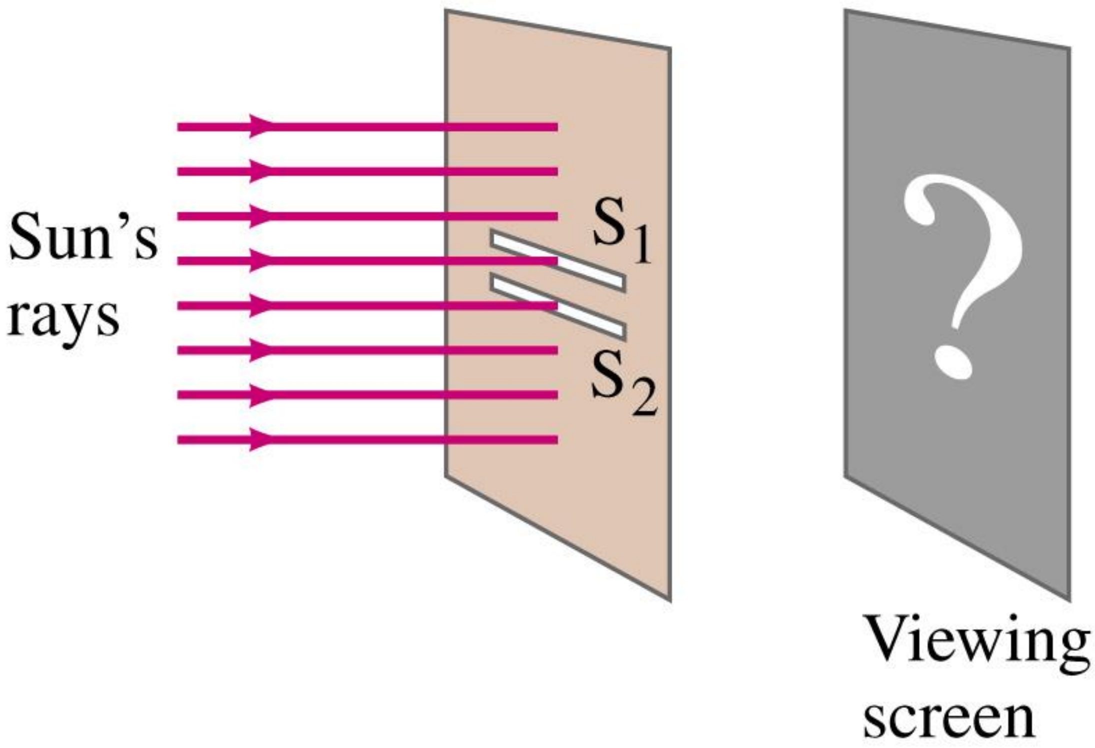
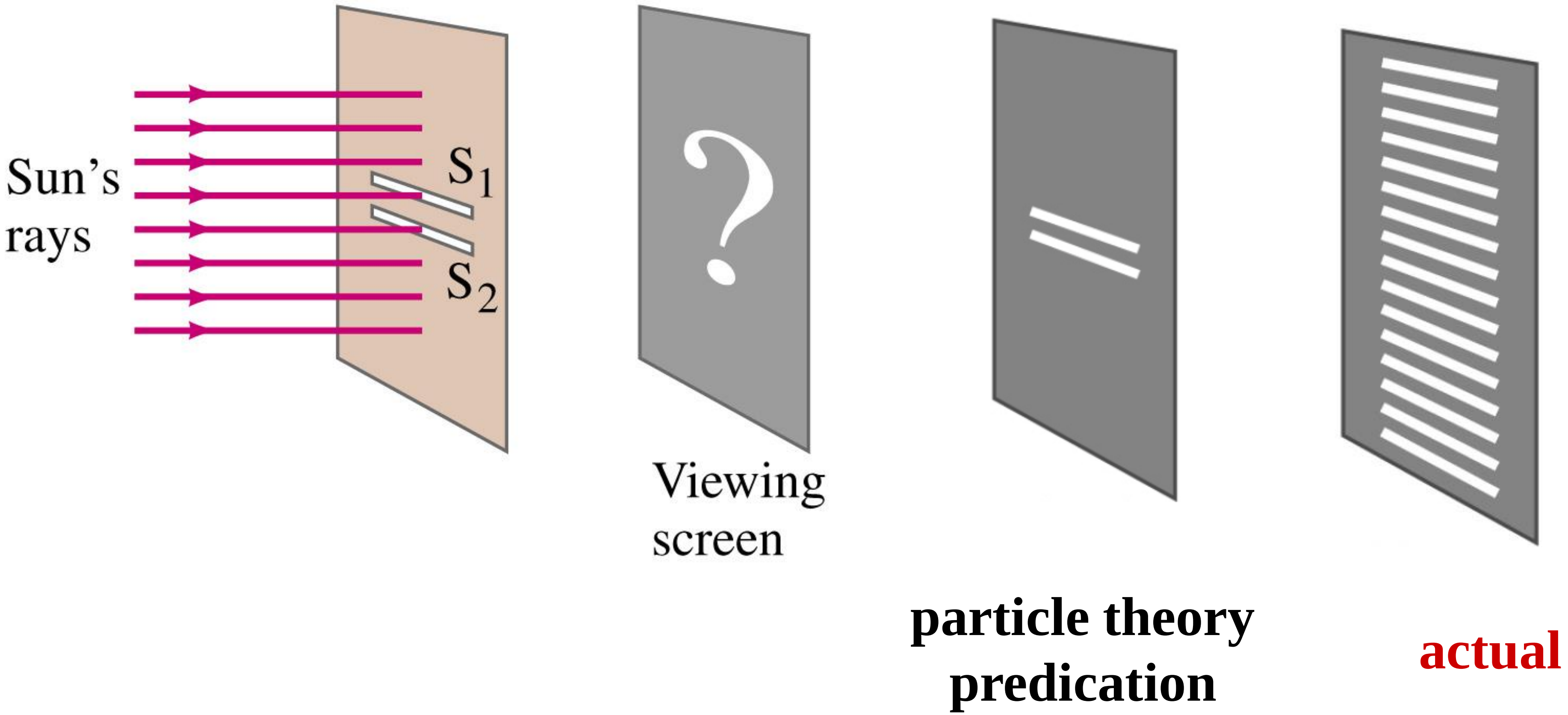
§ 1 Young's Double-Slit Experiment



In 1801

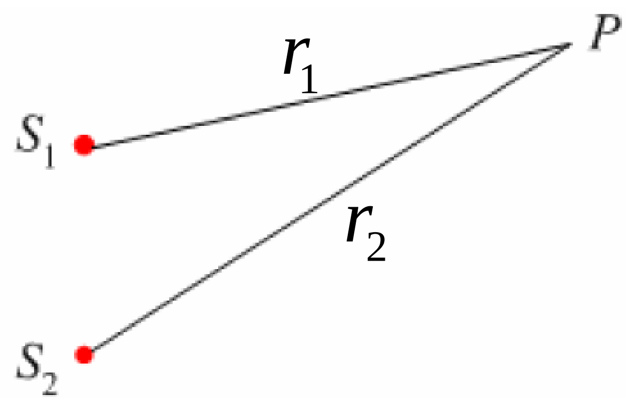
Thomas Young,
English physician
(1773 – 1829)

Young's Double-Slit Experiment



Review: Two-source interference

- **Two identical monochromatic waves from two sources overlap in a region**



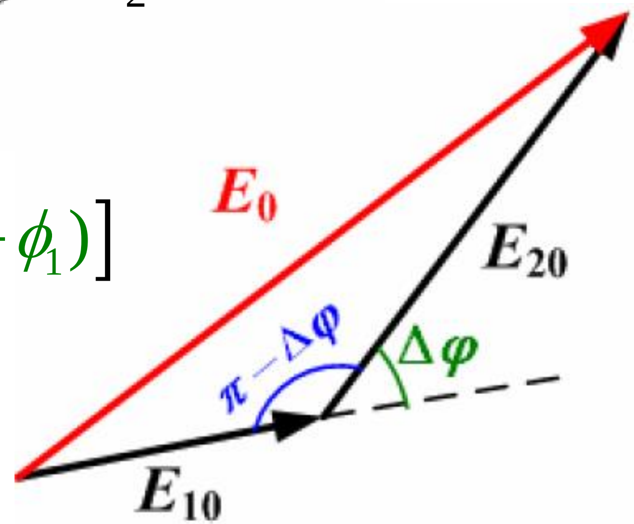
$$E_1 = E_{10}\cos(\omega t - kr_1 + \phi_1)$$

$$E_2 = E_{20}\cos(\omega t - kr_2 + \phi_2)$$

$$E = E_1 + E_2 = E_0\cos(\omega t + \varphi)$$

$$E_0^2 = E_{10}^2 + E_{20}^2 + 2E_{10}E_{20}\cos\left[k(r_2 - r_1) - (\phi_2 - \phi_1)\right]$$

$$\boxed{I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\Delta\varphi}$$



Phasor Diagram

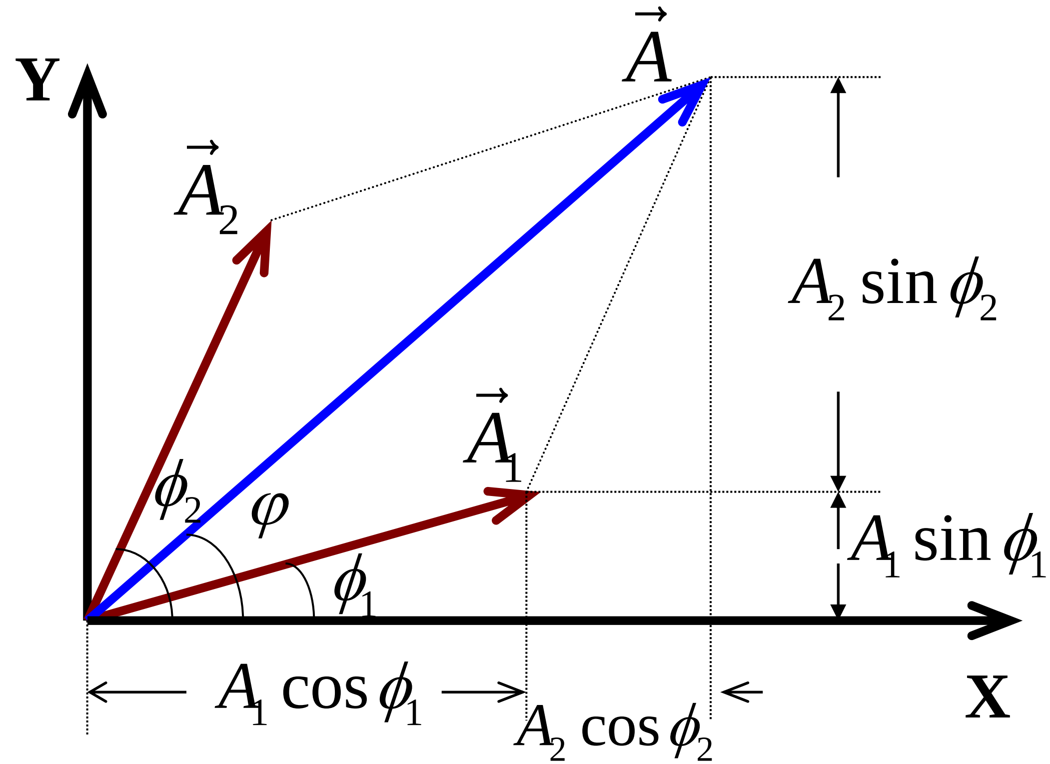
Phase difference: $\Delta\varphi = k(r_2 - r_1) - (\phi_2 - \phi_1)$

Depending on two factors:

(1) the location of point P

(2) the difference of two initial phase angles.

Using phasors,



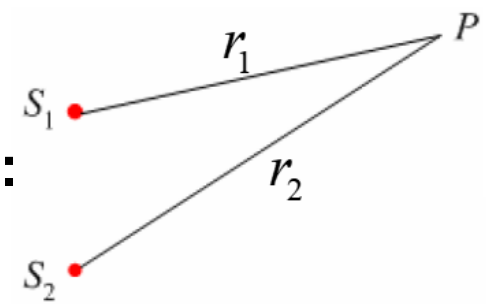
$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\phi_2 - \phi_1)}$$

$$\varphi = \arctan\frac{A_1\sin\phi_1 + A_2\sin\phi_2}{A_1\cos\phi_1 + A_2\cos\phi_2}$$

Review: Two-source interference

$$\boxed{I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\Delta\varphi,} \quad \Delta\varphi = k(r_2 - r_1) - (\phi_2 - \phi_1)$$

- **Constructive and destructive interference:**



Suppose the two sources are identical $\phi_1 = \phi_2$

For some points, maximum intensity occurs:

$$\Delta\varphi = \frac{2\pi}{\lambda}(r_2 - r_1) = \pm 2m\pi, \quad m = 0, 1, 2, \ldots$$ **in phase**

or path difference: $\delta = r_2 - r_1 = \pm m\lambda, \quad m = 0, 1, 2\ldots$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} = \xrightarrow{I_1 = I_2} I_{max} = 4I_1$$ **constructive interference**

For some points, minimum intensity occurs :

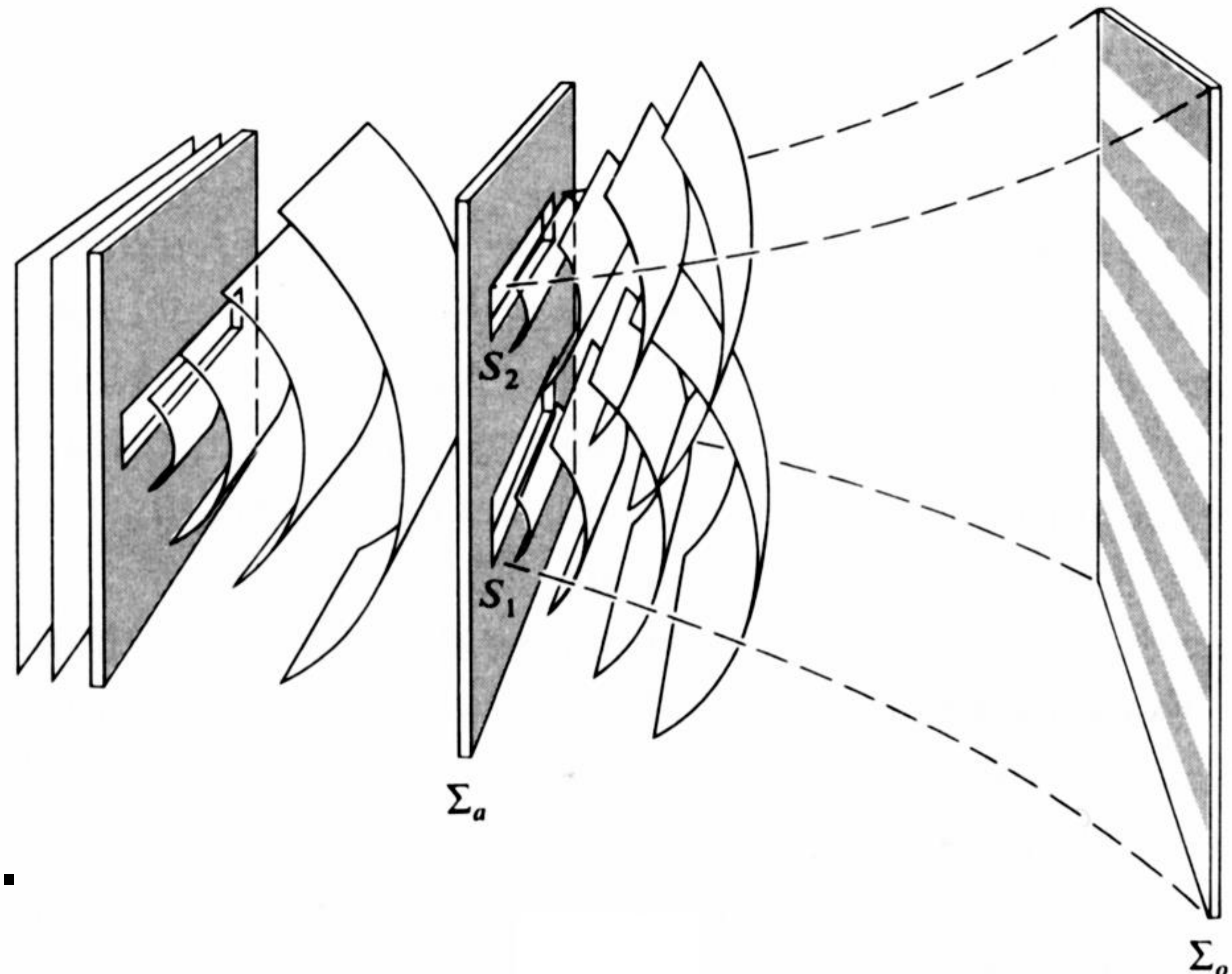
$$\Delta\varphi = \frac{2\pi}{\lambda}(r_2 - r_1) = \pm(2m+1)\pi, \quad m = 0, 1, 2, \ldots$$ **out of phase**

or path difference: $\delta = r_2 - r_1 = \pm(2m+1)\frac{\lambda}{2}, \quad m = 0, 1, 2\ldots$

$$I = I_1 + I_2 - 2\sqrt{I_1 I_2} = \xrightarrow{I_1 = I_2} I_{min} = 0$$ **destructive interference**

- Young's double-slit experiment —— wavefront-splitting interference

➡ A light source emits monochromatic light. The light is directed at a screen with a narrow slit S_0. The light from slit S_0 falls on a screen with two other narrow slits S_1 and S_2, with distance d apart.

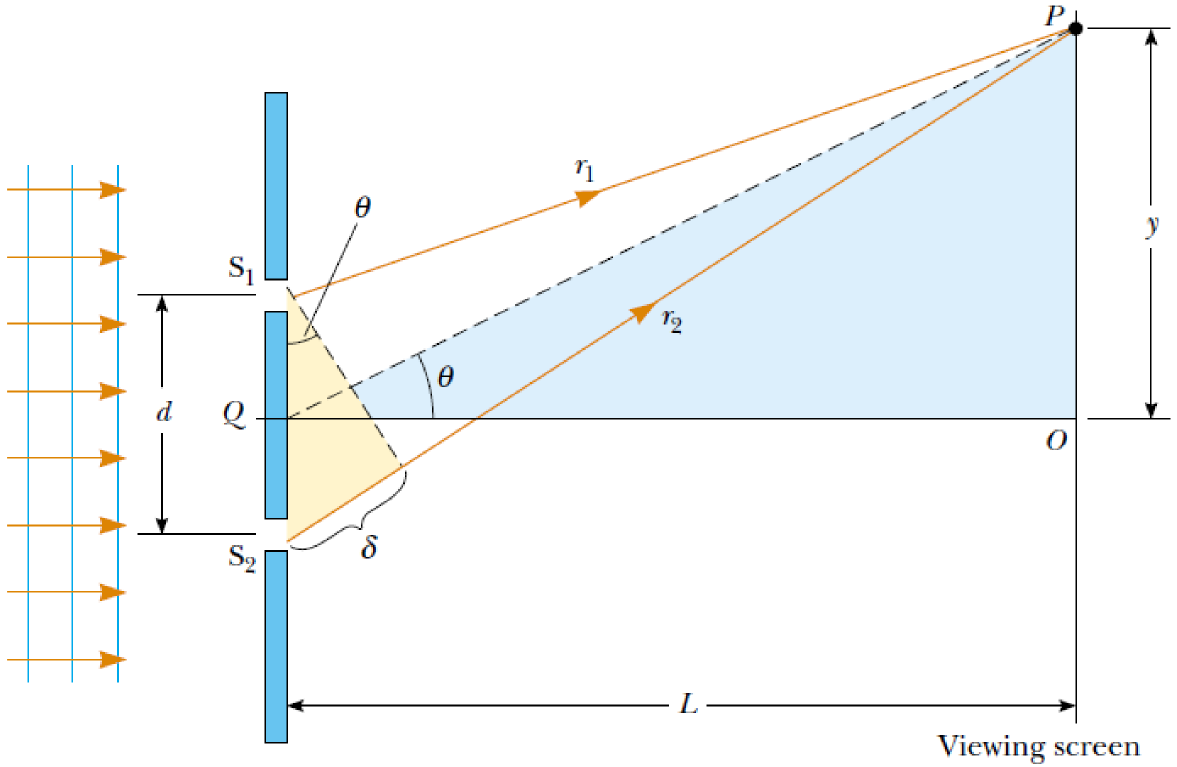


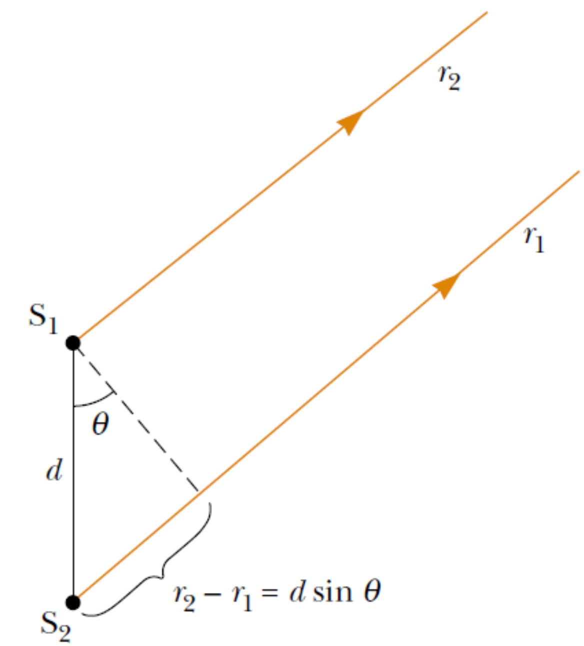
Wave (optical) path length difference:

Generally: $\begin{cases} d: 0.1 \sim 1 \text{ mm} \\ L: 1 \sim 10 \text{ m} \\ |y| \leq 1 \sim 10 \text{ cm} \end{cases}$

$$L \gg d, \quad L \gg |y|$$

$$\delta = r_2 - r_1 \approx d \sin\theta$$





Bright and dark fringes

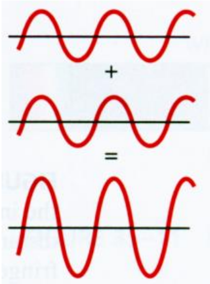
- **Bright fringes —— constructive interference**

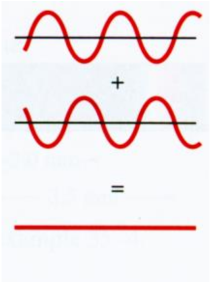
$$\delta = d\sin\theta_{\text{bright}} = \pm m\lambda, \quad m = 0, 1, 2, \cdots$$

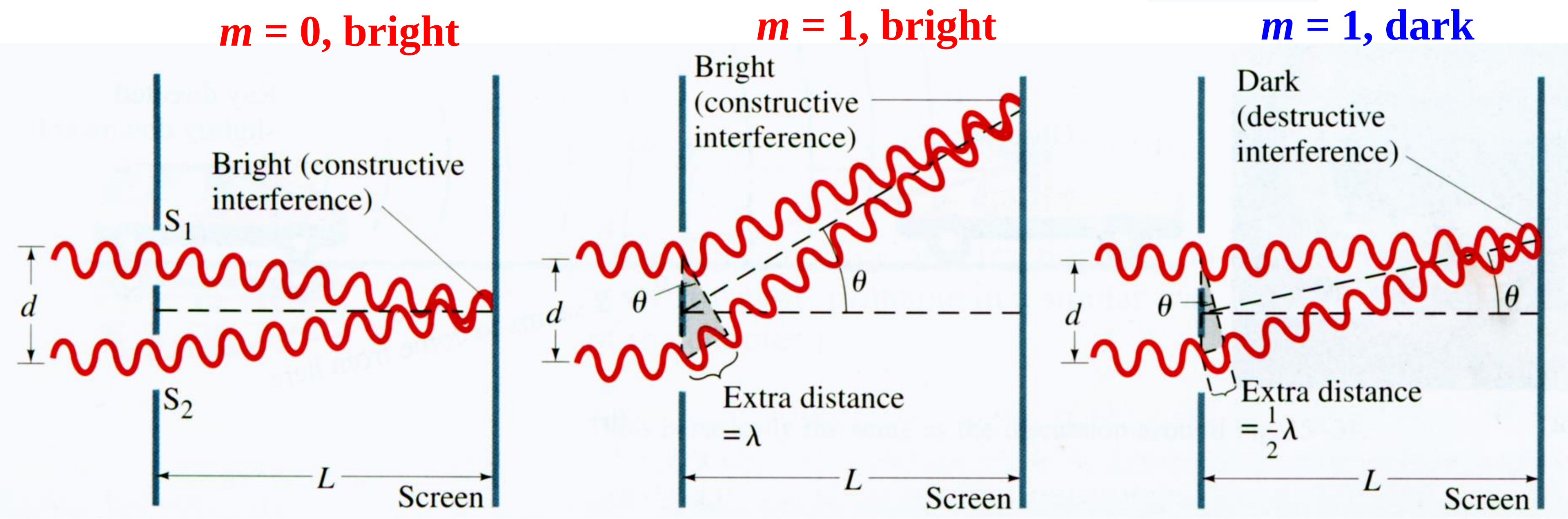
- **Dark fringes —— destructive interference:**

$$\delta = d\sin\theta_{\text{dark}} = \pm\left(m - \frac{1}{2}\right)\lambda, \quad m = 1, 2, \cdots$$

***m* is the order of the fringe**







Double-slit interference

- The positions of bright and dark fringes:

  θ is so small that $\sin\theta \approx \tan\theta$, $\delta = d\sin\theta \approx d\tan\theta = d\dfrac{y}{L}$, $y = \dfrac{L}{d}\delta$

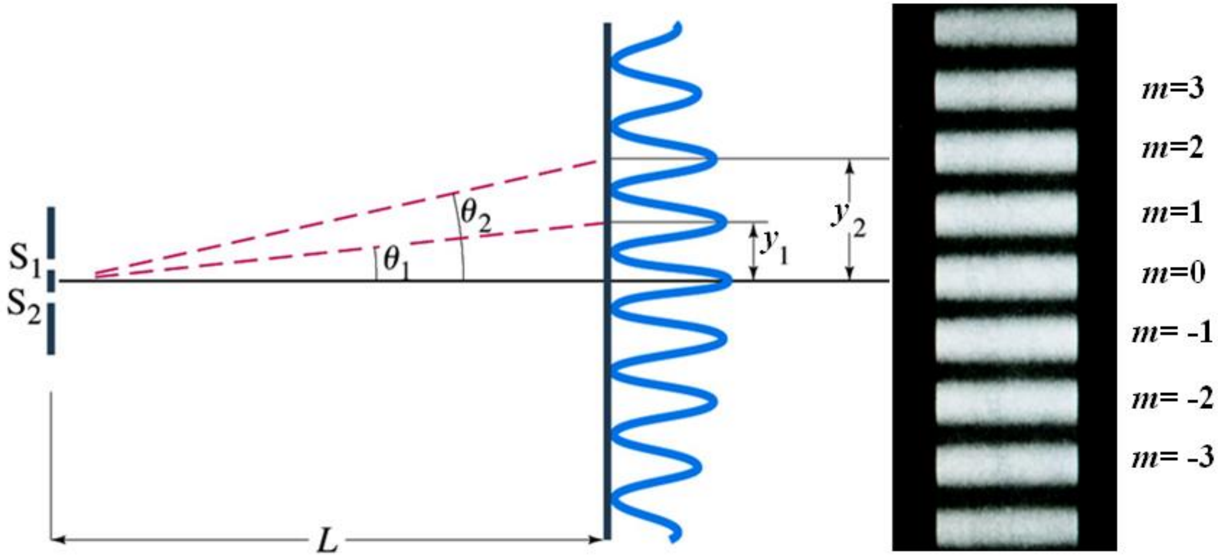
  - Bright fringes:

$$y_{\text{bright}} = \pm m\frac{L}{d}\lambda, \quad m = 0, 1, 2, \ldots$$

  - Dark fringes :

$$y_{\text{dark}} = \pm\left(m - \frac{1}{2}\right)\frac{L}{d}\lambda, \quad m = 1, 2, \ldots$$

  - Spacing of the fringes: $\Delta y = y_{m+1} - y_m = (m+1)\dfrac{L}{d}\lambda - m\dfrac{L}{d}\lambda = \dfrac{L}{d}\lambda$



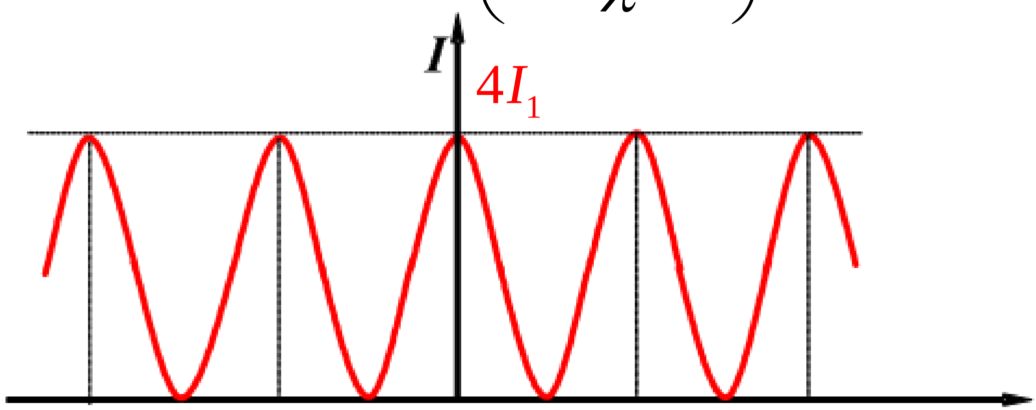
Equal fringe spacing

Intensity in the double-slit interference pattern

- Intensity distribution: $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\Delta\varphi, \quad \Delta\varphi = \dfrac{2\pi}{\lambda} d\sin\theta$
  - Suppose the width of two slit are the same

$$I_1 = I_2,\ \ I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\Delta\varphi = 2I_1(1+\cos\Delta\varphi) = 4I_1\cos^2\left(\frac{\Delta\varphi}{2}\right)$$

$$= 4I_1\cos^2\left(\frac{\pi d\sin\theta}{\lambda}\right) \approx 4I_1\cos^2\left(\frac{\pi d}{\lambda L}y\right)$$



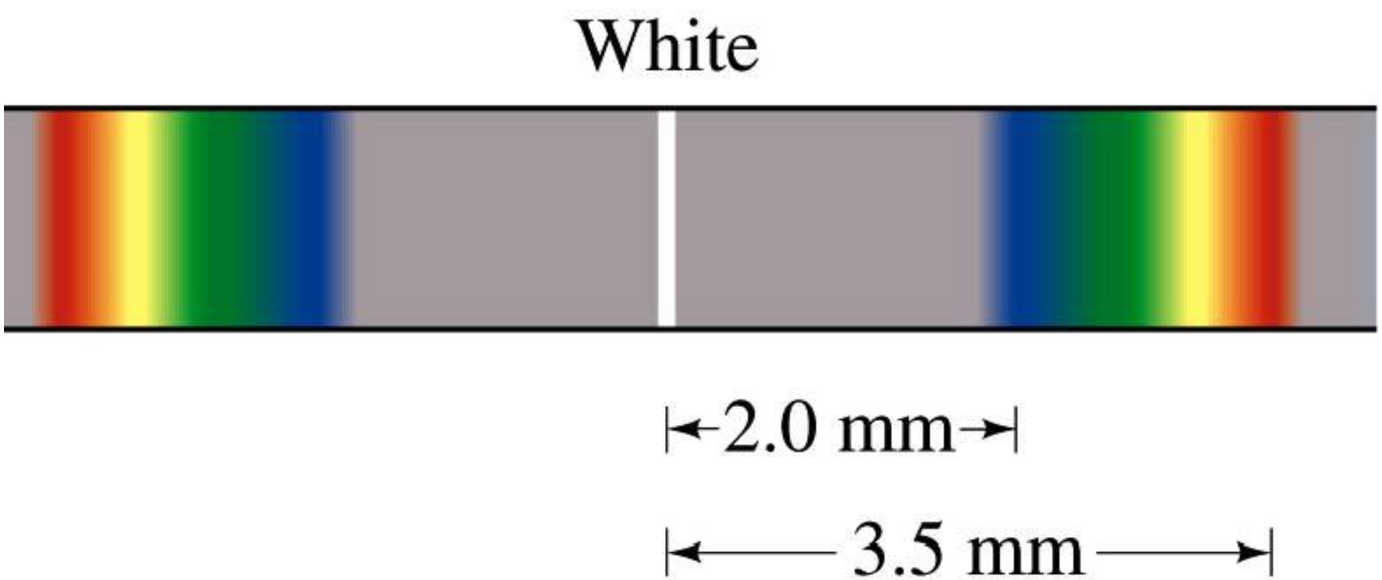
$$\sin\theta \approx \tan\theta = y/L$$

-4π	-2π	0	2π	4π	$\Delta\varphi$	Phase difference
-2	-1	0	1	2	m	Order of fringes
$-2\lambda/d$	$-\lambda/d$	0	λ/d	$2\lambda/d$	$\sin\theta$	Angular position
$-2\lambda L/d$	$-\lambda L/d$	0	$\lambda L/d$	$2\lambda L/d$	y	Position

Example

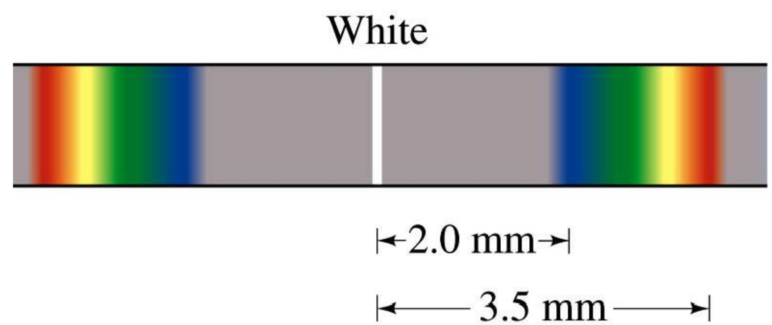
Wavelength from **double-slit interference**

White light passes through two slits 0.50mm apart and an interference pattern is observed on a screen 2.5m away. The first-order fringe resembles a rainbow with violet and red light at either end. The violet light falls about 2.0mm and the red 3.5mm from the center of the central white fringe. Estimate the wavelengths of the violet light and the red light.



Example

White light passes through two slits **0.50mm** apart and an interference pattern is observed on a screen **2.5m** away. The first-order fringe resembles a rainbow with violet and red light at either end. The violet light falls about **2.0mm** and the red **3.5mm** from the center of the central white fringe. Estimate the wavelengths of the violet light and the red light



Solution:

$$y_{\text{bright}} = \pm m \frac{L}{d}\lambda, \quad m = 1$$

For violet light y =2.0 mm

$$\lambda_{\text{violet}} = \frac{d}{L}\frac{y}{m} = \frac{(5.0\times10^{-4}\,\text{m})(2.0\times10^{-3}\,\text{m})}{2.5\text{m}} = 4.0\times10^{-7}\,\text{m} = 400\ \text{nm}$$

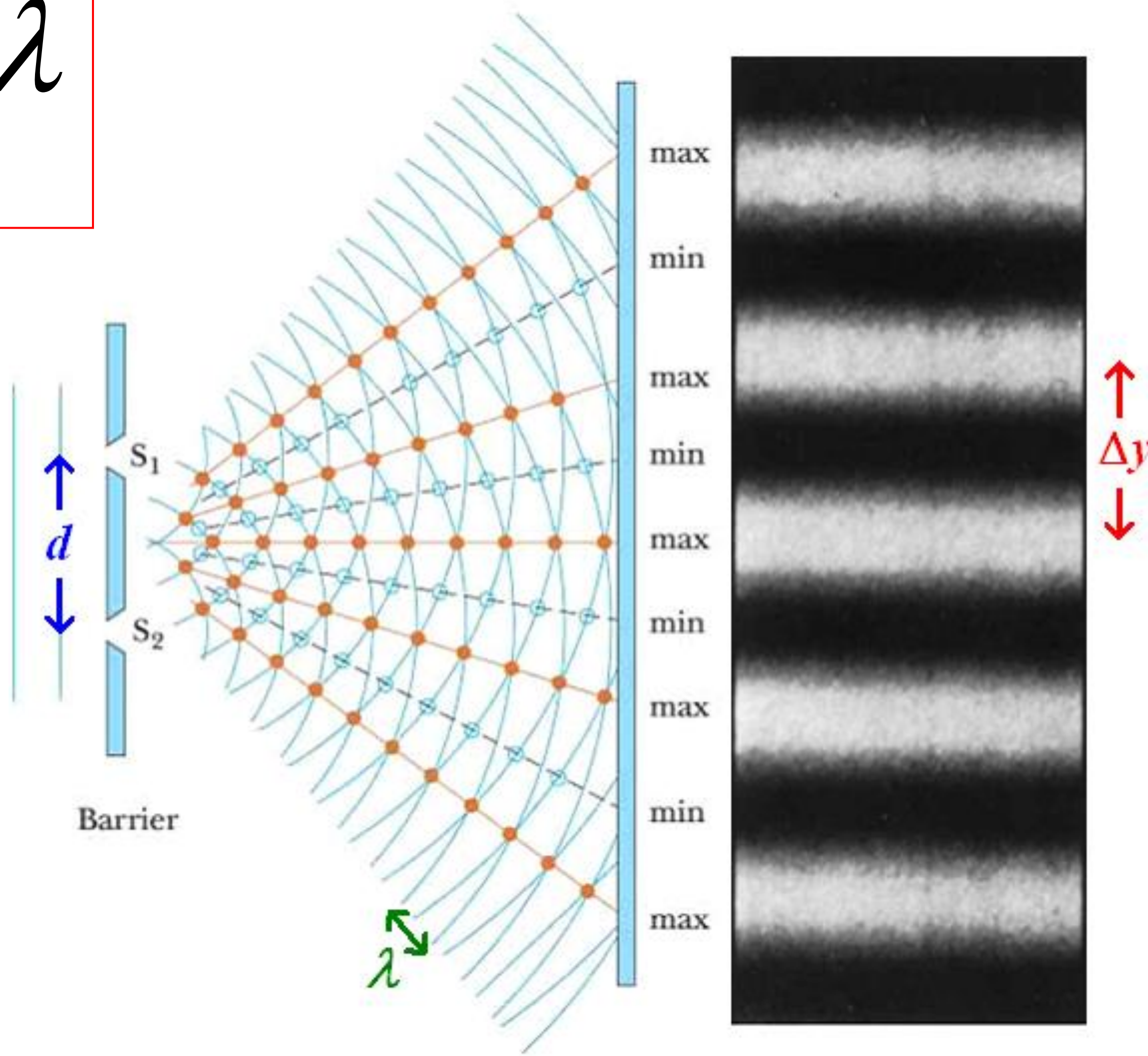
For red light y =3.5 mm

$$\lambda_{\text{red}} = \frac{d}{L}\frac{y}{m} = \frac{(5.0\times10^{-4}\,\text{m})(3.5\times10^{-3}\,\text{m})}{2.5\text{m}} = 7.0\times10^{-7}\,\text{m} = 700\ \text{nm}$$

Example (Cont'd)

$$\Delta y = \frac{L}{d}\lambda$$

Fringe spacing is proportional to λ, L, and inverse proportional to d.



Problems

P698, Prob. 6, 7, 9

(a) Consider three equally spaced and equal-intensity coherent sources of light (such as adding a third slit to the two slits). Use the phasor method to obtain the intensity as a function of the phase difference $\Delta\varphi$.

(b) Determine the positions of maxima and minima.

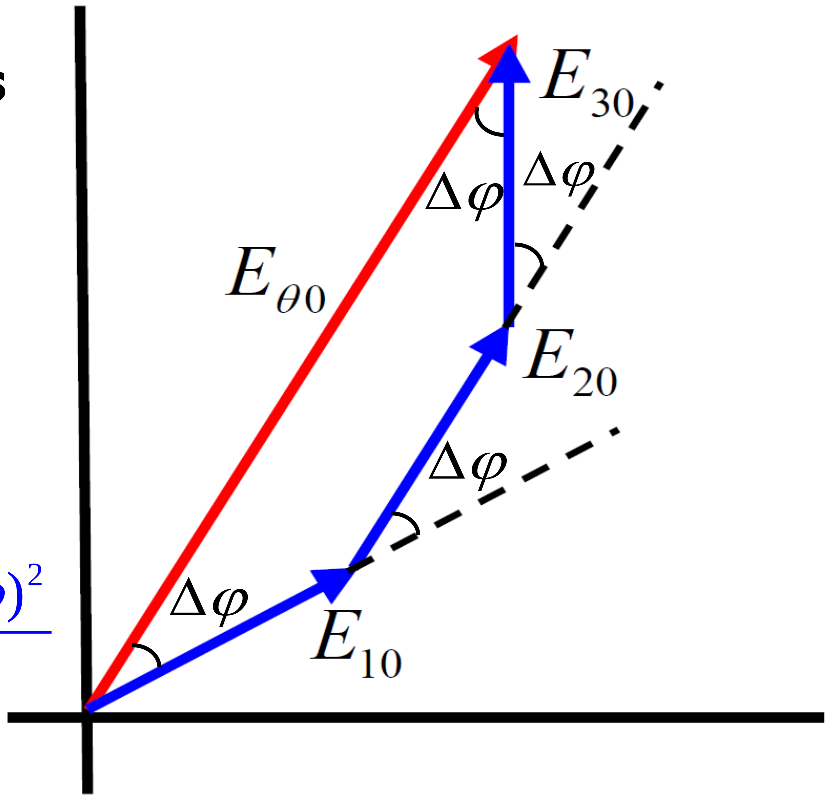
Solution:

(a) As shown, the magnitude of $E_{\theta 0}$ is

$$E_{\theta 0} = E_{10}\cos\Delta\varphi + E_{20} + E_{30}\cos\Delta\varphi$$

$$= E_{10}(1+2\cos\Delta\varphi)$$

$$\Delta\varphi = \frac{2\pi}{\lambda} d\sin\theta$$



The relative intensity is

$$\frac{I_\theta}{I_0} = \frac{E_{\theta 0}^2}{E_{00}^2} = \frac{[E_{10}(1+2\cos\Delta\varphi)]^2}{[E_{10}(1+2\cos 0)]^2} = \frac{(1+2\cos\Delta\varphi)^2}{9}$$

where I_0 is the maximum intensity.

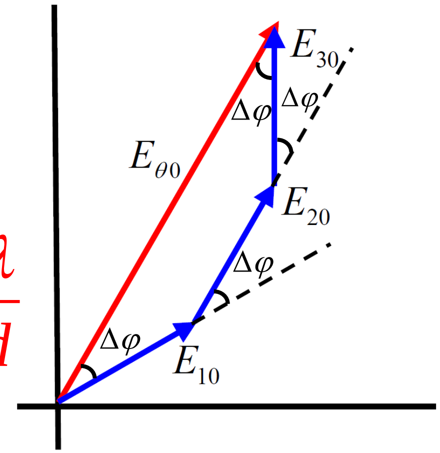
(b) Determine the positions of maxima and minima.

Solution:

$$I_\theta = \frac{(1+2\cos\Delta\varphi)^2}{9} I_0$$

(b) When the three phasors are all in line, the intensity will be at its maximum I_0.

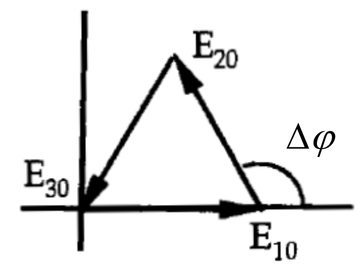
$$(\Delta\varphi)_{\max} = \frac{2\pi}{\lambda} d\sin\theta = (2\pi)m, \ \ m = 0, \pm1, \pm2\dots \qquad \sin\theta_{\max} = m\frac{\lambda}{d}$$



The intensity will be a minimum when

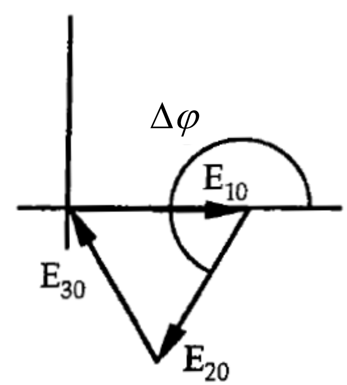
$$1 + 2\cos\Delta\varphi = 0$$

$$(\Delta\varphi)_{\min} = \arccos(-\frac{1}{2}) = \begin{cases} \frac{2}{3}\pi + 2m\pi = 2\pi(m+\frac{1}{3}) \\ \frac{4}{3}\pi + 2m\pi = 2\pi(m+\frac{2}{3}) \end{cases}, m = 0, \pm1, \pm2\dots$$



$$(\Delta\varphi)_{\min} = 2\pi(m + \frac{1}{3}k) = \frac{2\pi}{\lambda} d\sin\theta_{\min}, \ k = 1, 2; \ m = 0, \pm1, \pm2\dots$$

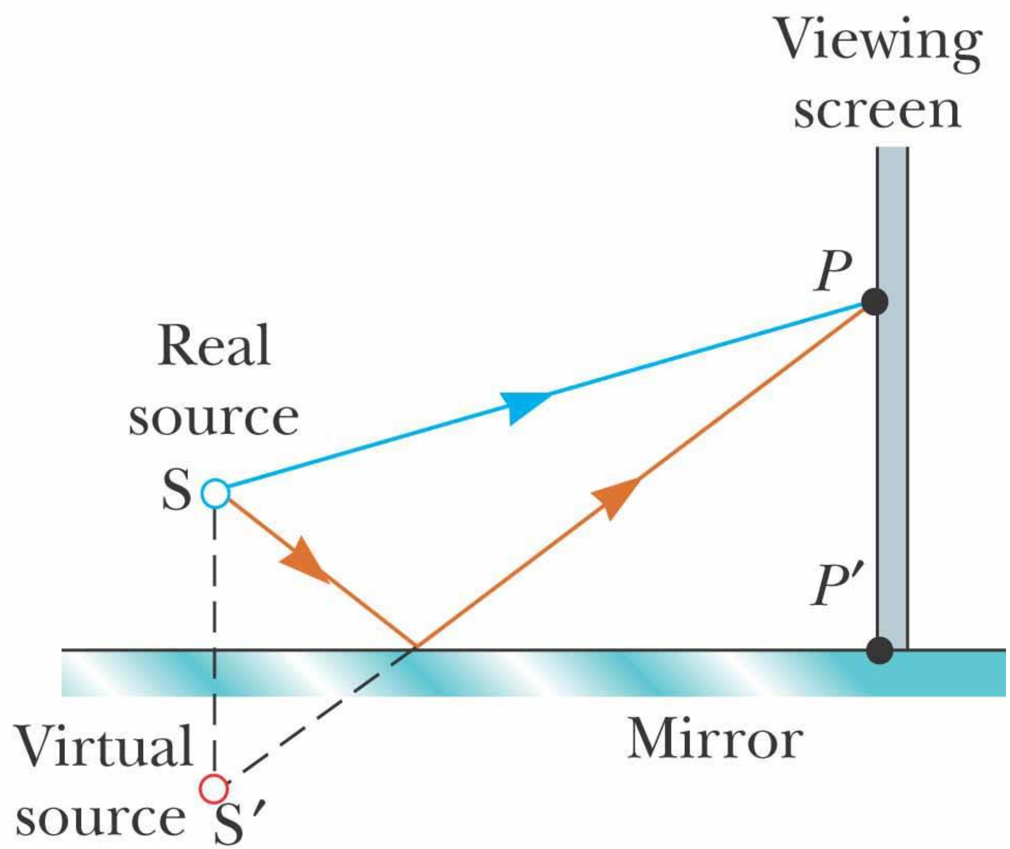
$$\sin\theta_{\min} = \frac{\lambda}{d}(m + \frac{1}{3}k)$$



Lloyd’s mirror (劳埃德镜)

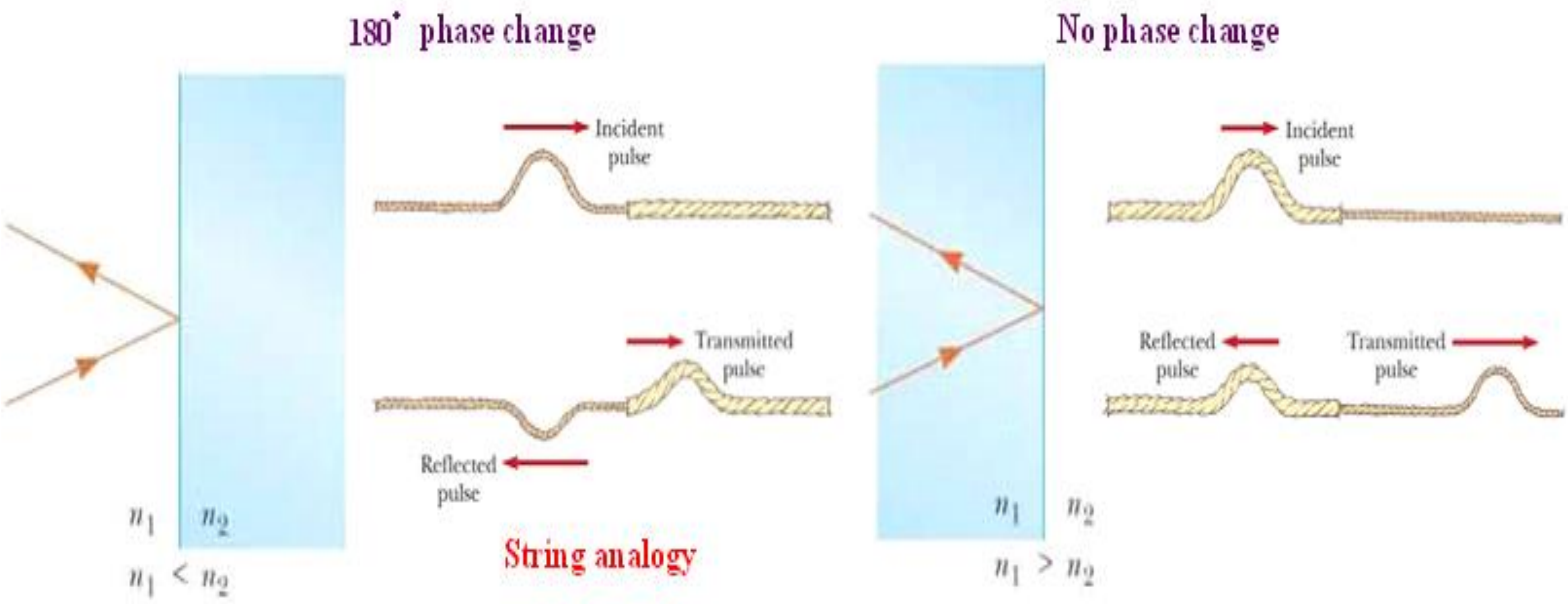
- Lloyd’s mirror —— another wavefront-splitting interference:

➧ The mirror reflects a portion of wavefront coming from slit *S*. This wave seems to come from the mirror image *S′*. Another portion of the wavefront proceeds directly from the slit to the screen. The interference pattern in the viewing screen is formed by coherent sources *S* and its image *S′*.



Lloyd's mirror

- **The phase change at the reflecting boundary:**

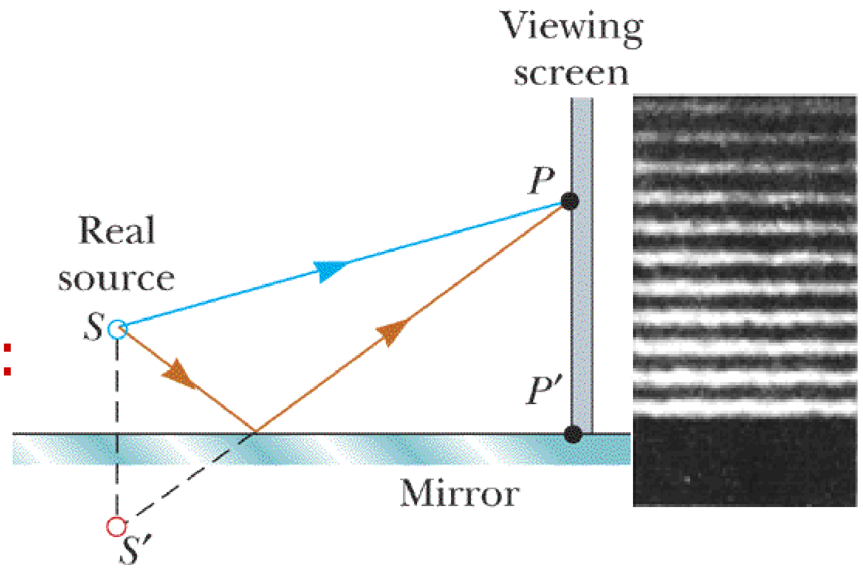


- The phase difference in Lloyd's mirror interference:
  - The distinguishing feature of Lloyd's mirror is that the reflected beam undergoes a 180° phase shift, corresponds to a additional optical path length difference λ/2.
  - The phase difference:

$$\Delta\varphi = \frac{2\pi}{\lambda}\delta = \frac{2\pi}{\lambda} d\sin\theta \underline{\pm\pi}$$

  - The optical path length difference:

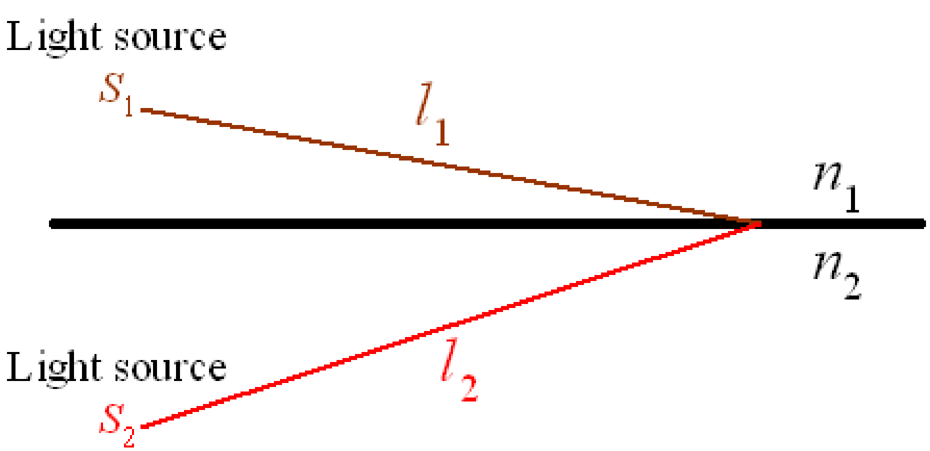
$$\delta = S'P - SP \underline{\pm\frac{\lambda}{2}} = d\sin\theta \underline{\pm\frac{\lambda}{2}}$$



- The interference pattern for Lloyd's mirror
  - Unlike we will see a bright fringe at ***P′*** in Young's double-slit experiment, instead we will observe a dark fringe at ***P′*** because of 180° phase shift on reflection. The whole pattern is a reversion of Young's pattern.

§ 2 Optical Path Length

Comparison of phase retardations when two light waves travel through two different routes.



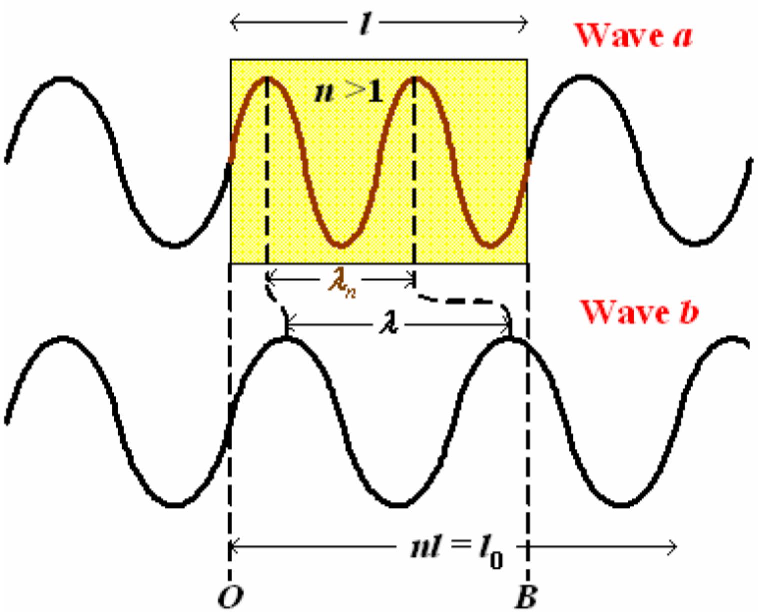
$$\Delta\varphi = 2\pi\frac{l_2}{\lambda_2} - 2\pi\frac{l_1}{\lambda_1}$$

$$= 2\pi\frac{l_2}{\frac{\lambda}{n_2}} - 2\pi\frac{l_1}{\frac{\lambda}{n_1}}$$

$$= \frac{2\pi}{\lambda}(n_2 l_2 - n_1 l_1)$$

Optical Path Length

- Why introducing the optical path length
  - Light travels with speed v in medium rather than c in vacuum.
  - The propagation of wave $\boldsymbol{a}$ undergoes a medium whose index is $\boldsymbol{n} > \mathbf{1}$.
  - When the waves arrive at point B, the phase retardation for wave $\boldsymbol{a}$ is



$$\varphi_a = \frac{2\pi}{\lambda_n} l$$

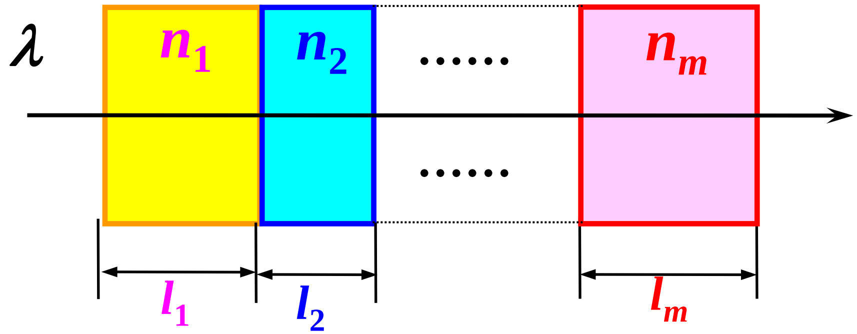
λ_n is the wavelength in the medium. $\lambda_n = \dfrac{\lambda}{n}$

$$\varphi_a = \frac{2\pi}{\lambda} nl = \frac{2\pi}{\lambda} L, \qquad L = nl$$

- The number of wavelength covered by the distance $\boldsymbol{l}$ in the medium is equal to the number of wavelength covered by the distance $\boldsymbol{L = nl}$ in the vacuum.

Optical path length

- **The optical path**
  - **When a light wave travel across a series of medium composed of m layers, for medium i the index is n_i, the optical path traversed by the wave is**

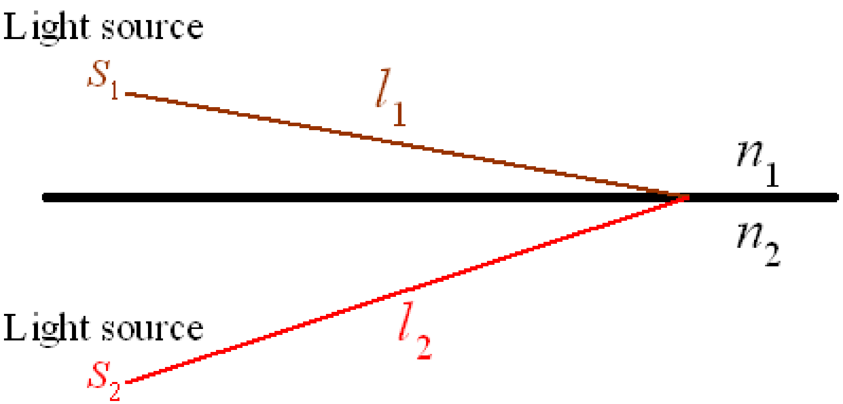


$$\boxed{L = \sum_{i=1}^{m} n_i l_i}$$

$$\varphi = \frac{2\pi}{\lambda} L$$

Applications of optical path length

- **The relationship between the optical path length and phase retardation.**



Comparison of phase retardations when two light waves travel through two different routes.

$$\text{phase difference} = \Delta\varphi = \varphi_2 - \varphi_1 = \frac{2\pi}{\lambda_2} l_2 - \frac{2\pi}{\lambda_1} l_1 = \frac{2\pi}{\lambda}(n_2 l_2 - n_1 l_1) = \frac{2\pi}{\lambda}\delta$$

$$= \frac{2\pi}{\lambda} \times \text{difference of optical path length}$$

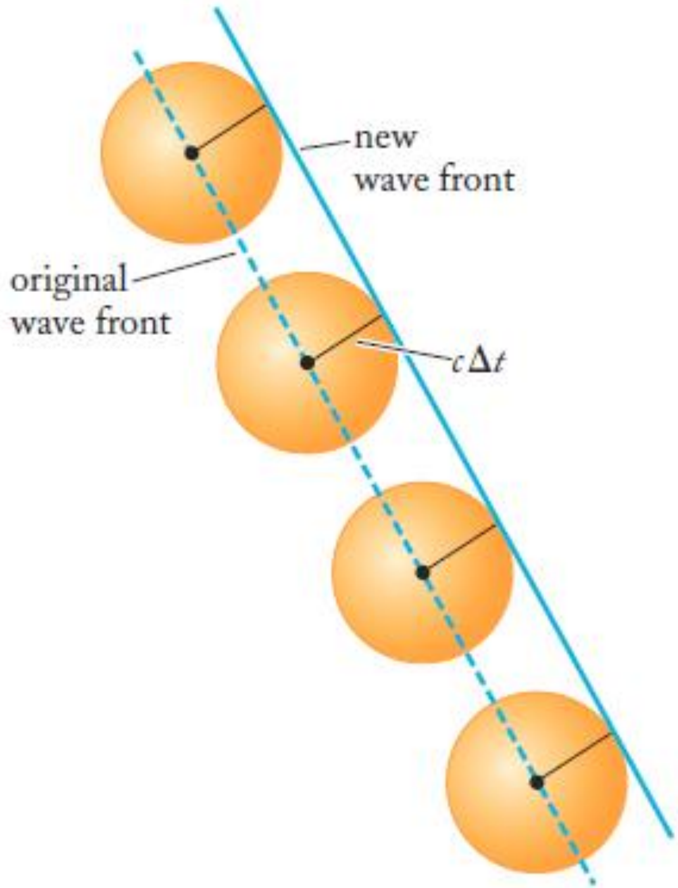
- **The relationship between the optical path length and traveling time.**

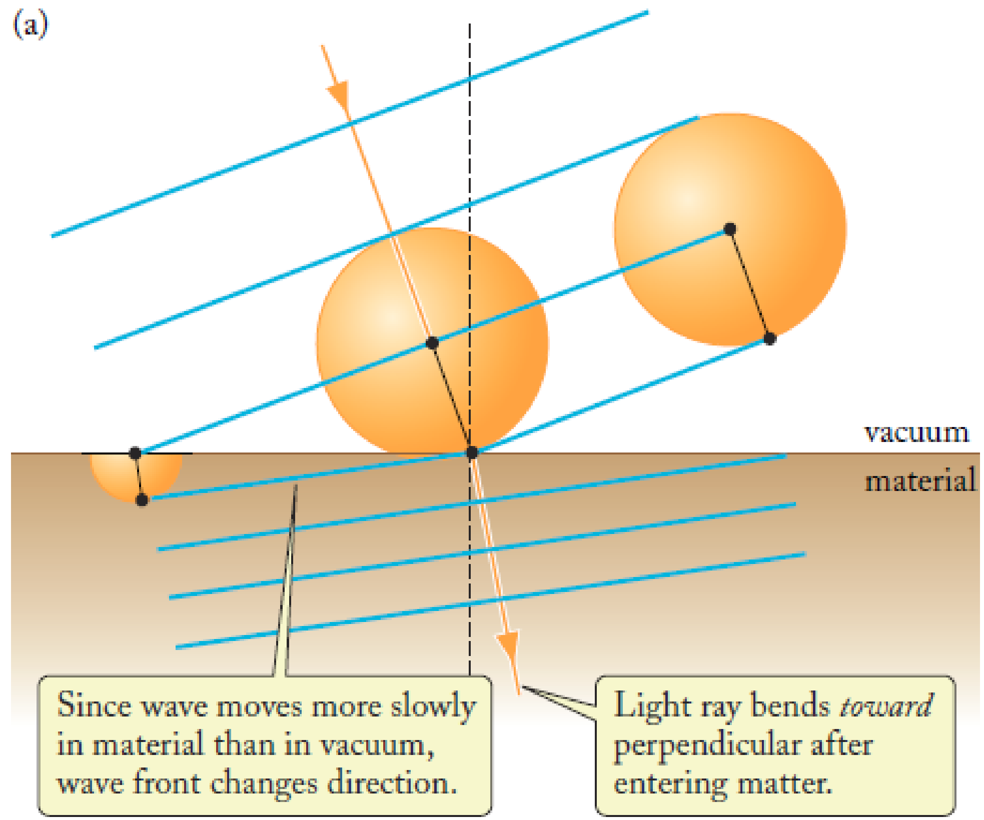
Comparison of time consumed when two light rays traverse through two different routes.

$$\text{time difference} = \Delta t = t_2 - t_1 = \frac{l_2}{v_2} - \frac{l_1}{v_1} = \frac{1}{c}(n_2 l_2 - n_1 l_1) = \frac{\delta}{c}$$

$$= \frac{\text{difference of optical path length}}{c}$$

Applications of optical path length

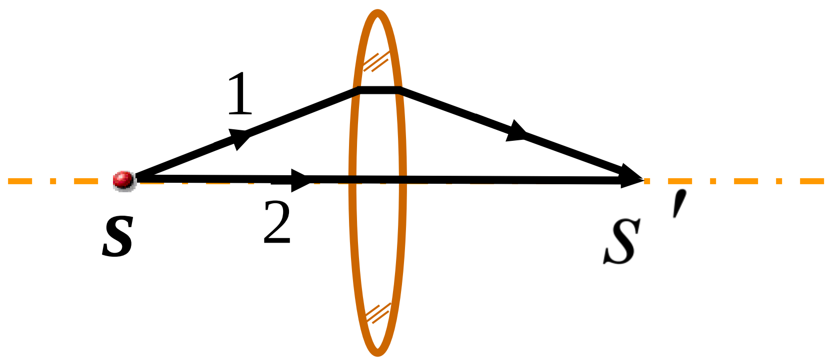


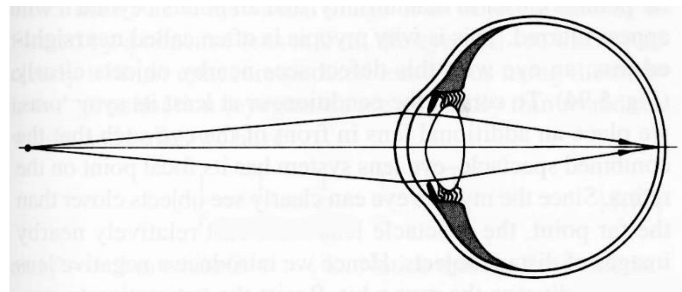


Applications of optical path length

- **No optical path length difference through lens.**

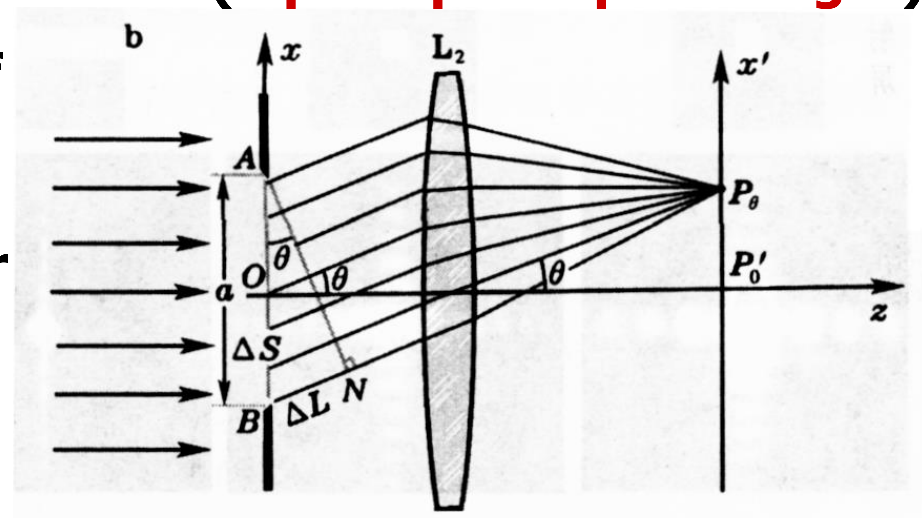
  The various light pass through the lens would introduce no additional optical path difference or phase shift.





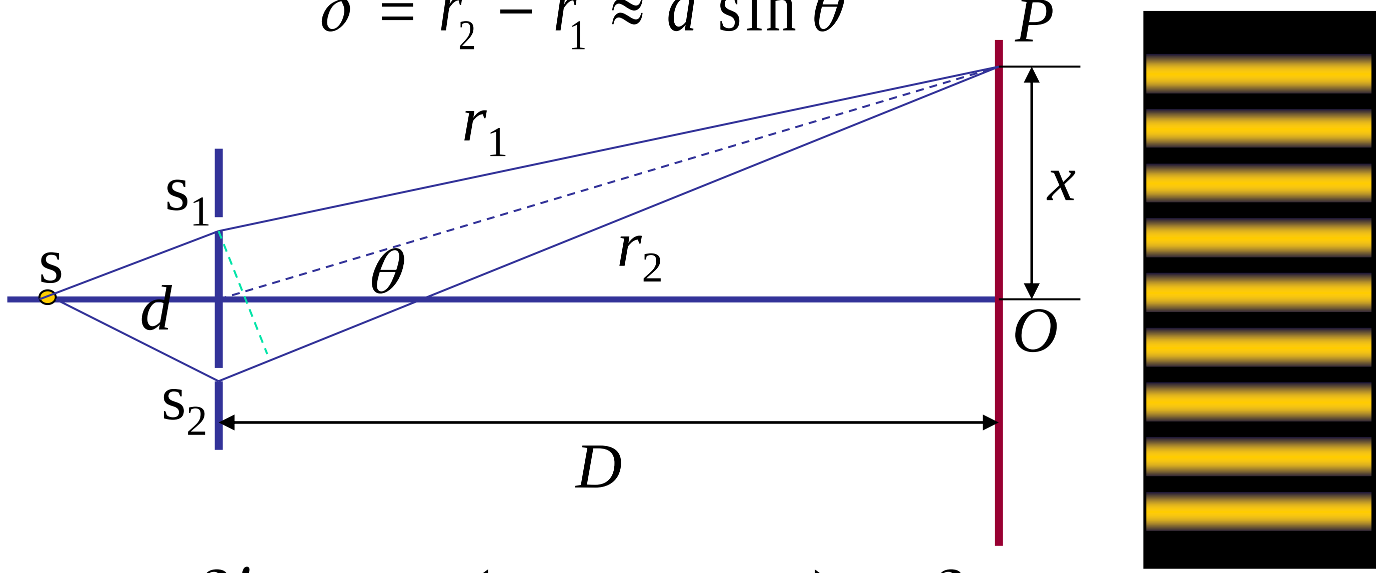
A human eye can get stable information of an object because each ray travels with the same time (equal optical path length).

When we make a comparison of optical differences among all rays that will focus on the point P_θ, we can only consider the portion of rays before the wavefront *AN*.

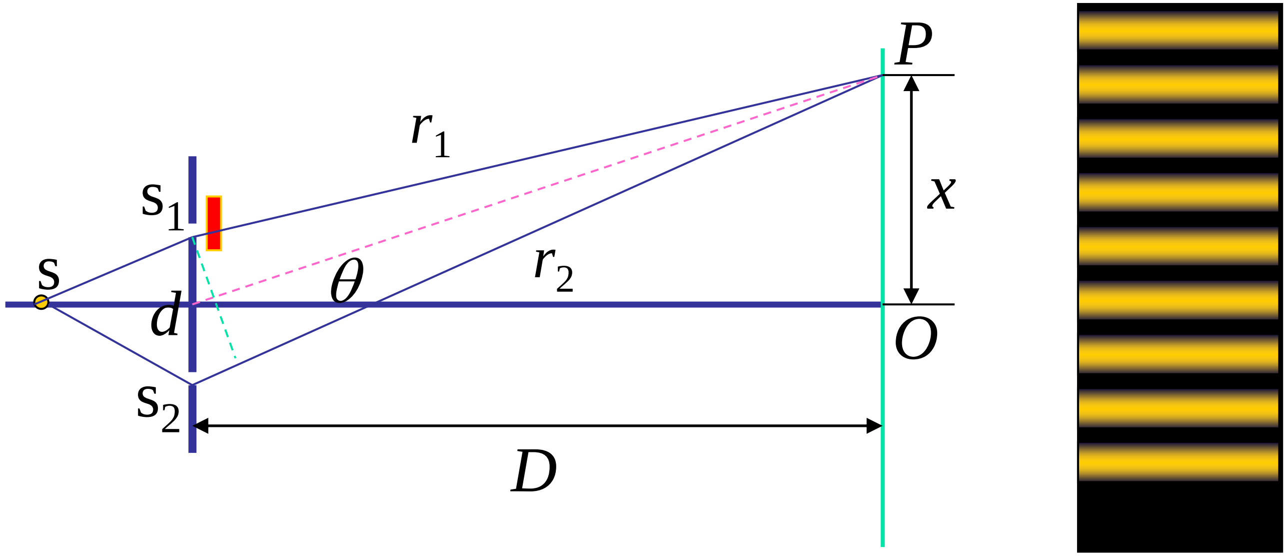


Example

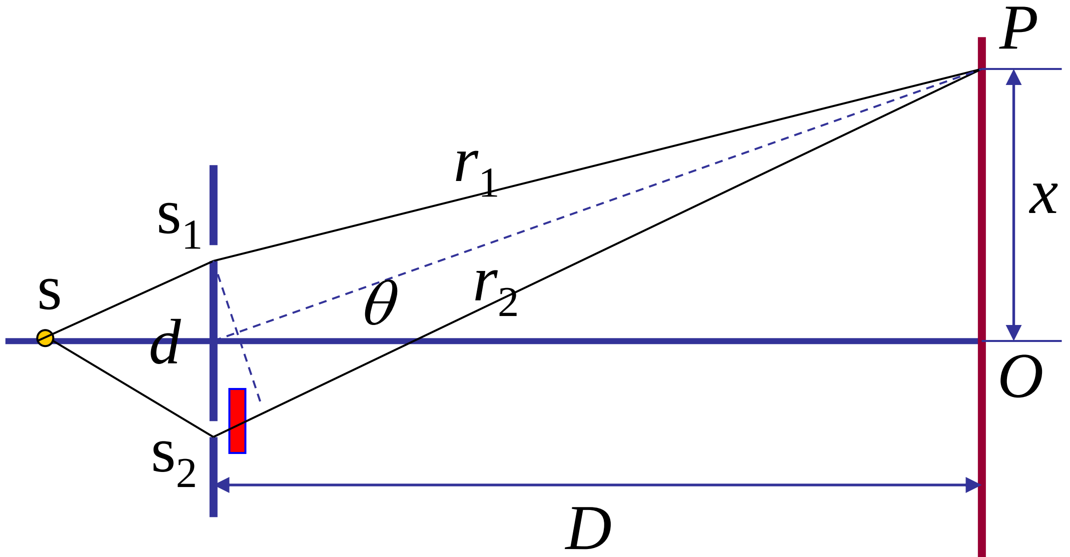
$$\delta = r_2 - r_1 \approx d \sin\theta$$

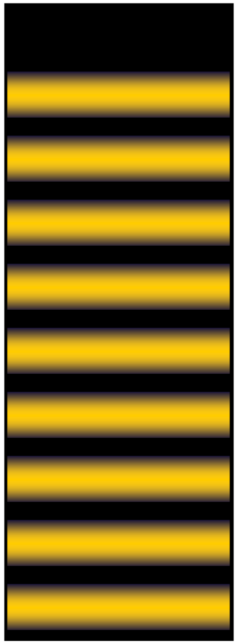


$$\delta' = r_2 - (r_1 - e + ne) < \delta$$



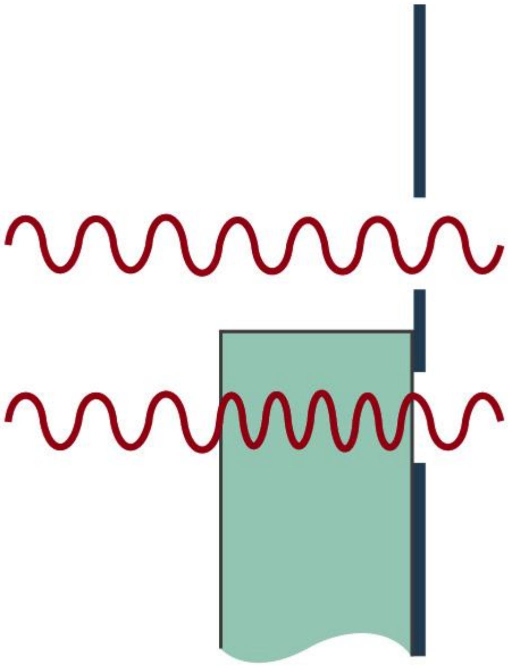
Example





Prob. 8 (P698)

Suppose a thin piece of glass were placed in front of the lower slit so that the two waves enter the slits 180° out of the phase. Describe in detail the interference pattern on the screen. What is the minimum thickness of the glass? (n, λ)



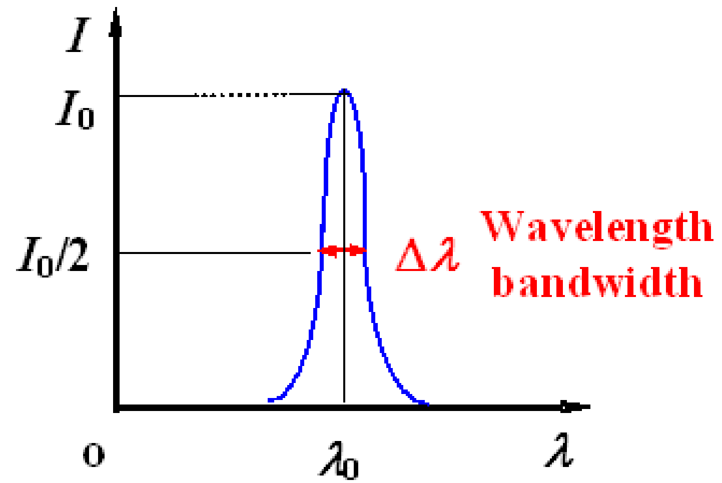
Solution:

$$\delta_{\min} = nt_{\min} - t_{\min} = \frac{\lambda}{2}, \qquad t_{\min} = \frac{\lambda}{2(n-1)}$$

Problems

P698, Prob. 10, 11; 14, 16

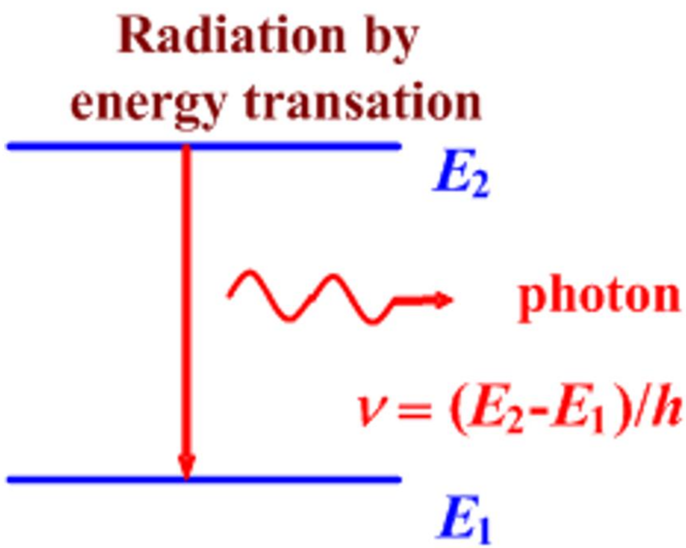
- **Monochromatic light and quasi-monochromatic light:**
  - **Monochromatic light means single-color light having a single wavelength.**
  - **Absolutely monochromatic light is an unattainable idealization. Real light have a range of wavelength bandwidth $\Delta\lambda$ around its central wavelength λ_0. A light with a narrow band $\Delta\lambda << \lambda_0$ is called quasi-monochromatic light.**

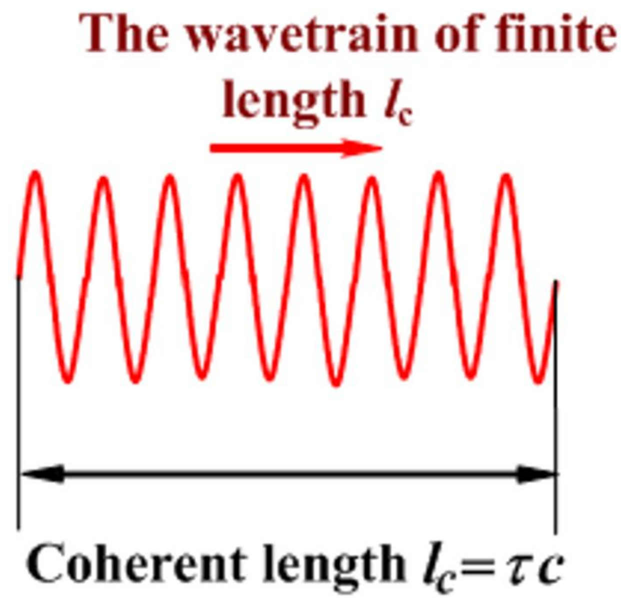


quasi-monochromatic light

Coherence and incoherence

- **Wavetrains and coherence length:**
  - **In the viewpoint of quantum mechanics, an atom emits a photon by energy transition from upper energy level to lower level within a short duration time τ.**
  - **In the viewpoint of classical physics, the same thing is described as: a dipole emits a wavetrain in time τ called coherence time. The wavetrain has a corresponding coherence length $l_c = \tau c$**





Coherence and incoherence

Coherence Lengths of Several Sources			
Source	λ_0(nm)	$\Delta\lambda$(nm)	Coherence length l_c
White Light	550	≈ 300	≈ 900 nm
Mercury arc	546.1	≈ 1.0	$\sim < 0.03$ cm
Kr^{86} discharge lamp	605.6	0.0012	0.3 m
Stabilized He-Ne Laser	632.8	$\approx 10^{-6}$	$\sim < 400$ m
Special He-Ne Laser	1153	8.9×10^{-11}	15×10^{6} m

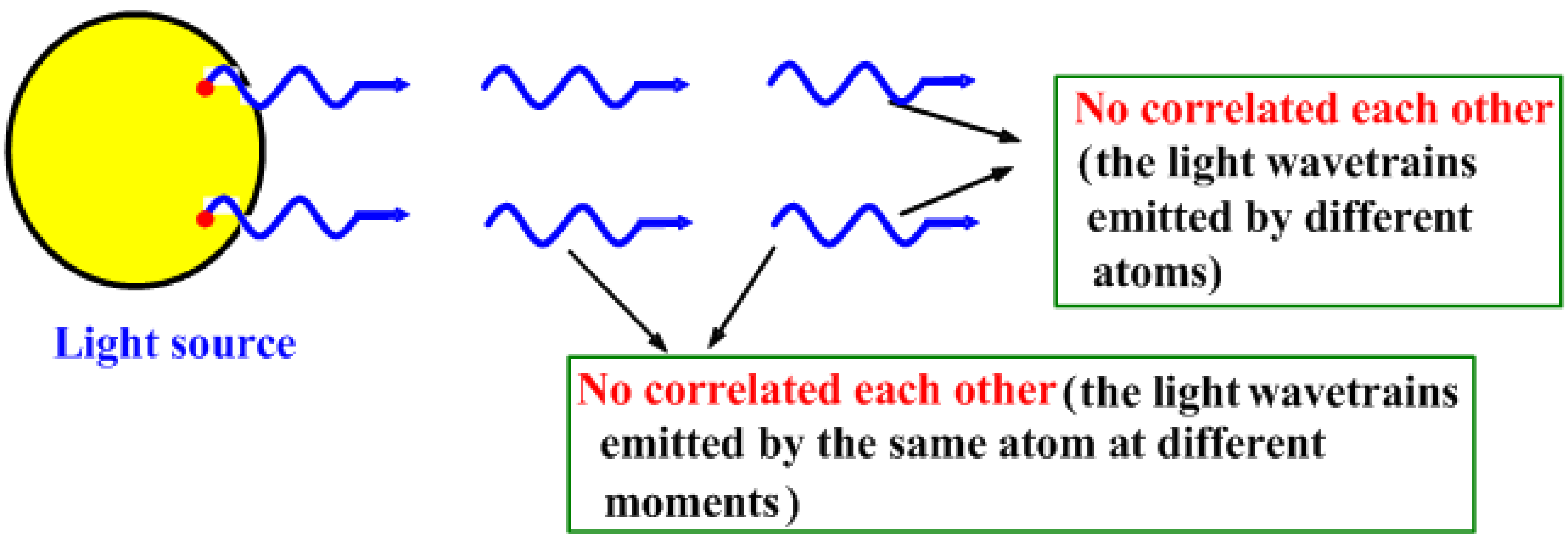
- Kr——krypton
- He——helium
- Ne——neon

(coherence length) $$l_c = \frac{\lambda_0^2}{\Delta\lambda}$$

(coherence time) $$\tau = \frac{l_c}{c}$$

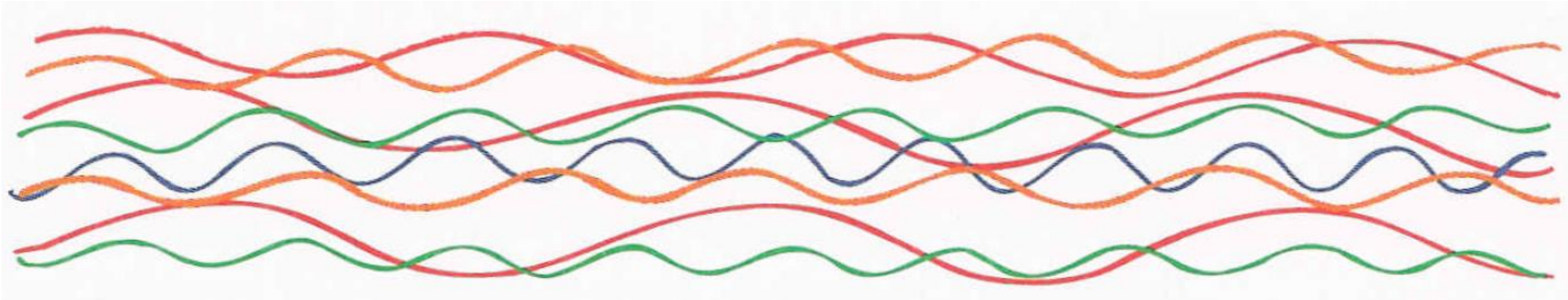
Coherence and incoherence

- Atoms emit wavetrains with random phase angles at different moments. $\phi = \phi(t)$
- There is no correlation—no definite phase relation—between the wavetrains emitted by different atoms.
- There is no correlation—no definite phase relation—between the wavetrains emitted by the same atoms at different moments.

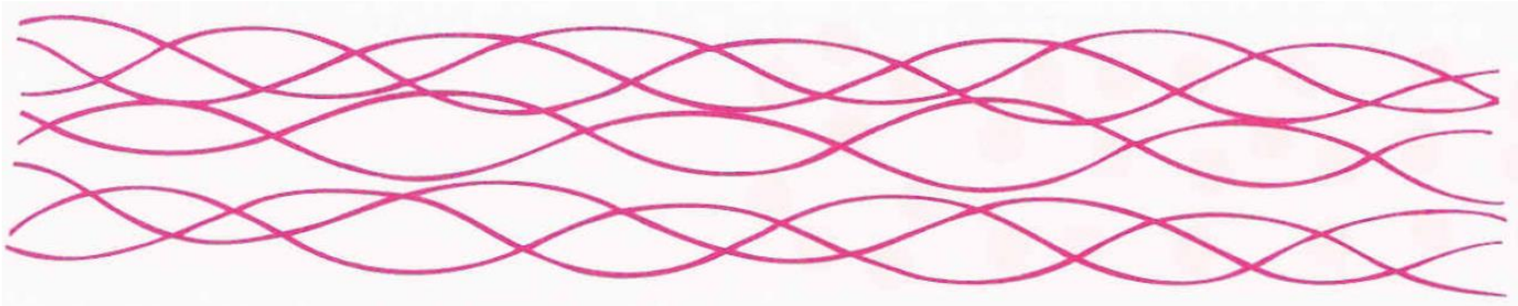


Coherence and incoherence

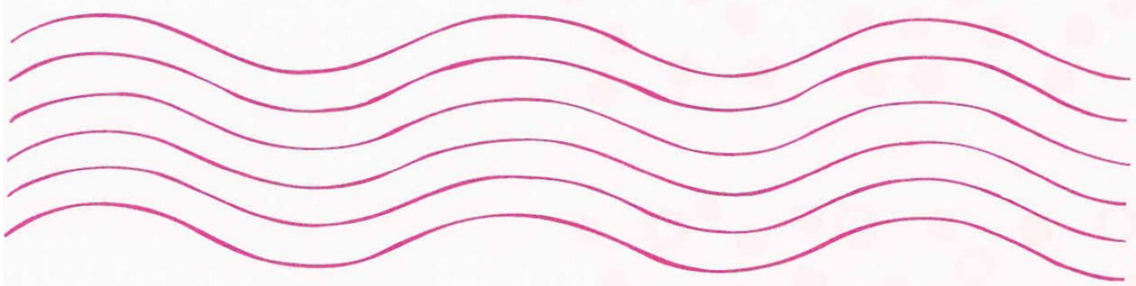
- Incoherent white light contains waves of many frequencies (and of many wavelengths) that are out of phase with one another.



- Light of a single frequency and wavelength still contains a mixture of phases.



- Coherent light: All the waves are identical and in phase .



Coherence and incoherence:

- The intensity of light is the time average of the Poynting vector

$$I = \langle (E_1 + E_2)^2 \rangle = \langle E_1^2 \rangle + \langle E_2^2 \rangle + 2 \langle E_1 E_2 \rangle$$

$$= I_1 + I_2 + 2\sqrt{I_1 I_2} \langle \cos\Delta\varphi \rangle$$

- The phase difference is generally random due to random phase angles.

$$\Delta\varphi = k(r_2 - r_1) - \left[\phi_2(t) - \phi_1(t)\right] = \Delta\varphi(t)$$

- **Incoherence**: $\langle \cos\Delta\varphi \rangle = 0 \Longrightarrow I_{\text{incoh}} = I_1 + I_2$

- **Coherence**:

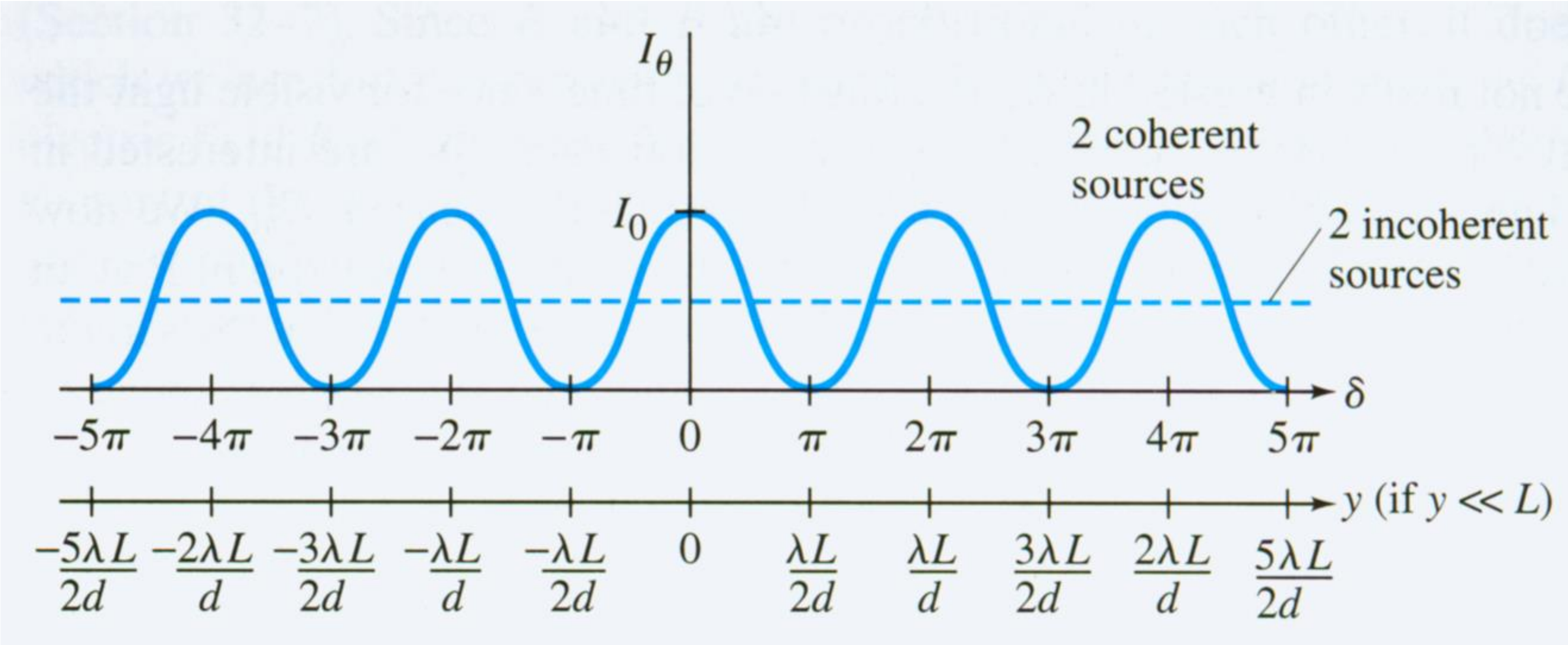
$$I_{\text{coh}} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\Delta\varphi = \begin{cases} I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2} \\ I_{\min} = I_1 + I_2 - 2\sqrt{I_1 I_2} \end{cases}$$

Coherence and incoherence

- **The energy (intensity) is spread evenly over the screen and there is no interfering maxima and minima, when the sources produce incoherent light.** $I_{\text{incoh}} = I_1 + I_2$

- **Whereas the energy is distributed in peaks and valleys when the light from two sources is coherent.**

$$I_{\text{coh}} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\Delta\varphi$$



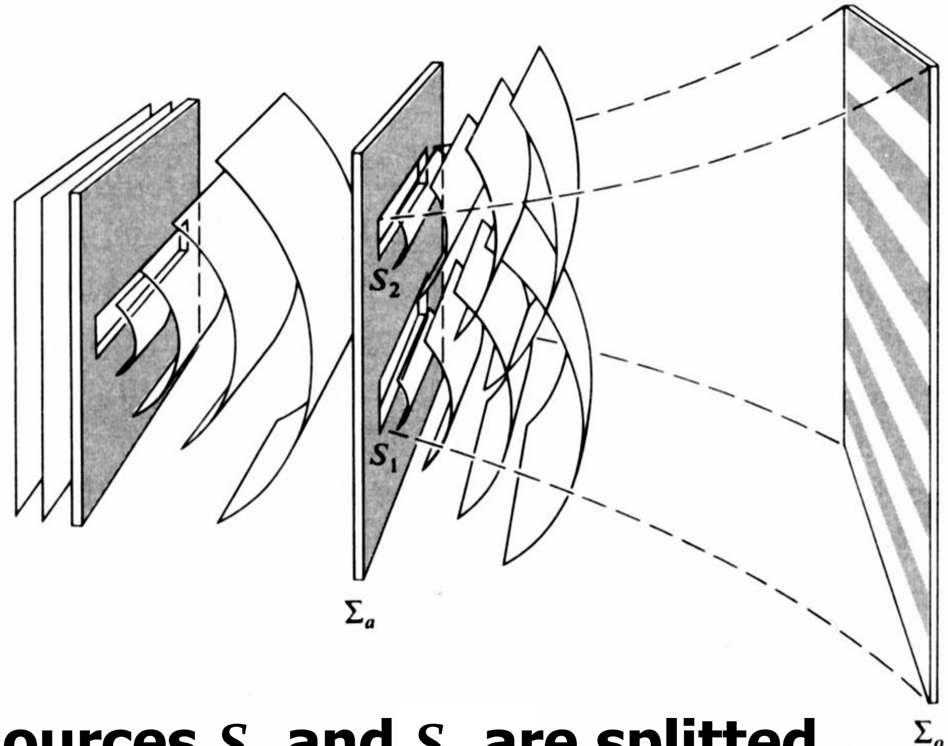
- The **clever** design of Young's double-slit experiment
  - The two sources S_1 and S_2 are coherent because they come from the same source S. Therefore their phase angles are the **same**.

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} < \cos\Delta\varphi >$$

$$\phi_1(t) = \phi_2(t) = \phi(t)$$

$$\Delta\varphi = k(r_2 - r_1) - \underbrace{\left(\phi_2(t) - \phi_1(t)\right)}_{\text{zero}}$$
$$= k(r_2 - r_1)$$

$$< \cos\Delta\varphi > \neq 0$$

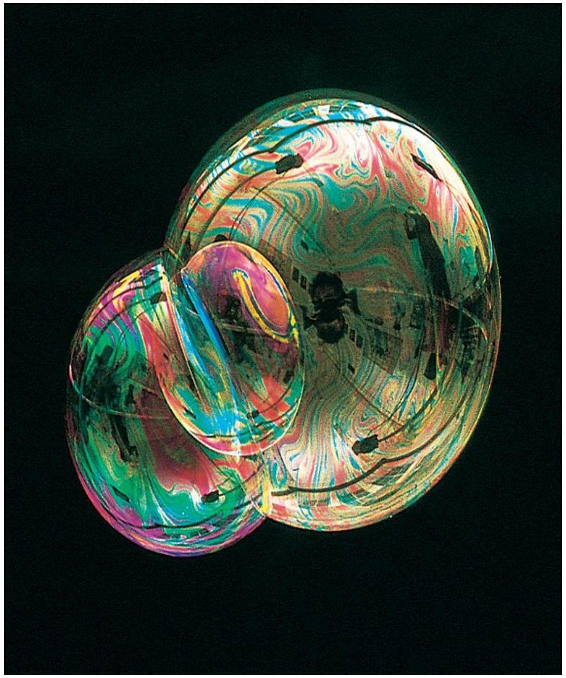


- Because the two coherent sources S_1 and S_2 are splitted from wavefront of S, so Young's experiment was in the category of **wavefront-splitting** interference.

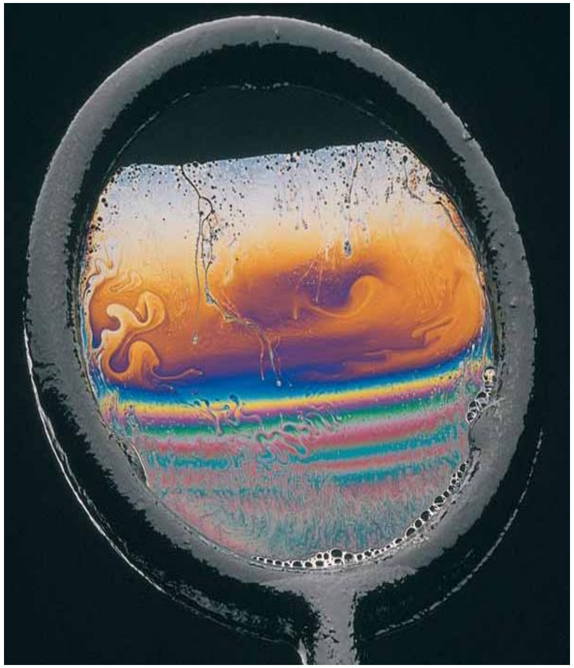
§ 4 Interference in Thin Films

Amplitude-splitting interference

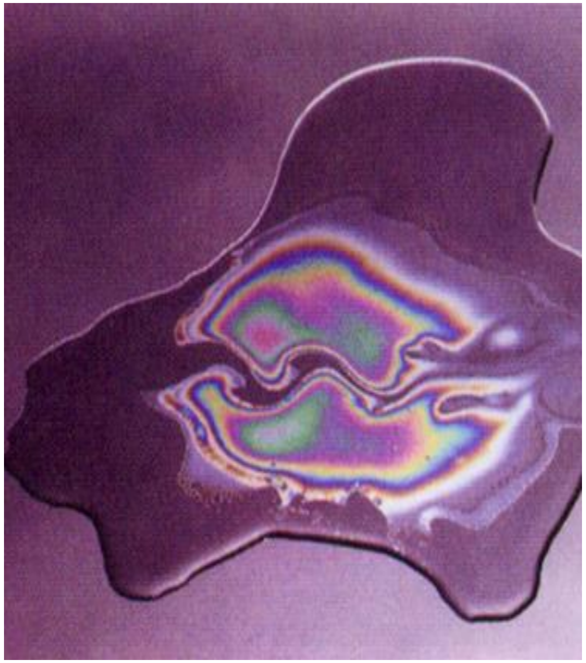
- **The everyday phenomena due to the interference in thin films**



Soap bubbles



Thin film of soapy water

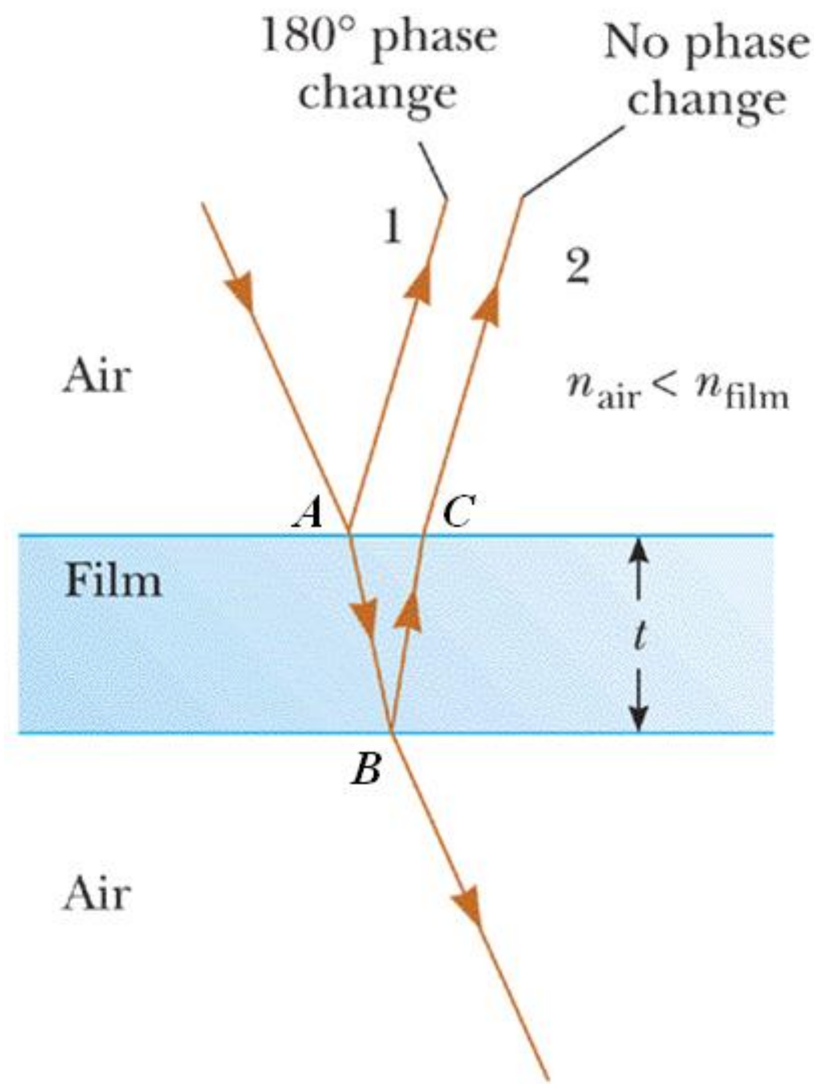


Thin layer of gasoline on water

- **Dielectric films —— Double-beam interference**
  - **Part of the incident light is reflected at A on the top surface, and part reflected at the bottom surface must travel the extra distance ABC.**
  - **The optical path length difference is:**

$$\delta \approx 2n_{\text{film}}t + \left(\frac{\lambda}{2}\right)$$

$\lambda/2$ is a additional path depending on index relations at interfaces of the film.



- **The conditions for constructive and destructive interference**
  - **Bright fringes —— constructive interference**

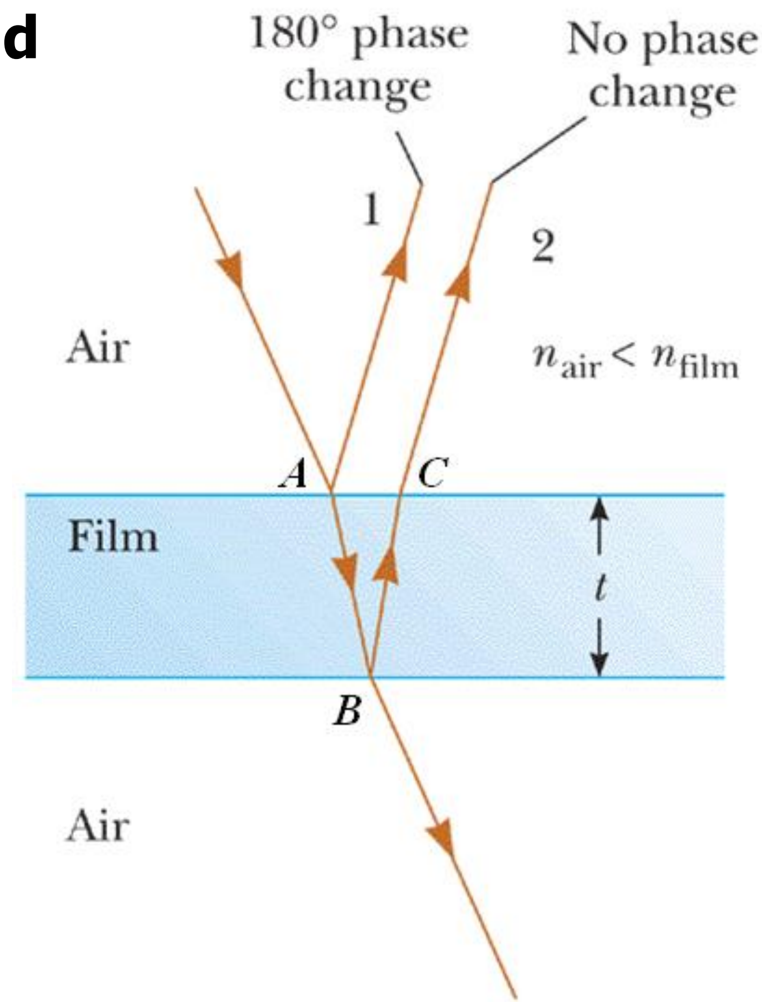
$$\delta = 2nt + \left(\frac{\lambda}{2}\right) = m\lambda$$

$$m = 0, 1, 2, \cdots$$

  - **Dark fringes —— destructive interference**

$$\delta = 2nt + \left(\frac{\lambda}{2}\right) = (2m+1)\frac{\lambda}{2}$$

$$m = 0, 1, 2, \cdots$$

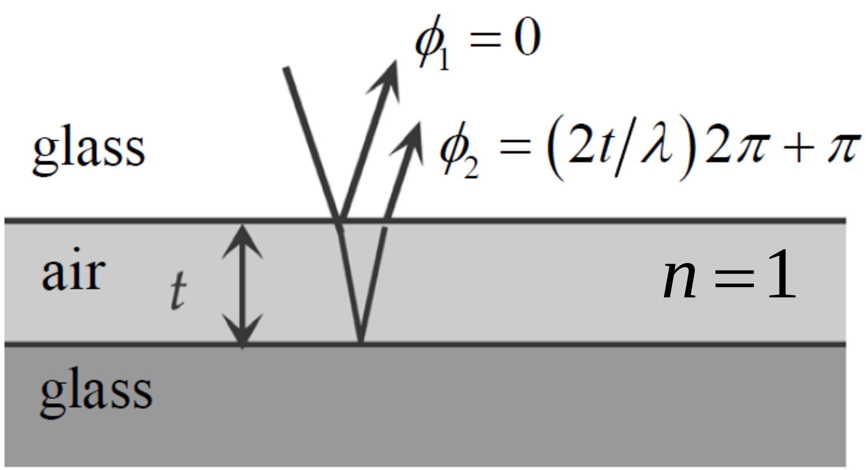


What is the minimum (non-zero) thickness for the air layer between two flat glass surfaces if the glass is to appear dark when 640-nm light is incident normally? What if the glass is to appear bright?

Solution:

Dark, $\delta_{\min} = 2nt_{\min} + \frac{\lambda}{2} = \frac{3\lambda}{2}$

$$t_{\min} = \frac{\lambda}{2} = \frac{640}{2} = 320 \text{ nm}$$



Bright, $\delta_{\min} = 2nt_{\min} + \frac{\lambda}{2} = \lambda,$

$$t_{\min} = \frac{\lambda}{4} = \frac{640}{4} = 160 \text{ nm}$$

How to generate fringes?

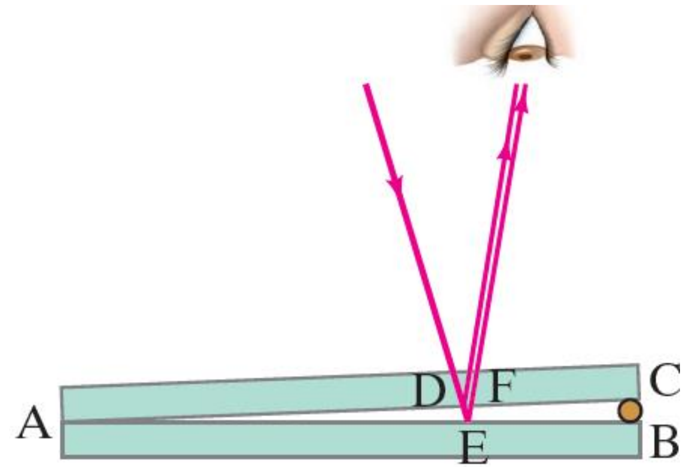
- **Air wedge**
  - **Air wedge is the wedge of air between the two glass plates.**
    - **Bright fringes:**

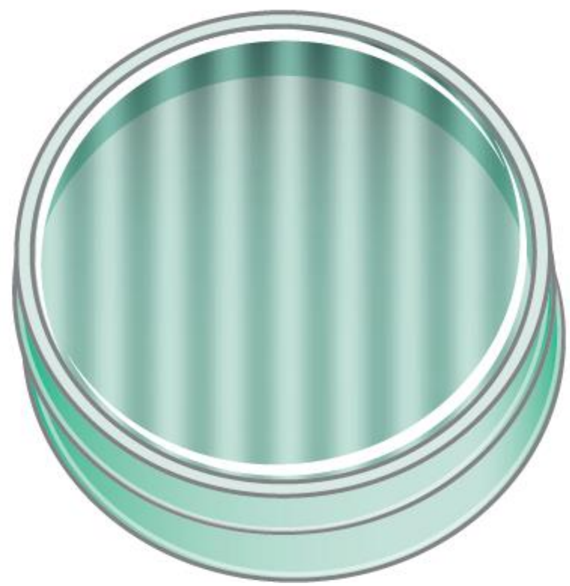
$$\delta = 2t + \frac{\lambda}{2} = m\lambda, \quad m = 1, 2, \cdots$$

    - **Dark fringes:**

$$\delta = 2t + \frac{\lambda}{2} = (2m+1)\frac{\lambda}{2}, \quad m = 0, 1, 2, \cdots$$

  - **Each fringe is the locus (轨迹) of all points in the film for which the optical thickness is constant. For air wedge, n=1, so that the fringes correspond to regions of equal film thickness.**

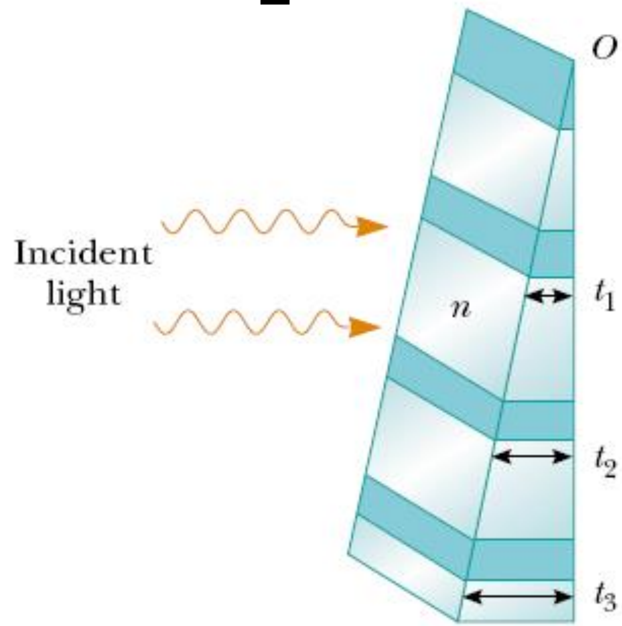




- **Why can we see different colors in soap bubbles and other films of varying thickness?**

Interference in a vertical film of variable thickness.

$$\delta = 2nt + \frac{\lambda}{2} = m\lambda \ (\text{bright})$$



The top of the film appears darkest where the film is thinnest.

If white light is used, bands of different colors are observed at different points, corresponding to the different wavelengths of light.

Air Wedge

- **The applications of air wedge**
  - **At the end where the two plates meet**

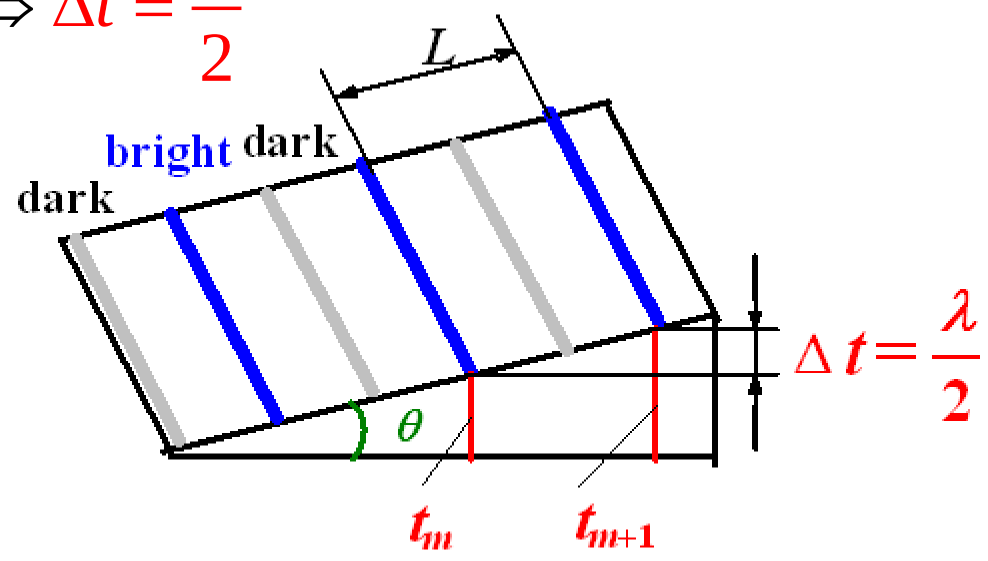
$$\delta = 2t + \frac{\lambda}{2}, \quad t = 0, \quad \delta = \frac{\lambda}{2} \qquad \textbf{(dark fringe)}$$

  - **The thickness difference between the adjacent fringes**

$$\left.\begin{aligned} \delta_m &= 2t_m + \frac{\lambda}{2} = m\lambda \\ \delta_{m+1} &= 2t_{m+1} + \frac{\lambda}{2} = (m+1)\lambda \end{aligned}\right\} \Rightarrow \Delta t = \frac{\lambda}{2}$$

  - **The spacing of fringes:**

$$L = \frac{\Delta t}{\sin\theta} \approx \frac{\Delta t}{\theta} = \frac{\lambda}{2\theta}$$

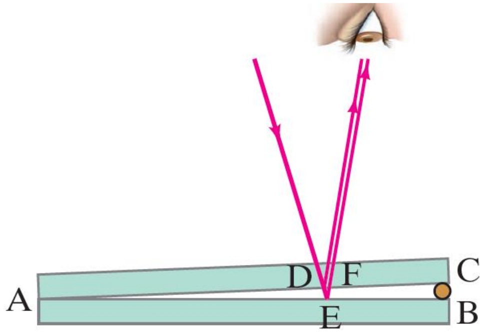


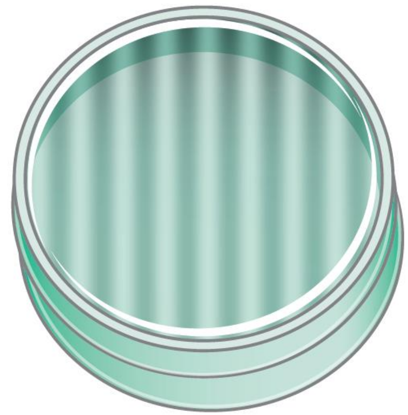
Air Wedge

- **The applications of air wedge (cont'd)**
  - **We can use air wedge to determine the surface features of optical elements (lenses, prisms).**

  - **If we want examine whether a surface of an optical element is flat or not, we put it into contact with an *optical flat*. If the test surface is perfect flat, a series of straight, equal spaced fringes will be appeared near the surface.**

  - **Now, mirrors that are flat to better than 5 percent of one wavelength, or about (500×5%=) 25nm, are available commercially.**





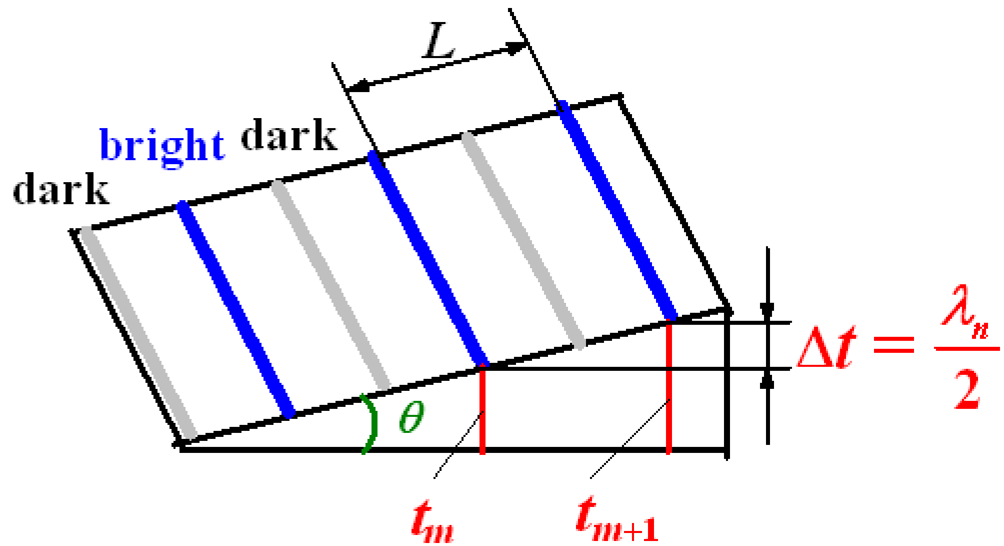
Air Wedge

- **The applications of air wedge (cont'd)**
  - **If the wedge between the two glass plates is filled with some transparent substance other than air — say, water — the pattern shifts because**

$$\lambda_n = \frac{\lambda}{n}$$

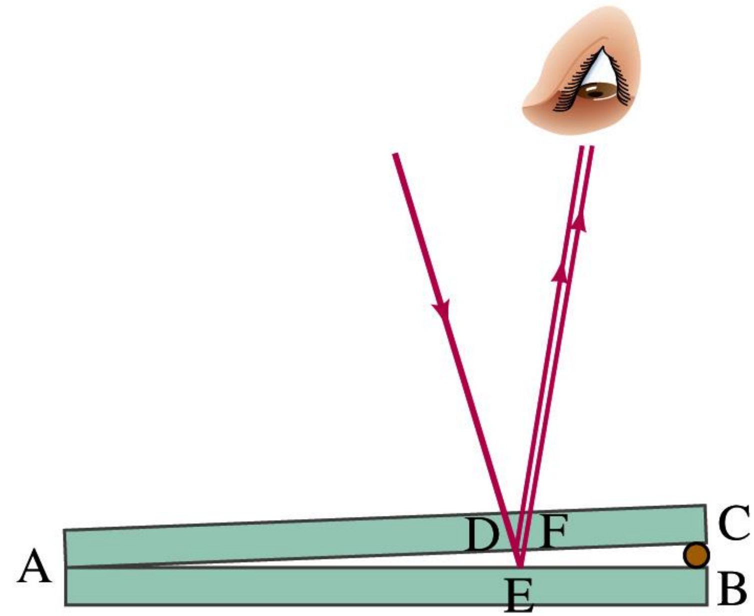
  - **The thickness difference between the adjacent fringes now is**

$$\boxed{\Delta t = \frac{\lambda_n}{2} = \frac{\lambda}{2n}}$$

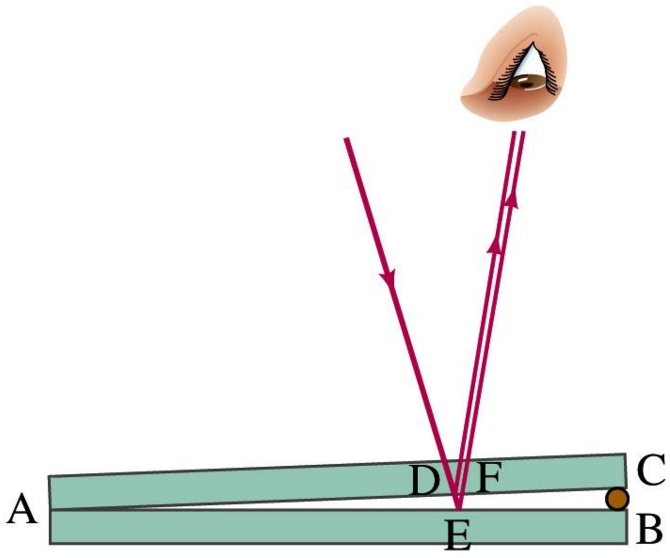


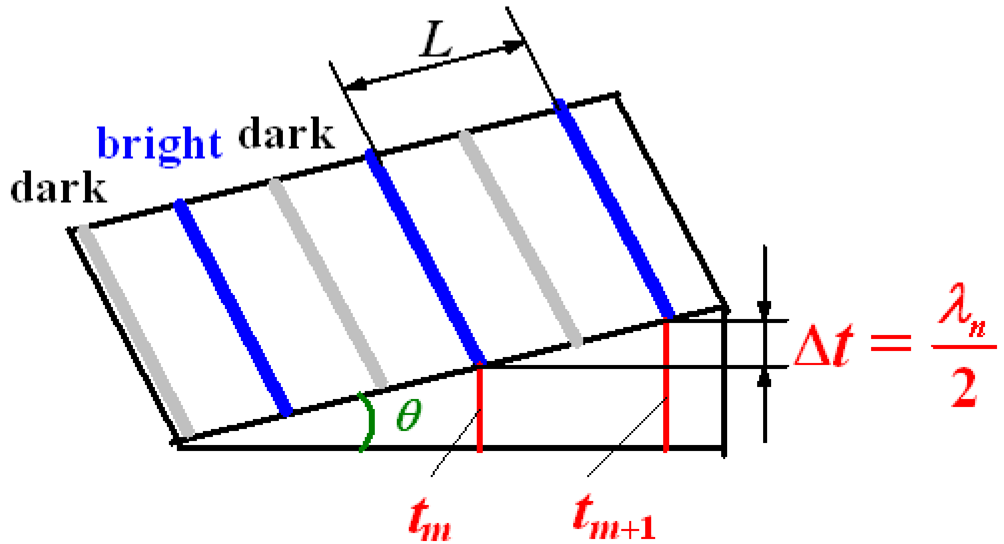
Thin film of air, wedge-shaped.

A very fine wire 7.35×10^{-3} mm in diameter is placed between two flat glass plates as seen. Light whose wavelength in air is 600 nm falls (and is viewed) perpendicular to the plates, and a series of bright and dark bands is seen. How many light and dark bands will there be in this case? Will the area next to the wire be bright or dark?



Ex. 30-6 (P692)



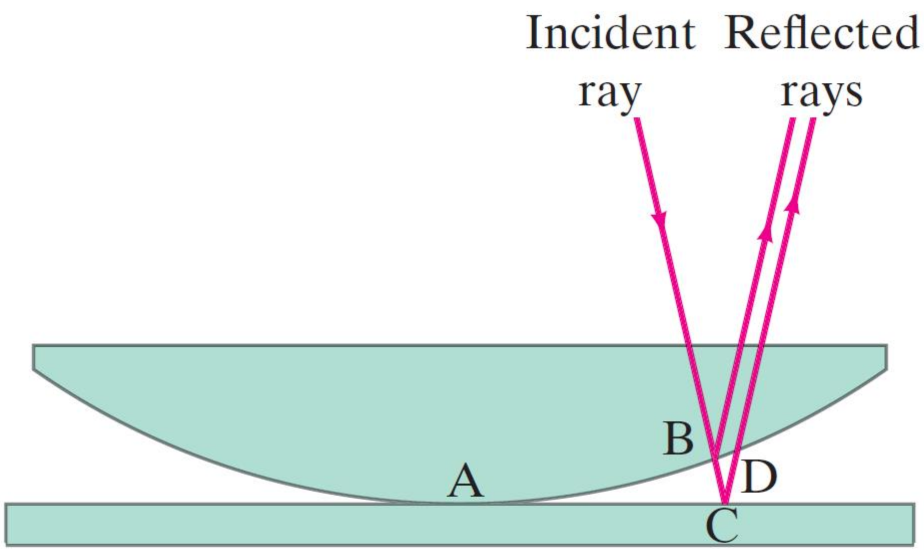


Solution:

$$\frac{t}{\lambda/2} = \frac{7.35\times10^{-6}}{600\times10^{-9}/2} = 24.5$$

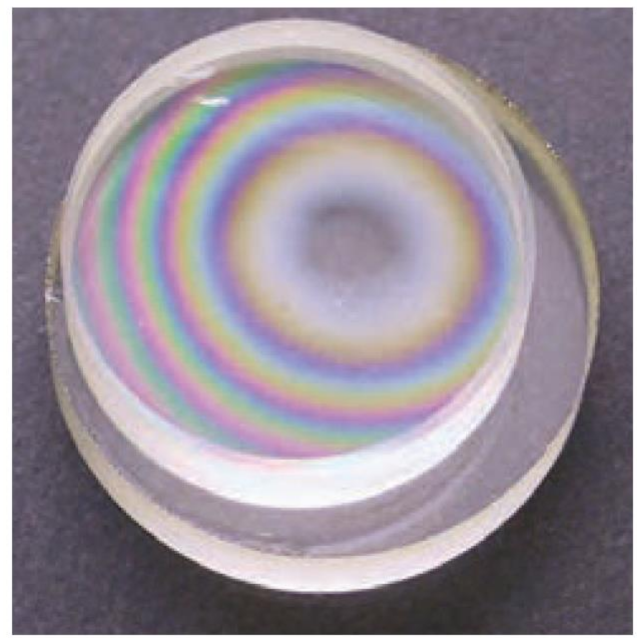
There will be a 25 dark lines and 25 bright lines.

- **Newton's rings**
  - **When a convex (凸起的) surface of a lens is placed in contact with a flat glass surface, a series of concentric rings is seen when illuminated from above by monochromatic light.**



(a)

(b)



Newton's rings

- **The radii of Newton's ring:**

$$r=\sqrt{R^2-(R-t)^2}=\sqrt{2Rt-t^2}\approx\sqrt{2Rt}\quad (t\ll R)$$

  - **For bright fringes:**

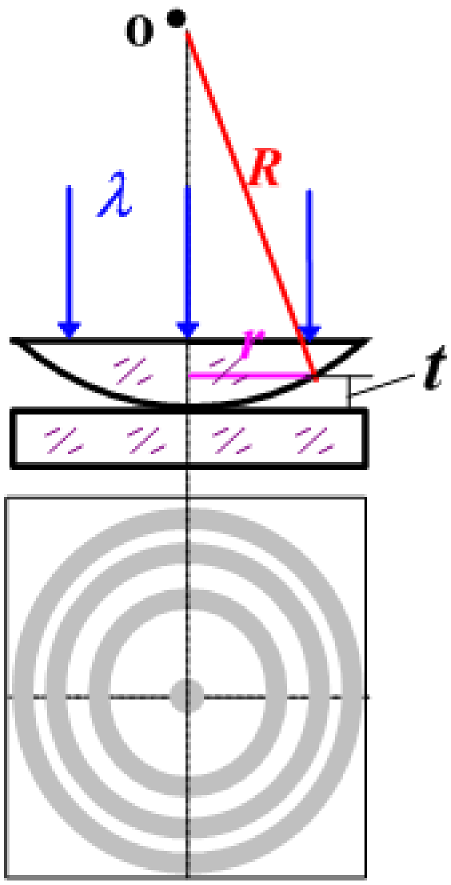
$$2t+\frac{\lambda}{2}=m\lambda,\qquad m=1,2,3,\cdots$$

$$r_m=\sqrt{\left(m-\frac{1}{2}\right)\lambda R},\quad m=1,2,3,\cdots$$

  - **For dark fringes:**

$$2t+\frac{\lambda}{2}=(2m+1)\frac{\lambda}{2},\qquad m=0,1,2,\cdots$$

$$r_m=\sqrt{m\lambda R},\quad m=0,1,2,\cdots$$

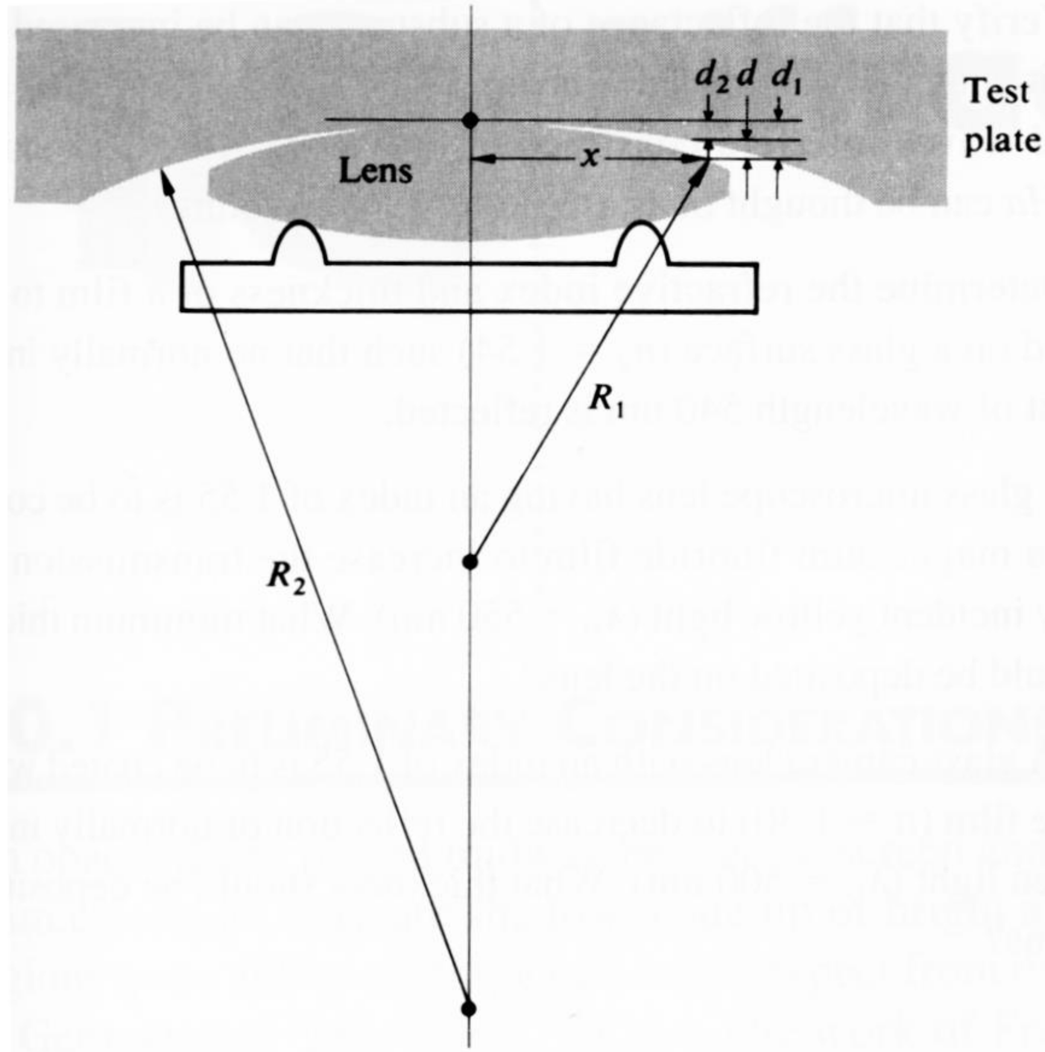


Example

The figure illustrates a setup used for testing lenses.

Show that:

$$d \approx \frac{x^2(R_2 - R_1)}{2R_1R_2}$$



Prove that the radius of m-th dark fringe is then

$$x_m = \sqrt{\frac{R_1R_2m\lambda}{(R_2 - R_1)}}$$

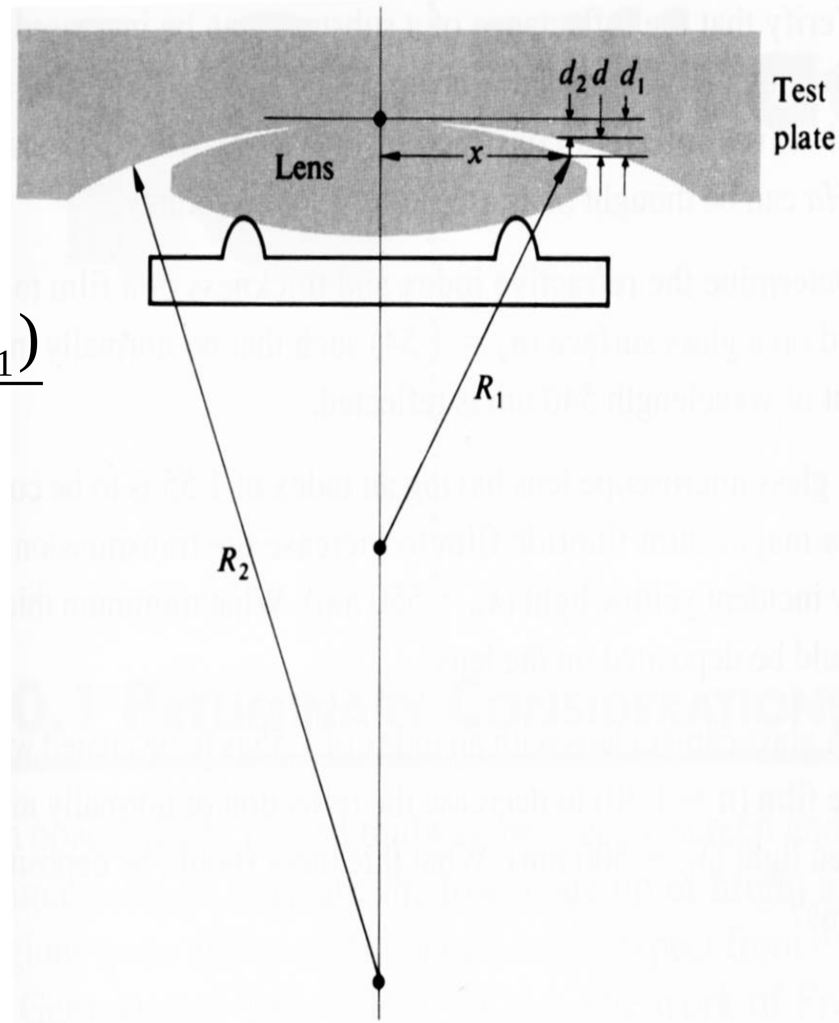
Example

Solution:

$$x \approx \sqrt{2R_1 d_1} \approx \sqrt{2R_2 d_2}$$

$$d_1 = \frac{x^2}{2R_1}, \quad d_2 = \frac{x^2}{2R_2}$$

$$d = d_1 - d_2 = \frac{x^2}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{x^2 (R_2 - R_1)}{2R_1 R_2}$$



For dark fringes:

$$2d + \frac{\lambda}{2} = (2m+1)\frac{\lambda}{2} \quad \Rightarrow \quad d = m\frac{\lambda}{2}$$

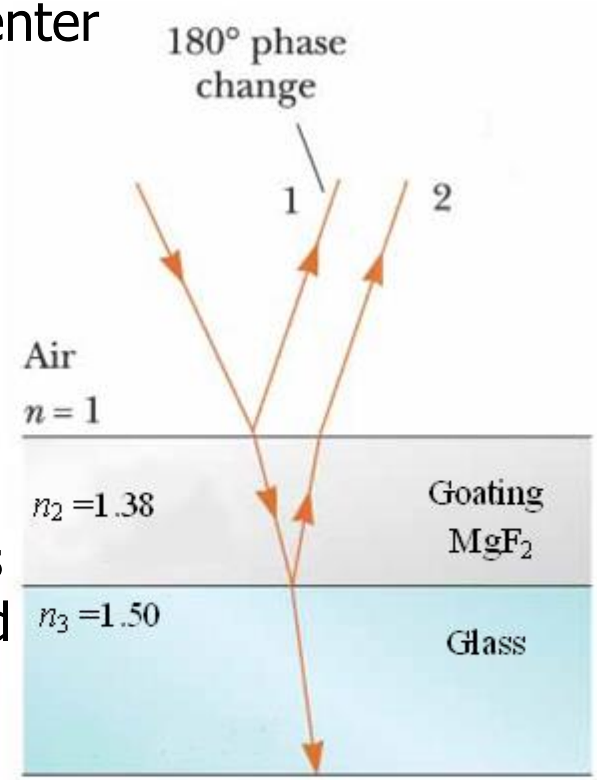
$$x_m = \sqrt{\frac{2R_1 R_2 d}{(R_2 - R_1)}} = \sqrt{\frac{R_1 R_2 m\lambda}{(R_2 - R_1)}}$$

Thin non-reflecting coating

- Thin non-reflecting coating (减反镀膜)
  - A glass surface reflect about 4% of light incident on it. Lenses are often coated with thin films of transparent substance such as MgF_2 (n_2=1.38) to reduce the reflection from the glass surface.

© 2004 Thomson - Brooks/Cole

- Often the coating is designed to eliminate the center of reflected spectrum, λ=550nm.

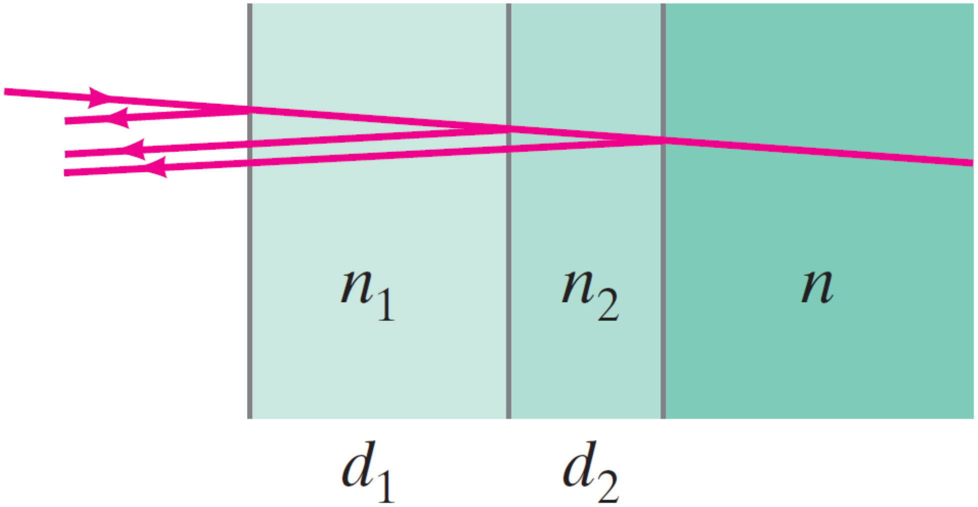


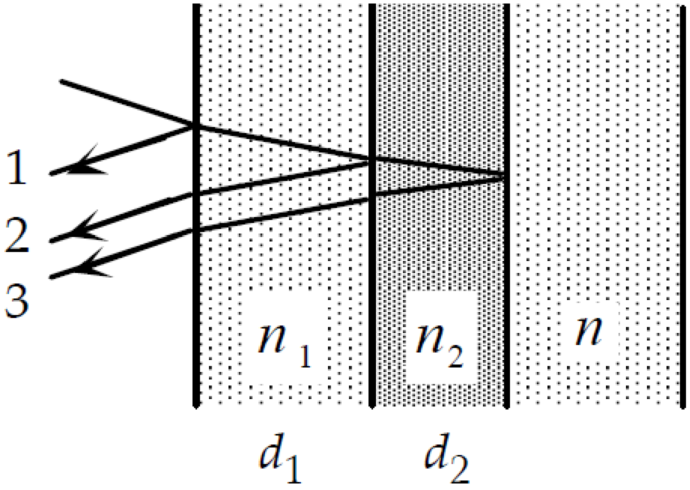
  - For MgF_2, n_2=1.38. For glass n_3=1.50.

$$\delta = 2n_2 t = (2m+1)\frac{\lambda}{2}, \qquad \boldsymbol{m=0}$$

  - The thickness of coating is: $t = \dfrac{\lambda}{4n_2}$
  - In reflected light, yellow light (around 550nm) is reduced, but two extremes of spectrum — red and violet — will not be reduced as much, so such coated lenses is purple (mixture of red and violet).

Very highly reflective mirrors





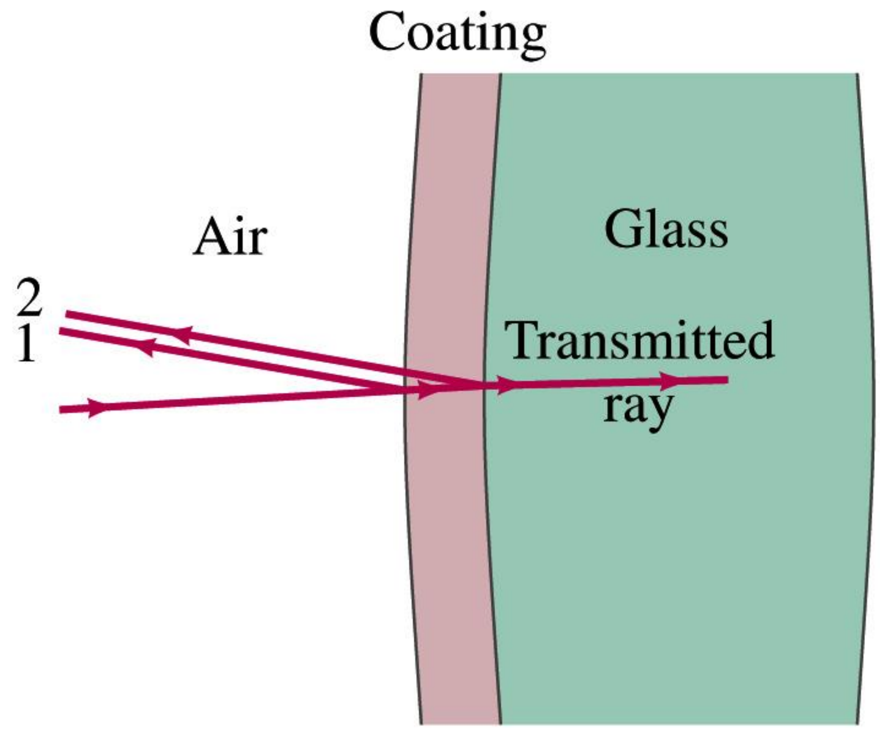
$$n_1 < n_2 < n$$

$$d_1 = \frac{\lambda}{2n_1}, d_2 = \frac{\lambda}{2n_2}$$

Ex. 30-8 (P694)

Non-reflective coating

What is the **smallest thickness** of an optical coating of MgF_2, whose index of reflection is $n_2=1.38$, which is designed to eliminate reflected light at wavelengths centered at 550 nm when incident normally an glass for which $n_3=1.50$?



Solution:

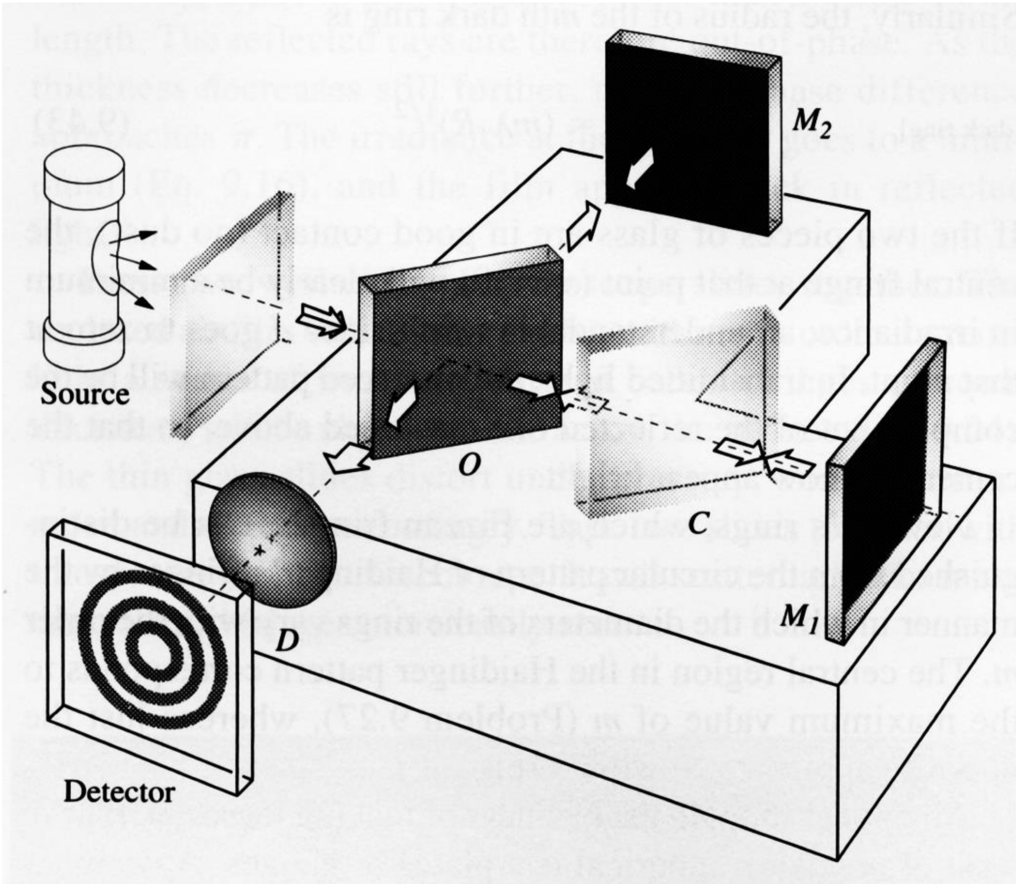
$$t=\frac{\lambda}{4n_2}=\frac{550}{4\times1.38}=99.6\text{ nm}$$

Prob.2

P699, Prob. 22, 26 (only the radius of curvature of the lens surface)

P700, Prob. 44, 46

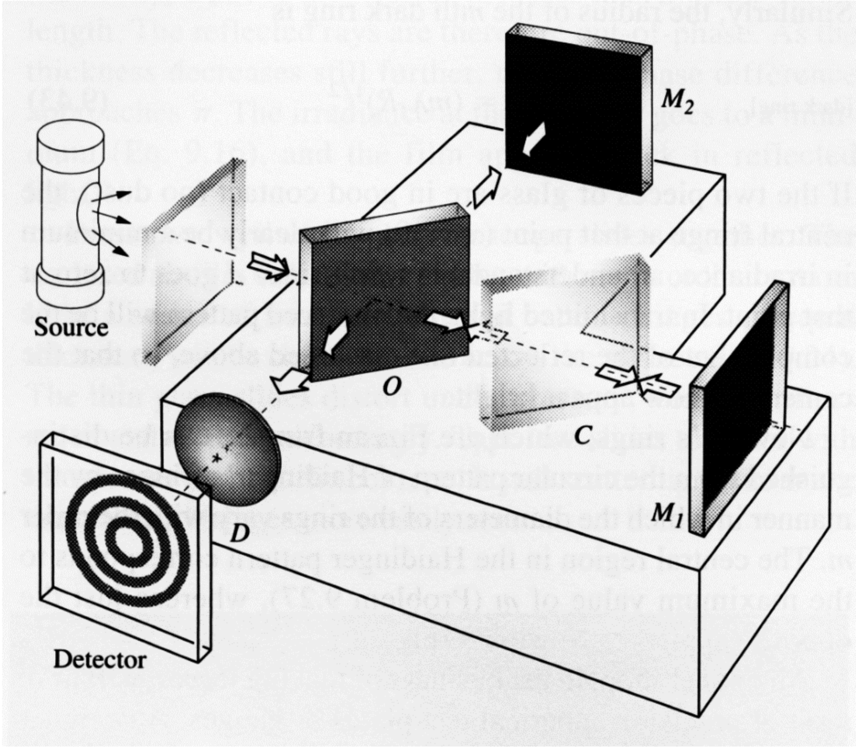
§ 5 Michelson Interferometer



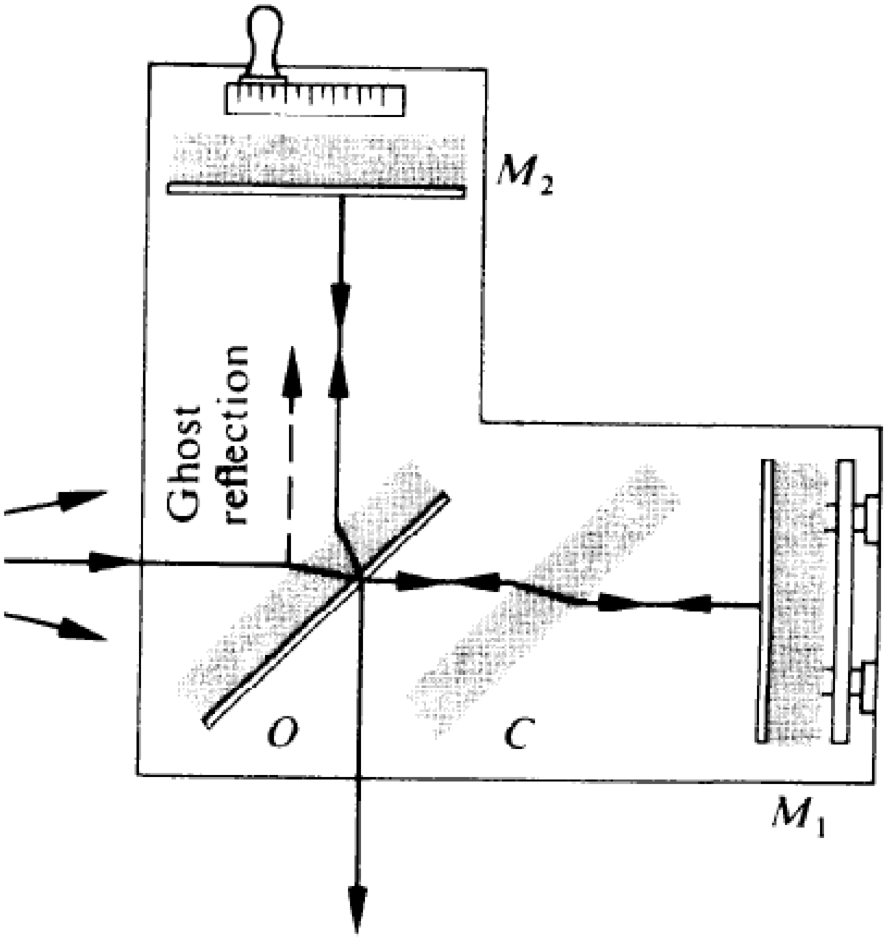
- An extended **source** emits a wave. The **beam splitter** with a half-silvered mirror divides the wave into two, one segment traveling to the mirror M_1 and one to the mirror M_2. The two reflected wave are united at the region where the eye locates, and the **interference** can be expected.

- The role of **compensator plate**: one beam passes through beam splitter **three** times, whereas the other traverses it only **once**. Each beam will pass through equal thicknesses of glass only when a compensator plate is inserted in the arm 1.

Michelson Interferometer



Top view



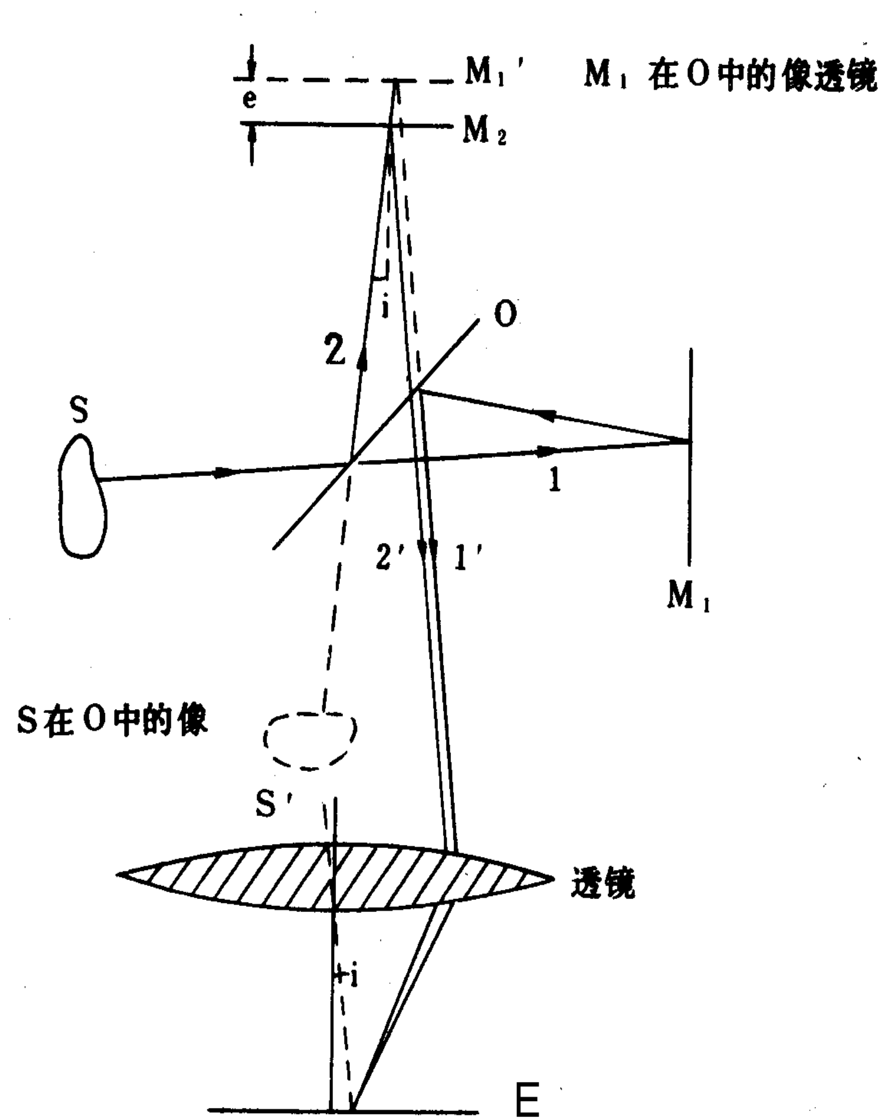
Circular fringes are centered on the lens.

Michelson Interferometer

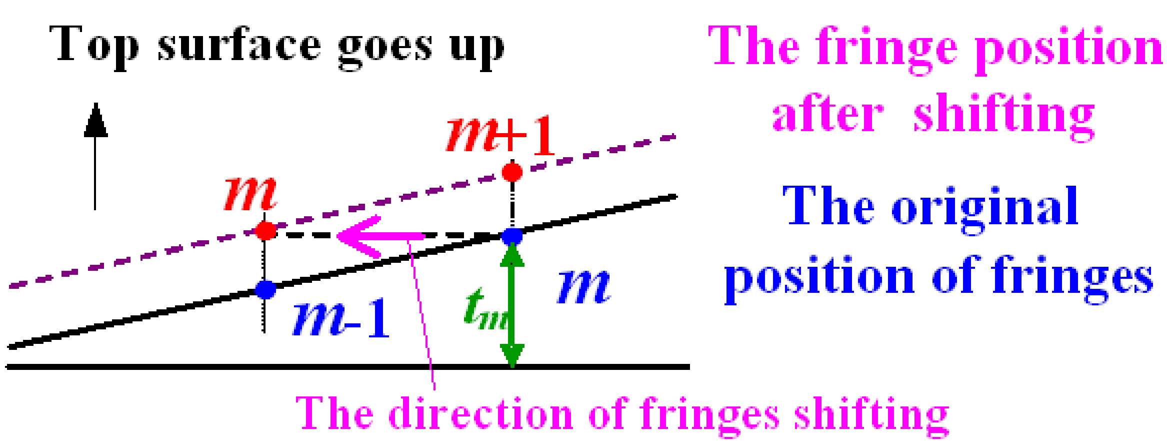
- To understand how fringes are formed
  - Re-draw the interferometer as if all the elements were in a straight line. The fringes are formed just like the thin film with two surface M_2 and M'_1 (the image of mirror M_1)
  - The optical path length difference:

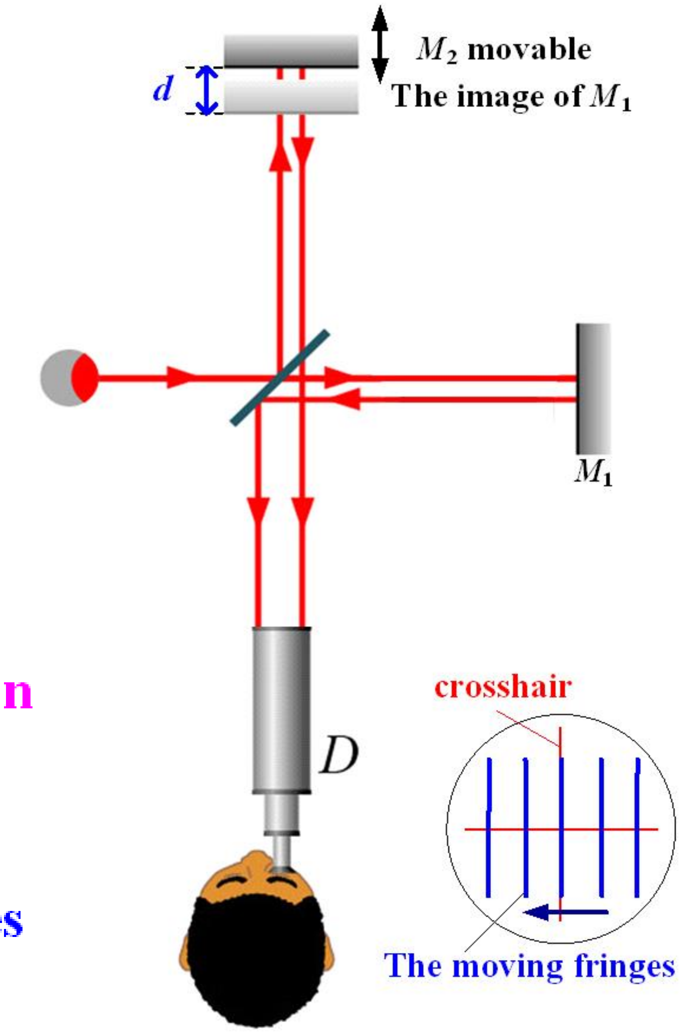
$$\delta = 2(L_2 - L_1)$$

$$i \approx 0$$



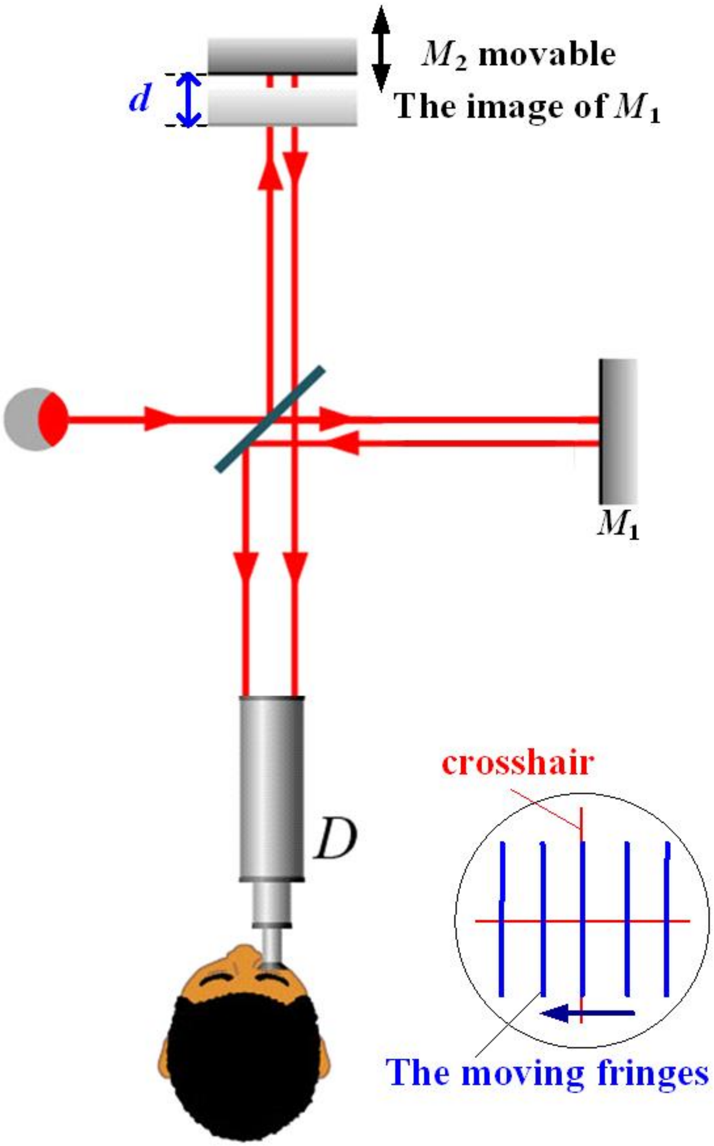
- **Very precise length measurement**
  - **For a thin film, if the top surface shifts up a distance $\lambda/2$, the fringes will move to the left with a distance of one fringe spacing. If we move M_2 slowly either backward or forward a distance $\lambda/2$, each fringe moves to the left or right a distance equal to one fringe spacing.**





The application of Michelson interferometer

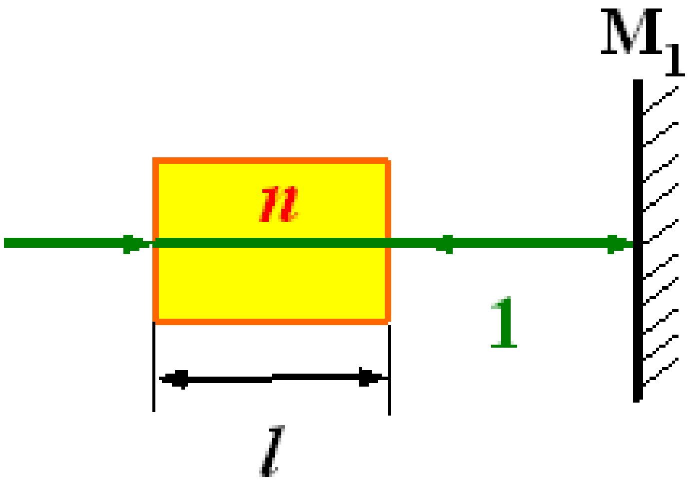
- **If we observe the fringes positions through a telescope with a crosshair eyepieces and N fringes cross the crosshair when we move the mirror M_2 a distance d**



$$d = N\frac{\lambda}{2}$$

- **Measurement of index of an object**

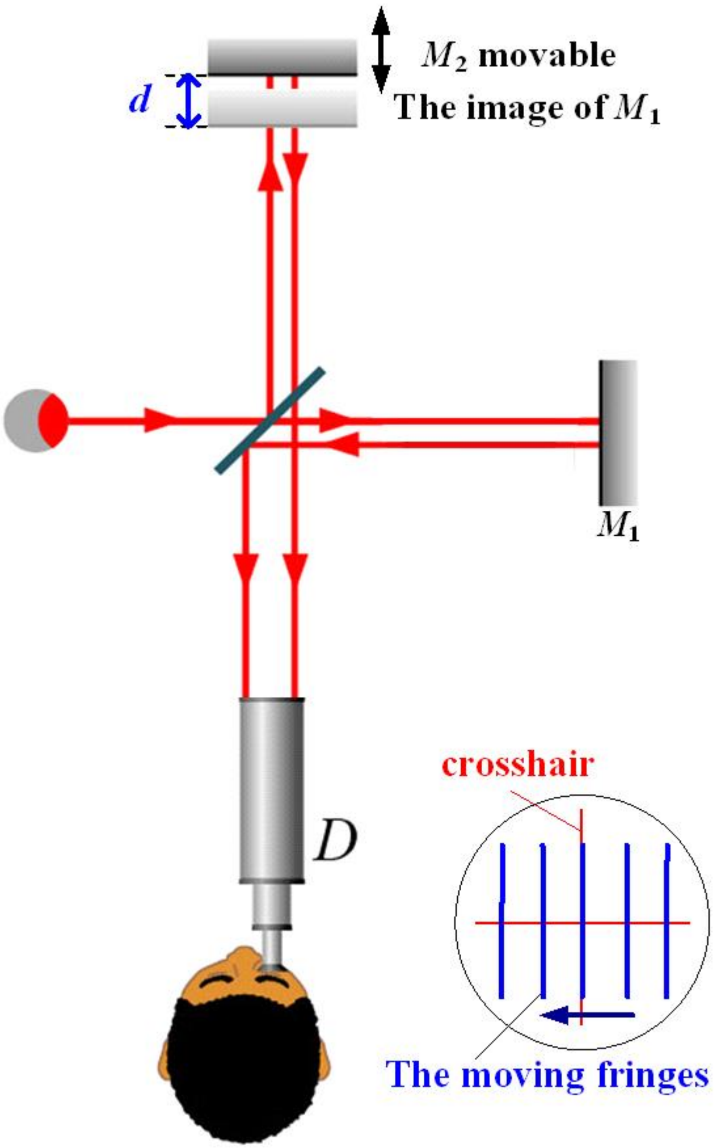
  - **We can insert an transparent object with index of refraction *n* in one arm, we observe *N* fringes cross the crosshair.**



  - **The difference of optical path length will change**

$$\Delta\delta = 2(nl - l) = N\lambda, \qquad n = 1 + \frac{N\lambda}{2l}$$

What is the wavelength of the light entering an interferometer if 344 bright fringes are counted when the movable mirror moves 0.125 mm?



Solution:

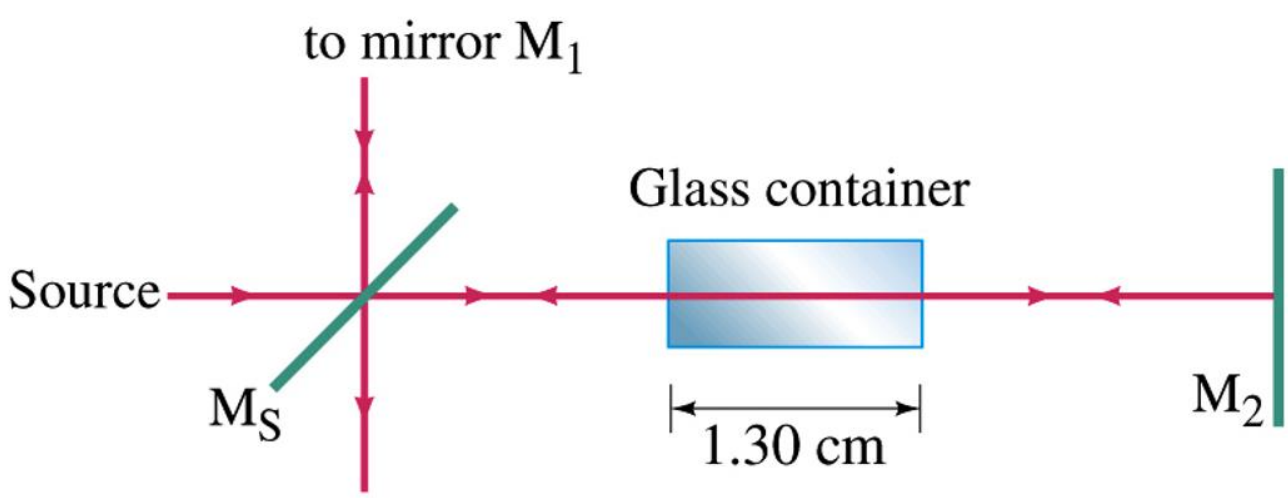
$$d = N\frac{\lambda}{2}$$

$$\lambda = \frac{2d}{N}$$

$$= \frac{2\times 0.125\times 10^{-3}}{344} = 727\ \text{nm}$$

Prob. 32 (P699)

One of the beams of an interferometer passes through a small glass container containing a cavity 1.30 cm deep. When a gas is allowed to slowly fill the container, a total of 186 dark fringes are counted to move past a reference line. The light used has a wavelength of 610 nm. Calculate the index of refraction of the gas at its final density, assuming that the interferometer is in vacuum.



Prob.3

P699, Prob.32

P701, Prob.51