1 Introduction to Solitons and Vorticies

Something or other

2 Question 1

3 Question 2

Consider the following potential

$$V(\phi) = \frac{\lambda}{4}(\phi^2 - v^2)^2$$

where $\lambda > 0$ and $v > 0$. Plot this potential, and show that the minima of this potential is at $\phi_{min} = \pm v$.

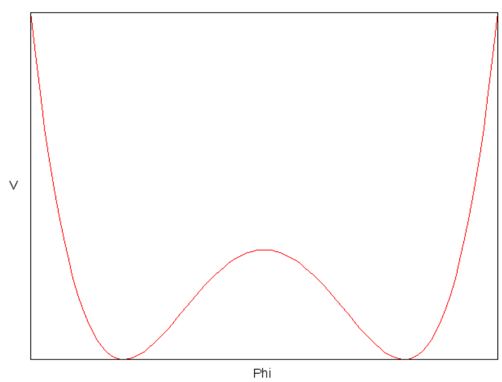

Figure 1: Potential

Turning points of a function may be found when the first derivative of the function is equal to zero.

$$\frac{\partial V(\phi)}{\partial \phi} = \frac{\lambda}{4} 2(\phi^2 - v^2) \times 2\phi = \lambda(\phi^2 - v^2)\phi$$

It can be seen that when this derivative is equal to zero that there are three possible solutions at $\phi = 0$ and $\phi = \pm v^2$. In order to identify which points are minima it is necessary to evaluate the second derivative of the potential at each of these points.

$$\frac{\partial^2 V(\phi)}{\partial \phi^2} = \lambda(\phi^2 - v^2) + 2\lambda\phi$$

Evaluating the second derivateive at these points shows that the minima of this function are at $\phi = \pm v^2$ and that there is a maximum at $\phi = 0$.

Considering the static solutions, where $\dot{\phi} = \ddot{\phi} = 0$. The Klein-Gordon Equation in this limit is then

$$\frac{\partial^2 \phi}{\partial x^2} - \lambda(\phi^2 - v^2)\phi = 0$$