UC Berkeley
Department of Electrical Engineering and Computer Sciences

EECS 126: Probability and Random Processes

Discussion 01
Spring 2023

1. **Properties of Probability Measures**

   Use the axioms of probability to show the following facts. Clearly identify which axioms are used, and where. You may also use facts from lecture, but identify where you use them.

   a. (Subadditivity, also known as the "union bound") If $A_1, A_2, \dots \in \mathcal{F}$, then

   $$P\left(\cup_{i\geq 1} A_i\right) \leq \sum_{i\geq 1} P(A_i).$$

   b. (Continuity from below) If $A_1 \subset A_2 \subset \dots \in \mathcal{F}$, then

   $$P\left(\cup_{i\geq 1} A_i\right) = \lim_{i\to\infty} P(A_i).$$

   Here and throughout the course, the notation $A \subset B$ means that A is a subset of B (not necessarily a proper subset).
   (Hint: If $a_i \geq 0$ for each $i \geq 1$, then we have *monotone convergence*: $\sum_{i\geq 1} a_i = \lim_{n\to\infty} \sum_{i=1}^{n} a_i$.)

2. **Independence**

Events $A, B \in \mathcal{F}$ are said to be **independent** if $P(A \cap B) = P(A)P(B)$.

a. Show that if events A, B are independent, then the probability exactly one of the events occurs is
$$P(A) + P(B) - 2P(A)P(B).$$

b. Show that if event A is independent of itself, then $P(A) = 0$ or 1.

3. **Balls and Bins**

Suppose n bins are arranged from left to right. You sequentially throw the n balls, and each ball lands in a bin chosen uniformly at random, independent of all other balls.

a. Formulate an appropriate probability space for modeling the outcome of this experiment.

b. Let A_i denote the event that exactly i bins are empty, for $0 \leq i \leq n$. Compute the probability of the event:

$$\{\text{all empty bins sit to the left of all bins containing at least one ball}\}$$

in terms of the $P(A_i)$'s.

c. Practice your CS70 skills by computing $P(A_1)$.