EE120 SIGNALS AND SYSTEMS Fall 2019

Homework #1

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1. a) proof: $x*h = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau$ and $h*x = \int_{-\infty}^{+\infty} h(\tau)x(t-\tau)d\tau$

let $s=t-\tau \Rightarrow \tau = t-s$ then we have the conculusion that

$h*x = \int_{-\infty}^{+\infty} h(t-s)x(s)ds = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau = x*h$

b) proof: $x*(h_1+h_2) = \int_{-\infty}^{+\infty} x(\tau)[h_1(t-\tau)+h_2(t-\tau)]d\tau$

$= \int_{-\infty}^{+\infty} x(\tau)h_1(t-\tau)d\tau + \int_{-\infty}^{+\infty} x(\tau)h_2(t-\tau)d\tau$

$= x*h_1 + x*h_2$

c) proof: $x*(h_1*h_2) = \int_{-\infty}^{+\infty} x(\tau)(h_1*h_2)(t-\tau)d\tau = \int_{-\infty}^{+\infty} x(\tau)\int_{-\infty}^{+\infty} h_1(s)h_2(t-\tau-s)ds$

$= \int_{-\infty}^{+\infty} x(\tau)h_1(s)h_2(t-\tau-s)d\tau ds$ and $(x*h_1)*h_2 = \int_{-\infty}^{+\infty}(x*h_1)(s)h_2(t-s)ds$

$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} x(\tau)h_1(s-\tau)h_2(t-s)d\tau ds$ let $s-\tau=p$ we have $s=\tau+p$ so

Original $= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} x(\tau)h_1(p)h_2(t-p-\tau)d\tau dp$. they equal to each other.

2. a) $h[n] = u[n]-u[n-10] = \sum_{k=0}^{9}\delta[n-k]$ and $\sum_{k=0}^{9}|\delta[n-k]| = 10 < \infty$

so it is stable, and for $h[n]$, all the related components are less than n.

so it is causal and a FIR system

b) $h[n] = 2^n u[n]$, which is a stable. causal IIR system

c) $h[n] = 2^n u[-n]$, which is a stable, not causal, IIR system

d) $h[n] = n(0.8)^n u[n]$, which is a stable, causal, IIR system

3. a) if signals $x[n], y[n]$ satisfy the equation. we have when $\hat{x}[n]=\lambda x[n]$, $\hat{y}[n]=\lambda y[n]$

assume: $\lambda y[n] + \lambda a_1 y[n-1] + \cdots + a_N\lambda y[n-N] = \lambda b_0 x[n] + \cdots + \lambda b_M x[n-M]$

① when $\lambda \neq 0$, both side devided λ, we gain the condition happen in the test.

② when $\lambda = 0$, obviously, it satisfy.

In the above, $\hat{x}[n]=\lambda x[n]$, $\hat{y}[n]=\lambda y[n]$ also satisfy the equation.

b) if $x_1[n]$, $y_1[n]$, and $x_2[n]$, $y_2[n]$ satisfy the difference equation. we have

$y_1[n] + a_1 y_1[n-1] + \cdots + a_N y_1[n-N] = b_0 x_1[n] + \cdots + b_M x_1[n-M]$ ①

$y_2[n] + a_2 y_2[n-1] + \cdots + a_N y_2[n-N] = b_0 x_2[n] + \cdots + b_M x_2[n-M]$ ②

add the equation ① and ②, then let $\hat{x}[n] = (x_1+x_2)[n]$, $\hat{y}[n] = (y_1+y_2)[n]$.

we have $\hat{y}[n] + a_1\hat{y}[n-1] + \cdots + a_N \hat{y}[n-N] = b_0 \hat{x}[n] + \cdots + b_M \hat{x}[n-M]$

c) if $x[n]$, $y[n]$ satisfy the difference equation, then let $k = n-L$, we have

$y[k] + a_1 y[k-1] + \cdots + a_N y[k-N] = b_0 x[k] + \cdots + b_M x[k-M]$ for k is any

and then substitute it. we can verify the $\hat{x}[n] = x[n-L]$, $\hat{y}[n] = y[n-L]$ also satisf

d) when $x[n]$ is unit impulse, $x[n] = \delta[n]$. we have $y[n] = y[n-1] + \delta[n]$

$\Rightarrow y[n] = \sum_{k=0}^{n} \delta[n-k] + 1$ while when $x[n] = 2\delta[n]$. we have $y[n] = y[n-1] + 2\delta[n]$

$\Rightarrow y'[n] = 2\sum_{k=0}^{n} \delta[n-k] + 1$ but it should have the linear property.

$2x[n] \rightarrow 2y[n] = 2\sum_{k=0}^{n} \delta[n-k] + 2 \neq y'[n]$, so the response is not scaled by the same constant.

(e) when $y[n] = \begin{cases} 0, & n<0 \\ y[n-1]+x[n], & n \geq 0 \end{cases}$ From the above analysis, we can easily verify its linearity and time-invariant

4. a) let $x[n] = \delta[n]$, we have $y[n] = h[n] = 0.25\,\delta[n-1] + 0.5\,\delta[n] + 0.25\,\delta[n+1]$

b) $H(e^{j\omega}) = \sum_{k=-\infty}^{+\infty} h[k] e^{j\omega n} = \sum_{n=-\infty}^{+\infty} [0.25\,\delta[n-1] + 0.5\,\delta[n] + 0.25\,\delta[n+1]]\, e^{j\omega n}$

$= 0.25 e^{j\omega} + 0.25 e^{-j\omega} + 0.5 = 0.5\cos\omega + 0.5$

$\Rightarrow H(e^{j\omega}) = 0.5(1+\cos\omega) = \cos^2\frac{\omega}{2}$

c) the type of filter is low-pass.

5 a) let $x(t) = e^{j\omega t}$, $y(t) = H(j\omega) e^{j\omega t}$ so substitute them and we have

$RC\frac{dy(t)}{dt} + y(t) = x(t) \Rightarrow (RC\,j\omega + 1) H(j\omega) e^{j\omega t} = e^{j\omega t} \Rightarrow H(j\omega) = \frac{1}{1+RCj\omega}$

b) $H(j\omega) = \frac{1}{1+R^2C^2\omega^2}(1 - jRC\omega)$, when $\omega\uparrow$, $|H(j\omega)|\downarrow$ confirm a low-pass