Classical Mechanics

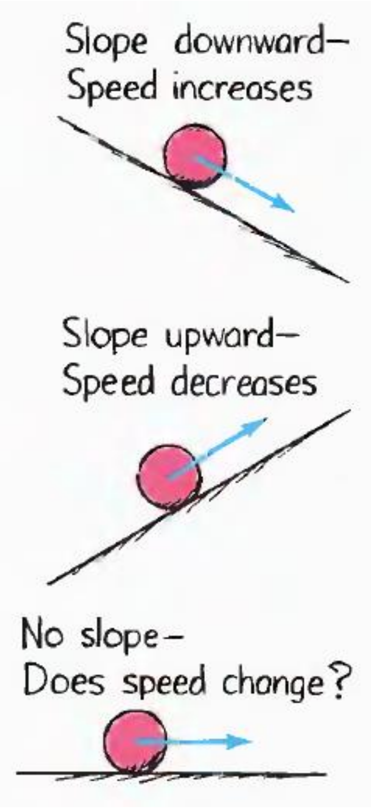
- **Kinematics**
- **Dynamics**

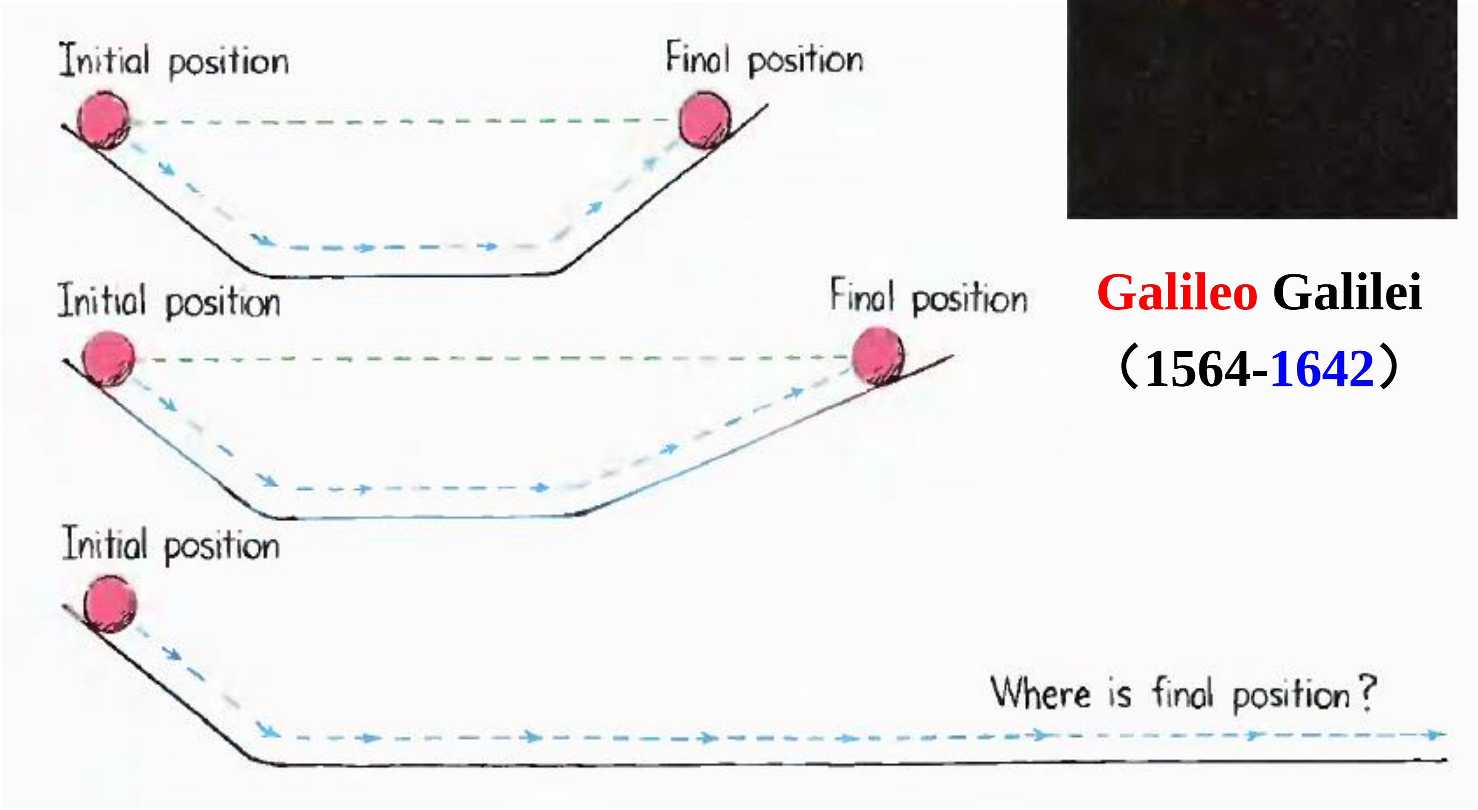
  It studies the rule on the interaction force between bodies and examines the causes of motion.

  - **Newtonian Mechanics**
    - **Newton's Laws**
  - **Analytical mechanics**

Chapter 4, 5 Newton's laws and their applications

§ 1 Newton's First Law





Galileo Galilei
（1564-1642）

Newton’s First Law — Law of Inertia

- **In the absence of external forces (or no net force), an object at rest remains at rest, and an object in motion continues in motion with a constant velocity (i.e. with a constant speed in a straight line).**
  - **Thought experiments**
  - **Velocity**
  - **Force —— An interaction that can cause an acceleration of a body.**
  - **An object has a tendency to maintain its original state of motion in the absence of a force. — This tendency is called inertia.**

Inertial Frames

- Newton's first law defines a special set of reference frames called inertial frames — An inertial frame of reference is one in which Newton's first law (also second law) is valid.
- Any reference frame that moves with constant velocity with respect to an inertial frame is itself an inertial frame.
- Reference frames where the law of inertia does not hold, such as the accelerating reference frames are called noninertial frames.

The Earth as a Inertia Frame

- The Earth is not an inertial frame because it is connected with two kinds of motions.
  - Rotational motion about its own axis:

    $$a_n \approx 3.4\times10^{-2}\mathrm{m/s^2}$$

  - Orbital motion about the sun:

    $$a_n \approx 6.0\times10^{-3}\mathrm{m/s^2}$$

  - Very small compared with $g = 9.8\mathrm{m/s^2}$
- In most situations, we consider a reference frame connected to the Earth as the approximate inertial frame.

§ 2 Newton's Second Law P79

- **The acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass.**

$$\boxed{\sum \vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = m\frac{d^2\vec{r}}{dt^2}}$$

- **An instantaneous relation:** $\sum \vec{F}(t) = m\vec{a}(t)$

- **The component expressions:**

In Cartesian coordinate $\sum F_x = ma_x, \quad \sum F_y = ma_y, \quad \sum F_z = ma_z$

In natrual coordinate $\sum F_t = ma_t = m\dfrac{dv}{dt}, \quad \sum F_n = ma_n = m\dfrac{v^2}{\rho}$

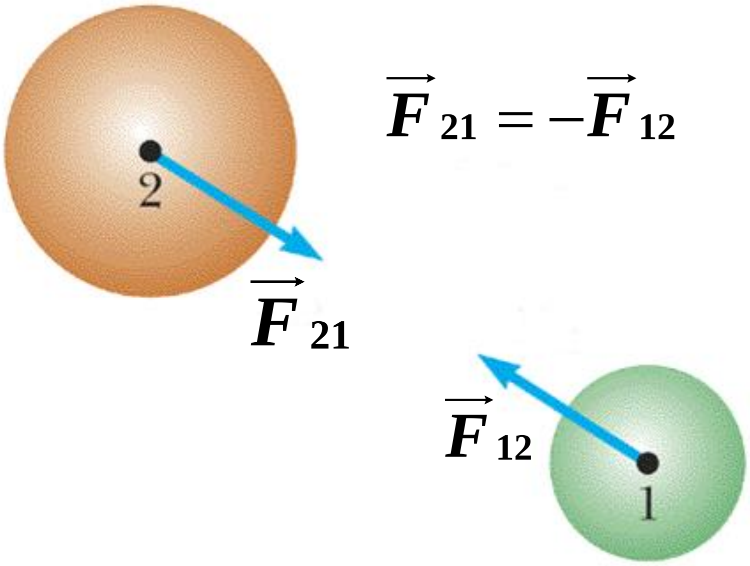
§ 3 Newton's Third Law

P82

- **If two objects interact, the force F_{21} exerted by object 1 on object 2 is equal in magnitude but opposite in direction to the force F_{12} exerted by object 2 on object 1.**

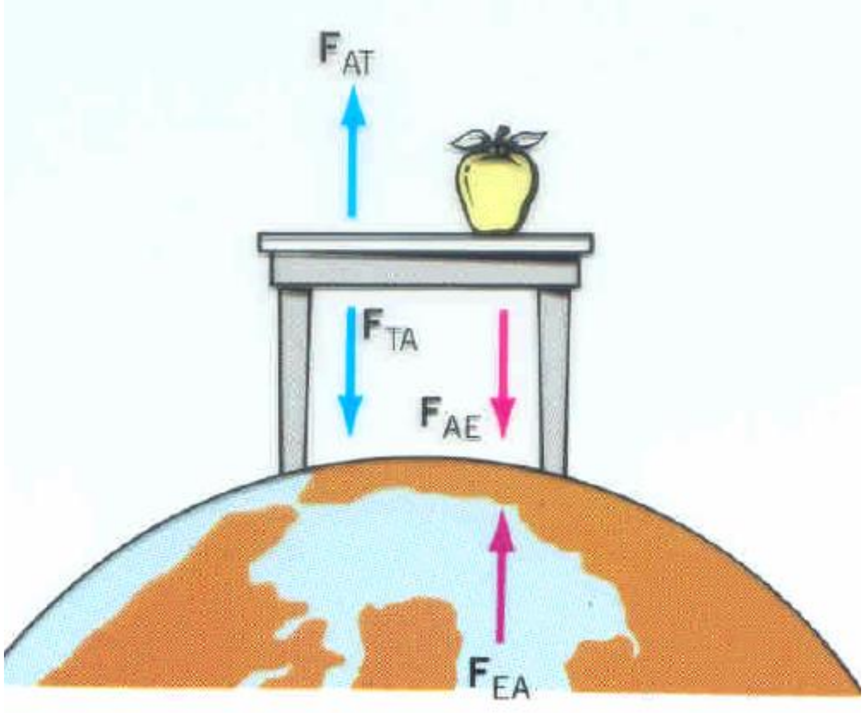
$$\vec{F}_{12} = -\vec{F}_{21}$$

  - F_{ba} means "the force exerted by a on b".



Newton's Third Law

- Simultaneous
- Same type of force
- The two forces in action-reaction pair always act on two different objects, and can not be counteracted.



§4 Some Particular Forces

物理学并不仅仅满足于把各式各样的力罗列出来，因为，物理学认为客观世界的现象虽是复杂的，但原因却是简单的，从本质上讲，自然界并不存在如此多种类型的力，我们希望寻求各种现象的统一。

- **Strong Force (1)**
- **Electromagnetic Force (10^{-2})**
- **Weak Force (10^{-5})**
- **Gravitational Force (10^{-39})**

Some Particular Forces

- **Action-at-a-Distance Forces**
  - **Gravitational Force**
  - **Electrical Force**
  - **Magnetic Force**

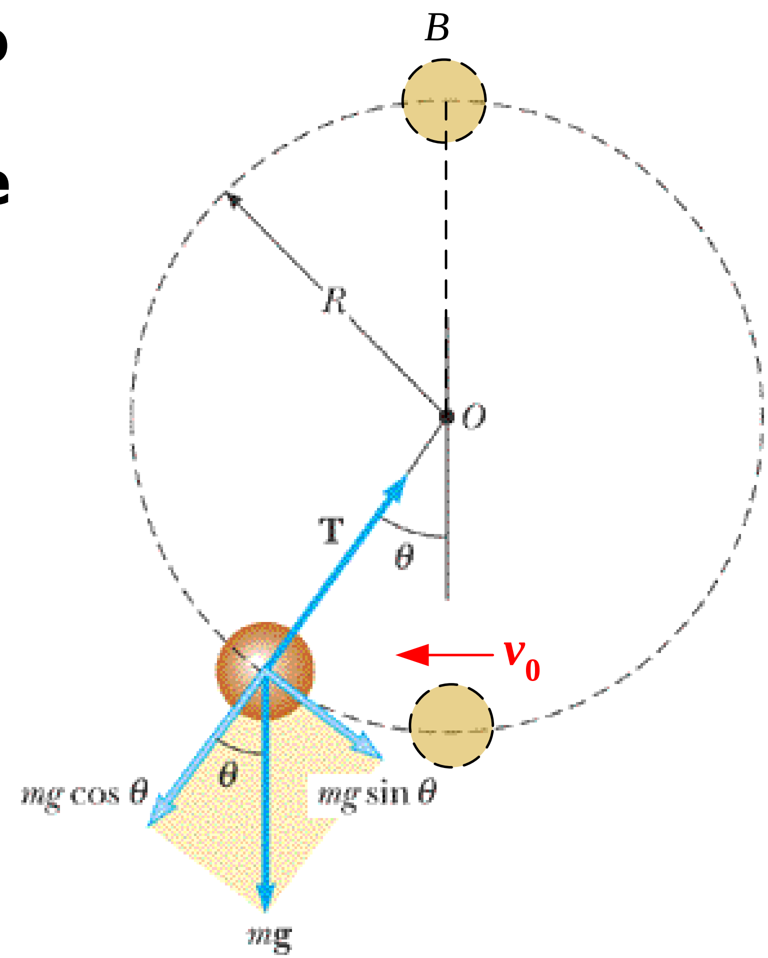
- **Contact Force**
  - **Frictional Force**
  - **Air Resistance Force**
  - **Spring Force**

Example

A small ball of mass m is attached to the end of a cord of length R, which rotates under the influence of gravitational force in vertical circle about a fixed point O.

(1) Determine the tension T in the cord at any angle θ. ($v = v_0$ when $\theta = 0$)

(2) When the ball starts motion in the bottom of the circle, in order to pass point B which is the top of the circle, find the minimum value of initial velocity v_0.



Solution

Reference frame: the Earth

$$\vec{T} + m\vec{g} = m\vec{a}$$

Coordinate system: natural coordinate

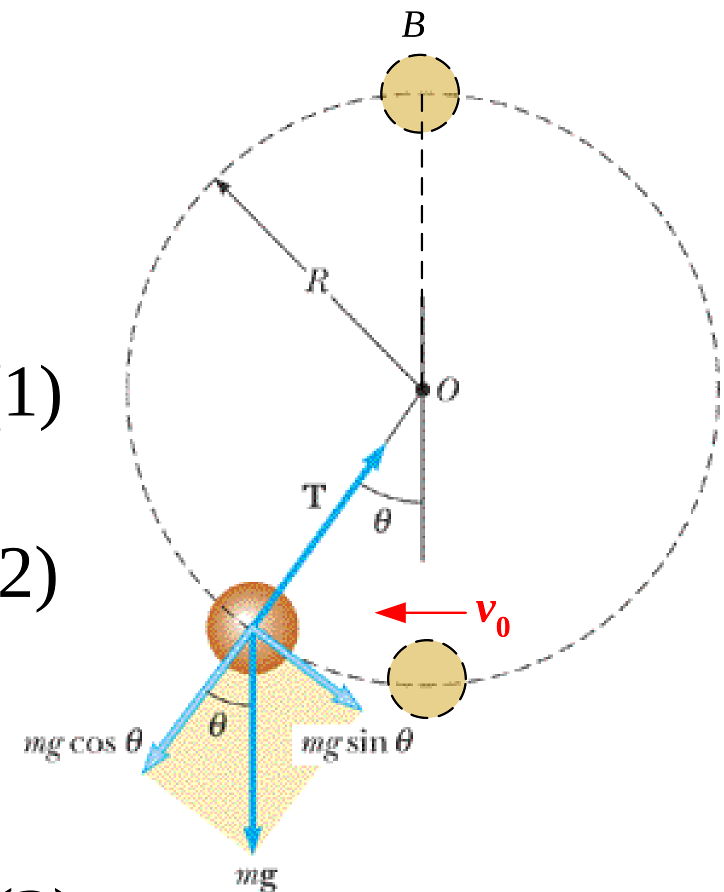
Normal: $$T - mg\cos\theta = m\frac{v^2}{R} \qquad (1)$$

Tangential: $$-mg\sin\theta = m\frac{dv}{dt} \qquad (2)$$

Unknown (θ, v, t).

Additional equation to be found.

Circular motion: $$v = R\omega = R\frac{d\theta}{dt} \qquad (3)$$

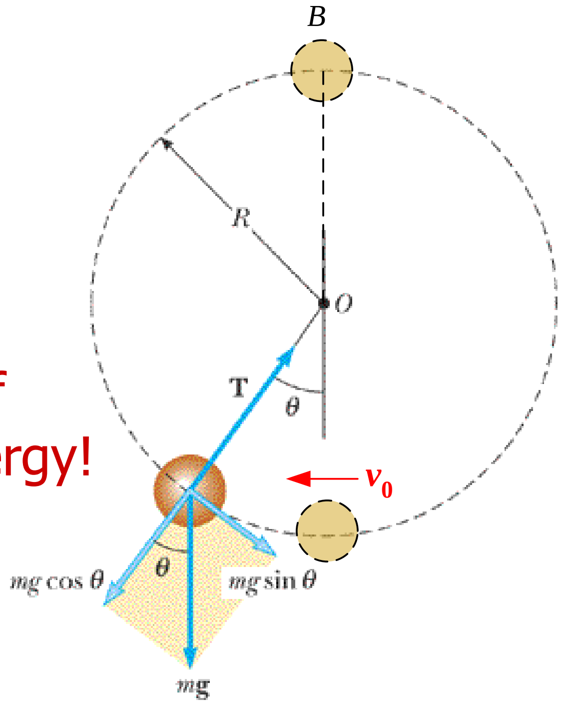


Example (continued)

Change the independent variable t to θ.

(2)→ $$-mg\sin\theta = m\frac{dv}{d\theta}\frac{d\theta}{dt} = m\omega\frac{dv}{d\theta} = m\frac{v}{R}\frac{dv}{d\theta}$$

$$\int_{v_0}^{v} mv dv = \int_0^{\theta} -mgR\sin\theta d\theta$$



$$\frac{1}{2}mv_0^2 - \frac{1}{2}mv^2 = mgR - mgR\cos\theta$$

Conservation of mechanical energy!

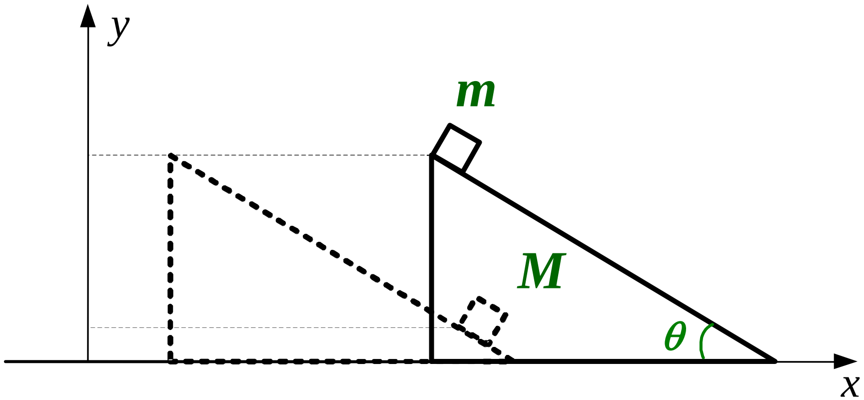
$$\boxed{T = \frac{mv_0^2 - mgR(2-3\cos\theta)}{R}}$$

At the point B, $T \geq 0$, and $\theta = 180°$

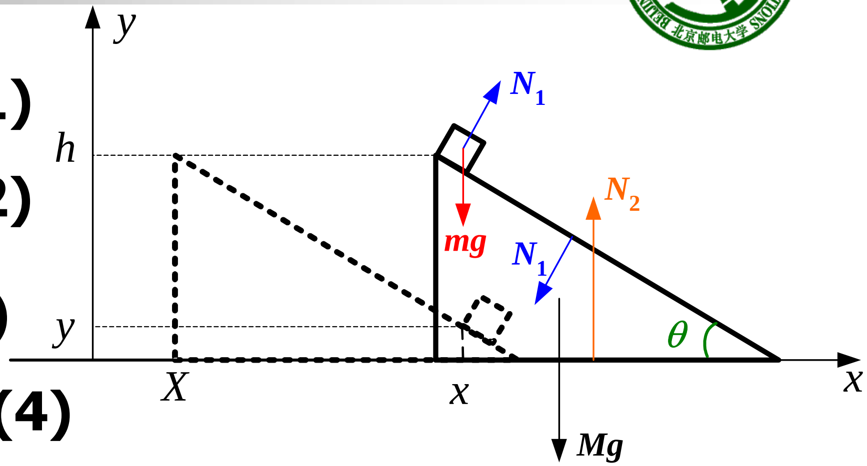
$$\frac{mv_0^2 - mgR(2+3)}{R} \geq 0 \qquad \boxed{v_0 \geq \sqrt{5gR}}$$

Example

A block of mass m is put on a wedge M, which, in turn, is put on a horizontal table. The incline angle of wedge is θ. All surfaces are frictionless. Determine the accelerations of block m and wedge M.



Solution



m: **Horizontal:** $N_1 \sin\theta = m a_x$ **(1)**

Vertical: $N_1 \cos\theta - mg = m a_y$ **(2)**

M: **Horizontal:** $-N_1 \sin\theta = M a_0$ **(3)**

Vertical: $N_2 - N_1 \cos\theta - Mg = 0$ **(4)**

(unknown: N_1, N_2, a_x, a_y, a_0)

Motion relation: $\tan\theta = \dfrac{h-y}{x-X}$ **(5)** $-\ddot{y}\cos\theta = \ddot{x}\sin\theta - \ddot{X}\sin\theta$

$\Rightarrow -a_y \cos\theta = a_x \sin\theta - a_0 \sin\theta$

$$a_x = \frac{g\sin\theta\cos\theta}{1+\frac{m}{M}\sin^2\theta}, \quad a_y = -\frac{\left(1+\frac{m}{M}\right)g\sin^2\theta}{1+\frac{m}{M}\sin^2\theta}, \quad a_0 = -\frac{\frac{m}{M}g\sin\theta\cos\theta}{1+\frac{m}{M}\sin^2\theta}$$

Example (continued)

Check the results:

(1) by dimensional analysis. ——reasonable

(2) directions are correct.

(3) by introducing extreme cases.

$\theta = 0,\ \theta = \pi/2$

If $M >> m$

$$a_x \to (g\sin\theta)\cos\theta,$$

$$a_y \to -(g\sin\theta)\sin\theta,$$

$$a_0 \to 0,$$

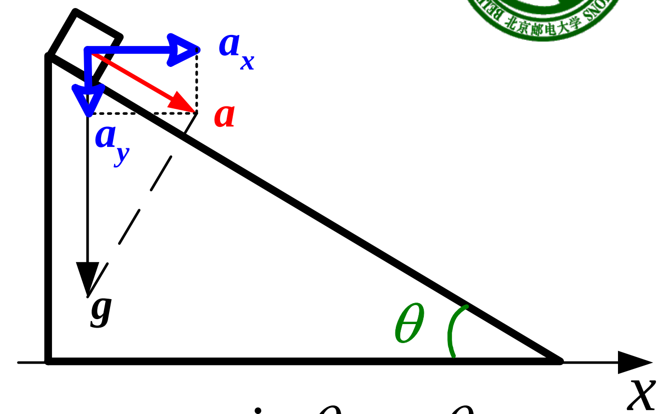
If $m >> M$

$$a_x \to 0$$

$$a_y \to -g$$

$$a_0 \to \frac{g}{\tan\theta}$$

The results are reasonable.



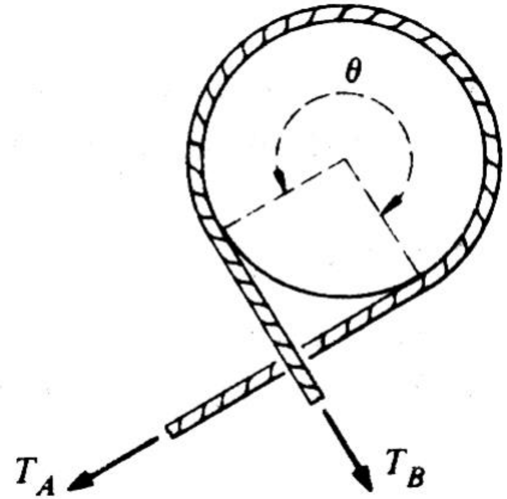
$$a_x = \frac{g\sin\theta\cos\theta}{1+\frac{m}{M}\sin^2\theta}$$

$$a_y = -\frac{\left(1+\frac{m}{M}\right)g\sin^2\theta}{1+\frac{m}{M}\sin^2\theta}$$

$$a_0 = -\frac{\frac{m}{M}g\sin\theta\cos\theta}{1+\frac{m}{M}\sin^2\theta}$$

Example

A device called a capstan (绞盘) is used aboard ships in order to control a rope which is under great tension. The rope is wrapped around a fixed drum, usually for several turns. The load on the rope (end B) pulls it with a force T_B, and the sailor (end A) holds it with a much smaller force T_A. Show that $T_A = T_B \exp(-\mu\theta)$, where μ is the coefficient of friction and θ is the total angle subtended by the rope on the drum.



The end for sailor holds The end attached to ship

Example

Solution: isolate a element of rope to consider.

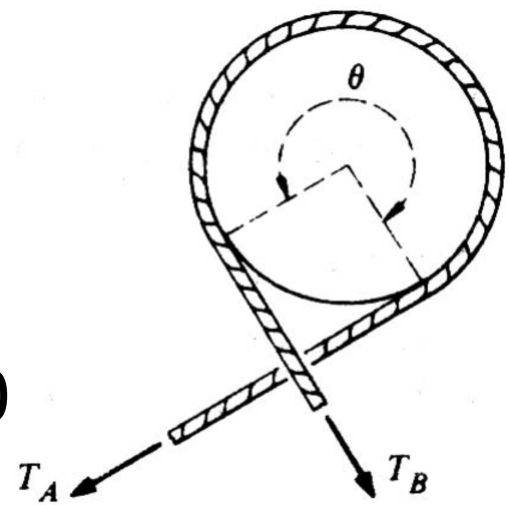
Tangential: $(T+dT)\cos(d\theta/2)-T\cos(d\theta/2)-\mu dN=0$

Normal: $(T+dT)\sin(d\theta/2)+T\sin(d\theta/2)-dN=0$

$$\sin(d\theta/2)\rightarrow d\theta/2$$

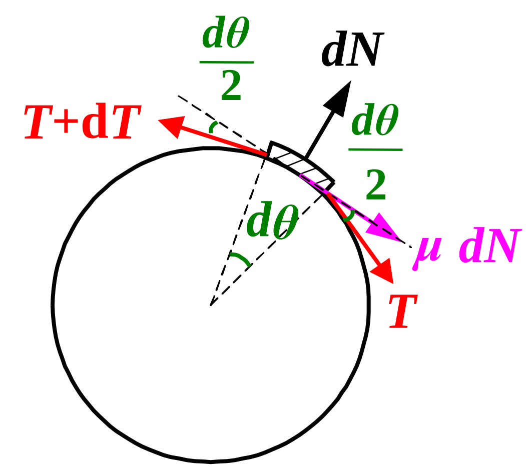
$$\cos(d\theta/2)\rightarrow 1$$

$$\begin{cases} dT-\mu dN=0 \\ Td\theta+\dfrac{1}{2}dTd\theta-dN=0 \end{cases}$$



The end for sailor holds

The end attached to ship



Neglect the second order infinitesimal ***dTdθ***

Example (continued)

$$\begin{cases} dT - \mu dN = 0 \\ Td\theta - dN = 0 \end{cases}$$

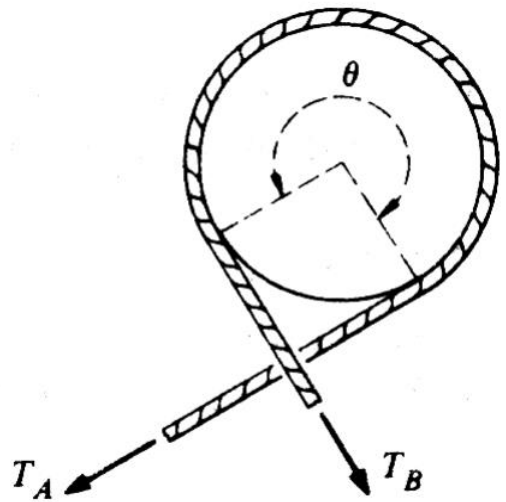
$$Td\theta = \frac{dT}{\mu}$$

$$\int_{T_A}^{T_B} \frac{dT}{T} = \int_0^{\theta} \mu d\theta$$

$$T_A = T_B \exp(-\mu\theta)$$

As long as the θ is large enough, we can get

$$T_A \ll T_B$$



The end for sailor holds

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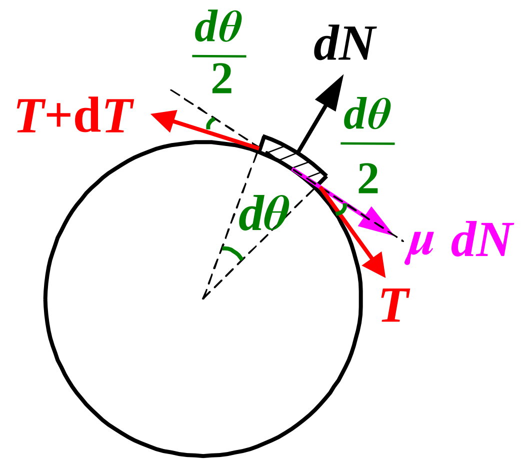

§ 5 Solving Problems using Newton's Laws

P88, P105

Problem-Solving Strategy

- **Isolate the object whose motion is being analyzed. Draw a separate free-body diagram for each object.**
  - **Be sure to include all the forces acting on the object, but be equally careful not to include any force exerted by this object on other object.**
  - **Never include the quantity $m\vec{a}$ in you free-body diagram. It's not a force.**
- **Establish a convenient reference frame and an appropriate coordinate system attached to it.**

Problem-Solving Strategy (continued)

- **For each object, write the equations for Newton's second law in component manner.**
  - **Generally, the number of unknowns must be equal to number of equations.**
  - **If number of unknowns < number of equations, there must be equivalent equations.**
  - **If number of unknowns > number of equations of Newton's second law, find relationship between motions. (this situation mostly occurs to many objects whose motions are dependent.)**
- **Solve the equations to find unknowns.**
- **Check the result**
  - **by introducing particular or extreme cases of quantities, when possible, and compare the results with your intuitive expectations. Ask, "Does this result make sense?"**
  - **by dimensional analysis.**

Problem

- Ch4 (P101)
  - 56，58
- Ch5 (P127)
  - 49，56