第五次作业答案

一、3.50(a,b,d,e)

3.50. Use the defining equation for the FS coefficients to evaluate the FS representation for the following signals.

(a) $T_1 = \frac{2}{3}$, $T_2 = \frac{1}{2}$, $T = lcm(T_1, T_2) = 2$, $\omega_o = \pi$

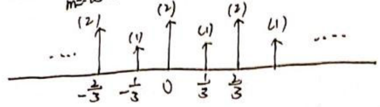
lcm is the least common multiple.

$$\begin{aligned} x(t) &= \sin(3\pi t) + \cos(4\pi t) \\ &= \frac{1}{2j}e^{j(3)\pi t} - \frac{1}{2j}e^{j(-3)\pi t} + \frac{1}{2}e^{j(4)\pi t} + \frac{1}{2}e^{j(-4)\pi t} \end{aligned}$$

By inspection

$$X[k] = \begin{cases} \frac{1}{2} & k = \pm 4 \\ \frac{1}{2j} & k = 3 \\ \frac{-1}{2j} & k = -3 \\ 0 & \text{otherwise} \end{cases}$$

(b) 解: $x(t)=\sum_{m=-\infty}^{+\infty}\left(\delta(t-\frac{m}{3})+\delta(t+\frac{2m}{3})\right)$



$\therefore T=\frac{2}{3}, \quad w_0=\frac{2\pi}{T}=3\pi$

$\therefore X[k]=\frac{1}{T}\int_0^T x(t)\,e^{-jkw_0t}\,dt$

$=\frac{3}{2}\int_0^{(\frac{2}{3})^-}\left(2\cdot\delta(t)+\delta(t-\frac{1}{3})\right)e^{-jkw_0t}\,dt$

$=\frac{3}{2}\int_0^{(\frac{2}{3})^-}2\cdot\delta(t)+e^{-jkw_0\frac{1}{3}}\delta(t-\frac{1}{3})\,dt$

$=\frac{3}{2}(2+e^{-jk\pi})$

$=3+\frac{3}{2}\cos k\pi$

注:

①画图困难时,可先画子图再合并.
如此题可先分别画 $\sum_{m=-\infty}^{+\infty}\delta(t-\frac{m}{3})$ 与 $\sum\delta(t+\frac{2m}{3})$ 的图再合并.

②一个周期的区间一定是一边开一边闭的,
如此题周期区间为 $[0+\frac{2}{3}k, \frac{2}{3}+\frac{2}{3}k)$

$\therefore$ 一个周期只包含两个冲激函数.

(d) 解: $x(t)=|\sin\pi t|$

$\therefore T=1, \quad w_0=2\pi$

$\therefore X[k]=\frac{1}{T}\int_0^T x(t)e^{-jkw_0t}\,dt$

$=\int_0^1 \sin\pi t\, e^{-jkw_0t}\,dt$

$=\frac{1}{2j}\int_0^1\left(e^{j\pi t}-e^{-j\pi t}\right)e^{-jkw_0t}\,dt$

$=\frac{1}{2j}\left\{\left[\frac{e^{j\pi t}\cdot e^{-jk\cdot2\pi t}}{(j\pi-jk\cdot2\pi)}\right]\Bigg|_0^1-\left[\frac{e^{-j\pi t-jkw_0t}}{-j\pi-jk\cdot2\pi}\right]\Bigg|_0^1\right\}$

$=\frac{1}{2j}\left\{\frac{e^{j\pi}e^{-jk\cdot2\pi}-1}{j\pi(1-2k)}+\frac{e^{-j\pi}\cdot e^{-jk2\pi}-1}{j\pi(1+2k)}\right\}$

$\cancel{=\frac{1}{2j}\left\{\frac{\ }{j\pi(1-2k)}\right.}=\frac{-1\times1-1}{2\pi(2k-1)}-\frac{-1\times1-1}{2\pi(2k+1)}$

$=\frac{1}{\pi(2k+1)}-\frac{1}{\pi(2k-1)}$

$=\frac{-2}{\pi(4k^2-1)}$

(e) $x(t)$ as depicted in Figure P3.50(b)

$T=2,\ \omega_o=\pi$

$$\begin{aligned}X[k] &= \frac{1}{2}\int_0^1 e^{-t}e^{-jk\pi t}\,dt\\ &= \frac{1}{2}\int_0^1 e^{-t(1+jk\pi)}\,dt\\ &= \frac{1-e^{-1(1+jk\pi)}}{2(j\pi k+1)}\end{aligned}$$

二、3.51(b,d,e)

(b) $X[k] = j\delta[k-1] - j\delta[k+1] + \delta[k-3] + \delta[k+3], \quad \omega_o = 4\pi$

$$\begin{aligned} x(t) &= je^{j(1)4\pi t} - je^{j(-1)4\pi t} + e^{j(3)4\pi t} + e^{j(-3)4\pi t} \\ &= -2\sin(4\pi t) + 2\cos(12\pi t) \end{aligned}$$

(d) $X[k]$ as depicted in Figure P3.51(a).

$\omega_o = \pi$

$$\begin{aligned} x(t) &= \sum_{m=-\infty}^{\infty} X[k]e^{j\pi kt} \\ &= 2e^{-j0.25\pi}e^{j(-4)\pi t} + e^{j0.25\pi}e^{j(-3)\pi t} + e^{-j0.25\pi}e^{j(3)\pi t} \\ &\quad +2e^{j0.25\pi}e^{j(4)\pi t} \\ &= 4\cos(4\pi t + 0.25\pi) + 2\cos(3\pi t - 0.25\pi) \end{aligned}$$

(e) $X[k]$ as depicted in Figure P3.51(b).

$\omega_o = 2\pi$.

$$X[k] = e^{-j2\pi k} \qquad -4 \le k < 4$$

$$\begin{aligned} x(t) &= \sum_{m=-4}^{4} e^{j2\pi k(t-1)} \\ &= \frac{\sin(9\pi t)}{\sin(\pi t)} \end{aligned}$$

三、3.48(a,c,e)

3.48. Use the defining equation for the DTFS coefficients to evaluate the DTFS representation for the following signals.

(a) $N = 17$, $\Omega_o = \frac{2\pi}{17}$

$$\begin{aligned} x[n] &= \cos(\frac{6\pi}{17}n + \frac{\pi}{3}) \\ &= \frac{1}{2}\left(e^{j(\frac{6\pi}{17}n+\frac{\pi}{3})} + e^{-j(\frac{6\pi}{17}n+\frac{\pi}{3})}\right) \\ &= \frac{1}{2}\left(e^{j\frac{\pi}{3}}e^{j(3)\frac{2\pi}{17}n} + e^{-j\frac{\pi}{3}}e^{j(-3)\frac{2\pi}{17}n}\right) \end{aligned}$$

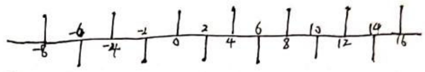
By inspection

$$X[k] = \begin{cases} \frac{1}{2}e^{j\frac{\pi}{3}} & k = 3 \\ \frac{1}{2}e^{-j\frac{\pi}{3}} & k = -3 \\ 0 & \text{otherwise on } k = \{-8, -7, ..., 8\} \end{cases}$$

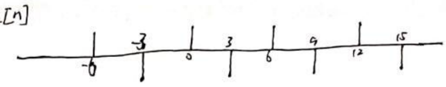
（c）答案有误

3.48 (c) 解: $x[n]=\sum_{m=-\infty}^{\infty}[(-1)^m(\delta[n-2m]+\delta[n+3m])]$

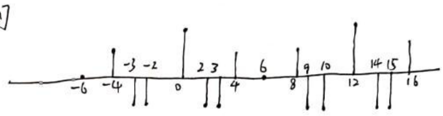
画图: $\sum_m(-1)^m\delta(n-2m)=x_1[n]$



$\sum_m(-1)^m\delta(n+3m)=x_2[n]$



$x[n]=x_1[n]+x_2[n]$



$\therefore x[n]: T=12,\ \omega_0=\frac{\pi}{6}$

$\therefore X[k]=\frac{1}{12}\sum_{n=-5}^{6}x[n]\cdot e^{-jk\frac{\pi}{6}n}$

$=\frac{1}{12}[e^{-jk\frac{\pi}{6}\cdot(-4)}-e^{j\frac{\pi}{2}k}-e^{j\frac{\pi}{3}k}+2\cdot e^{-jk\frac{\pi}{6}\cdot 0}-e^{-j\frac{\pi}{3}k}-e^{-j\frac{\pi}{2}k}+e^{-\frac{2\pi}{3}k\cdot j})$

$=\frac{1}{12}[2\cos(\frac{2}{3}\pi k)-2\cos\frac{\pi}{2}k-2\cos\frac{\pi}{3}k+2]$

$=\frac{1}{6}\cos(\frac{2}{3}\pi k)-\frac{1}{6}\cos(\frac{2}{3}\pi k)-\frac{1}{6}\cos(\frac{\pi}{3}k)+\frac{1}{6})$

(e) $x[n]$ as depicted in Figure P3.48(b)

N = 10, $\Omega_o=\frac{\pi}{5}$

$$
\begin{aligned}
X[k] &= \frac{1}{10}\sum_{n=-5}^{4}x[n]e^{-jk\frac{\pi}{5}n} \\
&= \frac{1}{10}\left[\frac{1}{4}e^{-j(-4)\frac{\pi}{5}k}+\frac{1}{2}e^{-j(-3)\frac{\pi}{5}k}+\frac{3}{4}e^{-j(-2)\frac{\pi}{5}k}+e^{-j(-1)\frac{\pi}{5}k}\right] \qquad k\in\{-5,-4,...,4\}
\end{aligned}
$$

四、3.49(b,d,e)

(b) $N=19$, $\Omega_o=\frac{2\pi}{19}$

$$
\begin{aligned}
X[k] &= \cos(\frac{10\pi}{19}k)+2j\sin(\frac{4\pi}{19}k) \\
&= \frac{1}{2}[e^{-j(-5)\frac{2\pi}{19}k}+e^{-j(5)\frac{2\pi}{19}k}]+e^{-j(-2)\frac{2\pi}{19}k}-e^{-j(2)\frac{2\pi}{19}k}
\end{aligned}
$$

By inspection

$$
x[n]=\begin{cases}\frac{19}{2} & n=\pm5\\ -19 & n=2\\ 19 & n=-2\\ 0 & \text{otherwise on } n\in\{-9-8,...,9\}\end{cases}
$$

3.49 (d) 解：由图P3.49(a) 得：

$N=14$ $\Omega_0=\frac{\pi}{7}$

$$\therefore x[n]=\sum_{k=-6}^{7} X[k]\cdot e^{jk\frac{\pi}{7}n}$$

$$=\underbrace{e^{-j\frac{\pi}{2}}}_{(-j)}e^{j(-4)\frac{\pi}{7}n}+e^{j\frac{\pi}{2}}\cdot e^{j(-3)\frac{\pi}{7}n}+e^{j\frac{\pi}{2}}e^{j(3)\frac{\pi}{7}n}+e^{-j\frac{\pi}{2}}\cdot e^{j(4)\frac{\pi}{7}n}$$

$$=-j\left(e^{j-\frac{4}{7}\pi n}+e^{j\frac{4\pi}{7}n}\right)+j\left(e^{j-\frac{3\pi}{7}n}+e^{j\frac{3\pi}{7}n}\right)$$

$$=-2j\cos\left(\frac{4}{7}\pi n\right)+2j\cos\left(\frac{3\pi}{7}n\right)$$

(e) $X[k]$ as depicted in Figure P3.49(b)
$N=7$, $\Omega_o=\frac{2\pi}{7}$

$$\begin{aligned} x[n] &= \sum_{k=-3}^{3} X[k]e^{jk\frac{2\pi}{7}n} \\ &= e^{j(-1)\frac{2\pi}{7}n}+e^{j(1)\frac{2\pi}{7}n}-\frac{1}{2} \\ &= 2\cos\left(\frac{2\pi}{7}n\right)-\frac{1}{2} \end{aligned}$$