1. Pochodne

$$\sqrt{x} \to \frac{1}{2\sqrt{x}}$$
$$\sin x \to \cos x$$
$$\cos x \to -\sin x$$
$$\operatorname{tg} x \to \frac{1}{\cos^2 x}$$
$$ctg\, x \to \frac{-1}{\sin^2 x}$$
$$\ln x \to \frac{1}{x}$$
$$\frac{1}{x} \to \frac{-1}{x^2}$$
$$f\left(g\left(x\right)\right) \to f'\left(g\left(x\right)\right) \cdot g'\left(x\right)$$

2. Całki

$$\int x^a\, dx = \frac{x^{a+1}}{a+1} + c$$
$$\int \frac{dx}{x} = \ln|x| + c$$
$$\int \frac{dx}{x^2 + a_2} = \frac{1}{a} \operatorname{arc}\tan\frac{x}{a} + c$$
$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \operatorname{arc}\sin\frac{x}{a} + c$$
$$\int e^x\, dx = e^x + c$$
$$\int a^x\, dx = \frac{a^x}{\ln a} + c$$
$$\int \cos x\, dx = \sin x + c$$
$$\int \sin x\, dx = -\cos x + c$$
$$\int \frac{dx}{\cos^2 x} = \operatorname{tg} x + c$$
$$\int \frac{dx}{\sin^2 x} = -ctg\, x + c$$
$$\int \frac{dx}{ax+b} = \frac{1}{x}\ln|ax+b| + c$$
$$\int \operatorname{tg} x\, dx = -\ln|\cos x| + c$$
$$\int ctg\, x\, dx = \ln|\sin x| + c$$
$$\int \left(ax+b\right)^n\, dx = \frac{\left(ax+b\right)^{n+1}}{a\left(n+1\right)} + c$$
$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + c$$
$$\int \sqrt{a^2 - x^2}\, dx = \frac{a^2}{2}\operatorname{arc}\sin\frac{x}{a} + \frac{x}{2}\sqrt{a^2 - x^2} + c$$

Całkowanie przez części

$$\int f\left(x\right) \cdot g'\left(x\right)\, dx = f\left(x\right) \cdot g\left(x\right) - \int f'\left(x\right) \cdot g\left(x\right)\, dx$$

Całkowanie przez podstawianie

$$\int\limits_a^b f\left(g\left(x\right)\right) \cdot g'\left(x\right)\, dx = \int\limits_{g(a)}^{g(b)} f\left(t\right)\, dt, \quad t = g\left(x\right)$$

3. Fourier

$$f\left(x\right) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{2\pi nx}{T} + b_n \sin\frac{2\pi nx}{T}\right)$$

Gdzie:

$$a_0 = \frac{1}{T}\int\limits_T^T f\left(x\right)\, dx$$
$$a_n = \frac{1}{T}\int\limits_T^T f\left(x\right)\cos\frac{\pi nx}{T}\, dx$$
$$b_n = \frac{1}{T}\int\limits_T^T f\left(x\right)\sin\frac{\pi nx}{T}\, dx$$

Tożsamość Parsevala

$$\frac{1}{\pi}\int\limits_{-\pi}^{\pi} f^2\left(x\right)\, dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty}\left(a_n^2 + b_n^2\right)$$

4. funkcja dwóch zmiennych

Jeśli $f_{xy}^{"} \neq f_{yx}^{"}$ to zjebałeś.

$$W\left(P\right) = \begin{vmatrix} f_{xx}^{"}\left(P\right) & f_{xy}^{"}\left(P\right) \\ f_{xy}^{"}\left(P\right) & f_{yy}^{"}\left(P\right) \end{vmatrix}$$

Jeśli $W(P) > 0$ to ekstremum.

Jeśli $W(P) < 0$ to nie ekstremum, w innym przypadku chuj wie.

Jeżeli $f^{"}_{xx}(P) > 0$ to maksimum.

Jeżeli $f^{"}_{xx}(P) < 0$ to minimum.

5. Laplace

$$F(s) = \int_0^\infty f(t)\, e^{-st}\, dt$$

Własności

$$\begin{aligned} af(t) + bg(t) &\risingdotseq aF(s) + bG(s) \\ f(at) &\risingdotseq \frac{1}{a} F\left(\frac{s}{a}\right) \\ f(t - t_0) H(t - t_0) &\risingdotseq e^{-t_0 s} F(s), \quad t_0 > 0 \\ e^{s_0 t} f(t) &\risingdotseq F(s - s_0) \end{aligned}$$

Transformata pochodnej

$$\begin{aligned} f'(t) &\risingdotseq sF(s) - f\left(0^+\right) \\ f''(t) &\risingdotseq s^2 F(s) - sf\left(0^+\right) - f'\left(0^+\right) \end{aligned}$$

Pochodna transformaty

$$tf(t) \risingdotseq -F'(s)$$

Transformata całki

$$\int_0^t f(u)\, du \risingdotseq \frac{F(s)}{s}$$

Całka transformaty

$$\frac{f(t)}{t} \risingdotseq \int_s^\infty F(u)\, du$$

Transformaty podstawowych funkcji

$$\begin{aligned} 1 &\risingdotseq \frac{1}{s} \\ t &\risingdotseq \frac{1}{s^2} \\ t^n &\risingdotseq \frac{n!}{s^{n+1}} \\ e^{s_0 t} &\risingdotseq \frac{1}{s - s_0} \\ te^{s_0 t} &\risingdotseq \frac{1}{(s - s_0)^2} \\ t^n e^{s_0 t} &\risingdotseq \frac{n!}{(s - s_0)^{n+1}} \\ \sin \omega t &\risingdotseq \frac{\omega}{s^2 + \omega^2} \\ \cos \omega t &\risingdotseq \frac{s}{s^2 + \omega^2} \\ t \sin \omega t &\risingdotseq \frac{2\omega s}{(s^2 + \omega^2)^2} \\ t \cos \omega t &\risingdotseq \frac{s^2 - \omega^2}{(s^2 + \omega^2)^2} \\ e^{s_0 t} \sin \omega t &\risingdotseq \frac{\omega}{(s - s_0)^2 + \omega^2} \\ e^{s_0 t} \cos \omega t &\risingdotseq \frac{s - s_0}{(s - s_0)^2 + \omega^2} \end{aligned}$$

Dodatkowe wzorki tu

6. Transformata odwrotna do Laplace'a

$$f(t) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} F(s)\, e^{st}\, ds$$

7. Sploty

$$f(t) * g(t) = \int_0^t f(u)\, g(t - u)\, du$$

Twierdzenia

$$
\begin{aligned}
f(t) * g(t) &\risingdotseq F(s) \cdot G(s) \\
\mathcal{L}^{-1}\left[F(s) \cdot G(s)\right] &= f(t) * g(t) \\
\left[f(t) * g(t)\right]' &= f(t) * g'(t) + f(t)\, g(0) \\
\left[f(t) * g(t)\right]' &\risingdotseq sF(s)\, G(s)
\end{aligned}
$$

8. transformata $\mathcal{Z}$

$$F(s) = \sum_{n=0}^{\infty} f(n)\, e^{-sn}\, dt$$

Przyjmujemy $z = e^s$, zatem:

$$F(s) = \sum_{n=0}^{\infty} f(n)\, z^{-n}\, dt$$

Własności

$$
\begin{aligned}
af(n) + bg(n) &\risingdotseq aF(z) + bG(z) \\
a^n f(n) &\risingdotseq F\left(\frac{z}{a}\right) \\
f(n-k) &\risingdotseq z^{-k} F(z)\,, \quad n \le k \\
f(n+k) &\risingdotseq z^k \cdot \left[F(z) - \sum_{i=0}^{k-1} f(i)\, z^{-i}\right]
\end{aligned}
$$

Transformata ciągu sum

Jeżeli $g(n) = \sum_{i=0}^{n} f(i)$, to:

$$g(n) \risingdotseq \frac{z}{z-1} F(z)$$

Różniczkowanie transformaty

$$nf(n) \risingdotseq -zF'(z)$$

Twierdzenia

$$
\begin{aligned}
\lim_{z\to\infty} F(z) &= f(0) \\
\lim_{z\to 1^+} (z-1)\, F(z) &= \lim_{n\to\infty} f(n)
\end{aligned}
$$

Transformaty podstawowych ciągofunkcji

$$
\begin{aligned}
\mathcal{Z}[0] &= 0 \\
\mathcal{Z}[\delta(n)] &= 1 \\
&\text{gdzie:} \quad \delta(n) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases} \\
\mathcal{Z}[\delta(n-k)] &= z^{-k}, \quad n \le k \\
\mathcal{Z}[1] &= \frac{z}{z-1}, \quad |z| > 1 \\
\mathcal{Z}[n] &= \frac{z}{(z-1)^2} \\
\mathcal{Z}\left[n^2\right] &= \frac{z(z+1)}{(z-1)^3} \\
\mathcal{Z}\left[n^3\right] &= \frac{z\left(z^2+4z+1\right)}{(z-1)^4} \\
\mathcal{Z}[a^n] &= \frac{z}{z-a}, \quad |z| > |a| \\
\mathcal{Z}[(-a)^n] &= \frac{z}{z+a} \\
\mathcal{Z}[na^n] &= \frac{az}{(z-a)^2} \\
\mathcal{Z}\left[n^2 a^n\right] &= \frac{az(z+a)}{(z-a)^3} \\
\mathcal{Z}[\sin \omega n] &= \frac{z \sin \omega}{z^2 - 2z\cos\omega + 1} \\
\mathcal{Z}[\cos \omega n] &= \frac{z \cos \omega}{z^2 - 2z\cos\omega + 1}
\end{aligned}
$$

9. Transformata $\mathcal{Z}^{-1}$

$$f(n) = \frac{1}{2\pi i} \oint_K F(z)\, z^{n-1}\, dz$$

10. Splot ciągów

$$h(n) = f(n) * g(n) = \sum_{i=0}^{n} f(i)\, g(n-i)$$

Twierdzenie

$$f(n) * g(n) \risingdotseq F(z) \cdot G(z)$$