Year 1 – Relativity
Lecture 9

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Overview of lectures

- Lecture 1: Introduction, concepts and classical results
- Lecture 2: The postulates of Relativity
- Lecture 3: Length contraction and simultaneity
- Lecture 4: The Lorentz transformations
- Lecture 5: Space-time diagrams and world lines
- Lecture 6: Four-vectors and causality
- Lecture 7: Energy and momentum
- Lecture 8: Rest mass energy and particle decays
- Lecture 9: Particle reactions
- Lecture 10: The relativistic Doppler effect

Previously on Relativity

- Saw energy, momentum and mass conservation
  - E and $\underline{p}$ are conserved, as is the case classically
  - But mass conservation is very different from the classical situation; initial $\Sigma_i m_i \neq$ final $\Sigma_i m_i$
  - The conserved quantity is the Lorentz invariant from total four-momentum ($E_T, \underline{p}_T c$)
- Energy and momentum conservation allows calculation of kinematics of particles after decay
  - For particle decay $0 \rightarrow 1 + 2$, need $m_0 > m_1 + m_2$
  - $E_1 = (m_0^2 + m_1^2 - m_2^2)c^2/2m_0$
  - E_2 has 1,2 swapped

What we will do today

- Clarify "invariant", "conserved" and "constant"
  - These are often confused
- Define the "centre-of-mass" frame
  - Convenient frame for calculations
- Look at particle reactions
  - Energy and momentum conservation still holds
  - Allows us to again work out the kinematics of the reactions
  - Figure out if/when reactions require a minimum energy (a "threshold") to proceed

Constant, invariant, conserved

- CONSTANT: a quantity which has a fixed value
  - E.g. speed of light c, Planck's constant h, electron mass m_e
- INVARIANT: a quantity which has the same value before and after a Lorentz transformation
  - E.g. any four-vector length-squared such as $(mc^2)^2$ for an object (so m is invariant), or $(c\tau)^2$ between two events (so τ is invariant)
  - It may depend on time
- CONSERVED: a quantity which does not change with time within one frame
  - Can have different (but still conserved) values in different frames
  - E.g. total energy and momentum
- The speed of light c is constant, invariant and conserved
- E_T and $\underline{p}_T$ are conserved, while m_T is invariant and conserved

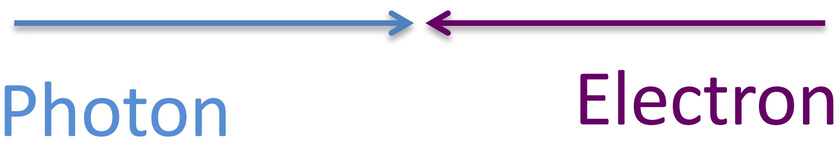
Centre-of-mass (CM) frame

- Any system has a total energy E_T and momentum $\underline{p}_T$
  - Simply found by adding all the separate contributions to the energy and momentum
  - The Lorentz invariant of $(E_T, \underline{p}_T c) = m_T^2 c^4$
- An inertial frame can always be found which has $\underline{p}_T = \underline{0}$
  - This particular frame is called the "centre-of-mass" frame for the system
  - It is alternatively known as the "centre-of-momentum" frame
  - In this frame, the total energy $E_{CM} = E_T = m_T c^2$
- Calculations are typically simplest in this frame
  - E.g. for a decay, this frame is the rest frame of the decaying particle, which is the frame we worked in for the last lecture

Compton scattering in CM

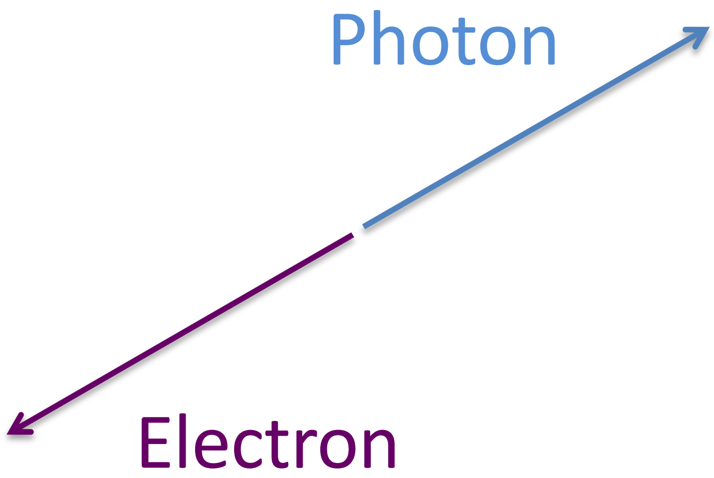
- Simple reaction: $e + \gamma \rightarrow e + \gamma$ = "scattering"
  - Sum of initial masses = sum of final masses
  - No energy converted to or from mass

Before reaction



$$E_{CM} = E_\gamma + E_e \qquad \underline{p}_T = \underline{p}_\gamma + \underline{p}_e = 0$$

After reaction

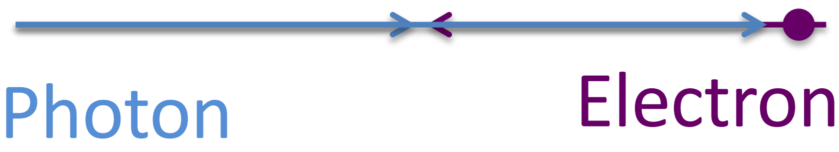


- Both the photon and electron have same energy before and after (but $E_\gamma \neq E_e$)

Compton scattering in lab

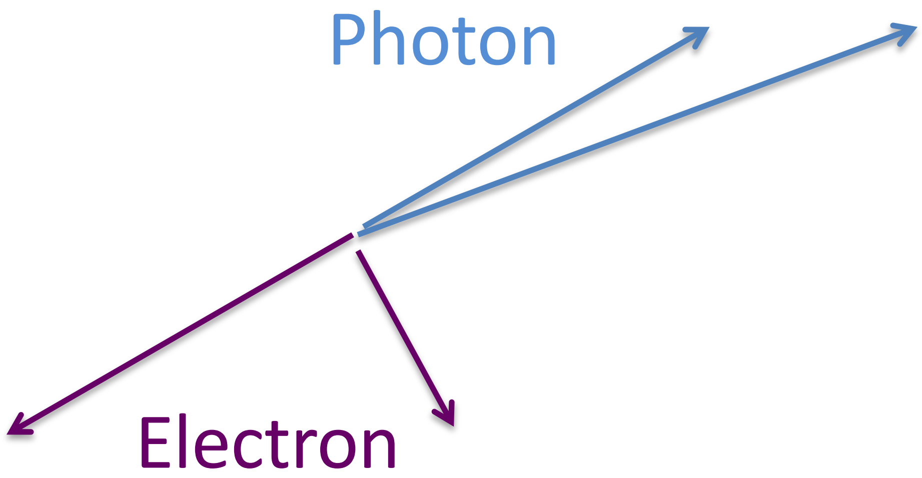
- Experiment typically done in "lab frame"
  - Bombard block of material with high energy gamma rays
  - Lorentz transform from CM frame to frame with $\beta_e = 0$

Before reaction



After reaction

- Photon and electron both change energies
- Angle randomly chosen according to QM



“Pretend particle” trick

- For any 2 particle → 2 particle reaction
  - Pretend a single particle is created by the incoming particles and decays to the outgoing particles
  - Mass of pretend particle must be $m_T = E_{CM}/c^2$
- We got the equations for the energies of the final particles for a decay mass m_0 in the last lecture
  - Can apply this formula to the final particles using $m_0c^2 = E_{CM}$
- E and <u>p</u> conservation does not depend on time order, so which is initial and which is final does not matter
  - E, <u>p</u> conservation for creation or decay is the same
  - Can also apply the formula to the initial particles, again using $m_0c^2 = E_{CM}$ and note this part works even for 2 → N reactions

Example – Compton scattering

- The CM energy for a Compton scattering experiment is $E_{CM} = m_T c^2 = 1$ MeV
  - Reminder of formula: $E_1 = (E_{CM}{}^2 + m_1{}^2c^4 - m_2{}^2c^4)/2E_{CM}$
  - Approximate $m_e = 0.5$ MeV/c^2

1. What are the incoming and outgoing photon and electron energies in the CM frame?
2. To get the same CM energy, what incoming photon energy is needed in the lab frame, where the electron is at rest?
3. Cross check the total energy and momentum in the lab frame give the correct m_T
4. What are the highest and lowest outgoing photon energies in the lab frame ?

Reactions can change mass sum

- More generally
  - Sum of initial masses ≠ sum of final masses
- If initial sum > final sum
  - Extra rest mass energy is converted into kinetic energy of the final particles
- If initial sum < final sum
  - Missing rest mass energy is taken from kinetic energy of initial particles
  - Requires there to be enough initial kinetic energy
  - Reaction will have a minimum energy “threshold”

Matter and antimatter

- Matter and antimatter can annihilate
  - Can convert all mass into energy
  - Extreme case of change of mass sum

Pro: would wipe out My Little Pony

Con: 100% conversion of the mass of two horses would be ~ 10^{19} J so would also wipe out much of SE England

Electron + positron annihilation

- Example reaction: $e^- + e^+ \rightarrow \gamma + \gamma$
  - Initial mass sum = $2m_e$ but final mass sum = 0
- In CM frame
  - Initial masses are the same so $E_{e-} = E_{e+}$
  - Final masses are the same so E_γ is the same for both photons
  - E conservation means $E_{e-} = E_{e+} = E_\gamma$
  - All electron rest energy goes to photon kinetic energy
- Reaction can happen even when $K_e \sim 0$
  - Photons created with energy $E_\gamma = m_e c^2 = 0.511$ MeV

e^+ and e^- annihilation signal

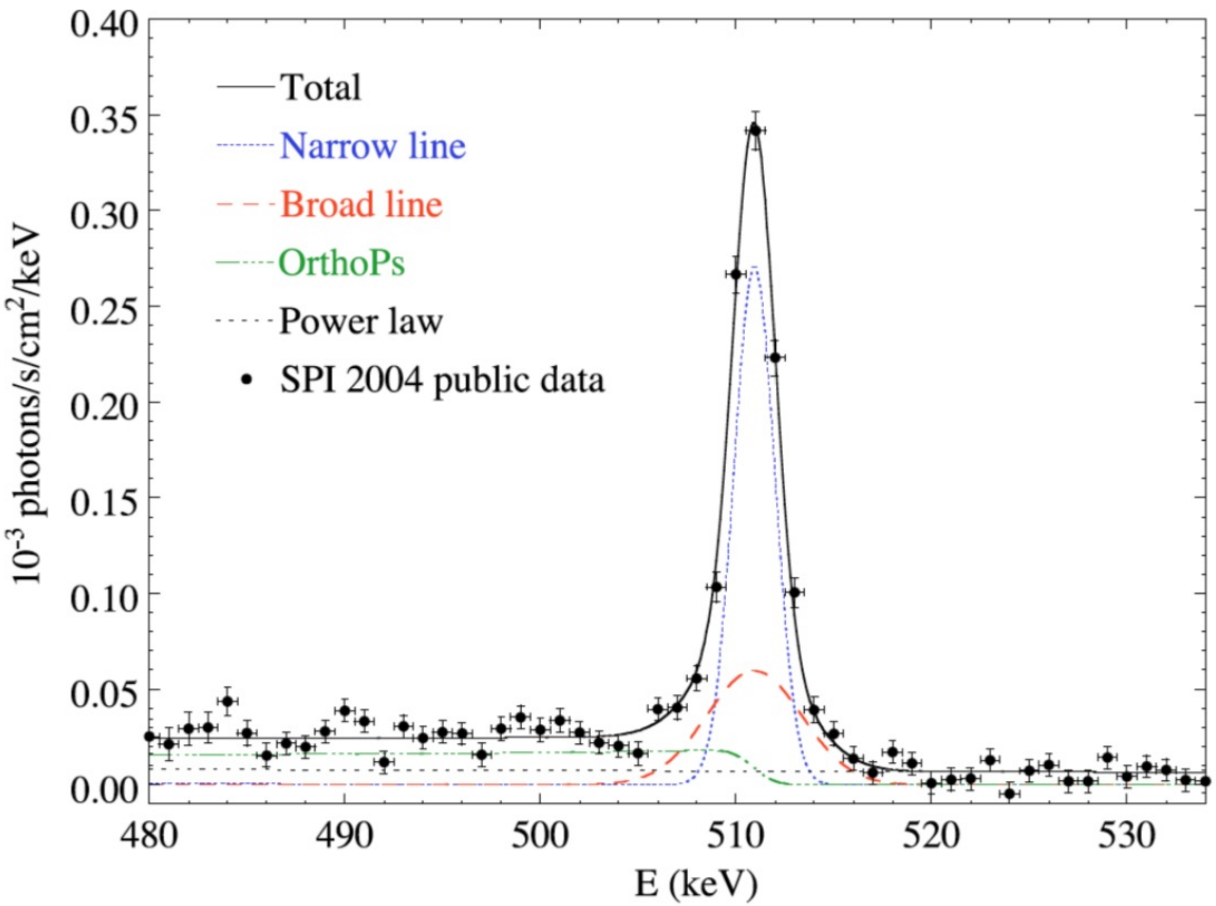


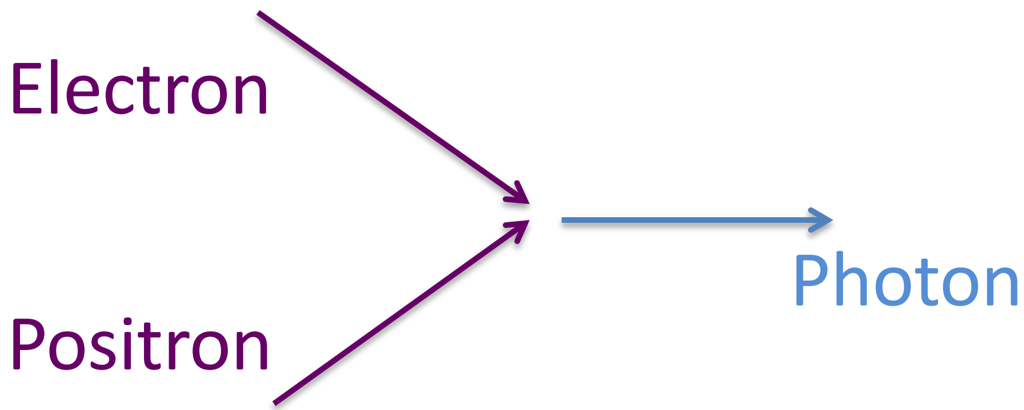
Fig. 4 The spectrum of the Galactic center $e^- e^+$ annihilation observed by INTEGRAL/SPI (Jean et al.2006). The fit of the spectrum – i.e. the narrow and a broad Gaussian line – is constraining the physical conditions in the sites where annihilation occurs, i.e. the neutral and ionized warm phases of the interstellar medium [10].

Gamma-ray astronomy measurement of centre of galaxy

Menti question

- Go to www.menti.com

- Question 1: An electron and positron can annihilate to give a single photon: $e^- + e^+ \rightarrow \gamma$
  A. True
  B. False



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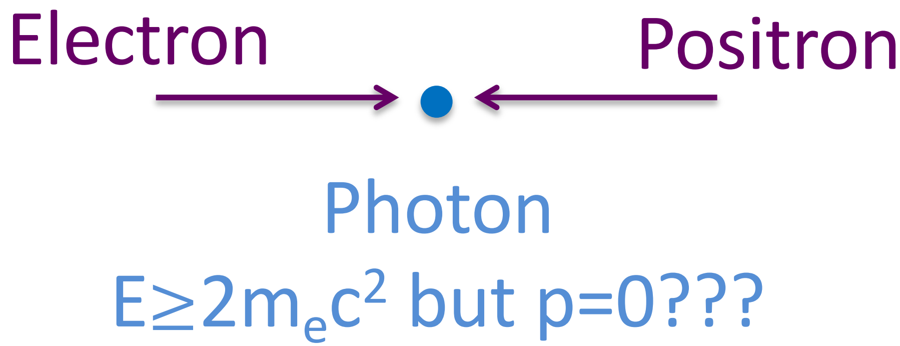
- Question 1: An electron and positron can annihilate to give a single photon: $e^- + e^+ \rightarrow \gamma$
  A. True
  B. False ✔

- Consider CM frame



Pair Production and Annihilation

- Picture shows pair-production:

  $\gamma + \gamma \rightarrow e^{+} + e^{-}$

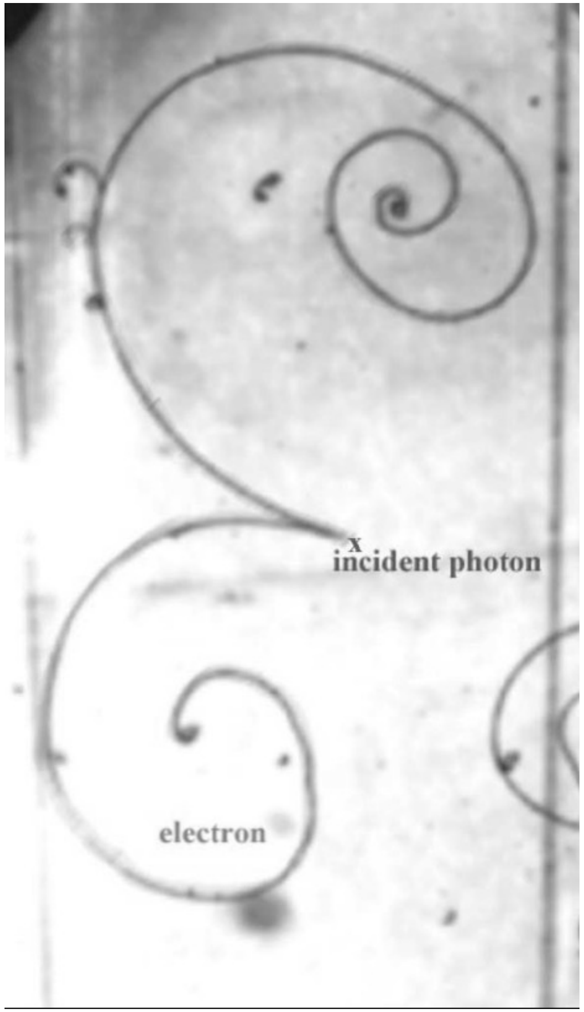
- Observe that particle and antiparticle are always created in pairs

- Just as annihilation also occurs in pairs:

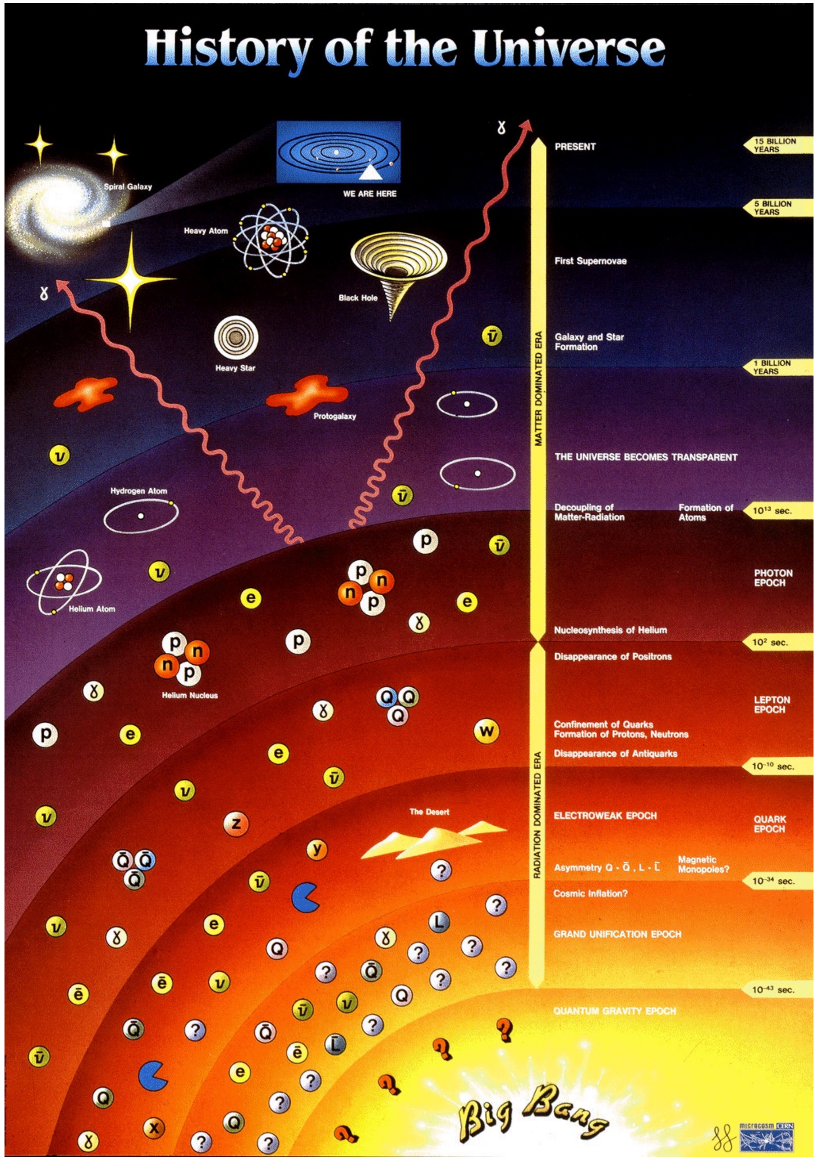
  $e^{+} + e^{-} \rightarrow \gamma + \gamma$

- Hence,

  Particles – Antiparticles = 0



The History of the Universe



- t = 13.7 × 10^{9} yrs

- All energy in Universe confined in a tiny region → extremely hot and dense

- ‘Soup’ of basic particles

Where did the antimatter go?

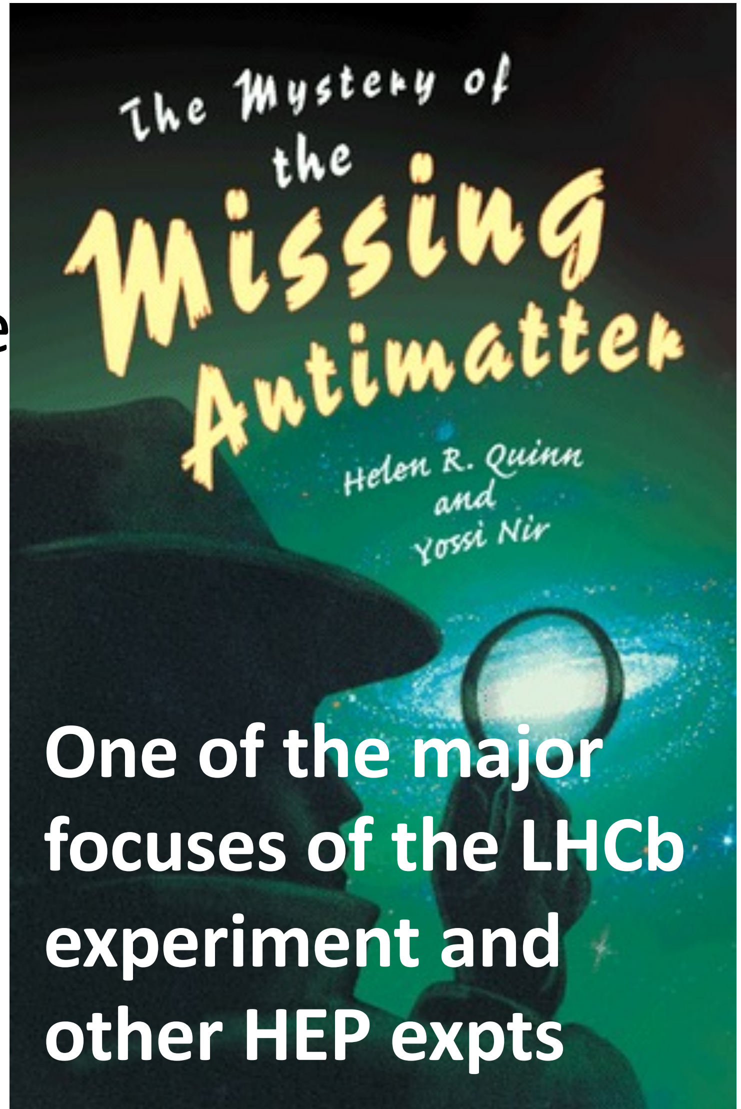
- Shortly after the Big Bang (extremely dense/hot) → equal amounts of matter and antimatter were created from the available energy

- Where did the antimatter go?

- Particle Physics – smallest of scales

↕

Big Bang – largest of scales



One of the major focuses of the LHCb experiment and other HEP expts

e^+ and e^- annihilation again

- Another reaction: $e^- + e^+ \rightarrow \mu^- + \mu^+$
  - Muon and antimuon have mass 106 MeV ~ 207 m_e
  - Initial mass sum = $2m_e$ < final mass sum = $2m_\mu$
  - Electron kinetic energy needed to make muon masses; there is a threshold
- Can you compute the threshold energy?

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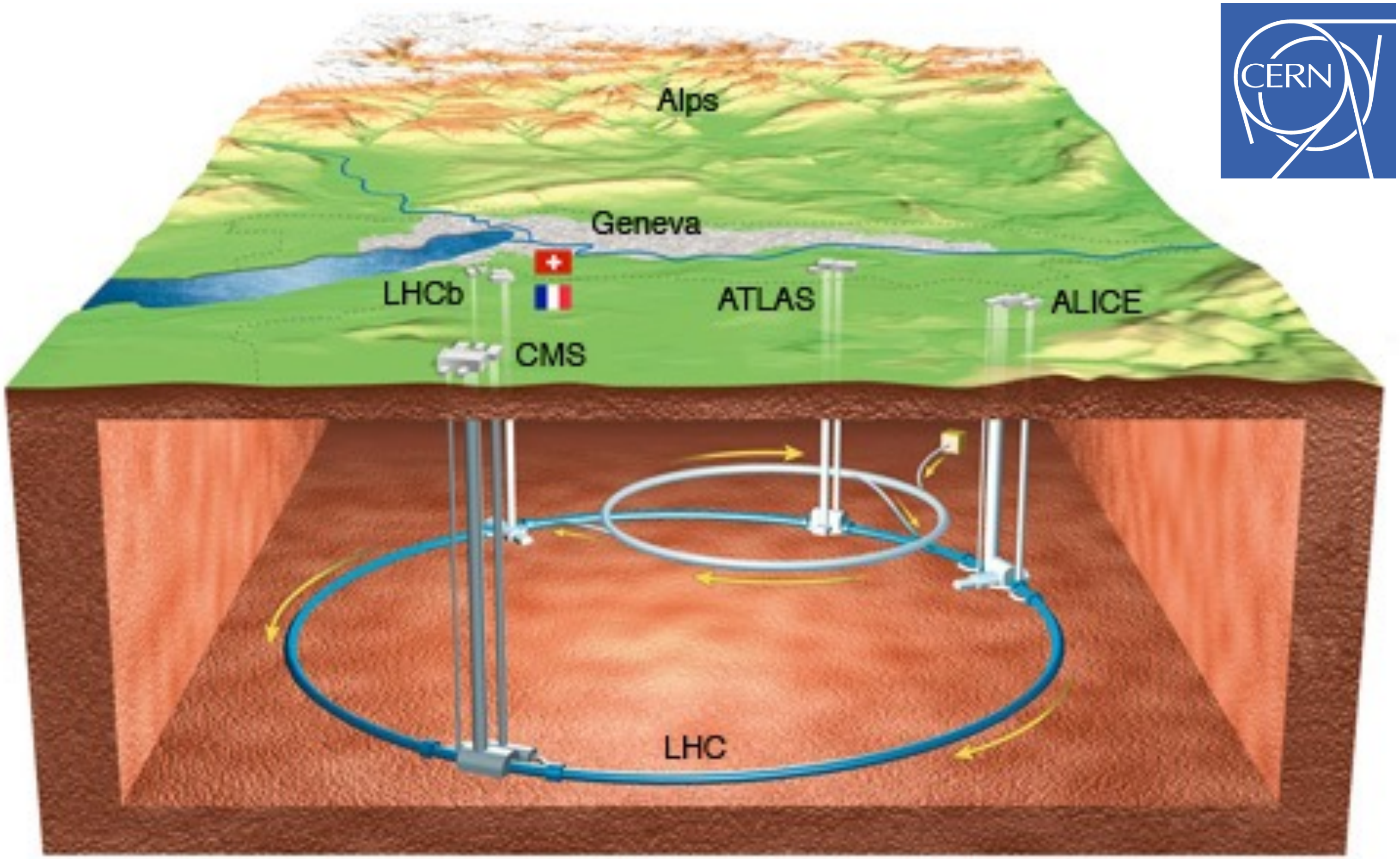
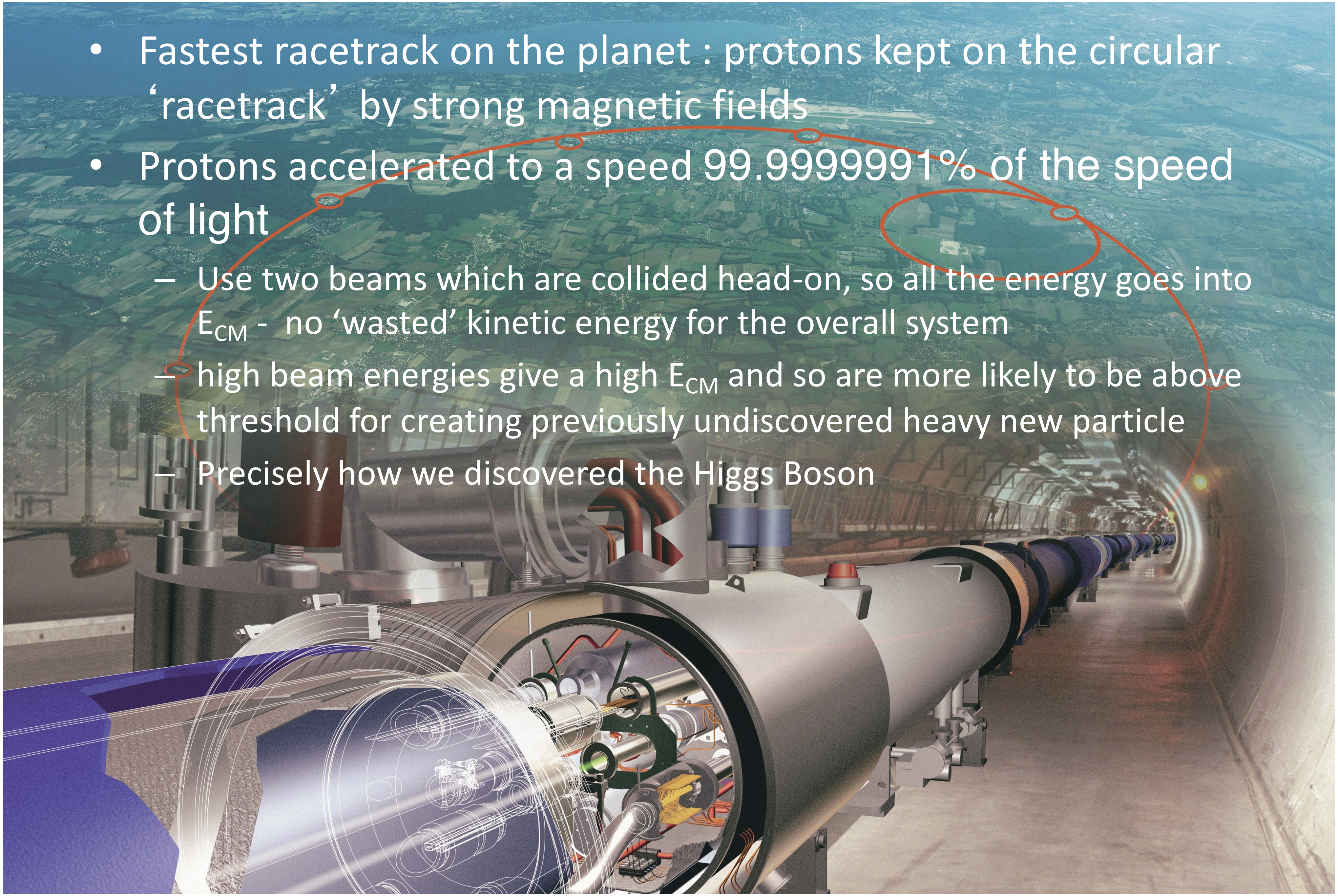
  - Initial mass sum = $2m_e$ < final mass sum = $2m_\mu$
  - Electron kinetic energy needed to make muon masses; there is a threshold
- In CM frame
  - Again by symmetry, E conservation means all particle energies are equal: $E_{e-} = E_{e+} = E_{\mu-} = E_{\mu+}$
  - Threshold is when muons have no kinetic energy so $E_{\mu-} = E_{\mu+} = m_\mu c^2$ and so need $E_{CM} \geq 2m_\mu c^2$
  - For N final particles, generalises to $E_{CM} \geq \Sigma_i^N m_i$

The Large Hadron Collider

- World's highest energy particle accelerator



- Fastest racetrack on the planet : protons kept on the circular 'racetrack' by strong magnetic fields
- Protons accelerated to a speed 99.9999991% of the speed of light
  - Use two beams which are collided head-on, so all the energy goes into E_{CM} - no 'wasted' kinetic energy for the overall system
  - high beam energies give a high E_{CM} and so are more likely to be above threshold for creating previously undiscovered heavy new particle
  - Precisely how we discovered the Higgs Boson



What we did today

- Clarified invariant, conserved and constant
  - Some quantities are more than one of these
  - They tend to be the most useful
- Introduced the centre-of-mass frame
  - Often the easiest to do calculations in
- Looked at particle reactions
  - Energy and momentum conservation still holds
  - The mass sum can change from initial to final state
  - The centre-of-mass energy is critical in determining if a reaction can occur