北京郵電大學 Queen Mary University of London

BBC4111 A

For examiners' use only

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Joint Programme Examinations 2022/23

BBC4111 Engineering Mathematics

Paper A

Time allowed 2 hours

Answer ALL EIGHT questions

Complete the information below about yourself very carefully.

QM student number

BUPT student number

Class number

NOT allowed: electronic calculators and electronic dictionaries.

INSTRUCTIONS

1. **You must NOT take answer books, used or unused, from the examination room.**
2. Write only with a black or blue pen **and in English**.
3. Do all rough work in the answer book – **do not tear out any pages**.
4. If you use Supplementary Answer Books, tie them to the end of this book.
5. Write clearly and legibly.
6. **Read the instructions on the inside cover.**

Examiners

Dr Lihua Zhang, Dr Xia Shi

Filename: 2223_BBC4111_A No answer book required

Instructions

Before the start of the examination

1) Place your BUPT and QM student cards on the corner of your desk so that your picture is visible.

2) Put all bags, coats and other belongings at the back/front of the room. All small items in your pockets, including wallets, mobile phones and other electronic devices must be **placed in your bag in advance**. **Possession of mobile phones, electronic devices and unauthorised materials is an offence.**

3) Please ensure your mobile phone is switched off and that no alarm will sound during the exam. **A mobile phone causing a disruption is also an assessment offence.**

4) Do not turn over your question paper or begin writing until told to do.

During the examination

1) You must not communicate with or copy from another student.

2) If you require any assistance or wish to leave the examination room for any reason, please raise your hand to attract the attention of the invigilator.

3) If you finish the examination early you may leave, but not in the first 30 minutes or the last 10 minutes.

4) For 2 hour examinations you may **not** leave temporarily.

5) For examinations longer than 2 hours you **may** leave temporarily but not in the first 2 hours or the last 30 minutes.

At the end of the examination

1) You must stop writing immediately – **if you continue writing after being told to stop, that is an assessment offence.**

2) Remain in your seat until you are told you may leave.

Question 1. [24 marks total, 2 marks for each blank]

Fill in all the following blanks. Only the final results are required to be written down.

a). The exponential form of $\dfrac{(\cos 5\varphi + i\sin 5\varphi)^2}{(\cos 3\varphi - i\sin 3\varphi)^3}$ is ( ).

b). Suppose that $Arg(z+2)=\dfrac{\pi}{3}$ and $Arg(z-2)=\dfrac{5\pi}{6}$. Determine that $z=($).

c). $\lim\limits_{z\to i}\dfrac{z-i}{z(1+z^2)}=($).

d). If $\cos(2+z)=3,$ then $z=($).

e). Let C denote the semicircle $|z|=1$ from 1 to -1. Then $\int_C \left(z^2+z\bar{z}\right)dz=($).

f). $\int_{-\infty}^{+\infty}\dfrac{1}{\left(1+x^2\right)^2}dx=($).

g). The partial differential equation (i.e. PDE) $\dfrac{\partial^2 u}{\partial x^2}+5\dfrac{\partial^2 u}{\partial x\partial t}+6xu+5t=0$ is of ( ) type.

h). The general solution of the equation $(1-x^2)y''(x)-2xy'(x)+12y(x)=0$ is ( ).

i). The characteristic curves of $\dfrac{\partial^2 u}{\partial x^2}-3\dfrac{\partial^2 u}{\partial x\partial y}+2\dfrac{\partial^2 u}{\partial y^2}=\sin(x^2+y^2)$ are ( ).

j). The eigenvalues of the problem

$$\begin{cases} X''(x)+\lambda X(x)=0, & 0<x<l \\ X'(0)=0, \quad X(l)=0 \end{cases}$$

are ( ), and the corresponding eigenfunctions are ( ).

k). $\int_0^x x^4 J_1(x)dx=($), where $J_1(x)$ is the first kind of 1st order Bessel function.

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		24 marks

Question 2. [6 marks total, 2 marks for each one]
Please choose the correct answers for the following questions.

a). The convergence domain of the power series $\sum_{n=1}^{+\infty}\frac{(z-i)^n}{n^3}$ is ( ).

A. $|z-i|<\frac{1}{n^3}$ B. $|z-i|<1$ C. $|z|<1$ D. $|z-i|<\frac{1}{n}$

b). *Which one of the following four equation is* **NOT** *correct*? ( )

A. $\int_{|z|=2}\frac{3z-1}{z(z-1)}dz=6\pi i$ B. $\oint_{|z-i|=0.5}\frac{e^z dz}{z^2+1}=\pi(\cos 1+i\sin 1)$

C. $\int\limits_{|z|=1}\frac{\cos z dz}{z^3}=-\pi i$ D. $\int_0^i (z-1)e^{-z}dz=-\sin 1+i\cos 1$

c). *Which one of the following four statements is* ***correct?*** ( )

A. $J_\nu(x)$ and $J_{-\nu}(x)$ are linearly dependent.

B. The first kind of *n* order Bessel function $J_n(x)$ and the Bessel function of second kind $Y_n(x)$ are linearly independent.

C. $\lim\limits_{x\to\infty} J_n(x)=0$ when n is a positive integer.

D. $J_n(0)=0$ for all positive integers, and $J_\upsilon(0)=\infty$ when ν is nonnegative.

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		6 marks

Question 3. [18 marks total, 6 marks for each part]

a). Find out all points at which the function $f(z)$ is differentiable and analytic (please give the explanation), when $f(z)=\dfrac{\sin z\cdot \mathrm{Log}(1+z)}{z}$.

b). If the real part of entire function $f(z)$ is $u(x,y)=e^{x}(x\cos y-y\sin y),$ and $f(0)=0$, then find the imaginary part of $f(z)$ and calculate the value of $f'(1)$.

c). Give the Laurent series expansions for the function $f(z)=\dfrac{1}{z(z+1)}$ in the following annular domain $1<|z-1|<2$.

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	18 marks

Question 4. [10 marks total, 5 marks for each part]

Determine all the isolated singular points of the following two functions and identify their types, explaining each type. Hence, select **ONE** isolated point and calculate its residue.

a). $f(z) = \dfrac{1}{z\sin\left(\dfrac{1}{z}\right)}$; b). $f(z) = \dfrac{\sin z - z}{\cos z - 1}$.

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	10 marks

Question 5. [10 marks]

Solve the following problem of small oscillation of semi-infinite unloaded string with rigidly free end $x = 0$.

$$\begin{cases} u_{tt} - 4u_{xx} = 0, \quad 0 < x < +\infty, \ t > 0, \\ u_x(0,t) = 0, \quad t > 0, \\ u(x,0) = x^2, u_t(x,0) = x, \quad 0 < x < +\infty. \end{cases}$$

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	10 marks

Question 6. [10 marks]

Determine the type of the PDE $u_{xx} - 6u_{xy} + 9u_{yy} = xy^2$ and transform it into its standard form.

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	10 marks

Question 7. [12 marks]

Solve the following problem by means of separation of variables:

$$\begin{cases} u_{tt} - 4u_{xx} = 0, \quad 0 < x < 1, \ \ t > 0, \\ u_x(0,t) = 0, \ u_x(1,t) = 0, \quad t \geq 0, \\ u(x,0) = \sin \pi x, \ \ u_t(x,0) = 0, \quad 0 \leq x \leq 1. \end{cases}$$

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	12 marks

Question 8. [10 marks]

Solve the following vibration problem of a half infinite string:

$$\begin{cases} u_{tt} = c^2 u_{xx}, \quad 0 < x < \infty, \quad t > 0, \\ u(x,0) = 0, \quad u_t(x,0) = 0, \\ u(0,t) = f(t), \quad \lim\limits_{x \to +\infty} u_x(x,t) = 0. \end{cases}$$

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		10 marks

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2022-2023

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