

# Hw2

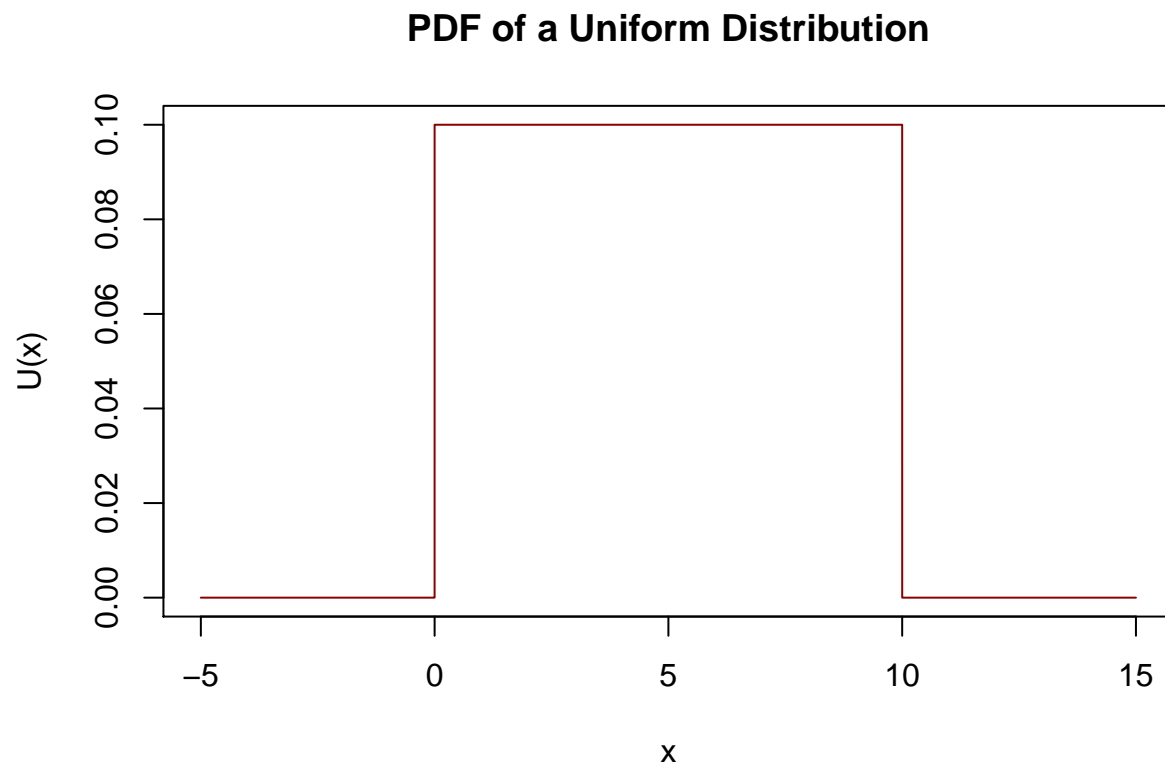
Akeem Ajede

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## Q. 1b(i)

Sketch the pdf of  $U(0,10)$

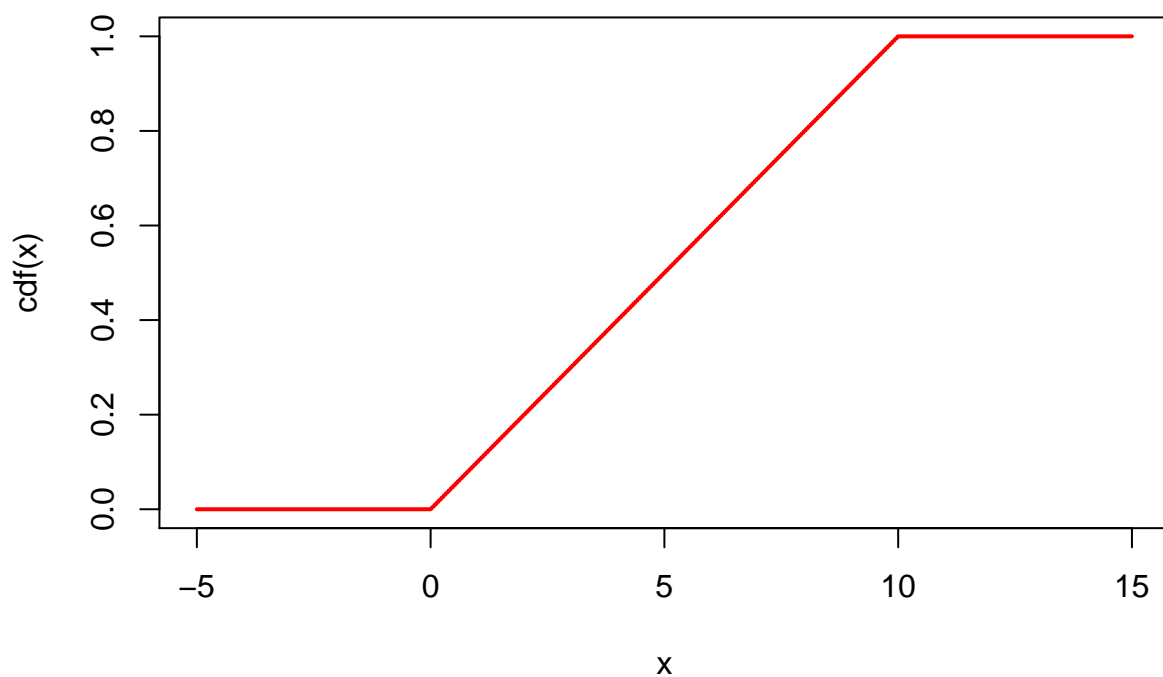
```
set.seed(7001)
U = function(x){dunif(x,0,10)}
x <- seq(-5,15,0.001)
plot(x, U(x), type = 'l', col = 'dark red', main = 'PDF of a Uniform Distribution')
```



## Q.1b(ii)

Sketch the cdf of  $U(0,10)$

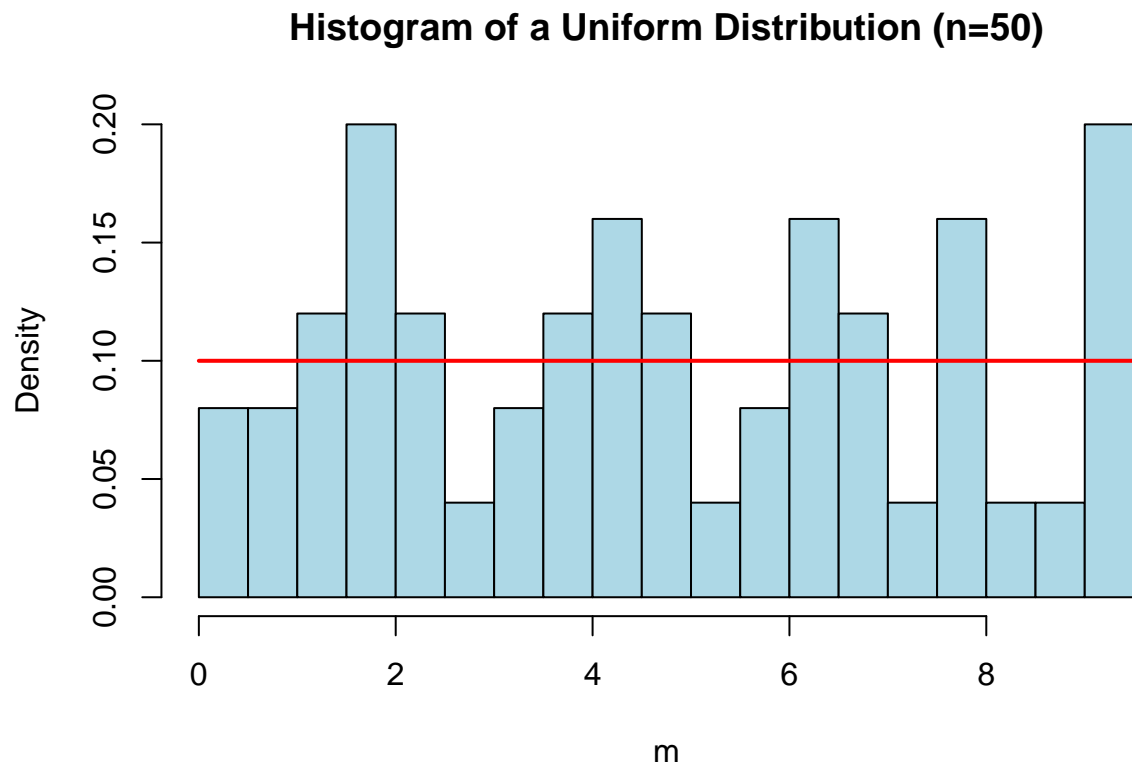
```
set.seed(7001)
cdf = function(x){punif(x,0,10)}
plot(x,cdf(x),type='l', col = 2, lwd = 2)
```



### Q. 1c(i)

Use R to generate a random sample of size 50 from  $\text{Uniform}(0, 10)$  and plot the histogram (overlaid with the true pdf).

```
set.seed(7001)
m = runif(50,0,10)
hist(m, probability = T, breaks = 15, main = "Histogram of a Uniform Distribution (n=50)", col = 'lightblue')
curve(dunif(x,0,10), col= 2, lwd=2, add=TRUE)
```

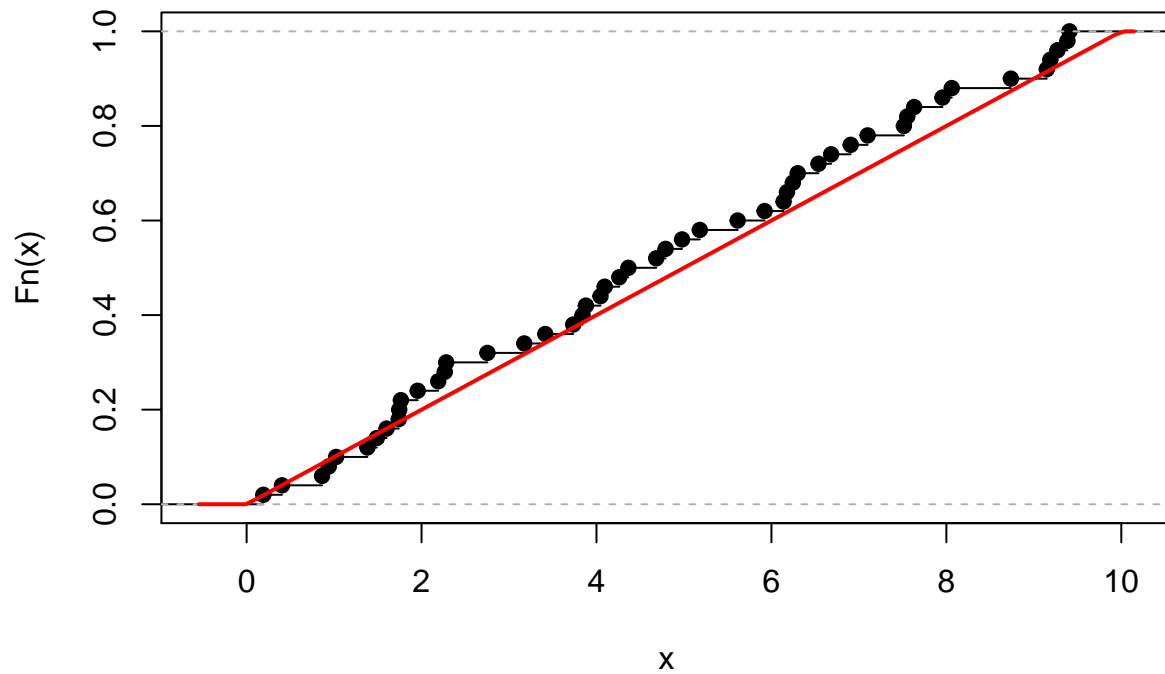


#### Q. 1c(ii)

plot the empirical cdf overlaid with the theoretical cdf.

```
set.seed(7001)
plot(ecdf(m), main = "ECDF of a Uniform Distribution (n=50)")
curve(punif(x,0,10), col= 2, lwd=2, add=TRUE)
```

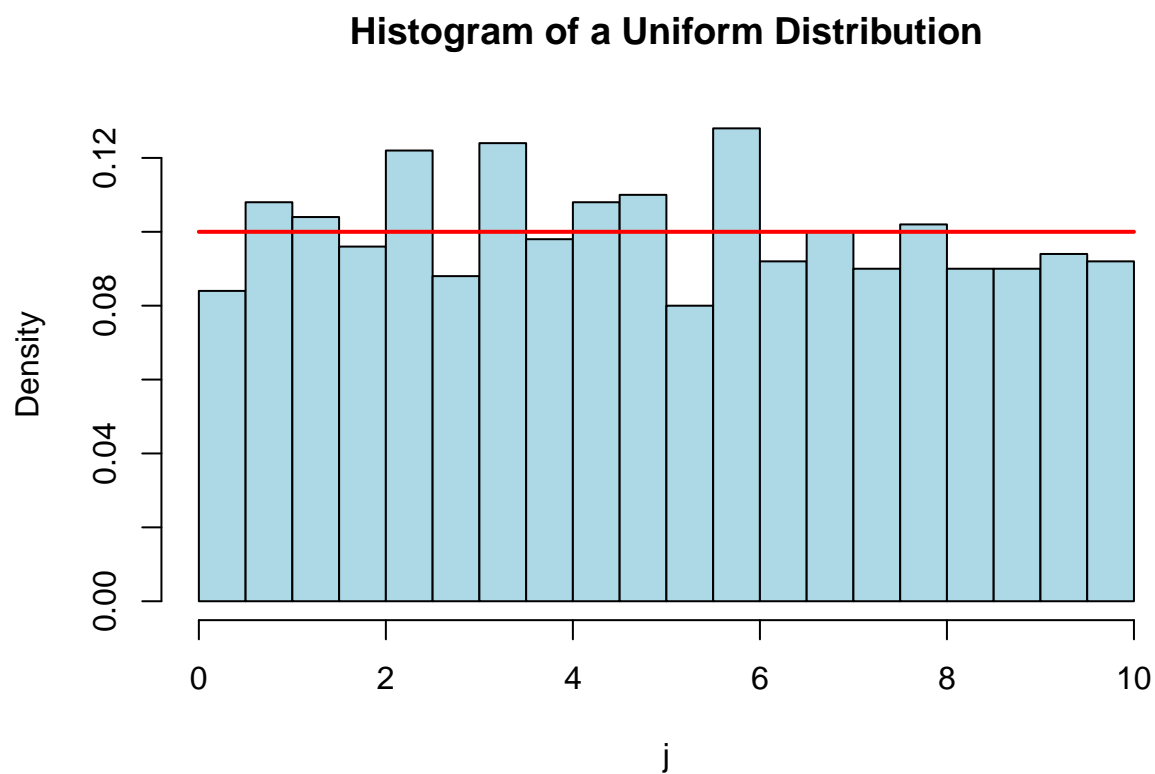
### ECDF of a Uniform Distribution (n=50)



### Q. 1d(i)

Use R to generate a random sample of size 1000 from Uniform(0, 10) and plot the histogram (overlaid with the true pdf).

```
set.seed(7001)
j = runif(1000,0,10)
hist(j, probability = T, breaks = 15, col = "light blue", main = "Histogram of a Uniform Distribution")
curve(dunif(x,0,10), col= 2, lwd=2, add=TRUE)
```

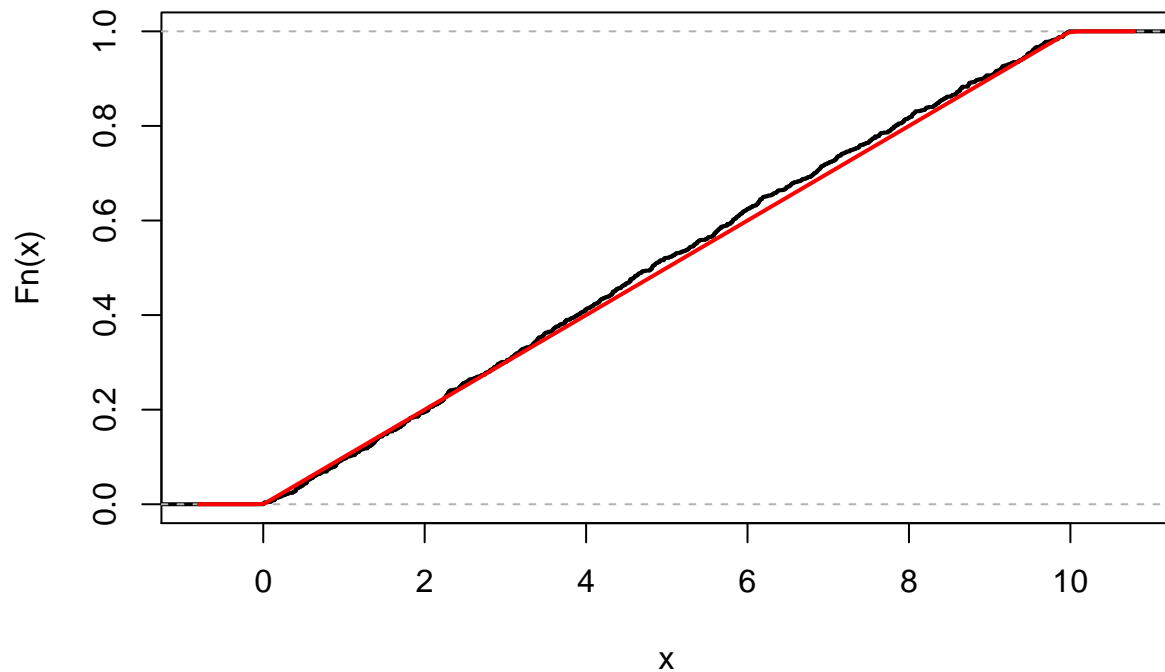


#### Q. 1d(ii)

Plot the empirical cdf overlaid with the theoretical cdf.

```
plot(ecdf(j), main = "ECDF of a Uniform Distribution", lwd = 2)
curve(punif(x,0,10), col= 2, lwd=2, add=TRUE)
```

## ECDF of a Uniform Distribution



### Q. 2d

```
set.seed(7001)
#generic //my.samples = matrix(runif(n*r,min,max),r)
# r = Number of random samples; n = sample size; min = a and max = b for a uniform distribution.
n <- 30
r <- 1000
sample.space = matrix(runif(n*r,0,10),r)

theta1 = 2*(apply(sample.space,1,mean)) #compute the means of each row

# l = mean(sample.space[1,])
# l

theta2 = apply(sample.space,1,max) #Compute the max of each row

theta3 = ((n+1)/n)*(apply(sample.space,1,max)) #Compute the max of each row
```

## Q. 2d(i)

```
set.seed(7001)
par(mfrow = c(2,2))

hist(sample.space[1,],col="light blue",main="Distribution of the Sample in the First Row")

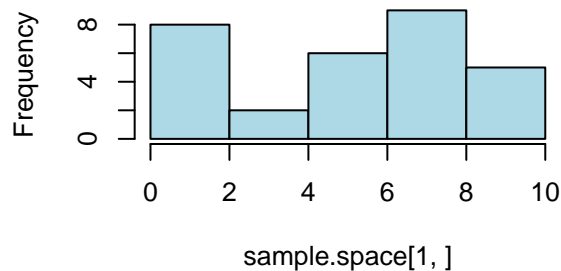
#The expression in eacy curve() is the respective "pdf" parameter

hist(theta1,col="light blue",main="Sampling Distribution of theta1",probability = T)
curve(dnorm(x, mean=10,sd=1.054093), col="red", lwd=2, add=TRUE)

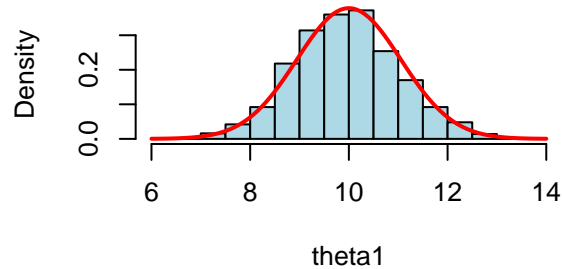
hist(theta2,col="light blue",main="Sampling Distribution of theta2",freq=F)
curve(((30*(x^29))/10^30), col=2, lwd=2, add=TRUE)

hist(theta3,col="light blue",main="Sampling Distribution of Theta3",freq=F)
curve(((30*(x^29))/10^30)*((30/31)^30), col= 2, lwd=2, add=TRUE)
```

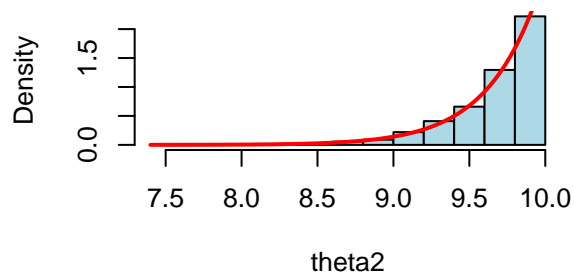
**Distribution of the Sample in the First R**



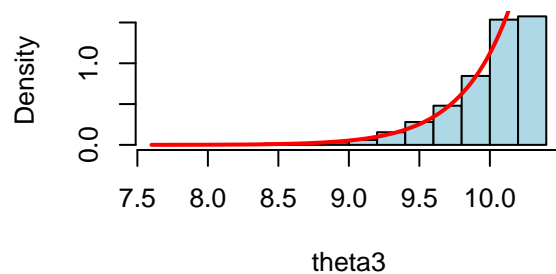
**Sampling Distribution of theta1**



**Sampling Distribution of theta2**



**Sampling Distribution of Theta3**



## Q. 2d(ii)

```
set.seed(7001)
```

```

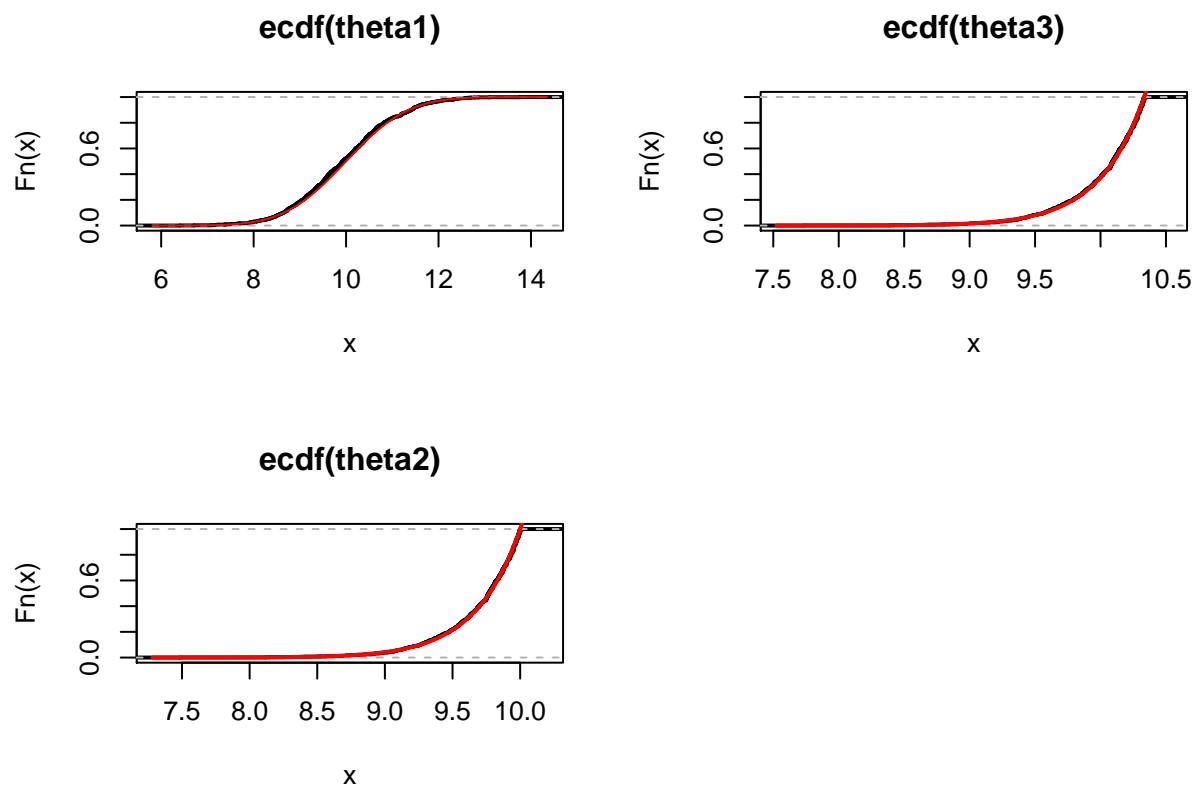
par(mfcol = c(2,2)) #2 rows by 2 col output space

plot(ecdf(theta1), lwd = 2)
curve(pnorm(x,10,1.054093),
      add=TRUE,
      col="red")

plot(ecdf(theta2), lwd = 2)
curve((x^30)/(10^30), col="red", lwd=2, add=TRUE)

plot(ecdf(theta3), lwd = 2)
curve(((x^30)/(10^30))*((30/31)^30), col="red", lwd=2, add=TRUE)

```



Q. 2d(iii)

```

bias1 = mean(theta1) - 10 #from bias = E(theta) - theta, where theta = 10 in U(0,10)

var1 <- var(theta1) #Computes the variance of theta 1

MSE1 = var1 + (bias1^2)

bias2 = mean(theta2) - 10

```



```

var2 = var(theta2) #Compute the variance of theta 2
MSE2 = var2+(bias2^2)
bias3 = mean(theta3) - 10
var3 = var(theta3) #Compute the variance of each row
MSE3 = var3+(bias3^2)
MSE1 > MSE2 #i.e., MSE2 is less than MSE1

```

```
## [1] TRUE
```

```
MSE1 > MSE3 #i.e., MSE3 is also less than MSE1
```

```
## [1] TRUE
```

```
MSE2 > MSE3 #MSE3 is less than both MSE1 and MSE2
```

```
## [1] TRUE
```

Since MSE3 is less than both MSE1 and MSE2, MSE3 (i.e., the mean square error of the modified MLE) has the least residual error for a 1000 uniform random samples of size 30.