ECE537: Lab 3 Report

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# ECE537: Lab 3 Report
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```
using Distributions, StatsBase, StatsPlots, LinearAlgebra, LaTeXStrings, PlutoUI,
Statistics
```

```
PlotlyBackend()
```

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plotly()
```

Question 1

A discrete random walk process, $Z(n)$ is given by,

$$Z(n) = \sum_{i=1}^{n} X_i \ ,$$

where X_i are i.i.d. random variables with pmf $p_X(1) = p$ and $p_X(-1) = 1 - p$. So we can easily have the mean and variance of the random variable

$$E(X_i) = 1 * p + (-1) * (1 - p) = 2p - 1$$

$$Var(X_i) = p * (1 - 2p + 1)^2 + (1 - p) * (-1 - 2p + 1)^2 = 4 - 4p^2$$

Q1(a)

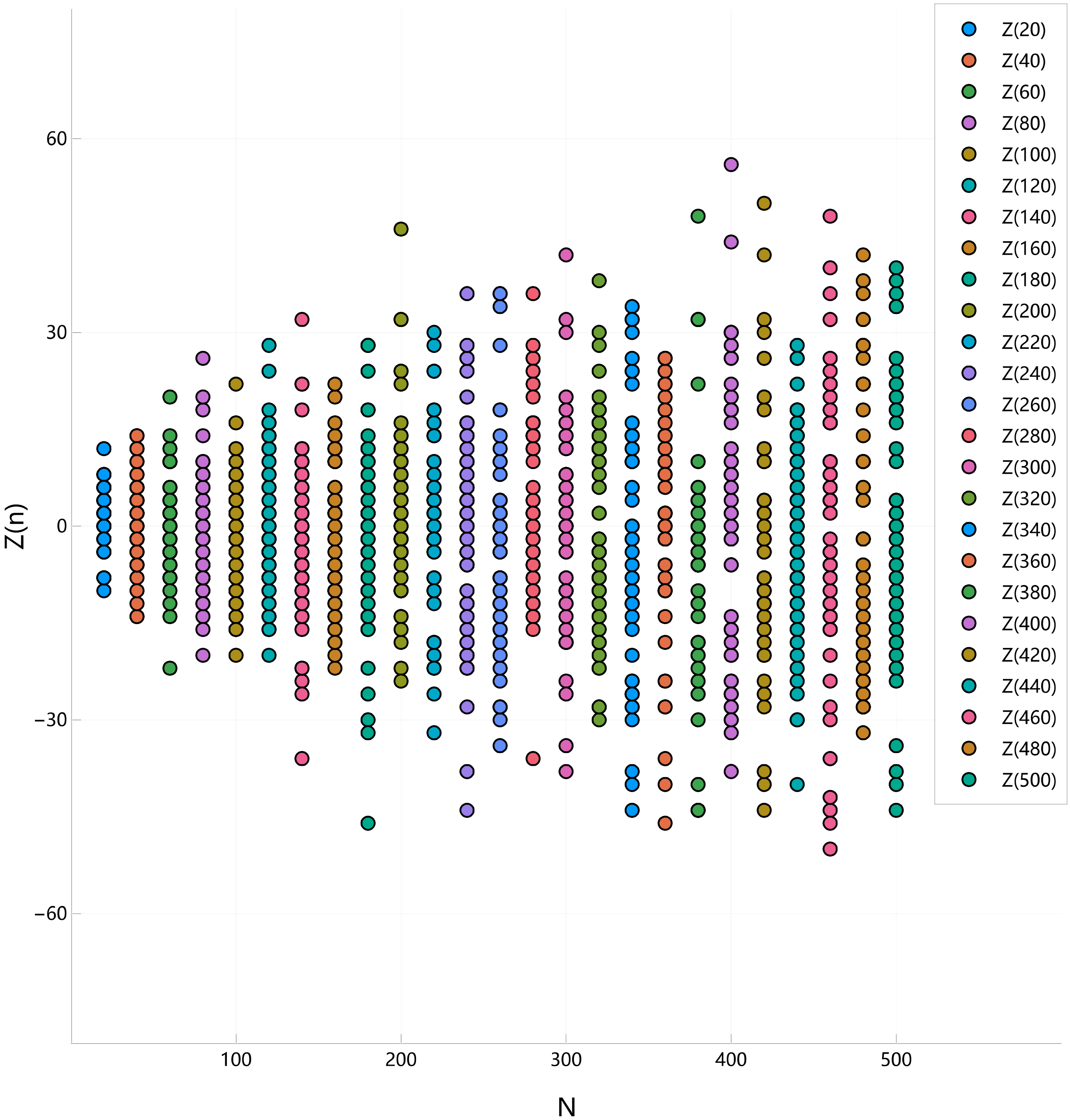
Assume $n = 1, \cdots, 500$ and $p = 0.5$, Generate 50 independent traces of the random process $Z(n)$ and plot them all together in the same figure as a function of n.

```
sample_rdw (generic function with 1 method)
```

```
function sample_rdw(p)
    val = rand()
    if val < p
        return 1
    else
        return -1
    end
end
```

Sample (generic function with 1 method)

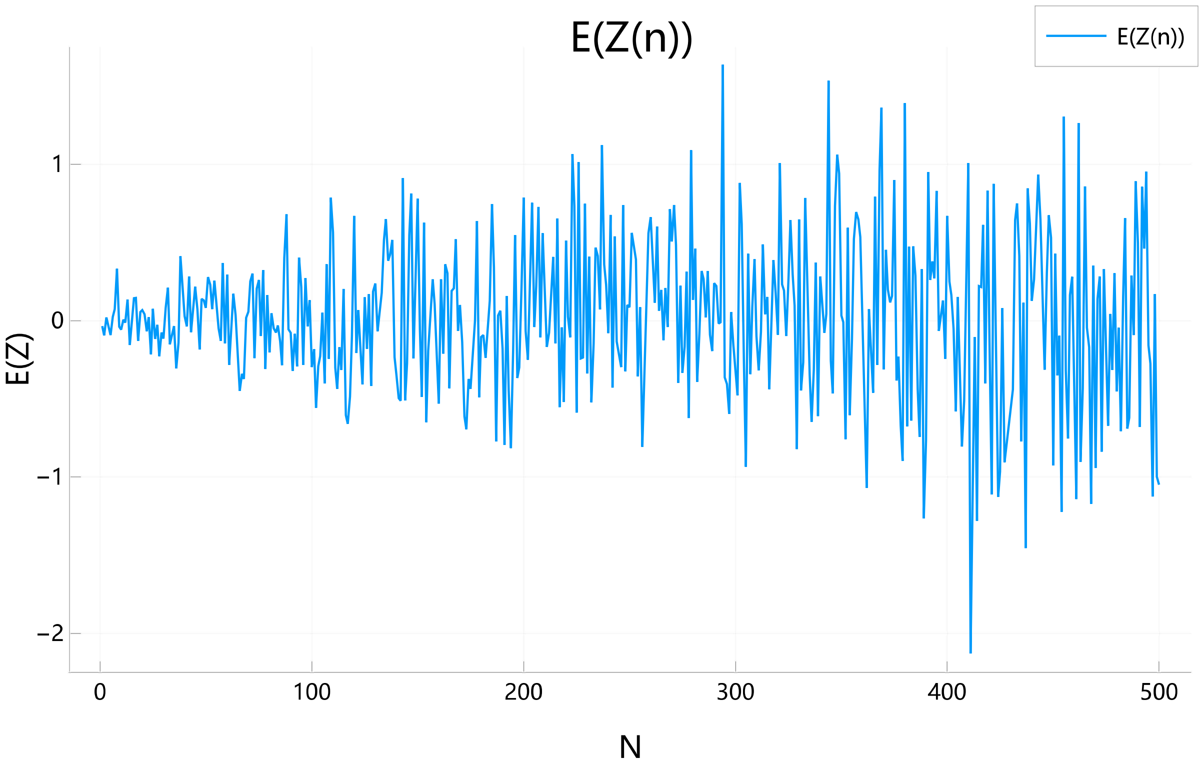
```
#define the sample function which has two parameters(n, times)
function Sample(n, times, p)
    group = Float64[]
    for time = 1:times
        each = [sample_rdw(p) for _ = 1:n ]
        push!(group, sum(each))
    end
    return group
end
```




```
begin
    scatter(0, 0, xlabel="N", ylabel="Z(n)", title="random walk", xlim=[1, 600], ylim=[-80, 80], size=(650, 700))
    for each = 20:20:500
        y = Sample(each, 50, 0.5)
        x = [each for _ = 1:50]
        scatter!(x, y, label="Z($each)")
    end
    scatter!(0, 0)
end
```

Q1(b)

Estimate the expected value of $Z(n)$ for all $1, \cdots, 500$ and plot as a function of n. Explain your observation.

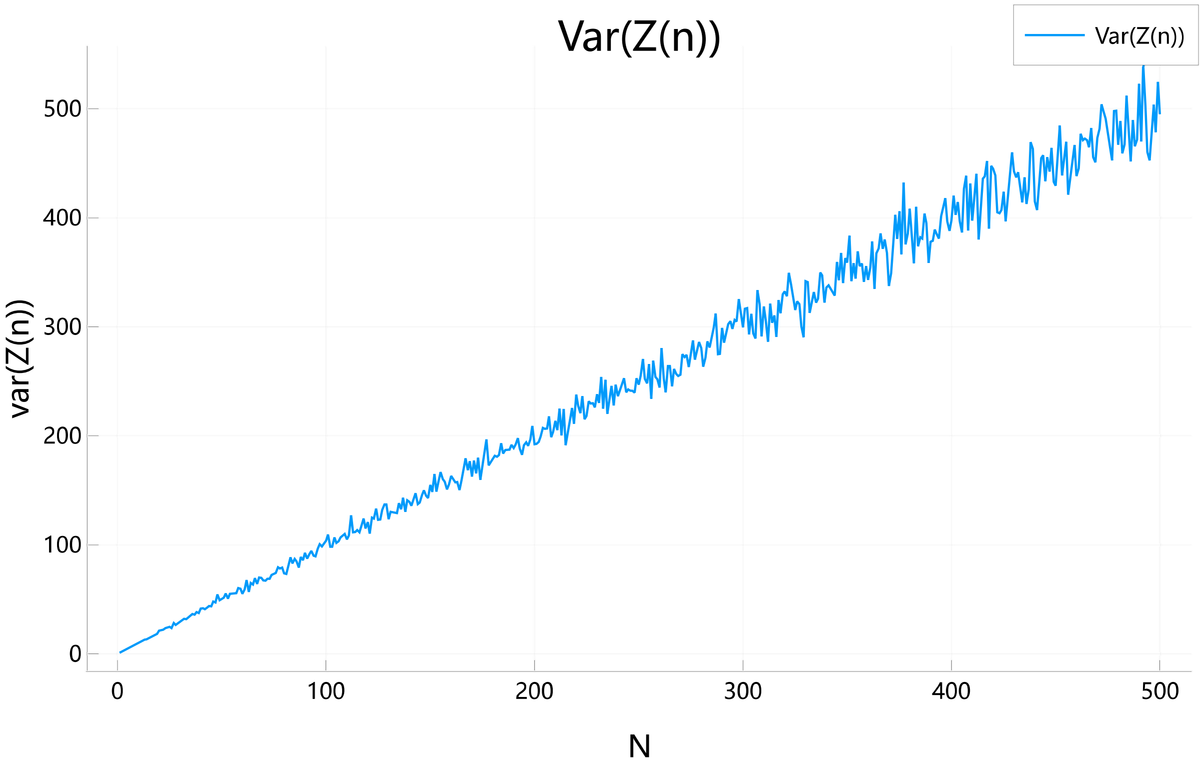



```
begin
    N = collect(1:500)
    E = zeros(500)
    for each = 1:500
        p = Sample(N[each], 1000, 0.5)
        E[each] = mean(p)
    end
    plot(N, E, xlabel="N", ylabel="E(Z)", title="E(Z(n))", label="E(Z(n))")
end
```

From the plot, we can have the view that with n goes up, the expected value of the value of $Z(n)$ shake is more noticeable, which can be explained that when the times of the sampling fixed, and n goes up, the uncertainty of the $Z(n)$ has become large, what it reflects on the plot is the shake becoming steeper.

Q1(c)

Estimate the variance of $Z(n)$ for all $1, \cdots, 500$ and plot as a function of n. Explain your observation.




```
begin
    Var = zeros(500)
    for each = 1:500
        p = Sample(N[each], 1000, 0.5)
        Var[each] = var(p)
    end
    plot(N, Var, xlabel="N", ylabel="var(Z(n))", title="Var(Z(n))",
    label="Var(Z(n))")
end
```

From the plot, we can have the view that the variance of the $Z(n)$ is linearly related to n approximately, which means the $Z(n)$ is not a stationary process.

Q1(d)

Repeat part (a) with $p = 0.6$, Explain what you see in the plot.

```
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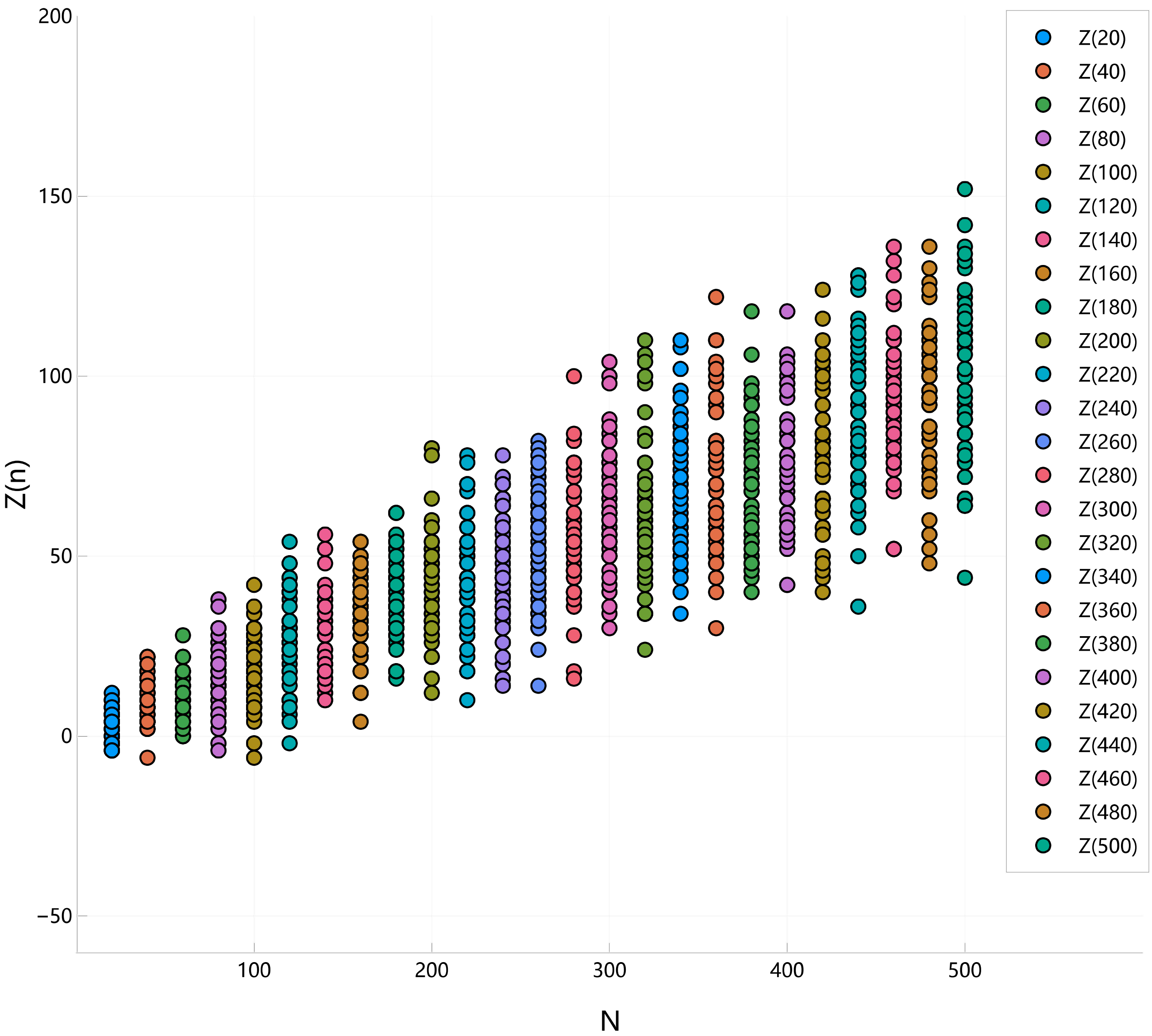
"
```

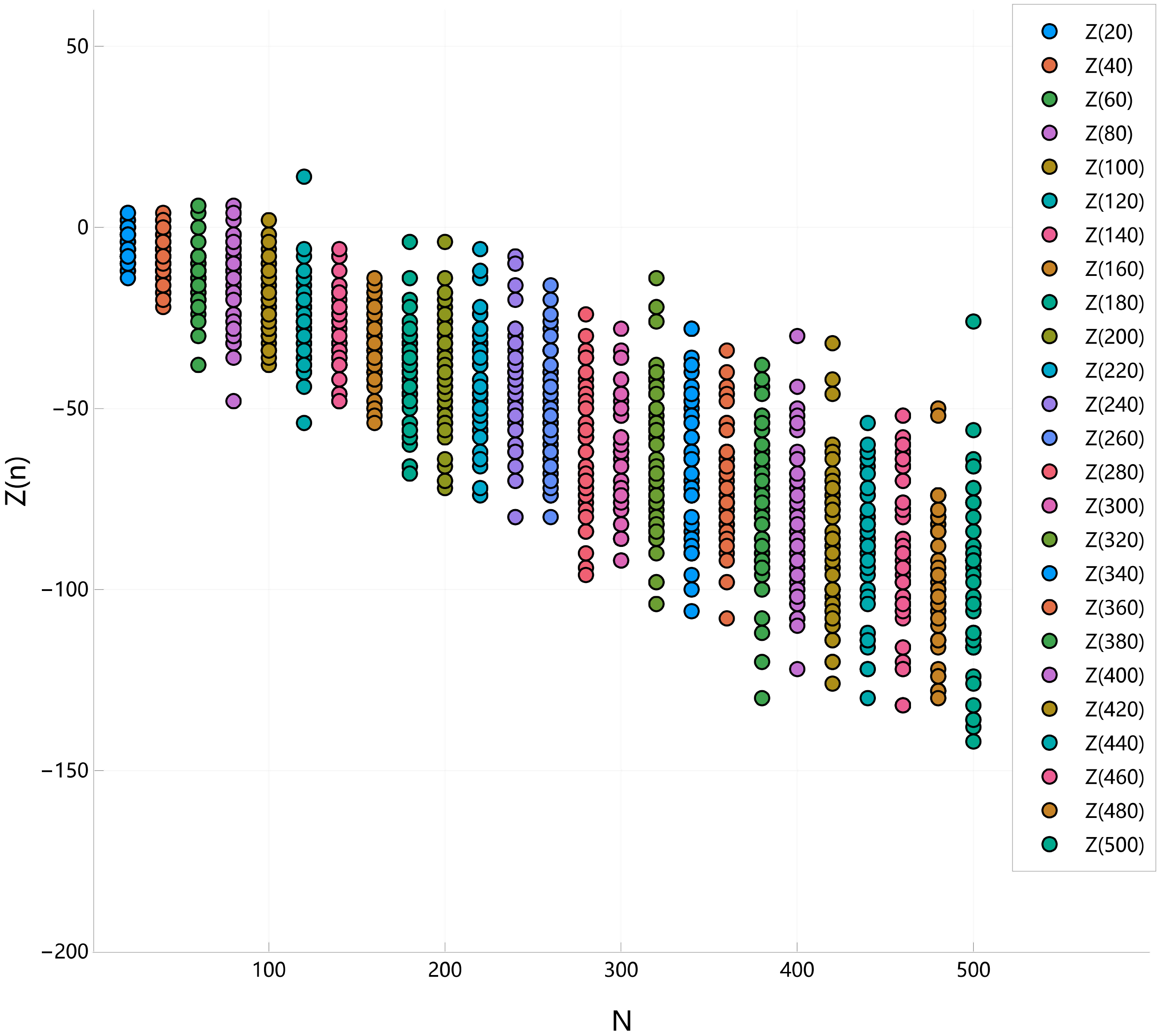



```
begin
    scatter(0, 0, xlabel="N", ylabel="Z(n)", title="random walk with p = 0.6", xlim=
    [1, 600], ylim=[-60, 200], size=(650, 600))
    for each = 20:20:500
        y = Sample(each, 50, 0.6)
        x = [each for _ = 1:50]
        scatter!(x, y, label="Z($each)")
    end
    scatter!(0, 0)
end
```

From the plot, we can find that all the image has gone up， and with the n goes up, the maximum of the $Z(n)$ for each n has be a linearly related to n.

Q1(e)

Repeat part (a) with $p = 0.4$, Explain what you see in the plot.




```
begin
    scatter(0, 0, xlabel="N", ylabel="Z(n)", title="random walk with p = 0.4", xlim=
    [1, 600], ylim=[-200, 60], size=(650, 600))
    for each = 20:20:500
        y = Sample(each, 50, 0.4)
        x = [each for _ = 1:50]
        scatter!(x, y, label="Z($each)")
    end
    scatter!(0, 0)
end
```

From the plot, we can find that all the image has gone down, and with the n goes up, the minimum of the $Z(n)$ for each n has be a linearly related to n.

Question 2

Let $N(t)$ be defined as

$$N(t) = \sum_{i=1}^{\infty} I(X_1 + \cdots + X_i < t)$$

where X_i's are independent exponentially distributed random variables with parameter λ, the identifier function $I(\zeta)$ is defined as

$$I(\zeta) = \begin{cases} 1 & \text{if the predicate } \zeta \text{ is true} \\ 0 & \text{if the predicate } \zeta \text{ is false} \end{cases}$$

Q2(a)

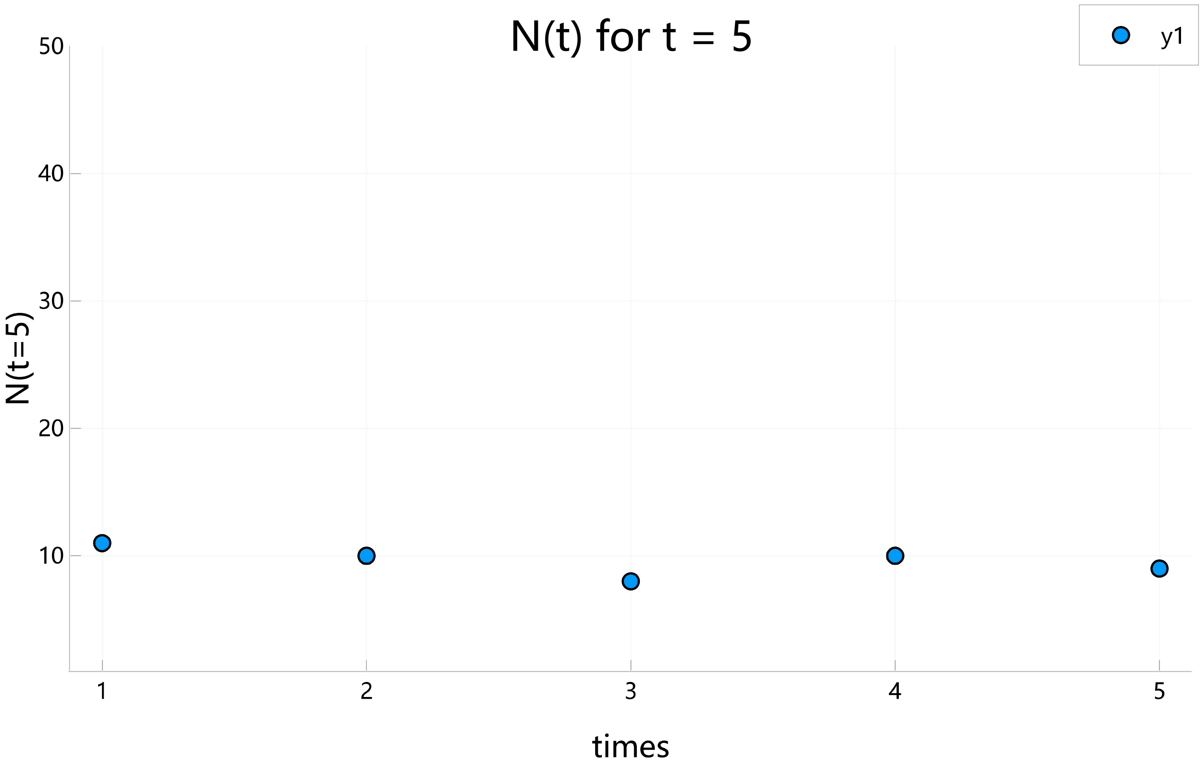
let $\lambda = 2$ and $t = 5$. Generate 5 independent traces of $N(t)$ and plot them all together in the same figure.

we can generate the exponential distributed random variables by the following ways:

1. we generate a variable obey the Uniform distribution named s
2. using the inverse transform of the $F(x = m) = 1 - e^{-\lambda m}$
3. finally, we can have the $m = \frac{-ln(1-s)}{\lambda}$

```
Sample_exp (generic function with 1 method)
```

```julia
function Sample_exp(lambda)
    s = rand()
    rv = - log(1 - s) / lambda
    return rv
end
```



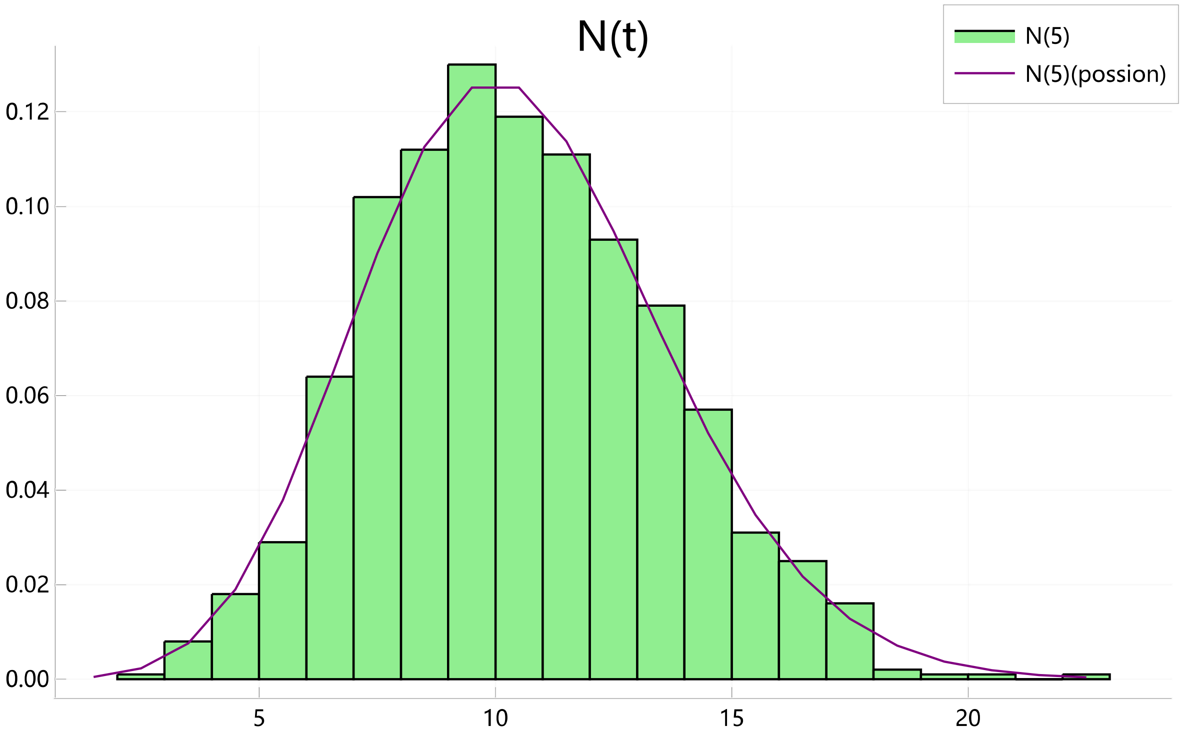

```
begin
    times = 1:5
    lambda = 2
    t = 5
    D = zeros(5)
    for time = times
        count = 0
        tot = 0
        while 1 != 0
            new_x = Sample_exp(lambda)
            tot = tot + new_x
            if tot >= 5
                break
            end
            count += 1
        end
        D[time] = count
    end
    scatter(times, D, xlabel = "times", ylabel = "N(t=5)", ylim = [1, 50],
    title="N(t) for t = 5")

end
```

Q2(b)

Generate 1000 independent traces of $N(t)$ for $t = 5$ and $\lambda = 2$. Plot the histogram of $N(t)$. in the same Plot, drwa the PDF of a possion distributon with mean λt, as

$$P[\hat{N(t)} = k] = \frac{(\lambda t)^k}{k!} e^{-\lambda t}, k = 0, 1, \cdots$$

Compare the histogram with the Possion distribution and explain your observations.




```
begin
    Distribution_N = zeros(1000)
    for time = 1:1000
        count = 0
        tot = 0
        while 1 != 0
            new_x = Sample_exp(lambda)
            tot = tot + new_x
            if tot >= 5
                break
            end
            count += 1
        end
        Distribution_N[time] = count
    end
    bins_dis = Int64(maximum(Distribution_N))
    histogram(Distribution_N, normalize=true, bins=bins_dis, label="N(5)",
    color="lightgreen", title="N(t)")

    possion_dis = zeros(bins_dis)
    tot_mult = 1.0
    for i = 1:bins_dis
        possion_dis[i] = (1.0 * lambda * t)^i / tot_mult * exp(-lambda * t)
        tot_mult *= i + 1
    end
    plot!(1.5:1:bins_dis + 1, possion_dis, label="N(5)(possion)", color="purple")
end
```

From the plot, we find that both of the images are corresponding totally, from the meaning of the random variable X_i, we can know that it is just the time interval of each event.

Q2(c)

Estimate the expected value of N(t) from the generated data and compare with the theoretical value.

From the theoretical value, we have $E(N(t)) = \lambda t$

```julia
begin
    mean_N = mean(Distribution_N)
    println("The predictable expected value of N(t) is $mean_N.")
    println("The theoretical expected value of N(t) is $(lambda * t).")
end
```

```
The predictable expected value of N(t) is 9.993.
The theoretical expected value of N(t) is 10.
```

Q2(d)

Estimate the variance of $N(t)$ from the generated data and compare with the theoretical value.

From the theoretical value, we have the $var(N(t)) = \lambda t$

```julia
begin
    var_N = var(Distribution_N)
    println("the predictable variance of N(t) is $(round(var_N, digits=3)).")
    println("the theoretical variance of N(t) is $(lambda * t).")
end
```

```
the predictable variance of N(t) is 9.376.
the theoretical variance of N(t) is 10.
```