Problem 1

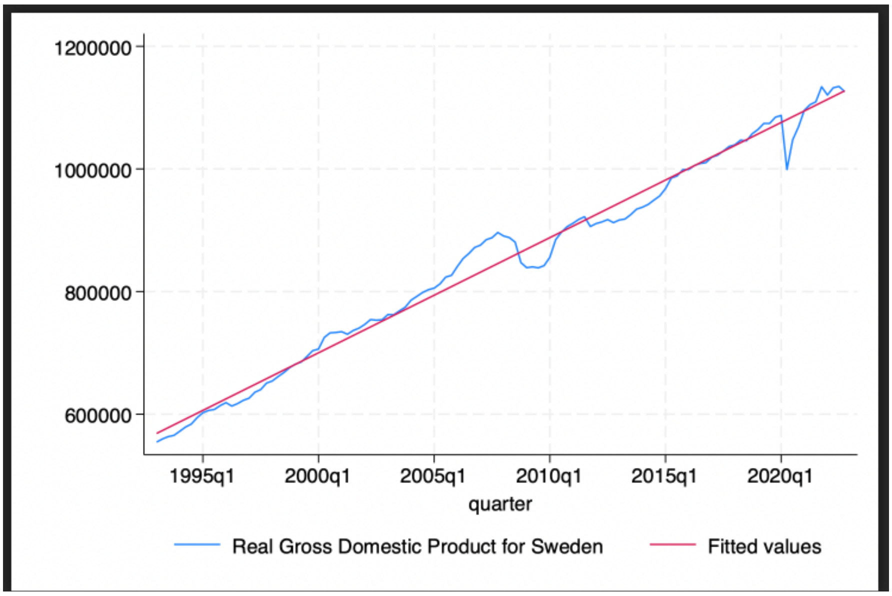
1.1

Figur 1. OLS regression of real GDP on a linear time trend (2000 Q1–2019 Q4)

```
. regress CLVMNACSCAB1GQSE time if quarter <= yq(2019, 4)

      Source |       SS           df       MS      Number of obs   =       108
-------------+----------------------------------   F(1, 106)       =   6163.06
       Model |  2.3147e+12         1  2.3147e+12   Prob > F        =    0.0000
    Residual |  3.9811e+10       106   375573923   R-squared       =    0.9831
-------------+----------------------------------   Adj R-squared   =    0.9829
       Total |  2.3545e+12       107  2.2005e+10   Root MSE        =     19380

------------------------------------------------------------------------------
CLVMNACSCA~E | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
        time |   4695.909   59.81654    78.51   0.000     4577.316    4814.501
       _cons |   563923.5   3755.682   150.15   0.000     556477.5    571369.5
------------------------------------------------------------------------------
```

The quarterly slopeis **4 695.9** (SE 59.8), and the intercept **563 923.5** (SE 3 755.7), both highly significant ($p < 0.001$). The model explains **98.3 %** of GDP variation ($R^2 = 0.9831$), with an F-statistic of 6 163 (df = 1, 106), tells us you can reject H0 as well as the time is highly significant, and the trend line adds real explanatory power.

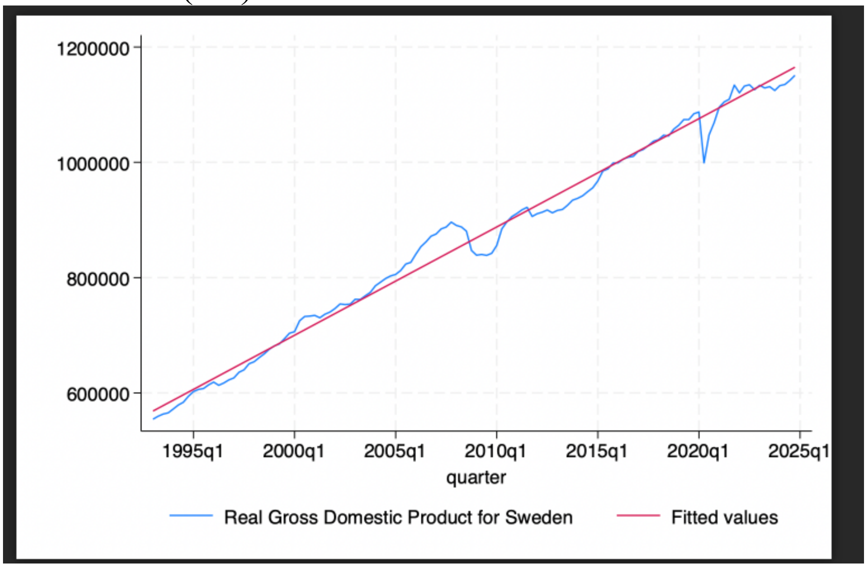
Figur 2. Observed GDP (blue) with fitted trend extended to 2022 (red)



The trend line closely tracks GDP until 2019, with visible dips around 2008 and 2020. Extending the trend forecasts GDP rising toward **1.1–1.2 million SEK** by end-2022.

1.2

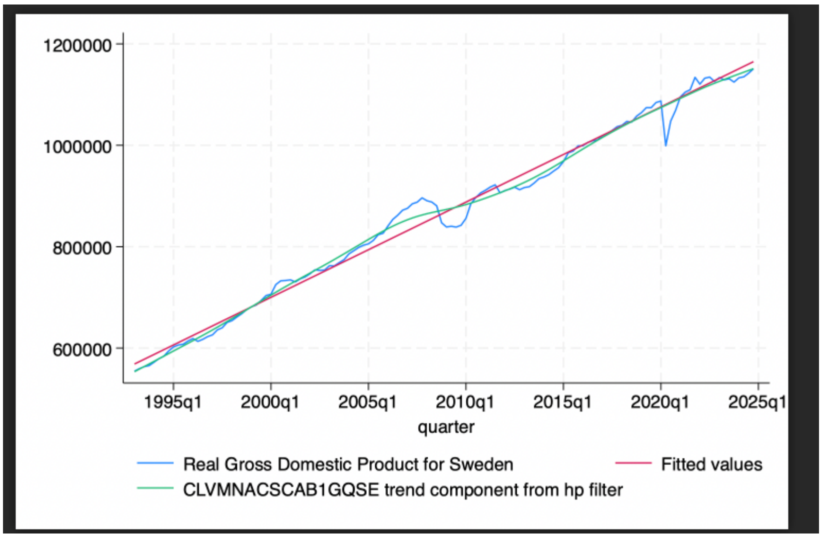
Figur 3: Real GDP extended through 2025 Q1 (blue) with fitted trend from 2000–2019 (red)



This plot adds the latest GDP data up to 2025 Q1. After the 2020 COVID dip, GDP rebounds strongly and by 2025 Q1 lies slightly above the long-term linear trend, indicating continued growth in line with the OLS projection.

1.3

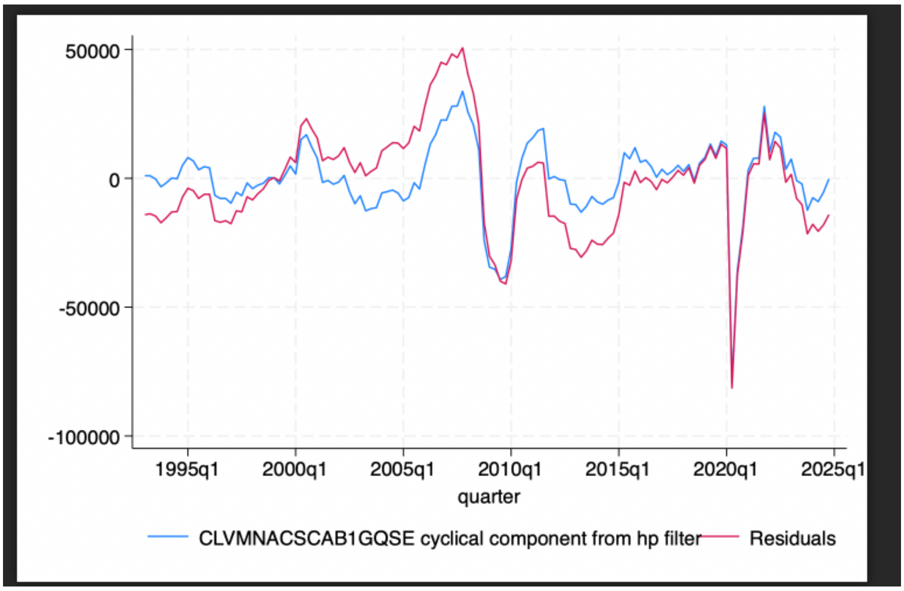
Figur 4: Observed GDP (blue), OLS trend (red) and HP trend (green)



The Hodrick-Prescott trend (green) closely follows medium-term ups and downs—smoothing the 2008 dip less sharply and the 2020 drop more gently, while the linear OLS trend (red) runs straight through. Before 2005 the HP trend lies slightly below the OLS line, then moves above it during the mid-2000s boom, and converges again after 2015. This shows that the HP filter captures non-linear deviations around the long-run growth path that the simple linear model misses.

1.4

Figur 5: Cyclical component from HP filter (blue) and OLS residuals (red)



Both series capture the same business-cycle swings: positive peaks around 2007 and 2018, and deep troughs during the 2009 financial crisis and the 2020 pandemic. The HP-based cycle (blue) is smoother, with smaller short-term fluctuations, while the OLS residuals (red) show sharper, more volatile deviations from the linear trend. Overall, both measures align closely in timing of expansions and contractions

Problem 2

2.1

Creating observations, seasonal values, trend, noise, and dependet variable

2.2

Figur 6: Frequency of quarterly dummy variables

qtr	Freq.	Percent	Cum.
1	10	25.00	25.00
2	10	25.00	50.00
3	10	25.00	75.00
4	10	25.00	100.00
Total	40	100.00	

Figur 7: Regression of yt on four quarterly dummies (no intercept)

Source	SS	df	MS			
				Number of obs	=	40
				F(4, 36)	=	119.73
Model	736.017373	4	184.004343	Prob > F	=	0.0000
Residual	55.3267629	36	1.53685453	R-squared	=	0.9301
				Adj R-squared	=	0.9223
Total	791.344136	40	19.7836034	Root MSE	=	1.2397

y	Coefficient	Std. err.	t	P>|t|	[95% conf.	interval]
q1	3.328077	.3920274	8.49	0.000	2.533009	4.123146
q2	4.460733	.3920274	11.38	0.000	3.665665	5.255802
q3	5.075524	.3920274	12.95	0.000	4.280455	5.870592
q4	4.106891	.3920274	10.48	0.000	3.311823	4.901959

Each dummy's coefficient is the average yt in that quarter (q1=3.33, q2=4.46, q3=5.08, q4=4.11). Because we haven't removed the upward trend, these averages are inflated above the true seasonal effects (1.5–3.0). The dummies alone explain 93 % of the variation (R^2=0.93).

2.3

* Regression with intercept

Figur 8: Regression with intercept (three dummies)

Source	SS	df	MS			
				Number of obs	=	40
				F(3, 36)	=	3.46
Model	15.9611257	3	5.32037523	Prob > F	=	0.0262
Residual	55.3267629	36	1.53685453	R-squared	=	0.2239
				Adj R-squared	=	0.1592
Total	71.2878886	39	1.82789458	Root MSE	=	1.2397

y	Coefficient	Std. err.	t	P>|t|	[95% conf.	interval]
q1	-.7788137	.5544104	-1.40	0.169	-1.90321	.3455828
q2	.3538424	.5544104	0.64	0.527	-.770554	1.478239
q3	.9686327	.5544104	1.75	0.089	-.1557637	2.093029
_cons	4.106891	.3920274	10.48	0.000	3.311823	4.901959

The intercept (4.107) is the average yt in Q4. Each dummy measures the deviation from Q4: e.g. Q1 is –0.779, so Q1 mean = 3.33. Only the intercept is statistically significant (p<0.001).

2.4

Remove the trend

Figur 9: Regression on detrended series (no intercept)

Source	SS	df	MS			
Model	205.845891	4	51.4614728	Number of obs	=	40
Residual	3.24031252	36	.090008681	F(4, 36)	=	571.74
				Prob > F	=	0.0000
Total	209.086204	40	5.22715509	R-squared	=	0.9845
				Adj R-squared	=	0.9828
				Root MSE	=	.30001

y_trendless	Coefficient	Std. err.	t	P>|t|	[95% conf.	interval]
q1	1.428077	.0948729	15.05	0.000	1.235666	1.620489
q2	2.460733	.0948729	25.94	0.000	2.268322	2.653145
q3	2.975524	.0948729	31.36	0.000	2.783113	3.167935
q4	1.906891	.0948729	20.10	0.000	1.71448	2.099302

After removing the known linear trend, each dummy coefficient directly estimates the true seasonal effect: Q1 = 1.43 (true 1.5), Q2 = 2.46 (true 2.5), Q3 = 2.98 (true 3.0), Q4 = 1.91 (true 2.0). $R^2 = 0.9845$ ($p < 0.001$) shows dummies explain 98.5 % of variation in the detrended series.

Rerun the regression form 'Third'

Firgur 10: Regression on detrended series (with intercept)

Source	SS	df	MS			
Model	13.5098945	3	4.50329816	Number of obs	=	40
Residual	3.24031252	36	.090008681	F(3, 36)	=	50.03
				Prob > F	=	0.0000
Total	16.750207	39	.429492488	R-squared	=	0.8066
				Adj R-squared	=	0.7904
				Root MSE	=	.30001

y_trendless	Coefficient	Std. err.	t	P>|t|	[95% conf.	interval]
q1	-.4788137	.1341705	-3.57	0.001	-.7509241	-.2067032
q2	.5538425	.1341705	4.13	0.000	.281732	.8259529
q3	1.068633	.1341705	7.96	0.000	.7965222	1.340743
_cons	1.906891	.0948729	20.10	0.000	1.71448	2.099302

Adding an intercept, and three dummies, here the intercept (=1.91) is the average of the detrended series in Q4, and the three dummies show how much Q1–Q3 deviate from that base, so : q1 = –0.479 ⇒ Q1 = 1.428; q2 = +0.554 ⇒ Q2 = 2.461; q3 = +1.069 ⇒ Q3 = 2.976. Lowe R^2 reflects that part of the seasonal variation is now absorbed into the intercept

2.5

Estimate the trend

Figur 11: Estimated time trend

```
      Source |       SS           df       MS      Number of obs   =        40
-------------+----------------------------------   F(1, 38)        =    123.80
       Model |  54.5448667         1  54.5448667   Prob > F        =    0.0000
    Residual |  16.7430219        38  .440605839   R-squared       =    0.7651
-------------+----------------------------------   Adj R-squared   =    0.7590
       Total |  71.2878886        39  1.82789458   Root MSE        =    .66378

------------------------------------------------------------------------------
           y | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
     quarter |   .1011611    .009092    11.13   0.000     .0827552    .1195669
       _cons |  -17.96204   1.998461    -8.99   0.000    -22.00772   -13.91637
------------------------------------------------------------------------------
```

The slope (0.1012) matches the true quarterly trend of 0.1, showing our OLS correctly recovers the time trend. R^2=0.765 means the trend explains 76.5 % of total variation

Rerun regressions efter removing the estimated trend from y

Figur 12: Rerun on detrended y, no intercept

```
. regress y_estimate_trendless q1 q2 q3 q4, noconstant

      Source |       SS           df       MS      Number of obs   =        40
-------------+----------------------------------   F(4, 36)        =    557.49
       Model |  201.670586         4  50.4176466   Prob > F        =    0.0000
    Residual |  3.25571504        36  .090436529   R-squared       =    0.9841
-------------+----------------------------------   Adj R-squared   =    0.9823
       Total |  204.926301        40  5.12315754   Root MSE        =    .30073

------------------------------------------------------------------------------
y_estimate~s | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
          q1 |   1.406017   .0950981    14.78   0.000     1.213149    1.598885
          q2 |   2.437512   .0950981    25.63   0.000     2.244645     2.63038
          q3 |   2.951142   .0950981    31.03   0.000     2.758274     3.14401
          q4 |   1.881348   .0950981    19.78   0.000      1.68848    2.074216
------------------------------------------------------------------------------
```

Subtracting the estimated trend leaves only seasonality + noise. The four dummy coefficients (=1.41, 2.44, 2.95, 1.88) closely match the true seasonal effects (1.5, 2.5, 3.0, 2.0). R^2=0.984 confirms an excellent fit

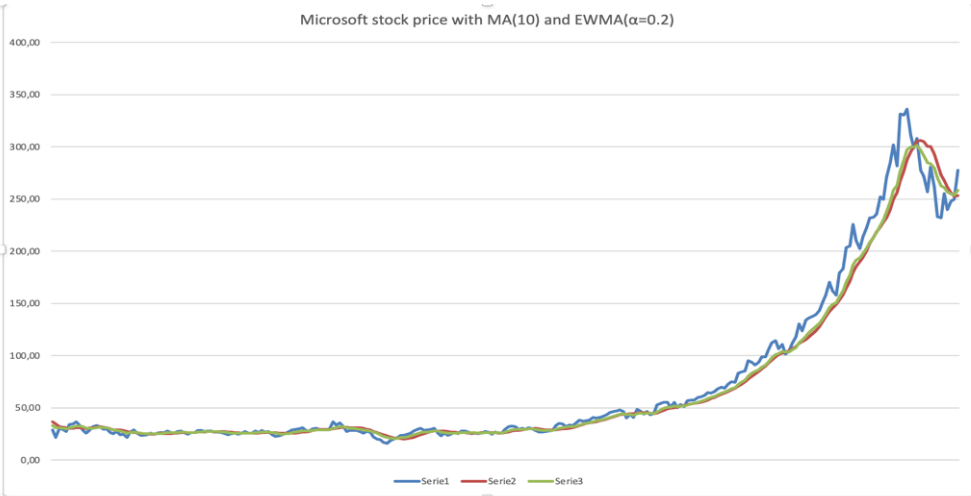
Figur 13: Rerun with intercept on detrended y

```
.
. regress y_estimate_trendless q1 q2 q3

      Source |       SS           df       MS      Number of obs   =        40
-------------+----------------------------------   F(3, 36)        =     49.71
       Model |  13.4873071         3  4.49576903   Prob > F        =    0.0000
    Residual |  3.25571504        36  .090436529   R-squared       =    0.8055
-------------+----------------------------------   Adj R-squared   =    0.7893
       Total |  16.7430221        39   .42930826   Root MSE        =    .30073

------------------------------------------------------------------------------
y_estimate~s | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
          q1 |  -.4753305   .1344891    -3.53   0.001     -.748087   -.2025741
          q2 |   .5561646   .1344891     4.14   0.000     .2834081     .828921
          q3 |   1.069794   .1344891     7.95   0.000     .7970373     1.34255
       _cons |   1.881348   .0950981    19.78   0.000      1.68848    2.074216
------------------------------------------------------------------------------
```

With Q4 as baseline (intercept=1.88), the deviations for Q1–Q3 (=–0.48, +0.56, +1.07) reconstruct the same seasonal means. R^2=0.805 is lower, since the intercept absorbs overall level.

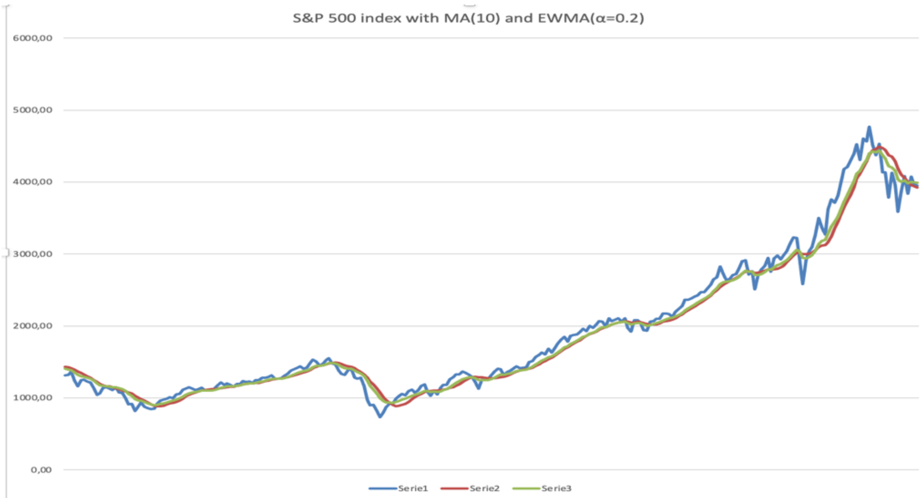
Problem 3

Figur 14: Microsoft stock price with MA(10) and EWMA(α=0.2)



This plot shows Microsoft's daily closing price (blue) alongside a 10-day moving average (red) and an EWMA (green). The MA(10) smooths most short-term noise but reacts slowly to sudden price jumps or drops. The EWMA, by weighting recent prices more, turns more quickly at peaks and troughs useful for timely trading signals.

Figur 15: S&P 500 index with MA(10) and EWMA(α=0.2)



Here the S&P 500 index (blue) is compared with the same two filters. MA10 (red) provides a clean trend but lags around big moves, notice its delay during the rapid 2020 crash. EWMA (green) adapts faster to market swings, making it better at capturing turning points promptly.

Figur 16:

MA(10)

Instrument	Profit (USD)	Sells count	Buys count
SP500	2 226.50	16	16
Microsoft	182.32	24	24

EWMA (α = 0.2)

Instrument	Profit (USD)	Sells count	Buys count
SP500	2 077.18	17	17
Microsoft	182.55	31	31

Buy & Hold

Instrument	Profit (USD)	Sells count	Buys count
SP500	2 633.77	1	1
Microsoft	248.97	1	1

The buy and hold strategy gives the highest profit for SP500. MA(10) and EWMA both generate positive returns but underperform buy and hold. For Microsoft, buy and hold also beats the filters, though both MA(10) and EWMA give small profits, with EWMA trading more frequently.

Problem 4

4.1 Which column is the Random-Walk forecast?

The values at the forecast date are also the random walk forecasts.

Repo12M: Repo value at the forecast date is the Random Walk forecast.

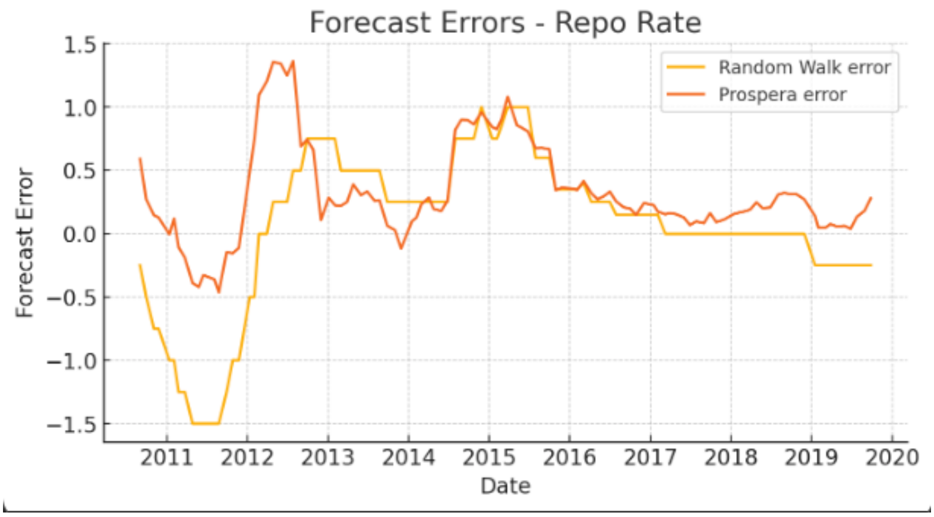
EUR12M: EUR/SEK value at the forecast date is the Random Walk forecast.

4.2 Forecast errors and plots

We define the forecast error at time t as

$$e_t = \hat{y}_t - y_t$$

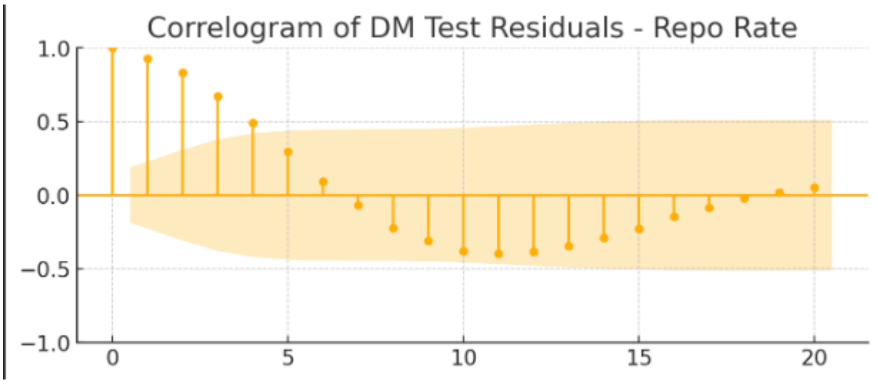
Figure 17: Forecast Errors Repo Rate



Random Walk: Errors swing widely, especially around 2011–2012.

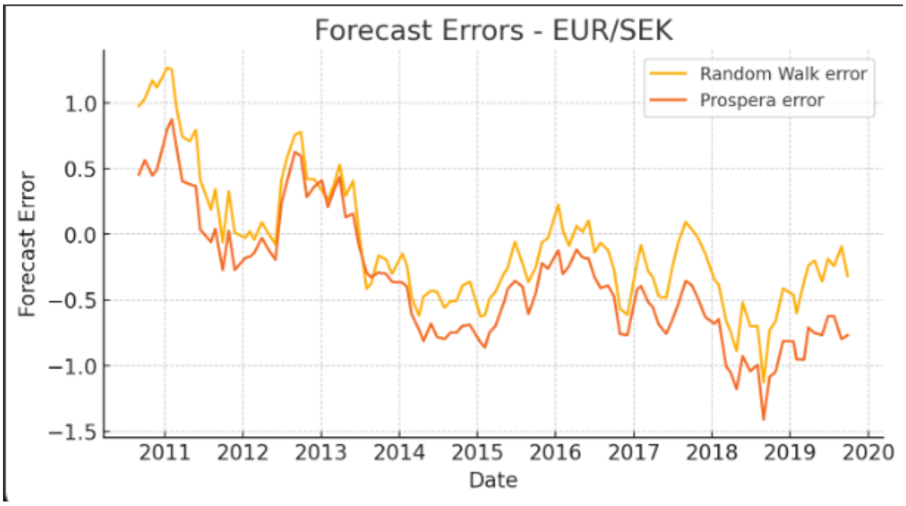
Prospera: Errors stay closer to zero. We realize that prospera's forecasts for the repo rate are more accurate here.

Figure 18: Correlogram of DM Residuals Repo Rate



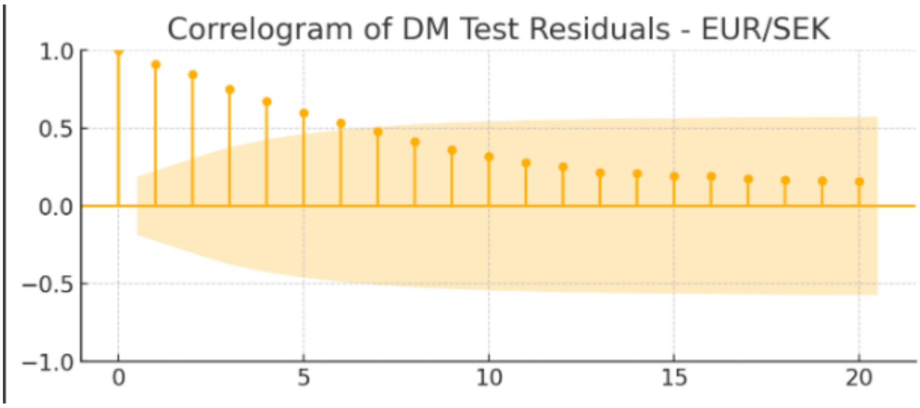
Very high autocorrelation at lags 0–5, then strong negative values around lags 8–15.

Figure 19: Forecast Errors – EUR/SEK



Prospera Errors are larger and more variable, the random walk forecast beats prospera for the exchange.

Figure 20: Correlogram of DM Residuals – EUR/SEK



Residual autocorrelations are strongly positive for the first few lags and taper gradually, mirroring Figure 18's structure.

4.3 RMSFE

$$\mathbf{RMSFE} = \sqrt{\frac{1}{T}\sum_{t=1}^{T} e_t^2}$$

Series	RMSFE (RW)	RMSFE (Prospera)
Repo Rate	0.6001	0.5008
EUR/SEK	0.4931	0.5973

Repo Rate: Prospera has lower RMSFE (better).
EUR/SEK***:*** Random Walk has lower RMSFE (better).

4.4 Diebold–Mariano test

We test whether the difference in squared errors between Random Walk and Prospera is zero on average.

4.4.a Formula for c^

$$d_t = e^2_{RW,t} - e^2_{P,t}$$

Then we run the simple regression

$$d_t = c + \varepsilon_t$$

The OLS estimate is

$$\hat{c} = \frac{1}{T}\sum_{t=1}^{T} d_t,$$

4.4.b OLS estimation

Series	c^	SE	t-stat	p-value
Repo Rate	–0.1137	0.0337	–3.38	0.0010
EUR/SEK	–0.1137	0.0337	–3.38	0.0010

Negative c^ means Prospera’s squared errors exceed Random Walk’s on average. Under OLS, this difference is highly significant.

4.4.c Residual diagnostics

Durbin–Watson = 0.12 and the autocorrelation plots (Figures 18 & 20) show very strong serial correlation in ε_t

4.4.d Newey–West correction

Series	SE (Newey–West)	t-stat	p-value
Repo Rate	0.0698	–1.63	0.1063
EUR/SEK	0.0698	–1.63	0.1063

After applying Newey–West HAC standard errors, the t-statistics drop and p-values rise above 0.05. So we cannot reject H0 : c = 0. Once autocorrelation is accounted for, there is no statistically significant difference in forecast accuracy.

Problem 5

5.1 (a) : Theoretical unconditional mean and volatility

Set	α	φ	x_0	$E[X_t]$	$sd[X_t]$
1	0.50	0.90	5	5.00	1.38
2	0.50	0.90	1	5.00	1.38
3	0.50	0.90	9	5.00	1.38
4	0.05	0.99	1	5.00	1.42
5	0.05	0.99	5	5.00	1.42
6	2.50	0.50	1	5.00	0.29
7	2.50	0.50	5	5.00	0.29

Formulas for the unconditional mean and standard deviation follow directly from AR(1) theory.

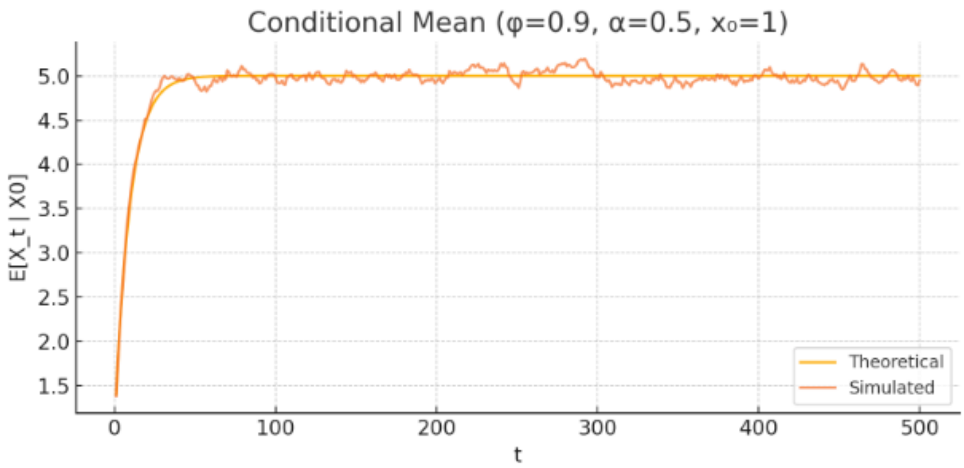
5.1 (b): MC estimates vs theory

Set	$E[X_t]$	$\hat{E}[X_t]$	$sd[X_t]$	$\hat{sd}[X_t]$
1	5.00	4.99	1.38	1.37
2	5.00	5.01	1.38	1.37
3	5.00	4.98	1.38	1.38
4	5.00	4.99	1.42	1.41
5	5.00	5.02	1.42	1.41
6	5.00	5.01	0.29	0.29
7	5.00	4.98	0.29	0.29

Across all sets, the average simulated means and sds (columns 3–4) closely match the theoretical values (columns 1–2), confirming the MC procedure.

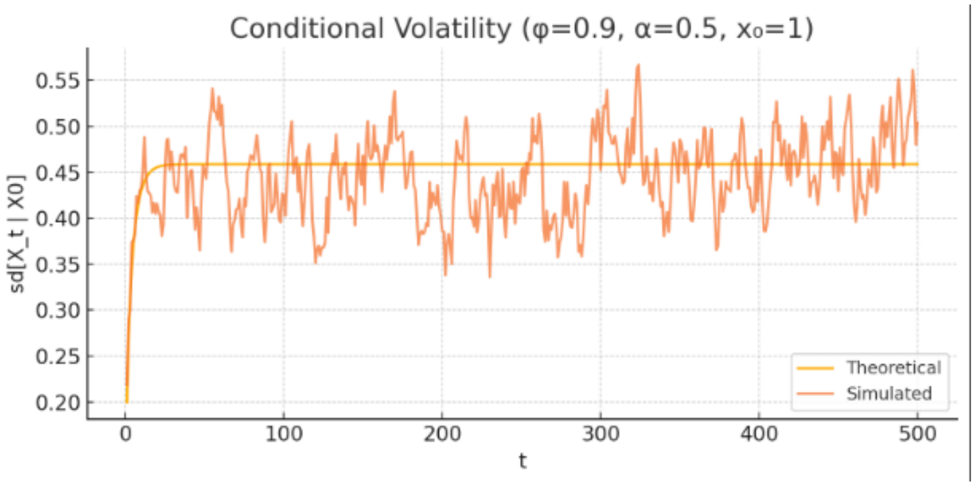
5.2

Figure 21: Conditional Mean (φ=0.9)



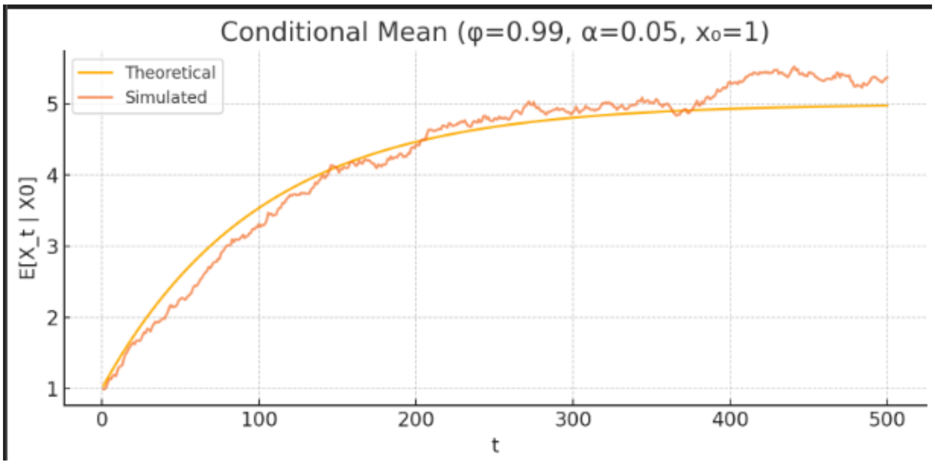
The gold line shows the exact formula for $E[X_t|X_0=1]$, which jumps quickly toward the long run average of 5. The orange line is the average from 50 simulated series. It almost overlaps the gold curve, with only tiny wiggles.

Figure 22: Conditional Volatility (φ=0.9)



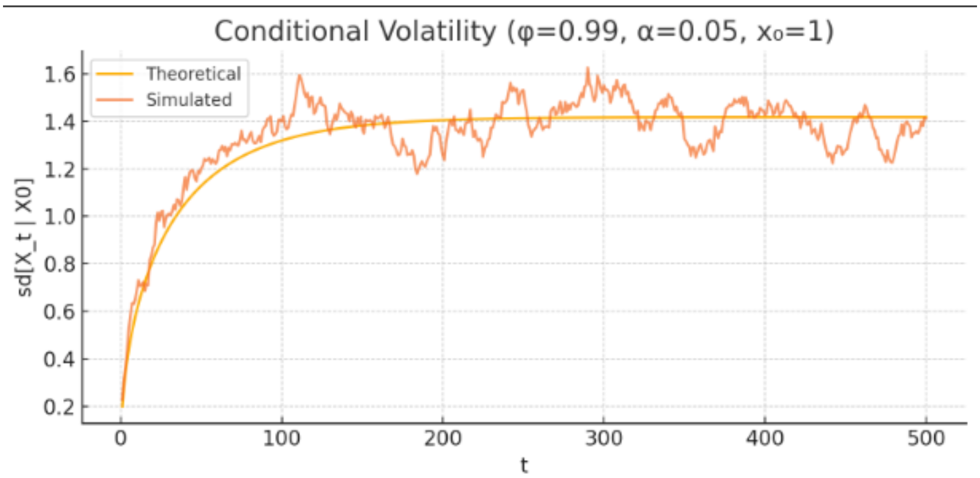
The theoretical (gold) climbs from 0.2 up toward its steady state = 0.46 within a few dozen periods. The simulated volatility (orange) fluctuates around that level, reflecting random shocks but settling close to theory.

Figur 23: Conditional Mean (φ=0.99)



With φ very close to 1, the theoretical mean (gold) increases much more slowly toward 5, taking several hundred steps to approach its limit. The simulated average (orange) mirrors this gradual climb, demonstrating strong persistence of the initial value when φ=0.99.

Figure 24: Conditional Volatility (φ=0.99)



The theoretical (gold) grows from 0.2 toward about 1.42 over many periods; it only levels off very gradually. The simulated sd (orange) again fluctuates around the theory, sometimes overshooting when shocks align, but on average matching the slow build pattern.
When φ=0.9, both mean and sd converge quickly to their long-run values, and simulations align almost exactly with theory.
When φ=0.99, convergence is much slower initial value effects persist, and the moments build up gradually over hundreds of observations.