University Physics

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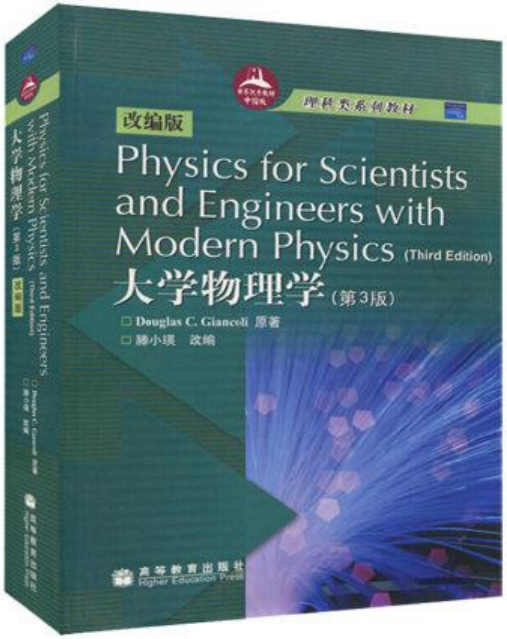
Reference Books

- "Physics for Scientists and Engineers with Modern Physics", D. C. Giancoli, 高等教育出版社（第三版，改编版）



- Sears; Halliday; Serway …
- 《物理学教程》，马文蔚，高等教育出版社
- 《大学物理通用教程》，钟锡华，陈熙谋，北京大学出版社
- 习题指导书

(I) Mechanics (Ch2 ～11) (35%)

(II) Oscillations and Waves (Ch12, 13) (15%)

(III) Electromagnetics (Ch19 ～ 29) (50%)

- 期中 **10%**
- 平时 **30%**
- 期末 **60%**

Newtonian Mechanics

The science of motion and its causes.

- **Kinematics**（运动学） → **description of motion. (Chapter 2 & 3)**
- **Dynamics**（动力学）→ **causes of motion. (Chapter 4 ~ 11)**

Motion
- **Translational (P16)**
- **Rotational**
- **Vibrational**

Chapter 2 & 3 Kinematics (from 1D to 3D)

- Description of motion
  - Reference frames and coordinate systems
  - Position, velocity, and acceleration
- Two categories of problems in kinematics
- Projectile Motion
- Natural coordinate
- Relative Motion

Idealized Models

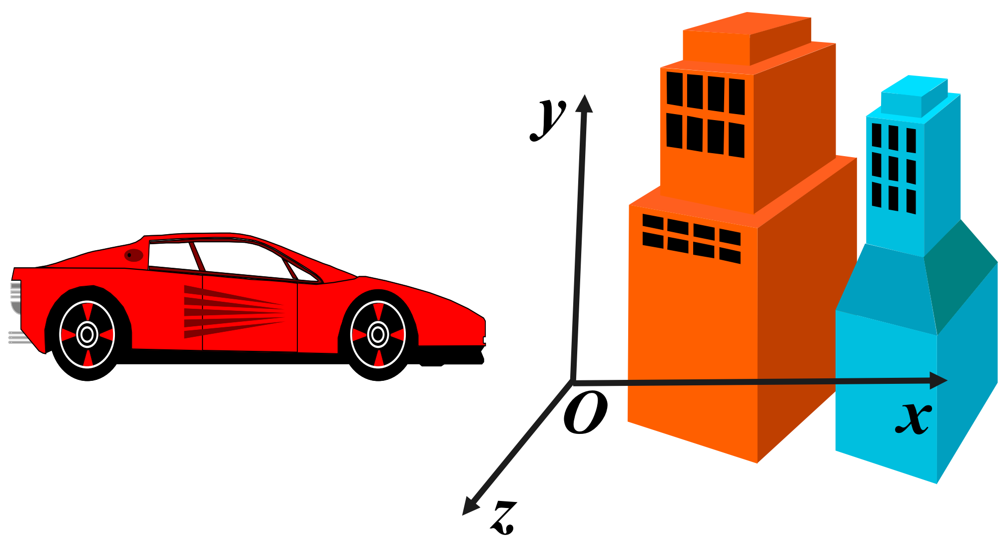
- **Idealized models (crucial role in science and technology)**
  - **A simplified version of a physical system that would be too complicated to analyze in full detail**
  - **To overlook quite a few minor effects and to concentrate on the most important features of the system**
- **The idealized model of particle (质点) (P16)**
  - **The replacement of an extended object with a particle which has mass, but zero size**
  - **Two conditions :**
    - **The size of the actual object is of no consequence in the analysis of its motion**
    - **Any internal processes occurring in the object are of no consequence in the analysis of its motion**
- **Other examples**
  - **Rigid body , point charge , ideal gas , ...**

Reference frames and coordinate systems (P17)

- The world, and everything in it, moves. Even seemingly stationary things, such as a house moves with the Earth, the Earth's orbit around the Sun, the Sun's orbit around the center of the Milky Way galaxy, and that galaxy's migration relative to other galaxies.

- To describe the position of an object, the other object which is referred to (reference frame) should be chosen.

- To determine the location of a body at the reference object quantitatively, a coordinate system is built on it.

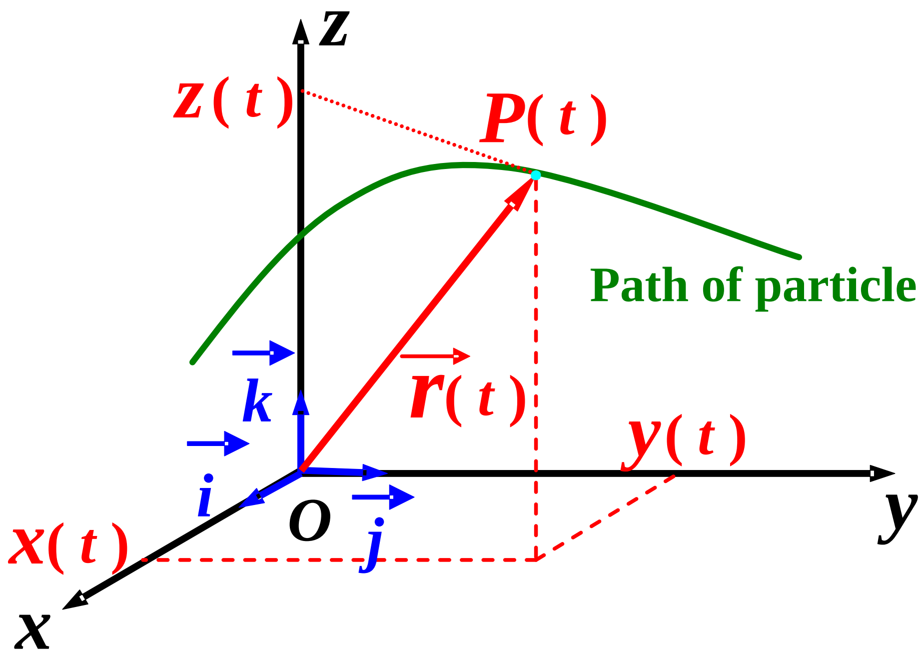


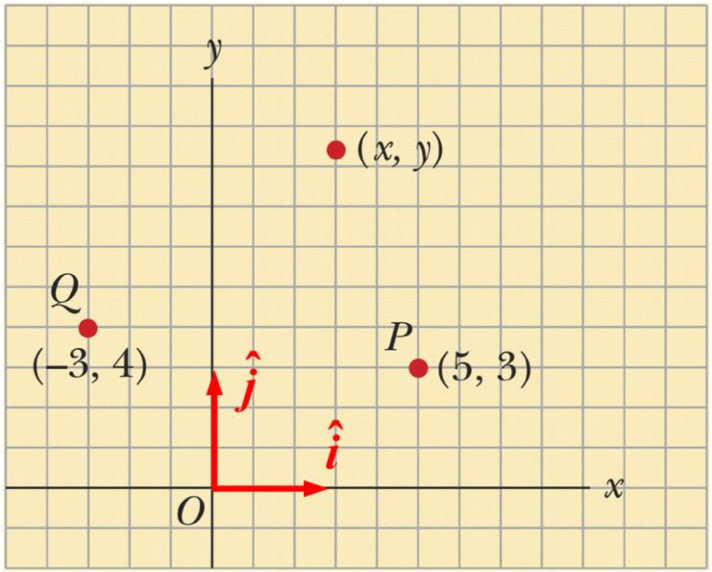
Coordinate Systems

- **Cartesian coordinate system (rectangular coordinate system)**

$\{\hat{i}, \hat{j}, \hat{k}\}$ **is a set of orthogonal bases.**

A point is described by (x, y, z)**.**

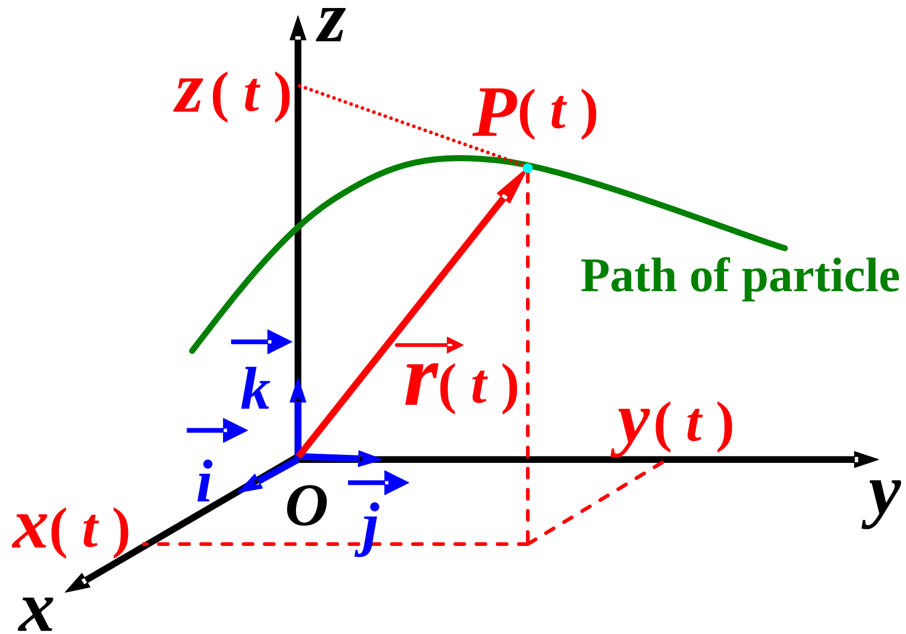




Position, displacement, velocity, and acceleration vectors in 3D (P52)

➢ **Position vector of a particle**

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$



Magnitude:

$$r = |\vec{r}| = \sqrt{x^2 + y^2 + z^2}$$

Direction:

$$\cos\alpha = \frac{x}{r}, \quad \cos\beta = \frac{y}{r}, \quad \cos\gamma = \frac{z}{r}$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

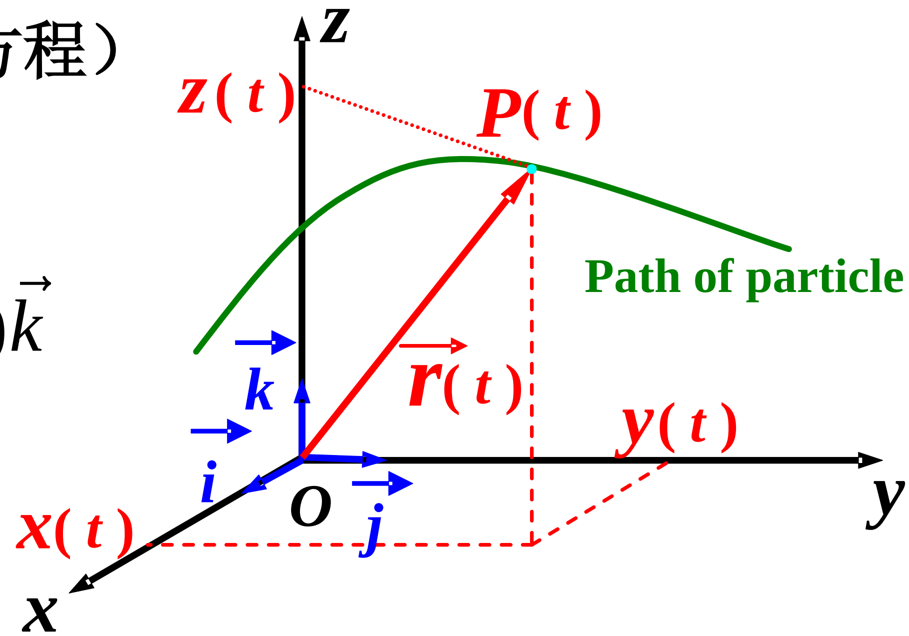
Motional equation

- **Motional equation**（运动方程）

$$\vec{r} = \vec{r}(t)$$

$$\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$$

$$\begin{cases} x = x(t) \\ y = y(t) \\ z = z(t) \end{cases}$$



- **Path (or trajectory) equation**（轨迹方程）

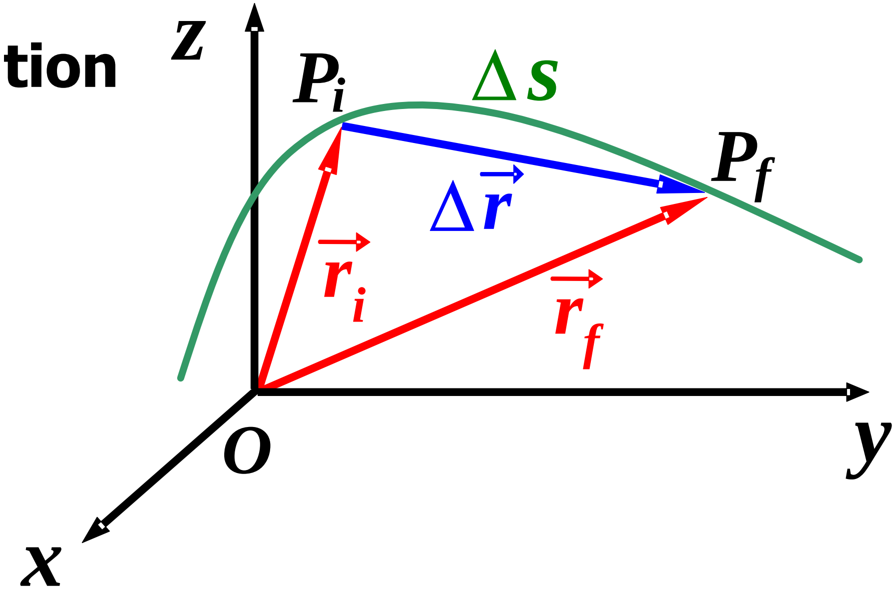
2D, $\begin{cases} x = x(t) \\ y = y(t) \end{cases}$ **by canceling time** t**,** $y = y(x)$

Displacement vector

- **Displacement**: change in position

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$

(a change in direction as well as in magnitude)

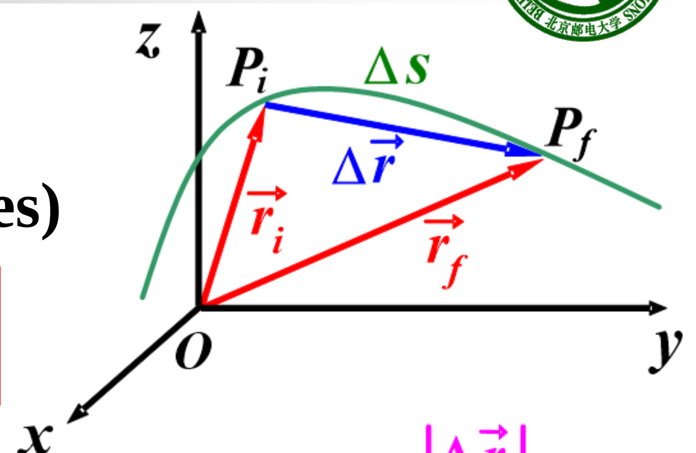


Δs（路程）

Comparison among some physical quantities

$$\vec{r}, \quad |\vec{r}|, \quad r \qquad r \equiv |\vec{r}|$$

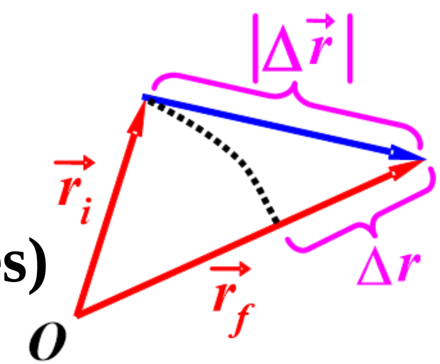
$\Delta s, \quad |\Delta\vec{r}|, \quad \Delta r$ (**finite** quantities)



$$\begin{cases} \Delta s \\ |\Delta\vec{r}| = |\vec{r}_f - \vec{r}_i| \\ \Delta r = \Delta|\vec{r}| = |\vec{r}_f| - |\vec{r}_i| = r_f - r_i \end{cases}$$

$$\boxed{\Delta s \neq |\Delta\vec{r}| \neq \Delta r}$$

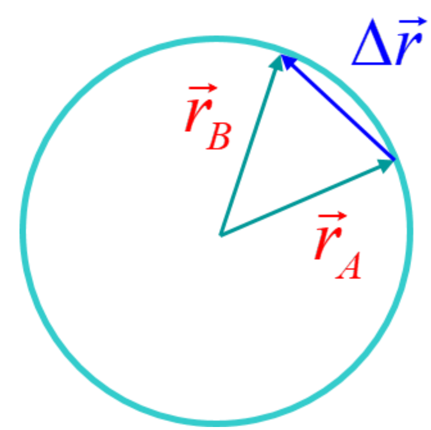
$$\Delta s \geq |\Delta\vec{r}| \geq \Delta r$$



$ds, \quad |d\vec{r}|, \quad dr$ (**infinitesimal** quantities)

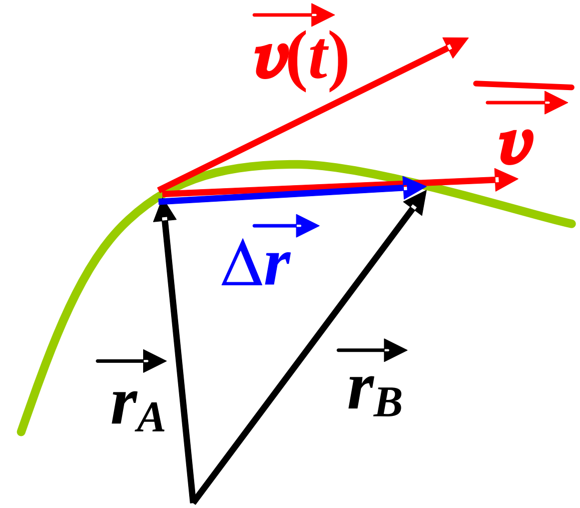
$$|d\vec{r}| \equiv \sqrt{dx^2 + dy^2} = ds \equiv \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

circular motion: $|d\vec{r}| \neq 0, \ dr = d|\vec{r}| = 0$



$$|d\vec{r}| \neq dr \qquad \boxed{ds = |d\vec{r}| \neq dr}$$

Velocity and speed



- **Average velocity:**

$$\bar{\vec{v}} = \frac{\Delta\vec{r}}{\Delta t}$$

- **Instantaneous velocity:**

$$\boxed{\vec{v} = \lim_{\Delta t\to 0}\frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}}$$

$$\vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

direction: limiting direction of $\Delta\vec{r}$ (tangent to the path curve)
magnitude: speed at that instant

- **Speed**

$$v = |\vec{v}| = \left|\frac{d\vec{r}}{dt}\right| = \frac{|d\vec{r}|}{dt} = \frac{ds}{dt} \neq \frac{dr}{dt}$$

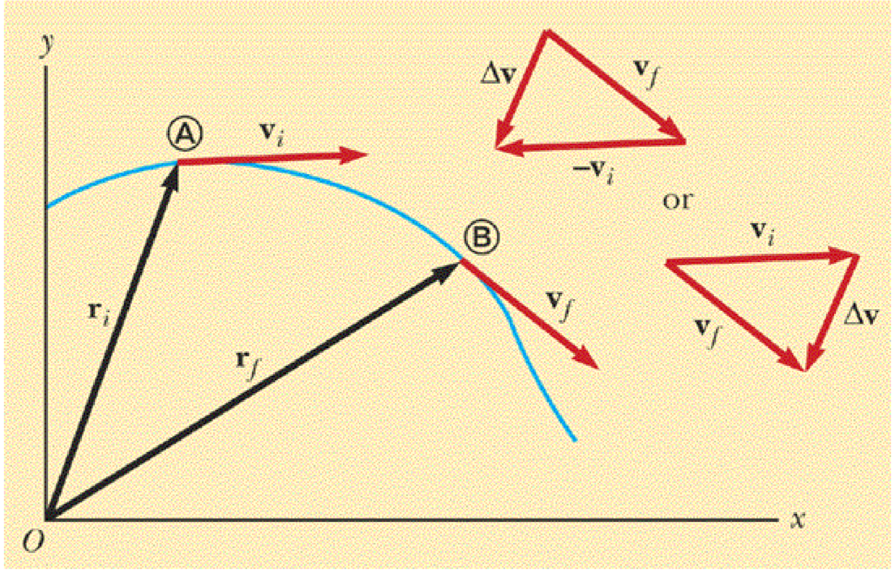
Acceleration

- **Average acceleration**

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$$

- **Instantaneous acceleration**

$$\boxed{\vec{a} = \lim_{\Delta t\to 0}\frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}}$$



direction: limiting direction of $\Delta\vec{v}$

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

$$= \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k} = \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j} + \frac{d^2z}{dt^2}\hat{k}$$

Two categories of problems in kinematics

- The position of particle is known quantity, find its velocity and acceleration——**By way of derivatives.**
- The acceleration of particle is known quantity, find its velocity and position——**By way of integrals.**

Known quantities		Unknown quantities
$\vec{r}(t)$	$\xrightarrow{\text{derivative}}$	$\vec{v}(t),\quad \vec{a}(t)$
$\vec{a}(t)$ $(\vec{v}_i, \vec{r}_i)$	$\xrightarrow{\text{integral}}$	$\vec{v}(t),\quad \vec{r}(t)$

Example

A particle moves in *xy*-plane. Its motional equations are:

$$x(t) = R\cos\omega t, \qquad y(t) = R\sin\omega t$$

where R and ω are constant.

(1) Show that the particle moves in a circle of radius R.

(2) Show that the magnitude of the particle's speed is constant and equals ωR.

(3) Show that the particle's acceleration is always opposite to its position vector and has the magnitude of $\omega^2 R$.

Solution:

Example

$$\begin{cases} x(t) = R\cos\omega t, \\ y(t) = R\sin\omega t, \end{cases} \qquad \vec{r} = x(t)\hat{i} + y(t)\hat{j}$$
$$= \left(R\cos\omega t\right)\hat{i} + \left(R\sin\omega t\right)\hat{j}$$

Solution:

(1) Its path equation $x^2 + y^2 = R^2$. So it moves in a circle of radius $\boldsymbol{R}$.

(2) $$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = \left(-\omega R\sin\omega t\right)\hat{i} + \left(\omega R\cos\omega t\right)\hat{j}$$

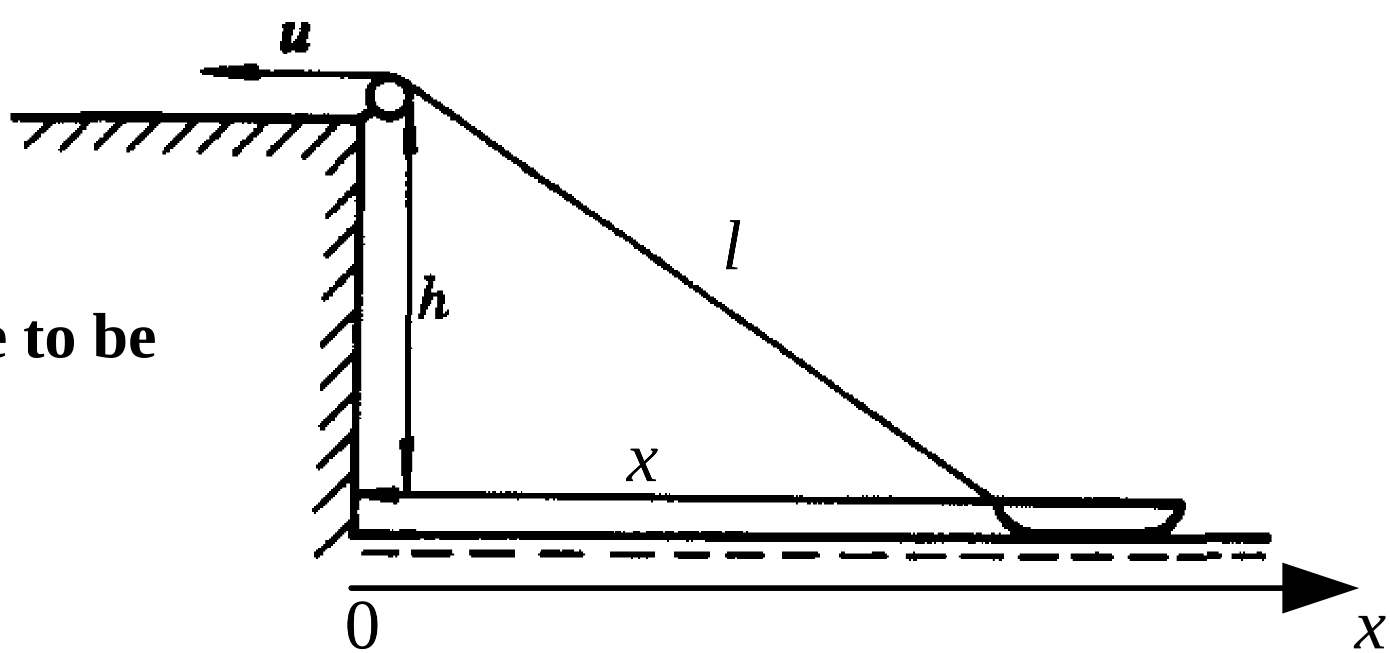
speed: $v = \sqrt{v_x^2 + v_y^2} = \omega R$, direction: $\vec{v}\cdot\vec{r} = 0$

(3) $$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} = \left(-\omega^2 R\cos\omega t\right)\hat{i} + \left(-\omega^2 R\sin\omega t\right)\hat{j}$$
$$= -\omega^2(R\cos\omega t\,\hat{i} + R\sin\omega t\,\hat{j}) = -\omega^2\vec{r}$$

$a = \sqrt{a_x^2 + a_y^2} = \omega^2 R$ opposite to the position vector

Example

A person on a cliff pulls a boat floating in water with a constant speed *u* through a rope over a pulley fixed on the edge of the cliff. The height of cliff above water is *h*, and the horizontal distance between the cliff and the boat is *x*. Find the **velocity** and **acceleration** of the boat in water.



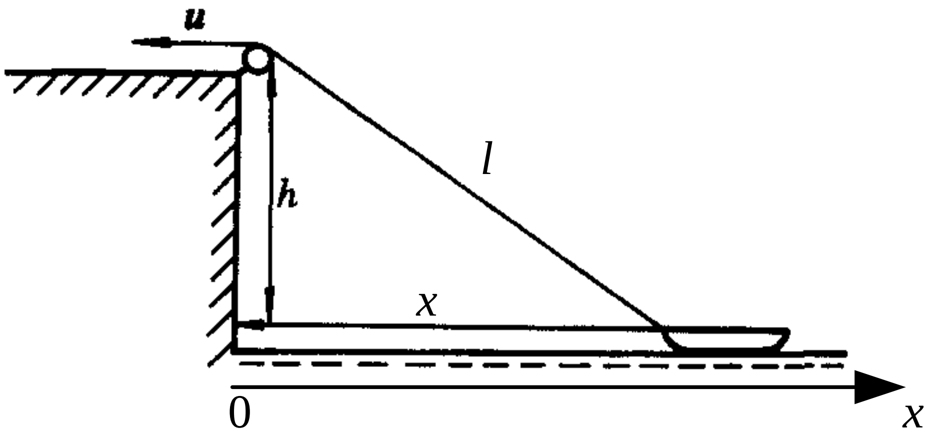
Solution :

Take right side to be positive.

Example

Solution :

$$x=\sqrt{l^2(t)-h^2}=\sqrt{(l_0-ut)^2-h^2}$$



$$v_x=\frac{dx}{dt}=\frac{d}{dt}\sqrt{(l_0-ut)^2-h^2}$$

$$=\frac{1}{2}\frac{1}{\sqrt{(l_0-ut)^2-h^2}}\cdot 2(l_0-ut)\cdot(-u)$$

$$=\frac{(l_0-ut)\cdot(-u)}{\sqrt{(l_0-ut)^2-h^2}}=\frac{-l}{\sqrt{l^2-h^2}}u=-\frac{\sqrt{h^2+x^2}}{x}u$$

Or $v_x=\dfrac{dx}{dt}=\dfrac{d}{dt}\sqrt{l^2-h^2}=\dfrac{d\sqrt{l^2-h^2}}{dl}\dfrac{dl}{dt}=\dfrac{1}{2}\dfrac{1}{\sqrt{l^2-h^2}}\cdot 2l\cdot(-u)=\dfrac{-l}{\sqrt{l^2-h^2}}u=-\dfrac{\sqrt{h^2+x^2}}{x}u$

Or $x^2=l^2-h^2$

$$2x\frac{dx}{dt}=2l\frac{dl}{dt},\quad xv_x=l(-u),\quad v_x=-\frac{l}{x}u=-\frac{u}{\cos\theta}=-\frac{\sqrt{h^2+x^2}}{x}u$$

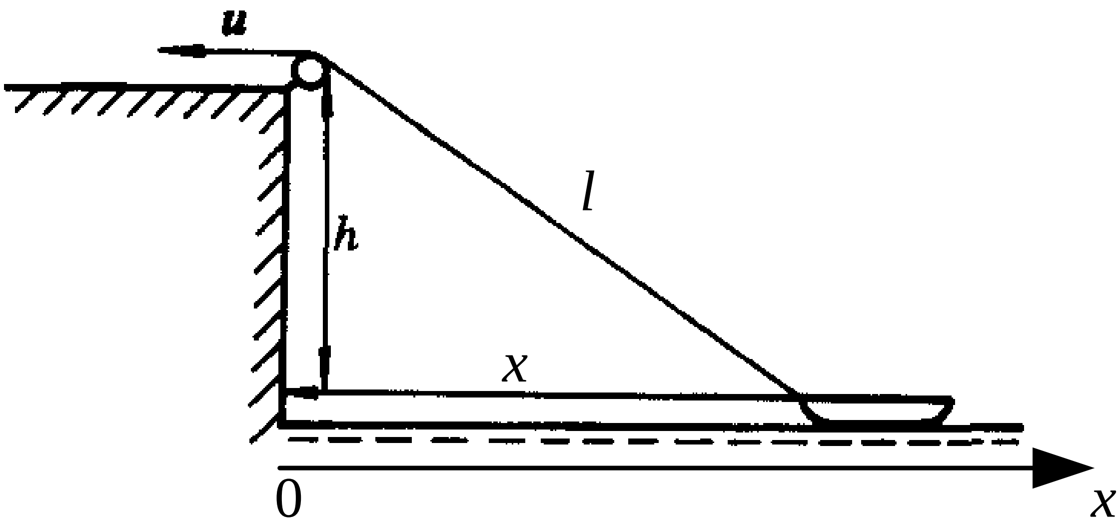
Example

Solution :

$$v_x = -\frac{\sqrt{h^2+x^2}}{x}u$$

$$a_x = \frac{dv_x}{dt} = \frac{dv_x}{dx}\frac{dx}{dt}$$

$$= v_x\frac{dv_x}{dx} = -\frac{h^2}{x^3}u^2$$



Example

For uniformly accelerated rectilinear motion, find the relationships between (1) velocity and time, (2) position and time, (3) velocity and position. $(a = \text{constant}$ and $x = x_0, v = v_0$ when $t = 0)$

Solution:

(1) Starting with $\dfrac{dv}{dt} = a,$ **or** $dv = a\,dt$ **(separation of variables)**

By integration $\int_{v_0}^{v} dv = a\int_0^t dt,$ $\boxed{v - v_0 = at}$

(2) Starting with $\dfrac{dx}{dt} = v$

$$\frac{dx}{dt} = v_0 + at, \quad \int_{x_0}^{x} dx = \int_0^t (v_0 + at)dt, \quad \boxed{x - x_0 = v_0 t + \frac{1}{2}at^2}$$

(3) Starting with

$$a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = v\frac{dv}{dx}, \quad \int_{v_0}^{v} v\,dv = a\int_{x_0}^{x} dx, \quad \boxed{v^2 - v_0^2 = 2a(x - x_0)}$$

(Introducing x as intermediate variable)

Air resistance acting on a falling body (1D, variable a) (P41, Prob. 67)

Air resistance on a falling body can be taken into account by the approximate relation for the acceleration:

$$a = g - kv,$$

where k is a constant. (a) Derive a formula for the velocity and the position of the body as a function of time assuming it starts from rest (v=0, x=0 at t=0). (b) Determine an expression for the terminal velocity, which is the maximum value the velocity reaches.

Solution:

Air resistance acting on a falling body (1D, variable a) (P41, Prob. 67)

Solution:

(a)

$$a = \frac{\mathrm{d}\upsilon}{\mathrm{d}t} = g - k\upsilon$$

Using the method of separation of variables

$$\int_0^{\upsilon} \frac{\mathrm{d}\upsilon}{g-k\upsilon} = \int_0^t \mathrm{d}t \quad \Rightarrow \quad \upsilon = \frac{g}{k}(1 - e^{-kt})$$

$$\upsilon = \frac{\mathrm{d}x}{\mathrm{d}t}, \qquad \int_0^x \mathrm{d}x = \int_0^t \upsilon \mathrm{d}t$$

$$\Rightarrow \quad x = \int_0^t \frac{g}{k}(1 - e^{-kt})\mathrm{d}t = \frac{g}{k}t - \frac{g}{k^2}(1 - e^{-kt})$$

(b) Terminal velocity $t \to \infty$, $\upsilon \to g/k$

Example

$$a = 3 + 4x \ (\mathrm{m \cdot s^{-2}}), \qquad x_0 = 0,\ v_0 = 0, \qquad v(x) = ?$$

Solution: $$a = \frac{\mathrm{d}v}{\mathrm{d}t} = 3 + 4x, \qquad (v,\ t,\ x)$$

$$a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}v}{\mathrm{d}x}\frac{\mathrm{d}x}{\mathrm{d}t} = v\frac{\mathrm{d}v}{\mathrm{d}x} \Rightarrow v\frac{\mathrm{d}v}{\mathrm{d}x} = 3 + 4x$$

$$v\mathrm{d}v = (3 + 4x)\mathrm{d}x$$

$$\int_0^v v\mathrm{d}v = \int_0^x (3 + 4x)\,\mathrm{d}\,x \Rightarrow \frac{1}{2}v^2 = 3x + 2x^2$$

$$v = \sqrt{6x + 4x^2}\ \mathrm{m \cdot s^{-1}}$$

Problem

- **Ch2 （P41）**
  - **66, 67, 68 (第2类问题)**
- **Ch3 （P70）**
  - **18, 20, 25 (第1类问题)**
  - **23，24 (第2类问题)**

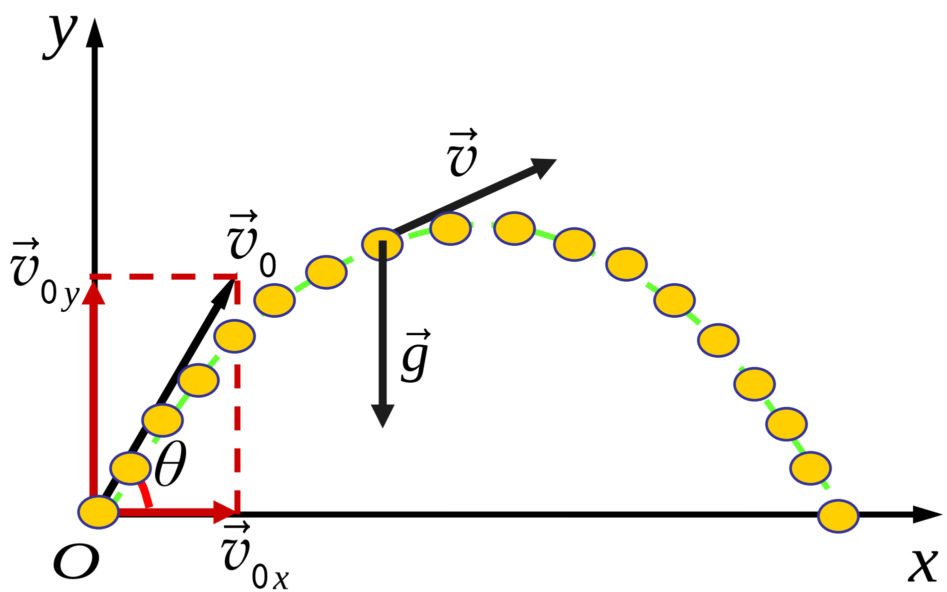
Projectile motion
(2D, a=const.) (P54, §3-7, 3-8)

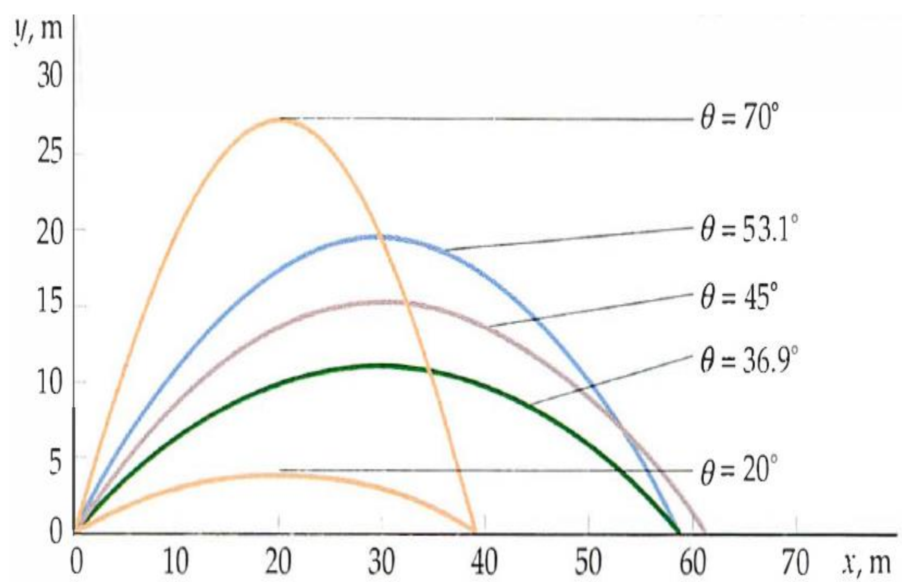
$$\vec{a} = \vec{g} = g(-\vec{j})$$

$$\begin{cases} v_x = v_0 \cos\theta \\ v_y = v_0 \sin\theta - gt \end{cases}$$

$$\begin{cases} x = (v_0 \cos\theta)t \\ y = (v_0 \sin\theta)t - \dfrac{1}{2}gt^2 \end{cases}$$

$$y = x(\tan\theta) - \frac{g}{2{v_0}^2 \cos^2\theta} x^2$$

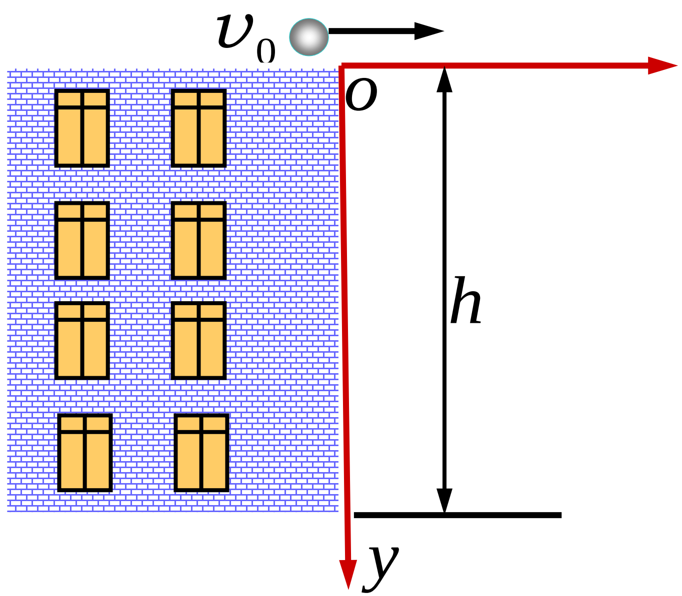




Example

$$\vec{a} = \vec{g} - k\vec{\upsilon}$$

$t = 0,\ x = 0,\ \upsilon_{0x} = v_0,\ \upsilon_{0y} = 0$



Solution:

$$\begin{cases} a_x = \dfrac{d\upsilon_x}{dt} = -k\upsilon_x, \\ a_y = \dfrac{d\upsilon_y}{dt} = g - k\upsilon_y, \end{cases}$$

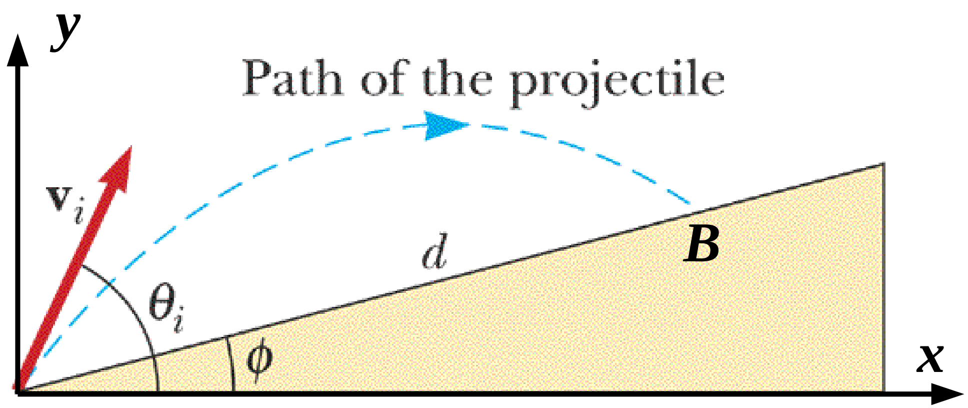
$$\begin{cases} \displaystyle\int_{\upsilon_0}^{\upsilon_x} \frac{d\upsilon_x}{\upsilon_x} = -\int_0^t k\,dt \\ \displaystyle\int_0^{\upsilon_y} \frac{d\upsilon_y}{g - k\upsilon_y} = \int_0^t dt \end{cases}$$

$$\begin{cases} \upsilon_x = \upsilon_0 e^{-kt} \\ \upsilon_y = \dfrac{g}{k}(1 - e^{-kt}) \end{cases}$$

$$\begin{cases} x = \dfrac{\upsilon_0}{k}(1 - e^{-kt}) \\ y = \dfrac{g}{k}t - \dfrac{g}{k^2}(1 - e^{-kt}) \end{cases}$$

Example

A projectile is lunched up an incline (incline angle ϕ) with an initial speed v_i at the angle θ_i with respect to the horizontal ($\theta_i > \phi$).



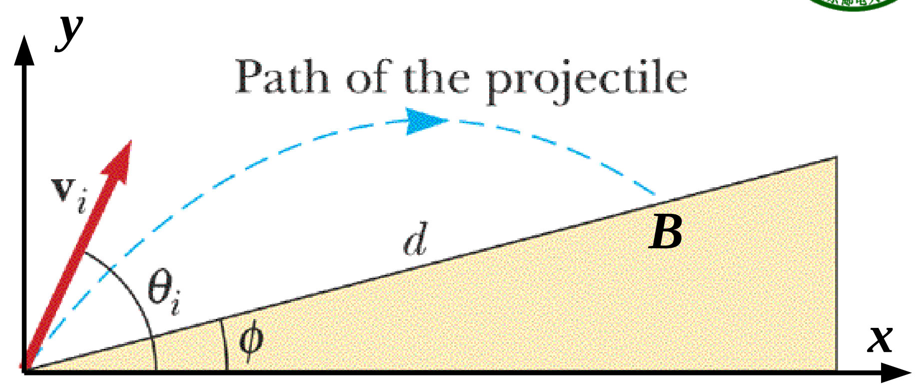
(a) Show that the projectile travels a distance d up the incline, where

$$d = \frac{2v_i^2 \cos\theta_i \sin(\theta_i - \phi)}{g\cos^2\phi}$$

(b) For what value of θ_i is d a maximum, where is maximum value?

Example

Solution:



$$\begin{cases} a_x = 0 \\ a_y = -g \end{cases}$$

$$\begin{cases} v_x = v_i \cos\theta_i \\ v_y = v_i \sin\theta_i - gt \end{cases}$$

$$\begin{cases} x = \left(v_i \cos\theta_i\right) t \\ y = \left(v_i \sin\theta_i\right) t - \dfrac{1}{2} g t^2 \end{cases}$$

For point B:

$$\begin{cases} x_B = d\cos\phi = (v_i \cos\theta_i) t_B \\ y_B = d\sin\phi = (v_i \sin\theta_i) t_B - \dfrac{1}{2} g t_B^2 \end{cases}$$

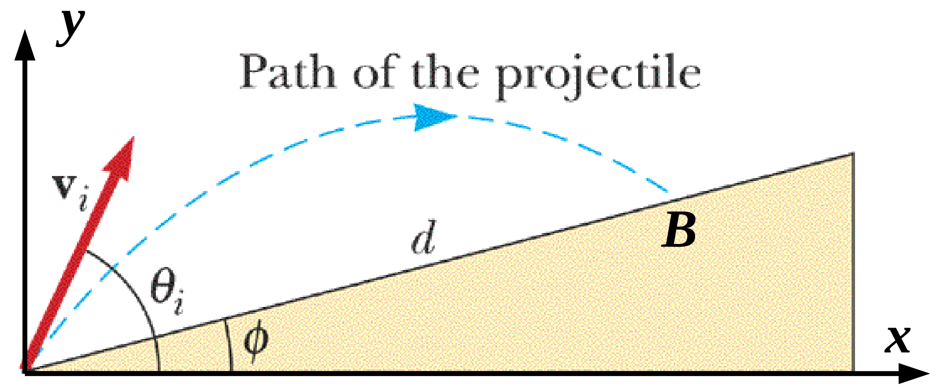
By canceling t_B

$$d = \frac{2v_i^2 \cos\theta_i \sin(\theta_i - \phi)}{g\cos^2\phi}$$

Example

d **takes the maximum value which is found from:**

$$\frac{d}{d\theta_i}(d) = \frac{2v_i^2\cos(2\theta_i - \phi)}{g\cos^2\phi} = 0$$



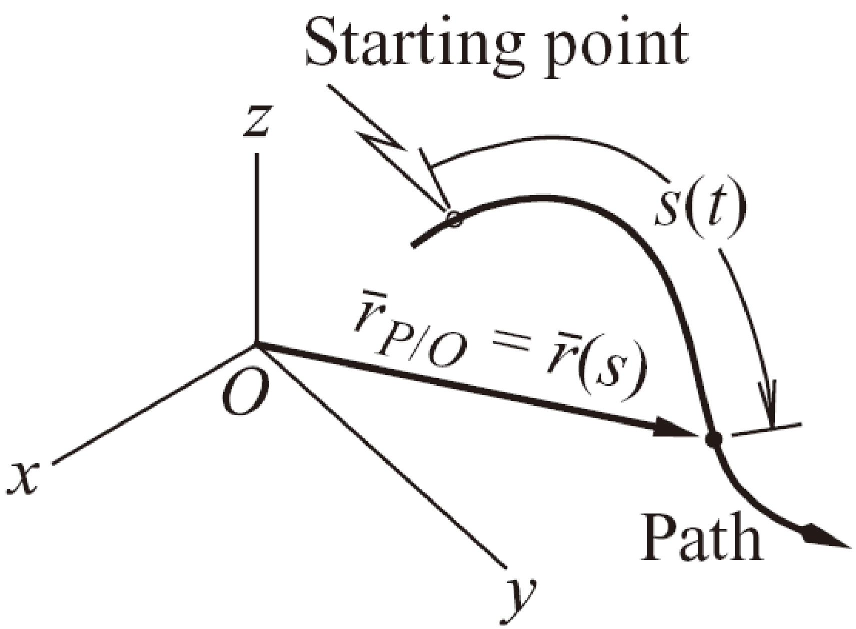
Leads to

$$\theta_i = 45° + \frac{\phi}{2}$$

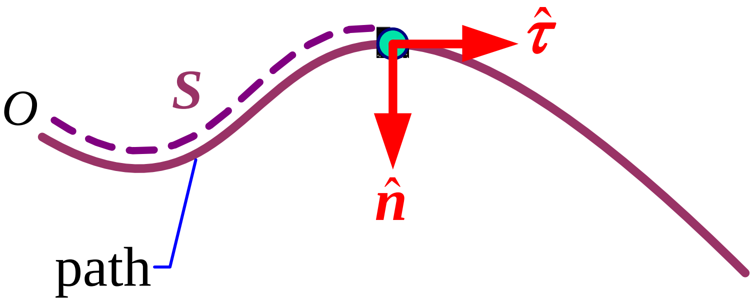
and

$$d_{\max} = \frac{v_i^2(1 - \sin\phi)}{g\cos^2\phi}$$

Natural coordinate



Orthogonal bases:
tangential and normal $\Big\} \quad \hat{\tau}, \hat{n}$



$$\hat{\tau} = \hat{\tau}(t), \hat{n} = \hat{n}(t)$$

$$\boxed{s = s(t)}$$

$$\boxed{\vec{v} = \frac{\mathrm{d}s}{\mathrm{d}t}\hat{\tau}}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}\left(v\hat{\tau}\right) = \frac{dv}{dt}\hat{\tau} + v\frac{d\hat{\tau}}{dt}$$

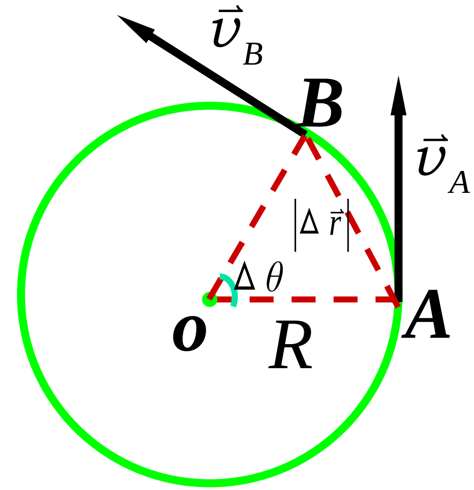
Uniform Circular Motion (2D, variable a)

(P62, §3-9)

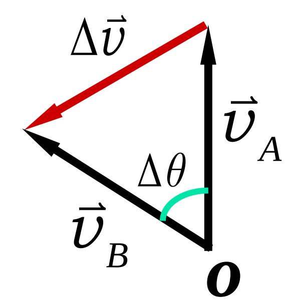
- Characteristics
  - Moves in a circle with constant speed:

$$|\vec{v}| = v = \text{constant}$$

  - Change in direction, has an acceleration.



$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}, \qquad \Delta \vec{v} = \vec{v}_B - \vec{v}_A$$



Magnitude: $$a = |\vec{a}| = \lim_{\Delta t \to 0} \frac{|\Delta \vec{v}|}{\Delta t}$$

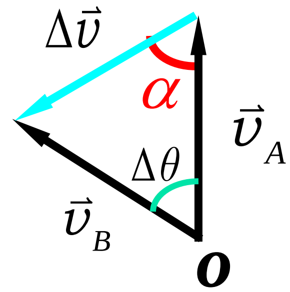
$$\frac{|\Delta \vec{v}|}{|\Delta \vec{r}|} = \frac{v}{R}, \qquad \frac{|\Delta \vec{v}|}{\Delta t} = \frac{v}{R}\frac{|\Delta \vec{r}|}{\Delta t}, \qquad a = \frac{v}{R}\lim_{\Delta t \to 0}\frac{|\Delta \vec{r}|}{\Delta t} = \frac{v^2}{R}$$

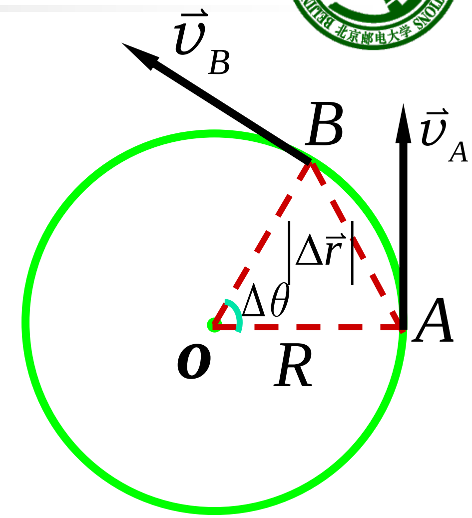
Uniform Circular Motion

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$$

Direction:

$$\alpha = \frac{1}{2}(\pi - \Delta\theta)$$





When $\Delta t \to 0$, $\Delta\theta \to 0$, $\alpha \to \pi/2$, $\Delta\vec{v} \perp \vec{v}_A$, $\vec{a} \perp \vec{v}_A$

In natural coordinate,

$$\vec{a} = \frac{v^2}{R}\hat{n}$$

Centripetal acceleration

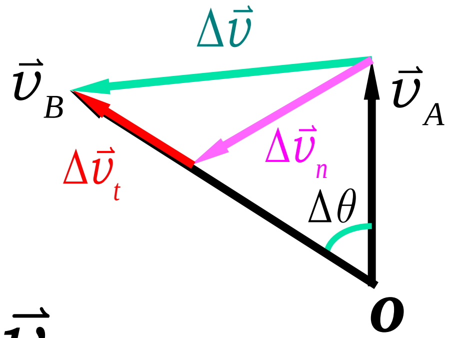
(meaning "seeking center")

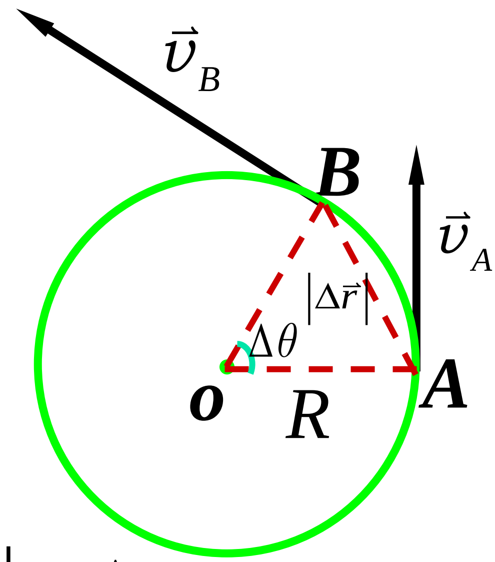
Non-Uniform Circular Motion (2D, variable a)

(P119, §5-4)

- Characteristics
  - Changes both in magnitude and direction

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\upsilon}}{\Delta t}$$





$$\Delta\vec{\upsilon} = \vec{\upsilon}_B - \vec{\upsilon}_A = \Delta\vec{\upsilon}_{\mathrm{n}} + \Delta\vec{\upsilon}_\tau$$

$$\left|\Delta\vec{\upsilon}_\tau\right| = \left|\vec{\upsilon}_{\mathrm{B}}\right| - \left|\vec{\upsilon}_{\mathrm{A}}\right| \equiv \Delta\left|\vec{\upsilon}\right| = \Delta\upsilon$$

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\upsilon}}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\upsilon}_{\mathrm{n}}}{\Delta t} + \lim_{\Delta t \to 0} \frac{\Delta\vec{\upsilon}_\tau}{\Delta t}$$

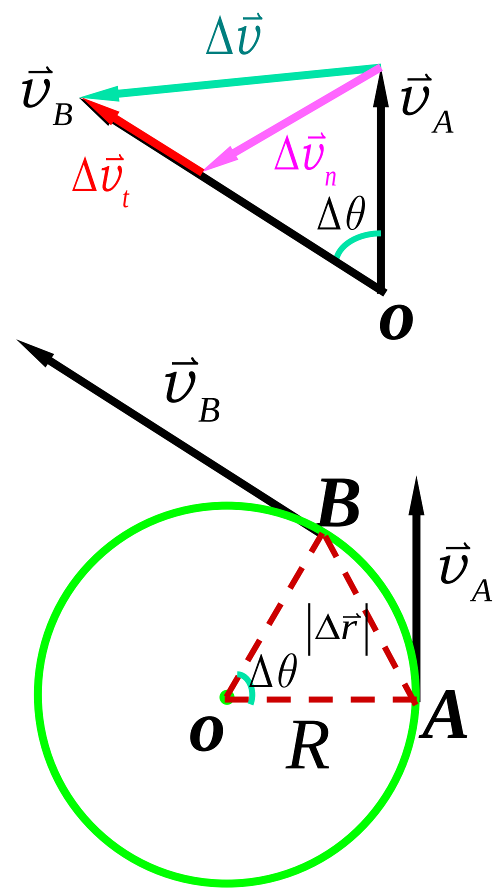
Non-Uniform Circular Motion

$$\vec{a} = \vec{a}_{\mathrm{n}} + \vec{a}_{\tau} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\upsilon}_{\mathrm{n}}}{\Delta t} + \lim_{\Delta t \to 0} \frac{\Delta \vec{\upsilon}_{\tau}}{\Delta t}$$

$$a_{\mathrm{n}} = \lim_{\Delta t \to 0} \frac{|\Delta \vec{\upsilon}_{\mathrm{n}}|}{\Delta t} = \frac{\upsilon^2}{R}$$

$$a_{\tau} = \lim_{\Delta t \to 0} \frac{|\Delta \vec{\upsilon}_{\tau}|}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta \upsilon}{\Delta t} = \frac{\mathrm{d}\upsilon}{\mathrm{d}t} = \frac{\mathrm{d}^2 s}{\mathrm{d}t^2}$$

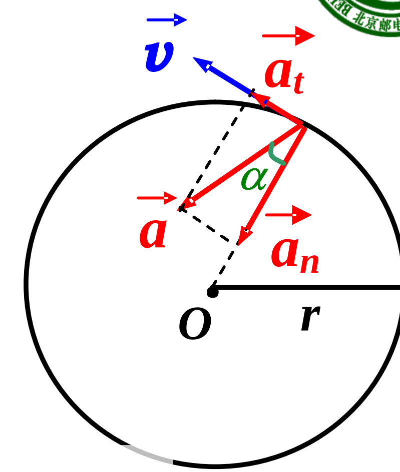
$$\vec{a} = \vec{a}_{\tau} + \vec{a}_{n} = \frac{dv}{dt}\hat{\tau} + \frac{v^2}{r}\hat{n}$$



Tangential and normal acceleration

Circular Motion

Total acceleration vector: $\vec{a} = \vec{a}_n + \vec{a}_t = \dfrac{v^2}{r}\hat{n} + \dfrac{dv}{dt}\hat{\tau}$

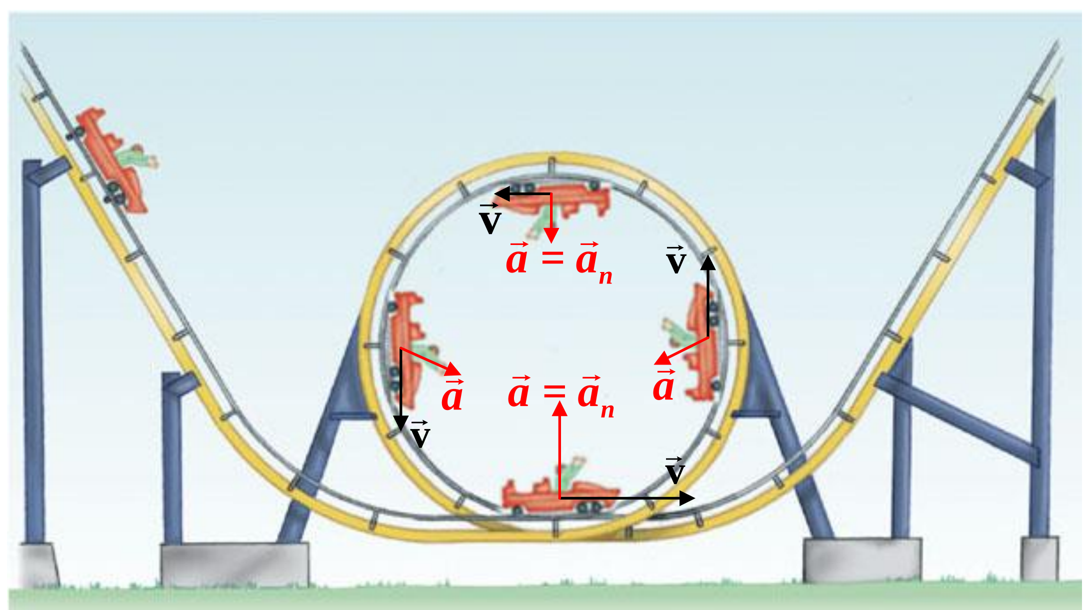


$$a = \sqrt{a_n^2 + a_t^2} = \sqrt{\left(\frac{v^2}{r}\right)^2 + \left(\frac{dv}{dt}\right)^2}$$

$$\alpha = \arctan\frac{a_t}{a_n}$$

- >0, also a_t>0, if the speed increases.
- <0, also a_t<0, if the speed decreases.

Example: a roller coaster slides freely with negligible friction in a vertical circular track.

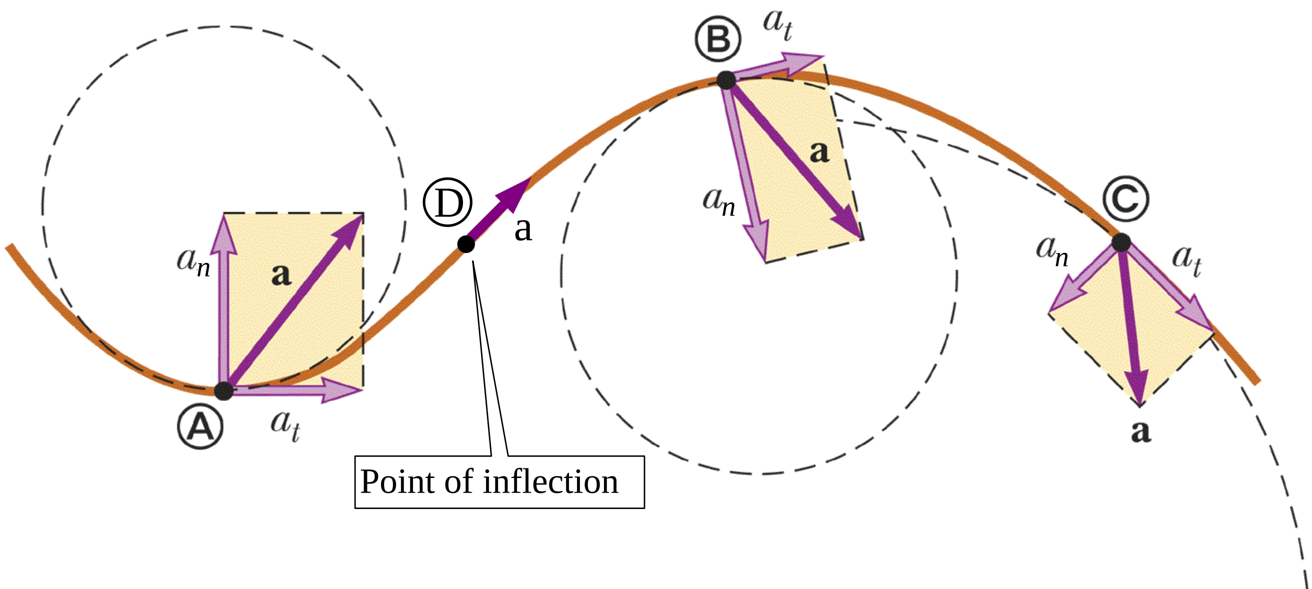


Motion along an arbitrary curved path

- Tangential acceleration and normal acceleration

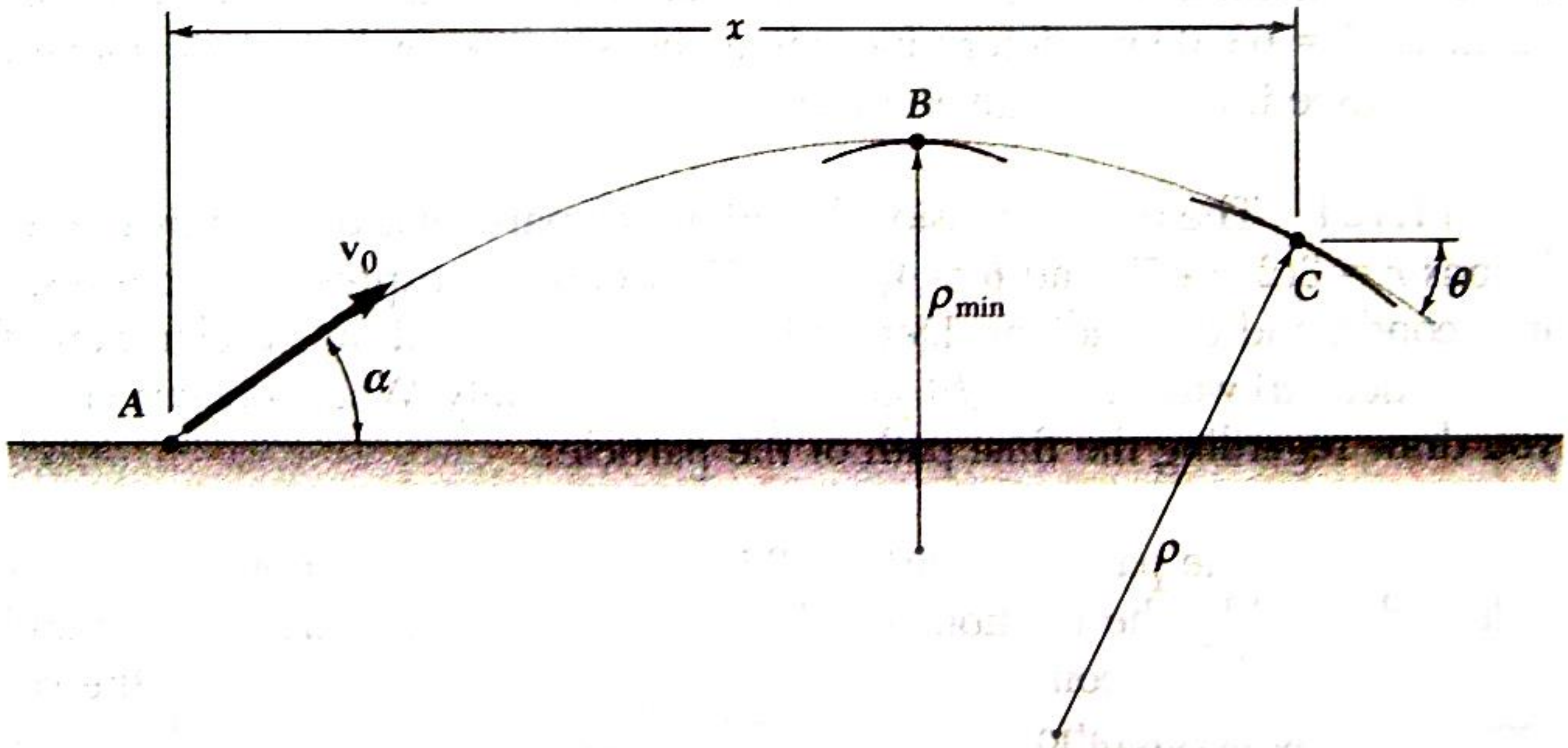
$$\vec{a} = \vec{a}_t + \vec{a}_n = \frac{dv}{dt}\hat{\tau} + \frac{v^2}{\rho}\hat{n}$$

  - Tangential acceleration— same as circular motion.
  - Normal acceleration— same as circular motion except that ρ is the radius of curvature of the path at the point.— always directs toward the center of the curvature. — be zero when particle passes through a point of inflection.



Example

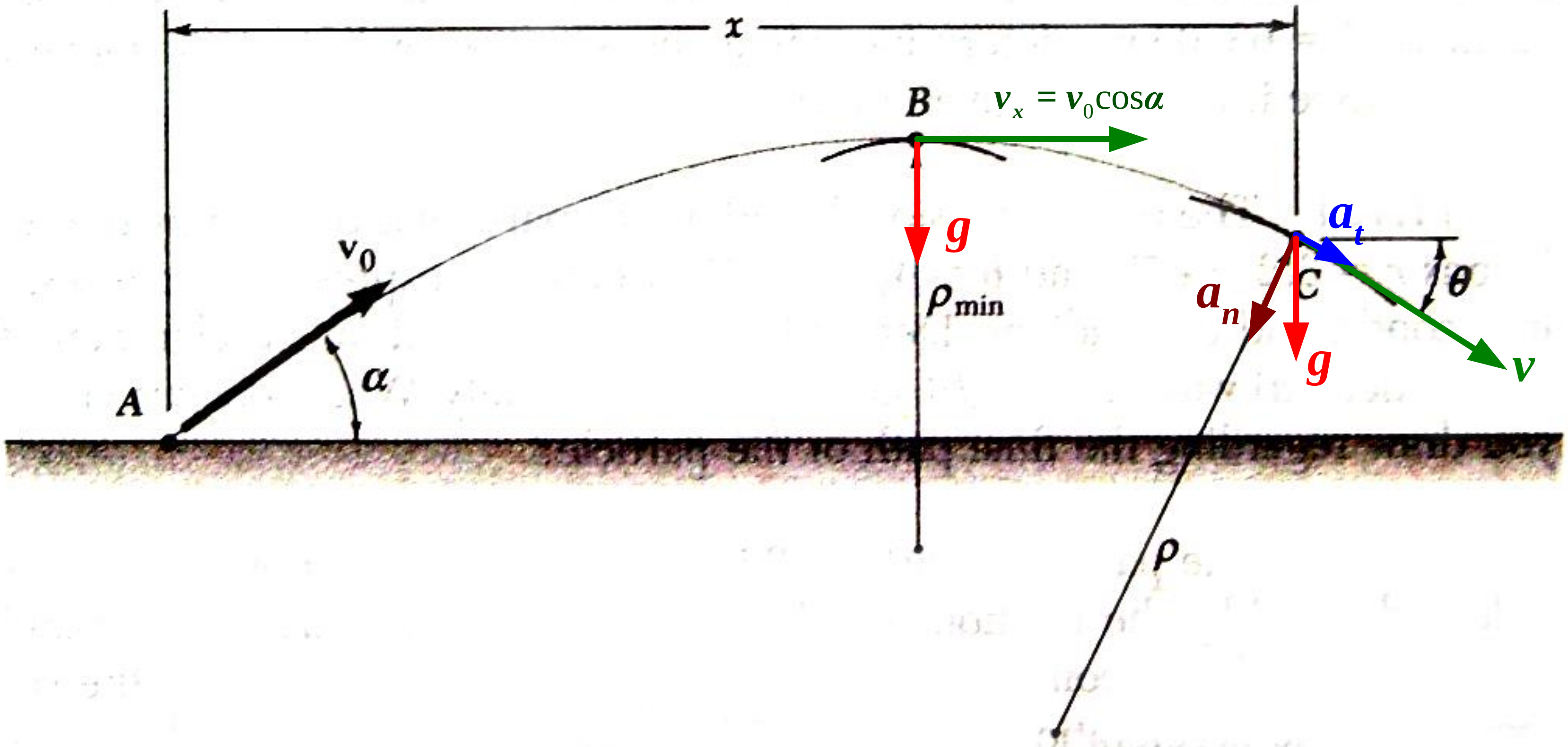
A projectile is fired from point *A* with an initial velocity v_0 which forms an angle α with the horizontal. Find the radii of curvature of the trajectory of the projectile at point *B* and *C*.



Solution

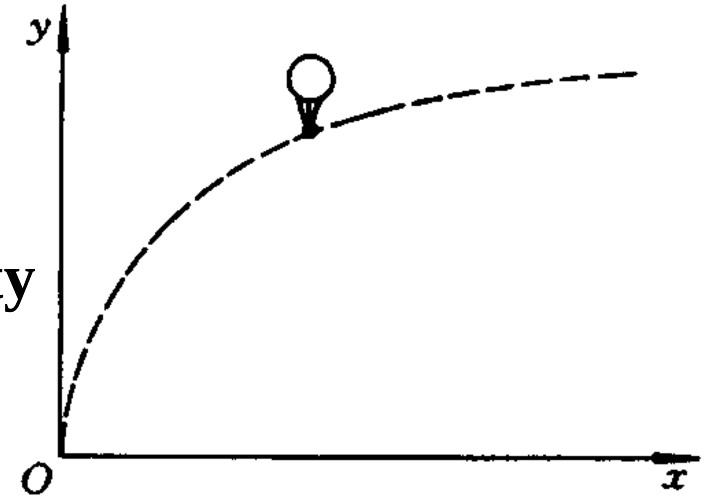
At point B, $v_B = v_0 \cos\alpha$, $a_{nB} = g$, $\rho_B = \dfrac{v_B^2}{a_{nB}} = \dfrac{v_0^2 \cos^2\alpha}{g}$

At point C, $v_C = \dfrac{v_0 \cos\alpha}{\cos\theta}$, $a_{nC} = g\cos\theta$, $\rho_C = \dfrac{v_C^2}{a_{nC}} = \dfrac{v_0^2 \cos^2\alpha}{g\cos^3\theta}$



Example

A balloon moves up from ground with an initial vertical velocity of v_0. For the reason of wind, in the air the balloon is blew to the right with horizontal velocity $v_x = by$ (b is a positive constant, y is the height of the balloon). Choose the right side to be positive for x axis.



(1) Find the motional equation of the balloon.

(2) Find the path (trajectory) equation of balloon.

(3) Determine the tangential acceleration and the radius of the curvature of the trajectory with respect to height y.

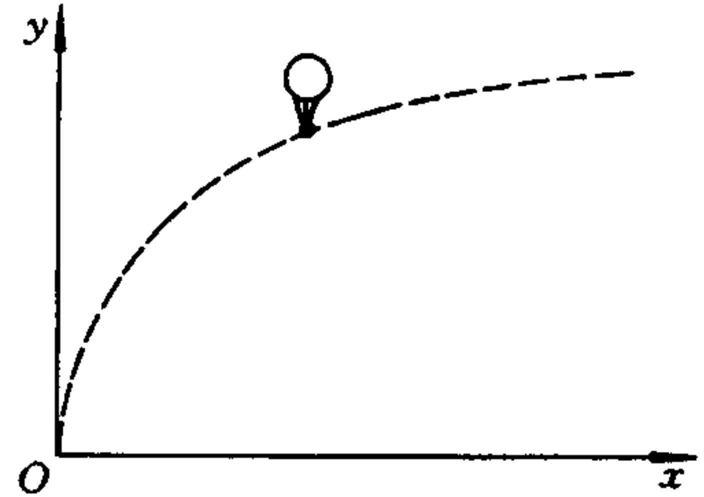
Solution: Establish a coordinate system shown in the figure. Let the balloon locates at origin point O when t=0.

Solution

(1) $v_y = v_0, \qquad y = v_0 t$

$$v_x = by, \quad \frac{dx}{dt} = by = bv_0 t, \quad \int_0^x dx = bv_0 \int_0^t t\,dt, \quad x = \frac{1}{2} b v_0 t^2$$

(2) $x = \dfrac{b}{2v_0} y^2$



(3) $a_\tau = \dfrac{dv}{dt} = \dfrac{d}{dt}\sqrt{v_x^2 + v_y^2}$

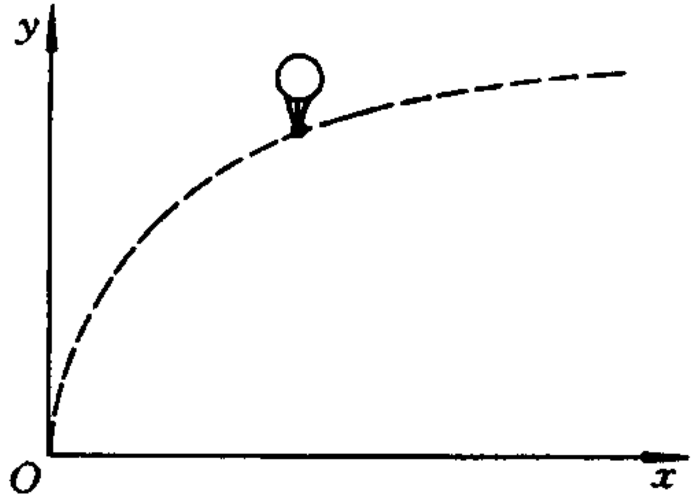
$$= \frac{d}{dt}\sqrt{b^2 y^2 + v_0^2} = \frac{d}{dt}\sqrt{b^2 v_0^2 t^2 + v_0^2}$$

$$= v_0 \frac{d}{dt}\sqrt{b^2 t^2 + 1} = \frac{b^2 v_0 t}{\sqrt{b^2 t^2 + 1}} = \frac{b^2 v_0 y}{\sqrt{b^2 y^2 + v_0^2}}$$

Solution

(3) $a = \sqrt{a_x^2 + a_y^2} = \sqrt{a_n^2 + a_\tau^2}$

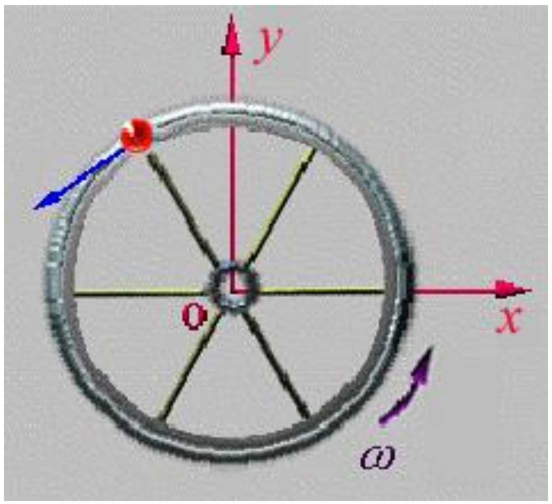
$$a_y = \frac{dv_y}{dt} = 0, \quad a_x = \frac{dv_x}{dt} = bv_0$$

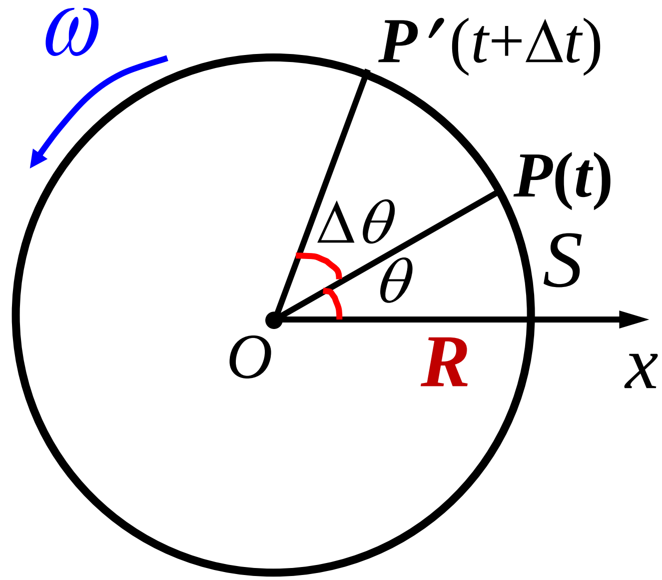


$$a_n = \frac{bv_0^2}{\sqrt{b^2 y^2 + v_0^2}}$$

$$\rho = \frac{v^2}{a_n} = \frac{(b^2 y^2 + v_0^2)^{\frac{3}{2}}}{bv_0^2}$$

Circular Motion





1. Angular position θ rad

2. Angular displacement $\Delta\theta$

counter-clockwise vs. clockwise

3. Angular velocity

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t} \ (\mathrm{rad \cdot s^{-1}})$$

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{\mathrm{d}\theta}{\mathrm{d}t}$$

$\vec{\omega}$

4. Angular acceleration

$$\bar{\alpha} = \frac{\Delta\omega}{\Delta t} \quad (\mathrm{rad \cdot s^{-2}})$$

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}$$

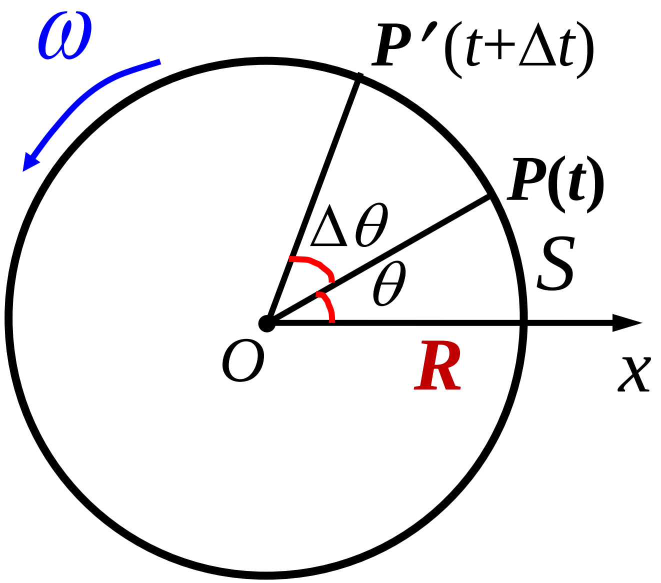
Circular Motion

5. $s = R\theta$

$$v = \frac{\mathrm{d}s}{\mathrm{d}t} = R\frac{\mathrm{d}\theta}{\mathrm{d}t} = R\omega$$

$$a_\tau = \frac{\mathrm{d}v}{\mathrm{d}t} = R\frac{\mathrm{d}\omega}{\mathrm{d}t} = R\alpha$$

$$a_n = \frac{v^2}{R} = \frac{(R\omega)^2}{R} = R\omega^2 = v\omega$$



6. For uniform circular motion

$$\omega = \frac{2\pi}{T}$$

6. Circular Motion (α = const.)

$$\omega = \omega_0 + \alpha t$$

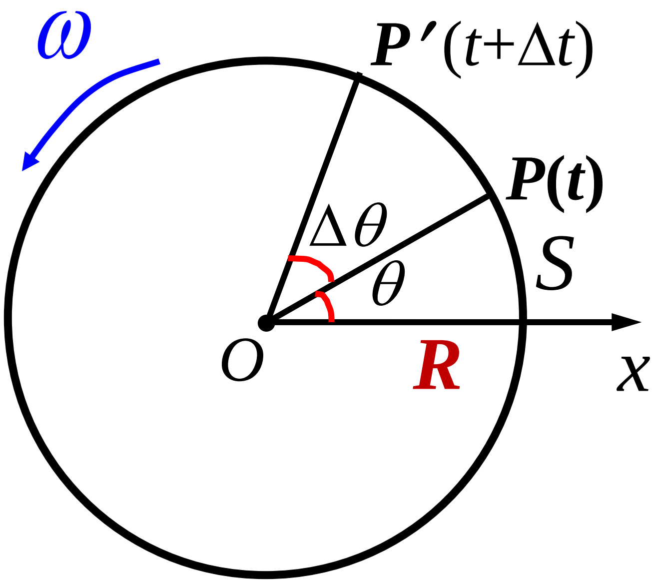
$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\left(\theta - \theta_0\right)$$

$$v = v_0 + a t$$

$$S = S_0 + v_0 t + \frac{1}{2} a t^2$$

$$v^2 = v_0^2 + 2a\left(S - S_0\right)$$



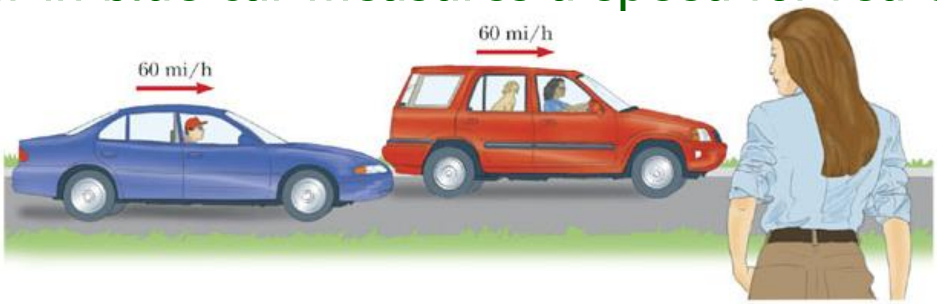
Relative Velocity

(P64, § 3-10)

- The descriptions of the motion are different in different frames of reference.

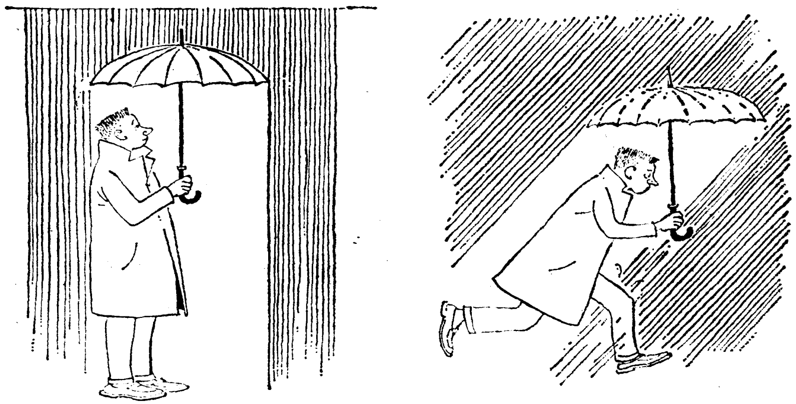
Example 1.

- The lady observer measures a speed for red car of 60mi/h.
- The observer in blue car measures a speed for red car of zero.



Example 2.

- The man in rest feels that the rain falling vertically.
- The man in motion feels that the rain inclines towards him.

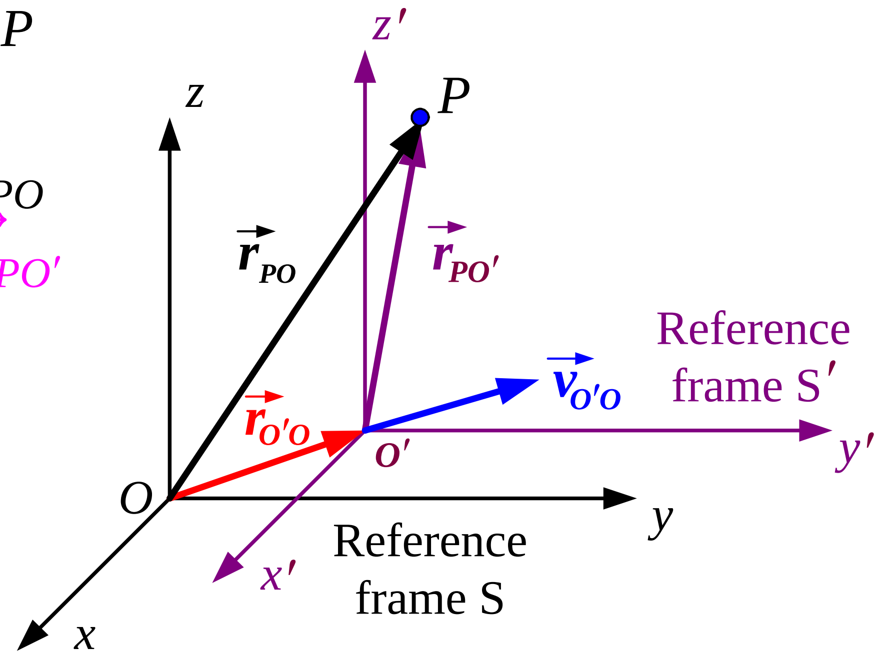


The Relative Motion

The relative motion respect to two the frames in translation

- The relationship between positions of P in two reference frames:
  - The position of P relative to S is $\vec{r}_{PO}$
  - The position of P relative to S' is $\vec{r}_{PO'}$

$$\vec{r}_{PO} = \vec{r}_{PO'} + \vec{r}_{O'O}$$



- The relationship between velocities of the particle in the two frames:

$$\frac{d}{dt}(\vec{r}_{PO}) = \frac{d}{dt}(\vec{r}_{PO'}) + \frac{d}{dt}(\vec{r}_{O'O})$$

$$\vec{v}_{PO} = \vec{v}_{PO'} + \vec{v}_{O'O}$$

absolute velocity

relative velocity

attached velocity

Subscript rule

- Conventional subscript rule for the equation relating velocities in different reference frame:
  - On the right-hand side: inner subscripts are the same,
  - Whereas the outer subscripts on the right are the same as the two subscripts for the "absolute vector"

$$\vec{v}_{PO} = \vec{v}_{PO'} + \vec{v}_{O'O}$$

The first

The last

$$\vec{r}_{AO} = \vec{r}_{AB} + \vec{r}_{BC} + \vec{r}_{CD} + \vec{r}_{DO}$$

$$\vec{v}_{AO} = \vec{v}_{AB} + \vec{v}_{BC} + \vec{v}_{CD} + \vec{v}_{DO}$$

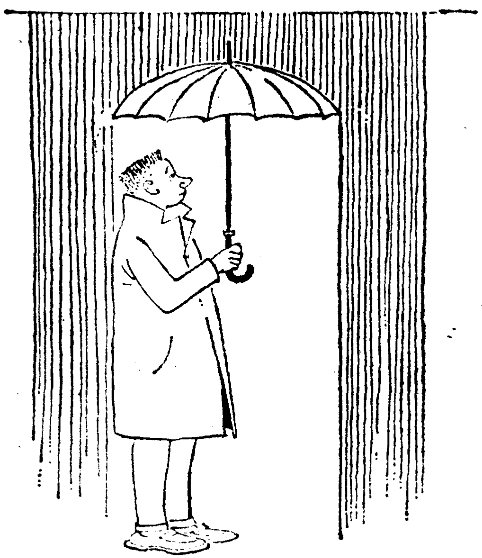
- Also valid for Position Vectors and Acceleration Vectors

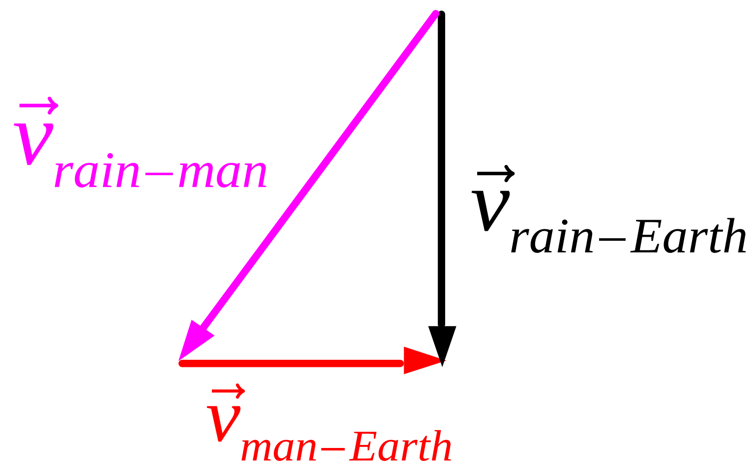
$$\vec{r}_{PO} = \vec{r}_{PO'} + \vec{r}_{O'O}$$

Example

The man in the rain:

$$\vec{v}_{rain-earth} = \vec{v}_{rain-man} + \vec{v}_{man-earth}$$



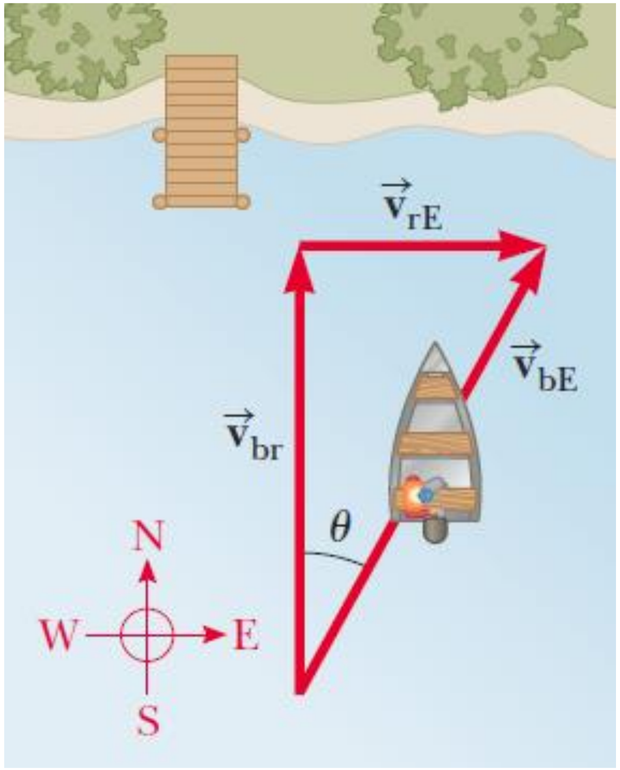


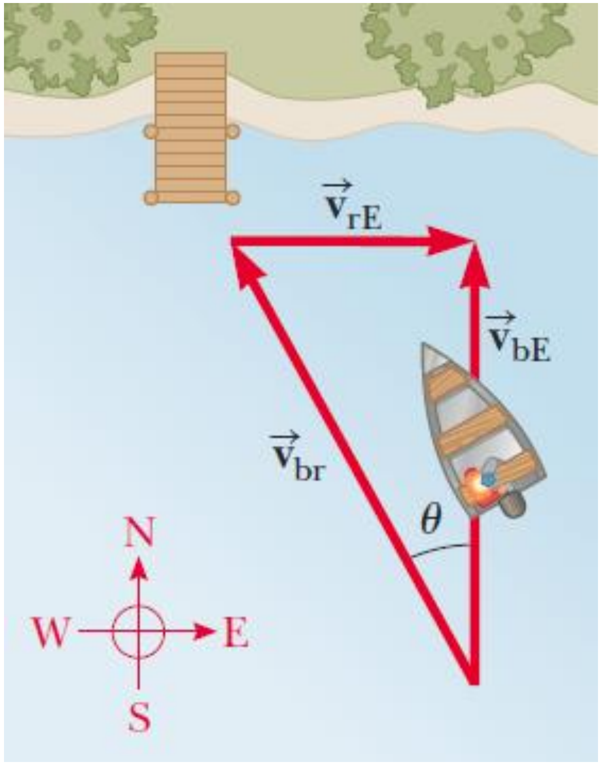
Example

A boat crossing a wide river moves with a speed of 10.0 km/h relative to the water. The water in the river has a uniform speed of 5.00 km/h due east relative to the Earth.

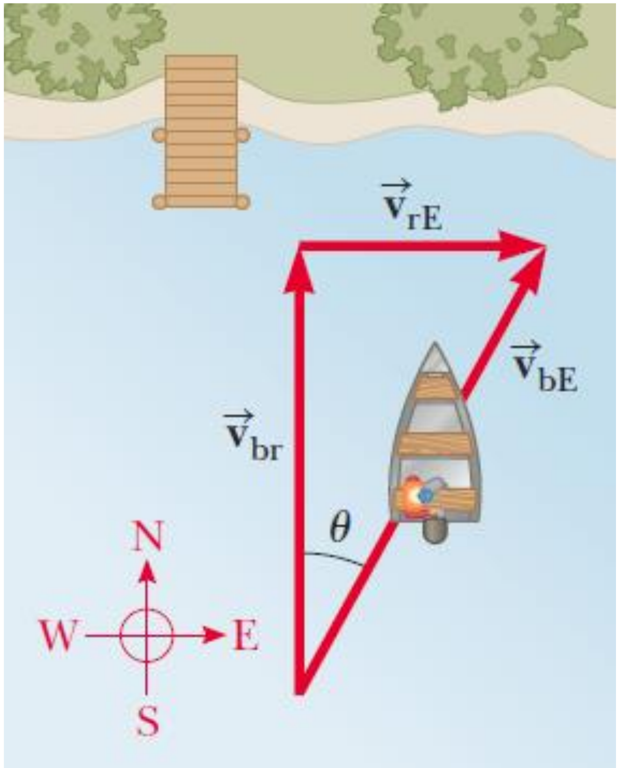
(1) If the boat heads due north, determine the velocity of the boat relative to an observer standing on either bank.

(2) If the boat travels with the same speed of 10.0 km/h relative to the river and is to travel due north, what should its heading be?

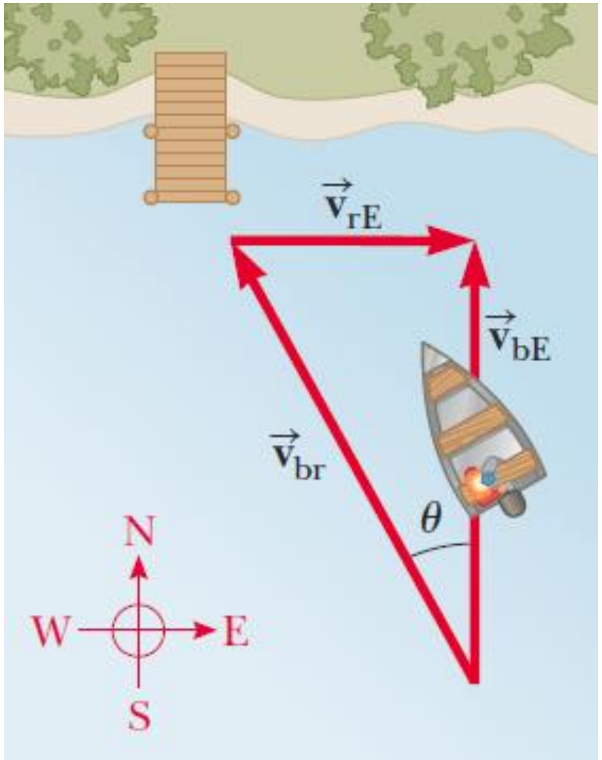




Solution



$$\vec{v}_{bE} = \vec{v}_{br} + \vec{v}_{rE}$$



(1)

$$v_{bE} = \sqrt{v_{br}^2 + v_{rE}^2} = 11.2 \ \text{ km/h}$$

$$\theta = \tan^{-1}\left(\frac{v_{rE}}{v_{br}}\right) = 26.6^\circ$$

(2)

$$v_{bE} = \sqrt{v_{br}^2 - v_{rE}^2} = 8.66 \ \text{ km/h}$$

$$\theta = \tan^{-1}\left(\frac{v_{rE}}{v_{bE}}\right) = 30^\circ$$

Problem

- **Ch3 (P72)**
  - **58 (Circular motion)**
  - **62, 71 (Relativity velocity)**