Advanced Mathematics (I) Midterm

- Time: Nov 21, 2021

Question 1

1. $\lim\limits_{n\to+\infty}(1+\dfrac{2}{n})^n$ = _____

2. $\lim\limits_{n\to+\infty}(\dfrac{1}{n^2+n+1}+\dfrac{2}{n^2+n+1}+\cdots+\dfrac{n}{n^2+n+1})$ = _____

3. $\lim\limits_{x\to+\infty}\dfrac{x-ln(e^x+x)}{x\sin(\frac{1}{x})}$ = _____

4. $\lim\limits_{x\to0^+}(ln\dfrac{1}{x})^x$ = _____

5. The equation of the line tangent to the gragh of $y=4x^3-2x^2+1$ at the point (1,3) is _____

6. Suppose that the function $y=arctan(2x)$. Then $\left.\dfrac{d^2y}{dx^2}\right|_{x=0}$ = _____

7. Suppose that the function $f(x)$ is deriavble, and $y=f(e^x)e^{f(x)}$. Then dy = _____

Question 2

Discuss the continuity of the function $f(x)=\begin{cases}\dfrac{x(2x+\pi)}{2\cos x} & x<0,\\ 1 & x=0\\ \dfrac{\sin x}{x} & x>0\end{cases}$ in the interval$[-\pi,\pi]$.

Question 3

Suppose that $f(x)=\begin{cases}\dfrac{2}{x^2}(1-\cos x) & x<0,\\ 1 & x=0,\\ \dfrac{\sin x}{x} & x>0.\end{cases}$ Discuss the derivability of the function $f(x)$ at $x=0$.

Question 4

Determine the constants a,b such that $f(x)=e^x-\dfrac{1+ax}{1+bx}$ is a 3-order infinitesimal quanity as $x\to 0$.

Question 5

Find the derivative $\left.\dfrac{dy}{dx}\right|_{t=0}$ and the second derivative $\left.\dfrac{d^2y}{dx^2}\right|_{t=0}$ of the function $y = y(x)$ defined by the parametrix equitions

$$\begin{cases} x = 3t^2 + 2t + 3, \\ e^y \sin t - y + 1 = 0. \end{cases}$$