Ch.2 *Time Domain Representations of Linear Time-Invariant Systems* (II)

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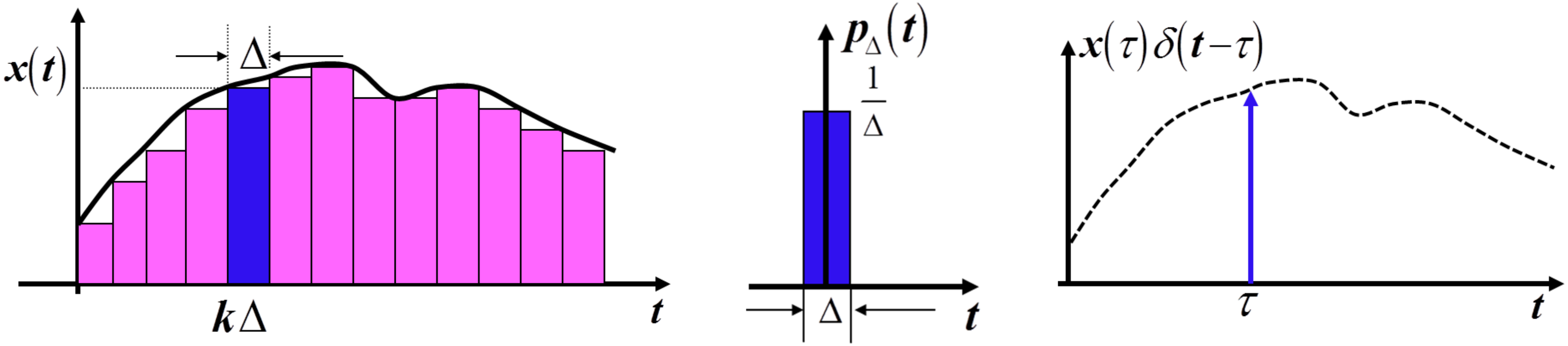
Outline

- Linear Time-invariant systems (LTI)
  - The Convolution Integral
  - Convolution Integral Evaluation Procedure
  - Relations between LTI System Properties and the Impulse Response
  - Step Response

Representing CT Signals

- A continuous-time signal can be expressed as a weighted superposition of time-shifted impulses



$$\hat{x}(t)=\sum_{k=-\infty}^{\infty} x(k\Delta)\,p_\Delta(t-k\Delta)\,\Delta$$

$$x(t)=\lim_{\Delta\to 0}\hat{x}(t)=\lim_{\Delta\to 0}\sum_{k=-\infty}^{\infty} x(k\Delta)\,p_\Delta(t-k\Delta)\,\Delta \quad =\int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau$$

since $\Delta\to\mathbf{0}, \quad p_\Delta(t)\to\delta(t), \quad k\Delta\to\tau, \quad \Delta\to d\tau$

The Convolution Integral

$$\text{Input } x(t) \longrightarrow \boxed{\begin{array}{c}\text{LTI system}\\ H\end{array}} \longrightarrow \text{Output } y(t)$$

- A continuous-time input signal: $x(t)=\int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)\,d\tau$
- Impulse response of the LTI system H: $h(t)=H\{\delta(t)\}$
- Output of the LTI system H

$$y(t)=H\{x(t)\}=H\left\{\int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)\,d\tau\right\}$$

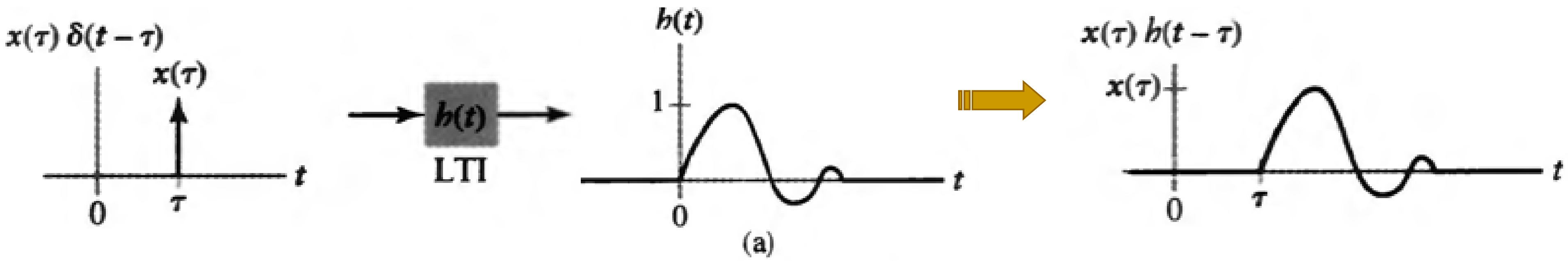
$$\xrightarrow{\textbf{Linearity}} \quad y(t)=\int_{-\infty}^{\infty} x(\tau)H\{\delta(t-\tau)\}\,d\tau$$

$$\xrightarrow{\textbf{Time invariant}} \quad y(t)=\int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$$

~ the integral of weighted and shifted unit-impulse responses.

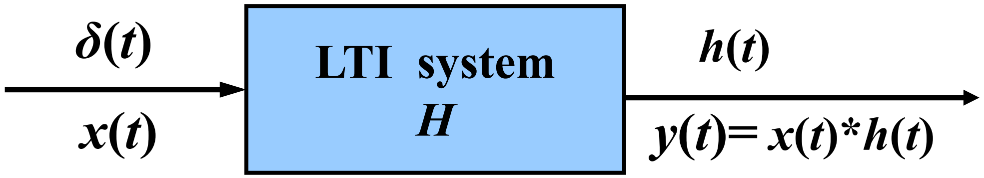
The Convolution Integral

$$y(t) = \int_{-\infty}^{\infty} \underline{x(\tau)h(t-\tau)}\, d\tau$$



- Convolution integral

$$\boxed{y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\, d\tau}$$



Convolution Integral Evaluation Procedure

- Convolution integral

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\boldsymbol{\tau}) h(t-\boldsymbol{\tau}) d\boldsymbol{\tau}$$

  - Define the intermediate signal:

$$w_t(\tau) = x(\tau) h(t-\tau)$$

Where τ = independent variable
t = constant.

$$\Rightarrow \quad h(t-\tau) = h(-(\tau - t))$$

~ a reflected and time-shifted version of $h(\tau)$.

$$\Rightarrow \quad y(t) = \int_{-\infty}^{\infty} w_t(\tau) d\tau$$

The time shift t determines the time at which we evaluate the output of the system.

Convolution Integral Evaluation Procedure

- ***Procedure : Reflect and Shift Convolution Integral Evaluation***
  - Graph both $x(\tau)$ and $h(t-\tau)$ as a function of the independent variable τ. To obtain $h(t-\tau)$, reflect $h(\tau)$ to obtain $h(-\tau)$ and then shift by $-t$.
  - Begin with the shift t large and negative. That is, shift $h(-\tau)$ to the far left on the time axis.
  - Write the mathematical representation for the intermediate signal $w_t(\tau)$.
  - Increase the shift t (i.e., move $h(t-\tau)$ toward the right) until the mathematical representation for $w_t(\tau)$ changes. The value of t at which the change occurs defines the end of the current set and the beginning of a new set.
  - Let t be in the new set. Repeat step 3 and 4 until all sets of shifts t and the corresponding mathematical representations for $w_t(\tau)$ are identified. This usually implies increasing t to a very large positive number.
  - For each sets of shifts t, integrate $w_t(\tau)$ from $\tau = -\infty$ to $\tau = \infty$ to obtain $y(t)$.

Convolution Integral Evaluation Procedure

Example 2.6 ***Reflect-and-shift Convolution Evaluation***

Given $x(t)=u(t-1)-u(t-3)$ **and** $h(t)=u(t)-u(t-2)$ **as depicted in Fig. 2-10, Evaluate the convolution integral** $y(t) = x(t) * h(t)$.

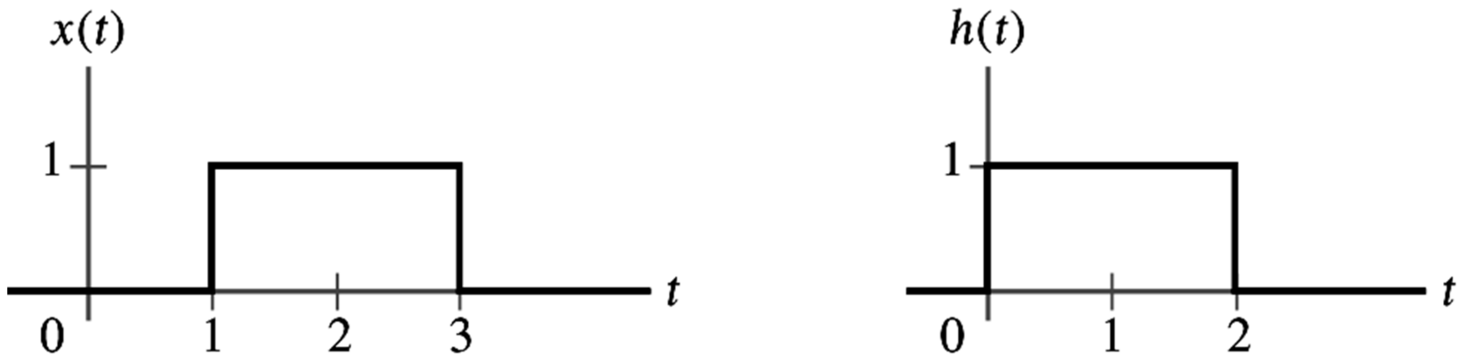
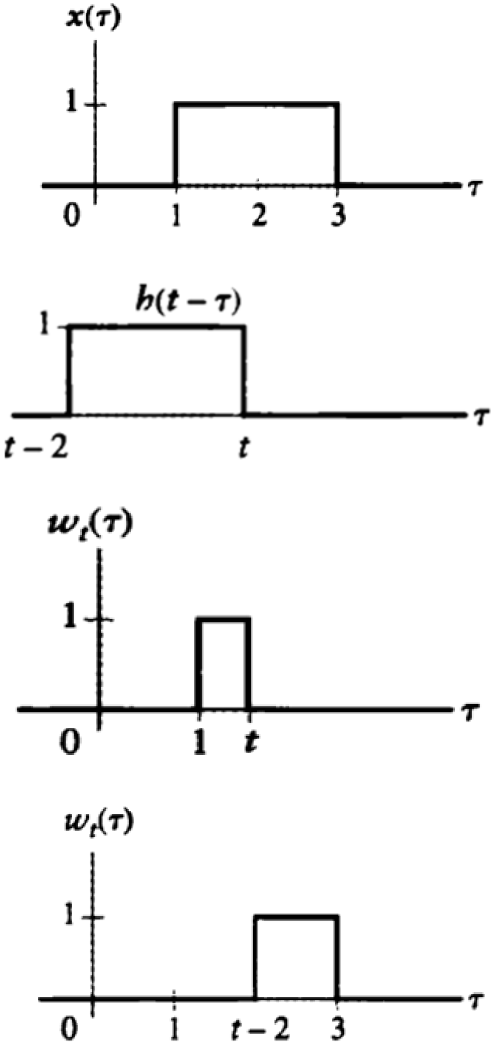


Figure 2.10 Input signal and LTI system impulse response for Example 2.6.

Convolution Integral Evaluation Procedure



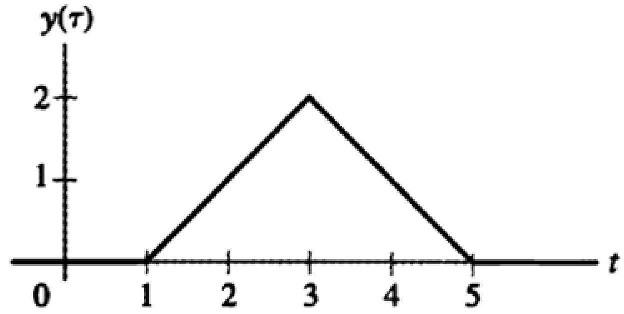
- **Four intervals**
  - $t < 1$: $w_t(\tau) = 0 \quad y(t) = 0$
  - $1 \le t \le 3$: $w_t(\tau) = 1, \quad 1 \le \tau \le t$

    $$y(t) = \int_{-\infty}^{\infty} w_t(\tau)\,d\tau = \int_1^t 1 \cdot d\tau = t - 1$$

  - $3 < t \le 5$: $w_t(\tau) = 1, \quad t-2 \le \tau \le 3$

    $$y(t) = \int_{t-2}^{3} 1 \cdot d\tau = 5 - t$$

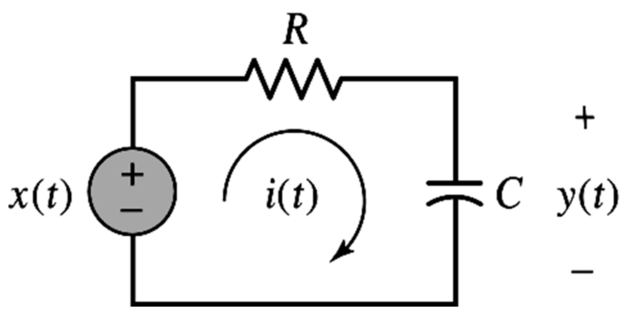
  - $t > 5$: $w_t(\tau) = 0 \quad y(t) = 0$



Convolution Integral Evaluation Procedure

Example 2.7 *RC* Circuit Output

For the *RC* circuit in Fig. 2.12, assume that the circuit's time constant is $RC = 1$s. Ex. 1.21 shows that the impulse response of the circuit is $h(t) = e^{-t}u(t)$. Use convolution to determine the capacitor voltage, $y(t)$, resulting from an input voltage $x(t) = u(t) - u(t - 2)$.



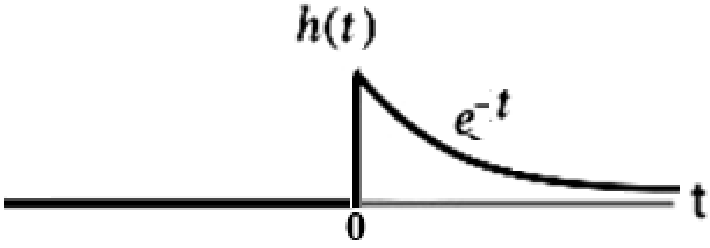
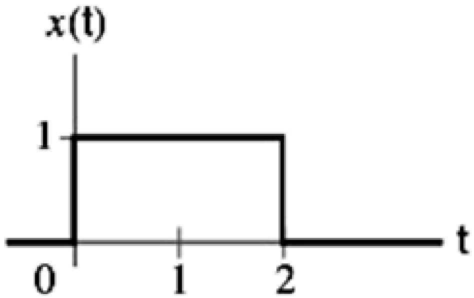
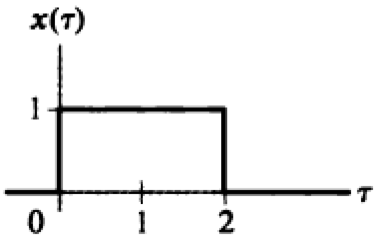
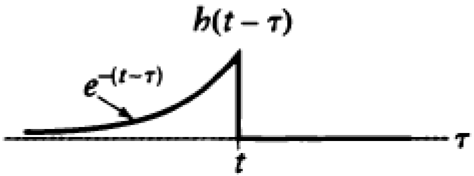


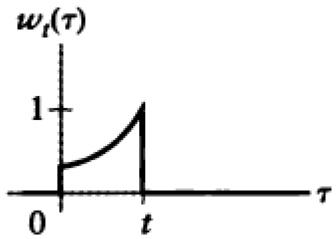
Figure 2.12 *RC* circuit system

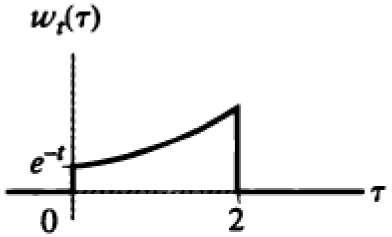


Convolution Integral Evaluation Procedure









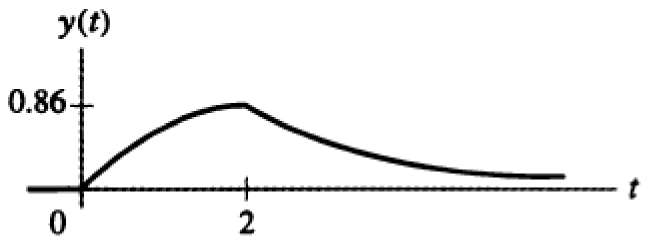
- **Three intervals**
  - $t < 0$ **:** $w_t(\tau) = 0 \quad y(t) = 0$
  - $0 \le t < 2$**:** $w_t(\tau) = e^{-(t-\tau)}, \quad 0 \le \tau \le t$

$$y(t) = \int_0^t e^{-(t-\tau)} d\tau = e^{-t} \cdot \left(e^{\tau} \Big|_0^t \right) = 1 - e^{-t}$$

  - $t \ge 2$**:** $w_t(\tau) = e^{-(t-\tau)}, \quad 0 \le \tau \le 2$

$$y(t) = \int_0^2 e^{-(t-\tau)} d\tau$$

$$= e^{-t} \cdot \left(e^{\tau} \Big|_0^2 \right) = \left(e^2 - 1 \right) e^{-t}$$



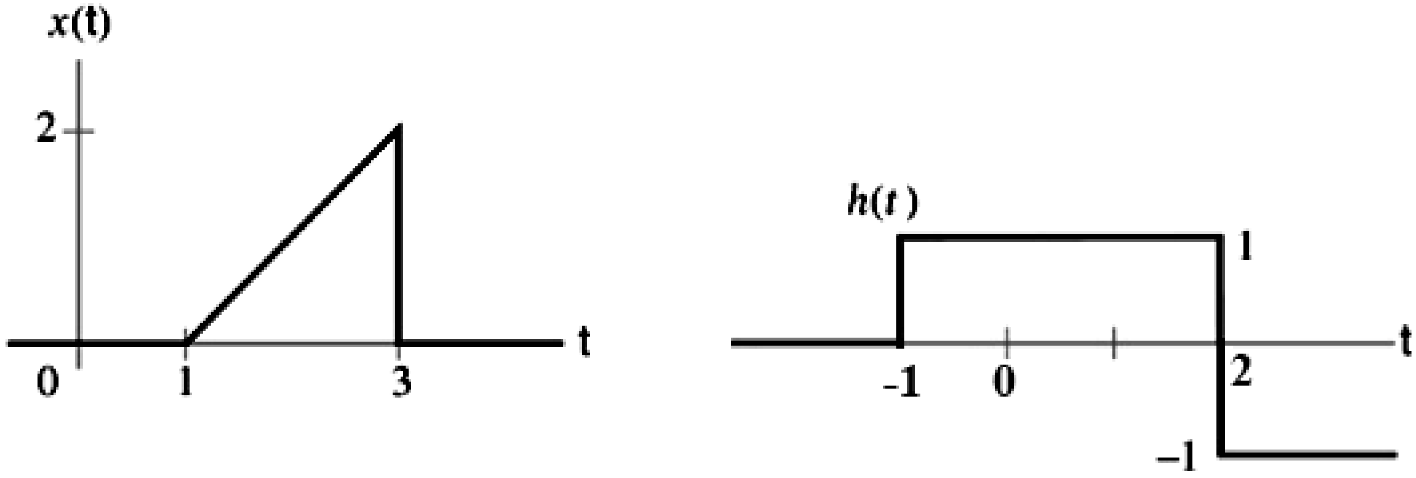
Convolution Integral Evaluation Procedure

Example 2.8 ***Another Reflect-and-Shift Convolution Evaluation***

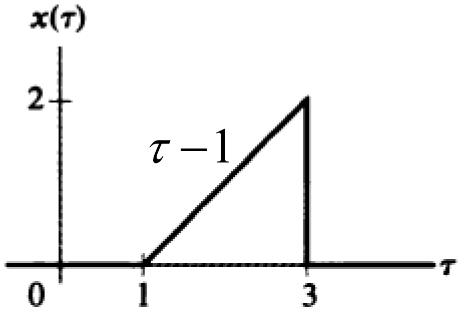
Suppose that the input $x(t)$ and impulse response $h(t)$ of an *LTI* system are, respectively, given by

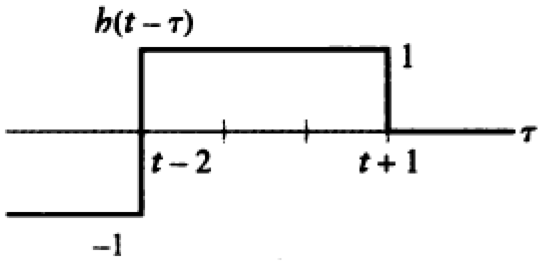
$$x(t) = (t-1)\left[u(t-1) - u(t-3)\right] \quad \textbf{and} \quad h(t) = u(t+1) - 2u(t-2)$$

Find the output of the system.

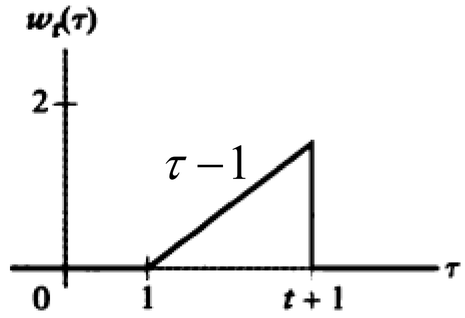


Convolution Integral Evaluation Procedure

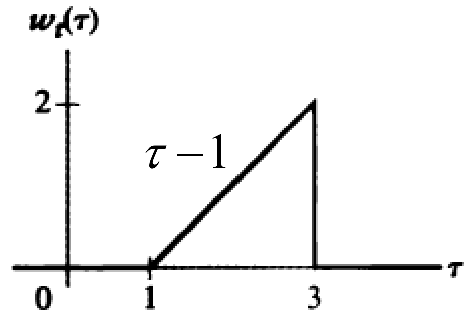




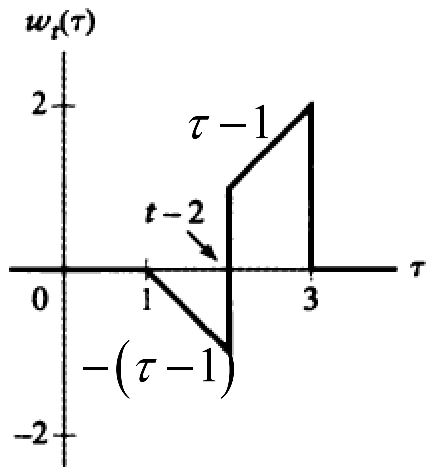
- **$1 \le t+1 < 3$, i.e. $0 \le t < 2$**



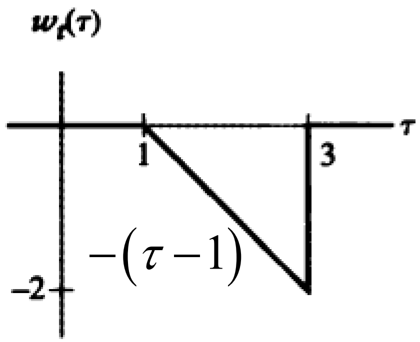
- **$0 \le t-2 < 1$, i.e. $2 \le t < 3$**



- **$1 \le t-2 < 3$, i.e. $3 \le t < 5$**

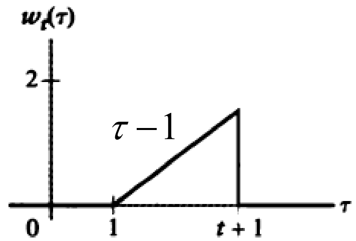


- **$t \ge 5$**



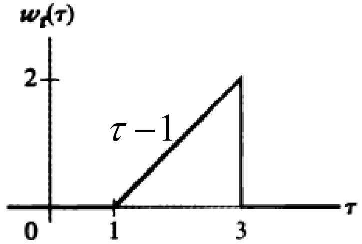
Convolution Integral Evaluation Procedure

- **Five intervals**
  - $\boldsymbol{t<0}$**:** $w_t(\tau)=0 \qquad y(t)=0$
  - $\boldsymbol{0\le t<2}$**:** $w_t(\tau)=\tau-1,\quad 1\le\tau\le t+1$



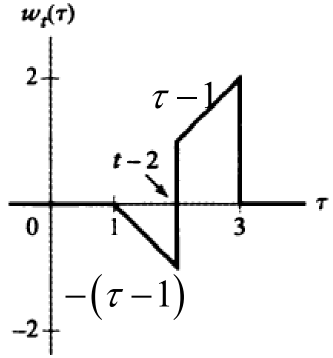
$$y(t)=\int_1^{t+1}(\tau-1)\,d\tau=\left(\frac{\tau^2}{2}-\tau\right)\Bigg|_1^{t+1}=\frac{t^2}{2}$$

  - $\boldsymbol{2\le t<3}$**:** $w_t(\tau)=\tau-1,\quad 1\le\tau\le 3$



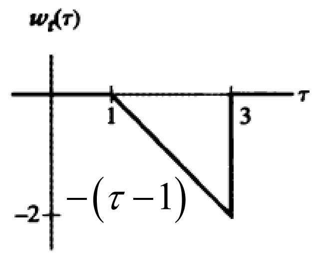
$$y(t)=\int_1^{3}(\tau-1)\,d\tau=\frac{(3-1)^2}{2}=2$$

  - $\boldsymbol{3\le t<5}$**:** $w_t(\tau)=\begin{cases}-(\tau-1), & 1\le\tau\le t-2\\ \tau-1, & t-2\le\tau\le 3\end{cases}$



$$y(t)=-\int_1^{t-2}(\tau-1)\,d\tau+\int_{t-2}^{3}(\tau-1)\,d\tau=-t^2+6t-7$$

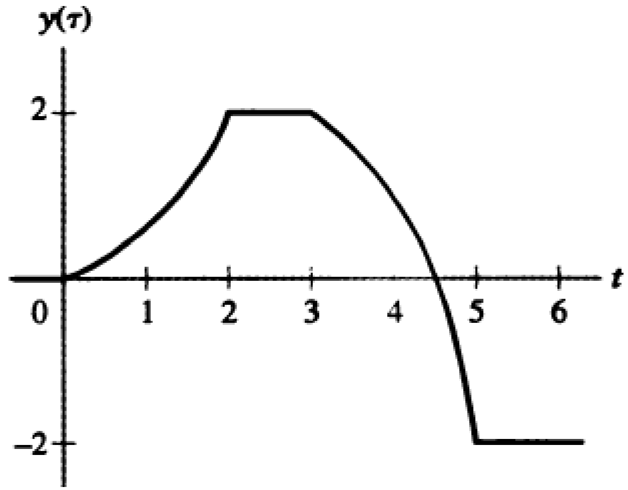
  - $\boldsymbol{t\ge 5}$**:** $w_t(\tau)=-(\tau-1),\quad 1\le\tau\le 3$



$$y(t)=\int_1^{3}-(\tau-1)\,d\tau=-2$$

Convolution Integral Evaluation Procedure

$$\therefore \quad y(t)=\begin{cases} 0, & t<0 \\ \dfrac{t^2}{2}, & 0\le t<2 \\ 2, & 2\le t<3 \\ -t^2+6t-7, & 3\le t<5 \\ -2, & t\ge 5 \end{cases}$$



- **Convolution with an impulse**

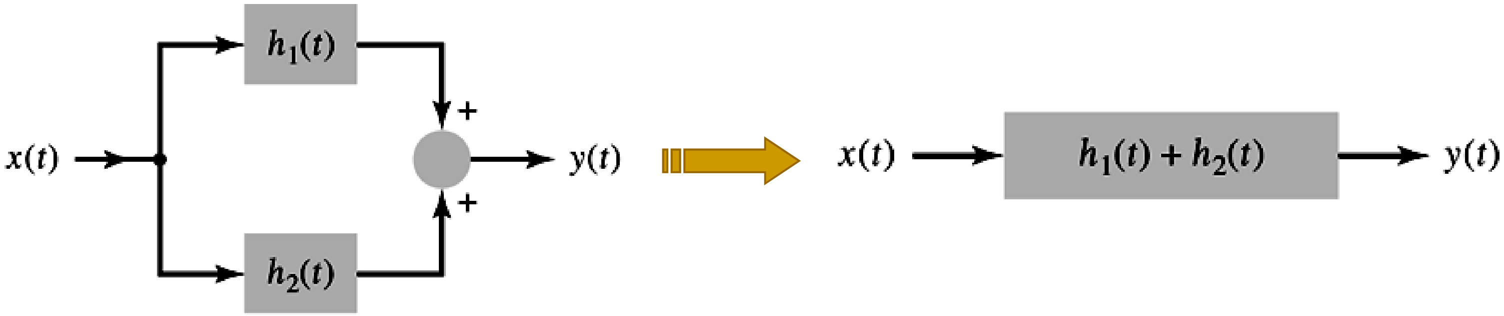
$$x(t)*\delta(t)=\int_{-\infty}^{\infty}x(\tau)\delta(t-\tau)d\tau \;=x(t)$$

$$x(t-T_1)*\delta(t-T_2)\;=\int_{-\infty}^{\infty}x(\tau-T_1)\delta(t-\tau-T_2)d\tau$$

$$=\int_{-\infty}^{\infty}x(\tau')\delta(t-\tau'-T_1-T_2)d\tau' \;=x(t-T_1-T_2)$$

Interconnections of LTI Systems

- **Parallel Connection of LTI Systems**



$$y(t) = y_1(t) + y_2(t) = x(t) * h_1(t) + x(t) * h_2(t) = \int_{-\infty}^{\infty} x(\tau)h_1(t-\tau)d\tau + \int_{-\infty}^{\infty} x(\tau)h_2(t-\tau)d\tau$$

$$= \int_{-\infty}^{\infty} x(\tau)\{h_1(t-\tau) + h_2(t-\tau)\}d\tau$$

$h(t) = h_1(t) + h_2(t)$ $\Longrightarrow$ $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = x(t) * h(t)$

- Distributive property for Continuous-time case

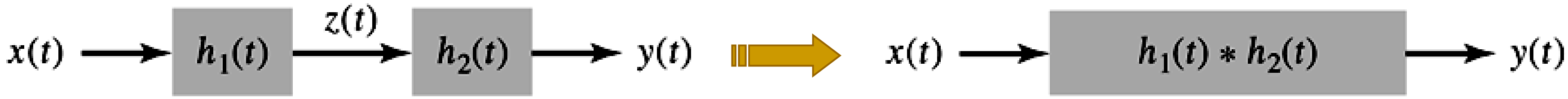
$$\boxed{x(t) * h_1(t) + x(t) * h_2(t) = x(t) * \{h_1(t) + h_2(t)\}}$$

- Distributive property for Discrete-time case

$$\boxed{x[n] * h_1[n] + x[n] * h_2[n] = x[n] * \{h_1[n] + h_2[n]\}}$$

Interconnections of LTI Systems

- **Cascade Connection of Systems**



$$y(t) = z(t) * h_2(t) = \int_{-\infty}^{\infty} z(\tau)h_2(t-\tau)\,d\tau = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(\nu)h_1(\tau-\nu)h_2(t-\tau)\,d\nu d\tau$$

$$\overset{\eta=\tau-\nu}{=} \int_{-\infty}^{\infty} x(\nu)\left[\int_{-\infty}^{\infty} h_1(\eta)h_2(t-\nu-\eta)\,d\eta\right]d\nu$$

Define $h(t) = h_1(t) * h_2(t)$, then $h(t-v) = \int_{-\infty}^{\infty} h_1(\eta)h_2(t-v-\eta)d\eta$

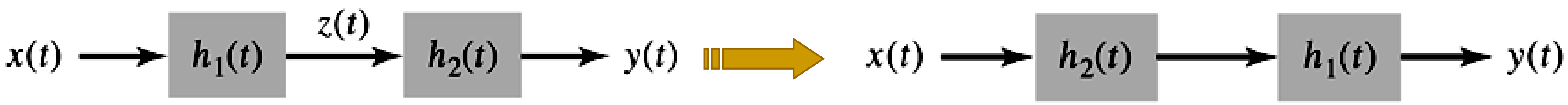
$$\Rightarrow\quad y(t) = \int_{-\infty}^{\infty} x(\nu)h(t-\nu)\,d\nu = x(t) * h(t)$$

- Associative property for Continuous-time case

$$\boxed{\{x(t) * h_1(t)\} * h_2(t) = x(t) * \{h_1(t) * h_2(t)\}}$$

Interconnections of LTI Systems

- Commutative property for Continuous-time case



$$h(t) = h_1(t) * h_2(t) = \int_{-\infty}^{\infty} h_1(\tau) h_2(t-\tau)\, d\tau$$

$$\xrightarrow{\nu = t - \tau} \quad h(t) = \int_{-\infty}^{\infty} h_1(t-\nu) h_2(\nu)\, d\nu = h_2(t) * h_1(t)$$

$$\boxed{h_1(t) * h_2(t) = h_2(t) * h_1(t)}$$

- Associative property for Discrete-time case

$$\{x[n] * h_1[n]\} * h_2[n] = x[n] * \{h_1[n] * h_2[n]\}$$

- Commutative property for Discrete-time case

$$h_1[n] * h_2[n] = h_2[n] * h_1[n].$$

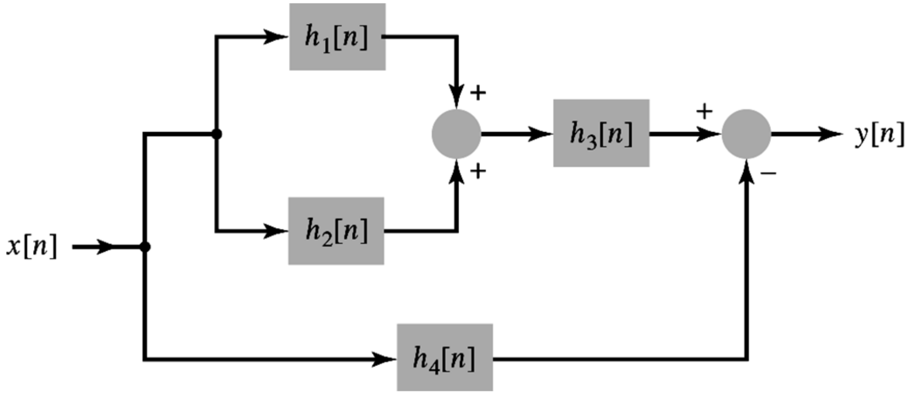
Interconnections of LTI Systems

Example 2.11 ***Equivalent System to Four Interconnected Systems***

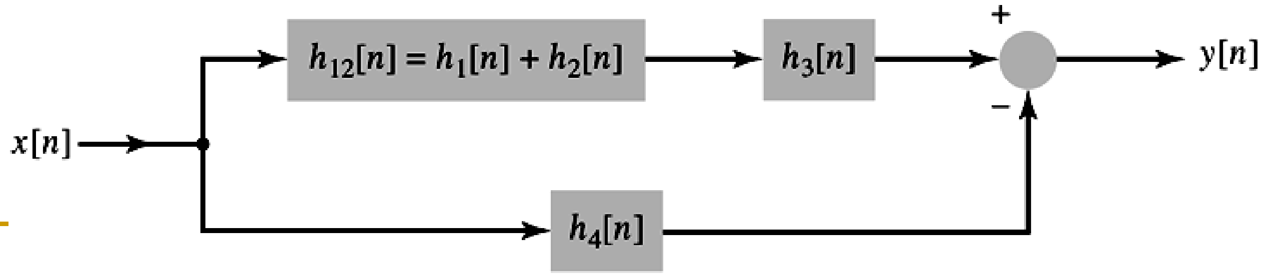
Consider the interconnection of four *LTI* systems, as depicted in Fig. 2.20. The impulse responses of the systems are

$h_1[n] = u[n]$, $h_2[n] = u[n+2] - u[n]$, $h_3[n] = \delta[n-2]$, **and** $h_4[n] = \alpha^n u[n]$.

Find the impulse response $h[n]$ of the overall system.



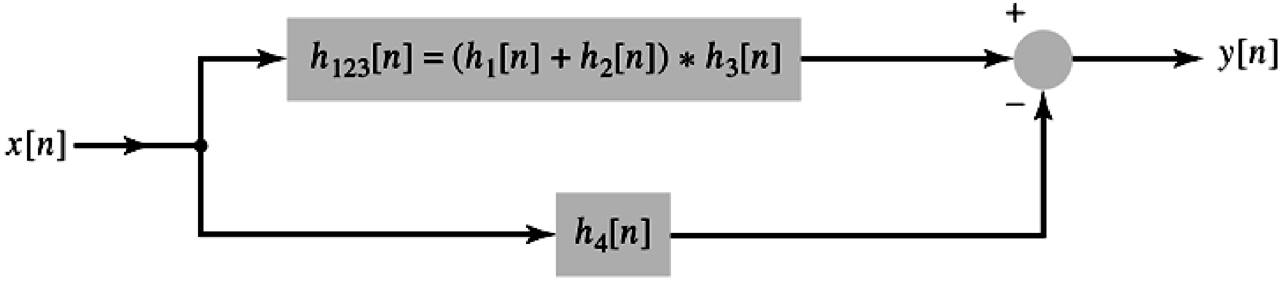
<Sol.> ❑ **Parallel combination of $h_1[n]$ and $h_2[n]$:** $h_{12}[n] = h_1[n] + h_2[n]$



(a)

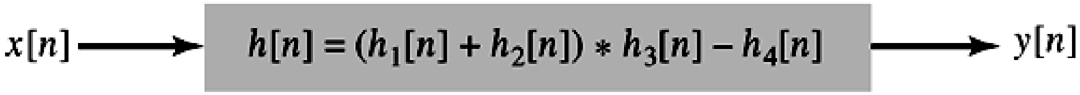
Interconnections of LTI Systems

- $h_{12}[n]$ is in series with $h_3[n]$:



(b)

- $h_{123}[n]$ is in parallel with $h_4[n]$:

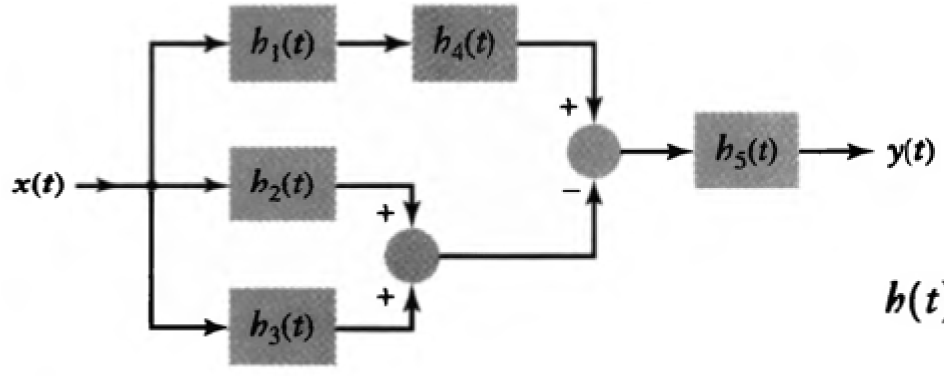


(c)

$$
\begin{aligned}
h[n] &= (h_1[n] + h_2[n]) * h_3[n] - h_4[n] \\
&= \left(u[n] + u[n+2] - u[n]\right) * \delta[n-2] - \alpha^n u[n] \\
&= u[n+2] * \delta[n-2] - \alpha^n u[n] \\
&= u[n] - \alpha^n u[n] \qquad = \left\{1 - \alpha^n\right\} u[n].
\end{aligned}
$$

Interconnections of LTI Systems

Problem 2.8 ***Equivalent System to Five Interconnected Systems***



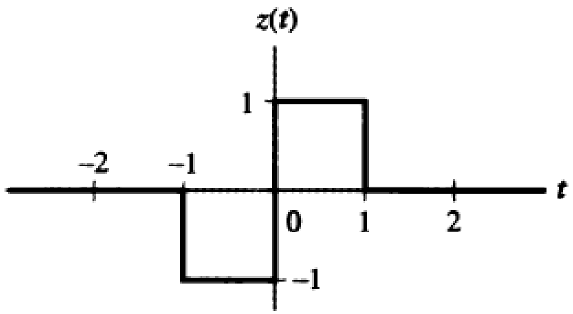
$$h(t) = [h_1(t) * h_4(t) - h_2(t) - h_3(t)] * h_5(t).$$

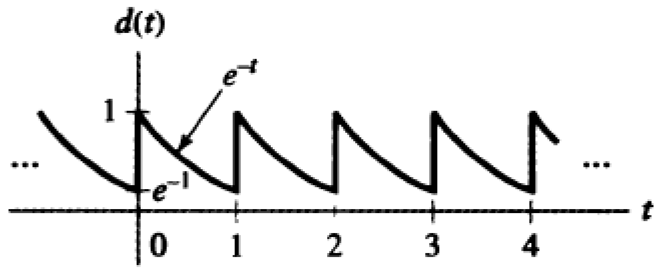
Table 2.1 Interconnection Properties for LTI Systems

Property	Continuous-time system	Discrete-time system
Distributive	$x(t)*h_1(t)+x(t)*h_2(t)=$ $x(t)*\{h_1(t)+h_2(t)\}$	$x[n]*h_1[n]+x[n]*h_2[n]=$ $x[n]*\{h_1[n]+h_2[n]\}$
Associative	$\{x(t)*h_1(t)\}*h_2(t)=x(t)*\{h_1(t)*h_2(t)\}$	$\{x[n]*h_1[n]\}*h_2[n]=x[n]*\{h_1[n]*h_2[n]\}$
Commutative	$h_1(t)*h_2(t)=h_2(t)*h_1(t)$	$h_1[n]*h_2[n]=h_2[n]*h_1[n]$

2.40 Consider the continuous-time signals depicted in Fig. P2.40. Evaluate the following convolution integrals:

(p) $m(t) = z(t) * d(t)$





let $$d'(t) = \begin{cases} e^{-t} & 0 \le t \le 1 \\ 0 & \text{otherwise} \end{cases} \Longrightarrow d(t) = \sum_{k=-\infty}^{\infty} d'(t-k)$$

consider $$m'(t) = z(t) * d'(t)$$

$$m'(t) = \begin{cases} 0 & t < -1 \\ e^{-(t+1)} - 1 & -1 \le t < 0 \\ 1 + e^{-1} - 2e^{-t} & 0 \le t < 1 \\ e^{-(t-1)} - e^{-1} & 1 \le t < 2 \\ 0 & t \ge 2 \end{cases} \Longrightarrow m(t) = \sum_{k=-\infty}^{\infty} m'(t-k)$$

Relation between LTI system properties and impulse response

- **Memoryless LTI Systems**
  - **discrete-time case:**

$$y[n] = h[n] * x[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

$$= \cdots + h[-2]x[n+2] + h[-1]x[n+1] + \underline{h[0]x[n]} + h[1]x[n-1] + h[2]x[n-2] + \cdots$$

To be memoryless, $y[n]$ must depend only on $x[n]$ and therefore cannot depend on $x[n-k]$ for $k \neq 0$.

A discrete-time LTI system is *memoryless* if and only if

$h[k] = c\delta[k]$ where c is an arbitrary constant.

  - **continuous-time case:**

$$y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau,$$

A continuous-time LTI system is *memoryless* if and only if

$h(\tau) = c\delta(\tau)$ where c is an arbitrary constant.

Relation between LTI system properties and impulse response

- **Causal LTI Systems**
  - **discrete-time case:**

$$y[n]=\sum_{k=-\infty}^{\infty} h[k]x[n-k]=\underline{\cdots+h[-2]x[n+2]+h[-1]x[n+1]}+h[0]x[n]$$

$$+h[1]x[n-1]+h[2]x[n-2]+\cdots.$$

For a discrete-time ***causal*** LTI system,

$$h[k]=0 \quad \text{for} \quad k<0$$

Convolution sum in new form: $y[n]=\sum_{\boxed{k=0}}^{\infty} h[k]x[n-k].$

  - **continuous-time case:**

$$y(t)=\int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau.$$

For a continuous-time ***causal*** LTI system, $h(\tau)=0 \quad \text{for} \quad \tau<0$

Convolution integral in new form: $y(t)=\int_{\boxed{0}}^{\infty} h(\tau)x(t-\tau)d\tau.$

Relation between LTI system properties and impulse response

- **Stable LTI Systems**

  A system is BIBO stable if the output is guaranteed to be bounded for every bounded input.

  - **discrete-time case:** $|x[n]| \le M_x \le \infty \Longrightarrow |y[n]| \le M_y \le \infty$

$$|y[n]| = |h[n] * x[n]| = \left| \sum_{k=-\infty}^{\infty} h[k]x[n-k] \right| \le \sum_{k=-\infty}^{\infty} |h[k]x[n-k]|$$

$$\le \sum_{k=-\infty}^{\infty} |h[k]||x[n-k]| \xrightarrow{|x[n]| \le M_x} |y[n]| \le M_x \sum_{k=-\infty}^{\infty} |h[k]|$$

Condition for impulse response of a *stable* discrete-time LTI system:

$$\sum_{k=-\infty}^{\infty} |h[k]| < \infty.$$

  - **continuous-time case:** $\int_{-\infty}^{\infty} |h(\tau)| d\tau < \infty$

Example 2.12 ***Properties of the First-Order Recursive System***

The first-order system is described by the difference equation

$$y[n] = \rho y[n-1] + x[n]$$

and has the impulse response

$$h[n] = \rho^n u[n]$$

Is this system causal, memoryless, and BIBO stable?

<Sol.>

- **The system is causal, since $h[n] = 0$ for $n < 0$.**
- **The system is not memoryless, since $h[n] \neq 0$ for $n > 0$.**
- **The system is stable, provided that $|\rho| < 1$.**

$$\sum_{k=-\infty}^{\infty} |h[k]| = \sum_{k=0}^{\infty} \left|\rho^k\right| = \sum_{k=0}^{\infty} |\rho|^k < \infty \quad \text{if and only if } |\rho| < 1.$$

Relation between LTI system properties and impulse response

- **Note: A system can be unstable even though the impulse response has a finite value.**
  - **Ideal integrator:** $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$

    Impulse response: $h(t) = \int_{-\infty}^{t} \delta(\tau)d\tau = u(t)$ **~ not absolutely integrable**

    ⇒ **Ideal integrator is *not stable*!**

  - **Ideal accumulator:** $y[n] = \sum_{k=-\infty}^{n} x[k]$

    Impulse response: $h[n] = \sum_{k=-\infty}^{n} \delta[k] = u[n]$ **~ not absolutely summable**

    ⇒ **Ideal accumulator is *not stable*!**

- **Invertible Systems and Deconvolution**

A system is *invertible* if the input to the system can be recovered from the output except for a constant scale factor.

- **continuous-time case:**

$$x(t) \longrightarrow \boxed{h(t)} \xrightarrow{y(t)} \boxed{h^{\text{inv}}(t)} \longrightarrow x(t)$$

***h*(*t*)** = impulse response of LTI system

$h^{\text{inv}}(t)$ = impulse response of LTI inverse system

$$x(t) * (h(t) * h^{inv}(t)) = x(t). \quad \Longrightarrow \quad h(t) * h^{inv}(t) = \delta(t)$$

- **discrete-time case:**

$$h[n] * h^{inv}[n] = \delta[n]$$

- **Deconvolution: the process of recovering *x*(*t*) from *h*(*t*)**x*(*t*).**

- **An inverse system performs deconvolution.**

Example 2.13 ***Multipath Communication Channels: Compensation by means of an Inverse System***

Consider designing a discrete-time inverse system to eliminate the distortion associated with multipath propagation in a data transmission problem. Assume that a discrete-time model for a two-path communication channel is

$$y[n] = x[n] + ax[n-1].$$

Find a causal inverse system that recovers $x[n]$ from $y[n]$. Check whether this inverse system is stable.

<Sol.>

- **Impulse response:** $h[n] = \delta[n] + a\delta[n-1] = \begin{cases} 1, & n = 0 \\ a, & n = 1 \\ 0, & \text{otherwise} \end{cases}$

- **The inverse system $h^{\text{inv}}[n]$ must satisfy $h[n] * h^{\text{inv}}[n] = \delta[n]$.**

$$h^{inv}[n] * \left(\delta[n] + a\delta[n-1]\right) = \delta[n]$$

$$\Rightarrow \quad h^{inv}[n] + ah^{inv}[n-1] = \delta[n].$$

Relation between LTI system properties and impulse response

$$h^{inv}[n] + a h^{inv}[n-1] = \delta[n].$$

- **in order to obtain a *causal* inverse system, we have**

$$h^{inv}[n] = 0, \quad for \quad n < 0.$$

- **For $n = 0$:** $h^{inv}[0] + a h^{inv}[-1] = \delta[0] = 1 \Rightarrow h^{inv}[0] = 1$
- **For $n > 0$:** $h^{inv}[n] + a h^{inv}[n-1] = 0 \Rightarrow h^{inv}[n] = -a h^{inv}[n-1]$

Since $h^{inv}[0] = 1$, we have

$$h^{inv}[1] = -a, \quad h^{inv}[2] = a^2, \quad h^{inv}[3] = -a^3, \cdots\cdots$$

$$\Rightarrow \quad h^{inv}[n] = (-a)^n u[n]$$

- **To check for stability,**

$$\sum_{k=-\infty}^{\infty} \left| h^{inv}[k] \right| = \sum_{k=0}^{\infty} |a|^k < \infty \qquad \textbf{if and only if } |a| < 1.$$

For $|a| < 1$, the system is stable.

Relation between LTI system properties and impulse response

- **Summary**

Table 2.2 *Properties of the Impulse Response Representation for LTI Systems*

Property	Continuous-time system	Discrete-time system
Memoryless	$h(t)=c\delta(t)$	$h[n]=c\delta[n]$
Causal	$h(t)=0 \quad \text{for} \quad t<0$	$h[n]=0 \quad \text{for} \quad n<0$
Stability	$\int_{-\infty}^{\infty}\lvert h(t)\rvert dt<\infty$	$\sum_{n=-\infty}^{\infty}\lvert h[n]\rvert<\infty$
Invertibility	$h(t)*h^{inv}=\delta(t)$	$h[n]*h^{inv}[n]=\delta[n]$

Step Response (阶跃响应)

- ***Step response***: **output due to a unit step input signal**
  - **discrete-time LTI system: $s[n]$**

$$s[n] = h[n] * u[n] = \sum_{k=-\infty}^{\infty} h[k]u[n-k] = \sum_{k=-\infty}^{n} h[k]$$

The step response is the running sum of the impulse response.

  - **continuous-time LTI system: $s(t)$**

$$s(t) = \int_{-\infty}^{t} h(\tau)d\tau$$

The step response is the running integral of the impulse response.

- **Express the impulse response in terms of the step response as**

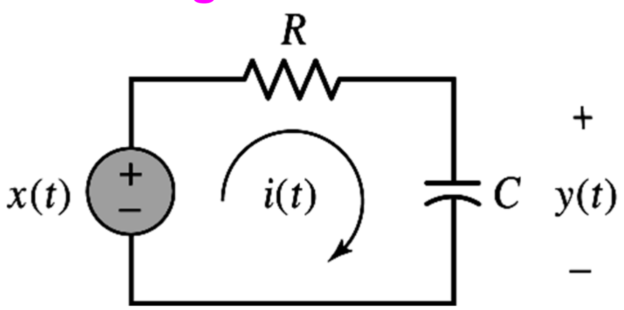
$$h[n] = s[n] - s[n-1] \qquad \text{and} \qquad h(t) = \frac{d}{dt}s(t)$$

Step Response

Example 2.14 *RC* Circuit: Step Response

The impulse response of the *RC* circuit depicted in Fig. 2.12 is

$$h(t) = \frac{1}{RC} e^{-\frac{t}{RC}} u(t)$$



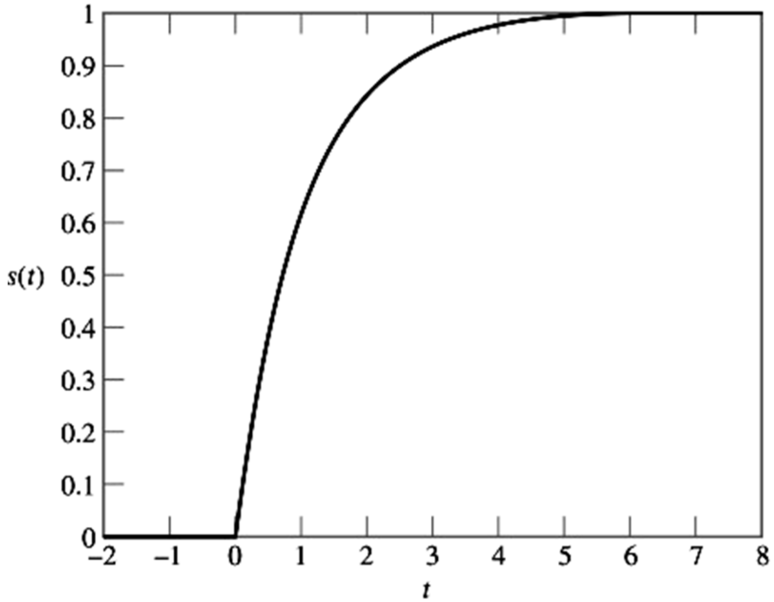
Find the step response of the circuit.

<Sol.>

$$s(t) = \int_{-\infty}^{t} \frac{1}{RC} e^{-\frac{\tau}{RC}} u(\tau) d\tau.$$

$$= \begin{cases} 0, & t < 0 \\ \frac{1}{RC} \int_{0}^{t} e^{-\frac{\tau}{RC}} d\tau, & t \geq 0 \end{cases}$$

$$= \begin{cases} 0, & t < 0 \\ 1 - e^{-\frac{t}{RC}}, & t \geq 0 \end{cases}$$



Summary

- Linear Time-invariant systems (LTI)
  - The Convolution Integral
  - Convolution Integral Evaluation Procedure
  - Interconnections of LTI Systems
  - Relations between LTI System Properties and the Impulse Response
  - Step Response

- Reference in textbook: 2.4~2.8
- Homework: 2.39(a,b,i,m), 2.40(g,k), 2.46, 2.48,
  2.49(a,f,h,k), 2.50(a,c,e,f)