Part I Multiple Choice (30%, each multiple choice question has a choice of multiple answers, **only one** of which is correct, 3 marks for each question)

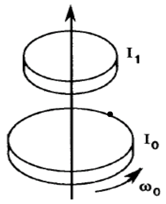
1. An object is moving in the $\boldsymbol{x}$ direction with velocity $\boldsymbol{v_x(t)}$, and $\mathbf{d}\boldsymbol{v_x}/\mathbf{d}\boldsymbol{t}$ is a nonzero constant. With $\boldsymbol{v_x = 0}$ when $\boldsymbol{t = 0}$, then for $\boldsymbol{t > 0}$ the quantity $\boldsymbol{v_x}\mathbf{d}\boldsymbol{v_x}/\mathbf{d}\boldsymbol{t}$ is: 【C】

(A) negative (B) zero (C) positive (D) not determined from the information given.

2. A single conservative force $\vec{F} = (6.0x - 12)\hat{i}$ N, where $\boldsymbol{x}$ is in meters, acts on a particle moving along an $\boldsymbol{x}$ axis. The potential energy $\boldsymbol{U}$ associated with this force is assigned a value of **27 J** at $\boldsymbol{x = 0}$. The maximum positive potential energy is: 【B】

(A) 38 **J** (B) 39 **J** (C) 40 **J** (D) 41 **J**

3. A cylinder with a moment of inertia I_0 rotates with angular velocity ω_0, second cylinder with moment of inertial I_1 initially not rotating drops onto the first cylinder and the two reach the same final angular velocity ω_f. Find ω_f 【B】



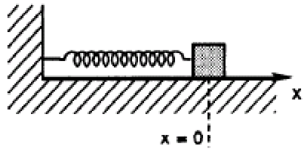
(A) $\omega_f = \omega_0 I_0/I_1$ (B) $\omega_f = \omega_0 I_0/(I_0 + I_1)$

(C) $\omega_f = \omega_0 I_1/I_0$ (D) $\omega_f = \omega_0 (I_0 + I_1)/I_0$

4. A wave has an amplitude 20.0 cm, a wavelength 3.00 m, and the wave travels 60.0 m in 12.0 s. What is the frequency of the wave? 【D】

(A) 250 Hz (B) 150 Hz (C) 8.33 Hz (D) 1.67 Hz

5. A 100g mass attached to a spring moves on a horizontal frictionless table in simple harmonic motion with amplitude 16cm and period 2s. Assuming that the mass is released from rest at t=0s and x=-16cm, find the displacements as a function of time. 【C】



(A) $x = 16\cos(\pi t)$ (B) $x = -16\cos(\pi t + \pi)$

(C) $x = 16\cos(\pi t + \pi)$ (D) $x = -16\cos(2\pi t + \pi)$

6. A certain parallel-plate capacitor has a capacitance of 5.0 μF. After it is charged to 5.0 μC and isolated, the plates are brought closer together so its capacitance becomes 10 μF. The work done by the agent is about 【B】

(A) 1.25×10^{-6} J. (B) -1.25×10^{-6} J. (C) 8.3×10^{-7} J. (D) -8.3×10^{-7} J.

7. A point charge is placed at the center of a spherical Gaussian surface. The electric flux Φ_E is changed if 【C】

(A) the sphere is replaced by a cube of one-tenth the volume.

(B) the point charge is moved off center (but still inside the original sphere).

(C) the point charge is moved to just outside the sphere.

(D) a second point charge is placed just outside the sphere.

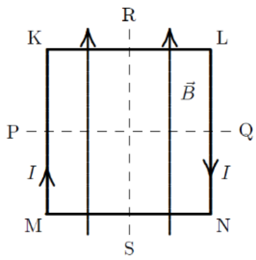
8. A square loop of wire lies in the plane of the page and carries a current I as shown. There is a uniform magnetic field $\vec{B}$ parallel to the side MK as indicated. The loop will tend to rotate 【A】

(A) about PQ with KL coming out of the page.

(B) about PQ with KL going into the page.

(C) about RS with MK coming out of the page.

(D) about RS with MK going into the page.



9. Two parallel long wires carry the same current and repel each other with a force F per unit length. If both these currents are doubled and the wire separation is tripled, the force per unit length becomes 【D】

(A) $2F/9$. (B) $4F/9$. (C) $2F/3$. (D) $4F/3$.

10. Suppose you are looking into one end of a long cylindrical tube in which there is a uniform electric field, pointing away from you. If the magnitude of the field is decreasing with time the direction of the induced magnetic field is: 【D】

(A) toward you. (B) away from you.

(C) clockwise. (D) counterclockwise.

Part II Filling the Blanks (30%, 3 marks for each blank)

11. The angular acceleration of a wheel, as a function of time, is $\alpha = 5.0t^2 - 3.5t$, where α is in **rad/s²** and t in seconds. If the wheel starts from rest ($\theta = \omega = 0$ at $t = 0$). The angular position θ at t = 1.0s is $-\frac{1}{6}$ rad.

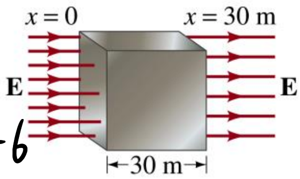
12. One force acting on a machine part is $\vec{F} = (-5.00\ \mathrm{N})\vec{i} + (4.00\ \mathrm{N})\vec{j}$. The vector from the origin to the point where the force is applied is $\vec{r} = (-0.450\ \mathrm{m})\vec{i} + (0.150\ \mathrm{m})\vec{j}$. The vector torque produced by this force is $-1.05\hat{k}$ N·m.

13. A solid cylinder with a radius of 10 cm and a mass of **3.0** kg is rotating about its center with an angular speed of **3.5** rad/s. Its kinetic energy is 0.092 (J)

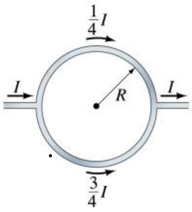
14. A **12** cm radius, **6.0** kg uniform ball is rotating about an axis through its center at **10** rev/s. The magnitude of angular momentum of the ball is 2.2 kg.m/s².

15. Two traveling waves have the forms $z_1 = A\sin(kx - \omega t)$, $z_2 = A\sin(kx + \omega t)$. Their super-position in the form of a standing wave can be expressed as $2A \sin kx \cos \omega t$

16. In a certain region of free space, the electric field is constant in the x direction, but its magnitude increases from E= 510 N/C at x=0 to E= 660 N/C at x=30 m. A cubical box of side l=30 m is oriented so that four of its sides are parallel to the field lines as shown in the figure. The charge within the box is 1.2×10^{-6} C. (The permittivity of free space is 8.85×10^{-12} $\mathrm{C^2/N\cdot m^2}$.)



17. A circular conducting ring of radius R is connected to two exterior straight wires ending at two ends of a diameter. The current I splits into unequal portions while passing through the ring as shown. The magnitude of the magnetic field at the center of the ring is $\frac{\mu_0 I}{8R}$.



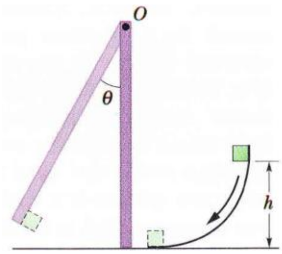
18. An electron (charge = -1.6×10^{-19} C) experiences a force $\vec{F} = (3.8\vec{i} - 2.7\vec{j})\times10^{-13}$ N when passing through a magnetic field $\vec{B} = -0.35\vec{k}$ T. The electron's velocity is $(4.8\vec{i} + 6.8\vec{j})\times10^{-6}$ m/s.

19. The radius of an elastic circular loop increases at a constant rate, dr/dt = 7.00 cm/s. The loop is in a magnetic field B = 0.48 T whose direction is perpendicular to the plane of the loop. At t=0, the loop has area A = 0.285 m². The emf induced in the loop at t = 1.00 s is 0.078 V.

20. At a given instant, a 1.8-A current flows in the wires connected to a parallel-plate capacitor. If the square plates of the capacitor are 1.60 cm on a side, the rate dE/dt at which the electric field is changing between the pates is 7.9×10^{14} V/(m·s).

Part III Problems (40%)

21.(10 marks) As shown in the figure, a small **50 g** block slides down a frictionless surface through height ***h*** **= 20 cm** and then sticks to a uniform rod of mass **100 g** and length **40 cm**. The rod pivots about point ***O*** through angle ***θ*** before momentarily stopping. Find ***θ***.



Solution:

Mechanical energy conservation applied to the particle (before impact) leads to

$$mgh = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gh}$$

for its speed right before undergoing the completely inelastic collision with the rod. The collision is described by angular momentum conservation:

$$mvd = \left(I_{\text{rod}} + md^2\right)\omega,$$

where

$$I_{\text{rod}} = \frac{1}{3}Md^2.$$

Thus, we obtain the angular velocity of the system immediately after the collision:

$$\omega = \frac{md\sqrt{2gh}}{\left(\frac{Md^2}{3}\right) + md^2}.$$

Mechanical energy conservation leads to

$$\frac{1}{2}\left(I_{\text{rod}} + md^2\right)\omega^2 = mgd(1-\cos\theta) + Mg\frac{d(1-\cos\theta)}{2}$$

from which we obtain

$$\cos\theta = 1 - \frac{h/d}{(1+M/2m)(1+M/3m)} = 0.85 \Rightarrow \theta = 32^\circ.$$

22. (10marks)A string oscillates according to the equation

$$y = (0.50 \text{ cm})\sin\left[\left(\frac{\pi}{3}\text{ cm}^{-1}\right)x\right]\cos\left[\left(40\pi \text{ s}^{-1}\right)t\right].$$

What are the (a) amplitude and (b) speed of the two waves whose superposition gives this oscillation? (c) What is the distance between nodes? (d) What is the transverse speed of a particle of the string at the position ***x*** **= 1.5 cm** when ***t*** **= 9/8 s**?

Solution:

(a) The amplitude of each of the traveling waves is half the maximum displacement of the string when the standing wave is present, or 0.25 cm.

(b) Each traveling wave has an angular frequency of

$$\omega = 40\pi$$

and an angular wave number of

$$k = \frac{\pi}{3}\text{ cm}^{-1}.$$

The wave speed is

$$v = \frac{\omega}{k} = 120 \text{ cm/s}.$$

(c) The distance between nodes is half a wavelength:

$$d = \frac{\lambda}{2} = \frac{\pi}{k} = 3.0 \text{ cm}.$$

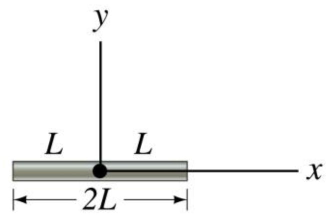
(d) The string speed is given by

$$u(x,t) = \frac{\partial y}{\partial t} = -\omega A\sin(kx)\sin(\omega t).$$

For the given coordinate and time,

$$u = -(40\pi \text{ rad/s})(0.50 \text{ cm})\sin\left[\left(\frac{\pi}{3}\text{ cm}^{-1}\right)(1.5 \text{ cm})\right]\sin\left[\left(40\pi \text{ s}^{-1}\right)\left(\frac{9}{8}\text{ s}\right)\right] = 0.$$

23. (10marks)A thin rod of length $2L$ is centered on the x axis as shown in the following figure. The rod carries a uniformly distributed charge Q. Determine the electric field E and the potential V as a function of x for points along the x axis outside the rod. Let V=0 at infinity.



Solution:

We choose a differential element of the rod dx' a distance x' from the origin of the coordinate system. The charge of the elements is

$$dq = \frac{Q}{2L}dx'.$$

We find the electric field by integrating along the rod

$$E = \int dE = \frac{1}{4\pi\varepsilon_0}\frac{Q}{2L}\int_{-L}^{L}\frac{dx'}{(x-x')^2}$$

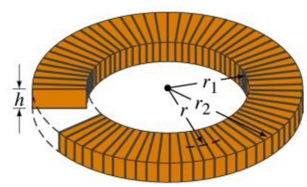
$$= \frac{1}{8\pi\varepsilon_0}\frac{Q}{L}\left(\frac{1}{x-L}-\frac{1}{x+L}\right) = \frac{Q}{4\pi\varepsilon_0}\frac{1}{x^2-L^2}.$$

The potential from the rod is

$$V = \int dV = \frac{1}{4\pi\varepsilon_0}\frac{Q}{2L}\int_{-L}^{L}\frac{dx'}{x-x'}$$

$$= \frac{1}{8\pi\varepsilon_0}\frac{Q}{L}\ln\left(\frac{x+L}{x-L}\right).$$

24. (10marks)A tightly wrapped toroid has a rectangular cross-section. (a) Determine a formula for the self-inductance L of this toroid. (b) Determine the energy density in the magnetic field as a function of r ($r_1<r<r_2$) and the total energy stored in the toriod, which carries a current I in each of its N loops. (r_1, r_2 and h are the dimensions as shown in the following figure.)

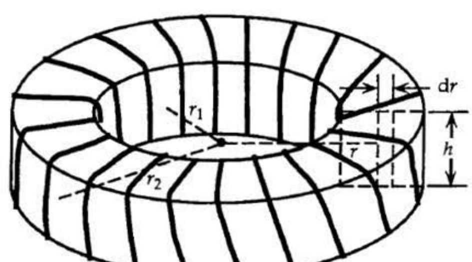


16. From the symmetry of the toroid, the magnetic field is circular. We apply Ampere's law to a circular path to find the magnetic field inside the toroid:

$\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0 I_{\text{enclosed}};$

$B2\pi r = \mu_0 NI$, which gives $B = \mu_0 NI/2\pi r$.

To find the magnetic flux through one turn of the toroid, we integrate over the rectangular cross-section. For a differential element, we choose a vertical strip at a radius r with width dr:



$$\Phi = \int \mathbf{B}\cdot d\mathbf{A} = \int_{r_1}^{r_2}\frac{\mu_0 NI}{2\pi r}h\,dr = \frac{\mu_0 NIh}{2\pi}\ln\left(\frac{r_2}{r_1}\right).$$

The flux through the entire toroidal winding is N times this, so the self-inductance is

$L = N\Phi/I = (\mu_0 N^2 h/2\pi)\ln(r_2/r_1)$.

22. From the symmetry of the toroid, the magnetic field is circular. We apply Ampere's law to a circular path to find the magnetic field inside the toroid:

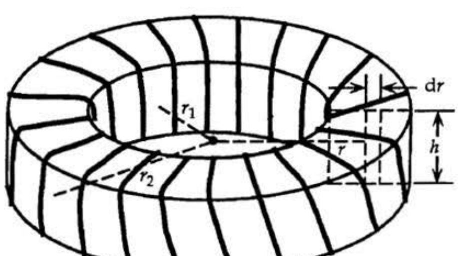
$\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0 I_{\text{enclosed}};$

$B2\pi r = \mu_0 NI$, which gives $B = \mu_0 NI/2\pi r$.

The energy density of this field is

$\boxed{u = \tfrac{1}{2}B^2/\mu_0 = \mu_0 N^2 I^2/8\pi^2 r^2.}$

To find the total energy stored in the magnetic field, we integrate over the volume. For a differential element, we choose a ring at a radius r with width dr and height h:



$$U = \int u\,dV = \int_{r_1}^{r_2}\frac{\mu_0 N^2 I^2}{8\pi^2 r^2}2\pi rh\,dr = \boxed{\frac{\mu_0 N^2 I^2 h}{4\pi}\ln\left(\frac{r_2}{r_1}\right).}$$