For examiners' use only

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3	
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BBC4111 A

Joint Programme Examinations 2021/22

BBC4111 Engineering Mathematics

Paper A

Time allowed 2 hours

Answer ALL questions

Complete the information below about yourself very carefully.

QM student number										
BUPT student number										
Class number										

NOT allowed: electronic calculators and electronic dictionaries.

INSTRUCTIONS

1. **You must NOT take answer books, used or unused, from the examination room.**
2. Write only with a black or blue pen **and in English**.
3. Do all rough work in the answer book – **do not tear out any pages**.
4. If you use Supplementary Answer Books, tie them to the end of this book.
5. Write clearly and legibly.
6. **Read the instructions on the inside cover.**

Examiners

Dr Xia Shi, Dr Huixia Mo

Filename: 2122_ BBC4111_A   No answer book required

Instructions

Before the start of the examination

1) Place your BUPT and QM student cards on the corner of your desk so that your picture is visible.

2) Put all bags, coats and other belongings at the back/front of the room. All small items in your pockets, including wallets, mobile phones and other electronic devices must be **placed in your bag in advance**. **Possession of mobile phones, electronic devices and unauthorised materials is an offence.**

3) Please ensure your mobile phone is switched off and that no alarm will sound during the exam. **A mobile phone causing a disruption is also an assessment offence.**

4) Do not turn over your question paper or begin writing until told to do.

During the examination

1) You must not communicate with or copy from another student.

2) If you require any assistance or wish to leave the examination room for any reason, please raise your hand to attract the attention of the invigilator.

3) If you finish the examination early you may leave, but not in the first 30 minutes or the last 10 minutes.

4) For 2 hour examinations you may **not** leave temporarily.

5) For examinations longer than 2 hours you **may** leave temporarily but not in the first 2 hours or the last 30 minutes.

At the end of the examination

1) You must stop writing immediately – **if you continue writing after being told to stop, that is an assessment offence.**

2) Remain in your seat until you are told you may leave.

Question1. [30 marks]

Fill in all the following blanks. Only the final results are required to be written down.

a) The modulus of the complex number $z = \frac{(3+i)(2-i)}{(2+i)(3-i)(1+i)}$ is ( ).**[3 marks]**

b) The function $f(z) = \begin{cases} 0, & z = 0 \\ \frac{(\bar{z})^2}{z}, & z \neq 0 \end{cases}$ is ( ) (continuous or discontinuous) at $z = 0$. **[3 marks]**

c) The period of the function $f(z) = e^{\frac{z}{5}}$ is ( ). **[3 marks]**

d) $\operatorname*{Res}_{z=0} \cot z$ =( ). **[3 marks]**

e) The Laurent series of $f(z) = \frac{1}{z^2-1}$ in the annular domain $0 < |z-1| < 2$ is ( ). **[3 marks]**

f) The standard form of the linear second order PDE $y^2u_{xx} - x^2u_{yy} = 0, (xy \neq 0)$ is ( ). **[3 marks]**

g) Given that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$, we can get $J_{\frac{3}{2}}(x) =$ ( ) by the recurrence formula. **[3 marks]**

h) Suppose that $\mathcal{F}[f(x)] = F(\lambda)$, where $\mathcal{F}[f(x)]$ is the Fourier integral transformation of $f(x)$, then for any constant $c \in R$, $\mathcal{F}[f(x-c)] =$ ( ). **[3 marks]**

i) The improper integral $\int_{-\infty}^{\infty} \frac{1}{(x^2+1)^2} dx =$ ( ). **[3 marks]**

j) The Laplace integral transformation of $f(t) = e^t$ is ( ). **[3 marks]**

	Do not write in this column
a)	
b)	
c)	
d)	
e)	
f)	
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h)	
i)	
j)	
	30 marks

Question 2. [10 marks]

Please determine whether the following statements are true. Put "T" if the statement is true or "F" if it's wrong.

a) If $\boldsymbol{f(z) = u(x,y) + iv(x,y)}$ is an analytic function, then $\boldsymbol{-u(x,y)}$ is the harmonic conjugate of $\boldsymbol{v(x,y)}$. ( ) **[2 marks]**

b) $\overline{\boldsymbol{e^{z}}} = \boldsymbol{e^{\bar{z}}}$. ( ) **[2 marks]**

c) $\boldsymbol{z = 0}$ is a pole of order 3 of the function $\boldsymbol{f(z)} = \frac{\boldsymbol{e^{z}-1}}{\boldsymbol{z^{3}}}$. ( ) **[2 marks]**

d) The Strurm-Liourville eigenvalue problem must have a finite number of real eigenvalues and eigenfunctions. ( ) **[2 marks]**

e) Let $\boldsymbol{J_n(x)}$ be the first kind of Bessel function of order $\boldsymbol{n}$ and $\boldsymbol{Y_n(x)}$ be the second kind of Bessel function of order $\boldsymbol{n}$, then $\boldsymbol{J_n(x)}$ and $\boldsymbol{Y_n(x)}$ both have finite values at $\boldsymbol{x = 0}$. ( ) **[2 marks]**

	Do not write in this column
a)	
b)	
c)	
d)	
e)	
	10 marks

Question 3. [10 marks]

Please choose the correct answers for the following questions. Only one is correct.

(1) The coefficients of the Laurent series of $\boldsymbol{f(z)}$ in the annular domain $\boldsymbol{0 < |z-b| < 2}$ is ( ).

A. $\boldsymbol{C_k = \frac{f^{(k)}(b)}{k!}}$ B. $\boldsymbol{C_k = \frac{1}{2\pi i}\oint_C \frac{f(z)}{(z-b)^{k+1}}dz}$

C. $\boldsymbol{C_k = \frac{1}{2\pi i}\oint_C \frac{f(z)}{z-b}dz}$ D. $\boldsymbol{\frac{k!}{2\pi i}\oint_C \frac{f(z)}{(z-b)^{k+1}}dz}$

(2) For the eigenvalue problem $\begin{cases} (1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + \lambda y(x) = 0, |x| < 1 \\ |y(x)|\big|_{x=\pm 1} < +\infty \end{cases}$, the eigenvalues are ( ).

A. $\boldsymbol{\lambda = n(n+1), n = 0, 1, 2, \cdots\cdots}$ B. $\boldsymbol{\lambda = n, n = 0, 1, 2, \cdots\cdots}$

C. $\boldsymbol{\lambda = n(n+1), n = 1, 2, \cdots\cdots}$ D. $\boldsymbol{\lambda = n, n = 1, 2, \cdots\cdots}$

(3) The type and the characteristic curves of the equation $\boldsymbol{y^2u_{xx} - x^2u_{yy} = 0, (x^2+y^2 \neq 0)}$ are ( ).

A. hyperbolic, $\boldsymbol{\frac{1}{2}y^2 + \frac{1}{2}x^2 = C}$ B. elliptic, $\boldsymbol{\frac{1}{2}y^2 + \frac{1}{2}x^2 = C}$

C. hyperbolic, $\boldsymbol{\frac{1}{2}y^2 \pm \frac{1}{2}x^2 = C}$ D. elliptic, $\boldsymbol{\frac{1}{2}y^2 \pm \frac{1}{2}x^2 = C}$

(4) Suppose that $\boldsymbol{\mathcal{F}[f(x)] = F(\lambda)}$, where $\boldsymbol{\mathcal{F}[f(x)]}$ is the Fourier integral transformation of $\boldsymbol{f(x)}$, then $\boldsymbol{\mathcal{F}[f'(x) - 3f(x)]}$ is ( ).

A. $\boldsymbol{i\lambda F(\lambda) - 3F(\lambda)}$ B. $\boldsymbol{-i\lambda F(\lambda) + 3F(\lambda)}$

C. $\boldsymbol{i\lambda F(\lambda) + 3F(\lambda)}$ D. $\boldsymbol{-i\lambda F(\lambda) - 3F(\lambda)}$

(5) Suppose that $\boldsymbol{f(t) = e^{-2t}\cos 3t}$, then its Laplace integral transformation $\mathcal{L}[f(t)]$ is ( ).

(Given that $\mathcal{L}[\cos 3t] = \frac{s}{s^2+9}$)

A. $\boldsymbol{\frac{3}{(s+2)^2+9}}$ B. $\boldsymbol{\frac{s+2}{(s+2)^2+9}}$

C. $\boldsymbol{\frac{3s}{(s+2)^2+9}}$ D. $\boldsymbol{\frac{3(s+2)}{(s+2)^2+9}}$

		Do not write in this column
a)		
b)		
c)		
d)		
e)		
		10 marks

Question 4. [12 marks]
Evaluate the following contour integral

a) $\oint_C (|z| - e^z \sin z)\,dz$, where C is the positively oriented circle $|z| = a\ (a > 0)$.

b) $\oint_{|z|=3} \frac{z^5}{(z^2+1)(z^4+2)}\,dz$ with positive orientation.

	Do not write in this column
a)	
b)	
	10 marks

Question 5. [8 marks]

Find out all points at which the function $\boldsymbol{f(z) = x^3 - y^3 + 2x^2y^2 i}$ is **differentiable** and **analytic** (give the explanation), and then find its derivatives.

	Do not write in this column
	6 marks

Question 6. [8 marks]

Solve the following problem by D'Alembert's formula:

$$\begin{cases} u_{tt} - a^2 u_{xx} = 0, & t \geq 0, -\infty < x < \infty, \\ u(x,0) = \cos x\,,\ u_t(x,0) = e^{-1}, & -\infty < x < \infty. \end{cases}$$

	Do not write in this column
	6 marks

Question 7. [12 marks]

Solve the following problem by means of separation of variables:

$$\begin{cases} \boldsymbol{u_t - a^2 u_{xx} = 0}, & \boldsymbol{t > 0, 0 < x < \pi}, \\ \boldsymbol{u(0,t) = u(\pi,t) = 0}, & \boldsymbol{t \geq 0} \\ \boldsymbol{u(x,0) = \sin x + 7 \sin 5x}, & \boldsymbol{0 \leq x \leq \pi}. \end{cases}$$

	Do not write in this column
	10 marks

Question 8. [10 marks]

Evaluate the following integral:

a) $\boldsymbol{I = \int_0^x x^4 J_1(x)\, dx}$ with $\boldsymbol{J_1(x)}$ being the first order of the first kind Bessel function.

b) $\boldsymbol{I = \int_{-1}^{1} x^3\, P_2(x) dx}$ with $\boldsymbol{P_2(x)}$ being the second order Legendre polynomial.

	Do not write in this column
a)	
b)	
	8 marks

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