Øving 1

Oppgave 2.1.1

a) $f \cdot g = \int_0^{2\pi} \sin(x) \cdot \cos(x)\, dx$

$u = \sin(x)$

$du = \cos(x)\, dx$

$$\int_{x=0}^{x=2\pi} u\, du = \tfrac{1}{2} u^2 \Big|_{x=0}^{x=2\pi} = \tfrac{1}{2} \sin^2 x \Big|_0^{2\pi}$$

$$= \tfrac{1}{2}\left(\sin^2(2\pi) - \sin^2(0)\right) = \underline{\underline{0}}$$

Oppgave 2.1.2

b) Trigonometrisk identitet:

$$\sin^2 u = \frac{1 - \cos(2u)}{2}$$

$$\underline{\underline{\sin^2(\pi x) = \tfrac{1}{2} - \tfrac{1}{2}\cos(2\pi x)}}$$

Oppgave 2.1.5

a) $\int_{-L}^{L} \cos\left(\frac{n\pi x}{L}\right) \cos\left(\frac{m\pi x}{L}\right) dx$

$= \int_{-L}^{L} 1/2 \left[\cos\left(\frac{(n-m)\pi x}{L}\right) + \cos\left(\frac{(n+m)\pi x}{L}\right)\right] dx$

Deler opp i to integral:

$(*) \int_{-L}^{L} \cos\left(\frac{(n-m)\pi x}{L}\right)$

$(**) \int_{-L}^{L} \cos\left(\frac{(n+m)\pi x}{L}\right)$

Løser først $(*)$:

$(*) \int_{-L}^{L} \cos\left(\frac{(n-m)\pi x}{L}\right) dx$

$u = \frac{(n-m)\pi x}{L}$

$du = \frac{(n-m)\pi}{L} dx$

$dx = \frac{L}{(n-m)\pi}$

Setter inn i $(*)$:

$\frac{L}{(n-m)\pi} \int_{x=-L}^{x=L} \cos(u)\, du$

$$= \frac{L}{(n-m)\pi} \left[\sin\left(\frac{(n-m)\pi x}{L}\right) \right]_{-L}^{L}$$

$$= \frac{L}{(n-m)\pi} \left(\sin\left(\frac{(n-m)\pi L}{L}\right) - \sin\left(\frac{(n-m)\pi(-L)}{L}\right)\right)$$

$$= \frac{L}{(n-m)\pi} \left(\sin((n-m)\pi) - \sin(-(n-m)\pi) \right)$$

$$= \underline{\frac{2L}{(n-m)\pi} \cdot \sin((n-m)\pi)}$$

Tilsvarende for $(**)$:

$$\int_{-L}^{L} \cos\left(\frac{(n+m)\pi x}{L}\right) dx$$

$$= \underline{\frac{2L}{(n+m)\pi} \sin((n+m)\pi)}$$

$*$ Case I, $m \neq n$:

$$\frac{1}{2} \int_{-L}^{L} \underbrace{\cos\left(\frac{(n-m)\pi x}{L}\right)}_{(*)} + \underbrace{\cos\left(\frac{(n+m)\pi x}{L}\right)}_{(**)} dx$$

Setter inn løsning for $(*)$ og $(**)$

$$\frac{1}{2}\left(\frac{2L}{(n-m)\pi} \sin((n-m)\pi) + \frac{2L}{(n+m)\pi} \sin((n+m)\pi)\right)$$

$$= \frac{L}{(n-m)\pi} \sin((n-m)\pi) + \frac{L}{(n+m)\pi} \sin((n+m)\pi)$$

n og m er begge heltall, så
$\sin((n+m)\pi) = 0$, derfor får vi:

$$\frac{L}{(n-m)\pi} \cdot 0 + \frac{L}{(n+m)\pi} \cdot 0 = \underline{0}$$

$*$ Case 2, $m = n$:

$$\frac{1}{2} \int_{-L}^{L} \cos(0) + \cos\left(\frac{(n+m)\pi x}{2}\right) dx$$

$(**)$

$$= \frac{1}{2}\left([x]_{-L}^{L} + \frac{2L}{(n+m)\pi} \underbrace{\sin((n+m)\pi)}_{=0} \right)$$

$$= \frac{1}{2}(L + L + 0) = \frac{1}{2}(2L) = \underline{\underline{L}}$$

Oppgave 2.1.6

b) $a_0 = \frac{1}{2} \int_{-1}^{1} f(x)\, dx$

$$= \frac{1}{2} \left(\int_{-1}^{0} -1\, dx + \int_{0}^{1} 1\, dx \right)$$

$$= \frac{1}{2} \left([-x]_{-1}^{0} + [x]_{0}^{1} \right)$$

$$= \frac{1}{2} (1 + 1) = \underline{\underline{0}}$$

$$a_n = \int_{-1}^{1} f(x) \cdot \cos(n\pi x)\, dx$$

$$= \int_{-1}^{0} -\cos(n\pi x)\, dx + \int_{0}^{1} \cos(n\pi x)\, dx$$

$$= \left[-\sin(n\pi x) \cdot \frac{1}{n\pi} \right]_{-1}^{0} + \left[\sin(n\pi x) \cdot \frac{1}{n\pi} \right]_{0}^{1}$$

$$= \sin(-n\pi) \cdot \frac{1}{n\pi} + \sin(n\pi) \cdot \frac{1}{n\pi}$$

$$= -\sin(n\pi) \cdot \frac{1}{n\pi} + \sin(n\pi) \cdot \frac{1}{n\pi}$$

$$\underline{\underline{= 0}}$$

$$b_n = \int_{-1}^{1} f(x) \cdot \sin(n\pi x)\, dx$$

$$= \int_{-1}^{0} -\sin(n\pi x)\, dx + \int_{0}^{1} \sin(n\pi x)\, dx$$

$$= \left[\cos(n\pi x) \cdot \frac{1}{n\pi} \right]_{-1}^{0} + \left[-\cos(n\pi x) \cdot \frac{1}{n\pi} \right]_{0}^{1}$$

$$= \frac{1}{n\pi} - \cos(-n\pi) \cdot \frac{1}{n\pi} - \cos(n\pi) \cdot \frac{1}{n\pi} + \frac{1}{n\pi}$$

$$= \frac{1}{n\pi} - (-1)^n \frac{1}{n\pi} - (-1)^n \frac{1}{n\pi} + \frac{1}{n\pi}$$

$$\underline{= \frac{2}{n\pi} - (-1)^n \frac{2}{n\pi}}$$

Oppgave 2.1.6, forts.

$$f \sim \sum_{n=1}^{\infty} \frac{2-(-1)^n 2}{n\pi} \sin(n\pi x)$$

Oppgave 2.2.2

b) $y(x) = e^x$

$$y(x) = \frac{y(x)}{2} + \frac{y(-x)}{2} + \frac{y(x)}{2} - \frac{y(-x)}{2}$$

$$y(x) = \underbrace{\frac{y(x)+y(-x)}{2}}_{\text{Jevn}} + \underbrace{\frac{y(x)-y(-x)}{2}}_{\text{odde}}$$

$$e^x = \frac{e^x + e^{-x}}{2} + \frac{e^x - e^{-x}}{2}$$

Oppgave 2.2.3

b) $f(x)$ og $g(x)$ er jevne. Har at:

$$f(x) = f(-x) \quad \text{og} \quad g(x) = g(-x)$$

$$h(x) = f(x)\,g(x) = f(-x)\,g(-x)$$
$$= \underline{h(-x)}$$

Altså er $h(x) = h(-x)$, og er derfor også jevn.

Oppgave 2.2.5

a) $$b_n = \frac{2}{\pi/2} \int_0^{\pi/2} \sin\left(\frac{n\pi x}{\pi/2}\right) \cdot \cos(x)\,dx$$

$$= 4/\pi \int_0^{\pi/2} \sin(2nx) \cdot \cos(x)\,dx$$

$$= 4/\pi \int_0^{\pi/2} 1/2 \left(\sin(2nx + x) + \sin(2nx - x) \right) dx$$

$$= 2/\pi \left(\int_0^{\pi/2} \sin(2nx + x)\,dx + \int_0^{\pi/2} \sin(2nx - x)\,dx \right)$$

$$= 2/\pi \left(\left[\frac{-\cos(2nx + x)}{2n+1} \right]_0^{\pi/2} + \left[\frac{-\cos(2nx - x)}{2n-1} \right]_0^{\pi/2} \right)$$

$$= 2/\pi \left(\frac{-\cos(n\pi + \pi/2) + 1}{2n+1} + \frac{-\cos(n\pi - \pi/2) + 1}{2n-1} \right)$$

$$= \frac{2}{\pi} \left(\frac{1}{2n+1} + \frac{1}{2n-1} \right) = \frac{2}{\pi} \left(\frac{(2n-1) + (2n+1)}{(2n-1)(2n+1)} \right)$$

$$= \underline{\frac{8n}{\pi(2n-1)(2n+1)}}$$

$$\underline{\underline{f \sim \frac{8}{\pi} \sum_{n=1}^{\infty} \frac{n}{(2n-1)(2n+1)} \sin(2nx)}}$$