第四次作业答案

一、2.39(a,b,i,m)

2.39. Evaluate the continuous-time convolution integrals given below.

(a) $y(t) = (u(t) - u(t-2)) * u(t)$

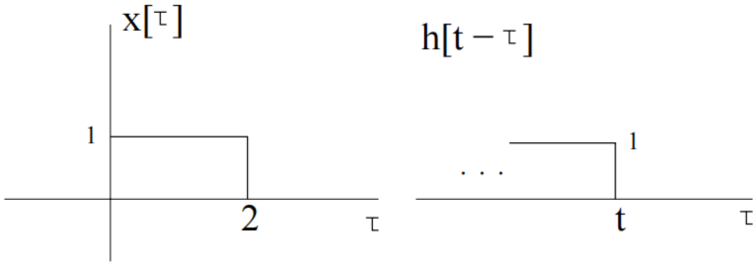

Figure P2.39. (a) Graph of $x[\tau]$ and $h[t-\tau]$

for $t < 0$

$$y(t) = 0$$

for $t < 2$

$$y(t) = \int_0^t d\tau = t$$

for $t \geq 2$

$$y(t) = \int_0^2 d\tau = 2$$

$$y(t) = \begin{cases} 0 & t < 0 \\ t & 0 \leq t < 2 \\ 2 & t \geq 2 \end{cases}$$

(b) $y(t) = e^{-3t}u(t) * u(t+3)$

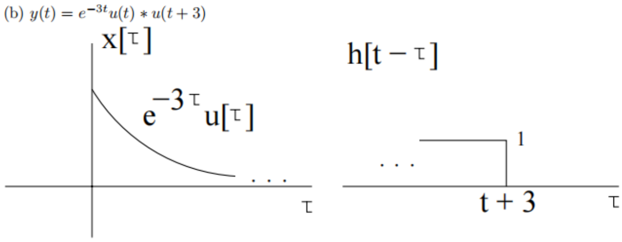

Figure P2.39. (b) Graph of $x[\tau]$ and $h[t-\tau]$

for $t+3 < 0$ $\quad t < -3$

$$y(t) = 0$$

for $t \geq -3$

$$y(t) = \int_0^{t+3} e^{-3\tau} d\tau$$

$$y(t) = \frac{1}{3}\left[1 - e^{-3(t+3)}\right]$$

$$y(t) = \begin{cases} 0 & t < -3 \\ \frac{1}{3}\left[1 - e^{-3(t+3)}\right] & t \geq -3 \end{cases}$$

(i) $y(t) = (2\delta(t+1) + \delta(t-5)) * u(t-1)$

$$\begin{aligned}
\text{for } t-1 < -1 \quad & t < 0 \\
& y(t) = 0 \\
\text{for } t-1 < 5 \quad & 0 \le t < 6 \\
& \text{By the sifting property.} \\
& y(t) = \int_{-\infty}^{t-1} 2\delta(t+1)d\tau = 2 \\
\text{for } t-1 \ge 5 \quad & t \ge 6 \\
& y(t) = \int_{-\infty}^{t-1} (2\delta(t+1) + \delta(t-5))\, d\tau = 3
\end{aligned}$$

$$y(t) = \begin{cases} 0 & t < 0 \\ 2 & 0 \le t < 6 \\ 3 & t \ge 6 \end{cases}$$

(m) $y(t) = (2\delta(t) + \delta(t-2)) * \sum_{p=0}^{\infty} \left(\frac{1}{2}\right)^p \delta(t-p)$

$$\text{let} \quad x_1(t) = \sum_{p=0}^{\infty} \left(\frac{1}{2}\right)^p \delta(t-p)$$

for $t < 0$

$$y(t) = 0$$

for $t < 2$

$$y(t) = 2\delta(t) * x_1(t) = 2x_1(t)$$

for $t \ge 2$

$$y(t) = 2\delta(t) * x_1(t) + \delta(t-2) * x_1(t) = 2x_1(t) + x_1(t-2)$$

$$y(t) = \begin{cases} 0 & t < 0 \\ 2\sum_{p=0}^{\infty} \left(\frac{1}{2}\right)^p \delta(t-p) & 0 \le t < 2 \\ 2\sum_{p=0}^{\infty} \left(\frac{1}{2}\right)^p \delta(t-p) + \sum_{p=0}^{\infty} \left(\frac{1}{2}\right)^p \delta(t-p-2) & t \ge 0 \end{cases}$$

二、2.40(g,k)

(g) $m(t) = y(t) * g(t)$

for $t < -1$

$$m(t) = 0$$

for $t < 1$ $\quad -1 \le t < 1$

$$m(t) = \int_{-1}^{t} \tau d\tau = 0.5[t^2 - 1]$$

for $t - 2 < 1$ $\quad 1 \le t < 3$

$$m(t) = \int_{-1}^{t-2} 2\tau d\tau + \int_{t-2}^{1} \tau d\tau = 0.5t^2 + 0.5(t-2)^2 - 1$$

for $t - 4 < 1$ $\quad 3 \le t < 5$

$$m(t) = \int_{t-4}^{1} 2\tau d\tau = 1 - (t-4)^2$$

for $t \ge 5$

$$m(t) = 0$$

$$m(t) = \begin{cases} 0 & t < -1 \\ 0.5[t^2 - 1] & -1 \le t < 1 \\ \cancel{0.5t^2 + 0.5(t-2)^2 - 1} & 1 \le t < 3 \\ 1 - (t-4)^2 & 3 \le t < 5 \\ 0 & t \ge 5 \end{cases}$$

$\frac{1}{2}(t-2)^2 - 0.5$，1≤t≤3

(k) $m(t) = z(t) * b(t)$

for $t + 1 < -3$ $\quad t < -4$

$$m(t) = 0$$

for $t + 1 < -2$ $\quad -4 \le t < -3$

$$m(t) = -\int_{-3}^{t+1} (\tau + 3) d\tau = -0.5(t+1)^2 + \frac{9}{2} - 3(t+1) - 9$$

for $t < -2$ $\quad -3 \le t < -2$

$$m(t) = \int_{-3}^{t} (\tau + 3) d\tau - \int_{t}^{-2} (\tau + 3) d\tau - \int_{-2}^{\cancel{t}} d\tau = t^2 + 5t + \frac{11}{2}$$

t+1

for $t - 1 < -2$ $\quad -2 \le t < -1$

$$m(t) = \int_{t-1}^{\cancel{2}} (\tau + 3) d\tau + \int_{-2}^{t} d\tau - \int_{t}^{t+1} d\tau = 12 - \frac{1}{2}(t-1)^2 - 2t$$

-2

for $t - 1 \ge -2$ $\quad t \ge -1$

$$m(t) = 0$$

$$\begin{aligned} m(t) &= 0, \quad t < -4 \\ &= -0.5(t+1)^2 - 3(t+1) - 4.5 = -0.5(t+4)^2, -4 \le t < 3 \\ &= t^2 + 5t + 5.5, -3 \le t < -2 \\ &= -0.5(t+1)^2, -2 \le t < -1 \\ &= 0, t \ge -1 \end{aligned}$$

三、2.46

2.46. Find the expression for the impulse response relating the input $x[n]$ or $x(t)$ to the output $y[n]$ or $y(t)$ in terms of the impulse response of each subsystem for the LTI systems depicted in

(a) Fig. P2.46 (a)

$$y(t) \quad = \quad x(t) * \{h_1(t) - h_4(t) * [h_2(t) + h_3(t)]\} * h_5(t)$$

(b) Fig. P2.46 (b)

$$y[n] \quad = \quad x[n] * \{-h_1[n] * h_2[n] * h_4[n] + h_1[n] * h_3[n] * h_5[n]\} * h_6[n]$$

(c) Fig. P2.46 (c)

$$y(t) \quad = \quad x(t) * \{[-h_1(t) + h_2(t)] * h_3(t) * h_4(t) + h_2(t)\}$$

四、2.48

$$\mathrm{h}(t) = -e^{-3t}u(t) + (e^{-2t-2} - e^{-3t-3})u(t+1) + e^{-2t}u(t)$$

详细解析见第四次作业-详细解答

五、2.49(a,f,h,k)

2.49. For each impulse response listed below, determine whether the corresponding system is (i) memoryless, (ii) causal, and (iii) stable.

(i) Memoryless if and only if $h(t) = c\delta(t)$ or $h[n] = c\delta[k]$
(ii) Causal if and only if $h(t) = 0$ for $t < 0$ or $h[n] = 0$ for $n < 0$
(iii) Stable if and only if $\int_{-\infty}^{\infty} |h(t)|dt < \infty$ or $\sum_{k=-\infty}^{\infty} |h[k]| < \infty$

(a) $h(t) = \cos(\pi t)$
(i) has memory
(ii) not causal
(iii) not stable

(f) $h[n] = (-1)^n u[-n]$
(i) has memory
(ii) not causal
(iii) not stable

(h) $h[n] = \cos(\frac{\pi}{8}n)\{u[n] - u[n-10]\}$

(i) has memory

(ii) causal

(iii) stable

(k) $h[n] = \sum_{p=-1}^{\infty} \delta[n-2p]$

(i) has memory

(ii) not causal

(iii) not stable

六、2.50(a,c,e,f)

2.50. Evaluate the step response for the LTI systems represented by the following impulse responses:

(a) $h[n] = (-1/2)^n u[n]$

for $n < 0$

$$s[n] = 0$$

for $n \geq 0$

$$s[n] = \sum_{k=0}^{n} (-\frac{1}{2})^k$$

$$s[n] = \frac{1}{3}\left(2 + \left(-\frac{1}{2}\right)^n\right)$$

$$s[n] = \begin{cases} \frac{1}{3}\left(2 + (-\frac{1}{2})^n\right) & n \geq 0 \\ 0 & n < 0 \end{cases}$$

(c) $h[n] = (-1)^n\{u[n+2] - u[n-3]\}$

for $n < -2$

$$s[n] = 0$$

for $-2 \leq n \leq 2$

$$s[n] = \begin{cases} 1 & n = \pm 2,\ 0 \\ 0 & n = \pm 1 \end{cases}$$

for $n \geq 3$

$$s[n] = 1$$

(e) $h(t) = e^{-|t|}$

for $t < 0$

$$s(t) = \int_{-\infty}^{t} e^{\tau} d\tau = e^{t}$$

for $t \geq 0$

$$s(t) = \int_{-\infty}^{0} e^{\tau} d\tau + \int_{0}^{t} e^{-\tau} d\tau = 2 - e^{-t}$$

$$s(t) = \begin{cases} e^{t} & t < 0 \\ 2 - e^{-t} & t \geq 0 \end{cases}$$

(f) $h(t) = \delta^{(2)}(t)$

for $t < 0$

$$s(t) = 0$$

for $t \geq 0$

$$s(t) = \int_{-\infty}^{t} \delta^{(2)}(t) d\tau = \cancel{\delta(t)}$$

$$s(t) = \delta(t)$$

$$s(t) = \delta^{(1)}(t)$$