Let us proceed with the pool hopping as follows. In the experiment, there will be only two pools that form the whole BitCoin network: pool\_1 with power  $p^*$  and pool\_2 with power  $1 - p^*$ . Try small values for pool\_1, e.g.  $p^* = 0.2$ , for instance (therefore, power of pool\_2 will be 0.8). Assume, that there is only one attacking miner with power  $p < p^*$  (total power of pool\_1 includes the power of that miner), and, it is very important that the miner starts in pool\_1 at the beginning of every mining round. The miner spends time  $\alpha\Delta$ , in pool\_1 and time  $(1 - \alpha)\Delta$  in pool\_2, where  $\alpha = \frac{1+p^*}{2}$ , time interval (or number of cycles)  $\Delta$  should be quite small, but if you use cycles,  $\alpha\Delta$  and  $(1 - \alpha)\Delta$  should be quite small integer numbers, like 3 and 2, for example. After mining in pool\_2 miner returns to pool\_1 and spends there the same time as he did previously, and, so on until the full solution is found. That should work and the miner should get reward greater than p (in long run).

While shifting periodically between pool\_1 and pool\_2, in the long run miner allocates power  $\alpha p$  and  $(1 - \alpha)p$  for pool\_1 and pool\_2, respectively. This is true for any sufficiently long time. Therefore, miner should receive extra profit not only under Proportional reward scheme, but under Pay Per Last N shares too. You can try this as well, or, we can discuss it next time.