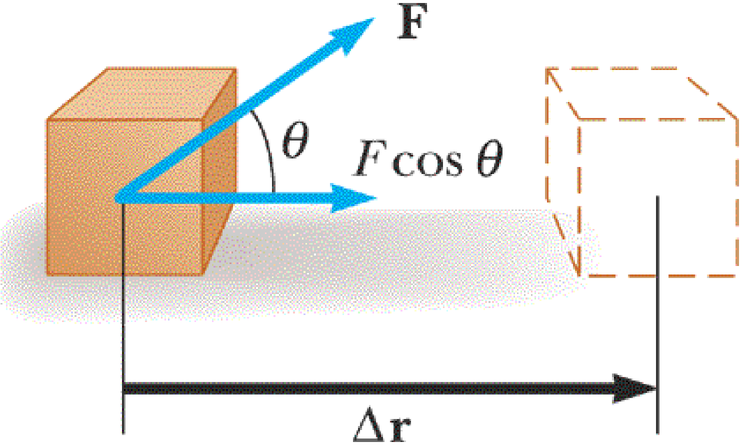
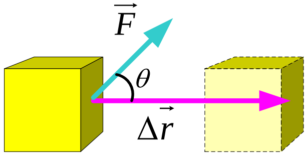
§ 1 Work and Power

- Work done by a constant force

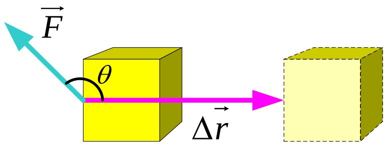
$$W = \vec{F} \cdot \Delta\vec{r} = F\,|\Delta\vec{r}|\cos\theta$$



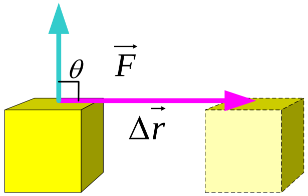
W is positive when θ <90°



W is negative when θ >90°



W is zero when θ =90°

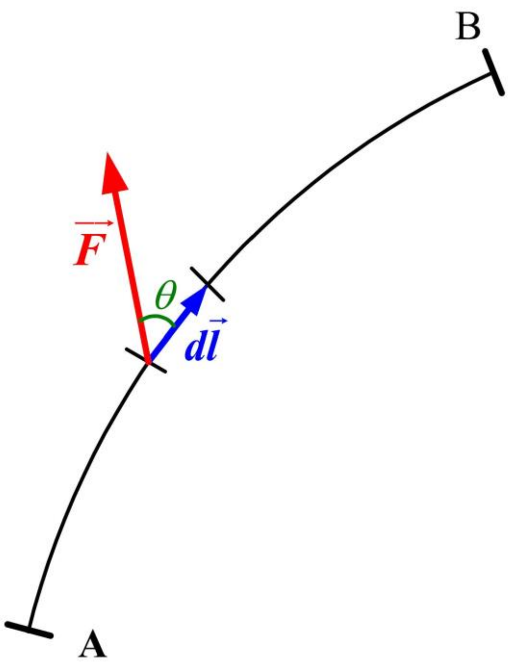


Work

- Work done by a varying force along a curve path
  - Divide the path into a large number of small displacement $d\vec{l}$

$$\boxed{W = \int_L \vec{F} \cdot d\vec{l}}$$

Line integral or path integral



The SI unit of work: Newton•meter or Joule

  - Work is a process quantity.
  - Calculation of work relates to the reference frame.

- **Work done by multiple forces.**

Total work done is the scalar addition of the work done by each force.

$$W_{net} = \int_A^B \vec{F}_{net} \cdot d\vec{l} = \int_A^B \left(\sum_i \vec{F}_i \right) \cdot d\vec{l} = \sum_i \int_A^B \vec{F}_i \cdot d\vec{l} = \sum_i W_i$$

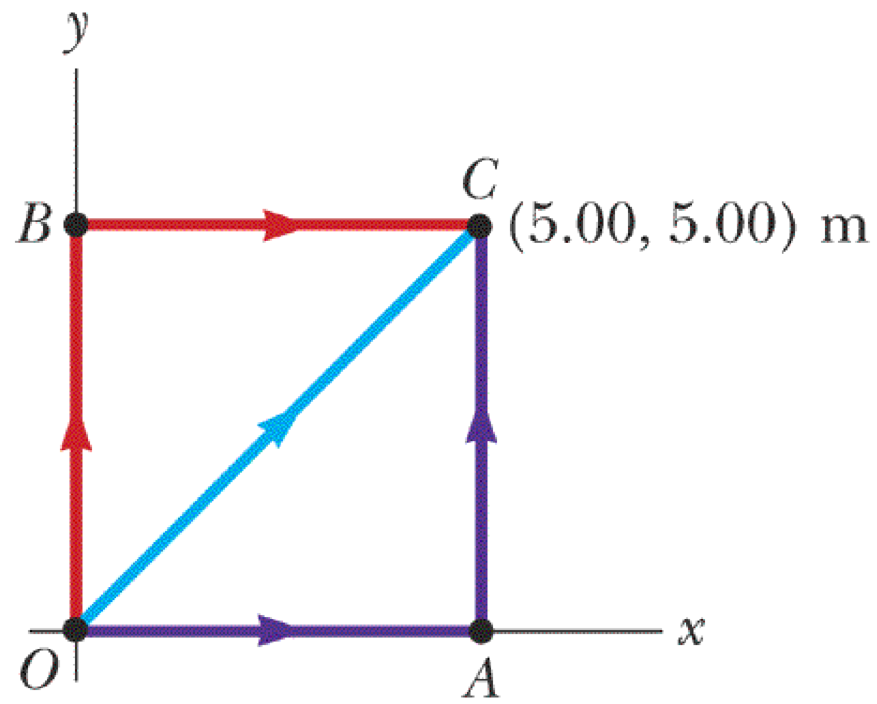
- **The power: The rate at which work is done (P186)**
  - **Average power:** $$\overline{P} = \frac{\Delta W}{\Delta t}$$
  - **Instantaneous power:** $$P = \frac{dW}{dt} = \frac{\vec{F} \cdot d\vec{r}}{dt} = \vec{F} \cdot \vec{v}$$
  - **SI unit: watt.**

Example

A force acting on a particle moving in the xy plane is given by

$$\vec{F} = 2y\hat{i} + x^2\hat{j} \qquad \textbf{(SI)}$$

The particle moves from the origin to a final position C (5.00m, 5.00m). Calculate the work done by $\vec{F}$ along (1) OC, (2) OAC, (3) OBC.



Example (continued)

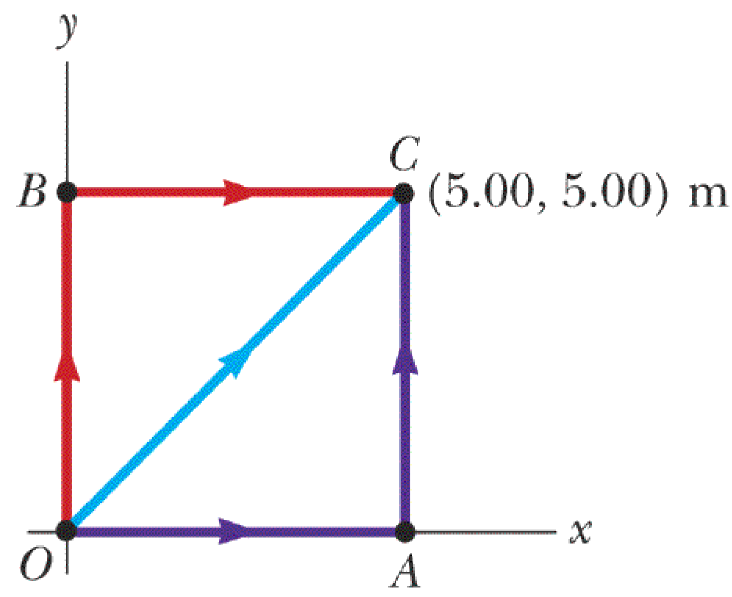
Solution:

$$\vec{F}\cdot d\vec{l} = (F_x\hat{i} + F_y\hat{j})\cdot(dx\hat{i} + dy\hat{j})$$

$$= F_x dx + F_y dy$$

$$\vec{F} = 2y\hat{i} + x^2\hat{j}$$

$$\vec{F}\cdot d\vec{l} = 2ydx + x^2 dy$$



(1) Along path *OC*:

$$\int_{OC} \vec{F}\cdot d\vec{l} = \int_{OC} (2ydx + x^2 dy) = \int_0^5 2xdx + \int_0^5 x^2 dx = 66.7\ \mathrm{J}$$

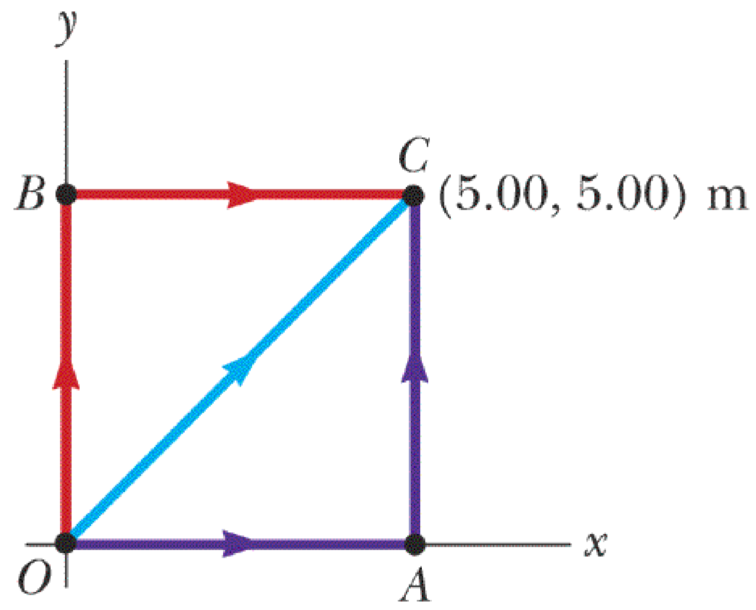
$$OC: y = x$$

Example

$$\vec{F} = 2y\hat{i} + x^2\hat{j}$$

$$\vec{F}\cdot d\vec{l} = F_x dx + F_y dy$$

$$= 2ydx + x^2 dy$$



(2) Along path *OAC*:

$$\int_{OAC} \vec{F}\cdot d\vec{l} = \int_{OA} \vec{F}\cdot d\vec{l} + \int_{AC} \vec{F}\cdot d\vec{l}$$

$$= \int_{OA} (2ydx + x^2 dy) + \int_{AC} (2ydx + x^2 dy)$$

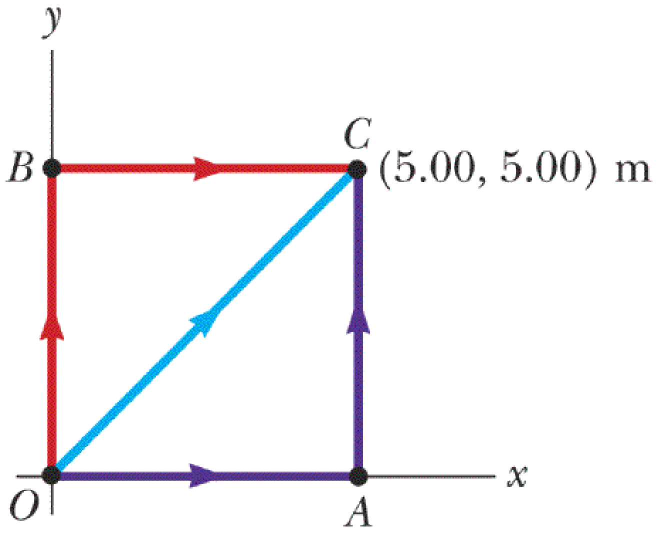
$$= \int_{AC} x^2 dy = \int_0^5 5^2 dy = 125\ \text{J}$$

Example (continued)

$$\vec{F} = 2y\hat{i} + x^2\hat{j}$$

$$\vec{F}\cdot d\vec{l} = F_x dx + F_y dy$$

$$= 2ydx + x^2dy$$



(3) Along path *OBC*:

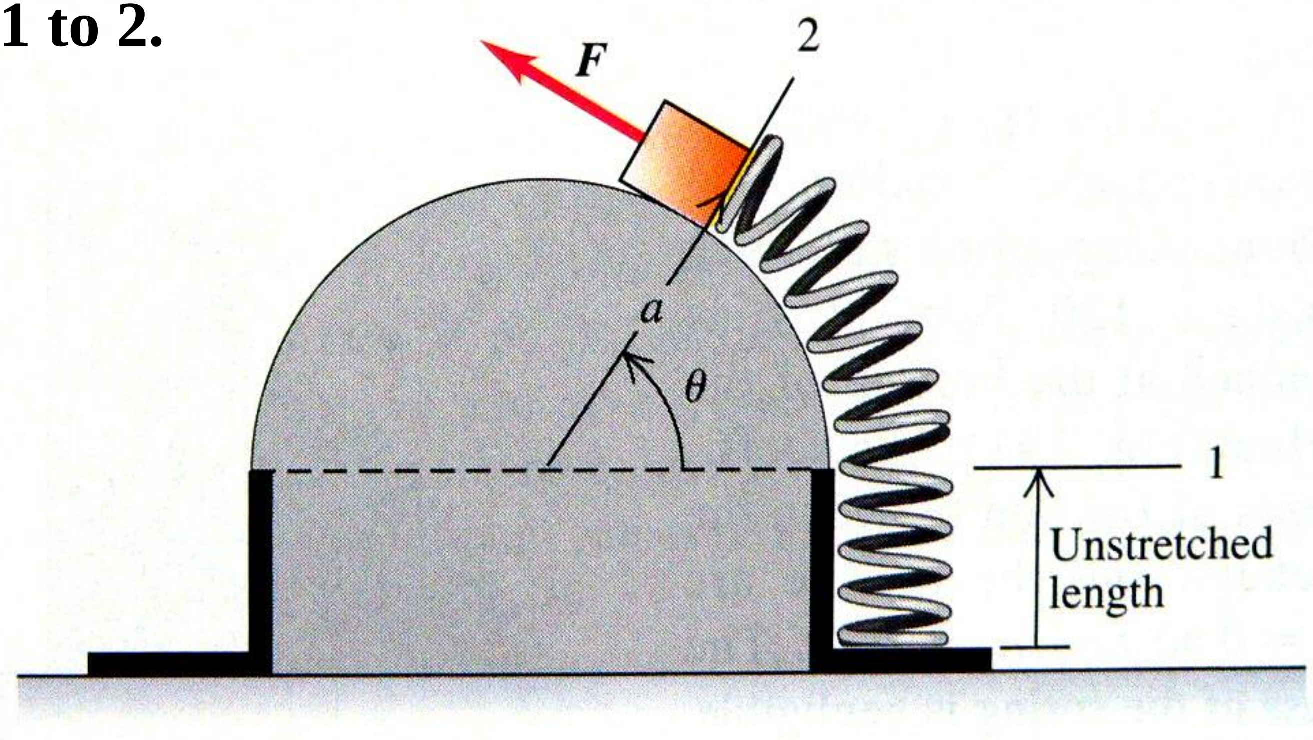
$$\int_{OBC}\vec{F}\cdot d\vec{l} = \int_{OB}\vec{F}\cdot d\vec{l} + \int_{BC}\vec{F}\cdot d\vec{l}$$

$$= \int_{OB}(2ydx + x^2dy) + \int_{BC}(2ydx + x^2dy)$$

$$= \int_{BC}2ydx = \int_0^5(2\times 5)dx = 50\text{ J}$$

Example

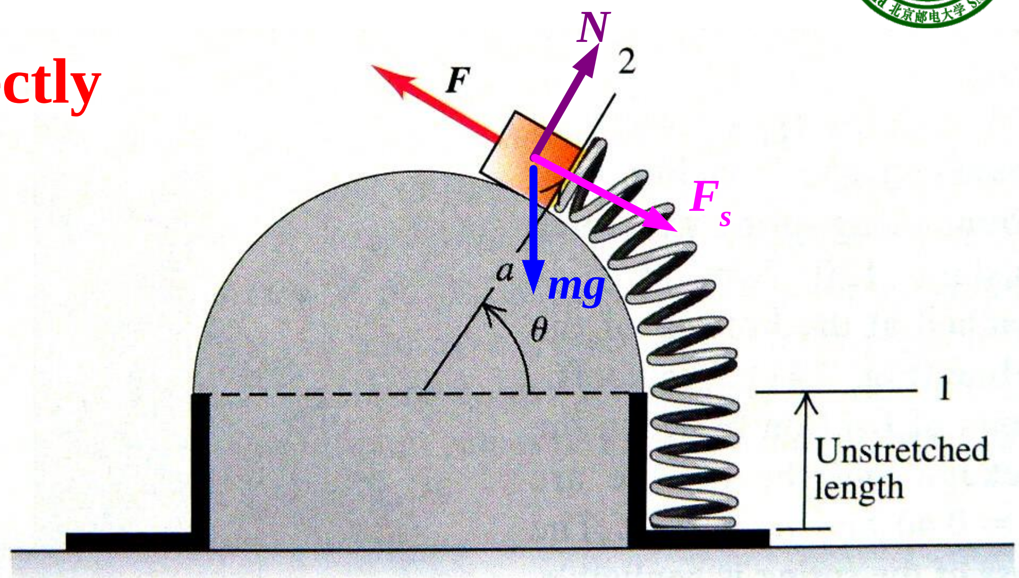
Variable force ***F*** is maintained tangent to a frictionless semicircular surface. By a slowly varying force *F*, a block with mass of ***m*** is moved, and spring to which it is attached is stretched from position 1 (unstretched length) to position 2 (***θ***). The spring has negligible mass and force constant ***k***. The end of the spring moves in an arc of radius ***a***. Calculate the **work** done by the force ***F*** from position 1 to 2.



Solution

Solution I: by integration directly

The block is in equilibrium in tangential direction:



$$F = k(a\theta) + mg\cos\theta$$

$$W_F = \int_1^2 \vec{F}\cdot d\vec{l} = \int_1^2 F\left|d\vec{l}\right|\cos\phi = \int_1^2 F ds$$

$$= \int_0^\theta [k(a\theta) + mg\cos\theta]d(a\theta)$$

$$= ka^2\int_0^\theta \theta d\theta + mga\int_0^\theta \cos\theta d\theta = \frac{1}{2}ka^2\theta^2 + mga\sin\theta$$

§ 2 Work – kinetic energy theorem (P156)

$$W_{net} = \int_A^B \sum_i \vec{F}_i \cdot d\vec{r} = \int_A^B \sum_i F_{it} ds = \int_A^B m \frac{dv}{dt} ds = \int_{v_A}^{v_B} mvdv = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2$$

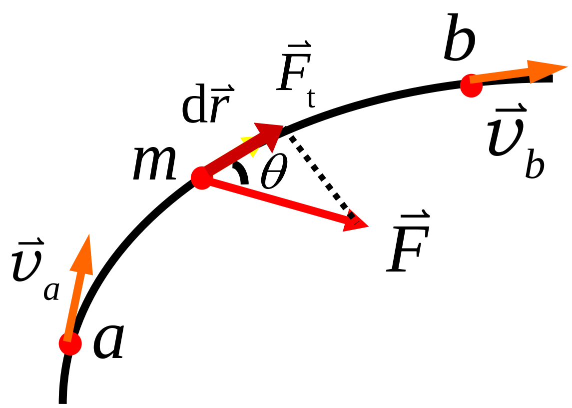
- Kinetic energy: $K = \frac{1}{2}mv^2$

Process quantity

The change of state quantity

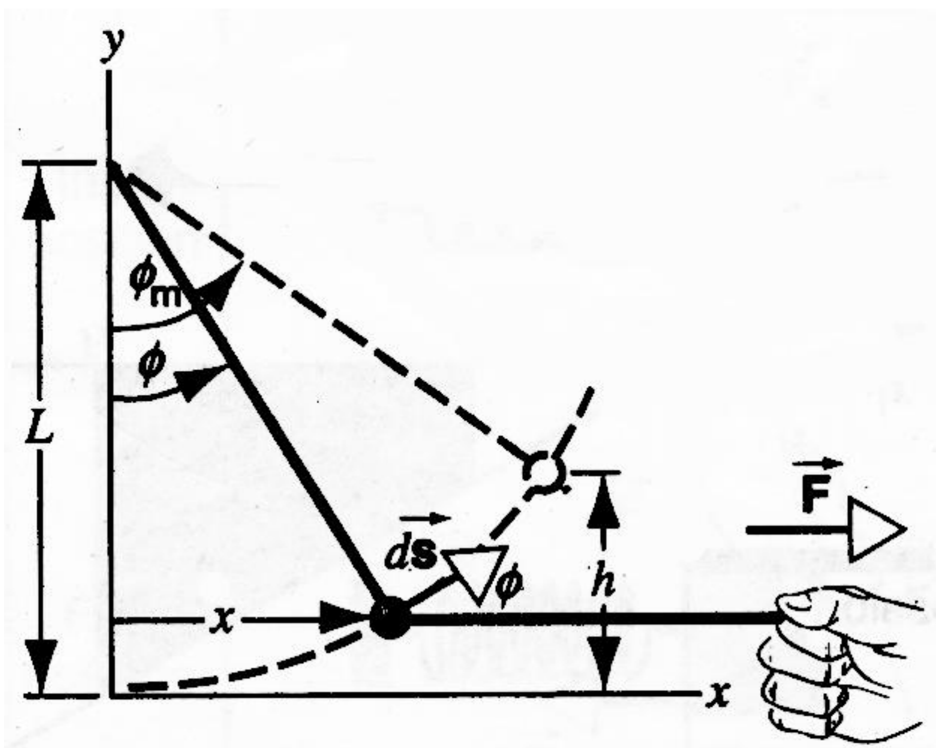
- Work – kinetic energy theorem: $\boxed{W_{net} = K_f - K_i}$

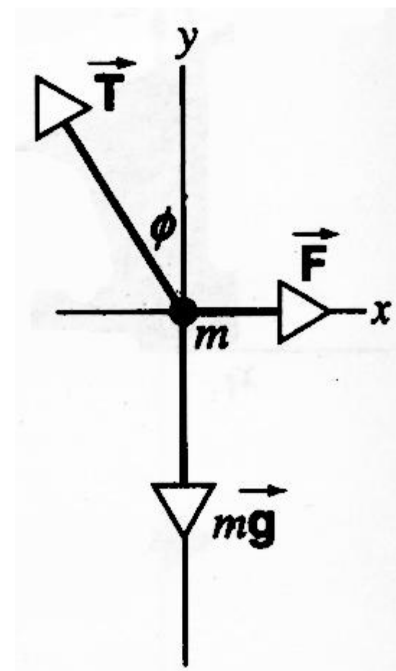
  - **The work done by the net force on a particle equals the change in kinetic energy (valid in the inertial frame of reference).**



Example

A small object of mass m is suspended from a string of length of L. The object is pulled sideways by a force F that is always horizontal, until the string finally makes an angle ϕ_m. The displacement is accomplished at a very small constant speed. Find the work done by all the forces that act on the object.



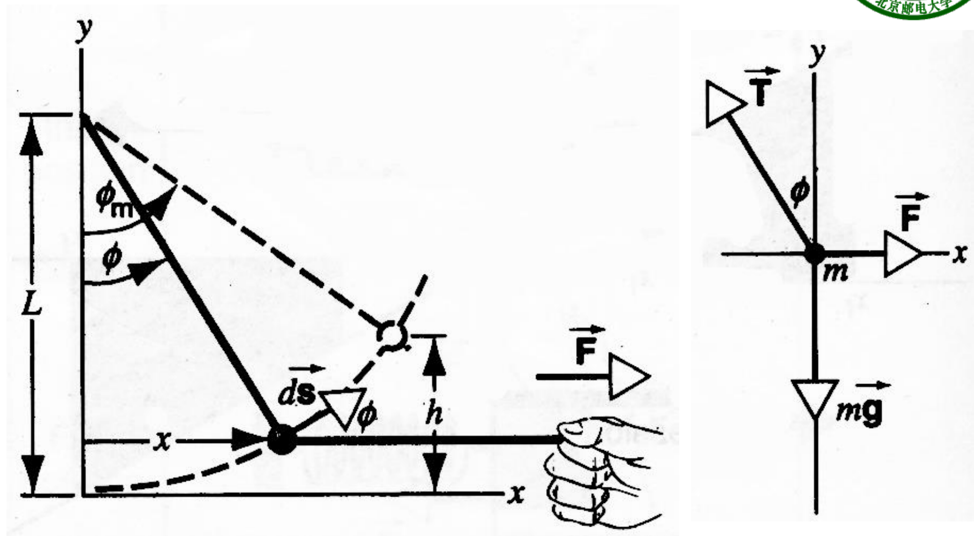


Solution

Solution I: Integration directly

x component: $F - T\sin\phi = 0$

y component: $T\cos\phi - mg = 0$



$$F = mg\tan\phi$$

$$W_F = \int_i^f \vec{F}\cdot d\vec{s} = \int_i^f F(\cos\phi)ds$$

$$W_F = \int_0^{\phi_m}(mg\tan\phi)\cos\phi(Ld\phi) = mgL\int_0^{\phi_m}\sin\phi\, d\phi = mgL(1-\cos\phi_m) = mgh$$

$$W_g = \int_i^f(-mg\hat{j})\cdot(dx\hat{i}+dy\hat{j}) = \int_0^h -mgdy = -mgh$$

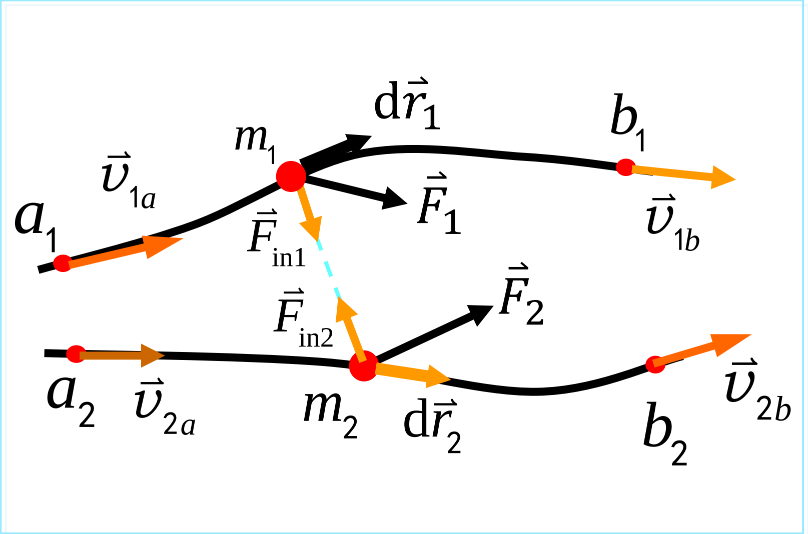
$W_T = 0$ $\qquad \vec{T}$ is perpendicular to the displacement $d\vec{s}$ at every point of the motion.

Solution II: Work – kinetic energy theorem

$$W_{net} = W_F + W_g + W_T = mgh - mgh + 0 = 0$$

Work – kinetic energy theorem

- Work – kinetic energy theorem for the system of particles



For m_1

$$W_1 = \int_{a_1}^{b_1} \vec{F}_1 \cdot \mathrm{d}\vec{r}_1 + \int_{a_1}^{b_1} \vec{F}_{\text{in}1} \cdot \mathrm{d}\vec{r}_1$$

$$= \frac{1}{2} m_1 \upsilon_{1b}^2 - \frac{1}{2} m_1 \upsilon_{1a}^2$$

For m_2

$$W_2 = \int_{a_2}^{b_2} \vec{F}_2 \cdot \mathrm{d}\vec{r}_2 + \int_{a_2}^{b_2} \vec{F}_{\text{in}2} \cdot \mathrm{d}\vec{r}_2$$

$$= \frac{1}{2} m_2 \upsilon_{2b}^2 - \frac{1}{2} m_2 \upsilon_{2a}^2$$

$$\left(\int_{a_1}^{b_1} \vec{F}_1 \cdot \mathrm{d}\vec{r}_1 + \int_{a_2}^{b_2} \vec{F}_2 \cdot \mathrm{d}\vec{r}_2\right) + \left(\int_{a_1}^{b_1} \vec{F}_{\text{in}1} \cdot \mathrm{d}\vec{r}_1 + \int_{a_2}^{b_2} \vec{F}_{\text{in}2} \cdot \mathrm{d}\vec{r}_2\right)$$

$$= \left(\frac{1}{2} m_1 \upsilon_{1b}^2 + \frac{1}{2} m_2 \upsilon_{2b}^2\right) - \left(\frac{1}{2} m_1 \upsilon_{1a}^2 + \frac{1}{2} m_2 \upsilon_{2a}^2\right)$$

Work – kinetic energy theorem

For a particle

$$\boxed{W_{net} = K_f - K_i}$$

- Work – kinetic energy theorem for the system of particles

$$\boxed{\sum W_{i-\text{external}} + \sum W_{i-\text{internal}} = \sum K_f - \sum K_i}$$

  - Generally, the works done by internal forces between particles cannot be canceled (the displacements of particles are different).

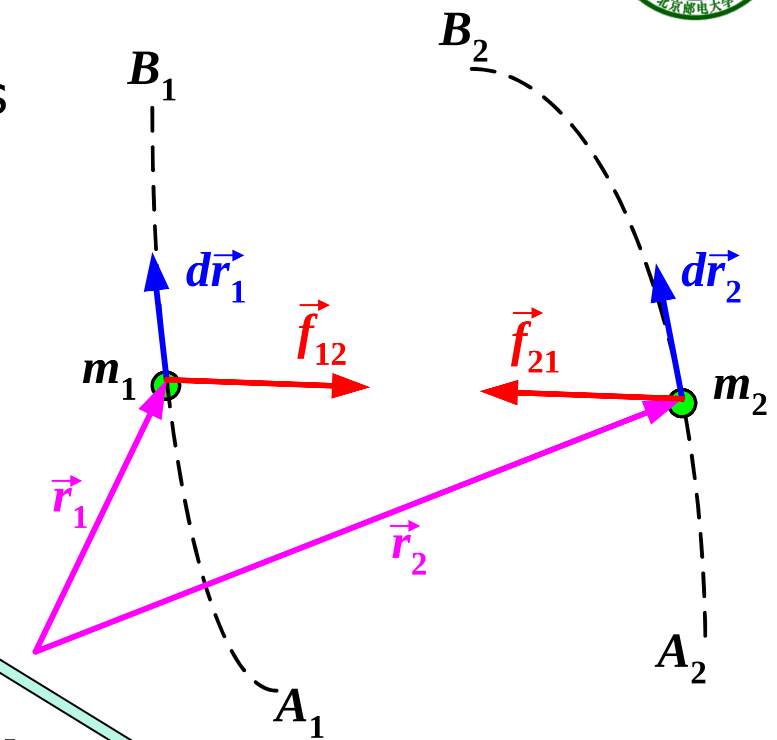
The work done by a pair of internal forces

- The work done by a pair of internal forces

$$\vec{f}_{12} = -\vec{f}_{21}$$

For a infinitesimal process

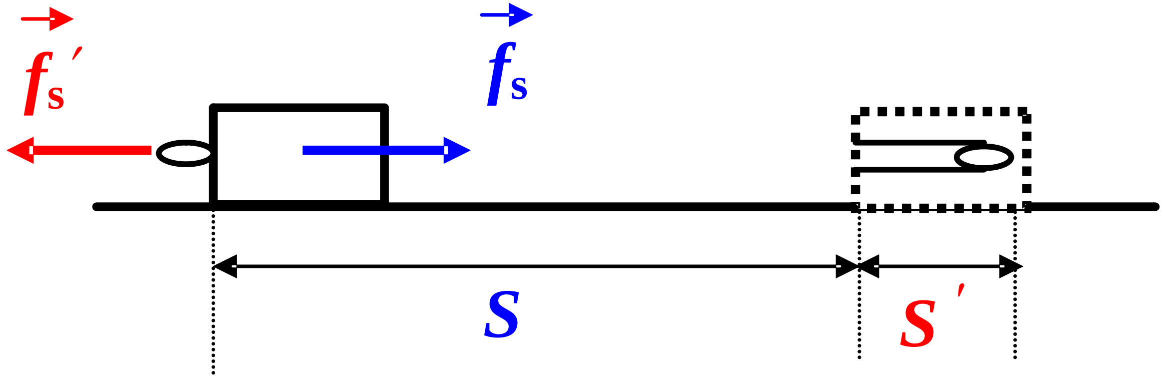
$$\begin{aligned} \mathrm{d}W &= \vec{f}_{12}\cdot d\vec{r}_1 + \vec{f}_{21}\cdot d\vec{r}_2 \\ &= \vec{f}_{21}\cdot(d\vec{r}_2 - d\vec{r}_1) = \vec{f}_{21}\cdot d\underbrace{(\vec{r}_2 - \vec{r}_1)}_{\text{The displacement of 2 relative to 1}} \\ &= \vec{f}_{21}\cdot d\vec{r}_{21} \end{aligned}$$



- **The calculation of net work done by a pair of internal forces on two particles is equivalent to —— in the reference frame of particle 1, the calculation of work done by one force acting on particle 2.**

Example

A bullet coming from left is shot into a wooden block and passes through a length of S' in the block. The system of bullet-block comes to a halt after sliding a distance of S. Calculate the net work done by a pair of friction forces f_s and f_s' between the bullet and the block.



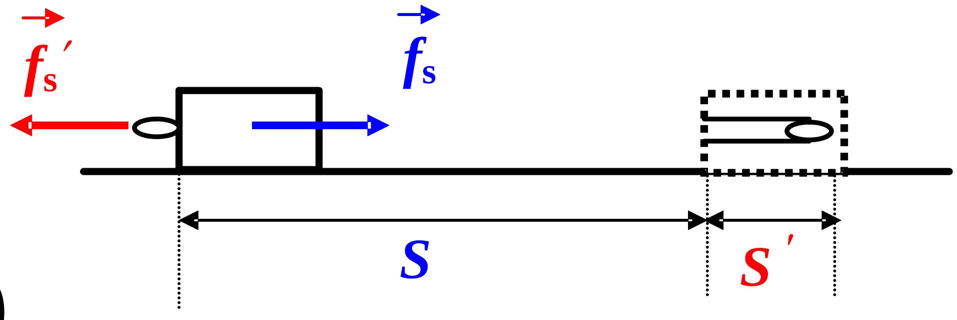
Example

$$\vec{f}_s = -\vec{f}_s', \qquad |\vec{f}_s| = |\vec{f}_s'| = f_s$$

Solution I:

For the block:

$$W_s = f_s S$$



For the bullet:

$$W_{s'} = -f_s(S + S')$$

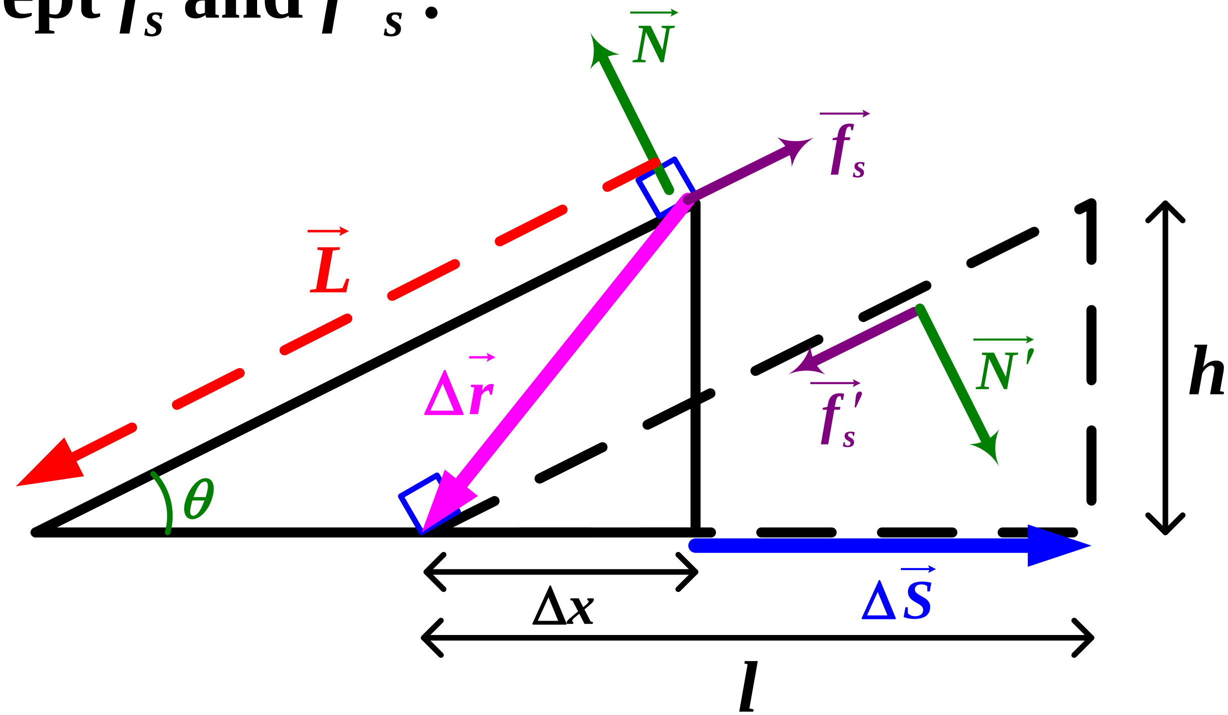
The net work:

$$W_s^{\text{net}} = W_s + W_{s'} = f_s S - f_s(S+S') = -f_s S'$$

Solution II, III:

Example

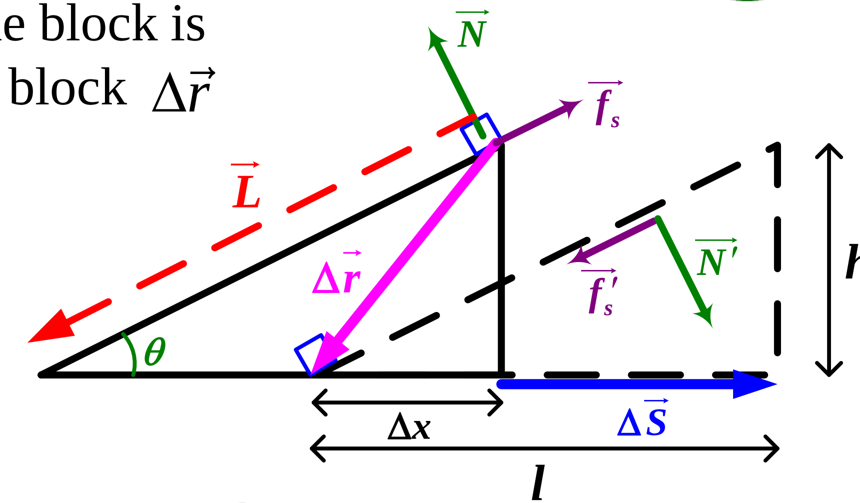
Calculate the net works done respectively by a pair of normal forces N and N' , f_s and f'_s between the block and the wedge. Neglecting the frictions except f_s and f'_s .



Example

Solution: The normal force $\vec{N}$ acting on the block is not perpendicular to the displacement of the block $\Delta\vec{r}$

Therefore: $W_N \neq 0$



The normal force $\vec{N}'$ acting on the wedge is not perpendicular to the displacement of the wedge $\Delta\vec{S}$. Therefore:

$$W_{N'} \neq 0$$

$$\Delta\vec{r} = -\Delta x\,\hat{i} - h\,\hat{j}, \qquad \vec{N} = -N\sin\theta\,\hat{i} + N\cos\theta\,\hat{j}$$

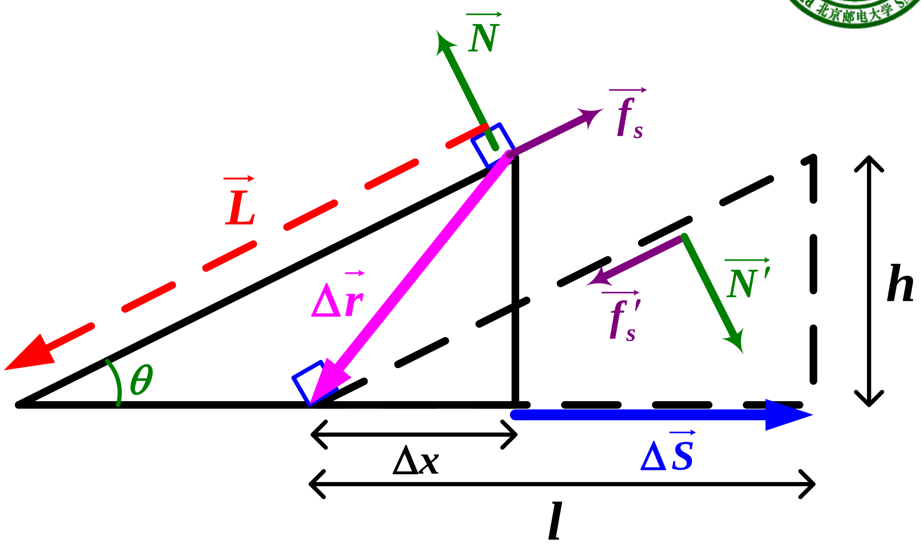
$$\Delta\vec{S} = (l - \Delta x)\,\hat{i}, \qquad \vec{N}' = N\sin\theta\,\hat{i} - N\cos\theta\,\hat{j}$$

$$W_N = \vec{N}\cdot\Delta\vec{r} = \Delta x N\sin\theta - hN\cos\theta, \qquad \tan\theta = \frac{h}{l}, \quad h\cos\theta = l\sin\theta$$

$$W_{N'} = \vec{N}'\cdot\Delta\vec{S} = lN\sin\theta - \Delta x N\sin\theta,$$

$$W_N^{\text{net}} = W_N + W_{N'} = 0$$

Example (continued)



$$\Delta\vec{r} = -\Delta x\,\hat{i} - h\,\hat{j}$$

$$\Delta\vec{S} = (l - \Delta x)\,\hat{i}$$

$$\vec{f}_s = f_s\cos\theta\,\hat{i} + f_s\sin\theta\,\hat{j}$$

$$\vec{f}' = -f_s\cos\theta\,\hat{i} - f_s\sin\theta\,\hat{j}$$

$$W_{f_s} = \vec{f}_s\cdot\Delta\vec{r} = -\Delta x\, f_s\cos\theta - h\, f_s\sin\theta$$

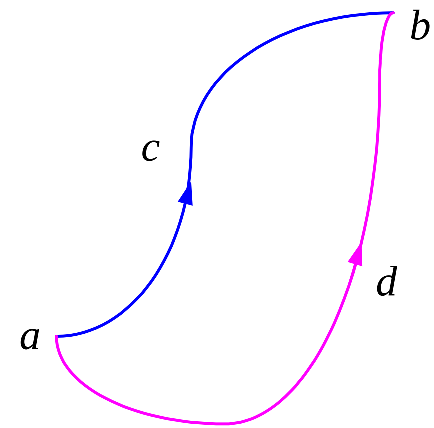
$$W_{f_s'} = \vec{f}_s'\cdot\Delta\vec{S} = -l\, f_s\cos\theta + \Delta x\, f_s\cos\theta$$

$$W_{f_s}^{\text{net}} = W_{f_s} + W_{f_s'} = -(l\cos\theta + h\sin\theta)\, f_s$$

$$= -(L\cos^2\theta + L\sin^2\theta)\, f_s = -f_s L$$

§ 3 Conservative Forces and Potential Energy

$$W = \int_{(L)} \vec{F} \cdot d\vec{r}$$

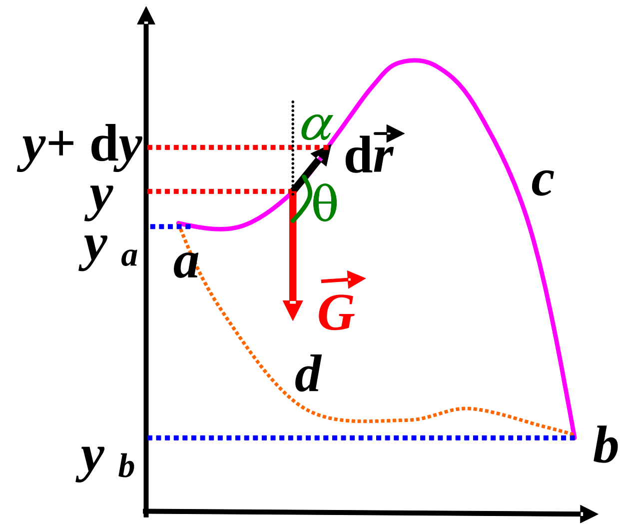


Work done by a force is a line integral or path integral. Generally, depends on the path followed by the particle. Different path corresponds to different work done by the same force.

A category of forces which have the special property, that the work done by such a force is independent of the path —— are conservative forces.

Work done by weight

$$\vec{G} = m\vec{g}$$



$$dW = \vec{G} \cdot d\vec{r} = G\cos\theta\, ds$$

$$= mg\cos(\pi - \alpha)ds$$

$$= -mg\,ds\cos\alpha = -mg\,dy$$

$$W = \int_a^b dW = \int_{y_a}^{y_b} -mg\,dy = -(mgy_b - mgy_a)$$

Only depends on the initial and final positions, and does not depend on the path taken by the particle.

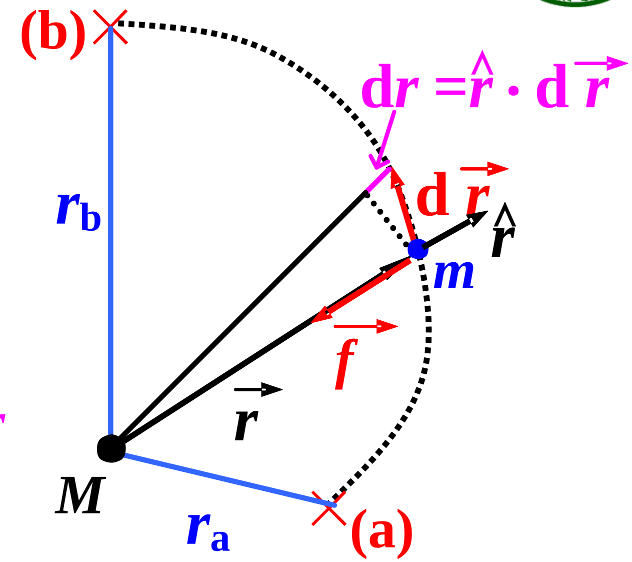
Work done by the universal gravitational force

$$\vec{f} = -G\frac{Mm}{r^2}\hat{r}$$

$$W = \int_a^b \vec{f}\cdot d\vec{r} = -\int_{r_a}^{r_b} G\frac{Mm}{r^2}\hat{r}\cdot d\vec{r}$$

$$= -\int_{r_a}^{r_b} G\frac{Mm}{r^2}|d\vec{r}|\cos\theta = -\int_{r_a}^{r_b} G\frac{Mm}{r^2}dr$$

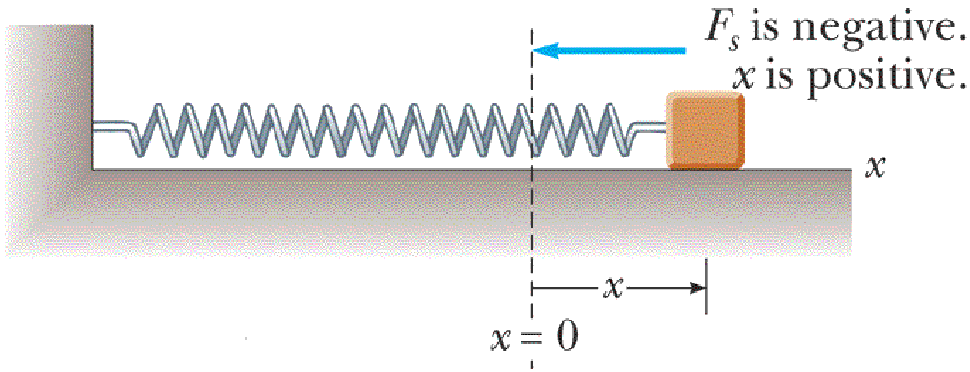
$$= -\left[\left(-G\frac{Mm}{r_b}\right) - \left(-G\frac{Mm}{r_a}\right)\right]$$



- **Only depends on the initial and final positions, and does not depend on the path taken by the particle.**

Work done by the spring force

$$\vec{F}_s = -kx\,\hat{i}$$

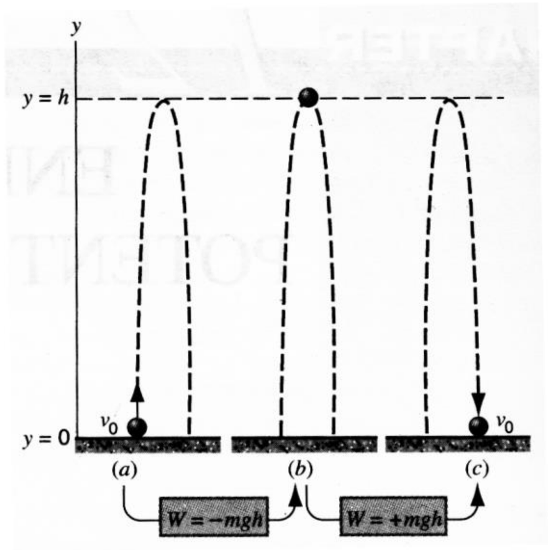


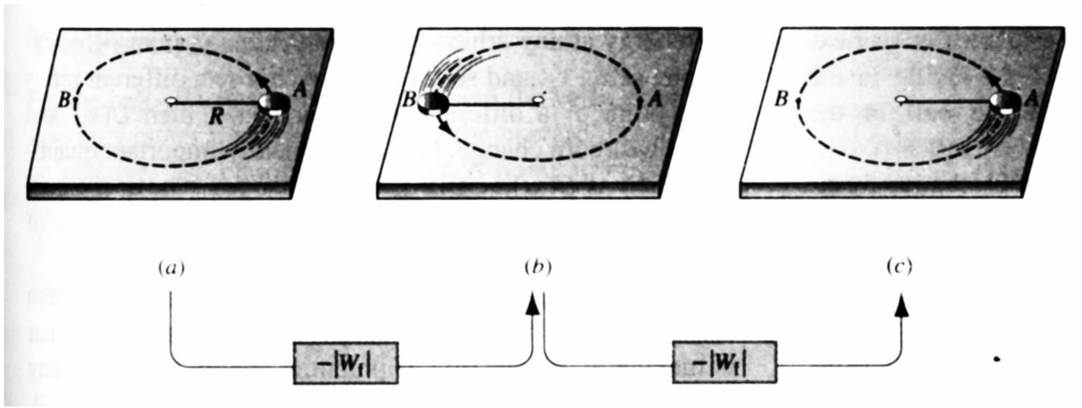
$$W = \int_{x_a}^{x_b} \vec{F}_s \cdot d\vec{r} = \int_{x_a}^{x_b} \left(-kx\,\hat{i}\right)\cdot\left(dx\,\hat{i}\right)$$

$$= -\int_{x_a}^{x_b} kx\,dx = -\left(\frac{1}{2}kx_b^2 - \frac{1}{2}kx_a^2\right)$$

- **Only depends on the initial and final positions, and does not depend on the path taken by the particle.**

- **The total work done by a conservative force is zero as the particle moves along a round trip.**
- **But when the particle moves along a round trip, the total work done by a nonconservative force is not zero.**





For the force of gravity:

$$W_{\text{round trip}} = 0$$

For the friction force:

$$W_{f\ \text{round trip}} = -2\pi R\mu_k mg$$

The conservative force

- Conclusion: The conservative force has properties that

  - **The work done by a conservative force dose not depend on the path followed by the particle, and depends only on the initial and final positions.**

$$\boxed{W=\int_a^b \overrightarrow{F_c}\cdot d\vec{r}=-\left[U(\vec{r}_b)-U(\vec{r}_a)\right]}$$

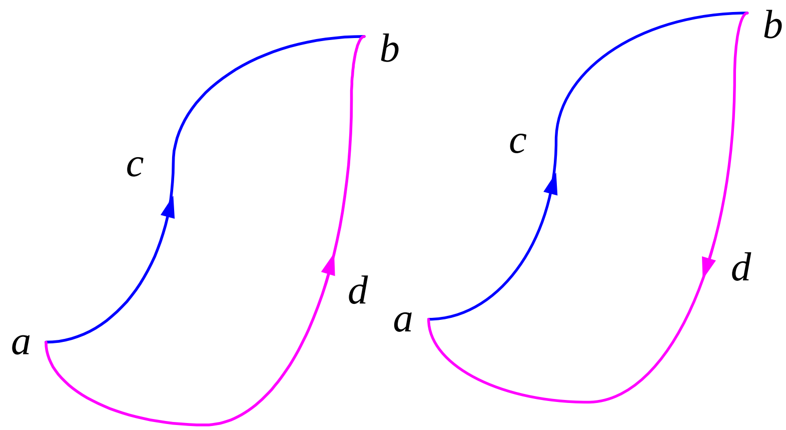
Equivalent statement:

- **The total work done by a conservative force is zero, as the particle moves around a close path and returns to its starting point (round trip).**

$$\int_{acb}\overrightarrow{F_c}\cdot d\vec{r}=\int_{adb}\overrightarrow{F_c}\cdot d\vec{r}$$

$$\int_{acb}\overrightarrow{F_c}\cdot d\vec{r}-\int_{adb}\overrightarrow{F_c}\cdot d\vec{r}=\int_{acb}\overrightarrow{F_c}\cdot d\vec{r}+\int_{bda}\overrightarrow{F_c}\cdot d\vec{r}=0$$

$$\oint\overrightarrow{F_c}\cdot d\vec{r}=0$$



How to get the **absolute** value of potential energy?

$$U(\vec{r}_b)-U(\vec{r}_a)=-\int_a^b \overrightarrow{F_c}\cdot d\vec{r}$$ the definition of potential energy only gives the change in potential energy, or the **relative** value of potential energy. We can choose a position $\vec{r}_0=\vec{r}_a$ as the reference point, define $U(\vec{r}_0)=0$ at the reference point. The choice of reference point is arbitrarily.

New definition of potential energy: $\boxed{U(\vec{r})=U(\vec{r})-0=-\int_{\vec{r}_0}^{\vec{r}} \overrightarrow{F_c}\cdot d\vec{r}=\int_{\vec{r}}^{\vec{r}_0} \overrightarrow{F_c}\cdot d\vec{r}}$

- For gravitational potential energy near the Earth's surface, it is accustomed to choose the reference point $\boldsymbol{y_0=0}$ as surface of the Earth.

$$U(y)=mgy$$

- For gravitational potential energy associate with two particles, it is accustomed to take $U(r_0=\infty)=0$.

$$U(r)=-G\frac{Mm}{r}$$

- For elastic potential energy, it is accustomed to choose the reference position to be that in which the spring is in its relaxed state.

$$U(x)=\frac{1}{2}kx^2$$

Why introduce potential energy?

$$\Delta U = U(\vec{r}_b) - U(\vec{r}_a) = -W = -\int_a^b \overrightarrow{F_c} \cdot d\vec{r}$$

- The work done by a conservative force can be represented in terms of the change in potential energy.
- Notice:
  - The potential energy belongs to the **system**. We should properly speak of “the elastic potential energy of the block-spring system” or “the gravitational potential energy of the ball-Earth system”, not “the elastic potential energy of the spring” or “the gravitational energy of the ball”.
  - The potential energy U is the energy associated with the configuration of a system. Here “***configuration***” means how the parts of a system are located or arranged with respect to one another (the compression or stretching of the spring in the block-spring system, or height of the ball in the ball-Earth system.)

The conservative force and potential energy

For an infinitesimal process,

$$-dU = -\left(\frac{\partial U}{\partial x}dx + \frac{\partial U}{\partial y}dy + \frac{\partial U}{\partial z}dz\right),$$

$$-dU = \vec{F}\cdot d\vec{r} = F_x dx + F_y dy + F_z dz,$$

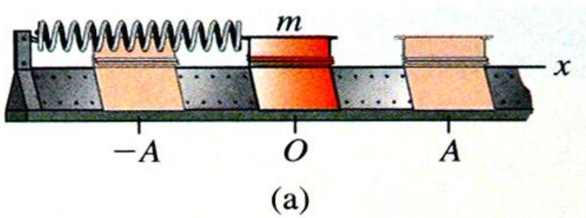
$$\begin{cases} F_x = -\dfrac{\partial U}{\partial x} \\ F_y = -\dfrac{\partial U}{\partial y} \\ F_z = -\dfrac{\partial U}{\partial z} \end{cases}$$

$$\boxed{\vec{F} = -\left(\hat{i}\frac{\partial U}{\partial x} + \hat{j}\frac{\partial U}{\partial y} + \hat{k}\frac{\partial U}{\partial z}\right) = -\nabla U}$$

∇U means the **gradient** of the potential-energy function. The gradient of a scalar function is a vector function. ∇ is a gradient operator.

$$\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

Energy Diagrams



- For force of gravity

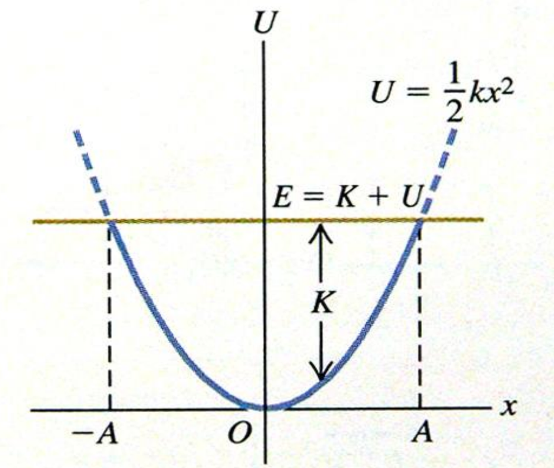
$$U(y)=mgy,\qquad F_y=-\frac{\partial U}{\partial y}=-mg$$

- For universal gravitational force

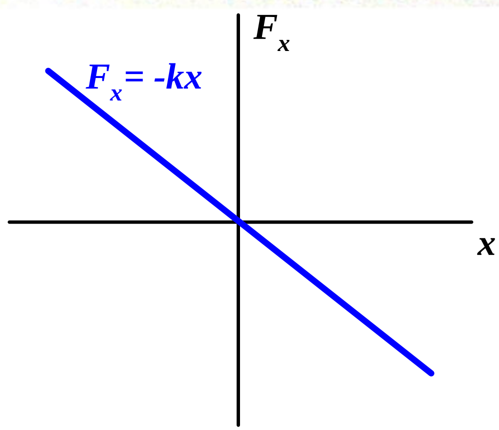
$$U(r)=-\frac{GMm}{r},\qquad F_r=-\frac{\partial U}{\partial r}=-\frac{GMm}{r^2}$$

- For spring force

$$U(x)=\frac{1}{2}kx^2,\qquad F_x=-\frac{\partial U}{\partial x}=-kx$$

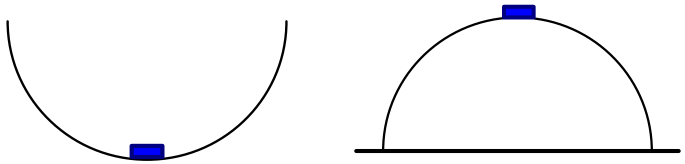


The force is equal to the negative of the slope of *U*(*x*)



- **Because of conservation of mechanical energy, *E* as a function of *x* is a straight horizontal line** $E=K+U$
- **The glider can only move in the range between $x=\pm A$, since the kinetic energy in this range is positive.**
- **At $x=0$, the slope of *U*(*x*) and the force are zero, so it is an equilibrium position.**

Stable and unstable equilibrium



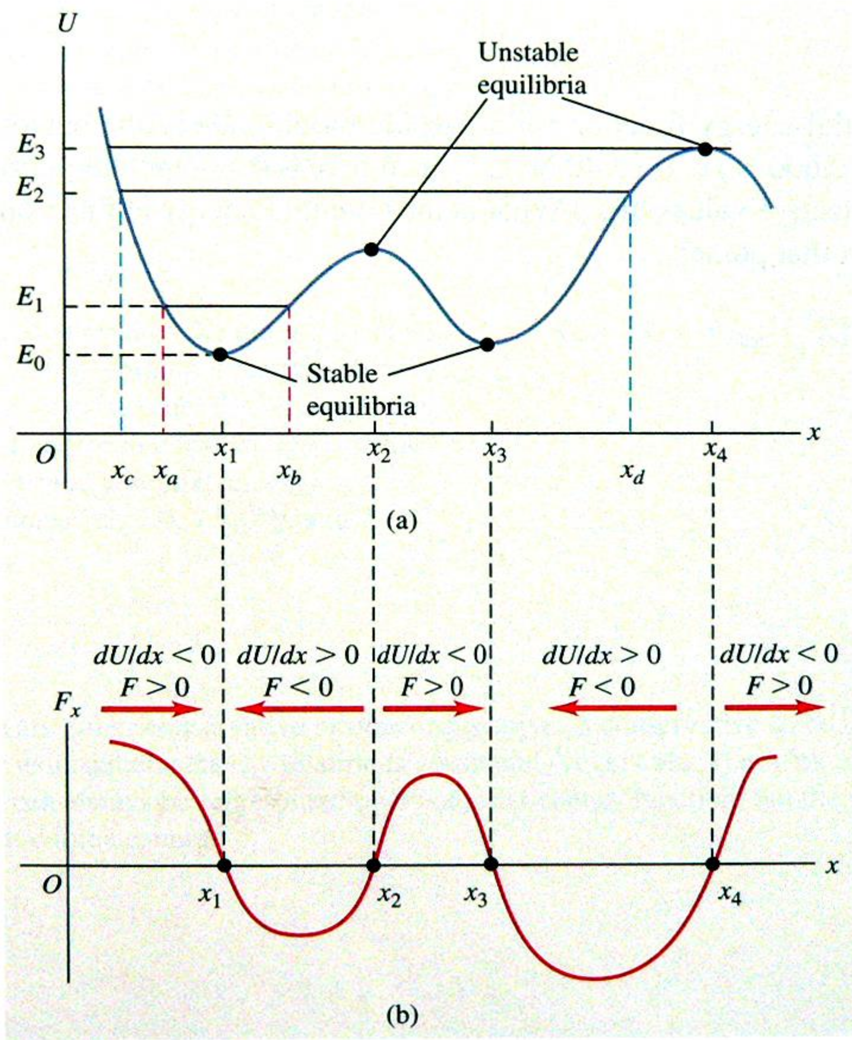
The particle is in stable equilibrium (left) and in unstable equilibrium (right).

- Any **minimum** in a potential-energy curve is a **stable** equilibrium position.

  Points x_1 and x_3 are stable equilibrium points. When the particle is displaced to either side, the force pushes back toward the equilibrium point.

- Any **maximum** in a potential-energy curve is an **unstable** equilibrium position.

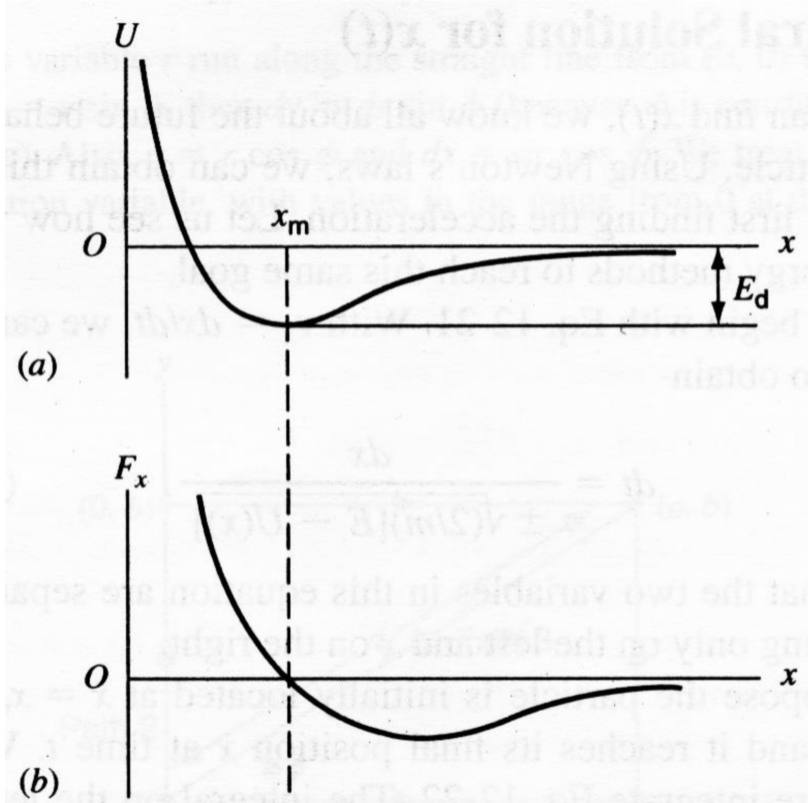
  Points x_2 and x_4 are unstable equilibrium points. When the particle is displaced to either side, the force pushes away from the equilibrium point.



Example

A commonly used potential function to describe the interaction between the two atoms in a diatomic molecule is the Lennard-Jones 6-12 potential

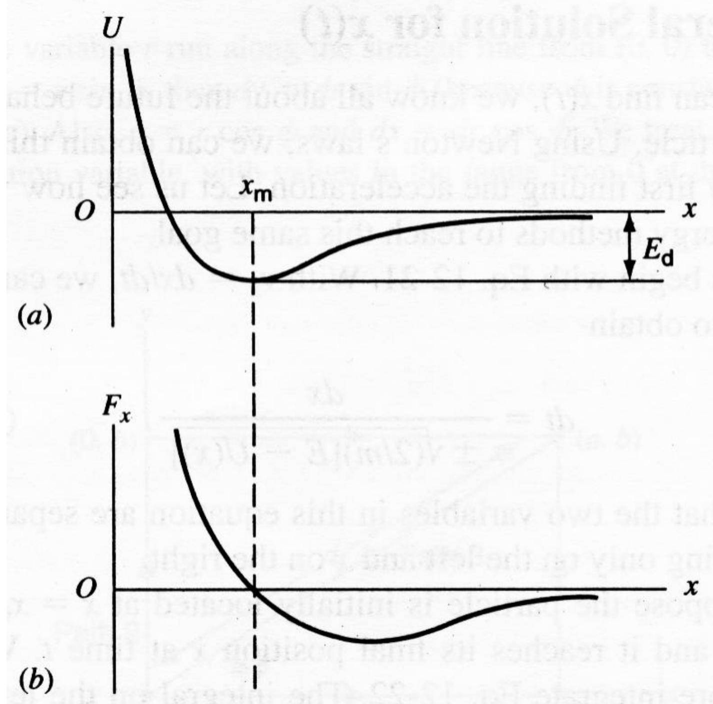
$$U(x) = \varepsilon\left[\left(\frac{x_0}{x}\right)^{12} - 2\left(\frac{x_0}{x}\right)^{6}\right]$$



Find (a) the equilibrium separation between the atoms, (b) the force between the atoms, (c) the minimum energy necessary to break the molecule apart.

Example

$$U(x)=\varepsilon\left[\left(\frac{x_0}{x}\right)^{12}-2\left(\frac{x_0}{x}\right)^{6}\right]$$



Solution: (a) Equilibrium occurs at the position where *U*(*x*) is minimum which is found from

$$\left(\frac{dU(x)}{dx}\right)_{x=x_m}=0,\qquad \varepsilon\left(-12\frac{x_0^{12}}{x_m^{13}}+12\frac{x_0^{6}}{x_m^{7}}\right)=0$$

$$x_m=x_0$$

(b) $$F(x)=-\frac{dU(x)}{dx}=12\varepsilon\left(\frac{x_0^{12}}{x^{13}}-\frac{x_0^{6}}{x^{7}}\right)$$

(c) The minimum energy needed to break up the molecule into separate atoms is called *dissociation energy*, E_d.

$$U(x_0)+E_d=0,\qquad E_d=-U(x_0)=\varepsilon$$

§ 5 Work-Energy Theorem and Conservation of Mechanical Energy

- Starting with work – kinetic energy theorem for the system of particles

$$\sum W_{i-\text{external}} + \sum W_{i-\text{internal}} = K_f - K_i$$

  - The internal forces can be divided into conservative and non-conservative.

$$\sum W_{i-\text{external}} + \sum W_{i-\text{internal-conserv}} + \sum W_{i-\text{internal-nonconserv}} = K_f - K_i$$

  - The work done by conservative forces can be described by the change in potential energy $\sum W_{i-\text{internal-conserv}} = -(U_f - U_i)$

$$\sum W_{i-\text{external}} + \sum W_{i-\text{internal-nonconserv}} = (K_f + U_f) - (K_i + U_i)$$

  - Define $E_{\text{mech}} = K + U$ to be total mechanical energy of the system.

- Work – energy theorem:

$$\sum W_{i-\text{external}} + \sum W_{i-\text{internal-nonconserv}} = \Delta E_{\text{mech}}$$

  - The work done by all the external forces and internal forces other than internal conservative forces acting in a system of particles equals the change in total mechanical energy of the system.

Conservation of Mechanical Energy

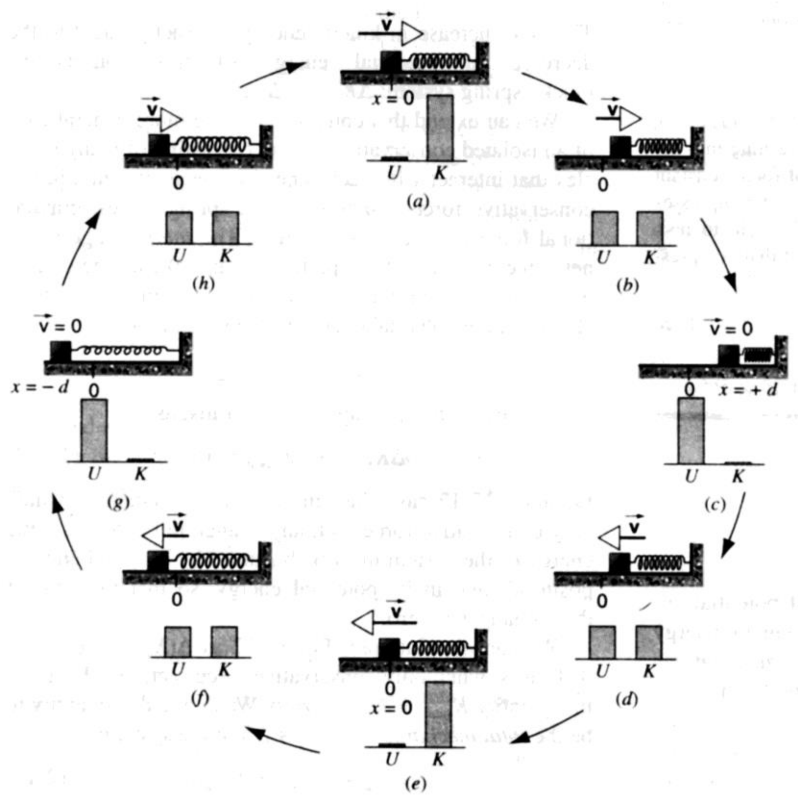
- For a system, if $\sum W_{i-\text{external}} + \sum W_{i-\text{internal-nonconserv}} = 0$

  then $\Delta E_{\text{mech}} = 0$ or $K_f + U_f = K_i + U_i = \text{constant}$

- In a system in which only internal conservative forces act, the total mechanical energy remains constant.

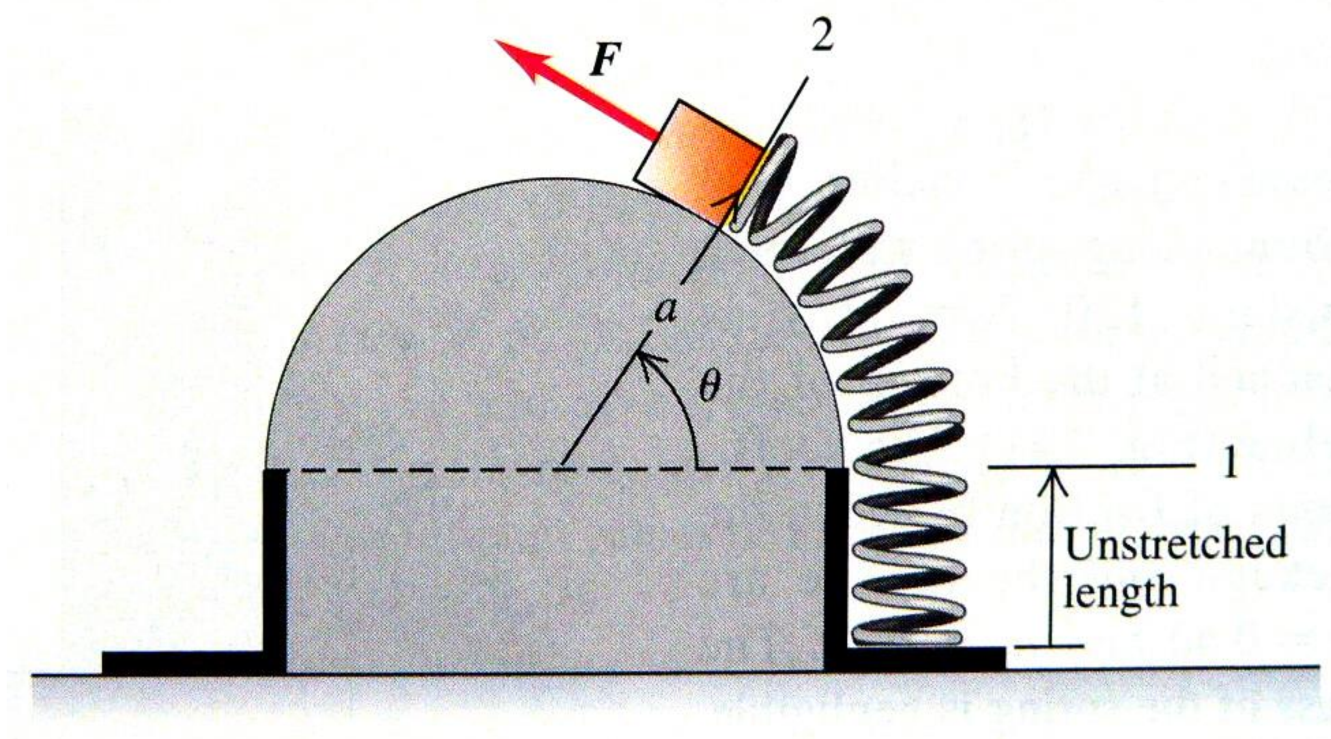
- When $\mathbf{\Delta E_{mech}=0}$, it is the internal conservative forces acting within the system that change kinetic into potential or potential into kinetic energy.

$$U \underset{W_{\text{conservative}}<0}{\overset{W_{\text{conservative}}>0}{\rightleftarrows}} K$$



Example

Variable force F is maintained tangent to a frictionless semicircular surface. By a slowly varying force F, a block with mass of m is moved, and spring to which it is attached is stretched from position 1 to position 2. The spring has negligible mass and force constant k. The end of the spring moves in an arc of radius a. Calculate the work done by the force F from position 1 to 2.(θ)



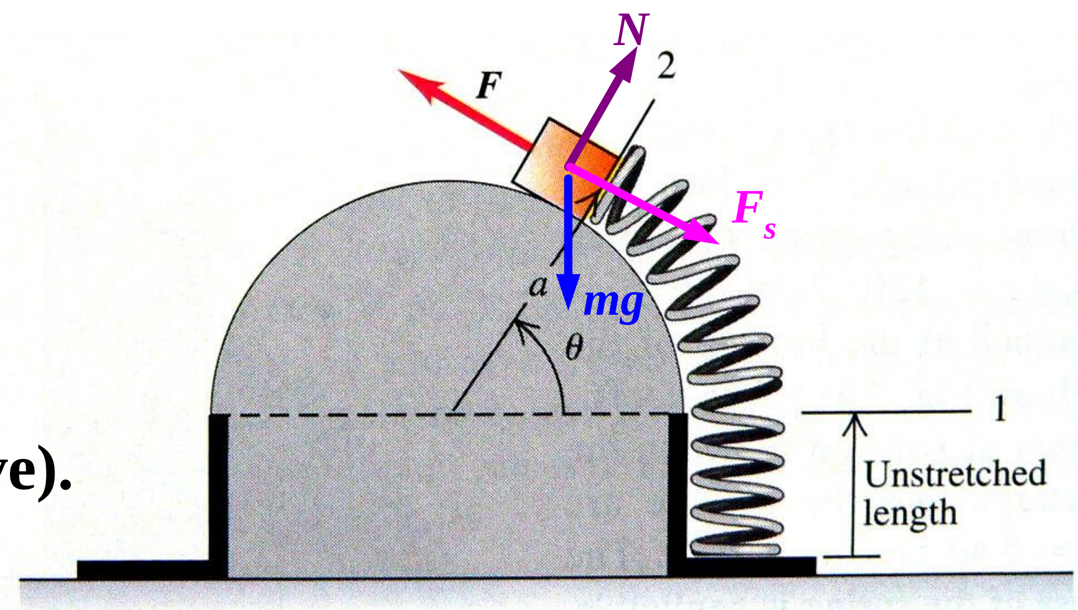
Solution II: by using work-energy theorem.

External force: F;

Internal forces:

N (non-conservative);

mg and F_s (conservative).

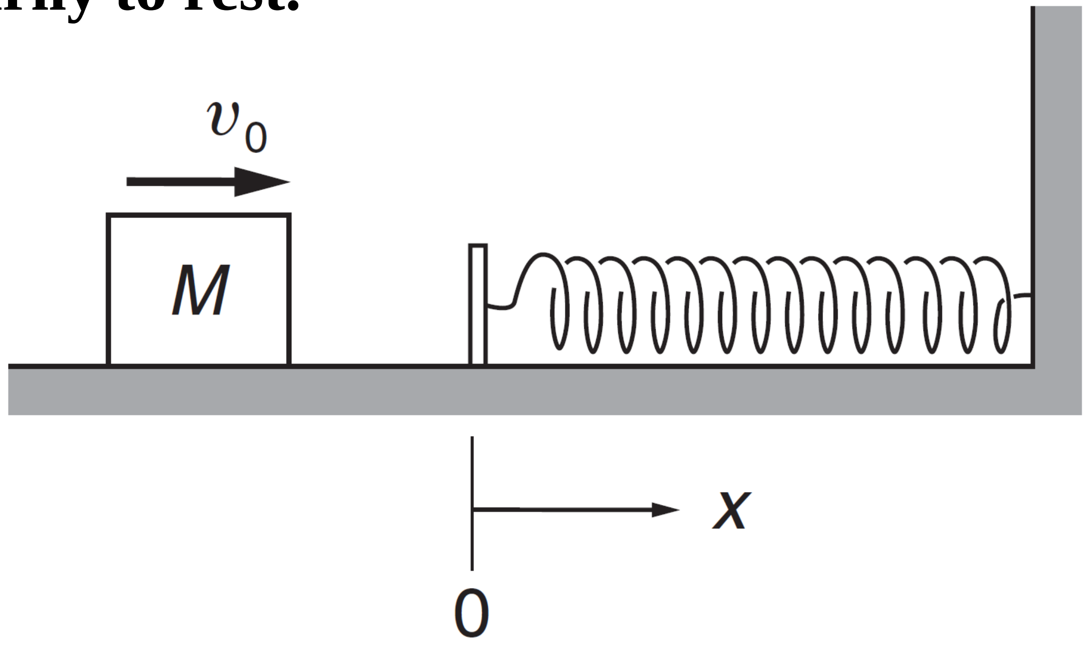


Choose the reference point at position 1 both for gravitational and elastic energy of block-spring-Earth system.

$$W_F = \Delta E = \Delta U = \frac{1}{2}ks^2 + mga\sin\theta = \frac{1}{2}ka^2\theta^2 + mga\sin\theta$$

Example

A block of mass M slide along a horizontal table with speed v_0. At $x=0$ it hits a spring with spring constant k and begins to experience a friction force. The coefficient of friction is variable and is given by $\mu=bx$, where b is a constant. Find the loss in mechanical energy when the block has first come momentarily to rest.



Solution

Take block-spring-Earth as a system. No external force exists.

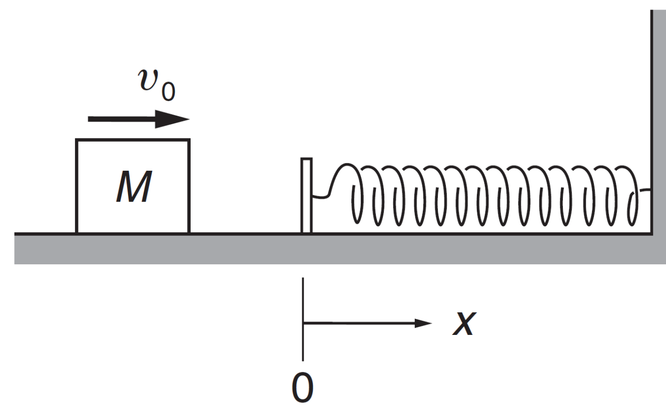
Internal conservative forces: spring force, gravitational force

Internal non-conservative forces: normal force (does no work), friction force.

Using work-energy theorem: $W_{f_s} = E_f - E_i = -E_{loss}$

Suppose the block's position is x_f at the moment when it first come to rest.

$$W_{f_s}(x=0 \to x_f) = \int_0^{x_f} -bxMg\,dx = -\frac{1}{2}bMgx_f^2$$



$$-\frac{1}{2}bMgx_f^2 = \frac{1}{2}kx_f^2 - \frac{1}{2}Mv_0^2, \quad x_f^2 = \frac{Mv_0^2}{k+bMg}$$

$$E_{loss} = E_i - E_f = -W_{f_s} = \frac{1}{2}bMgx_f^2 = \frac{bgM^2v_0^2}{2(k+bMg)}$$

Problem

- **Ch7 (P164)**
  - **38, 69, 70**
- **Ch8 (P194)**
  - **35, 67, 79**