Review: 大学物理D（上）

Mechanics

Particles

Newton’s Second Law

Electromagnetism

Fields

Maxwell's Equations

Oscillations & Waves

Mechanical

Electromagnetic

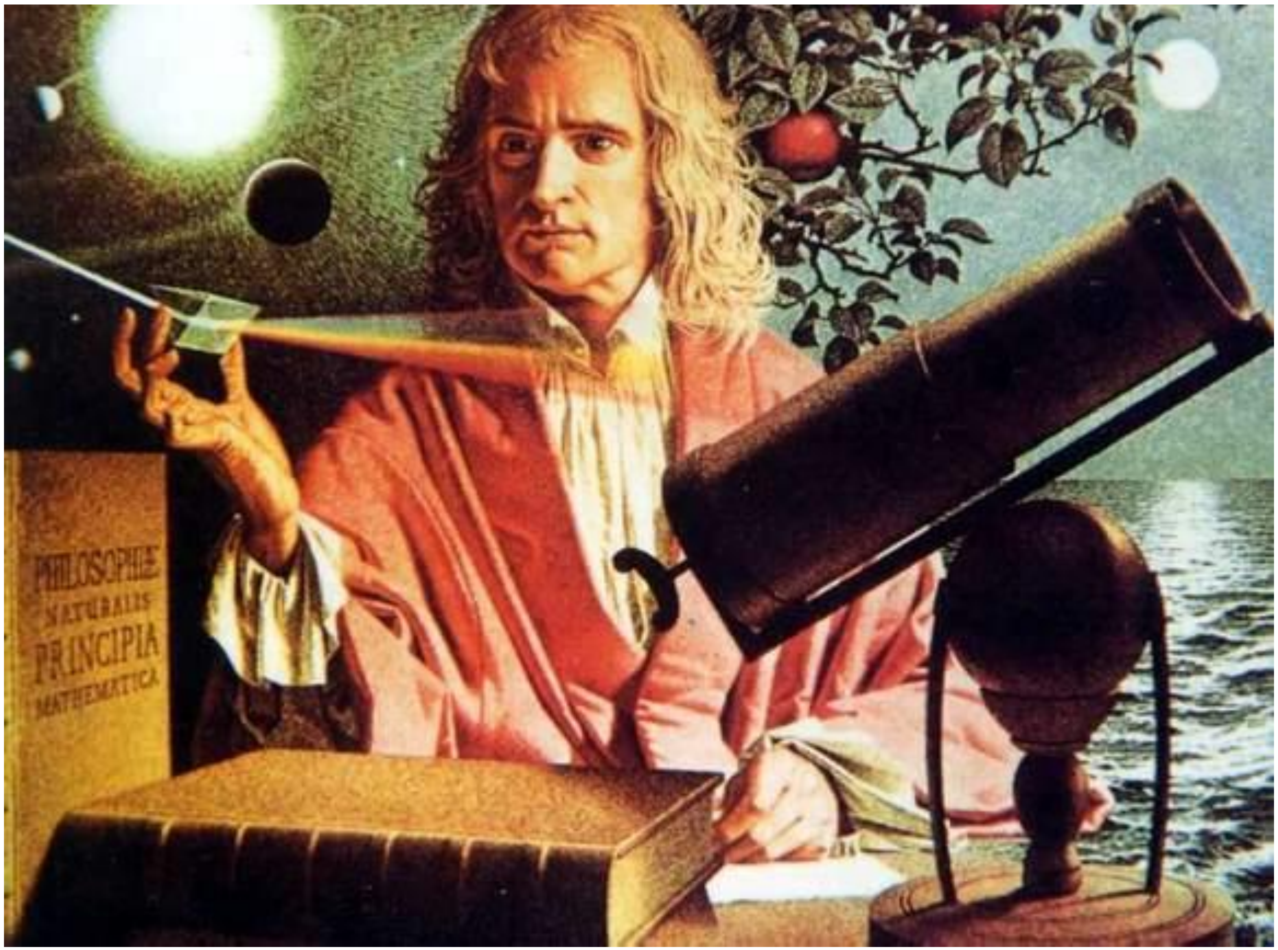
Electromagnetic Waves

↓

Wave Optics

- Interference
- Diffraction
- Polarization

We all know light, but we don't really know what is light.



§ 1 The Nature of Light

- **What is light ?**
- **Models**
  - **Physicists devise various conceptual models to understand nature.**
  - **The worth of a model lies not in whether it is “true” but in whether it is useful.**
  - **A good model not only is consistent with and explains observations, but also predicts what may happen.**

The Nature of Light

- **What is light —— the particle model or the wave model?**
  - **Huygens' wave model:**

    Light is a wave that has the characteristic properties of interference and diffraction.

  - **Newton's particle model:**

    Light is a stream of particles (called corpuscles) emitted by light sources.

  - **Maxwell's viewpoint:**

    Light is a form of high-frequency electromagnetic waves.

    No need to travel through a medium. Light travel through vacuum with the same speed:

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 3\times 10^8\,\text{m/s}$$

The Nature of Light

- **In modern viewpoint: wave-particle duality**
  - **In some cases light acts like a wave and in others it acts like a particle.**
  - **The propagation of light is best described by the wave model, but understanding emission and absorption requires the particle approach.**

Albert Einstein
(1879 ~ 1955)

Louis Victor de Broglie
(1892 ~ 1987)

Erwin Schrödinger
(1887 ~ 1961)

Review: Maxwell's Equations

$$\oiint_S \vec{E}\cdot d\vec{A} = \frac{q_{\mathrm{encl}}}{\varepsilon_0}$$

Gauss's law for electricity

$$\oiint_S \vec{B}\cdot d\vec{A} = 0$$

Gauss's law for magnetism

$$\oint_L \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt} = -\iint_S \frac{\partial \vec{B}}{\partial t}\cdot d\vec{A}$$

Faraday's law of induction

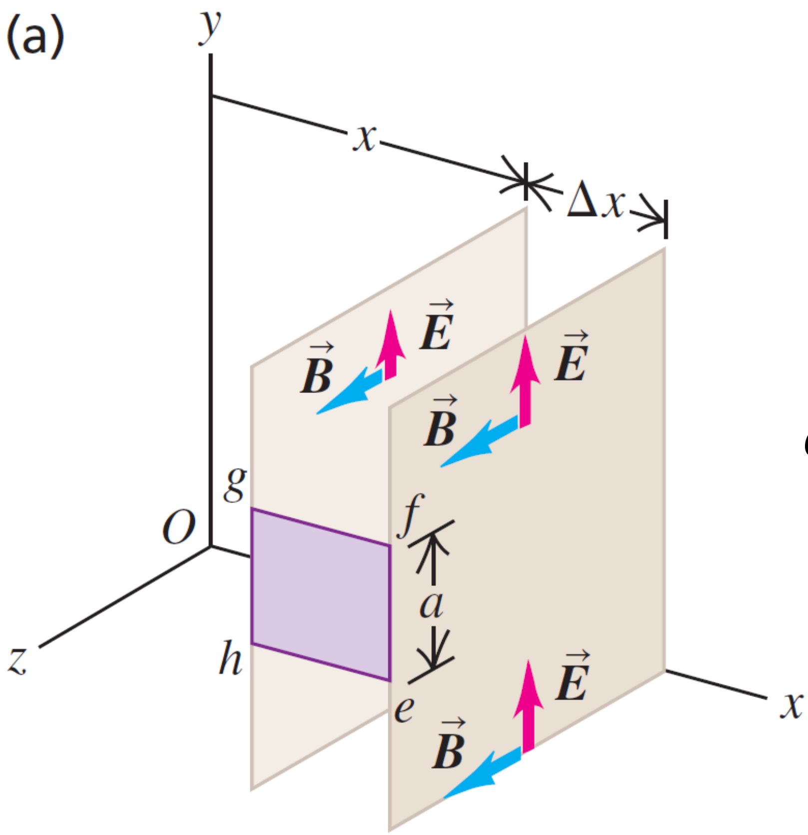
$$\oint_L \vec{B}\cdot d\vec{l} = \mu_0 I_{\mathrm{encl}} + \varepsilon_0\mu_0 \frac{d\Phi_E}{dt} = \mu_0 I_{\mathrm{encl}} + \varepsilon_0\mu_0 \iint_S \frac{\partial \vec{E}}{\partial t}\cdot d\vec{A}$$

Ampére-Maxwell law

Maxwell's equations and Lorentz force give the fundamental relations of electromagnetism! They are fundamental in the sense that Newton's three laws are for mechanics.

Derivation of the electromagnetic wave equation

Consider a plane wave.



$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt},$$

Faraday's law applied to a rectangle with height a and width Δx parallel to the xy-plane.

$$\oint \vec{E}\cdot d\vec{l} = E_y(x+\Delta x,t)a - E_y(x,t)a$$

$$= a[E_y(x+\Delta x,t) - E_y(x,t)]$$

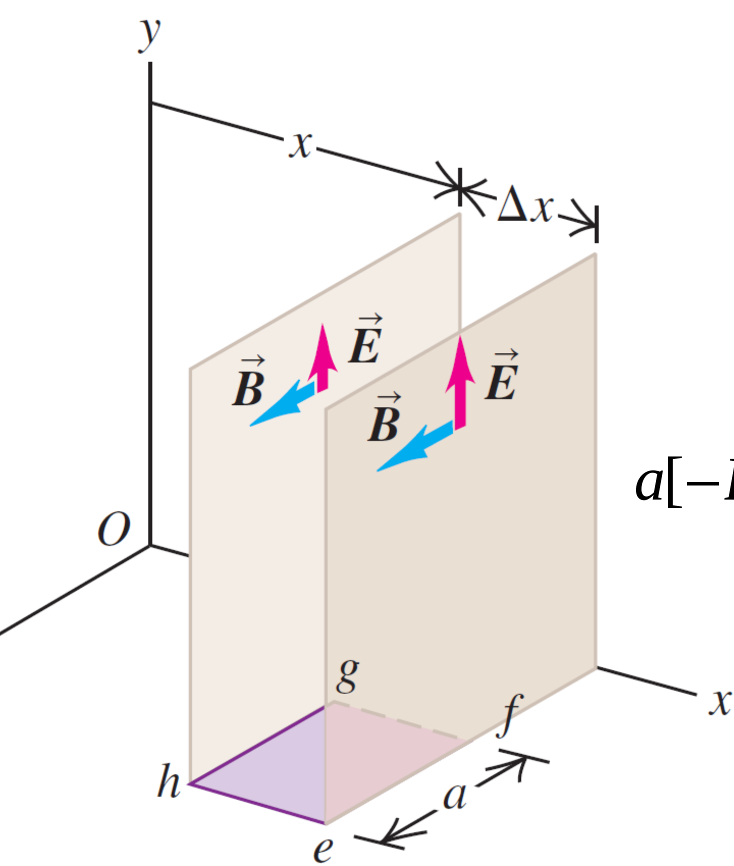
$$-\frac{d\Phi_B}{dt} = -\frac{\partial B_z(x,t)}{\partial t}a(\Delta x)$$

$$a[E_y(x+\Delta x,t) - E_y(x,t)] = -\frac{\partial B_z(x,t)}{\partial t}a(\Delta x)$$

$$\frac{E_y(x+\Delta x,t) - E_y(x,t)}{\Delta x} = -\frac{\partial B_z(x,t)}{\partial t}$$

$$\frac{\partial E_y(x,t)}{\partial x} = -\frac{\partial B_z(x,t)}{\partial t}$$

(a)



$$\oint \vec{B}\cdot d\vec{l} = -B_z(x+\Delta x,t)a + B_z(x,t)a$$

$$= a[-B_z(x+\Delta x,t) + B_z(x,t)]$$

$$\varepsilon_0\mu_0 \frac{d\Phi_E}{dt} = \varepsilon_0\mu_0 \frac{\partial E_y(x,t)}{\partial t} a(\Delta x)$$

$$a[-B_z(x+\Delta x,t) + B_z(x,t)] = \varepsilon_0\mu_0 \frac{\partial E_y(x,t)}{\partial t} a(\Delta x)$$

$$\frac{-B_z(x+\Delta x,t) + B_z(x,t)}{\Delta x} = \varepsilon_0\mu_0 \frac{\partial E_y(x,t)}{\partial t}$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}} + \varepsilon_0\mu_0 \frac{d\Phi_E}{dt},$$

$$-\frac{\partial B_z(x,t)}{\partial x} = \varepsilon_0\mu_0 \frac{\partial E_y(x,t)}{\partial t}$$

Ampere's law applied to a rectangle with height a and width Δx parallel to the xz-plane.

Derivation of the electromagnetic wave equation

$$\frac{\partial E_y(x,t)}{\partial x} = -\frac{\partial B_z(x,t)}{\partial t}, \qquad -\frac{\partial B_z(x,t)}{\partial x} = \varepsilon_0 \mu_0 \frac{\partial E_y(x,t)}{\partial t}$$

$$\frac{\partial^2 E_y(x,t)}{\partial x^2} = -\frac{\partial^2 B_z(x,t)}{\partial x \partial t}, \qquad -\frac{\partial^2 B_z(x,t)}{\partial x \partial t} = \varepsilon_0 \mu_0 \frac{\partial^2 E_y(x,t)}{\partial t^2}$$

$$\frac{\partial^2 E_y(x,t)}{\partial x^2} = \varepsilon_0 \mu_0 \frac{\partial^2 E_y(x,t)}{\partial t^2}$$

$$\frac{\partial^2 B_z(x,t)}{\partial x^2} = \varepsilon_0 \mu_0 \frac{\partial^2 B_z(x,t)}{\partial t^2}$$

Electromagnetic wave equation in vacuum!

$$\frac{1}{v^2} = \varepsilon_0 \mu_0, \qquad v = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

The important features of electromagnetic waves

- **The wave equation:**

From Maxwell's equations, we can obtain the wave equation for a wave which propagates in x-direction

$$\frac{\partial^2 E}{\partial x^2} = \varepsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2}, \qquad \frac{\partial^2 B}{\partial x^2} = \varepsilon_0 \mu_0 \frac{\partial^2 B}{\partial t^2}$$

- **The wave speed:**

Generally, the wave equation

$$\frac{\partial^2 \Psi(x,t)}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \Psi(x,t)}{\partial t^2}$$

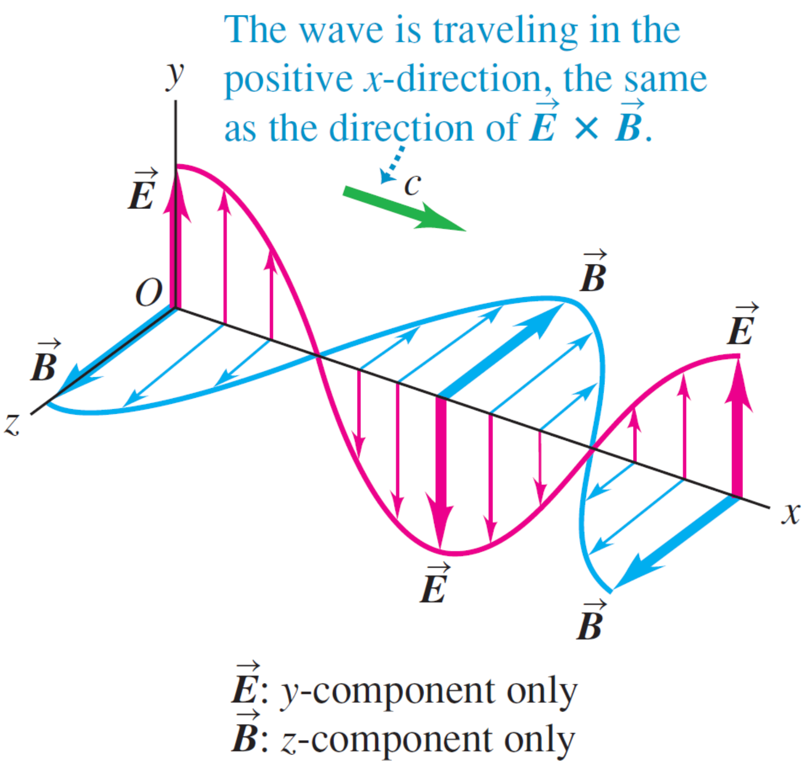
$$\boxed{v = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 2.997 \times 10^8 \ \text{m/s} = c}$$

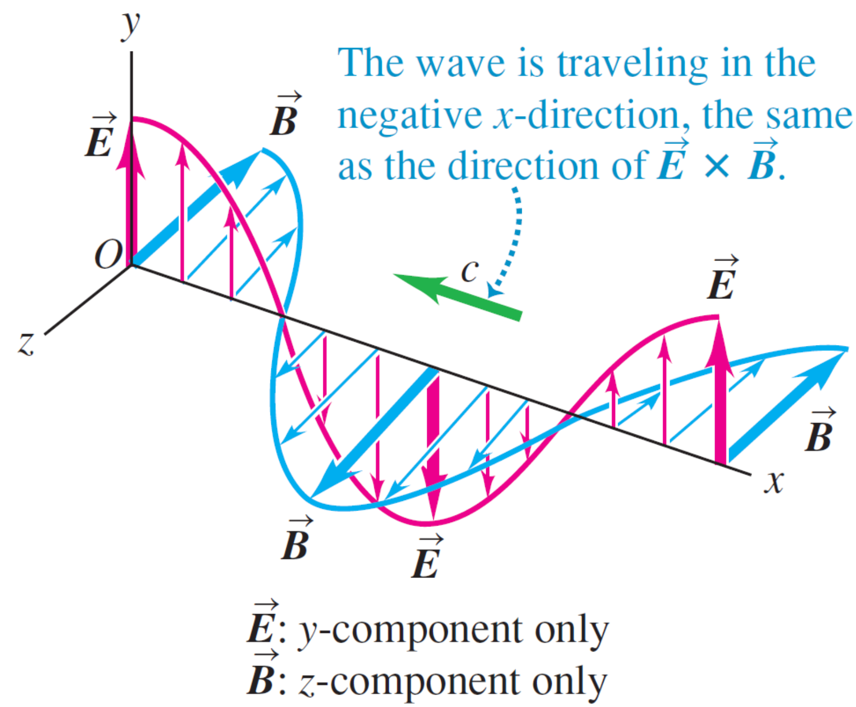
This speed is precisely the same as the speed of light in empty space.

The important features of electromagnetic waves

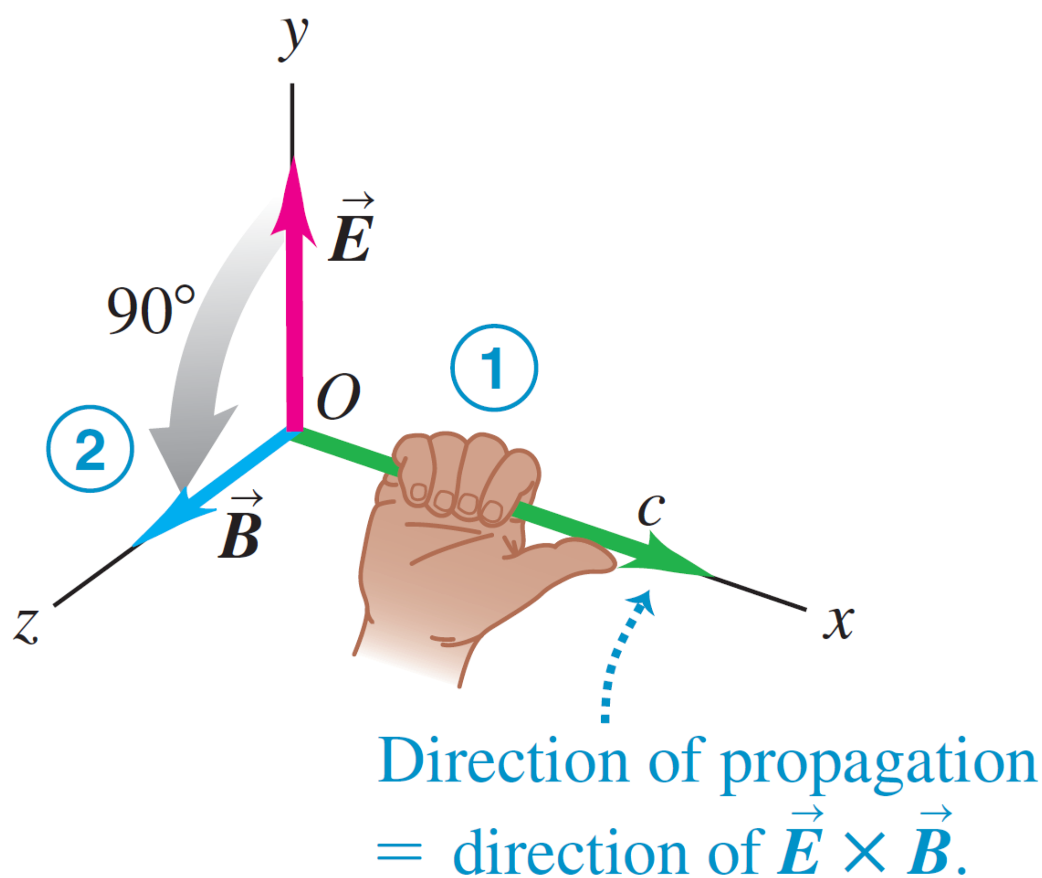
- **The sinusoidal plane wave is the simplest solution of the wave equations**

$$E = E_{max}\cos(\omega t - kx), \qquad B = B_{max}\cos(\omega t - kx)$$





- **Right-hand rule for an electromagnetic wave**



The important features of electromagnetic waves

$$E = E_{\max}\cos(\omega t - kx), \qquad B = B_{\max}\cos(\omega t - kx)$$

- **The wave is transverse.**

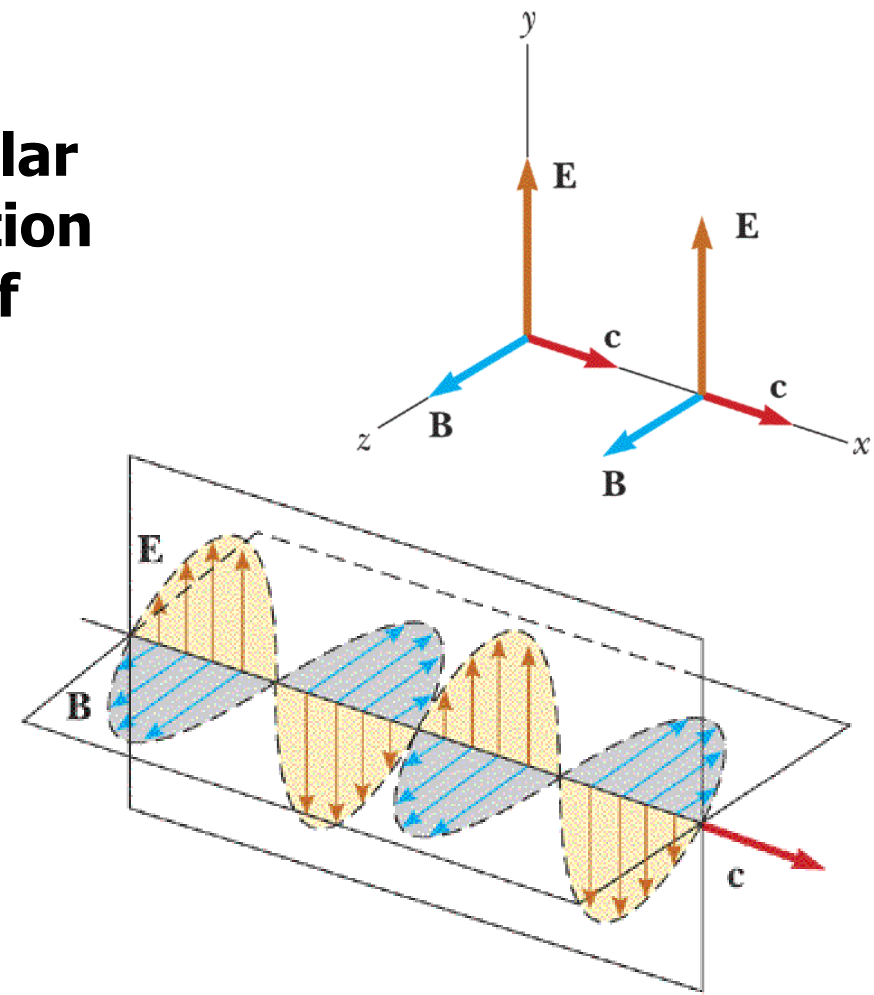
Both $\vec{E}$ and $\vec{B}$ are perpendicular to each other, and to the direction of propagation. The direction of propagation is $\vec{E}\times\vec{B}$

- **$\vec{E}$ and $\vec{B}$ are in phase, and has a definite ratio**

$$\frac{E}{B} = \frac{E_{\max}}{B_{\max}} = c,$$

$$\boxed{E = cB}$$

$$\sqrt{\varepsilon_0}E = \frac{B}{\sqrt{\mu_0}}$$

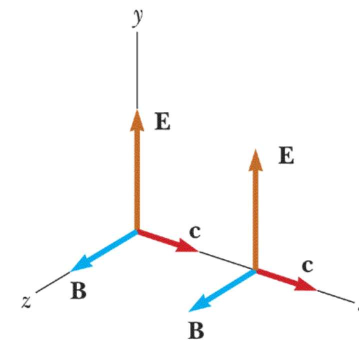


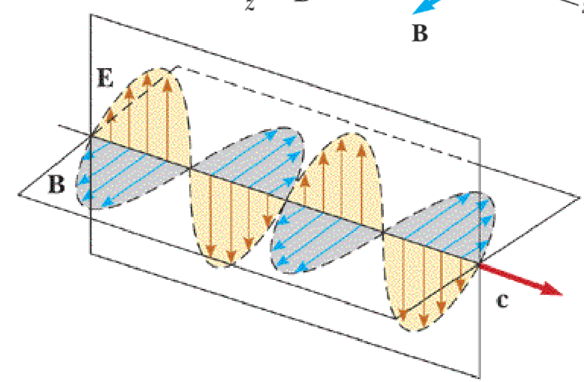
The important features of electromagnetic waves

- **Poynting vector: energy current density vector.**

The total energy density:

$$u = u_E + u_B = \frac{1}{2}\varepsilon_0 E^2 + \frac{B^2}{2\mu_0} = \frac{EB}{\mu_0 c}$$

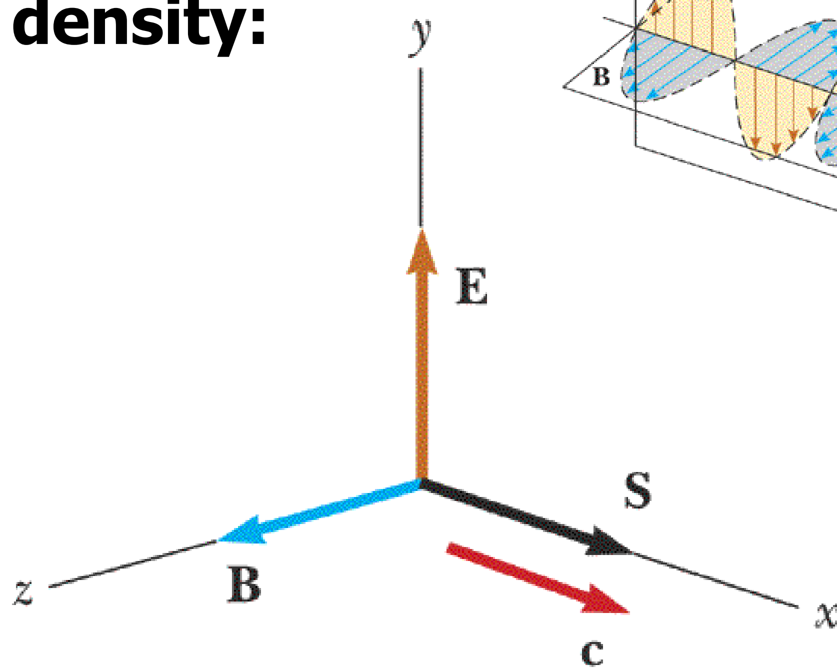




The energy current density:

$$S = uc = \frac{EB}{\mu_0}$$

$$\boxed{\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}}$$

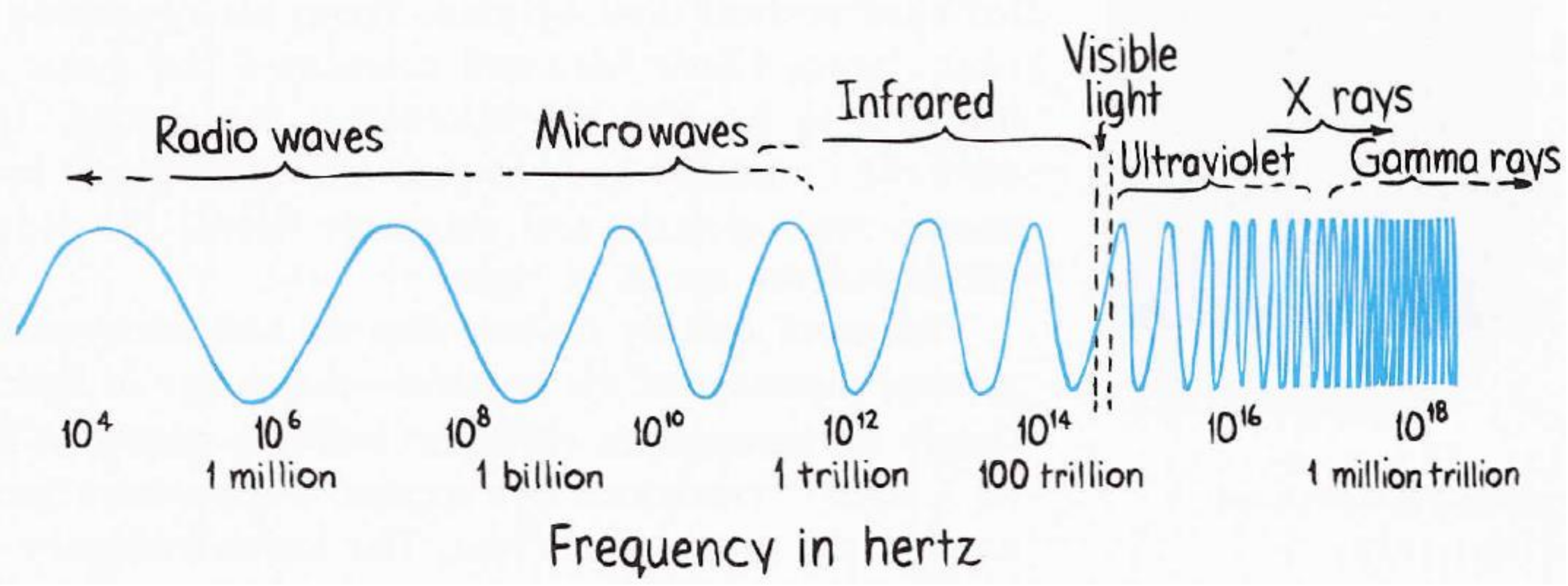


§ 2 The Electromagnetic Spectrum

- **A broad range of different kinds of radiation from a variety of sources.**

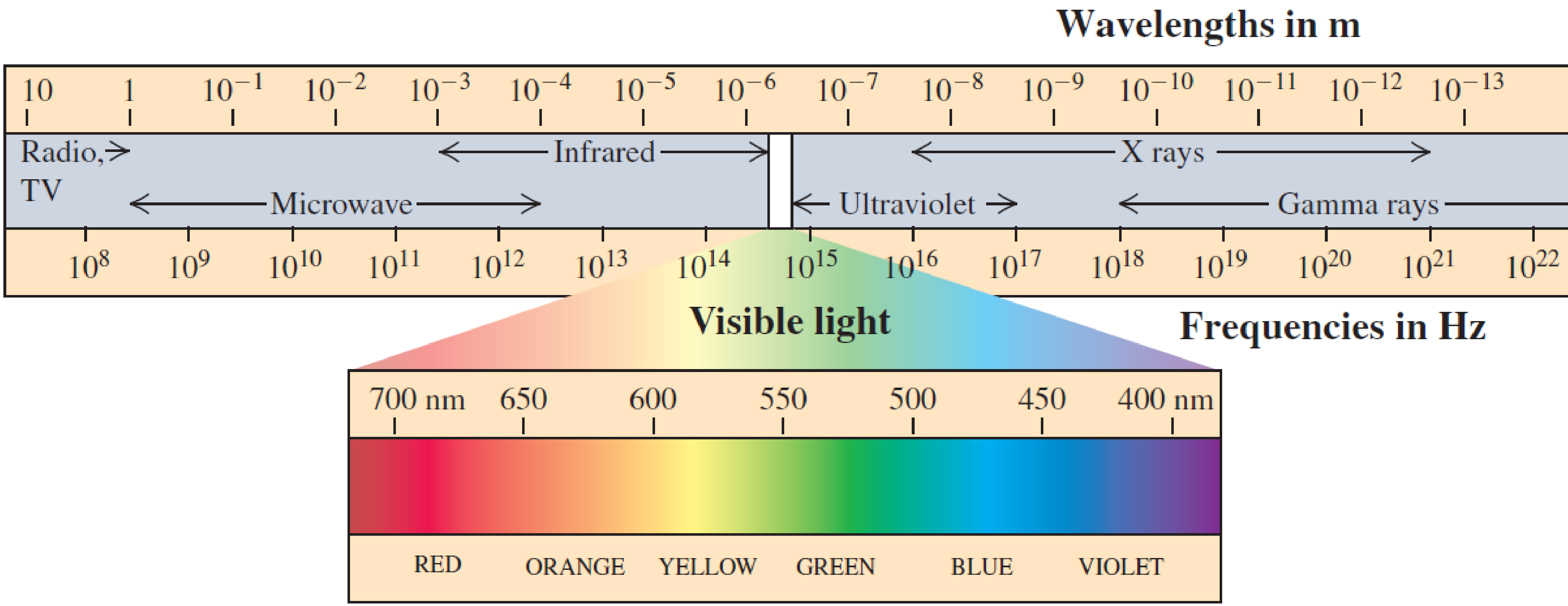
  Even though these radiations differ greatly in their properties, in their means of production, and in the ways we observe them, they share other features in common: they all can be described in term of electric and magnetic fields, and they all travel through vacuum with the same speed *c*.

- **They differ only in wavelength or frequency.**



The Electromagnetic Spectrum

$$\lambda f = c$$



The Visible Light

color	λ/nm	f /Hz	λ_{centre}/nm
red	760~622	$3.9\times10^{14} \sim 4.8\times10^{14}$	660
orange	622~597	$4.8\times10^{14} \sim 5.0\times10^{14}$	610
yellow	597~577	$5.0\times10^{14} \sim 5.4\times10^{14}$	570
green	577~492	$5.4\times10^{14} \sim 6.1\times10^{14}$	540
cyan	492~470	$6.1\times10^{14} \sim 6.4\times10^{14}$	480
blue	470~455	$6.4\times10^{14} \sim 6.6\times10^{14}$	460
violet	455~400	$6.6\times10^{14} \sim 7.5\times10^{14}$	430

§ 3 Optical Intensity

To describe energy flow of electromagnetic wave (energy current density), we use Poynting vector

$$\vec{S} = \frac{1}{\mu}\vec{E}\times\vec{B}$$

At optical frequencies (≈10^{15}Hz), S is an extremely rapid varying function of time, its instantaneous value would be an impractical quantity to measure directly.

Generally we must employ an average procedure.

Any optical device, such as a photocell, a film plate, or the retina (视网膜) of a human eye, can only sense the light energy during some finite interval of time.

Optical intensity

- **We define the light intensity as the average value of S over a period of electromagnetic wave T :**

$$I \equiv < S > = \frac{1}{T}\int_{-T/2}^{T/2} S\,dt = \frac{1}{T}\int_{-T/2}^{T/2} \frac{EB}{\mu}\,dt$$

- **Recall that for a electromagnetic wave** $\sqrt{\varepsilon}E = \frac{B}{\sqrt{\mu}}$

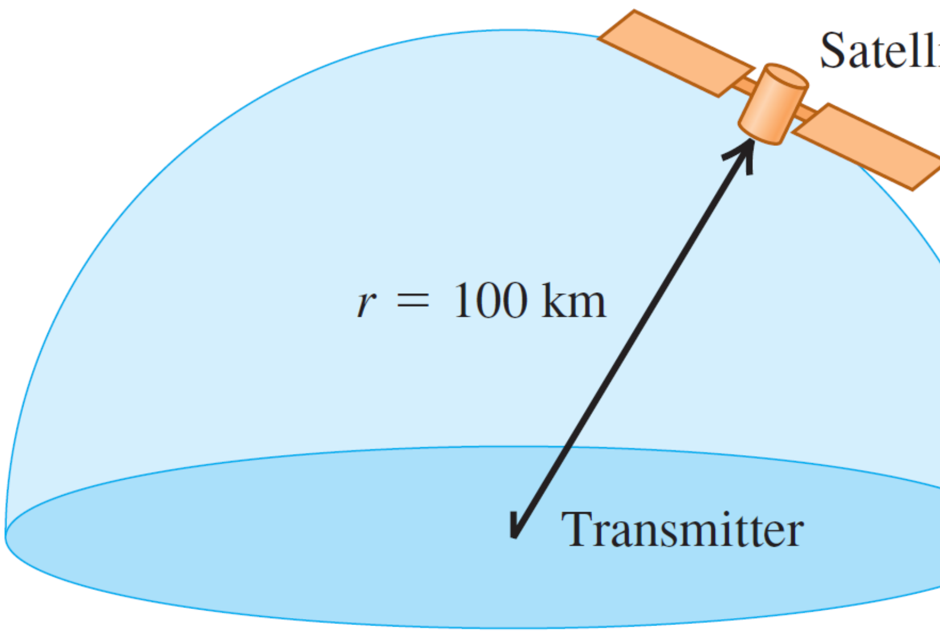
$$\frac{EB}{\mu} = \frac{\sqrt{\varepsilon\mu}}{\mu}E^2 = \frac{n}{c}\frac{1}{\mu}E^2 = \frac{n}{c\mu_0}E^2,$$ **for optical medium** $\mu_r \approx 1$

$$v = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{1}{\sqrt{\varepsilon_0\varepsilon_r\mu_0\mu_r}} = \left(\frac{1}{\sqrt{\varepsilon_0\mu_0}}\right)\frac{1}{\sqrt{\varepsilon_r\mu_r}} = \frac{c}{n}$$

$$I = \frac{n}{c\mu_0}\frac{1}{T}\int_{-T/2}^{T/2} E_0^2\cos^2(\omega t - kz + \phi)dt = \frac{1}{2}\frac{n}{c\mu_0}E_0^2 \propto E_0^2$$

Ex. energy in a sinusoidal wave

A radio station on the earth's surface emits a sinusoidal wave with average total power 50 kW. Assuming that the transmitter radiates equally in all directions above the ground (which is unlikely in real situations), find the electric-field and magnetic-field amplitudes E_{max} and B_{max} detected by a satellite 100 km from the antenna.

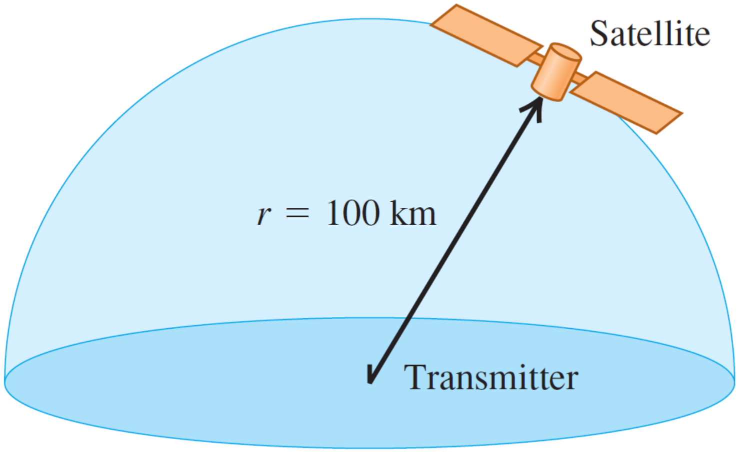


Ex. Energy in a sinusoidal wave

Solution:

$$I=\frac{P}{A}=\frac{P}{2\pi R^2}=\frac{5.00\times10^4\ \text{W}}{6.28\times(1.00\times10^5\ \text{m})^2}=7.96\times10^{-7}\ \text{W/m}^2$$

$$I=\frac{1}{2}\frac{n}{c\mu_0}E_0^2,\quad n=1$$



$$E_{\text{max}}=\sqrt{2\mu_0 cI}$$

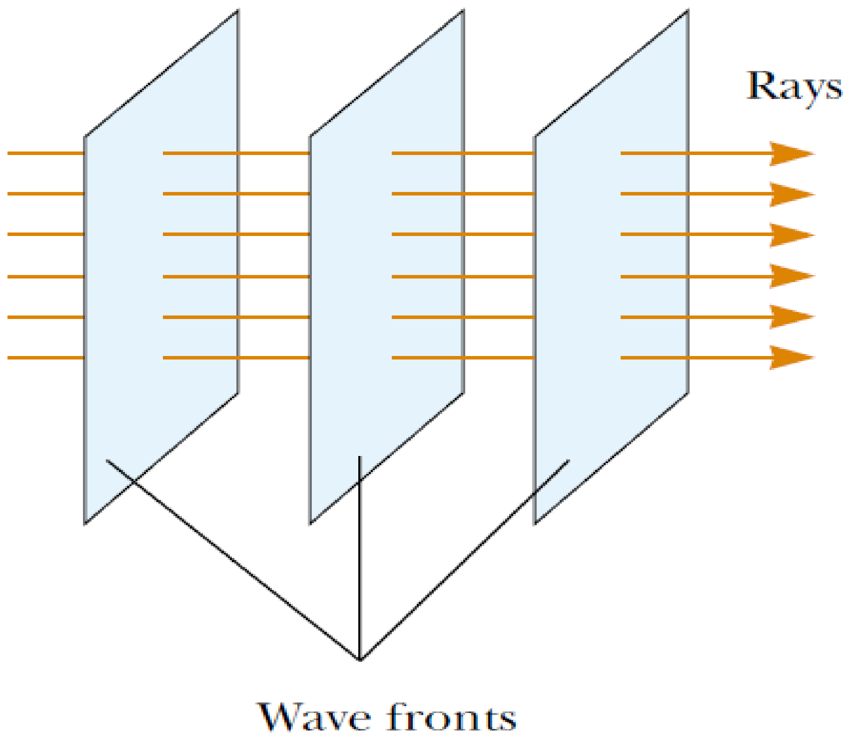
$$=\sqrt{2(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(3.00\times10^8\ \text{m/s})(7.96\times10^{-7}\ \text{W/m}^2)}$$

$$=2.45\times10^{-2}\ \text{V/m}$$

$$B_{\text{max}}=\frac{E_{\text{max}}}{c}=8.17\times10^{-11}\ \text{T}$$

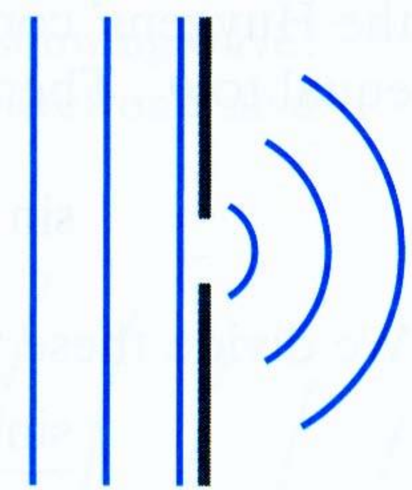
§ 4 Huygens' Principle

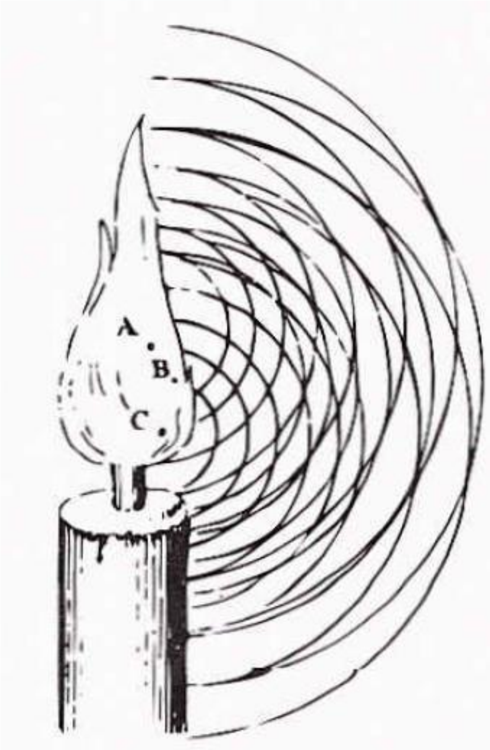
- A **geometric construction** for using knowledge of an earlier **wave front** to determine the position of a new wave front at some instant.



Christian Huygens, Dutch Physicist and Astronomer (1629–1695)

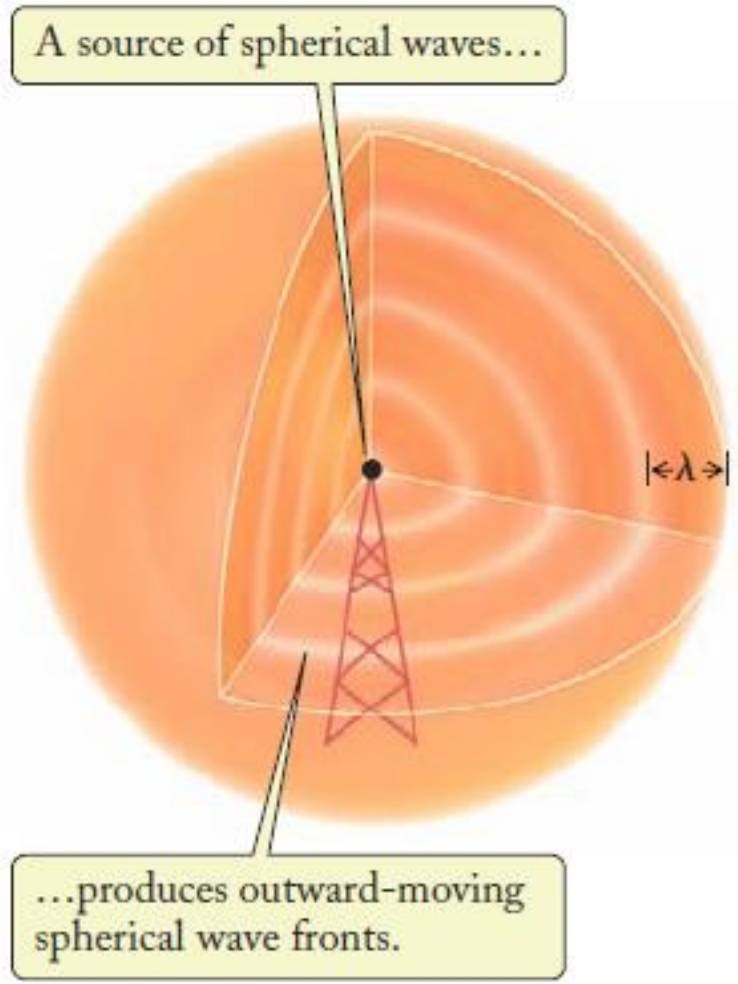
Huygens' Principle

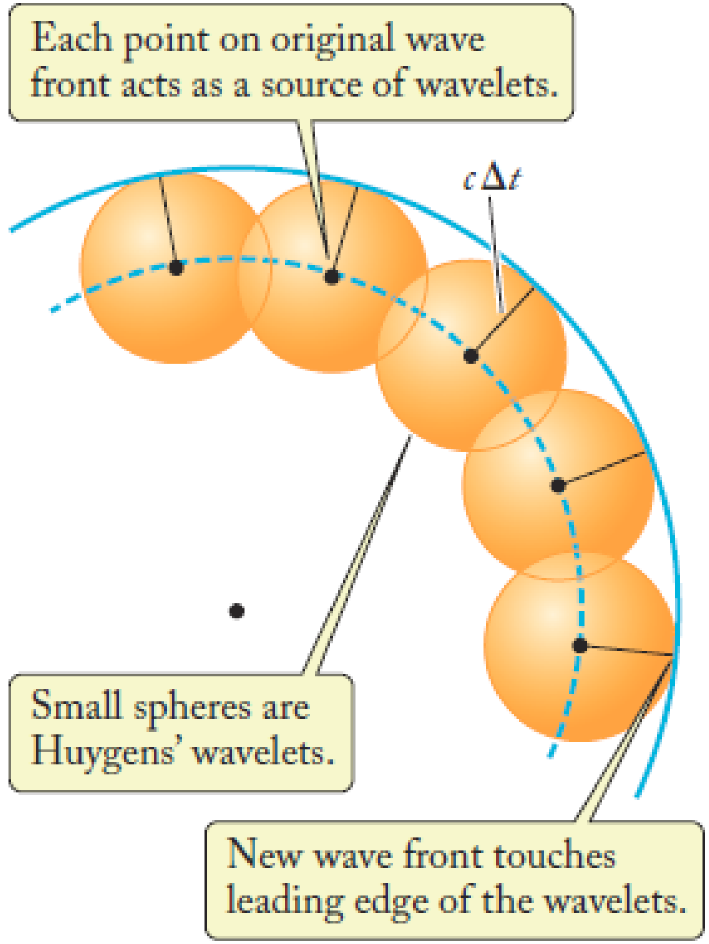




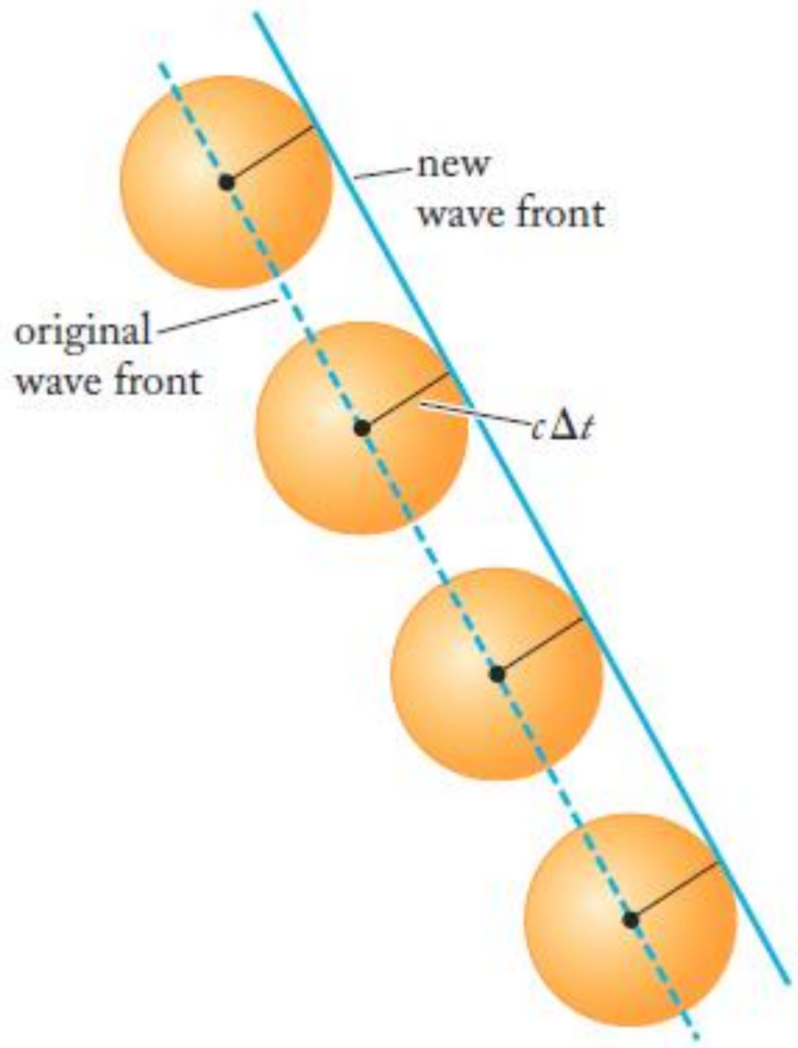
- **Every point on a given wavefront can be considered as a source of tiny spherical secondary waves, called wavelets (子波) that spread out in the forward direction at the speed of the wave itself. After a time Δt the new wavefront is the envelope of all the wavelets—that is the tangent to all these secondary wavelets. (1678)**

Huygens's construction

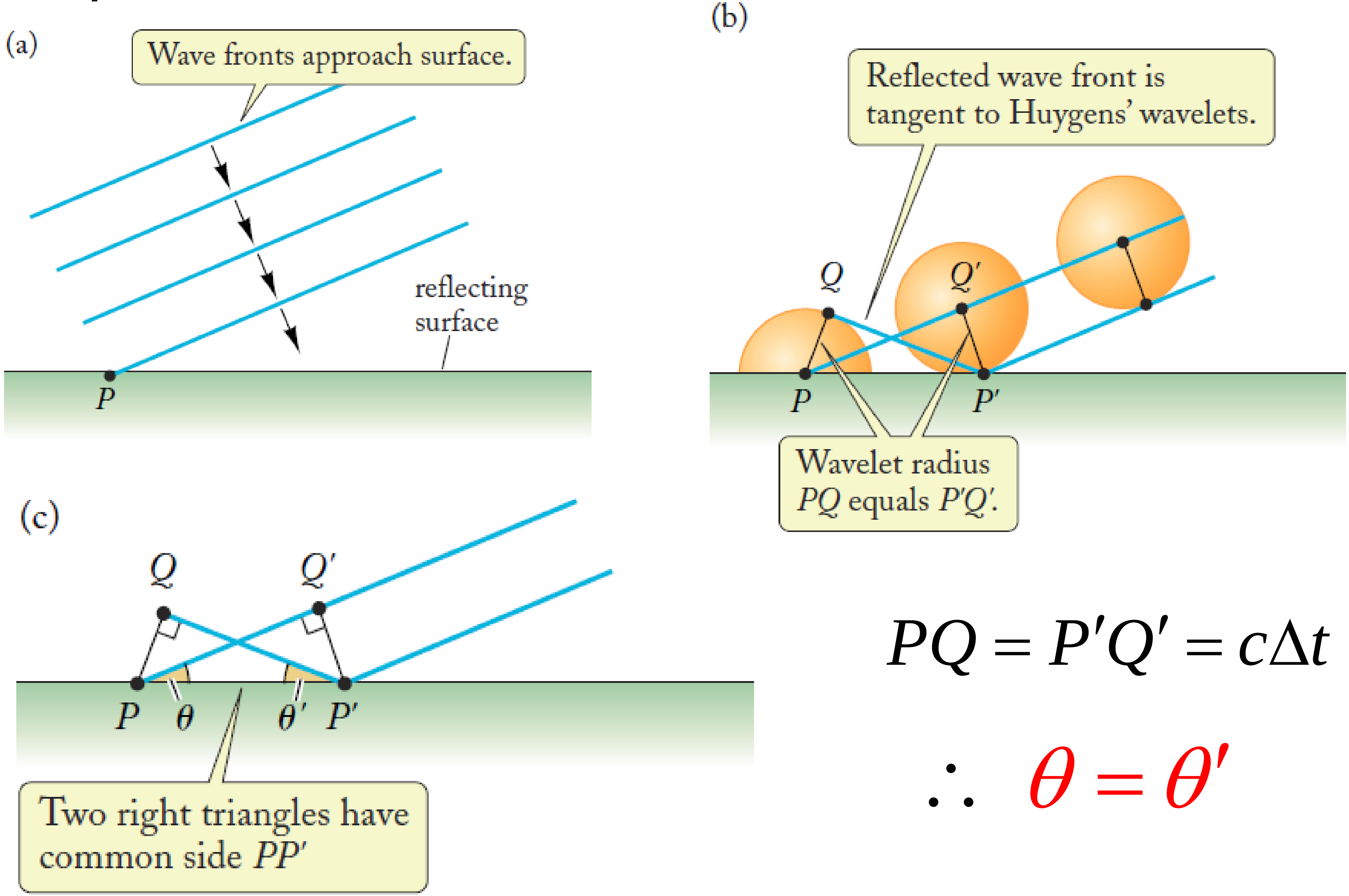




Huygens's construction



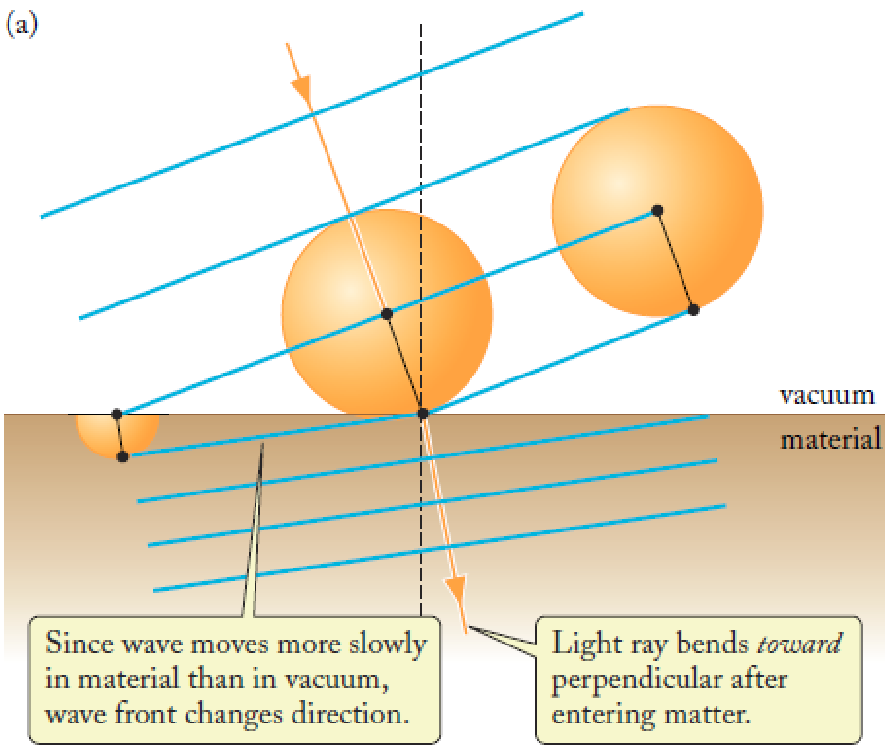
Reflection and Huygens's Principle

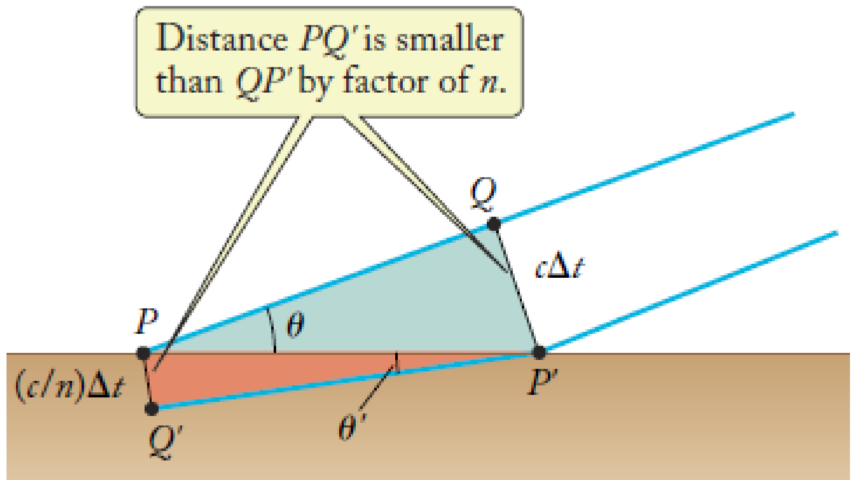


$$PQ = P'Q' = c\Delta t$$

$$\therefore \quad \theta = \theta'$$

Refraction and Huygens's Principle





$$\sin\theta = \frac{c\Delta t}{PP'},$$

$$\sin\theta' = \frac{(c/n)\Delta t}{PP'},$$

$$\frac{\sin\theta}{\sin\theta'} = \frac{c}{c/n},$$

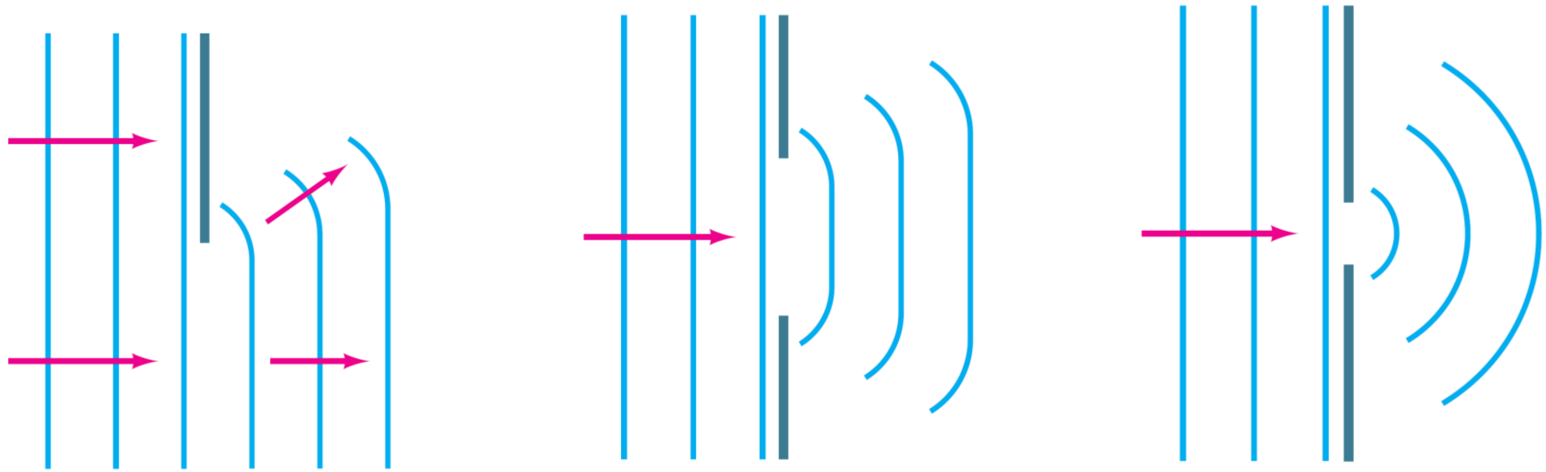
Snell's law of refraction

$$\sin\theta = n\sin\theta'$$

$$n_1\sin\theta_1 = n_2\sin\theta_2$$

Huygens' principle

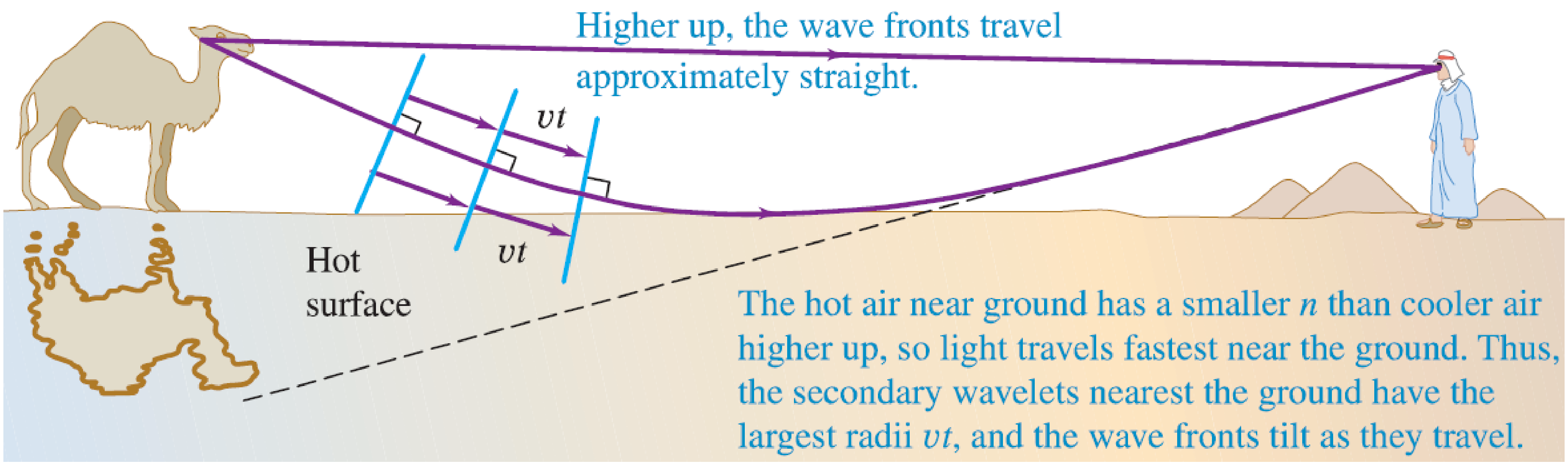
- **Huygens' principle used to explain diffraction phenomenon**
  - **Diffraction: When the waves impinge on an obstacle, the waves will bend behind the obstacle into the "shadow region".**



(a) (b) (c)

- **The diffraction is one of unique phenomena for all kinds of waves, not for particles.**

Mirages

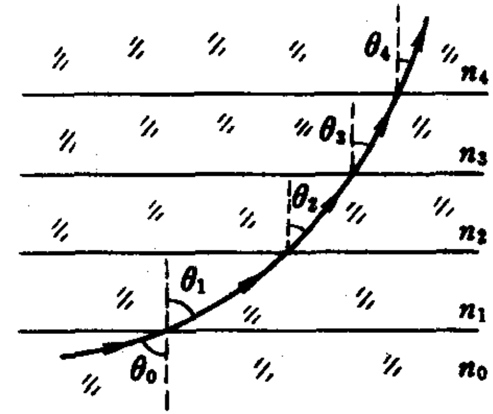

Higher up, the wave fronts travel approximately straight.
vt
Hot surface
vt
The hot air near ground has a smaller n than cooler air higher up, so light travels fastest near the ground. Thus, the secondary wavelets nearest the ground have the largest radii vt, and the wave fronts tilt as they travel.

Applications of optical path length

- The bending of rays through inhomogeneous medium
  - Bending trend of rays.

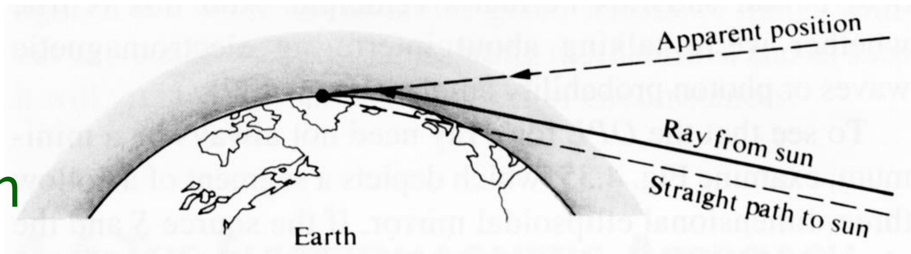
    Considering a material composed of a series of layers with index $n_0<n_1<n_2<n_3<n_4<\ldots$,

    a ray tends to bend towards the direction with higher index of refraction.

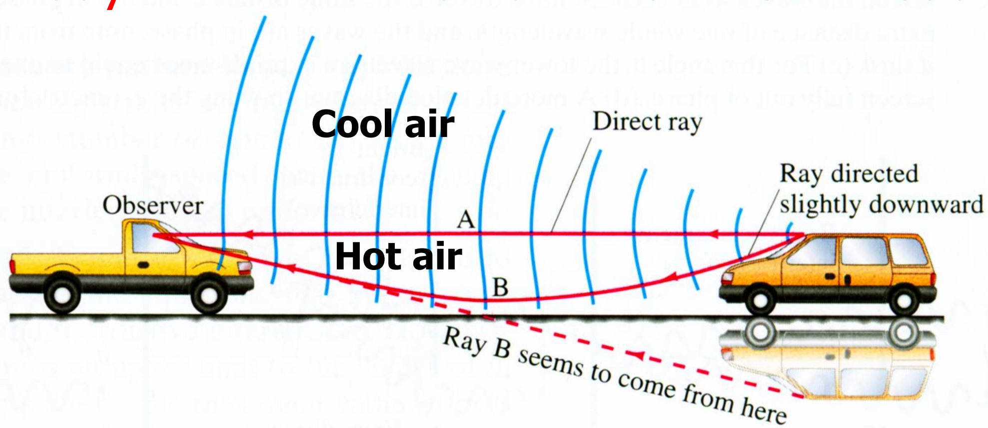


  - The phenomena of bending of rays

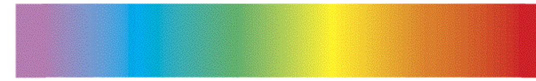
    Sunset: At the moment we see the sun getting down, the sun has been already down in a period of time.



    Highway mirage on a hot day: It is seem to have water over roadway.

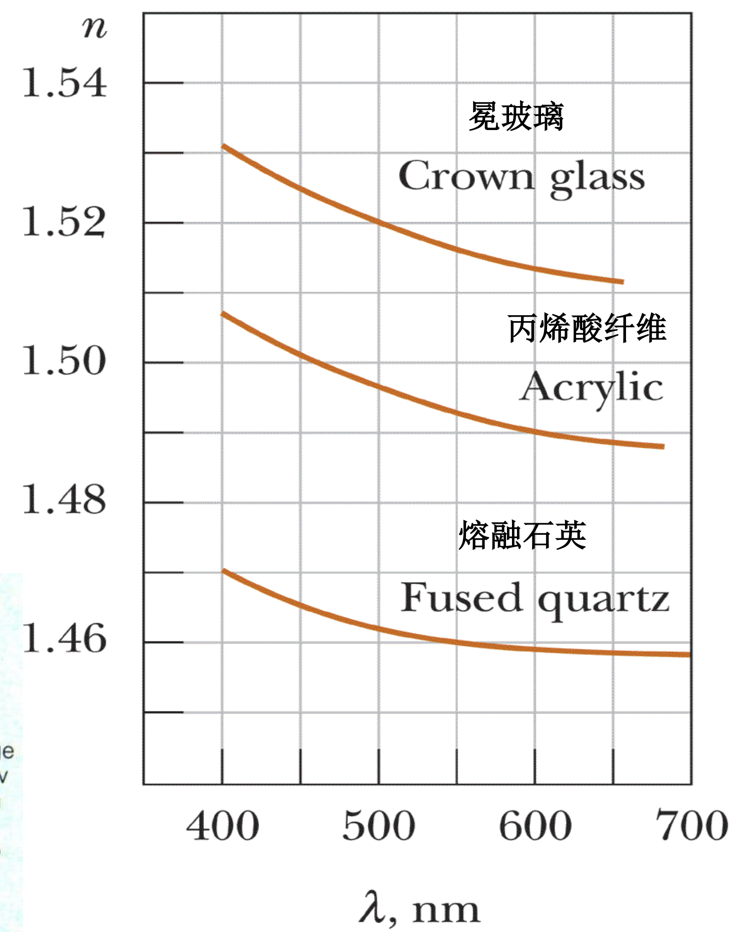


§ 5 Dispersion and Prisms



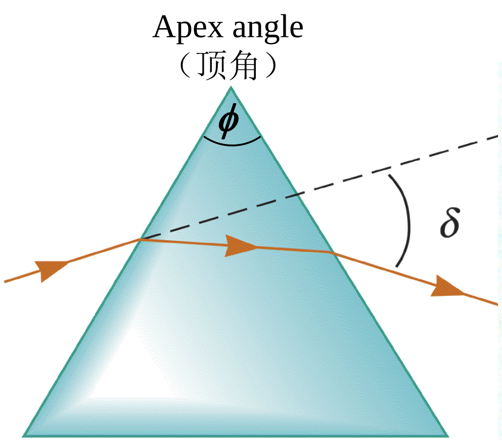
- **Dispersion:**
  - The dependence of the index of refraction on wavelength.

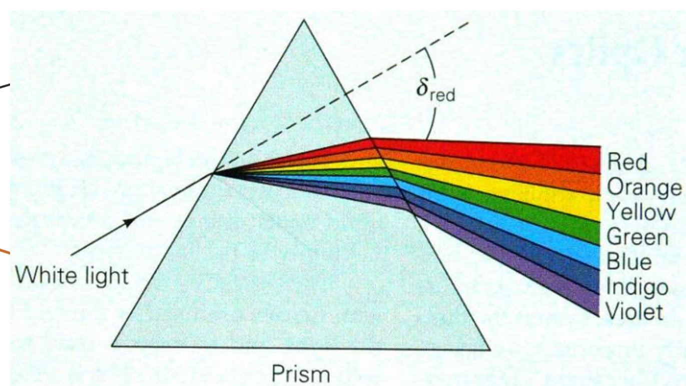
$$\boxed{n = n(\lambda)} \qquad v = \frac{c}{n(\lambda)} = v(\lambda)$$



  - Generally, $\frac{dn}{d\lambda} < 0$ (normal dispersion)

- **Prism**

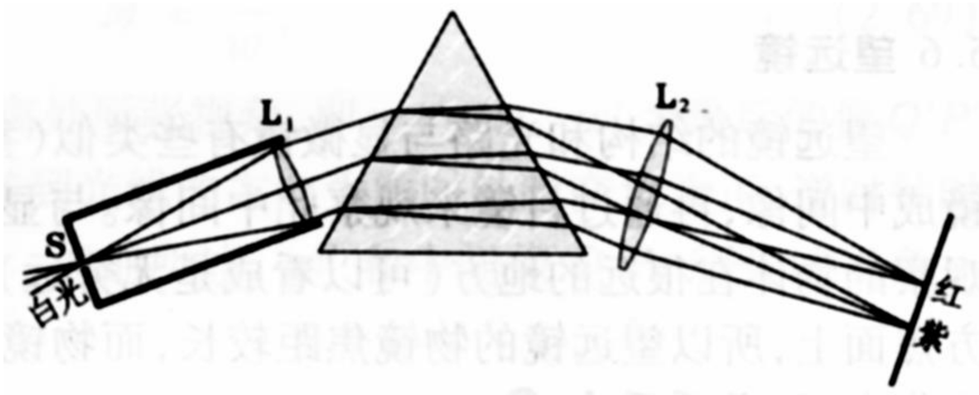




angle of deviation δ

Prism spectrum analyzer

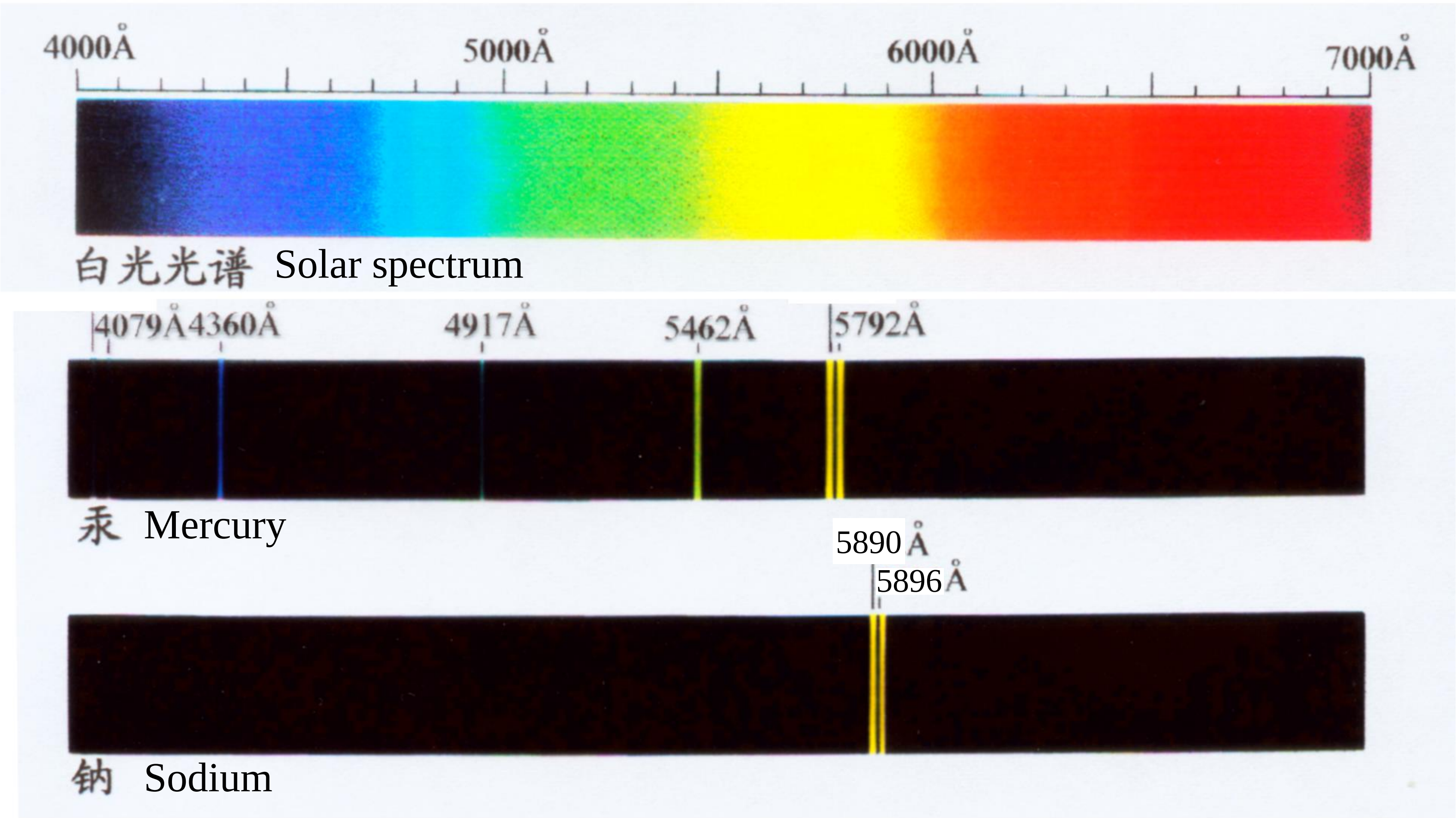
- Prism (Cont'd)
  - The prism can be employed as the key component in spectrum analyzer.



Prism spectrum analyzer

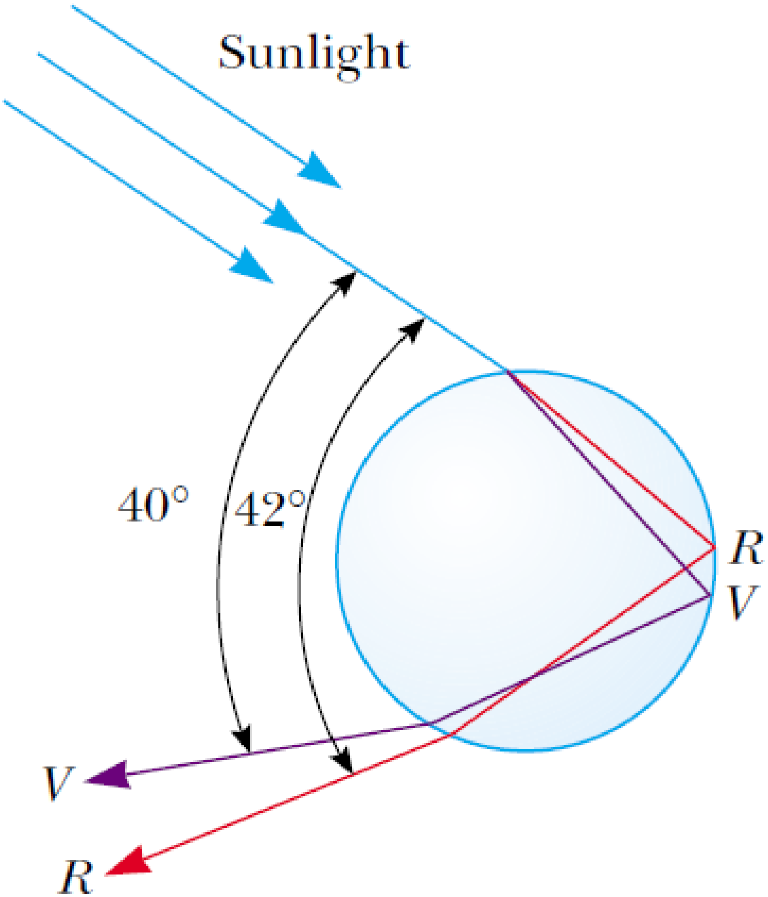
Some typical spectrums

- Some typical spectrums in visible range of light sources measured by Prism spectrum analyzer:

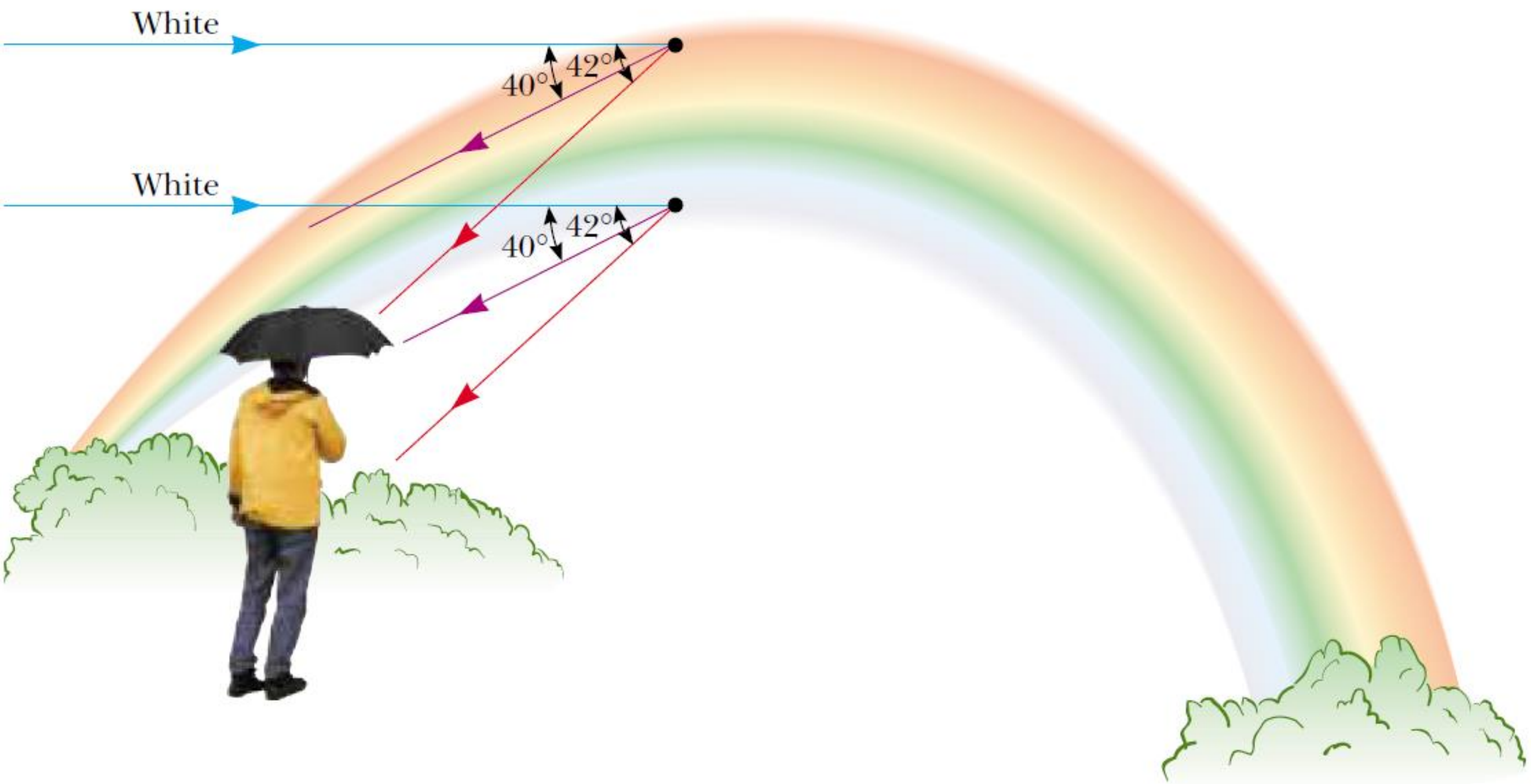


Rainbow

- The formation of rainbow can be explained by the **dispersion** of light.
- A ray of light passing overhead strikes a spherical drop of water in the atmosphere. The exit angle is 42° for the red light and 40° for the violet light.



Rainbow



- The red portion of the rainbow seen by an observer is supplied by drops upper in the sky, and the violet portion of the rainbow is supplied by drops lower in the sky.

* § 6 Fermat's Principle

"Nature always acts by the shortest course."

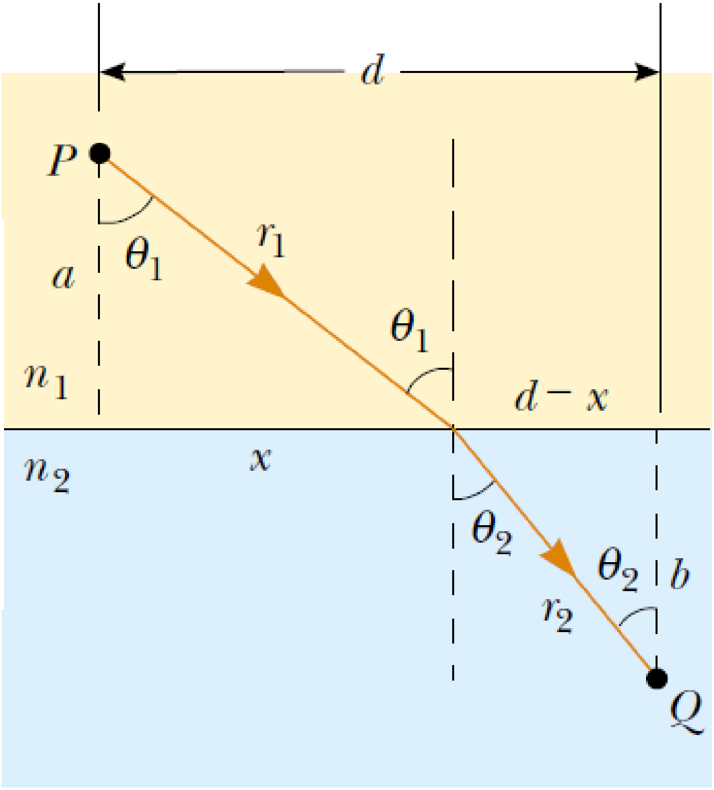
- When a light ray travels between any two points, its path is the one that requires the smallest time interval.

Pierre de Fermat,
French lawyer
(1607—1665)

Fermat's principle and Snell's law

$$t = \frac{r_1}{v_1} + \frac{r_2}{v_2} = \frac{\sqrt{a^2 + x^2}}{c/n_1} + \frac{\sqrt{b^2 + (d-x)^2}}{c/n_2}$$

$$\begin{aligned}\frac{dt}{dx} &= \frac{n_1}{c}\frac{d}{dx}\sqrt{a^2+x^2} + \frac{n_2}{c}\frac{d}{dx}\sqrt{b^2+(d-x)^2}\\ &= \frac{n_1}{c}\left(\tfrac{1}{2}\right)\frac{2x}{(a^2+x^2)^{1/2}} + \frac{n_2}{c}\left(\tfrac{1}{2}\right)\frac{2(d-x)(-1)}{[b^2+(d-x)^2]^{1/2}}\\ &= \frac{n_1 x}{c(a^2+x^2)^{1/2}} - \frac{n_2(d-x)}{c[b^2+(d-x)^2]^{1/2}} = 0\end{aligned}$$



$$\frac{n_1 x}{(a^2+x^2)^{1/2}} = \frac{n_2(d-x)}{[b^2+(d-x)^2]^{1/2}}$$

$$\sin\theta_1 = \frac{x}{(a^2+x^2)^{1/2}} \qquad \sin\theta_2 = \frac{d-x}{[b^2+(d-x)^2]^{1/2}}$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$