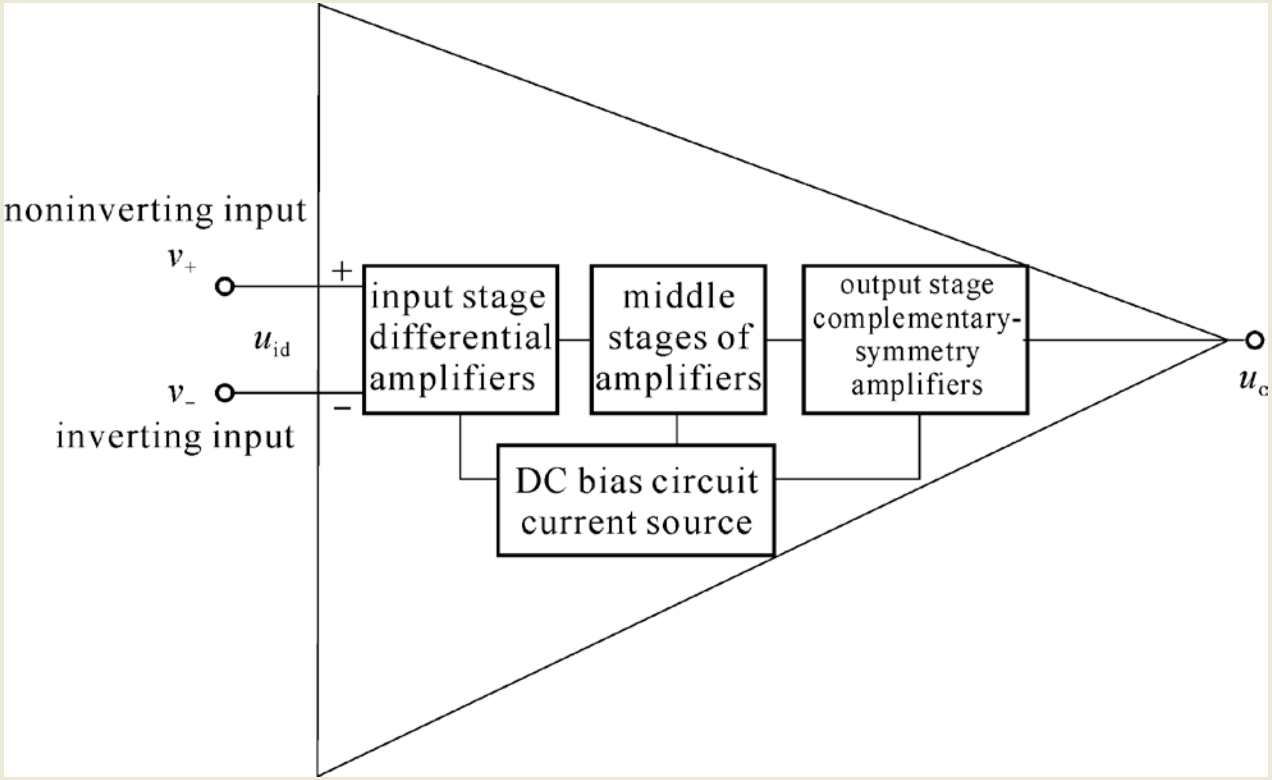
Chapter 8

Operational Amplifiers and Comparators

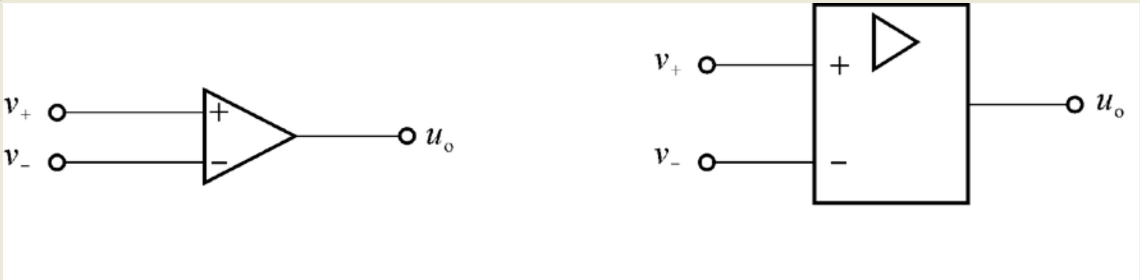
8.1 Construction and Basic Features of Operational Amplifiers

Construction of the op-amp:

- ***Input stage***
- ***Middle stage***
- ***Output stage***
- ***DC bias***



Symbol of the op-amp:



(a) the international symbol (b) the Chinese domestic symbol

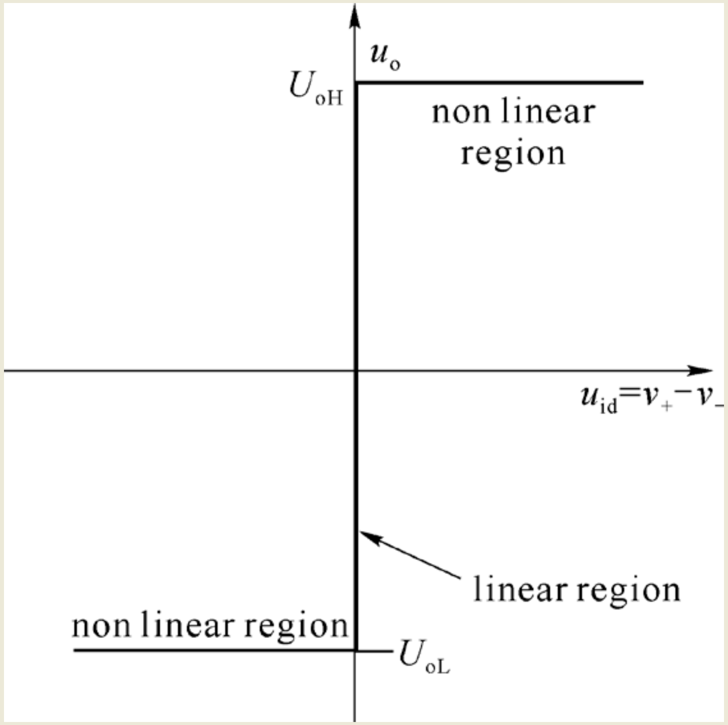
Note that the Op-amp has two inputs and one output.

8.2 Characteristics and Analyses on Ideal Operational Amplifiers

Characteristics of ideal op-amps:

- **Infinite gain for the differential signal: $A_{ud} = \infty$, $A_{uc}=0$**
- **Infinite Common Mode Rejection Ratio: $K_{CMRR} = \infty$**
- **Infinite input resistance: $R_i = \infty$**
- **Zero output resistance: $R_o = 0$**
- **Infinite bandwidth**

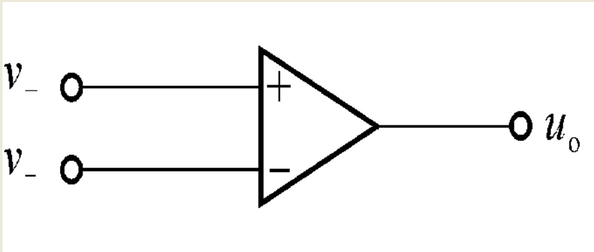
Transfer characteristics of ideal op-amps:



In the linear region, "virtue short" and "virtue open" are features for ideal op-amps

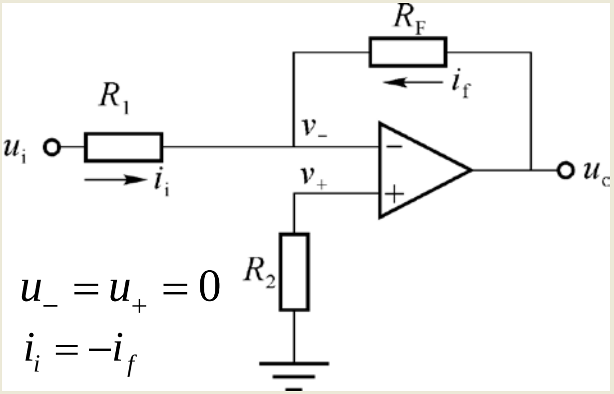
$$u_{id} = \frac{u_o}{A_{ud}} \Rightarrow$$ **virtual short: u_{id}=0, or $u_+=u_-$**

$$i_{id} = \frac{u_{id}}{R_i} \Rightarrow$$ **virtual open: i_{id}=0**



8.3.1 Multiplication with a Constant

(1) Inverting amplifiers



$$u_- = u_+ = 0$$
$$i_i = -i_f$$

- The input signal is applied to the **inverting (–) input.**
- The **non-inverting input (+)** is grounded.
- The resistor R_F is the **feedback resistor.**

The *non-inverting input* pin is at ground. *The inverting input* pin is also at 0V for an AC signal due to *virtual short* concept. *The inverting input* is at virtual ground.

$$\frac{u_i}{R_1} = -\frac{u_o}{R_f} \qquad u_o = -\frac{R_f}{R_1}u_i \qquad A_{uf} = -\frac{R_f}{R_1} \qquad R_{if} = R_1 \qquad R_{of} = 0$$

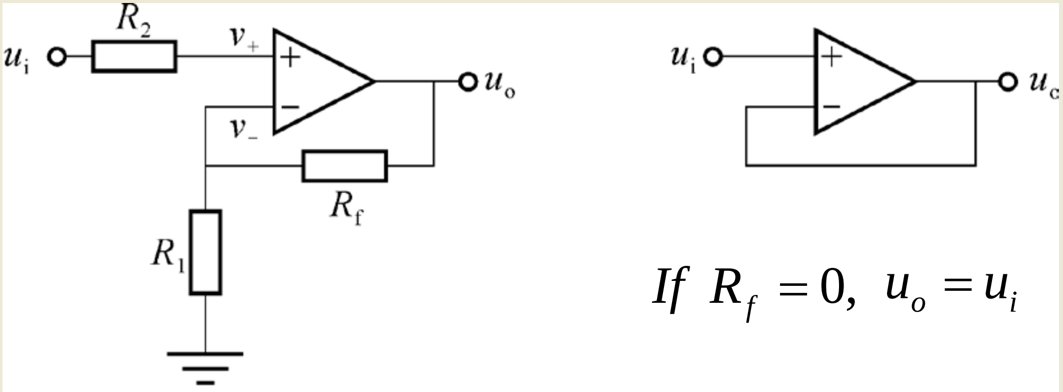
(2) Non-inverting amplifiers

$$u_+ = u_i \qquad u_- = \frac{R_1}{R_1 + R_f}u_o$$

$$u_+ = u_-$$

$$u_i = \frac{R_1}{R_1 + R_f}u_o$$

$$u_o = \left(1 + \frac{R_f}{R_1}\right)u_i \qquad A_{uf} = 1 + \frac{R_f}{R_1}$$



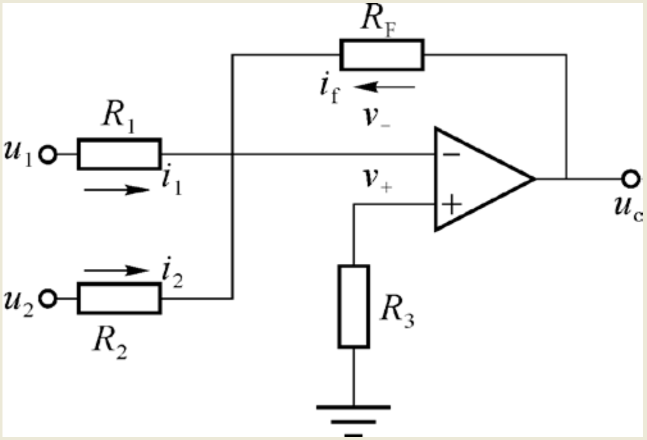
$$If \;\; R_f = 0, \;\; u_o = u_i$$

(a) Noninverting amplifier (b) voltage follower

For symmetric requirements: $(R_2 = R_1 \,||\, R_f)$

8.3.2 Summing Amplifiers

(1) Inverting Summing amplifiers



$$u_- = u_+ = 0$$

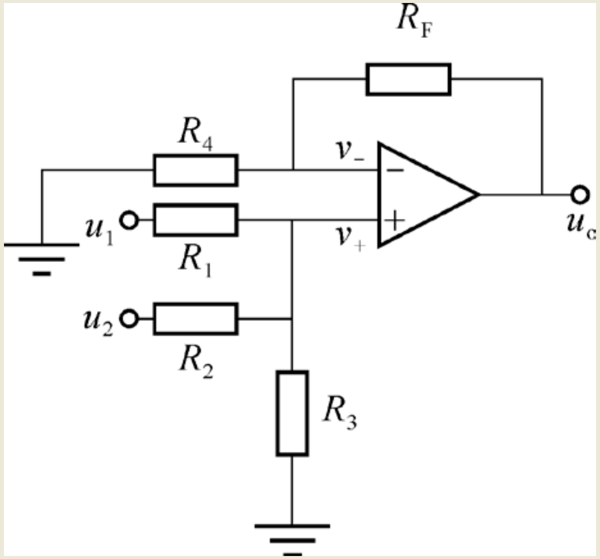
$$i_1 + i_2 = -i_f$$

$$\frac{u_1}{R_1} + \frac{u_2}{R_2} = -\frac{u_o}{R_f}$$

$$u_o = -\left(\frac{R_f}{R_1}u_1 + \frac{R_f}{R_2}u_2\right)$$

$$(R_3 = R_1 \parallel R_2 \parallel R_f)$$

(2) Non-inverting Summing



$$u_+ = u_-$$

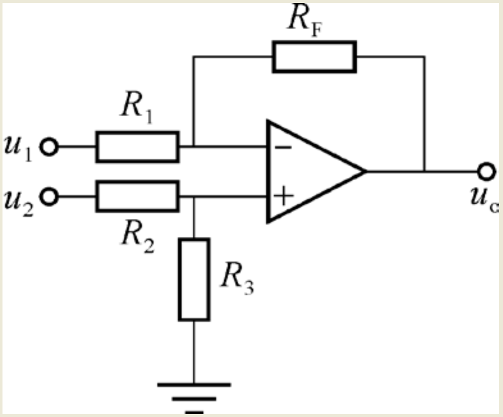
$$u_+ = \frac{R_2 \parallel R_3}{R_1 + R_2 \parallel R_3}u_1 + \frac{R_1 \parallel R_3}{R_2 + R_1 \parallel R_3}u_2$$

$$u_- = \frac{R_4}{R_4 + R_f}u_o$$

$$\frac{R_4}{R_4 + R_f}u_o = \frac{R_2 \parallel R_3}{R_1 + R_2 \parallel R_3}u_1 + \frac{R_1 \parallel R_3}{R_2 + R_1 \parallel R_3}u_2$$

$$u_o = \left(1 + \frac{R_f}{R_4}\right)\left(\frac{R_2 \parallel R_3}{R_1 + R_2 \parallel R_3}u_1 + \frac{R_1 \parallel R_3}{R_2 + R_1 \parallel R_3}u_2\right)$$

8.3.3 Subtraction Amplifiers

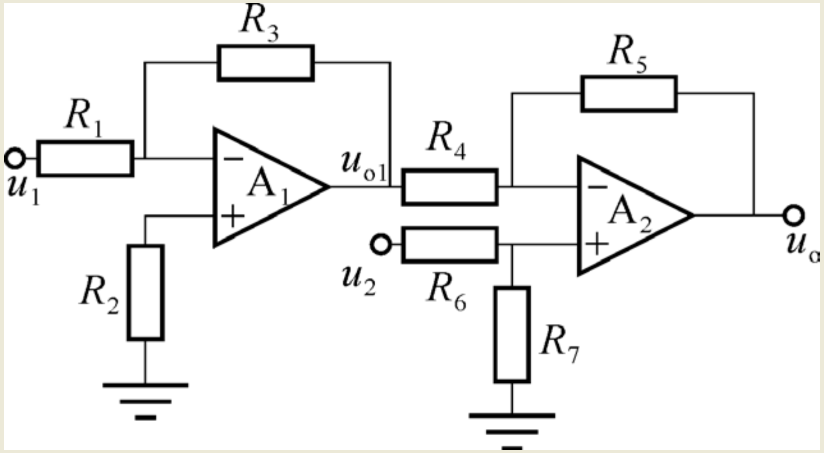


$$u_1 = 0 \qquad u_{o2} = \left(1 + \frac{R_f}{R_1}\right)\left(\frac{R_3}{R_2 + R_3}\right)u_2$$

$$u_2 = 0 \qquad u_{o1} = -\frac{R_f}{R_1}u_1$$

$$u_o = \left(1 + \frac{R_f}{R_1}\right)\left(\frac{R_3}{R_2 + R_3}\right)u_2 - \frac{R_f}{R_1}u_1$$

Example 8.1 $(R_4 = R_5 = R_6 = R_7)$



$$u_{o1} = -\frac{R_3}{R_1}u_1$$

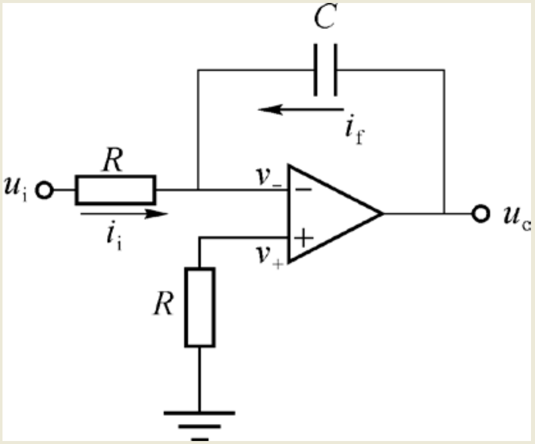
$$u_o = \frac{R_4 + R_5}{R_4} \bullet \frac{R_7}{R_6 + R_7}u_2 - \frac{R_5}{R_4}u_{o1} = u_2 - u_{o1}$$

$$u_o = u_2 + \frac{R_3}{R_1}u_1$$

$$(R_2 = R_1 \parallel R_3)$$

8.3.4 Integrators

(1) Inverting Integrators



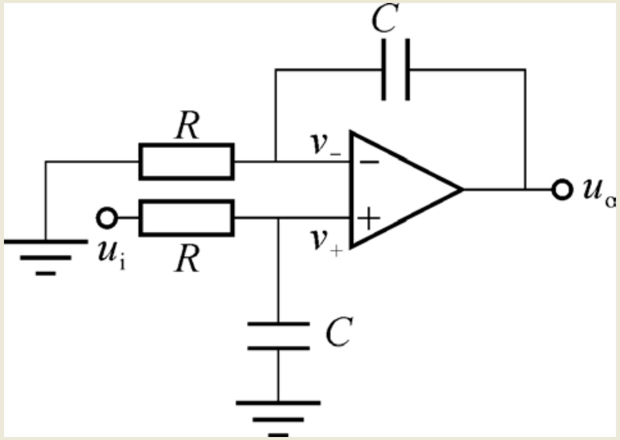
$$u_- = u_+ = 0$$

$$i_i = -i_f$$

$$\frac{u_i}{R} = -C\frac{du_o(t)}{dt}$$

$$u_o(t) = -\frac{1}{RC}\int_{t_0}^{t} u_i(t)dt + u_o(t_0)$$

(2) Non-inverting Integrators



$$u_+(j\omega) = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} u_i(j\omega)$$

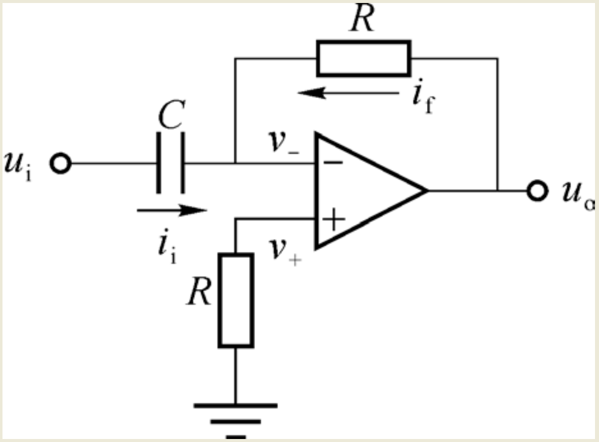
$$u_-(j\omega) = \frac{R}{R + \frac{1}{j\omega C}} u_o(j\omega)$$

$$u_+ = u_- \qquad u_o(j\omega) = \frac{1}{j\omega CR} u_i(j\omega)$$

$$u_o(t) = \frac{1}{CR}\int_{t_0}^{t} u_i(t)dt + u_o(t_0)$$

8.3.5 Differentiators

(1) Inverting Differentiators



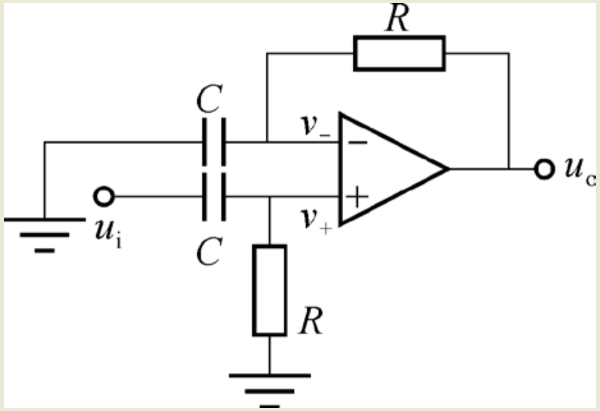
$$u_- = u_+ = 0$$

$$i_f = -i_i$$

$$\frac{u_o}{R} = -C\frac{du_i(t)}{dt}$$

$$u_o(t) = -RC\frac{du_i(t)}{dt}$$

(2) Non-inverting Differentiators



$$u_-(j\omega) = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} u_o(j\omega)$$

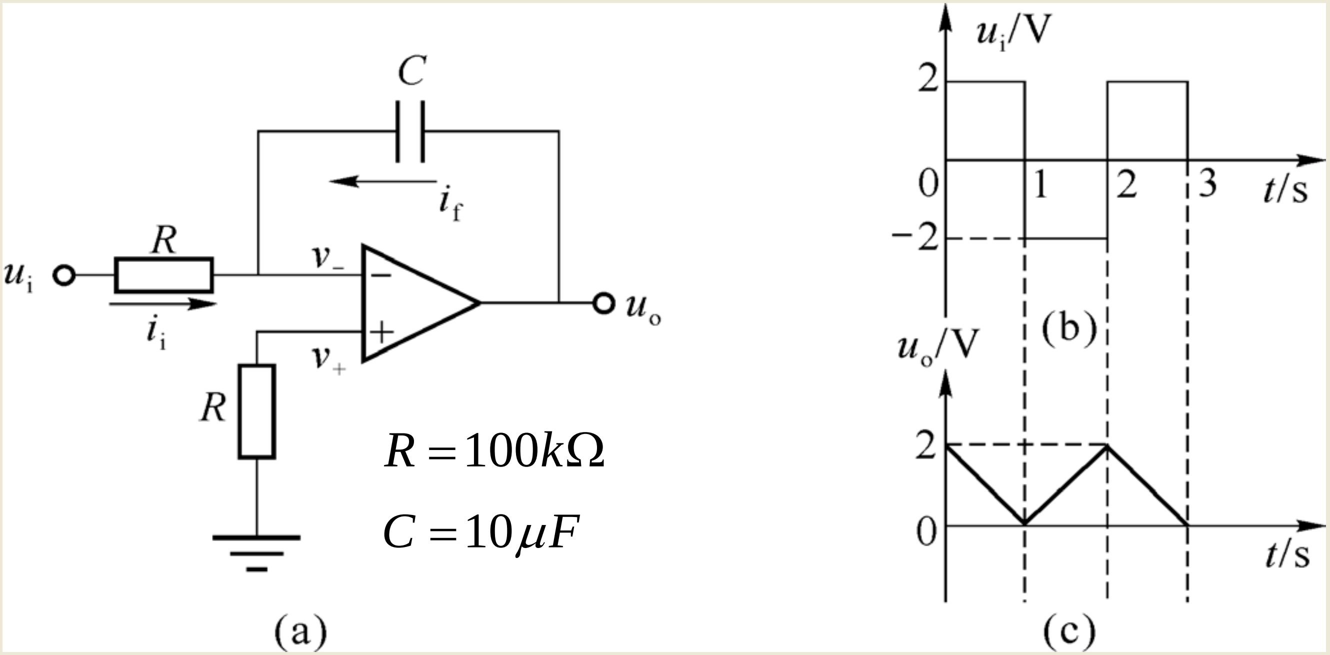
$$u_+(j\omega) = \frac{R}{R + \frac{1}{j\omega C}} u_i(j\omega)$$

$$u_+ = u_- \qquad u_o(j\omega) = j\omega CR \cdot u_i(j\omega)$$

$$u_o(t) = RC\frac{du_i(t)}{dt}$$

Example 8.2:

Sketch the output waveform



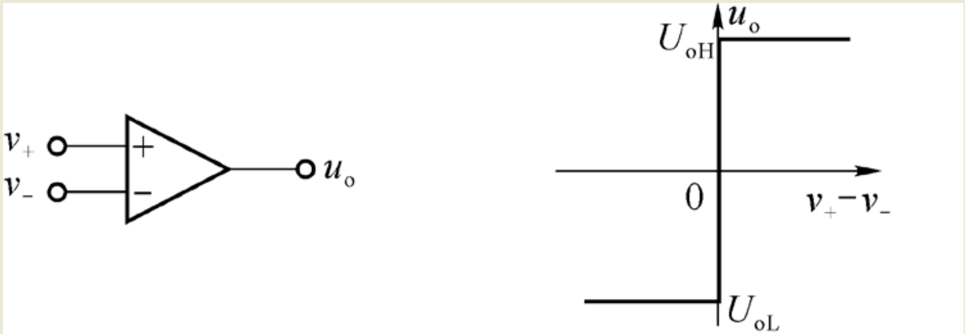
$$u_o(t) = -\frac{1}{RC}\int_{t_0}^{t} u_i(t)dt + u_o(t_0)$$

In the time duration of 0~1s $u_i = 2\text{V}$ $u_o(0) = 2\text{V}$ $u_o(t) = -2t + 2$ $u_o(1) = 0\text{V}$

In the time duration of 1~2s $u_i = -2\text{V}$ $u_o(1) = 0\text{V}$ $u_o(t) = 2t$ $u_o(2) = 2\text{V}$

8.4 Comparators

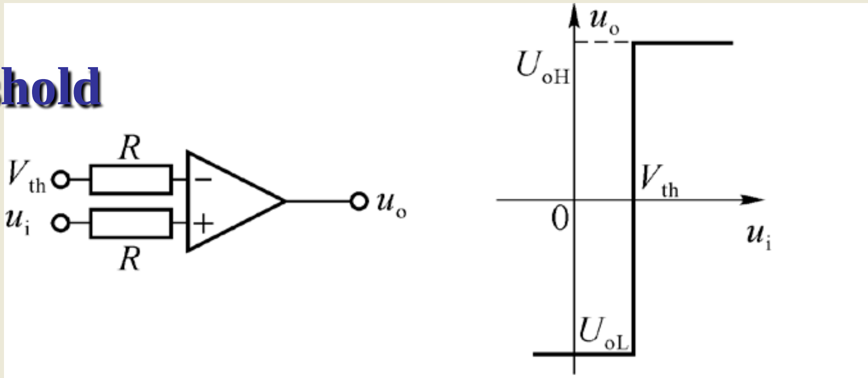
Symbol and voltage transfer characteristics of comparators



(a) symbol (b) Transfer characteristics

The operation is a basic comparator. The output swings between its maximum and minimum voltage, depending upon whether one input is greater or less than the other.

8.4.1 Basic Comparators with a single threshold



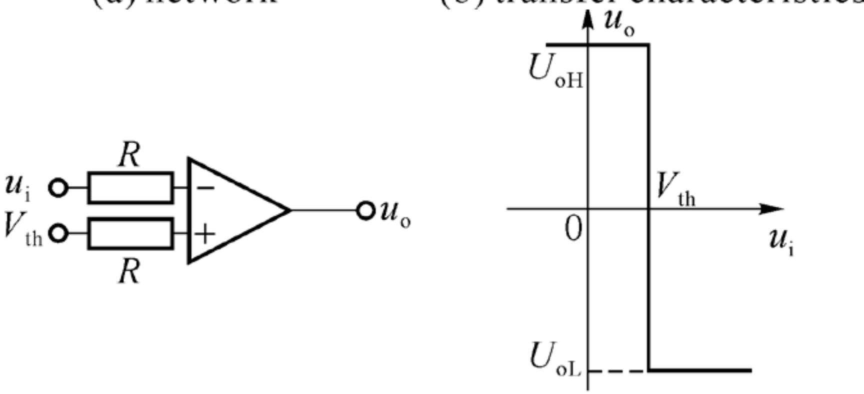
(a) network (b) transfer characteristics

For *non-inverting input comparator*:

- $u_o = U_{oH},\ if\ u_i > V_{th}$
- $u_o = U_{oL},\ if\ u_i < V_{th}$

For *inverting input comparator*:

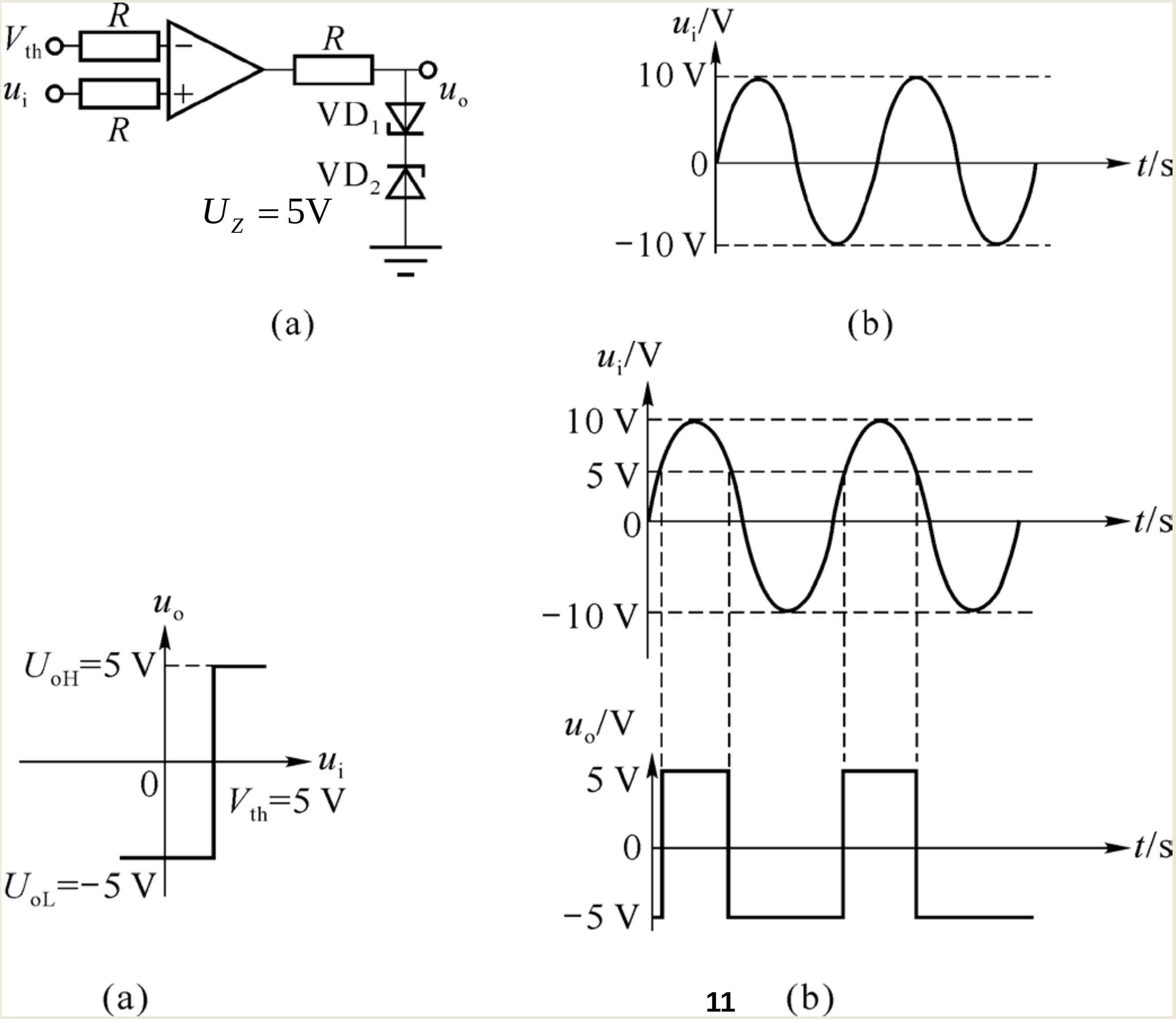
- $u_o = U_{oH},\ if\ u_i < V_{th}$
- $u_o = U_{oL},\ if\ u_i > V_{th}$



(a) network (b) transfer characteristics

Examples of Comparators

Example 8.3: **Sketch the transfer characteristics and the waveform of the output.**



Summary

- **Characteristics of op-amps and comparators**
    - ***Ideal assumptions***
    - ***Virtual short and Virtual open***
- **Application of op-amps in the linear region**
    - ***Inverting vs. non-inverting***
    - ***Summing***
    - ***Subtraction***
    - ***Integrator***
    - ***Differentiator***
- **Comparator: application of op-amps in the nonlinear region**
    - ***Basic comparator with single threshold***