第八次作业答案

一、4.18(a,c)

4.18. An LTI system has the impulse response

$$h(t) = 2\frac{\sin(2\pi t)}{\pi t}\cos(7\pi t)$$

Use the FT to determine the system output for the following inputs, $x(t)$.

$$\text{Let } a(t) = \frac{\sin(2\pi t)}{\pi t} \xleftrightarrow{FT} A(j\omega) = \begin{cases} 1 & |\omega| < 2\pi \\ 0 & \text{otherwise} \end{cases}$$

$$h(t) = 2a(t)\cos(7\pi t) \xleftrightarrow{FT} H(j\omega) = A(j(\omega - 7\pi)) + A(j(\omega + 7\pi))$$

(a) $x(t) = \cos(2\pi t) + \sin(6\pi t)$

$$\begin{aligned}
X(j\omega) &= 2\pi \sum_{k=-\infty}^{\infty} X[k]\delta(\omega - k\omega_o) \\
X(j\omega) &= \pi\delta(\omega - 2\pi) + \pi\delta(\omega + 2\pi) + \frac{\pi}{j}\pi\delta(\omega - 6\pi) - \frac{\pi}{j}\pi\delta(\omega + 6\pi) \\
Y(j\omega) &= X(j\omega)H(j\omega) \\
&= \frac{\pi}{j}\delta(\omega - 6\pi) - \frac{\pi}{j}\delta(\omega + 6\pi) \\
y(t) &= \sin(6\pi t)
\end{aligned}$$

(c) $x(t)$ as depicted in Fig. P4.18 (a).

$$\begin{aligned}
T = 1 \qquad & \omega_o = 2\pi \\
X(j\omega) &= 2\pi \sum_{k=-\infty}^{\infty} \left(\frac{\sin(k\frac{\pi}{4})}{k\pi}(1 - e^{-jk\pi}) \right) \delta(\omega - k2\pi) \\
Y(j\omega) &= X(j\omega)H(j\omega) \\
&= 2\pi \left[\frac{\sin(3\frac{\pi}{4})}{3\pi}(1 - e^{-j3\pi})\delta(\omega - 6\pi) + \frac{\sin(-3\frac{\pi}{4})}{-3\pi}(1 - e^{j3\pi})\delta(\omega + 6\pi) \right] \\
&= \frac{4\sin(3\frac{\pi}{4})}{3}\delta(\omega - 6\pi) + \frac{4\sin(3\frac{\pi}{4})}{3}\delta(\omega + 6\pi) \\
y(t) &= \frac{4\sin(\frac{3\pi}{4})}{3\pi}\cos(6\pi t)
\end{aligned}$$

二、4.29(a,c,d)

4.29. For each of the following signals sampled with sampling interval T_s, determine the bounds on T_s that gaurantee there will be no aliasing.

(a) $x(t) = \frac{1}{t}\sin 3\pi t + \cos(2\pi t)$

$$\frac{1}{t}\sin(3\pi t) \xleftrightarrow{FT} \begin{cases} \frac{1}{\pi} & |\omega| \le 3\pi \\ 0 & \text{otherwise} \end{cases}$$

$$\cos(2\pi t) \xleftrightarrow{FT} \pi\delta(\Omega - 2\pi) + \pi\delta(\Omega + 2\pi)$$

$$\begin{aligned}
\omega_{max} &= 3\pi \\
T &< \frac{\pi}{\omega_{max}} \\
T &< \frac{1}{3}
\end{aligned}$$

(c) $x(t) = e^{-6t}u(t) * \frac{\sin(Wt)}{\pi t}$

$$
\begin{aligned}
X(j\omega) &= \frac{1}{6+j\omega}[u(\omega+W) - u(\omega-W)] \\
\omega_{max} &= W \\
T &< \frac{\pi}{\omega_{max}} \\
T &< \frac{\pi}{W}
\end{aligned}
$$

(d) $x(t) = w(t)z(t)$, where the FTs $W(j\omega)$ and $Z(j\omega)$ are depicted in Fig. P4.29.

$$
\begin{aligned}
X(j\omega) &= \frac{1}{2\pi}W(j\omega) * G(j\omega) \\
\omega_{max} &= 4\pi + w_a \\
T &< \frac{\pi}{\omega_{max}} \\
T &< \frac{\pi}{4\pi + w_a}
\end{aligned}
$$

三、4.30

4.30. Consider the system depicted in Fig. P4.30. Assume $|X(j\omega)| = 0$ for $|\omega| > \omega_m$. Find the largest value of T such that $x(t)$ can be reconstructed from $y(t)$. Determine a system that will perform the reconstruction for this maximum value of T.

For reconstruction, we need to have $w_s > 2w_{max}$, or $T < \frac{\pi}{\omega_{max}}$. A finite duty cycle results in distortion.

$$
\begin{aligned}
W[k] &= \frac{\sin(\frac{\pi}{2}k)}{k\pi}e^{-j\frac{\pi}{2}k} \\
W(j\omega) &= 2\pi \sum_{k=-\infty}^{\infty} W[k]\delta(\omega - k\frac{2\pi}{T})
\end{aligned}
$$

After multiplication:

$$
Y(j\omega) = \sum_{k=-\infty}^{\infty} \frac{\sin(\frac{\pi}{2}k)}{k\pi}e^{-j\frac{\pi}{2}k}X(j(\omega - k\frac{2\pi}{T}))
$$

To reconstruct:

$$
\begin{aligned}
H_r(j\omega)Y(j\omega) &= X(j\omega), \quad |\omega| < \omega_{max}, \quad \frac{2\pi}{T} > 2\omega_{max} \\
k &= 0 \\
H_r(j\omega)\frac{1}{2}X(j\omega) &= X(j\omega) \\
H_r(j\omega) &= \begin{cases} 2 & |\omega| < \omega_{max} \\ \text{don't care} & \omega_{max} < |\omega| < \frac{2\pi}{T} - \omega_{max} \\ 0 & |\omega| > \frac{2\pi}{T} - \omega_{max} \end{cases}
\end{aligned}
$$