BBC4111 A

Joint Programme Examinations 2023/24

BBC4111 Engineering Mathematics

Paper A

Time allowed 2 hours

Answer ALL questions

For examiners' use only

1	
2	
3	
4	
5	
6	
7	
8	
Total	

Complete the information below about yourself very carefully.

QM student number										
BUPT student number										
Class number										

NOT allowed: electronic calculators and electronic dictionaries.

INSTRUCTIONS

1. **You must NOT take answer books, used or unused, from the examination room.**
2. Write only with a black or blue pen **and in English**.
3. Do all rough work in the answer book – **do not tear out any pages**.
4. If you use Supplementary Answer Books, tie them to the end of this book.
5. Write clearly and legibly.
6. **Read the instructions on the inside cover.**

Examiners

Dr Ting Mei

Filename: 2324_ BBC4111_A No answer book required

Instructions

Before the start of the examination

1) Place your BUPT and QM student cards on the corner of your desk so that your picture is visible.

2) Put all bags, coats and other belongings at the back/front of the room. All small items in your pockets, including wallets, mobile phones and other electronic devices must be **placed in your bag in advance**. **Possession of mobile phones, electronic devices and unauthorised materials is an offence.**

3) Please ensure your mobile phone is switched off and that no alarm will sound during the exam. **A mobile phone causing a disruption is also an assessment offence.**

4) Do not turn over your question paper or begin writing until told to do.

During the examination

1) You must not communicate with or copy from another student.

2) If you require any assistance or wish to leave the examination room for any reason, please raise your hand to attract the attention of the invigilator.

3) If you finish the examination early you may leave, but not in the first 30 minutes or the last 10 minutes.

4) For 2 hour examinations you may **not** leave temporarily.

5) For examinations longer than 2 hours you **may** leave temporarily but not in the first 2 hours or the last 30 minutes.

At the end of the examination

1) You must stop writing immediately – **if you continue writing after being told to stop, that is an assessment offence.**

2) Remain in your seat until you are told you may leave.

Question1. [30 marks total, 3 marks for each blank]

Fill in all the following blanks. Only the final results are required to be written down.

a) The principal root of $(-2+2\sqrt{3}\mathrm{i})^{\frac{1}{4}}$ is (   ).

b) Let x, y be real numbers. If the function $f(z)=(x^3-3xy^2)+\mathrm{i}v(x,y)$ is analytic, then $f'(z)=$ (   ).

c) $\left[\frac{1}{\sqrt{2}}(1-i)\right]^{-i}$ = (   ).

d) Suppose $f(z)=\int\limits_{|s|=2}\frac{s^3-2s^2-1}{(s-z)^3}\mathrm{d}s,$ then $f'(1)=$ (   ).

e) $\operatorname*{Res}\limits_{z=0} z^2\sin\frac{1}{z}=$ (   ).

f) The solution of the initial problem $\begin{cases} u_{tt}-4u_{xx}=0, & -\infty<x<+\infty,\ t>0, \\ u(x,0)=x^2-x, & -\infty<x<+\infty, \\ u_t(x,0)=\sin x, & -\infty<x<+\infty, \end{cases}$ is (   ).

g) The eigenvalues of the eigenvalue problem $\begin{cases} u''(x)+\lambda u(x)=0, 0<x<1, \\ u(0)=u'(1)=0, \end{cases}$ are (   ), and the corresponding eigenfunctions are (   ).

h) Let $P_n(x)$ be the Legendre polynomial of degree n, then $\int_{-1}^{1}(x^4-2x^3+x)P_3(x)\mathrm{d}x=$ (   ).

i) Suppose that $\mathcal{F}[f(x)]=F(\lambda)$, where $\mathcal{F}[f(x)]$ is the Fourier integral transformation of $f(x)$, then for any constant $a,b\in R, a>0$, $\mathcal{F}[f(ax+b)]=$ (   ).

	Do not write in this column
a)	
b)	
c)	
d)	
e)	
f)	
g)	
h)	
i)	
	30 marks

Question 2. [10 marks]

Please determine whether the following statements are true. Put "T" if the statement is true or "F" if it's wrong.

a) The function $f(z) = \mathrm{Log}(z - 2i)$ is analytic in the domain $\{(x, y) : x > 0, y = 2\}$. ( )

b) If $f(z)$ is analytic at the point z_0, then $f(z)$ is analytic in some neighbourhood of z_0. ( )

c) Suppose $\sum_{n=1}^{\infty} c_n$ converges and $\sum_{n=1}^{\infty} |c_n|$ diverges, then the radius of convergence of the power series $\sum_{n=1}^{\infty} c_n z^n$ is $R = 1$. ( )

d) The general solution of the Legendre equation $(1 - x^2)y''(x) - 2xy'(x) + 6y(x) = 0$ is $y(x) = C_1 P_2(x) + C_2 Q_2(x)$. ( )

e) Let $J_v(x)$ be the first kind of Bessel function of order v. Then for all v, $J_v(x)$ have finite values at $x = 0$. ( )

	Do not write in this column
a)	
b)	
c)	
d)	
e)	
	10 marks

Question 3. [12 marks]

Find the Laurent series expansions for the function $f(z) = \dfrac{2z-1}{z(z+1)}$ in the following annular domains

a) $1 < |z| < \infty$; b) $1 < |z-1| < 2$.

	Do not write in this column
	12 marks

Question 4. [12 marks]

Suppose the function $f(z)=\dfrac{e^z}{z^2(z^2+1)}$, then

a) find out all the singular points of $f(z)$, and point out their types;

b) evaluate the residues of $f(z)$ at those singular points;

c) evaluate the integral $\displaystyle\oint_{|z-i|=\frac{3}{2}} f(z)\,dz$.

	Do not write in this column
	12 marks

Question 5. [8 marks]

Evaluate the integral $I = \int_0^{+\infty} \frac{1}{(x^2+1)(x^2+4)} \mathrm{d}x.$

	Do not write in this column

	8 marks

Question 6. [8 marks]

Determine the type of the linear partial differential equation $y^2 u_{xx} + 4xyu_{xy} + 4x^2 u_{yy} = x^2 y$ and reduce it to the normal type.

	Do not write in this column
	8 marks

Question 7. [12 marks]

Solve the following problem by means of separation of variables:

$$\begin{cases} u_t = 4u_{xx}, & 0 < x < \pi, t > 0, \\ u_x(0,t) = u_x(\pi,t) = 0, & t \geq 0, \\ u(x,0) = \cos 2x - 3\cos x, & 0 \leq x \leq \pi. \end{cases}$$

	Do not write in this column
	12 marks

Question 8. [8 marks]

Use Laplace transformation to solve the ordinary differential equation

$$\begin{cases} x'''(t)+2x''(t)-x'(t)-2x(t)=1, \\ x(0)=x'(0)=x''(0)=0. \end{cases}$$

	Do not write in this column
	8 marks

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2023-2024
Rough Working
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