Year 1 – Relativity
Lecture 10

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Overview of lectures

- Lecture 1: Introduction, concepts and classical results
- Lecture 2: The postulates of Relativity
- Lecture 3: Length contraction and simultaneity
- Lecture 4: The Lorentz transformations
- Lecture 5: Space-time diagrams and world lines
- Lecture 6: Four-vectors and causality
- Lecture 7: Energy and momentum
- Lecture 8: Rest mass energy and particle decays
- Lecture 9: Particle reactions
- Lecture 10: The relativistic Doppler effect

Previously on Relativity

- Clarified invariant, conserved and constant
  - Some quantities are more than one of these
  - They tend to be the most useful
- Introduced the centre-of-mass frame
  - Often the easiest to do calculations in
- Looked at particle reactions
  - Energy and momentum conservation still holds
  - The mass sum can change from initial to final state
  - The centre-of-mass energy is critical in determining if a reaction can occur

Previously on Relativity

- Saw time slows down for moving objects
  - Time dilation: $T' = \gamma T$
- Looked at the energy and momentum four-vector for photons
  - Satisfies $E = |\underline{p}|c$
- Quantum mechanics gives
  - Planck-Einstein: $E = hf$
  - de Broglie: $p = h/\lambda$

What we will do today

- Look at the Doppler shift in relativity
  - Two physical effects
    - Time dilation: only in Relativity
    - Wavelength squeeze and stretch: same as for the classical Doppler effect
- Reproduce the Doppler equations using photon four-momentum
  - Considering it as a Lorentz transform from the frame where the light source is stationary
- Combines both the space-time and the energy-momentum parts of Relativity

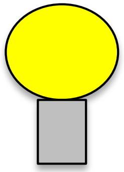
Classical Doppler shift

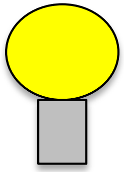
- Waves get "squeezed" as a sound source comes towards you and "stretched" as it moves away

Lower frequency

Higher frequency

Time dilation effect





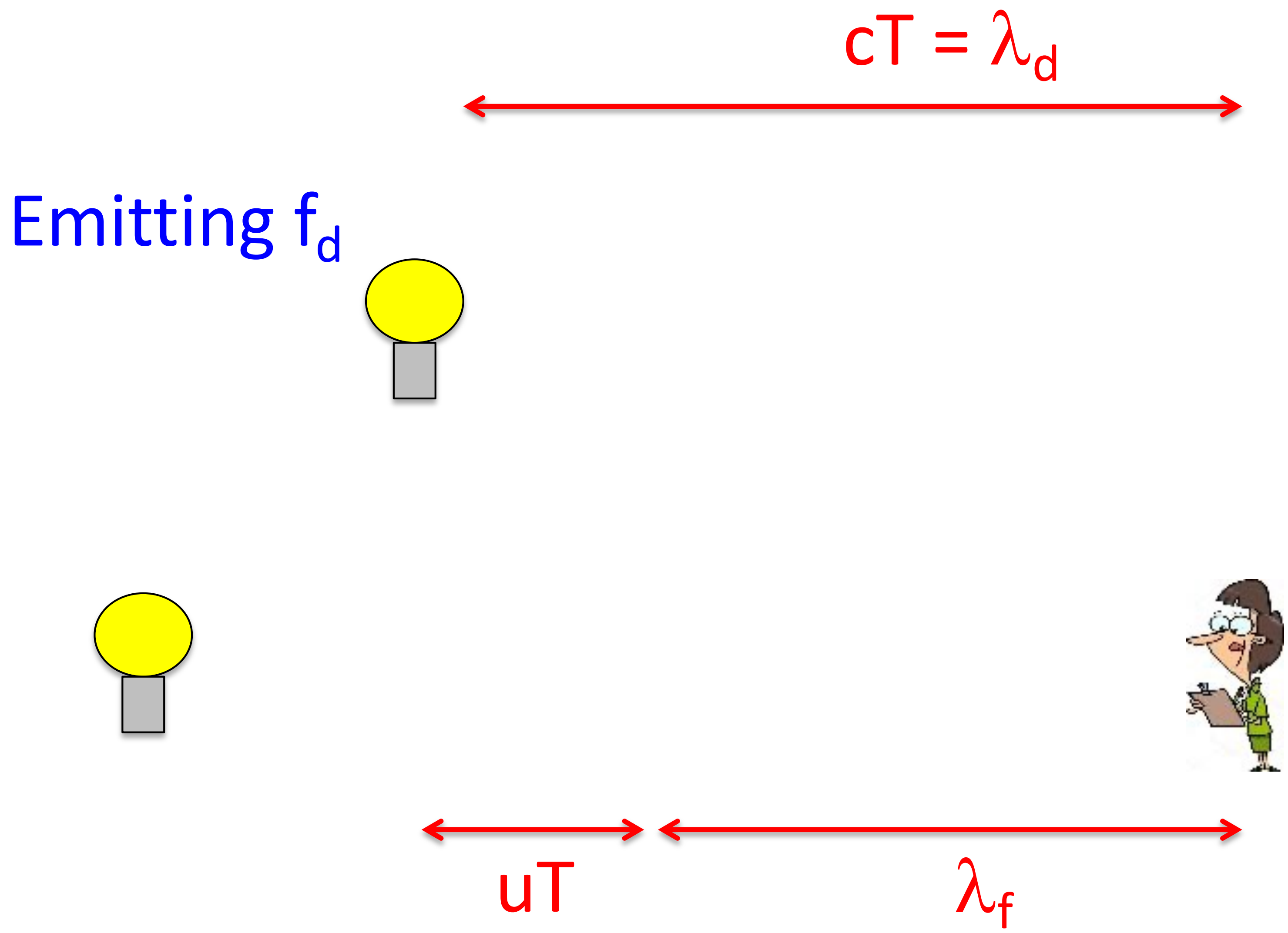
Speed u

Light pulse

Frequency f

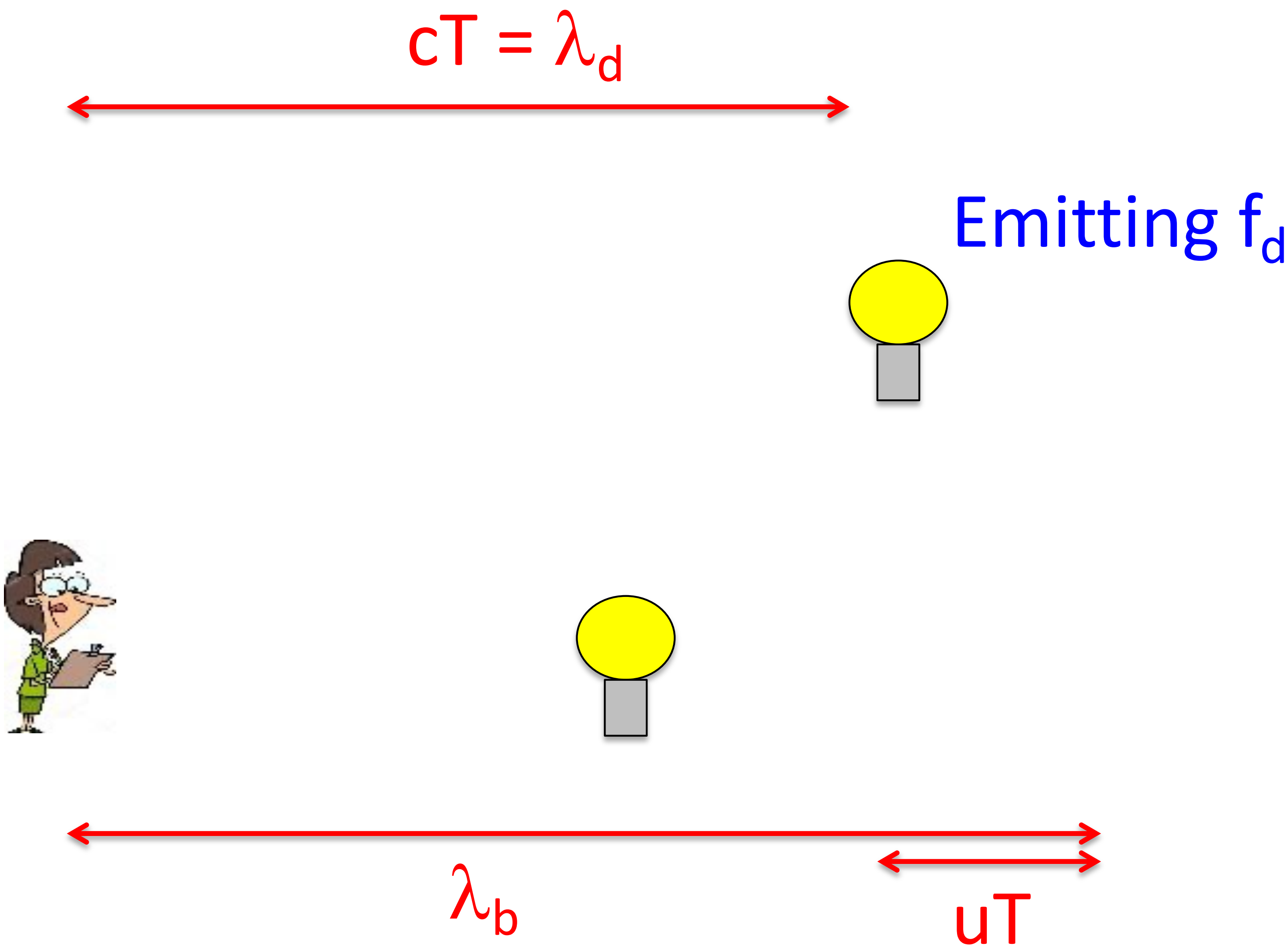
Frequency
$f_d = f/\gamma_u$

Wavelength squeeze



Derivation of resulting frequency

Wavelength stretch



Derivation of resulting frequency

Menti question

- Go to www.menti.com

- Two effects: time dilation always decreases f but forward wavelength squeeze always increases f
- Question: Is total $f_f > f$ (blue-shifted)

  or $f_f < f$ (red-shifted)

  A. Always blue-shifted
  B. Always red-shifted
  C. Depends on the value of β_u

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Some useful expressions

Expansion of the Universe

- Universe is expanding – galaxies further from us moving away faster than those closer to us

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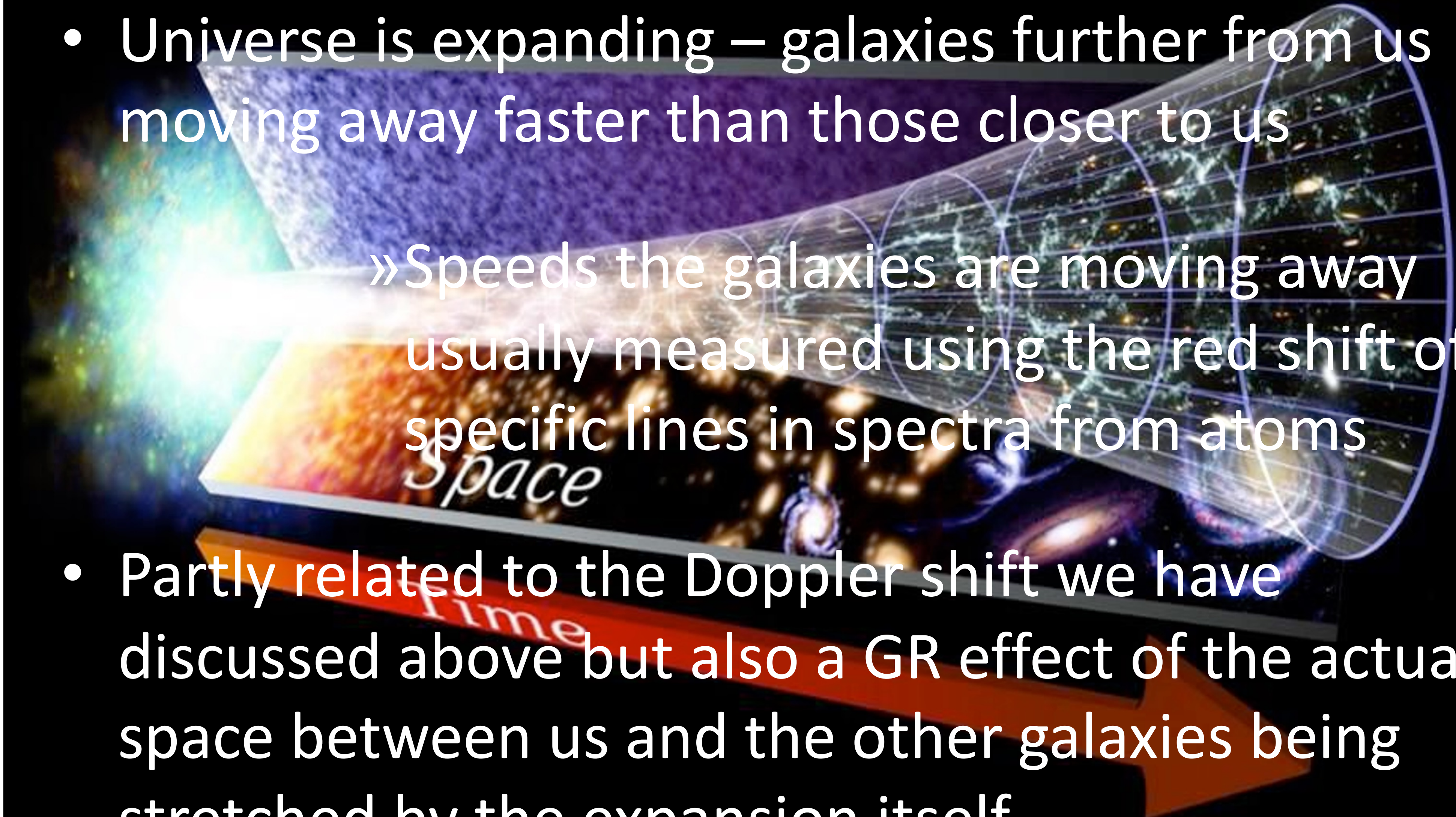
  - »Speeds the galaxies are moving away usually measured using the red shift of specific lines in spectra from atoms



- Partly related to the Doppler shift we have discussed above but also a GR effect of the actual space between us and the other galaxies being stretched by the expansion itself

Doppler shift from four-momentum

- A tank travelling at speed v that fires a shell with speed u will classically give a shell with speed u+v
- Shell “thrown forwards” by tanks motion
- If instead tank were firing photons, we could not increase their speed – always travel with c – but can still give extra momentum and hence energy

Lorentz transformation of photon

Rest frame of source

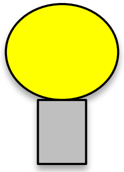
Moving frame of source = rest frame of observer

Requires Lorentz transformation in –x direction

Lorentz transformation of photon

Time dilation effect again

Moving frame of source



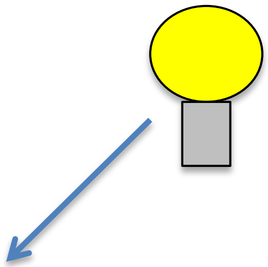
Photon

$(E', \underline{p}'c)$

Rest frame of source

Photon

$(E, \underline{p}c)$



Time dilation effect again

Change in angle - an Example

- A beam of light is emitted at an angle θ to the x-axis in a reference frame, S

- Show that in a reference frame S' moving with velocity v in the positive x-direction, the beam of light still has speed c

- Show that it is emitted at an angle θ' to the x' axis, where, $\cos\theta' = (\cos\theta - \beta)/(1-\beta\cos\theta)$

What we did today

- Looked at the Doppler shift in relativity
  - Saw there were two physical effects
  - Time dilation: only in Relativity
  - Wavelength squeeze and stretch: same as for the classical Doppler effect
- Reproduced the Doppler equations using photon four-momentum
  - Is easier to do but does not make the separate physical effects as clear


OMG – we
survived the
whole lecture...