Signals and Systems

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BUPT

微信联系方式

群聊：信号与系统 2024秋
18-19-20班

该二维码7天内(9月16日前)有效，重新进入将更新

课程群

教师：杨少石

助教：张宇炘

Outline of Today's Lecture

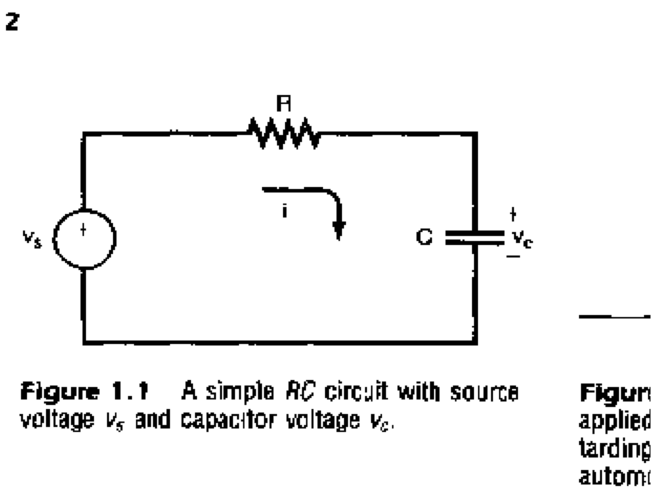
- Introduction of signals and systems
- Overview of our course
- Classification of signals
- Operation on signals
- Summary

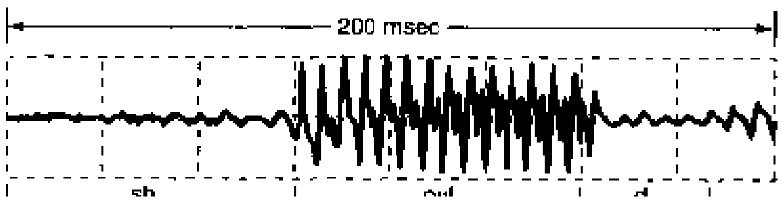
Introduction of Signals and Systems

- What are signals?
  - **A signal is formally defined as a function of one or more variables that conveys information on the nature of a physical phenomenon**
  - Our world is full of signals, both natural and man-made.
    - Variation in air pressure when we speak.
    - Voltage waveform in a circuit.
    - The periodic electrical signals generated by the heart.
    - Stock prices
    - Radar/infrared/optical images about outer space from a probe

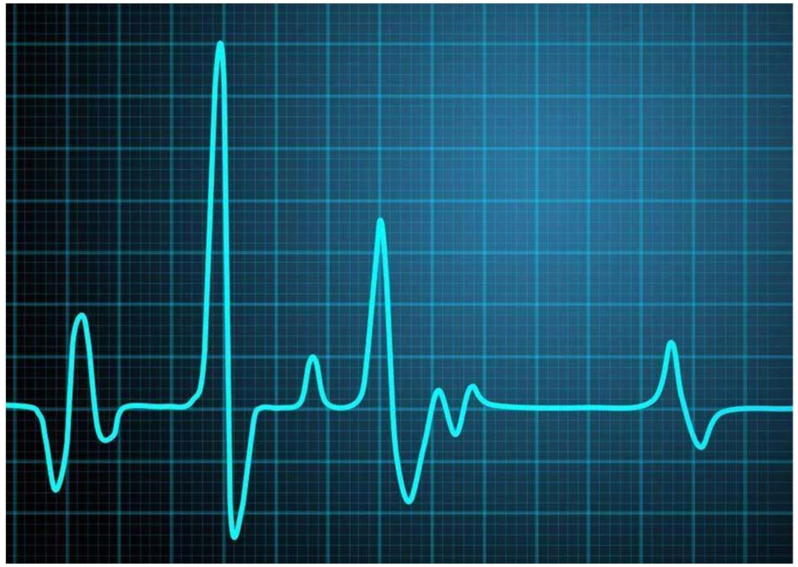
Introduction of Signals and Systems

- Examples of Signal





Should we chase?



Cardiogram

Introduction of Signals and Systems

- What are systems?
  - A system is a generator of signals or a transformer of signals.
  - A system is formally defined as an entity that manipulates one or more signals to accomplish a function, thereby yielding new signals.

Figure 1.1 Block diagram representation of a system.

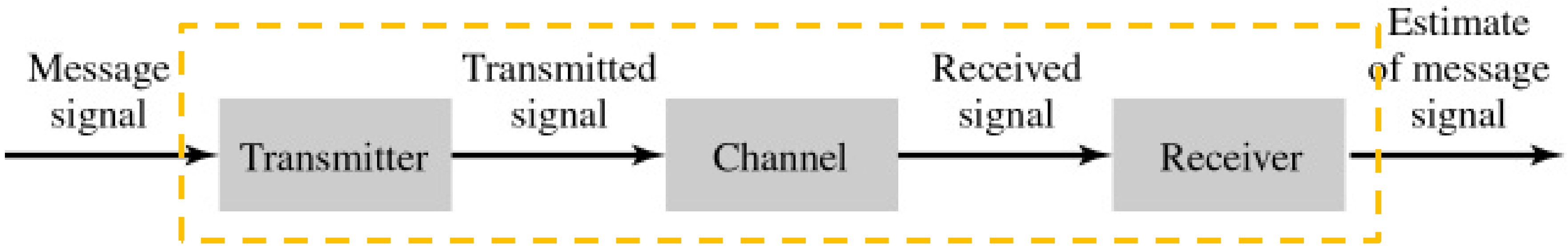
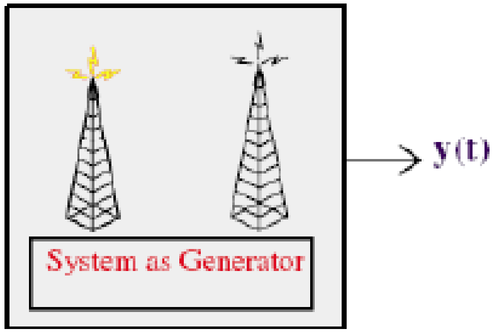


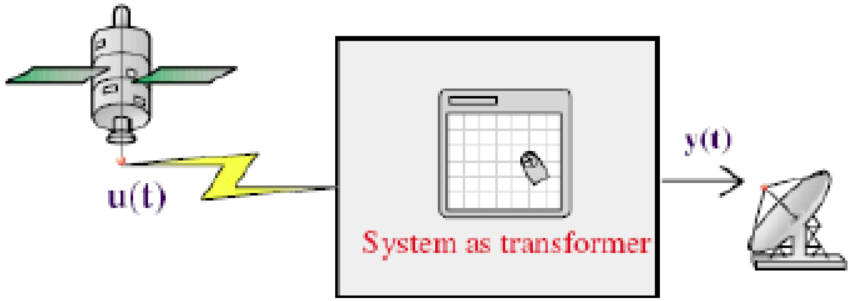
Figure 1.2 Elements of a communication system.

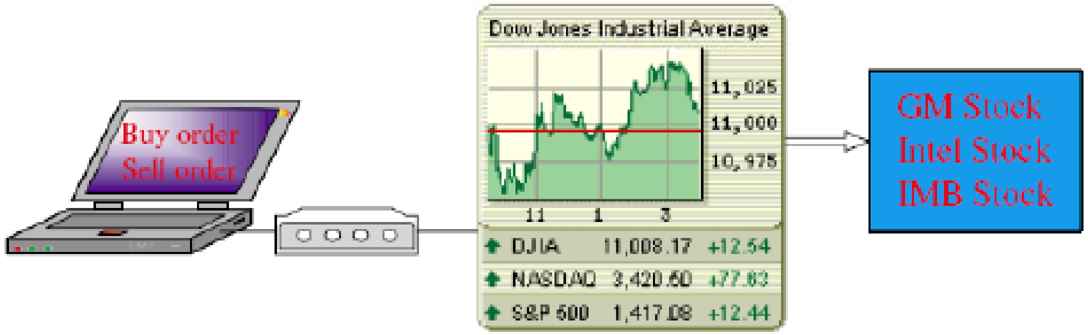
The transmitter changes the message signal into a form suitable for transmission over the channel. The receiver processes the channel output to produce an estimate of the message signal.

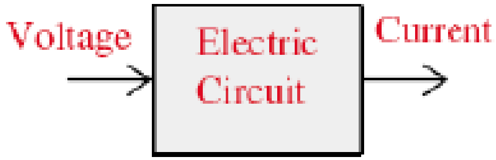
Introduction of Signals and Systems

- Examples of systems







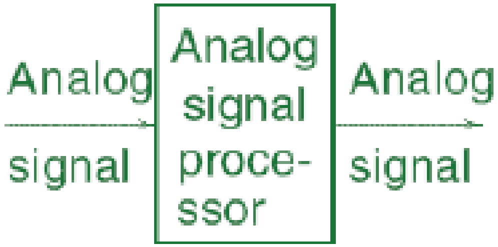


Introduction of Signals and Systems

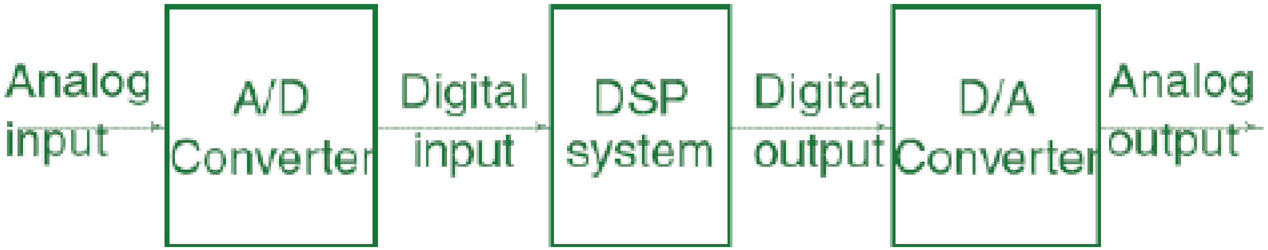
- Typical systems in electrical and electronic engineering
  - Communication systems
  - Control systems
  - Computer systems

Introduction of Signals and Systems

- Signal processing involves enhancing, extracting, storing and transmitting useful information. Electrical signals are best suited for such manipulations. It is common to convert signals to electrical form for processing.
- Analog signal processing: relies on the use of analog circuit elements(resistors, capacitors, inductors, transistor amplifiers)
- Digital signal processing: relies on three basic digital computer elements: adders, multipliers and memory
  - Flexibility
  - Repeatability



Analog signal processing



Digital processing of analog signals

Introduction of Signals and Systems

- Why study Signals and Systems?
  - Signals and Systems are fundamental to all of engineering!
  - Steps involved in engineering are:
    - **Model system:** Involves writing a mathematical description of input and output signals.
    - **Analyze system:** Study of the various signals associated with the system.
    - **Design system:** Requires deciding on a suitable system architecture, as well as finding suitable system parameters.
    - **Implement system/test system:** Check system, and the input and output signals, to see that the performance is satisfactory.

Overview of our course

- This course is about **signals** and their **processing by systems**. It involves:
  - Modelling of signals by mathematical functions
  - Modelling of systems by mathematical equations
  - Solution of the equations when excited by the functions
  - Stability of the systems
- The course will serve as the prerequisites for additional coursework in the study of communications, signal processing and control.

Overview of our course

- Contents
  - Introduction
  - Time-domain representations of linear time-invariant systems
  - Fourier representations of signals and linear time-invariant systems
  - Applications of Fourier representations to mixed signal classes
  - Representing signals by using continuous-time complex exponentials: the Laplace transform
  - Representing signals by using discrete-time complex exponentials: the z-Transform.

Overview of our course

- Textbook
  - Signals and Systems, 2nd edition, by S. Haykin and B. Van Veen, John Wiley & sons, Inc 2003
- Reference books
  - Signals and Systems, 2nd edition, by By Alan V.Oppenheim, Alan S.Willsky and S.Hamid Nawab, 清华大学出版社，影印版 2002
  - 《信号与系统》，郑君里，应启珩，杨为理，高等教育出版社，2000
  - Schaum's outline of signals and systems, Hwei P. Hsu, McGraw-Hill, 1995. Website: http://issuu.com/ek.korat/docs/schaum_s_outline_of_signals_and_systems
- Teaching hours：16W×3hr = 48 hours

Overview of our course

Grading Scheme

Homework and attendance	**20%**
Final examination	**80%**
Total	**100%**

Lectures slides

教学云-云邮空间

Classification of Signals

- Methods used for processing a signal or analysing the response of a system to a signal significantly depend on the characteristic attributes of the signal.
- Certain techniques apply to only specific types of signals – hence the need for classification
  - continuous-time & discrete-time signals
  - even and odd signals
  - periodic and aperiodic signals
  - deterministic and random signals
  - energy and power signals

Continuous-time and discrete-time signals

- A continuous-time signal is defined for all time t, except at some discontinuous point.
- A discrete-time signal is defined only at discrete instants of time.

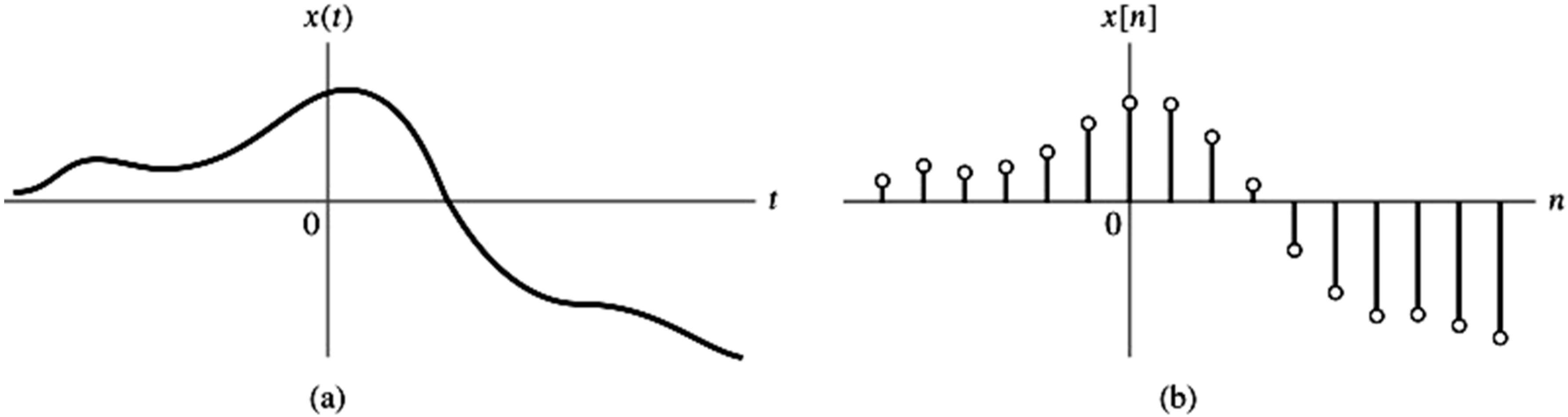


Figure 1.12 (a) Continuous-time signal $x(t)$.
(b) Representation of $x(t)$ as a discrete-time signal $x[n]$.

Continuous-time and discrete-time signals

- A discrete-time signal is often derived from A continuous-time signal by sampling it at a uniform rate.

$$x[n] = x(t)|_{t=nTs} = x(nT_s)$$

T_s: *sampling period;* n *denote an integer*

In this lecture, we use t to denote time for a continuous- time signal, and n to denote time for a discrete-time signal.

Continuous-time signals: $x(t)$

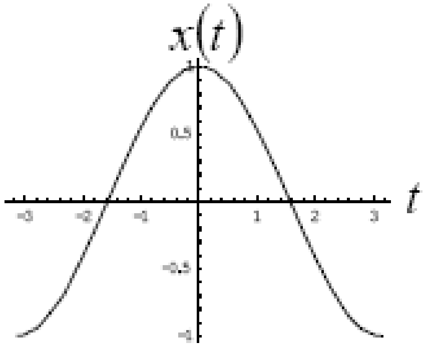
Parentheses (·)

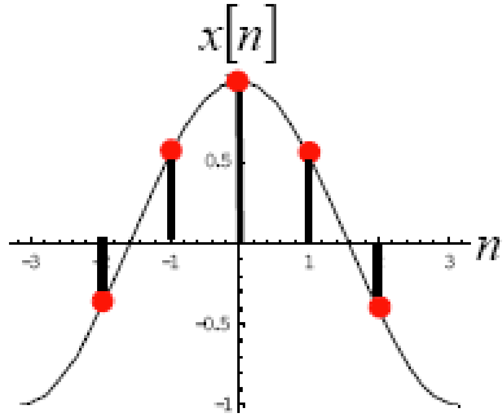
Discrete-time signals: $x[n] = x(nT_s), \quad n = 0, \ \pm 1, \ \pm 2, \ \ldots\ldots$ **where $t = nT_s$**

Brackets [·]

Even and odd signals

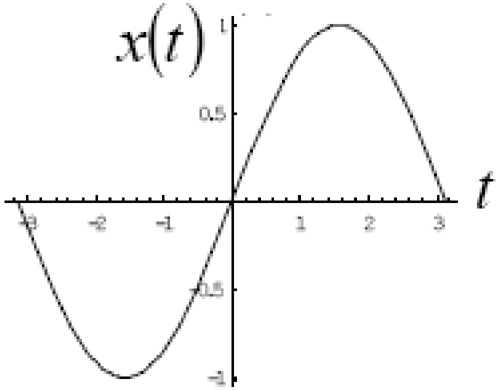
- Even signals: $x(-t)=x(t),\quad x[-n]=x[n]\quad \text{for all}\quad t$

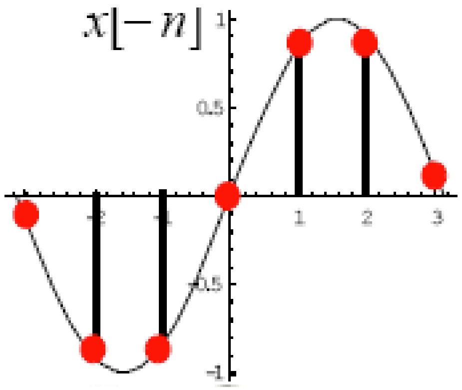




Symmetric **about vertical axis**

- odd signals: $x(-t)=-x(t),\quad x[-n]=-x[n]\quad \text{for all}\quad t$





Antisymmetric **about origin**

Even and odd signals

Example 1.1 **Consider the signal**

$$x(t)=\begin{cases}\sin\left(\dfrac{\pi t}{T}\right), & -T\le t\le T\\ 0\quad, & \text{otherwise}\end{cases}$$

Is the signal $x(t)$ an even or an odd function of time?

<Sol.>

$$x(-t)=\begin{cases}\sin\left(-\dfrac{\pi t}{T}\right), & -T\le t\le T\\ 0\quad, & \text{otherwise}\end{cases}=\begin{cases}-\sin\left(\dfrac{\pi t}{T}\right), & -T\le t\le T\\ 0\quad, & \text{otherwise}\end{cases}$$

$$=-x(t),\quad \text{for all t}$$ ⇒ ***odd function***

Even and odd signals

- **Even-odd decomposition of $x(t)$:**

$$x(t) = x_e(t) + x_o(t)$$

where $x_e(-t) = x_e(t)$

$$x_o(-t) = -x_o(t)$$

$$\Rightarrow x(-t) = x_e(-t) + x_o(-t)$$

$$= x_e(t) - x_o(t)$$

$$\Rightarrow \boxed{x_e(t) = \frac{1}{2}\left[x(t) + x(-t)\right]}$$

$$\boxed{x_o(t) = \frac{1}{2}\left[x(t) - x(-t)\right]}$$

- **Example 1.2** **Find the even and odd components of the signal**

$$x(t) = e^{-2t}\cos t$$

<Sol.>

$$x(-t) = e^{2t}\cos(-t) = e^{2t}\cos t$$

Even component:

$$x_e(t) = \frac{1}{2}(e^{-2t}\cos t + e^{2t}\cos t)$$

$$= \cosh(2t)\ \cos t$$

Odd component: $x_o(t) = \frac{1}{2}(e^{-2t}\cos t - e^{2t}\cos t) = -\sinh(2t)\ \cos t$

Even and odd signals

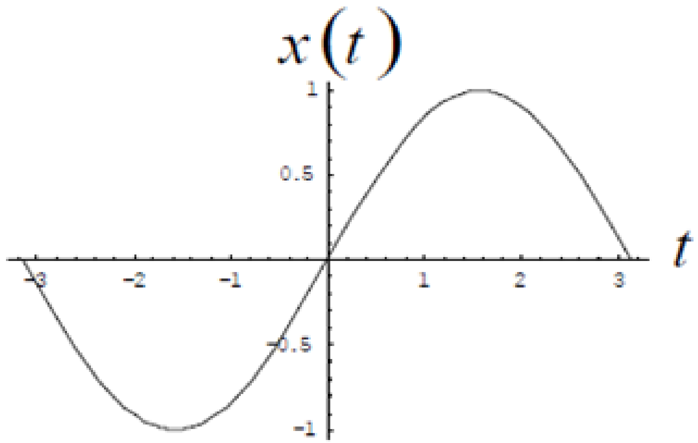
- **PRODUCT rule**

ODD * ODD = EVEN

EVEV * EVEN = EVEN

EVEN * ODD = ODD

ODD * EVEN = ODD



$$s = \int_{-T}^{T} x(t)\,dt = 0, \quad \text{always if } x(t) \text{ is odd.}$$

$$s = \int_{-T}^{T} x(t)dt = 2\int_{0}^{T} x(t)dt, \quad \text{for } x(t) \text{ is even.}$$

Periodic and Aperiodic Signals

- **Continuous-Time Case**
  - **Periodic signals:**

$$x(t) = x(t+T) \quad \forall t, \quad \text{where T is a positive constant.}$$

$$T = T_0, \quad 2T_0, \quad 3T_0, \quad ......$$

- Fundamental period: $T = T_0$
- Fundamental frequency: $f = \dfrac{1}{T}$, measured in hertz(Hz).

cycles per second. how frequent the periodic signal repeats itself.

- Angular frequency: $\omega = 2\pi f = \dfrac{2\pi}{T}$, measured in radians per second (rad/s)

  - **Aperiodic signals:** $\boldsymbol{x(t)}$ where $\boldsymbol{T_0}$ does not exist.

Periodic and Aperiodic Signals

- **Continuous-Time Case**

Example of periodic and nonperiodic signals.

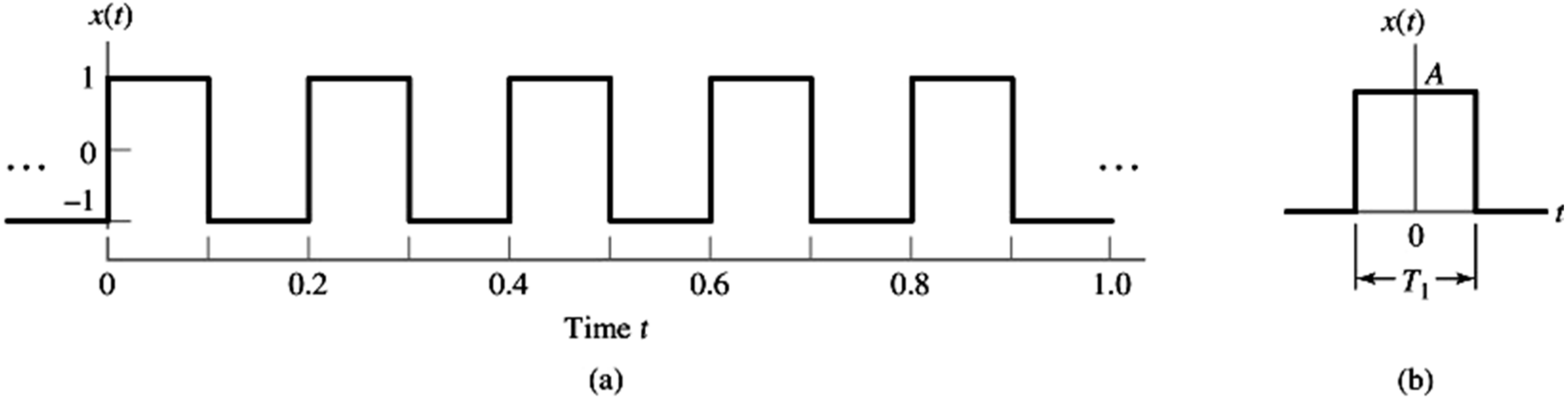


Figure 1.14
(a) Square wave with amplitude $A = 1$ and period $T = 0.2$s.
(b) Rectangular pulse of amplitude A and duration T_1.

$$\boldsymbol{T = 0.2\text{s}} \Rightarrow f = \frac{1}{T} = 5Hz, \quad \omega = \frac{2\pi}{T} = 10\pi \text{ rad/s}$$

Periodic and Aperiodic Signals

- **Discrete-Time Case**
  - **Periodic signals:** $x[n] = x[n+N]$ for integer n
    - Fundamental period: The smallest integer value of *N* for which the periodicity holds
    - Fundamental (angular) frequency: $\Omega = \dfrac{2\pi}{N}$, measured in radians.

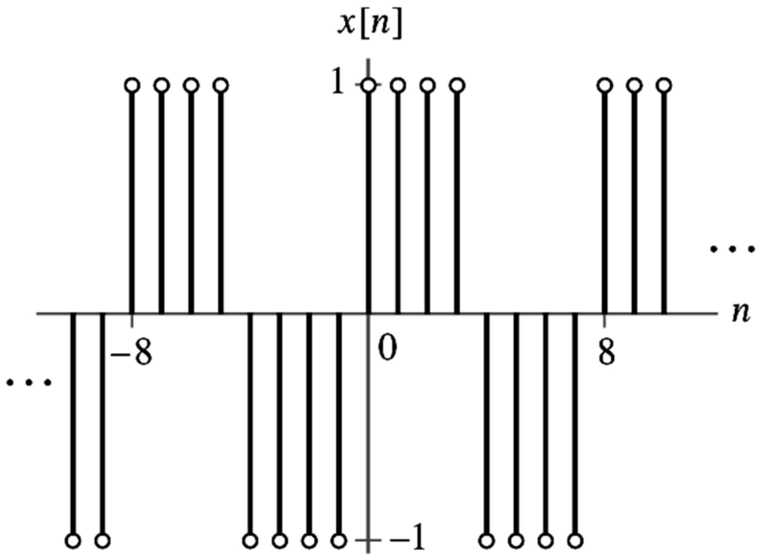


Figure 1.16 Discrete-time square wave alternative between –1 and +1.

$$N = 8 \Rightarrow \Omega = \frac{2\pi}{8} = \frac{\pi}{4} \text{radians.}$$

Periodic and Aperiodic Signals

- **Discrete-Time Case**
  - **Aperiodic signals:**

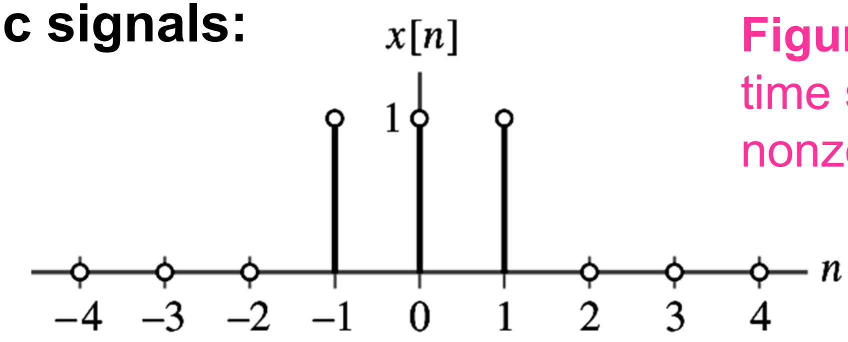


Figure 1.17 Aperiodic discrete-time signal consisting of three nonzero samples.

- **Notes**
  - A sequence obtained by uniform sampling of a periodic continuous-time signal may not be periodic.
  - The sum of two continuous-time periodic signals may not be periodic.
  - The sum of two periodic sequences is always periodic.

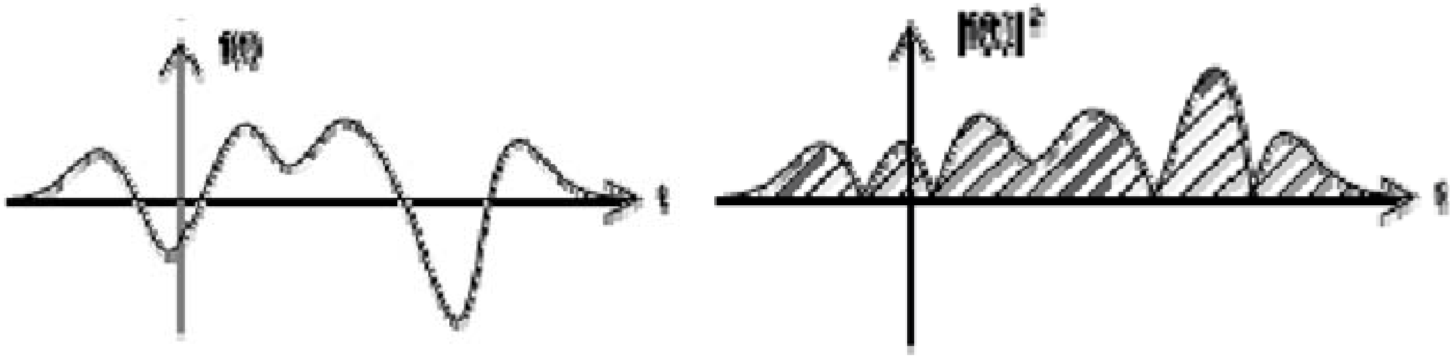
Deterministic signals and random signals

- *Deterministic signals* are those signals whose values are completely specified for any given time. Thus, a deterministic signal can be modeled by a known function of time
- *Random signals* are those signals that take random values at any given time and must be characterized statistically.
- Random signals will not be discussed in this text.

Energy and Power Signals

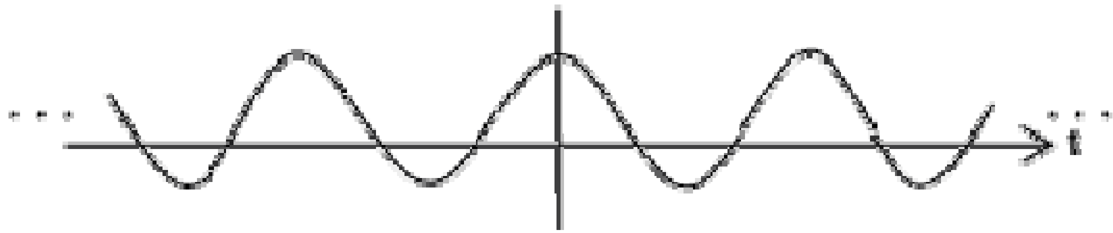
- Strength of a signal: area under the curve.
- ***Energy*** of a signal: area under the squared signal.

$$E = \int_{-\infty}^{\infty} |f(t)|^2 \, dt$$



Energy and Power Signals

- If the signal does not decay: infinite energy



A simple, common signal with infinite energy.

- **Instantaneous power:** $p(t)=\dfrac{v^2(t)}{R}=Ri^2(t)$

If $R=1\Omega$ and $x(t)$ represents a current or a voltage,

$$\boxed{p(t)=x^2(t)}$$

- **Power**: a time average of energy(energy per unit time).

Energy and Power Signals

- **continuous-time signal $x(t)$**
  - **Total energy:**

$$E = \lim_{T\to\infty}\int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t)dt = \int_{-\infty}^{\infty} x^2(t)dt$$

  - **Time-averaged/average power**

$$P = \lim_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t)dt$$

  - **For periodic signal,**

$$P = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t)dt$$

- **discrete-time signal $x(n)$**
  - **Total energy:**

$$E = \sum_{n=-\infty}^{\infty} x^2[n]$$

  - **average power**

$$P = \lim_{n\to\infty}\frac{1}{2N}\sum_{n=-N}^{N} x^2[n]$$

  - **For periodic signal,**

$$P = \frac{1}{N}\sum_{n=0}^{N-1} x^2[n]$$

Energy and Power Signals

- **Energy signal**

  If and only if the total energy of the signal satisfies the condition

  $$0 < E < \infty$$

- **Power signal**

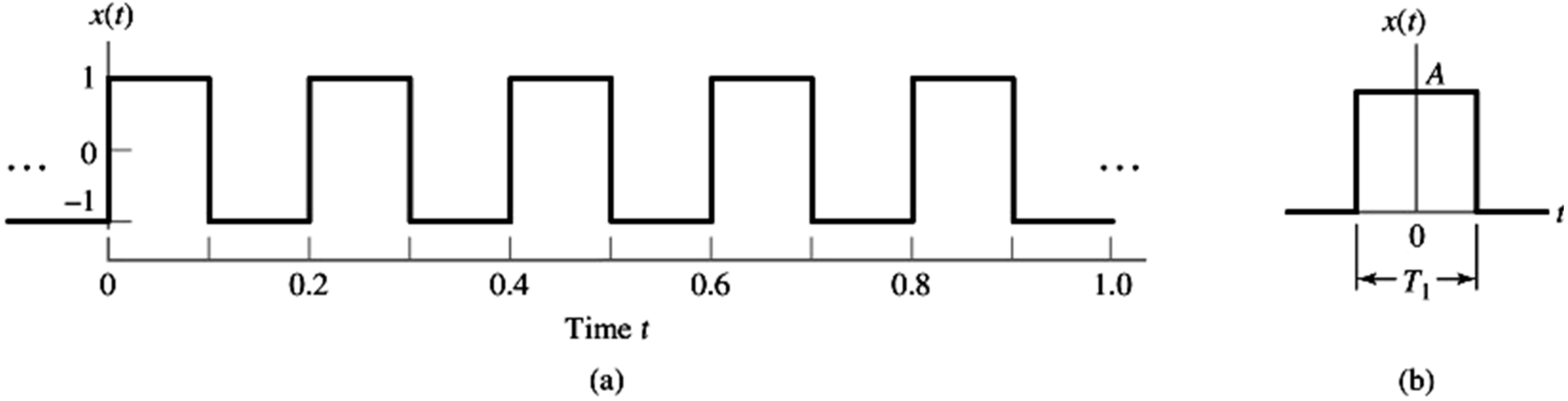
  If and only if the average power of the signal satisfies the condition

  $$0 < P < \infty$$

- Energy signal has zero time-average power (why?)
- Power signal has infinite energy (why?)
- Energy signal and power signal are mutually exclusive.
- Periodic signal and random signal are usually viewed as power signal.
- Nonperiodic and deterministic signal are usually viewed as energy signal.

Energy and Power Signals

Problem 1.6



- **(a) square wave: power signal**

$$P = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x^2(t)dt \ = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} 1^2 dt = 1$$

- **(b) rectangular pulse: energy signal**

$$E = \int_{-\infty}^{\infty} x^2(t)dt \ = \int_{-\frac{T_1}{2}}^{\frac{T_1}{2}} A^2 dt = A^2 T_1$$

Orthogonality (正交性)

- Orthogonality is fundamental to almost everything that is subsequent in signals and systems theory.
  - For discrete signals, if the product of two signals averages to zero over the period T, then those two signals are ORTHOGONAL in that interval (T).

$$\frac{1}{T}\sum_{n=0}^{N} x_1[n]x_2[n]=0$$

  - For continuous signals, if the product of two signals integrates to zero over the period T, then those two signals are ORTHOGONAL in that interval (T).

$$\int_0^T x_1(t)x_2(t)dt=0$$

Operation on Signals

- An issue of fundamental importance in the signals and systems is the use of systems to process or manipulate signals. This issue usually involves a combination of some basic operations in signals.
  - Three transformation in amplitude
    - ***Amplitude scaling***
    - ***Addition***
    - ***Multiplication***
  - Three transformations in time domain
    - ***Time Scaling*** **(尺度变换)**
    - ***Time Reflection*** **(折叠)**
    - ***Time Shifting*** **(时移)**

Transformation in Amplitude

- ***Amplitude scaling***: $y(t) = cx(t)$ c: **scaling factor**

  $$y[n] = cx[n]$$

  - Performed by amplifier or resistor
- ***Addition***: $y(t) = x_1(t) + x_2(t)$

  $$y[n] = x_1[n] + x_2[n]$$

  - E.g. audio mixer
- ***Multiplication***: $y(t) = x_1(t)x_2(t)$

  $$y[n] = x_1[n]x_2[n]$$

  - E.g. AM radio signal

Transformation in Amplitude

- **Differentiation**: $y(t) = \frac{d}{dt} x(t)$
  - E.g. inductor $v(t) = L \frac{d}{dt} i(t)$
- **Integration**: $y(t) = \int_{-\infty}^{t} x(\tau) d\tau$
  - E.g. capacitor $v(t) = \frac{1}{C} \int_{-\infty}^{t} i(\tau) d\tau$

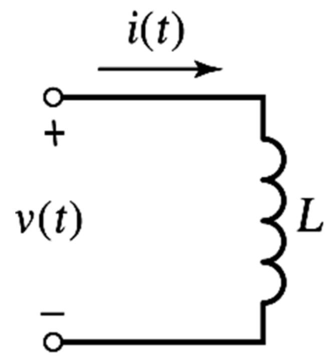


Figure 1.18 Inductor

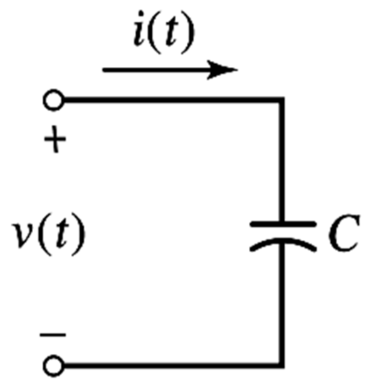


Figure 1.19 Capacitor

Time Scaling

- **Continuous-Time Case**

$$y(t) = x(at), \quad a > 0 \quad \Rightarrow \quad \begin{cases} a > 1: \text{ compressed} \\ 0 < a < 1: \text{ expanded} \end{cases}$$

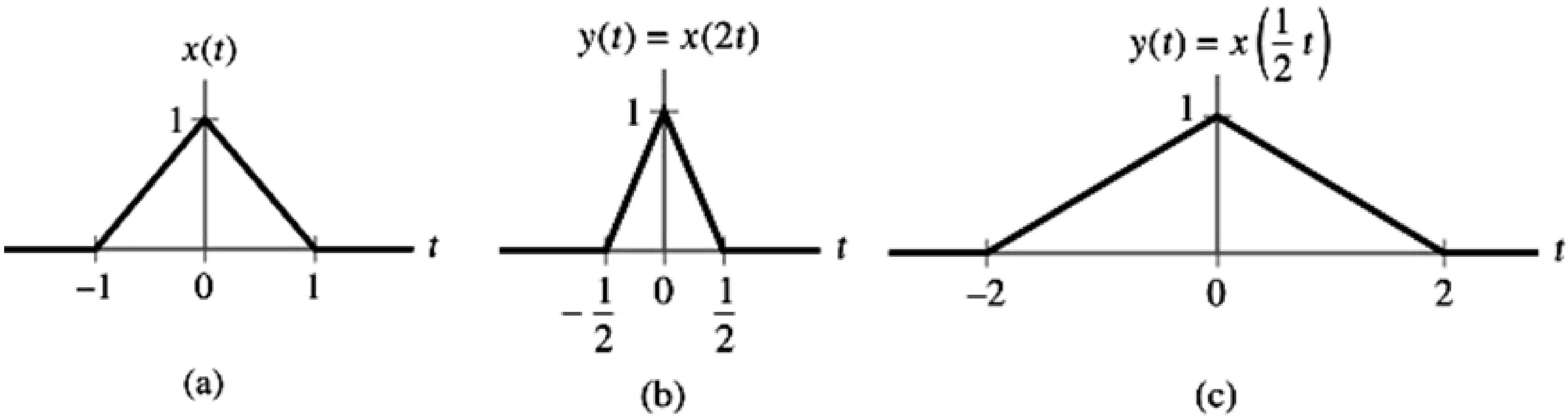


Figure 1.20
Time-scaling operation; (a) continuous-time signal $x(t)$, (b) version of $x(t)$ compressed by a factor of 2, and (c) version of $x(t)$ expanded by a factor of 2.

Time Scaling

- **Discrete-Time Case**

$$y[n] = x[kn], \quad k > 0$$

***k* = integer** ⇒ **Some values lost!**

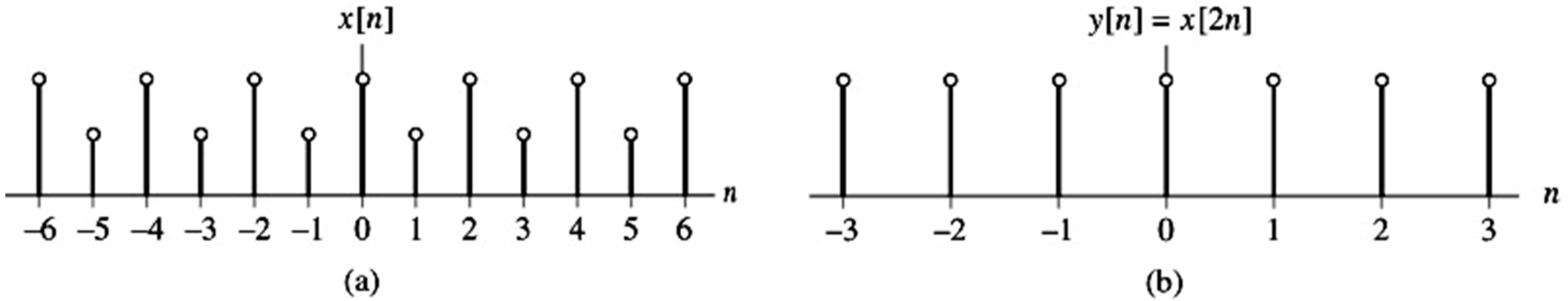


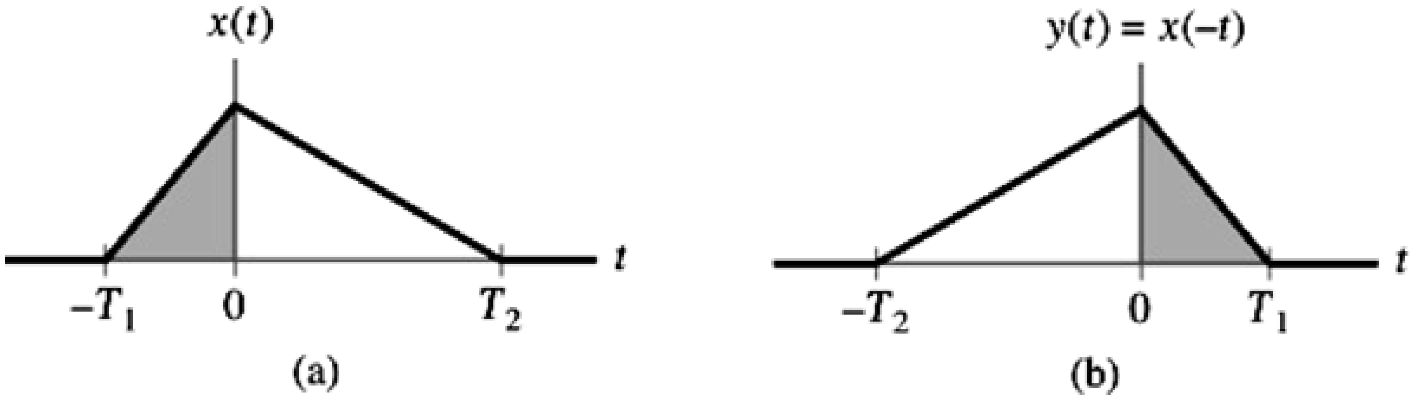
Figure 1.21
Effect of time scaling on a discrete-time signal: (a) discrete-time signal $x[n]$ and (b) version of $x[n]$ compressed by a factor of 2, with some values of the original $x[n]$ lost as a result of the compression.

Time Reflection

$y(t) = x(-t)$ → **$y(t)$ represents a reflected version of $x(t)$ about $t = 0$.**

- An even signal is the same as its reflected version: $x(-t) = x(t)$
- An odd signal is the negative of its reflected version: $x(-t) = -x(t)$

Ex. 1-3 Consider the triangular pulse x(t) shown in Fig. 1-22(a). Find the reflected version of x(t) about the amplitude axis (i.e., the origin).



$x(t) = 0 \quad \text{for} \quad t < -T_1 \quad \text{and} \quad t > T_2$ → $y(t) = 0 \quad \text{for} \quad t > T_1 \quad \text{and} \quad t < -T_2$

Time Shifting

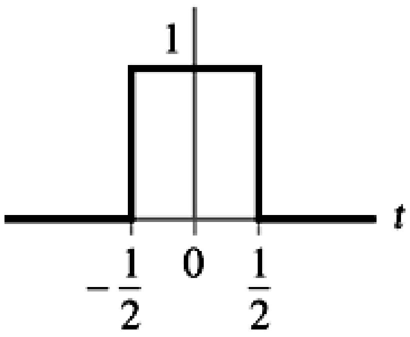
- **Continuous-Time Case**

$$y(t) = x(t - t_0)$$

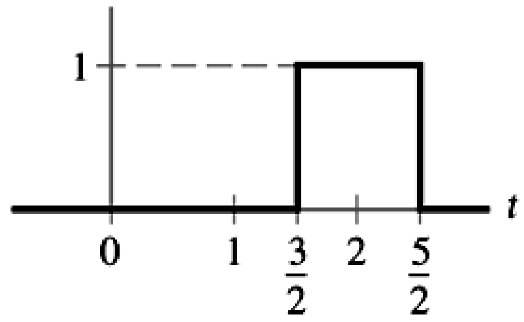
t_0 > 0: shift toward right

t_0 < 0: shift toward left

Ex.1-4 $y(t) = x\,(t - 2)$



(a)



(b)

- **Discrete-Time Case**

$$y[n] = x[n - m]$$ **where *m* is a positive or negative integer**

Precedence Rule for Time Shifting and Time Scaling

- **Combination of time shifting and time scaling**

$$y(t) = x(at - b) \Rightarrow \begin{cases} y(0) = x(-b) \\ y\left(\dfrac{b}{a}\right) = x(0) \end{cases}$$

- Operation order :

  1st step: time shifting $v(t) = x(t - b)$

  2nd step: time scaling $y(t) = v(at) = x(at - b)$

  or

  1st step: time scaling $v(t) = x(at)$

  2nd step: time shifting

$$y(t) = v(t - \frac{b}{a}) = x(a(t - \frac{b}{a})) = x(at - b)$$

Precedence Rule for Time Shifting and Time Scaling

Ex. 1-5. Consider the rectangular pulse $x(t)$ in Fig. 1-24(a). Find

$$y(t) = x(2t + 3).$$

<Sol.> Case 1: Shifting first, then scaling

$$v(t) = x(t+3)$$

$$y(t) = v(2t) = x(2t+3)$$

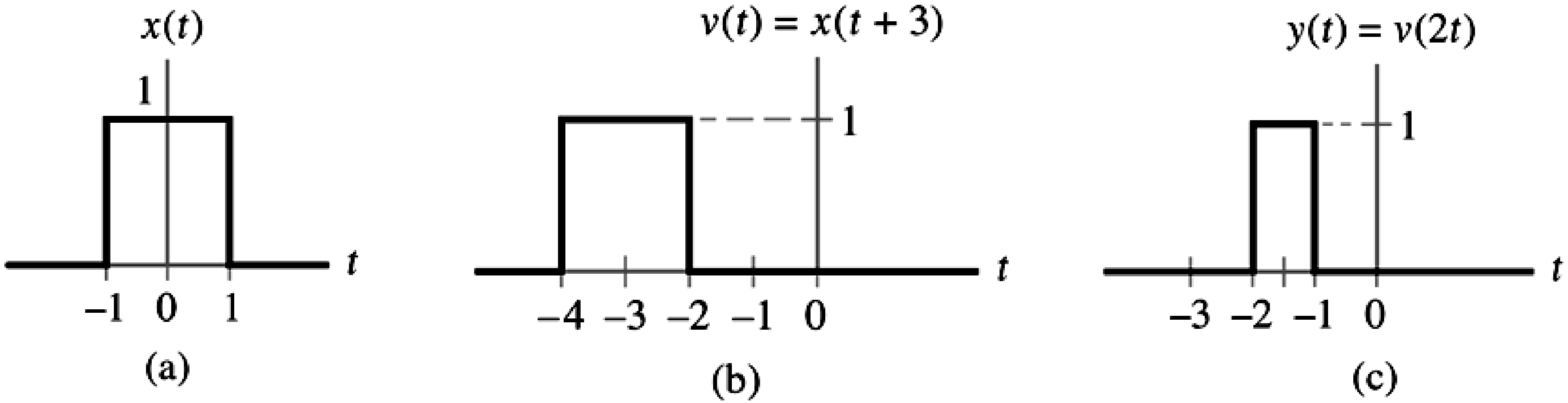


Figure 1.24 The proper order of time scaling and time shifting operations

Precedence Rule for Time Shifting and Time Scaling

Case 2: Scaling first, then shifting

$$v(t) = x(2t)$$

$$y(t) = v(t+3) = x\left(2\left(t+3\right)\right) \neq x\left(2t+3\right)$$

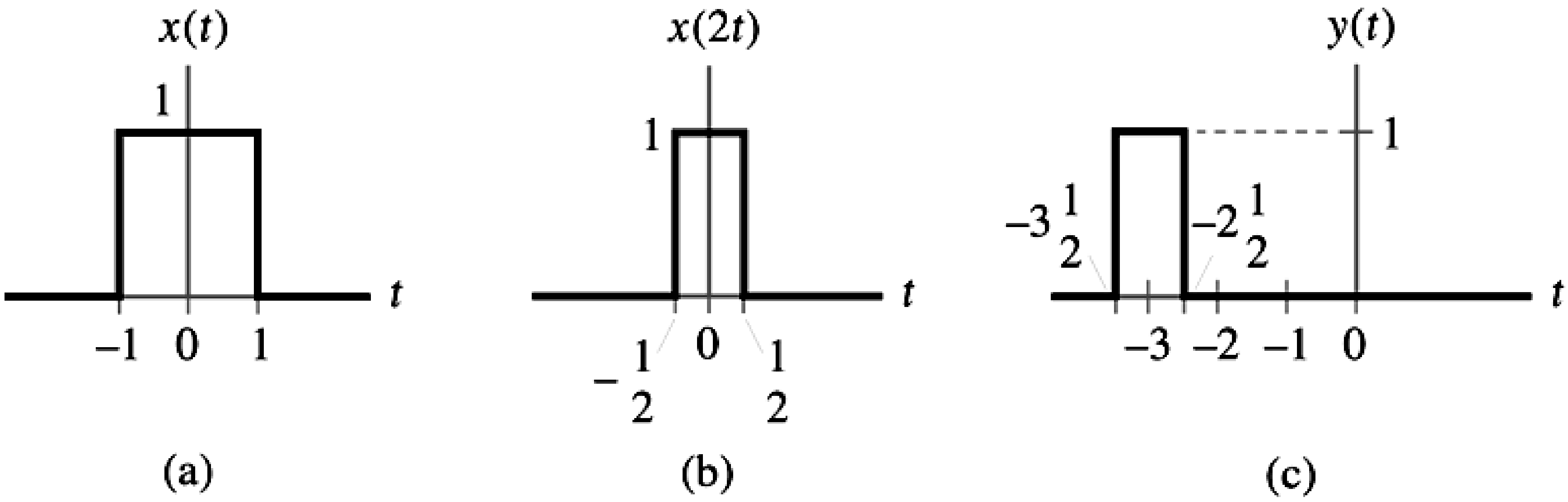


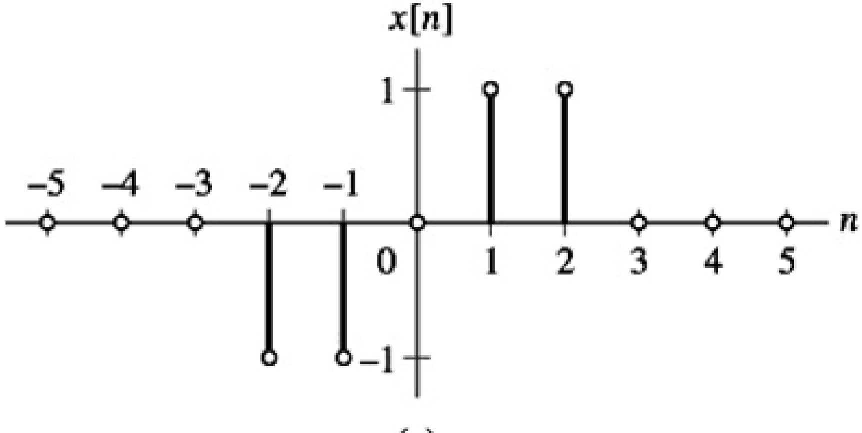
Figure 1.25 The incorrect way of applying the precedence rule.

Precedence Rule for Time Shifting and Time Scaling

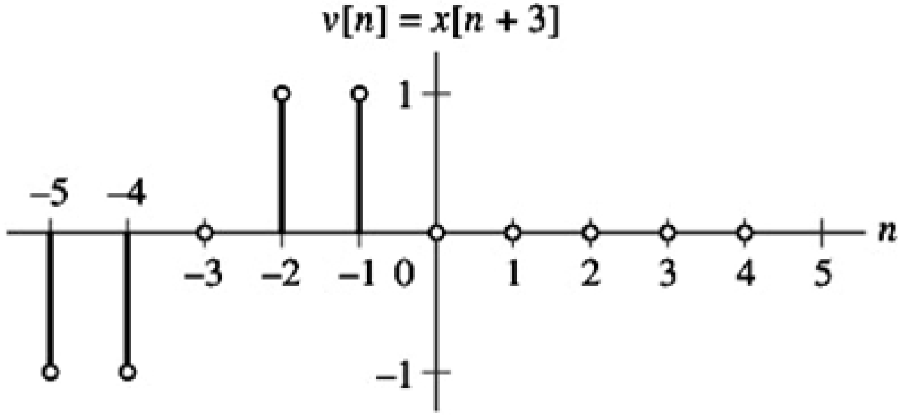
Ex. 1-6 A discrete-time signal is defined by

$$x[n]=\begin{cases}1, & n=1,2\\ -1, & n=-1,-2\\ 0, & n=0 \text{ and } |n|>2\end{cases}$$

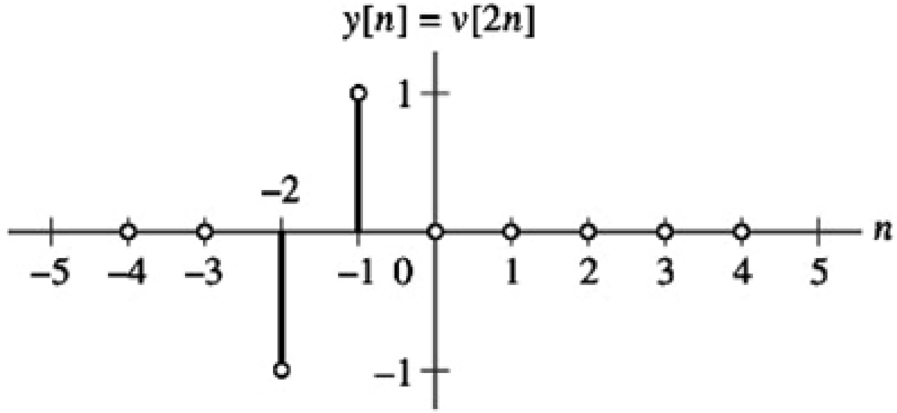
Find $y[n]=x[2n+3]$.



(a)



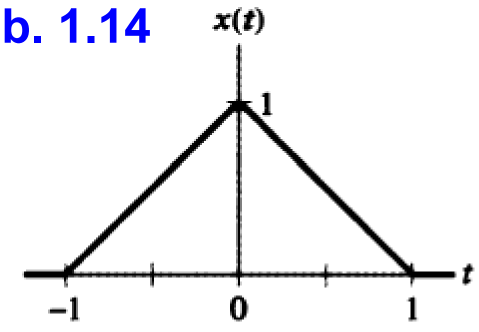
(b)



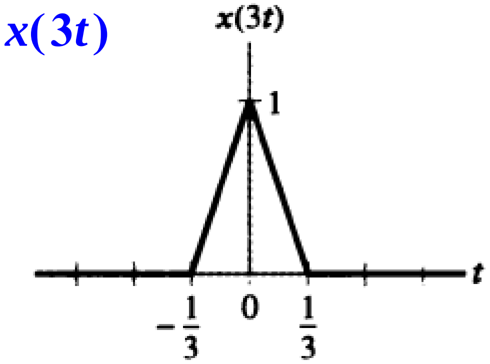
(c)

Precedence Rule for Time Shifting and Time Scaling

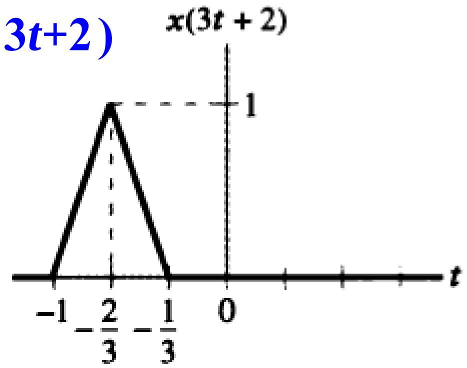
Prob. 1.14



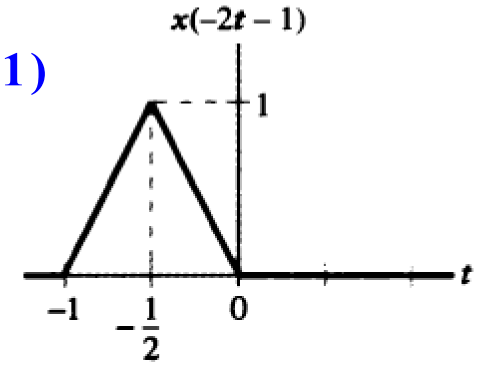
a) $x(3t)$



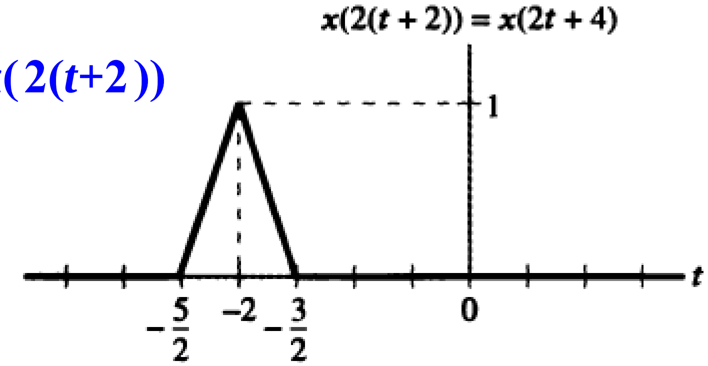
b) $x(3t+2)$



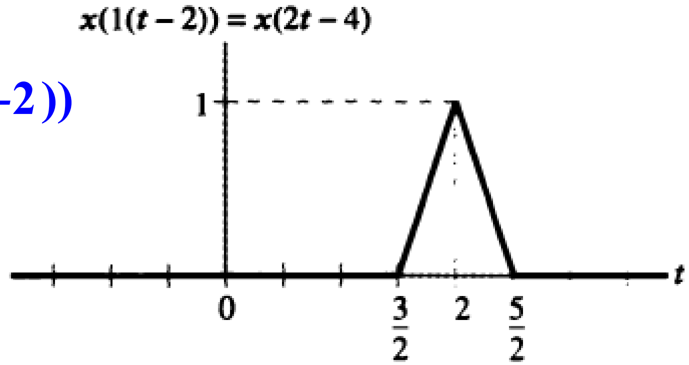
c) $x(-2t-1)$



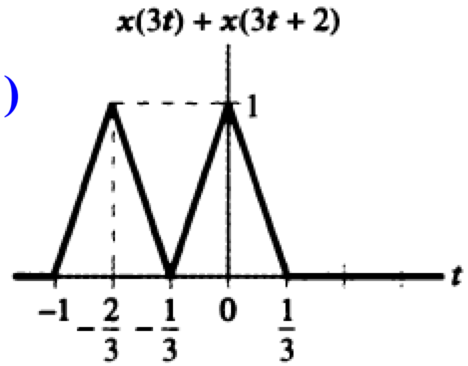
d) $x(2(t+2))$



e) $x(2(t-2))$



f) $x(3t) + x(3t+2)$



Summary

- Signals and systems introduction
- Overview of our course
- Classification of signals
- Operation on Signals

- Reference in textbook:
  - 1.1,1.2,1.4,1.5
  - 1.3 (optional)
- Homework: 1.42, 1.44, 1.46, 1.51, 1.56