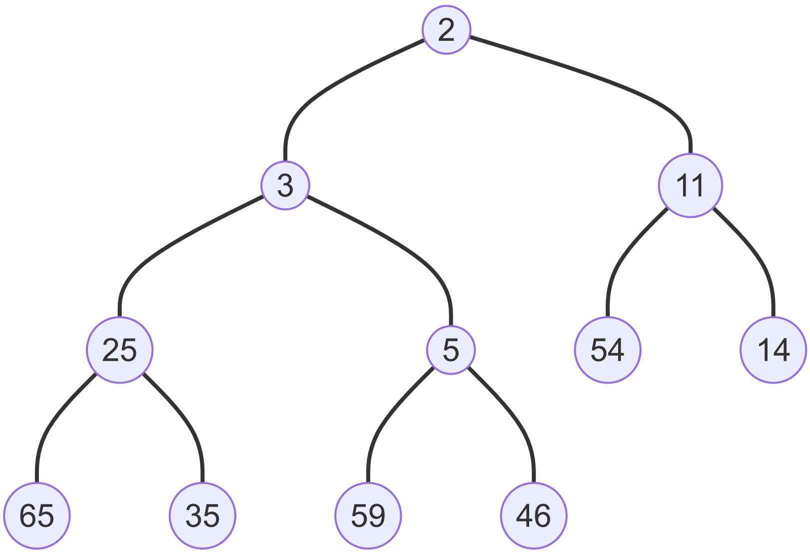
# problem1

## a heapsort

### start:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 3 | 11 | 25 | 5 | 54 | 14 | 65 | 35 | 59 | 46 | |

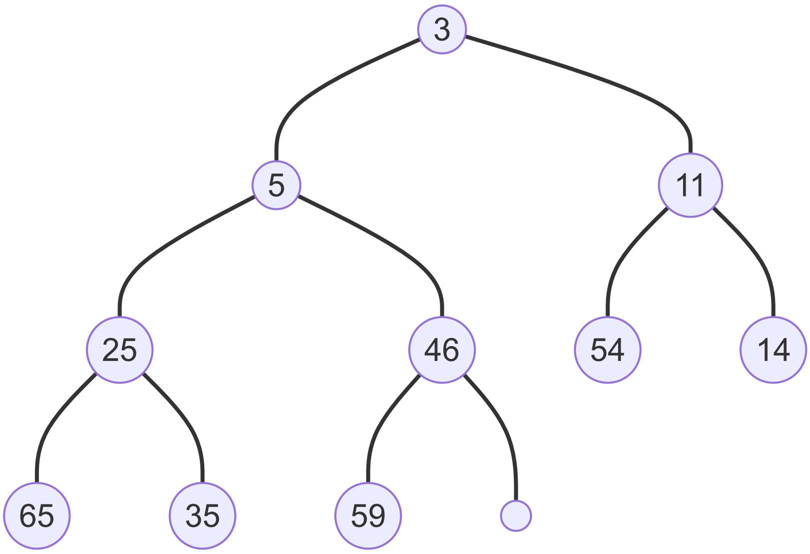



###

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | |

### delete2

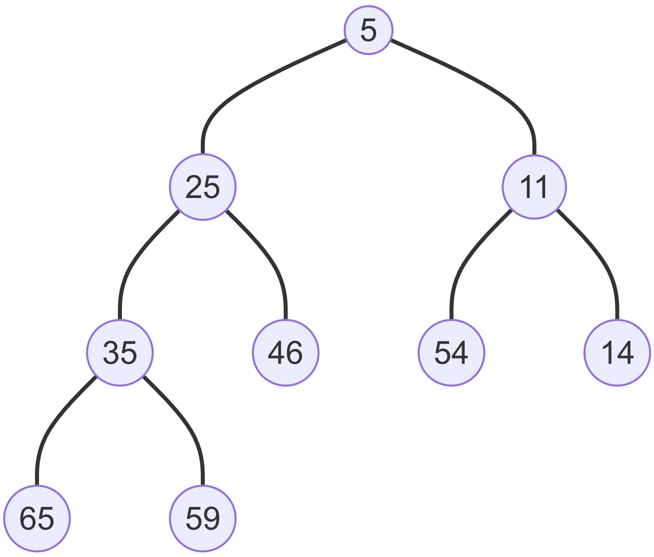
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 5 | 11 | 25 | 46 | 54 | 14 | 65 | 35 | 59 | | |




| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | | | | | | | | | | | |

## delete3

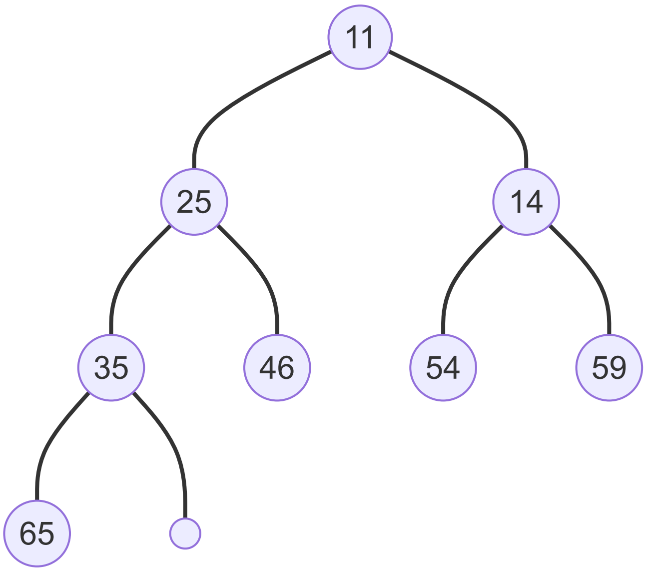
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 5 | 25 | 11 | 35 | 46 | 54 | 14 | 65 | 59 | | | |




| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 3 | | | | | | | | | | |

## delete5

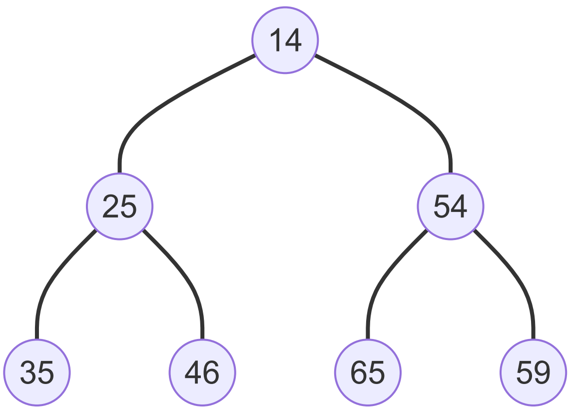
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 11 | 25 | 14 | 35 | 46 | 54 | 59 | 65 | | | | |




| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 3 | 5 | | | | | | | | | |

## delete11

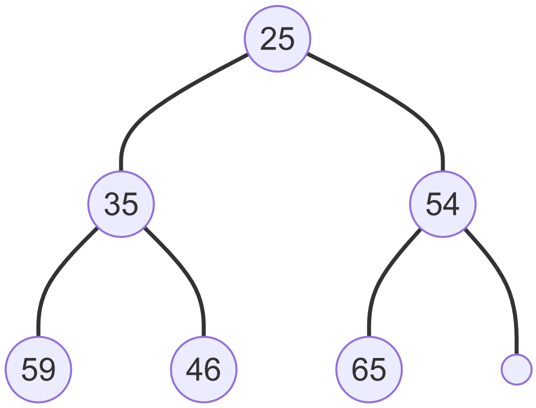
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 14 | 25 | 54 | 35 | 46 | 65 | 59 | | | | | |




| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 3 | 5 | 11 | | | | | | | | |

## delete14

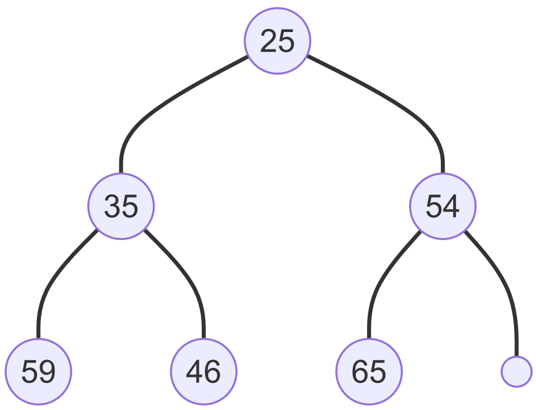
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 25 | 35 | 54 | 59 | 46 | 65 | | | | | | |




| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 3 | 5 | 11 | 14 | | | | | | | |


## delete14

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 25 | 35 | 54 | 59 | 46 | 65 | | | | | | |

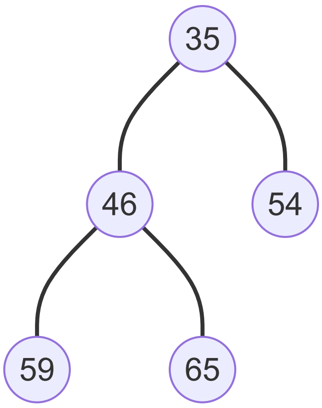





| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 3 | 5 | 11 | 14 | | | | | | | |

## delete25

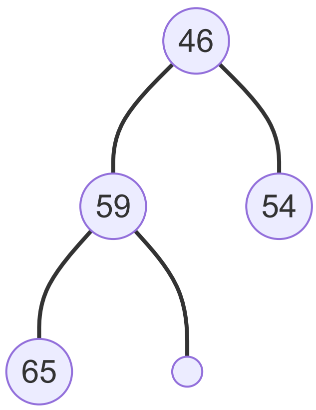
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 35 | 46 | 54 | 59 | 65 | | | | | | | |




| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 3 | 5 | 11 | 14 | 25 | | | | | | |

## delete35

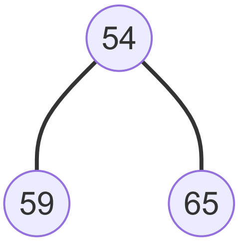
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 46 | 59 | 54 | 65 | | | | | | | | |




| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 3 | 5 | 11 | 14 | 25 | 35 | | | | | |

## delete46

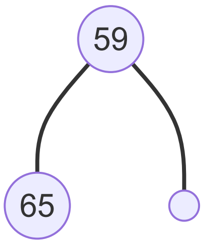
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 54 | 59 | 65 | | | | | | | | | |




| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 3 | 5 | 11 | 14 | 25 | 35 | 46 | | | | |

## delete54

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 59 | 65 | | | | | | | | | | |




| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 3 | 5 | 11 | 14 | 25 | 35 | 46 | 54 | | | |

## delete59

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 65 | | | | | | | | | | | |

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 3 | 5 | 11 | 14 | 25 | 35 | 46 | 54 | 59 | | |

### delete65

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | |

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 3 | 5 | 11 | 14 | 25 | 35 | 46 | 54 | 59 | 65 | |

heapsort finished

## b quicksort

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 25 | 11 | 54 | 35 | 46 | 5 | 14 | 65 | 2 | 59 | 3 | |

### use 25 to partition

| 1 | 2 | 3 | 4 | 5 | 6(partition) | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 5 | 11 | 3 | 2 | 14 | 25 | 46 | 65 | 35 | 59 | 54 | |

### use 5,46 to partition

| 1 | 2 | 3(partition) | 4 | 5 | 6(partition) | 7 | 8(partition) | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 2 | 5 | 11 | 14 | 25 | 35 | 46 | 65 | 59 | 54 | |

### use 3， 11， 65 to partition

| 1 | 2(partition) | 3(partition) | 4(partition) | 5 | 6(partition) | 7 | 8(partition) | 9 | 10 | 11(partition) | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 3 | 5 | 11 | 14 | 25 | 35 | 46 | 54 | 59 | 65 | |

### now all item is returned, sort finished

## c mergesort

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|
| 25 | 11 | 54 | 35 | 46 | 5 | 14 | 65 | 2 | 59 | 3 |

## Split

| 1 | 2 | 3 | 4 | 5 | 6 | | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 25 | 11 | 54 | 35 | 46 | 5 | | 14 | 65 | 2 | 59 | 3 |

## Split

| 1 | 2 | 3 | | 4 | 5 | 6 | | 7 | 8 | 9 | | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 25 | 11 | 54 | | 35 | 46 | 5 | | 14 | 65 | 2 | | 59 | 3 |

## Split

| 1 | 2 | | 3 | | 4 | 5 | | 6 | | 7 | 8 | | 9 | | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 25 | 11 | | 54 | | 35 | 46 | | 5 | | 14 | 65 | | 2 | | 59 | 3 |

## Split

| 1 | | 2 | | 3 | | 4 | | 5 | | 6 | | 7 | | 8 | | 9 | | 10 | | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 25 | | 11 | | 54 | | 35 | | 46 | | 5 | | 14 | | 65 | | 2 | | 59 | | 3 |

## Merge

| 1 | 2 | | 3 | | 4 | 5 | | 6 | | 7 | 8 | | 9 | | 10 | | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 11 | 25 | | 54 | | 35 | 46 | | 5 | | 14 | 65 | | 2 | | 59 | | 3 |

## Merge

| 1 | 2 | 3 | | 4 | 5 | 6 | | 7 | 8 | 9 | | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 11 | 25 | 54 | | 5 | 35 | 46 | | 2 | 14 | 65 | | 3 | 59 |

## Merge

| 1 | 2 | 3 | 4 | 5 | 6 | | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 5 | 11 | 25 | 35 | 46 | 54 | | 2 | 3 | 14 | 59 | 65 |

### Merge

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 3 | 5 | 11 | 14 | 25 | 35 | 46 | 54 | 59 | 65 |

## d insertion sort

### insert 25

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|
| 25 | | | | | | | | | | |

### insert 11

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|
| 11 | 25 | | | | | | | | | |

### insert 54

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|
| 11 | 25 | 54 | | | | | | | | |

### insert 35

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|
| 11 | 25 | 35 | 54 | | | | | | | |

### insert 46

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|
| 11 | 25 | 35 | 46 | 54 | | | | | | |

### insert 5

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|
| 5 | 11 | 25 | 35 | 46 | 54 | | | | | |

### insert 14

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|
| 5 | 11 | 14 | 25 | 35 | 46 | 54 | | | | |

### insert 65

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|
| 5 | 11 | 14 | 25 | 35 | 46 | 54 | 65 | | | |

### insert 2

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 5 | 11 | 14 | 25 | 35 | 46 | 54 | 65 | | |

### insert 59

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 5 | 11 | 14 | 25 | 35 | 46 | 54 | 59 | 65 | |

### insert 3

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 3 | 5 | 11 | 14 | 25 | 35 | 46 | 54 | 59 | 65 |

Insertion sort completed

## e selection sort

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|
| 25 | 11 | 54 | 35 | 46 | 5 | 14 | 65 | 2 | 59 | 3 |

### min->2

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 11 | 54 | 35 | 46 | 5 | 14 | 65 | 25 | 59 | 3 |

## min->3

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 3 | 54 | 35 | 46 | 5 | 14 | 65 | 25 | 59 | 11 |

## min->5

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 3 | 5 | 35 | 46 | 54 | 14 | 65 | 25 | 59 | 11 |

## min->11

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 3 | 5 | 11 | 46 | 54 | 14 | 65 | 25 | 59 | 35 |

## min->14

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 3 | 5 | 11 | 14 | 54 | 46 | 65 | 25 | 59 | 35 |

## min->25

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 3 | 5 | 11 | 14 | 25 | 46 | 65 | 54 | 59 | 35 |

## min->35

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 3 | 5 | 11 | 14 | 25 | 35 | 65 | 54 | 59 | 46 |

## min->46

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 3 | 5 | 11 | 14 | 25 | 35 | 46 | 54 | 59 | 65 |

## min->54

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 3 | 5 | 11 | 14 | 25 | 35 | 46 | 54 | 59 | 65 |

### min->59

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 3 | 5 | 11 | 14 | 25 | 35 | 46 | 54 | 59 | 65 |

### min->65

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 3 | 5 | 11 | 14 | 25 | 35 | 46 | 54 | 59 | 65 |

selection sort finished

# problem2

## a

### Minimum prefix

```
struct recursiveResult
begin
    float leftMin;
    int leftMinMarker;
    float total;
end recursiveResult

function getMinimumPrefix(x[1:n])
begin
    recursiveResult result;
    if (n==1) then
        result.leftMin = x[0];
        result.leftMinMarker = 1;
        result.total = x[0];
        return (result);
    endif
    recursiveResult leftResult;
    recursiveResult rightResult;
    int mid;
    mid = floor(n/2);
    //leftside
    leftResult = getMinimumPrefix(x[1:mid])
    //rightside
    rightResult = getMinimumPrefix(x[mid+1:n])
    //get the smallest prefix in the two prefix
    if (leftResult.leftMin <= leftResult.total * rightResult.leftMin) then
        result.leftMin = leftResult.leftMin;
        result.leftMinMarker = leftResult.leftMinMarker;
    else
        result.leftMin = leftResult.total * rightResult.leftMin;
        result.leftMinMarker = rightResult.leftMinMarker + mid;
    endif
```

```
    //add the total for outer level usage
    result.total = leftResult.total * rightResult.total;
    return(result)
end getMinimumPrefix

function main()
begin
    float x[1:n]=[0.1,0.2,0.3,.....]
    int k;
    recursiveResult result;
    recursiveResult = getMinimumPrefix(x);
    k = recursiveResult.leftMinMarker;
    print(k);
end main
```

## Minimum suffix

```
struct recursiveResult
begin
    float rightMin;
    int rightMinMarker;
    float total;
end recursiveResult

function getMinimumSuffix(x[1:n])
begin
    recursiveResult result;
    if (n==1) then
        result.rightMin = x[0];
        result.rightMinMarker = 1;
        result.total = x[0];
        return (result);
    endif
    recursiveResult leftResult;
    recursiveResult rightResult;
    int mid;
    mid = floor(n/2);
    //leftside
    leftResult = getMinimumSuffix(x[1:mid])
    //rightside
    rightResult = getMinimumSuffix(x[mid+1:n])
    //get the smallest suffix in the two suffix
    if (rightResult.rightMin <= rightResult.total * leftResult.rightmin)
then
        result.rightMin = rightResult.rightMin;
        result.rightMinMarker = rightResult.rightMinMarker;
    else
        result.rightMin = rightResult.total * leftResult.rightmin;
        result.rightMinMarker = leftResult.rightMinMarker + mid;
    endif
    //add the total for outer level usage
    result.total = leftResult.total * rightResult.total;
    return(result)
end getMinimumPrefix

function main()
begin
```

```
    float x[1:n]=[0.1,0.2,0.3,.....]
    int k;
    recursiveResult result;
    recursiveResult = getMinimumSuffix(x);
    k = recursiveResult.rightMinMarker;
    print(k);
end main
```

## Minimum subarray

```
struct recursiveResult
begin
    float rightMin;
    int rightMinMarker;
    float leftMin;
    int leftMinMarker;
    float middleMin;
    int middleMinLeftMarker;
    int middleMinRightMarker;
    float total;
end recursiveResult

function getMinimumSubarray(x[1:n])
begin
    recursiveResult result;
    if (n==1) then
        result.rightMin = x[0];
        result.rightMinMarker = 1;
        reuslt.leftMin = x[0];
        result.leftMinMarker =1;
        result.middleMin = x[0];
        result.middleMinLeftMarker = 1;
        result.middleMinRightMarker = 1;
        result.total = x[0];
        return (result);
    endif
    recursiveResult leftResult;
    recursiveResult rightResult;
    int mid;
    mid = floor(n/2);
    //leftside
    leftResult = getMinimumSubarray(x[1:mid])
    //rightside
    rightResult = getMinimumSubarray(x[mid+1:n])
    //get the smallest suffix in the two suffix
    if (rightResult.rightMin <= rightResult.total * leftResult.rightmin) then
        result.rightMin = rightResult.rightMin;
        result.rightMinMarker = rightResult.rightMinMarker;
    else
        result.rightMin = rightResult.total * leftResult.rightmin;
        result.rightMinMarker = leftResult.rightMinMarker + mid;
    endif
    //get the smallest prefix in the two prefix
    if (leftResult.leftMin <= leftResult.total * rightResult.leftMin) then
        result.leftMin = leftResult.leftMin;
        result.leftMinMarker = leftResult.leftMinMarker;
```

```
    else
        result.leftMin = leftResult.total * rightResult.leftMin;
        result.leftMinMarker = rightResult.leftMinMarker + mid;
    endif
    //get the smallest subarray in the two subarray and left suffix* right prefix
    if (leftResult.middleMin <= rightResult.middleMin && leftResult.middleMin <= leftResult.rightMin*rightResult.leftMin) then
        result.middleMin = leftResult.MiddleMin;
        result.middleMinLeftMarker = leftResult.middleMinLeftMarker;
        result.middleMinRightMarker = leftResult.middleMinRightMarker;
    else if (rightResult.middleMin <= leftResult.middleMin && rightResult.middleMin <= leftResult.rightMin*rightResult.leftMin) then
        result.middleMin = leftResult.rightMin*rightResult.leftMin;
        result.middleMinLeftMarker = leftResult.middleMinLeftMarker;
        result.middleMinRightMarker = rightResult.middleMinRightMarker + mid;
    else
        result.middleMin = rightResult.MiddleMin;
    endif
    //add the total for outer level usage
    result.total = leftResult.total * rightResult.total;
    return(result);
end getMinimumSubarray

function main()
begin
    float x[1:n]=[0.1,0.2,0.3,.....]
    int k,r;
    recursiveResult result;
    recursiveResult = getMinimumSubarray(x);
    k = recursiveResult.middleMinLeftMarker;
    r = recursiveResult.middleMinRightMarker;
    print(k,r);
end main
```

## c

About Time Complexity, the three algorithms above are all acting in a same way, that is recursively split the input into two half and calls themselves, then each recursive call it self does constant operations. So

$$\begin{gathered} T(1) = c \\ T(n) = T(\frac{n}{2}) + c \\ Therefore \\ T(n) = O(logn) \end{gathered} \tag{1}$$

# problem3

```
struct recursiveResult
begin
    float maxTrough;
    int maxTroughMarker;
end recursiveResult
```

```

struct minHeapNode
begin
    float value;
    int location;
end minHeapNode

function buildMinHeap(x[1:n])
begin
    int i;
    minHeapNode minHeap[1:n];
    int mapArray[1:n];
    for i=1 to n do
        //insert into min heap
        minHeapNode heapNode;
        heapNode.value = x[i];
        heapNode.location = i;
        minHeap[i]= heapNode;
        mapArray[i]=i;
        int currentLocation;
        currentLocation = i;
        while (currentLocation != 1 and minHeap[currentLocation].value < minHeap[floor(currentLocation/2)].value) do
            //swap both minheap and the minheap mapping array
            swap (minHeap[currentLocation],minHeap[floor(currentLocation/2)]);
            swap (mapArray[minHeap[currentLocation].location],mapArray[minHeap[floor(currentLocation/2)].location]);
            currentLocation = floor(currentLocation/2);
        endwhile
    endfor
    return (minHeap,mapArray);
end buildMinHeap

function heapify(x[1:n],mapArray[1:n],location)
begin
    while((location<n/2)&&(x[location].value>x[location*2].value ||x[location].value>x[location*2+1].value)||((location!=1)&& (x[location].value < x[floor(location/2)].value)) do
        if (x[location].value>x[location*2].value ||x[location].value>x[location*2+1].value) then
            //swap down
            if x[location*2].value > x[location*2+1].value then
                swap (x[location],x[location*2+1]);
                swap (mapArray[x[location].location],mapArray[x[location*2+1].location]);
                location = floor(location*2+1);
            else
                swap (x[location],x[location*2]);
                swap (mapArray[x[location].location],mapArray[x[location*2].location]);
                location = floor(location*2);
        else
            //swap up
            //swap both minheap and the minheap mapping array
            swap (x[location],x[floor(location/2)]);
```

```
            swap
(mapArray[x[location].location],mapArray[x[floor(location/2)].location]);
            location = floor(location/2);
        endif
    endwhile
    return x,mapArray;
end heapify

function getTrough(x[1:n],l)
begin
    int mid = floor(n/2);
    float midTrough;
    int midTroughMarker;
    int i;
    float arrayToCheck[];
    int arrayLength;
    if 2*l>n then
        //the basic unit of recursive
        arrayToCheck = x;
        arrayLength = n;
    else
        //the middle unit of recursive
        arrayToCheck = x[mid-l+1:mid+l]
        arrayLength = 2*l;
    endif
    //build a l size min heap, with mapping to the element's location in a
    //this minheap also has a location value for it to map to the array
when doing heapify
    //also return a l size location mapArray for each element in array x to
find it's location in the min heap.
    minHeapNode minHeap[1:l];
    int mapArray[1:l];
    minHeap,mapArray := buildMinHeap(arrayToCheck[1:l]);
    midTroughMarker = 1;
    midTrough = minHeap[1].value;
    for i=l+1 to arrayLength do
        // replace old value with new one
        minHeap[mapArray[i%l]].value = arrayToCheck[i];
        // rebuild the heap, return the new heap and the heap mapArray
        minHeap,mapArray =
heapify(arrayToCheck[1:arrayLength];mapArray[1:l],mapArray[i%l]);
        // check for trough, the 1st element in min heap is always min
        if midTrough<minHeap[1].value then
            //record the new max trough
            midTrough = minHeap[1].value;
            midTroughMarker = i-l;
        endif
    endfor
    recursiveResult result;
    if 2*l>n then
        //the basic unit of recursive
        result.maxTrough = midTrough;
        result.maxTroughMarker = midTroughMarker;
    else
        //start recursive here
        recursiveResult leftResult;
        recursiveResult rightResult;
        leftResult = getTrough(x[1:mid],l);
```

```
        rightResult = getTrough(x[mid+1:n],l);
        if (midTrough >= leftResult.maxTrough && midTrough >= rightResult.maxTrough) then
            result.maxTrough = midTrough;
            //need to add offset to marker
            result.maxTroughMarker = mid-l+midTroughMarker-1;
        else if leftResult.maxTrough >= rightResult.maxTrough then
            result.maxTrough = leftResult.maxTrough;
            result.maxTroughMarker = leftResult.maxTroughMarker;
        else
            result.maxTrough = rightResult.maxTrough;
            result.maxTroughMarker = rightResult.maxTroughMarker + mid;
        endif
    return (result);
end getTrough


func main()
begin
    //populate x and l here
    float x[1:n]=[1,2,3,4,5,6,7,8...]
    int l = 22;
    print(getTrough(x,l).maxTroughMarker)
end main
```

For Time complexity, the algorithm calls itself on half recursively. And each recursive operation will have 1 time of building min-heap(O(l)) and l times of heapify on l elements (log l for each heapify) operations. So

$$\begin{gathered}T(2*l) = c*l\log l \\ T(n) = T(\frac{n}{2}) + c*l\log l \\ Therefore \\ T(n) = O(l\log l\log n)\end{gathered} \tag{2}$$

# problem 4

For the `Knapsack` problem, a greedy solution on `price per weight` is needed

Calculate `price per weight` first

| weight | 8 | 14 | 6 | 4 | 2 | 10 |
|---|---|---|---|---|---|---|
| price | 40 | 14 | 24 | 12 | 12 | 20 |
| price/weight | 5 | 1 | 4 | 3 | 6 | 2 |
| x | | | | | | |

M=14

### firstly choose the highest `price/weight` which is 6

| weight | 8 | 14 | 6 | 4 | 2 | 10 |
|---|---|---|---|---|---|---|
| price | 40 | 14 | 24 | 12 | 12 | 20 |
| price/weight | 5 | 1 | 4 | 3 | 6 | 2 |
| x | | | | | 2 | |

M=12

### choose the highest `price/weight` which is other than 6 (5)

| weight | 8 | 14 | 6 | 4 | 2 | 10 |
|---|---|---|---|---|---|---|
| price | 40 | 14 | 24 | 12 | 12 | 20 |
| price/weight | 5 | 1 | 4 | 3 | 6 | 2 |
| x | 8 | | | | 2 | |

M=4

### choose the highest `price/weight` which is other than 6，5 (4)

| weight | 8 | 14 | 6 | 4 | 2 | 10 |
|---|---|---|---|---|---|---|
| price | 40 | 14 | 24 | 12 | 12 | 20 |
| price/weight | 5 | 1 | 4 | 3 | 6 | 2 |
| x | 1 | | 2/3 | | 1 | |

M=0

because the remaining capacity is only 4, only 4/6 of the total weight can be taken. so x =2/3, and now the pack is full.

So the $x_n$ is shown below

| n | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $x_n$ | 1 | 0 | 2/3 | 0 | 1 | 0 |

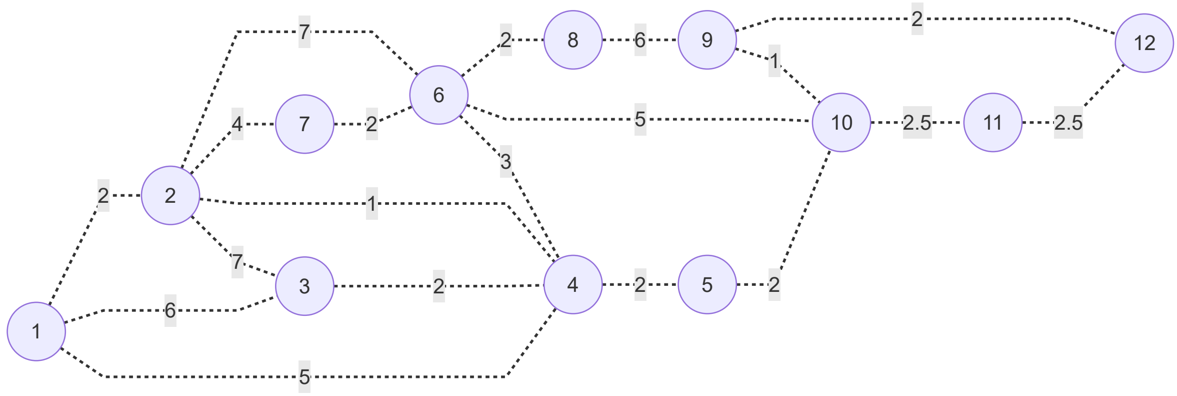
total price should be

$x_1P_1+x_3P_3+x_5P_5$= 40+16+12=68

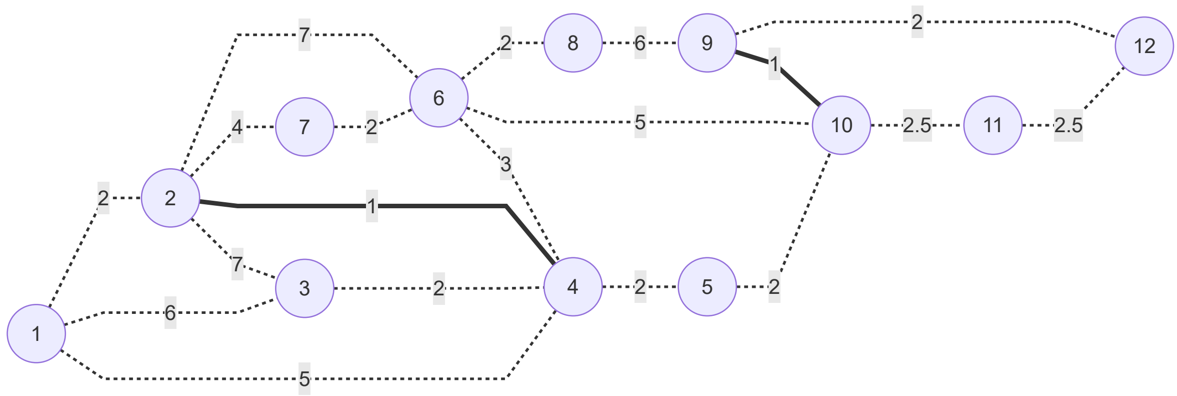
# problem 5

## a

We have the graph G as below




## Search for the smallest weight (1), add to the tree(the bold line)



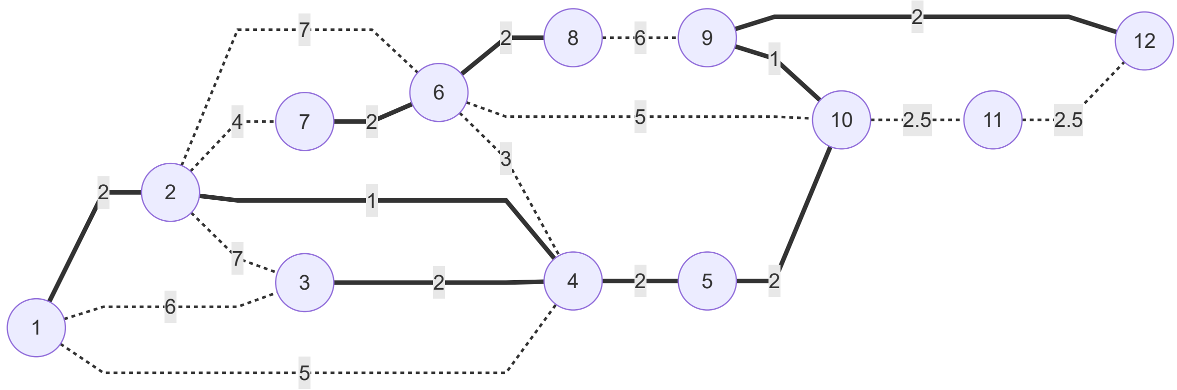

add E(2,4),E(9,10)

check for circle, no circle

check for count(E)=2!=11

ok continue

## Search for the smallest weight (2), add to the tree



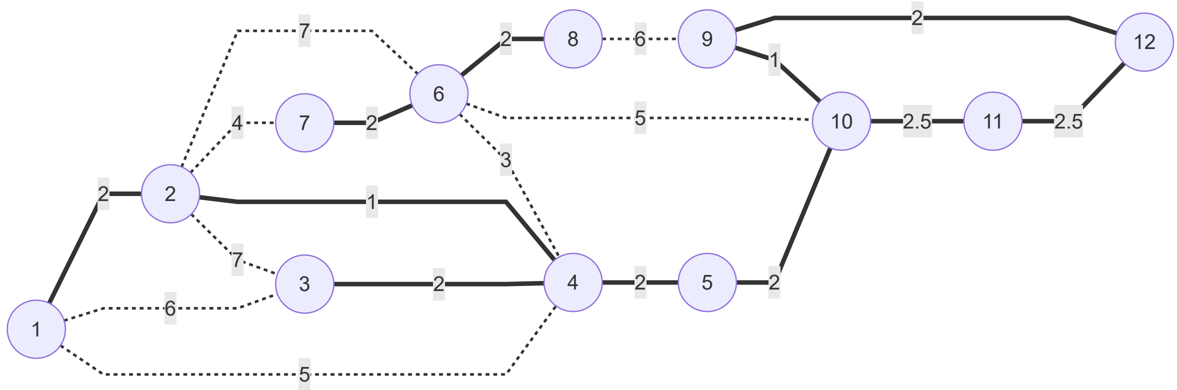

Add E(1,2),E(3,4),E(6,7),E(4,5),E(5,10),E(9,10),E(6,8)

check for circle, no circle

check for count(E)=9!=11

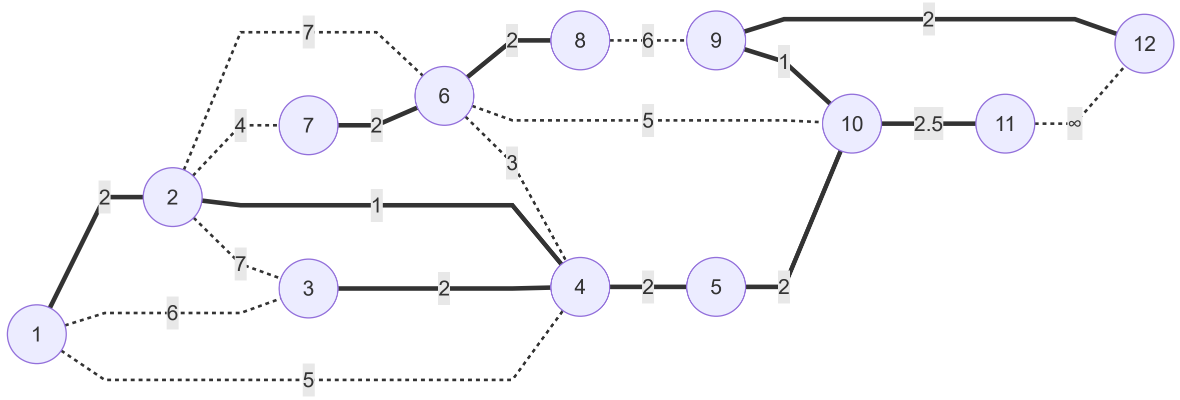
ok continue

# Search for the smallest weight (2.5), add to the tree




Add E(10,11),E(11,12)

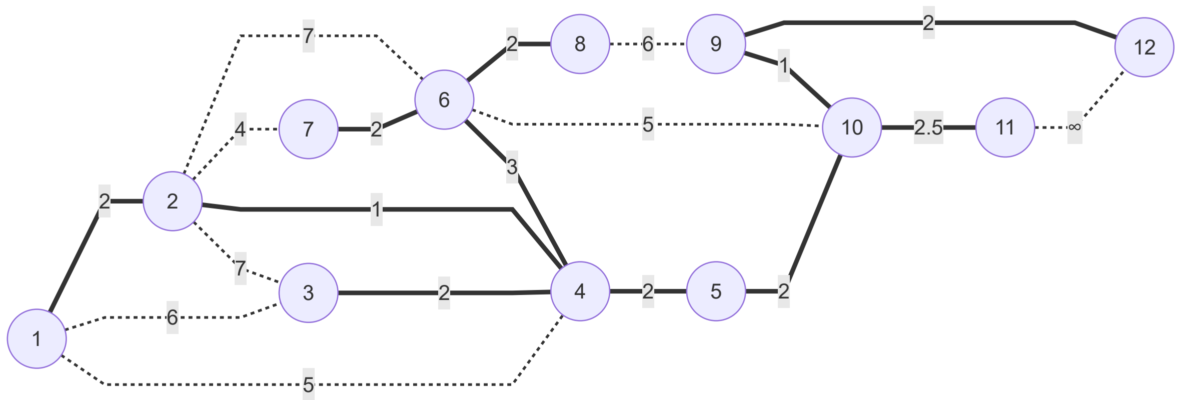
check for circle, adding E(11,12), vertex 11,12 is on the same tree, 9,10,11,12 is a circle, so remove E(11,12)




check for count(E)=10!=11

ok continue

## Search for the smallest weight (3), add to the tree




check for circle, adding E(4,6) is on the different tree, no circle

check for count(E)=11==11

so now the tree should be a `MST` of G

## b

## Add 1 to group

distance from 1

| i | 1+ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| D[i] | 0 | 2 | 6 | 5 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ |

## choose the shortest distance (D[2]=2)to add to group

refresh distance from 1

D(1,2)+D(2,4)=3>D(1,4)=5

| i | 1+ | 2+ | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| D[i] | 0 | 2 | 6 | 3 | ∞ | 9 | 6 | ∞ | ∞ | ∞ | ∞ | ∞ |

## choose the shortest distance (D[4]=3)to add to group

refresh distance from 1

D(1,4)+D(4,6)=6>D(1,6)=9

D(1,4)+D(4,3)=5>D(1,3)=6

| i | 1+ | 2+ | 3 | 4+ | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| D[i] | 0 | 2 | 5 | 3 | 5 | 6 | 6 | ∞ | ∞ | ∞ | ∞ | ∞ |

## choose the shortest distance (D[5]=5)to add to group

refresh distance from 1

| i | 1+ | 2+ | 3 | 4+ | 5+ | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| D[i] | 0 | 2 | 5 | 3 | 5 | 6 | 6 | ∞ | ∞ | 7 | ∞ | ∞ |

## choose the shortest distance (D[3]=5)to add to group

refresh distance from 1

| i | 1+ | 2+ | 3+ | 4+ | 5+ | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| D[i] | 0 | 2 | 5 | 3 | 5 | 6 | 6 | ∞ | ∞ | 7 | ∞ | ∞ |

## choose the shortest distance (D[6]=6)to add to group

refresh distance from 1

| i | 1+ | 2+ | 3+ | 4+ | 5+ | 6+ | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| D[i] | 0 | 2 | 5 | 3 | 5 | 6 | 6 | 8 | ∞ | 7 | ∞ | ∞ |

## choose the shortest distance (D[7]=6)to add to group

refresh distance from 1

| i | 1+ | 2+ | 3+ | 4+ | 5+ | 6+ | 7+ | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| D[i] | 0 | 2 | 5 | 3 | 5 | 6 | 6 | 8 | ∞ | 7 | ∞ | ∞ |

## choose the shortest distance (D[10]=7)to add to group

refresh distance from 1

| i | 1+ | 2+ | 3+ | 4+ | 5+ | 6+ | 7+ | 8 | 9 | 10+ | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| D[i] | 0 | 2 | 5 | 3 | 5 | 6 | 6 | 8 | 8 | 7 | 9.5 | ∞ |

## choose the shortest distance (D[8]=8)to add to group

refresh distance from 1

| i | 1+ | 2+ | 3+ | 4+ | 5+ | 6+ | 7+ | 8+ | 9 | 10+ | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| D[i] | 0 | 2 | 5 | 3 | 5 | 6 | 6 | 8 | 8 | 7 | 9.5 | ∞ |

## choose the shortest distance (D[9]=8)to add to group

refresh distance from 1

| i | 1+ | 2+ | 3+ | 4+ | 5+ | 6+ | 7+ | 8+ | 9 | 10+ | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| D[i] | 0 | 2 | 5 | 3 | 5 | 6 | 6 | 8 | 8 | 7 | 9.5 | 10 |

## choose the shortest distance (D[11]=9.5)to add to group

refresh distance from 1

| i | 1+ | 2+ | 3+ | 4+ | 5+ | 6+ | 7+ | 8+ | 9+ | 10+ | 11+ | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| D[i] | 0 | 2 | 5 | 3 | 5 | 6 | 6 | 8 | 8 | 7 | 9.5 | 10 |

## choose the shortest distance (D[12]=10)to add to group

refresh distance from 1

| i | 1+ | 2+ | 3+ | 4+ | 5+ | 6+ | 7+ | 8+ | 9+ | 10+ | 11+ | 12+ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| D[i] | 0 | 2 | 5 | 3 | 5 | 6 | 6 | 8 | 8 | 7 | 9.5 | 10 |

now that all the elements are added to the group, finished.

## c

2: 1->2

3: 1->2->4->3

4: 1->2>4

5: 1->2->4->5

6: 1->2->4->6

7: 1->2->7

8: 1->2->4->6->8

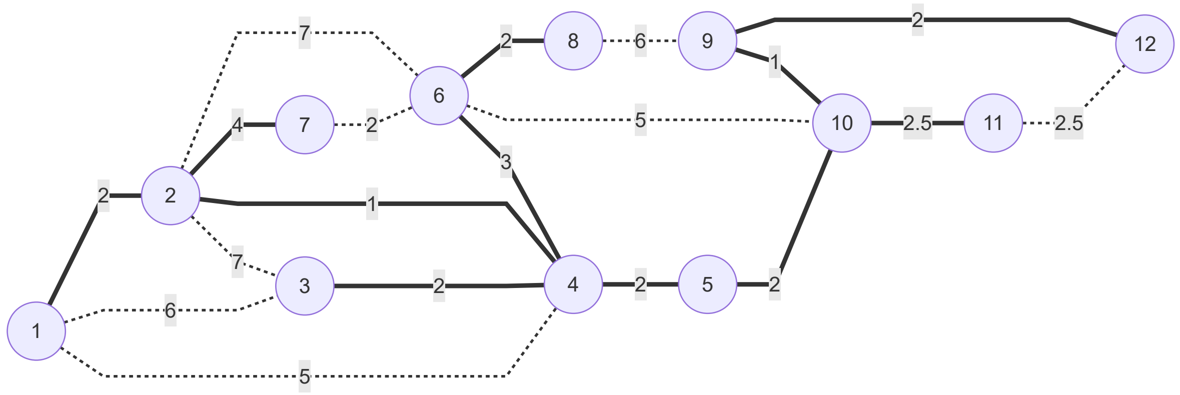
9: 1->2->4->5->10->9

10:1->2->4->5->10

11:1->2->4->5->10->11

12: 1->2->4->5->10->9->12

To add these paths all together, got the spanning tree below

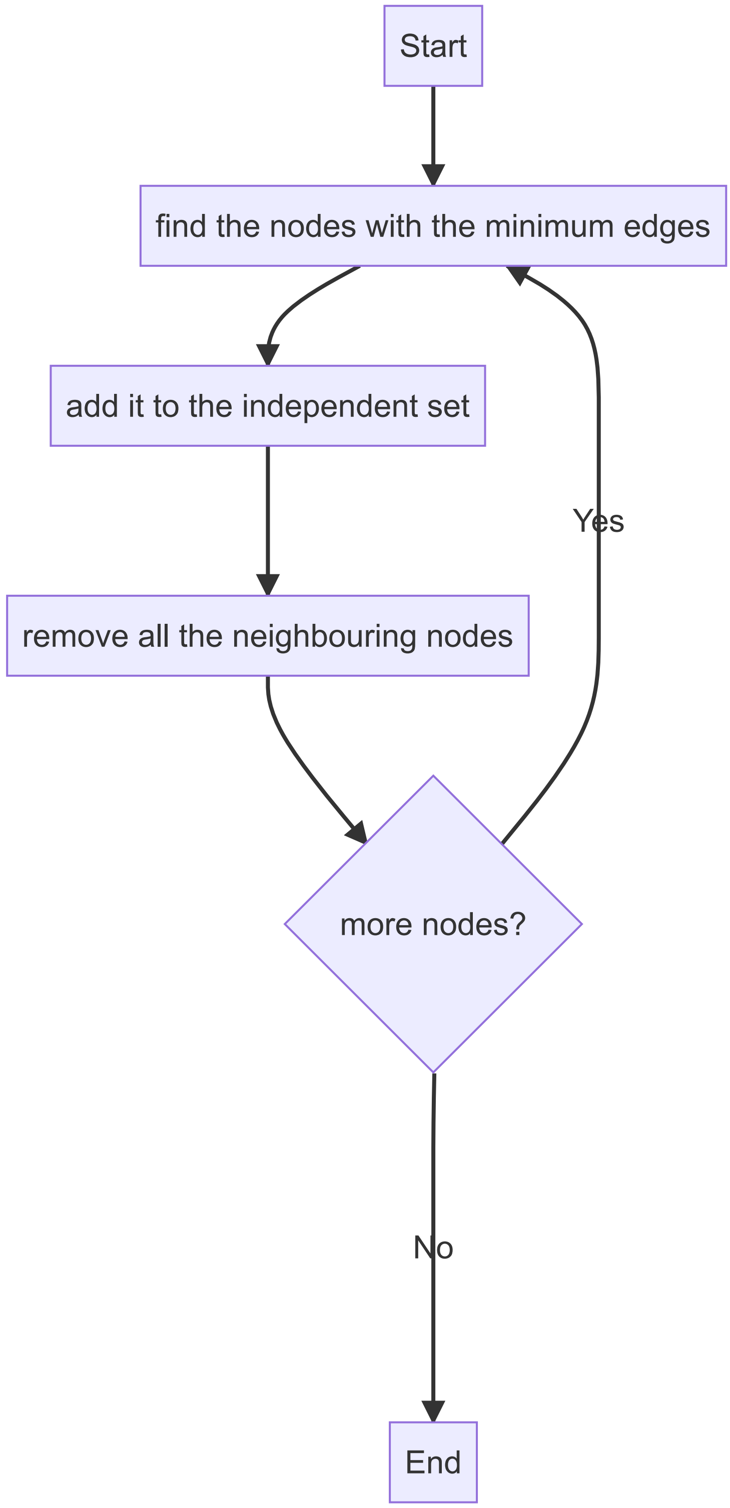



This is not a minimum spanning tree, for this tree added E(2,7) =4 instead of the minimum edge E(6,7)=2

# problem6

## a

basic idea




By removing the node with minimum neighbours one by one, hope this algorithem will have the maximum removal times, in order to bring out the maximum independent set

pesudo code

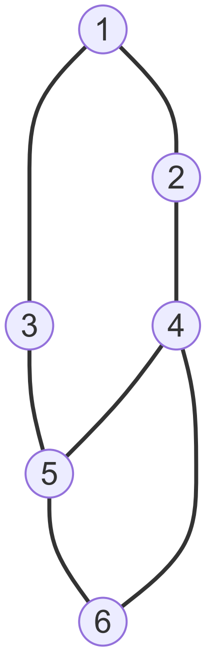
```
function min(A[1:n])
begin
    int min = A[1];
    for int i=2 to n
        if min>a[i] then
            min = a[i]
        endif
```

```
    endfor
    return min
end min

function minLocation(A[1:n])
begin
    int min = A[1];
    int mark = 1;
    for int i=2 to n
        if min>a[i] then
            min = a[i]
            mark = i;
        endif
    endfor
    return mark
endmin

//assume G is an adjacency matrix of n node graph
function greedyGetMaxIndependentSet(G[1:n][1:n])
begin
    //count edge for each node
    int edgeNum[1:n];
    int independentSet[1:n];
    for int i=1 to n do
        for int j=i+1 to n do
            if G[i][j]=1 then
                edgeNum[i]++;
                edgeNum[j]++;
            endif
        endfor
    endfor
    int counter = 0;
    for min(edgeNum[1:n])!=0 do
        //get node with min edges
        int currentLocation := minLocation(edgeNum[1:n]);
        //add to independent set
        independentSet[counter]=currentLocation;
        counter ++;
        //remove neighbours
        for int i=1 to n do
            if G[currentLocation][i] == 1 then
                for int j=1 to n do
                    if G[i][j]!=0 then
                        edgeNum[i]--;
                        edgeNum[j]--;
                        G[i][j]=0;
                        G[j][i]=0;
                    endif
                endfor
            endif
       endfor
       edgeNum[currentLocation]=0;
    endfor
    return independentSet[1:counter];
end greedyGetMaxIndependentSet
```

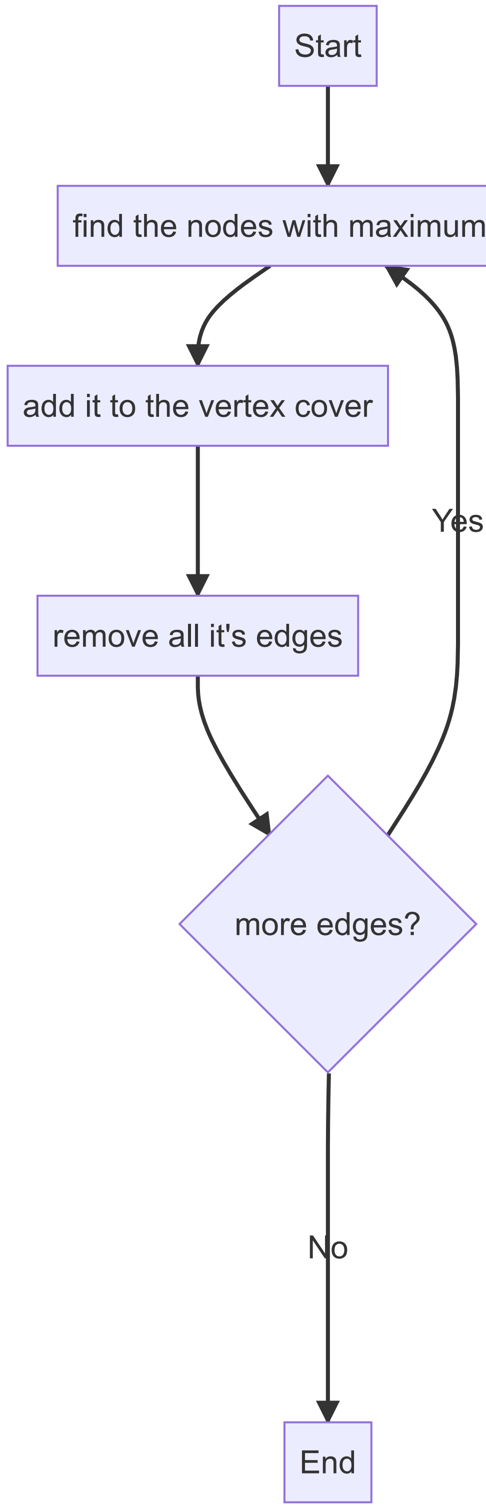
Counter Example:




In this case, 1,2,3,6 all have 2 edges, but if we select node 1 as the first node to remove, we will result in (1,4) as the maximum set, while choosing 2,3,6 at start node will have a maximum set of 3 (2,3,6). So in this case, greedy is not always optimal.

## b

basic idea




For edges removed in each step is the current maximum, after removed all edges, we hope this algorithem will bring out the minimum vertex cover

pesudo code

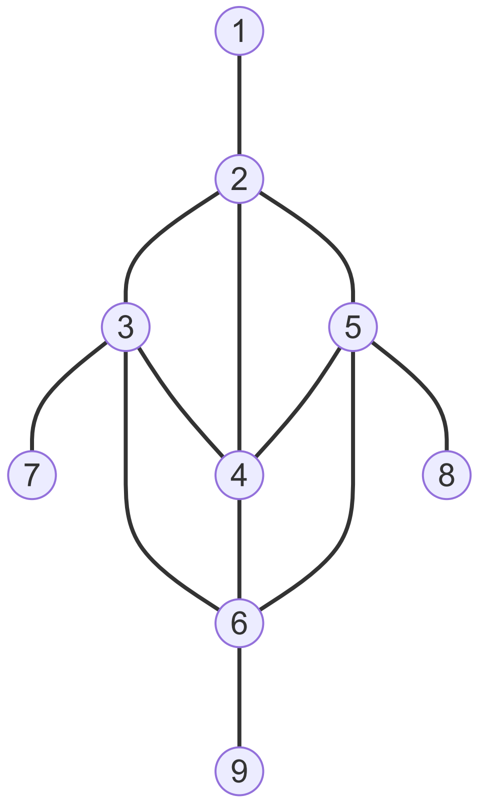
```
function max(A[1:n])
begin
    int max = A[1];
    for int i=2 to n
        if max<a[i] then
            max = a[i]
        endif
    endfor
    return max
```

```
end min

function maxLocation(A[1:n])
begin
    int max = A[1];
    int mark = 1;
    for int i=2 to n
        if max<a[i] then
            max = a[i]
            mark = i;
        endif
    endfor
    return mark
endmin

//assume G is an adjacency matrix of n node graph
function greedyGetMinVertexCover(G[1:n][1:n])
begin
    //count edge for each node
    int edgeNum[1:n];
    int vertexCover[1:n];
    for int i=1 to n do
        for int j=i+1 to n do
            if G[i][j]=1 then
                edgeNum[i]++;
                edgeNum[j]++;
            endif
        endfor
    endfor
    int counter = 0;
    for max(edgeNum[1:n])!=0 do
        //get node with max edges
        int currentLocation = maxLocation(edgeNum[1:n]);
        //add to vertex cover set
        vertexCover[counter]=currentLocation;
        counter ++;
        //remove edges
        for int i=1 to n do
            if G[currentLocation][i] == 1 then
                        edgeNum[i]--;
                        G[i][currentLocation]=0;
                        G[currentLocation][i]=0;
            endif
        endfor
        edgeNum[currentLocation] = 0;
    endfor
    return vertexCover[1:counter];
end greedyGetMinVertexCover
```

Counter Example:




In this case, we can see 2,3,4,5,6 all have 4 edges, if we remove 4 first, the result would be a 5 node vertex cover like(1,4,7,8,9), but if we remove 2 first, the result would be a 4 node vertex (2,3,5,6) so this counter case proved that greedy is not always optimal for vertex cover problem

# bonus

```
function getMultiplyResult(arr[1:n])
begin
    //by defination
    if (n == 1) then
        return arr;
    endif
    int mid;
    mid = n/2;
    //start compute
    int rightUpArr[1:mid];
    int leftDownArr[1:mid];
    //assume we are using deep copy here, start recursion
    leftDownArr = getMultiplyResult(arr[1:mid]);
    rightUpArr  = getMultiplyResult(arr[mid+1:n]);
    //leftUpArr,rightDownArr are identity matrix, so should return same arr value directly
    int i;
    for i=1 to mid do
        arr[i]     += rightUpArr[i];
        arr[i+mid] += leftDownArr[i];
    endfor
    return (arr);
end getMultiplyResult

func main()
begin
```

```
    int x[1:n]=[1,2,3,4,5,6,7,8...]
    //we can assume n=len(x) here, so n is not passed into function
    print(getMultiplyResult(x))
end main
```

For Time complexity, the algorithm calls it self on half recursively. And will have cn operations per recursive call. So

$$\begin{gathered} T(1) = c \\ T(n) = T(\frac{n}{2}) + cn \\ Therefore \\ T(n) = O(nlogn) \end{gathered} \tag{3}$$