Bases of random models

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Outline

1. Independent Increment Processes
2. Wiener(-Levy) Processes
3. Counting Processes

1. Independent Increment Processes

- Increments

  the increment of a stochastic process $\{X(t), t \in T\}$ with respect to the interval $[t_1, t_2]$ is the difference $X(t_2) - X(t_1)$.

1. Independent Increment Processes

Def.1 Independent Increment Process

A stochastic process X(t) is said to be an independent increment process if $X(t_{i+1})-X(t_i)$, where i=0,1,2,…, is statistically independent .

(and thereby statistically uncorrelated).

1. Independent Increment Processes

Def.2 Stationary Increment Process

A stochastic process X(t) is said to be a **stationary increment process** if its increments $X(t_2+\tau)-X(t_1+\tau)$ have the same probability distribution for all τ with $t_1+\tau \in T, t_2+\tau \in T$; t_1, t_2 fixed, but arbitrary.

1. Independent Increment Processes

Def.2 Stationary Increment Process

an equivalent definition:

A stochastic process X(t) is said to be a **stationary increment process** if the probability distribution of $X(t+\tau)-X(t)$ does not depend on t for any fixed τ ; $t, t+\tau \in T$

1. Independent Increment Processes

- A stochastic process with stationary increments need not be stationary in any sense.(strictly or weakly)

1. Independent Increment Processes

Def.3

Stationary Independent Increment Process

A stochastic process X(t) which possess both stationary as well as independent increments properties is called a stationary independent increment process.

1. Independent Increment Processes

e.g.1.

Given: $X(t)$ is an independent increment process, $X(t_0)=0$.

Prove: Its autocovariance function is equal to variance function.

Sln:

$$C_{XX}(t_1,t_2)=Var(X(t_1))$$

$$C_{XX}(t_1,t_2)=Var(X(t_2)) \quad ?$$

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2. Wiener(-Levy) Processes

(Brownian motion process)

- A special Gaussian process.

Def.1 Wiener-Levy process

a stochastic process $X(t)$ is said to be a Wiener-Levy process if

(i) $X(t)$ has stationary independent increment.

(ii) Every independent increment is normally distributed.

(iii) $E[X(t)]=0$ for all time.

(iv) $X(0)=0$.

2. Wiener(-Levy) Processes

- Increments distribution of a Wiener process

$$X(t_2) - X(t_1) \sim N(0, \sigma^2 |t_2 - t_1|)$$

- The distribution of a Wiener process

$$X(t) \sim N(0, \sigma^2 |t|)$$

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3. Counting Processes

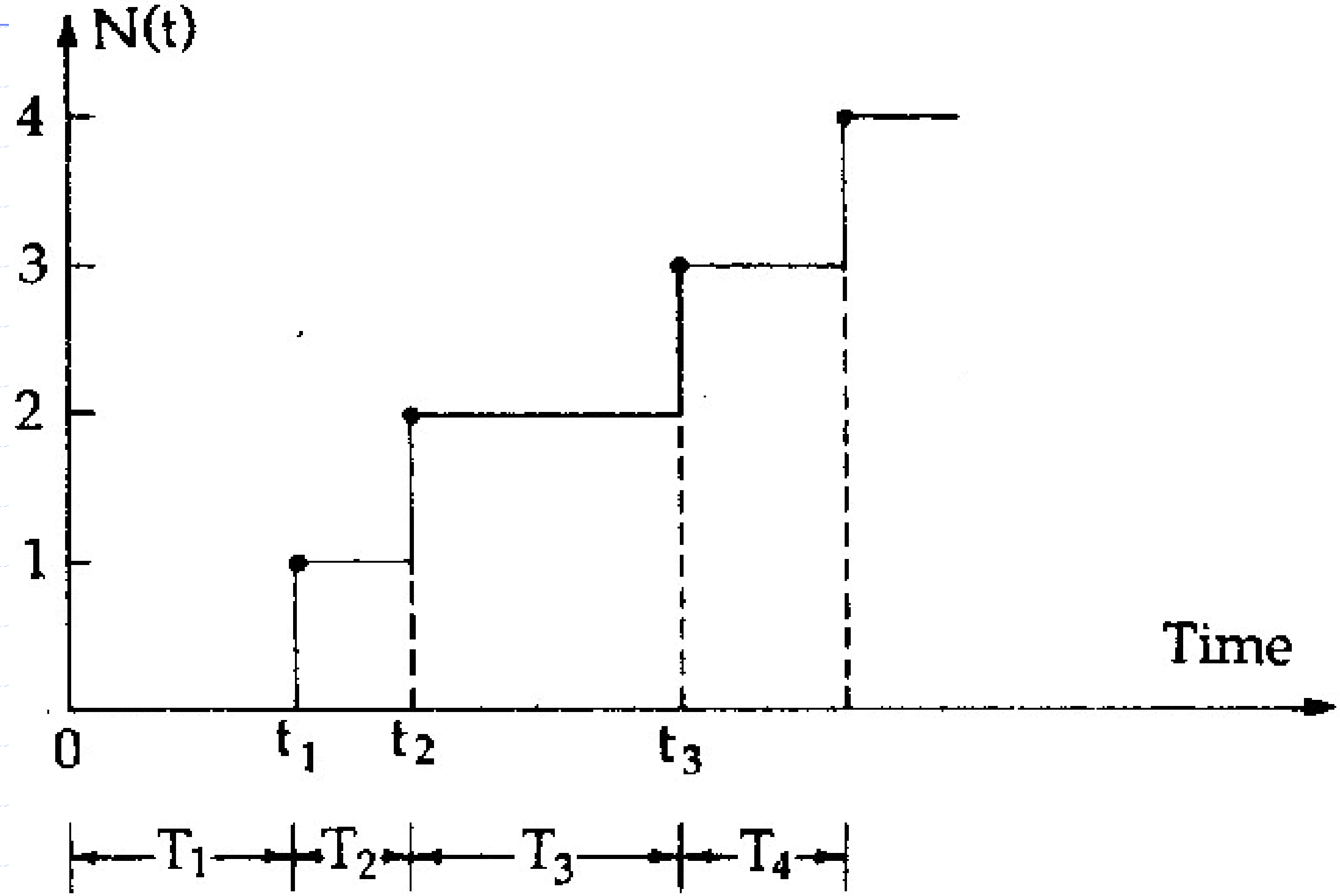
- Counting processes deal with the frequency of occurrence of random events.

Def.1 Counting Process

- An integer-valued continuous-time stochastic process N(t) is called a counting process of the series of events if N(t) represents the total number of occurrences of the event in the time interval t=0 to t.

3. Counting Processes

A sample wave of a counting process N(t)



$T_1=t_1, T_2=t_2-t_1, T_3=t_3-t_2, \ldots$ Interarrival times

3. Counting Processes

- If the interarrival times are independent, identically distributed random variables, then the process is called a renewal process.

- If the interarrival times(are independent, identically distributed random variables) obey an exponential distribution, the process is called a Poisson process.