① $\left(1-\frac{1}{4}\right)+\left(\frac{1}{3}-\frac{1}{8}\right)+\cdots+\left(\frac{1}{2n-1}-\frac{1}{4n}\right)+\cdots$ 敛散性.

② $\sum_{n=2}^{\infty}\frac{(-1)^n}{n\ln^{\alpha}n}$ 敛散性 $\sum_{n=0}^{\infty}a_n(x-1)^n$

① $\frac{1}{2n-1}-\frac{1}{4n}\sim\frac{1}{4n}$ $S_n \geq C\sum_{k=1}^{n/2}\frac{1}{4k}$ 发散

② $\sum_{n=2}^{\infty}\frac{1}{n\ln^{\alpha}n}\sim\int_2^{\infty}\frac{1}{x\ln^{\alpha}x}dx$ $t=\ln x$ $\int\frac{1}{n\ln n}$

$\sim\int^{\infty}\frac{1}{t^{\alpha}}dt$ $\frac{\text{conv} \quad \alpha>1}{\text{div} \quad \alpha\leq 1}$

$\sum_{n=2}^{\infty}\frac{(-1)^n}{n\ln^{\alpha}n}$ $\frac{1}{n\ln^{\alpha}n}\downarrow$ conv conditionally

$\alpha\leq 1$ 条件 $\alpha>1$ 绝对.

$\sum_{n=0}^{\infty}a_n(x-1)^n$ conv $x=-\frac{1}{2}$ div $x=\frac{5}{2}$ $\sum a_n x^n$

(i) $\sum_{n=1}^{\infty}n a_n x^{n-1}$ $R=?$ $\frac{3}{2}$ $\sum(n^2+3n+\frac{1}{n})a_n x^n$

$t=(x-1)$ $x=-\frac{1}{2}$ $x-1=-\frac{3}{2}$ $\sum_{n=0}^{\infty}a_n t^n$ $\boxed{R=\frac{3}{2}}$

$x=\frac{5}{2}$ $x-1=\frac{3}{2}$

$\left(\sum_{n=0}^{\infty}a_n x^n\right)'=\sum_{n=0}^{\infty}n a_n x^{n-1}$ $R=\frac{3}{2}$

$\sum_{n=0}^{\infty}\frac{x^{2n+1}}{2n+1}$ 和函数: $\underline{S(x)}$

$S'(x)=\sum_{n=0}^{\infty}x^{2n}=\frac{1}{1-x^2}$ $(x^2<1)$ $q=x^2<1$ $\frac{1}{1-q}$

$$=\frac{1}{2}\left(\frac{1}{1-x}+\frac{1}{1+x}\right)$$

$$S(x)=\frac{1}{2}\int\frac{1}{1-x}dx+\frac{1}{2}\int\frac{1}{1+x}dx$$

$$=-\frac{1}{2}\ln|1-x|+\frac{1}{2}\ln|1+x|+C \qquad x\in(-1,1)$$

$$=\frac{1}{2}\ln\frac{1+x}{1-x}+C$$

$$S(0)=0 \Rightarrow C=0 \qquad S(x)=\frac{1}{2}\ln\frac{1+x}{1-x} \quad x\in(-1,1)$$

$$\sum_{n=0}^{\infty}\frac{n+1}{n!}\left(\frac{1}{2}\right)^n=\underline{\frac{3}{2}\sqrt{e}} \qquad x\cdot e^x=\sum_{n=0}^{\infty}\frac{1}{n!}x^n\cdot x$$

$$S(x)=\sum_{n=0}^{\infty}\frac{n+1}{n!}x^n \qquad \int S(x)dx=\sum_{n=0}^{\infty}\frac{1}{n!}x^{n+1}=xe^x$$

$$S(x)=(xe^x)'=xe^x+e^x \qquad S\left(\frac{1}{2}\right)=\frac{3}{2}e^{\frac{1}{2}}=\frac{3}{2}\sqrt{e}.$$

$f(x)=\int_0^x\frac{\ln(1+t)}{t}dt$. 麦克劳林级数.

$$\ln(1+t)=t-\frac{t^2}{2}+\frac{t^3}{3}-\cdots=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}t^n$$

$$\frac{\ln(1+t)}{t}=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}t^{n-1}}{n} \qquad \int_0^x t^{n-1}dt=\left.\frac{t^n}{n}\right|_0^x=\frac{x^n}{n}$$

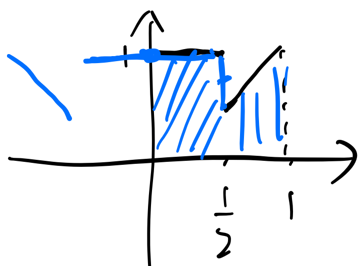
$$\int_0^x\frac{\ln(1+t)}{t}dt=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}x^n}{n^2},\quad -1\le x\le 1$$

$$f(x)=\begin{cases}1 & 0\le x<\frac{1}{2}\\ x & \frac{1}{2}\le x\le 1\end{cases} \qquad f(x)\sim\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos n\pi x \;\; \text{“}S(x)\text{”} \;/$$

$$a_n = 2\int_0^1 f(x)\cos n\pi x \, dx \quad n \geq 0$$

$$\sum_{n=1}^{\infty} a_n = \frac{1}{8}$$

$$\sum_{n=1}^{\infty} a_n = S(0) - \frac{a_0}{2} = 1 - \frac{a_0}{2}$$



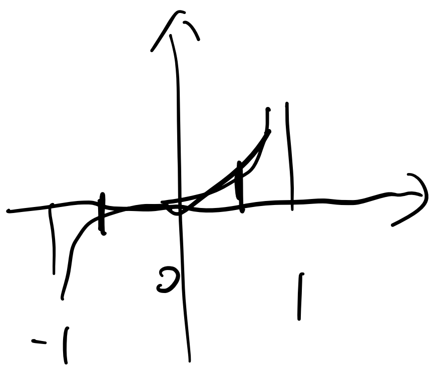
$$S\left(\frac{1}{2}\right) = \frac{3}{4}$$

$$a_0 = 2\int_0^1 f(x)\,dx$$

$$= 2\left(1 - \frac{1}{8}\right)$$

$$\sum_{n=1}^{\infty} a_n = 1 - \frac{1}{2}\cdot 2\left(1 - \frac{1}{8}\right) = \frac{1}{8}$$

$f(x) = x^2 \ (x \in [0, 1])$ 正弦级数 $\to S(x)$. $S\left(-\frac{5}{2}\right) = ?$



$$S\left(-\frac{5}{2}\right) = S\left(-\frac{1}{2}\right) = -S\left(\frac{1}{2}\right)$$

$$= -f\left(\frac{1}{2}\right) = -\frac{1}{4}$$

$$\lim_{(x,y)\to(1,0)} \frac{xy - \ln(1+xy)}{(2x-1)y^2} \neq \lim_{x\to 1}\lim_{y\to 0}$$

$$\ln(1+x) = x - \frac{x^2}{2} + o(x^2)$$

$$\frac{0-0}{1\cdot 0}$$

$$xy - \ln(1+xy) = \frac{(xy)^2}{2} + o((xy)^2)$$

$$\boxed{xy \to 0}$$

$$\frac{\frac{(xy)^2}{2} + o((xy)^2)}{y^2} = \frac{x^2}{2} = \frac{1}{2} \quad (o(1) = 0)$$

$$\lim_{y\to 0}\left(\qquad\right)x$$

$$f(x,y)=e^{\frac{x}{y}}+(y+1)\arctan\frac{x-2}{x^2 y} \qquad f_x(1,-1)=\underline{-\frac{1}{e}}$$

$y=-1$

$$f(x,y)=\begin{cases}\dfrac{\sin xy-y}{xy^2} & xy\neq 0\\ 0 & xy=0.\end{cases} \qquad f_y(1,0)=\underline{-1/6}$$

$$f(1,y)=\begin{cases}\dfrac{\sin y-y}{y^2} & y\neq 0.\\ 0 & y=0\end{cases} \qquad \sin x=x-\frac{x^3}{6}+o(x^3)$$

$$f(1,y)\Big|_{y=0}=\lim_{\Delta y\to 0}\frac{f(1,y+\Delta y)-f(1,y)}{y+\Delta y-y}\Bigg|_{y=0}=\lim_{\Delta y\to 0}\frac{\frac{\sin(\Delta y)-\Delta y}{\Delta y^2}-0}{\Delta y}$$

$$=\frac{1}{6}$$

$$u=\int_{xyz}^{x-y-z}e^{t^2}dt \qquad du\Big|_{(1,0,1)}=\underline{\qquad}$$

$$u=\int_0^{x-y-z}e^{t^2}dt-\int_0^{xyz}e^{t^2}dt$$

$$\frac{\partial u}{\partial x}=\frac{\partial(x-y-z)}{\partial x}\cdot e^{(x-y-z)^2}-\frac{\partial(xyz)}{\partial x}\cdot e^{(xyz)^2}$$

$$=e^{(x-y-z)^2}-yz\,e^{(xyz)^2}$$

$\frac{\partial u}{\partial x}\Big|_{(1,0,1)} = 1 \qquad -2 \qquad -1$

$$d\int_0^{w} e^{t^2}dt \Big/ \cancel{dw} = e^{w^2}dw$$

$$du = d\int_{xyz}^{x-y-z} e^{t^2}dt$$

$$= d\int_0^{x-y-z} e^{t^2}dt - d\int_0^{xyz} e^{t^2}dt$$

$$= e^{(x-y-z)^2}d(x-y-z) - e^{(xyz)^2}d(xyz)$$

$$du\big|_{(1,0,1)} = \left(1\cdot d(x-y-z) - 1\cdot d(xyz)\right)\Big|_{(1,0,1)}$$

$$\left(dx - dy - dz - yz\,dx - xz\,dy - xy\,dz\right)\Big|_{(1,0,1)}$$

$$\to 1\cdot dx - 2dy - 1dz$$

$z = f(x,y) \qquad dz = (axy^3 - y^2\cos x + x)\,dx + (by\sin x + 3x^2y^2)\,dy$

$a = \_\_ \qquad b = \_\_\_ \qquad \frac{\partial z}{\partial x} \qquad \frac{\partial z}{\partial y}$

$$\int \frac{\partial z}{\partial x}\,dx = \_\_ + C(y)$$

$$\frac{\partial \frac{\partial z}{\partial x}}{\partial y} = \frac{\partial z}{\partial y\partial x} \qquad \frac{\partial \frac{\partial z}{\partial y}}{\partial x} = \frac{\partial z}{\partial x\partial y}$$

$$\frac{\partial z}{\partial y\partial x} = 3axy^2 - 2y\cos x = \frac{\partial z}{\partial x\partial y} = by\cos x + 6xy^2$$

$\forall\, xy$

$3a = 6 \qquad -2 = b$

$a = 2 \qquad b = -2$

$z = f(x, u, v) \quad \underline{u = u(x) \quad v = v(x)}$

$$\begin{cases} \int_v^u e^{t^2} dt = x \\ u + v = x^2 \end{cases}$$

$z \begin{cases} u - x \\ v - x \\ x \end{cases}$

$\underline{f_x} \quad f_1$

$$\frac{dz}{dx} = f_1' + f_2' \cdot \frac{du}{dx} + f_3' \frac{dv}{dx}$$

$$d \int_v^u e^{t^2} dt = dx = e^{u^2} du - e^{v^2} dv$$

$$d(u+v) = dx^2 \qquad du + dv = 2x\,dx$$

$$\begin{cases} e^{u^2} du - e^{v^2} dv = dx \\ du + dv = 2x\,dx \end{cases}$$

$$\frac{du}{dx} = \frac{\begin{vmatrix} 1 & -e^{v^2} \\ 2x & 1 \end{vmatrix}}{\begin{vmatrix} e^{u^2} & -e^{v^2} \\ 1 & 1 \end{vmatrix}} = \frac{1 + 2xe^{v^2}}{e^{u^2} + e^{v^2}}$$

$$\frac{dv}{dx} = \frac{\begin{vmatrix} e^{u^2} & 1 \\ 1 & 2x \end{vmatrix}}{\begin{vmatrix} e^{u^2} & -e^{v^2} \\ 1 & 1 \end{vmatrix}} = \frac{2xe^{u^2} - 1}{e^{u^2} + e^{v^2}}$$

$$\begin{cases} z = x^2 + y^2 \\ x + y + z = 4 \end{cases}$$ 在 $M(1, 1, 2)$ tangent normal plane

$\underline{y(x)}, \ \underline{z(x)} \quad \left(\frac{dx}{dx}, \frac{dy}{dx}, \frac{dz}{dx}\right)$ tangent vector

$F(x, y, z) = x^2 + y^2 - z$

$G(x, y, z) = x + y + z - 4$

$$\frac{\partial(F,G)}{\partial(y,z)} = \begin{vmatrix} 2y & -1 \\ 1 & 1 \end{vmatrix} = 2y+1 \Big|_{(1,1,2)} = 3$$

$$\frac{\partial(F,G)}{\partial(z,x)} = \begin{vmatrix} -1 & 2x \\ 1 & 1 \end{vmatrix} = \qquad -3$$

$$\frac{\partial(F,G)}{\partial(x,y)} = \begin{vmatrix} 2x & 2y \\ 1 & 1 \end{vmatrix} \qquad 0$$

$$\left[\ \frac{x-1}{3} = \frac{y-1}{-3} = \frac{z-2}{0} \qquad z = \cancel{0}\ 2\right.$$