Embodied Constructive Mathematics for Immersive Analytics

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CHI 2020 Extended Abstracts, April 25–30, 2020, Honolulu, HI, USA.

Abstract
Taking inspiration from the nature of abstractions in scientific diagrams and the constructionist learning philosophy, we envision a future of immersive analytics that hinges on mathematical exploration capabilities using explainable frameworks and personalized artifacts. We discuss the need for redefining basic mathematical operations from an embodied and constructionist perspective, operations that underpin the construction of more complex analytics.

Author Keywords
Immersive Analytics; Embodied Interactions; Constructionism; Diagrams; Mathematics.

CSS Concepts
•Human-centered computing~Interaction design~Interaction design theory, concepts and paradigms •Human-centered computing~Visualization~Visualization theory, concepts and paradigms

Introduction
"In current professional definitions, physicists think about how to do physics, educators think about how to teach it. There is no recognized place for people whose research is really physics, but physics oriented in

directions that will be educationally meaningful.” -- Seymour Papert. Mindstorms (1980).

The constructionist philosophy of learning [8], which was Seymour Papert's influential work, relies on representations that are personalized and meaningful to the learner. The essence of the above quote from his book Mindstorms can be summed up in a simple diagram (Figure 1).

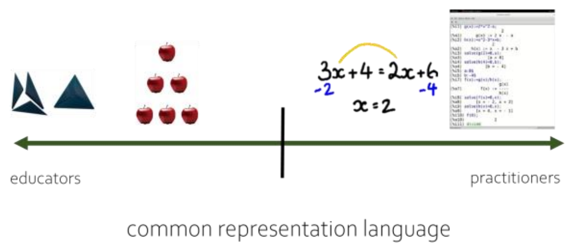

Figure 1. In terms of representations used to convey symbols and concepts, educators and scientists/practitioners often use different ones. Educators tend to use real world objects, embodied representations, and interactions to convey ideas, practitioners use abstract symbolic representations and associated software. Seymour Papert talked about powerful representations that are useful for both ends of the spectrum.

The lack of people Papert talks about stems from his discussions in the book on the lack of explainable, personalized representations to play and construct knowledge. Immersive analytics and visualization open up an important opportunity to create and manipulate abstractions in terms of embodied experiences, using everyday objects to create and understand analytics and associated mathematical operations. Other than education and learning, such embodied representations has been shown to be useful in personal analytics [9] using objects of one’s interest.

It is however, hard to define a computational language and syntax where embodied experiences can become useful for abstract symbol manipulation. There are many efforts in the information visualization and HCI community that gear towards designing new primitives [7] and also tangible, embodied interactions [1,3,10] to create new analytics and presentation systems. The designs of these systems could incorporate lessons from scientific diagrams.

As a possible way to integrate design lessons, we can look into standard scientific diagrams – the quintessential element of pre- mixed reality, pen and paper era – and some of the common design principles behind some widely used diagrams, to start thinking about constructive frameworks for immersive analytics.

We discuss three common features of the chosen diagrams: ladder of abstractions, ease of moving between the abstractions, and compositionality. We believe discussions around these features will contribute to the future directions of immersive analytics frameworks.

Other Diagram Studies

There are several other diagram studies that are noteworthy. Diagrams involving time and categories [4], discrete or continuous data [2], space, enumeration, and symbolic abstraction [6] etc. are studied previously. We add to the discussion from the perspective of lack of embodied interactions in scientific diagrams, and at the same time how immersive analytics could also be aided by the three features commonly present in widely used scientific diagrams. The three features found in this study and the

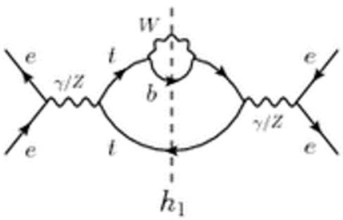

Figure 2. A Feynman Diagram, widely used in particle physics.

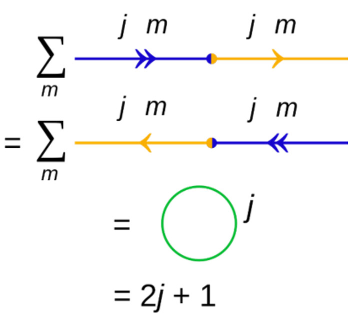

Figure 3. Angular Momentum diagram used in Quantum Mechanics.

constructionist perspective, to our knowledge, have not been explored in any other diagrams studies.

Choice of Diagrams

We have studied a total of 24 diagrams from different scientific disciplines and mathematics. We chose the diagrams by consulting with a number of physicists, mathematicians, engineers, and chemists. Examples of each field include: Physics: Feynman diagram, Hertzsprung-Russel diagram, Free body/kinematics diagram etc. Mathematics: commutative diagrams, Hasse diagram etc. Engineering: circuit diagrams, Mohr circles etc. Chemistry: Lewis Structure diagram, Grotrian diagram etc. Due to space limitations, we withhold details of each diagram, and present the three common features by using some example diagrams.

Ladder of Abstractions

Feynman Diagrams (Figure 2) are widely used in particle physics, and are perfect examples of moving between abstraction layers. These diagrams describe interaction between particles. Each edge and vertex of the diagram represents equations that are very long symbolic expressions, inaccessible to physicists, and would require painstaking symbol pushing to simplify and work, in the pre computer algebra systems days. These diagrams are made from edges and vertices that represent the symbolic expressions and let a scientist work in the diagram space.

A somewhat different diagram is the angular momentum diagram (Figure 3) where the symbolic abstraction is not completely hidden. A user still works in symbolic expression level, but instead of regular symbols we typically see in equations, visual markers are used to make the equations more explainable and intuitive. Commutative diagrams (Figure 6) use a similar kind of abstraction.

Another kind of visual abstraction mostly uses words and picture based symbols (Penrose diagrams, Figures 4 and 5). These also represent physical processes (different cosmology models), but the usual equation based abstraction (including alphabet based symbols) is not its focus. Because of that, this diagram is arguably more readable compared to the other two.

Regardless of the kind of abstraction, layers of abstractions are present in all of these chosen diagrams.

- Each element in the diagrams represent either a set of symbolic expressions, physical process, or a physical object itself (e.g., circuit diagrams representing resistors and capacitors).
- Each unit diagram may create further layers of abstraction by composing more complex structures. Penrose diagrams or circuit diagrams (components within electronic components) are prime examples of this kind of abstraction.

It is to be noted that we use the term “ladder of abstraction” in a somewhat separate sense than works like Bret Victor’s ladder of abstraction [12] and explainable mathematics [11]. Our aim is to show the abstraction layers commonly used in diagrams in the daily workflow of scientists and engineers. Our agenda overlaps with [12] in terms of retaining explainability in each step of the ladder.

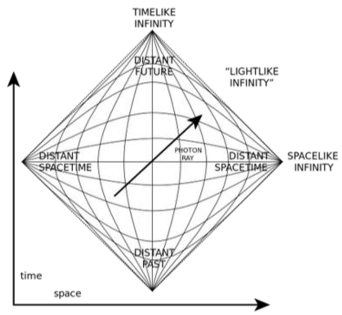

Figure 4. Penrose Diagram, a unit element.

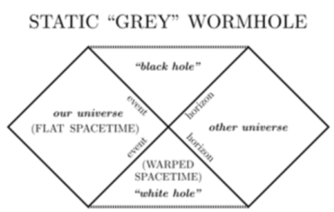

Figure 5. A Penrose Diagram, with multiple units composed according to the rules of the diagram.

Ease of Moving Along the Ladder

In standard scientific/engineering workflow, the diagrams described so far all tend to follow a similar goal: making the symbol pushing or symbol interpretation burden easier by using a diagrammatic representation of the process. It is to be noted that often these diagrams serve the purpose of interpretation only. In other words, they are used as the equivalent of plotting data to understand the data, and not necessarily working in the plot space to generate further insights. Thus, moving along an abstraction ladder may not serve further purpose other than interpretation.

Often times, these are computationally expensive processes, and making them interactive possesses significant challenge.

Directionality

Most of the diagrams we have studied are uni-directional in their nature of abstraction, because they are mostly used for interpretation purpose. A truly bi-directional abstraction mechanism would provide a way (either mathematically or by visual means) to move between the symbolic and visual space seamlessly. Feynman Diagrams or Penrose diagrams can be called bi-directional in this sense, because it is possible to compute a Feynman or Penrose diagram, and go back to the changed symbolic representations if a change in the visual representation is made. This partly depends on the compositionality aspect of the diagram design.

Compositionality

Some diagrams like Feynman, Penrose or circuit diagrams have specific rules/grammar defined for constructing them, so the composition of more complex structures are possible without resorting to more symbol pushing and symbol manipulation. The compositionality of any iconic or symbolic structure, in a linguistics sense, is the ability to form consistent meaning for complex structures composed from simpler unit elements for whom we have established definition and meaning. It is in fact very important to be able to define a grammar for manipulating visual primitives that make up these diagrams, informed by the underlying mathematics or science, in order to truly utilize the ladder of abstractions. In scientific communities, traditionally such grammar were developed by the scientists based on their practical needs, and these grammars may not always take account of HCI and information visualization research.

Connections to Constructionism and Embodied Interactions

The three characteristics of scientific diagrams detailed so far have important takeaways for immersive analytics, in how they contribute to the design of embodied and constructionist properties of immersive manipulation of virtual or real elements.

Papert's constructionism based learning philosophy hypothesizes that we construct knowledge by free play with objects that we care about (defined as "personalized artifacts"). In works like constructive visualization [5], researchers have applied such a constructionist approach for non-experts to build visualizations from manipulable, physical building blocks. Immersive analytics with AR and VR gives us immense opportunities for embodied mathematical operations such as counting, grouping, filtering datasets by gestures etc.

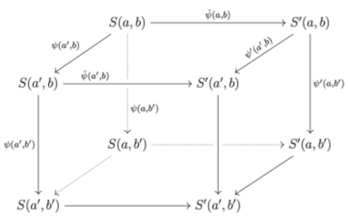

Figure 6. Commutative Diagrams used in category theory.

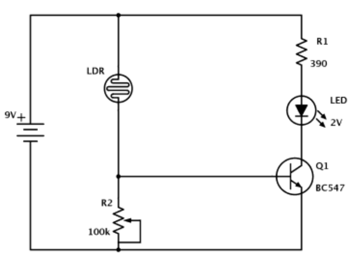

Figure 7. A typical circuit diagram.

Following the need for common powerful representations as Papert suggested, which retain explainable and explorable aspects of difficult concepts, the community can discuss and focus on designing such representations by taking lessons from the scientific diagrams designed for the pen and paper era.

Use of Ladder of Abstractions

Most ladders of abstractions in scientific diagrams go from an obscure, symbolic space to more interpretable, visual space. In immersive analytics, a true constructionist approach will be to start with the real world physical objects, and model abstractions on top of them, which is the opposite direction of traditional abstractions. This is not to say that the traditional abstraction direction, i.e., going from symbolic expressions or numbers to more interpretable and understandable visual forms is to be discarded in immersive analytics, but mixed reality opens up new opportunities for exploring the bi-directionality of abstraction forms typically present in mathematics, engineering, and sciences, which are not usually explored bi-directionally. This method of thinking should prove useful for scenarios where live modeling on top of physical structures are useful. Example applications include factory machines or tools modeling, doing analytics on top of moving objects, cars, people etc. The abstractions may not be symbolic expressions, but are visual diagrams. Yet, the construction of such primitives may start from the embodied experience, as opposed to starting from numbers and other symbols.

It is to be noted that traditional systems such as Wolfram Mathematica or GeoGebra can move between symbolic and graphical layers. However, their top-down abstraction design approach (start with symbolic, and then move to graphical) do not present favorable ways to construct abstract mathematics by manipulating real world objects. We advocate for bottom-up construction using gestures, sketching, and other actions that are ubiquitous in embodied interactions and immersive environments.

Compositionality

In order to work with such bi-directional abstractions, we may need to define some fundamental mathematical operations as embodied interactions, either through gestures or by drawing/sketching. Such operations will then facilitate a grammar for composing more complex diagrams or abstraction, using the unit abstractions present in a scene.

For every traditional diagram that has the capacity of composing more complex structures from unit primitives (Feynman, Penrose etc.), it has a specific set of grammar derived from the basic principles of the underlying mathematics. In the case of immersive analytics, the community may need to rethink some basic mathematical operations in order to build any set of abstractions constructively on top of a scene. Depending on the application and context, these could be as simple as grouping and filtering, to attributing algebraic operators to designated actions and gestures on manipulable objects.

Embodied Constructive Mathematics

Following these connections to constructionism and embodied experiences, we propose that a framework be explored for fundamental mathematical operations in immersive analytics that build up abstractions by letting the user work on manipulable real world objects first. We define any such framework as *embodied* *constructive mathematics.* This retains clarity and explainability of the construction process better. For specific applications, such a framework for basic mathematical operations might change around. However, it is important that we think about such bottom-up construction approach instead of assuming a top-down knowledge (symbolic to visual/graphical) of visualization and analytics for every user.

Such frameworks may also prove to be useful to redefine the traditional scientific diagrams, and may spark discussions for redesigning the fundamental approach towards doing mathematics and sciences, which tend to have mostly top-down direction of abstractions. For example, what would a pipeline for constructing Feynman diagrams look like where one starts with tangible and understandable elements of the underlying physics, such as physical or virtual models of electrons and muons, and then through grammars defined for each layer of abstraction, construct the current symbolic or graphical representations of Feynman diagrams gradually in immersive environments?

Challenges

Some real challenges are involved when building abstractions constructively on top of real world objects in augmented reality. Dynamic video in-painting algorithms maybe required to show layers of abstraction or hide such layers, as the layers may involve real world objects and artifacts. On top of that, as discussed, moving between abstraction layers maybe computationally very expensive for real-time interactions on a standard GPU used in mixed reality headsets.

Despite the challenges involved, which are mostly hardware performance issues or rely on advancement in computer vision algorithms, we believe that immersive analytics frameworks that explore ladders of abstractions, ease of movement along the ladder, and compositionality (i.e., expressive and flexible grammar) in each step of the ladder based on embodied interactions have a lot of potential in creating intuitive, explainable insights.

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