Ch 4 Applications of Fourier Representations to Mixed Signal Classes

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Introduction

- Two cases of mixed signals to be studied
  - Periodic and nonperiodic signals

    Eg. a periodic signal to a stable LTI system
  - Continuous-time and discrete-time signals

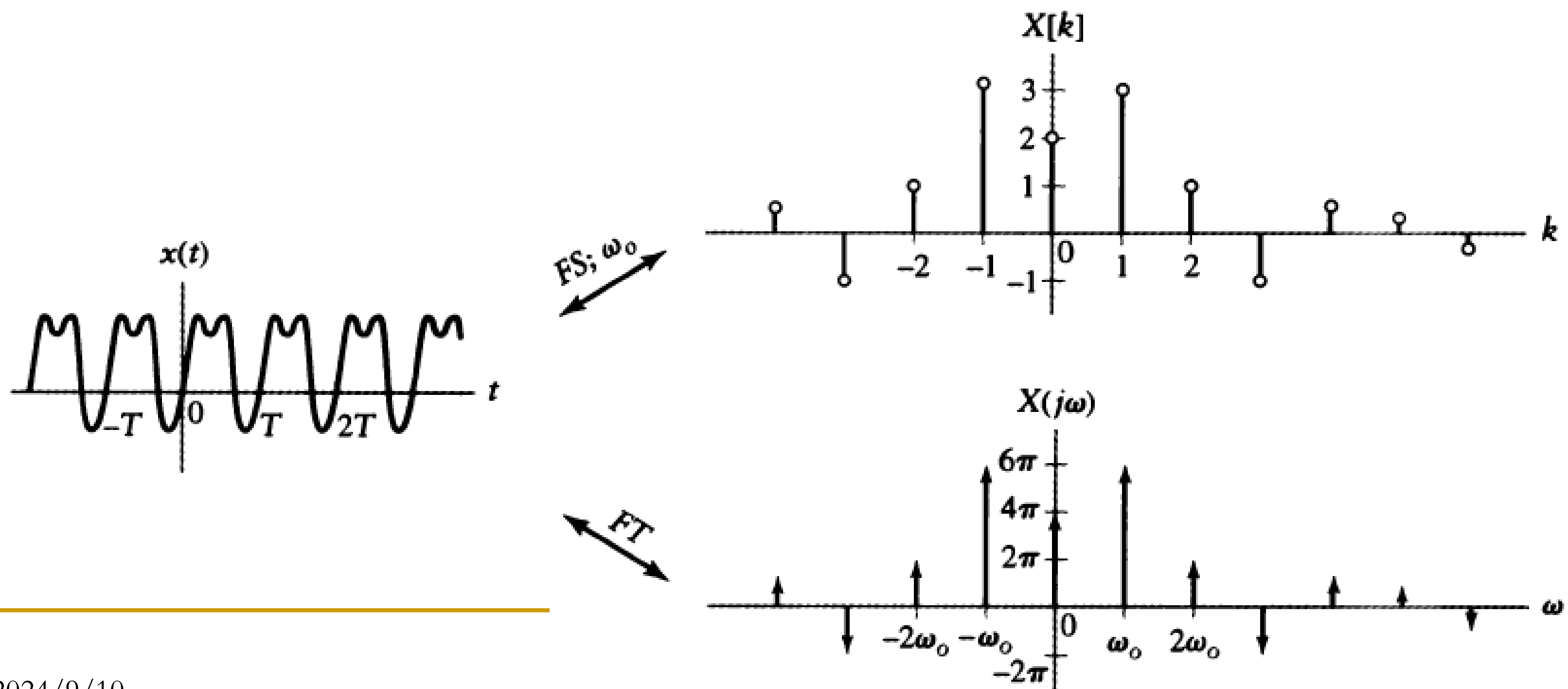
    Eg. a system that samples continuous-time signals
- Applications of Fourier Representations to Mixed Signal Classes
  - Fourier Transform Representations of Periodic Signals
  - Convolution and Multiplication with Mixtures of Periodic and Nonperiodic Signals
  - Fourier Transform Representation of Discrete-Time Signals
  - Sampling

Relating the FT to the FS

- The FT of a periodic continuous-time signal is a series of impulses spaced by the fundamental frequency ω_0.

$$1 \xleftrightarrow{FT} 2\pi\delta(\omega) \Rightarrow e^{jk\omega_0 t} \xleftrightarrow{FT} 2\pi\delta(\omega - k\omega_0)$$

$$x(t) = \sum_{k=-\infty}^{\infty} X[k] e^{jk\omega_0 t} \xleftrightarrow{FT} X(j\omega) = 2\pi \sum_{k=-\infty}^{\infty} X[k]\delta(\omega - k\omega_0)$$



Relating the FT to the FS

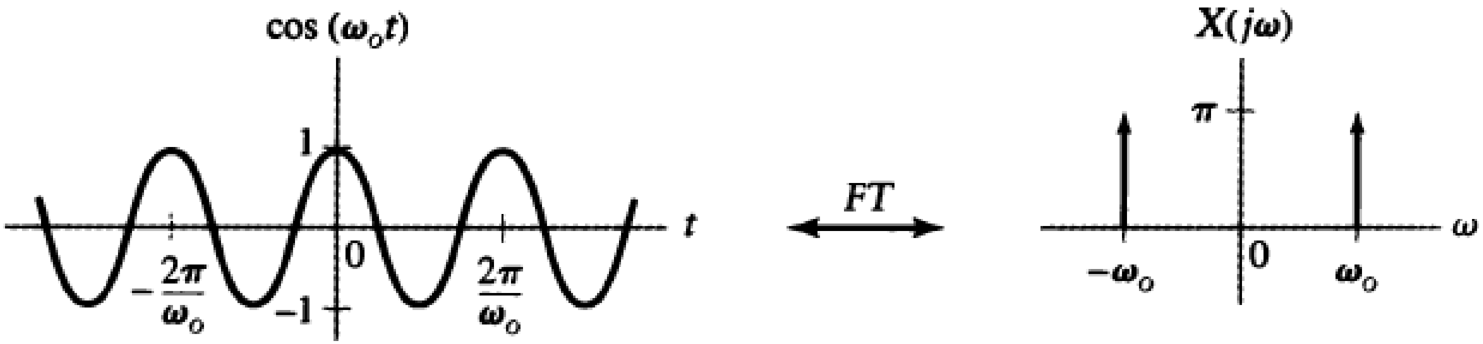
Example 4.1 ***FT of a Cosine***

Find the FT representation of $x(t) = \cos(\omega_0 t)$.

<Sol.>

$$\cos(\omega_0 t) \xleftrightarrow{FS;\omega_0} X[k] = \begin{cases} \frac{1}{2}, & k = \pm 1 \\ 0, & k \neq 1 \end{cases}$$

$$\Rightarrow \cos(\omega_0 t) \xleftrightarrow{FT} X(j\omega) = \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$$



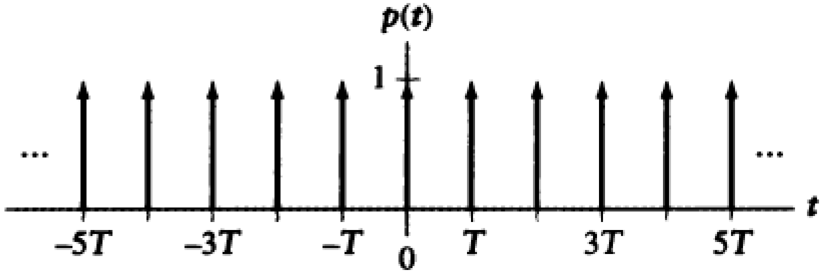
Relating the FT to the FS

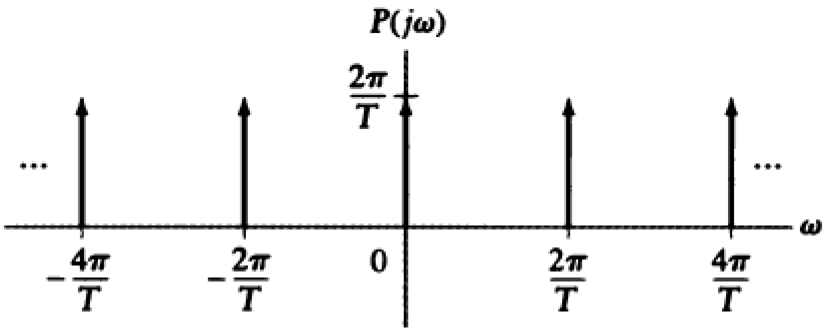
Example 4.2 ***FT of a Unit Impulse Train***

Find the FT of the impulse train $p(t)=\sum_{n=-\infty}^{\infty}\delta(t-nT)$

<Sol.> **Fundamental period =** T ⟹ $\omega_0 = 2\pi/T$

$$P[k]=\frac{1}{T}\int_{-T/2}^{T/2}\delta(t)e^{-jk\omega_0 t}dt=1/T \quad \Longrightarrow \quad P(j\omega)=\frac{2\pi}{T}\sum_{k=-\infty}^{\infty}\delta(\omega-k\omega_0)$$





♣ **The FT of $p(t)$ is also an impulse train.** Impulse spacing is inversed each other; the strength of impulses differ by a factor of $2\pi/T$.

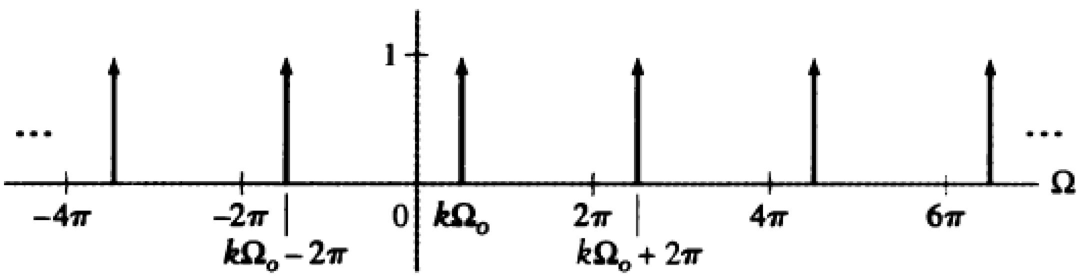
Relating the DTFT to the DTFS

- The DTFT of a periodic discrete-time signal

$$1 \xleftrightarrow{DTFT} 2\pi\delta(\Omega)$$

$$e^{jk\Omega_0 n} \xleftrightarrow{DTFT} 2\pi\delta(\Omega - k\Omega_0), -\pi < \Omega \le \pi, -\pi < k\Omega_0 \le \pi$$

$$\Rightarrow \quad e^{jk\Omega_0 n} \xleftrightarrow{DTFT} 2\pi \sum_{m=-\infty}^{\infty} \delta(\Omega - k\Omega_0 - m2\pi),$$



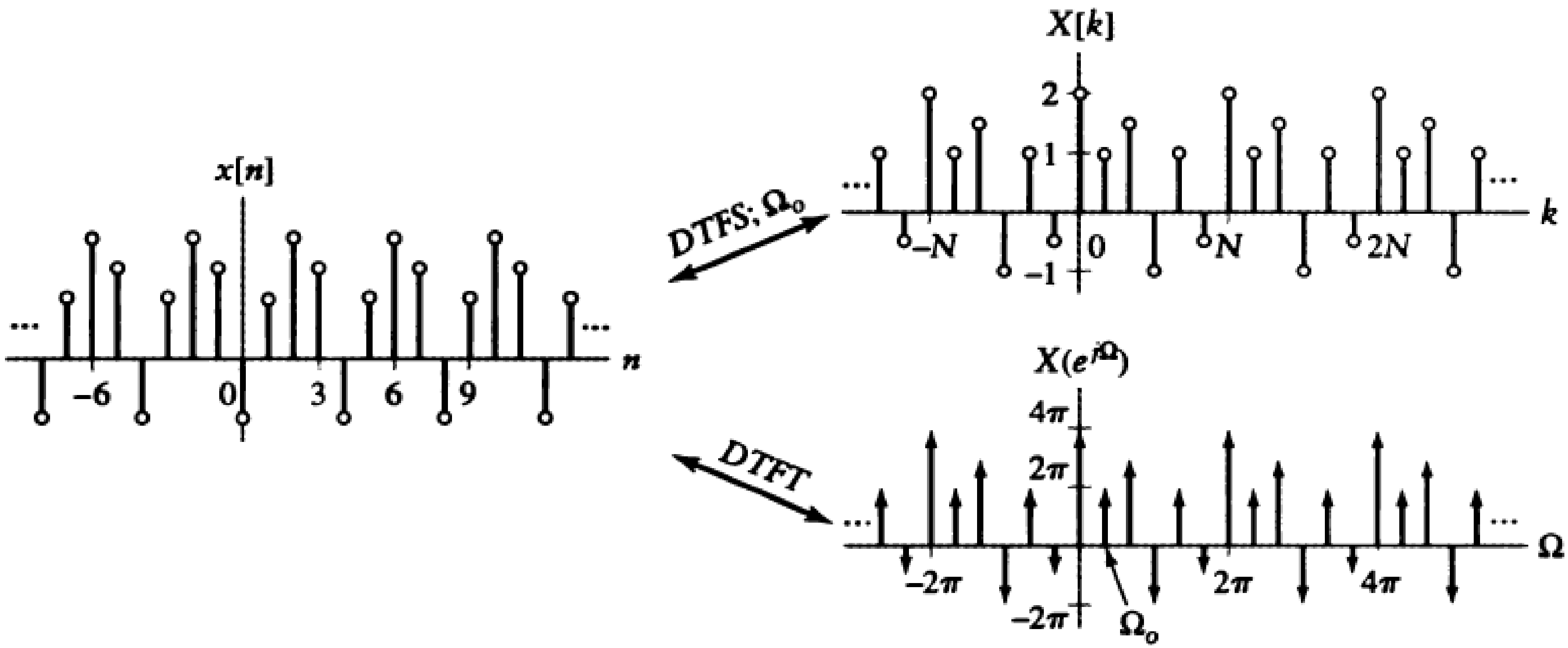
$$x[n] = \sum_{k=0}^{N-1} X[k] e^{jk\Omega_0 n} \xleftrightarrow{DTFT} X(e^{j\Omega}) = 2\pi \sum_{k=0}^{N-1} X[k] \sum_{m=-\infty}^{\infty} \delta(\Omega - k\Omega_0 - m2\pi)$$

$$\xRightarrow{N\Omega_0 = 2\pi} \quad X(e^{j\Omega}) = 2\pi \sum_{k=-\infty}^{\infty} X[k] \delta(\Omega - k\Omega_0)$$

Relating the DTFT to the DTFS

- The DTFT of a periodic discrete-time signal is a series of impulses spaced by the fundamental frequency Ω_0.

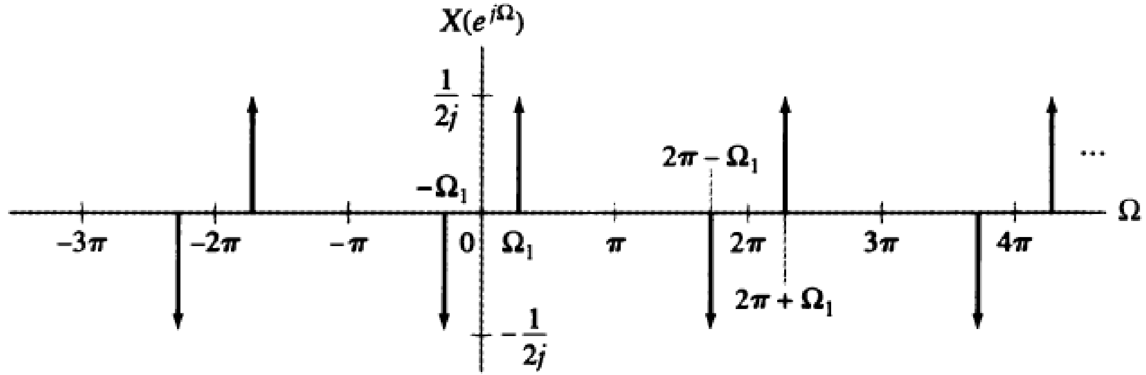
$$x[n]=\sum_{k=0}^{N-1}X[k]e^{jk\Omega_0 n}\xleftrightarrow{DTFT}X\left(e^{j\Omega}\right)=2\pi\sum_{k=-\infty}^{\infty}X[k]\delta\left(\Omega-k\Omega_0\right)$$



Relating the DTFT to the DTFS

Example 4.3 ***DTFT of a Periodic Signal***

Determine the inverse DTFT of the frequency-domain representation depicted in Fig. 4.7, where $\Omega_1 = \pi / N$.



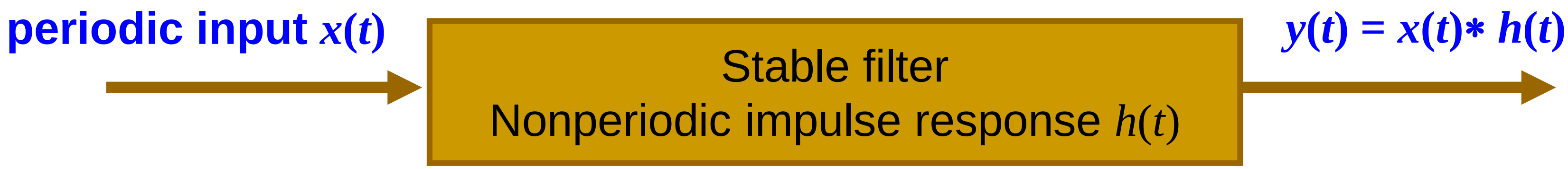
$$X\left(e^{j\Omega}\right) = \frac{1}{2j}\delta\left(\Omega - \Omega_1\right) - \frac{1}{2j}\delta\left(\Omega + \Omega_1\right), \quad -\pi < \Omega \le \pi$$

$$X[k] = \begin{cases} 1/\left(4\pi j\right), & k = 1 \\ -1/\left(4\pi j\right), & k = -1 \\ 0, & \text{otherwise} \quad \text{on} \quad -1 \le k \le N-2 \end{cases}$$

$$x[n] = \frac{1}{2\pi}\left[\frac{1}{2j}\left(e^{j\Omega_1 n} - e^{-j\Omega_1 n}\right)\right] = \frac{1}{2\pi}\sin\left(\Omega_1 n\right)$$

$$x[n] = \sum_{k=0}^{N-1} X[k] e^{jk\Omega_0 n} \xleftrightarrow{DTFT} X\left(e^{j\Omega}\right) = 2\pi \sum_{k=-\infty}^{\infty} X[k]\delta\left(\Omega - k\Omega_0\right)$$

Convolution of Periodic and Nonperiodic Signals



$$x(t) \xleftrightarrow{FT} X(j\omega) = 2\pi \sum_{k=-\infty}^{\infty} X[k]\delta(\omega - k\omega_0)$$

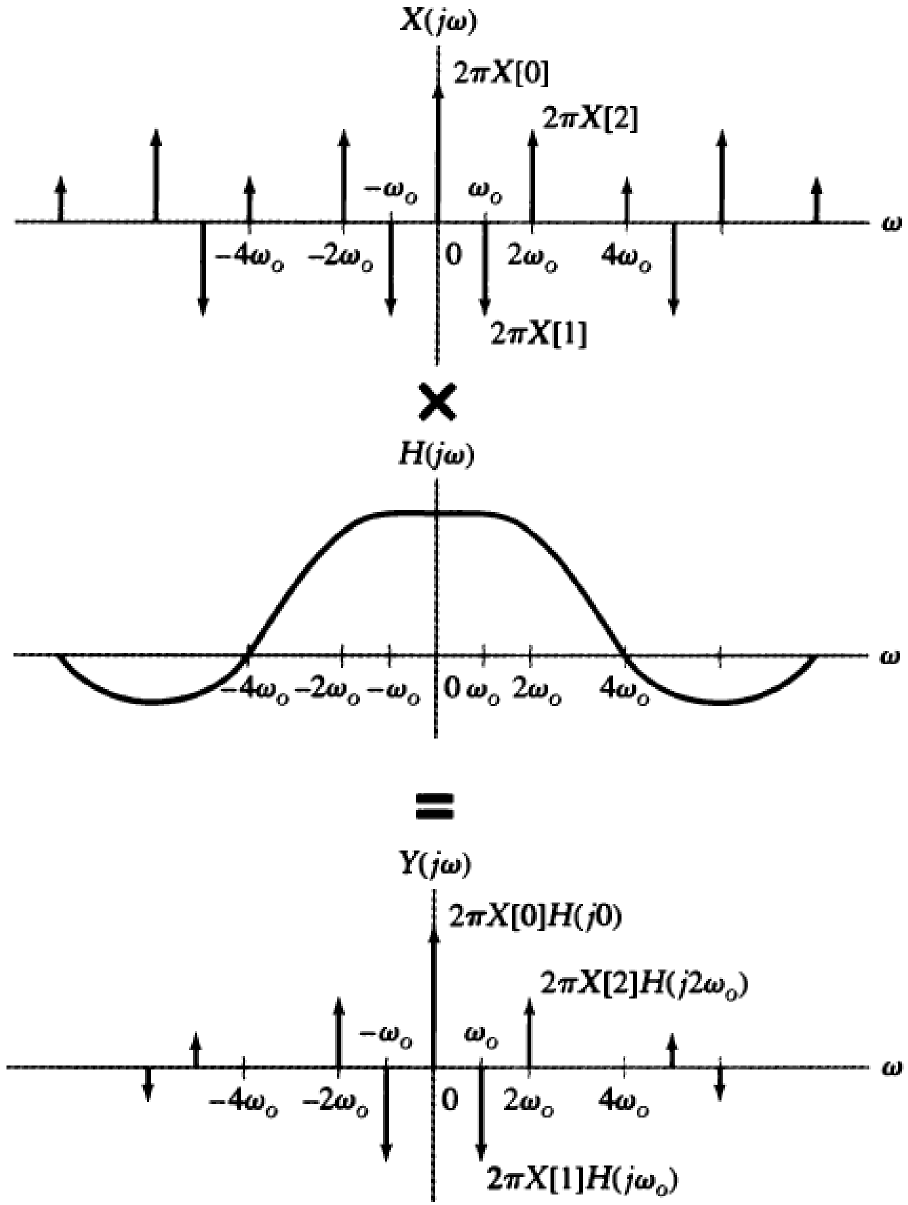
$$y(t) = x(t) * h(t) \xleftrightarrow{FT} Y(j\omega) = X(j\omega)H(j\omega)$$

$$= 2\pi \sum_{k=-\infty}^{\infty} X[k]\delta(\omega - k\omega_0)H(j\omega)$$

$$\boxed{y(t) = x(t) * h(t) \xleftrightarrow{FT} Y(j\omega) = 2\pi \sum_{k=-\infty}^{\infty} H(jk\omega_0)X[k]\delta(\omega - k\omega_0)}$$

$$\boxed{y[n] = x[n] * b[n] \xleftrightarrow{DTFT} Y(e^{j\omega}) = 2\pi \sum_{k=-\infty}^{\infty} H(e^{jk\Omega_0})X[k]\delta(\Omega - k\Omega_0).}$$

Convolution of Periodic and Nonperiodic Signals



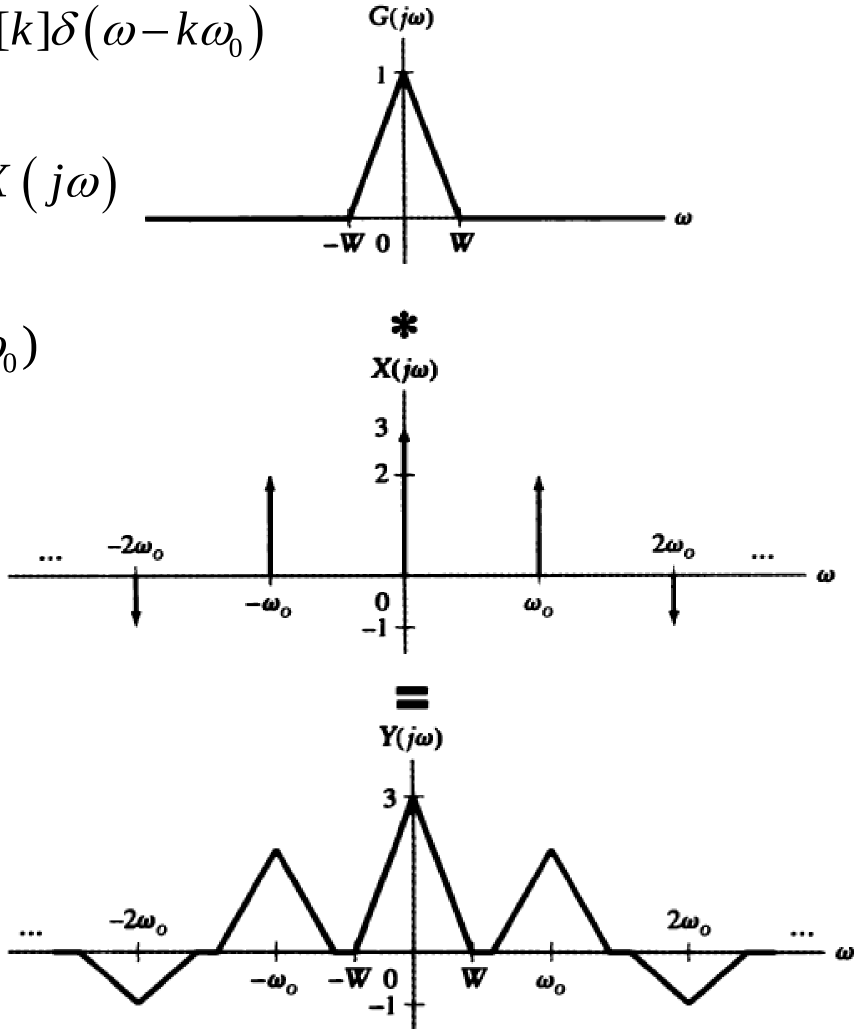
Multiplication of Periodic and Nonperiodic Signals

$$x(t)=\sum_{k=-\infty}^{\infty} X[k]e^{jk\omega_0 t} \xleftrightarrow{FT} X(j\omega)=2\pi\sum_{k=-\infty}^{\infty} X[k]\delta(\omega-k\omega_0)$$

$$y(t)=g(t)x(t) \xleftrightarrow{FT} Y(j\omega)=\frac{1}{2\pi}G(j\omega)*X(j\omega)$$

$$\Rightarrow Y(j\omega)=\frac{1}{2\pi}G(j\omega)*2\pi\sum_{k=-\infty}^{\infty} X[k]\delta(\omega-k\omega_0)$$

$$=\sum_{k=-\infty}^{\infty} X[k]G\big(j(\omega-k\omega_0)\big)$$



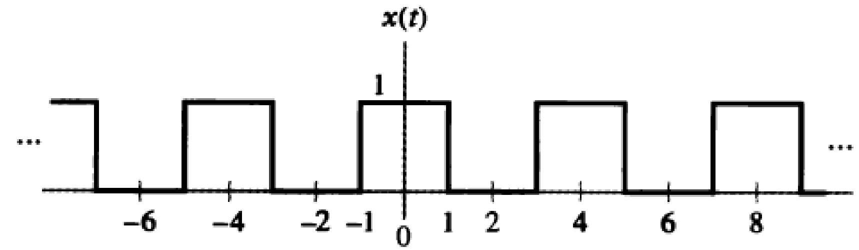
$y(t)$ becomes nonperiodic signal !

Multiplication of Periodic and Nonperiodic Signals

Example 4.5 ***Multiplication with a Square Wave***

Consider a system with output $y(t) = g(t)x(t)$. Let $x(t)$ be the square wave in Fig. 4.4. (a) Find $Y(j\omega)$ in terms of $G(j\omega)$. (b) Sketch $Y(j\omega)$ if $g(t) = \cos(t/2)$.

<Sol.> $T = 4 \Rightarrow \omega_0 = \pi/2$

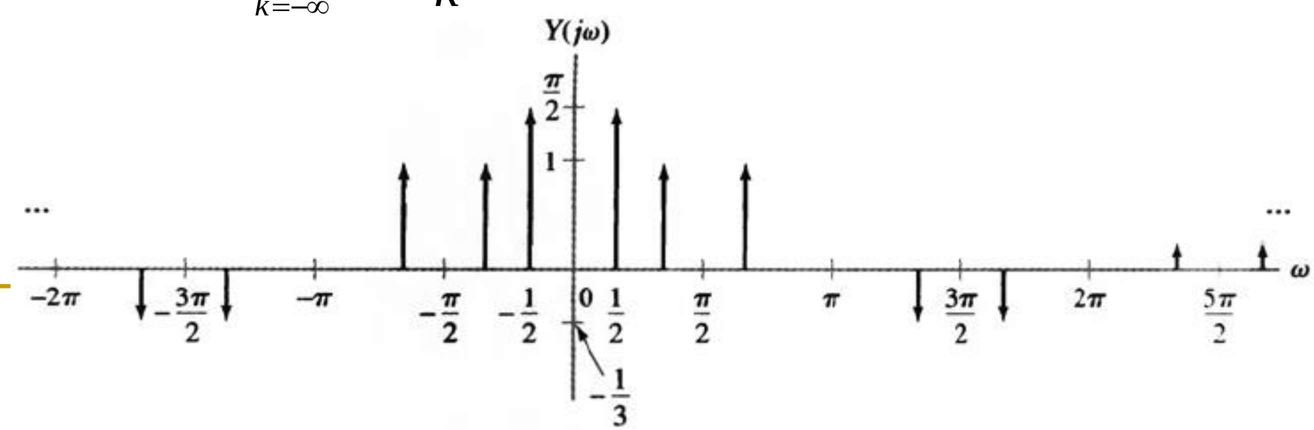


$$x(t) \xleftrightarrow{FS;\pi/2} X[k] = \frac{\sin(k\pi/2)}{\pi k}$$

$$Y(j\omega) = \sum_{k=-\infty}^{\infty} X[k]G(j(\omega - k\omega_0)) = \sum_{k=-\infty}^{\infty} \frac{\sin(k\pi/2)}{\pi k} G(j(\omega - k\pi/2))$$

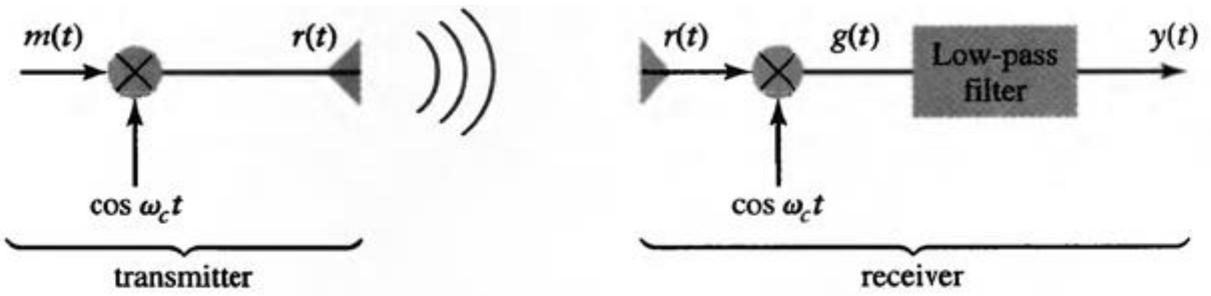
$$G(j\omega) = \pi\delta(\omega - 1/2) + \pi\delta(\omega + 1/2)$$

$$\Rightarrow \quad Y(j\omega) = \sum_{k=-\infty}^{\infty} \frac{\sin(k\pi/2)}{k}\left[\delta(\omega - 1/2 - k\pi/2) + \delta(\omega + 1/2 - k\pi/2)\right]$$



Multiplication of Periodic and Nonperiodic Signals

Example 4.6 ***AM Radio***



$$r(t)=m(t)\cos(\omega_c t) \xleftrightarrow{FT} R(j\omega)=\frac{1}{2\pi}M(j\omega)*\left[\pi\delta(\omega-\omega_c)+\pi\delta(\omega+\omega_c)\right]$$

$$\Rightarrow R(j\omega)=(1/2)M(j(\omega-\omega_c))+(1/2)M(j(\omega+\omega_c))$$

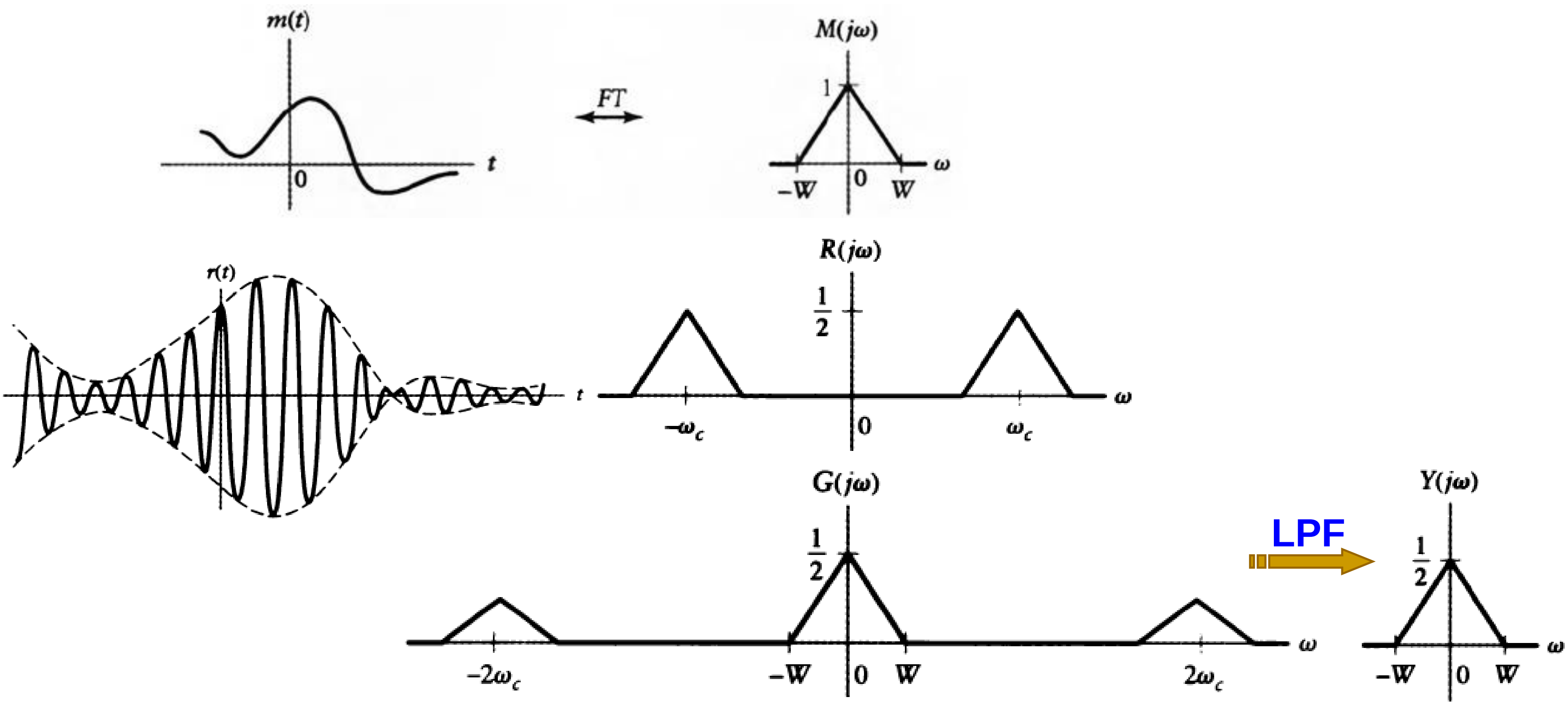
$$g(t)=r(t)\cos(\omega_c t) \xleftrightarrow{FT} G(j\omega)=(1/2)R(j(\omega-\omega_c))+(1/2)R(j(\omega+\omega_c))$$

$$\Rightarrow G(j\omega)=(1/4)M(j(\omega-2\omega_c))+(1/2)M(j(\omega))+(1/4)M(j(\omega+2\omega_c))$$

Multiplication of Periodic and Nonperiodic Signals

$$R(j\omega) = (1/2)M\left(j(\omega-\omega_c)\right) + (1/2)M\left(j(\omega+\omega_c)\right)$$

$$G(j\omega) = (1/4)M\left(j(\omega-2\omega_c)\right) + (1/2)M\left(j(\omega)\right) + (1/4)M\left(j(\omega+2\omega_c)\right)$$



Fourier Transform of Discrete-time Signals

- Using FT representation of discrete-time signals by incorporating impulses into the description of the signal in the appropriate manner.

Complex sinusoids: $x(t) = e^{j\omega t}$ and $g[n] = e^{j\Omega n}$

Suppose we force $\boldsymbol{g[n] = x(nT_s)}$ ⇒ $e^{j\Omega n} = e^{j\omega T_s n}$ i.e. $\boldsymbol{\Omega = \omega T_s}$

♣ The dimensionless discrete-time frequency Ω corresponds to the continuous-time frequency ω, multiplied by the sampling interval T_s.

$$x[n] \xleftrightarrow{DTFT} X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n}$$

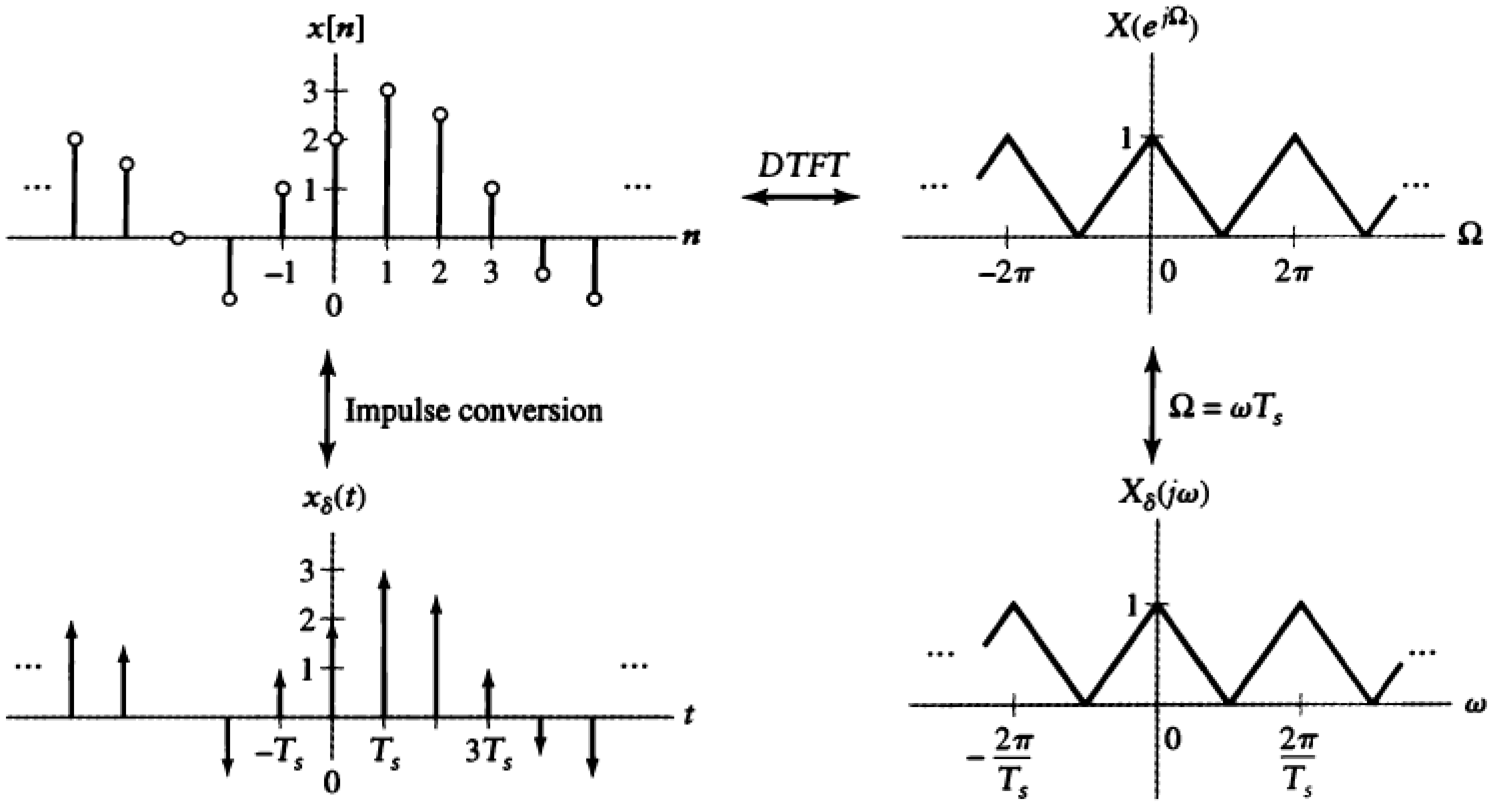
$$x_\delta(t) = \sum_{n=-\infty}^{\infty} x[n]\delta(t - nT_s) \xleftrightarrow{FT} X_\delta(j\omega) = X\left(e^{j\Omega}\right)\Big|_{\Omega=\omega T_s} = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega T_s n}$$

- $\boldsymbol{x_\delta(t)}$ ≡ a continuous-time representation of $\boldsymbol{x[n]}$;
- Relationship between continuous- and discrete-time frequency: $\boldsymbol{\Omega = \omega T_s}$

Fourier Transform of Discrete-time Signals

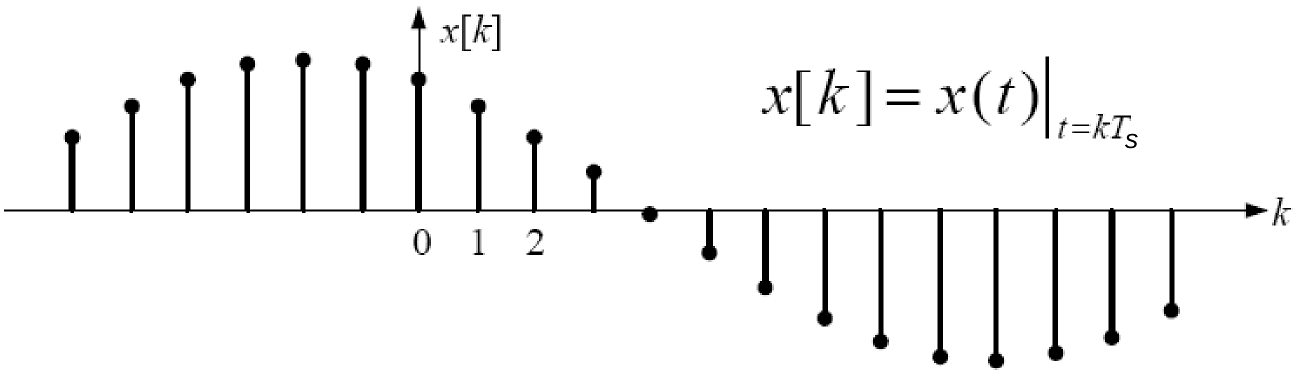
$$x[n] \xleftrightarrow{DTFT} X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n}$$

$$x_\delta(t) = \sum_{n=-\infty}^{\infty} x[n]\delta(t - nT_s) \xleftrightarrow{FT} X_\delta(j\omega) = X\left(e^{j\Omega}\right)\Big|_{\Omega=\omega T_s} = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega T_s n}$$



Sampling (抽样)

- Sampling： taking snap shots of $\boldsymbol{x(t)}$ *every* $\boldsymbol{T_s}$ seconds， where T_s is the sampling period. After the signal sampling, we can get the samples $\boldsymbol{x[k]}$.



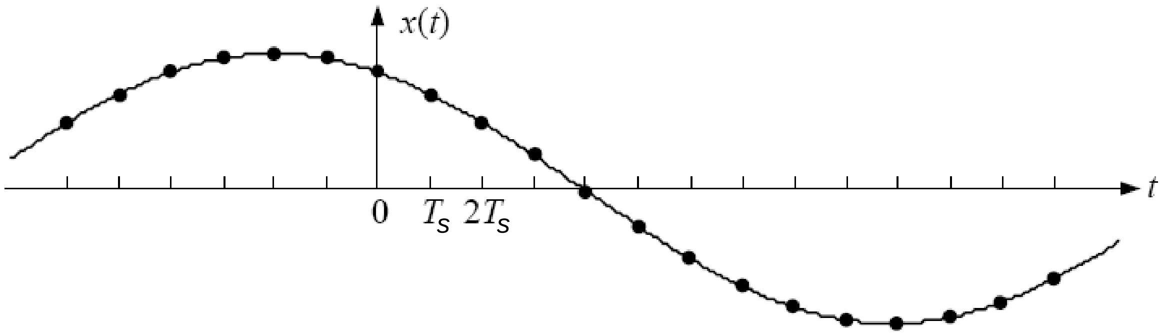
$$x[k] = x(t)\big|_{t=kT_s}$$

Why to Sample?

Discrete-time processing of continuous-time signals

- Advantages
  - Easily performed by computer
  - Implementing a system only involving programming
  - Easily changed by modifying program

How to Sample? – The choice of sampling period



Should the sampling period be small or large? In other words, how many samples would be adequate for a specific signal?

Theoretical Derivation of Sampling Theorem

- $x(t)$ = CT signal, $x[n]$ = DT signal that is equal to the "samples " of $x(t)$ at integer multiples of a sampling interval T_s , i.e.

$$x[n] = x(nT_s)$$

- $x_\delta(t)$ = CT representation of the DT signal $x[n]$

$$x_\delta(t) = \sum_{n=-\infty}^{\infty} x[n]\delta(t-nT_s) = \sum_{n=-\infty}^{\infty} x(nT_s)\delta(t-nT_s) = x(t)\,p(t)$$

where

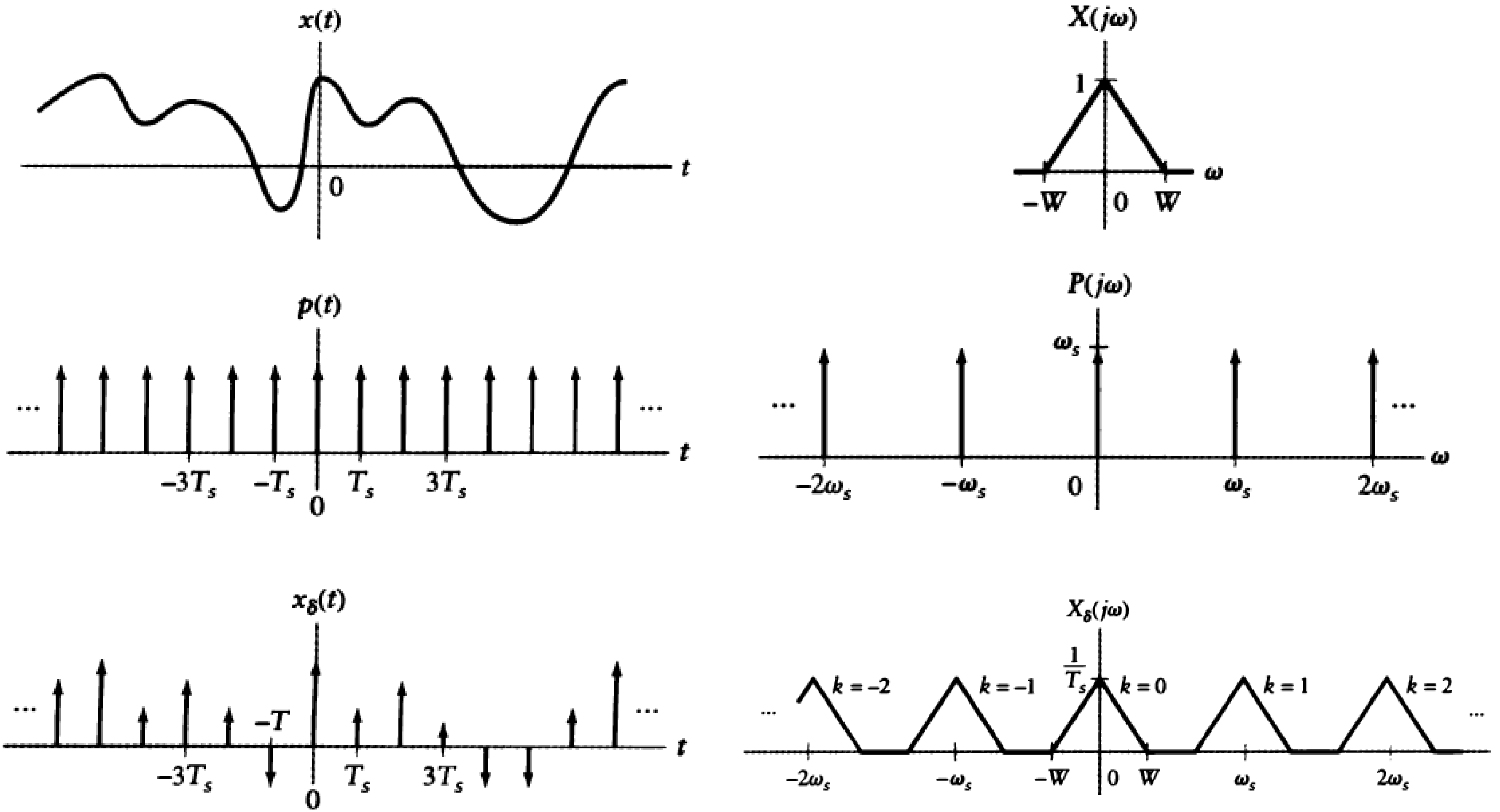
Impulse sampling !

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT_s) \xleftrightarrow{FT} P(j\omega) = \omega_s \sum_{k=-\infty}^{\infty} \delta(\omega-k\omega_s), \quad \omega_s = \frac{2\pi}{T_s}.$$

$$\Rightarrow X_\delta(j\omega) = \frac{1}{2\pi} X(j\omega) * P(j\omega) = \frac{1}{2\pi} X(j\omega) * \frac{2\pi}{T_s} \sum_{k=-\infty}^{\infty} \delta(\omega-k\omega_s)$$

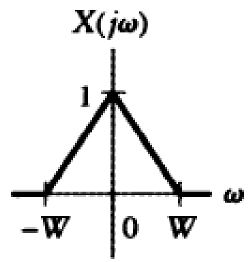
$$= \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X(j\omega - jk\omega_s)$$

Theoretical Derivation of Sampling Theorem



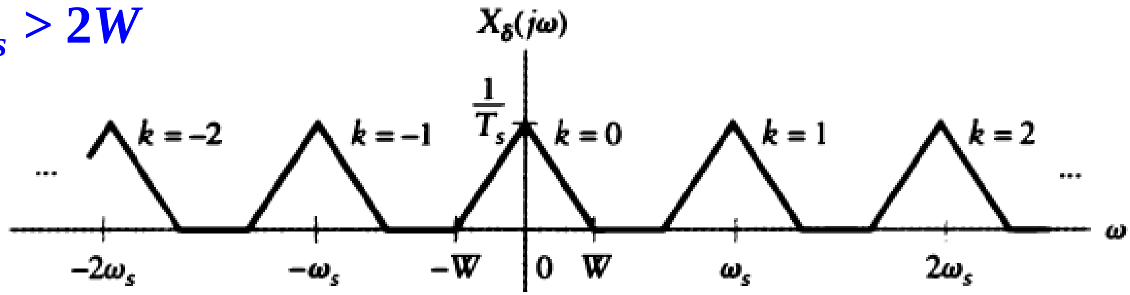
$$X_\delta(j\omega) = \frac{1}{T_s}\sum_{k=-\infty}^{\infty} X(j\omega - jk\omega_s)$$

Theoretical Derivation of Sampling Theorem

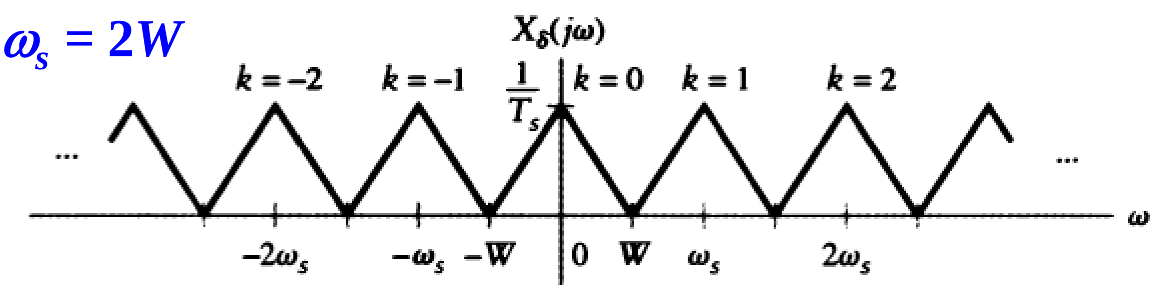


$$X_\delta(j\omega)=\frac{1}{T_s}\sum_{k=-\infty}^{\infty}X(j\omega-jk\omega_s)$$

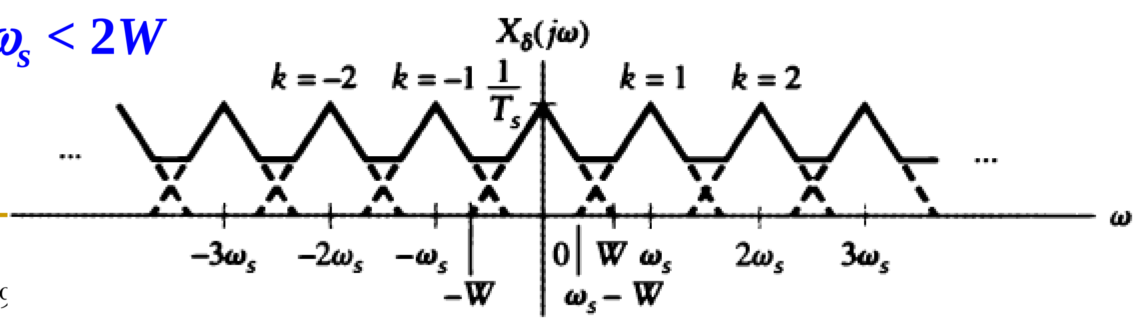
- $\omega_s > 2W$



- $\omega_s = 2W$



- $\omega_s < 2W$



Overlap in the shifted replicas of the original spectrum is termed *aliasing* (混叠)！

Theoretical Derivation of Sampling Theorem

- Aliasing distorts the spectrum of the original signal, and destroys the one-to-one relationship between the FT's of the CT signal and the sampled signal.
- To prevent aliasing, choose the sampling interval T_s so that

$$\omega_s > 2W$$

where W is the highest nonzero frequency component in the signal.

➡ ***No distortion! Reconstruction of the original signal to be feasible!***

- **Sampling Theorem**

Let $x(t) \xleftrightarrow{FT} X(j\omega)$ **represents a band-limited signal, so that $X(j\omega) = 0$ for $|\omega| > \omega_m$. If $\omega_s > 2\omega_m$, where $\omega_s = 2\pi/T_s$ is sampling frequency, then $x(t)$ is uniquely determined by its samples $x(nT_s)$, $n = 0, \pm 1, \pm 2, \ldots$**

  - **Nyquist sampling rate/Nyquist rate:** $2\omega_m$
  - **The actual sampling frequency $f_s > 2f_m$, where $f_s = 1/T_s$.**

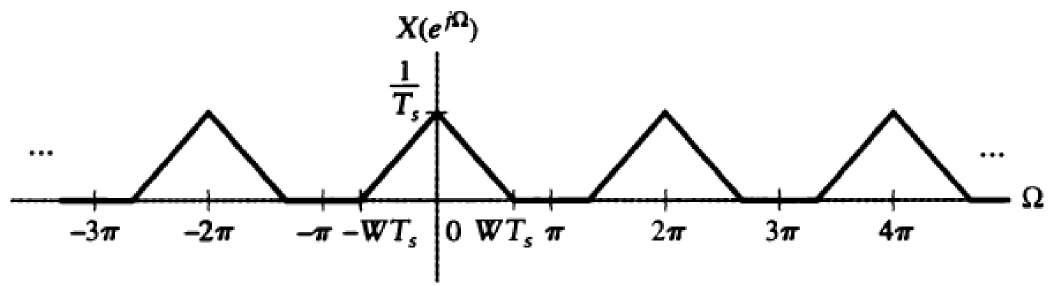
➡ $$T_s < 1/(2f_m) = \pi/\omega_m$$

Theoretical Derivation of Sampling Theorem

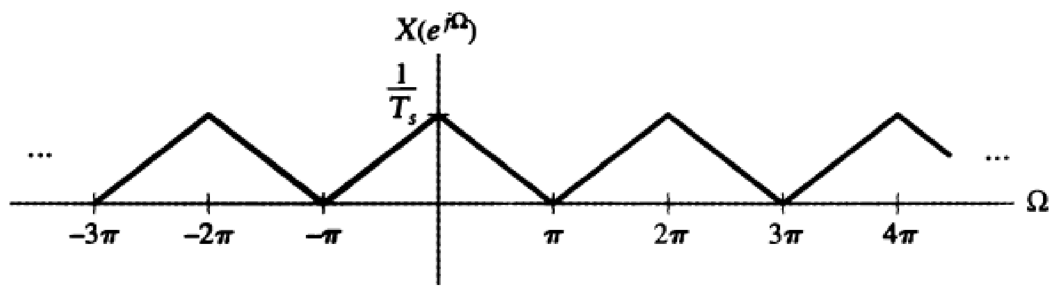
- The DTFT of the sampled signal is obtained from $X_\delta(j\omega)$ by using the relationship $\Omega = \omega T_s$, i.e.,

$$x[n] \xleftrightarrow{DTFT} X\left(e^{j\Omega}\right) = X_\delta\left(j\omega\right)\Big|_{\omega=\Omega/T_s}$$

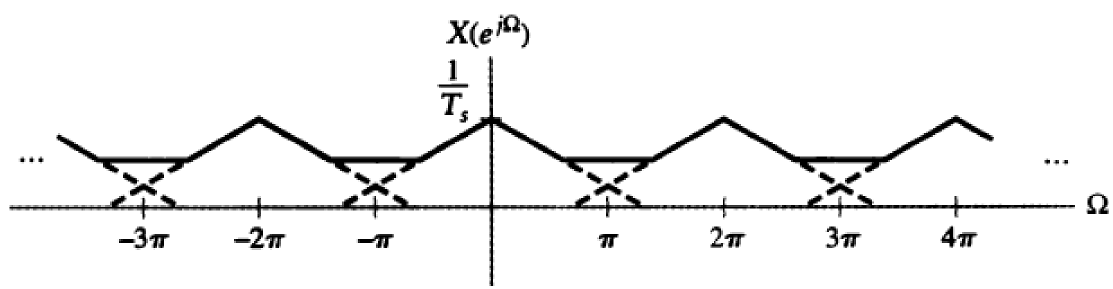
  - $\omega_s > 2W$



  - $\omega_s = 2W$



  - $\omega_s < 2W$



Sampling Theorem

Example 4.9 ***Sampling a Sinusoid***

Consider the effect of sampling the sinusoidal signal

$$x(t) = \cos(\pi t)$$

Determine the FT of the sampled signal for the following sampling intervals: (i) T_s = 1/4, (ii) T_s = 1, and (iii) T_s = 3/2.

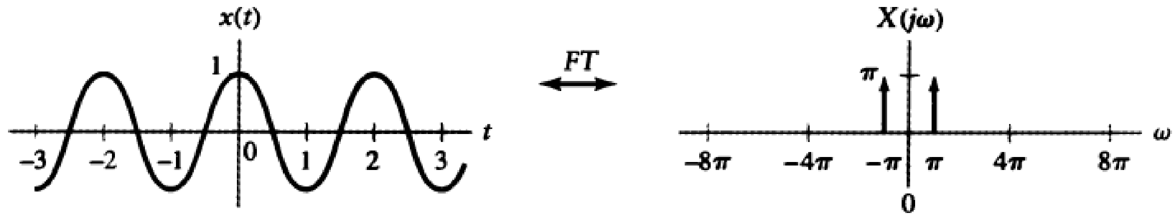
<Sol.> $x(t) = \cos(\pi t) \xleftrightarrow{FT} X(j\omega) = \pi\delta(\omega+\pi) + \pi\delta(\omega-\pi)$

$$\omega_m = \pi \quad \Rightarrow \quad T_s < \pi / \omega_m = 1$$

$$X_\delta(j\omega) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X(j\omega - jk\omega_s)$$

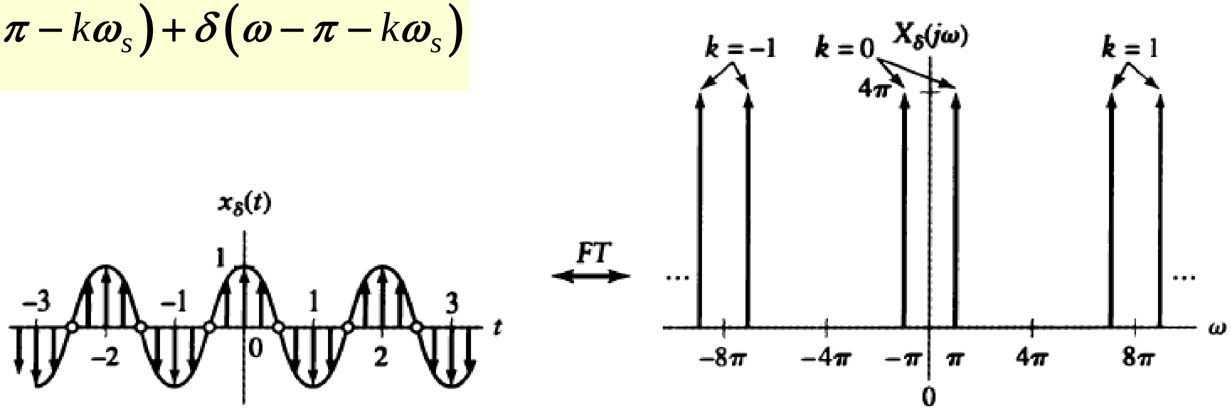
$$= \frac{\pi}{T_s} \sum_{k=-\infty}^{\infty} \delta(\omega + \pi - k\omega_s) + \delta(\omega - \pi - k\omega_s)$$

Sampling Theorem

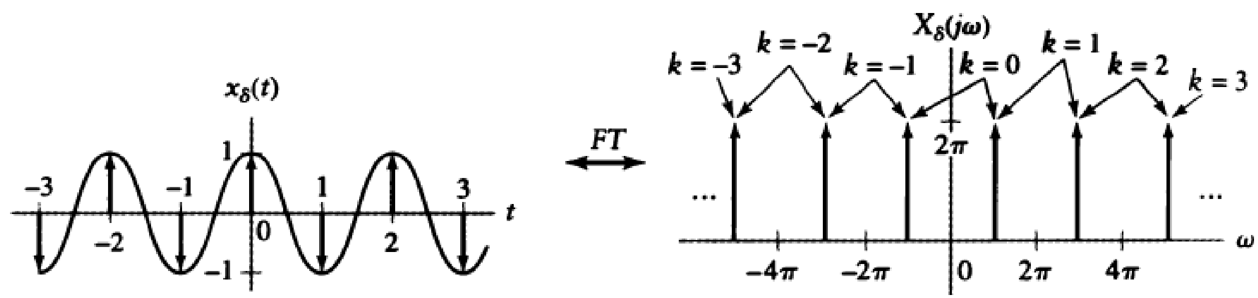


$$X_\delta(j\omega)=\frac{\pi}{T_s}\sum_{k=-\infty}^{\infty}\delta(\omega+\pi-k\omega_s)+\delta(\omega-\pi-k\omega_s)$$

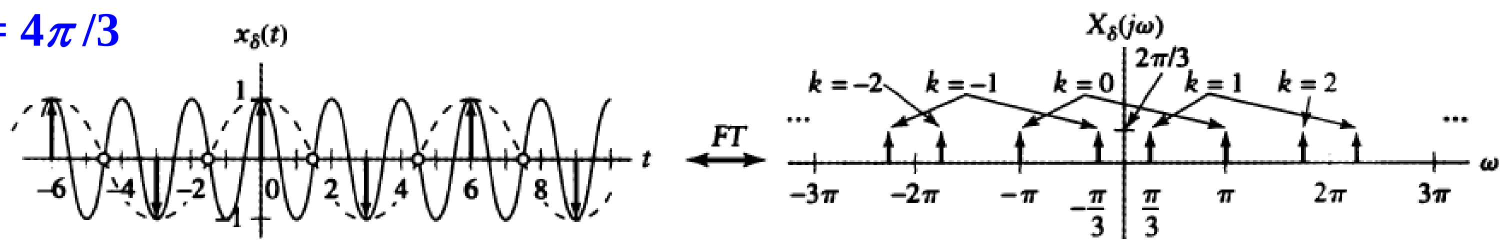
- **$T_s=1/4,\ \omega_s=8\pi$**



- **$T_s=1,\ \omega_s=2\pi$**



- **$T_s=3/2,\ \omega_s=4\pi/3$**



Sampling Theorem

Example 4.12 ***Selecting the Sampling Interval***

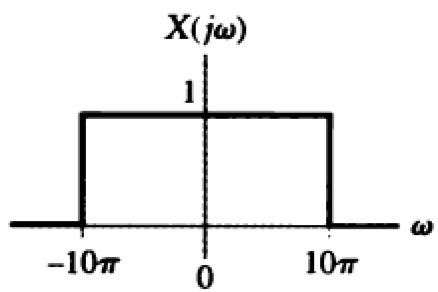
Suppose $x(t) = \sin(10\pi t)/(\pi t)$. Determine the condition on the sampling interval T_s, so that $x(t)$ uniquely represented by the discrete-time sequence $x[n] = x(nT_s)$.

<Sol.>

$$x(t) = \frac{1}{\pi t}\sin(Wt) \quad \xleftrightarrow{FT} \quad X(j\omega) = \begin{cases} 1, & -W < \omega < W \\ 0, & |\omega| > W \end{cases}$$

$$\Rightarrow \quad x(t) = \frac{\sin(10\pi t)}{\pi t} \quad \xleftrightarrow{FT} \quad X(j\omega) = \begin{cases} 1, & |\omega| \le 10\pi \\ 0, & |\omega| > 10\pi \end{cases}$$

$$\omega_m = 10\pi \quad \Rightarrow \quad \omega_s = \frac{2\pi}{T_s} > 20\pi \quad \Rightarrow \quad T_s < (1/10)$$



Sampling Theorem

Example The highest frequency of a real-valued signal $x(t)$ is ω_m. For each of the following signals, determine the smallest sampling frequency which guarantee that there will be no aliasing:
(i) $x(2t)$; (ii) $x(t)*x(2t)$; and (iii) $x(t)\cdot x(2t)$.

<Sol.>

- **$x(2t)$:** $x(2t) \xleftrightarrow{FT} \dfrac{1}{2}X\left(\dfrac{j\omega}{2}\right)$

  the highest frequency of $x(2t)$ is $2\omega_m$ ➡ $\omega_s > 4\omega_m$

- **$x(t)*x(2t)$:** $x(t)*x(2t) \xleftrightarrow{FT} X(j\omega)\cdot\dfrac{1}{2}X\left(\dfrac{j\omega}{2}\right)$

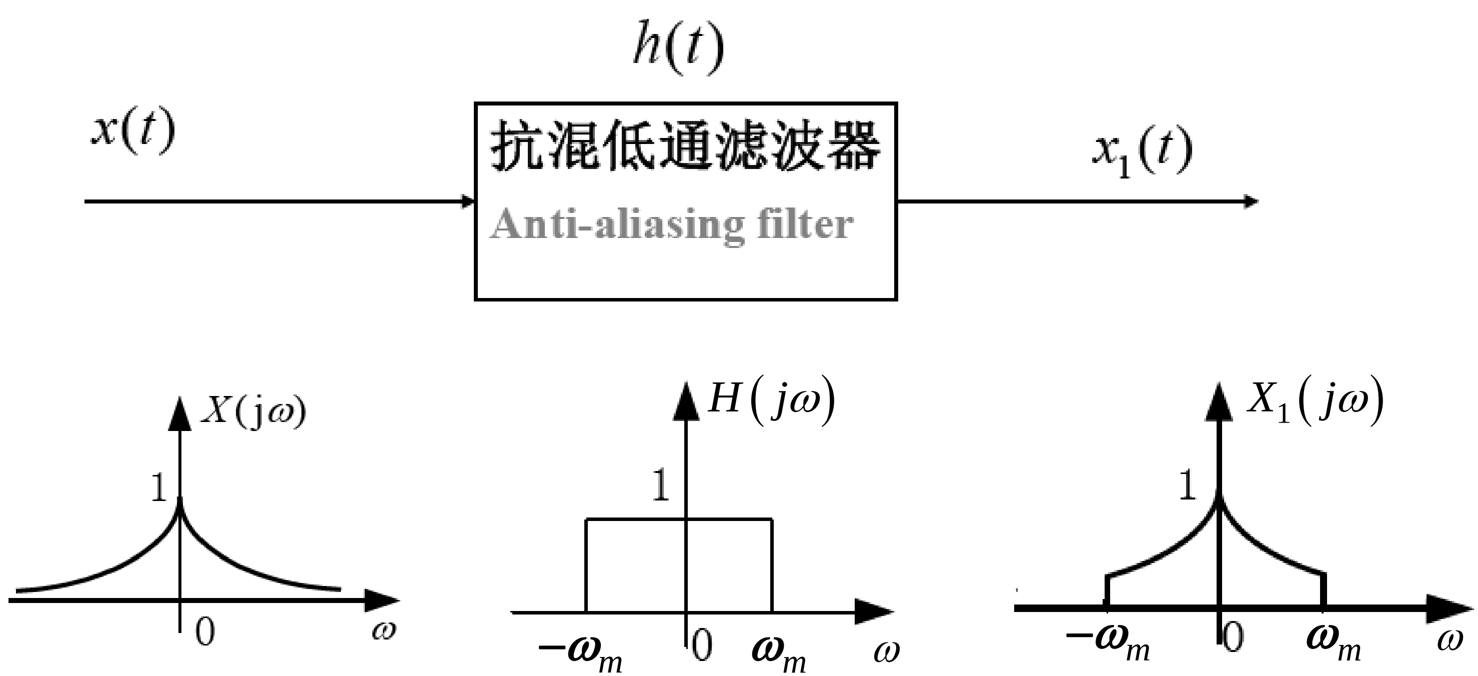
  the highest frequency of $x(t)*x(2t)$ is ω_m ➡ $\omega_s > 2\omega_m$

- **$x(t)\cdot x(2t)$:** $x(t)\cdot x(2t) \xleftrightarrow{FT} X(j\omega)*\dfrac{1}{2}X\left(\dfrac{j\omega}{2}\right)$

  the highest frequency of $x(t)\cdot x(2t)$ is $3\omega_m$ ➡ $\omega_s > 6\omega_m$

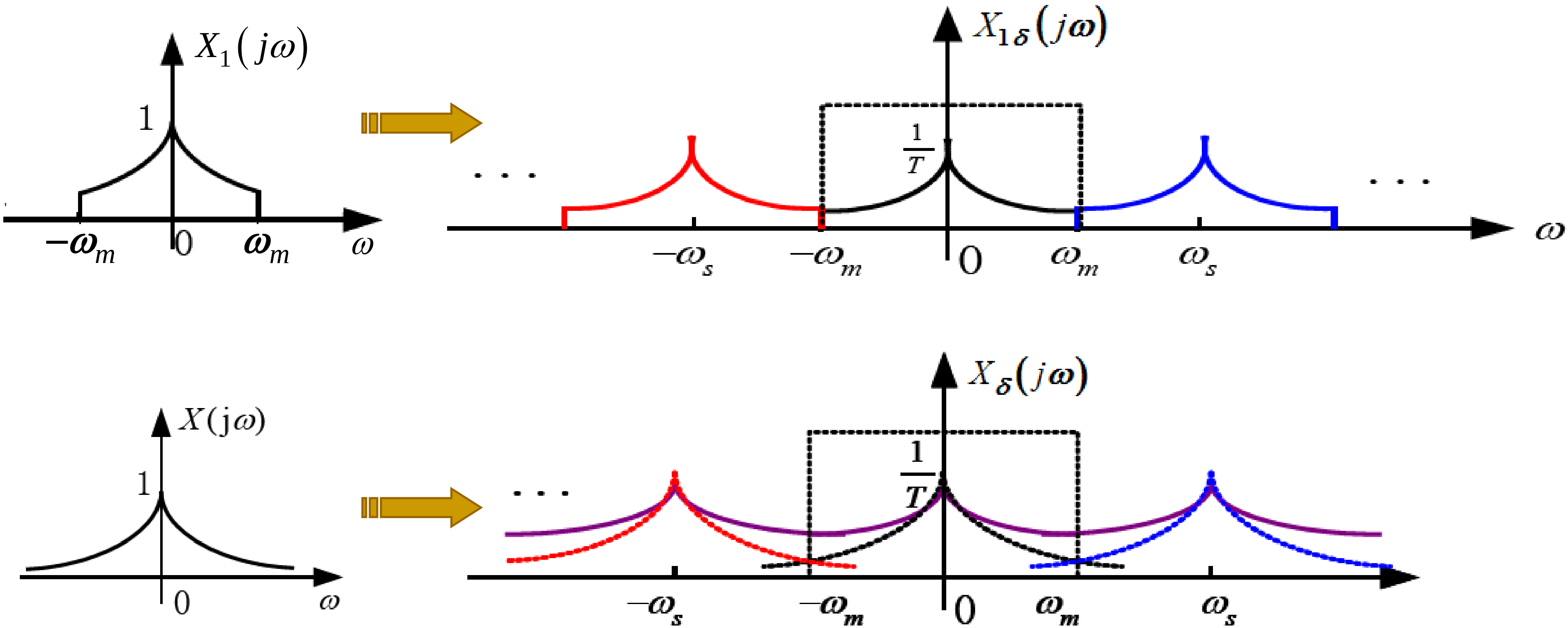
Practical Applications of Sampling Theorem

- **If the practical signal is not a band-limited signal, an anti-aliasing(抗混叠) filter is used before sampling.**
- **Anti-aliasing filter can pass frequency components below $\omega_s/2$ (ω_m) without distortion and suppress any frequency components above $\omega_s/2$.**



Practical Applications of Sampling Theorem

- **Comparison between aliasing error and truncation error**

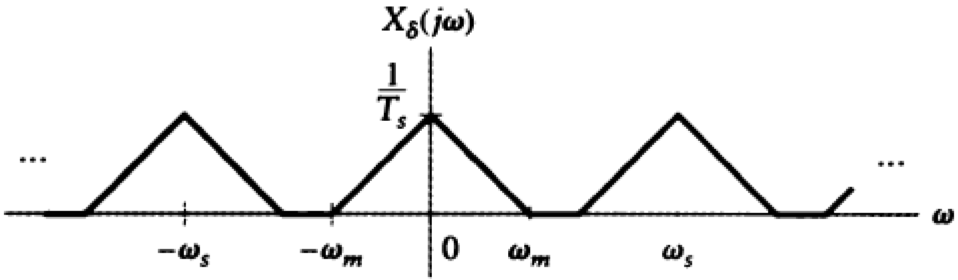


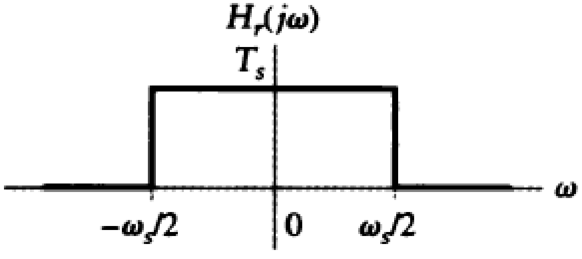
Sampling Theorem - Conclusions

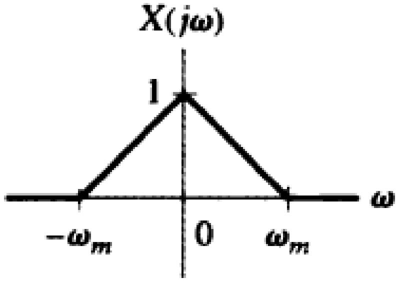
- **Sampling in time domain will result in periodicity in frequency domain. The spectra of DT signal $x[k]$ is the periodicity of CT signal $x(t)$*'s* spectra.**
- **Nyquist sampling theory: if a CT signal is band-limited, and it is sampled with sampling frequency up to the threshold, the time function can be recovered perfectly from the samples. The sampling frequency is no less than 2 times bandwidth of the CT signal.**
- **Engineering applications of Sampling: if the practical signal is not a band-limited signal, $x(t)$ can be recovered by passing the CT signal through an anti-aliasing filter with cutoff frequency at $\omega = \omega_m$.**

Reconstruction of Continuous-time Signals from Samples

- **Ideal reconstruction**







$$X_\delta(j\omega) = \frac{1}{T_s}\sum_{k=-\infty}^{\infty} X(j\omega - jk\omega_s)$$

$$H_r(j\omega) = \begin{cases} T_s, & |\omega| \le \omega_s/2 \\ 0, & |\omega| > \omega_s/2 \end{cases}$$

$$X(j\omega) = X_\delta(j\omega)H_r(j\omega)$$

Reconstruction of Continuous-time Signals from Samples

- **Ideal reconstruction**

$$H_r(j\omega) = \begin{cases} T_s, & |\omega| \le \omega_s/2 \\ 0, & |\omega| > \omega_s/2 \end{cases}$$

$$\Rightarrow \quad h_r(t) = \frac{T_s \sin(\omega_s t/2)}{\pi t}$$

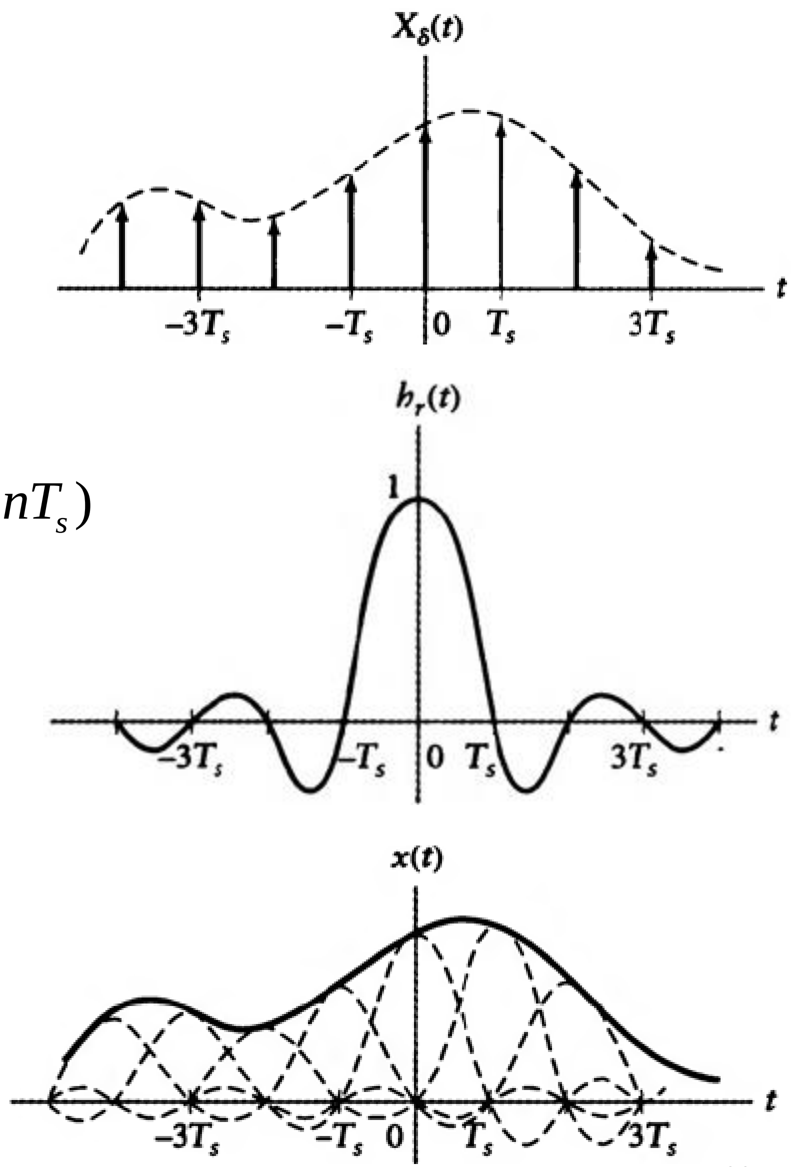
$$x(t) = x_\delta(t) * h_r(t) = h_r(t) * \sum_{n=-\infty}^{\infty} x[n]\delta(t - nT_s)$$

$$= \sum_{n=-\infty}^{\infty} x[n] h_r(t - nT_s)$$

$$\boxed{x(t) = \sum_{n=-\infty}^{\infty} x[n] \operatorname{sinc}(\omega_s(t - nT_s)/(2\pi))}$$

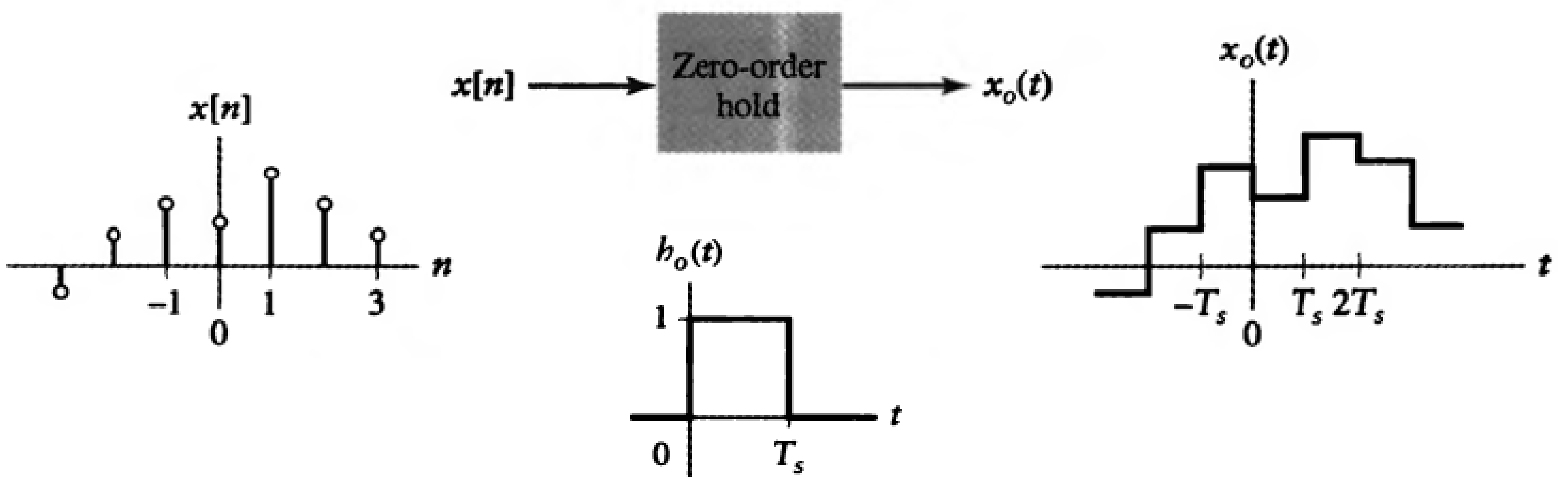
~ Ideal band-limited interpolation

Cannot be implemented!



Reconstruction of Continuous-time Signals from Samples

- **Zero-order Hold (零阶保持)**



$$h_o(t) \xleftrightarrow{FT} H_o(j\omega) = 2e^{-j\omega T_s/2}\frac{\sin(\omega T_s/2)}{\omega}$$

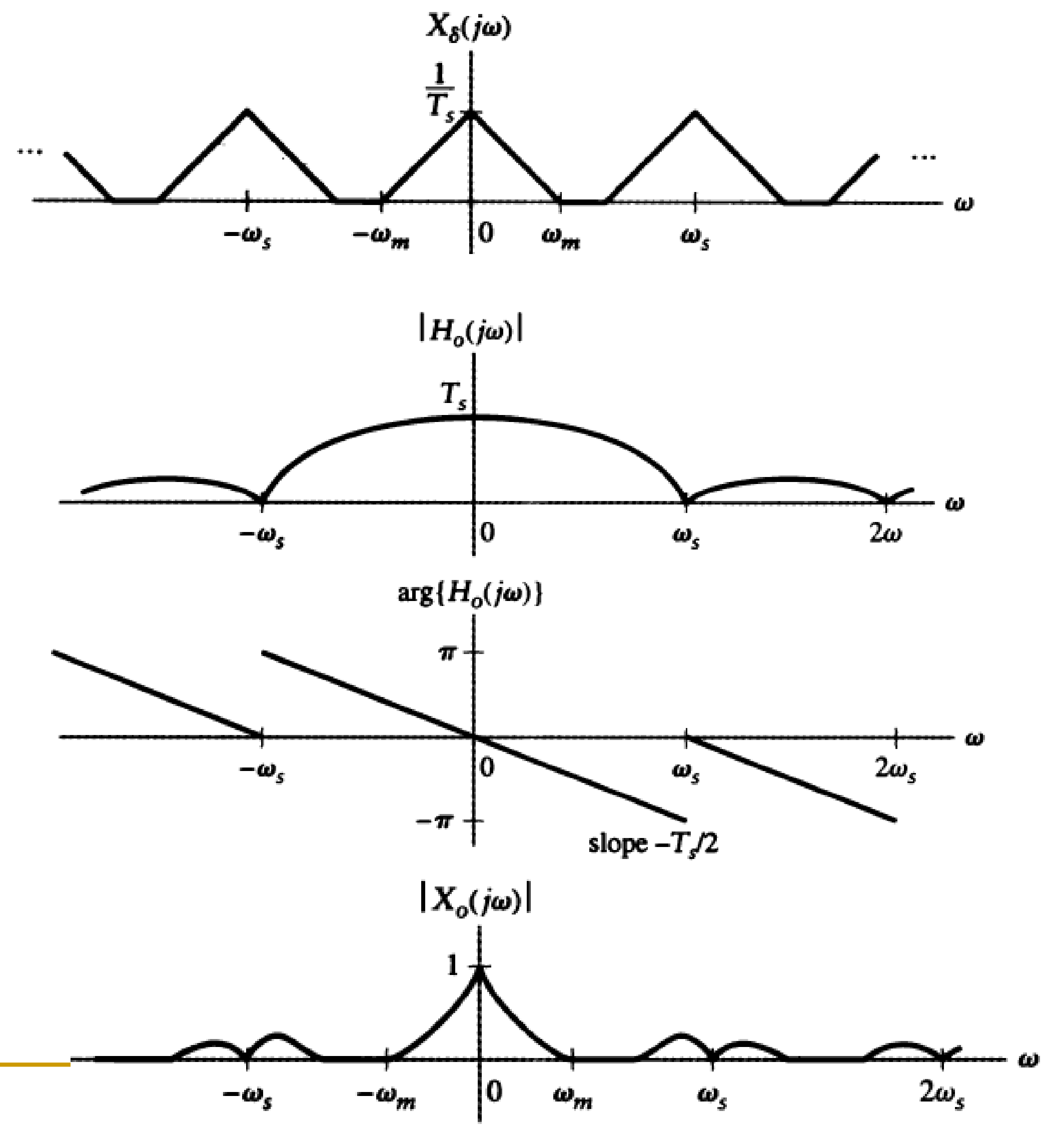
$$X_o(j\omega) = H_o(j\omega)X_\delta(j\omega)$$

Reconstruction of Continuous-time Signals from Samples

- **Zero-order Hold**

$$H_o(j\omega) = 2e^{-j\omega T_s/2}\frac{\sin(\omega T_s/2)}{\omega}$$

$$X_o(j\omega) = H_o(j\omega)X_\delta(j\omega)$$



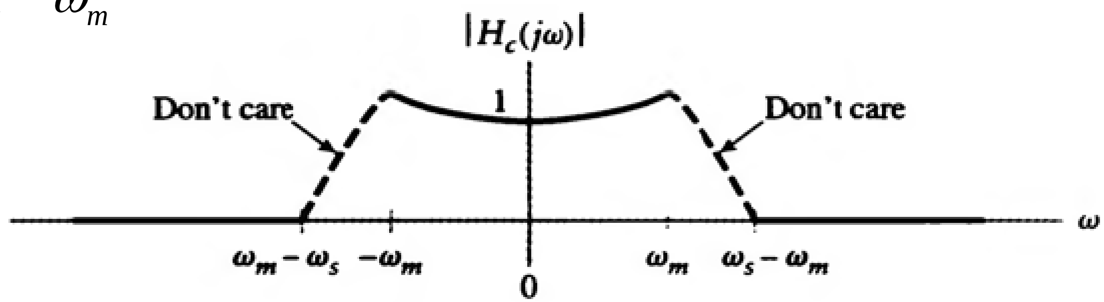
- **A linear phase shift corresponding to a time delay of $T_s/2$ seconds.**
- **A distortion of the portion of $X_\delta(j\omega)$ between $-\omega_m$ and ω_m.**
- **Distorted and attenuated versions of the images of $X(j\omega)$, centered at nonzero multiples of ω_s**

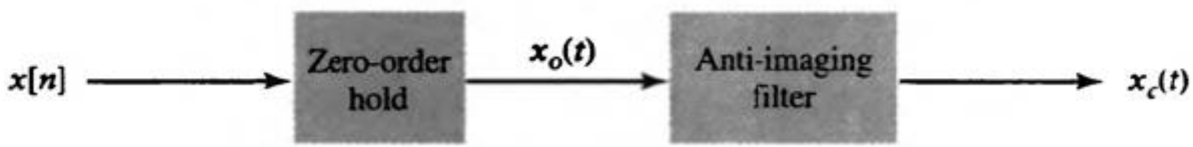
Reconstruction of Continuous-time Signals from Samples

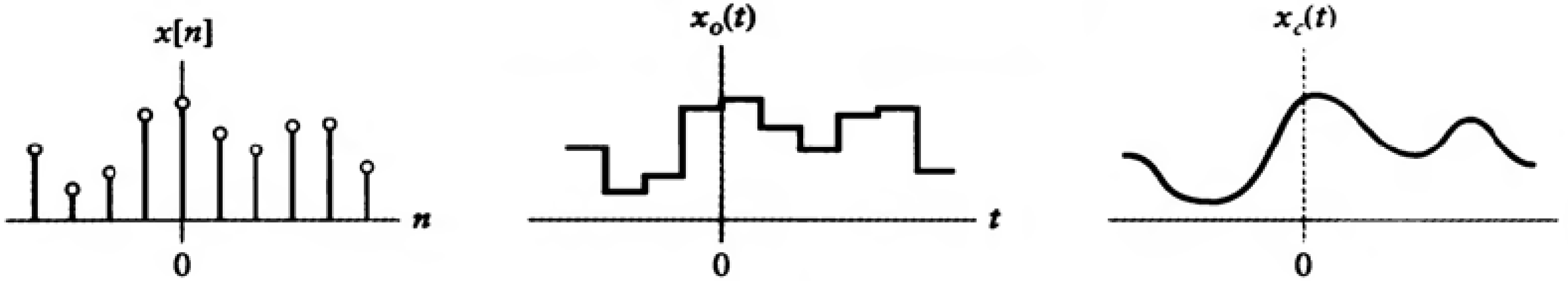
- Modification 2&3 may be eliminated by passing $x_o(t)$ through a CT compensation filter with frequency response:

$$H_c(j\omega)=\begin{cases}\dfrac{\omega T_s}{2\sin(\omega T_s/2)}, & |\omega|<\omega_m \\ 0, & |\omega|>\omega_s-\omega_m\end{cases} \Rightarrow |H_o(j\omega)||H_c(j\omega)|=T_s,\quad |\omega|<\omega_m$$

~ *Anti-imaging filter*
(反像滤波器)

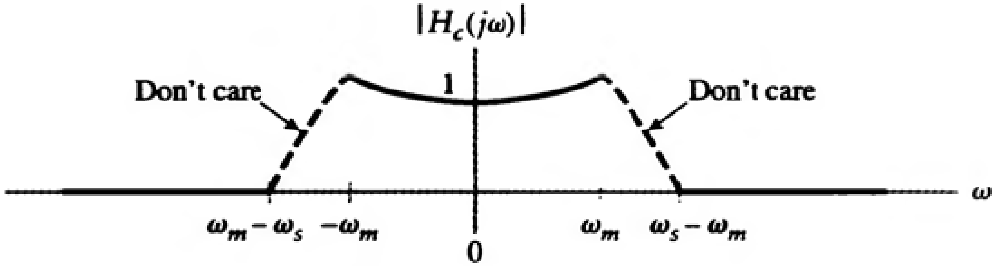






Reconstruction of Continuous-time Signals from Samples

- **Oversampling(过抽样):** increase the effective sampling rate of the DT signal prior to the zero-order hold to relax the requirements on the anti-imaging filter.

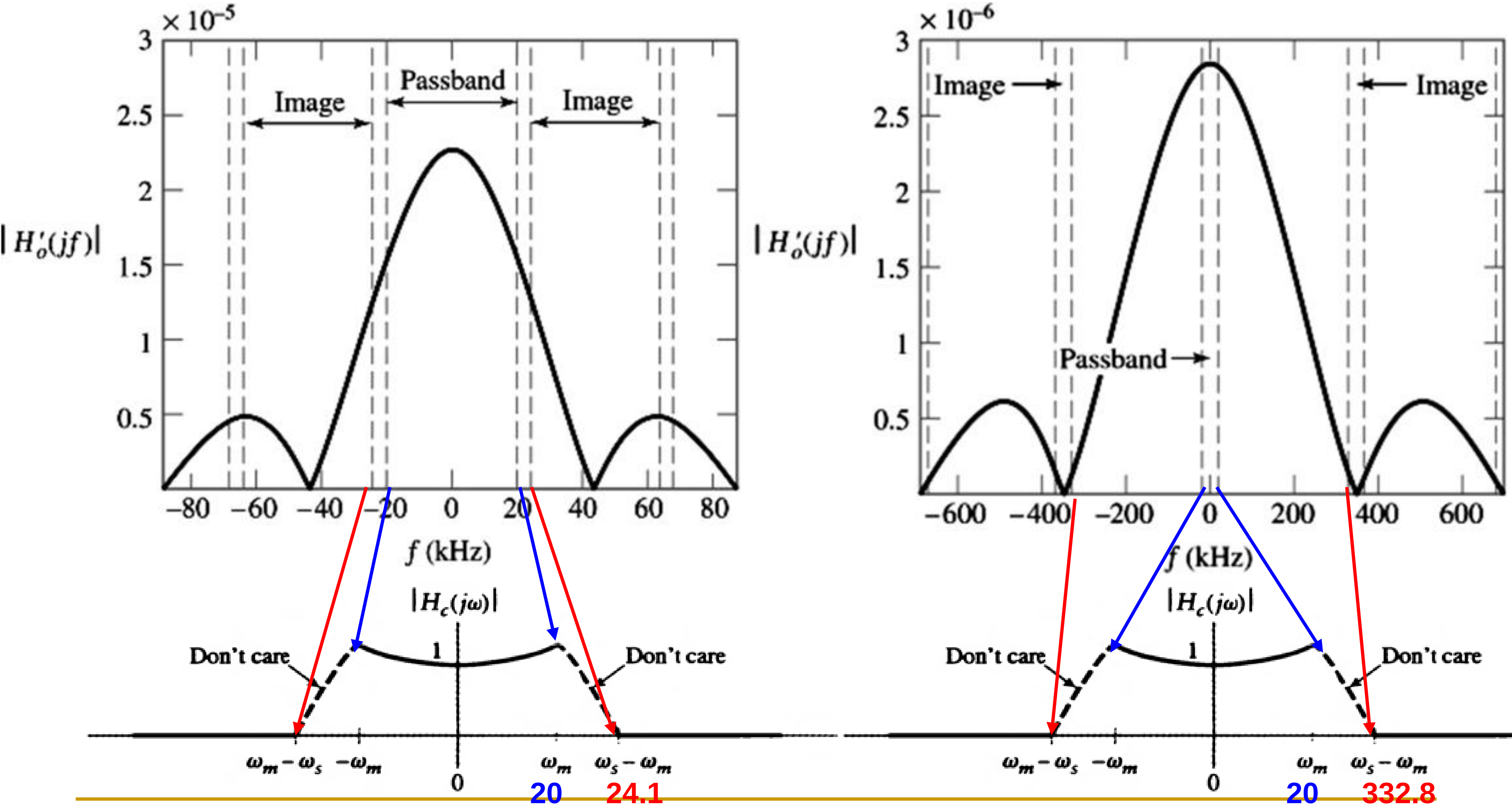


Example 4.13 ***Oversampling in CD Players***

Assume that the maximum signal frequency is f_m=20kHz. Consider two cases: (a) reconstruction using the standard digital audio rate of $1/T_{s1}$=44.1kHz, and (b) reconstruction using eight-times oversampling, for an effective sampling rate of $1/T_{s2}$=352.8kHz. In each case, determine the constraints on the magnitude response of an anti-imaging filter so that the overall magnitude response of the zero-order hold reconstruction system is between 0.99 and 1.01 in the signal passband and the images of the original signal's spectrum centered at multiples of the sampling frequency are attenuated by a factor of 10^{-3} or more.

Reconstruction of Continuous-time Signals from Samples

$1/T_{s1}$=44.1kHz $\qquad$ $1/T_{s2}$=352.8kHz



$$\left|H_o(j\omega)\right|\left|H_c(j\omega)\right| = T_s, \quad \left|\omega\right| < \omega_m$$

Reconstruction of Continuous-time Signals from Samples

- The passband constraint is $0.99 < |H_o(jf)||H_c(jf)| < 1.01$, i.e.

$$\frac{0.99}{|H_o(jf)|} < |H_c(jf)| < \frac{1.01}{|H_o(jf)|}, \quad -20\,\text{kHz} < f < 20\,\text{kHz}$$

Case (a): $1.4257 < T_{s1}|H_c(jf_m)| < 1.4545, \quad f_m = 20\,\text{kHz}$

Case (b): $0.9953 < T_{s2}|H_c(jf_m)| < 1.0154, \quad f_m = 20\,\text{kHz}$

- The image-rejection constraint implies that $|H_o(jf)||H_c(jf)| < 10^{-3}$ for all frequencies at which images are present.

Case (a): $T_{s1}|H_c(jf)| < 0.0017, \quad f > 24.1\,\text{kHz}$

Case (b): $T_{s2}|H_c(jf)| < 0.0167, \quad f > 332.8\,\text{kHz}$

Oversampling not only increases transition width by a factor of almost 80, but also relaxes the stopband attenuation constraint by a factor of more than 10.

Summary

- Applications of Fourier Representations to Mixed Signal Classes
  - Fourier Transform Representations of Periodic Signals
  - Convolution and Multiplication with Mixtures of Periodic and Nonperiodic Signals
  - Fourier Transform Representation of Discrete-Time Signals
  - Sampling

- Reference in textbook: 4.1~4.6
- Homework: 4.18(a,c), 4.29(a,c,d); 4.30