Sample Questions

Question 3 [25 marks]

a) Please list and explain the two types of errors in hypothesis testing. Usually, a hypothesis test aims to keep which type of error below the prespecified significance level?

[7 marks]

Answer
a)

• Type I error (1 mark): The mistaken rejection of a null hypothesis that is actually true as the result of a test procedure. (or, Deciding the alternative (H_1) is true when actually (H_0) is true.) (2 marks) • Type II error (1 mark): The failure to reject a null hypothesis that is actually false as the result of a test procedure. (Deciding the null (H_0) is true when actually (H_1) is true.) (2 marks) Usually, a hypothesis test is designed to keep the type I error rate below the prespecified significance level. (1 mark)	[7 marks]
TOTAL	**[7 marks]**

b) A researcher is testing the effect of a drug on response time by injecting 100 rats with a unit dose of the drug, and recording the neurological response time of each rat. The researcher knows that the mean response time for rats which are NOT injected with the drug is 1.2 seconds. The mean of the 100 injected rats' response time is 1.05 seconds with a standard deviation of 0.5 seconds. The researcher wants to use a hypothesis test to test if the drug has a significant effect on reducing the rats' response time.

[18 marks]

i) What type of hypothesis test should be used? (two-sample or one-sample test? two-tailed or one-tailed test?)

(2 marks)

ii) State the null (H_0) and alternative (H_1) hypotheses.

(4 marks)

iii) Based on the information in the question description, summarise the statistics for the sample of the response time of the injected rats in **Table 3.1**.

Table 3.1

Sample	Sample Size (n)	Sample mean ($\bar{x}$)	Sample standard deviation (s)
Response time of the rats injected with the drug			

(3 marks)

iv) Suppose the significance level in this experiment is set to be $\alpha = 0.05$, and all assumptions for the t-test are met. Perform a t-test to test the hypothesis you set up in question ii) using the statistics you summarised in question iii). Provide the degree of freedom for this t-test in your answer. You can refer to the t-table in the **Appendix** at the end of this paper.

(9 marks)

Answer

i)

One-sample (1 mark) one-tailed test (1 mark) OR One-sample (1 mark) left-tailed (1 mark) test	[2 marks]
TOTAL	**[2 marks]**

ii)

$H_0: \mu \geq 1.2$ *(2 marks, 1 mark for correct notions (μ), 1 mark for ≥)* $H_1: \mu < 1.2$ *(2 marks, 1 mark for correct notions (μ), 1 mark for <)*	[4 marks]
TOTAL	**[4 marks]**

iii)

Sample	Sample Size (n)	Sample mean ($\bar{x}$)	Sample standard deviation (s)	
Response time of the rats injected with the drug	100 (1 mark)	1.05 (1 mark)	0.5 (1 mark)	[3 marks]
TOTAL				**[3 marks]**

iv)

The standard deviation of the null distribution $$SE = \frac{s}{\sqrt{n}} \ (1\ mark)$$ $$= \frac{0.5}{\sqrt{100}} = 0.05\ (1\ mark)$$	[2 marks]
t-value $$t = \frac{\bar{x} - 1.2}{SE} (1\ mark)$$ $$= \frac{1.05 - 1.2}{0.05} (1\ mark)$$ $$= -3 (1\ mark)$$	[3 marks]
The hypothesis test is a one-sample one-tailed (left-tailed) t-test. So, the degree of freedom for this t-test is df = 100 – 1 = 99 *(1 mark)* by looking up the t-table with $\alpha = 0.05$ and df=99 we can get the critical value $$t_{0.05,99} = 1.66\ (1\ mark)$$	[2 marks]
Because $\|t\| = 3 > t_{0.05,99} = 1.66$ *(1 mark)*, we reject the null in favour of the alternative. *(1 mark)* The drug has a significant effect on reducing the rats' response time.	[2 marks]
TOTAL	**[9 marks]**

Appendix

Two-sample t-Test

t-value

$$t = \frac{\bar{x}_1 - \bar{x}_2}{SE_{diff}}$$

where SE_{diff} is the standard error of the null distribution of the mean difference

1) If two populations have <u>**equal**</u> **variances**

$$SE_{diff} = s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

where the pooled variance:

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

where n_1 and n_2 are the sample sizes, and s_1 and s_2 are the standard deviation of the two samples, and the degree of freedom:

$$df = n_1 + n_2 - 2$$

2) If two populations have <u>**unequal**</u> **variances**

$$SE_{diff} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

the approximate degree of freedom

$$df = \frac{(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2})^2}{\frac{1}{(n_1 - 1)}(\frac{s_1^2}{n_1})^2 + \frac{1}{(n_2 - 1)}(\frac{s_2^2}{n_2})^2}$$

t-Table

Table entry for p and C is the point t^* with probability p lying above it and probability C lying between $-t^*$ and t^*.

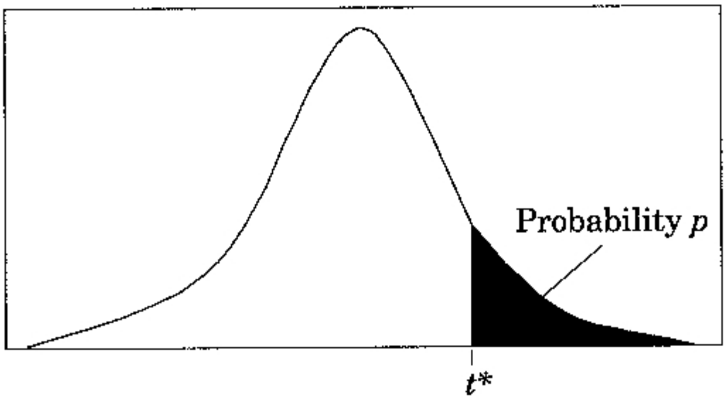

Table B t distribution critical values

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											