

Task 5-6

Defines file path

```
addpath('classes');
```

Task description

Bruk implementeringen av RK4 eller RKF45 til å løse systemet. Benytt treghetsmomentent til T-nøkkelen som ble utregnet i tidligere oppgave. Bruk $X(0) = I$ (3×3 identitetsmatrisen). Beregn L slik at $\omega(0)$ har lengde ca lik 1 ved tiden 0. Kjør eksperimentet med:

- a) $\omega(0) = [1, 0.05, 0]^T$
- b) $\omega(0) = [0, 1, 0.05]^T$
- c) $\omega(0) = [0.05, 0, 1]^T$

Steglengden lar dere være så liten at energen ikke endrer seg nevneverdig

Tegn opp komponentene til løsningene X fra oppgave 5 som ni funksjoner av tiden. Gi en tolkning av disse grafene. Alternativt kan dere lage en 3-D animasjon av T-nøkkelen som roterer

Parameter initialization

```
R1 = 1; R2 = 1; % Radius
L1 = 8; L2 = 4; % Length
p = 6.7;      % Mass density
M = 12*pi * p; % Mass

X0 = eye(3); % X-matrix

h = 0.01;     % Initial step length
n = 10000;    % Number of iterations
TOL = 1e-30;  % Tolerance
```

Calculates moment of inertia

```
tHandle = THandle(R1, R2, L1, L2, M, p);
I = tHandle.calculateMomentOfInertia();
```

Task a

Parameter initialization

```
w0 = [1, 0.05, 0]'; % Rotation vector
L = X0 * I * w0;    % Torque vector
```

RKF45

Approximates solution

```
rkf45 = RKF45(h, n, TOL);
[t, W, E, h] = rkf45.solve(X0, I, L);
fprintf('Max Error: %d', max(E));
```

Max Error: 9.795761e-31

Calculates energy

```
energy = zeros(length(W), 1);

for i = 1:length(W)
    w = (W{i} * I)^(-1) * L;
    energy(i) = Energy.calculate(L, W{i} * w);
end

fprintf('Initial energy: %i \nFinal energy:    %i \nDifference:        %i', energy(1), ener
```

Initial energy: 4.920390e+02
 Final energy: 4.920390e+02
 Difference: 1.149231e-05

Visualization

```
v = Visualization(tHandle, '6a/RKF45');
v.plotX(t, W);
v.plotH(t, h);
v.plotHX(t, h, W, 1);
v.plotError(t, E);
v.plotErrorX(t, W, 1, E);
v.plotEnergy(t, energy);
v.plotErrorEnergy(t, energy, E, 8.0*10^(25));
v.plotAccumulativeError(t, E);
v.plotAccumulativeErrorX(t, W, 1, E);
v.plotAccumulativeErrorEnergy(t, energy, E, 9*10^(20));
v.animateTHandle(W, t(end) / length(W));
```

RK4, h = 0.1

Approximates solution

```
h = 0.1; n = 1000;  
rk4 = RK4(h, n);  
[t, W, E] = rk4.solve(X0, I, L);  
fprintf('Max Error: %d', max(E));
```

Max Error: 5.597257e-15

Calculates energy

```
energy = zeros(n, 1);  
  
for i = 1:length(W)  
    w = (W{i} * I)^(-1) * L;  
    energy(i) = Energy.calculate(L, W{i} * w);  
end  
  
fprintf('Initial energy: %i \nFinal energy:    %i \nDifference:        %i', energy(1), ener
```

Initial energy: 4.920390e+02
Final energy: 4.921088e+02
Difference: 6.982597e-02

Visualization

```
v = Visualization(tHandle, '6a/RK4/0.1');  
v.plotX(t, W);  
v.plotError(t, E);  
v.plotErrorX(t, W, 1, E);  
v.plotEnergy(t, energy);  
v.plotErrorEnergy(t, energy, E, 1.15*10^(13));  
v.plotAccumulativeError(t, E);  
v.plotAccumulativeErrorX(t, W, 1, E);  
v.plotAccumulativeErrorEnergy(t, energy, E, 5.7*10^(10));
```

RK4, h = 0.01

Approximates solution

```
h = 0.01; n = 10000;  
rk4 = RK4(h, n);  
[t, W, E] = rk4.solve(X0, I, L);  
fprintf('Max Error: %d', max(E));
```

Max Error: 5.567912e-25

Calculates energy

```

energy = zeros(n, 1);

for i = 1:length(W)
    w = (W{i} * I)^(-1) * L;
    energy(i) = Energy.calculate(L, W{i} * w);
end

fprintf('Initial energy: %i \nFinal energy:    %i \nDifference:        %i', energy(1), ener

Initial energy: 4.920390e+02
Final energy:   4.920391e+02
Difference:     8.296730e-05

```

Visualization

```

v = Visualization(tHandle, '6a/RK4/0.01');
v.plotX(t, W);
v.plotError(t, E);
v.plotErrorX(t, W, 1, E);
v.plotEnergy(t, energy);
v.plotErrorEnergy(t, energy, E, 1.0*10^(21));
v.plotAccumulativeError(t, E);
v.plotAccumulativeErrorX(t, W, 1, E);
v.plotAccumulativeErrorEnergy(t, energy, E, 5.5*10^(16));

```

RK4, h = 0.001

Approximates solution

```

h = 0.001; n = 100000;
rk4 = RK4(h, n);
[t, W, E] = rk4.solve(X0, I, L);
fprintf('Max Error: %d', max(E));

```

Max Error: 4.949640e-32

Calculates energy

```

energy = zeros(n, 1);

for i = 1:length(W)
    w = (W{i} * I)^(-1) * L;
    energy(i) = Energy.calculate(L, W{i} * w);
end

fprintf('Initial energy: %i \nFinal energy:    %i \nDifference:        %i', energy(1), ener

Initial energy: 4.920390e+02
Final energy:   4.920390e+02

```

Difference: 2.603802e-07

Visualization

```
v = Visualization(tHandle, '6a/RK4/0.001');
v.plotX(t, W);
v.plotError(t, E);
v.plotErrorX(t, W, 1, E);
v.plotEnergy(t, energy);
v.plotErrorEnergy(t, energy, E, 5.0*10^(25));
v.plotAccumulativeError(t, E);
v.plotAccumulativeErrorX(t, W, 1, E);
v.plotAccumulativeErrorEnergy(t, energy, E, 1.1*10^(21));
```

Task b

Parameter initialization

```
h = 0.01; n = 10000;
w0 = [0, 1, 0.05]'; % Rotation vector
L = X0 * I * w0;    % Torque vector
```

RKF45

Approximates solution

```
rkf45 = RKF45(h, n, TOL);
[t, W, E, h] = rkf45.solve(X0, I, L);
fprintf('Max Error: %d', max(E));
```

Max Error: 8.369184e-31

Calculates energy

```
energy = zeros(length(W), 1);

for i = 1:length(W)
    w = (W{i} * I)^(-1) * L;
    energy(i) = Energy.calculate(L, W{i} * w);
end

fprintf('Initial energy: %i \nFinal energy: %i \nDifference: %i', energy(1), energy(end), energy(end) - energy(1));
```

Initial energy: 3.633088e+02
Final energy: 3.633088e+02
Difference: 8.599500e-07

Visualization

```
v = Visualization(tHandle, '6b/RKF45');
v.plotX(t, W);
v.plotH(t, h);
v.plotError(t, E);
v.plotEnergy(t, energy);
v.plotAccumulativeError(t, E);
v.animateTHandle(W, t(end) / length(W));
```

RK4

Approximates solution

```
h = 0.01;
rk4 = RK4(h, n);
[t, W, E] = rk4.solve(X0, I, L);
fprintf('Max Error: %d', max(E));
```

Max Error: 3.716739e-28

Calculates energy

```
energy = zeros(n, 1);

for i = 1:length(W)
    w = (W{i} * I)^(-1) * L;
    energy(i) = Energy.calculate(L, W{i} * w);
end

fprintf('Initial energy: %i \nFinal energy:    %i \nDifference:        %i', energy(1), energy(end), energy(end) - energy(1));
```

Initial energy: 3.633088e+02
Final energy: 3.633088e+02
Difference: 1.697041e-06

Visualization

```
v = Visualization(tHandle, '6b/RK4');
v.plotX(t, W);
v.plotError(t, E);
v.plotEnergy(t, energy);
v.plotAccumulativeError(t, E);
```

Task c

Parameter initialization

```
w0 = [0.05, 0, 1]'; % Rotation vector
L = X0 * I * w0;    % Torque vector
```

RKF45

Approximates solution

```
rkf45 = RKF45(h, n, TOL);
[t, W, E, h] = rkf45.solve(X0, I, L);
fprintf('Max Error: %d', max(E));
```

Max Error: 5.828470e-31

Calculates energy

```
energy = zeros(length(W), 1);

for i = 1:length(W)
    w = (W{i} * I)^(-1) * L;
    energy(i) = Energy.calculate(L, W{i} * w);
end

fprintf('Initial energy: %i \nFinal energy:    %i \nDifference:        %i', energy(1), energy(end), energy(end) - energy(1));
```

Initial energy: 7.905530e+02
Final energy: 7.905530e+02
Difference: 3.418527e-07

Visualization

```
v = Visualization(tHandle, '6c/RKF45');
v.plotX(t, W);
v.plotH(t, h);
v.plotError(t, E);
v.plotEnergy(t, energy);
v.plotAccumulativeError(t, E);
v.animateTHandle(W, t(end) / length(W));
```

RK4

Approximates solution

```
h = 0.01;
```

```
rk4 = RK4(h, n);
[t, W, E] = rk4.solve(X0, I, L);
fprintf('Max Error: %d', max(E));
```

Max Error: 3.018502e-25

Calculates energy

```
energy = zeros(n, 1);

for i = 1:length(W)
    w = (W{i} * I)^(-1) * L;
    energy(i) = Energy.calculate(L, W{i} * w);
end

fprintf('Initial energy: %i \nFinal energy:    %i \nDifference:        %i', energy(1), ener
```

Initial energy: 7.905530e+02
Final energy: 7.905530e+02
Difference: 5.764557e-06

Visualization

```
v = Visualization(tHandle, '6c/RK4');
v.plotX(t, W);
v.plotError(t, E);
v.plotEnergy(t, energy);
v.plotAccumulativeError(t, E);
```