3.54 (a) solution

$$X(j\omega)=\int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$
$$=\int_3^{\infty} e^{-(2+j\omega)t}dt$$
$$=\lim_{b\to\infty}\left[-\frac{e^{-(2+j\omega)t}}{2+j\omega}\right]_3^b$$
$$=\frac{e^{-(2+j\omega)3}}{2+j\omega}$$

(d) solution

$$X(j\omega)=\int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$
$$=\int_{-\infty}^{\infty}\left[\sum_{m=0}^{\infty}a^m\delta(t-m)\right]e^{-j\omega t}dt.$$
$$=\sum_{m=0}^{\infty}a^m\left[\int_{-\infty}^{\infty}\delta(t-m)e^{-j\omega t}\right]$$
$$=\sum_{m=0}^{\infty}(ae^{-j\omega})^m$$
$$=\lim_{m\to\infty}\frac{1-a^m e^{-j\omega m}}{1-ae^{-j\omega}}$$
$$=\frac{1}{1-ae^{-j\omega}}$$

(f) solution

$$X(j\omega)=\int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$
$$=\int_{-2}^{0}e^{t}e^{-j\omega t}dt+\int_0^2 e^{-t}e^{-j\omega t}dt$$
$$=\frac{e^{(1-j\omega)t}}{1-j\omega}\Big|_{-2}^{0}-\frac{e^{-(1+j\omega)t}}{1+j\omega}\Big|_0^2$$
$$=\frac{1}{1-j\omega}-\frac{e^{2(j\omega-1)}}{1-j\omega}-\frac{e^{-2(j\omega+1)}}{1+j\omega}+\frac{1}{1+j\omega}$$
$$=\frac{2-2e^{-2}\cos(2\omega)+2\omega e^{-2}\sin(2\omega)}{1+\omega^2}$$

3.55. (a) solution.

$$x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}X(j\omega)e^{j\omega t}d\omega$$
$$=\frac{1}{2\pi}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\cos(2\omega)e^{j\omega t}d\omega$$
$$=\frac{1}{2\pi}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{e^{2j\omega}+e^{-2j\omega}}{2}e^{j\omega t}d\omega$$
$$=\frac{1}{4\pi}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}e^{j\omega(2+t)}+e^{j\omega(t-2)}d\omega$$

注意：t等于±2的情况，应该从这一步开始分析

$$=\frac{1}{4\pi}\left[\frac{e^{j\omega(2+t)}}{j(2+t)}+\frac{e^{j\omega(t-2)}}{j(t-2)}\right]_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$$
$$=\begin{cases}\dfrac{\sin[\frac{\pi}{4}(2+t)]}{2\pi(2+t)}+\dfrac{\sin[\frac{\pi}{4}(t-2)]}{2\pi(t-2)}, & t\neq\pm 2\\ 1/8, & t=\pm 2\end{cases}$$

(c) solution.

$$x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}X(j\omega)e^{j\omega t}d\omega$$
$$=\frac{1}{2\pi}\left(\int_{-\infty}^{0}e^{2\omega}e^{j\omega t}d\omega+\int_0^{+\infty}e^{-2\omega}e^{j\omega t}d\omega\right)$$
$$=\frac{1}{2\pi}\left[\frac{e^{(2+jt)\omega}}{2+jt}\Big|_{-\infty}^{0}+\frac{e^{-(2-jt)\omega}}{jt-2}\Big|_0^{\infty}\right]$$
$$=\frac{1}{2\pi}\left(\frac{1}{2+jt}+\frac{1}{-jt+2}\right)$$
$$=\frac{2}{\pi(4+t^2)}$$

(e) solution

$$x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}X(j\omega)e^{j\omega t}d\omega$$

$$=\frac{1}{2\pi}\int_{-3}^{3}\frac{2j}{3}\omega e^{j\omega t}d\omega$$

$$=\frac{1}{2\pi}\int_{-3}^{3}\frac{2\omega}{3t}de^{j\omega t}$$

$$=\frac{1}{2\pi}\left(\frac{2\omega}{3t}e^{j\omega t}\Big|_{-3}^{3}-\int_{-3}^{3}\frac{2e^{j\omega t}}{3t}d\omega\right).$$

$$=\frac{1}{2\pi}\left(\frac{4}{t}\cos 3t-\frac{4}{3t^2}\sin 3t\right).$$

$$=\frac{2\cos 3t}{\pi t}-\frac{2\sin 3t}{3\pi t^2}.$$

$$x(t)=\begin{cases}\frac{2\cos 3t}{\pi t}-\frac{2\sin 3t}{3\pi t^2}, & t\neq 0\\ 0, & t=0.\end{cases}$$

3.52. (b) solution:

$$X(e^{j\Omega})=\sum_{n=-\infty}^{\infty}x[n]e^{-j\Omega n}$$

$$=\sum_{n=-\infty}^{\infty}a^{|n|}e^{-j\Omega n}$$

$$=\sum_{n=-\infty}^{0}a^{-n}e^{-j\Omega n}+\sum_{n=0}^{\infty}a^{n}e^{-j\Omega n}$$

$$=\sum_{n=-\infty}^{-1}(ae^{j\Omega})^{-n}+\sum_{n=0}^{\infty}(ae^{-j\Omega})^{n}$$

$$=\frac{ae^{j\Omega}}{1-ae^{j\Omega}}+\frac{1}{1-ae^{j\Omega}}$$

$$=\frac{1-a^2}{1+a^2-2a\cos(\Omega)}$$

(c) solution

$$X(e^{j\Omega})=\sum_{n=-\infty}^{\infty}x[n]e^{-j\Omega n}$$

$$=\sum_{n=-N}^{N}\left[\frac{1}{2}+\frac{1}{2}\cos\left(\frac{\pi}{N}n\right)\right]e^{-j\Omega n}.$$

$$=\sum_{n=-N}^{N}\frac{1}{2}e^{-j\Omega n}+\sum_{n=-N}^{N}\frac{e^{(\frac{\pi}{N}-j\Omega)n}}{4}+\sum_{n=-N}^{N}\frac{e^{-(\frac{\pi}{N}+j\Omega)n}}{4}$$

$$=\frac{1}{2}\frac{e^{-j\Omega N}(1-e^{j\Omega N})}{1-e^{j\Omega}}+\frac{1}{4}\frac{e^{-(\pi+j\Omega N)}(1-e^{\pi+j\Omega N})}{1-e^{(\frac{\pi}{N}+j\Omega)}}+\frac{1}{4}\frac{e^{(\pi-j\Omega N)}(1-e^{j\Omega N-\pi})}{1-e^{(-\frac{\pi}{N}+j\Omega)}}$$

$$=\frac{e^{-j\Omega N}-1}{2(1-e^{j\Omega})}+\frac{e^{-(\pi+j\Omega N)}-1}{4-4e^{(\frac{\pi}{N}+j\Omega)}}+\frac{e^{(\pi-j\Omega N)}-1}{4-4e^{(-\frac{\pi}{N}+j\Omega)}}$$

(e) solution

$$X(e^{j\Omega})=\sum_{n=-\infty}^{\infty}x[n]e^{-j\Omega n}$$

$$=e^{-j\Omega(-4)}+e^{-j\Omega(-2)}+e^{-j\Omega 2}-e^{-j\Omega 4}$$

$$=2j\sin 4\Omega+2\cos 2\Omega.$$

3.53. (a) solution

$$x[n]=\frac{1}{2\pi}\int_{-\pi}^{\pi}X(e^{j\Omega})e^{j\Omega n}d\Omega$$

$$=\frac{1}{2\pi}\int_{-\pi}^{\pi}e^{j2\Omega}e^{j\Omega n}d\Omega$$

$$=\frac{1}{2\pi}\cdot\frac{e^{j(2+n)\Omega}}{j(2+n)}\Big|_{-\pi}^{\pi}$$

$$=\frac{1}{2\pi}\frac{e^{j(2+n)\pi}-e^{-j(2+n)\pi}}{j(2+n)}$$

$$=\frac{\sin(n\pi+2\pi)}{\pi(2+n)}$$

$$=\delta(2+n).$$

$$\lim_{n\to -2}\frac{\sin(n\pi+2\pi)}{\pi(2+n)}=1$$

(d) solution.

$$x[n]=\frac{1}{2\pi}\left[\int_{-\pi}^{0}e^{(jn+1)\Omega}d\Omega+\int_{0}^{\pi}e^{(jn-1)\Omega}d\Omega\right]$$

$$=\frac{1}{2\pi}\left[\frac{e^{(jn+1)\Omega}}{jn+1}\Big|_{-\pi}^{0}+\frac{e^{(jn-1)\Omega}}{jn-1}\Big|_{0}^{\pi}\right]$$

$$=\frac{1}{2\pi}\left[\frac{1-e^{-(jn+1)\pi}}{jn+1}+\frac{e^{(jn-1)\pi}-1}{jn-1}\right].$$

$$=\frac{1}{2\pi}\frac{[-2+2e^{-\pi}(-1)^n]}{(-n^2-1)}$$

$$=\frac{1-e^{-\pi}(-1)^n}{\pi(n^2+1)}$$

(f) solution.

$$x[n] = \frac{1}{2\pi}\left[\int_{-\frac{\pi}{2}}^{0} e^{-j\pi} e^{j\Omega n}\, d\Omega + \int_{0}^{\frac{\pi}{2}} e^{j\Omega n}\, d\Omega\right]$$

$$= \frac{1}{2\pi}\left[\frac{e^{j(n\Omega-\pi)}}{j(n-\pi)}\Bigg|_{-\frac{\pi}{2}}^{0} + \frac{e^{j\Omega n}}{jn}\Bigg|_{0}^{\frac{\pi}{2}}\right]$$

$$= \frac{1}{2\pi} \cdot \frac{e^{-j\pi} - e^{j(-\frac{\pi}{2}n-\pi)}}{jn} + \frac{e^{j\frac{\pi}{2}n}-1}{jn}$$

$$= \frac{1}{2\pi} \times \frac{-1+e^{-j\frac{\pi}{2}n}}{jn} + \frac{e^{j\frac{\pi}{2}n}-1}{jn}$$

$$= \frac{\cos(\frac{\pi}{2}n)-1}{j\pi n}$$

$$x[n] = \begin{cases} \frac{\cos(\frac{\pi}{2}n)-1}{j\pi n}, & n \neq 0 \\ \frac{1}{4}(1+e^{-j\pi}), & n = 0. \end{cases}$$