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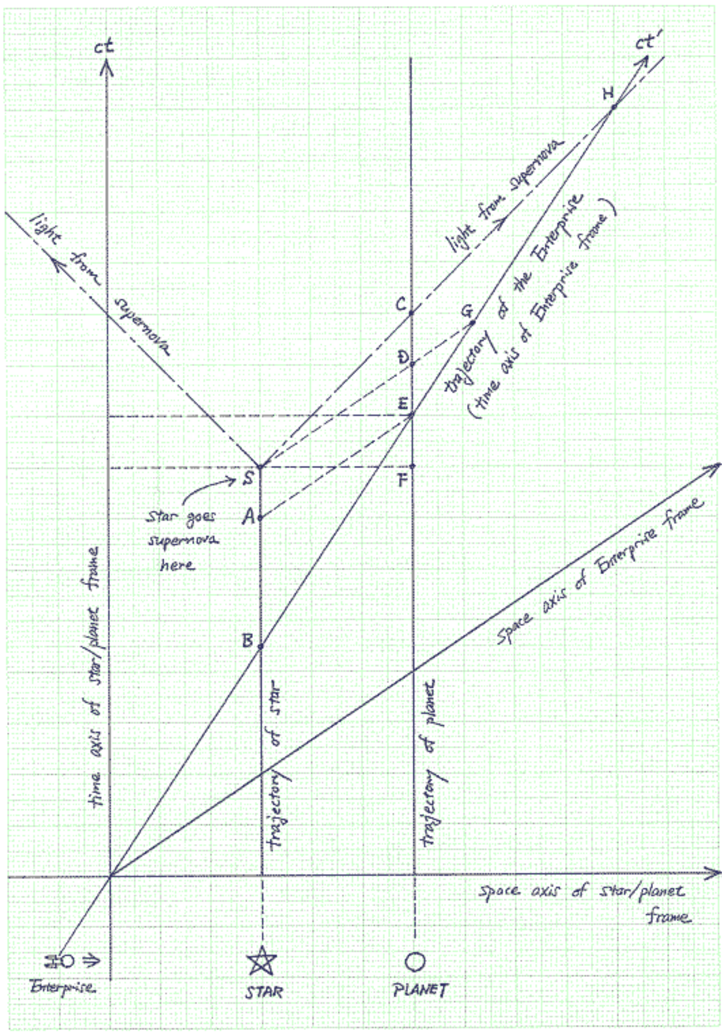
Relativity – Seminar 2, Supernova and CERN

A star is about to explode in a supernova and a planet orbiting it must be evacuated. The starship Enterprise is sent to the planet to pick up some Imperial College Physics students on a field trip to observe the star's final stages.

Assume that the planet is not moving relative to the star. The Enterprise will fly by at a constant velocity past the star and then the planet, and beam up the students without stopping. The supernova explosion occurs at spacetime point S.

1. Draw a spacetime diagram with axes for the frame of the star/planet and axes for the Enterprise. What does the trajectory of the enterprise look like in this diagram? Add to your diagram the world lines for the star and the planet.

   ANSWER: Note this question is adapted from Tatsu Takeuchi at Virginia Tech, (http://www1.phys.vt.edu/˜takeuchi/relativity/practice/problem13.htm)



2. Add the following events to your spacetime diagram, labelled A-H.

   - A In the Enterprise frame, when the Enterprise arrives at the planet the star (not yet gone supernova) is at this spacetime point.
   - B The Enterprise flies by the star.
   - C Light from the supernova reaches the planet.

- D In the Enterprise frame, the planet is at this spacetime point when the supernova occurs.
- E The Enterprise arrives at the planet.
- F In the planet's frame, the planet is at this spacetime point when the supernova occurs.
- G In the Enterprise frame, the Enterprise is at this spacetime point when the supernova occurs.
- H Light from the supernova reaches the Enterprise.

3. Rank in chronological order for an observer in the planet's frame:

   - 1. A spectator on the planet sees the supernova.
   - 2. The Enterprise arrives at the planet.
   - 3. The star explodes in a supernova.
   - 4. A spectator on the Enterprise sees the supernova.

   ANSWER: (3), (2), (1), (4).

4. Rank in chronological order for an observer in the Enterprise frame:

   - 1. A spectator on the planet sees the supernova.
   - 2. The Enterprise arrives at the planet.
   - 3. The star explodes in a supernova.
   - 4. A spectator on the Enterprise sees the supernova.

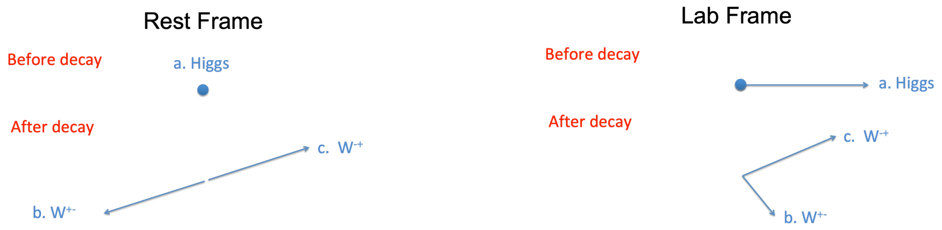
   ANSWER: The solution depends on the velocity that is chosen - for that used in the diagram above the time-ordering is: (2), (3), (1), (4).

5. In seminar 1, we found that the order depended on v. Is a different time-order possible here too?
   ANSWER: Yes!

Computing the kinematics of the Higgs to W^+W^- decay

1. For a two-body decay $a \to b + c$, draw a diagram of the decay in its rest frame and in the lab frame, assuming that the particle a is produced moving in the positive z direction in the lab-frame.

   ANSWER:



2. What is the expression for the energies of particle b and c in terms of the invariant masses, m_a, m_b and m_c [hint - see Lecture 8 notes!]

   ANSWER:

$$E_b + E_c = m_a c^2 \tag{1}$$

The two daughter particles must be back-to-back to conserve momentum, i.e. $\vec{p}_c = -\vec{p}_b$ so $p_c^2 = p_b^2$. Their momenta magnitudes are related to their energies by

$$p_b^2 c^2 = E_b^2 - m_b^2 c^4, \qquad p_c^2 c^2 = E_c^2 - m_c^2 c^4$$

so momentum conservation requires

$$E_b^2 - m_b^2 c^4 = E_c^2 - m_c^2 c^4$$

We can write

$$E_b^2 - E_c^2 = (E_b + E_c)(E_b - E_c) = m_b^2 c^4 - m_c^2 c^4$$

Using Eqn. 1 this becomes

$$E_b - E_c = \frac{m_b^2 c^4 - m_c^2 c^4}{m_a c^2} \tag{2}$$

Taking the sum and difference of Eqns. 1 and 2 gives

$$E_b = \frac{m_a c^2}{2} + \frac{m_b^2 c^4 - m_c^2 c^4}{2 m_a c^2} = \frac{m_a^2 c^4 + m_b^2 c^4 - m_c^2 c^4}{2 m_a c^2}$$

and

$$E_c = \frac{m_a c^2}{2} - \frac{m_b^2 c^4 - m_c^2 c^4}{2 m_a c^2} = \frac{m_a^2 c^4 + m_c^2 c^4 - m_b^2 c^4}{2 m_a c^2}$$

The equation is symmetric under interchange of $1 \leftrightarrow 2$, as would be expected.

3. Compute the magnitude of the momentum of one of the decay products b or c. Draw a diagram of the momentum vector of that decay product in spherical coordinates

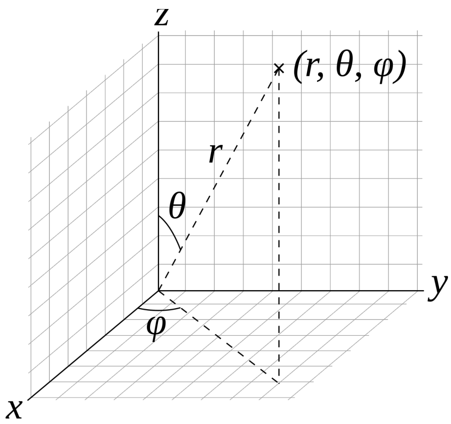
   ANSWER:

   As above,

$$p_b^2 c^2 = E_b^2 - m_b^2 c^4, \qquad p_c^2 c^2 = E_c^2 - m_c^2 c^4$$

which can be rearranged for the momentum.

The diagram should be standard, right-handed spherical polar coordinates i.e. are looking for x component to have $\sin\theta \times \cos\phi$, y component to have $\sin\theta \times \sin\phi$, and z component to have $\cos\theta$.



4. Use your diagram to derive the 3-momentum components and combine with your previous results to give the four-momentum in terms of the angles θ, ϕ

   ANSWER:

$$\begin{aligned} p_x &= p\sin\theta \times \cos\phi \\ p_y &= p\sin\theta \times \sin\phi \\ p_z &= p\cos\theta \end{aligned}$$

5. If the decay products lie in the plane $x = 0$, what must the value of ϕ be?

   ANSWER:

   $\phi = \pi/2$

6. Assuming the rest frame values of this decay product's E, p, what values are expected in the lab frame after a boost in the z-direction?

   ANSWER:

   Note that in the lectures we have been boosting in the x direction so you will get this wrong if you take the solution from there.

$$\begin{pmatrix} E' \\ p'_x c \\ p'_y c \\ p'_z c \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} E \\ p_x c \\ p_y c \\ p_z c \end{pmatrix}$$

7. For two example masses, sketch the allowed range of boosted values in the E, pc plane. Contrast your sketch to what happens to spacetime position values after a boost.

   ANSWER: