Chapter 5:
Frequency Response

5.1 The basis of Frequency Response

- **Analyses limited to particular frequencies.**
- **Need to investigate the frequency effects:**
  - ***Larger capacitive elements of the network at low frequency.***
  - ***Smaller capacitive elements of the active device at high frequency.***
- **The concept of the decibel (dB).**
- **Similar features for both BJT and FET.**

5.1 The basis of Frequency Response

The *Frequency Response* of an amplifier refers to the *frequency range* in which the amplifier will operate with negligible effects from *capacitors* and *capacitance in devices*.

This range of frequencies were usually called *mid-range*.

- ***At frequencies above and below the mid-range, Capacitance and Inductance will affect the gain of the amplifier.***
  - **At low frequencies, *coupling and bypass capacitors* lower the gain.**
  - **At high frequencies, *stray capacitances* associated with the active device lower the gain.**
- ***Also, cascading amplifiers limits the gain at high and low frequencies.***

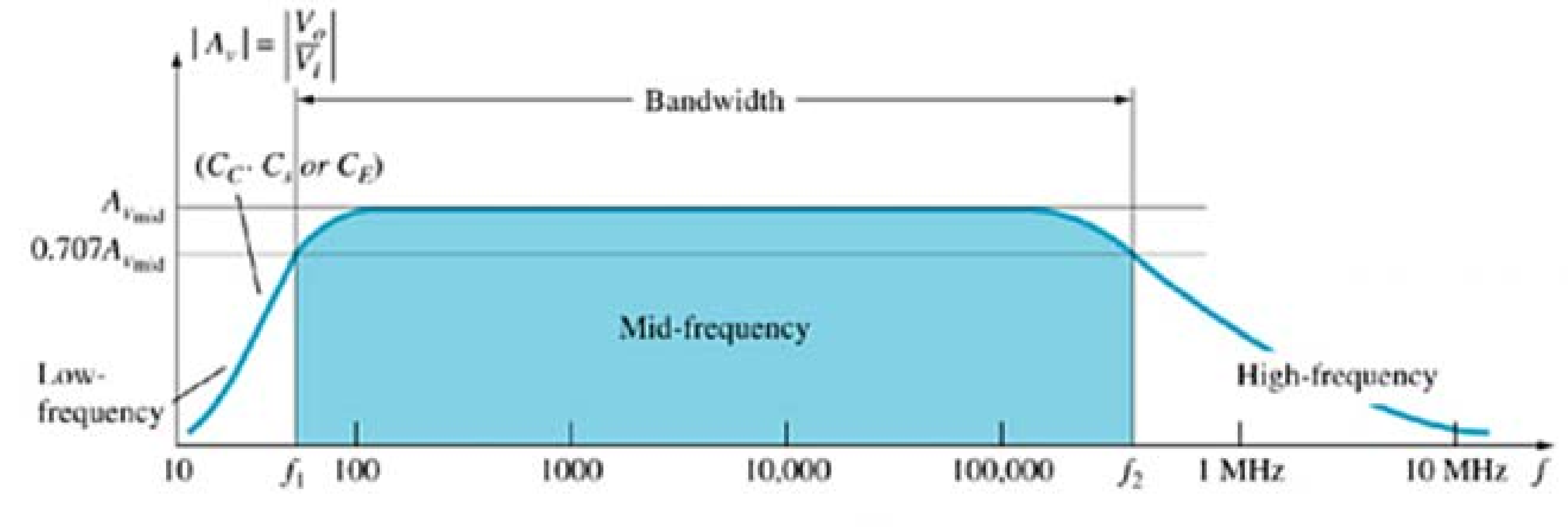
Bandwidth and Cutoff Frequencies

The mid-range frequency of an amplifier is called the ***bandwidth*** of the amplifier.

The bandwidth is defined by the ***lower*** and ***upper*** cutoff frequencies.

Cutoff - Frequency at which the gain has dropped by:

- 0.5 power
- -3dB
- 0.707 voltage



Bandwidth and Cutoff Frequencies

Mid-frequency:

$$P_{omid} = \frac{\left|V_o^2\right|}{R_o} = \frac{\left|A_{vmid} V_i\right|^2}{R_o}$$

Half-power frequency
f_H , f_L

$$P_{oHPF} = 0.5 P_{omid}$$

$$P_{oHPF} = 0.5 \frac{\left|A_{vmid} V_i\right|^2}{R_o} = \frac{\left|0.707 A_{vmid} V_i\right|^2}{R_o}$$

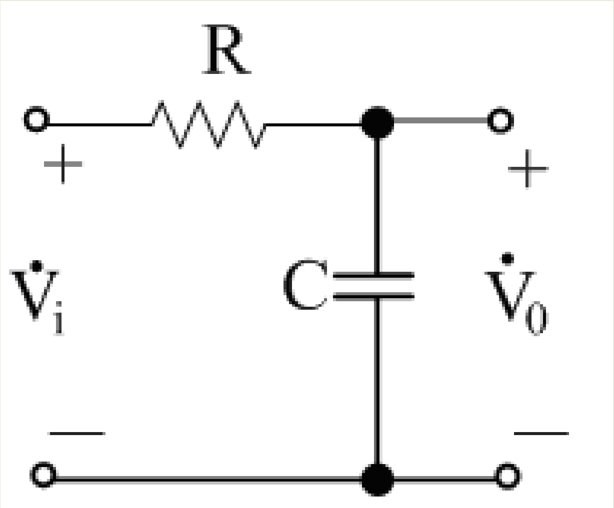
Bandwidth (BW) = f_H -f_L

Decibel definition:

$$\left.\frac{A_v}{A_{vmid}}\right|_{dB} = 20 \log_{10} \frac{A_v}{A_{vmid}}$$

(1) Low-Pass Circuit

(a) If the input signal is at low frequency, the reactance X_C of capacitor is very large. ***The output signal equals to the input signal approximately.***



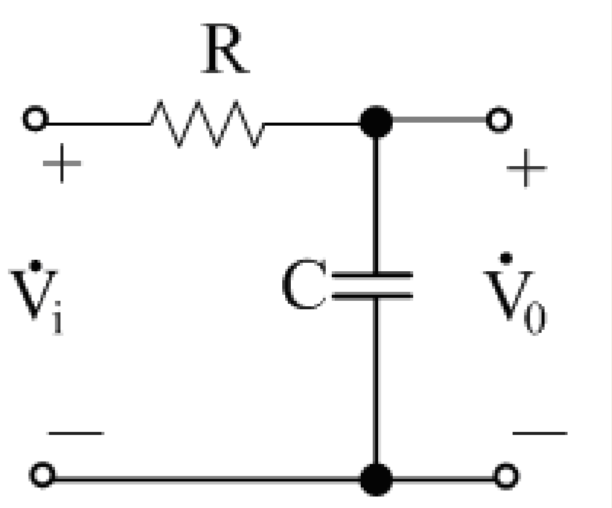
(b) If the frequency is very high, the reactance of capacitor will be shorted, ***the output is zero.***

$$X_C = \frac{1}{2\pi fC}$$

(c) So, it's ***Low-Pass*****, but with** ***high frequency cutoff*** **point.**

(1) Low-Pass Circuit

For the case when: $X_C = R$



The output is 3dB drop (0.707) of the input at the frequency for $X_C = R$.

The *high frequency cutoff point* is determined from:

$$f_H = \frac{1}{2\pi RC}$$

The gain of any frequency can be:

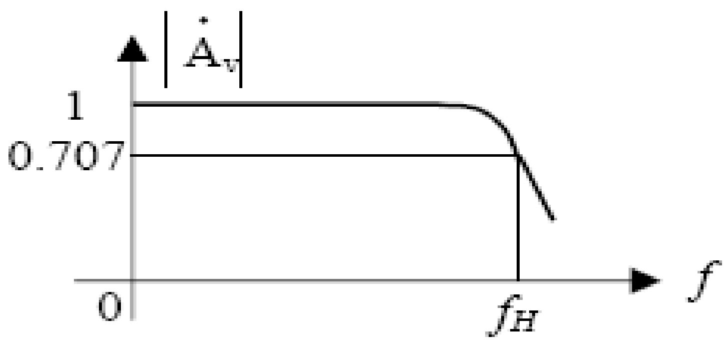
$$A_v = \frac{V_o}{V_i} = \frac{1}{1 + j\left(f / f_H\right)}$$

(1) Low-Pass Circuit

In the magnitude and phase form:

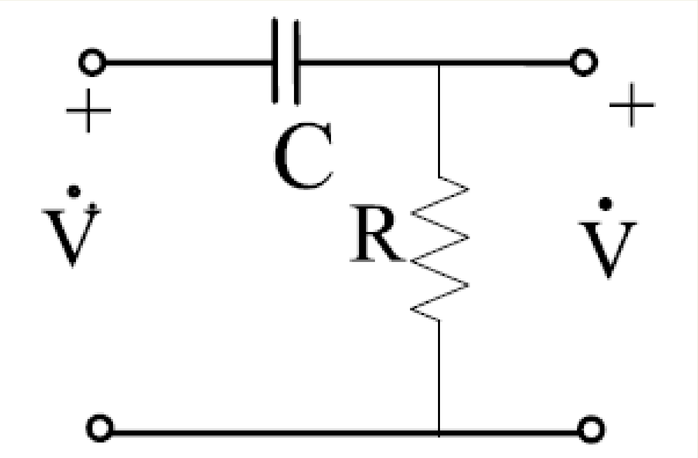
$$|A_v| = \left|\frac{V_o}{V_i}\right| = \frac{1}{\sqrt{1+(f/f_H)^2}}, \varphi = -\arctan(f/f_H)$$

The frequency response of low-pass circuit:



(a) Magnitude response

(2) High-Pass Circuit



(a) At low frequency, the reactance XC is very large. *The output is zero.*

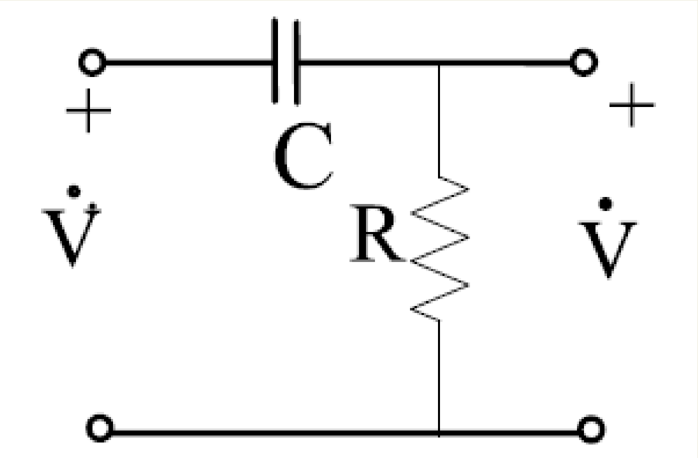
(b) At high frequency, the reactance will be shorted, *the output equals to the input.*

(c) So, it's *High-Pass*, but with *low frequency cutoff* point.

$$X_C = \frac{1}{2\pi fC}$$

(2) High-Pass Circuit

For the case where $X_C=R$



The output is 3dB drop of the input at the frequency for $X_C=R$.

The *low frequency cutoff point* is determined from:

$$f_L = \frac{1}{2\pi RC}$$

The gain of any frequency can be:

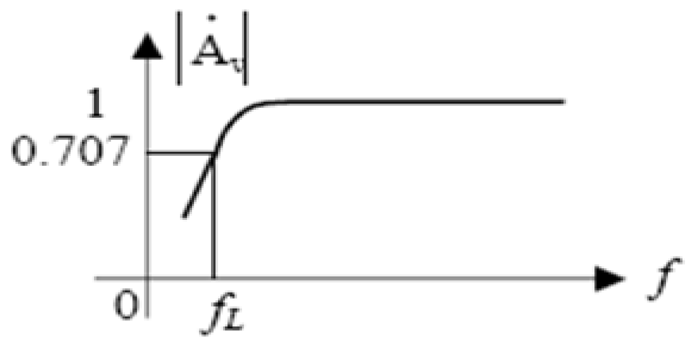
$$A_v = \frac{V_o}{V_i} = \frac{1}{1 - j\left(f_L / f\right)}$$

(2) High-Pass Circuit

In the magnitude and phase form:

$$\left|A_v\right| = \left|\frac{V_o}{V_i}\right| = \frac{1}{\sqrt{1+\left(f_L / f\right)^2}}, \varphi = \arctan\left(f_L / f\right)$$

The frequency response of high-pass circuit:



(a) amplitude response

(3) Bode Plot

- **If the frequency response is plotted by logarithmic scale, the plot is called the *Bode Plot*.**

In the logarithmic form, the gain of low-pass circuit in dB is

$$A_{v(dB)} = 20\log_{10}\frac{1}{\sqrt{1+(f/f_H)^2}} = -20\log_{10}\sqrt{1+(f/f_H)^2}$$

$$A_{v(dB)} = -20\log_{10}\sqrt{1+(f/f_H)^2} = -10\log_{10}\left[1+(f/f_H)^2\right]$$

For the frequencies where $f \gg f_H$, the equation above can be approximated by

$$A_{v(dB)} = -10\log_{10}(f/f_H)^2 = -20\log_{10}\frac{f}{f_H}\bigg|_{f \gg f_H}$$

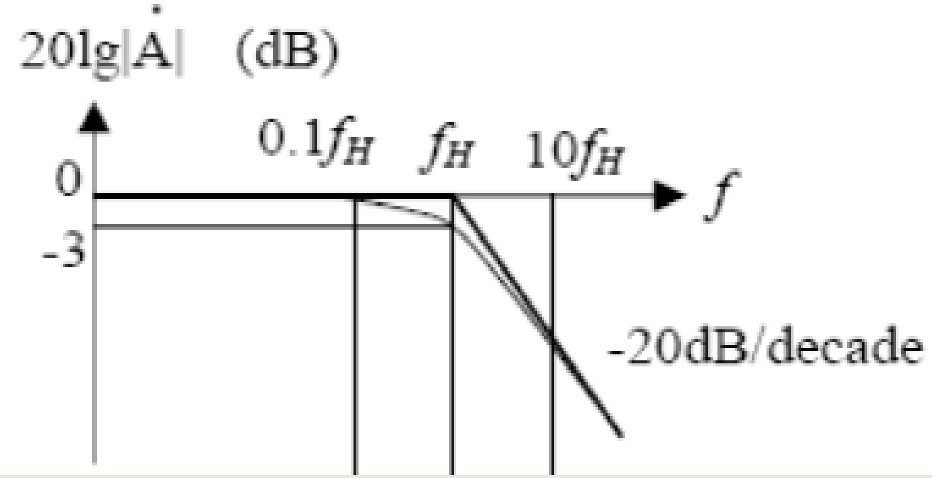
(3) Bode Plot

- A straight line is drawn for the condition 0dB for $f \ll f_H$
- But the straight line are only accurate for $f \ll f_H$, when $f = f_H$ there is actually a 3dB drop from the mid-range level.

At $f = f_H : \dfrac{f}{f_H} = 1, -20\log_{10} 1 = 0dB$

At $f = 2f_H : \dfrac{f}{f_H} = 2, -20\log_{10} 2 = -6dB$

At $f = 10f_H : \dfrac{f}{f_H} = 10, -20\log_{10} 10 = -20dB$



(3) Bode Plot

In the logarithmic form, the gain of high-pass circuit in dB is

$$A_{v(dB)} = 20\log_{10}\frac{1}{\sqrt{1+(f_L/f)^2}} = -20\log_{10}\sqrt{1+(f_L/f)^2}$$

For the frequencies where $f \ll f_L$, the equation above can be approximated by

$$A_{v(dB)} = -10\log_{10}(f_L/f)^2 = -20\log_{10}\frac{f_L}{f}\bigg|_{f \ll f_L}$$

At $f = \frac{1}{2}f_L : \frac{f_L}{f} = 2, -20\log_{10}2 = -6dB$

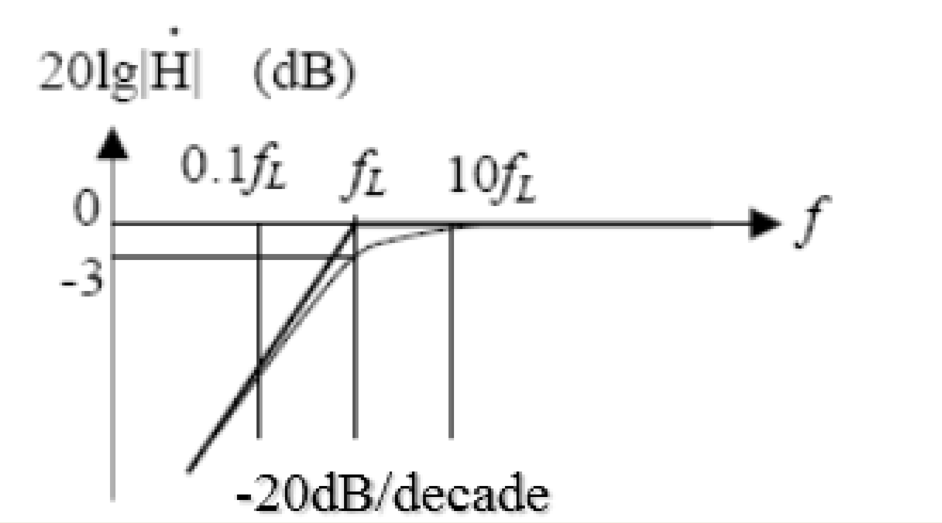
At $f = \frac{1}{10}f_L : \frac{f_L}{f} = 10, -20\log_{10}10 = -20dB$

(3) Bode Plot

- **A straight line is drawn for the condition 0dB for** $f \ll f_L$

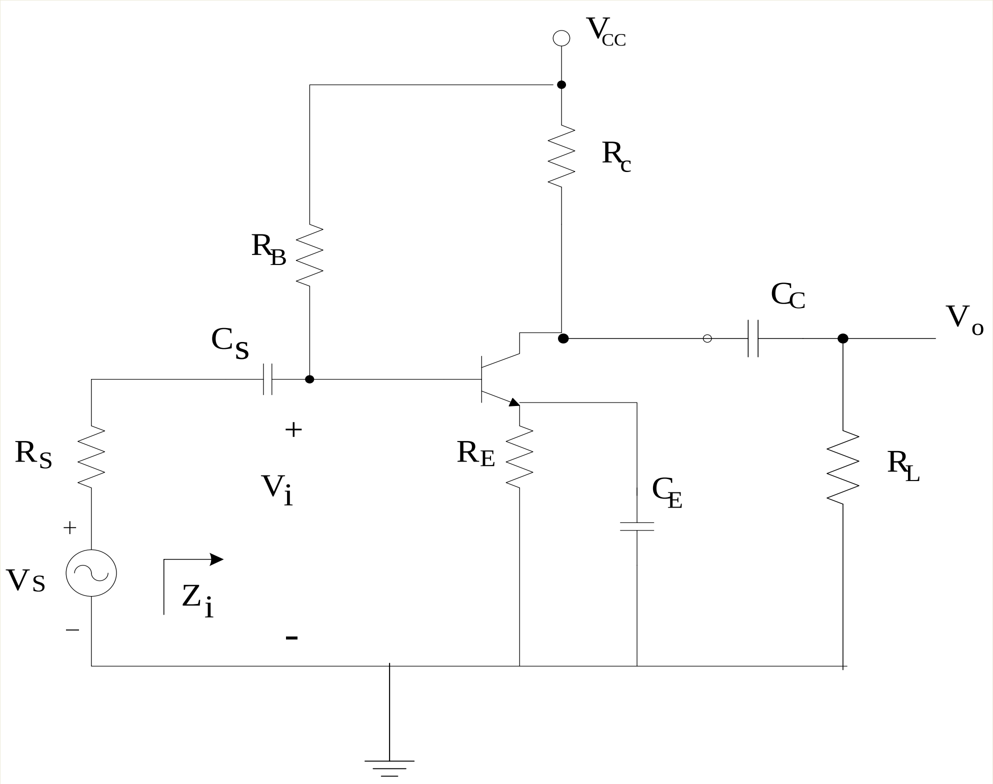
- **But the straight line are only accurate for** $f \ll f_L$ **,**

when $f = f_L$ **there is actually a 3dB drop from the mid-range level.**



5.2 Low-Frequency Response for BJT

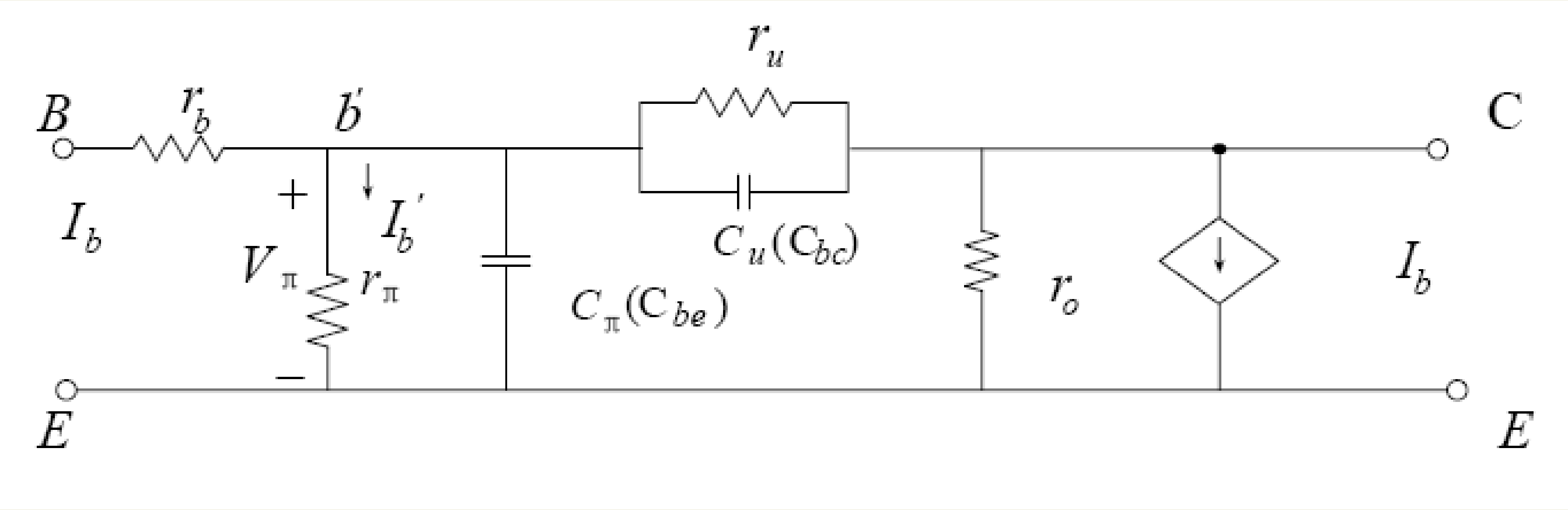
- Find the appropriate ***equivalent resistance*** for the high-pass circuit.
- The drop in gain at the low frequency is due to the increasing reactance of C_S, C_E and C_C.
- Consider the large capacitor ***independently.***



5.3 High-Frequency Response for BJT

Equivalent model for high frequency response

---- *hybrid π model*



r_b: Base contact, bulk, and spreading resistance.

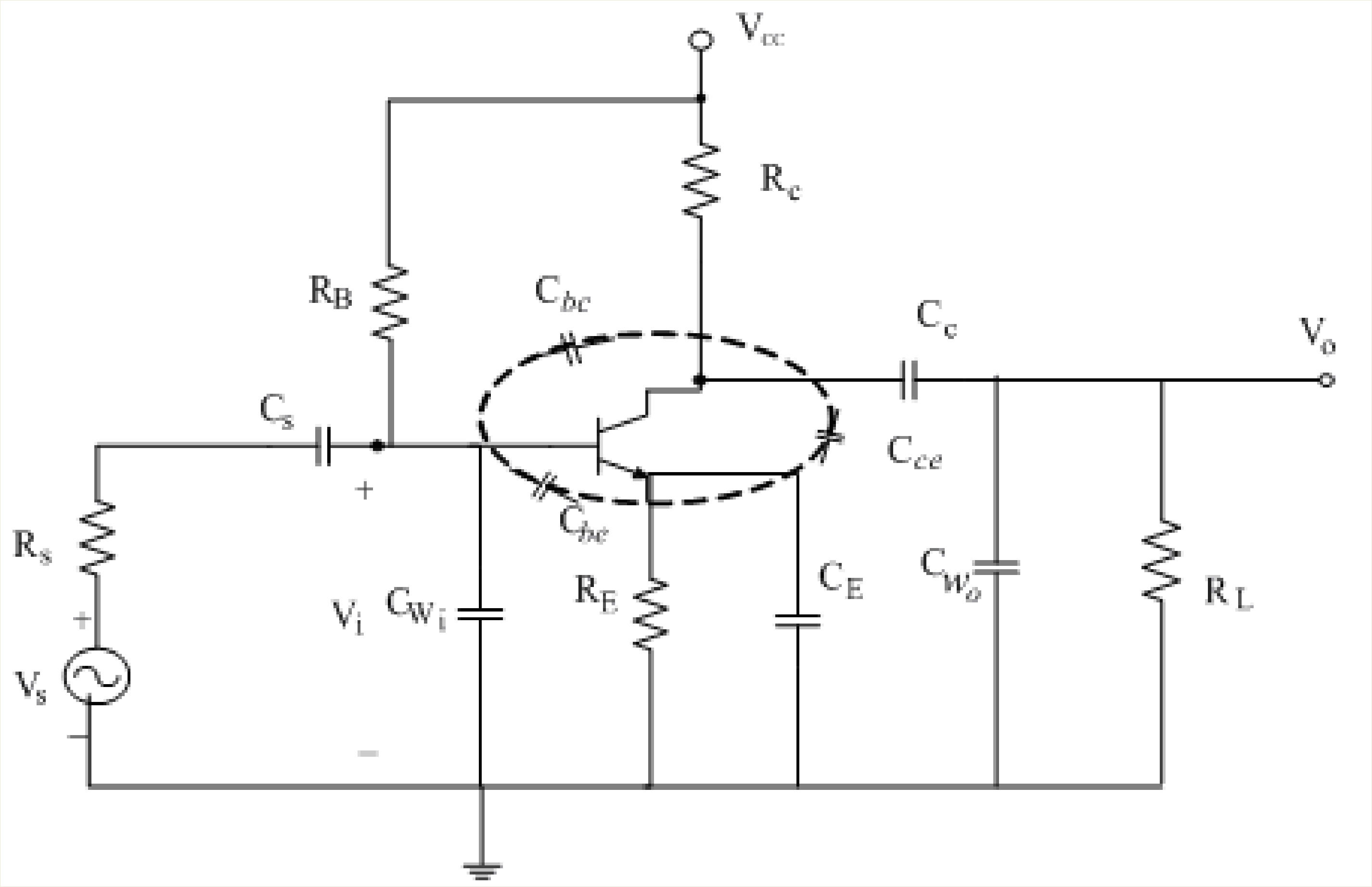
r_π, r_u, r_o: resistance between terminals

C_{bc} C_{be}: the inter-electrode capacitances.

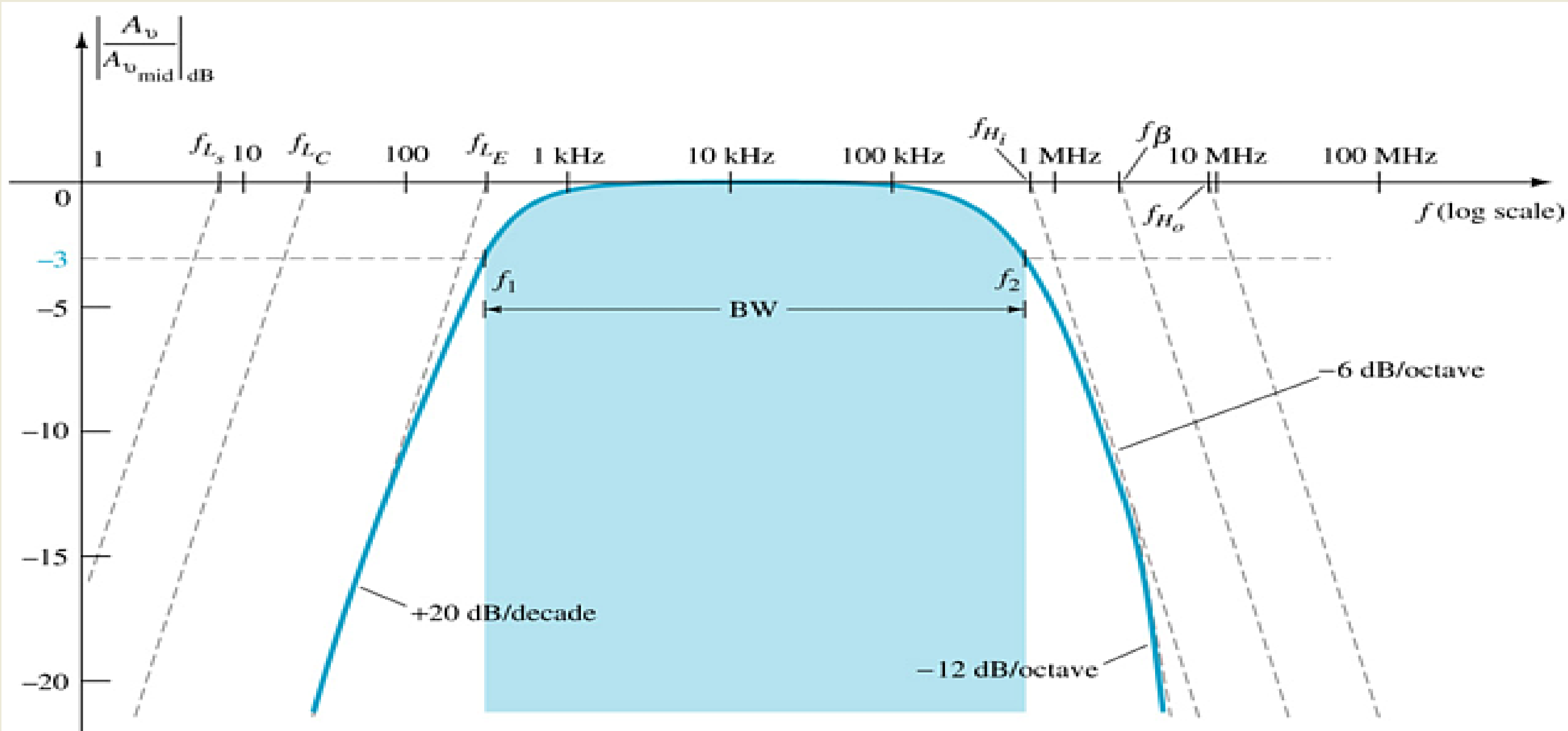
r_π: Base-to-Emitter resistance, is βr_e.

5.3 High-Frequency Response for BJT

The capacitor for high frequency response:



Full Frequency Response for BJT



Middle-frequency region: resistance features by short and open equivalents of capacitances

Low-frequency region: coupling and bypass capacitors determine lower cutoff frequency

High-frequency region: network capacitance (parasitic and introduced) determine the upper cutoff frequency

Summary

The bandwidth of an amplifies is determined by:

- **Larger capacitive elements**
- **Smaller parasitic capacitors**

Bode plot is required to present frequency response, and determine

- **Cutoff frequencies**
- **Bandwidth**

Capacitances that affect the low-frequency response are:

- **Coupling capacitances: C_S, C_C**
- **Bypass capacitance: C_E**

Capacitances that affect the high-frequency response are:

- **Junction and parasitic capacitances: C_{be}, C_{bc}, C_{ce}**
- **Wiring capacitances: C_{wi}, C_{wo}**