Year 1 – Relativity
Lecture 4

Mitesh Patel

Overview of lectures

- Lecture 1: Introduction, concepts and classical results
- Lecture 2: The postulates of Relativity
- Lecture 3: Length contraction and simultaneity
- Lecture 4: The Lorentz transformations
- Lecture 5: Space-time diagrams and world lines
- Lecture 6: Four-vectors and causality
- Lecture 7: Energy and momentum
- Lecture 8: Rest mass energy and particle decays
- Lecture 9: Particle reactions
- Lecture 10: The relativistic Doppler effect

Previously on Relativity

- Lorentz (or length) contraction
  - An observer sees a moving object get shorter along its direction of motion

- Non-simultaneity
  - Two occurrences that happen at the same time for one observer (i.e. are simultaneous) do not always occur at the same time for a moving observer
  - If they are at the same position, then they are simultaneous for all observers

Previously on Relativity

- Need to think in terms of "space-time" rather than space (length) and time separately
  - Measuring the length of a rod – can only measure both ends simultaneously in rest-frame

- Saw that the Galilean transformations must be only approximations
  - u' = u – v cannot be exact if c is a constant
  - Specifically, there must be some other relativistic transformations which reduce to the Galilean ones for low speeds

What we will do today

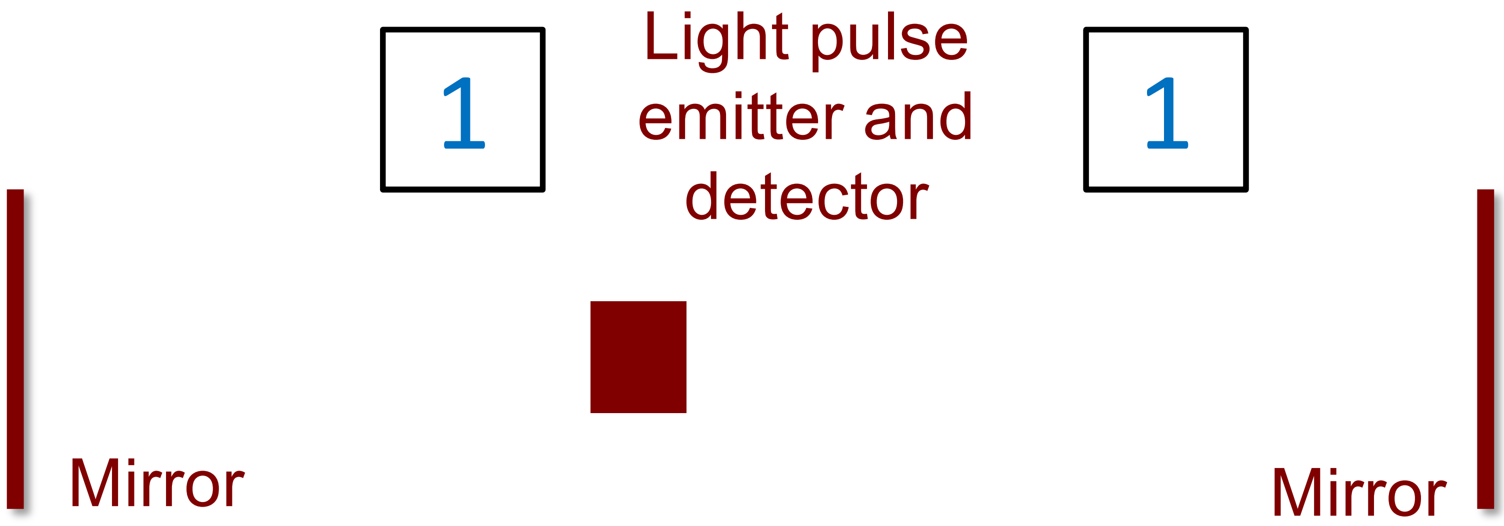
- Discuss the concept of an event
  - A coordinate point in space and time
- Introduce the Lorentz transformations
  - The exact transformations in Relativity
- Check they are consistent with the postulates
  - The speed of light must not be changed by the transformations
- See how velocities transform between frames
  - They must also give the Galilean transformations in the low speed approximation

The concept of an "event"

- For rotations we dealt with a position x,y
  - When we do a rotation, we get new values x',y'
- An event is just a "position" in space and time
  - In principle t,x,y,z (but will often drop y and z)
  - When we change frames, we get new values t',x'
  - Ideally an event = infinitesimally small point in space at an instant in time (or some approximation…)
- An event can be a considered both as
  - A position in space-time
  - Something happening at a particular time and place

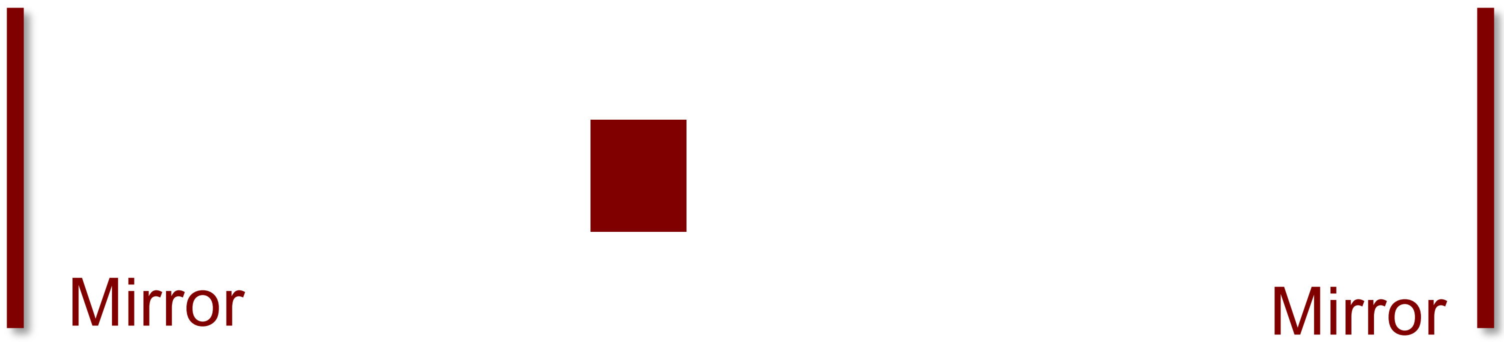
Menti question 1

- Go to www.menti.com



- How many “interesting” events are happening this asymmetric double light clock?

Menti answer 1

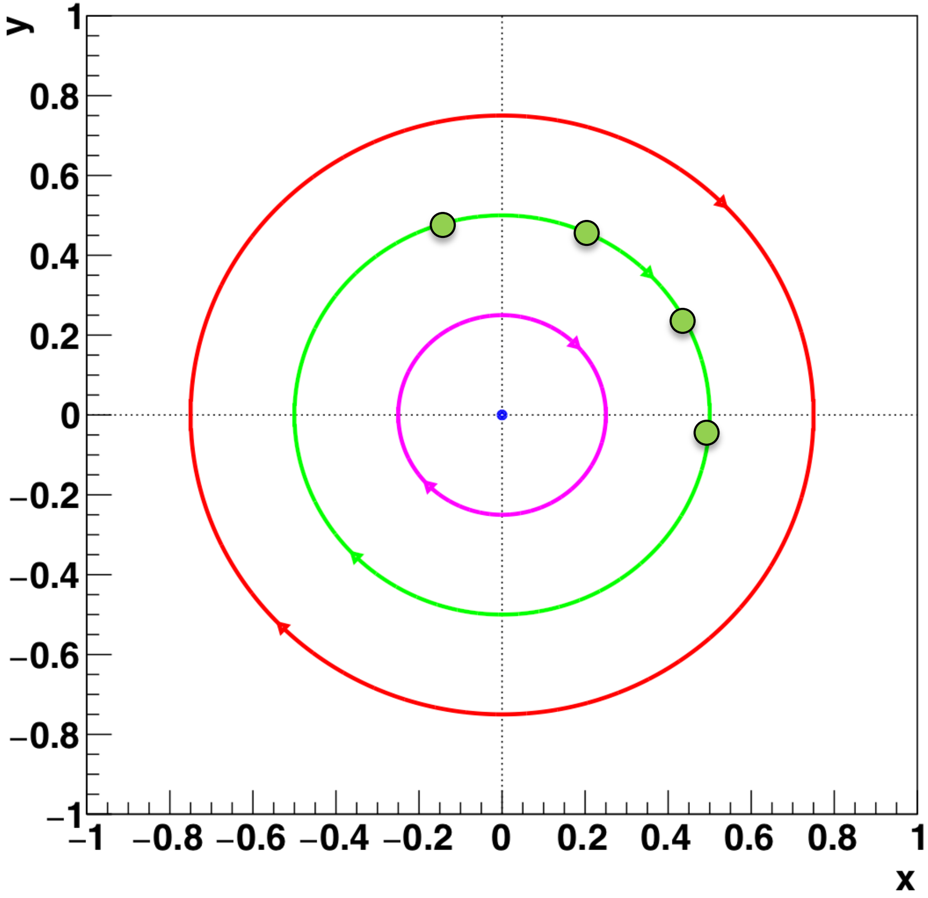


- There are 5 events
    1. Both light pulses are emitted
    2. Left pulse hits mirror
    3. Right pulse hits mirror
    4. Left pulse reaches centre
    5. Right pulse reaches centre

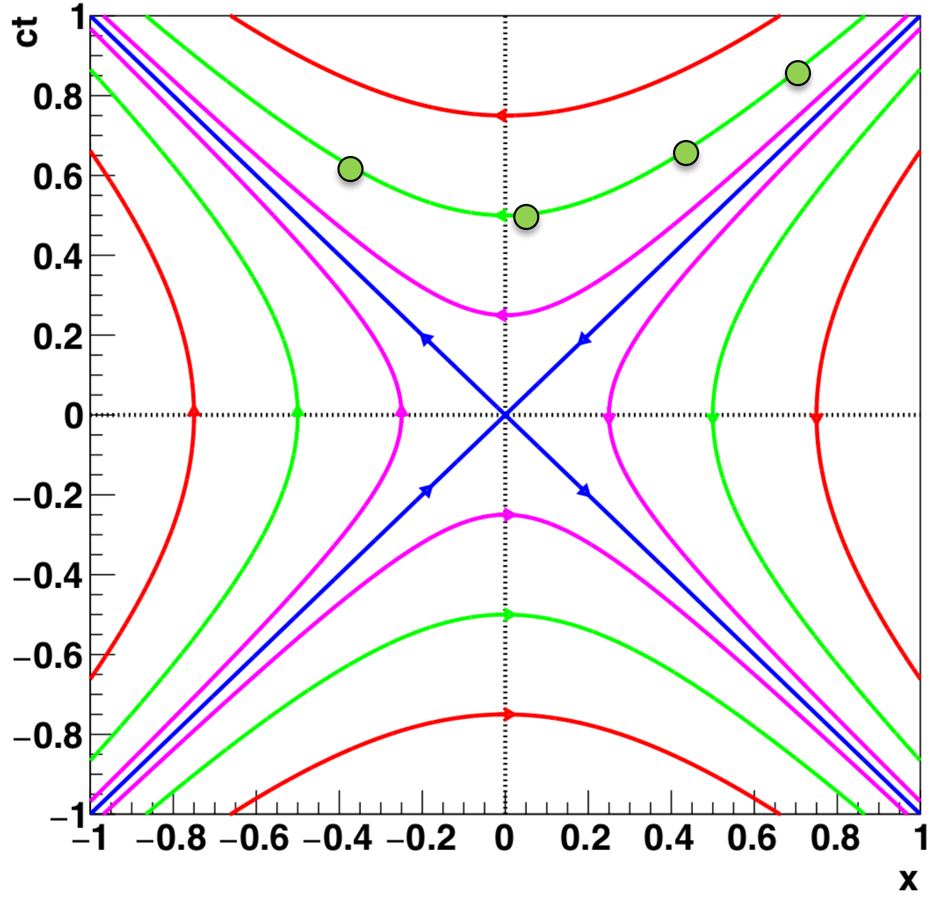
The Lorentz transformations

Changes to coordinates/events

Rotations



Lorentz transformations



The separation

- In 4d "space-time" what is conserved quantity equivalent to radius for spatial rotations?
- The separation, $S^2 = c^2t^2 - r^2$ [lecture 6]
- This is why our measurements of length or time change between coordinate frames
  - Our everyday experience tells us length (or time) are conserved but should be conserving S^2
  - Hence have to think of 4d "space-time" rather than space (length) and time

Menti question

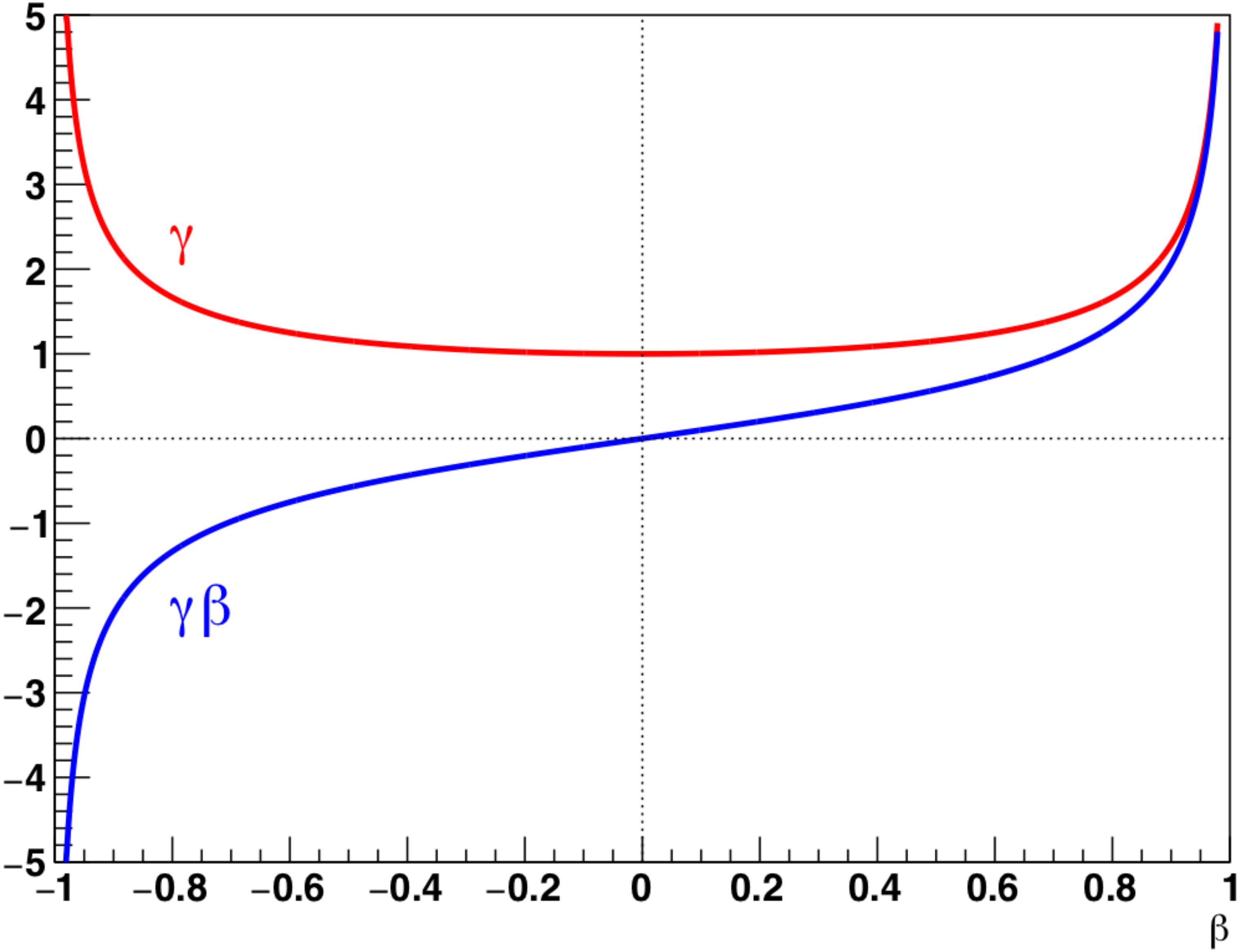
- Go to www.menti.com
- Consider two simultaneous events in frame 1
  - At $x_1=0$, $ct_1=0$ and $x_2=1$, $ct_2=0$
- Consider a transformation to frame 2
  - Moving at $\beta=3/5$ so $\gamma=5/4$
- What is the time difference of the events in frame 2?

Menti answer 2

- We only need to find the times in frame 2
- For $x_1=0,\ ct_1=0$
  - $ct_1' = (5/4)\ [0 - (3/5)\ 0] = 0$
  - 0,0 always transforms to 0,0
- For $x_2=1,\ ct_2=0$
  - $ct_2' = (5/4)\ [0 - (3/5)\ 1] = -(5/4)(3/5) = -3/4$
- Time difference is 3/4c
  - Not simultaneous in frame 2
  - Purely because events are not at the same x

Small speed approximation

γ and $\gamma\beta$ vs β



How big is $\Delta t = t' - t$?

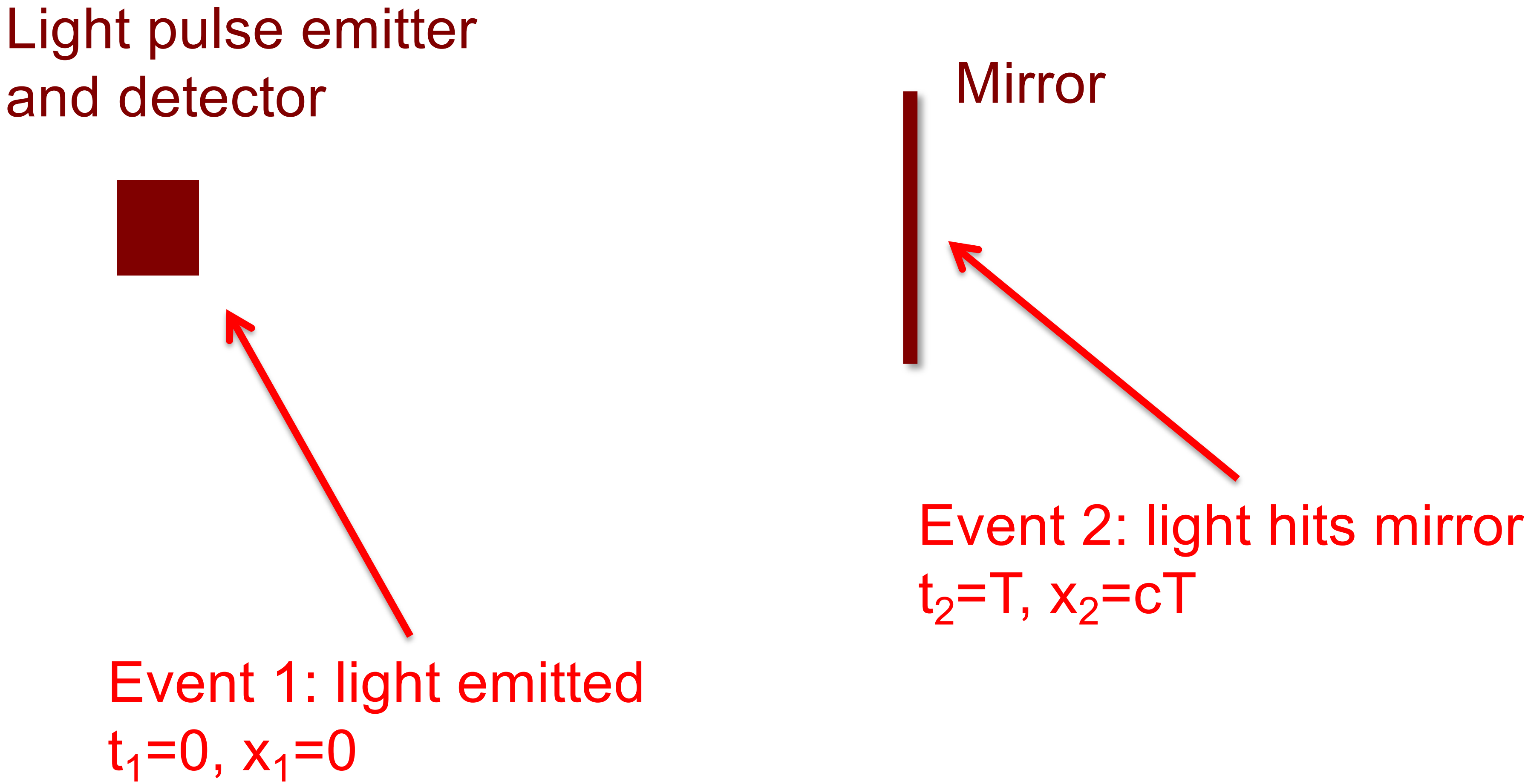
Fastest man-made object: Helios 2 space probe achieved 360,000 km/h ~ 100 km/s ~ 3×10^{-4} c

Light takes 0.04s to cross the world

~13000 km

For an experiment the size of the Earth and some of its apparatus going as fast as Helios 2, $\Delta t \sim 10\ \mu s$

Consistency of the speed of light



Consistency of the speed of light

What about other speeds?

Spaceship base

Spaceship speed u

Earth

Event 1: spaceship leaves base
$t_1=0,\ x_1=0$

Event 2: spaceship reaches Earth
$t_2=T,\ x_2=uT$

Velocity transformation

Example: velocity transformation

- An observer on Earth sees two spaceships approaching from opposite directions
  - Spaceship 1 is coming at $u_1 = 3c/4$
  - Spaceship 2 is coming at $u_2 = -c/2$
  - For observer on Earth, their relative velocity is $3c/4+c/2 = 5c/4$ i.e. more than the speed of light
  - But no individual object is moving at speed > c so this is allowed in the Earth's frame
- What speed does spaceship 2 appear to be moving for an observer on spaceship 1?

Example: velocity transformation

What we did today

- Saw the concept of an event
  - A coordinate point in space and time
- Introduced the Lorentz transformations
  - Move an event from t,x to t’,x’
  - Cleanest to express as ct,x
  - They also give the Galilean transformations in the low speed approximation
- Saw how velocities transform
  - The speed of light is not changed
  - This also approximates to Galilean transformations