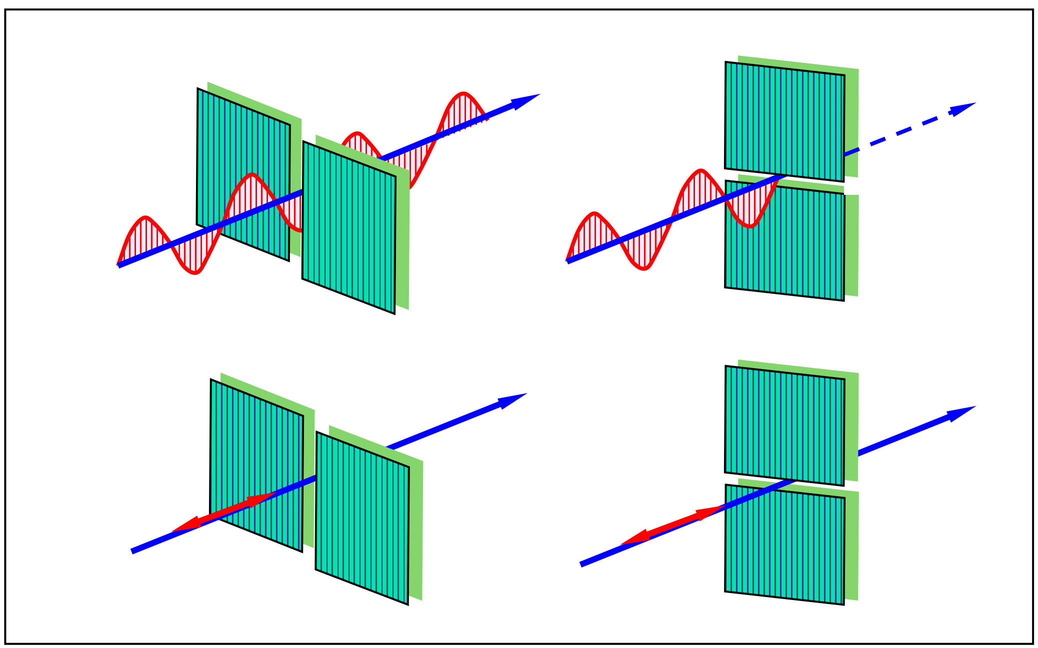
Chapter 31-B Polarization

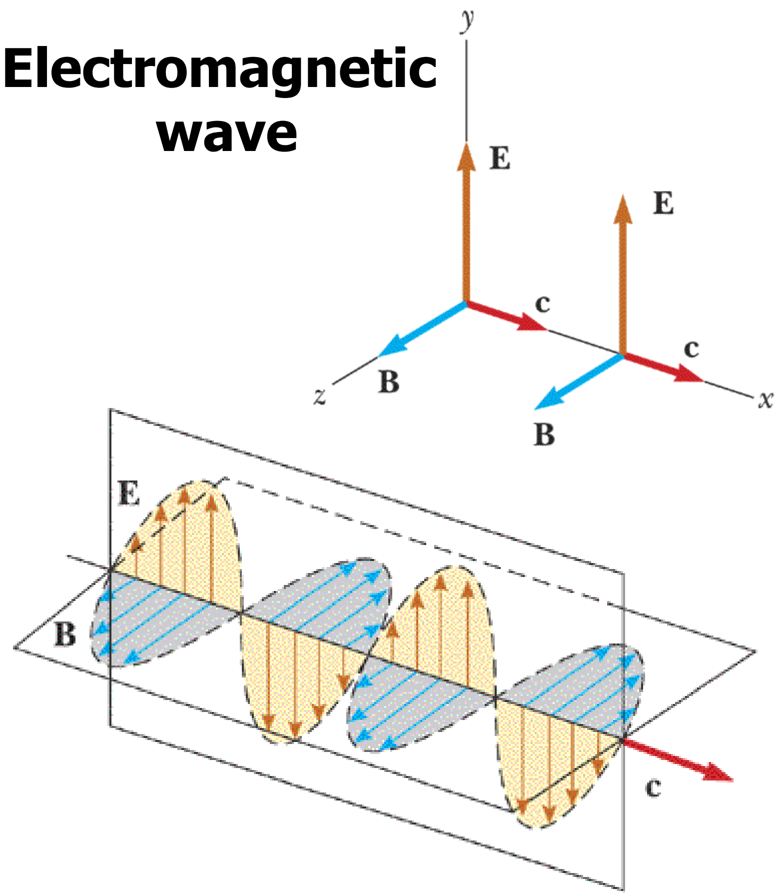
§ 1 Polarization of Light

An important and useful property of light is that it can be polarized, which shows the transverse property of electromagnetic wave.

mechanical wave

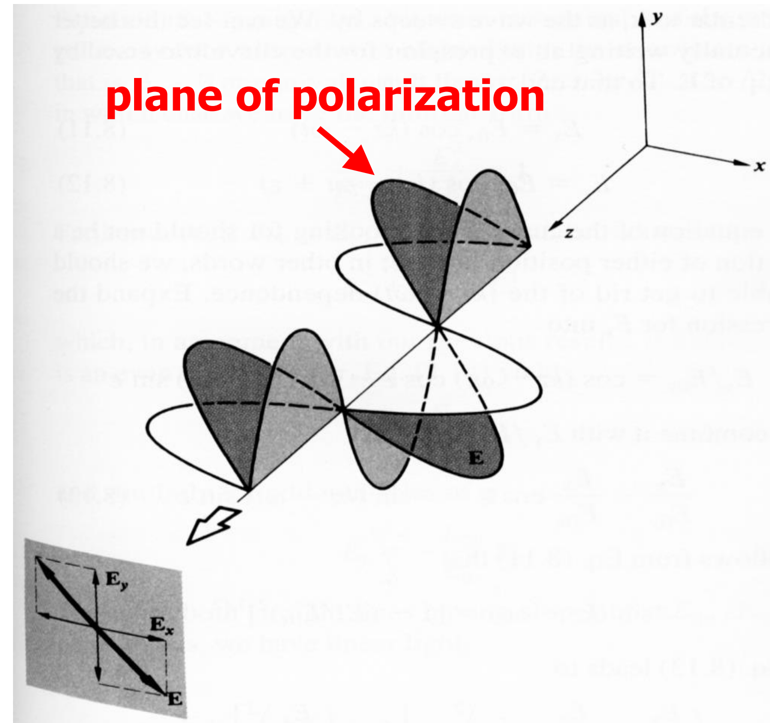


Electromagnetic wave



- **Linear polarization:**

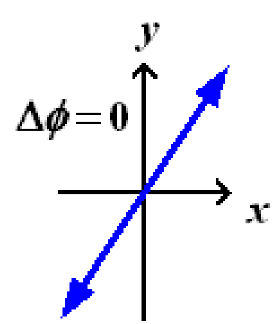
A light is said to be linearly polarized (also called plane polarized), if the electric field $\vec{E}$ remains in fixed plane (called plane of polarization). This fixed plane contains both electric field vector $\vec{E}$ and propagation vector $\vec{k}$.



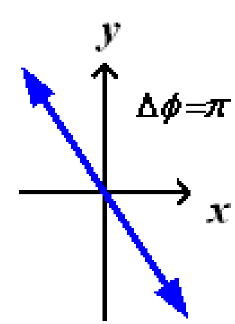
- **Any polarized wave can considered to be vector sum of two orthogonal components.**

$$\vec{E}(z,t) = \vec{E}_x(z,t) + \vec{E}_y(z,t)$$

$$\left.\begin{aligned} E_x(z,t) &= E_{0x}\cos(\omega t - kz + \phi_x) \\ E_y(z,t) &= E_{0y}\cos(\omega t - kz + \phi_y) \end{aligned}\right\} \quad \Delta\phi = \phi_y - \phi_x$$



Linear light in I, III quadrant



Linear light in II, IV quadrant

- **For linear polarization:** $\boxed{\Delta\phi = 0,\ \pi}$

3D figure of linear light

$\Delta\phi = 0$ $\Delta\phi = \pi$

Figure 8.1 Linear light.

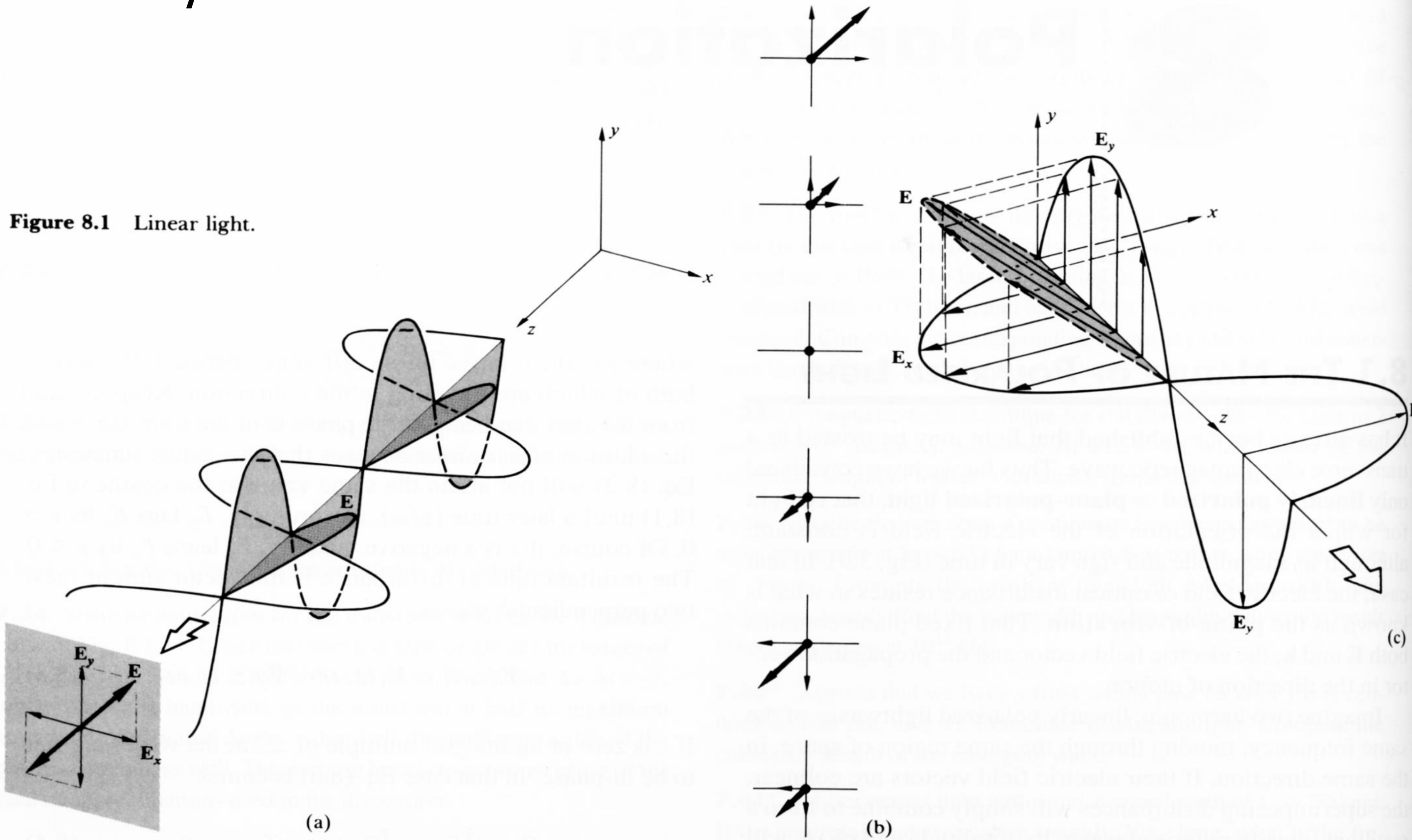


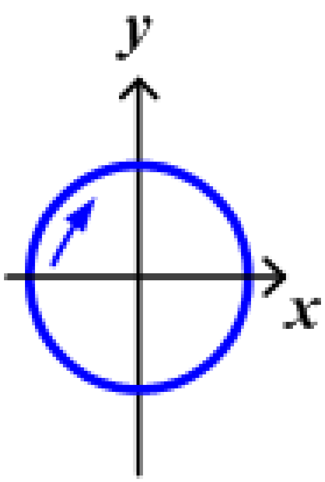
FIGURE 8.1 Linear light. (*a*) The *E*-field linearly polarized in the first and third quadrants. (*b*) That same oscillating field seen head on. (*c*) Light linearly polarized in the second and fourth quadrants.

State of polarization 2 —— circular polarization

- **Circular polarization:**

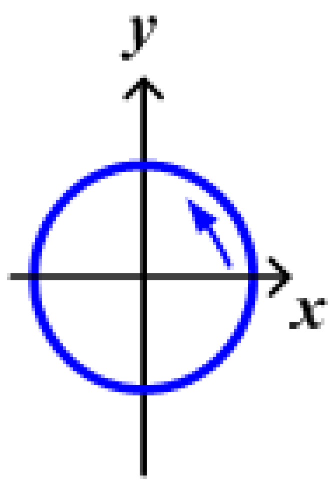
  A light is said to be circular polarized if the electric field $\vec{E}$ rotate uniformly in the plane perpendicular to the propagation direction at wave frequency.

- **The circular polarized light is said to be right-circular if rotating clockwise when $\Delta\phi = \pi/2$, and to be left-circular if rotating counterclockwise when $\Delta\phi = -\pi/2$.**



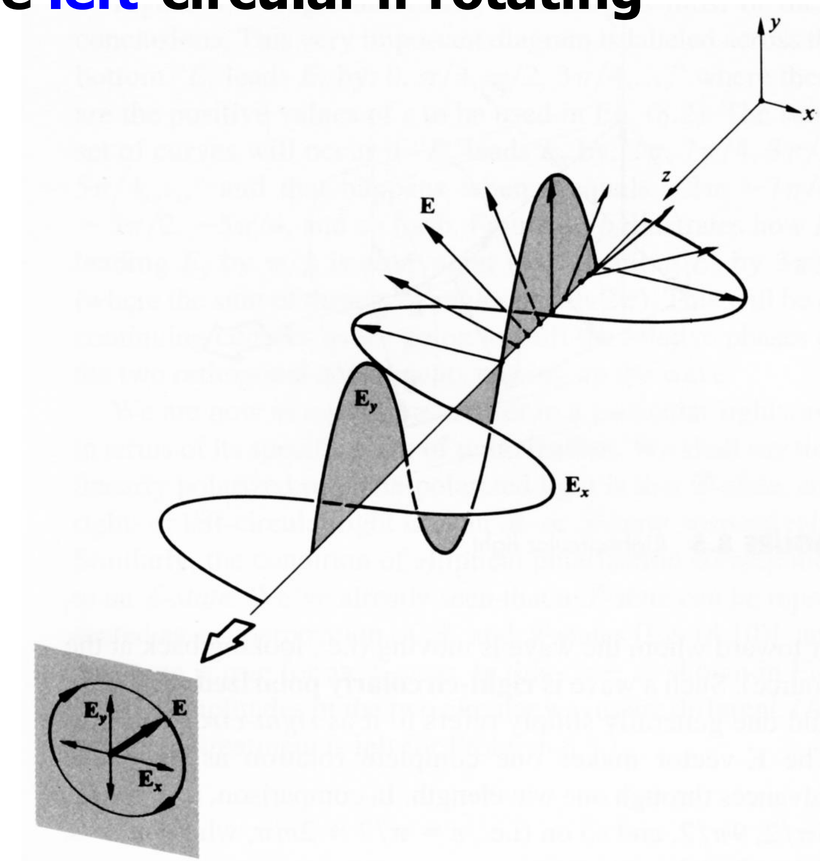
$\Delta\phi = \dfrac{\pi}{2}$

Right-circular light



$\Delta\phi = -\dfrac{\pi}{2}$

Left-circular light

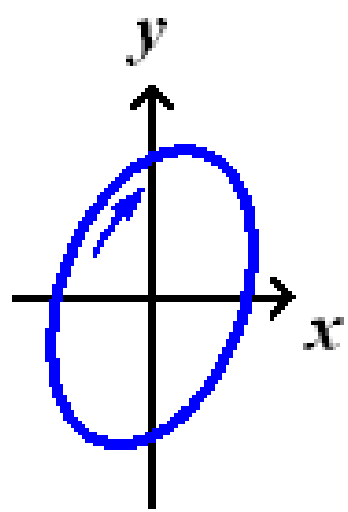


State of polarization 3 —— Elliptical polarization

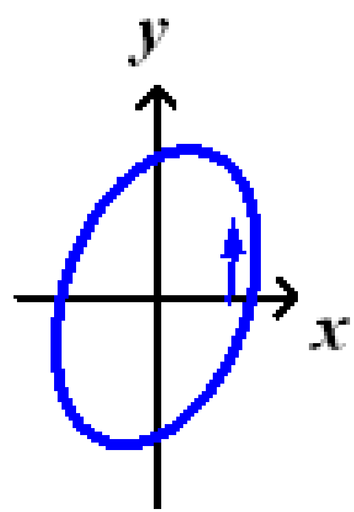
- **Elliptical polarization:**

  A light is said to be elliptically polarized (or simply elliptical light) if the end of electric field vector $\vec{E}$ draws an ellipse in the plane perpendicular to the propagation direction at wave frequency.

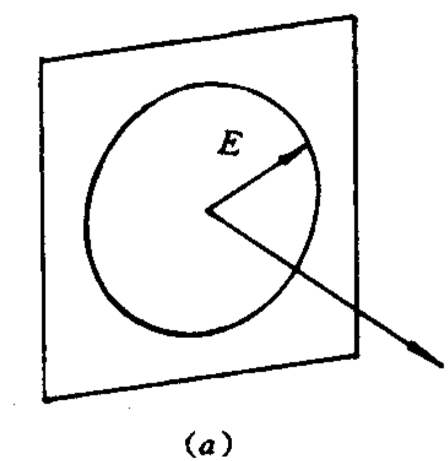
- **The elliptically polarized light is said to be right-handed when rotating clockwise, and to be left-handed when rotating counterclockwise.**



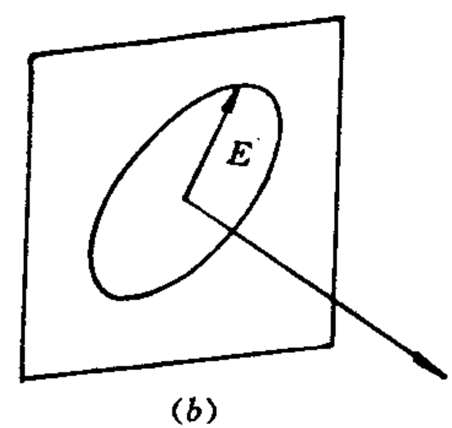
Right-handed elliptical light



Left-handed elliptical light



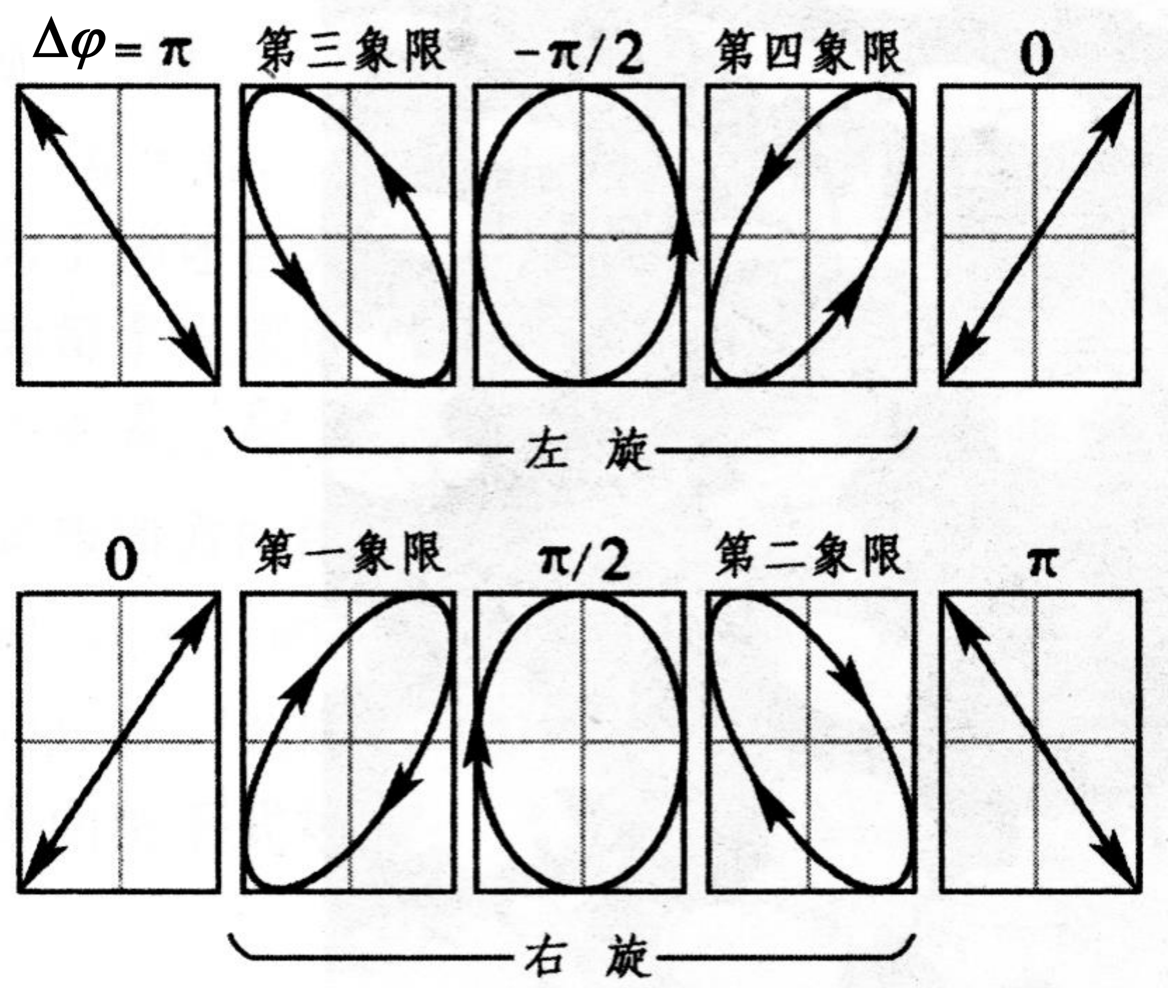
(a) 圆偏振光



(b) 椭圆偏振光

Polarization of Light

Each state of polarization is a special case of elliptical light according to the phase difference $\Delta\varphi$. $\Delta\phi = \phi_y - \phi_x$



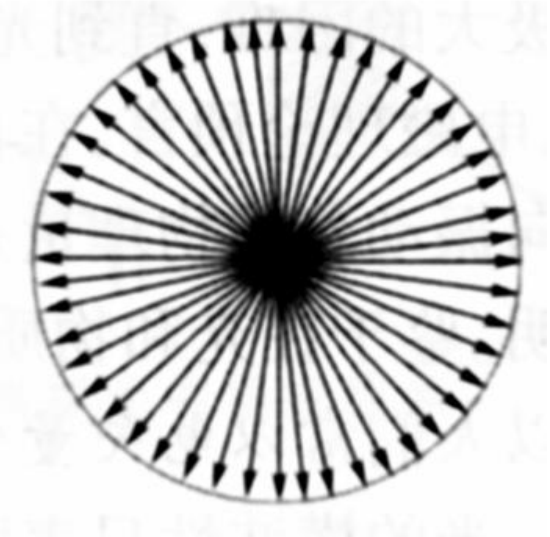
State of polarization 4

—— Natural light (or unpolarized light)

- **Unpolarized light:**

  A light is said to be unpolarized if the direction of $\vec{E}$ vector change randomly with time.

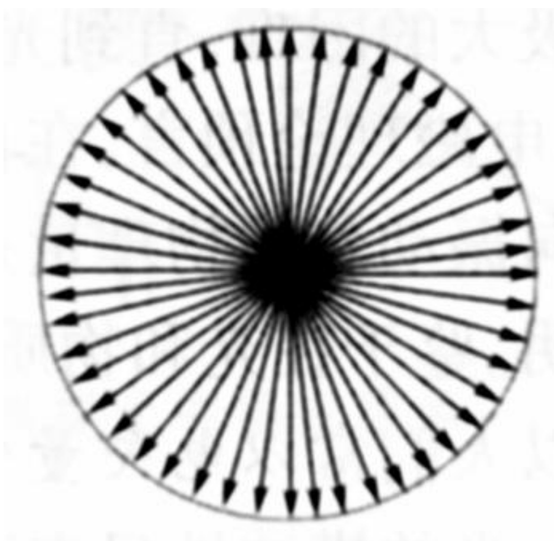
- **An ordinary light source consists of a very large number of randomly oriented atomic emitters. Each excited atom radiates a polarized wavetrain for roughly 10^{-8} second. All emissions having the same frequency will combine to form a single resultant polarized wave, which persists for no longer than 10^{-8} second. These new wavetrains are constantly emitted, and the overall polarization changes in a completely unpredictable fashion at so rapid a rate as to render any single resultant polarization state indiscernible.**

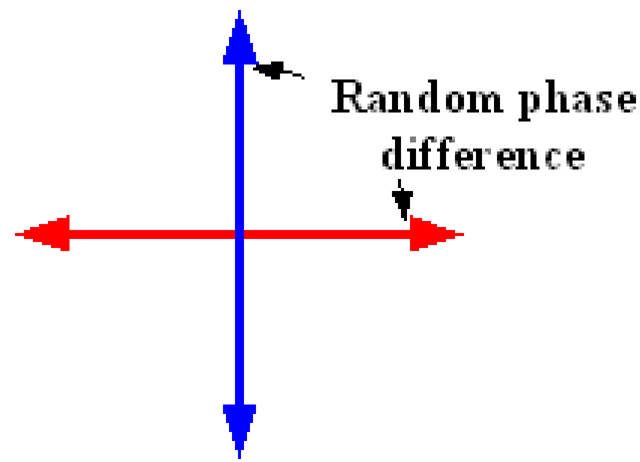


Natural light (or unpolarized light)

- **For a unpolarized light, all $\vec{E}$ vectors emitted in a short time distributed in all directions uniformly.**

- **We can mathematically represent natural light in terms of two arbitrary, incoherent, orthogonal, linearly polarized waves with equal amplitude but random phase difference between them.**

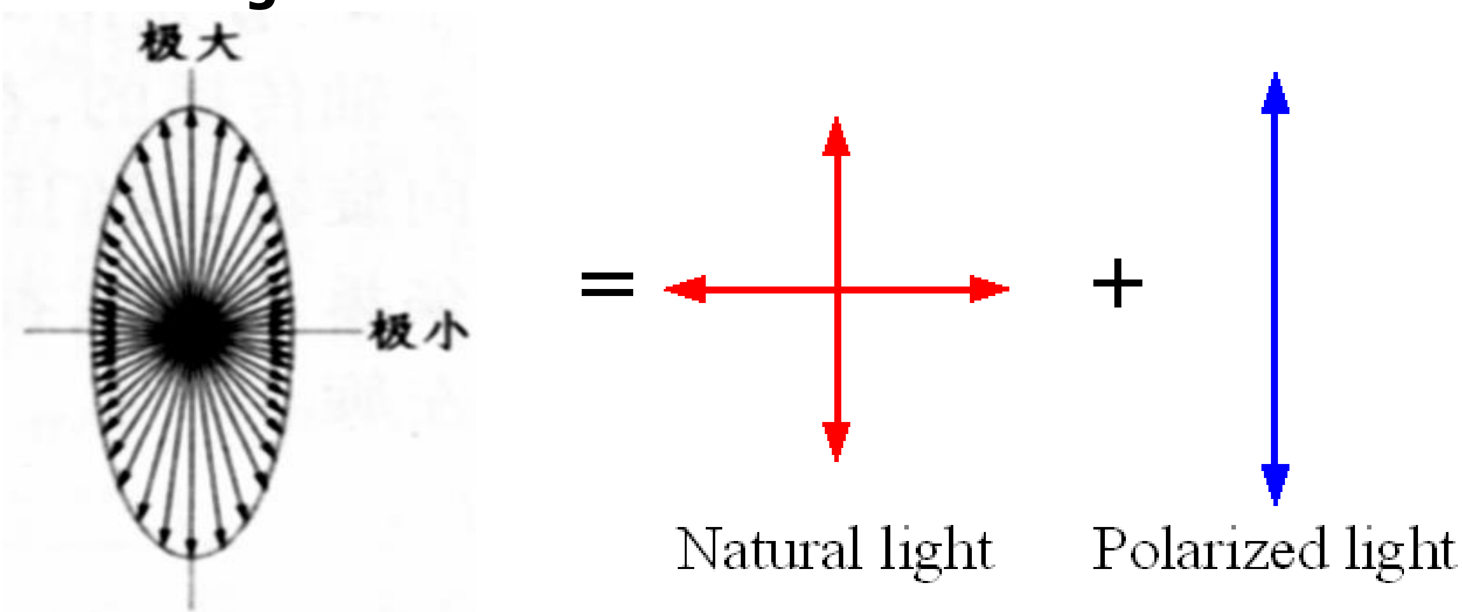




- **Partially polarized light:**

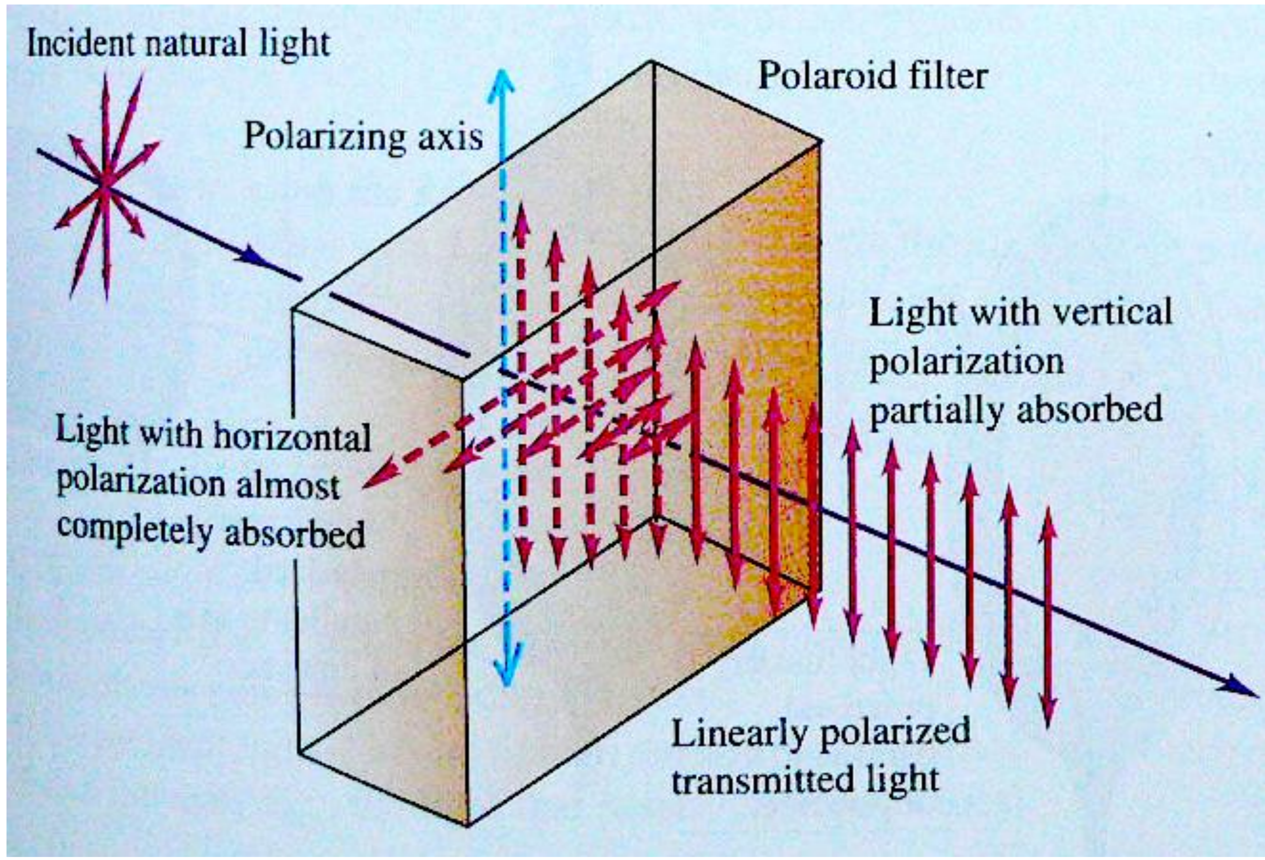
  A light is generally neither completely polarized nor completely unpolarized; both cases are extremes. More often, the electric field vectors $\vec{E}$ varies in a way that is neither totally regular nor totally irregular, and such light is said to be partially polarized light.

- **We can mathematically represent partially polarized light in terms of combination of specific amount of natural light and polarized light.**



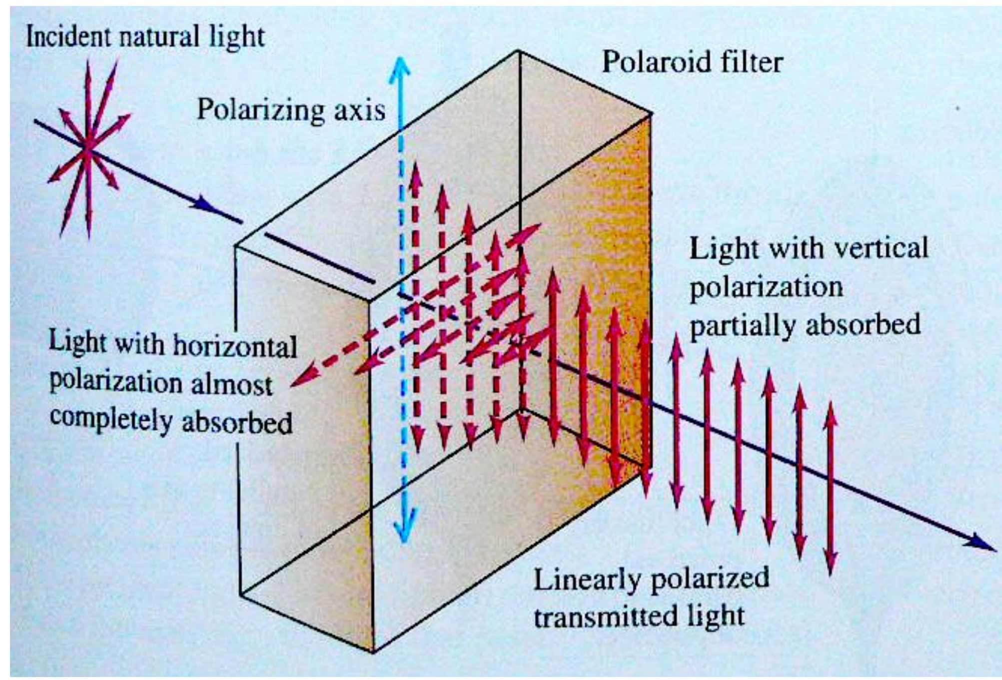
Polarizer (起偏器)

- **Dichroism (二向色性) refers to the selective absorption of one of the two orthogonal components of an incident light.**
- **Dichroic crystal: Certain materials like tourmaline (电气石) are inherently dichroic because of an anisotropy (各向异性) in their respective crystalline structure.**



Polarizer

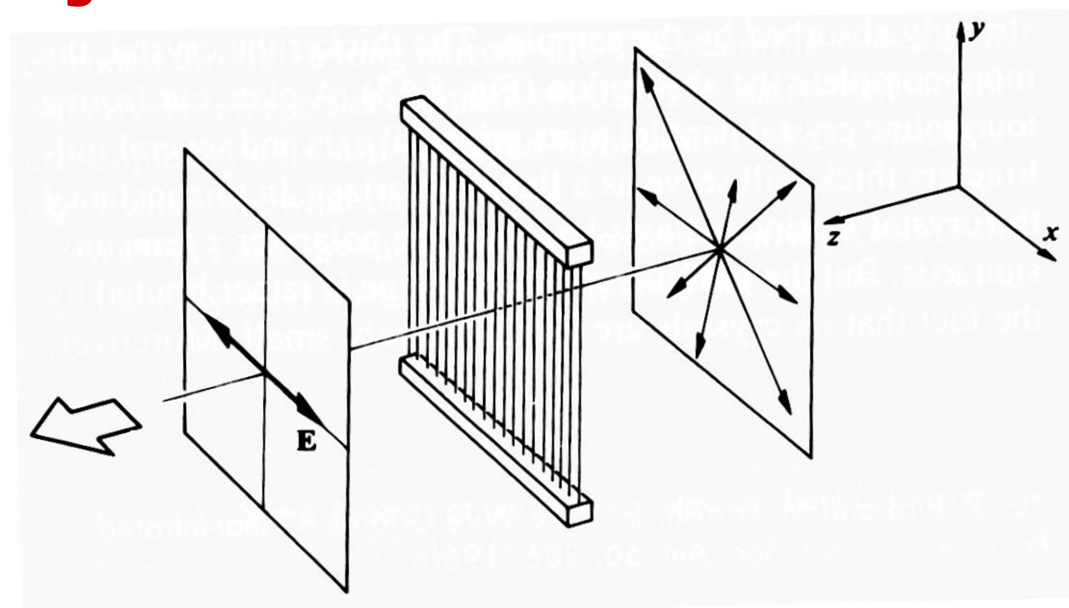
- The dichroic crystal, which has a optic axis called polarizing axis, possess a property that the electric-field component of an incident light that is perpendicular to the polarizing axis is strongly absorbed than another orthogonal component.
- When a natural light passes through such dichroic crystal, it will be predominantly polarized in the direction of crystal's polarizing axis. If the crystal is thick enough, the output light will linearly polarized in polarizing axis (also called transmission axis).



- This type of crystals serves as a linear polarizer.

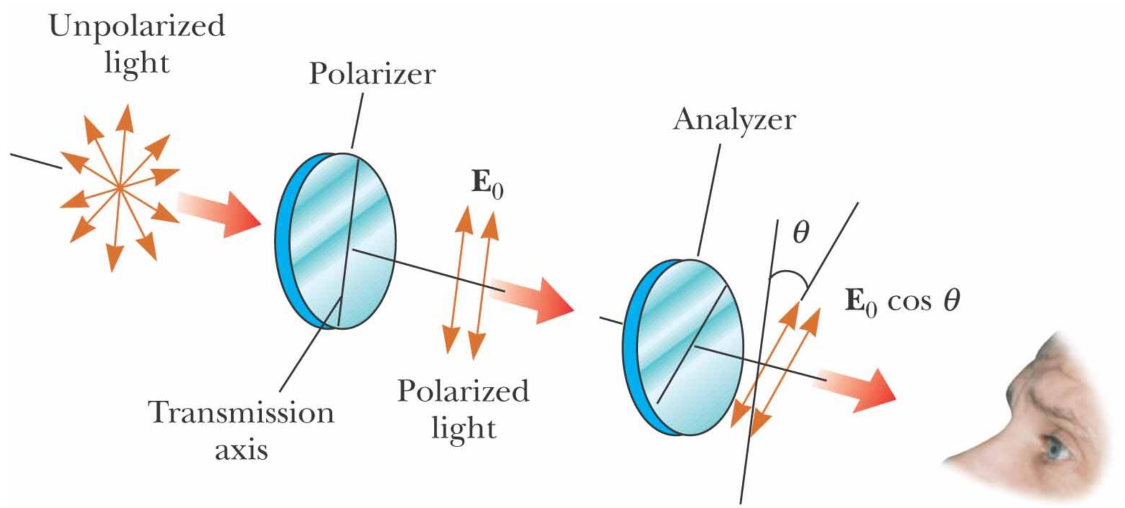
Polaroid sheet (偏振片)

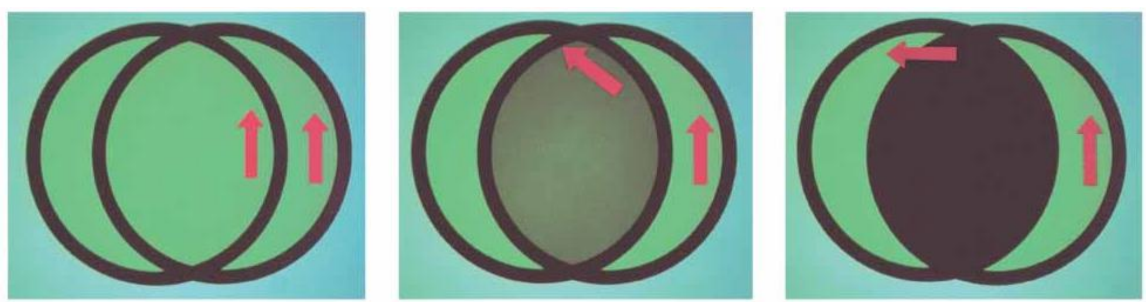
- The Polaroid sheet is an artificial device that has dichroism, which can be made in larger size.
- A Polaroid sheet consists of complicated long molecules chains, in a large transparent film plate, arranged parallel to one another. Such a Polaroid acts like a series of parallel slit, just as a grid of parallel conducting wire.
- For a incident electromagnetic wave, the y-component of field drives the conduction electrons along the length of each wire, generating current with loss of energy transferring into heat, resulting in little or no transmission of y-component of the field.



Marlus's law

- **Phenomenon：**
  If a natural light is incident two polarizers whose transmission axis make an angle θ with each other, the intensity of output light I_{out} we see through the second polarizer depends on the relative orientation θ of two polarizers.
- **In the range of 0~360°, we see maximum intensity twice and zero intensity twice.**

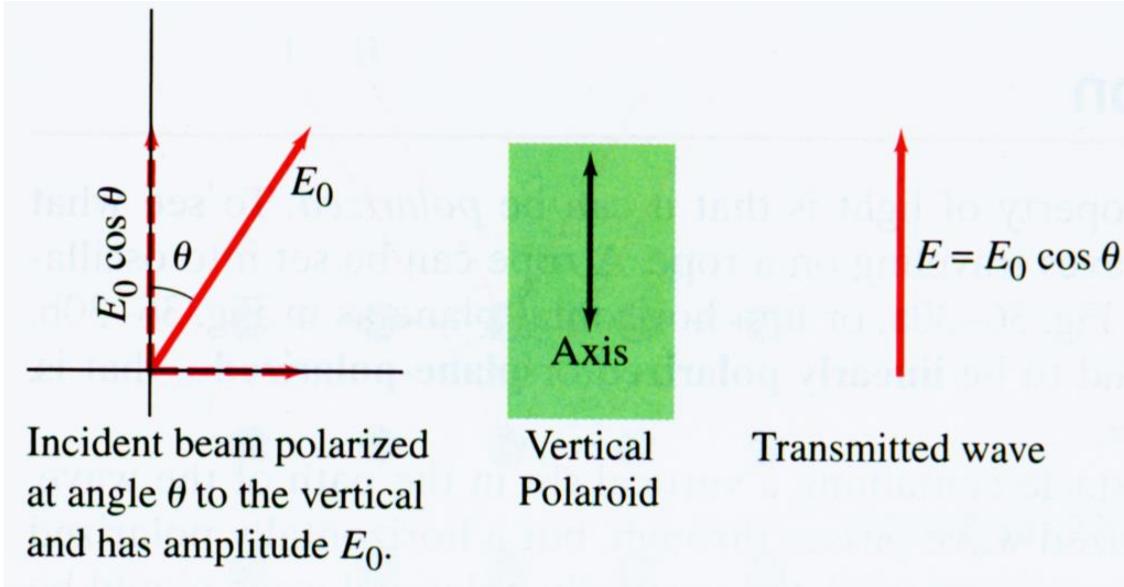




$I_{out}=I_{MAX}$ $\theta=0°, 180°$ 0< I_{out}< I_{MAX} other θ $I_{out}=0$ $\theta=90°, 270°$

Marlus's law (continued)

- **Explanation：**
  If a linear light strikes a Polaroid whose axis at an angle θ to the incident polarization direction, the perpendicular component amplitude vanished, and the parallel amplitude $E_0\cos\theta$ will be pass through the Polaroid.



$$E = E_0 \cos\theta$$

- **Marlus's law**

The intensity of the linear light transmitted by a polarizer is:

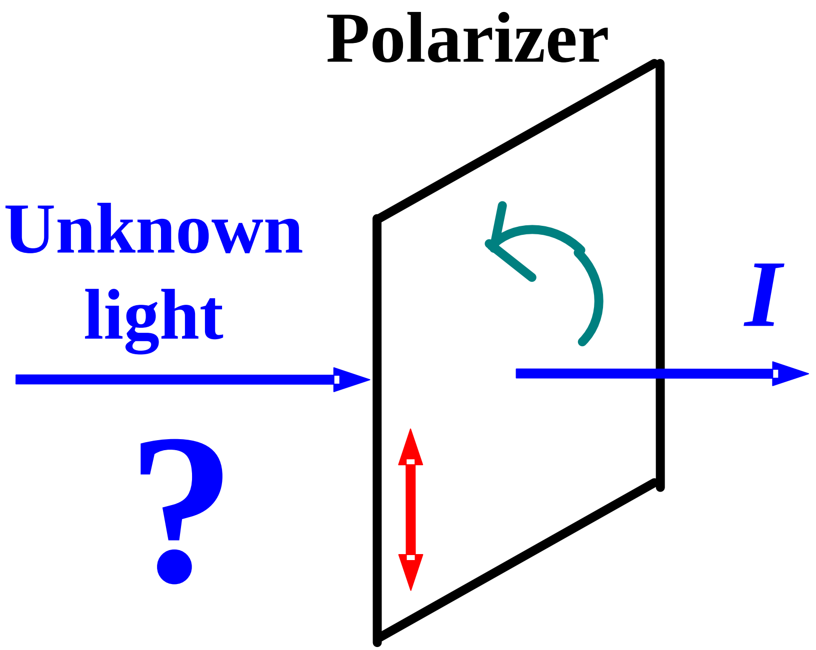
$$\boxed{I = I_0 \cos^2\theta}$$

- **When $\theta = 0°, 180°, I = I_0$.**
- **When $\theta = 90°, 270°, I = 0$.**

Question

- **How to distinguish a polarized light?**
  - **Can a Polaroid be used as polarization analyzer (检偏器)?**

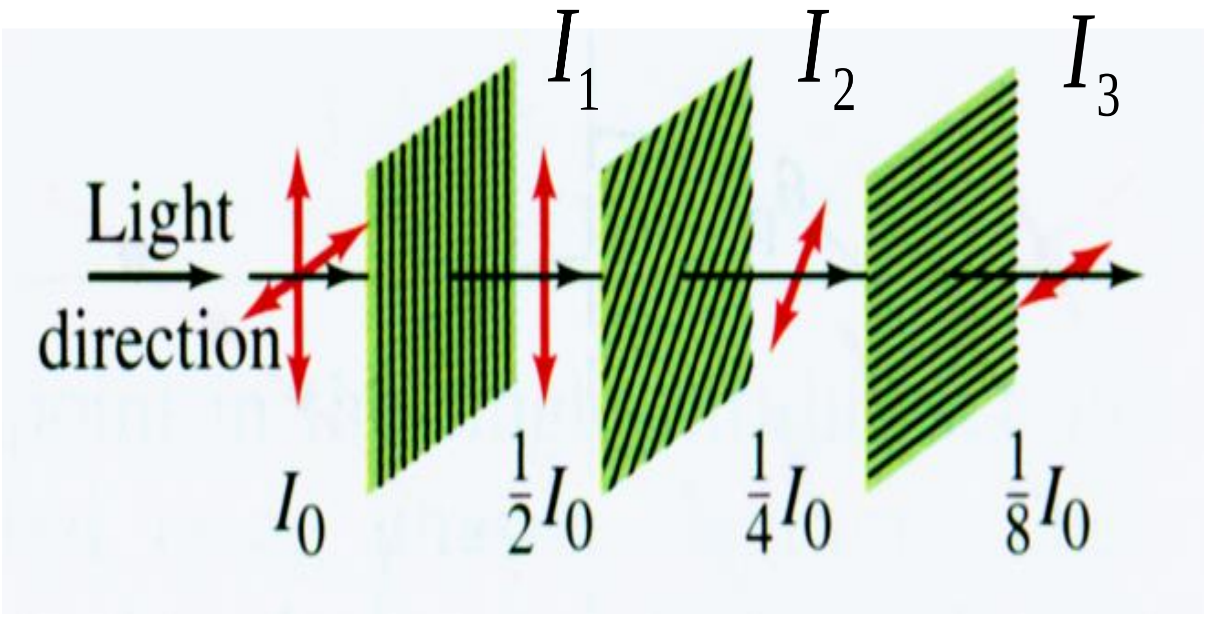
① Linear; ② unpolarized; ③ partially polarized light.



Example

Three Polaroids

When unpolarized light falls on two crossed Polaroids, no light passes through. What happens if a third Polaroid, with axis at 45° to each of the other two, is placed between them?



Example - Three Polaroids

When unpolarized light falls on two crossed Polaroids, no light passes through. What happens if a third Polaroid, with axis at 45º to each of the other two, is placed between them?

Solution:

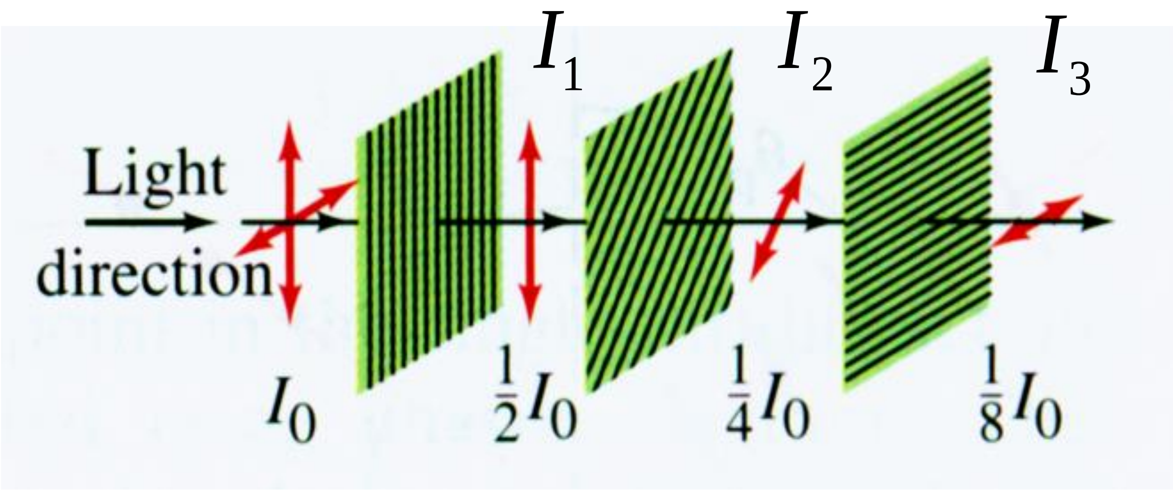
The first Polaroid changes the unpolarized light to plane-polarized and reduces the intensity to 1/2.

$$I_1 = \frac{1}{2} I_0$$

The light leaving the last polarizer:

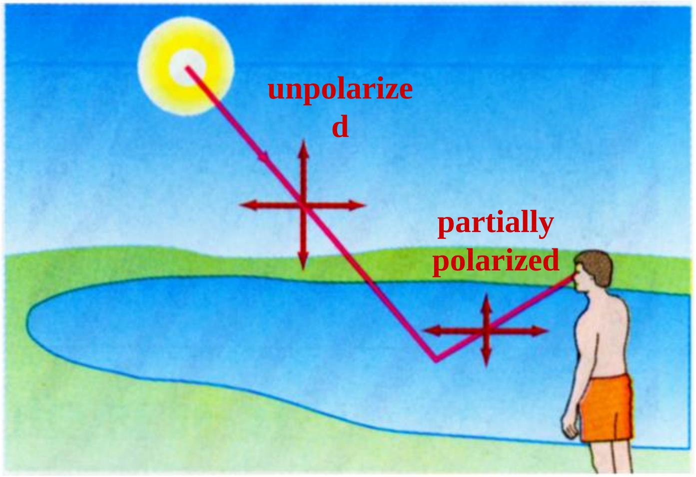
$$I_2 = I_1 \cos^2 45^\circ = \frac{1}{2} I_1 = \frac{1}{4} I_0$$

$$I_3 = I_2 \cos^2 45^\circ = \frac{1}{2} I_2 = \frac{1}{8} I_0$$

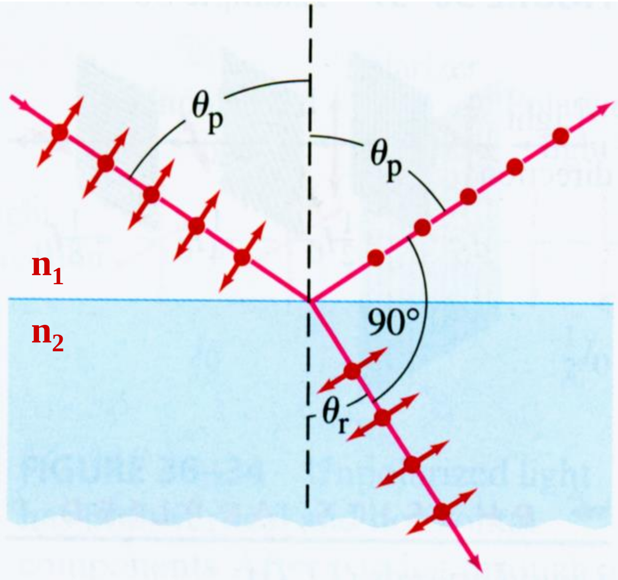


§ 2 Polarization by Reflection

- Polarization by reflection
  - Unpolarized light can be polarized, either partially or totally, by **reflection**.
  - Light reflected from the smooth surface of water in a lake is partially polarized parallel to the surface.



- **Brewster's law**
  - The degree of polarization of reflected light are varied with incident angle. At one particular angle of incidence, called polarizing angle, or **Brewster's angle**, the reflected light is **totally** polarized, with its plane of polarization **perpendicular** to the plane of incidence.
  - At the Brewster's angle, the reflected ray and the refracted ray are **perpendicular** to each other.

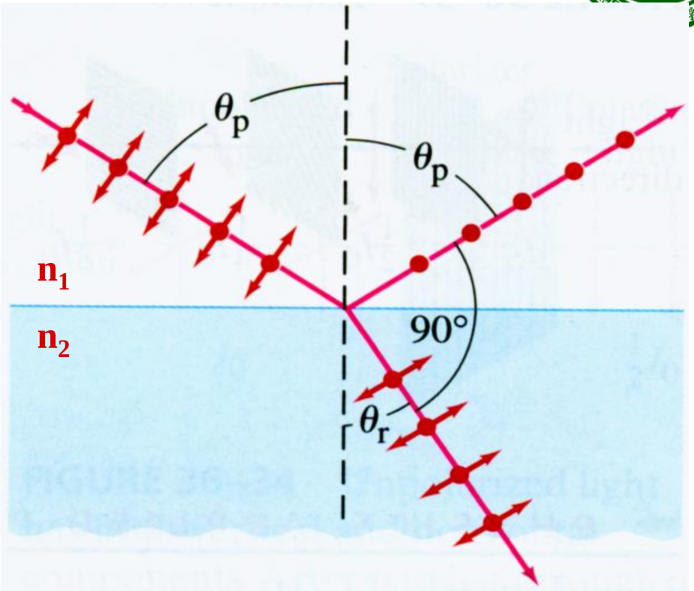


Brewster's law

$$\theta_p + \theta_r = 90^\circ$$

$$n_1 \sin\theta_p = n_2 \sin\theta_r$$

$$= n_2 \sin(90^\circ - \theta_p) = n_2 \cos\theta_p$$



- **Brewster's law:** $$\tan\theta_p = \frac{n_2}{n_1}$$

- **Application**

- **Glare reduction: When taking the picture outside of a shop window, you can use a polarizer before the lens.**

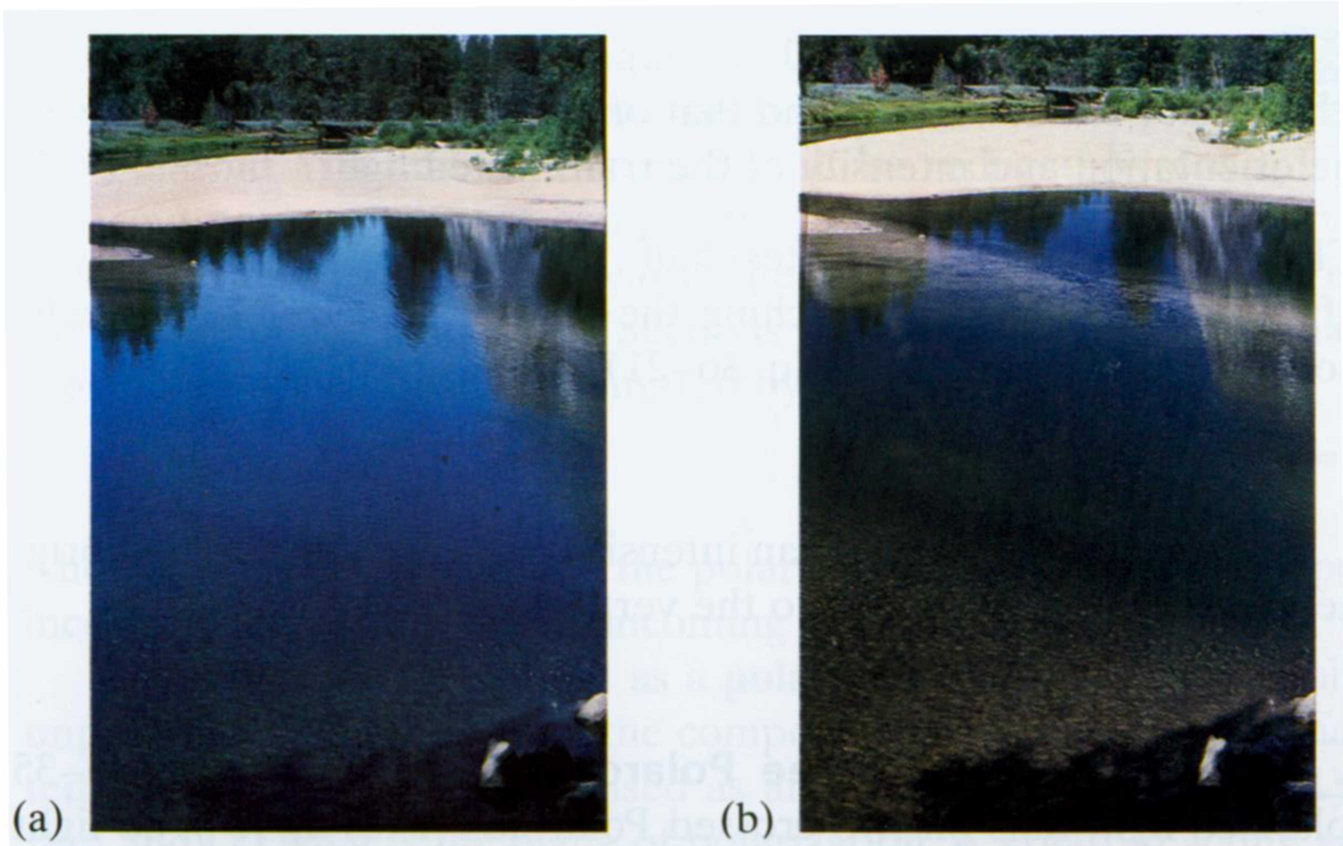
Without polarizer

Using a polarizer

Polarization by Reflection

(a) Without polarizer;

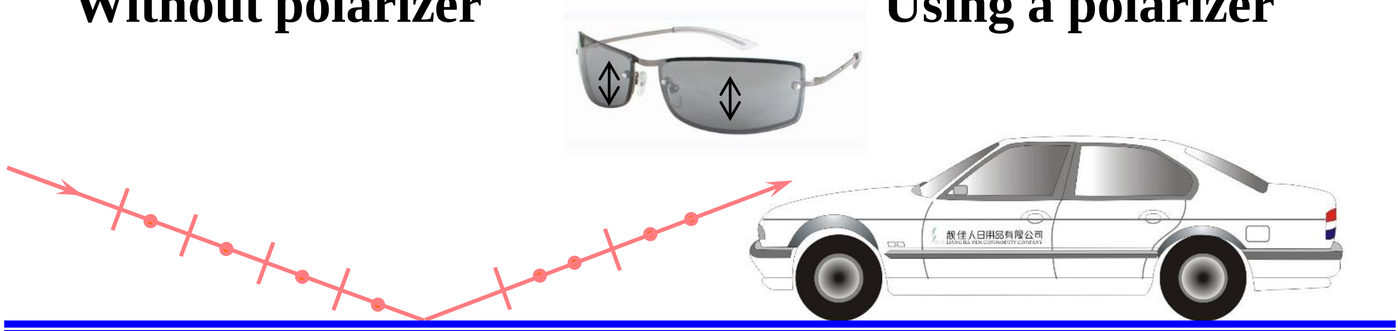
(b) Using a polarizer which is adjusted to absorb most of the polarized light reflected from the water's surface, and any fish to be seen more readily.



Polarization by Reflection

Without polarizer

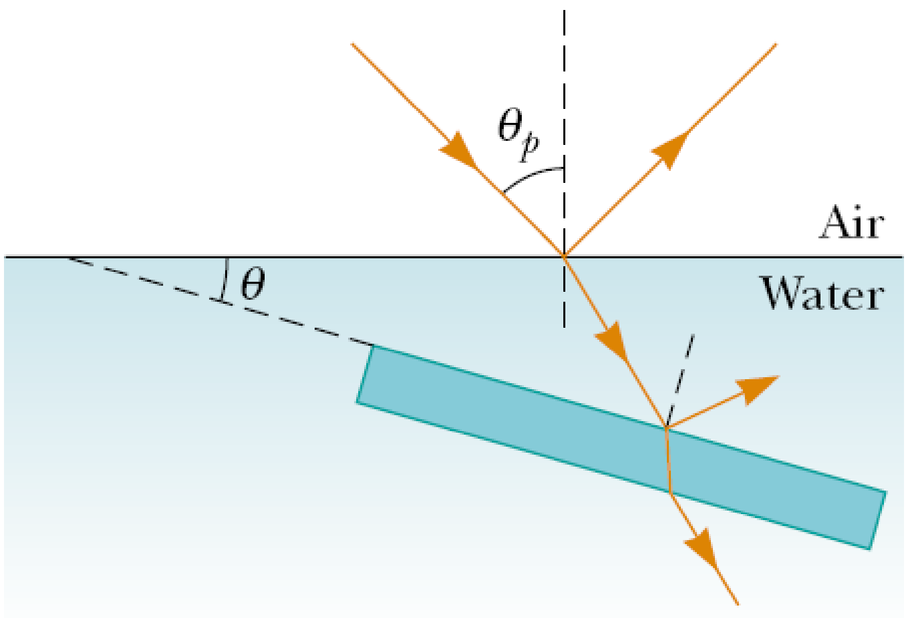
Using a polarizer

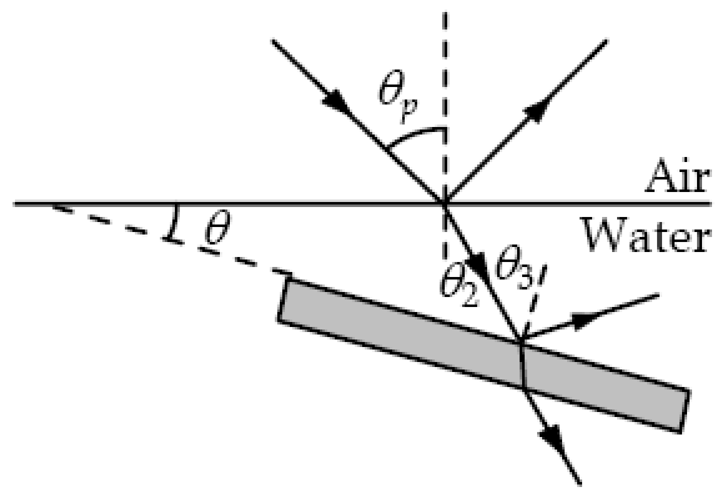


The driver can reduce the glare of the reflection of light from road.

Example

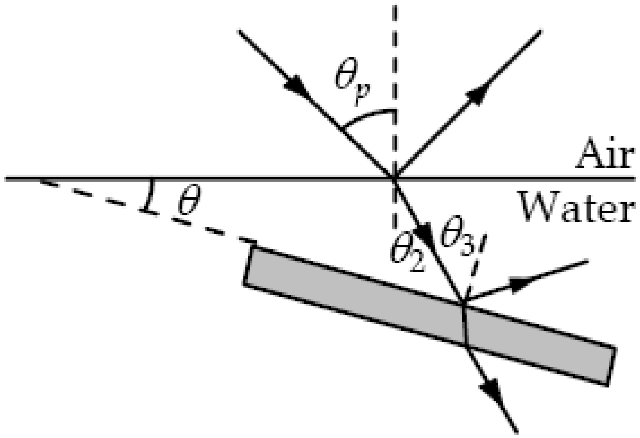
Light strikes a water surface at the polarizing angle. The part of the beam refracted into the water (index of refraction, $n_{water}=1.33$) strikes a submerged glass slab ($n_{glass}=1.50$), as shown in the figure. The light reflected from the upper surface of the slab is completely polarized. Find the angle between the water surface and the glass slab.





Solution:

For the air-to-water interface,



$$\tan\theta_p = \frac{n_{\text{water}}}{n_{\text{air}}} = \frac{1.33}{1.00}, \quad \theta_p = 53.1°$$

and

$$(1.00)\sin\theta_p = (1.33)\sin\theta_2, \quad \theta_2 = \arcsin\frac{\sin 53.1°}{1.33} = 36.9°$$

For the water-to-glass interface,

$$\tan\theta_3 = \frac{n_{\text{glass}}}{n_{\text{water}}} = \frac{1.50}{1.33}, \quad \theta_3 = 48.4°$$

The angle between surfaces is $\theta = \theta_3 - \theta_2 = 11.5°$

Prob.8

P729, Prob.47, 49, 52