Year 1 – Relativity
Lecture 5

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Overview of lectures

- Lecture 1: Introduction, concepts and classical results
- Lecture 2: The postulates of Relativity
- Lecture 3: Length contraction and simultaneity
- Lecture 4: The Lorentz transformations
- Lecture 5: Space-time diagrams and world lines
- Lecture 6: Four-vectors and causality
- Lecture 7: Energy and momentum
- Lecture 8: Rest mass energy and particle decays
- Lecture 9: Particle reactions
- Lecture 10: The relativistic Doppler effect

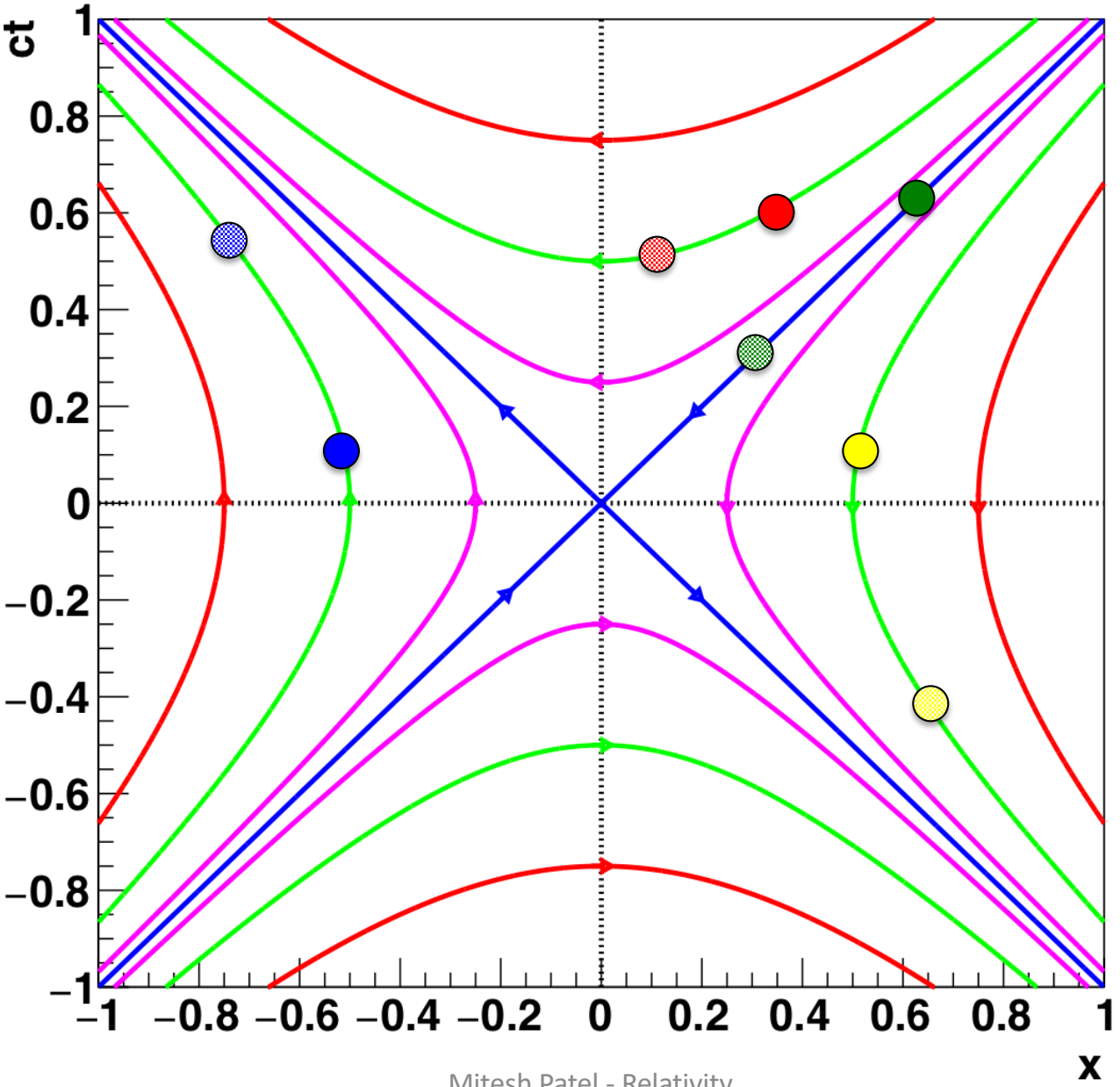
Previously on Relativity

- Saw the Lorentz transformations
  - Mathematically similar to rotations
  - Work with space-time positions = “events”
  - LTs move an event from (ct, x) to (ct’, x’)
  - $ct' = \gamma(ct - \beta x) \qquad x' = \gamma(x - \beta ct)$
- Derived the velocity transformation
  - Formula: $u' = (u-v)/(1-uv/c^2)$ where
  - u = speed of object in frame 1
  - v = relative speed of frame 2 in frame 1
  - u’ = speed of object in frame 2

What we will do today

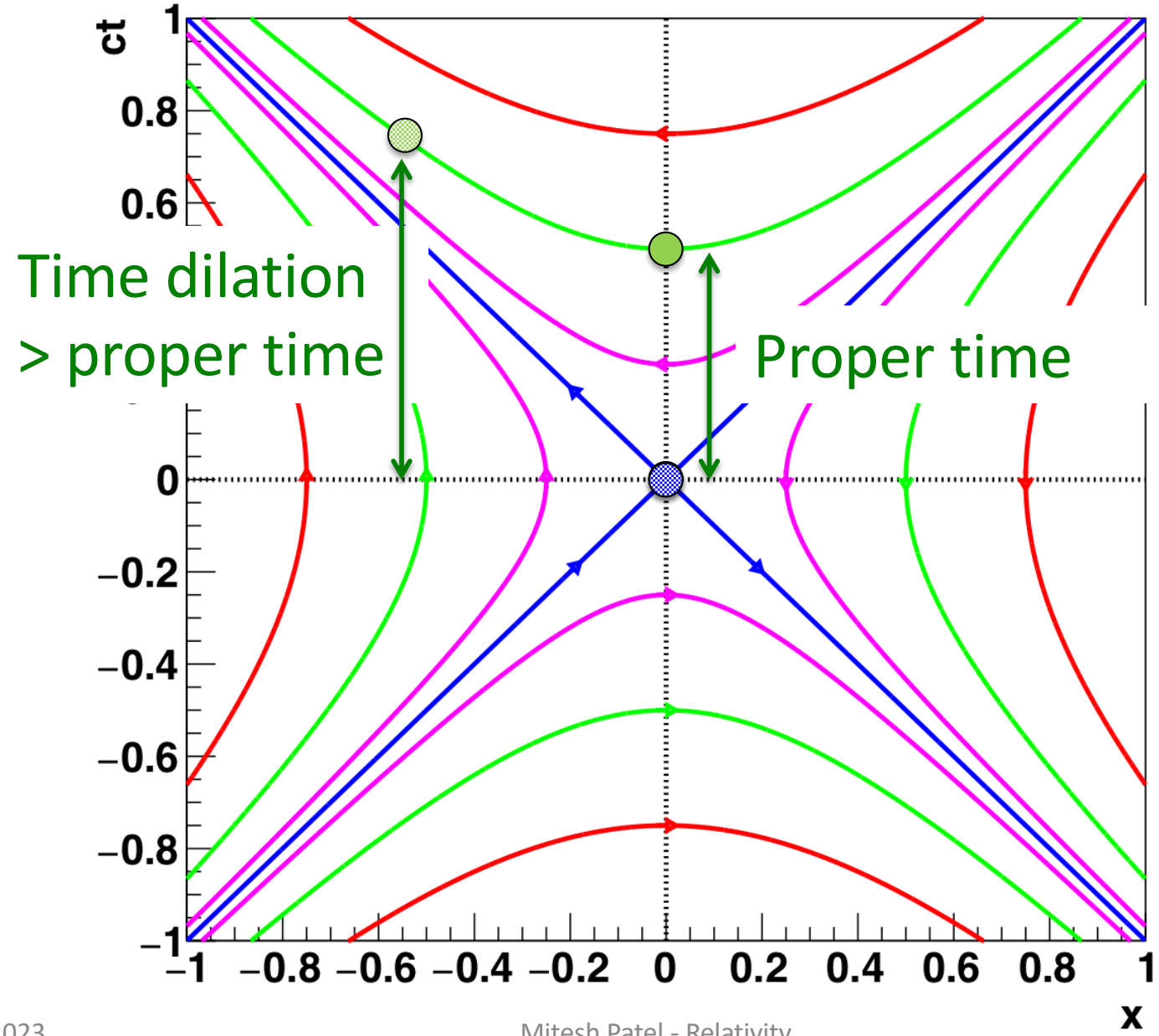
- Study space-time diagrams
  - A way to represent events and how they transform
- Understand world lines
  - These give a graphical picture of an object trajectory in a space-time diagram
- See how world lines change under Lorentz transformations
  - World lines change in position and gradient

LTs of events in space-time

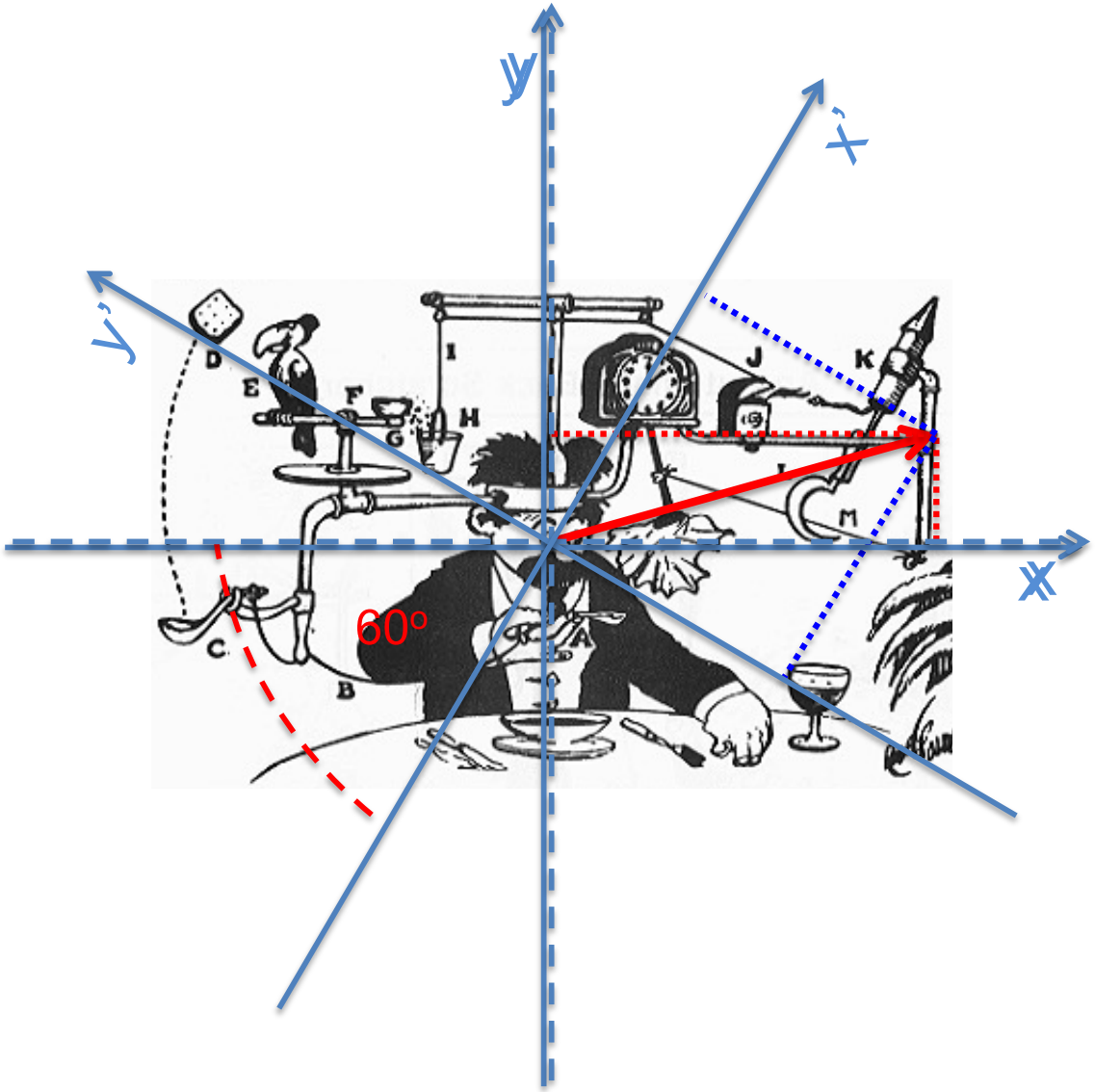


Frame 2

Two events at same position

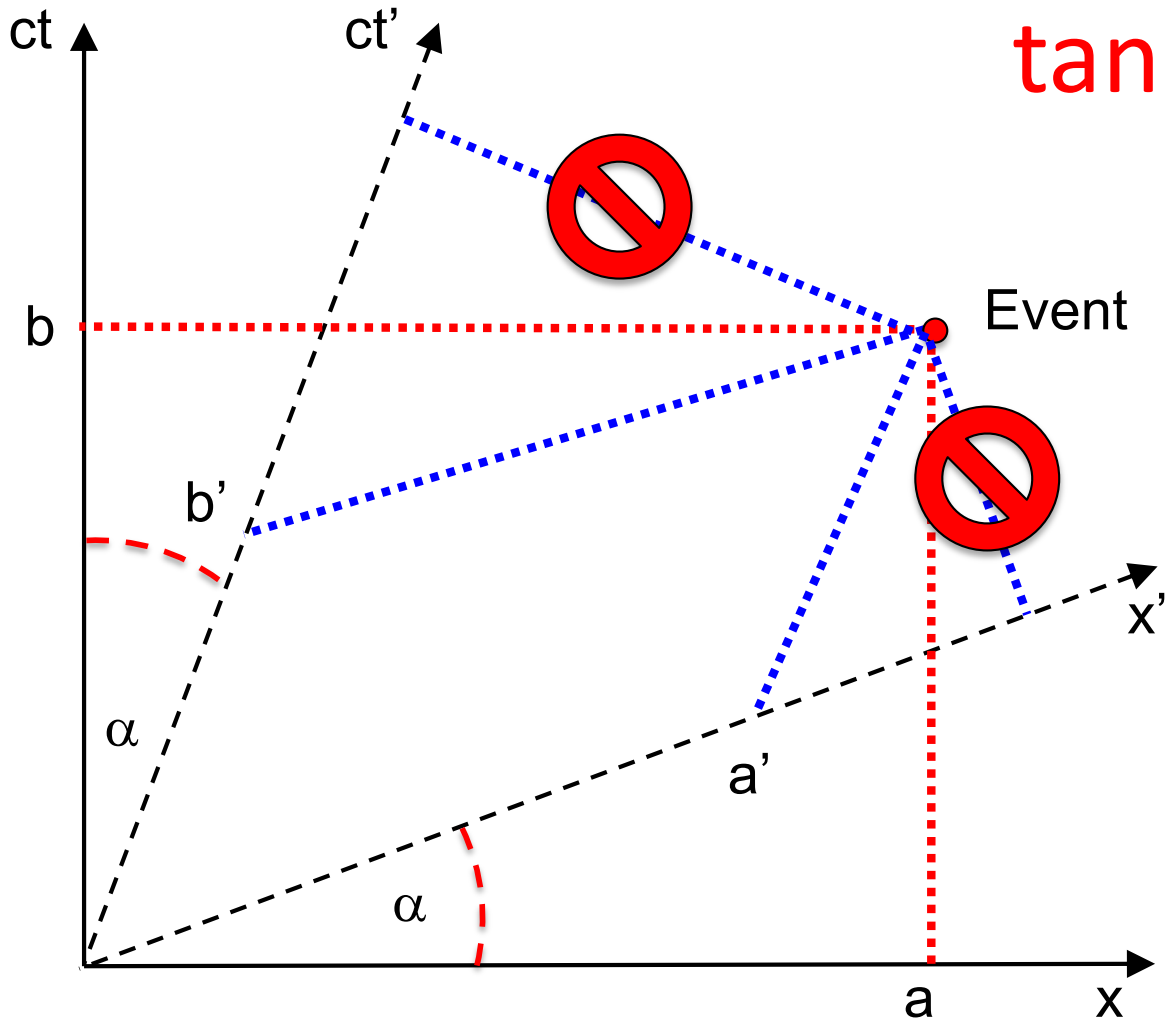


Passive rotations



Changing the ct and x axes under LT

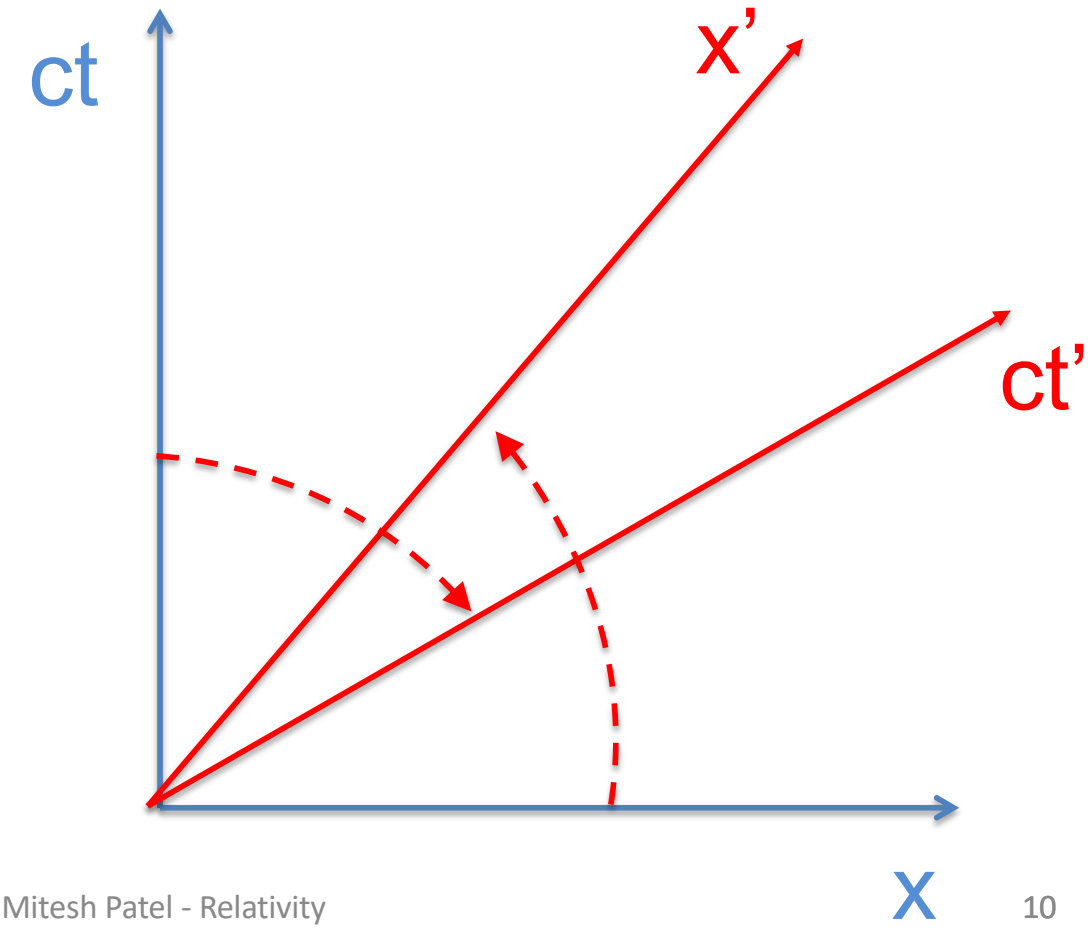
Passive Lorentz transformations



$$\tan \alpha = \beta$$

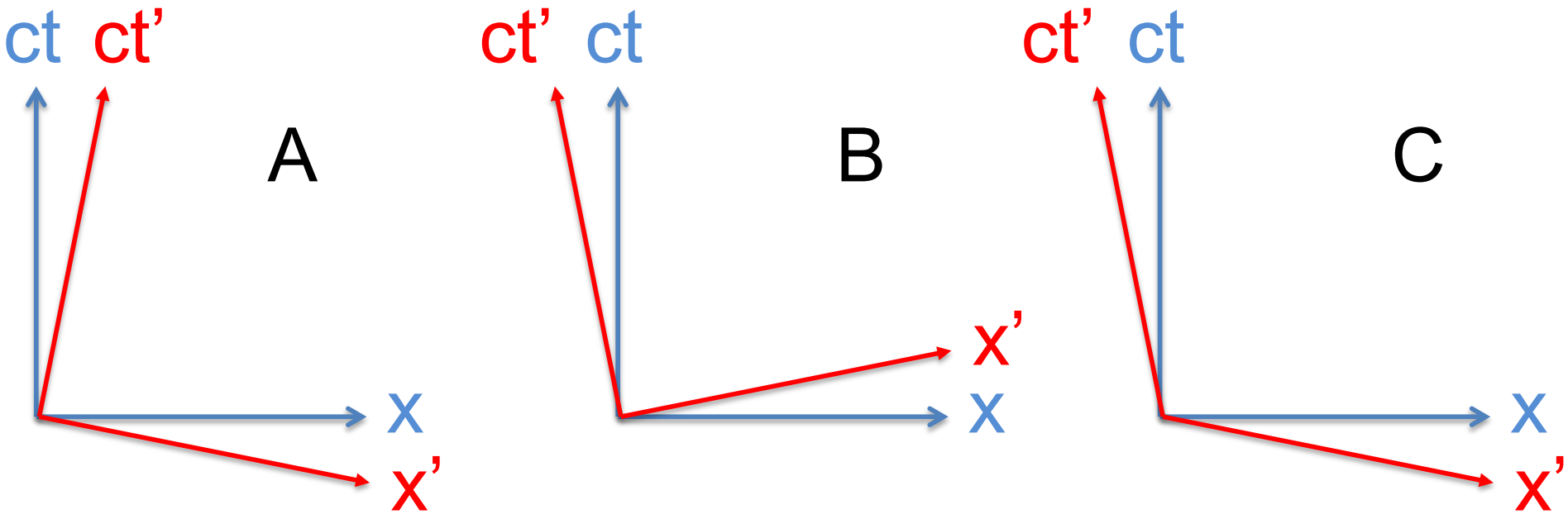
Can the axes swap, i.e. $\alpha > 45^{o}$?

- Go to www.menti.com
- Question 1



What does an LT by $-\beta$ look like?

- Go to www.menti.com
- Question 2



Can the axes swap, i.e. $\alpha > 45^{o}$?

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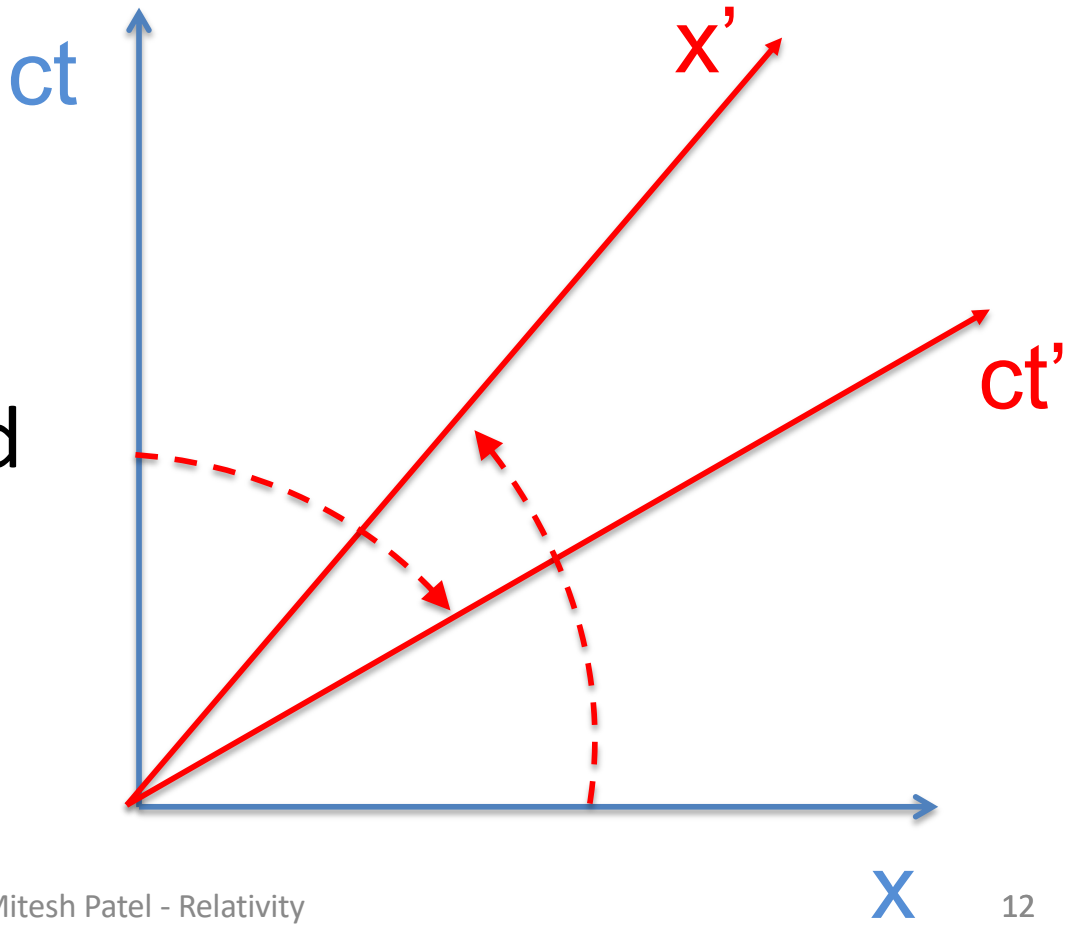
- Answer: False
- Tan(45°)=1 so more than 45° would need $\beta>1$, i.e. v>c



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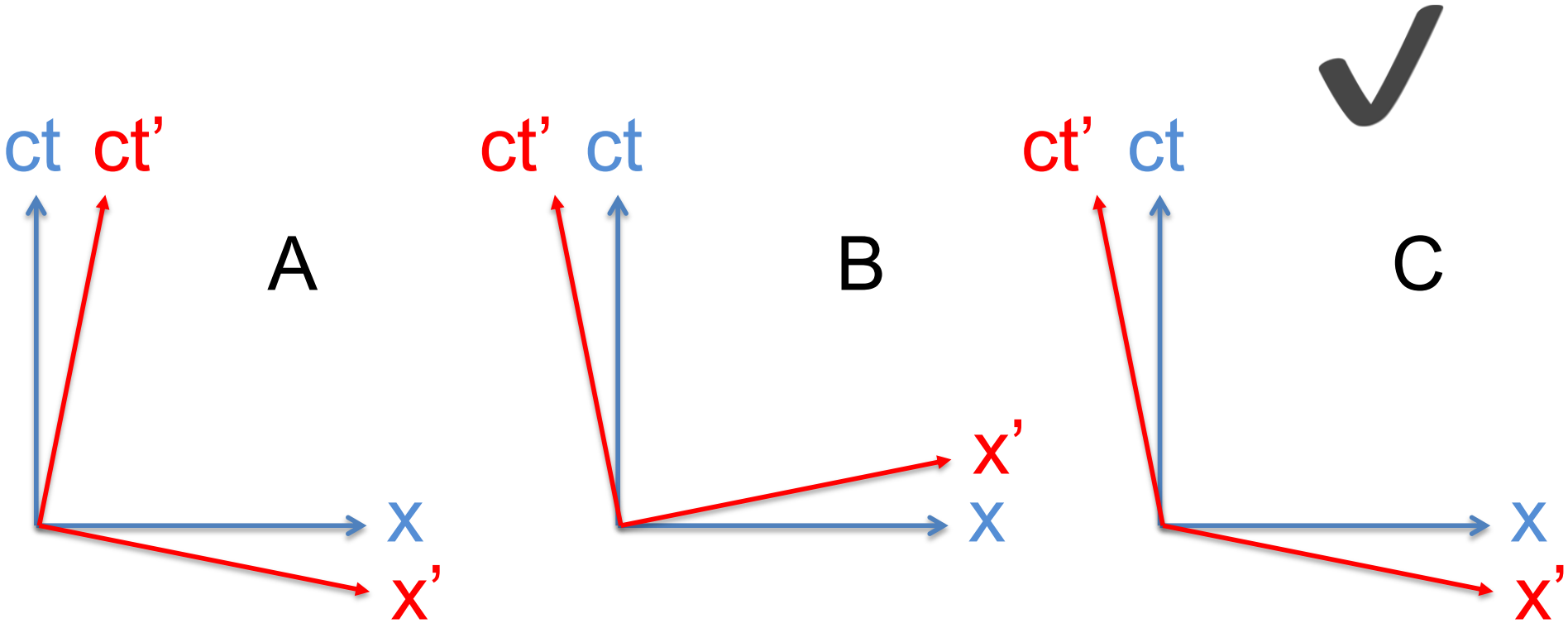
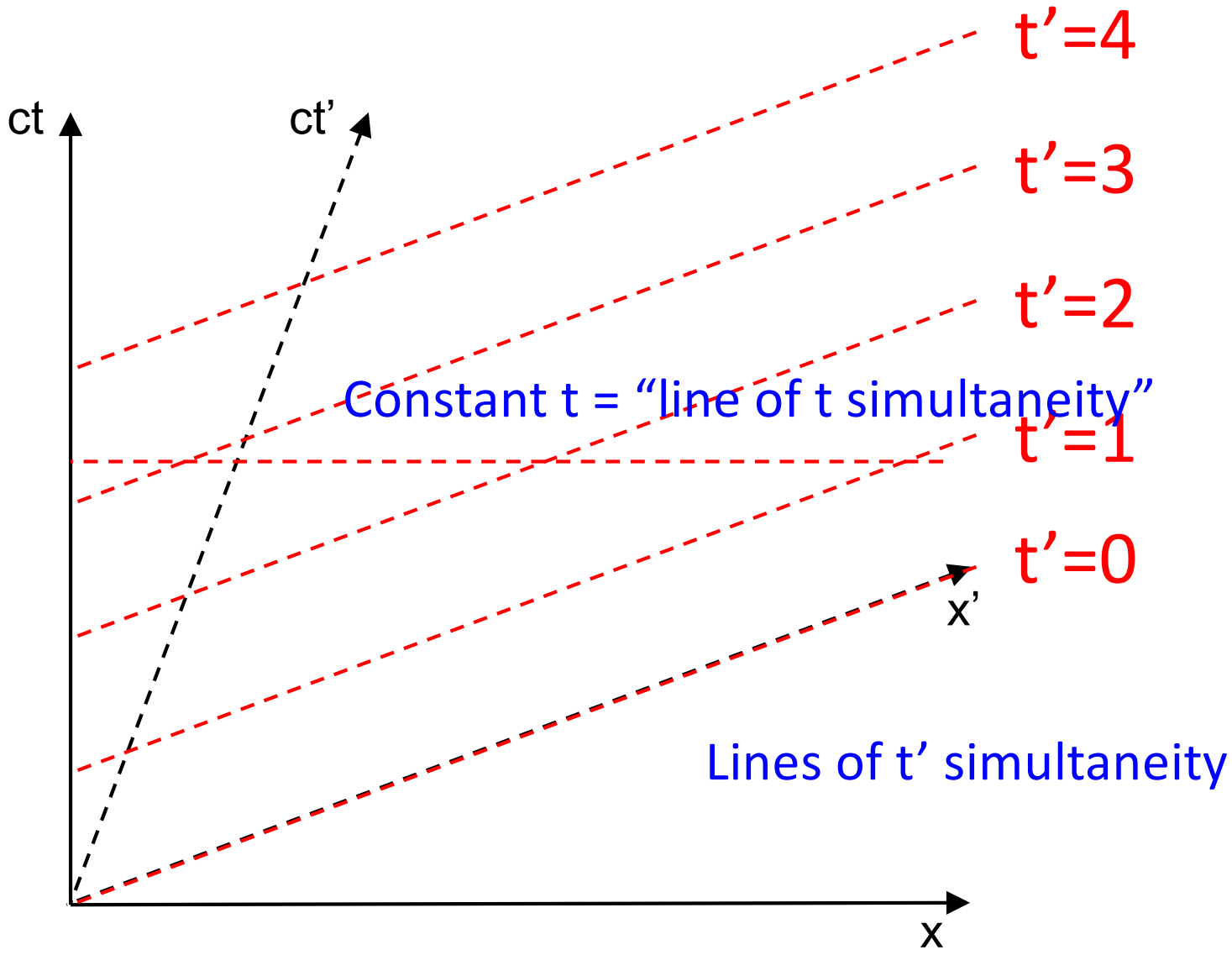
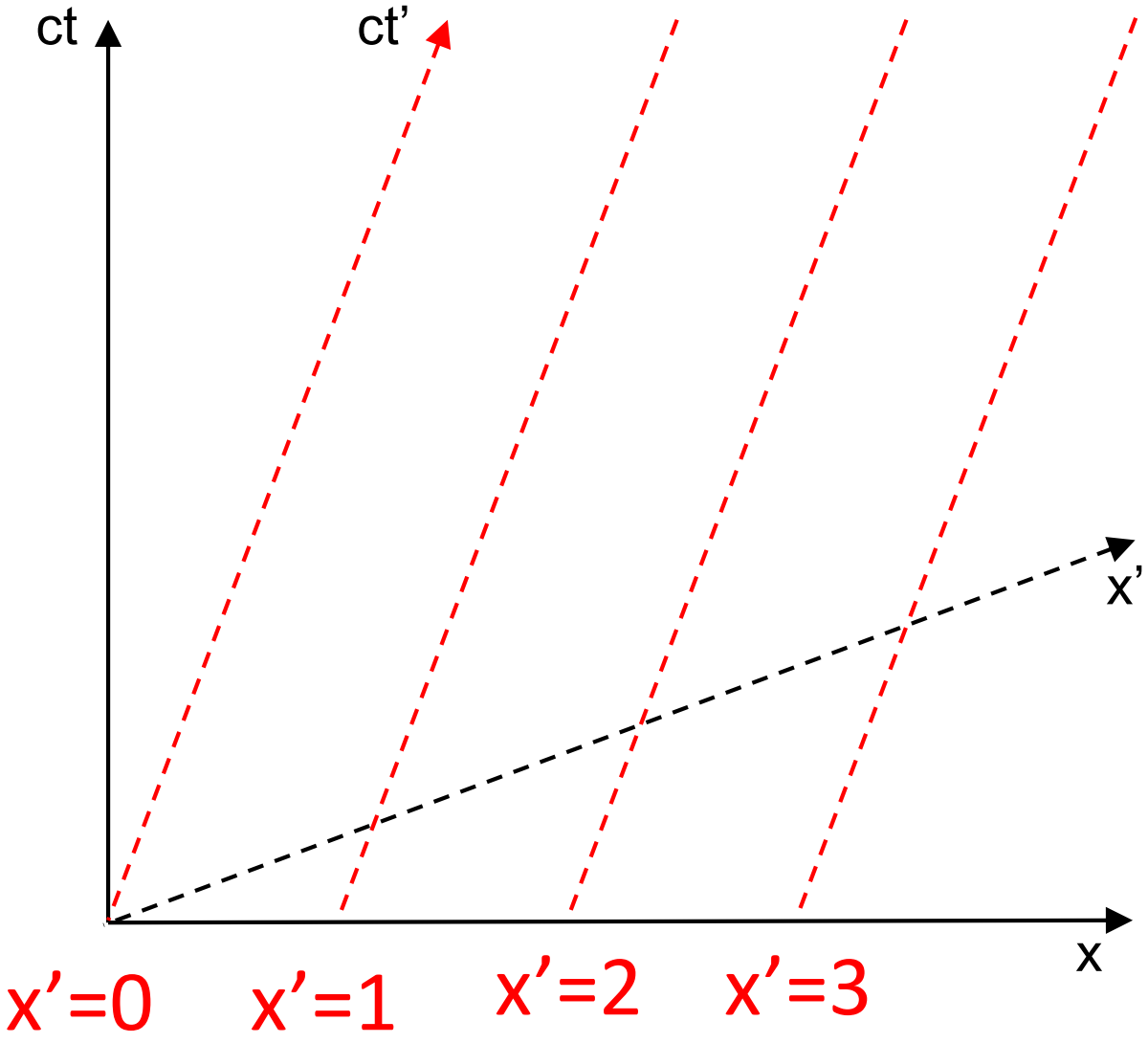


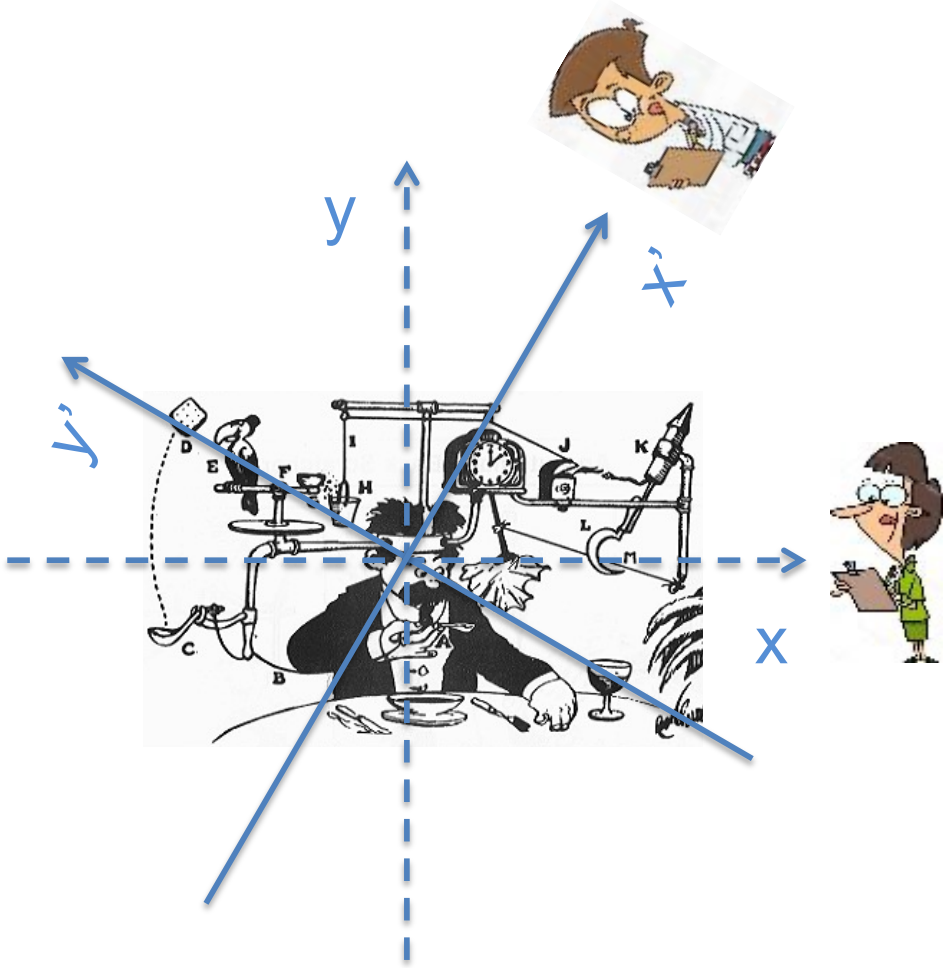
Lines of constant t and t’

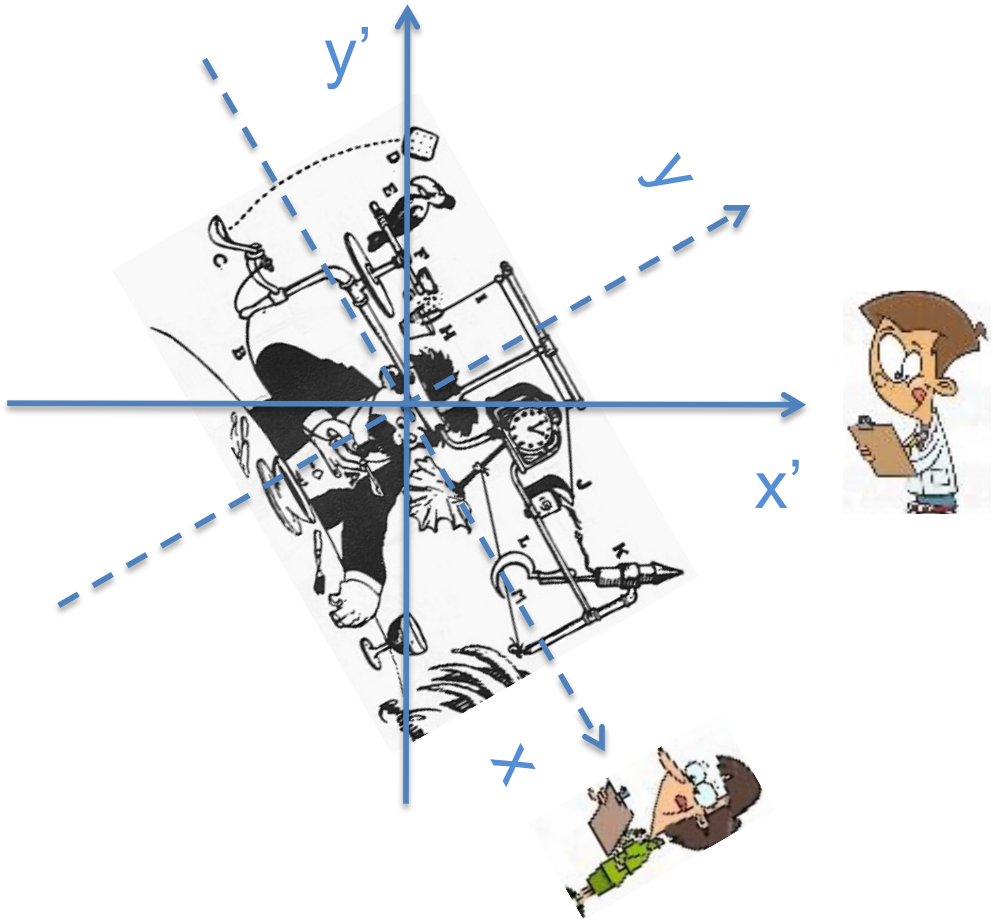


Lines of constant x’

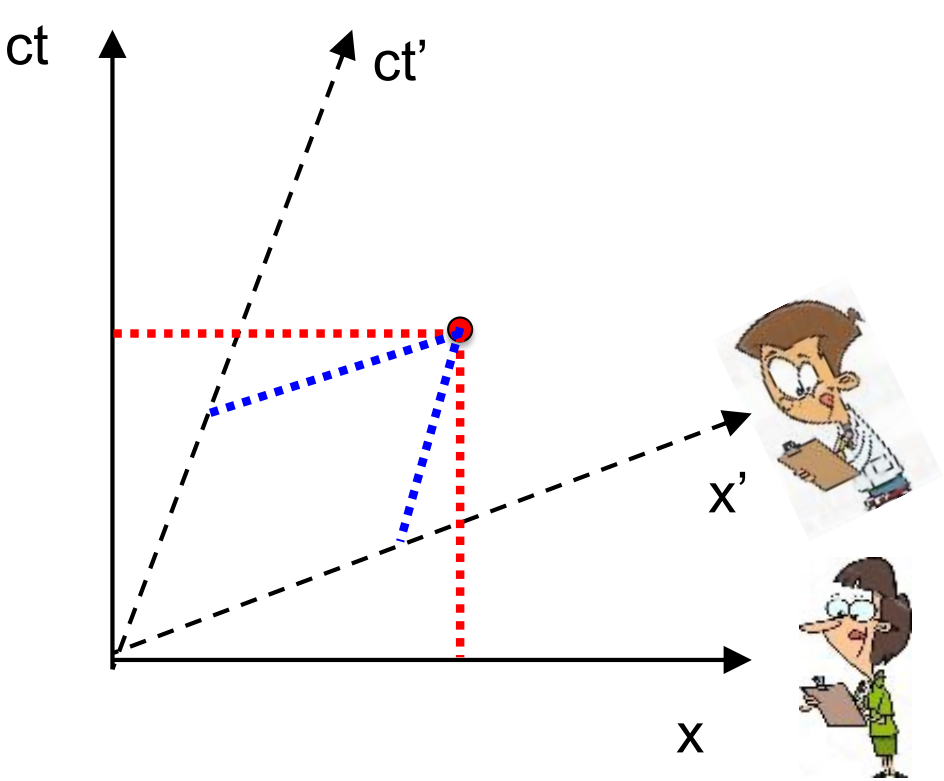


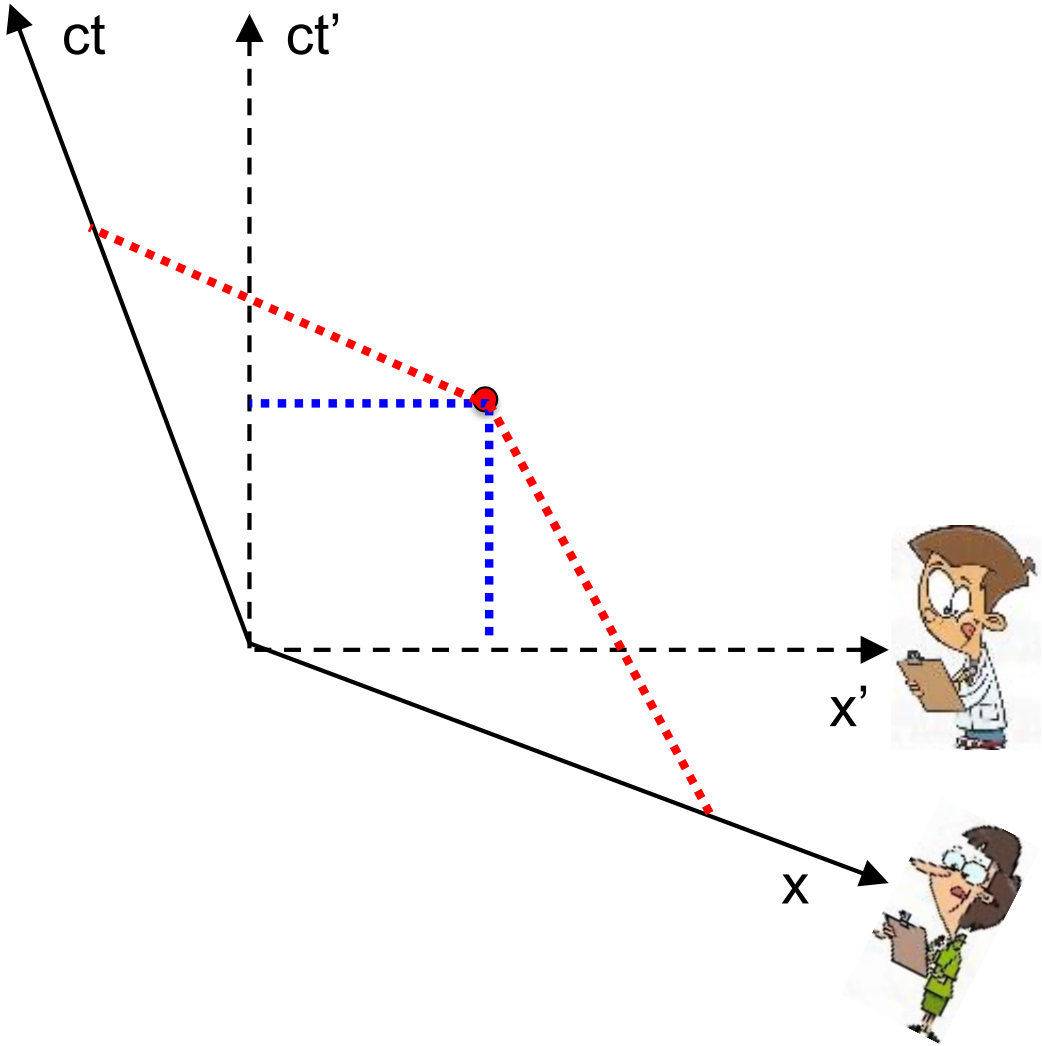
Passive rotation observers



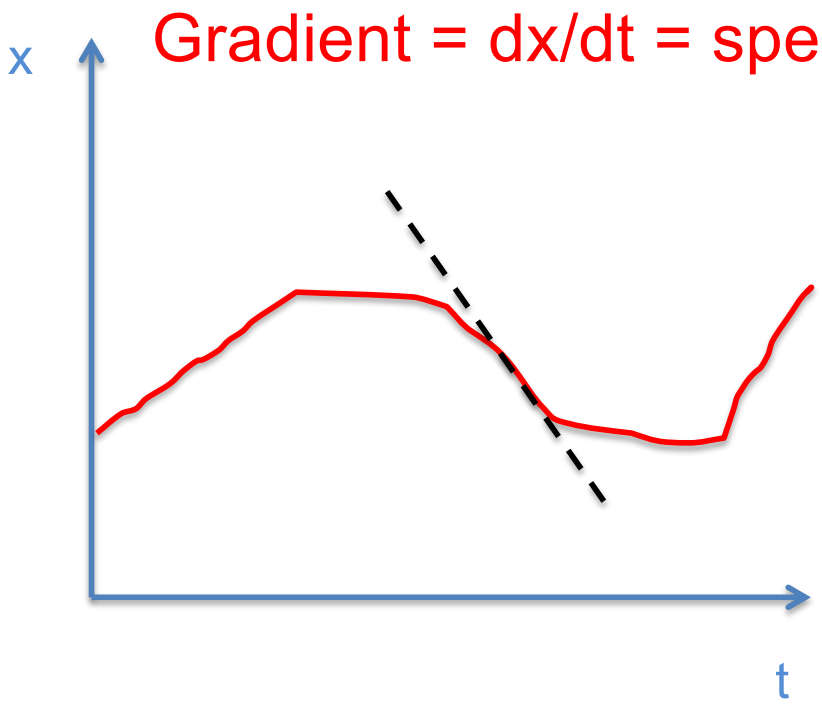


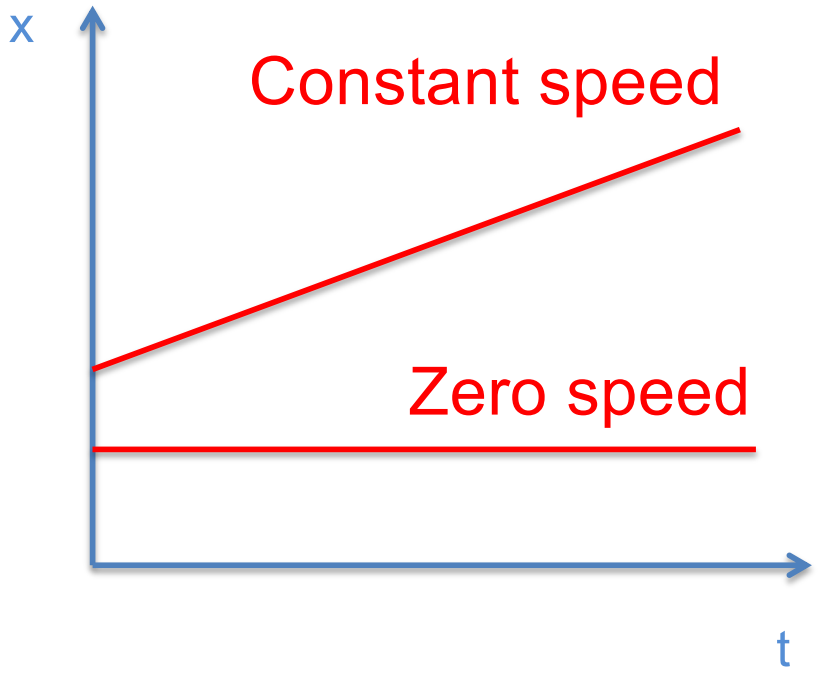
View from second frame





Mechanics problem: solve x(t)

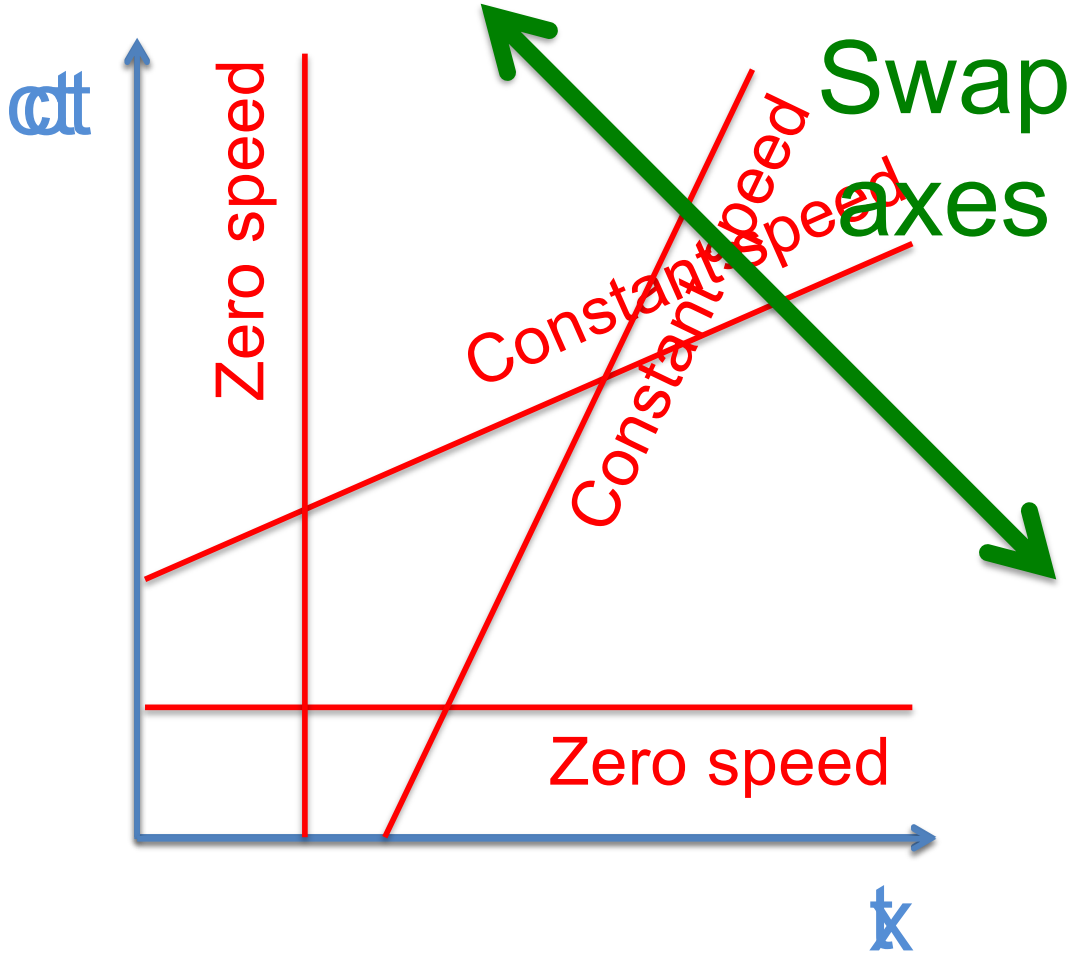




World lines

- Any object which exists for a finite time will form a line in our space-time diagrams
- Call this the ‘world line’ of the object
- In space-time diagrams, axes swapped cf our usual way of thinking

World lines in space-time diagrams



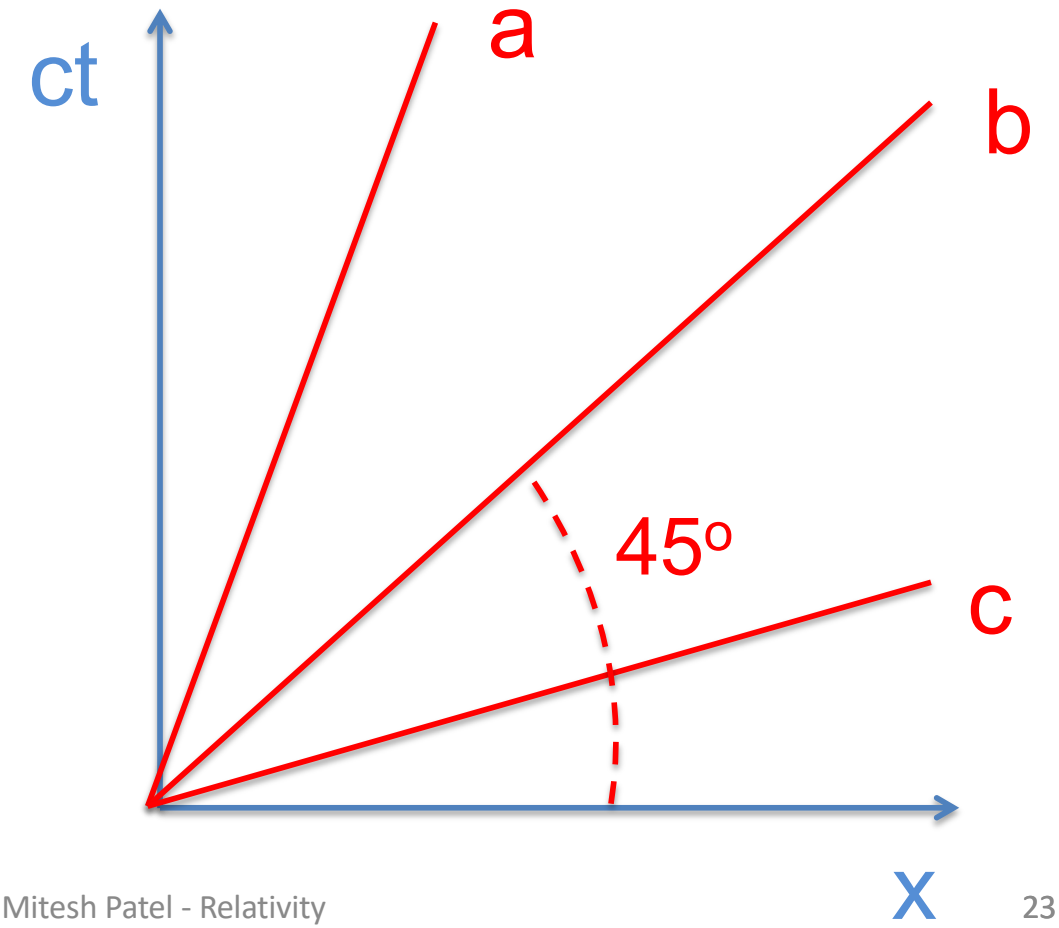
World lines

- Any object which exists for a finite time will form a line in our space-time diagrams
- Call this the ‘world line’ of the object
- In space-time diagrams, axes swapped cf our usual way of thinking
- Stationary object will give a vertical line
- Object moving at constant velocity u will give a straight line with some finite gradient

An object going through the origin

Which world line is unphysical?

- Go to www.menti.com
- Question 3



Which world line is unphysical?

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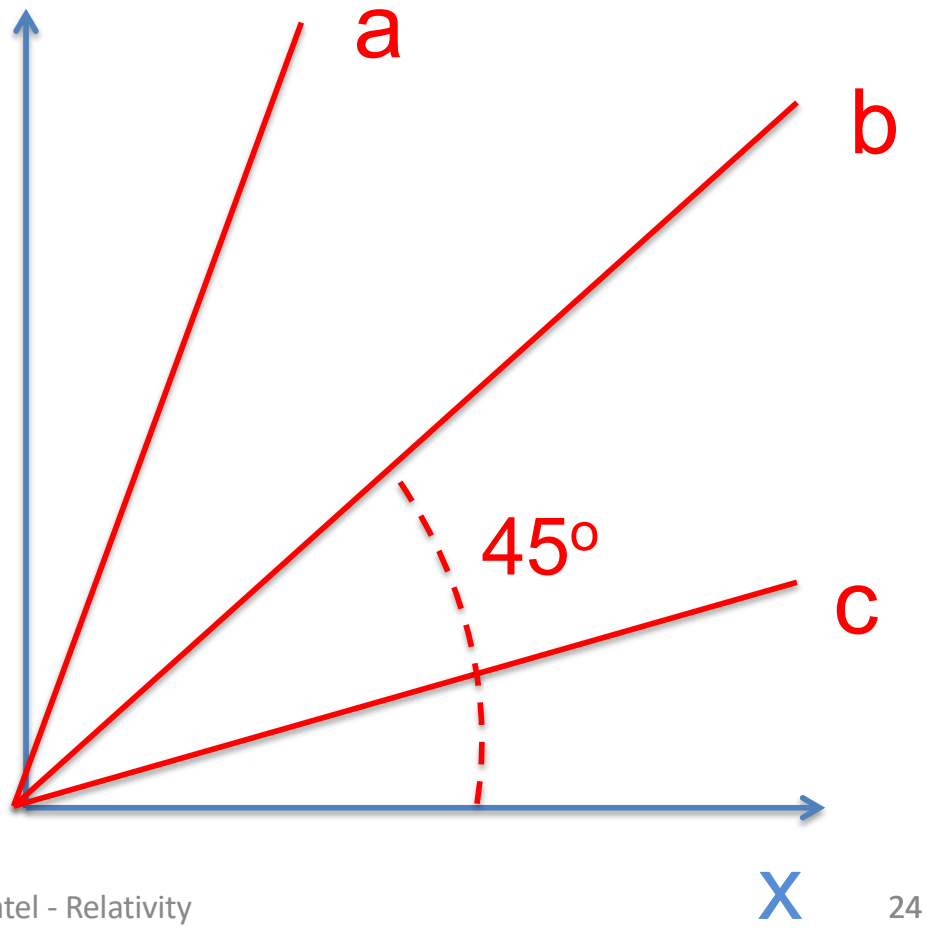


- Answer: c is unphysical
- The gradient is < 1 which would mean that $\beta_u > 1$
- Note, line b has $\beta_u = 1$ so only allowed for photons

World lines

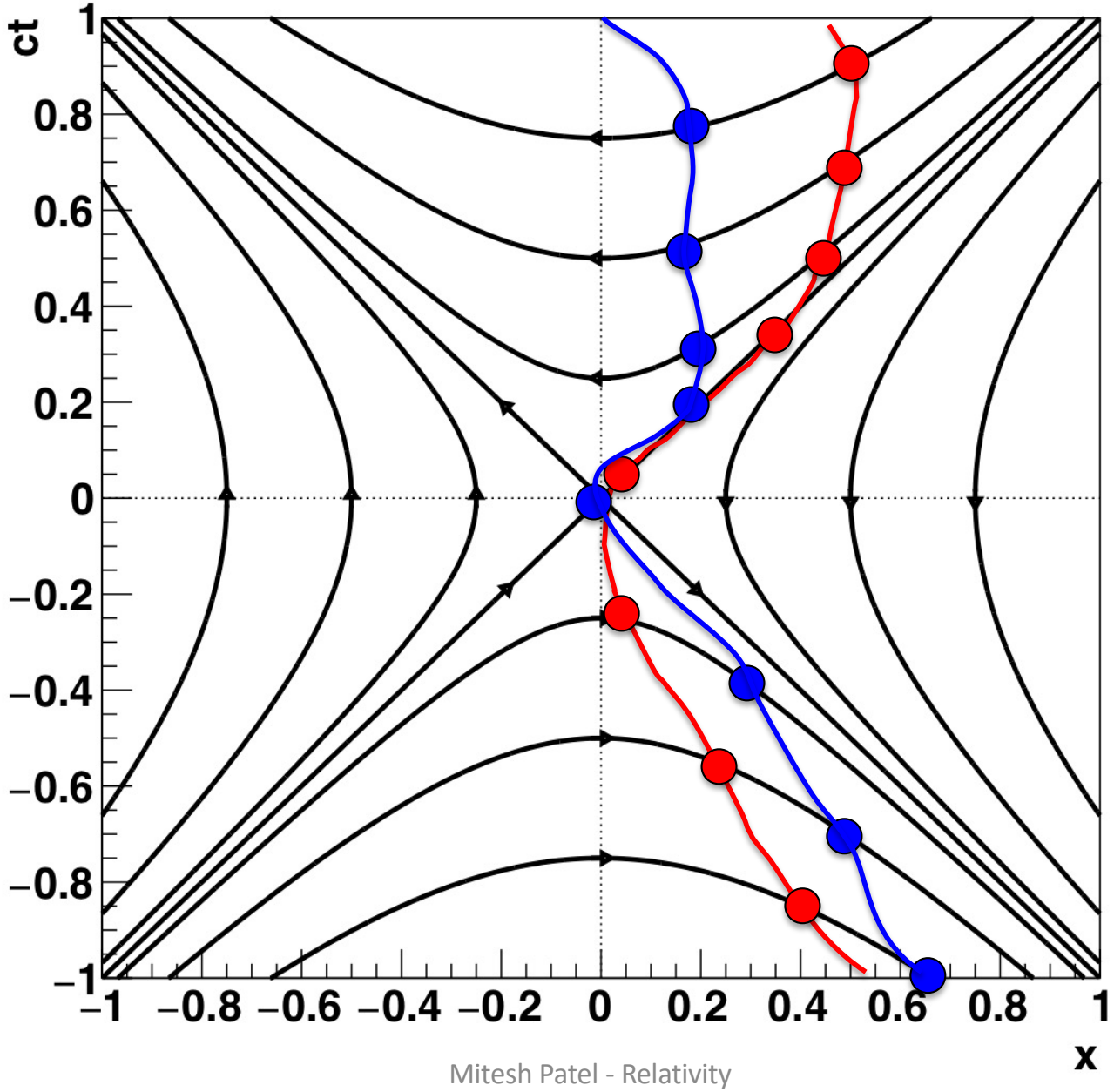
- Since we are limited to speeds $|u| \leq c$ for which $|\beta_u| \leq 1$ and hence $1/|\beta_u| \geq 1$, the magnitude of the gradient must be always at least 1

- Light will always travel along lines with a gradient = 1, which have an angle of 45◦

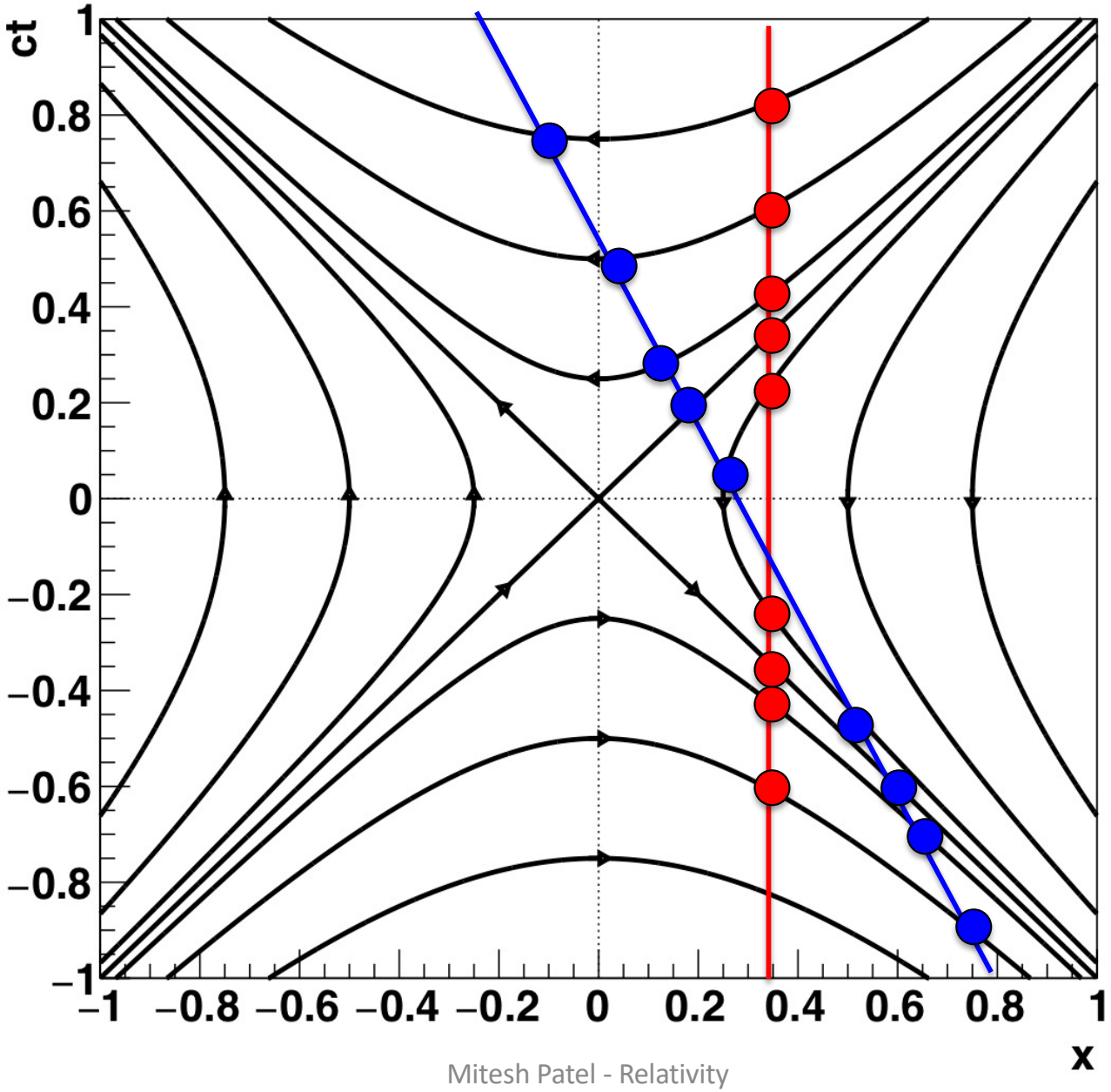
Example

- A light bulb is at rest at x=0
- At t=0, it briefly turns on and off
  - A light pulse is emitted in both the +x and −x directions
- Draw a space-time diagram showing world lines for the light bulb and both light pulses
- Redraw the space-time diagram in a frame moving at speed $v = \beta c$ with respect to the bulb

Transforming an arbitrary world line



Transforming a straight world line



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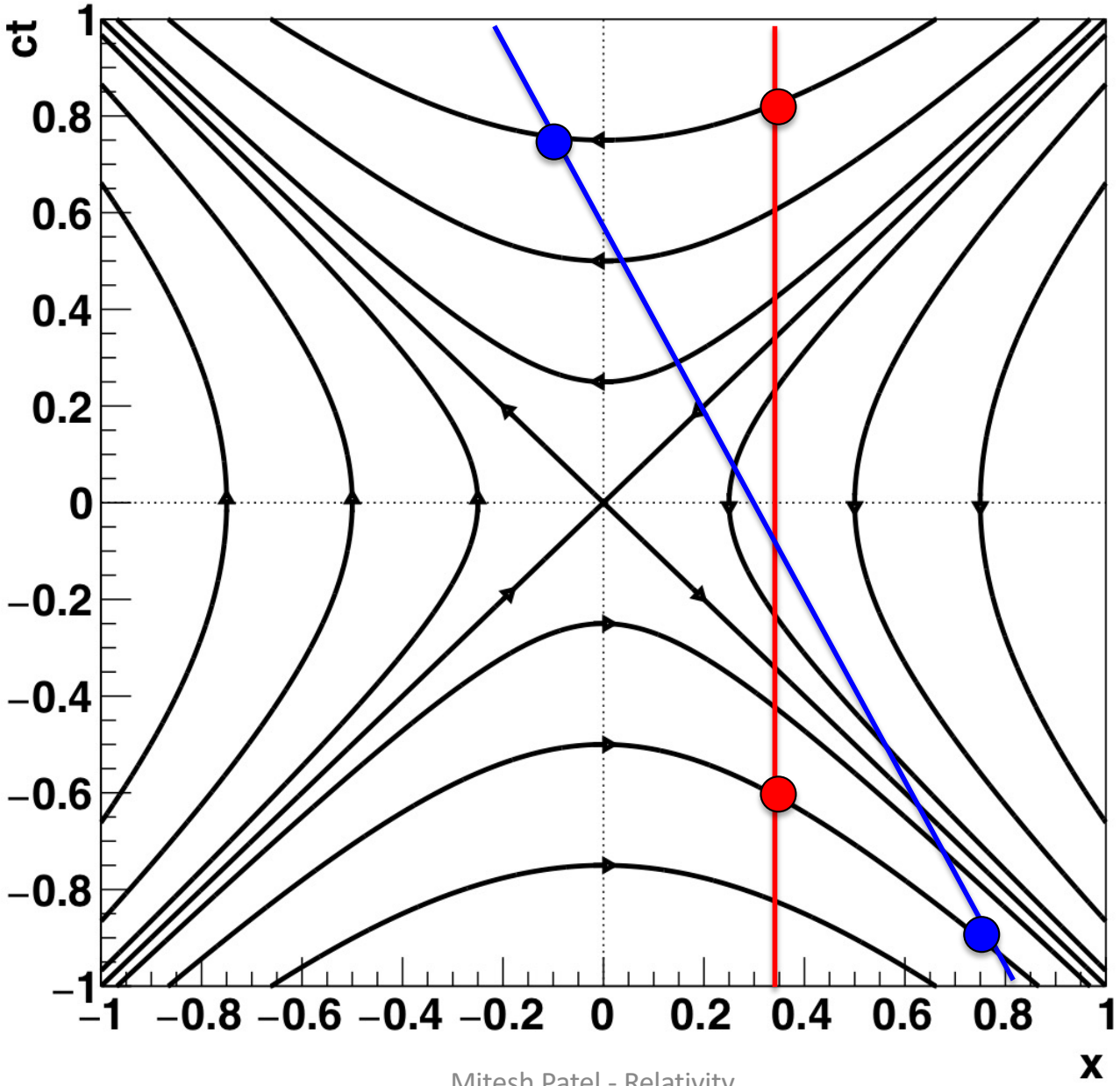
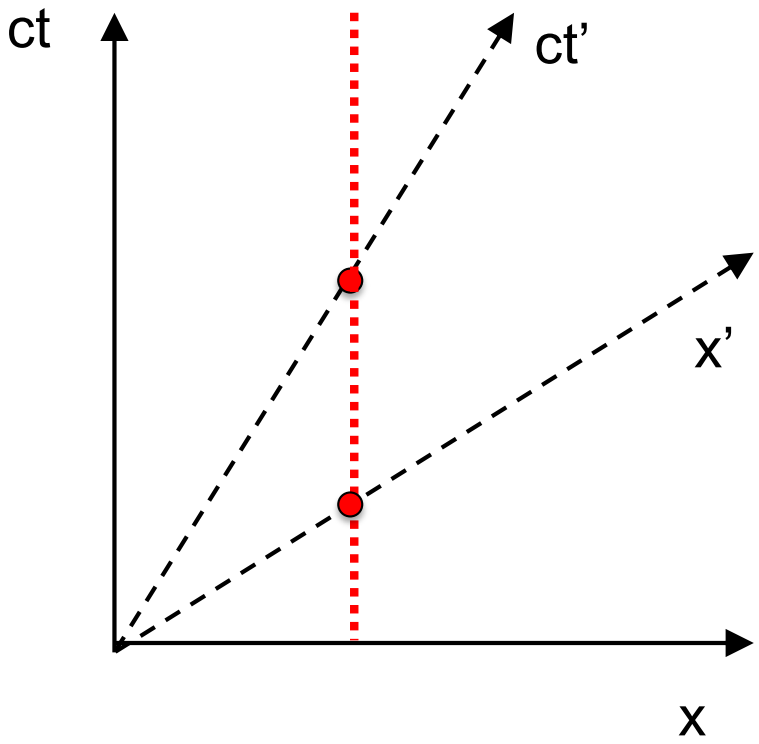
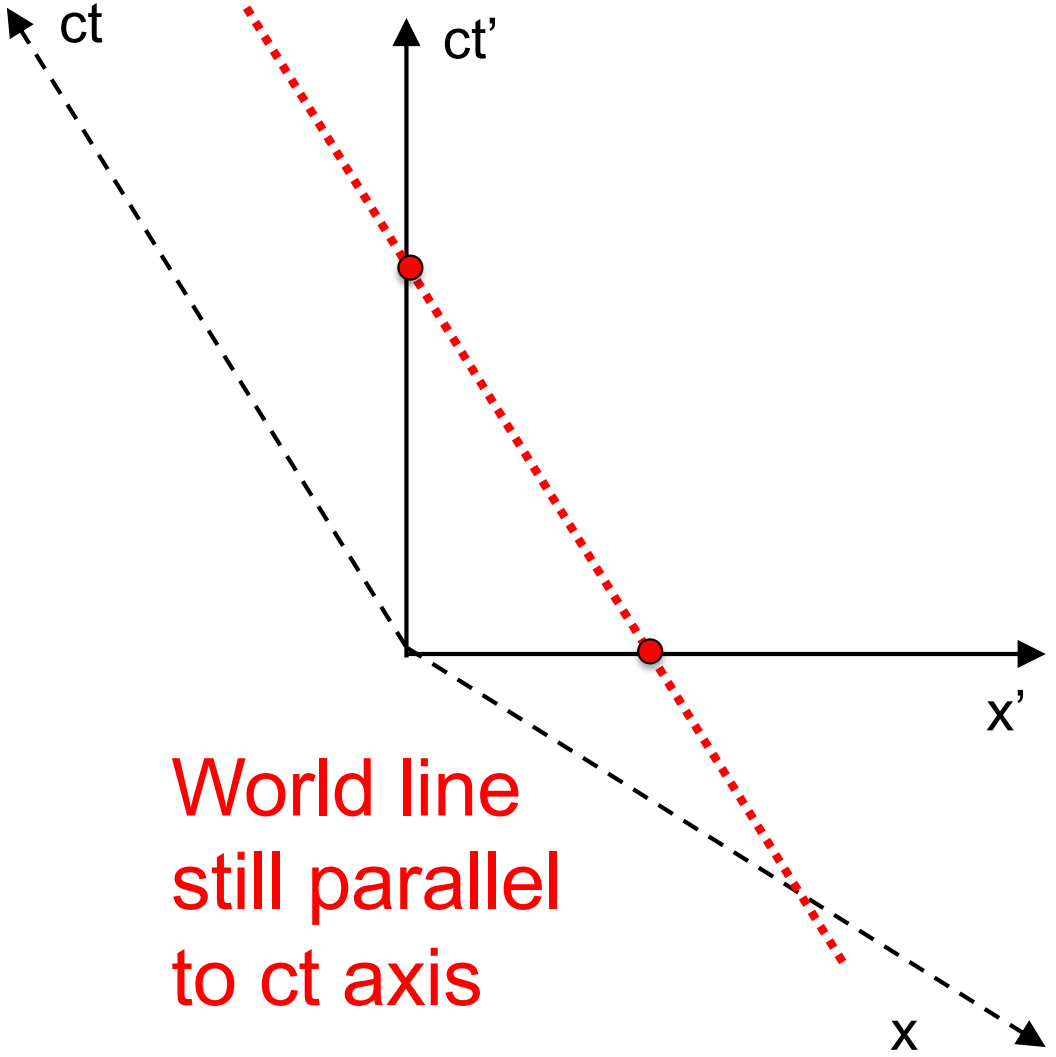


World line for object at rest



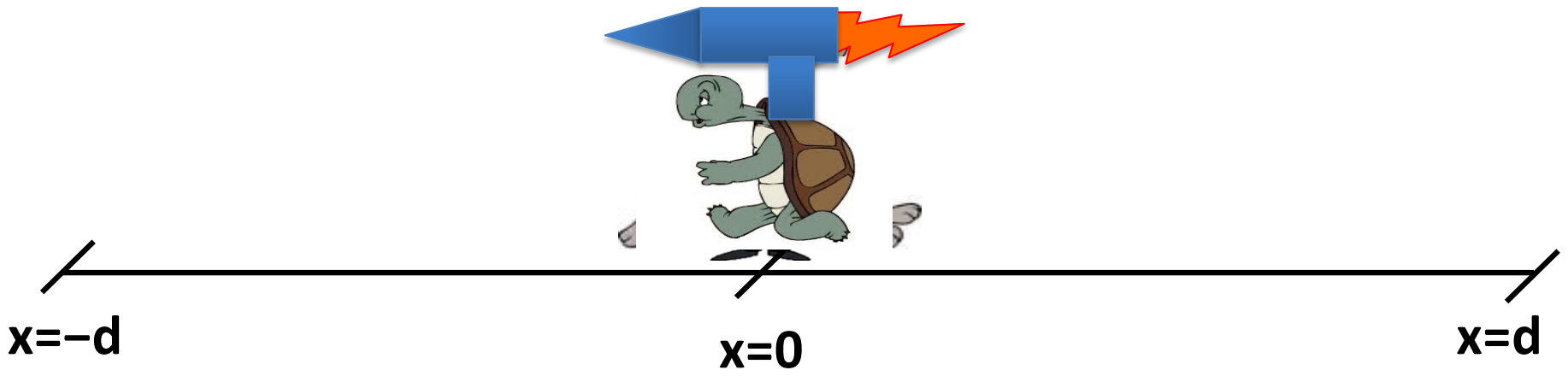
World line parallel to ct axis



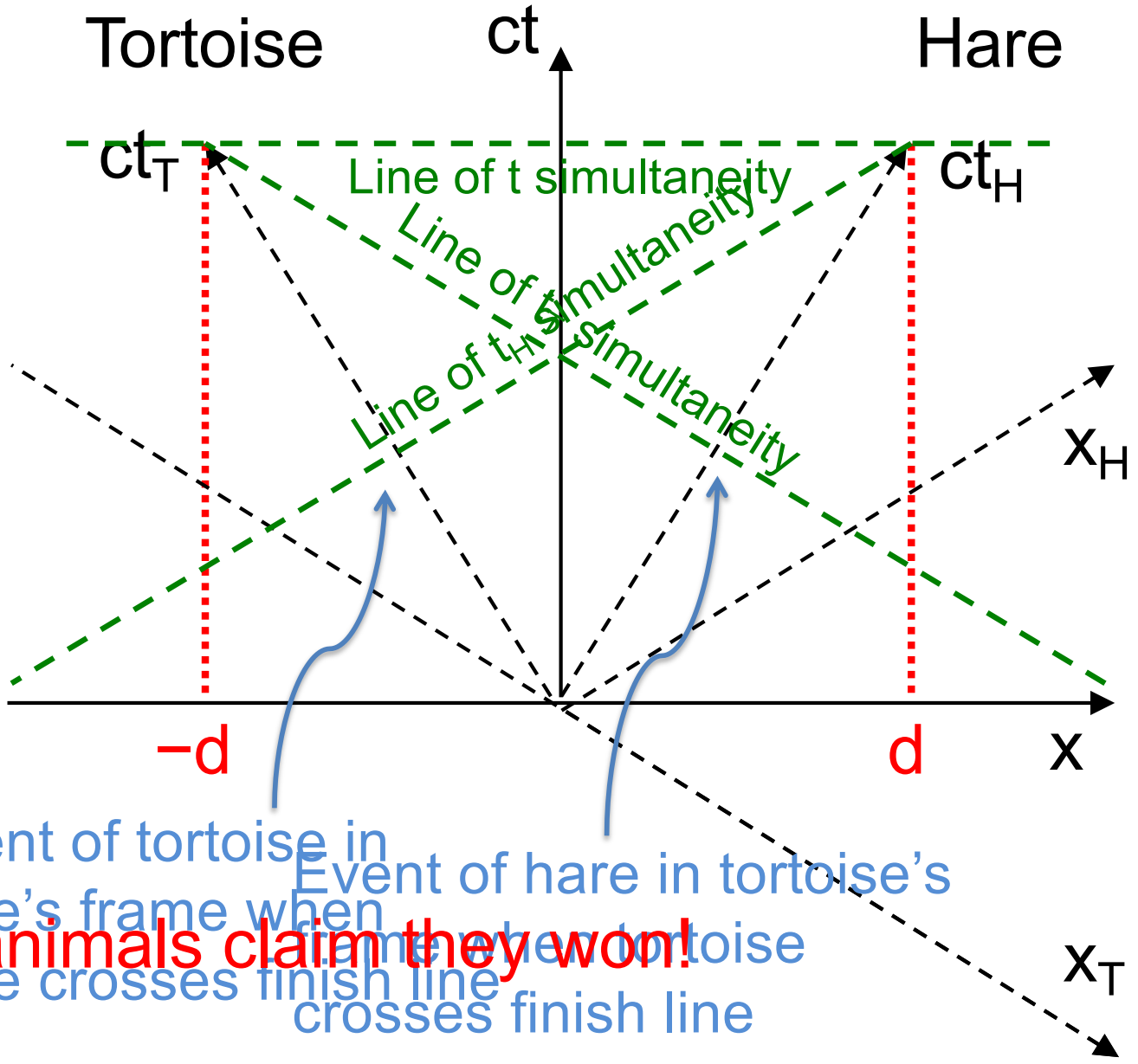
World line still parallel to ct axis

Example: the hare and the tortoise

- The hare and tortoise have a race
  - They both run the same distance
  - But they run in opposite directions
- The referee watches the race at rest
  - Both animals run at the same speed
  - The referee says the race is a draw
- Who do the animals think won?



Hare and tortoise



Both animals claim they won!

What we did today

- Looked at space-time diagrams
  - How events and axes move under Lorentz transformations
- Saw the concept of world lines
  - “History” of an object in space-time
- Saw how world lines change under Lorentz transformations
  - World lines change in position and gradient