Fundamental Constants:

Electron rest mass: $m_e = 9.11\times10^{-31}$ kg, Charge on electron: $e = -1.60\times10^{-19}$ C

Vacuum permittivity: $\varepsilon_0 = 8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}$, Vacuum permeability: $\mu_0 = 4\pi\times10^{-7}\ \mathrm{T\cdot m/A}$

Part I Multiple Choice (30%, each multiple choice question has a choice of multiple answers, **only one** of which is correct, 3 marks for each question)

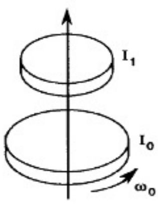
1. An object is moving along the x axis with position as a function of time given by $x = x(t)$. Point O is at $x = 0$. The object is definitely moving toward O when______. 【 D 】

(A) $\frac{dx}{dt} > 0$ (B) $\frac{dx}{dt} < 0$ (C) $\frac{d(x^2)}{dt} > 0$ (D) $\frac{d(x^2)}{dt} < 0$.

2. A thin hoop with a radius of 10 cm and a mass of 2.0 kg is rotating about its center with an angular speed of 3.0 rad/s. What is its kinetic energy? 【 C 】

(A) 0.27 J (B) 0.18 J (C) 0.09 J (D) 0.03 J

3. A cylinder with a moment of inertia I_0 rotates with angular velocity ω_0, second cylinder with moment of inertial I_1 initially not rotating drops onto the first cylinder and the two reach the same final angular velocity ω_f. Find ω_f 【 B 】



(A) $\omega_f = \omega_0 I_0/I_1$ (B) $\omega_f = \omega_0 I_0/(I_0+I_1)$

(C) $\omega_f = \omega_0 I_1/I_0$ (D) $\omega_f = \omega_0 (I_0+I_1)/I_0$

4. What is the length of a simple pendulum with a period of 2.0 s? 【 A 】

(A) 0.994 m (B) 1.22 m (C) 1.62 m (D) 19.6 m

5. Two waves travel simultaneously through the same medium. The first wave is described by $y_1 = A\cos(kx-\omega t)$ and the second wave by $y_2 = A\cos(kx-\omega t+\varphi)$. If the amplitude of the resulting superposition is $\sqrt{2}A$, what are the possible values of φ? 【 B 】

(A) π, -π (B) π/2, -π/2 (C) π/4, -π/4 (D) π/8, -π/8

6. A point particle with charge q is placed inside a cube but not at its center. The electric flux through any one side of the cube 【 B 】

(A)is zero (B) cannot be computed using Gauss' law (C) is $q/6\varepsilon_0$ (D) is q/ε_0

7. Choose the correct statement: 【 D 】

(A) The potential of a negatively charged conductor must be negative.
(B) If $\vec{E} = 0$ at a point P then V must be zero at P.
(C) If $V = 0$ at a point P then $\vec{E}$ must be zero at P.
(D) None of the above is correct.

8. The capacitance of a spherical capacitor with inner radius a and outer radius b is proportional to

(A) a/b (B) $b-a$ (C) $ab/(b-a)$ (D) $ab/(b^2-a^2)$ 【 C 】

9. A loop of current-carrying wire has a magnetic dipole moment of 5.0×10^{-4} A□m^2. The moment initially is aligned with a 0.50-T magnetic field. To rotate the loop so its dipole moment is perpendicular to the field and hold it in that orientation, you must do work of 【 A 】

(A) 2.5×10^{-4} J (B) -2.5×10^{-4} J (C) 1.0×10^{-3} J (D) -1.0×10^{-3} J

10. An electric field is directed out of the page within a circular region of radius $R = 3.0$ cm, with magnitude $E = (0.50t)(1 - r/R)$ V/m, where t is in seconds and r is the radial distance ($r < R$). What is the magnitude of the induced magnetic field at radial distances 2.00 cm? 【 D 】

(A) 9.31×10^{-20} T (B) 7.58×10^{-20} T (C) 6.14×10^{-20} T. (D) 3.09×10^{-20} T

Part II Filling the Blanks (30%, 3 marks for each blank)

11. The position of an object is $\vec{r} = 3.0t\hat{i} + t^2\hat{j}$m. The speed of the object at time $t = 2.0s$ is

5.0 m/s.

12. The potential energy function associated with a conservative force is $U(x) = U_0 + ax + bx^2$(J), and the conservative force at the position x is $-a-2bx$.

13. A 1.0 m radius, 6.0 kg uniform ball is rotating about an axis through its center at 2π rad/s. The magnitude of angular momentum of the ball is 15.1 or 4.8π kg· m²/s.

14. The position of an oscillator is given by $x = 2.0\cos(\pi t + \frac{\pi}{2})$ cm. The frequency of the motion is

0.5 Hz.

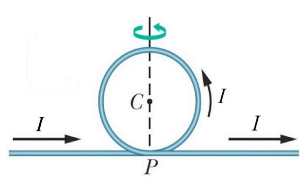
15. A 100g mass attached to a spring moves in simple harmonic motion with amplitude 16.0cm and period 2.0s. Assuming that the mass is released from rest at $t = 0$s and $x = -16.0$cm, the displacements as a function of time can be expressed as $x=16\cos(\pi t+\pi)$ cm.

16. An electric dipole consists of charges +2e and -2e separated by 0.78 nm. It is in an electric field of strength 3.4×10^6 N/C. What is the magnitude of the torque on the dipole when the dipole moment is perpendicular to the electric field? 8.5×10^{-22} N • m

17. A cylindrical copper rod has resistance 1.0×10^{-5} Ω. It is reformed to twice its original length with no change of volume. What is its new resistance? 4.0×10^{-5} Ω

18. An electron has an initial velocity of $12.0\hat{j} + 15.0\hat{k}$ km/s and a constant acceleration of $2.00\times10^{12}\hat{i}$ m/s² in a region in which uniform electric and magnetic fields are present. If $\vec{B} = 400\hat{i}$ μT, What is the electric field $\vec{E}$? $-11.4\vec{i} - 6.00\vec{j} + 4.80\vec{k}$ V/m

19. In the right figure, part of a long wire carrying current $I = 5.78$ mA is bent into a circular section of radius $R = 1.89$ cm. What is the magnitude of the magnetic field at the center of curvature C if the circular section is perpendicular to the plane of the page after being rotated 90° counter-clockwise as indicated? 2.02×10^{-7} T.



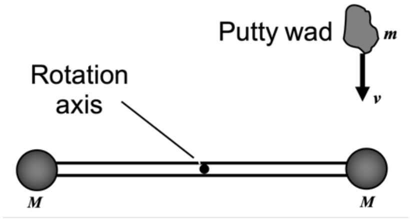
20. Two identical long wires of radius 1.53 mm are parallel and carry identical currents in opposite directions. Their center-to-center separation is 14.2 cm. Neglect the flux within the wires but consider the flux in the region between the wires. What is the inductance per unit length of the wires? 1.81×10^{-6} H/m

Part III Problems (40%)

21. (10 marks) Two balls with mass $\boldsymbol{M}$ are attached to the ends of a thin rod of length $\boldsymbol{d}$ and negligible mass. The rod is free to rotate in a vertical plane without friction about a horizontal axis through its centre. With the rod initially horizontal, as shown in the following figure, a wad of wet putty with mass $\boldsymbol{m}$ hits one of the balls with a speed $\boldsymbol{v}$ and then sticks to it.

(a) What is the angular speed of the system just after the putty wad hits?

(b) Through what angle will the system rotate before it momentarily stops?



Solution:

(1) Conservation of angular momentum yields:

$$mv\frac{d}{2} = I\omega,$$

where $$I = (2M+m)(d/2)^2,$$

Then the angular speed is $$\omega = \frac{2mv}{(2M+m)d}$$

(2) Conservation of energy provides the relation:

$$K_i + U_i = K_f + U_f,$$

where $$K_i = \frac{1}{2}I\omega^2 = \frac{1}{2}(2M+m)\left(\frac{d}{2}\right)^2\omega^2,$$

$$U_i = mg\frac{d}{2},\quad K_f = 0,\quad \text{and}\quad U_f = mg\frac{d}{2}(1-\cos\theta).$$

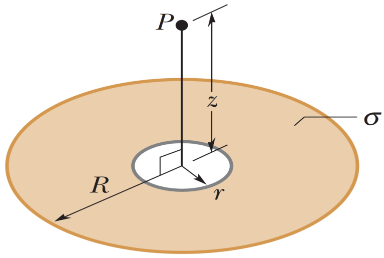
We take the lowest point on the path to be the zero of potential energy and the system swing up to the angular position θ, as measured from its lowest point, before it momentarily stops.

Therefore, $$\cos\theta = -\frac{1}{2}\left(\frac{2M+m}{mg}\right)\left(\frac{d}{2}\right)\omega^2$$

The total angle through which it has swung is. $$\vartheta = \frac{\pi}{2} + \theta$$

22. (10 marks) The following figure shows a ring of outer radius R, inner radius r, and uniform surface charge density σ. With $V = 0$ at infinity, find (a) the electric potential and (b) the electric field magnitude at point P on the central axis of the ring, at distance z from the center of the ring. (10 marks)

Solution:



(a) Consider a differential element consisting of a flat ring of radius r and radial width dr. Its charge has magnitude dq=σ(2πr)dr.

The contribution of this ring to the electric potential at P is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{\sqrt{z^2+r^2}} = \frac{1}{4\pi\varepsilon_0}\frac{\sigma(2\pi r)dr}{\sqrt{z^2+r^2}}.$$

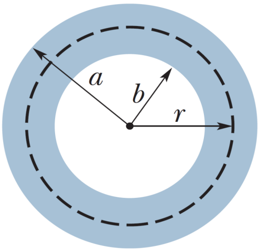
We find the net potential at P by adding the contributions of all the rings

$$V = \int dV = \frac{\sigma}{2\varepsilon_0}\int_r^R \frac{r dr}{\sqrt{z^2+r^2}} = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2+R^2} - \sqrt{z^2+r^2}\right).$$

(b) The direction of the electric field must be along the z axis because the ring has circular symmetry about that axis. Thus, the magnitude of the electric field at point P is

$$E = E_z = -\frac{\partial V}{\partial z} = \frac{\sigma}{2\varepsilon_0}\left(\frac{z}{\sqrt{z^2+r^2}} - \frac{z}{\sqrt{z^2+R^2}}\right).$$

23. (10marks) As shown, a cross section of a hollow cylindrical conductor of radii a and b, carries a uniformly distributed current I. Determine expressions for the magnetic field magnitude $B(r)$ with the radial distance r in the ranges (a) $r < b$, (b) $b < r < a$, and (c) $r > a$.



Solution:

The hollow conductor has cylindrical symmetry, so we can apply Ampere's law,

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{enc}},$$

to calculate the magnetic field due to the current distribution. We choose the Amperian loop to be a circle of radius r and concentric with the cylindrical shell.

(a) For $r < b$, the enclosed current is zero and the magnetic field magnitude is

$$B = 0.$$

(b) For $b < r < a$, since the current is uniformly distributed throughout the cross section of the shell, the enclosed current is

$$I_{\text{enc}} = I\frac{\pi(r^2 - b^2)}{\pi(a^2 - b^2)} = I\frac{r^2 - b^2}{a^2 - b^2}.$$

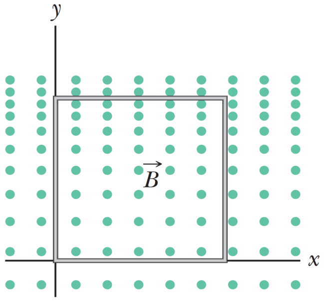
Thus, we have $$\oint \vec{B}\cdot d\vec{l} = (2\pi r)B = \mu_0 I_{\text{enc}} = \mu_0 I\frac{r^2 - b^2}{a^2 - b^2},$$

which gives $$B = \frac{\mu_0 I}{2\pi(a^2 - b^2)}\frac{r^2 - b^2}{r}.$$

(c) For $r > a$, the enclosed current is I. So $$(2\pi r)B = \mu_0 I.$$

The magnetic field magnitude is $$B = \frac{\mu_0 I}{2\pi r}.$$

24. (10marks)As seen in the following figure, a square loop of wire has sides of length l. A magnetic field is directed out of the page; its magnitude is given by $B = 4t^2y$. (a) What is the direction of the emf induced in the loop? (clockwise or counter-clockwise) (b) What is the magnitude of the emf as a function of time



Solution:

(a) Clockwise.

(b) Consider a thin strip of area of height dy and width l. The strip is located at some $0 < y < l$. The element of flux through the strip is

$$d\Phi_B = B dA = (4t^2 y)(l dy).$$

To find the total flux through the square loop, we integrate

$$\Phi_B = \int d\Phi_B = \int_0^l (4t^2 y)(l dy) = 2t^2 l^3.$$

Thus, Faraday's law yields

$$|\varepsilon| = \left|\frac{d\Phi_B}{dt}\right| = 4tl^3.$$