第二、三次作业答案

1.58

The fundamental period of the sinusoidal signal $x[n]$ is $N = 10$. Hence the angular frequency of $x[n]$ is

$$\Omega = \frac{2\pi m}{N} \qquad m\text{: integer}$$

The smallest value of Ω is attained with $m = 1$. Hence,

$$\Omega = \frac{2\pi}{10} = \frac{\pi}{5} \quad \text{radians/cycle}$$

1.60

Real part of $x(t)$ is

$$\mathrm{Re}\{x(t)\} = Ae^{\alpha t}\cos(\omega t)$$

Imaginary part of $x(t)$ is

$$\mathrm{Im}\{x(t)\} = Ae^{\alpha t}\sin(\omega t)$$

1.64(a, c, f, g)

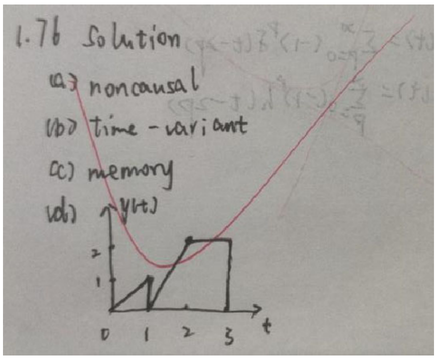
	memoryless	stable	causal	linear	Time-invariant
(a)	√	√	√	x	√
(c)	√	x	√	x	√
(f)	x	x	√	√	√
(g)	x	√	x	x	√

(a)	memoryless	stable	causal	Nonlinear	Time-invariant
(c)	memoryless	Nonstable	causal	Nonlinear	Time-invariant
(f)	memory	Nonstable	causal	linear	Time-invariant
(g)	memory	stable	Noncausal	Nonlinear	Time-invariant

积分器与微分器均不稳定。

1.76

若认为是一个系统：



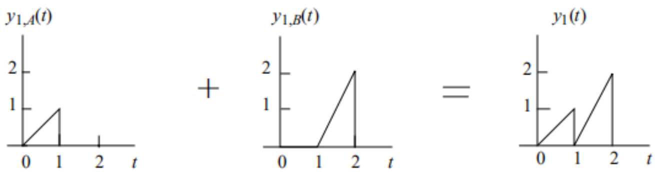
若认为是三个系统：

	causal	Time invariant	memoryless
H1	√	√	x
H2	x	x/ √	x
H3	√	√	x

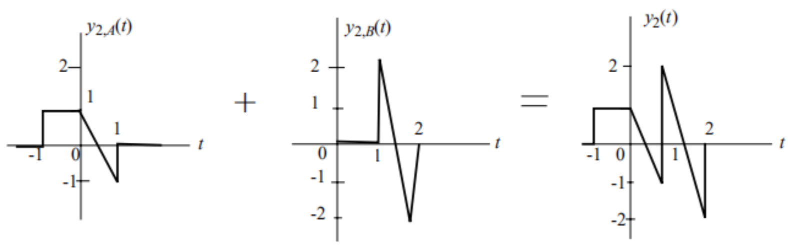
H1	causal	Time invariant	memory
H2	Noncausal	都行	memory
H3	causal	Time invariant	memory

(d)

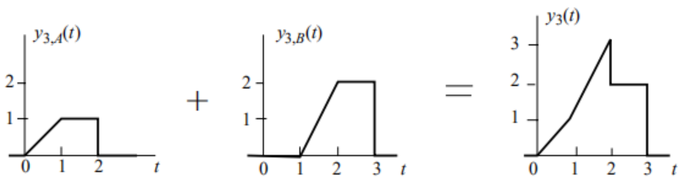
Response of H_1 to $x(t)$:



Response of H_2 to $x(t)$:



Response of H_3 to $x(t)$:



2.32

(a) $x[n] = 3\delta[n] - 2\delta[n-1]$

$$\begin{aligned} y[n] &= 3h[n] - 2h[n-1] \\ &= 3\delta[n+1] + 7\delta[n] - 7\delta[n-2] + 5\delta[n-3] - 2\delta[n-4] \end{aligned}$$

n	-1	0	2	3	4
y[n]	3	7	-7	5	-2

（b）答案有误，正确答案如下

$$x[n] = \delta(n+1) + \delta(n) + \delta(n-1) + \delta(n-2)$$
$$y[n] = h(n+1) + h(n) + h(n-1) + h(n-2)$$
$$= \delta(n+2) + 4\delta(n+1) + 6\delta(n) + 5\delta(n-1) + 5\delta(n-2) + 2\delta(n-3) + \delta(n-5)$$

n	-2	-1	0	1	2	3	5
y[n]	1	4	6	5	5	2	1

(c) $x[n]$ as given in Fig. P2.32 (b)

.

$$\begin{aligned} x[n] &= 2\delta[n-3] + 2\delta[n] - \delta[n+2] \\ y[n] &= 2h[n-3] + 2h[n] - h[n+2] \\ &= -\delta[n+3] - 3\delta[n+2] + 7\delta[n] + 3\delta[n-1] + 8\delta[n-3] + 4\delta[n-4] - 2\delta[n-5] + 2\delta[n-6] \end{aligned}$$

n	-3	-2	0	1	3	4	5	6
y[n]	-1	-3	7	3	8	4	-2	2

2.34(a,c,d)

(a) $m[n] = x[n] * z[n]$

$$
\begin{array}{ll}
\text{for } n+5<0 & n<-5 \\
 & m[n]=0 \\
\text{for } n+5<4 & -5\le n<-1 \\
 & m[n]=\displaystyle\sum_{k=0}^{n+5} 1 = n+6 \\
\text{for } n-1<1 & -1\le n<2 \\
 & m[n]=\displaystyle\sum_{k=0}^{3} 1 + 2\sum_{k=4}^{n+5} 1 = 2n+8 \\
\text{for } n+5<9 & 2\le n<4 \\
 & m[n]=\displaystyle\sum_{k=n-1}^{3} 1 + 2\sum_{k=4}^{n+5} 1 = 9+n \\
\text{for } n-1<4 & 4\le n<5 \\
 & m[n]=\displaystyle\sum_{k=n-1}^{3} 1 + 2\sum_{k=4}^{8} 1 = 15-n \\
\text{for } n-1<9 & 5\le n<10 \\
 & m[n]=2\displaystyle\sum_{k=n-1}^{8} 1 = 20-2n \\
\text{for } n-1\ge 9 & n\ge 10 \\
 & m[n]=0
\end{array}
$$

$$
m[n] = \begin{cases} 0 & n<-5 \\ n+6 & -5\le n<-1 \\ 2n+8 & -1\le n<2 \\ 9+n & 2\le n<4 \\ 15-n & 4\le n<5 \\ 20-2n & 5\le n<10 \\ 0 & n\ge 10 \end{cases}
$$

n	-5	-4	-3	-2	-1	0	1	2
m[n]	1	2	3	4	6	8	10	11
n	3	4	5	6	7	8	9	10
m[n]	12	11	10	8	6	4	2	0

(c) $m[n] = x[n] * f[n]$

$$\begin{array}{ll}
\text{for } n+5 < -5 & n < -10 \\
& m[n] = 0 \\
\text{for } n-1 < -5 & -10 \le n < -4 \\
& m[n] = \frac{1}{2}\sum_{k=-5}^{n+5} k = -5n - 55 + \frac{1}{2}(n+10)(n+11) \\
\text{for } n+5 < 6 & -4 \le n < 1 \\
& m[n] = \frac{1}{2}\sum_{k=n-1}^{n+5} k = \frac{7}{2}(n-1) + \frac{21}{2} \\
\text{for } n-1 < 6 & 1 \le n < 7 \\
& m[n] = \frac{1}{2}\sum_{k=n-1}^{5} k = \frac{1}{2}(7-n)\left[(n-1) + \frac{1}{2}(6-n)\right] \\
\text{for } n-1 \ge 6 & n \ge 7
\end{array}$$

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$$m[n] = 0$$

$$m[n] = \begin{cases} 0 & n < -10 \\ -5n - 55 + \frac{1}{2}(n+10)(n+11) & -10 \le n < -4 \\ \frac{7}{2}(n-1) + \frac{21}{2} & -4 \le n < 1 \\ \frac{1}{2}(7-n)\left[(n-1) + \frac{1}{2}(6-n)\right] & 1 \le n < 7 \\ 0 & n \ge 7 \end{cases}$$

注意：

$$m[n] = -2.5n - 27.5 + 0.25(n+10)(n+11)$$

$$= \frac{n^2 + 11n}{4}, -10 \le n < -4$$

n	-10	-9	-8	-7	-6	-5	-4	-3	-2
m[n]	-2.5	-4.5	-6	-7	-7.5	-7.5	-7	-3.5	0
n	-1	0	1	2	3	4	5	6	7
m[n]	3.5	7	7.5	7.5	7	6	4.5	2.5	

(d) $m[n] = x[n] * g[n]$

$$\text{for } n+5 < -8 \qquad n < -13$$

$$m[n] = 0$$

$$\text{for } n-1 < -7 \qquad -14 \le n < -6$$

$$m[n] = \sum_{k=-8}^{n+5} 1 = n + 14$$

$$\text{for } n+5 < 4 \qquad -6 \le n < -1$$

$$m[n] = \sum_{k=n-1}^{-2} 1 = -n$$

$$\text{for } n-1 < -1 \qquad -1 \le n < 0$$

$$m[n] = \sum_{k=n-1}^{-2} 1 + \sum_{k=4}^{n+5} 1 = -2$$

$$\boxed{2}$$

$$\text{for } n-1 < 4 \qquad 0 \le n < 5$$

$$m[n] = \sum_{k=4}^{n+5} 1 = n + 2$$

$$\text{for } n-1 < 11 \qquad 5 \le n < 12$$

$$m[n] = \sum_{k=n-1}^{10} 1 = 12 - n$$

$$\text{for } n-1 \ge 11 \qquad n \ge 12$$

$$m[n] = 0$$

$$m[n] = \begin{cases} 0 & n < -13 \\ n+14 & -13 \le n < -6 \\ -n & -6 \le n < -1 \\ -2 & -1 \le n < 0 \\ n+2 & 0 \le n < 5 \\ 12-n & 5 \le n < 12 \\ 0 & n \ge 12 \end{cases}$$

n	-13	-12	-11	-10	-9	-8	-7	-6	-5
M[n]	1	2	3	4	5	6	7	6	5
n	-4	-3	-2	-1	0	1	2	3	4
M[n]	4	3	2	2	2	3	4	5	6
n	5	6	7	8	9	10	11		
M[n]	7	6	5	4	3	2	1		

五、2.35

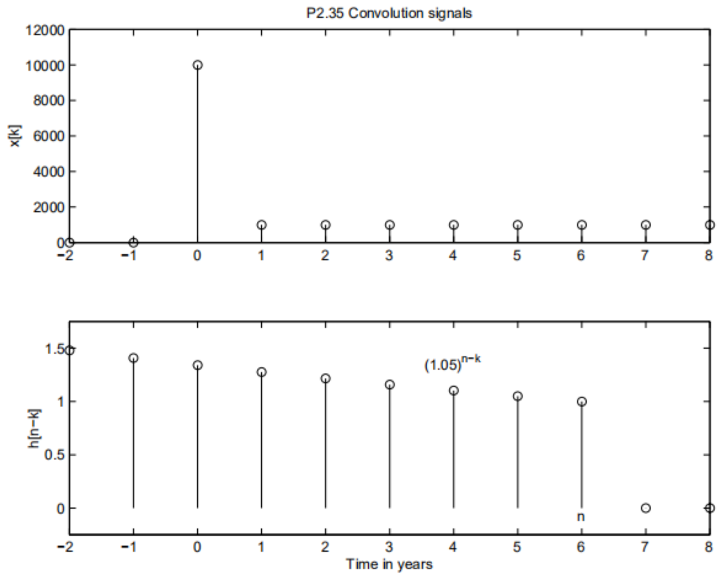


Figure P2.35. Graph of $x[k]$ and $h[n-k]$

for $n = -1$

$$y[-1] = \sum_{k=-1}^{-1} 10000(1.05)^{n-k} = 10000(1.05)^{n+1}$$

\$1000 is invested annually, similar to example 2.5

for $n \geq 0$

$$y[n] = 10000(1.05)^{n+1} + \sum_{k=0}^{n} 1000(1.05)^{n-k}$$

$$y[n] = 10000(1.05)^{n+1} + 1000(1.05)^{n} \sum_{k=0}^{n} (1.05)^{-k}$$

$$y[n] = 10000(1.05)^{n+1} + 1000(1.05)^{n} \frac{1 - \left(\frac{1}{1.05}\right)^{n+1}}{1 - \frac{1}{1.05}}$$

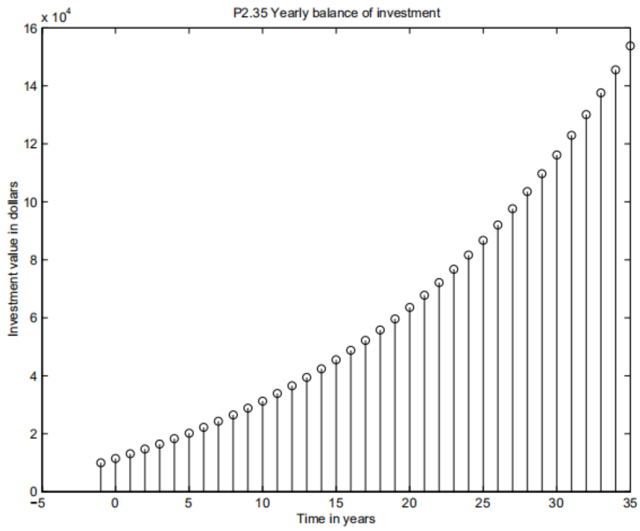
$$y[n] = 10000(1.05)^{n+1} + 20000\left[1.05^{n+1} - 1\right]$$

The following is a graph of the value of the account.

n 从 0 算起也算对，即

$$y[n] = 10000, n = 0$$

$$y[n] = 10000(1.05)^{n} + 20000[1.05^{n} - 1], n \geq 1$$



P2.35 Yearly balance of investment
x 10^4
Investment value in dollars
Time in years