Year 1 – Relativity
Lecture 3

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Overview of lectures

- Lecture 1: Introduction, concepts and classical results
- Lecture 2: The postulates of Relativity
- Lecture 3: Length contraction and simultaneity
- Lecture 4: The Lorentz transformations
- Lecture 5: Space-time diagrams and world lines
- Lecture 6: Four-vectors and causality
- Lecture 7: Energy and momentum
- Lecture 8: Rest mass energy and particle decays
- Lecture 9: Particle reactions
- Lecture 10: The relativistic Doppler effect

Previously on Relativity

- Saw the postulates of Relativity
  - All inertial frames (i.e. moving at constant speed) are equivalent
  - The speed of light c is the same for all observers
- An observer sees time as measured by a moving object slows down
  - Shown for a light clock, using just the postulates of Relativity
  - Must hold for all clocks; *intrinsic to time itself*
  - Proper time is the name for the time experienced by any object, i.e. in its rest frame

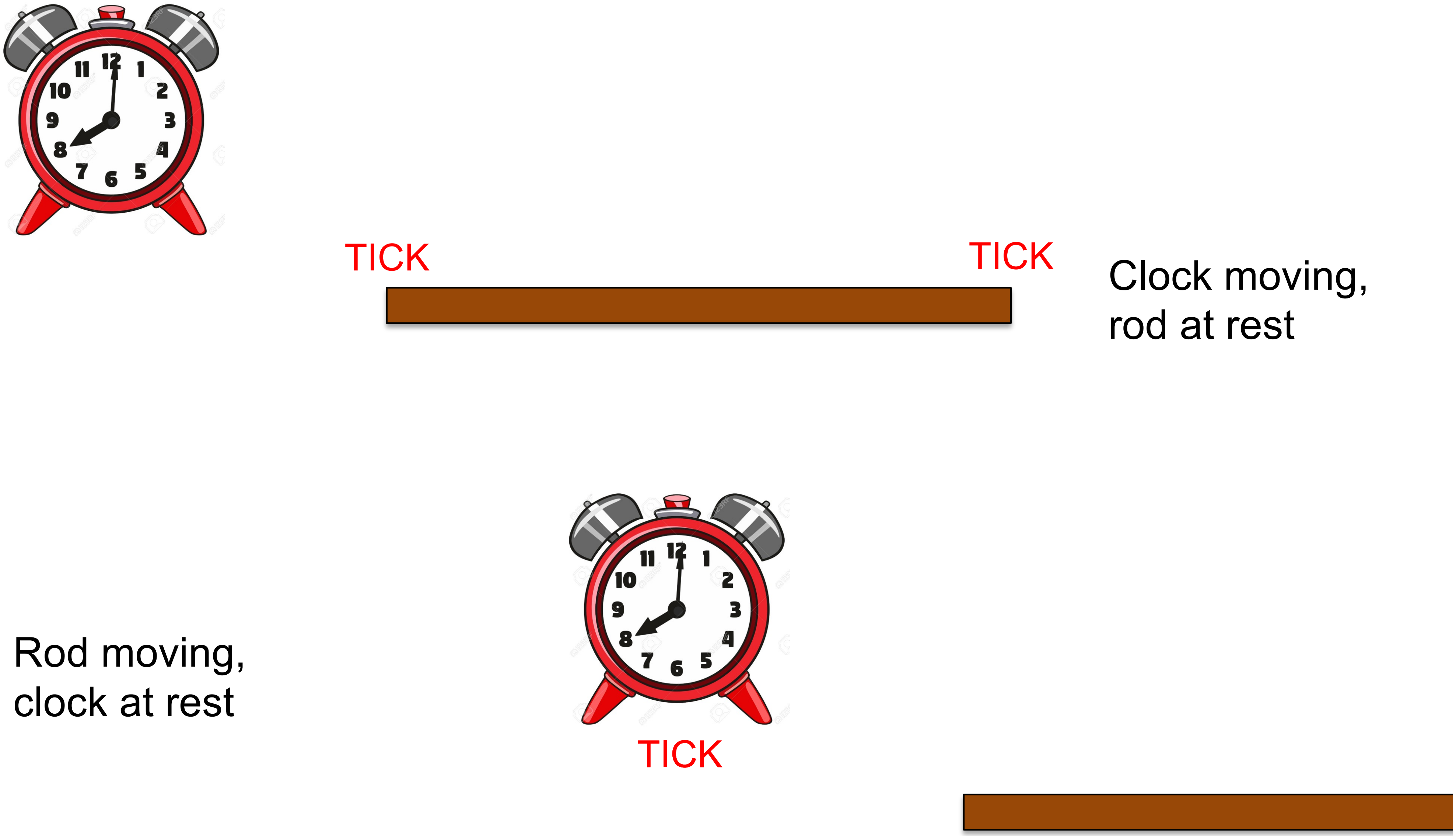
What we will do today

- Look at two other phenomena of Relativity
  – Both simply derived from the postulates

- Lorentz (or length) contraction
  – An observer sees a moving object get shorter along its direction of motion

- Non-simultaneity
  – Two things which happen at the same time for one observer (i.e. are simultaneous) do not always occur at the same time for a moving observer

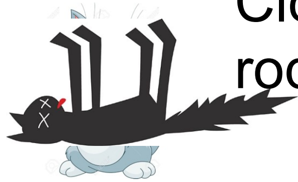
Lorentz contraction



Lorentz contraction

TICK

TICK



Clock moving,
rod at rest

TICK

Rod moving,
clock at rest

Measuring a train length?

- Lucy and Rick want to measure the length of a fast train they are on, relative to the track
  - At rest, the train is 100m long
  - When the train is moving, Lucy and Rick stand at either end of the train
  - Two friends by the track simultaneously each drop a dead cat, one by Lucy and one by Rick
  - Lucy and Rick get off at the next station, walk back and measure the distance between the cats
- Go to www.menti.com
  - Question 1: What distance do they measure?

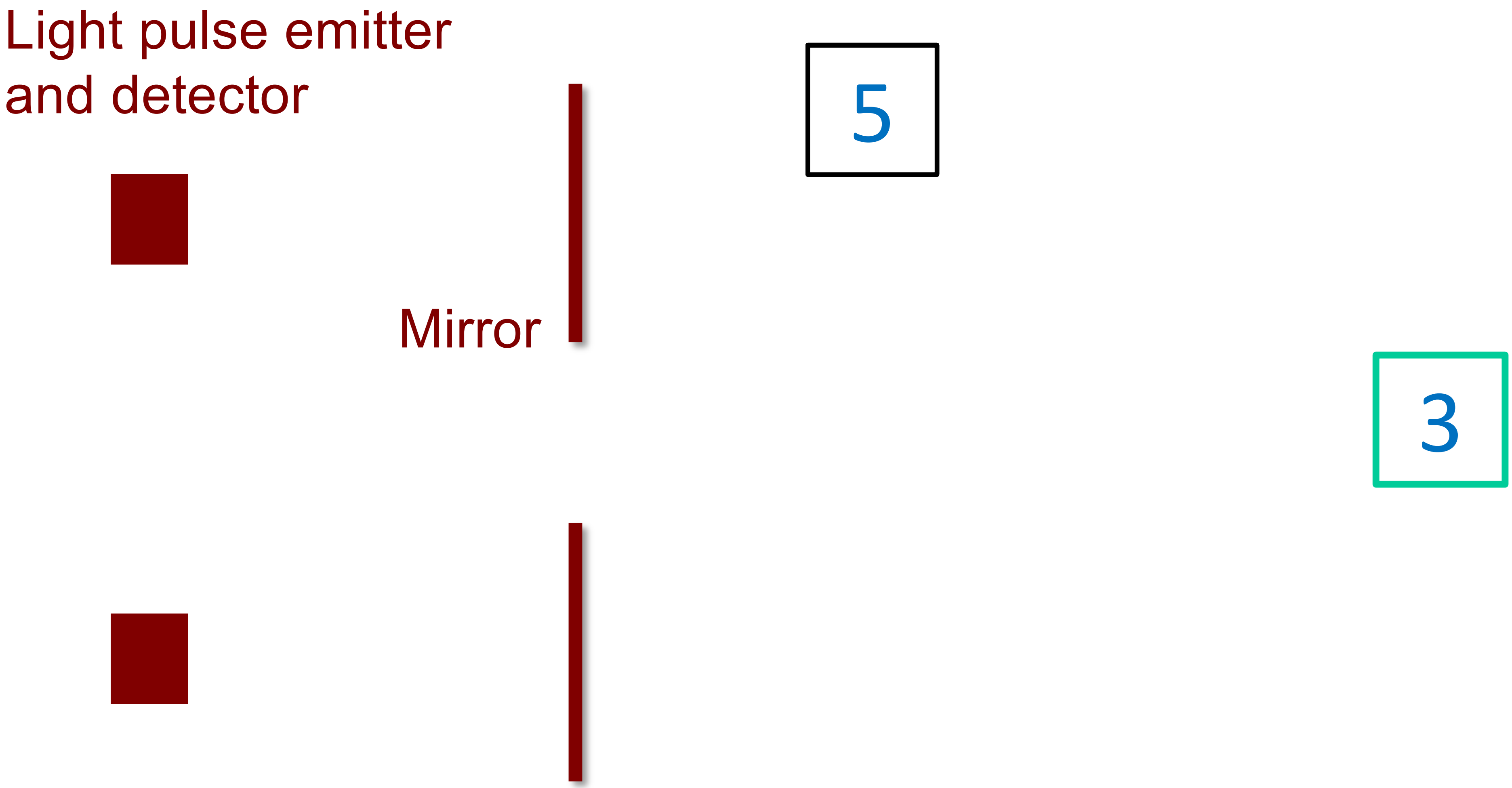
Measuring a train length?

- Lucy and Rick want to measure the length of a fast train they are on, relative to the track
  - Their friends are disgusted by dead cats
  - Hence Lucy and Rick have to do this alone
  - When the train is moving, Lucy and Rick stand at either end of the train, lean out of the window and drop a dead cat on the ground at the same time
  - They get off at the next station, walk back and measure the distance between the cats
- Go to www.menti.com
  - Question 2: What distance do they measure?

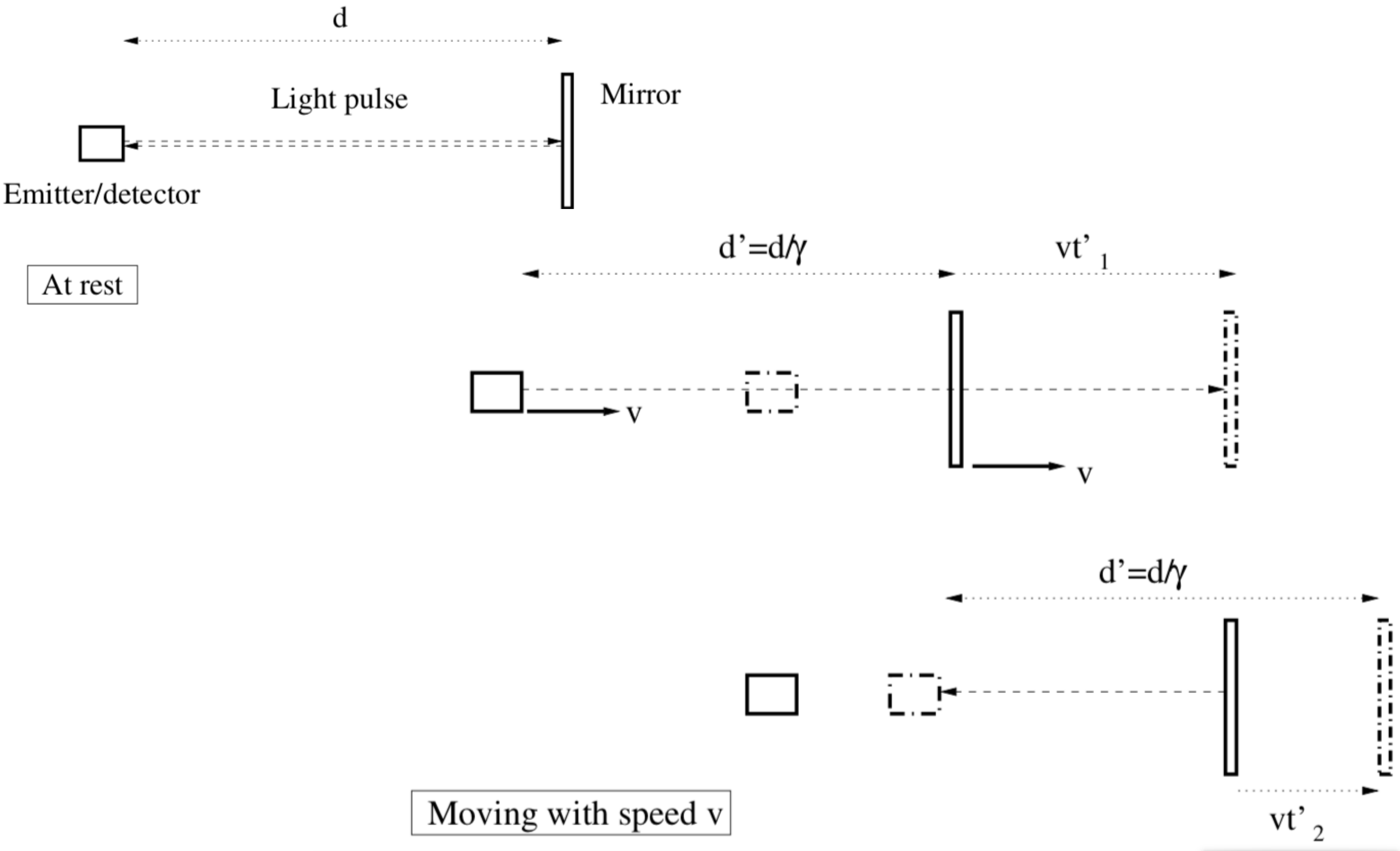
Measuring a train length?

- In the first scenario
  - Rick and Lucy's friends are making a length measurement of the moving train, just as for the rod earlier
  - The train is length contracted so they will therefore measure $100/\gamma$ i.e. less than 100m
- In the second scenario
  - Rick and Lucy are really making a length measurement of the ground, which is moving relative to them
  - The 100m they are apart must tie up with the contracted length of the ground
  - Hence 100m is the contracted length = l' so the distance between the cats will be $l = l'\gamma = 100\gamma$ i.e. more than 100m

More light clocks

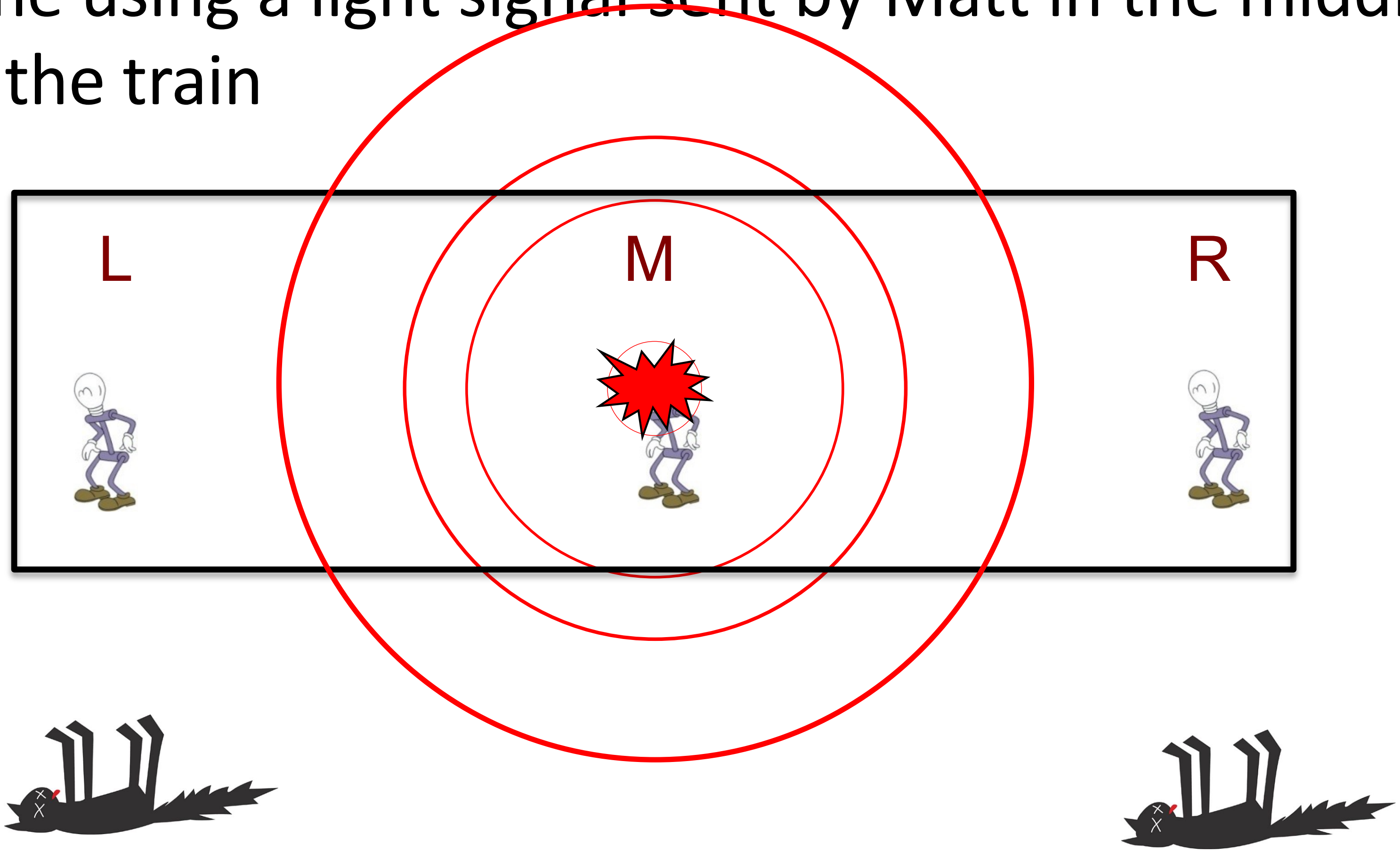


The light clock revisited



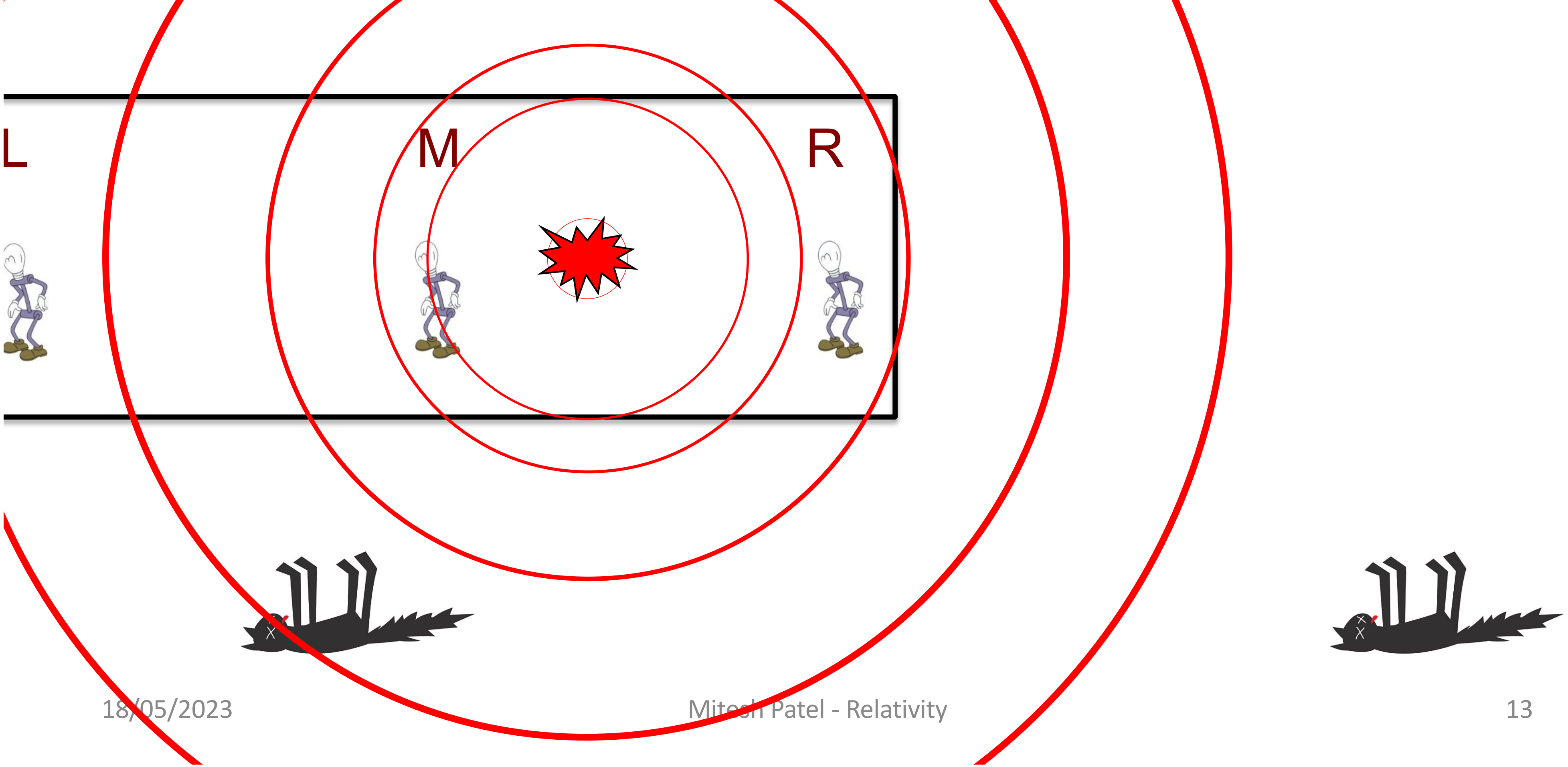
Measuring a train length?

- In the train inertial frame
  - Lucy and Rick ensure they drop the cats at the same time using a light signal sent by Matt in the middle of the train

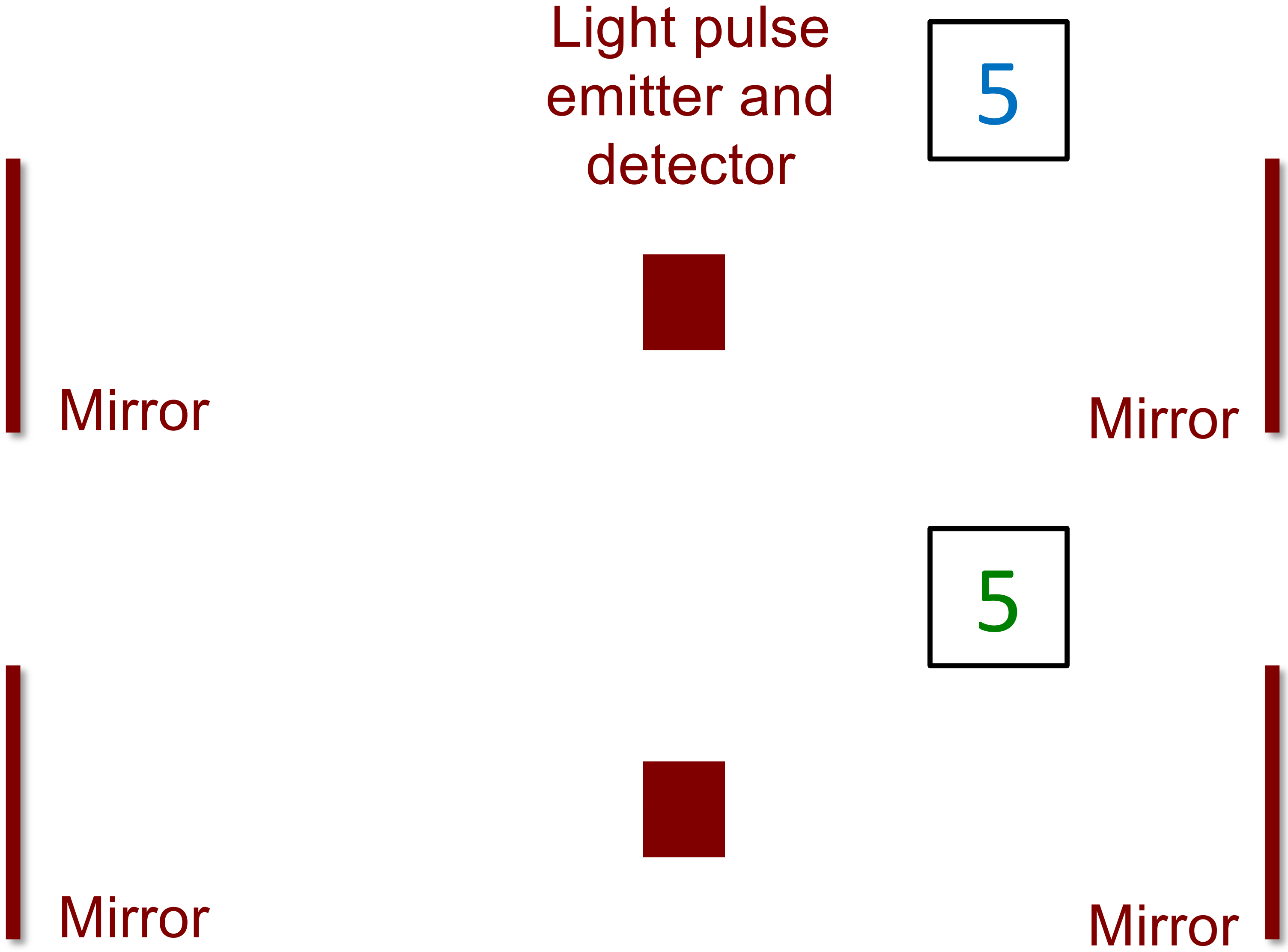


Measuring a train length?

- In the track inertial frame
  - Lucy and Rick get the signals at different times
  - The cats are not dropped simultaneously in this frame



Double light clock



Double light clock

- Clock rest frame
  - Mirrors same distance from source; light arrives at mirrors simultaneously and returns to detector at same time

- Moving frame
  - Just as with trains, light will no longer hit mirrors simultaneously

Double light clock

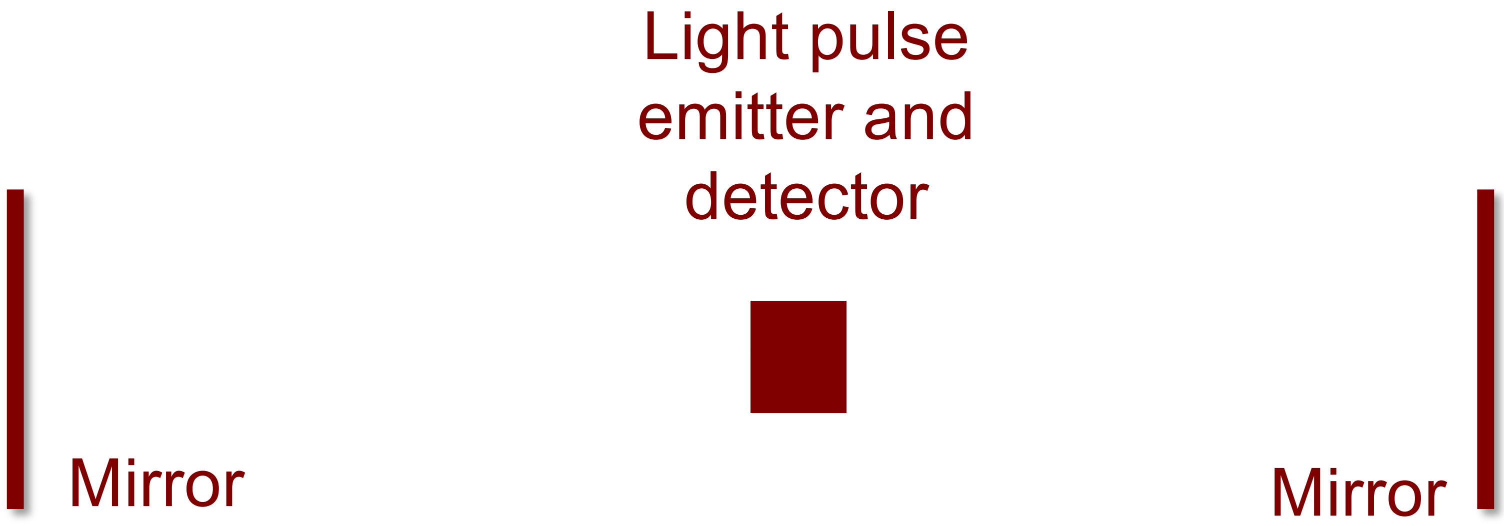
The "relativity of simultaneity"

- Arrival at mirrors (which are at diff. positions)
  - Simultaneous in clock frame
  - Not simultaneous in moving frame

- Arrival at detector (same position for both pulses)
  - Simultaneous in all inertial frames

- "The relativity of simultaneity"
  - Two simultaneous occurrences which are not in the same position are no longer simultaneous if viewed from any other inertial frame

Synchronising clocks



- e.g. if took tick of our clock to be arrival of light at the mirrors rather than detector, could set t=t'=0 when light leaves emitter

Synchronising clocks

Measuring lengths

- Similarly for measurement of lengths:
  - Measure two ends of rod simultaneously in rest frame -> proper length, l
  - Measurement *not* simultaneous in moving frame; if measure two ends of a moving rod at different times, of course find a different length l' !

- So is the rod really contracted or not?! Will it fit through a gap of size l' rather than l ?

Example

- Charged pions ($\pi^{\pm}$) are particles that decay with an average lifetime of τ = 26ns when at rest
  - They can be thought of as having an "internal" clock

- The CERN laboratory in Geneva can create and accelerate pions to $\beta \sim 0.9999995$, so $\gamma \sim 1000$
  1. What is the average pion lifetime in the laboratory frame?
  2. What is the average distance in the laboratory the pions will travel before decaying?
  3. How far does this distance appear to be (i.e. how far does the lab move) for an observer in the pion rest frame?

What we did today

- Lorentz (or length) contraction
  - An observer sees a moving object get shorter along its direction of motion by a factor of γ
- Non-simultaneity
  - Two occurrences that happen at the same time for one observer (i.e. are simultaneous) do not always occur at the same time for a moving observer
  - If they are at the same position, then they are simultaneous for all observers