1. Complex numbers

$z = x+iy = re^{i\theta} = r(\cos\theta + i\sin\theta)$

$r = |z|$ $\quad \theta = \arg z = \underline{\operatorname{Arg} z} + 2k\pi$, $k = 0, \pm 1, \cdots$

$z^n = (re^{i\theta})^n = r^n e^{in\theta}$

$\underline{z^{\frac{1}{n}}} = r^{\frac{1}{n}} e^{i\frac{\theta + 2k\pi}{n}}$, $k = 0, 1, \cdots, n-1$.

2. Analytic functions.

Derivatives. (definition, computation)

Conditions for differentiability: C-R equations

$w = f(z) = \underline{u(x,y)} + i\underline{v(x,y)}$

Analytic function.

Elementary functions. — $e^z = e^x \cdot e^{iy}$

$\underline{\operatorname{Log} z} = \ln|z| + i\arg z$. $\quad \operatorname{Log} z$

$\sin z$, $\cos z$, z^c.

Harmonic function — $u(x,y)$

$f(z) = u(x,y) + iv(x,y)$ — analytic.

harmonic conjugate.

3. Integral. $\quad C: z = z(t), a \le t \le b$.

Contour integral $\int_C f(z)dz = \int_a^b f(z(t)) z'(t) dt$

C — closed contour

Cauchy-Goursat Theorem. $\oint_C f(z)dz = 0$

Cauchy integral formula $\oint \frac{f(z)}{\ldots} dz = 2\pi i \cdot f(z_0)$

$$\oint_C \frac{f(z)}{(z-z_0)^{n+1}}dz = \frac{2\pi i}{n!} f^{(n)}(z_0)$$

4. Series.

power series $\sum_{n=0}^{\infty} a_n (z-z_0)^n$

↓

circle of convergence $|z-z_0| = \underline{\underline{R}}$.

domain of convergence $|z-z_0| < R$.

Taylor series

Laurent series $f(z) = \sum_{n=-\infty}^{+\infty} C_n (z-z_0)^n$. $r < |z-z_0| < R$.

5. Residue

isolated singular points —— types.

pole essential removable.

order m.

Residue —— $\operatorname{Res}_{z=z_0} f(z) = C_{-1}$

$\boxed{\text{① definition} \quad \text{② pole} \quad \text{③ zero.}}$

$$\oint_C f(z)dz = 2\pi i \sum_{k=1}^{n} \operatorname{Res}_{z=z_k} f(z)$$

$$\boxed{\oint_C f(z)dz = 2\pi i \operatorname{Res}_{z=0}\left[\frac{1}{z^2} f\left(\frac{1}{z}\right)\right]}$$

$f(z)$ — z_k, $k=1,\cdots,n \in C$.

II.

1. Basic concepts.

PDE — order.

— linear, quasilinear, nonlinear.

Three types of classical equation.

wave, heat, Laplace.

2. Classification and simplification of 2nd order PDEs.

↓

elliptic, parabolic, hyperbolic. △

3. D'Alembert's formula — infinite ✓

semi-infinite

4. Separation of variables.

① PDE + BC + IC

② Solution.

5. Eigenvalue problems

Bessel's equation
$$\begin{cases} x^2 y''(x) + x y'(x) + (x^2 - \nu^2) y(x) = 0 . \\ |y(0)| < +\infty . \end{cases}$$

$x = \sqrt{\lambda}\rho$.

$$\begin{cases} \rho^2 P''(\rho) + \rho P'(\rho) + (\lambda\rho^2 - \nu^2) P(\rho) = 0 . \\ |P(0)| < +\infty , \ P(R) = \cdots . \end{cases}$$

Legendre's equation

$$(1-x^2) y''(x) - 2x y'(x) + \nu(\nu+1) y(x) = 0 .$$

$$y(x) = C_1 P_\nu(x) + C_2 Q_\nu(x)$$

$$\begin{cases} \nu \ne n & y(x) = C_1 P_\nu(x) + C_2 Q_\nu(x) \\ \nu = n & y(x) = C_1 P_n(x) + C_2 Q_n(x). \\ |y(\pm 1)| < +\infty. \end{cases}$$

6. Special function.

Bessel function $\begin{cases} J_\nu(x) \qquad Y_\nu(x) \\ J_0(0)=1,\ J_n(0)=0,\ Y_n(0)=-\infty. \\ [x^\nu J_\nu(x)]' = x^\nu J_{\nu-1}(x) \\ [x^{-\nu} J_\nu(x)]' = -x^{-\nu} J_{\nu+1}(x) \end{cases}$

Legendre's polynomial $\underline{P_n(x)}$

$$P_0(x)=1,\quad P_1(x)=x.$$

$$(n+1)P_{n+1}(x) - (2n+1)xP_n(x) + nP_{n-1}(x) = 0.$$

$$f(x) = \sum C_n P_n(x)$$

$$\int_{-1}^{1} P_n(x)P_m(x)dx = \begin{cases} 0, & n \ne m \\ \frac{2}{2n+1}, & n = m \end{cases}$$

7. Integral transformation.

① Definition.

② Property.

delay, displacement, similarity,

Differentrial, Integral.