The Basics of Fuzzy Logic: A Tutorial Review

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Introduction

Fuzzy logic has emerged as a very powerful tool in dealing with complex problems. Recently the role of inference in handling uncertainty in engineering applications is gaining importance. Engineers and scientists are generally confronted with problems which are impossible to solve numerically using traditional mathematical rules. By making use of fuzzy logic , one can characterize and control a system whose model is not known or is ill- defined [11]. Fuzzy theory has the capability to capture the impreciseness of linguistic terms in statements of natural language. this provided with a greater capability to model human common-sense reasoning and decision making [14].

The idea of fuzzy logic was propounded by Lotfi Zadeh in 1965. His later works 'A Rationale for Fuzzy Control' and 'Linguistic Approach' (in 1972 and 1973 respectively) motivated the pioneering work done by other scientists. The first trial of fuzzy control was conducted by Mamdani on a laboratory steam engine (1974). In the 70's this motivated the researchers to develop a series of applications of fuzzy control [3]. the work on derivation of fuzzy control rules (1983) was carried out by Takagi and Sugeno. Since mid-eighties research has been directed towards incorporating fuzzy at the hardware level itself [1], [7]. [11]. The work on fuzzy chip done by Togai Watanabe (1985) in an important milestone in this direction.

An attempt has been made in this paper to inspire the reader to appreciate the use of fuzzy logic as a new logical tool that is instrumental in handling problems with ambiguities and where the information is imprecise and non numerical. the in adequacy of traditional Boolean logic to emulate the human thinking has also been highlighted.

Transition From Boolean To Fuzzy

Fuzzy logic can be viewed as the superset of Boolean logic. In Boolean logic, each element either belongs to or does not belong to a set. If an element is a member of a given set, Boolean logic will return a 'ONE' (representing complete membership) else a 'ZERO' (representing non-membership) will be returned. For example, consider the statement:

'The incandescent bulb glows at a supply voltage of 220V'. According to this statement, the bulb will glow (representing state 'ONE') at 220V and not otherwise.

Now consider the statement:
'The incandescent bulb glows when supply voltage is around 220V'. According to this, the bulb will glow even for voltage lower as well as higher than 220V.

The condition 'around 220V' cannot be represented by either '1' or '0' although the human mind can very well comprehend that it refers to voltages little below or higher than 220V. Thus 'around 200V' is not a binary condition, i.e. two distinct state '1' or '0' are not enough to characterize it. So, one might be inclined to say that we require more states or multiple states. But how many? Now we attempt to answer this question.

Fuzzy Sets

The phase 'around 220 V' can be represented by a set of points. Each point is a measure of the degree to which the phrase 'around 220 V' is true. The ordinate of any point lies between '0' (representing completely true condition) and '1' (representing completely true condition). The ordinate which expresses the degree of truth for a given value of the abscissa is called the Membership Function represented by μ .

Mathematically, $0 \leq \mu \leq 1$

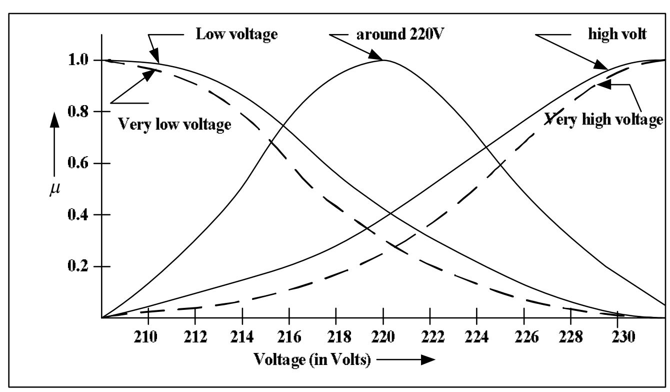

Figure 1- Membership functions for voltage

Thus, from figure 1, a voltage of 217 V would return a value of $\mu = 0.8$. The ordered pair (0.8, 217) is written as (0.8/217).

Now if we ask the question, **'To what degree is the voltage 217V around 220V?'**

The answer would be 0.8

The collection of points which determine the curve 'around 220V' can be written in the form $F = \int_U \mu F^{(x)} / X$ or

$$F = \sum_U \mu F(X) / X$$

It is to be noted here neither the integral sign denotes integration nor does the summation sign denote sum. They only denote a collection of points which form the set F(F is called a fuzzy set). [10].

F = around 220V = [0/210. 0.4/213, 0.6/215, 0.8/217, 1/220, 0.8/222, 0.6/226, 0/230]

The points on the x axis for which $\mu = 0$ are called the supports of fuzzy set. Also when the maximum height of the fuzzy set is 1.0then it is called a **normal fuzzy set**. The above fuzzy set F is an example of such fuzzy set.

Thus, we have seen that around 220V voltage is treated as a linguistic variable and expressed as a set of points called fuzzy set. Other curves or fuzzy sets such as 'low voltage' and 'high voltage' can also be defined and are shown in figure 1. The shape of curves that describe the fuzzy sets are generally triangular, trapezoidal or S and π shaped. In the diagram shown, the curves for 'around 220V' is π shaped whereas the curves for low and high are S-shaped.

Hedges

A linguistic hedge or modifier A linguistic hedge or modifier in an operation that modifies the meaning of a linguistic variable or a fuzzy set [10]. For example, if low current is a fuzzy set then very low current and extremely low current are examples of hedges. Hedges actually act upon a fuzzy set's membership function to modify it.

Hedges	Modification
very	μ_i^2
extremely	μ_i^3
more or less	$\mu_i^{0.5}$

Fuzzy Vs Boolean/Crisp

It is evident now that Fuzzy Logic approach departs from the strict limitations of Boolean Logic and allows a great deal of flexibility. It also allows us to handle vague and imprecise terms which give only an intuitive feeling. The boundary of a Fuzzy set is not required to be precise. Membership function in a fuzzy set is not, in general, a matter of affirmation or denial, but a matter of degree. To distinguish fuzzy sets from classical sets, the latter are referred to as crisp sets. A very simple example can highlight that Boolean Logic or crisp set is a special case of Fuzzy sets.

Consider the three standard definitions in Fuzzy Logic.

truth (not x) = 1.0 – truth (x)
truth (x and y) = min (truth(x), truth (y))
truth (x or y) = max (truth(x), truth (y))

where min and max denote the minimum and maximum operator.

In Boolean logic the value for truth (x) or truth (y) can be '0' or '1' only. if we insert the values '0' or '1' in the above expressions, we will obtain the truth tables for negation, AND, OR respectively. This forms the basis of the EXTENSION PRINCIPLE which states that classical results of Boolean Logic are recovered from fuzzy logic operations when all membership grades are restricted to the traditional set [0,1]. This effectively establishes Fuzzy logic as true generalization of classical Boolean Logic. The major difference between crisp sets have unique membership functions while fuzzy sets have infinite membership functions that may represent it. Also for crisp sets,

$AU \dot{A} = U$ and $A \cap \dot{A} = \phi$

but for fuzzy sets

$AU \dot{A} \neq U$ and $A \cap \dot{A} \neq \phi$

i.e. laws of contradiction and excluded middle are not valid for fuzzy sets which follows from the fact that

max[(A, 1-A)] is not equal to 1.
and
min[(A, 1-A)] is not equal to 0.

Fuzzy in everyday life

Suppose we want to raise the volume of radio set, would we say 'increase the volume by 5 dB' or would we say 'please increase the volume slightly.' The latter statement is easier to understand and to react to than the former even though the former gives precise information.

We quote an example from yesteryear, even before the study of fuzzy had formally started. Take for example the old, big, vacuum tube based radio set. It used to have a bulb which glowed the brightest when perfectly tuned to a particular frequency. From the relative brightness of the bulb, one could easily infer the degree to which the radio is tuned to the required frequency.

Now let us take an example from control systems. The roots of the characteristic equation of a system have negative real parts excepts for the presence of one or more non-repeated roots on the jw-axis. The above system as we know will not be absolutely stable nor will it be unstable. Then how stable is it? The degree of stability can be effectively expressed and understood by saying that the system is **marginally stable**. Such imprecise and vague terms facilitates the engineer understanding and gives him a feel of the system.

There are many such quantities in Electronics and Communication Engineering which when expressed in Fuzzy technology can give the information in a nutshell. Some typical quantities are listed in the table below, [10]

Field	Term Set	Values
1. Communication	i) SNR ii) noise iii) gain	Low, high Low, high Low, high
2.Solid-state Electronics	i) bof transistor ii) doping levels	Low, high, super Low, high, extremely
3. Control Systems	i) power output ii) stability	Low, high Marginally, limitedly

Fuzzy Vs Probability

Now, we highlight the difference between fuzzy logic and probability since a beginner is bound to be confronted with the problem of associating fuzzy with probability since he has a knowledge of the latter and the probability values also lies between 0 and 1.0.

Suppose we have two data modems A and B. Both have a value of 0.9 for probability and membership of being 'error free'. Membership value of 0.9 means that B is 'fairly similar' to being 'error free'. On the other hand, probability that A is error free = 0.9 means that A is expected to be error free in 90% of the trials. In the other 10 cases the modem will deliver unsuccessful results (i.e. 1 in 10 chances of being successful), Thus the user would opt for modem A.

Now consider a different scenario. Suppose we change the numerical values and consider that both membership as well as probability values are 0.5. In this case most users would switch to modem B because it offers a 50% chances of being successful whereas a membership value that low would indicate the unsuitability for such use [9].

Modelling of Fuzzy System

A typical fuzzy system is modelled on the basis of rules called fuzzy rules. These rules are the guiding force of the system which are implemented as a series of if.......then statements. Their role in fuzzy systems is analogous to that of transfer functions in classical control systems. The rules are based on experience, intuitive thinking and inference of humans. Some such rules are framed below [5].

1. Consider a simple mechanical system and its analogous electrical system as shown in figure 2.

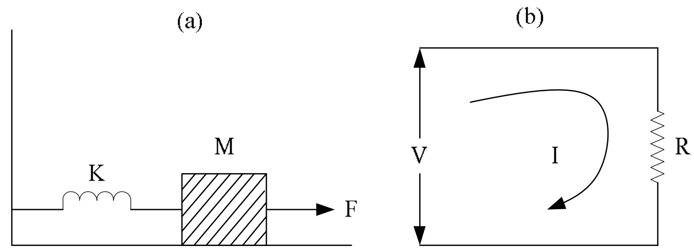

Figure 2- Analogous Systems

The rule corresponding to figure-2(a) will be * which will be equivalent to saying ** corresponding to figure-2(b).

*: If force F is high, coefficient is low, then velocity V is high.
** If voltage is high and impedance is low, then current is high.

2. Doppler effect can also be viewed in a fuzzy manner as

If the frequency (emitted by source) is high and the speed of observer (receding from the source) is low, then apparent frequency (heard by the observer) is medium. The inputs in the above system are frequency emitted and speed of the observer whereas the apparent frequency is the output.

3. Similarly in data communication involving retrieval of files we can have a rule

If data transmission error rate is medium and ftp data file size is high, then the attempts of repeat-request is medium.

Thus, the rules governing the behavior of a fuzzy system are written in simple language without any complicated equations. It is possible that the performance of a system varies with respect to different input variables. Certain combinations of inputs would lead to different outputs. All such possible outputs have a certain degree of truth to them. The defuzzification of these outputs determines the actual output of the system [14].

Fuzzy Relation and Composition: Towards inferencing

In order to relate two fuzzy sets, concept of fuzzy conditional statement is introduce as

'if A THEN B' or $A \Rightarrow B$

A fuzzy relation R from a set A to set B is expressed in terms of fuzzy subset of Cartesian product A*B and its membership function is defined by

$$\mu_R(u,v) = \mu_{A\times B}(u,v)$$

$$= \min\,[\,\mu_A(u) = \mu_B(v)\,] \quad u \in U, v \in V$$

where U and V are universal sets.

Now given a relation R, the subset [B'] induced by the application of the fuzzy sets [A'] is given by composition of R and A, that is

B' = A' o (A * B), Where 'o' is the operator for composition

The membership function is defined by

$$(v) = \max\min\,[\mu_A(u); \mu_B(u,v)]$$

The above procedure demonstrates the inference for a very simple fuzzy system and will be much more complicated if more than one premises is involved in the rule i.e. fuzzy conditional statements are nested.

e.g. If X is A and Y is B then Z is C.

Applications

The major applications of fuzzy logic may be listed as follows:

i) Fuzzy control in industry: The idea of fuzzy control is simply characterised by a control strategy expressed by a number of fuzzy control rules. Recent applications include water quality control, automatic train operating systems, elevator control [3], control of smart locomotives [12], cement kiln control [6], power electronics – speed control of DC motor, induction motor efficiency optimization control [2], [3].

ii) Fuzzy Logic Based Products: Fuzzy logic has been introduced in consumer goods like washing machines and television sets. Recent work has been directed towards developing fuzzy logic based hand held portable products [15].

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Conclusions and Future Work

We have thus seen in a nutshell that a fuzzy control rule is expressed by a fuzzy implication of the form 'if .... then....' which includes fuzzy variables often called linguistic variables. Fuzzy sets can effectively describe the process of human thinking where ill-defined and linguistic concepts of terms are usually treated. This is because fuzzy propositions in fuzzy implications are qualitative rather than quantitative [11]. Fuzzy sets and Fuzzy logic are powerful tools for managing complexities. Their use facilitates the bridge between mathematical models and the associated physical reality.

There is a tremendous scope of fuzzy logic based portable products in the near future. One major area where its use is being investigated is mobile cellular communication system [16].

This also opens an avenue for designing ASICs comprising integrated circuits for basic fuzzy computational modules. For this VHDL based simulation of integrated circuits for fuzzy operations is gaining popularity [1].

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