第七次作业答案

一、3.58(a,d,f)

3.58. Use the tables of transforms and properties to find the FT's of the following signals.

(a) $x(t) = \sin(2\pi t)e^{-t}u(t)$

$$\begin{aligned} x(t) &= \sin(2\pi t)e^{-t}u(t) \\ &= \frac{1}{2j}e^{j2\pi t}e^{-t}u(t) - \frac{1}{2j}e^{-j2\pi t}e^{-t}u(t) \end{aligned}$$

$$\begin{aligned} e^{-t}u(t) &\xleftrightarrow{FT} \frac{1}{1+j\omega} \\ e^{j2\pi t}s(t) &\xleftrightarrow{FT} S(j(\omega-2\pi)) \\ X(j\omega) &= \frac{1}{2j}\left[\frac{1}{1+j(\omega-2\pi)} - \frac{1}{1+j(\omega+2\pi)}\right] \end{aligned}$$

(d)

$$\begin{aligned} x(t) &= \frac{d}{dt}te^{-2t}\sin(t)u(t) \\ &= \frac{d}{dt}te^{-2t}u(t)\frac{e^{jt}-e^{-jt}}{2j} \end{aligned}$$

$$te^{-2t}u(t) \xleftrightarrow{FT} \frac{1}{(2+j\omega)^2}$$

$$\begin{aligned} e^{jt}s(t) &\xleftrightarrow{FT} S(j(\omega-1)) \\ \frac{d}{dt}s(t) &\xleftrightarrow{FT} j\omega S(j\omega) \\ X(j\omega) &= j\omega\frac{1}{2j}\left[\frac{1}{(2+j(\omega-1))^2} - \frac{1}{(2+j(\omega+1))^2}\right] \end{aligned}$$

(f) $x(t) = e^{-t+2}u(t-2)$

$$\begin{aligned} e^{-t}u(t) &\xleftrightarrow{FT} \frac{1}{1+j\omega} \\ s(t-2) &\xleftrightarrow{FT} e^{-j2\omega}S(j\omega) \\ X(j\omega) &= e^{-j2\omega}\frac{1}{1+j\omega} \end{aligned}$$

二、3.59(b,c,e)

(b) $X(j\omega) = \frac{4\sin(2\omega-4)}{2\omega-4} - \frac{4\sin(2\omega+4)}{2\omega+4}$

$$\begin{array}{rcl}
\frac{2\sin(\omega)}{\omega} & \xleftrightarrow{FT} & \mathrm{rect}(t) = \begin{cases} 1 & |t| \leq 1 \\ 0, & \text{otherwise} \end{cases} \\
S(j2\omega) & \xleftrightarrow{FT} & \frac{1}{2}s(\frac{t}{2}) \\
S(j(\omega-2)) & \xleftrightarrow{FT} & e^{j2t}s(t)
\end{array}$$

$$\begin{aligned}
x(t) &= \mathrm{rect}(\frac{t}{2})e^{j2t} - \mathrm{rect}(\frac{t}{2})e^{-j2t} \\
&= 2j\mathrm{rect}(\frac{t}{2})\sin(2t)
\end{aligned}$$

(c) $X(j\omega) = \frac{1}{j\omega(j\omega+2)} - \pi\delta(\omega)$

$$\begin{array}{rcl}
\frac{1}{j\omega} + \pi\delta(j\omega) & \xleftrightarrow{FT} & u(t) \\
\frac{1}{2+j\omega} & \xleftrightarrow{FT} & e^{-2t}u(t) \\
2\pi\delta(\omega) & \xleftrightarrow{FT} & 1 \\
X(j\omega) & = & -0.5\frac{1}{(j\omega+2)} + 0.5\frac{1}{j\omega} + 0.5\pi\delta(\omega) - 1.5\pi\delta(\omega) \\
X(j\omega) & \xleftrightarrow{FT} & x(t) = -0.5e^{-2t}u(t) + 0.5u(t) - \frac{3}{4}
\end{array}$$

(e) $X(j\omega)=\frac{2\sin(\omega)}{\omega(j\omega+2)}$

$$S_1(j\omega)=\frac{2\sin(\omega)}{\omega} \xleftrightarrow{FT} s_1(t)=\begin{cases} 1 & |t|\leq 1 \\ 0, & \text{otherwise} \end{cases}$$

$$\begin{aligned} S_2(j\omega) &= \frac{1}{(j\omega+2)} \xleftrightarrow{FT} s_2(t)=e^{-2t}u(t) \\ x(t) &= s_1(t) * s_2(t) \end{aligned}$$

$$x(t)=\begin{cases} 0 & t < \text{-}1 \\ \frac{1}{2}[1-e^{-2(t+1)}] & -1 \leq t < -1 \\ \frac{e^{-2t}}{2}[e^2-e^{-2}] & t \geq 1 \end{cases}$$

三、3.60(a,b)

3.60. Use the tables of transforms and properties to find the DTFT's of the following signals.

(a) $x[n]=\left(\frac{1}{3}\right)^n u[n+2]$

$$\begin{aligned} x[n] &= (\frac{1}{3})^n u[n+2] \\ &= (\frac{1}{3})^{-2}(\frac{1}{3})^{n+2}u[n+2] \\ (\frac{1}{3})^n u[n] &\xleftrightarrow{DTFT} \frac{1}{1-\frac{1}{3}e^{-j\Omega}} \\ s[n+2] &\xleftrightarrow{DTFT} e^{j2\Omega}S(e^{j\Omega}) \\ X(e^{j\Omega}) &= \frac{9e^{j2\Omega}}{1-\frac{1}{3}e^{-j\Omega}} \end{aligned}$$

(b) $x[n]=(n-2)(u[n+4]-u[n-5])$

$$\begin{aligned} u[n+4]-u[n-5] &\xleftrightarrow{DTFT} \frac{\sin(\frac{9\Omega}{2})}{\sin(\frac{\Omega}{2})} \\ ns[n] &\xleftrightarrow{DTFT} j\frac{d}{d\Omega}S(e^{j\Omega}) \\ x[n] &= j\frac{d}{d\Omega}\frac{\sin(\frac{9\Omega}{2})}{\sin(\frac{\Omega}{2})} - 2\frac{\sin(\frac{9\Omega}{2})}{\sin(\frac{\Omega}{2})} \end{aligned}$$

四、3.61(b,e)

$$\text{(b) } X(e^{j\Omega}) = \left[e^{-j2\Omega}\frac{\sin(\frac{15}{2}\Omega)}{\sin(\frac{\Omega}{2})}\right] \circledast \left[\frac{\sin(\frac{7}{2}\Omega)}{\sin(\frac{\Omega}{2})}\right]$$

Let the first part be $A((e^{j\Omega}))$, and the second be $B(e^{j\Omega})$.

$$\begin{aligned}
a[n] &= \begin{cases} 1 & |n-2| \le 7 \\ 0, & \text{otherwise} \end{cases} \\
b[n] &= \begin{cases} 1 & |n| \le 3 \\ 0, & \text{otherwise} \end{cases} \\
X((e^{j\Omega})) &= A((e^{j\Omega}))\circledast B(e^{j\Omega}) \xleftrightarrow{DTFT} x[n] = 2\pi a[n]b[n] \\
x[n] &= \begin{cases} 2\pi & |n| \le 3 \\ 0, & \text{otherwise} \end{cases}
\end{aligned}$$

$$\text{(e) } X(e^{j\Omega}) = e^{-j(4\Omega+\frac{\pi}{2})}\frac{d}{d\Omega}\left[\frac{2}{1+\frac{1}{4}e^{-j(\Omega-\frac{\pi}{4})}} + \frac{2}{1+\frac{1}{4}e^{-j(\Omega+\frac{\pi}{4})}}\right]$$

$$\begin{aligned}
S_1(e^{j\Omega}) = \frac{2}{1+\frac{1}{4}e^{-j\omega}} &\xleftrightarrow{DTFT} s_1[n] = 2(-\frac{1}{4})^n u[n] \\
S_1(e^{j(\Omega-\frac{\pi}{4})}) &\xleftrightarrow{DTFT} e^{j\frac{\pi}{4}n}s_1[n] \\
S_1(e^{j(\Omega+\frac{\pi}{4})}) &\xleftrightarrow{DTFT} e^{-j\frac{\pi}{4}n}s_1[n] \\
S(e^{j\Omega}) = S_1(e^{j(\Omega-\frac{\pi}{4})}) + S_1(e^{j(\Omega+\frac{\pi}{4})}) &\xleftrightarrow{DTFT} s[n] = 2\cos(\frac{\pi}{4}n)s_1[n] \\
-je^{j4\Omega}\frac{d}{d\Omega}S(e^{j\Omega}) &\xleftrightarrow{DTFT} -(n-4)s[n-4] \\
x[n] &= -4(n-4)\cos(\frac{\pi}{4}(n-4))(-\frac{1}{4})^{n-4}u[n-4]
\end{aligned}$$

五、3.73(b,c)

(b)

$$\begin{aligned}
X(j\omega) &= \frac{j\omega-2}{-\omega^2+5j\omega+4} \\
&= \frac{A}{4+j\omega} + \frac{B}{1+j\omega} \\
1 &= A+B \\
-2 &= A+4B \\
X(j\omega) &= \frac{2}{4+j\omega} - \frac{1}{1+j\omega} \\
x(t) &= (2e^{-4t}-e^{-t})u(t)
\end{aligned}$$

(c)

$$\begin{aligned}
X(j\omega) &= \frac{j\omega}{(j\omega)^2+6j\omega+8} \\
&= \frac{A}{4+j\omega}+\frac{B}{2+j\omega} \\
1 &= A+B \\
0 &= 2A+4B \\
X(j\omega) &= \frac{2}{4+j\omega}-\frac{1}{2+j\omega} \\
x(t) &= (2e^{-4t}-e^{-2t})u(t)
\end{aligned}$$

六、3.74(b,d)

(b)

$$\begin{aligned}
X(e^{j\Omega}) &= \frac{2+\frac{1}{4}e^{-j\Omega}}{-\frac{1}{8}e^{-j2\Omega}+\frac{1}{4}e^{-j\Omega}+1} \\
&= \frac{A}{1+\frac{1}{2}e^{-j\Omega}}+\frac{B}{1-\frac{1}{4}e^{-j\Omega}} \\
\frac{1}{4} &= -\frac{1}{4}A+\frac{1}{2}B \\
2 &= A+B \\
X(e^{j\Omega}) &= \frac{1}{1+\frac{1}{2}e^{-j\Omega}}+\frac{1}{1-\frac{1}{4}e^{-j\Omega}} \\
x[n] &= \left((-\frac{1}{2})^n+(\frac{1}{4})^n\right)u[n]
\end{aligned}$$

(d)

$$\begin{aligned}
X(e^{j\Omega}) &= \frac{6-2e^{-j\Omega}+\frac{1}{2}e^{-j2\Omega}}{(-\frac{1}{4}e^{-j2\Omega}+1)(1-\frac{1}{4}e^{-j\Omega})} \\
&= \frac{A}{1+\frac{1}{2}e^{-j\Omega}}+\frac{B}{1-\frac{1}{2}e^{-j\Omega}}+\frac{C}{1-\frac{1}{4}e^{-j\Omega}} \\
6 &= A+B+C \\
-2 &= -\frac{3}{4}A+\frac{1}{4}B \\
\frac{1}{2} &= \frac{1}{8}A-\frac{1}{8}B-\frac{1}{4}C \\
X(e^{j\Omega}) &= \frac{4}{1+\frac{1}{2}e^{-j\Omega}}+\frac{4}{1-\frac{1}{2}e^{-j\Omega}}-\frac{2}{1-\frac{1}{4}e^{-j\Omega}} \\
x[n] &= \left(4(-\frac{1}{2})^n+4(\frac{1}{2})^n-2(\frac{1}{4})^n\right)u[n]
\end{aligned}$$