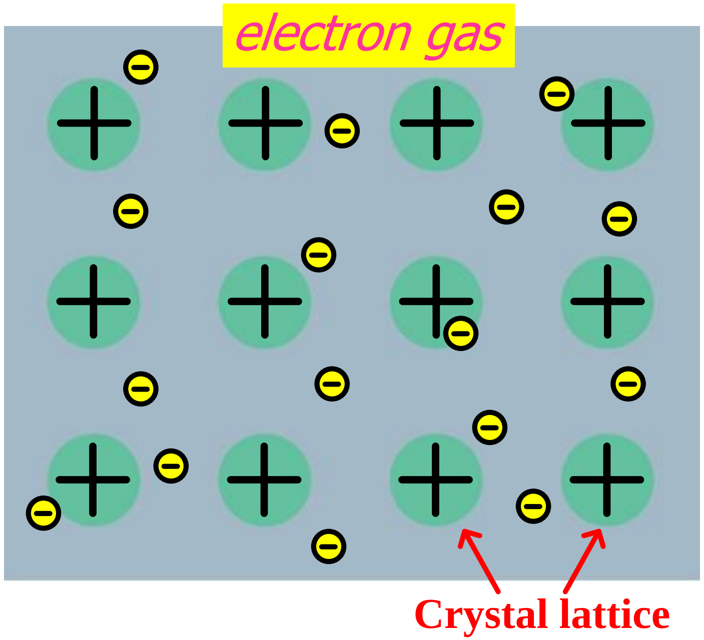
§ 5 Conductors in Electrostatic Equilibrium

(P473 § 19-9)

- **The characteristics of a electrical conductor**

  - **A good electrical conductor contains charges that are not bound to any atom and free to move about within the conductor —— called free charge.**

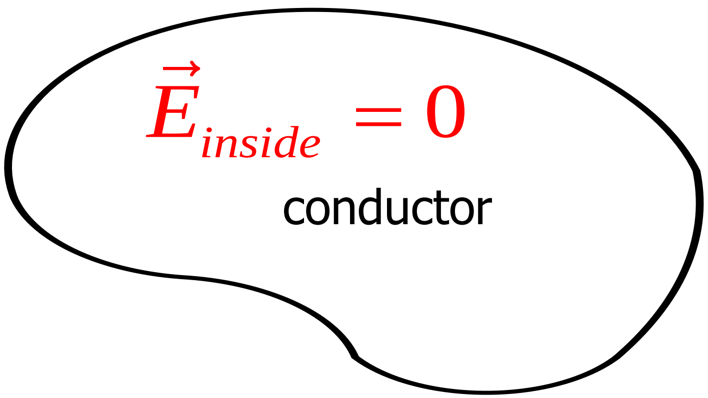
  - **When no motion of charge occurs within the conductor, the conductor is in electrostatic equilibrium.**



The properties

- **The properties that an isolated conductor in electrostatic equilibrium.**

① **The electric field is zero everywhere inside the conductor.**



If the field were not zero, free charges in the conductor would accelerate under the action of the electric field —— not the case in electrostatic equilibrium

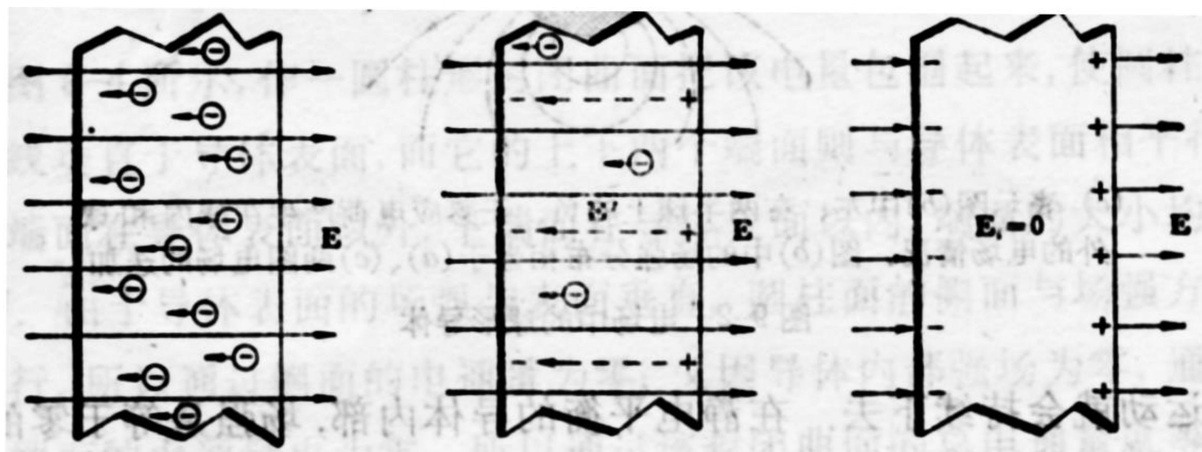
- ## The Mechanism.

If we apply a external field E, the free electrons inside the conductor will move under E and are accumulated on the surface of the conductor, and establish another field E' until the total field inside the conductor reach zero.

$$\overrightarrow{E}_{inside} = \vec{E} + \vec{E}'$$

$$= 0$$

Now the conductor is in electrostatic equilibrium.



(a) 导体中自由电子在外电场作用下作宏观的定向运动

(b) 导体的两个相对表面上出现的感应电荷在增加中

(c) 最后，导体处于静电平衡状态，感应电荷不再变化，导体内的总场强 $E_i=0$

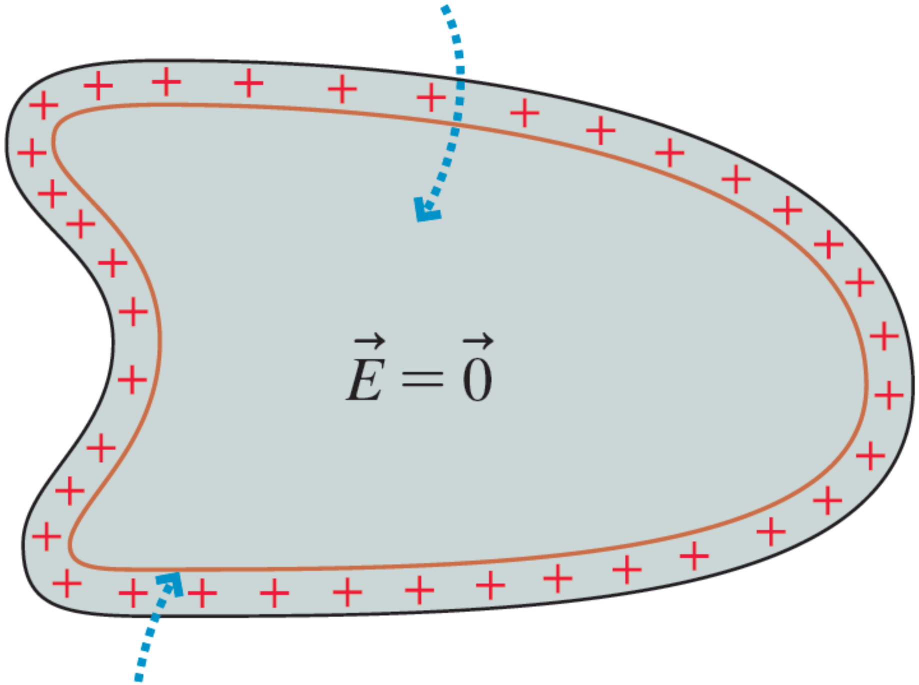
② **If the isolated conductor carries a net charge, the net charge resides entirely on its surface.**

$$\oiint_S \vec{E} \cdot d\vec{A} = \frac{q_{inside}}{\varepsilon_0},$$

Inside the conductor,

$$\vec{E}_{inside} = 0 \;\Rightarrow\; q_{inside} = 0$$

The electric field inside is zero.

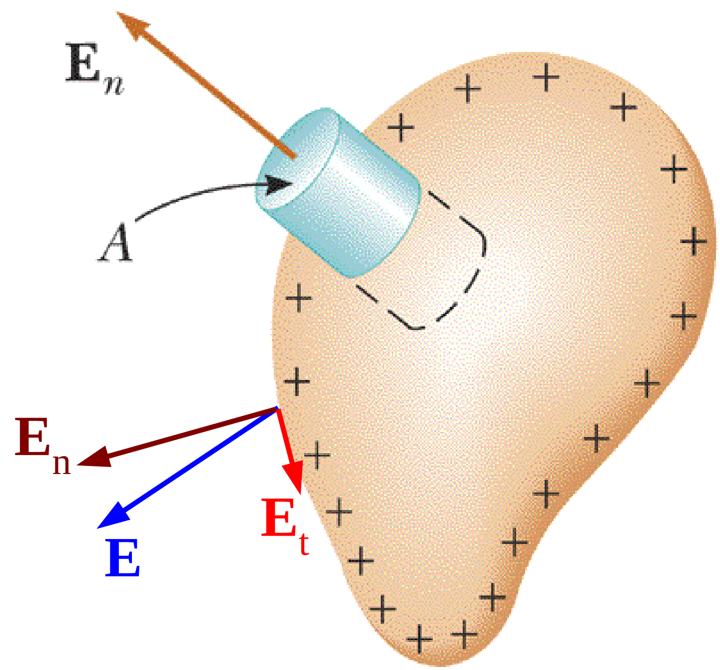


The flux through the Gaussian surface is zero. Hence all the excess charge must be on the surface.

③ The electric field just outside the charged conductor is perpendicular to the conductor surface and has a magnitude σ/ε_0, where σ is the surface charge density at that point.

If E had a component parallel to the surface, the free charges would move along the surface, and so the conductor would not be in equilibrium.

Draw a small cylinder just containing the surface of the conductor.



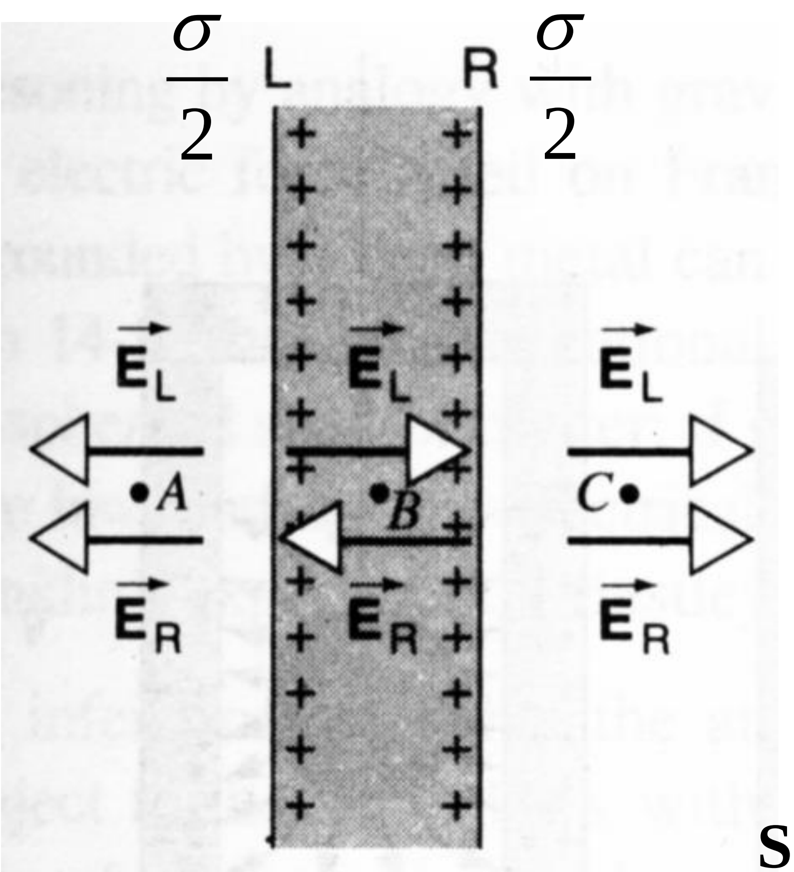
$$\oint\!\!\oint_S \vec{E}\cdot d\vec{A} = EA$$

$$= \frac{q_{in}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$$

$$\boxed{E = \frac{\sigma}{\varepsilon_0}}$$

Example

Find E_A, E_B, and E_C.



Solution (I): Superposition principle

$$E_A = -\frac{\sigma/2}{2\varepsilon_0} - \frac{\sigma/2}{2\varepsilon_0} = -\frac{\sigma}{2\varepsilon_0}$$

$$E_B = \frac{\sigma/2}{2\varepsilon_0} - \frac{\sigma/2}{2\varepsilon_0} = 0$$

$$E_C = \frac{\sigma/2}{2\varepsilon_0} + \frac{\sigma/2}{2\varepsilon_0} = \frac{\sigma}{2\varepsilon_0}$$

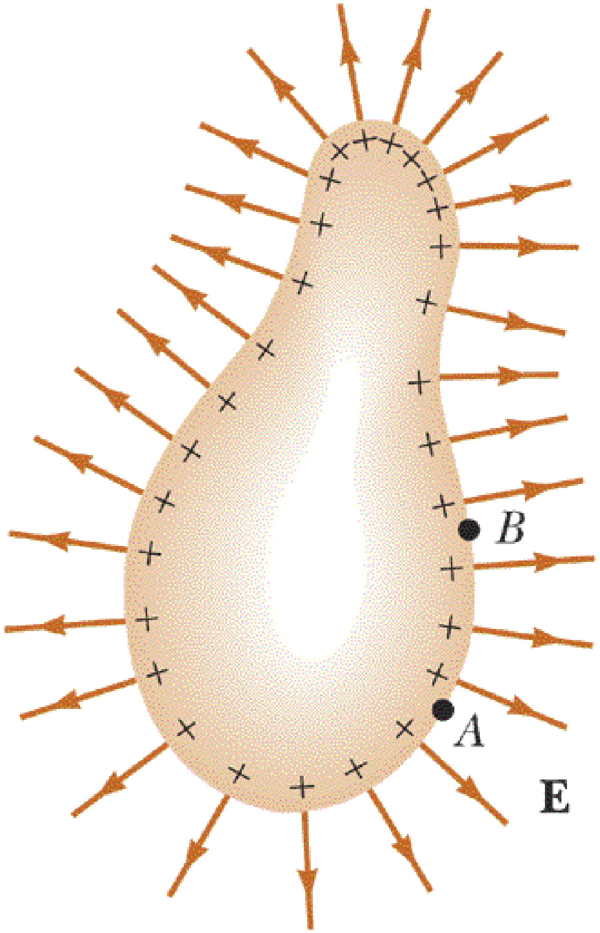
Solution (II): One infinite plane

$$E_A = -E_C = -\frac{\frac{\sigma}{2} + \frac{\sigma}{2}}{2\varepsilon_0} = -\frac{\sigma}{2\varepsilon_0}$$

Solution (III): Electrostatic equilibrium

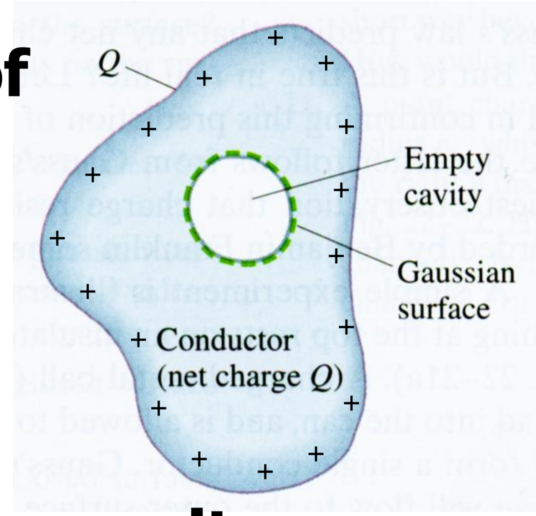
$$E_A = -E_C = -\frac{\sigma/2}{\varepsilon_0} = -\frac{\sigma}{2\varepsilon_0}, \quad E_B = 0$$

④ **On an irregularly shaped conductor, the surface charge density is highest at locations where the radius of curvature of the surface is smallest.**

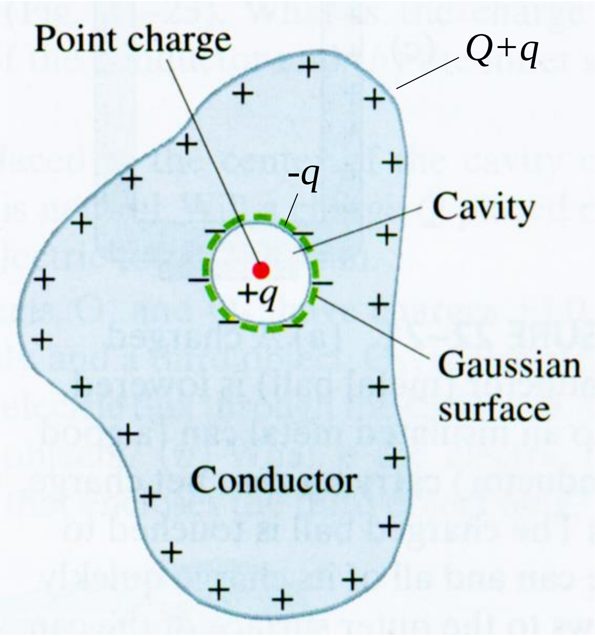


The charge distribution for a conductor cavity

- **No charge in the internal cavity of the conductor.**
  - **There is no charge at the inner surface of the cavity.**

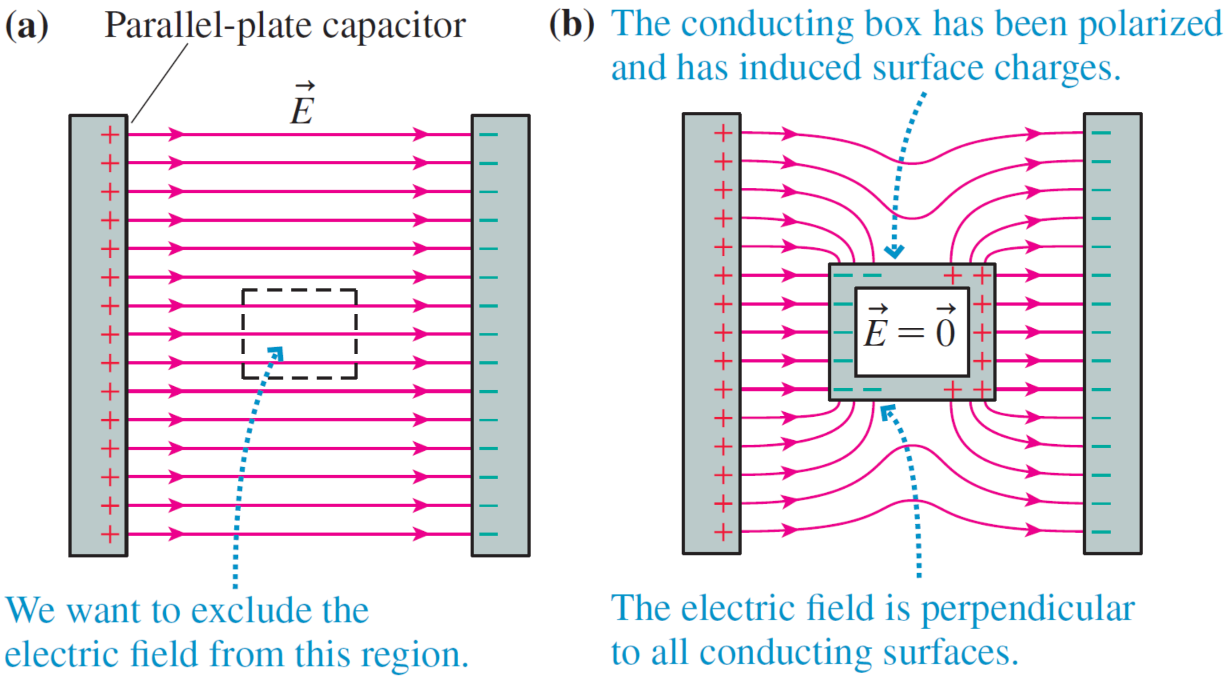


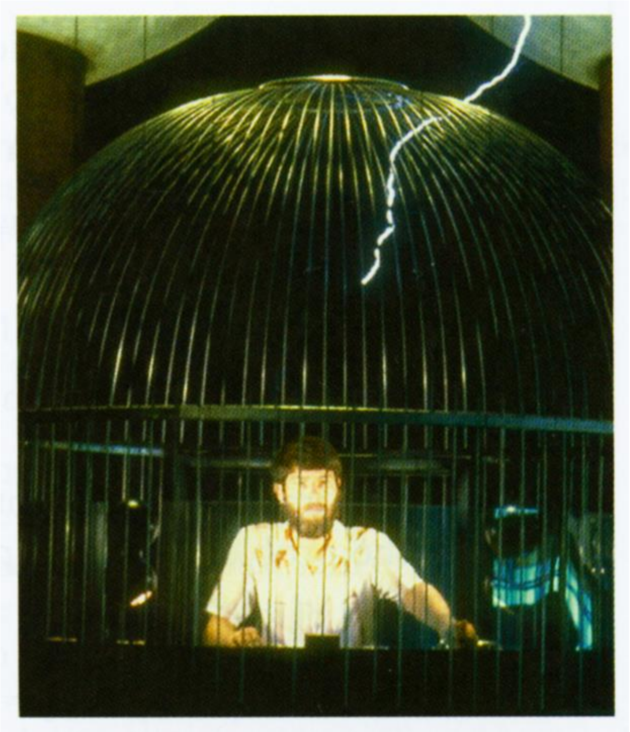
- **A point charge $+q$ is place inside the cavity.**
  - **A charge $-q$ must be attracted to the inner surface of the cavity to keep the net charge zero within the Gaussian surface.**
  - **A charge of $Q+q$ will appear on the outer surface of the cavity, so that the net charge of the conductor does not change.**



The application of conductor cavity

Shielding, and safety in a strong electric field

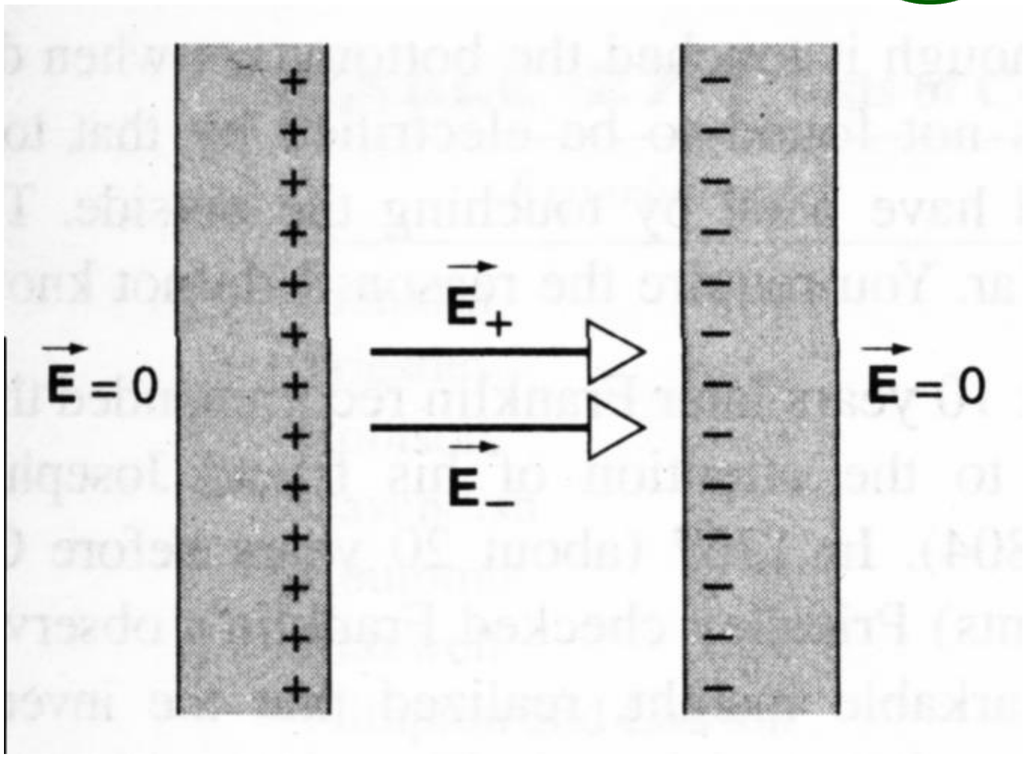




Faraday cage

Example

Two thin **conducting plates** carry equal and opposite charges $+q$ and $-q$. Find the **electric fields** between the two plates and at the two sides of the plates.



Example

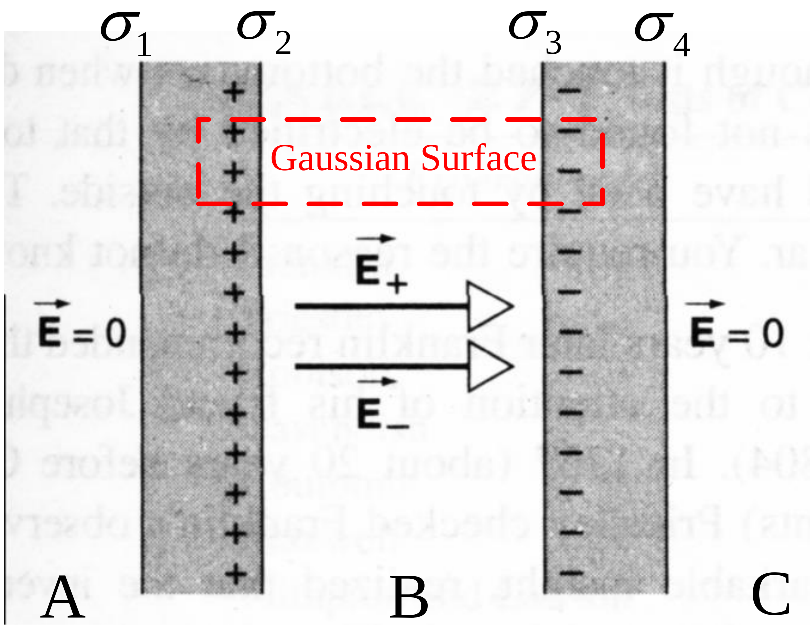
Solution: Conservation of net charge:

$$(\sigma_1 + \sigma_2)S = +q$$

$$(\sigma_3 + \sigma_4)S = -q$$

Gauss's law:

$$\oiint_S \vec{E} \cdot d\vec{A} = 0 = \frac{(\sigma_2 + \sigma_3)A}{\varepsilon_0} \Rightarrow \sigma_2 = -\sigma_3$$



The field inside the plate 2 is zero:

$$E_{2in} = \frac{\sigma_1}{2\varepsilon_0} + \frac{\sigma_2}{2\varepsilon_0} + \frac{\sigma_3}{2\varepsilon_0} - \frac{\sigma_4}{2\varepsilon_0} = 0$$

$$\Rightarrow \begin{cases} \sigma_1 = \sigma_4 = 0 \\ \sigma_2 = -\sigma_3 = \dfrac{q}{S} \end{cases}$$

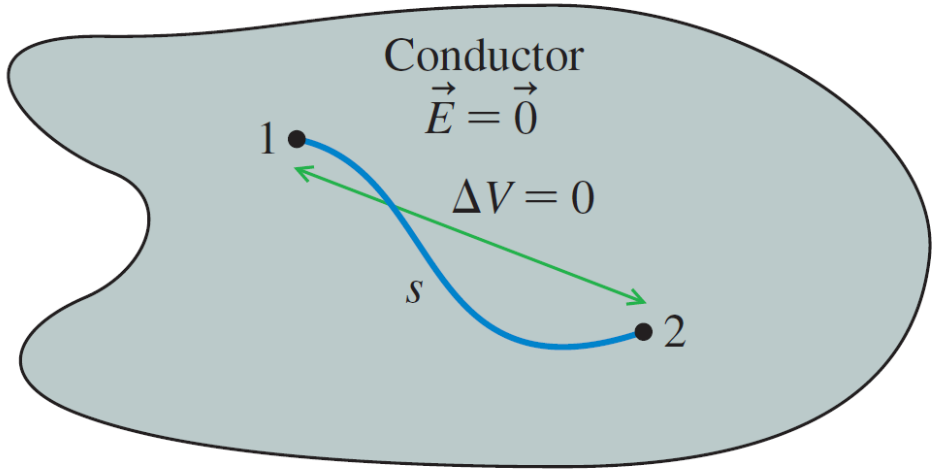
$$\boxed{E_A = \frac{\sigma_1}{\varepsilon_0} = 0, \quad E_B = \frac{\sigma_2}{\varepsilon_0} = \frac{q}{S\varepsilon_0}, \quad E_C = \frac{\sigma_4}{\varepsilon_0} = 0}$$

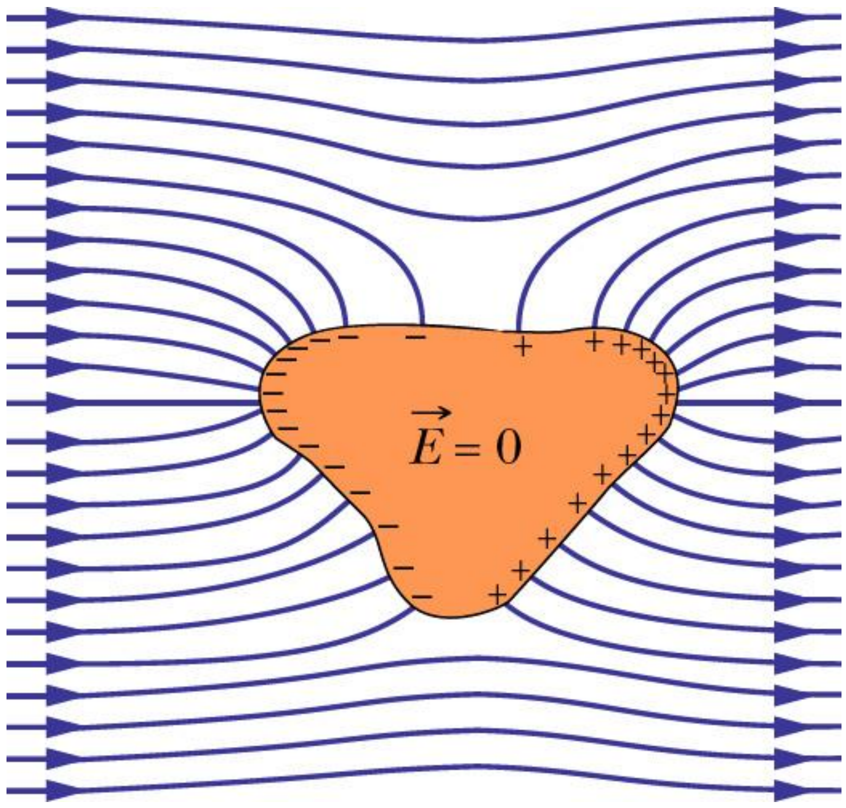
When $|+Q| \neq |-q|$,

$$\begin{cases} \sigma_1 = \sigma_4 = \dfrac{Q-q}{2S} \\ \sigma_2 = -\sigma_3 = \dfrac{Q+q}{2S} \end{cases}$$

- **The properties that an isolated conductor in electrostatic equilibrium**

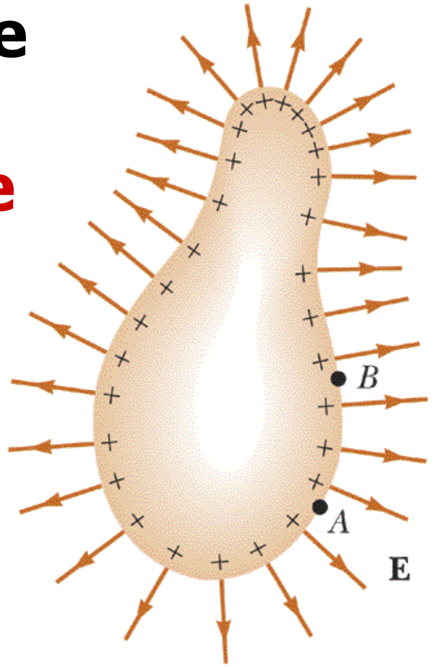
⑤ **The entire conductor is at the same potential. The surface of a conductor is always an equipotential surface.**





The validity of property ④

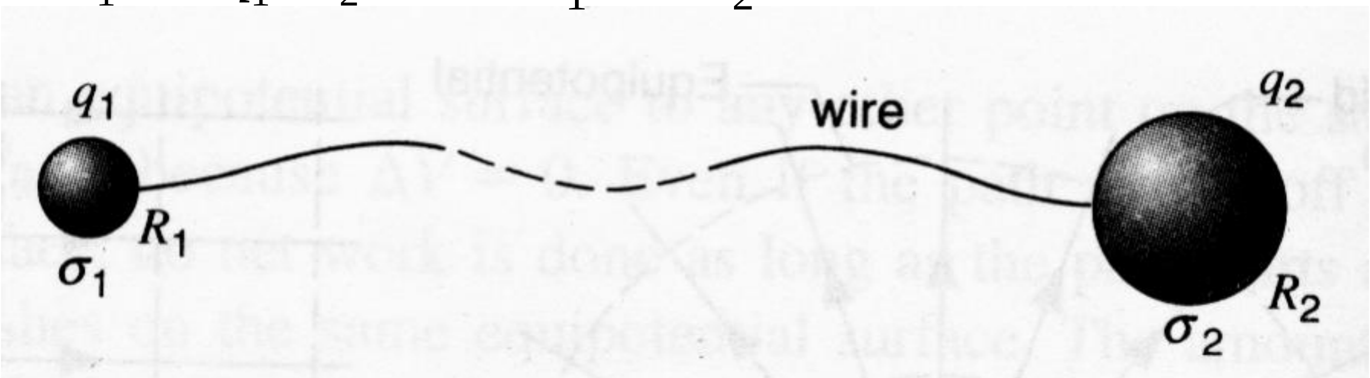
④ **On an irregularly shaped conductor, the surface charge density is highest at locations where the radius of curvature of the surface is smallest.**



Consider two conducting spheres of different radii connected by a fine wire, let the entire assembly be raised to same arbitrary potential V .

$$V=\frac{1}{4\pi\varepsilon_0}\frac{q_1}{R_1}=\frac{1}{4\pi\varepsilon_0}\frac{q_2}{R_2},\quad \text{which yields}\quad \frac{q_2}{q_1}=\frac{R_2}{R_1}$$

$$\frac{\sigma_2}{\sigma_1}=\frac{q_2/4\pi R_2^2}{q_1/4\pi R_1^2}=\frac{q_2}{q_1}\frac{R_1^2}{R_2^2},\qquad \frac{\sigma_2}{\sigma_1}=\frac{R_1}{R_2},\qquad R_1<R_2\Rightarrow\sigma_1>\sigma_2$$

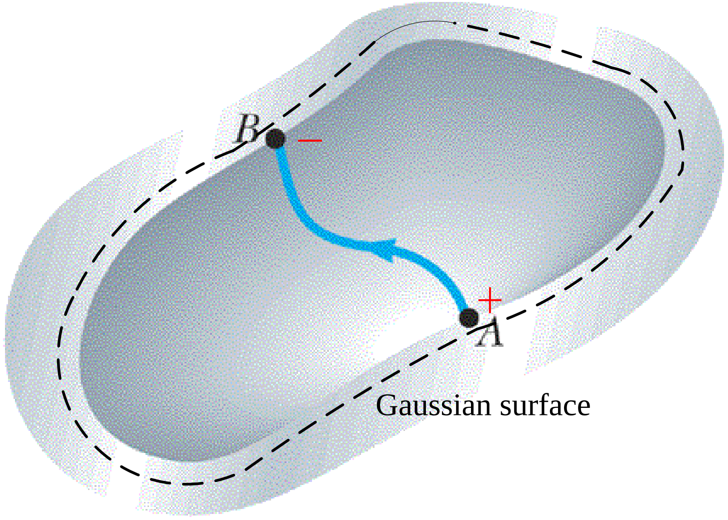


The property of an internal cavity in the conductor

- The validity of the statement "There is no charge at the inner surface of the cavity".

Draw a Gaussian surface just inside the inner surface.

$$\oint_S \vec{E} \cdot d\vec{A} = \frac{\sum q_{in}}{\varepsilon_0} = 0 \quad \Rightarrow \quad \sum q_{in} = 0$$



Is zero charge every where?

If not, then

$$V_A - V_B = \int_A^B \vec{E} \cdot d\vec{s} > 0$$

It is contradictory to the fact that the surface of a conductor is an equipotential surface.

Summary

Electric properties of a conductor in electrostatic equilibrium