Cookbook Data Science

June 17, 2019

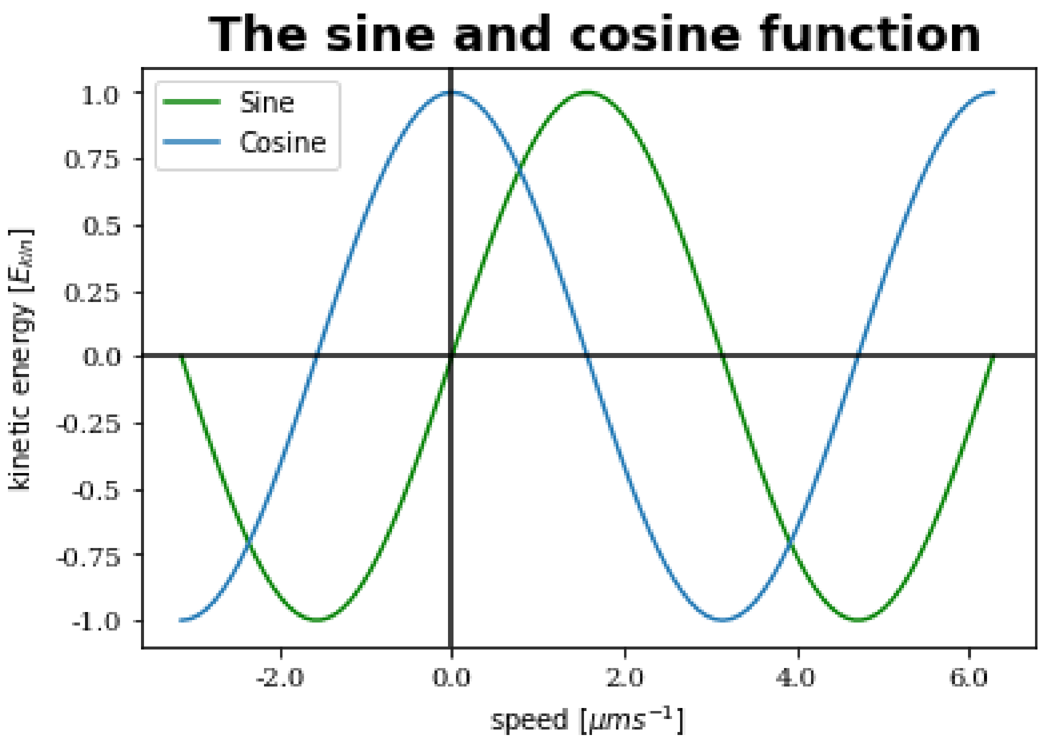
1 Plotting

1.1 A simple plot

In [1]:
```
# Simple plot
%pylab inline

import matplotlib.pyplot as plt
import numpy as np

num_points = 100                      # define once, and reuse often
x_min, x_max = -np.pi , 2*np.pi           # define once, and reuse often

x_values = np.linspace(x_min, x_max, num_points)
y_values = np.sin(x_values)
p_values = np.cos(x_values)

# checking if the two arrays have the same length
assert len(x_values) == len(y_values) , \
    "lenght-mismatch: {:d} versus {:d}".format(len(x_values),len(y_values))
plt.plot(x_values, y_values,'g')# label="Population A") #adds a label to the line
#plt.legend()    #plot the label
plt.plot(x_values, p_values)


# default color is blue, 'r' is red, 'k' is black, 'g' is green
# default solid line, ':' is dottet, '--' is dashed
# '.' adds dot points, 'o' adds open circle points


# plt.xlim(2,3)       # optional argument to set min and max x- values
# plt.ylim(5,20)      # optional argument to set min and max y- values

# plt.axis('tight')     # optional argument to make the axis just fit the range of data

# plt.axis ('equal')      # option argument to make both axis the same scale
```

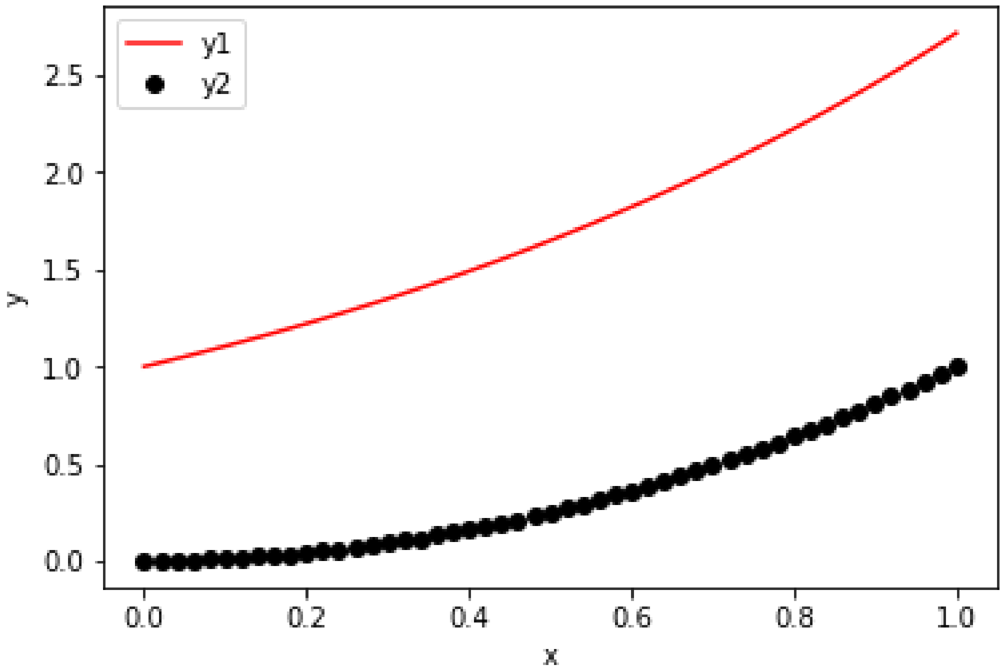
```
# optional graph modifications
ax = plt.gca()          # get current axis - returns the objext that controls as many

# Title of plot
ax.set_title( ' The sine and cosine function', size=18, weight='bold')
# plt.title( ' My first plot', size=24, weight='bold')  #alternative title on plot

# Axis lable of plot
ax.set_xlabel('speed [$\\mu m s^{-1}$]')
ax.set_ylabel('kinetic energy [$E_{kin}$]')
# Alternative axis lable of plot:
# plt.xlabel('speed')
# plt.ylabel('kinetic energy')

# Font
ax.set_xticklabels(ax.get_xticks(), family='serif', fontsize=10)
ax.set_yticklabels(ax.get_yticks(), family='serif', fontsize=10)

# Line style
# (changing after initial plot command)
lines = ax.get_lines()       # Lines is a list of line objects.
# Make the first line thick, dashed, and red
#plt.setp(lines[0], linestyle='--', linewidth=3, color='r')

# Legend
# adding a descriptive label for each line
#plt.plot(x_values, y_values, label="Population 1")
#plt.plot(x_values, y_values**3, label="Population 2")

# use line objects to set lables after plotting.
lines[0].set_label('Sine')
lines[1].set_label('Cosine')
#lines[1].set_label('Cured Poupulation')
ax.legend()

plt.axhline(0, color='black')
plt.axvline(0, color='black')


plt.show()
```

```
Populating the interactive namespace from numpy and matplotlib
```



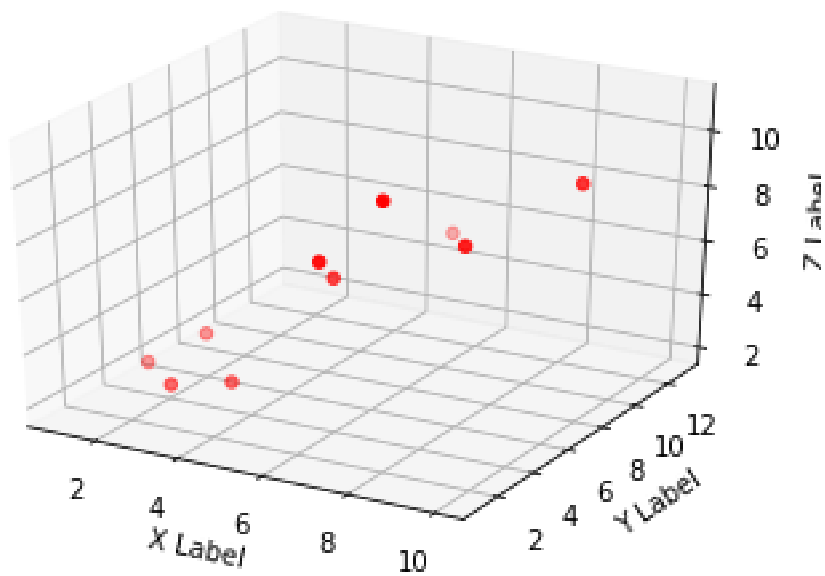
1.2 Plotting multiple equations in same graph

In [2]:
```
import matplotlib.pyplot as plt
import numpy as np


x = np.linspace(0,1,51)     # Define plotting range

y1 = np.exp(x)     # Define function y1
y2 = x**2          # Define function y2

plt.plot(x,y1, 'r', x, y2, 'ko')
plt.xlabel('x')
plt.ylabel('y')

ax = plt.gca()
lines = ax.get_lines()
lines[0].set_label('y1')
lines[1].set_label('y2')
ax.legend()


plt.show()
```



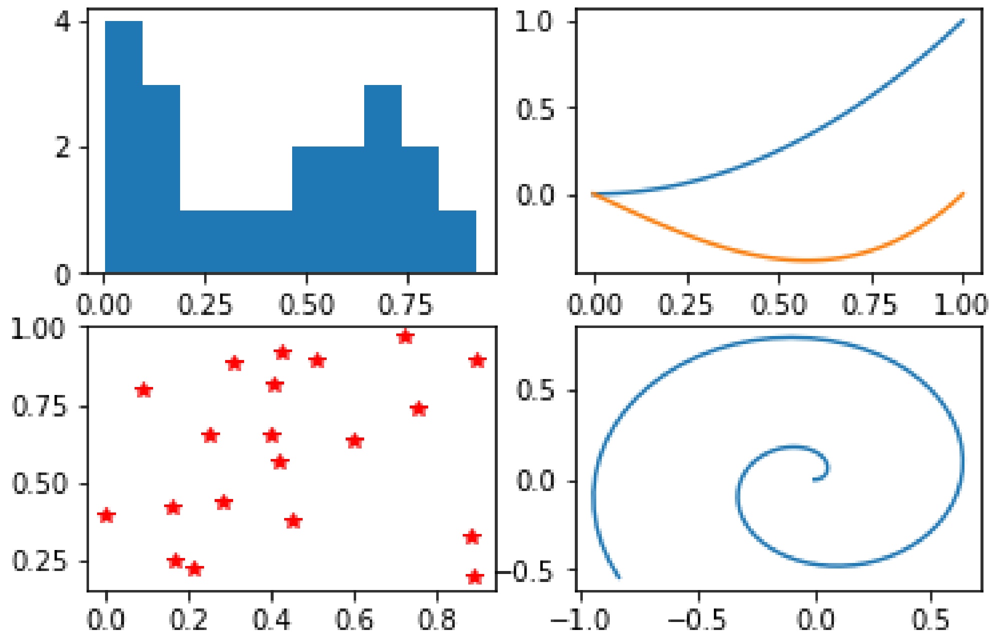
1.3 Linear equation plotting in 3D

In [3]:
```
from mpl_toolkits.mplot3d import Axes3D
import matplotlib.pyplot as plt



fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')

x =[1,2,3,4,5,6,7,8,9,10]
y =[5,6,2,3,13,4,1,2,4,8]
z =[2,3,3,3,5,7,9,11,9,10]



ax.scatter(x, y, z, c='r', marker='o')

ax.set_xlabel('X Label')
ax.set_ylabel('Y Label')
ax.set_zlabel('Z Label')

plt.show()
```



Subplots

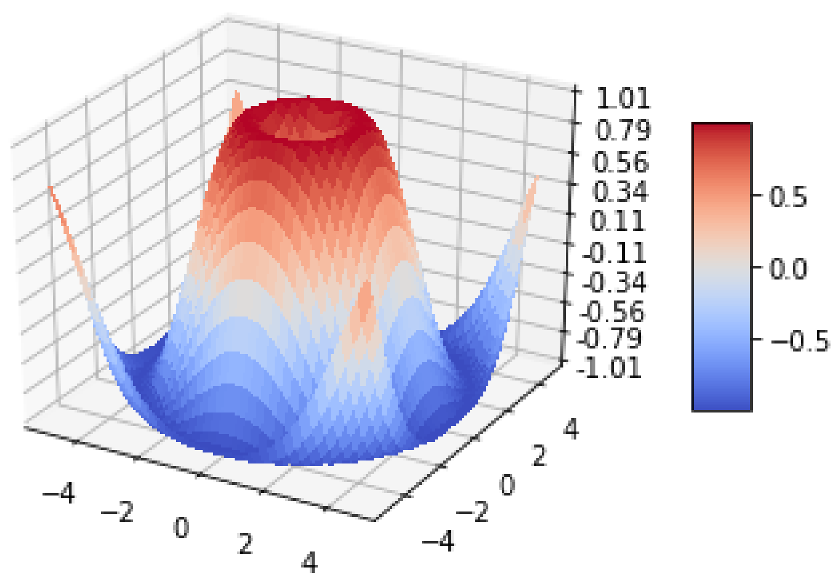
```
In [4]: import numpy as np
        from numpy.random import random
        import matplotlib.pyplot as plt

        t = np.linspace(0,1,100)

        plt.figure()
        plt.subplot(2,2,1); plt.hist(random(20))                        # Upper left
        plt.subplot (2,2,2); plt.plot (t, t**2, t, t**3 - t)            # Upper right
        plt.subplot (2,2,3); plt.plot(random(20), random(20), 'r*')     # Lower left
        plt.subplot (2,2,4); plt.plot (t*np.cos(10*t) , t*np.sin (10*t))    # Lower right

        plt.savefig('The_greatest_figure.pdf')

        plt.show()
```



1.4 3D color surface plot

In [5]:
```
'''
======================
3D surface (color map)
======================

Demonstrates plotting a 3D surface colored with the coolwarm color map.
The surface is made opaque by using antialiased=False.

Also demonstrates using the LinearLocator and custom formatting for the
z axis tick labels.
'''

from mpl_toolkits.mplot3d import Axes3D
import matplotlib.pyplot as plt
from matplotlib import cm
from matplotlib.ticker import LinearLocator, FormatStrFormatter
import numpy as np


fig = plt.figure()
ax = fig.gca(projection='3d')

# Make data.
```

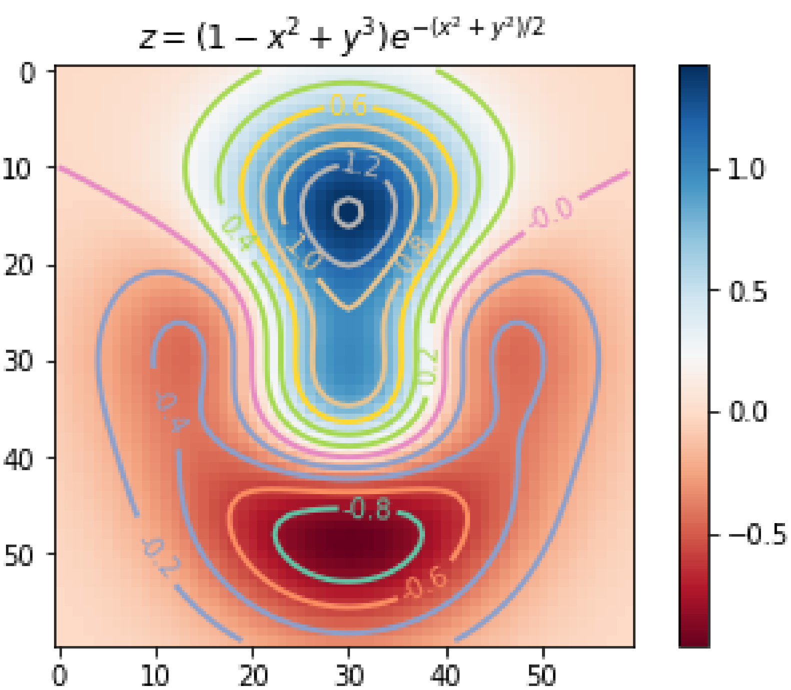
```
X = np.arange(-5, 5, 0.25)
Y = np.arange(-5, 5, 0.25)
X, Y = np.meshgrid(X, Y)
R = np.sqrt(X**2 + Y**2)
Z = np.sin(R)

# Plot the surface.
surf = ax.plot_surface(X, Y, Z, cmap=cm.coolwarm,
                       linewidth=0, antialiased=False)

# Customize the z axis.
ax.set_zlim(-1.01, 1.01)
ax.zaxis.set_major_locator(LinearLocator(10))
ax.zaxis.set_major_formatter(FormatStrFormatter('%.02f'))

# Add a color bar which maps values to colors.
fig.colorbar(surf, shrink=0.5, aspect=5)

plt.show()
```



1.5 A function with two variables

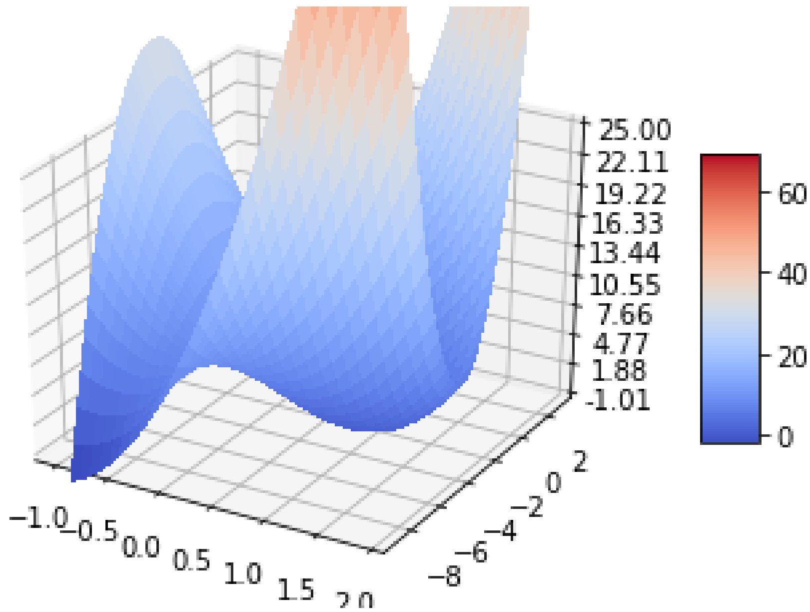
```
In [6]: from numpy import exp, arange
        from pylab import meshgrid, cm, imshow, contour, clabel, colorbar, axis, title, show


        # the function that I'm going to plot
```

```python
def z_func(x, y):
    return (1 - (x ** 2 + y ** 3)) * exp(-(x ** 2 + y ** 2) / 2)

# make data
x = arange(-3.0, 3.0, 0.1)
y = arange(-3.0, 3.0, 0.1)
X, Y = meshgrid(x, y)  # grid of point
Z = z_func(X, Y)  # evaluation of the function on the grid

im = imshow(Z, cmap=cm.RdBu)  # drawing the function
# adding the Contour lines with labels
cset = contour(Z, arange(-1, 1.5, 0.2), linewidths=2, cmap=cm.Set2)
clabel(cset, inline=True, fmt='%1.1f', fontsize=10)
colorbar(im)  # adding the colobar on the right
# latex fashion title
title('$z=(1-x^2+y^3) e^{-(x^2+y^2)/2}$')
show()
```




```python
In [7]: from mpl_toolkits.mplot3d import Axes3D
        import matplotlib.pyplot as plt
        from matplotlib import cm
        from matplotlib.ticker import LinearLocator, FormatStrFormatter
        import numpy as np
```

```python
fig = plt.figure()
ax = fig.gca(projection='3d')

# the function that I'm going to plot
def z_func(x, y):
    return (8*x**2 + x*y**2 + 6*x*y - 7*x +8)

# make data
x = np.arange(-1.0, 2.0, 0.1)
y = np.arange(-9.0, 3.0, 0.1)
X, Y = np.meshgrid(x, y)  # grid of point
Z = z_func(X, Y)  # evaluation of the function on the grid

# Plot the surface.
surf = ax.plot_surface(X, Y, Z, cmap=cm.coolwarm,
                       linewidth=0, antialiased=False)

# Customize the z axis.
ax.set_zlim(-1.01, 25)
ax.zaxis.set_major_locator(LinearLocator(10))
ax.zaxis.set_major_formatter(FormatStrFormatter('%.02f'))

# Add a color bar which maps values to colors.
fig.colorbar(surf, shrink=0.5, aspect=5)

plt.show()

X, Y, Z = axes3d.get_test_data(0.05)
ax.plot_surface(X, Y, Z, rstride=8, cstride=8, alpha=0.3)
cset = ax.contour(X, Y, Z, zdir='z', offset=-100, cmap=cm.coolwarm)
cset = ax.contour(X, Y, Z, zdir='x', offset=-40, cmap=cm.coolwarm)
cset = ax.contour(X, Y, Z, zdir='y', offset=40, cmap=cm.coolwarm)

ax.set_xlabel('X')
ax.set_xlim(-1, 1)
ax.set_ylabel('Y')
ax.set_ylim(-9, 3)
ax.set_zlabel('Z')
ax.set_zlim(-1, 50)

plt.show()
```

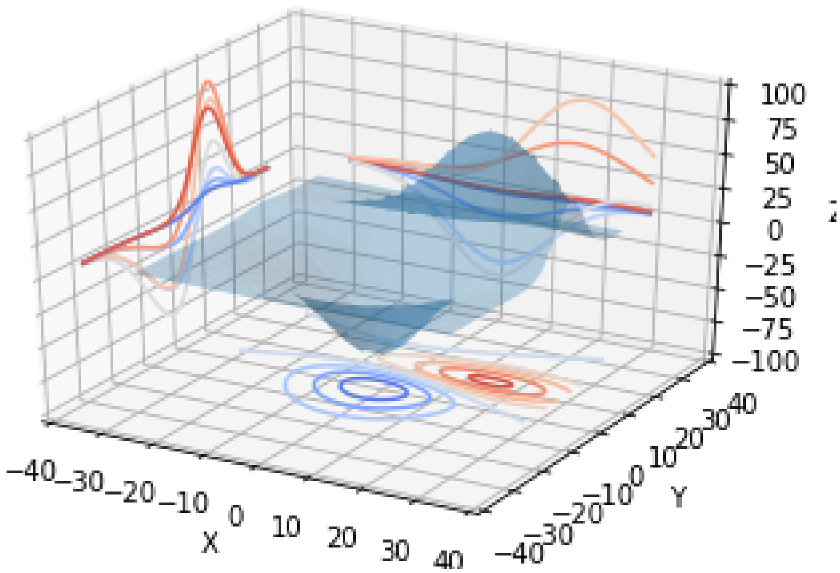



```
---------------------------------------------------------------------------

NameError                                 Traceback (most recent call last)

<ipython-input-7-d75ff393532f> in <module>
     32 plt.show()
     33
---> 34 X, Y, Z = axes3d.get_test_data(0.05)
     35 ax.plot_surface(X, Y, Z, rstride=8, cstride=8, alpha=0.3)
     36 cset = ax.contour(X, Y, Z, zdir='z', offset=-100, cmap=cm.coolwarm)


NameError: name 'axes3d' is not defined
```

1.6 Contour plot

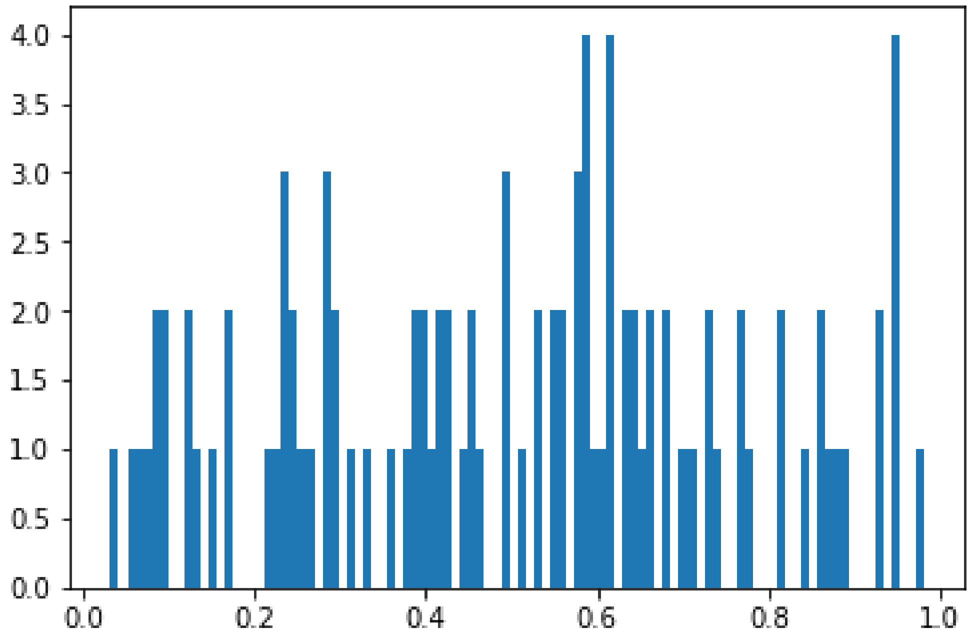
```
In [8]: from mpl_toolkits.mplot3d import axes3d
        import matplotlib.pyplot as plt
        from matplotlib import cm

        fig = plt.figure()
        ax = fig.gca(projection='3d')
        X, Y, Z = axes3d.get_test_data(0.05)
        ax.plot_surface(X, Y, Z, rstride=8, cstride=8, alpha=0.3)
        cset = ax.contour(X, Y, Z, zdir='z', offset=-100, cmap=cm.coolwarm)
```

```
cset = ax.contour(X, Y, Z, zdir='x', offset=-40, cmap=cm.coolwarm)
cset = ax.contour(X, Y, Z, zdir='y', offset=40, cmap=cm.coolwarm)

ax.set_xlabel('X')
ax.set_xlim(-40, 40)
ax.set_ylabel('Y')
ax.set_ylim(-40, 40)
ax.set_zlabel('Z')
ax.set_zlim(-100, 100)

plt.show()
```



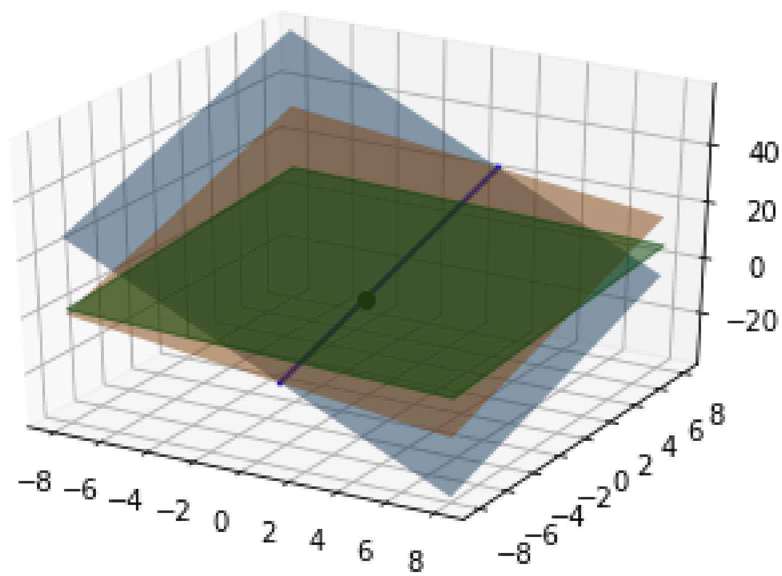
1.7 Histogram

```
In [9]: from numpy.random import random as rng
        import matplotlib.pyplot as plt

        data = rng(100)

        plt.hist(data, bins=100, align='mid')

        plt.show()
```



1.8 3D plot of linear euquations

In [10]:
```
''' demonstration of a 3D plot of linear equations'''

import numpy as np
import matplotlib.pyplot as plt

from mpl_toolkits.mplot3d import Axes3D
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')

x, y = np.linspace(-8,8,100), np.linspace(-8,8,100)
X, Y = np.meshgrid(x,y)
Z1 = 11 - 4*X + 2*Y
Z2 = (16 - 2*X + 4*Y) / 2
Z3 = (17 - X + 2*Y) / 4

ax.plot_surface(X,Y,Z1, alpha=0.5, rstride=100, cstride=100)
ax.plot_surface(X,Y,Z2, alpha=0.5, rstride=100, cstride=100)


ax.plot((1,1),(-8,8),(-9,23), lw=2, c='b')
ax.plot_surface(X,Y,Z3, alpha=0.5, facecolors='g', rstride=100, cstride=100)
ax.plot((1,),(-2,),(3,), lw=2, c='k', marker='o')

plt.show()
```



1.9 Importing and Exporting Data

In [11]:
```
"""
Created: 2018
Author: Stephan Goldberg (Python 3.4)

Description:
Importing and Exporting Data

"""

import numpy as np

# 1. import method:
# import data from local directory, specifying that it is a csv file
data_set = np.loadtxt("HIVseries.csv", delimiter=',')

# 2. import method:
# specifiying the complete path of a file on local computer
data_file = '/Users/stephangoldberg/Google Drive/Python Projects/MathProblems/PMLSdata
data_set02 = np.loadtxt(data_file, delimiter=',')

# 3. import method:
# specifying home directory, data directory once, to safe work later:
home_dir = '/Users/stephangoldberg/Google Drive/Python Projects/MathProblems/'
data_dir = home_dir + 'PMLSdata/01HIVseries/'
data_set03 = np.loadtxt(data_dir+ 'HIVseries.csv', delimiter=',')
```

```
# 4. importing other kinds of text
# to be used when line by line is to be imported
my_file = open('HIVseries.csv')
temp_data = []              # creates an object that can read the data file
for line in my_file:
    print(line)
    x, y = line.split(',')  # uses split line method to break the line into numbers u
    temp_data += [(float(x), float(y))] #converts text into numbers
my_file.close()
data_set04 = np.array(temp_data)

# 5. direct web import
import urllib.request
web_file = urllib.request.urlopen( " http://www.physics.upenn.edu/biophys/PMLS/Dataset
data_set05 = np.loadtxt(web_file, delimiter = ',')

# saving data
x = np.linspace(0,1,1001)
y = 3* np.sin(x)**3 - np.sin(x)

np.save('x_values', x)
np.save('y_values', y)

np.savetxt('x_values.dat',x)
np.savetxt('y_values.dat', y)

np.savez ('xy_values', x_vals=x, y_vals=y)

# recovering data
x2 = np.load('x_values.npy')
y2 = np.loadtxt('y_values.dat')
w = np.load('xy_values.npz')

print(w.keys())
x2 == x
y2 == y
w['x_vals'] == x
w['y_vals'] == y

# Writing information directly into a file:

my_file = open('power.txt', 'w')        # opens file and prepares it with 'w' writing
print('N \t\t2**N\t\t3**N')             # print labels for columns, also '\t' inserts
print('---\t\t----\t\t----')            # print separator.

my_file.write('N \t\t2**N\t\t3**N')     # write lables to file
my_file.write('---\t\t----\t\t----')    # write separators to file
```

```
#%% Loop over integers from 0 1- 10 and print/write results
for N in range(11):
    print('{:d}\t\t{:d}\t\t{:d}'.format(N, pow(2,N), pow(3,N)))
    my_file.write('{:d}\t\t{:d}\t\t{:d}'.format(N, pow(2,N), pow(3,N)))
my_file.close()                                   # the file must be closed again when finished

print(my_file)a
```

```
  File "<ipython-input-11-aa43a32f6a7b>", line 81
    print(my_file)a
                  ^
SyntaxError: invalid syntax
```

2 Statistics Basics

Importing the python libraries:

```
In [1]: import numpy as np
        import pandas as pd
        import matplotlib.pyplot as plt
        %pylab inline
```

```
Populating the interactive namespace from numpy and matplotlib
```

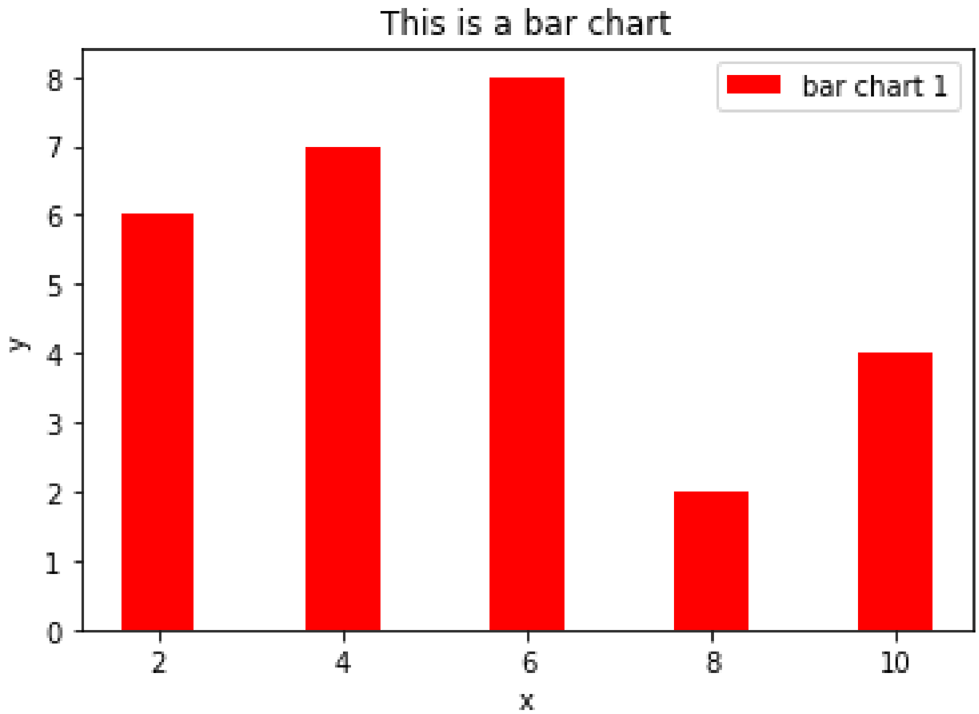
Terminology: (cited from page 10, unit 1) - Observations _ (or **cases**, or **sampling units**) refer to objects (people, countries, . . .) on which characteristics are recorded. - **Variables** are the characteristics recorded, and the pattern of variation of a variable is its distribution. - Variables are **linked** if they are each recorded for the same observations. - A variable is **continuous** if its values are numerical and all values in an interval are possible. - A variable is **discrete** if its values are numerical but only particular values (typically, integers) are possible. - A variable is **categorical** if its values indicate to which group an observation belongs. - A categorical variable is **ordinal** if its values correspond to labels which have a natural ordering. - A categorical variable is __nominal__if its values correspond to labels but the labels do not have a natural ordering.

2.1 A bar chart

A bar chart is usually used for categorical data or with numerical data that are discrete.

```
In [2]: x = [2,4,6,8,10]
        y = [6,7,8,2,4]

        plt.bar(x,y,label = 'bar chart 1', color='r')
```

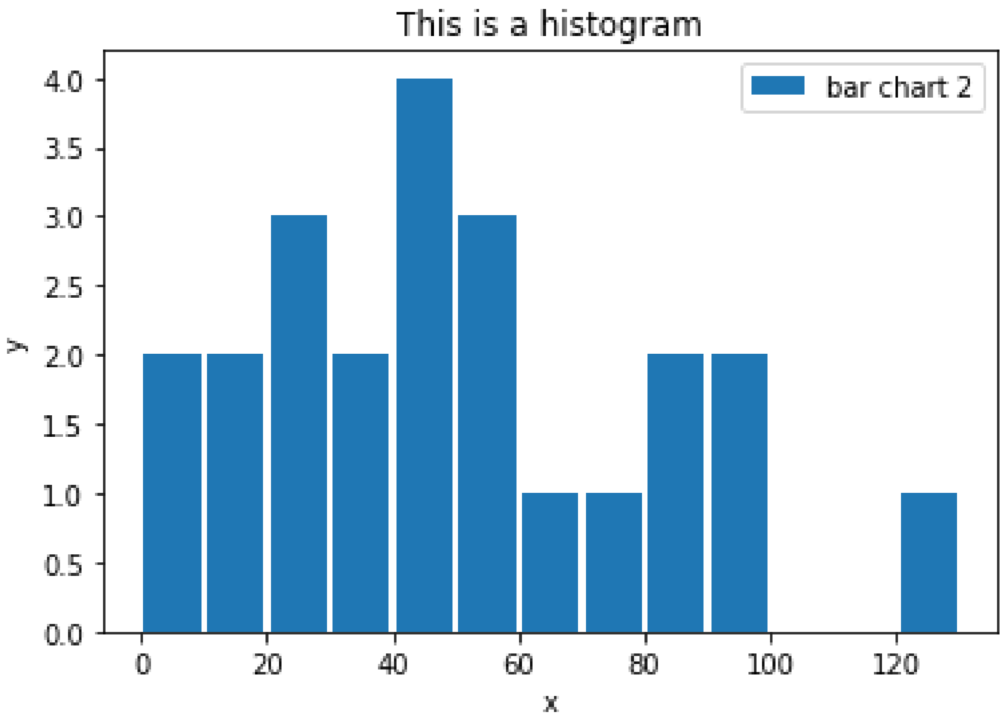
```
plt.xlabel('x')
plt.ylabel('y')
plt.title('This is a bar chart')
plt.legend()
plt.show()
```



2.2 A histogram chart

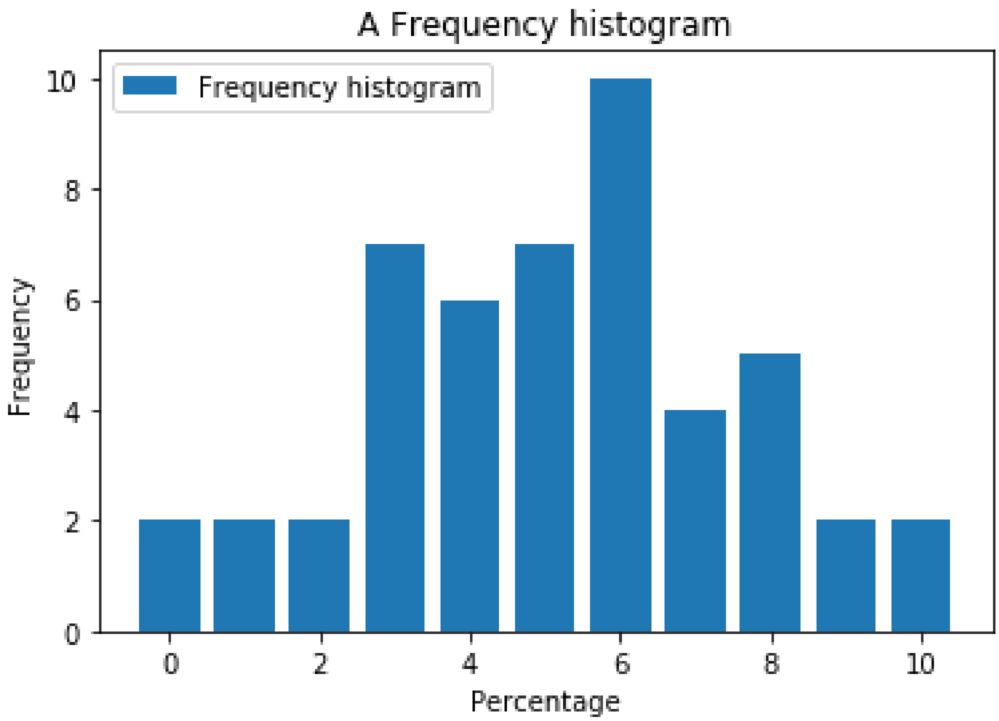
- A histogram chart is generally used with continuous data.
- Histograms need a reasonably large dataset and are sensitive to the choice of cutpoints.

```
In [41]: population_ages = [12,34,65,87,23,123,3,7,45,92,54,74,41,21,98,34,43,10,20,50,40,85,5

         bins = [0,10,20,30,40,50,60,70,80,90,100,110,120,130]

         plt.hist(population_ages, bins, histtype = 'bar', rwidth = 0.9, label = 'bar chart 2'

         plt.xlabel('x')
         plt.ylabel('y')
         plt.title('This is a histogram')
         plt.legend()
         plt.show()
```



2.3 A frequency histogram (according to M248)

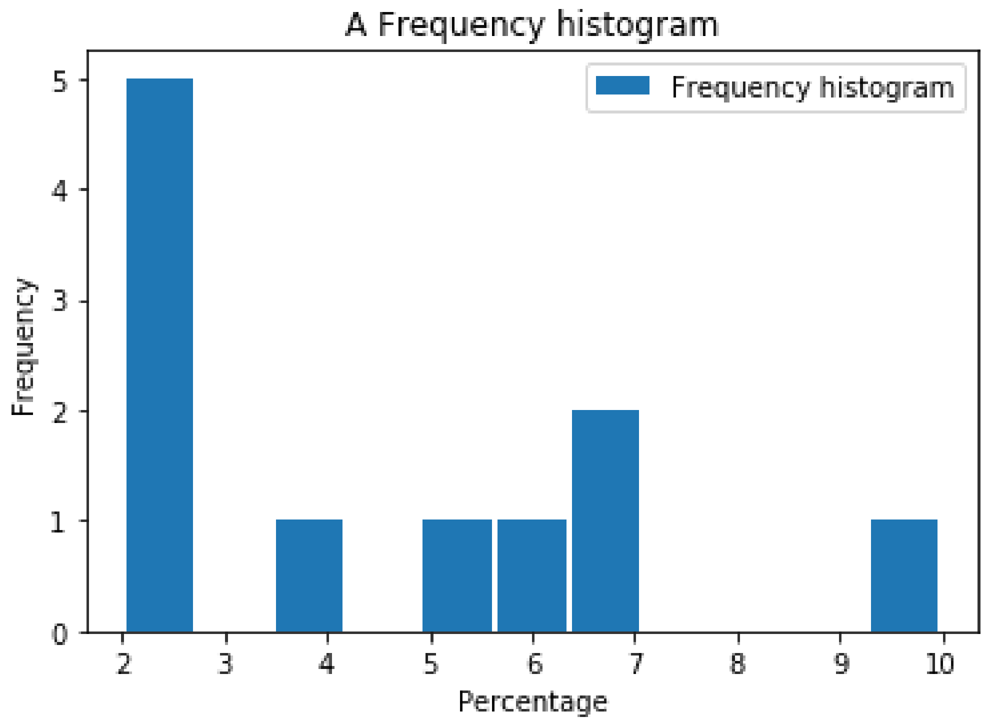
```
In [95]: frequency = [2, 2, 2, 7, 6, 7, 10, 4, 5, 2, 2]
         y_pos = np.arange(len(frequency))
         plt.bar(y_pos, frequency, label = 'Frequency histogram' )

         plt.xlabel('Percentage')
         plt.ylabel('Frequency')
         plt.title('A Frequency histogram')
         plt.legend()
         plt.show()
```



Frequency histogram according to https://www.mathsisfun.com/definitions/frequency-histogram.html

note: the occurances of values of same magnitude are stacked!!!

```
In [47]: plt.hist(frequency, bins=11, histtype = 'bar', rwidth = 0.9, label = 'Frequency histog

         plt.xlabel('Percentage')
         plt.ylabel('Frequency')
         plt.title('A Frequency histogram')
         plt.legend()
         plt.show()
```



Defining a new data set:

```
In [4]: data = [3,7,23,56,324,234,56,23,456,1223,78,432,23,456,756,85,345,234,234,]
```

2.4 Sample Size

The sample size is the number of observations a dataset contains:

```
In [5]: len(data)

Out[5]: 19
```

2.5 Sorting the data:

```
In [6]: data_sorted = np.sort(data)
        data_sorted

Out[6]: array([   3,    7,   23,   23,   23,   56,   56,   78,   85,  234,  234,
          234,  324,  345,  432,  456,  456,  756, 1223])
```

2.6 The sample mean:

If the n values in a dataset are denoted x_1, x_2, x_3,...x_n, then the **sample mean**, which is denoted $\bar{x}$ is given by

$$\bar{x} = \frac{x_1 + x_2 + x_3 + n_n}{n} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

it is also called the 'average'.

```
In [7]: x_bar = sum(data) /len(data)
        x_bar
```

```
Out[7]: 265.6842105263158
```

Mean in numpy:

```
In [8]: mean = np.mean(data)
        mean
```

```
Out[8]: 265.6842105263158
```

2.7 The population mean:

The population mean (or mean or expected value or expectation) of a random variable is given: - if X is discrete with p.m.f $p(x)$, by

$$\mu = E(X) = \sum_x xp(x)$$

- if X is continuous with p.d.f. $f(x)$, by

$$\mu = E(X) = \int xf(x)$$

where the integral is taken over all values x in the range of X.

2.8 The sample median:

The sample median is defined as 'the middle value' of the dataset.
$m = x_{\left(\frac{1}{2}(n+1)\right)}$

- if n is even, then the sample median lies exactly half way between the two centre values.
- the median, just like the interquartile range are more resistant to unusual values in the data than are the mean and the standard deviation.

Median in numpy:

```
In [9]: np.median(data)
```

```
Out[9]: 234.0
```

2.9 The sample quartiles:

Let a dataset x_1, x_2, x_n, $n = 3, 4, ...$ be recorded as $x_{(1)}$, $x_{(2)}$, $x_{(n)}$. Then the **sample lower quartile**, q_L, is given by

$q_L = x_{\left(\frac{1}{4}(n+1)\right)}$

and the **sample upper quartile**, q_U, is given by

$q_U = x_{\left(\frac{3}{4}(n+1)\right)}$

!!! Note: numpy and pandas define quartiles different than M248 or Minitab !!! (see page 37, unit 1)!!!

Upper and lower quartiles in numpy:

```
In [10]: print( np.percentile(data,25))
         print( np.percentile(data,75))
```

```
39.5
388.5
```

Upper and lower quartiles in pandas:

```
In [11]: df = pd.DataFrame(data)

         print( df.quantile(0.25))
         print( df.quantile(0.75))
```

```
0    39.5
Name: 0.25, dtype: float64
0    388.5
Name: 0.75, dtype: float64
```

```
In [12]: a = [19.1, 17.4, 23.7, 22.3, 16.7, 22.6]
```

Upper and lower quartiles in M248 (and Minitab):
First, ensure that the data is sorted in ascending order.

```
In [17]: data1 = [66,72,79,84,102,110,123,144,162,169,414]
```

Lower quartile for data-set with n+1 divisable by 4:

```
In [18]: len(data1)+1
```

```
Out[18]: 12
```

```
In [19]: q_L = data1[int(0.25*(len(data1)+1))-1]
         print (q_L)
```

```
79
```

Upper quartile for data-set with n+1 divisable by 4:

```
In [20]: q_U = data1[int(0.75*(len(data1)+1))-1]
         print (q_U)
```

```
162
```

Lower quartile for data-set with n+1 NOT divisible by 4:
removing the outlier in the data-set (to make it undividable by 4):

```
In [29]: del data1[-1]
         print(data1)
```

```
[66, 72, 79, 84, 102, 110, 123, 144, 162]
```

finding the position of the integer value x_n and the corresponding $\frac{3}{4}$-fraction of the lower quartile in the new data set and then finding the lower quartile:

```
In [30]: x_n = int((len(data1)+1)*0.25)-1
         fraction = 0.75* (data1[x_n+1] - data1[x_n])

         lower_quartile = data1[x_n]+fraction
         print(lower_quartile)
```

```
77.25
```

Upper quartile for n+1 NOT divisable by 4):
finding the position of the integer value x_n and the corresponding $\frac{1}{4}$-fraction of the upper quartile in the new data set, and then findin the upper quartile:

```
In [31]: x_n = int((len(data1)+1)*0.75)-1
         fraction = 0.25* (data1[x_n+1] - data1[x_n])

         upper_quartile = data1[x_n]+fraction
         print(upper_quartile)
```

```
128.25
```

2.10 The sample interquartile range

The__ sample interquartile range __ is defined as $q_U - q_L$, where q_U is the sample upper quartile and q_L is the sample lower quartile.

```
In [32]: sample_interquartile_range = q_U - q_L
         print(sample_interquartile_range)
```

```
83
```

2.11 The standard deviation

If n values in a dataset are denoted $x_1, x_2, ..., x_n$ and their sample mean is $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$, then the **sample standard deviation**, s is defined by: $s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$

```
In [33]: data = [ 19.1, 17.4, 23.7, 22.3, 16.7, 22.6]
```

```
In [37]: x_bar = sum(data) / len(data)
         total=0
         for i in range(0,len(data)):
            total = total + (data[i] - x_bar)**2
         s = np.sqrt(1/(len(data) - 1 ) * total)
         s
```

```
Out[37]: 2.9549957698785296
```

2.12 The sample variance

The square of the standard deviation is the sample variance. Hence it is given by: $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$

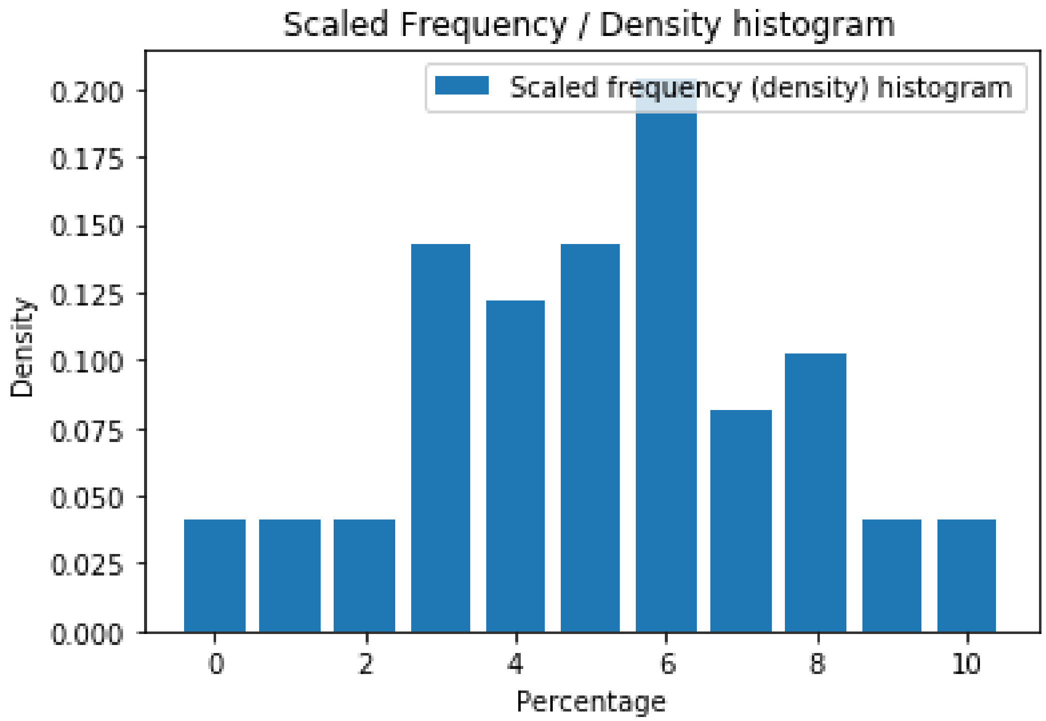
```
In [36]: x_bar = sum(data) / len(data)
         total=0
         for i in range(0,len(data)):
            total = total + (data[i] - x_bar)**2
         s_squared = 1/(len(data) - 1 ) * total
         s_squared
```

```
Out[36]: 8.732000000000005
```

2.13 Unit-area histogram

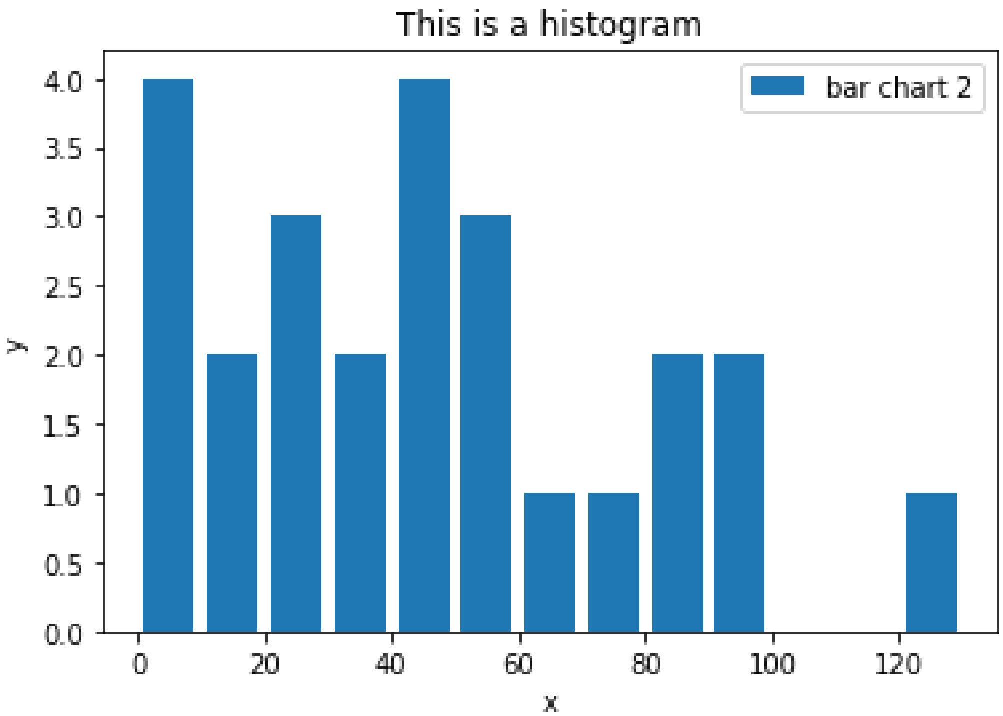
A **unit-area histogram** is a frequency histogram in which the frequencies are scaled so that the total area of the bars in the histogram is 1

```
In [53]: frequency = [2, 2, 2, 7, 6, 7, 10, 4, 5, 2, 2]
         y_pos = np.arange(len(frequency))

         scaled_frequency = frequency/np.sum(frequency)

         plt.bar(y_pos, scaled_frequency, label = 'Scaled frequency (density) histogram' )

         plt.xlabel('Percentage')
         plt.ylabel('Density')
         plt.title('Scaled Frequency / Density histogram')
         plt.legend()
         plt.show()
```



A non-scaled histogram:

```
In [71]: population_ages = [2,5,12,34,65,87,23,123,3,7,45,92,54,74,41,21,98,34,43,10,20,50,40,8

         bins = [0,10,20,30,40,50,60,70,80,90,100,110,120,130]

         plt.hist(population_ages, bins, histtype = 'bar', rwidth = 0.8, label = 'bar chart 2')
         plt.xlabel('x')
         plt.ylabel('y')
         plt.title('This is a histogram')
         plt.legend()
         plt.show()
```

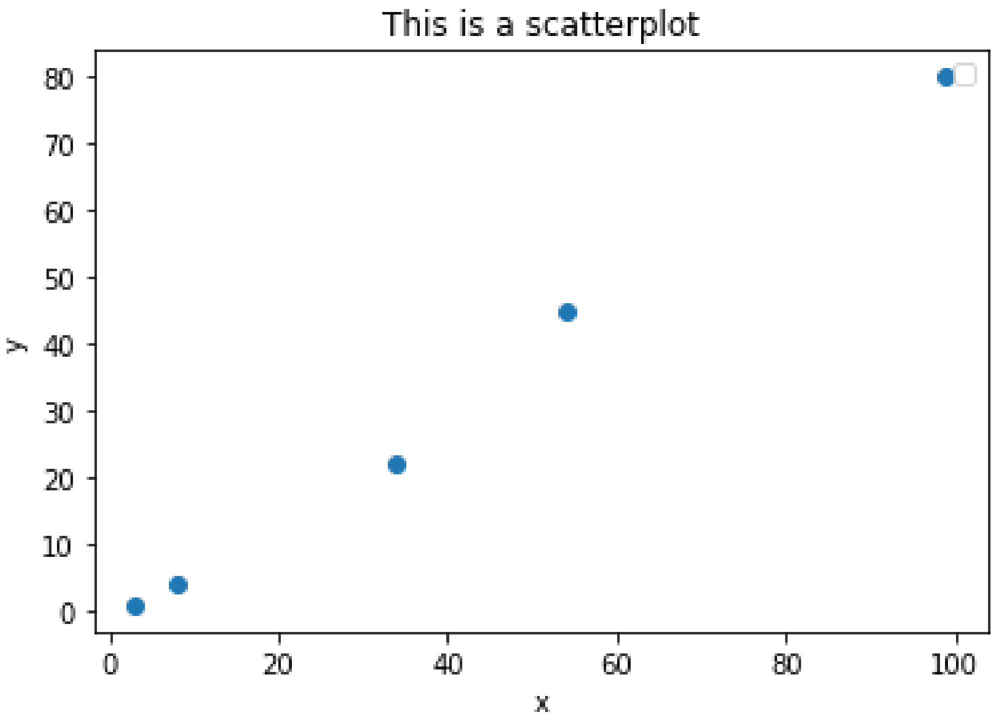


2.14 A Scatterplot

A scatterplots are used to investigate the relationship between two numercial variables.

```
In [73]: x = [3,8,34,54,99]
         y = [1,4,22,45,80]
         plt.xlabel('x')
         plt.ylabel('y')
         plt.title('This is a scatterplot')
         plt.legend()
         plt.scatter(x, y,label = 'scatterplot')
         plt.show()
```

```
No handles with labels found to put in legend.
```



Scatterplot Interpretation checklist

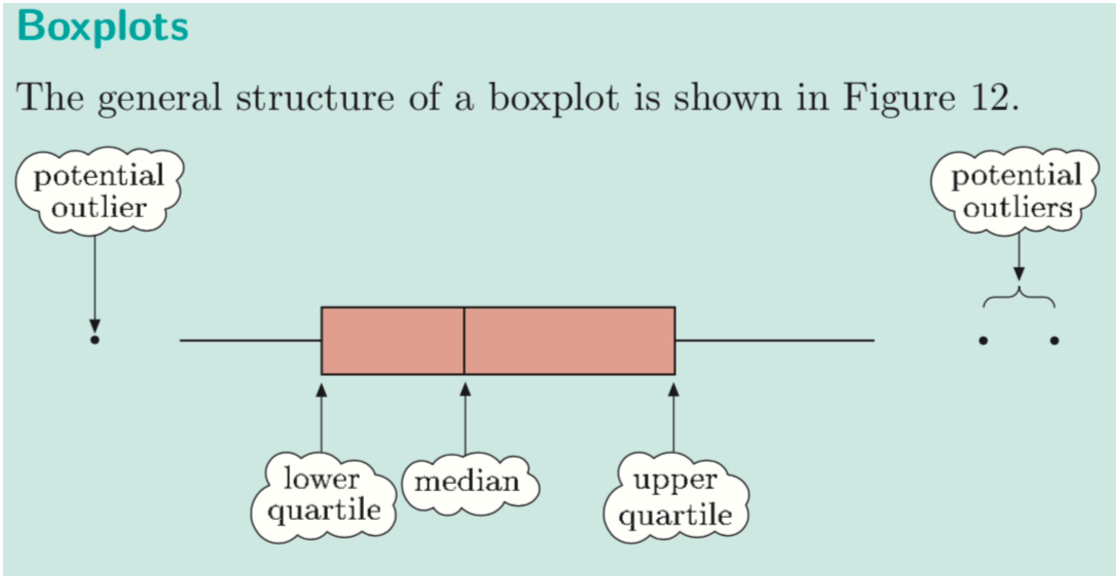
1. Is the relationship positive, negative or neither?
2. Is the relationship linear or non-linear?
3. Is the relationship strong or weak?
4. Are there any outliers?

2.15 A boxplot

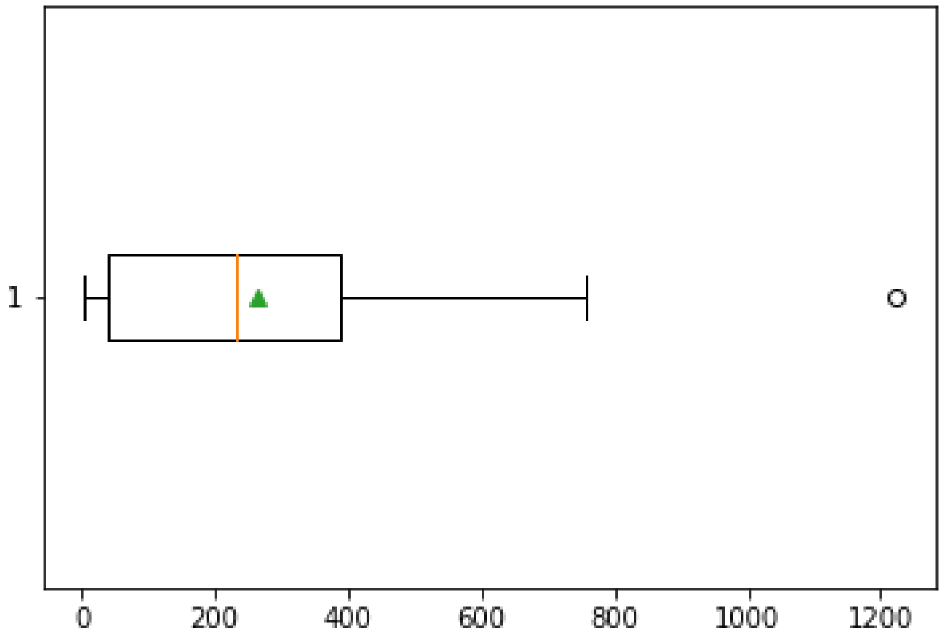
- A boxplot, just like the histogram is also used with continuous data.
- Boxplots cannot show how many modes a distribution has.
- Comperative boxplots allow more than one continuous variable to be displayed.

```
In [57]: from IPython.display import Image
         Image(filename = "images/boxplot.png" , width = 400, height = 200)
```

```
Out[57]:
```




```
In [58]: data_box = [3,7,23,56,324,234,56,23,456,1223,78,432,23,456,756,85,345,234,234,]

In [59]: plt.boxplot(data_box, showmeans=True, vert=False)

Out[59]: {'boxes': [<matplotlib.lines.Line2D at 0x118b8f208>],
          'caps': [<matplotlib.lines.Line2D at 0x118c19fd0>,
           <matplotlib.lines.Line2D at 0x118c19d30>],
          'fliers': [<matplotlib.lines.Line2D at 0x118c23898>],
          'means': [<matplotlib.lines.Line2D at 0x118c23518>],
          'medians': [<matplotlib.lines.Line2D at 0x118c19390>],
          'whiskers': [<matplotlib.lines.Line2D at 0x118b8f470>,
           <matplotlib.lines.Line2D at 0x118c19e48>]}
```



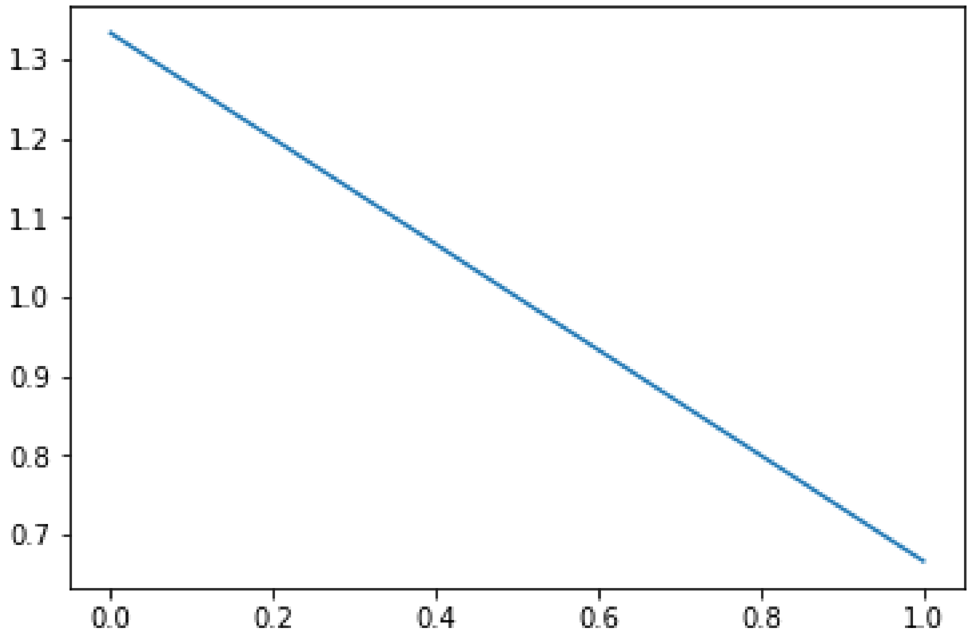
3 P.D.F. and C.D.F

3.1 P.d.f. Probability distribution functions (Excercise 8)

In [65]:
```
from scipy.integrate import quad

num_points = 100
x_min, x_max = 0 , 1

x = np.linspace(x_min, x_max, num_points)

def integrand(x):
    return 2/3 * (2-x)  # insert function here

ans, err = quad(integrand, x_min, x_max)
print ("The integral over the interval over the range (" + str(x_min) + ", " + str(x_m

plt.plot(x, integrand(x))
```

```
The integral over the interval over the range (0, 1) evaluates to 1.0
```

Out[65]:
```
[<matplotlib.lines.Line2D at 0x119fc1ac8>]
```



3.2 C.d.f. Comulative distribution function (Activity 24)

a)

```
In [70]: def F(x):
             return x**3
         a = F(3/4)-F(1/2)
         Fraction (a)

Out[70]: Fraction(19, 64)

In [72]: def F(x):
             return x**3
         a = 1- F(0.6)
         a

Out[72]: 0.784

In [75]: def F(x):
             return x**3
         a = F(0.6)-F(0.1)
         round(a,3)

Out[75]: 0.215
```

b
i)

```
In [77]: def F(x):
             return x/5 - x**2/500 -16/5
         a = 1-F(22)
         round(a,3)

Out[77]: 0.768
```

ii)

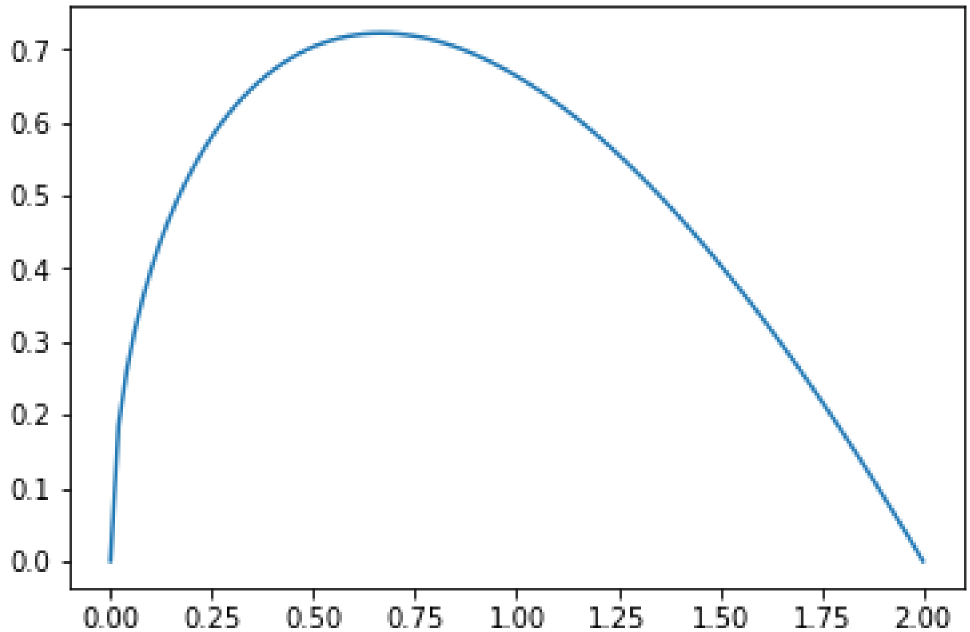
```
In [78]: def F(x):
             return x/5 - x**2/500 -16/5
         a = F(29)-F(21)
         round(a,3)

Out[78]: 0.8
```

3.3 P.d.f. and c.d.f.(Activity 25)

a)

As seen from the graph, f(x)>0 for all x. We test if the integral of f(x) with respect to x is 1:

```
In [8]: from scipy.integrate import quad
        import numpy as np
        import matplotlib.pyplot as plt
        %pylab inline

        num_points = 100
        x_min, x_max = 0 , 2
        x = np.linspace(x_min, x_max, num_points)

        def integrand(x):
            return 15/(16*np.sqrt(2)) * np.sqrt(x) * (2-x)   # insert function here

        ans, err = quad(integrand, x_min, x_max)
        print ("The integral over the interval over the range (" + str(x_min) + ", " + str(x_ma

        plt.plot(x, integrand(x))

Populating the interactive namespace from numpy and matplotlib
The integral over the interval over the range (0, 2) evaluates to 1.0


Out[8]: [<matplotlib.lines.Line2D at 0x113cb5358>]
```



c)

```
In [9]: def F(x):
            return (x*np.sqrt(x))/(8*np.sqrt(2))*(10- 3*x)
        1-F(1)

Out[9]: 0.381281566461771
```

the probability is 38% that the bulldozers return time is greater than a minute.
d)

```
In [10]: F(1)-F(0.5)

Out[10]: 0.353093433538229
```

the probabilit is 35.3% (to three s.f.) that the bulldozer returns between 30 seconds and one minute

3.4 P.d.f. and c.d.f. (Exercise 10)

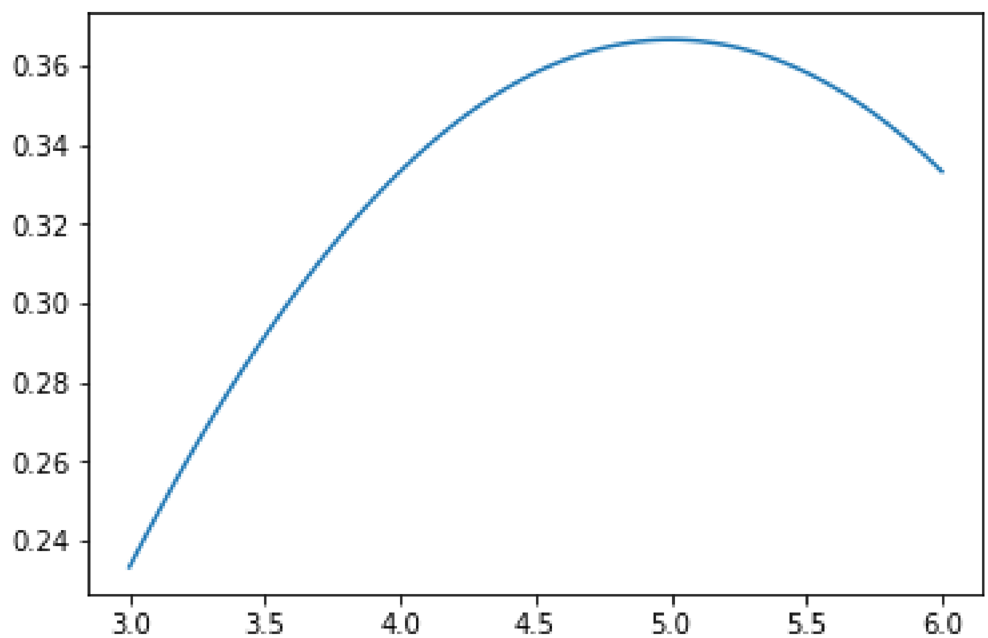
a)

```
In [11]: from scipy.integrate import quad
         import numpy as np
         import matplotlib.pyplot as plt
         %pylab inline
```

```
num_points = 100
x_min, x_max = 3 , 6                        # insert limits here
x = np.linspace(x_min, x_max, num_points)

def integrand(x):
    return 1/30 *(10*x- x**2 - 14)          # insert function here

ans, err = quad(integrand, x_min, x_max)
print ("The integral over the interval over the range (" + str(x_min) + ", " + str(x_m

plt.plot(x, integrand(x))
```

```
Populating the interactive namespace from numpy and matplotlib
The integral over the interval over the range (3, 6) evaluates to 1.0
```

Out[11]: [<matplotlib.lines.Line2D at 0x113d1d4e0>]



The integral over the interval is 1 and the function is non-negative over the interval, so yes, f is a valid p.f.d..

c)

```
In [12]: def F(x):
             return 1/30*(5*x**2 - 1/3* x**3 - 14*x +6)
         F(4)
```

Out[12]: 0.28888888888888903

In [13]: F(5) - F(4)

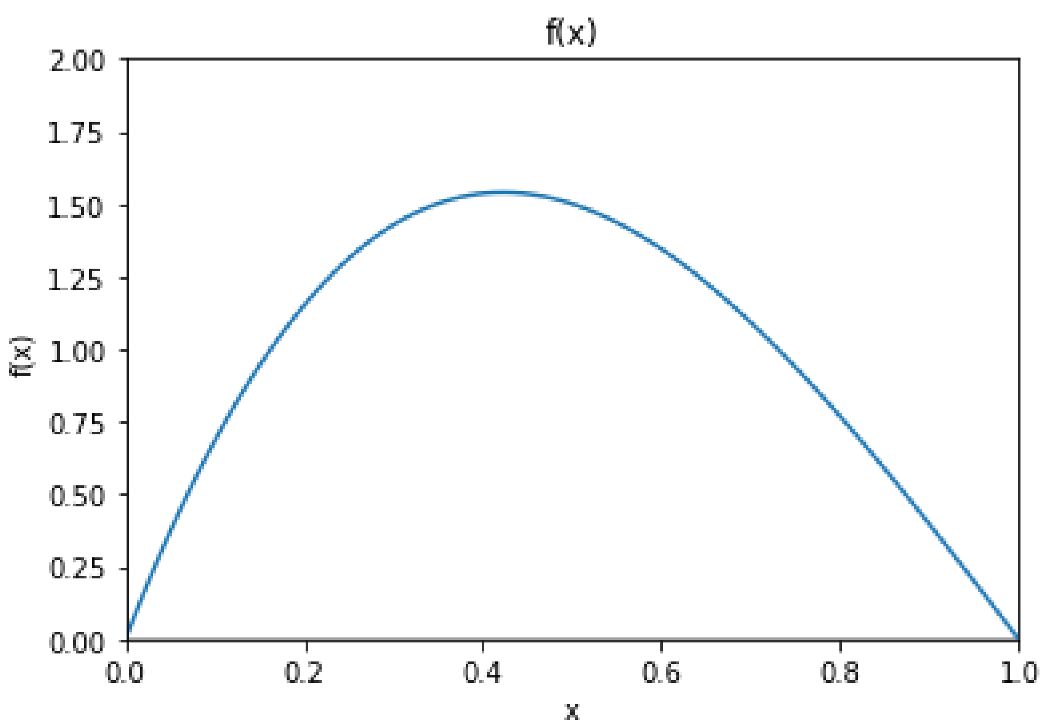
Out[13]: 0.3555555555555557

3.4.1 TMA 01, Question 2 b) ii)

We plot $f(x)$ in for the range {0,6}:

In [28]:
```
# defining the plotting limits:
xmin, xmax, ymin, ymax, number_of_dots = 0, 1, 0, 2, 100   #fill limits in here

# defining f(x) and x:
def f(x):
    return 4*x*(1-x)*(2-x)                                   #fill function in here
x = np.linspace(xmin , xmax , number_of_dots)

# plotting:
plt.xlim(xmin, xmax),plt.ylim(ymin, ymax),plt.axhline(0, color="grey"),plt.axvline(0,
plt.ylabel('f(x)'), plt.xlabel('x'),plt.title('f(x)'),
plt.plot(x, f(x))
```

Out[28]: [<matplotlib.lines.Line2D at 0x151dab03c8>]



In [ ]:

4 Discrete Probability Distributions

The mean of a discrete distribution with p.m.f. $p(x)$**:**

$$\mu = E(X) = \sum xp(x)$$

The variance of a discrete random variable with p.m.f. $p(x)$**:**

$$\sigma^2 = V(X) = E[(X-\mu)^2] = \sum (x-\mu)^2 p(x)$$

also:

$$\sigma^2 = V(X) = E(X^2) - \mu^2$$

("the variance equals the mean of the squares minus the square of the mean")

The standard variation:

$$\sigma = S(X) = \sqrt{V(X)}$$

Mean and variance of linear functions:

If Y is a linear function of X, so that $Y = aX + b$, where a and b are constants, then:

$$E(Y) = aE(X) + b$$

and

$$V(Y) = a^1 v(X)$$

4.1 Bernoulli distribution, Bernoulli (p)

A single bernoulli trial with probability p.

Examples: - the probability of obtaining heads on a fair coin toss. - whether it rains in the Atacama desert in a certain year, or not

The mean:

$$\mu = E(X) = p$$

The variance:

$$\sigma^2 = V(X) = p(1-p)$$

The p.m.f. $X \sim Bernoulli(p)$**:**

$$p(1) = 1, \quad p(0) = 0$$

or

$$p(x) = p^x(1-p)^{1-x} \qquad x = 0, 1.$$

4.2 Binomial distribution, $B(n, p)$

A binomial distribution is a number of events (bernoulli trials) with probability p happening in a given sample size n.

Examples: - the number of defective items coming from a production line in a sample of 100. - the number of arrows from an archer that hit the centre of the target - the number of matches a tennis player wins against his friend in a series of matches.

The mean:

$$\mu = E(X) = np$$

The variance:

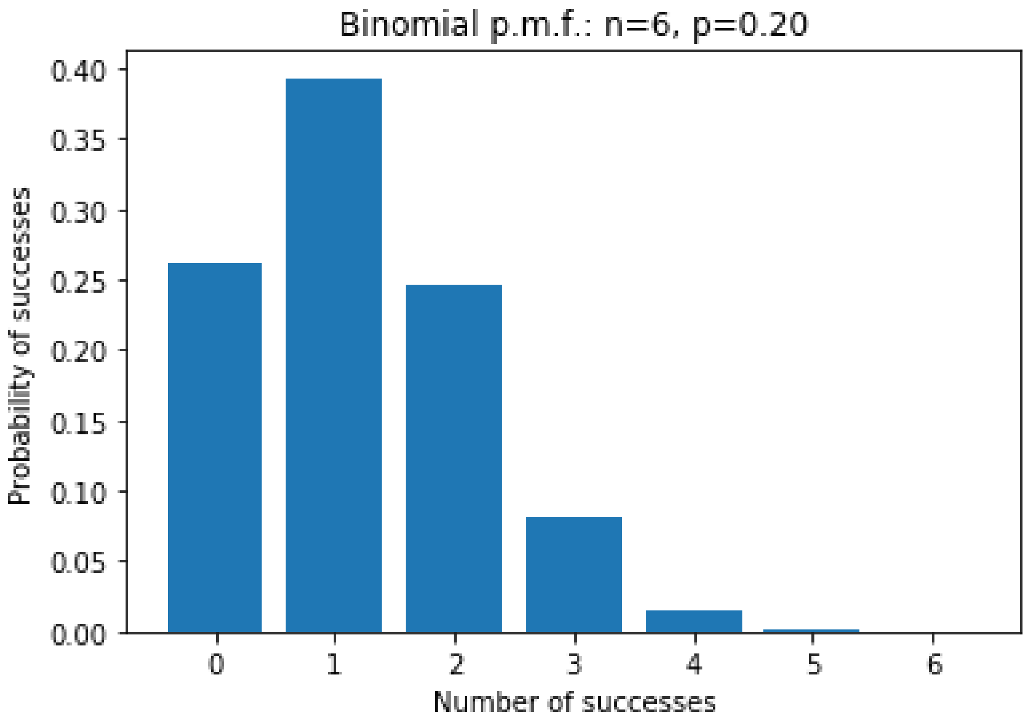
$$\sigma^2 = V(X) = np(1-p)$$

The p.m.f. $X \sim B(n, p)$**:**

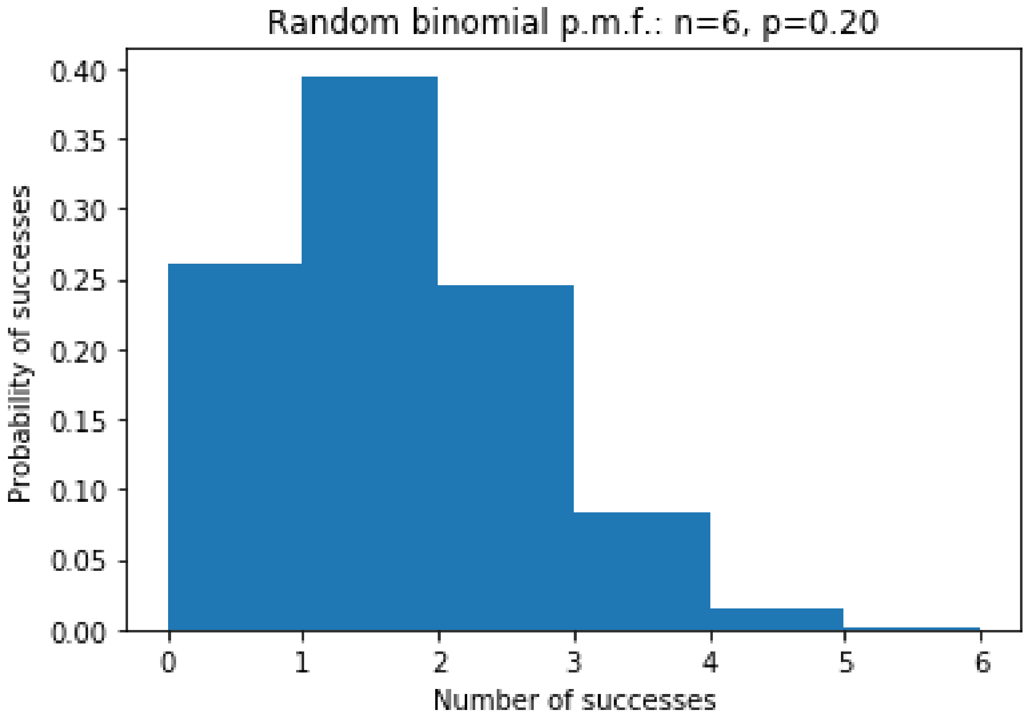
$$p(x) = \binom{n}{x} p^x (1-p)^{n-x} \qquad x = 0, 1, 3, ..., n$$

In [5]:
```
# p.m.f. of binomial distribution

def factorial(n):
    if n == 0:
        return 1
    else:
        return n * factorial(n-1)

# n = total number of Bernoulli trials
# p = probability of positive outcome
# X = number of trials with a positive outcome

def B(n,p ,X):
    return factorial(n) / factorial(X) / factorial (n-X) * p**X * (1-p)**(n-X)
```

In [4]:
```
# graph of p.m.f. of binomial distribution

import scipy, scipy.stats
import matplotlib.pyplot as plt

n = 6                                   # number of trials
p = 0.2                                 # probability of each successfull outcome

x = scipy.linspace(0,n,n+1)
binomial = scipy.stats.binom.pmf(x,n,p)

plt.bar(x, binomial)

plt.xlabel('Number of successes')
plt.ylabel('Probability of successes')
plt.title('Binomial p.m.f.: n=%i, p=%.2f' % (n,p))
plt.show()
```



In [34]:
```
# alternative graph of p.m.f.  of binomial distribution

import scipy, scipy.stats
import numpy as np
import matplotlib.pyplot as plt

n = 6                                # number of trials
p =  0.2                            # probability of each successfull outcome
number_of_runs = 10000               # number of simulation runs

binom_sim = scipy.stats.binom.rvs(n,p,size=number_of_runs)

plt.hist(binom_sim, bins = n, normed = True)

plt.xlabel('Number of successes')
plt.ylabel('Probability of successes')
plt.title('Random binomial p.m.f.: n=%i, p=%.2f' % (n,p))
plt.show()
```



4.3 Geometric distribution, $G(p)$

A geometric distribution is a series n of bernoulli trials until the outcome with probability p is positive.

Examples: - the number of times a fair coin needs to be tossed to obtain 'heads' - the number of times a die needs to be rolled to obtain a 6. - the number of times a tennis player needs to play against his friend, until he wins.

The mean:

$$\mu = E(X) = \frac{1}{p}$$

The variance:

$$\sigma^2 = V(X) = \frac{1-p}{p^2}$$

Dispersion: A geometric distribution is: - under-dispersed when $\dfrac{(1-p)}{p} < 1$ - equi-dispersed when $\dfrac{(1-p)}{p} = 1$ - over-dispersed when $\dfrac{(1-p)}{p} > 1$

The p.m.f. $X \sim G(p)$**:**

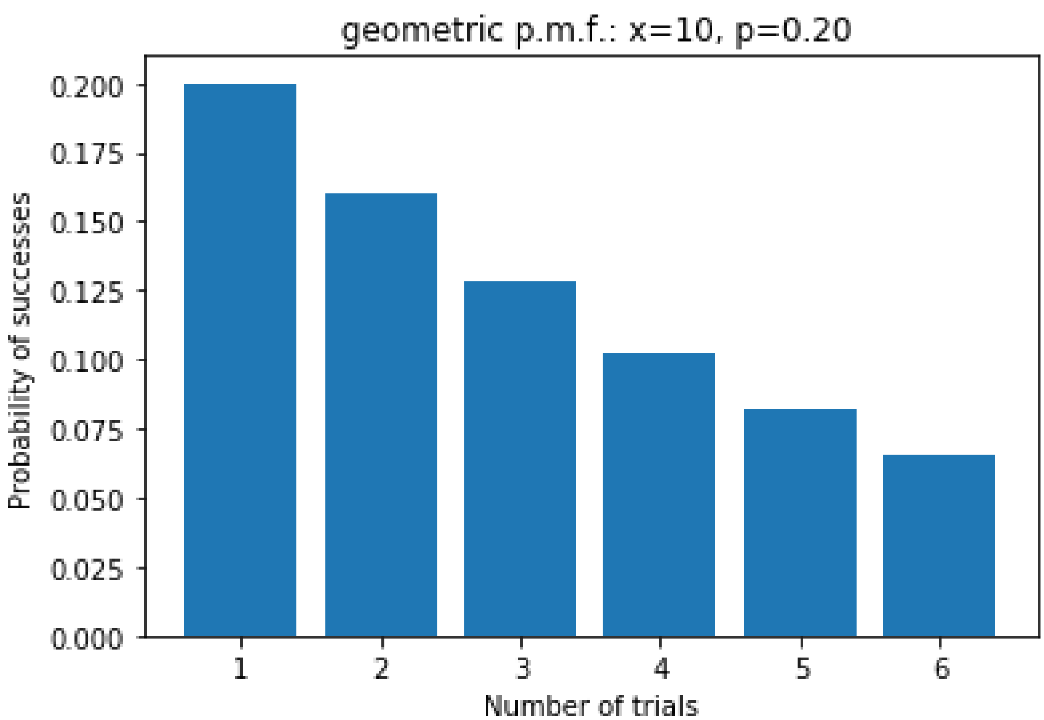
$$p(x) = P(X = x) = (1-p)^{x-1}p, \qquad x = 1, 2, 3, ...$$

The c.d.f. :

$$F(x) = P(X \leq x) = 1 - (1-p)^x, \qquad x = 1, 2, 3, ...$$

In [5]:
```
# geometric p.m.f.

def G(p,x):
    return (1-p)**(x-1) * p
```

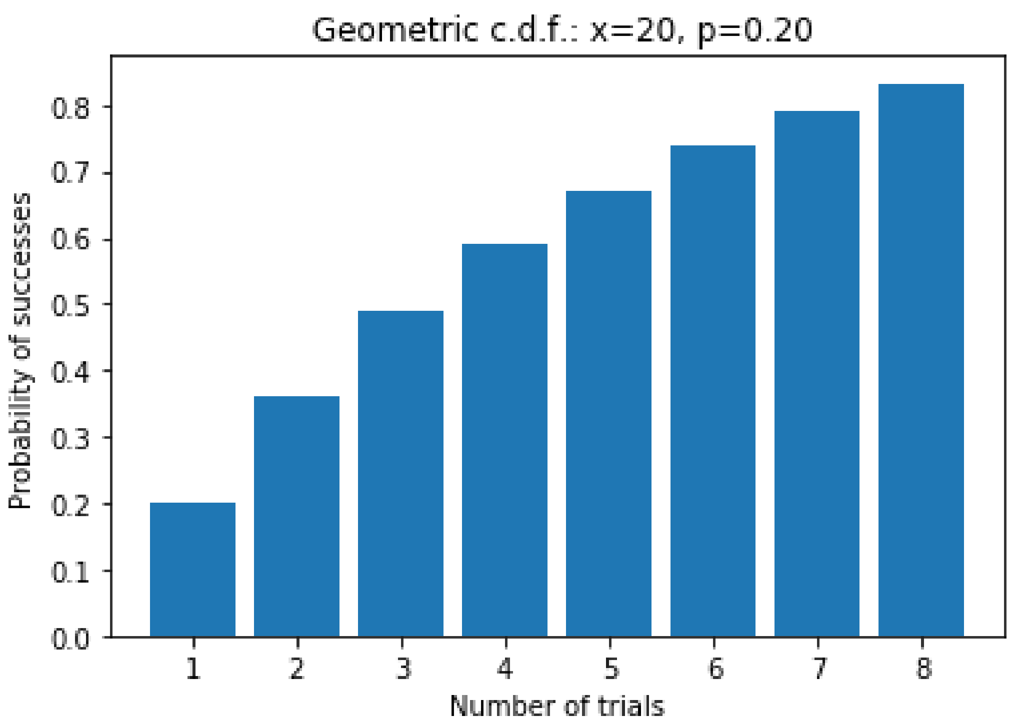
In [121]:
```
# graph of geometric p.m.f.

import scipy, scipy.stats
import matplotlib.pyplot as plt

X = 6                                   # number of trials
p = 0.2                                 # probability of each successfull outcome

x = scipy.linspace(1,X,X)
geometrical = scipy.stats.geom.pmf(x,p)

plt.bar(x, geometrical)

plt.xlabel('Number of trials')
plt.ylabel('Probability of successes')
plt.title(' geometric p.m.f.: x=%i, p=%.2f' % (n,p))
plt.show()
```



In [53]:
```
# geometric c.d.f.
```

```
    def F(p,x):
        return 1 - (1-p)**(x)
```

```
In [90]: # graph of geometric c.d.f.

         import scipy, scipy.stats
         import matplotlib.pyplot as plt

         X = 8                                # number of trials
         p = 0.2                               # probability of each successfull outcome

         x = scipy.linspace(1,X,X)
         geometrical = scipy.stats.geom.cdf(x,p)

         plt.bar(x, geometrical)

         plt.xlabel('Number of trials')
         plt.ylabel('Probability of successes')
         plt.title(' Geometric c.d.f.: x=%i, p=%.2f' % (n,p))
         plt.show()
```



4.4 Poisson distribution, (λ)

Examples: - the number of claims on a motor insurance policy over a certain period - the number of yeast cells found in a randomly chose small square on a microscope slide

The mean:

$$\mu = E(X) = \lambda$$

The variance:

$$\sigma^2 = V(X) = \lambda$$

Comment: whenever it is suggested that the poisson distribution model is to be used to model a data-set, calculate the mean and the variance of the sample data. If they are close to each other, a possion is likely to be a good model to be used.

The p.m.f $X \sim Poisson(\lambda)$**:**

$$p(x) = \frac{e^{-\lambda}\lambda^x}{x!} \qquad x = 0, 1, 2, ...$$

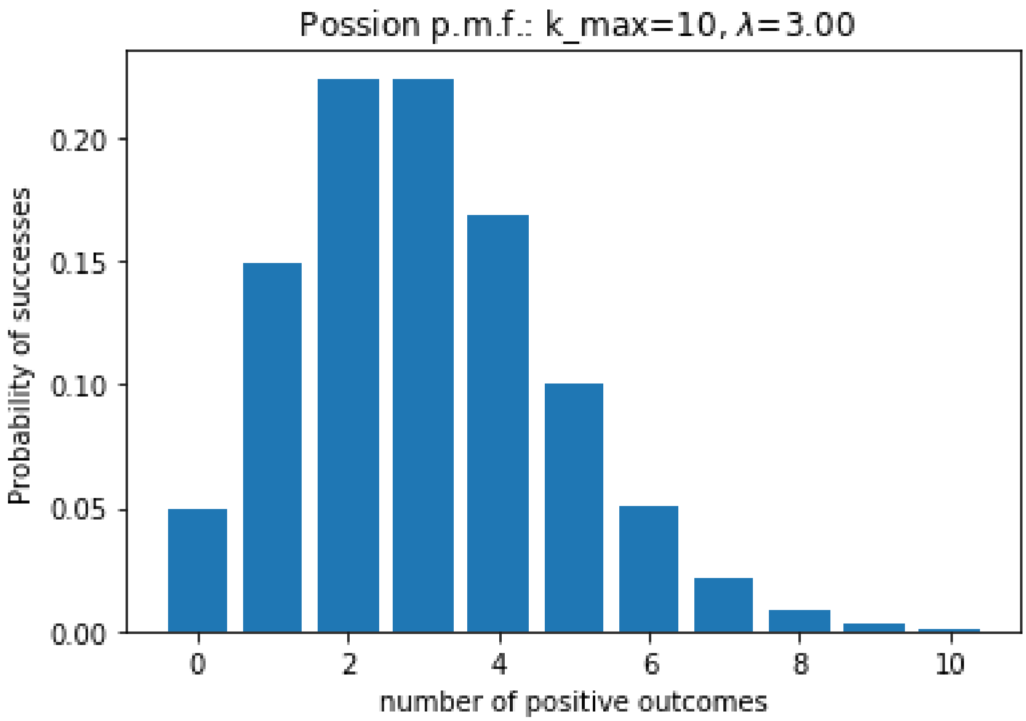
, where

$$\lambda = \frac{n}{t}$$

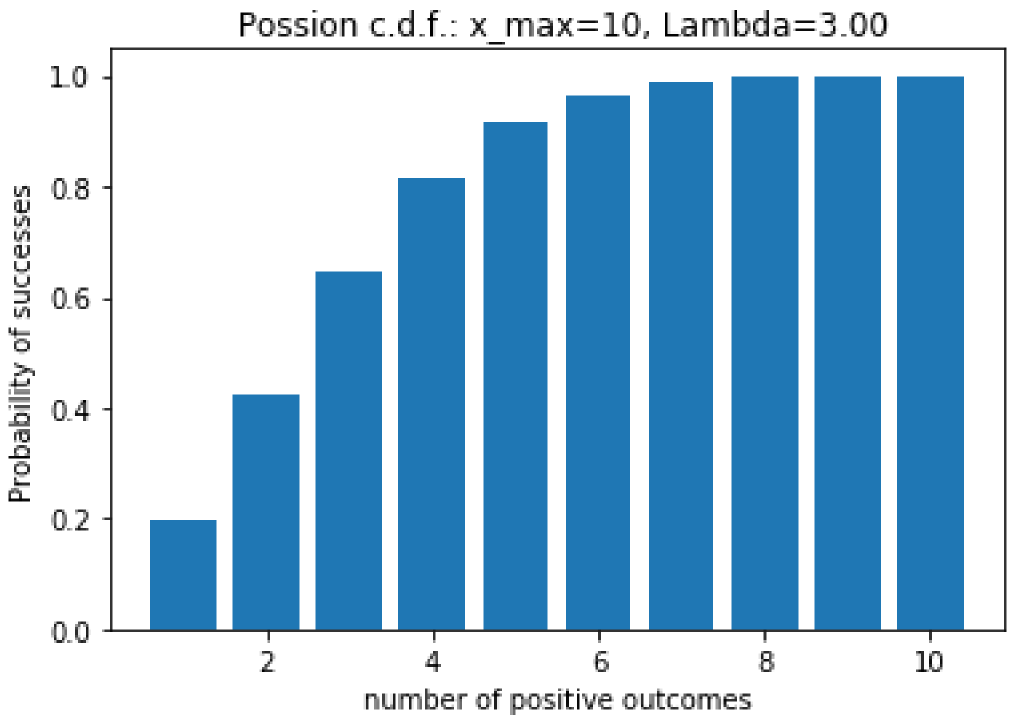
, where n is the number of observed occurances per unit of time/space, and t is the unit of time/space.

```
In [6]: # The Poisson p.m.f.

        import numpy as np

        def factorial(k):
            if k == 0:
                return 1
            else:
                return k * factorial(k-1)

        def P(x,Lambda):
            return (np.exp(-Lambda) * Lambda**x  ) / factorial(x)     # poisson p.m.f.
```

```
In [4]: # graph of poisson p.m.f.:

        import scipy, scipy.stats
        import numpy as np
        import matplotlib.pyplot as plt

        k = 10                                  # maximum number of occurances
        Lambda = 3                              # parameter lambda

        x = np.arange (0,k+1)
        y = scipy.stats.poisson.pmf(x,Lambda)

        plt.bar(x, y)

        plt.xlabel('number of positive outcomes')
        plt.ylabel('Probability of successes')
        plt.title(' Possion p.m.f.: k_max=%i, $\lambda$=%.2f' % (k,Lambda))
        plt.show()
```



In [5]:
```
# graph of poisson c.d.f.:

import scipy, scipy.stats
import matplotlib.pyplot as plt

N = 10                                  # maximum number of occurances
Lambda = 3                              # parameter lambda

x = scipy.linspace(1,N,N)
geometrical = scipy.stats.poisson.cdf(x,Lambda)

plt.bar(x, geometrical)

plt.xlabel('number of positive outcomes')
plt.ylabel('Probability of successes')
plt.title(' Possion c.d.f.: x_max=%i, Lambda=%.2f' % (N,Lambda))
plt.show()
```



4.5 Discrete uniform distribution on $m, m+1, ..., n$

The mean:

$$\mu = E(X) = \frac{m+n}{2}$$

The variance:

$$\sigma^2 = V(X) = \frac{1}{12}(n-m)(n-m+2)$$

The p.m.f.:

$$p(x) = \frac{1}{n-m+1} \qquad x = m, m+1, m+1, ..., n.$$

, where m is the mimimum and n is the maximum attainable value.

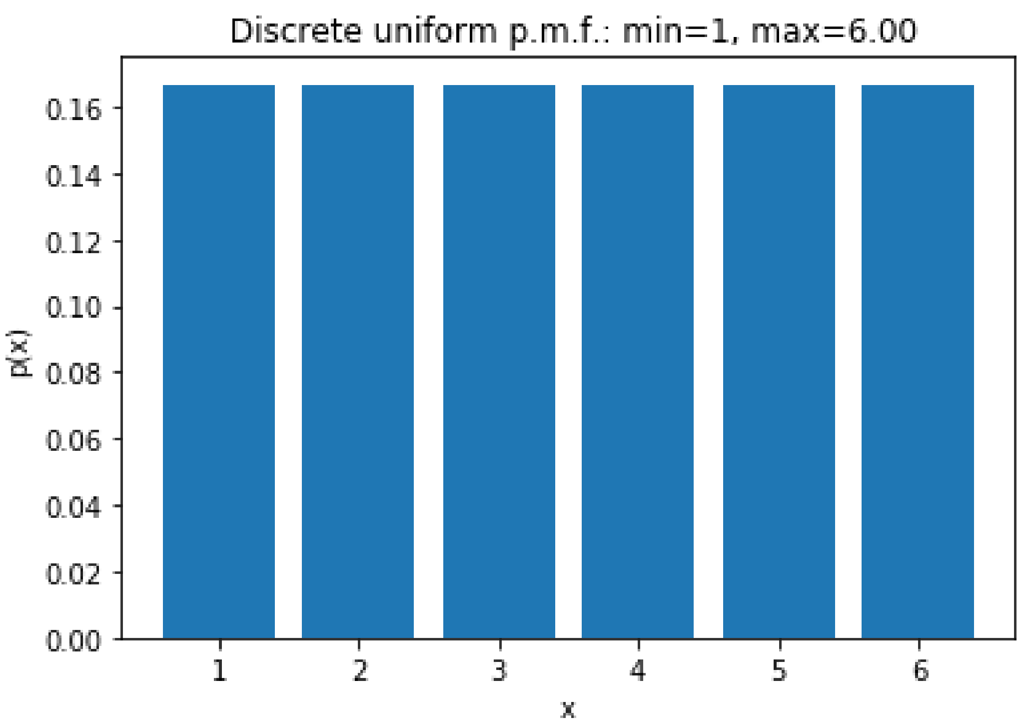
The c.d.f.:

$$F(x) = P(X \leq x) = \frac{x-m+1}{n-m+1} \qquad x = m, m+1, m+2, ..., n.$$

In [1]:
```
# p.m.f. of discrete uniform distribution

def p(m,n,x):
    return 1 / n-m+1
```

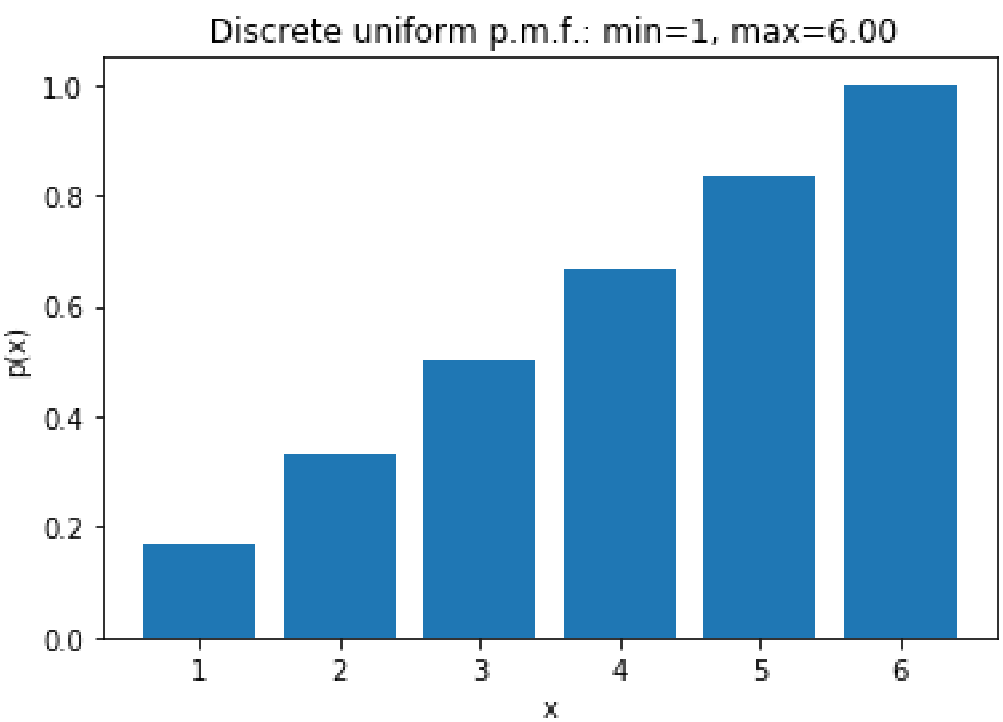
In [173]:
```
# graph for p.m.f. of discrete uniform distribution

import numpy as np
```

```python
import matplotlib.pyplot as plt


def p(m,n):
    return 1 / (n-m+1)

m = 1                                       # minimum attainable value
n = 6                                       # maximum attainable value


x = np.arange (m,n+1)
y = p(m,n)

plt.bar(x, y)

plt.xlabel('x')
plt.ylabel('p(x)')
plt.title(' Discrete uniform p.m.f.: min=%i, max=%.2f' % (m,n))
plt.show()
```




```python
In [2]: # c.d.f. of discrete uniform distribution:
        def F(m,n,x):
            return (x-m+1) / (n-m+1)
```

In [176]:
```python
# graph of c.d.f. of discrete uniform distribution
import numpy as np
import matplotlib.pyplot as plt


def F(m,n):
    return (x-m+1) / (n-m+1)

m = 1                                           # minimum attainable value
n = 6                                           # maximum attainable value


x = np.arange (m,n+1)
y = F(m,n)

plt.bar(x, y)

plt.xlabel('x')
plt.ylabel('p(x)')
plt.title(' Discrete uniform p.m.f.: min=%i, max=%.2f' % (m,n))
plt.show()
```



5 Continuous Probability Distributions

The mean of a continuous distribution with p.d.f. $f(x)$:

$$\mu = E(X) = \int x f(x) dx$$

The variance of a continuous random variable with p.d.f. $f(x)$:

$$\sigma^2 = V(X) = E[(X-\mu)^2] = \int (x-\mu)^2 f(x) dx$$

also:

$$\sigma^2 = V(X) = E(X^2) - \mu^2$$

("the variance equals the mean of the squares minus the square of the mean")

5.1 Continuous uniform distribution, $U(a, b)$

The mean:

$$\mu = E(X) = \frac{a+b}{2}$$

The variance:

$$\sigma^2 = V(X) = \frac{1}{12}(b-a)^2$$

The p.d.f. $X \sim U(a, b)$:

$$f(x) = \frac{1}{b-a}, \qquad a < x < b$$

The c.d.f. of $X \sim U(a, b)$:

$$F(x) = P(X \leq x) = \frac{x-a}{b-a}, \qquad a < x < b$$

5.2 Standard uniform distribution

The p.d.f. of the the standard uniform distribution $V \sim U(0, 1)$:

$$f(v) = 1, \quad 0 < v < 1$$

The C.d.f. of the the standard uniform distribution $V \sim U(0, 1)$:

$$F(v) = v, \quad 0 < v < 1$$

```
In [ ]:
```

5.3 Exponential distribution, $M(\lambda)$

A random variable X has an **exponential distribution** with parameter λ, where $\lambda > 0$, if it has:

Examples: -

The mean:

$$\mu = E(X) = \frac{1}{\lambda}$$

The variance:

$$\sigma^2 = V(X) = \frac{1}{\lambda^2}$$

The standard deviation:

$$\sigma = \sqrt{V(X)} = \frac{1}{\lambda}$$

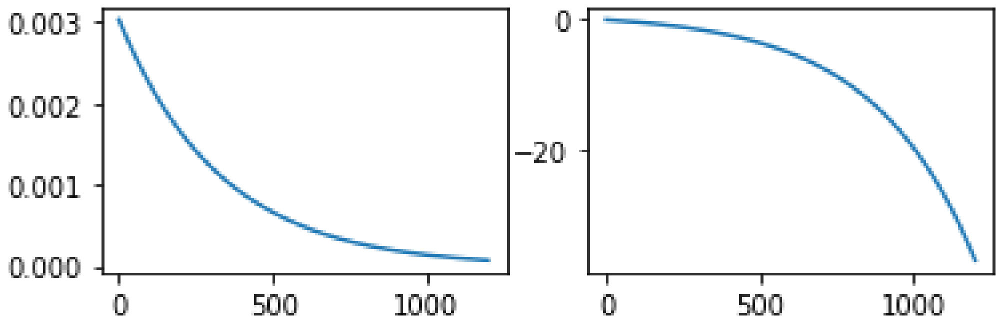
The p.d.f. $X \sim M(\lambda)$**:**

$$f(x) = \lambda e^{-\lambda x}, \qquad x > 0$$

The c.d.f.:

$$F(x) = 1 - e^{\lambda x}, \qquad x > 0$$

In [19]:
```
import numpy as np
import matplotlib.pyplot as plt

Lambda  = 67/22120
x = np.linspace(0, 1200, 1000)


# p.d.f. of eponential distribution
def exponential_pdf(Lambda,x):
        return Lambda * np.exp(-Lambda * x)


# c.d.f. of exponential distribution
def exponential_cdf(Lambda,x):
    return 1 - np.exp(Lambda * x)



plt.figure()
plt.subplot(2,2,1); plt.plot(x, exponential_pdf(Lambda, x))
plt.subplot (2,2,2); plt.plot(x, exponential_cdf(Lambda, x))                # Upper r

plt.show()
```




```
In [ ]: import scipy, scipy.stats
        import numpy as np
        import matplotlib.pyplot as plt

        n = 6                                 # number of trials
        p =  0.2                             # probability of each successfull outcome
        number_of_runs = 10000                # number of simulation runs

        binom_sim = scipy.stats.expon

        plt.hist(binom_sim, bins = n, normed = True)

        plt.xlabel('Number of successes')
        plt.ylabel('Probability of successes')
        plt.title('Random binomial p.m.f.: n=%i, p=%.2f' % (n,p))
        plt.show()
```

6 Linear regression

```
In [20]: import numpy as np
         import pandas as pd
         import matplotlib.pyplot as plt
         data = pd.read_csv("cholesterol.csv", header = [0])

         # the data
         N = data['Age'].count()
         response = data[["Cholesterol"]]
         predictor = data[["Age"]]


         # Create data points
         #N = 10
         #x = np.random.rand(N)
         #y = np.random.rand(N)
```

```python
# desired probability for confidence interval
probability = 0.95

# Regression function
xbar = np.mean(predictor)
ybar = np.mean(response)
Sxx = np.sum((predictor - xbar) ** 2)
Sxy = np.sum((predictor - xbar) * (response - ybar))
beta_hat = Sxy / Sxx
alpha_hat = ybar - beta_hat * xbar

# Least square line
x_values = np.linspace(min(x_values), max(x_values), 100)
y_values = alpha_hat + beta_hat * x_values
print(' ')
print('The least square line is given by:')
print('y = '+ str(alpha_hat) +' + ' + str(beta_hat) +' x')

# students t-value for two sided confidence interval
from scipy.stats import t
p = 0.5 + (probability / 2) # probability converted to lookup t-value
df = N - 2                  # degrees of freedom
t_value = t.ppf(p, df)

# Fitted values:
y_hat = alpha_hat + beta_hat * predictor

# Residuals:
w = response - y_hat

# unbiased estimator of variance:
variance  = (np.sum(y - y_hat)) / (N - 2)

# confidence interval for slope of beta_hat:
beta_lower = beta_hat - t_value * (np.sqrt(variance))/(np.sqrt(Sxx))
beta_upper = beta_hat + t_value * (np.sqrt(variance))/(np.sqrt(Sxx))
y_values_lower = alpha_hat + beta_lower * x_values
y_values_upper = alpha_hat + beta_upper * x_values


# Linear regression Plot
plt.scatter(predictor, response)
plt.plot(x_values, y_values,'r')
# plt.plot(x_values, y_values_lower, 'y')
# plt.plot(x_values, y_values_upper, 'y')
plt.title('Linear regression model')
plt.xlabel('Predictor')
```

```
plt.ylabel('Response')
ax = plt.gca()
lines = ax.get_lines()
lines[0].set_label('Least square line')
#lines[1].set_label('Confidence interval')
ax.legend()
plt.show()


# Resiual plot
plt.scatter(y_hat, w )
plt.title('Residual plot')
plt.axhline(0, color='black')
plt.xlabel('Fitted value $\hat{y}$')
plt.ylabel('Residuals $w$')
ax.legend()
plt.show()

# Normal probability plot of the residuals
from scipy import stats
w = stats.norm.rvs(loc=0, scale=1, size=N)
stats.probplot(w, plot=plt, dist='norm')
plt.show()

# unbiased estimator of the variance:
variance_estimate  = np.sum(response - y_hat)/( N - 2)
print('an unbiased estimator of the variacne is ' + str(variance_estimate))
```

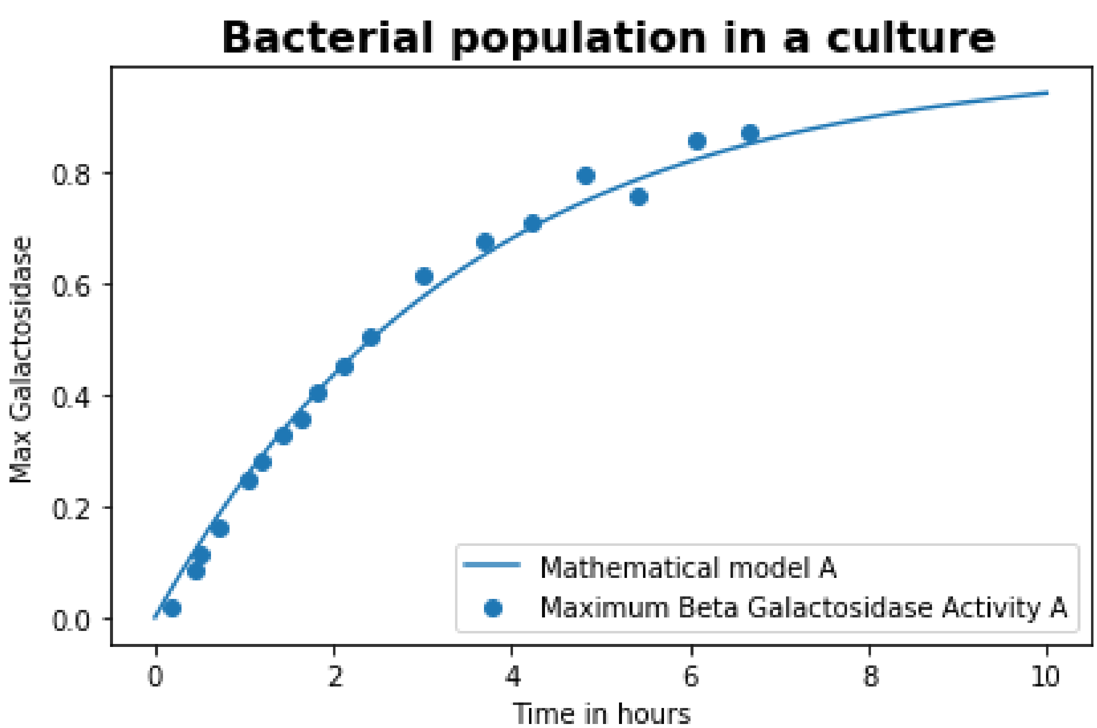
```
---------------------------------------------------------------------------

NameError                                 Traceback (most recent call last)

<ipython-input-20-140fd3031a5e> in <module>
     27
     28 # Least square line
---> 29 x_values = np.linspace(min(x_values), max(x_values), 100)
     30 y_values = alpha_hat + beta_hat * x_values
     31 print(' ')


NameError: name 'x_values' is not defined
```

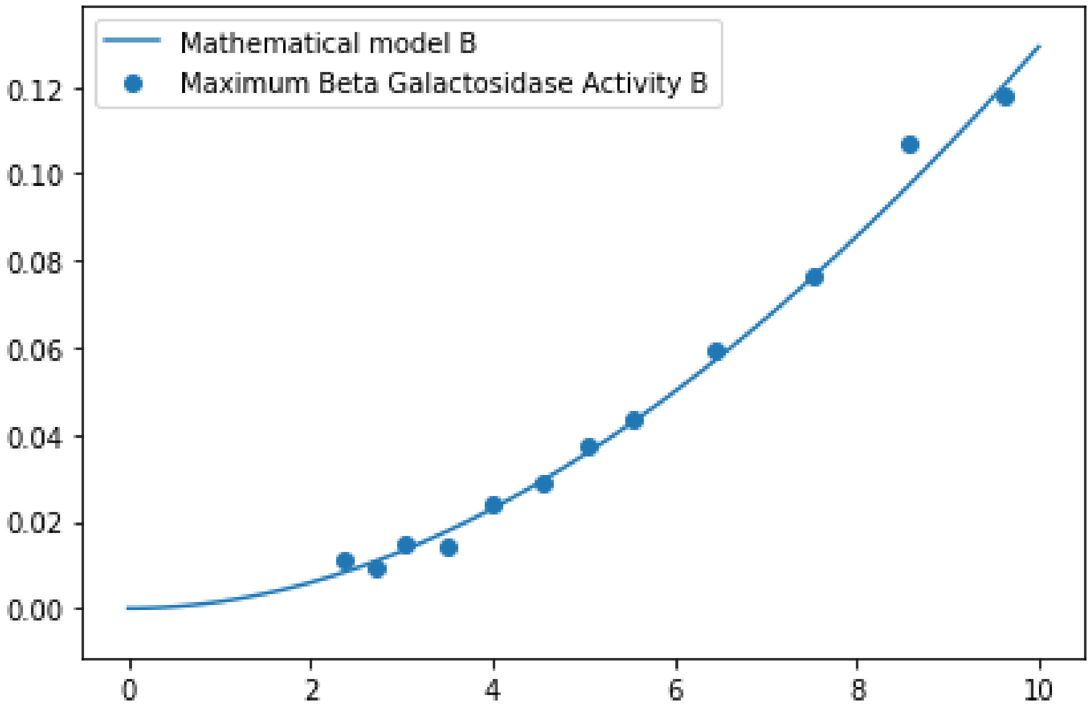
```
In [ ]: xbar
```

6.1 Fitting experimental data to a model

In [21]:
```
#!/usr/bin/env python3
# -*- coding: utf-8 -*-
"""
Created on Wed Feb 28 20:01:56 2018

Purpose: Exercise of fitting a mathematical model onto experimental data from two fil


@author: stephangoldberg
"""

import numpy as np
import matplotlib.pyplot as plt


# importing the datasets (each set has 2 columns)
data_set_A = np.loadtxt('g149novickA.csv', delimiter=',')
data_set_B = np.loadtxt('g149novickB.csv', delimiter=',')


# choosig from data_set_B only the entries where the first column is smaller than 10:
B = data_set_B[(data_set_B[:,0:1]<10).all(1)]

# slicing the two data sets(separating the columns):
time_in_hours_A = data_set_A[:, 0]  #first column
max_beta_galactosidase_activity_A = data_set_A[:, 1] # second column

time_in_hours_B = B[:, 0] #first column
max_beta_galactosidase_activity_B = B[:, 1] #second column

# creating a mathematical model, which represents the data
# Define parameters:
A = 1
tau_A = 3.5
tau_B = 18

# Defining the range of x-values for both models
time_model = np.linspace (0,10,101)

# Defining the y-values (These are the formulas for the models)
V_t = 1 - np.exp(- time_model / tau_A)
W_t = A * (np.exp(-time_model/tau_B) - 1 + time_model / tau_B)

# plotting the actual data and the model:
plt.scatter(time_in_hours_A, max_beta_galactosidase_activity_A, label = "Maximum Beta
plt.plot(time_model, V_t, label = "Mathematical model A")
```

```
ax = plt.gca()       # getting control of the axis
ax.set_xlabel("Time in hours")
ax.set_ylabel ("Max Galactosidase", size=10)
ax.set_title( " Bacterial population in a culture", size=16, weight="bold")
plt.legend()         # showing the labels of the plotted curves
plt.tight_layout()  # ensures that labels of x- or y-axis are not cut off
plt.figure()    # plotting the first figure with data_set_A

plt.scatter(time_in_hours_B, max_beta_galactosidase_activity_B, label = "Maximum Beta
plt.plot(time_model, W_t, label = "Mathematical model B")
ax.set_xlabel("Time in hours")
ax.set_ylabel ("Max Galactosidase", size=10)
ax.set_title( " Bacterial population in a culture", size=16, weight="bold")
plt.legend()         # showing the labels of the plotted curves
plt.tight_layout()  # ensures that labels of x- or y-axis are not cut off
plt.figure()    #plotting the second figure with data_set_B



plt.show()   # showing the plot
```





<Figure size 432x288 with 0 Axes>

In [ ]: