Year 1 – Relativity
Lecture 8

Mitesh Patel

Overview of lectures

- Lecture 1: Introduction, concepts and classical results
- Lecture 2: The postulates of Relativity
- Lecture 3: Length contraction and simultaneity
- Lecture 4: The Lorentz transformations
- Lecture 5: Space-time diagrams and world lines
- Lecture 6: Four-vectors and causality
- Lecture 7: Energy and momentum
- Lecture 8: Rest mass energy and particle decays
- Lecture 9: Particle reactions
- Lecture 10: The relativistic Doppler effect

Previously on Relativity

- Met the energy and momentum four-vector
  - Combined as $(E,\underline{p}c)$
  - Changes between frames using Lorentz transformations (like all four-vectors)
- Properties of energy and momentum
  - When at rest:

$$E = mc^2, \;\; \underline{p} = \underline{0}$$

  - When moving with velocity $\underline{u}$

$$E = \gamma_u mc^2, \;\; \underline{p} = \gamma_u m\underline{u}$$

  - The length-squared invariant is $(mc^2)^2 = E^2 - p^2c^2$
  - Photons are special case with $E = |\underline{p}|c$

What we will do today

- Look at what mass means
  – Not the same as in classical physics
- Discuss energy and momentum conservation
  – Both are conserved, as is the case classically
  – But mass conservation is very different from the classical situation
- Look at the kinematics of particle decays
  – Use energy and momentum conservation to find what happens

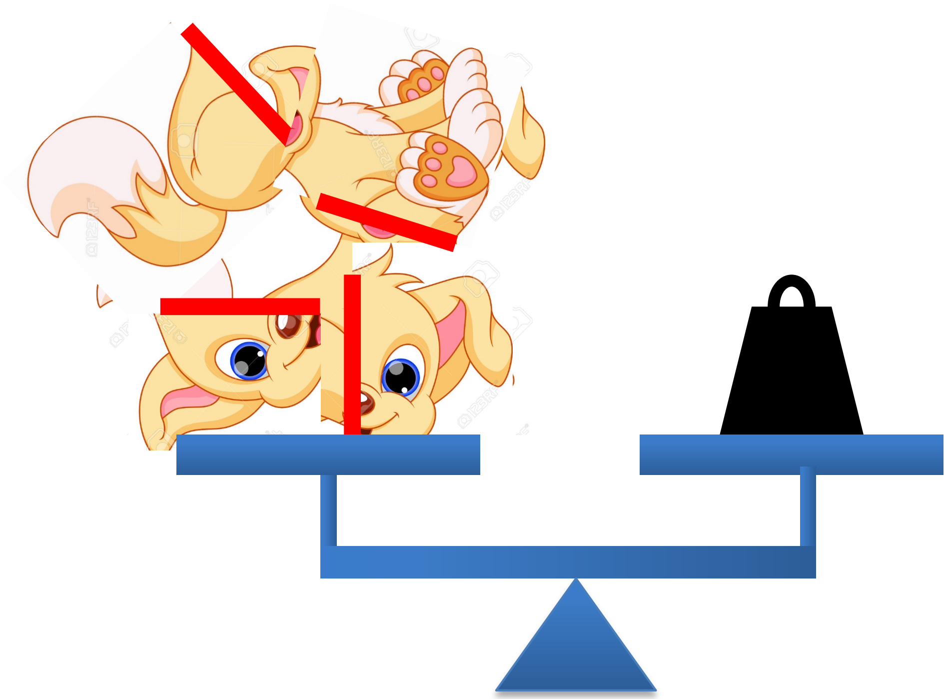
Scalars and vectors

- Before this course, you knew about scalars and vectors
  - These properties are defined by their rotations
  - These are often called "three-scalars" and "three-vectors" where "three" is shorthand for "property under rotation"
  - Often use the term "invariant" instead of "scalar"
- We now have four-scalars and four-vectors
  - Where "four" means "property under Lorentz transformations"
- Mapping one to the other is not obvious
  - Some three-scalars are also four-scalars; e.g. m
  - Some three-scalars are part of four-vectors; e.g. E, t
  - Some three-vectors are parts of four-vectors; e.g. $\underline{r}$, $\underline{p}$
  - Some three-vectors form other structures; e.g. $\underline{v}$, $\underline{E}$&$\underline{M}$ fields

Units

- Almost all experimental tests of Special Relativity have involved particle physics
  - Mass and hence energy scales are very small
  - SI units are inconvenient; use eV energy units
  - An electron gains 1 eV of energy in moving through 1V, so 1 eV = 1.6×10^{-19} J
  - Also use keV (10^3 eV), MeV (10^6 eV), GeV (10^9 eV)…
- Dimensions: $[p] = [E/c]$ and $[m] = [E/c^2]$
  - Can also give p in units of eV/c, m in eV/c^2
  - E.g. electron rest mass energy $mc^2 = 8.2 \times 10^{-14}$ J = 0.511 MeV
  - Therefore electron mass m = 0.511 MeV/c^2

Mass in classical physics



Su[illegible] masses of all objects
is conserved classically

Mass in relativity

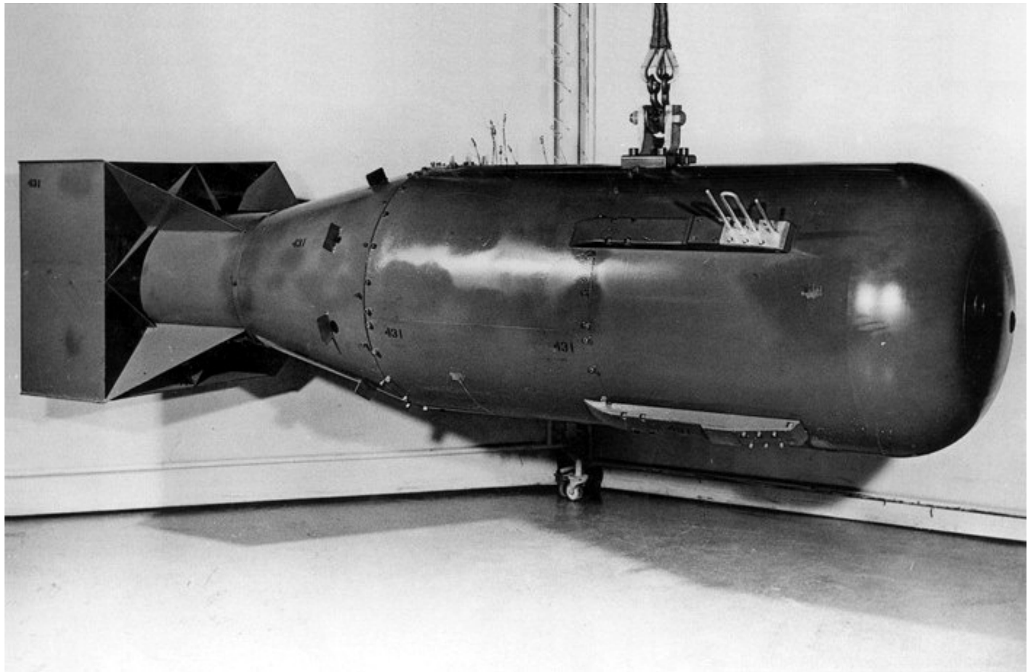
Classical vs Relativistic

- Bring e and p together to make a hydrogen atom
  - A 13 eV photon is radiated off a when e-p bind together
- Classical view
  - Mass of H atom = m_e+m_p
  - H atom has binding energy of 13 eV
- Relativistic view
  - Initial energy = $(m_e+m_p)c^2$
  - Energy lost through photon = 13 eV
  - Mass of hydrogen atom = $m_e+m_p-13\ \mathrm{eV}/c^2$
  - Atom is lighter than m_e+m_p
  - Reduction factor: $13\ \mathrm{eV}/(m_e+m_p)c^2 \sim 10\ \mathrm{eV}/10^9\ \mathrm{eV} \sim 10^{-8}$
  - Very hard to detect as "chemical" energies << m_pc^2

Example – uranium

- Uranium-235 undergoes induced decay ('fission') when it is hit by a slow neutron
  - It usually splits into two other nuclei and several neutrons
  - The average mass change is ~ 200 MeV/c^2
  - Uranium metal contains ~ 5×10^{28} nuclei per m^3
- Estimate the energy (in J) released if all the nuclei in a $10 \times 10 \times 10$ cm^3 block of uranium fissioned

Little Boy: first atomic bomb



- Contained a cylinder of uranium
  - 16cm diameter x 18 cm long
  - Total energy release = 63 TJ = 6.3×10^{13} J
  - Inefficient as not all U-235, not all fissioned, etc.

Mass in relativity

- Both classically and relativistically, energy and momentum are conserved

- BUT – the mass of an object can change, so the sum of the masses will not in general be conserved in reactions
  - initial $\Sigma_i m_i \neq$ final $\Sigma_i m_i$
  - Classically, mass appears to be conserved because we only rearrange electrons and nuclei – changes in mass then v small

- However, we know that

$$E_T^2 = p_T^2c^2 + m_T^2c^4 \quad \longrightarrow \quad m_T = \sqrt{(E_T^2 - p_T^2c^2)}/c^2$$

since E and p are conserved, m_T is conserved

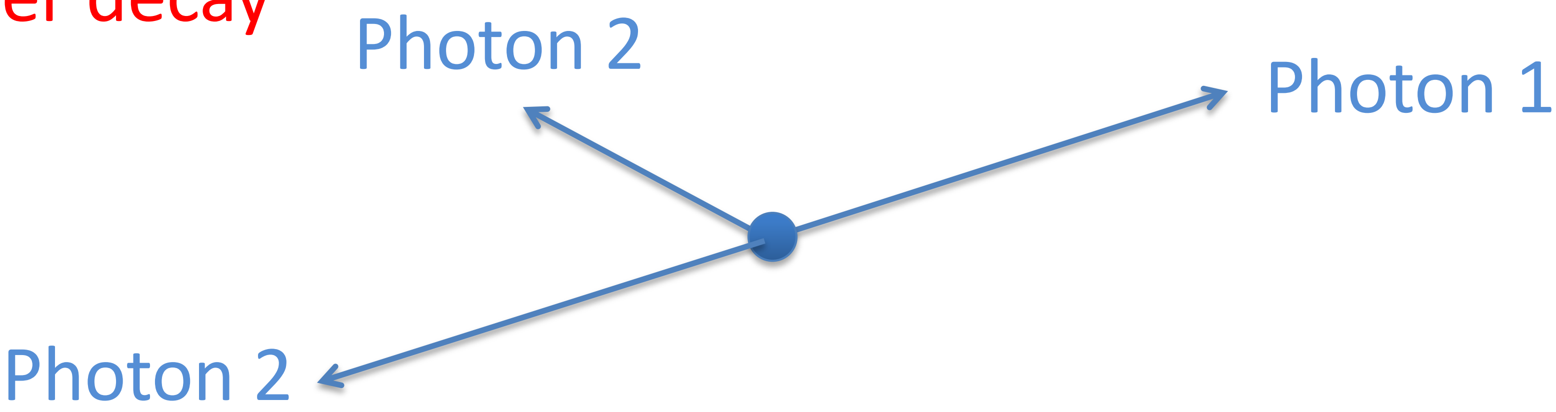
Higgs boson decay at rest

Before decay

Higgs m_H

$$E_T = E_H = m_H c^2, \;\; \underline{p}_T = \underline{p}_H = \underline{0}, \; m_T = m_H$$

After decay



$$E_T = E_{\gamma 1} + E_{\gamma 2}, \;\; \underline{p}_T = \underline{p}_{\gamma 1} + \underline{p}_{\gamma 2} = \underline{0}$$

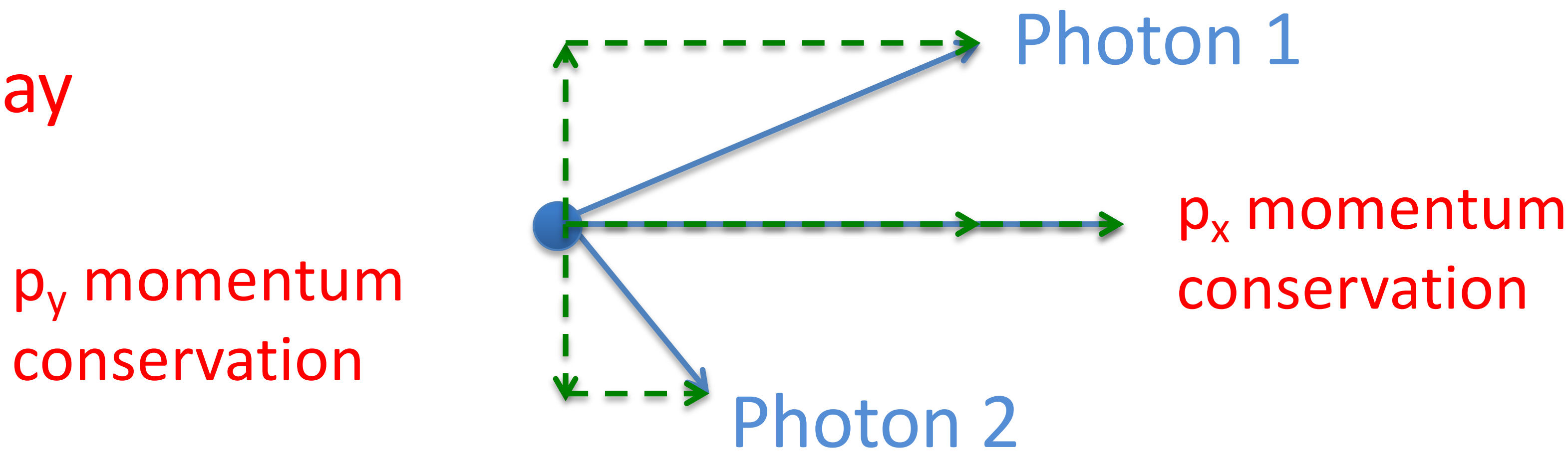
Momentum conservation

Moving Higgs boson decay

Before decay

$$E_T = E_H,\ \ \underline{p}_T = \underline{p}_H,\ m_T = m_H$$

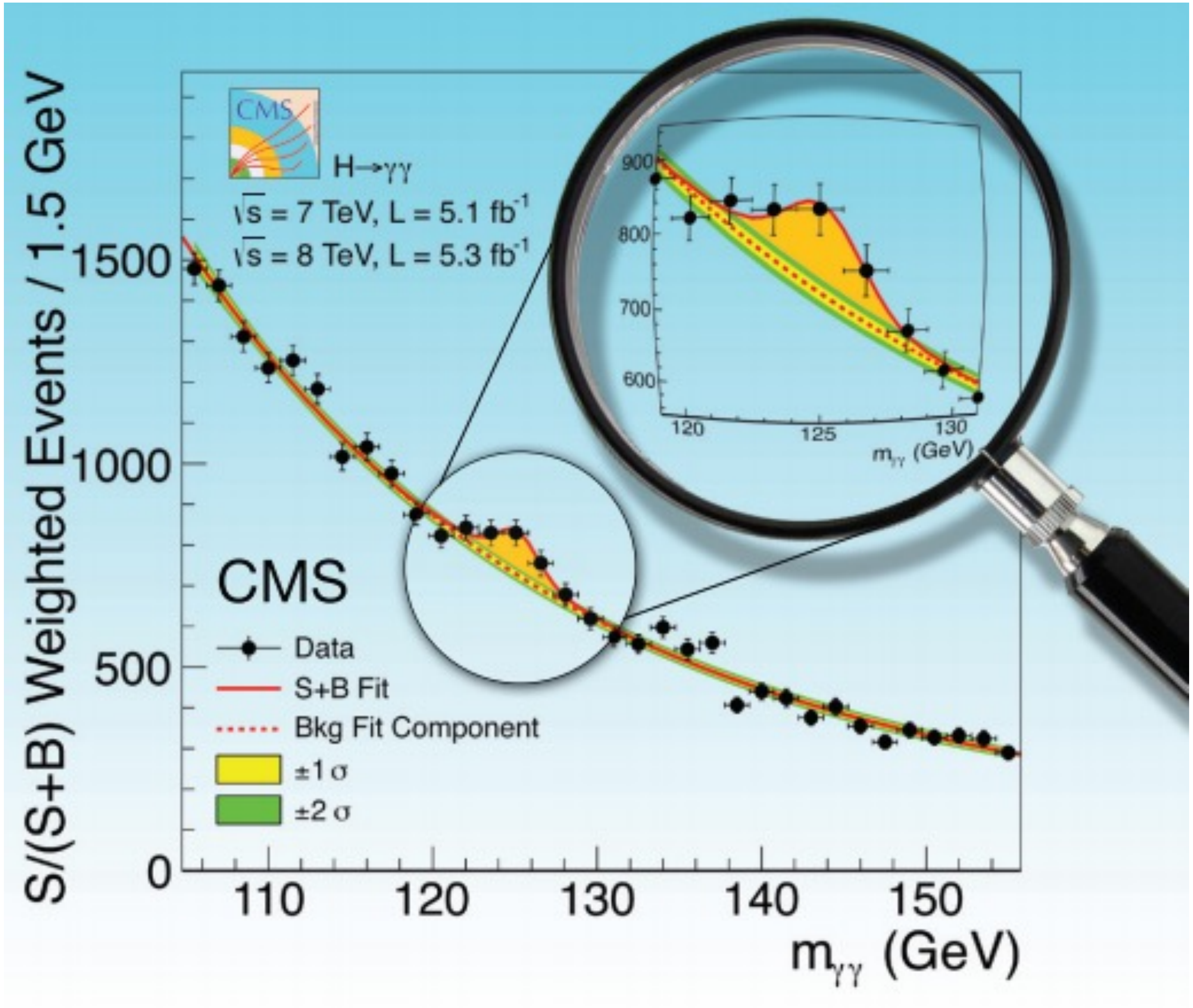
After decay



$$E_T = E_{\gamma 1} + E_{\gamma 2},\ \ \underline{p}_T = \underline{p}_{\gamma 1} + \underline{p}_{\gamma 2}$$

Solving for the energies

The Higgs discovery



Example – pion decay

- A charged pion decays to a muon and a neutrino
  - Decay formula: $\pi^+ \rightarrow \mu^+ + \nu$
- The neutrino mass is negligibly small and the other masses are
  - $m_\pi = 139.6\ \mathrm{MeV}/c^2$, $m_\mu = 105.7\ \mathrm{MeV}/c^2$
- Find the change in the sum of the masses
- Find the total and kinetic energies of the muon and the neutrino in the pion rest frame

Solution for example – pion decay

- The masses are
  - $m_\pi = 139.6$ MeV/c^2, $m_\mu = 105.7$ MeV/c^2 , $m_\nu = 0$ MeV/c^2
  - so the mass change is $\Delta m = 139.6 - 105.7 - 0 = 33.9$ MeV/c^2
- Using the formula, the total energy of the muon is
  - $E_\mu = [(139.6)^2 + (105.7)^2 - (0)^2]/(2 \times 139.6) = 109.8$ MeV
  - so $K_\mu = E_\mu - m_\mu c^2 = 109.8 - 105.7 = 4.1$ MeV
- Similarly, the total energy of the neutrino is
  - $E_\nu = [(139.6)^2 + (0)^2 - (105.7)^2]/(2 \times 139.6) = 29.8$ MeV
  - so $K_\nu = E_\nu - m_\nu c^2 = 29.8 - 0 = 29.8$ MeV
- Cross check: the total kinetic energy is
  - $K_\mu + K_\nu = 4.1 + 29.8 = 33.9$ MeV $= \Delta mc^2$ as expected

What we did today

- Discussed the meaning of mass
  - Arises from all energy in the system
  - Total of individual object masses is not conserved
  - But total mass of system is conserved
- Discussed energy and momentum conservation
  - E and p conservation allows us to solve kinematic problems in particle decays
  - The sum of the particle masses is not always conserved, but the total invariant mass is
  - This allows us to identify particles by only measuring their decay products