Random Processes

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EI HUST

- Stochastic Processes
- Random Processes

- Stochastic processes is a subject about modeling and analysis of random phenomena occurring over time.

- Prerequisites:
  - Circuits, Signals and Systems
  - Probability Theory
  - Queuing Systems

Chapter 1: Introduction

- OUTLINE:

  1.1 The Concept of the Random Process

  1.2 An Example of Random Processes

  1.3 Applications of Random Processes

Chapter 1: Introduction

1.1 The Concept of the Random Process

1.2 An Example of Random Processes

1.3 Applications of Random Processes

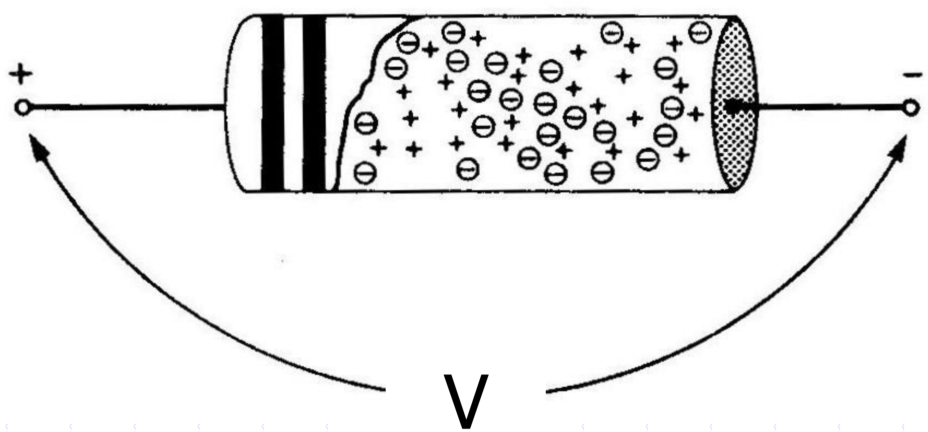
1.1 The Concept of Random Process

1.1.1. Random Experiments

- Rolling a die and observing the number that shows up
- Tossing a coin
- Strength of signal received by a Mobile Station
- Call arrivals to a Web server in a time interval
- Thermal noise in electronic devices
- …

1.1.1. Random Experiments

- Thermal Noise in a resistor



- Thermal Motion of Free Electrons in a Resistor
- It is one of main reason that disturbs the work of circuits and systems, such as radio receiver, mobile phone.

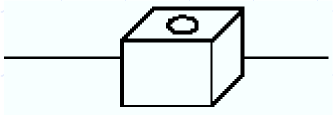
1.1.1. Random Experiments

3 stages:

In order to develop a useful theory of probability, it is important to separate 3 stages in the consideration of any real probability.

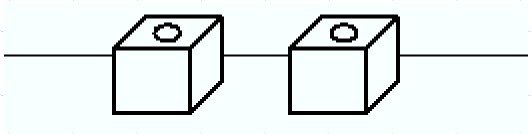
Stage 1. The association of an events with a probability by (i) experiments and (ii) reasoning.

e.g. $P(1) = 1/6$

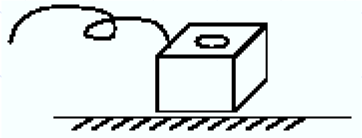


Stage 2. Development of the relationship of the probability of an event with the probabilities of some other events.

e.g. $P(1)\, P(1)$



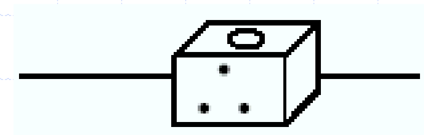
Stage 3. The application of the results of stage 1 & stage 2 to the real world. e.g.



1.1.1. Random Experiments

The concepts in Random Experiments:

1. A single performance is called a **trial** & at it we observe a single **outcome** *Si*.
2. The **event** *A* is said to have occurred in this trial if *Si*   *A*.
   e.g. If event A={1,2,3} and outcome Si =1,
   then we say event *A* occurred.



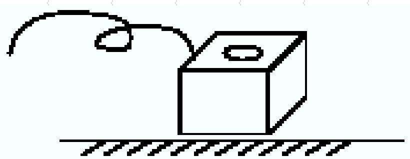
3. **Sample space** *S* = universal set =Set contains all possible experimental outcomes.
   e.g. To the experiment of rolling a die, S={1,2,…,6}

Φ = empty set = set contains impossible outcomes

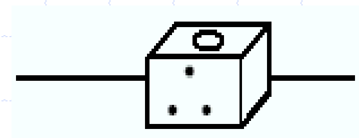
1.1.1. Random Experiments

Probabilities in Random Experiments:

a trial **Space S** **outcome = 1** **Event {1}**



{1,2,…,6}



occurred

Rolling a die

We assign to each event A a number P(A) which we call the probability of A. This number satisfies the 3 axioms:

1. $P(A) \geq 0$
2. $P(S) = 1$
3. $AB = \phi \rightarrow P(A+B) = P(A) + P(B)$

   (i.e. mutually exclusive)

1.1.1. Random Experiments

3 Elements of Random Experiments: definitions

- Sample Space
  - The set of all possible outcomes in any given experiments.
- Event
  - A subset of the sample space.
  - An element in sample space is called basic event.

    e.g. A={the number that shows up is an odd number in the experiment "rolling a die"}

    If the number that shows up is an odd number, we say the event A was happened.
- Probability
  - A measurement of the random nature of the experiment for each event defined on a sample space.

1.1.1. Random Experiments

- Example1: Tossing a coin
  - Sample space: { Head, Tail}
  - Set of basic event: {Head, Tail}
  - Stage 1: The association of an events with a probability.
  - Probabilities for each basic event: {0.5, 0.5}
- Example2: Tossing two coins
  - Sample space: { (H,H),(H,T),(T,H),(T,T)}
  - Probabilities for each basic event: {0.25,0.25,0.25,0.25}
  - Stage 2: Development of the relationship between probabilities of different events.
  - Event: A={Head appears at least one times}

    ={(H,H),(H,T),(T,H)}
  - Stage3: Probability of A: 0.75

1.1.1. Random Experiments

- Fill in the missing words in __

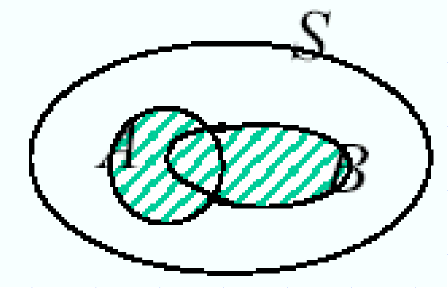
Definitions:-

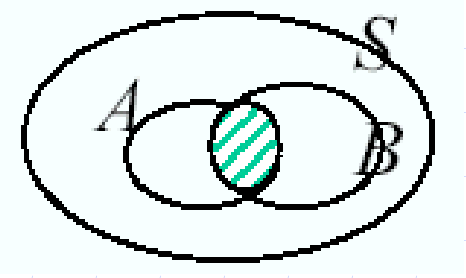
1) Event S(universal set) occurs at _____trial.

2) Event Φ(empty set) _____ occurs.

3) Event A+B occurs when event A_____event B occur.

4) Event AB occurs when event A_____event B occur.

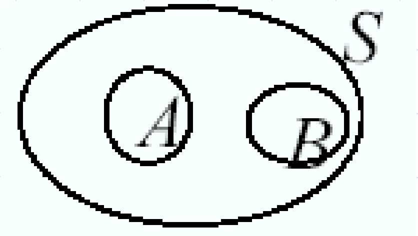




1.1.1. Random Experiments

- Fill in the missing words in ___

Properties:



1) $A \cdot B = 0 \longrightarrow$

event A & event B _____ occur in the same trial.

2) Event A occurs $\longrightarrow$ Event $\bar{A}$ _____ occurs.

1.1 The Concept of Random Process

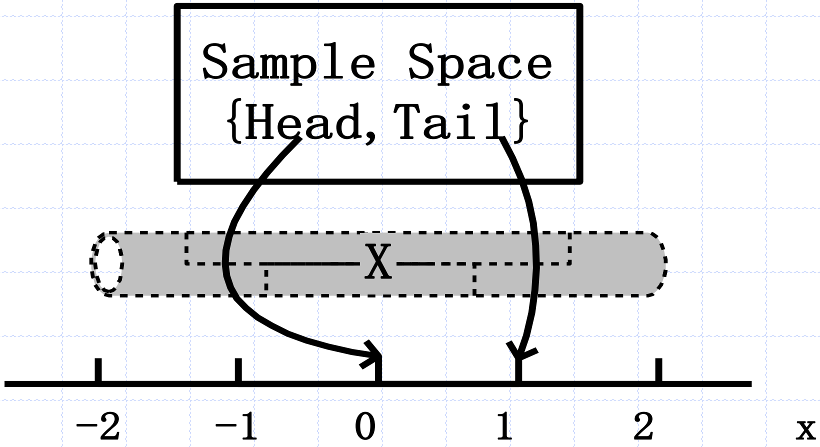
1.1.2. Random Variables

Tossing a coin

Sample space: { Head, Tail }

Rolling a die

Sample space: { 1,2,3,4,5,6 }



- According experiments to be performed, there are different types of Sample Space.
- In order to mathematically analysis practical problems, a consistent numerical representation of sample space is defined.
- A sample space is mapped to a numerical space: real axis, then a random Variable is obtained.

Definition: A random variable is a real one value function of the elements of a sample space.

1.1.2. Random Variables

- Example: Tossing a coin
  - Sample space: { Head, Tail}
  - Random Variable:

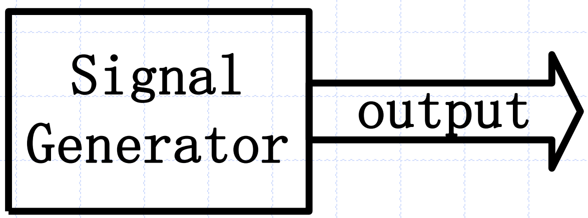
$$X = \begin{cases} 0, & Head \\ 1, & Tail \end{cases}$$

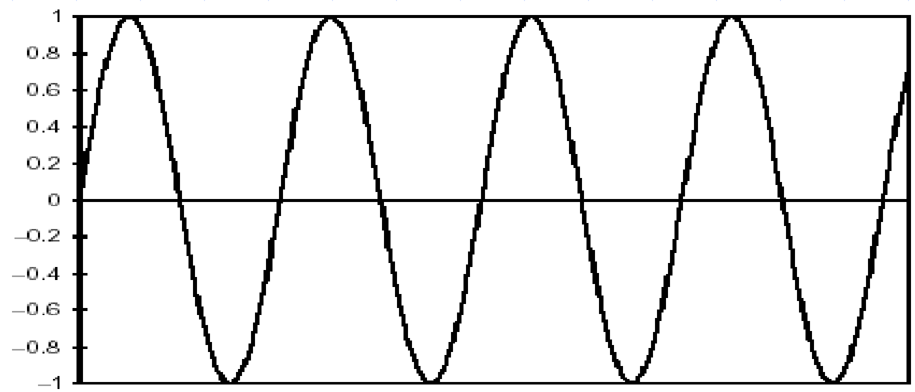
  - Probability:

$$P(X = 0) = 0.5, \quad P(X = 1) = 0.5$$

1.1 The Concept of Random Process

1.1.3. Deterministic Signals





Sine Wave: $A\sin(w_0 t+\varphi)$

Sawtooth wave ….

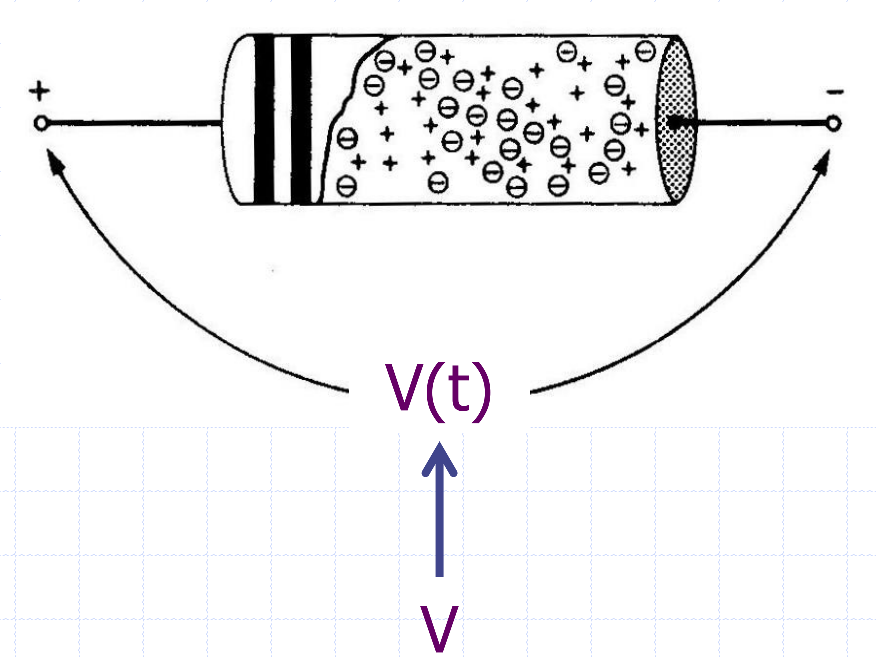
The signal's value at any moment is a fixed value decided by the Deterministic function.

Consideration of the time element is very important in many random phenomena also.

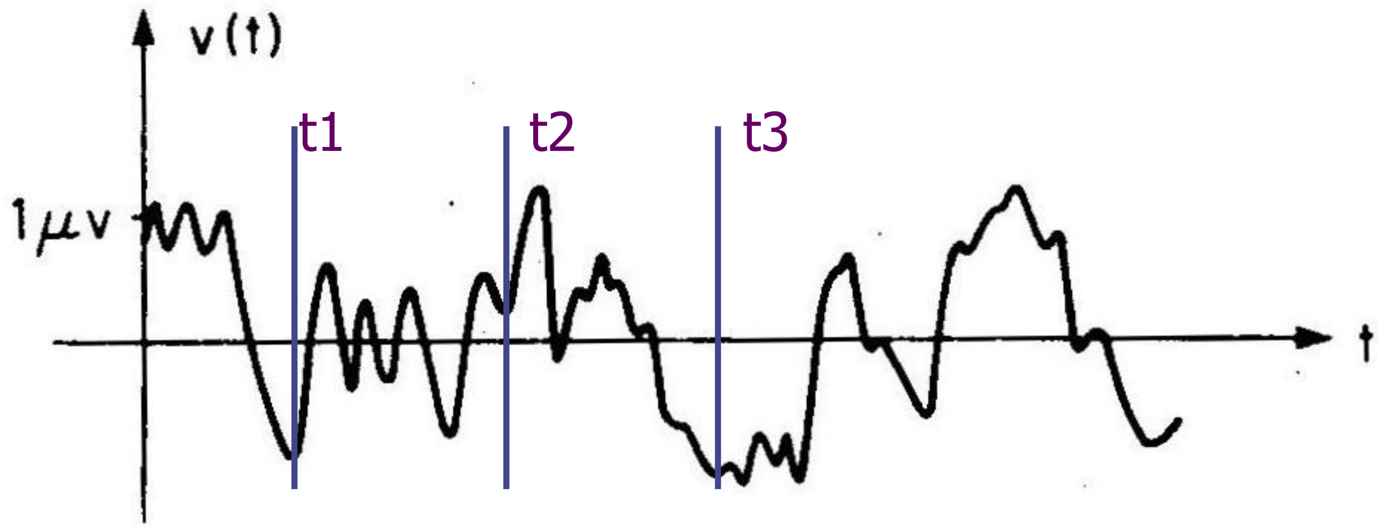
1.1 The Concept of Random Process

1.1.4. Random Signals

Random Experiments: Measure thermal noise voltage at a resistor for a period of time.



1.1.4. Random Signals



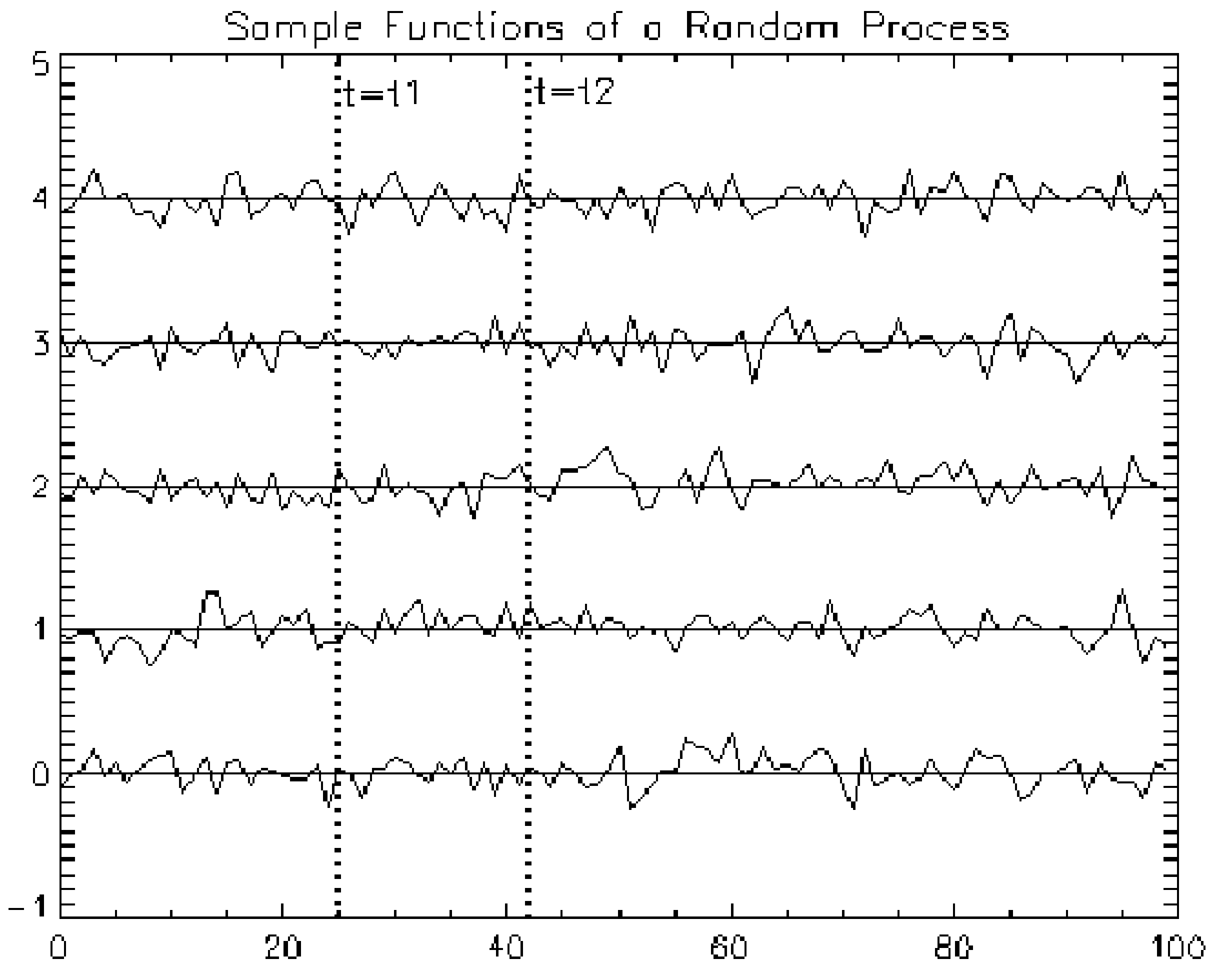
Thermal Noise Voltage at a Resistor

Because thermal noise voltage is a random variable at any moment, the time wave of thermal noise voltage is a Random Signal.

1.1.4. Random Processes

Sample Waves = Sample functions:

Every times the experiment was performed, a different random wave which is named sample Wave is obtained



1.1.4. Random Processes

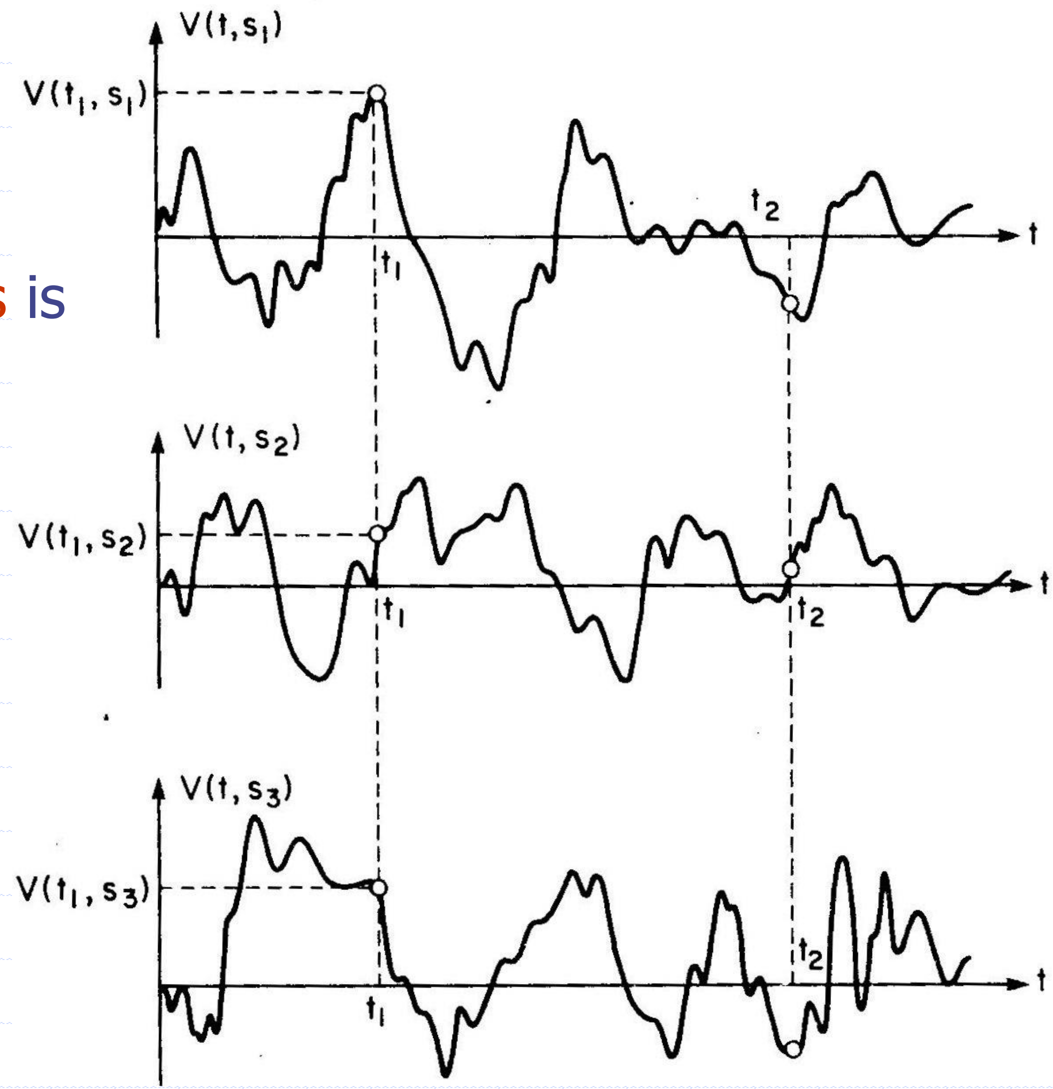
Definition:

The set of sample waves is a Random Process.

Features:

I. At any moment, such as t1, the value of the sample wave is a random variable.

II. Each sample wave has different values at t1.



Statistical Samples of a Thermal Noise Process

1.1.4. Random Processes

- Wind speed for a wind-powered generator
- Random signals received by a Mobile Station
- Bits stream in a digital communication link in a time interval
- Call arrivals to a Web server in a time interval
- …

Chapter 1: Introduction

- OUTLINE:

1.1 The Concept of Random Process

1.2 An Example of Random Processes

1.3 Applications of Random Process

1.2 An Example of Random Processes

1.2.1 Poisson Distribution

- "Occurrence of events"
  - λ = average rate of occurrence per second;
  - N = the number of occurrences of an event in an arbitrary period.

$$\Pr\left(N=k\right)=\frac{\lambda^{k}e^{-\lambda}}{k!},\ k=0,1,2,\ldots$$

$$E\left(N\right)=\lambda$$

$$\mathrm{Var}\left(N\right)=\lambda$$

$$Cv_{N}^{2}=1/\lambda$$

Coefficient of Variation

$$Cv_{X}=\sqrt{\mathrm{Var}\left(X\right)}\Big/E\left(X\right)$$

$$\text{(s.c.v.) }Cv_{X}^{2}=\mathrm{Var}\left(X\right)\Big/E^{2}\left(X\right)$$

1.2.2 Poisson Random Processes

Let N(t_1, t_2) be the number of occurrences of an event in the interval (t_1, t_2).

The probability of $N = n$ is a Poisson distributed:

$$P[N = n] = \frac{(\lambda\tau)^n e^{-\lambda\tau}}{n!}$$

- where $\tau = t_2 - t_1$.
- $E[N(t_1, t_2)] = \lambda\tau$.

1.2.2 Poisson Random Processes

A random process can be defined as the number of events in the interval $(0, t)$. Thus,

$$X(t) = N(0, t).$$

$X(t)$ is a Poisson Process.

The expected number of events in t is

$$E[X(t)] = \lambda t.$$

The average rate of occurrences of an event is λ events per second.

The variance of $X(t)$ is

$$E[(X(t) - \lambda t)^2] = E[X^2(t)] - (\lambda t)^2 = \lambda t$$

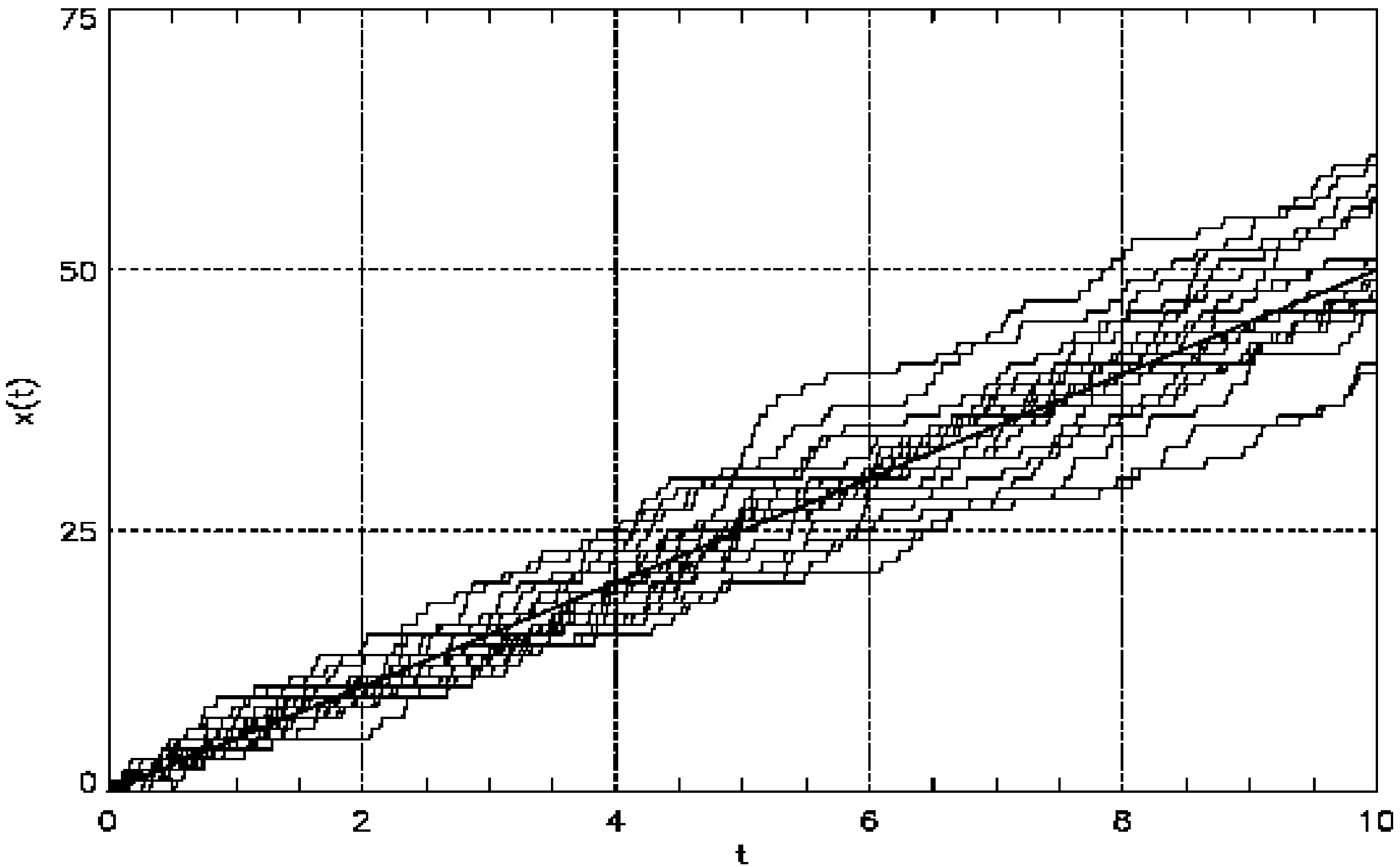
1.2.3 A Function to Simulate a Poisson Process

- ; FUNCTION PoissonProcess,t,lambda,p
- ; S=PoissonProcess(t,lambda,p)
- ; divides the interval [0,t] into
- ; intervals of size deltaT=p/lambda where
- ; p is sufficiently small so that the
- ; Poisson assumptions are satisfied.
- ;
- ; The interval (0,t) is divided into
- ; n=t*lambda/p intervals and the number of
- ; events in the interval (0,k*deltaT) is
- ; returned in the array S. The maximum length of
- ; S is 10000.

1.2.3 A Function to Simulate a Poisson Process

```
; USAGE:
;   S=PoissonProcess(10,1,0.1)
;   Plot,S
;   FOR m=1,10 DO OPLOT,PoissonProcess(10,1,0.1)
NP=N_PARAMS()
IF NP LT 3 THEN p=0.1
n=lambda*t/p
u=RANDOMN(SEED,n,POISSON=p)
s=INTARR(n+1)
FOR k=1,n DO s[k]=s[k-1]+u[k-1]
RETURN,s
END
```

1.2.3 A Function to Simulate a Poisson Process



A graph of $X(t)$ would show a function fluctuating about an average trend line with a slope λ.

Chapter 1: Introduction

- OUTLINE:

  1.1 The Concept of Random Process

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1.3 Applications of Random Processes

RP is a basic mathematical model of kinds of problems in communication and information engineering.

RP is a mathematical base of courses :

Digital Signal Processing

Information Theory and Coding

Principles of Communication

Image processing

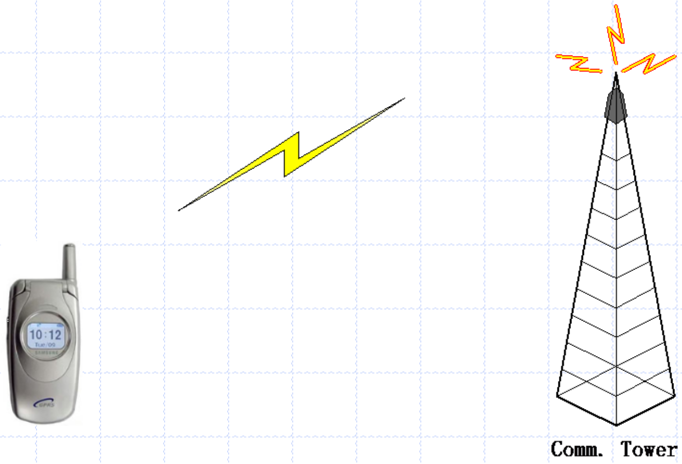
……

1.3 Applications of Random Processes

- 3 Typical applications in communication and information processing
- Example1:

  Signal-Processing Applications
  - Signal Detection
  - Signal Extraction

    (undesired noise)

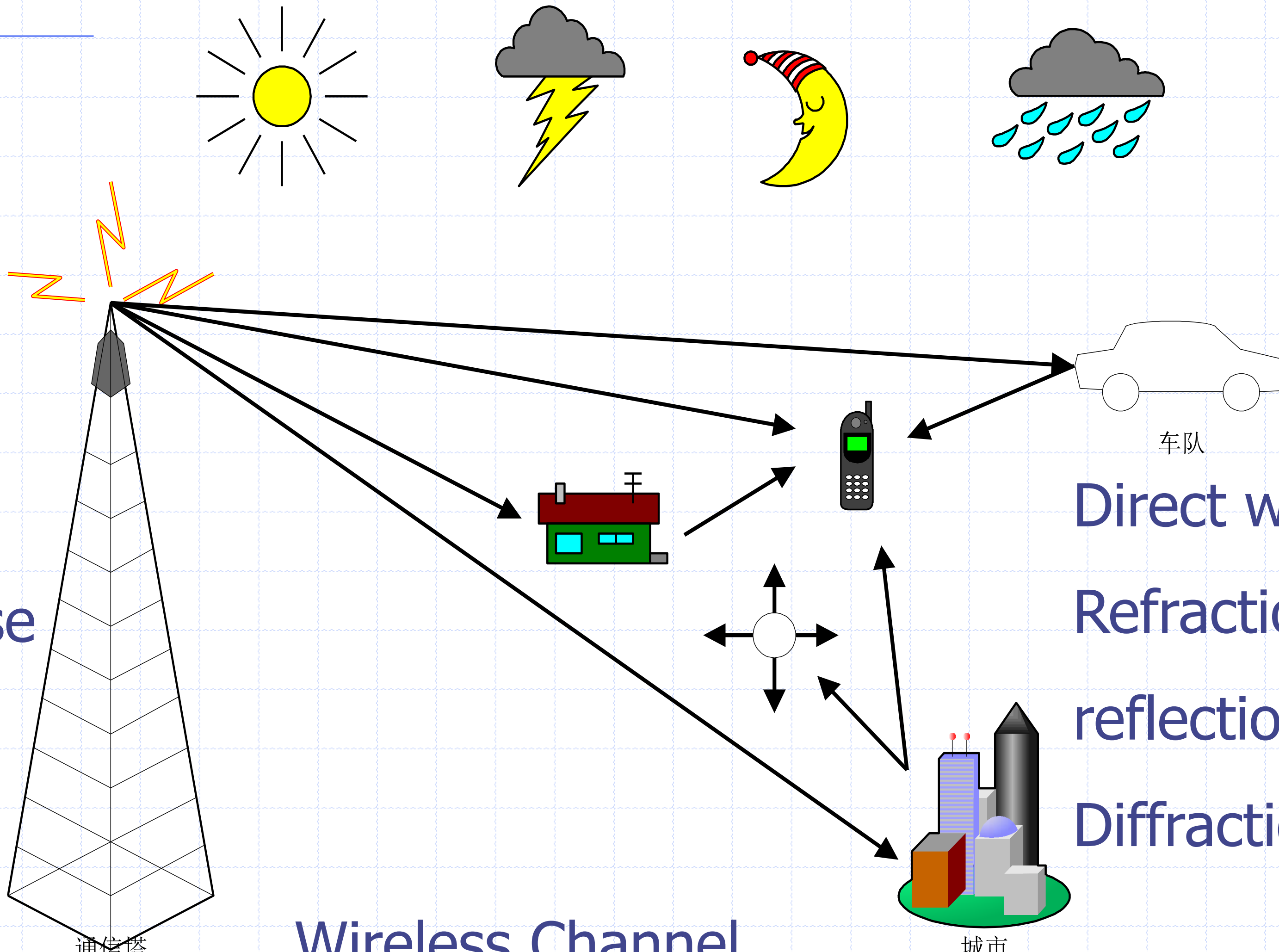


Thermal noise,
Other noise problems,
Interference,
Attenuation…

The noise disturbed the signal.

The useful signals have to be detected and extracted carefully from the noise.

Example1: Signal-Processing Applications



Sky noise
Thermal noise

Direct waves

Refraction

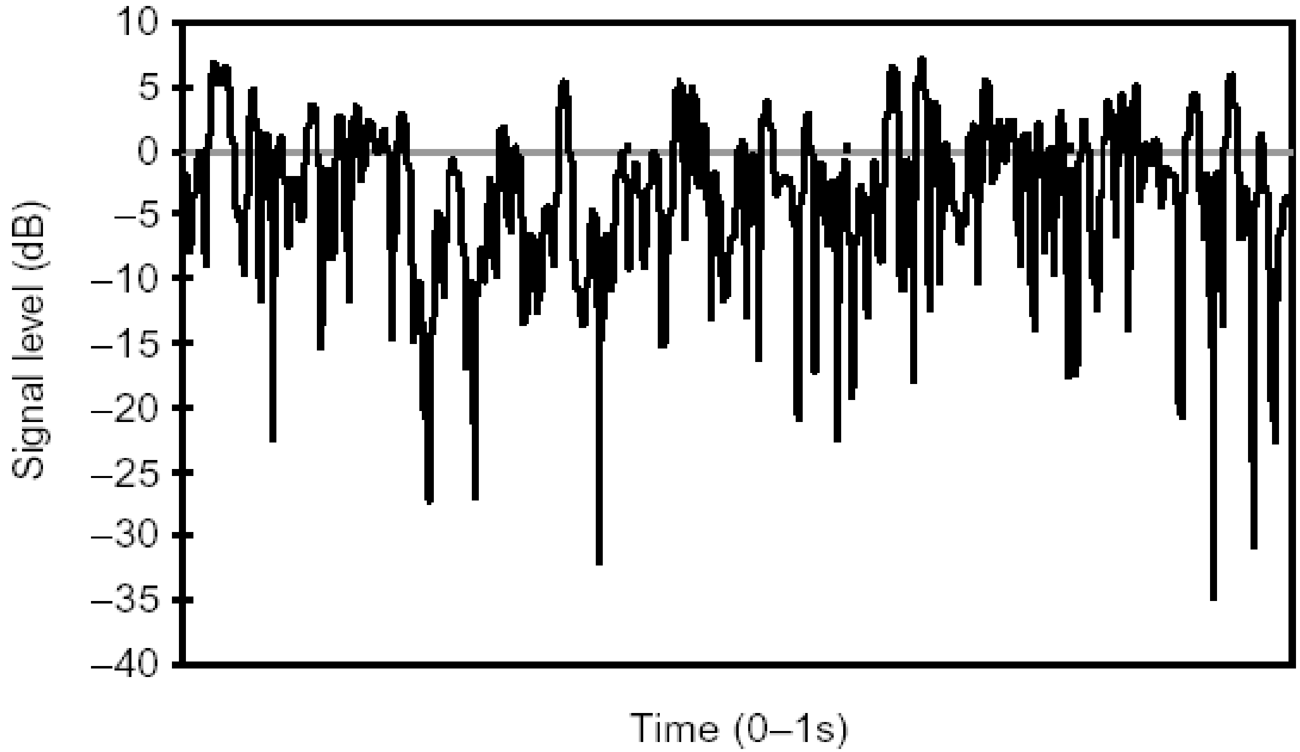
reflection

Diffraction

Wireless Channel

Example1: Signal-Processing Applications

A signal wave received by mobile station



Rayleigh fading waveform for a mobile moving at walking speed over 1 sec. (Rayleigh is one of random process models)

How can we detect and extract information from this disturbed signal?

1.3 Applications of Random Processes

Example2: Desired noise

- In order to save system resources, mute detection technology is used.
- Mute symbol is sent in system when A is not speaking. So, if B heard nothing, he will guess whether the link is break down.
- At this time, communication system will play some background noise.
- Mute is usually replaced by background noise so that the listener will feel more comfortable in the communication .

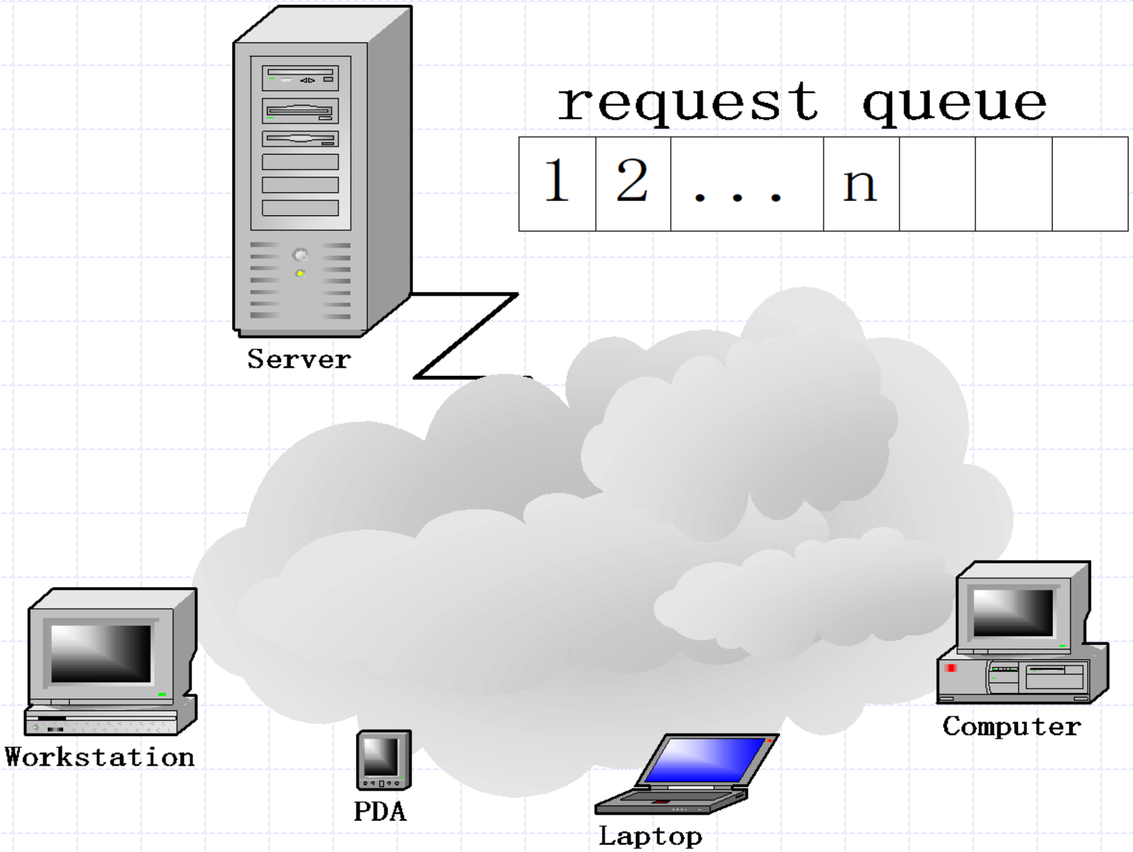
1.2 Applications of Random Processes

◆ Example3: Serving Model of a Web Server

- Poisson Random Processes

The number of arriving calls in a Web server is a Poisson Process.

The serving time of every request is an Exponential Process.



1.3 Applications of Random Processes

- Other Applications
  - Life Insurance
  - Finance
  - General Insurance
  - Production reliability……

Course Information

- Instructor information:
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Course Information

- Grading:
  - Homework/experiments: 20%
  - Examinations/ Attendance : 10%
  - Final examination: 70%

- How can you pass the course?
  - Try your best

Course Information

- Textbook:
  - Book1：
    - Edward P.C. Kao,
    - An introduction to stochastic processes
    - Duxbury Press
    - China Machine Press (机械工业出版社)
    - Chapter 2, chapter 4
  - Book2：
    - Michael B. Pursley,
    - Random processes in linear systems
    - Index entry: O211.6 104
    - Prentice Hall

Course Information

- Reference

[1] Michel K. Ochi,

- Applied Probability and Stochastic Processes In Engineering and Physical Sciences,
- Index Entry: TB114 W29

[2] Frank E. Beichelt, L. Paul Fatti

- Stochastic Processes and Their Applications,
- Index Entry: O211 W138

[3] William A. Gardner

- Introduction to Random Processes with Applications to Signals and Systems,
- Index Entry: O211.6 W77

[4] Geoffrey R. Grimmett, David R. Stirzaker

- Probability and Random Processes,
- Index Entry: O211 W134/3

Course Information

Course Contents

- Concepts , characteristics and classification of stochastic processes
  - (Book1-chap1,Book2-chap 2)
- Stationary processes (Book2-chap 2, chap 3)
  - Correlation function
  - Ergodic process
- Poisson processes (Book1-chap 6)
  - Definition and properties
  - Interarrival time and waiting time
  - Generalization of the Poisson process

Course Information

- Contents:
  - Markov chain (Book1-chap 5)
    - Definition and Chapman-Kolmogorov Equation
    - Stationary distribution
  - Spectral analysis of Stochastic processes (Book2-chap 4)
    - Spectral density function
    - Wiener-khintchine theorem
    - Cross-correlation function
    - Cross-spectral density function

Course Information

- Contents:
  - Narrow-band processes(Book2-chap 4)
    - Statistical properties of narrow-band processes
    - Joint probability distribution for narrow-band processes
  - Linear system (Book2-chap 4)
    - Input and output spectral relationship
  - Simulation

End of Chapter 1