EE 120: Signals and Systems
Bonus: Diff. Equations
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In this class, we've encountered a lot of ways to represent an LTI System, such as:

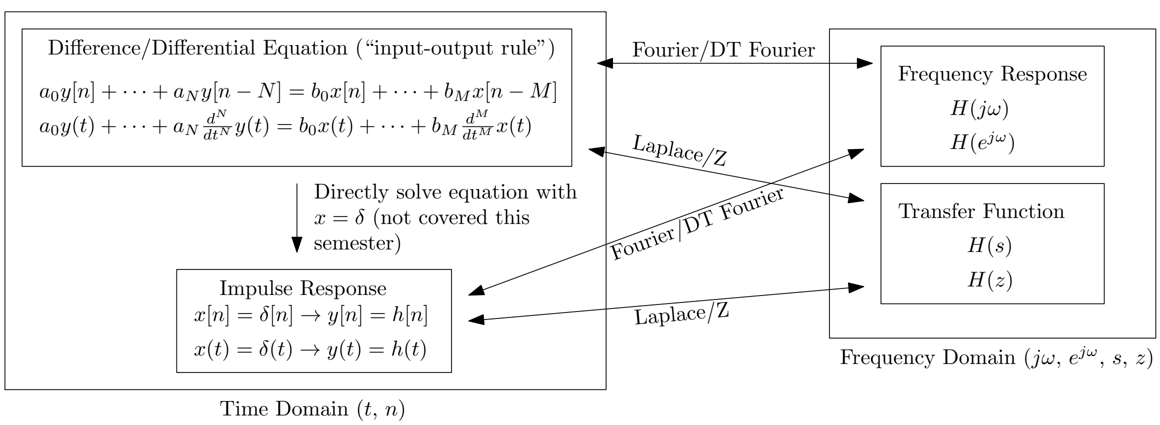
1. An *input-output rule* in the form of a linear differential equation (in continuous time) or difference equation (in discrete time) with constant coefficients;

2. An *impulse response*, a function of time which yields the output of the system when convolved with the input;

3. A *frequency response*, a rational function of an imaginary variable $j\omega$ (in continuous time) or $e^{j\omega}$ (in discrete time), which gives the output of the system when the input is a complex exponential;

4. A *transfer function*, a rational function of a complex variable s (in continuous time) or z (in discrete time) whose *poles* characterize the system behavior, especially its stability.

A system given in one form may be expressed in any other others using the transforms we have learned. This diagram shows how the representations and the transforms are related:



To go from the difference/differential equation representation to either of the frequency-domain transformations, you can just use the appropriate transform.

Going from the difference/differential equation to the impulse response is a little more tricky. There are two approaches. First, you can directly solve the equation with $x = \delta$ and using the method of homogeneous and particular solutions. On the other hand, You could first find the transfer function of the system using the Laplace transform or Z transform, solve for Y when $X = 1$, and take the inverse Laplace transform of Y to find h. Since we didn't really cover the direct method this semester, we recommend the Laplace/Z method.

Problem 1: [1] Consider an LTI system defined by the difference equation

$$y[n] = -x[n] - 2x[n-1] - x[n-2].$$

a) Determine the impulse response of this system.

b) Determine if the system is causal and/or stable.

c) Find the frequency response $H(e^{j\omega})$.

d) Determine the response of this system when the input is $x[n] = 1 + (-1)^n$. Your answer should be of the form $y[n] = a + b(-1)^n$.

[1] Adapted from EE120 Midterm 1, Spring 2013

Problem 2: Consider an LTI system defined by the differential equation

$$\frac{d^2}{dt^2}y(t) + 3\frac{d}{dt}y(t) + 2y(t) = x(t). \tag{1}$$

a) Find the impulse response and transfer function of this system.

b) Find the response of this system to $x(t) = e^t u(t)$.

Problem 3: [2] A system has the transfer function

$$H(s) = 4 + \frac{2s+3}{s^2+6s+9}.$$

1. What is the system's impulse response?

2. Draw a block diagram of this system as a parallel interconnection of two subsystems.

3. Is this system stable?

[2] Adapted from EE120 Final, fall 1993. This problem is old enough to drink!

Problem 4: [3] Consider an LTI system given defined by the frequency response

$$H(e^{j\omega}) = \frac{1 - 1.25e^{-j\omega}}{1 - 0.8e^{-j\omega}}.$$

a) Find a difference equation that represents the system defined by $H(e^{j\omega})$.

b) Determine the impulse response $h[n]$.

[3] Adapted from EE120 Midterm 1, Spring 2013

Problem 5: [4] Consider an LTI system whose response to the *step function* $u(t)$ is

$$s(t) = (1 - \tfrac{1}{2}e^{-t} + \tfrac{1}{2}e^{-2t})u(t).$$

Find its transfer function and impulse response.

[4] Adapted from EE120 Final, Spring 2000.