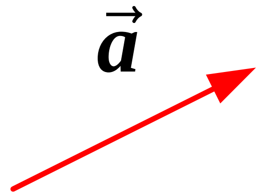
Vectors and scalars

(P45-51)

- A vector has magnitude as well as direction, and vectors follow certain (vector) rules of combination.
- A vector quantity is a quantity that has both a magnitude and a direction and thus can be represented with a vector.
  - Ex., displacement vector, velocity vector, and acceleration vector.
- A single value, with a sign, specifies a scalar quantity.
  - Ex., temperature, pressure, energy, mass, and time.

Description of vector

- **Graphical description** (using a arrow)

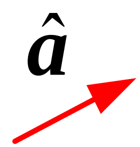


$$\vec{a} = |\vec{a}|\,\hat{a} = a\hat{a}$$

- **Magnitude** $a = |\vec{a}|$

- **Direction**
  - **A unit vector is a dimensionless vector that has a magnitude of exactly 1 and points in a particular direction.**

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|}$$

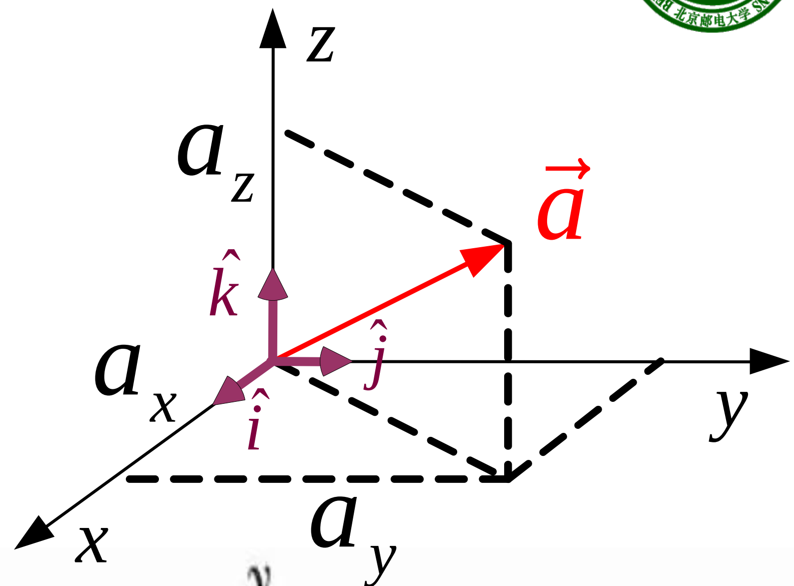


$$|\hat{a}| = 1$$

Description of vector

- **Component description**

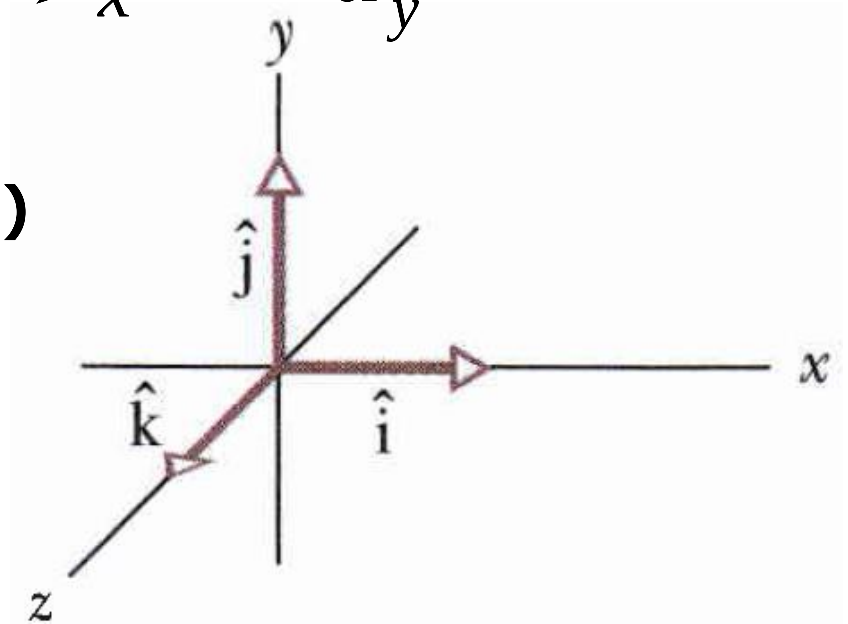
$$\boxed{\vec{a} = a_x\hat{\boldsymbol{i}} + a_y\hat{\boldsymbol{j}} + a_z\hat{\boldsymbol{k}}}$$



- **The unit vectors in the positive directions of the x, y, and z axes are labeled**

$$\hat{\boldsymbol{i}}, \hat{\boldsymbol{j}}, \hat{\boldsymbol{k}}$$

(right-handed coordinate system)



- **A component of a vector is the projection of the vector on an axis.**

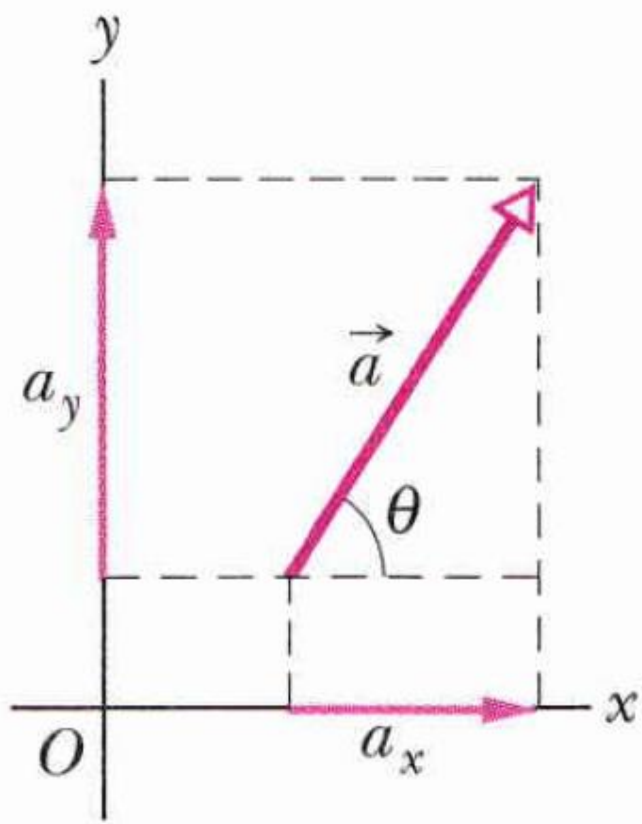
$$a_x = a\cos\alpha,\ \ a_y = a\cos\beta,\ \ a_z = A\cos\gamma$$

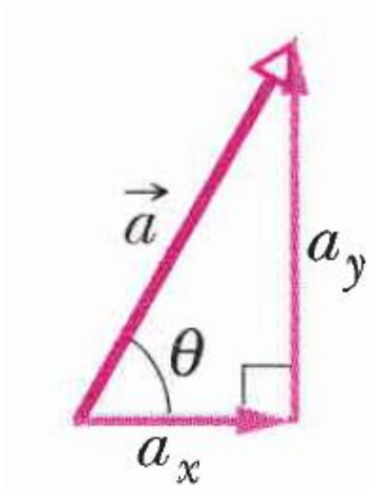
Description of vector

➢ **component notation and magnitude-angle notation**

$$\begin{cases} a_x = a\cos\theta \\ a_y = a\sin\theta \end{cases}$$

$$\begin{cases} a = \sqrt{a_x^2 + a_y^2} \\ \tan\theta = \dfrac{a_y}{a_x} \end{cases}$$

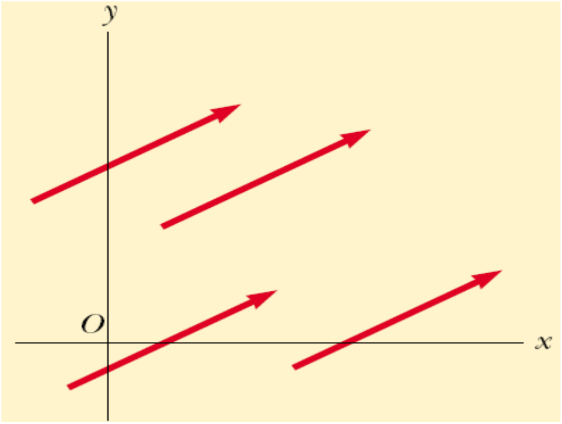




Vector addition

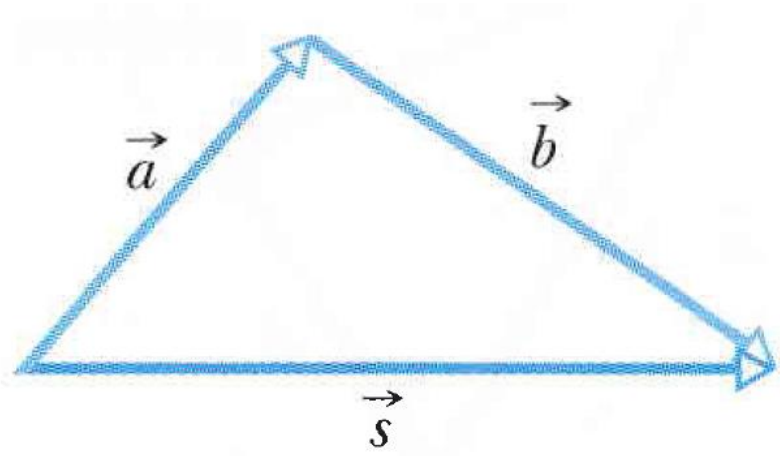
- **Equality of several vectors**

  magnitude and direction



- **Adding vectors geometrically**

$$\vec{s} = \vec{a} + \vec{b} \qquad \textbf{(head-to-tail)}$$



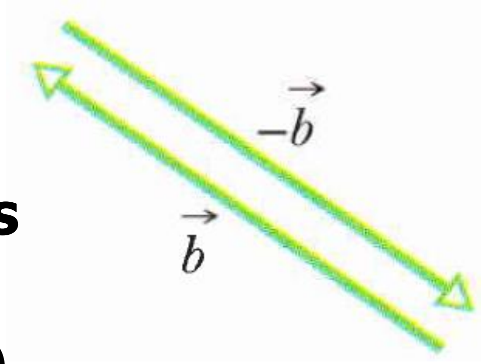
- **Adding vectors by components**

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

$$\vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$$

$$\boxed{\vec{s} = \vec{a} + \vec{b} = (a_x + b_x)\hat{i} + (a_y + b_y)\hat{j} + (a_z + b_z)\hat{k}}$$

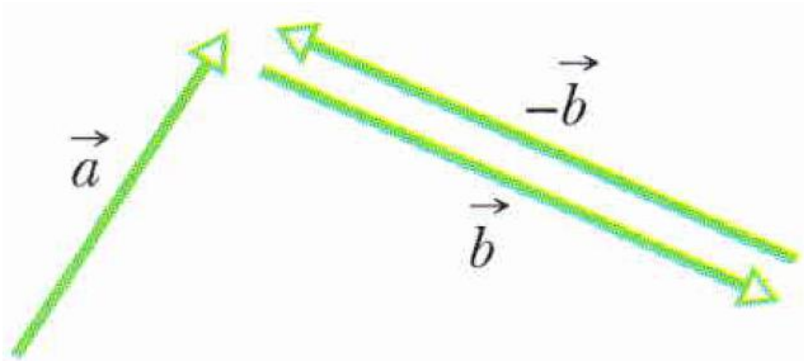
Vector subtraction

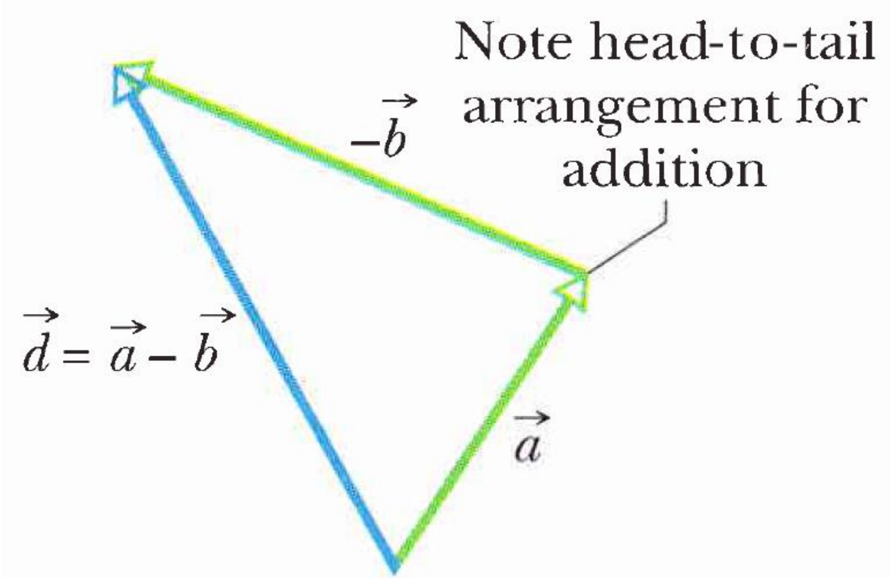


- **Negative of a vector**

  same magnitude and opposite directions

- **Vector subtraction** $\vec{d} = \vec{a} - \vec{b} = \vec{a} + (-\vec{b})$

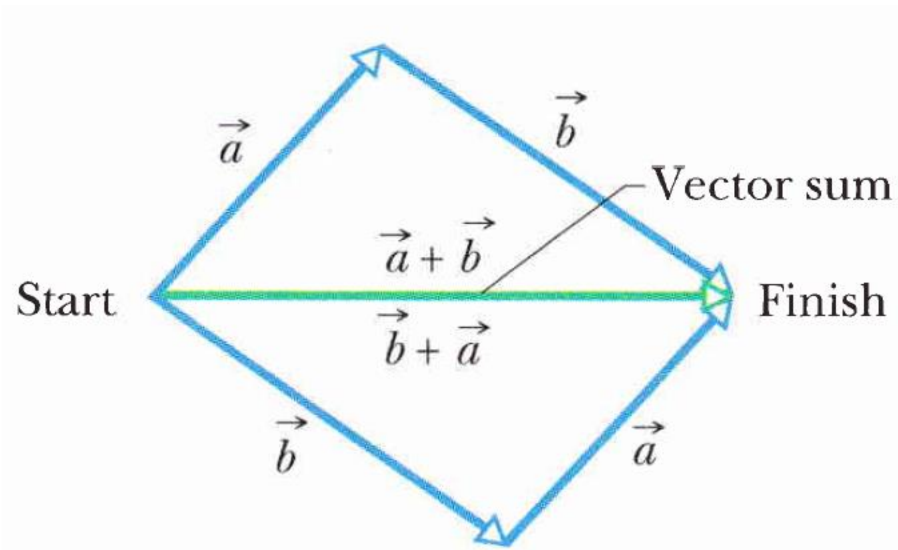




Laws of algebra for the vector addition

- **Commutative law**

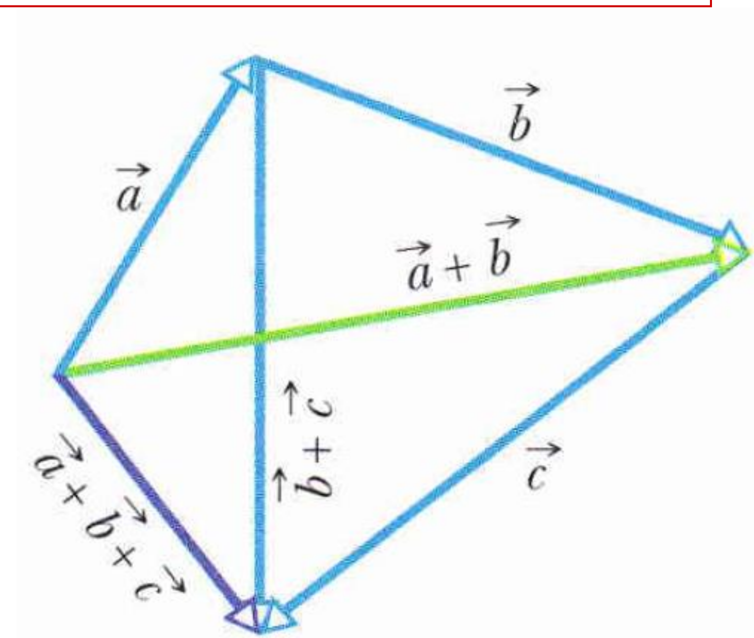
$$\boxed{\vec{a}+\vec{b}=\vec{b}+\vec{a}}$$



(parallelogram rule of addition)

- **Associative law**

$$\boxed{\vec{a}+(\vec{b}+\vec{c})=(\vec{a}+\vec{b})+\vec{c}}$$



(Addition of many vectors)

$$\vec{a}+\vec{b}=(a_x+b_x)\hat{i}+(a_y+b_y)\hat{j}+(a_z+b_z)\hat{k}$$

$$=(b_x+a_x)\hat{i}+(b_y+a_y)\hat{j}+(b_z+a_z)\hat{k}=\vec{b}+\vec{a}$$

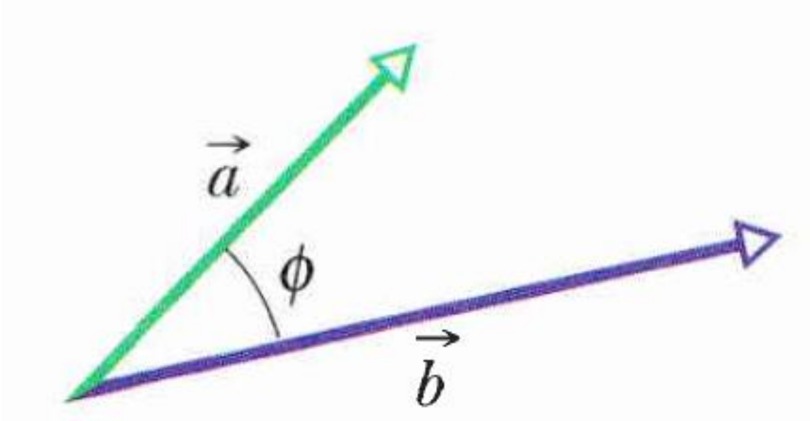
Vector multiplication

- **Multiplying a vector by a scalar,** $\lambda\vec{a},\ \vec{a}/\lambda$

$$\lambda(\mu\vec{a}) = (\lambda\mu)\vec{a} \qquad \text{(associative law)}$$

$$\lambda(\vec{a}+\vec{b}) = \lambda\vec{a}+\lambda\vec{b} \ \text{ and } \ (\lambda+\mu)\vec{a} = \lambda\vec{a}+\mu\vec{a} \qquad \text{(distribution laws)}$$

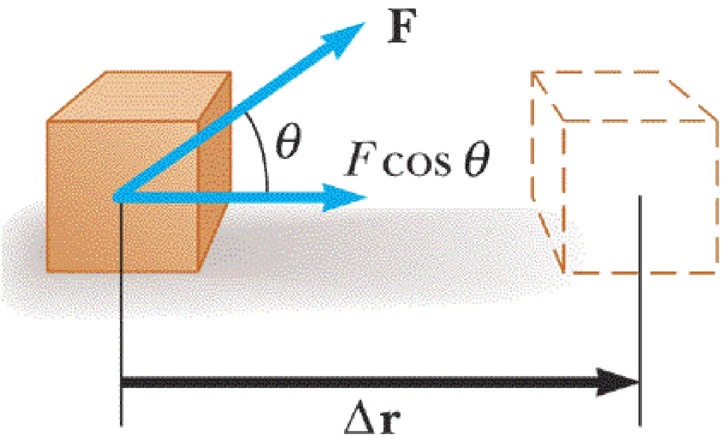
- **Scalar products (dot products)**



$$\boxed{\vec{a}\cdot\vec{b} = ab\cos\phi}$$

- **For example**

$$W = \vec{F}\cdot\Delta\vec{r} = F\,|\Delta\vec{r}|\cos\theta$$



Laws of algebra and properties of the scalar product

$$\boxed{\vec{a}\cdot\vec{b} = ab\cos\phi}$$

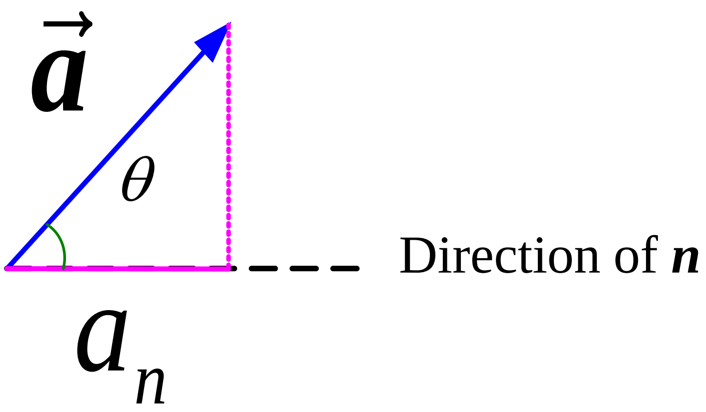
$$\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$$ **(Commutative law)**

$$\vec{a}\cdot(\vec{b}+\vec{c}) = \vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}$$ **(Distributive law)**

- **(i)** $\vec{a}\cdot\vec{a} = a^2 = |\vec{a}|^2$
- **(ii)** The scalar product $\vec{a}\cdot\vec{b} = \mathbf{0}$ if (and only if) $\vec{a}$ and $\vec{b}$ are perpendicular (or one of them is zero).
- **(iii)** $a_n = \vec{a}\cdot\hat{n}$

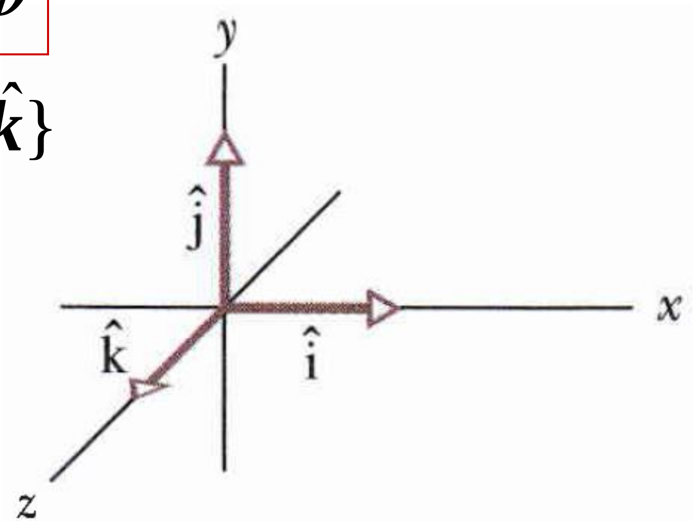
$$= a\cdot 1\cdot\cos\theta$$

$$= a\cos\theta$$



Laws of algebra and properties of the scalar product

$$\boxed{\vec{a}\cdot\vec{b}=ab\cos\phi}$$



- **(iv) For an orthonormal basis** $\{\hat{i}\ ,\hat{j}\ ,\hat{k}\}$

$$\hat{i}\cdot\hat{i}=\hat{j}\cdot\hat{j}=\hat{k}\cdot\hat{k}=1$$
$$\hat{i}\cdot\hat{j}=\hat{j}\cdot\hat{k}=\hat{k}\cdot\hat{i}=0$$

- **(v) Component form**

$$\vec{a}\cdot\vec{b}=(a_x\hat{i}+a_y\hat{j}+a_z\hat{k})\cdot(b_x\hat{i}+b_y\hat{j}+b_z\hat{k})$$
$$=a_xb_x+a_yb_y+a_zb_z$$

- **(vi) The magnitude of a vector**

$$|\vec{a}|=\sqrt{\vec{a}\cdot\vec{a}}=\sqrt{a_x^2+a_y^2+a_z^2}$$

Example

Find the angle between the vector $\vec{A} = 8\hat{i} + 3\hat{j}$ **and the vector** $\vec{B} = -5\hat{i} - 7\hat{j}$**.**

Solution: $\vec{A} \cdot \vec{B} = AB\cos\theta = \sqrt{8^2 + 3^2} \times \sqrt{(-5)^2 + (-7)^2}\cos\theta$

$$= 8.544 \times 8.60\cos\theta$$

$$= 73.5\cos\theta$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y$$

$$= 8 \times (-5) + 3 \times (-7)$$

$$= -61$$

Thus, $\theta = \arccos\left(\frac{-61}{73.5}\right) = 146.1^\circ$

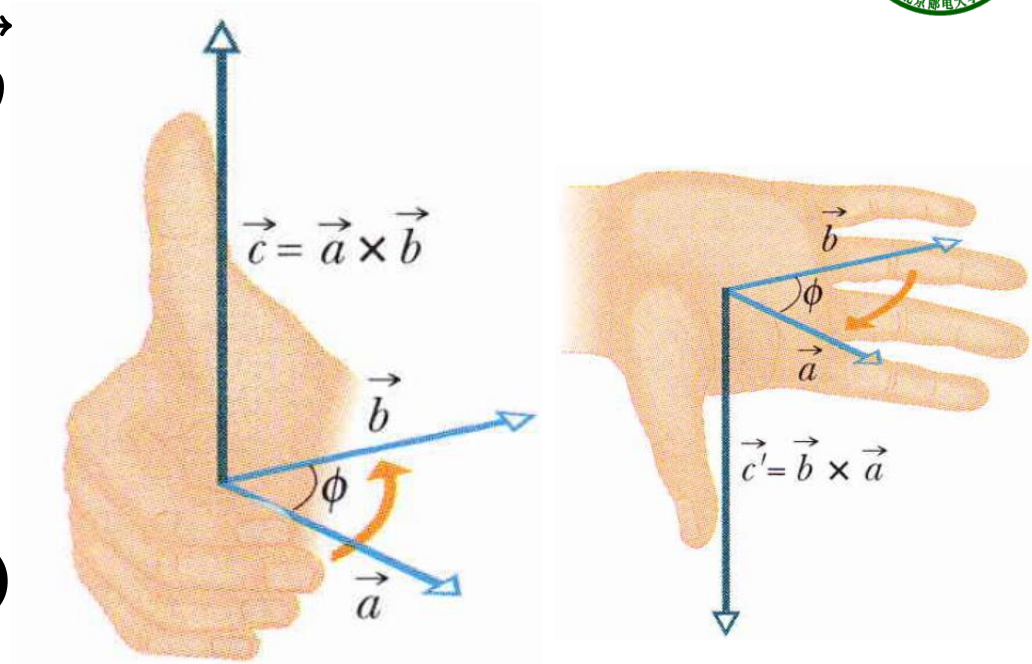
Vector products (cross products)

$$\vec{c} = \vec{a} \times \vec{b}$$

- **Magnitude**

$$\boxed{c = \left|\vec{a} \times \vec{b}\right| = ab\sin\phi}$$

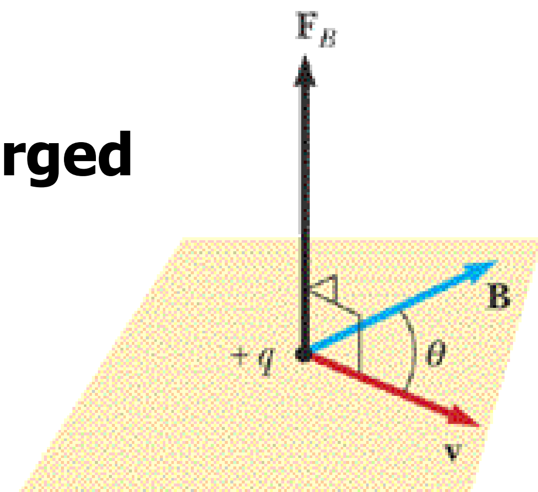
- **Direction (right-hand rule)**



- **For example**

  the magnetic force on a moving charged particle

$$\vec{F}_B = q\vec{v} \times \vec{B}$$



Laws of algebra and properties of the vector product

$$\boxed{\left|\vec{a}\times\vec{b}\right| = ab\sin\phi}$$

$$\vec{b}\times\vec{a} = -\vec{a}\times\vec{b}$$ **(Anti-commutative law)**

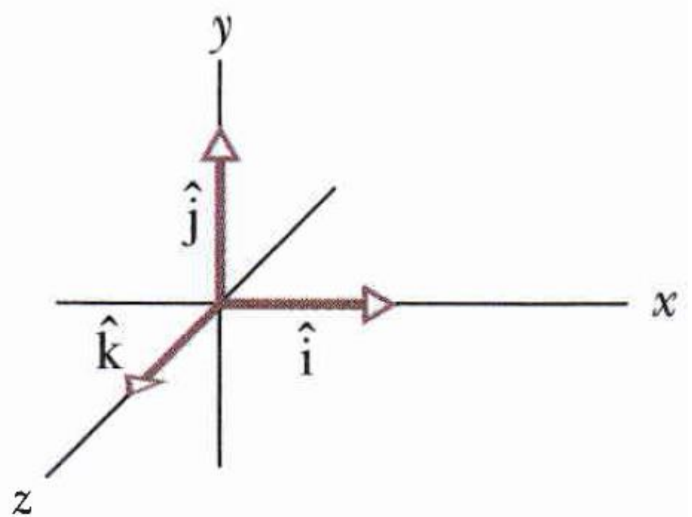
$$\vec{a}\times(\vec{b}+\vec{c}) = \vec{a}\times\vec{b}+\vec{a}\times\vec{c}$$ **(Distributive law)**

- **(i)** $\vec{a}\times\vec{a} = 0$
- **(ii) The vector product** $\vec{a}\times\vec{b} = 0$ **if (and only if)** $\vec{a}$ **and** $\vec{b}$ **are parallel (or one of them is zero).**
- **(iii) For an orthonormal basis (right-hand)** $\{\hat{i}\ ,\hat{j}\ ,\hat{k}\}$

$$\hat{i}\times\hat{j} = \hat{k} \qquad \hat{i}\times\hat{i} = 0$$

$$\hat{j}\times\hat{k} = \hat{i} \qquad \hat{j}\times\hat{j} = 0$$

$$\hat{k}\times\hat{i} = \hat{j} \qquad \hat{k}\times\hat{k} = 0$$



Properties of the vector product

$$\left|\vec{\boldsymbol{a}} \times \vec{\boldsymbol{b}}\right| = ab\sin\phi$$

- **(iv) Component form**

$$\vec{\boldsymbol{a}} \times \vec{\boldsymbol{b}} = (a_x\hat{\boldsymbol{i}} + a_y\hat{\boldsymbol{j}} + a_z\hat{\boldsymbol{k}}) \times (b_x\hat{\boldsymbol{i}} + b_y\hat{\boldsymbol{j}} + b_z\hat{\boldsymbol{k}})$$

$$= (a_yb_z - b_ya_z)\hat{\boldsymbol{i}} + (a_zb_x - b_za_x)\hat{\boldsymbol{j}} + (a_xb_y - b_xa_y)\hat{\boldsymbol{k}}$$

$$= \begin{vmatrix} \hat{\boldsymbol{i}} & \hat{\boldsymbol{j}} & \hat{\boldsymbol{k}} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

Example

Find the angle between the vector $\vec{A} = 8\hat{i} + 3\hat{j}$ and the vector $\vec{B} = -5\hat{i} - 7\hat{j}$.

Solution: $\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$

$$= \sqrt{8^2+3^2}\times\sqrt{(-5)^2+(-7)^2}\sin\theta = 73.5\sin\theta$$

$$\vec{A}\times\vec{B} = \left(8\hat{i}+3\hat{j}\right)\times\left(-5\hat{i}-7\hat{j}\right)$$

$$= -40\hat{i}\times\hat{i} - 56\hat{i}\times\hat{j} - 15\hat{j}\times\hat{i} - 21\hat{j}\times\hat{j}$$

$$= 0 - 56\hat{k} + 15\hat{k} + 0 = -41\hat{k}$$

$$\left|\vec{A}\times\vec{B}\right| = 41, \qquad \theta = \arcsin\left(\frac{41}{73.5}\right) = 33.91^\circ$$

Since $\sin\theta = \sin(180^\circ - \theta)$, then the angle between the two vectors could be either 33.91° or 146.1°.

You can find the correct answer by using a graphical method, to prove that the correct answer is $\theta = 146.1°$.

Differentiation of vectors

- **Definition**

$$\frac{d\vec{\boldsymbol{u}}}{d\alpha}=\lim_{\Delta\alpha\to 0}\frac{\vec{\boldsymbol{u}}(\alpha+\Delta\alpha)-\vec{\boldsymbol{u}}(\alpha)}{\Delta\alpha}$$

- **Differentiation rules**

$$\frac{d}{d\alpha}(\vec{\boldsymbol{u}}+\vec{\boldsymbol{v}})=\frac{d\vec{\boldsymbol{u}}}{d\alpha}+\frac{d\vec{\boldsymbol{v}}}{d\alpha},\qquad \frac{d}{d\alpha}(\lambda\vec{\boldsymbol{u}})=\frac{d\lambda}{d\alpha}\vec{\boldsymbol{u}}+\lambda\frac{d\vec{\boldsymbol{u}}}{d\alpha}$$

$$\frac{d}{d\alpha}(\vec{\boldsymbol{u}}\cdot\vec{\boldsymbol{v}})=\left(\frac{d\vec{\boldsymbol{u}}}{d\alpha}\right)\cdot\vec{\boldsymbol{v}}+\vec{\boldsymbol{u}}\cdot\left(\frac{d\vec{\boldsymbol{v}}}{d\alpha}\right)$$

$$\frac{d}{d\alpha}(\vec{\boldsymbol{u}}\times\vec{\boldsymbol{v}})=\left(\frac{d\vec{\boldsymbol{u}}}{d\alpha}\right)\times\vec{\boldsymbol{v}}+\vec{\boldsymbol{u}}\times\left(\frac{d\vec{\boldsymbol{v}}}{d\alpha}\right)$$

Chapter 1 Introduction, Measurement, Estimating

- Significant figures（有效数字）
  - How to denote the significant figures for a number?
  - Scientific notation.
  - How to treat the number of significant figures when multiplying or dividing, and adding or subtracting.
- SI unit system（单位制）
  - Base units & derived units; 7 base units for SI unit system.
  - The standards of Length, Time, and Mass.
  - Unit prefixes.
- Dimensions and Dimensional analysis（量纲与量纲分析）
  - Check an equation by dimensional consistency.
- Order-of-magnitude（数量级估计）

The Seven SI Base Units

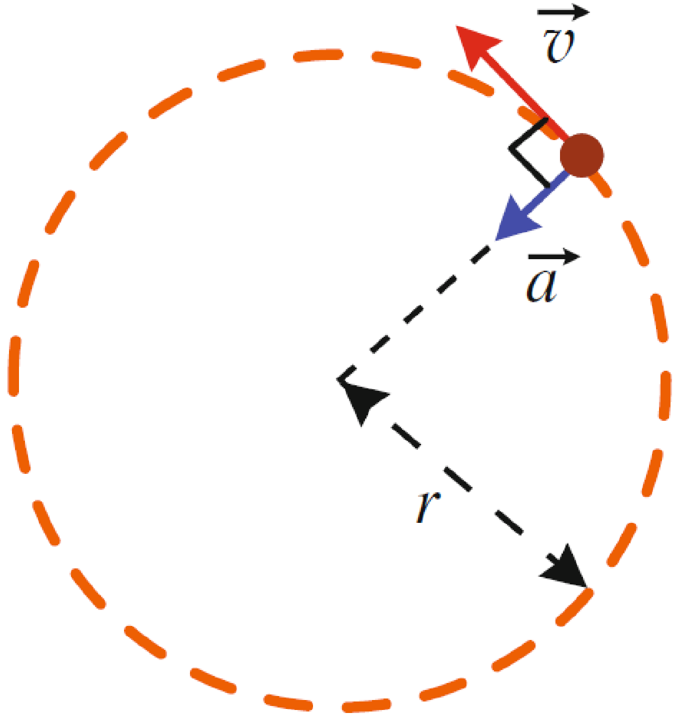
Quantity		SI Unit: Name	Symbol	中文
Time	*In Mechanics*	second	s	秒
Length		meter	m	米
Mass		kilogram	kg	千克
Electric current		ampere	A	安培
Thermodynamic temperature		kelvin	K	开尔文
Amount of substance		mole	mol	摩尔
Luminous intensity		candela	cd	坎德拉

SI Prefixes

Factor	Prefix	Symbol	中文	Factor	Prefix	Symbol	中文
10^{18}	exa-	E	艾	10^{-1}	deci-	d	分
10^{15}	peta-	P	拍	10^{-2}	centi-	c	厘
10^{12}	tera-	T	太	10^{-3}	milli-	m	毫
10^{9}	giga-	G	吉	10^{-6}	micro-	μ	微
10^{6}	mega-	M	兆	10^{-9}	nano-	n	纳
10^{3}	kilo-	k	千	10^{-12}	pico-	p	皮
10^{2}	hector-	h	百	10^{-15}	femto-	f	飞
10^{1}	deka-	da	十	10^{-18}	atto-	a	阿

Example

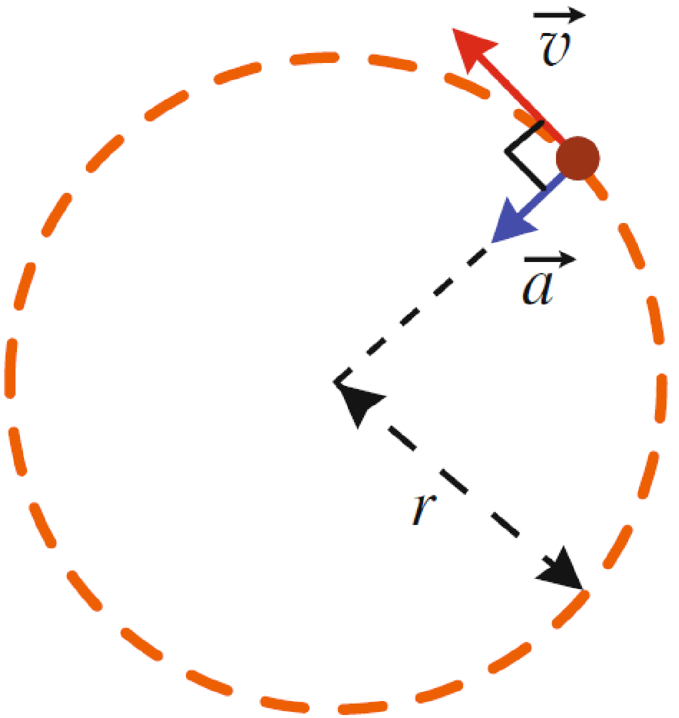
A particle moves with a constant speed v in a circular orbit of radius r. Given that the magnitude of the acceleration a is proportional to some power of r, say r^m, and some power of v, say v^n, then determine the powers of r and v.



Solution

Assume $$a = kr^m v^n$$

where k is a dimensionless proportionality constant.



Dimensional analysis,

$$\left[\frac{\mathrm{L}}{\mathrm{T}^2}\right] = \mathrm{L}^m \times \left[\frac{\mathrm{L}}{\mathrm{T}}\right]^n = \frac{\mathrm{L}^{m+n}}{\mathrm{T}^n}$$

Therefore, $$\begin{cases} m + n = 1 \\ n = 2, \qquad m = -1 \end{cases}$$

And the acceleration is $$a = kr^{-1}v^2 = k\frac{v^2}{r}$$

Enjoy your physics!

You know you can't enjoy a game unless you know its rules; whether it's a ball game, a computer game, or simply a party game.

Likewise, you can't fully appreciate your surroundings until you understand the rules of nature. Physics is the study of these rules, which show how everything in nature is beautifully connected. So the main reason to study physics is to enhance the way you see the physical world.