compute the corresponding 1000 values of $\hat{\theta}_{\mathbf{1}}, \hat{\theta}_{\mathbf{2}}, \hat{\theta}_{\mathbf{3}}$.

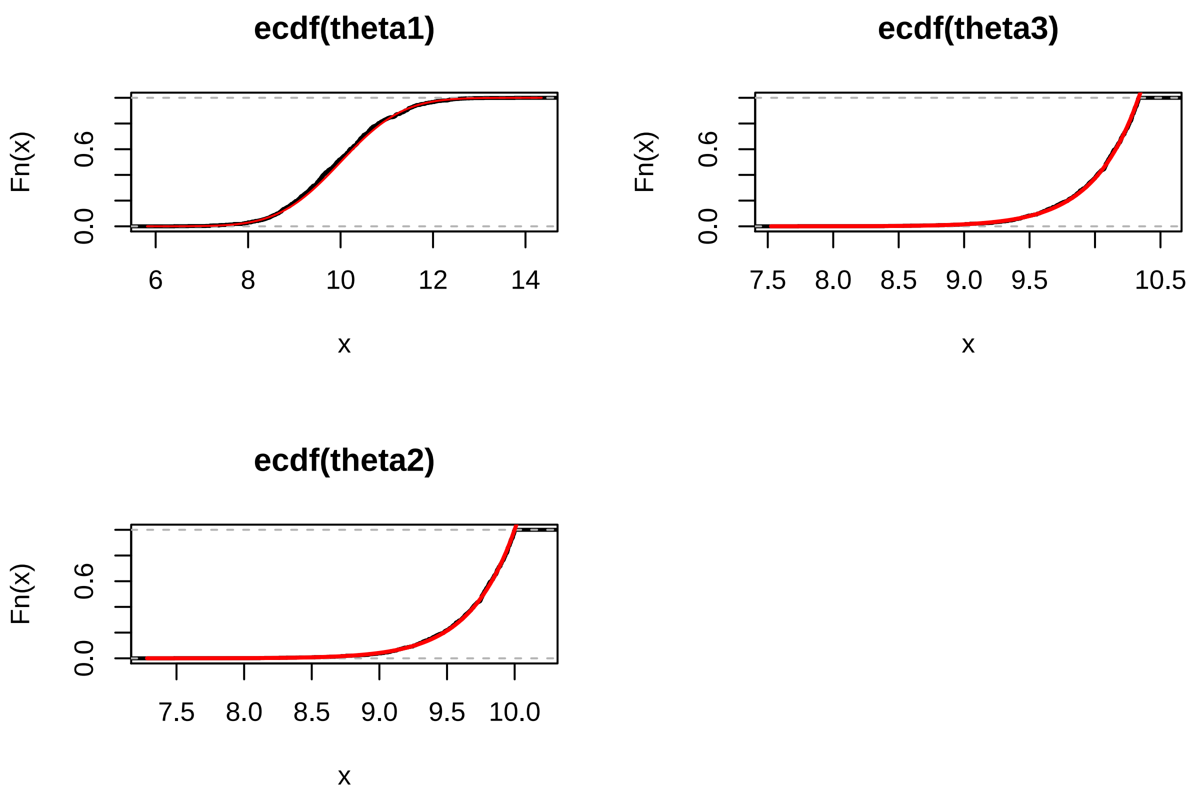
Q. 2d(i)

For each estimator, draw the histograms based on their 1000 estimates. Include the theoretical pdf overlayed. For $\hat{\theta}_{\mathbf{1}}$, you can use the approximate distribution in part (c).



Q. 2d(ii)

For each estimator, draw the ecdf based on their 1000 estimates. Include the theoretical cdf overlayed. For $\hat{\theta}_{\mathbf{1}}$, you can use the approximate distribution in part (c).



Q. 2d(iii)

Compute the empirical bias, variance and MSE for each of the estimators. Compare and contrast. Note: $bias(\hat{\theta}_1) = \bar{\hat{\theta}}_1$ - θ and MSE can be computed as above or recall, $MSE = Var + bias^2$.

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## [1] TRUE
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## [1] TRUE
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## [1] TRUE
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Since MSE_3 is less than both MSE_1 and MSE_2, MSE_3 (i.e.,the mean square error of the modified MLE) has the least residual error for a 1000 uniform random samples of size 30.

$$
\begin{aligned}
(bias(2\bar{X}) &= E(2\bar{X}) - \theta \\
&= 2E(\bar{X}) - \theta \\
&= 2\frac{\theta}{2} - \theta \\
&= 0 \\
) & & (Var(2\bar{X}) = 4Var(\bar{X}) \\
&= 4\frac{\theta^2}{12n} \\
&= \frac{\theta^2}{3n}
\end{aligned}
$$

$$
\begin{aligned}
MSE(2\bar{X}) &= Var(2\bar{X}) + bias^2(2\bar{X}) \\
&= Var(2\bar{X}) \\
&= \frac{\theta^2}{3n} \\
)
\end{aligned}
$$