School of Computing and Information Systems
COMP20007 Design of Algorithms
Semester 1, 2023
Sample Mid Semester Test

Instructions to Students

- **Do not start the exam until instructed to do so.** You may read this page now.
- You may fill out your student number now in the box provided at the bottom right of this page.
- You must have your student card on display during this test.
- The total time allowed for this test is 45 minutes, including 5 minutes of reading time. **NOTE: The actual MST will have a shorter duration, 30min, and will have less content.**
- This test contains 4 questions worth a total of 10 marks and contributes 10% of your final grade.
- This is a closed book exam. You should not have any study notes of any kind, including electronic devices (no calculators, phones, etc).
- Any student seen looking at their phone (or similar) or another student's paper during the test will have their paper removed immediately, and will be referred to the Academic Misconduct committee.
- At the end of the mid-semester test, you must stop writing and turn over the test paper. Otherwise, a mark deduction applies.
- Answer all questions on the provided blank space after each question.

Student Number:

Question 1 [2 Marks]

We know, from lectures, the following facts. For $0 < \varepsilon < 1 < c$,

$$1 \prec \log n \prec n^{\varepsilon} \prec n^{c} \prec n^{\log n} \prec c^{n} \prec n^{n},$$

where $f(n) \prec g(n)$ means both $f(n) \in O(g(n))$ and $g(n) \notin O(f(n))$ (i.e. $g(n)$ is not in $O(f(n))$).

For the following pairs of functions, $f(n), g(n)$, determine if $f(n) \in \Theta(g(n)), f(n) \in O(g(n))$, or $f(n) \in \Omega(g(n))$, making the strongest statement possible. That is, if both $f(n) \in O(g(n))$ and $f(n) \in \Omega(g(n))$ are true, you **must** answer with the form $f(n) \in \Theta(g(n))$. You must answer with $f(n)$ on the left and $g(n)$ on the right, for example, you may not answer $g(n) \in O(f(n))$.

You **must** show **all** working. A correct answer that does not show your working will result in 0 marks.

(a) $f(n) = (n+1)^3, \quad g(n) = (2n)^3$

(b) $f(n) = 3^{n+1}, \quad g(n) = (3+1)^n$

(c) $f(n) = n^3 + 1.1^n, \quad g(n) = (n^3)^2 + 1.1^n$

(d) $f(n) = \log(n^n), \quad g(n) = \sqrt{n}$

Question 1 - Answer

Question 2 [2 Marks]

A string of characters is called a palindrome if reading the characters from left to right gives the same sequence as reading the characters from right to left.

For example: `abccba`, `ahgiigha` and `abgllggllgba` are all palindromes while `abcdabcd`, `lghddgl` and `abcd` are not.

Write the pseudocode for an algorithm which, given a string and its length n, can identify a palindrome **using only one scan from left to right** (which means you can only access each letter once). Your algorithm **must employ the use of either a queue or a stack**, and you **must justify your choice**.

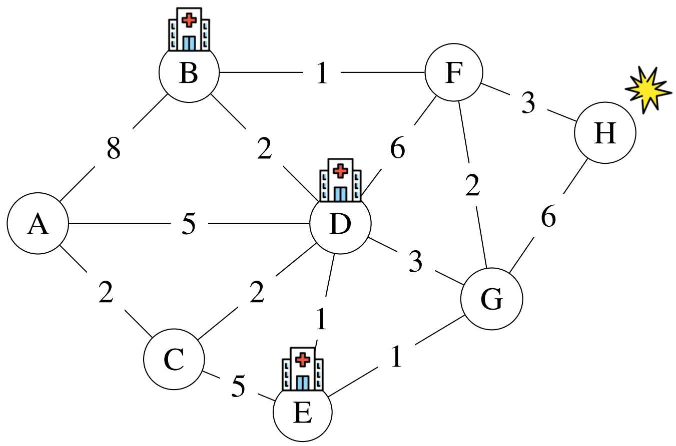
If you use a stack myou must use PUSH() and POP(), and if you use a queue you must use ENQUEUE() and DEQUEUE().

Assume for simplicity every input will be of even length.

Question 2 - Answer

Question 3 [3 Marks]

The following graph represents a road network where each edge presents a road segment. The costs of the road segments are given in the figure below. There are three hospitals at nodes B, D, and E, and an accident occurs at node H.



(a) Which algorithm would you use to find the nearest hospital from the accident. Explain how and why you would use this algorithm.

(b) Which is the nearest hospital, which path would you take and what is the cost of this path? You must show the execution of the algorithm described above.

(c) List the order of the nodes visited from H in the graph when a breadth-first search is used.

You should break ties in alphabetic order.

Question 3 - Answer

Question 4 [3 marks]

Consider the following recursive function, which takes an unordered array of integers, A, and an integer, k, and returns TRUE if k appears in A and FALSE otherwise.

function THIRDSSEARCH(A[0..n-1], k)
    if $n == 0$ **then**
        return FALSE
    else if $n == 1$ **then**
        return $(A[0] == k)$
    else
        $a \leftarrow \text{THIRDSSEARCH}(A[0..n/3-1], k)$
        $b \leftarrow \text{THIRDSSEARCH}(A[n/3..n \times 2/3-1], k)$
        $c \leftarrow \text{THIRDSSEARCH}(A[n \times 2/3..n-1], k)$
        return (a OR b OR c)

For simplicity, you may assume the input array is of length $n = 3^m$ for some positive integer m, so that each recursive call gets an exact third of the input array.

(a) Write down, and **solve**, a recursive formula $W(n) = \dots$ which describes the number of steps taken by THIRDSSEARCH on an array of length n, for the worst case input. Make sure you include the base case(s). Keep in mind that the basic operation is usually the most frequent or most expensive operation. Remember the formula for the geometric series states that

$$\sum_{i=0}^{k} x^i = \frac{x^k - 1}{x - 1}.$$

(b) Use the language of Ω, O, Θ to bound the time complexity of THIRDSSEARCH, using $T(n)$ for its runtime. You must include an upper **and** lower bound. Full marks are awarded only to the tightest possible bounds.

(c) Explain how THIRDSSEARCH could be slightly modified to improve its efficiency in the best case, and repeat the analysis from part (b), with justification.

Question 4 - Answer

END OF TEST