Ch 3.2 Fourier Series

Prof. Shaoshi Yang

E-mail: shaoshi.yang@bupt.edu.cn

Web: https://teacher.bupt.edu.cn/yangshaoshi

School of Information & Communication Engineering

BUPT

Outline

- Fourier series and transform
  - Continuous-Time Periodic Signals: Fourier series
  - Discrete-Time Periodic Signals: Discrete-Time Fourier Series

Fourier series (傅立叶级数)

- Periodic signals can be expressed as a sum of sinusoids. In this case, the **frequency spectrum** can be generated by computation of the ***Fourier series***.
- The Fourier series is named after the French physicist Jean Baptiste Fourier(1768-1830), who was the first one to propose that **periodic** waveforms could be represented by a sum of sinusoids (or complex exponentials).
- An example showing how the Fourier series work:

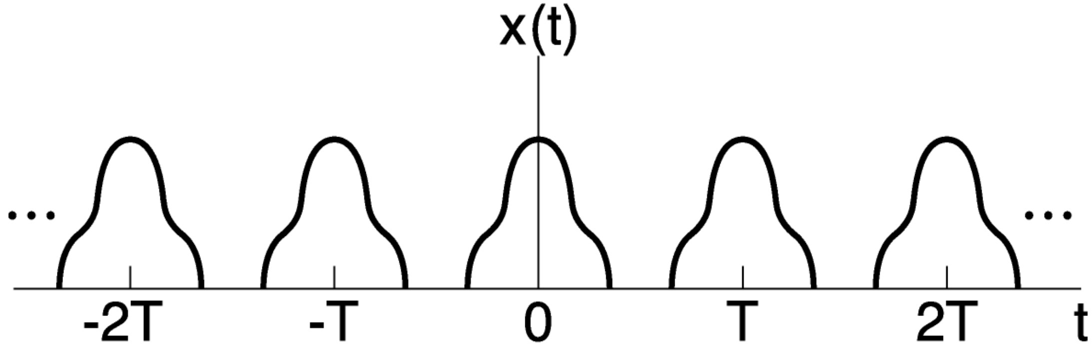
  http://www.falstad.com/fourier/

Periodic signal

$x(t) = x(t+T) \quad \forall t,$ where T is a positive constant.

$T = T_0, \quad 2T_0, \quad 3T_0, \quad ......$

- Fundamental period: $T = T_0$
- Fundamental frequency: $f = \dfrac{1}{T},$ measured in hertz(Hz).
- Angular frequency: $\omega_0 = 2\pi f = \dfrac{2\pi}{T},$ measured in radians per second.



$$x(t) = \cos(\omega_0 t + \phi)$$

$$x(t) = e^{j\omega_0 t}$$

Trigonometric Fourier Series

- A periodic signal $x(t)$ with period is T, can be represented by the appropriate sum of sine and cosine components:

$$\boxed{x(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos(k\omega_0 t) + \sum_{k=1}^{\infty} b_k \sin(k\omega_0 t)}$$

  - a_0 is the **mean value,** or **zero frequency** term.

$$\int_0^T x(t)\,dt = \int_0^T a_0\,dt + \int_0^T \left[\cancel{\sum_{k=1}^{\infty} a_k \cos(k\omega_0 t) + \sum_{k=1}^{\infty} b_k \sin(k\omega_0 t)}\right] dt$$

$$= \int_0^T a_0\,dt \quad = a_0 T$$

$$\boxed{a_0 = \frac{1}{T}\int_0^T x(t)\,dt}$$

Trigonometric Fourier Series

- **To find a_k,**

$$\int_0^T x(t)\cos(m\omega_0 t)dt = \int_0^T \cancel{a_0 \cos(m\omega_0 t)dt}$$

$$+\int_0^T \left[\sum_{k=1}^{\infty} a_k \cos(k\omega_0 t)\cos(m\omega_0 t) + \sum_{k=1}^{\infty} b_k \sin(k\omega_0 t)\cos(m\omega_0 t)\right]dt$$

$$= \sum_{k=1}^{\infty}\int_0^T \frac{a_k}{2}\left[\cos(m+k)\omega_0 t + \cos(m-k)\omega_0 t\right]dt$$

$$+\sum_{k=1}^{\infty}\int_0^T \frac{b_k}{2}\left[\sin(m+k)\omega_0 t + \sin(m-k)\omega_0 t\right]dt$$

$$= \int_0^T \frac{a_m}{2}\cos 0\cdot\omega_0 t dt \quad = \frac{a_m T}{2}$$

$$\Rightarrow \boxed{a_k = \frac{2}{T}\int_0^T x(t)\cos(k\omega_0 t)dt} \qquad \boxed{b_k = \frac{2}{T}\int_0^T x(t)\sin(k\omega_0 t)dt}$$

$$\cos A\cdot\cos B = \tfrac{1}{2}\left[\cos(A+B)+\cos(A-B)\right] \qquad \cos A\cdot\sin B = \tfrac{1}{2}\left[\sin(A+B)-\sin(A-B)\right]$$

Trigonometric Fourier Series

- Trigonometric FS:

$$\boxed{x(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos(k\omega_0 t) + \sum_{k=1}^{\infty} b_k \sin(k\omega_0 t)}$$

$$a_0 = \frac{1}{T}\int_0^T x(t)dt, \qquad a_k = \frac{2}{T}\int_0^T x(t)\cos(k\omega_0 t)dt$$

$$b_k = \frac{2}{T}\int_0^T x(t)\sin(k\omega_0 t)dt$$

- Trigonometric FS in the cosine-in-phase form:

$$\boxed{x(t) = a_0 + \sum_{k=1}^{\infty} A_k \cos(k\omega_0 t + \theta_k)}$$

$$A_k = \sqrt{a_k^2 + b_k^2}, \qquad \theta_k = -arctg\frac{b_k}{a_k}$$

Convergence (收敛性) *of Fourier Series*

- Fourier believed that any periodic signal could be expressed as a sum of sinusoids. However, this turned out not to be the case, although virtually all periodic signals arising in engineering do have a Fourier series representation.
- In particular, a periodic signal x(t) has a Fourier series if it satisfies the following *Dirichlet* (狄里赫利) *conditions*:
  - x(t) is absolutely integrable over any period;

$$\int_{a}^{a+T} |x(t)|dt < \infty \quad for\ any\ a$$

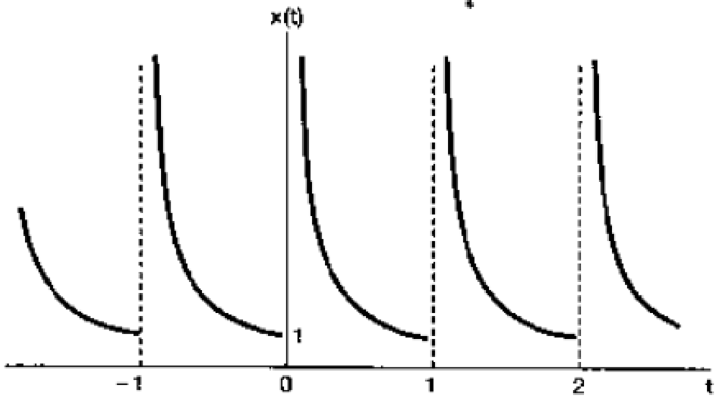
  - x(t) has only a finite number of maxima and minima over any period.
  - x(t) has only a finite number of discontinuities over any period.

Note that the Dirichlet conditions are sufficient but not necessary conditions for the Fourier series representation.

Convergence (收敛性) of Fourier Series

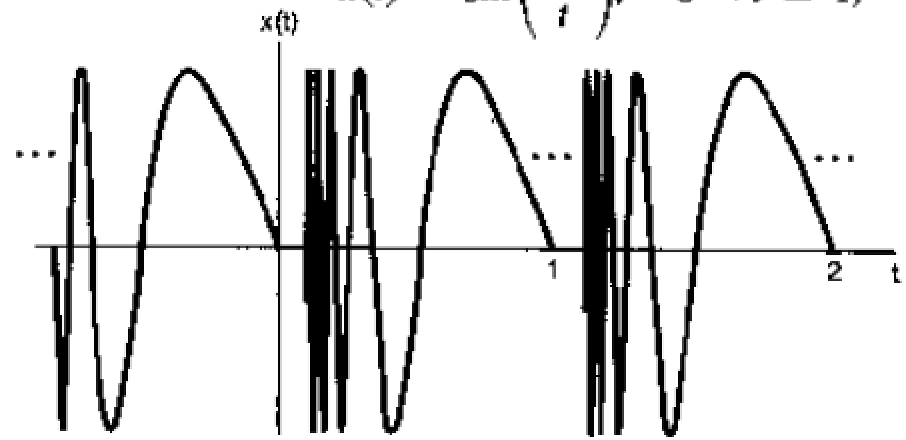
- Signals that violate the Dirichlet conditions

$$x(t) = \frac{1}{t}, \quad 0 < t \leq 1;$$

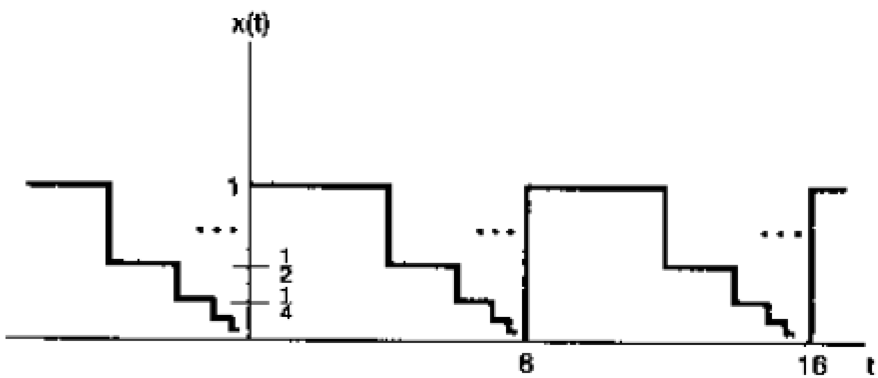


(a)

$$x(t) = \sin\left(\frac{2\pi}{t}\right), \quad 0 < t \leq 1,$$



(b)



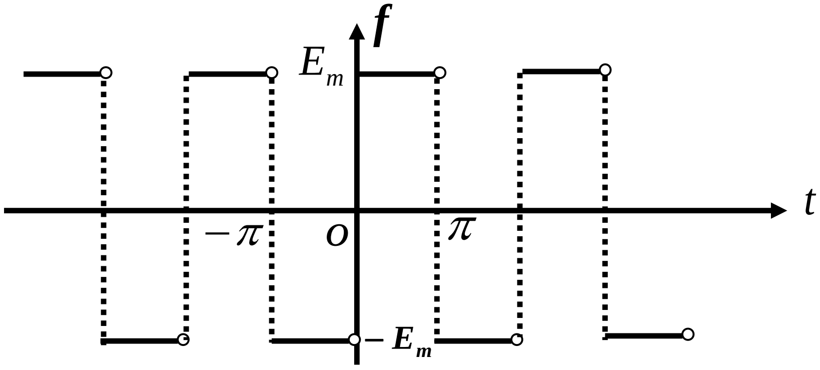
(c)

Signals that do not satisfy the Dirichlet conditions are generally pathological in nature and consequently do not typically arise in practical contexts.

Application of the Fourier Series

- **A rectangle impulse with period 2π**

$$f(t)=\begin{cases}-E_m, -\pi \le t < 0\\ E_m, 0 \le t < \pi\end{cases}$$



$$T = 2\pi \Rightarrow \omega_o = 2\pi/2\pi = 1$$

$$a_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos ktdt$$

$$= \frac{1}{\pi}\int_{-\pi}^{0}(-E_m)\cos ktdt + \frac{1}{\pi}\int_{0}^{\pi} E_m \cos ktdt \quad = 0 \quad (k=0,1,2,\cdots)$$

$$b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\sin ktdt = \frac{1}{\pi}\int_{-\pi}^{0}(-E_m)\sin ktdt + \frac{1}{\pi}\int_{0}^{\pi} E_m \sin ktdt$$

$$= \frac{2E_m}{k\pi}(1-\cos k\pi) \quad = \frac{2E_m}{k\pi}[1-(-1)^k] = \begin{cases}\dfrac{4E_m}{(2n-1)\pi}, & k = 2n-1,\ n = 1,2,\cdots\\ 0, & k = 2n,\ n = 1,2,\cdots\end{cases}$$

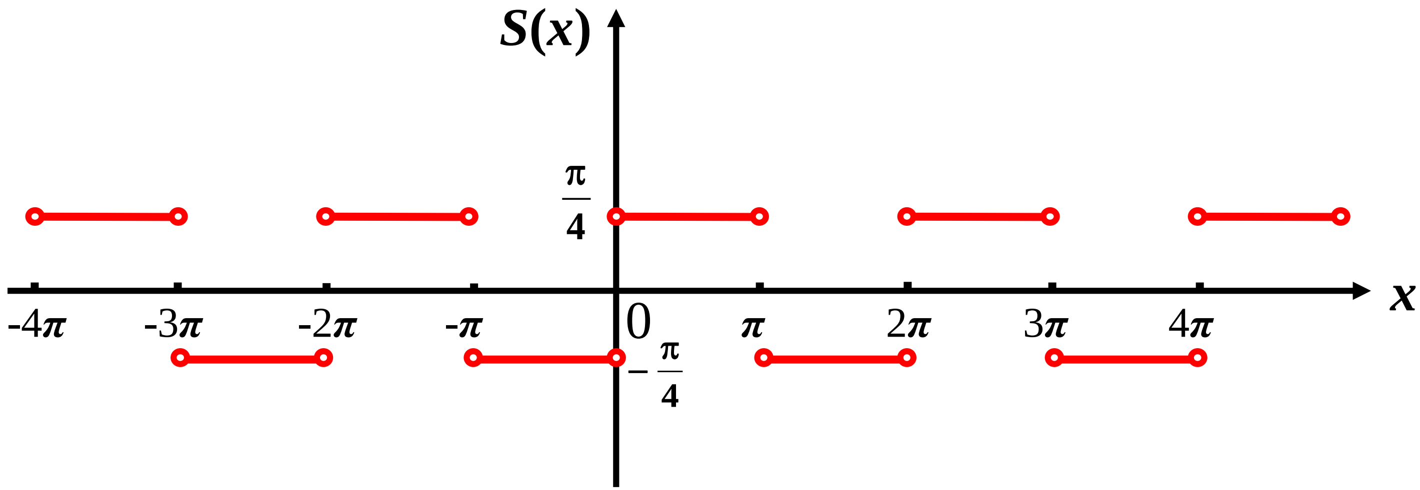
$$\Rightarrow f(t) = \sum_{k=1}^{\infty}\frac{4E_m}{(2k-1)\pi}\sin(2k-1)t$$ (- ¥ < t < +¥; t ¹ 0, ± p, ± 2p, ···)

Application of the Fourier Series

- **For E_m=1:**

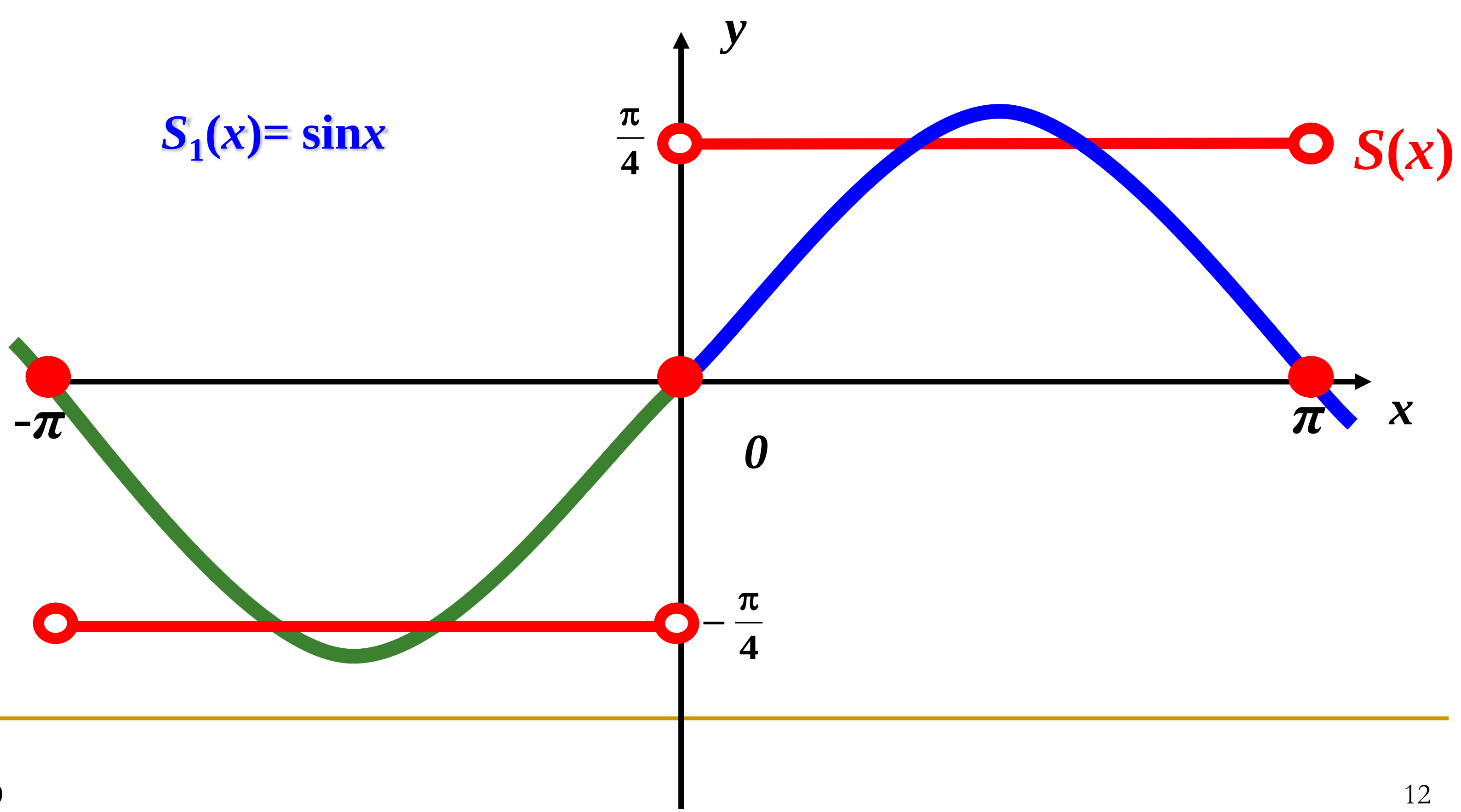
$$f(t)=\frac{4}{\pi}\sum_{k=1}^{\infty}\frac{\sin(2k-1)t}{2k-1}\qquad(-\infty<t<+\infty;\, t\neq 0,\pm\pi,\pm 2\pi,\cdots)$$

$$\Rightarrow\quad \sum_{k=1}^{\infty}\frac{\sin(2k-1)x}{2k-1}=\begin{cases}\dfrac{\pi}{4}, x\in(0,\pi)\\[2ex] -\dfrac{\pi}{4}, x\in(-\pi,0)\end{cases}\triangleq S(x)$$



Application of the Fourier Series

$$S(x)=\sum_{k=1}^{\infty}\frac{\sin(2k-1)x}{2k-1}=\begin{cases}\frac{\pi}{4}, x\in(0,\pi)\\ -\frac{\pi}{4}, x\in(-\pi,0)\end{cases}$$

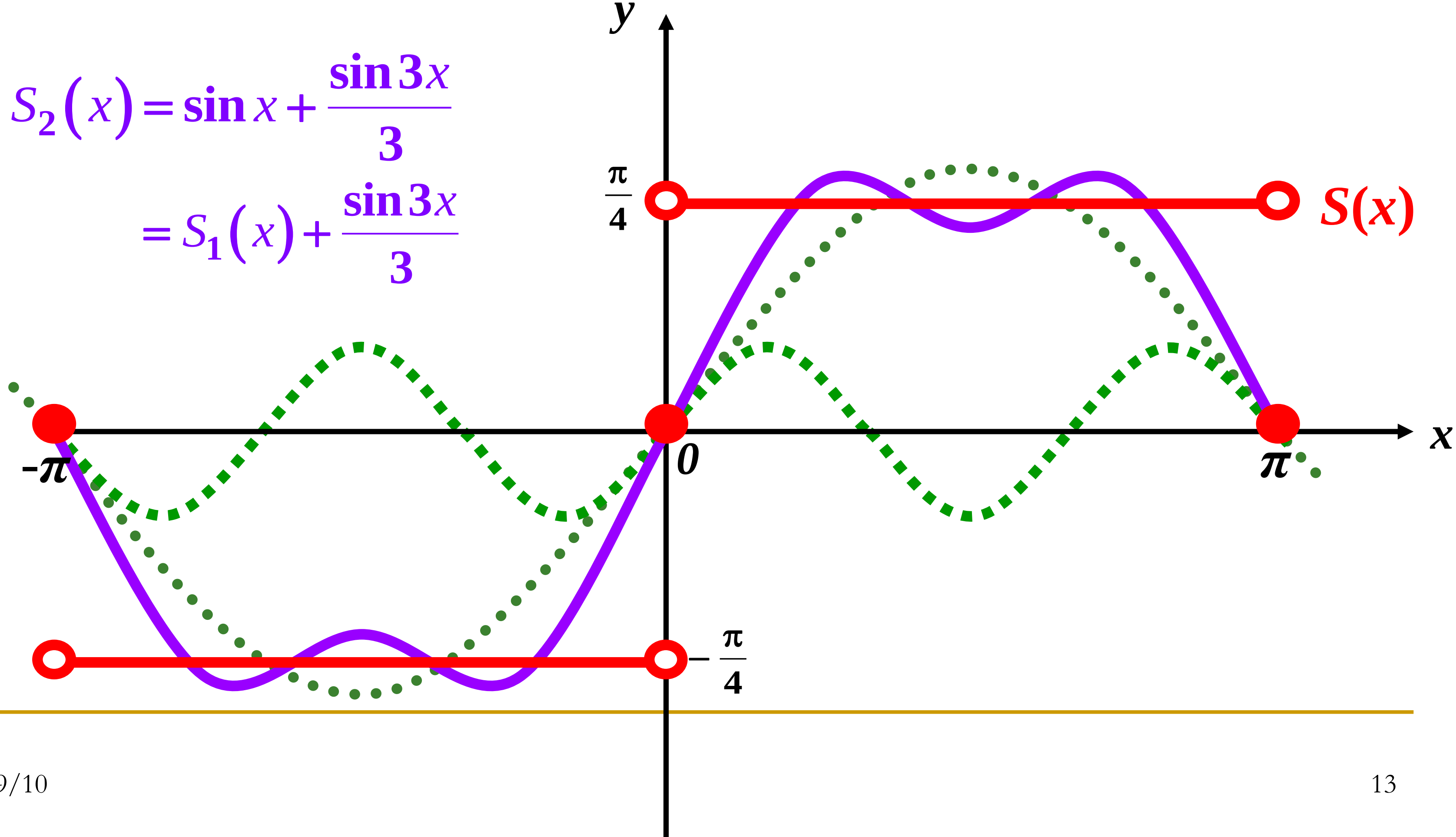


Application of the Fourier Series

$$S(x)=\sum_{k=1}^{\infty}\frac{\sin(2k-1)x}{2k-1}=\begin{cases}\dfrac{\pi}{4},x\in(0,\pi)\\ -\dfrac{\pi}{4},x\in(-\pi,0)\end{cases}$$

$$S_2(x)=\sin x+\frac{\sin 3x}{3}$$

$$=S_1(x)+\frac{\sin 3x}{3}$$

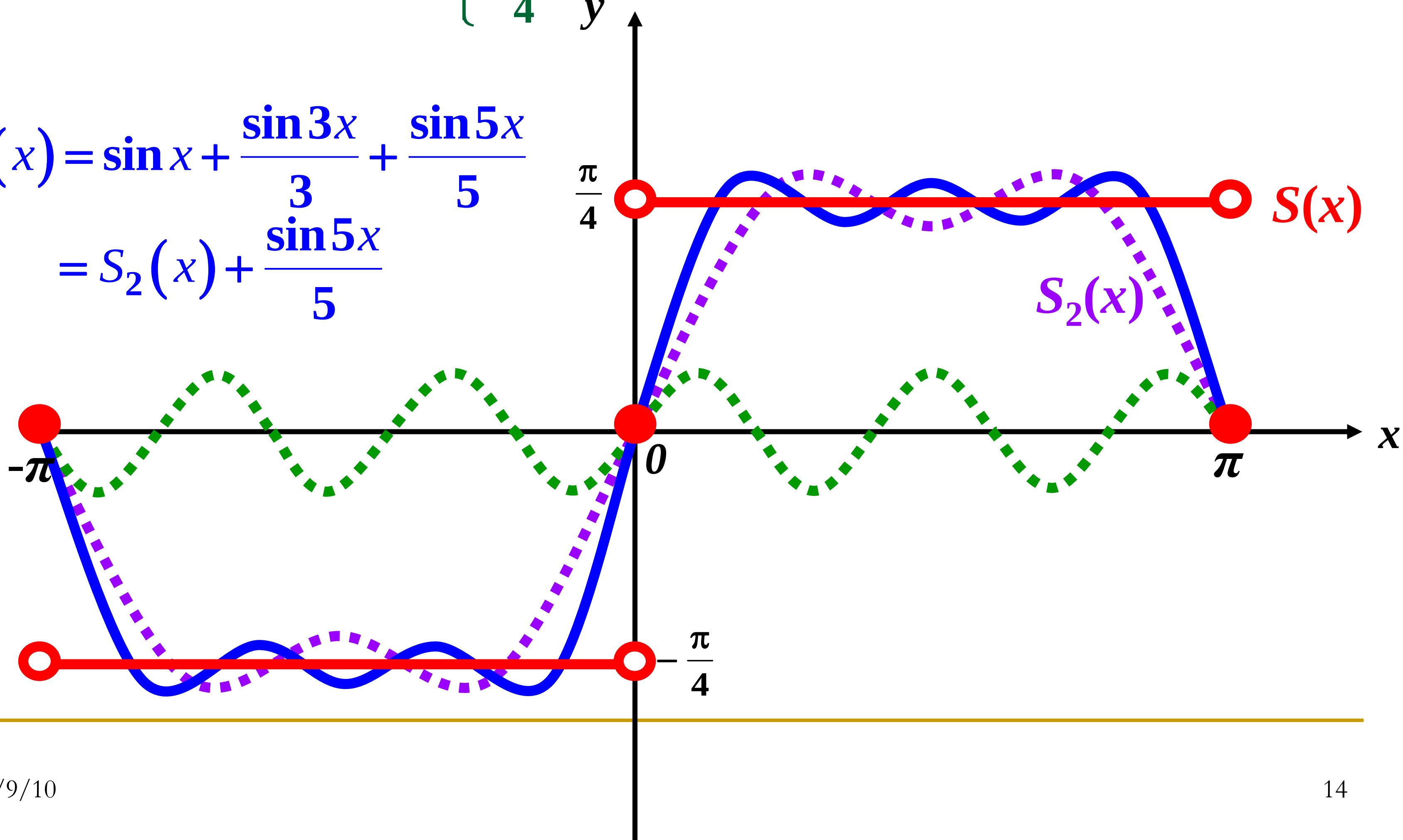


Application of the Fourier Series

$$S(x)=\sum_{k=1}^{\infty}\frac{\sin(2k-1)x}{2k-1}=\begin{cases}\dfrac{\pi}{4}, x\in(0,\pi)\\ -\dfrac{\pi}{4}, x\in(-\pi,0)\end{cases}$$

$$S_3(x)=\sin x+\frac{\sin 3x}{3}+\frac{\sin 5x}{5}$$

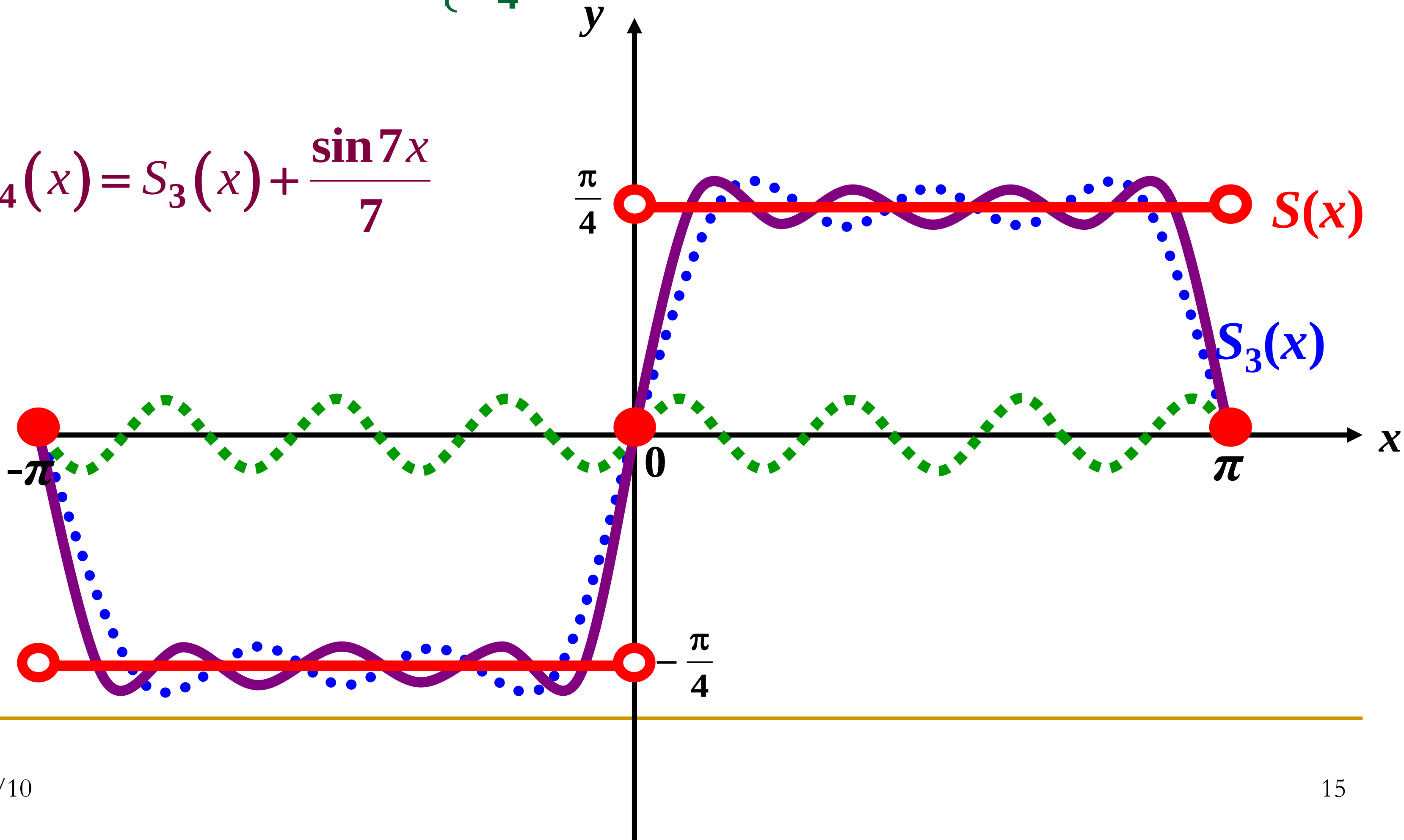
$$=S_2(x)+\frac{\sin 5x}{5}$$



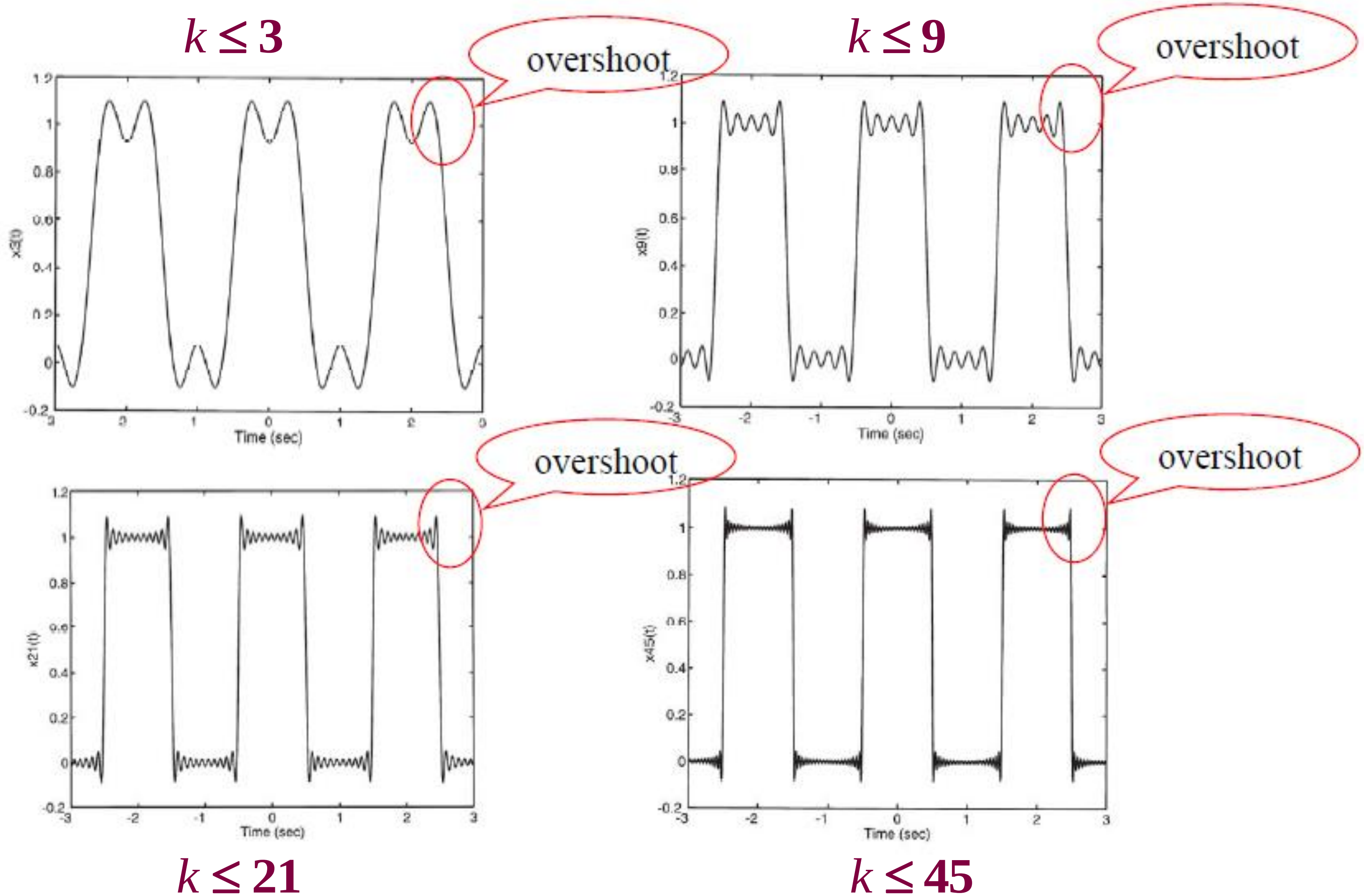
Application of the Fourier Series

$$S(x)=\sum_{k=1}^{\infty}\frac{\sin(2k-1)x}{2k-1}=\begin{cases}\dfrac{\pi}{4},x\in(0,\pi)\\ -\dfrac{\pi}{4},x\in(-\pi,0)\end{cases}$$

$$S_4(x)=S_3(x)+\frac{\sin 7x}{7}$$



Gibbs Phenomenon



Anything in common of the four diagrams?

Gibbs Phenomenon

- **The overshoot at the corners is till present even in the limit as N approaches to infinity.**
- **Gibbs phenomenon**
  - **The Fourier series representation of an arbitrary periodic signal $x(t)$ is not actually equal to the true value of $x(t)$ at any points where $x(t)$ is discontinuous.**
  - **If $x(t)$ is discontinuous at $t=t1$, the Fourier series representation is off by approximately 9% at $t1-$ and $t1+$.**

The exponential form of the Fourier Series

$$\boxed{x(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos(k\omega_0 t) + \sum_{k=1}^{\infty} b_k \sin(k\omega_0 t)}$$

$$a_k \cos(k\omega_0 t) = \frac{a_k}{2}\left[e^{jk\omega_0 t} + e^{-jk\omega_0 t}\right], \quad b_k \sin(k\omega t) = \frac{b_k}{2j}\left[e^{jk\omega_0 t} - e^{-jk\omega_0 t}\right]$$

$$a_k \cos(k\omega_0 t) + b_k \sin(k\omega_0 t) = \frac{a_k}{2}\left[e^{jk\omega_0 t} + e^{-jk\omega_0 t}\right] + \frac{b_k}{2j}\left[e^{jk\omega_0 t} - e^{-jk\omega_0 t}\right]$$

$$= X_k e^{jk\omega_0 t} + X_{-k} e^{-jk\omega_0 t}$$

where $X[k] = (a_k - jb_k)/2 = \frac{1}{T}\int_0^T x(t)\left[\cos(k\omega_0 t) - j\sin(k\omega_0 t)\right]dt$

$$= \frac{1}{T}\int_0^T x(t) e^{-jk\omega_0 t} dt$$

$$X[-k] = (a_k + jb_k)/2 = \frac{1}{T}\int_0^T x(t)\left[\cos(k\omega_0 t) + j\sin(k\omega_0 t)\right]dt = \frac{1}{T}\int_0^T x(t) e^{jk\omega_0 t} dt$$

$$X[0] = a_0 = \frac{1}{T}\int_0^T x(t)dt \Rightarrow x(t) = \sum_{k=-\infty}^{\infty} X[k] e^{jk\omega_0 t}, \quad X[k] = \frac{1}{T}\int_0^T x(t) e^{-jk\omega_0 t} dt$$

The exponential form of the Fourier Series

- **Frequency domain representation of $x(t)$**

$$x(t)=\sum_{k=-\infty}^{\infty} X[k]e^{jk\omega_0 t}$$

$$X[k]=\frac{1}{T}\int_0^T x(t)e^{-jk\omega_0 t}dt$$

~ **Fourier series coefficients** **or**
Spectral coefficients **of $x(t)$**

- **Notation**

$$x(t) \xleftrightarrow{FS;\omega_0} X[k] \quad \text{where } X[k]=a_k+jb_k=\left|X[k]\right|e^{j\arg\{X[k]\}}.$$

The variable k determines the frequency of the complex sinusoid associated with $X[k]$.

- **Magnitude spectrum of $x(t)$:** $\left|X[k]\right|=\sqrt{a_k^2+b_k^2}$
- **Phase spectrum of $x(t)$:** $\arg\{X[k]\}=arctg\dfrac{b_k}{a_k}$

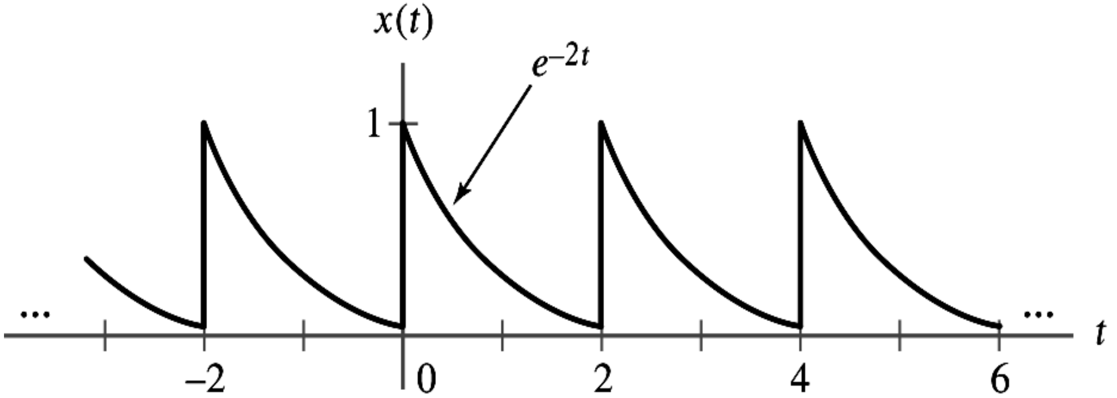
CT Periodic Signals: The Fourier Series

Example 3.9 ***Direct Calculation of FS Coefficients***

Determine the FS coefficients for the signal $x(t)$ depicted in Fig. 3.16.

<Sol.>

$T = 2$ ⇒ $\omega_o = 2\pi/2 = \pi$



$$X[k] = \frac{1}{T}\int_0^T x(t) e^{-jk\omega_0 t} dt$$

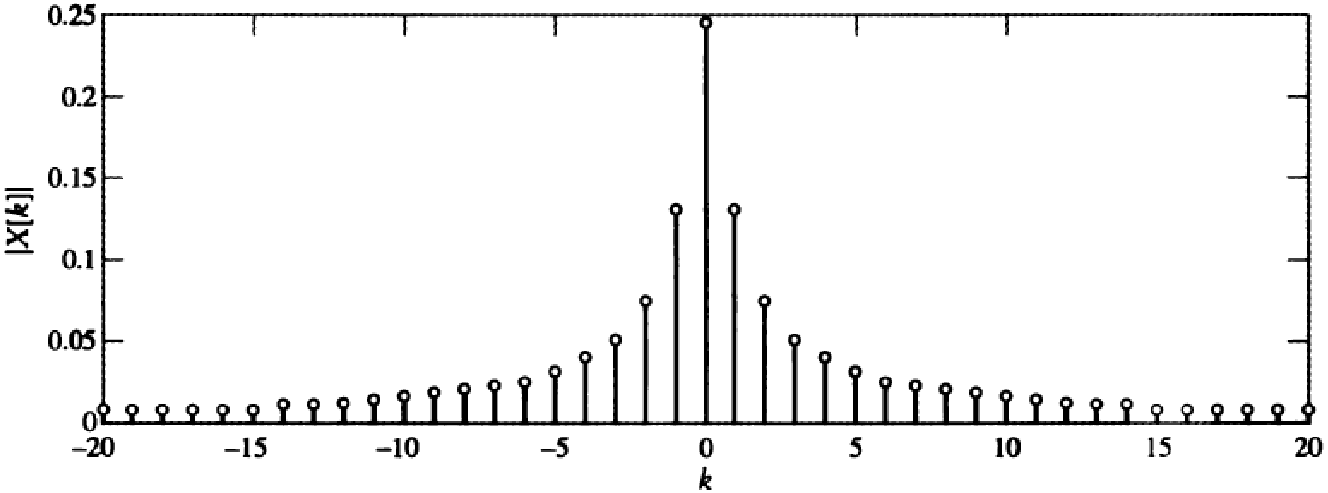
$$= \frac{1}{2}\int_0^2 e^{-2t} e^{-jk\pi t} dt$$

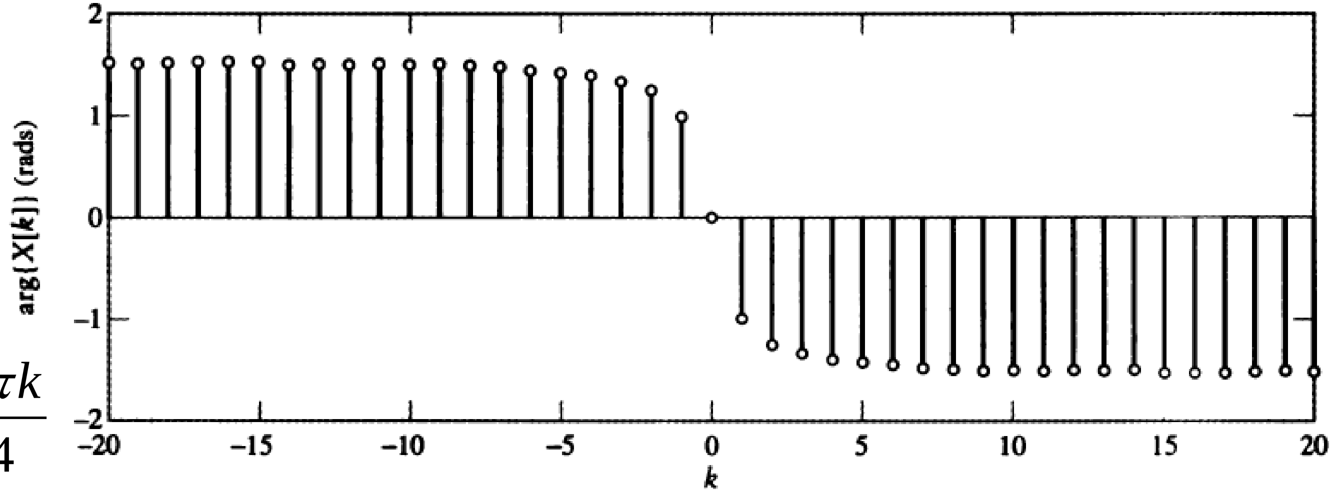
$$= \frac{1}{2}\int_0^2 e^{-(2+jk\pi)t} dt = \frac{-1}{2(2+jk\pi)} e^{-(2+jk\pi)t}\Big|_0^2 = \frac{1}{4+jk2\pi}\left(1-e^{-4}e^{-jk2\pi}\right) = \frac{1-e^{-4}}{4+jk2\pi}$$

⇒ $$\left|X[k]\right| = \frac{1-e^{-4}}{\sqrt{16+4\pi^2 k^2}}, \qquad \arg\{X[k]\} = -\arctan\frac{2\pi k}{4}$$

CT Periodic Signals: The Fourier Series

$$\left|X[k]\right| = \frac{1-e^{-4}}{\sqrt{16+4\pi^2 k^2}}$$





$$\arg\left\{X[k]\right\} = -\arctan\frac{2\pi k}{4}$$

CT Periodic Signals: The Fourier Series

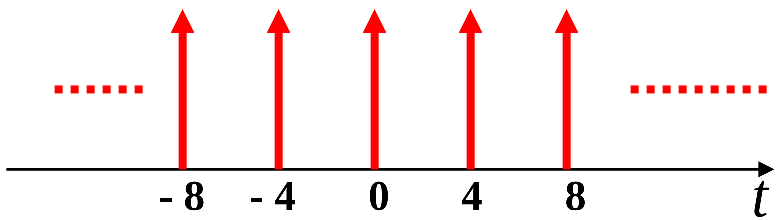
$$x(t)=\sum_{k=-\infty}^{\infty} X[k]e^{jk\omega_0 t}, \quad X[k]=\frac{1}{T}\int_0^T x(t)e^{-jk\omega_0 t}dt$$

- **The interval of integration may be chosen as any interval one period in length. Choosing the appropriate interval of integration often simplifies the problem.**

Example 3.10 ***FS Coefficients For an Impulse Train***

Determine the FS coefficients for the signal defined by

$$x(t)=\sum_{l=-\infty}^{\infty} \delta(t-4l)$$



<Sol.> $T=4$ $\omega_o = 2\pi/4 = \pi/2$

$$X[k]=\frac{1}{T}\int_0^T x(t)e^{-jk\omega_0 t}dt = \frac{1}{4}\int_{-2}^{2} \delta(t) e^{-jk(\pi/2)t}dt = \frac{1}{4}$$

CT Periodic Signals: The Fourier Series

Example 3.11 ***Calculation of FS Coefficients by Inspection***

Determine the FS coefficients for the signal defined by

$$x(t) = 1 - \cos(\pi t) + 2\sin(2\pi t) + \cos(3\pi t)$$

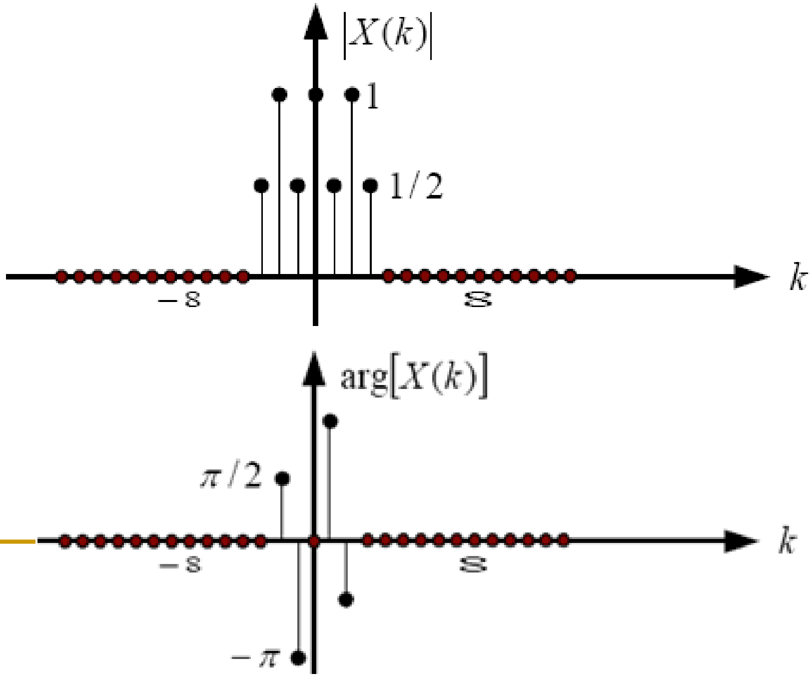
$$x(t) = \sum_{k=-\infty}^{\infty} X[k] e^{jk\omega_0 t}$$

<Sol.> $\omega_0 = \pi$

$$x(t) = 1 - \cos(\pi t) + 2\sin(2\pi t) + \cos(3\pi t)$$

$$= 1 - \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} + 2\frac{e^{j2\omega_0 t} - e^{-j2\omega_0 t}}{2j} + \frac{e^{j3\omega_0 t} + e^{-j3\omega_0 t}}{2}$$

$$\Rightarrow \quad X[k] = \begin{cases} 1, & k = 0 \\ -1/2, & k = \pm 1 \\ \mp j, & k = \pm 2 \\ 1/2, & k = \pm 3 \\ 0, & others \end{cases}$$



CT Periodic Signals: The Fourier Series

Example 3.12 ***Inverse FS***

Find the time-domain signal $x(t)$ corresponding to the FS coefficients

$X[k]=(1/2)^{|k|}e^{jk\pi/20}$ **Assume that the fundamental period is $T = 2$.**

<Sol.>

- **Fundamental frequency:** $\omega_0 = 2\pi/T = \pi$

$$x(t)=\sum_{k=-\infty}^{\infty}X[k]e^{jk\omega_0 t}=\sum_{k=0}^{\infty}(1/2)^k e^{jk\pi/20}e^{jk\pi t}+\sum_{k=-1}^{-\infty}(1/2)^{-k}e^{jk\pi/20}e^{jk\pi t}$$

$$=\sum_{k=0}^{\infty}(1/2)^k e^{jk\pi/20}e^{jk\pi t}+\sum_{l=1}^{\infty}(1/2)^l e^{-jl\pi/20}e^{-jl\pi t}$$

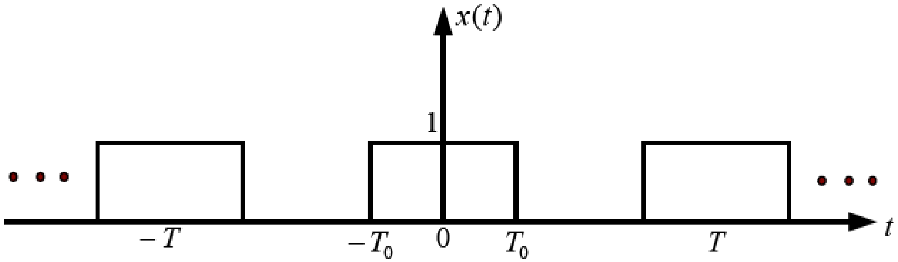
$$=\frac{1}{1-(1/2)e^{j(\pi t+\pi/20)}}+\frac{1}{1-(1/2)e^{-j(\pi t+\pi/20)}}-1$$

$$=\frac{3}{5-4\cos(\pi t+\pi/20)}$$

CT Periodic Signals: The Fourier Series

Example 3.13 ***FS for a Square Wave***

Determine the FS coefficients of the square wave depicted in Fig.3.21



<Sol.> $\omega_0 = 2\pi/T$

$$X[k] = \frac{1}{T}\int_{-T/2}^{T/2} x(t)e^{-jk\omega_0 t}dt = \frac{1}{T}\int_{-T_0}^{T_0} e^{-jk\omega_0 t}dt$$

$$= \frac{2\sin(k\omega_0 T_0)}{Tk\omega_0} = \frac{2\sin(2\pi k T_0/T)}{2\pi k}$$

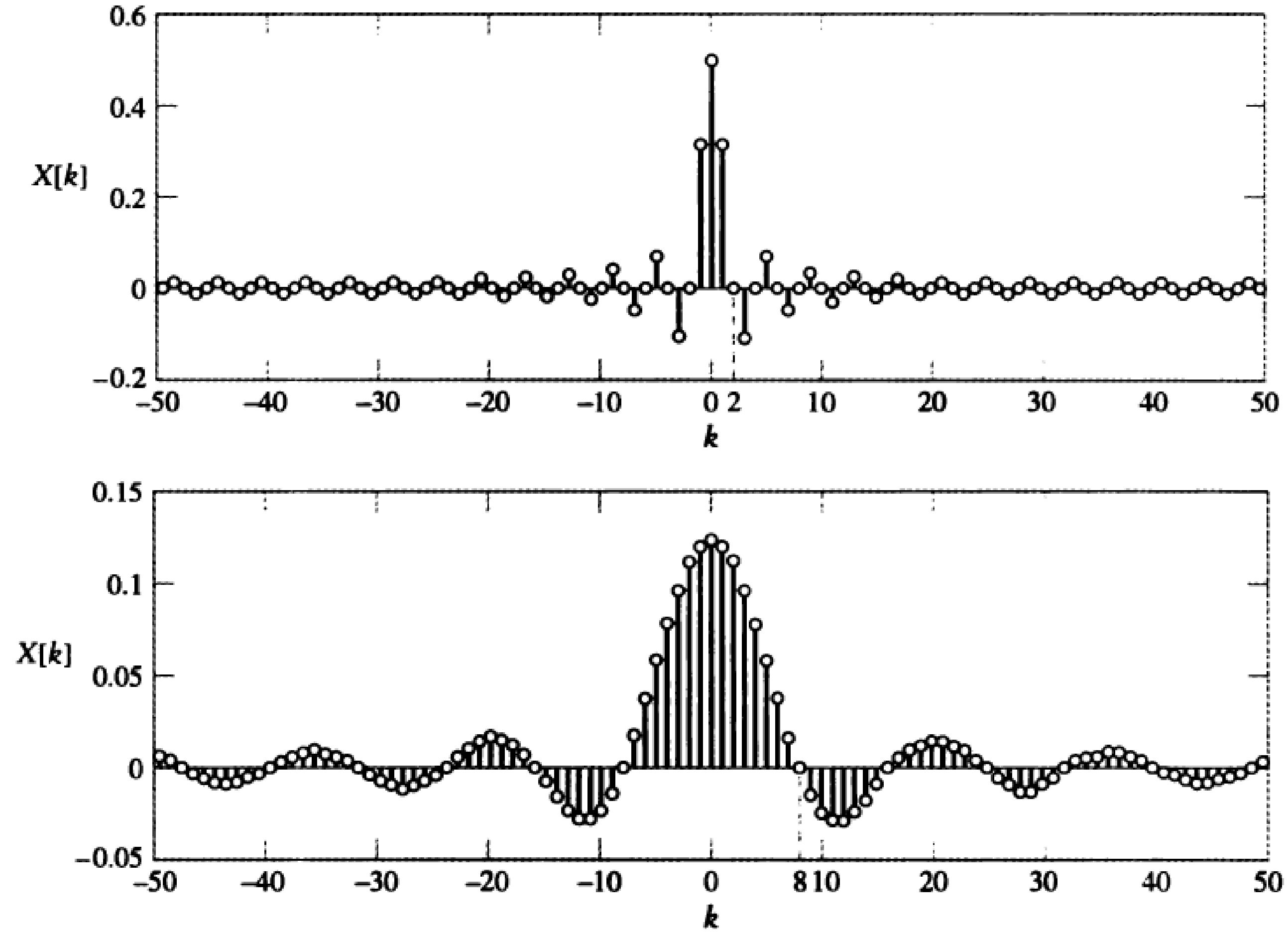
~ real value

CT Periodic Signals: The Fourier Series

$$X[k] = \frac{2\sin(2\pi k T_0/T)}{2\pi k}$$

T_o/T = 1/4

T_o/T = 1/16

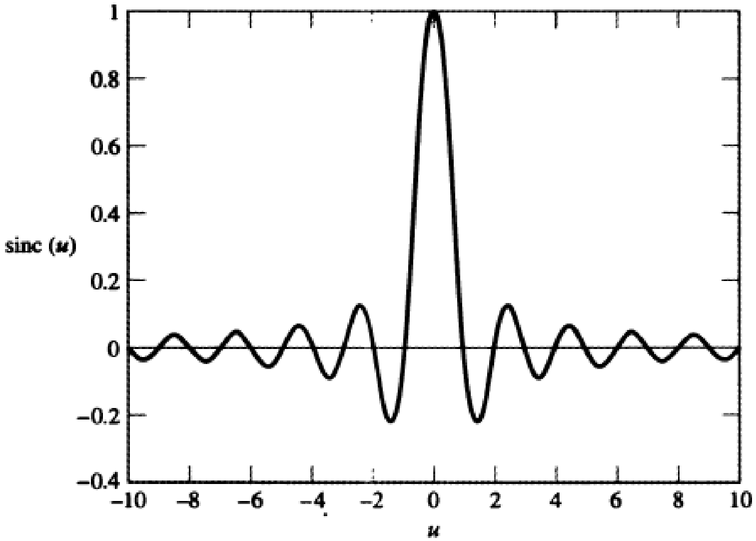


CT Periodic Signals: The Fourier Series

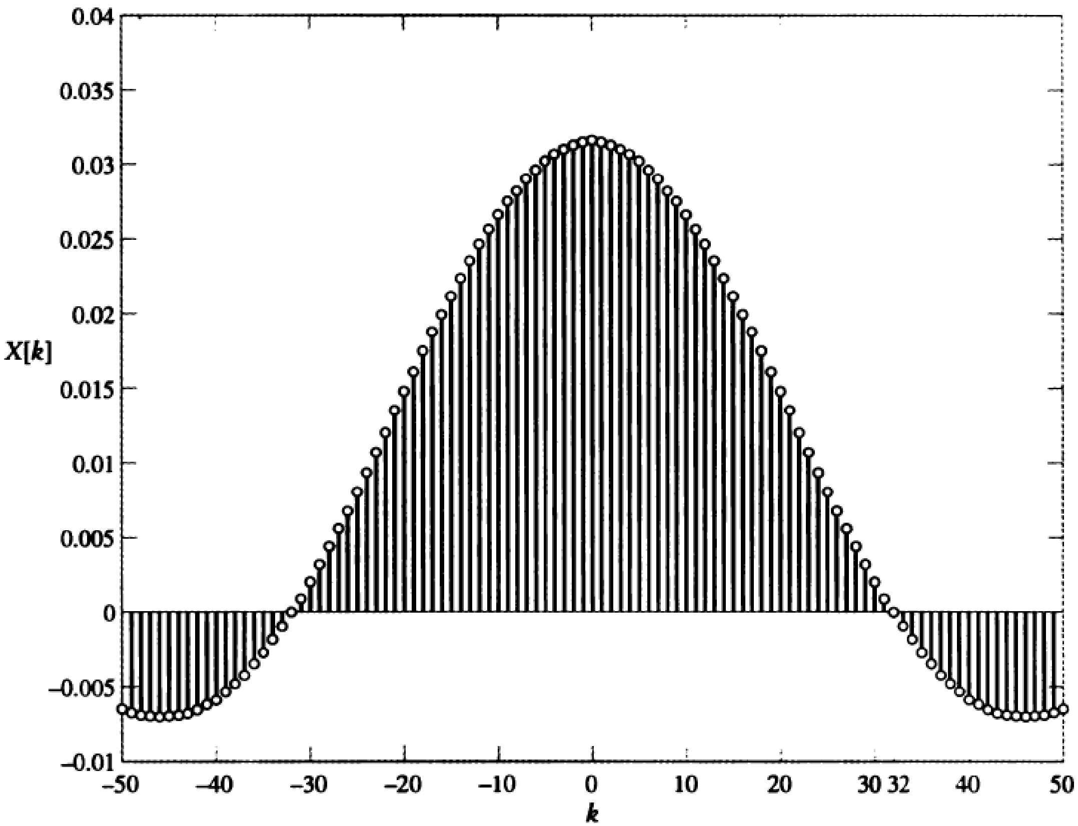
$$X[k] = \frac{2\sin(2\pi k T_0/T)}{2\pi k}$$

$$= \frac{2T_0}{T}\,\mathbf{sinc}\left(k\frac{2T_0}{T}\right)$$

$$\boxed{\mathbf{sinc}(u) = \frac{\sin(\pi u)}{\pi u}}$$



$T_o/T = 1/64$



DT Periodic Signals: The DT Fourier Series (DTFS)

- **The DTFS representation of a periodic signal $x[n]$ with fundamental period N and fundamental frequency $\Omega_o = 2\pi/N$**

$$x[n] = \sum_{k=0}^{N-1} X[k] e^{jk\Omega_0 n}$$

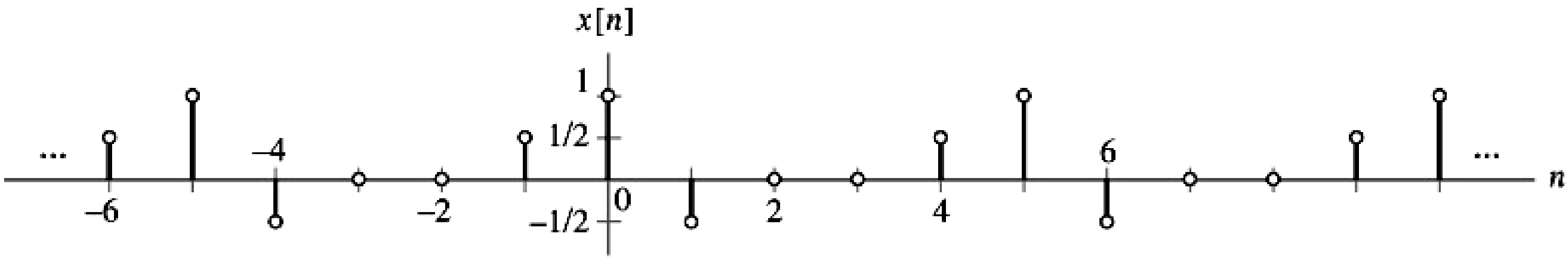
$$X[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-jk\Omega_0 n}$$

~ Fourier series coefficients or Spectral coefficients of $x[n]$

- **Notation** $x[n] \xleftrightarrow{DTFS;\ \Omega_0} X[k]$
  - **Magnitude spectrum of $x[n]$:** $\left|X[k]\right| = \sqrt{a_k^2 + b_k^2}$
  - **Phase spectrum of $x[n]$:** $\arg\left\{X[k]\right\} = arctg \dfrac{b_k}{a_k}$

where $X[k] = a_k + jb_k = \left|X[k]\right| e^{j\arg\{X[k]\}}$.

DT Periodic Signals: The DT Fourier Series (DTFS)



Example 3.2 ***Determining DTFS Coefficients***

Find the frequency domain representation of the signal depicted in Fig. 3.5.

<Sol.>

- **Period:** $N = 5$ ⟹ $\Omega_o = 2\pi/5$
- **Odd symmetry** ⟹ $n = -2$ **to** $n = 2$

$$X[k] = \frac{1}{5}\sum_{n=-2}^{2} x[n]e^{-jk2\pi n/5}$$

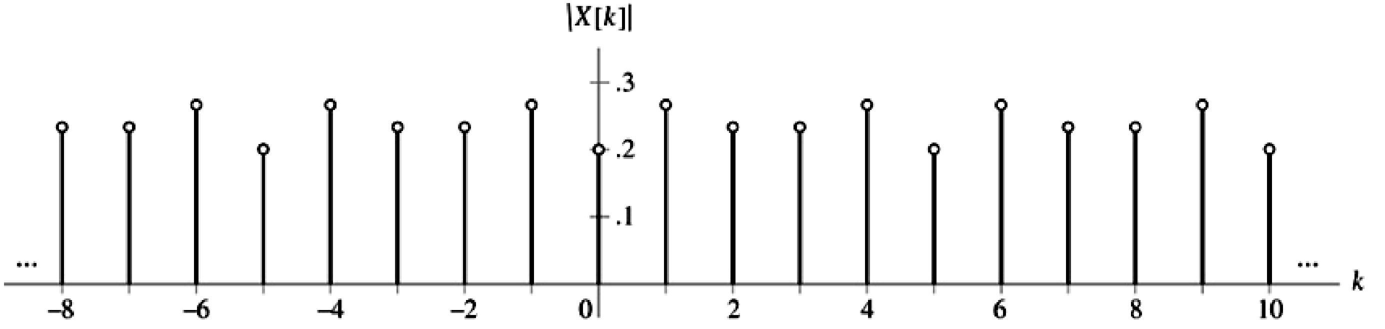
$$= \frac{1}{5}\left\{x[-2]e^{jk4\pi/5} + x[-1]e^{jk2\pi/5} + x[0]e^{j0} + x[1]e^{-jk2\pi/5} + x[2]e^{-jk4\pi/5}\right\}$$

$$= \frac{1}{5}\{1 + \frac{1}{2}e^{jk2\pi/5} - \frac{1}{2}e^{-jk2\pi/5}\} = \frac{1}{5}\{1 + j\sin(k2\pi/5)\}$$

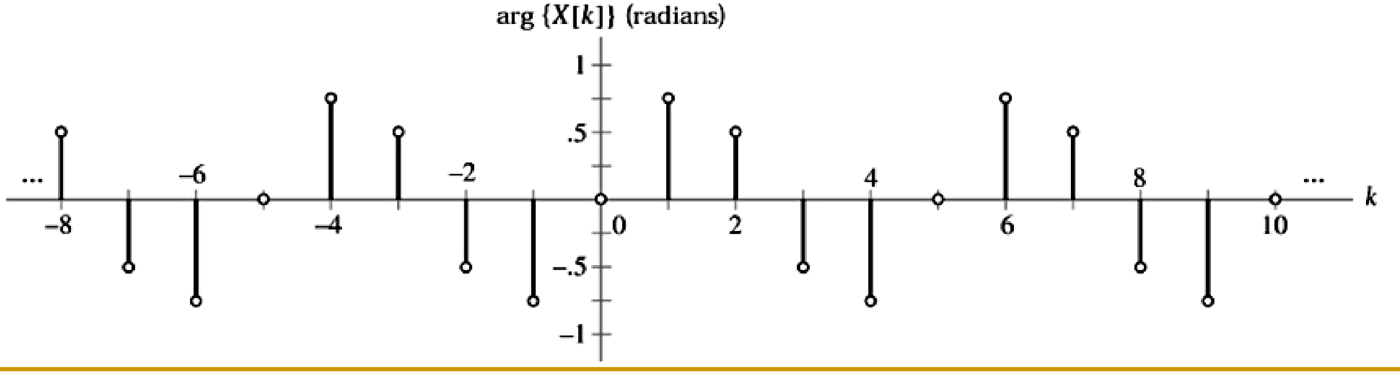
DT Periodic Signals: The DT Fourier Series (DTFS)

$$X[k] = \frac{1}{5}\{1 + j\sin(k2\pi/5)\}$$ ~ **Periodic with period=5**

$$|X[k]| = \sqrt{1+\sin^2(k2\pi/5)}\Big/5$$ ⇨ **Even function**



$$\arg\{X[k]\} = \arctan\{\sin(k2\pi/5)\}$$ ⇨ **Odd function**



DT Periodic Signals: The DT Fourier Series (DTFS)

Example 3.3 *Computation of DTFS by Inspection*

Determine the DTFS coefficients of $x[n] = \cos(n\pi/3 + \phi)$, using the method of inspection.

<Sol.>

- $\Omega_o = \pi/3$ $\Rightarrow$ **Period:** $N = 2\pi/\Omega_o = 6$

$$x[n] = \sum_{k=0}^{N-1} X[k]e^{jk\Omega_0 n} = \sum_{k=0}^{N-1} X[k]e^{jk\pi n/3} = \sum_{k=-2}^{3} X[k]e^{jk\pi n/3}$$

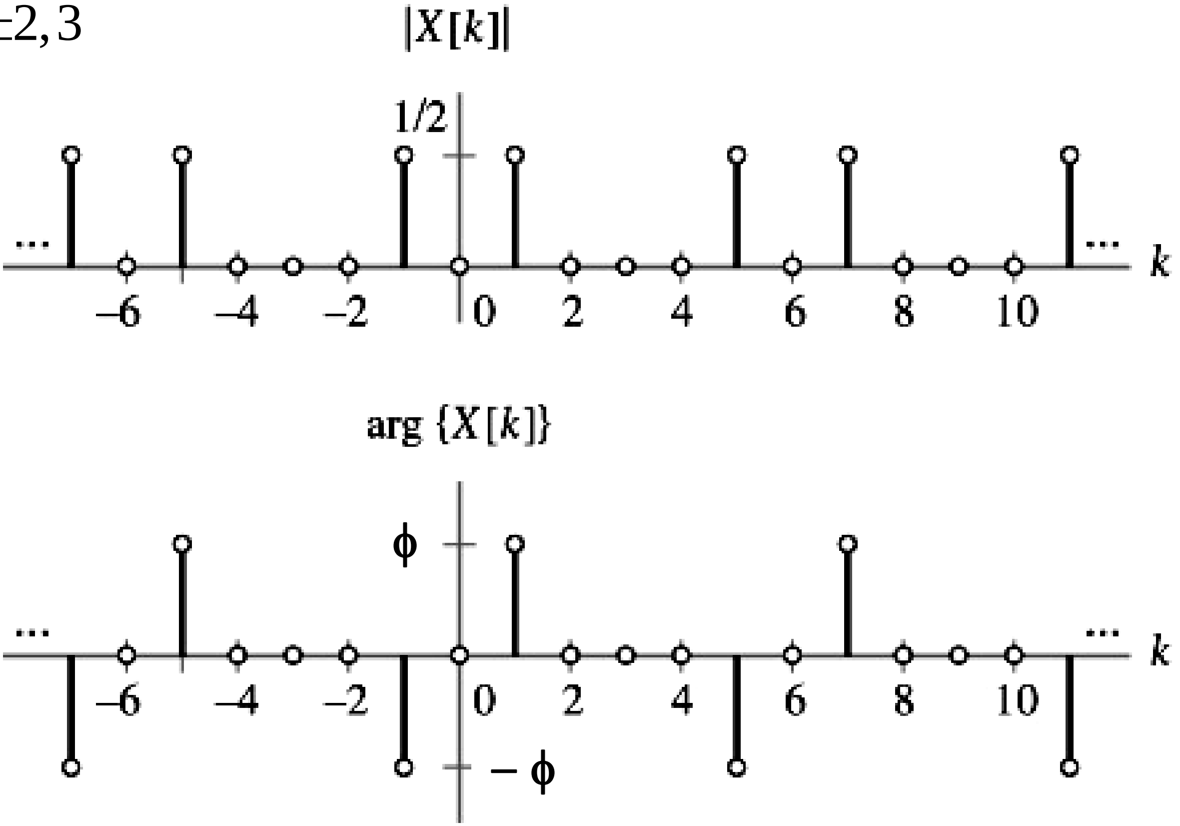
$$= X[-2]e^{-j2\pi n/3} + X[-1]e^{-j\pi n/3} + X[0] + X[1]e^{j\pi n/3} + X[2]e^{j2\pi n/3} + X[3]e^{j\pi n}$$

$$x[n] = \frac{1}{2}\left\{e^{j(\frac{\pi}{3}n+\phi)} + e^{-j(\frac{\pi}{3}n+\phi)}\right\} = \frac{1}{2}e^{-j\phi}e^{-j\frac{\pi}{3}n} + \frac{1}{2}e^{j\phi}e^{j\frac{\pi}{3}n}$$

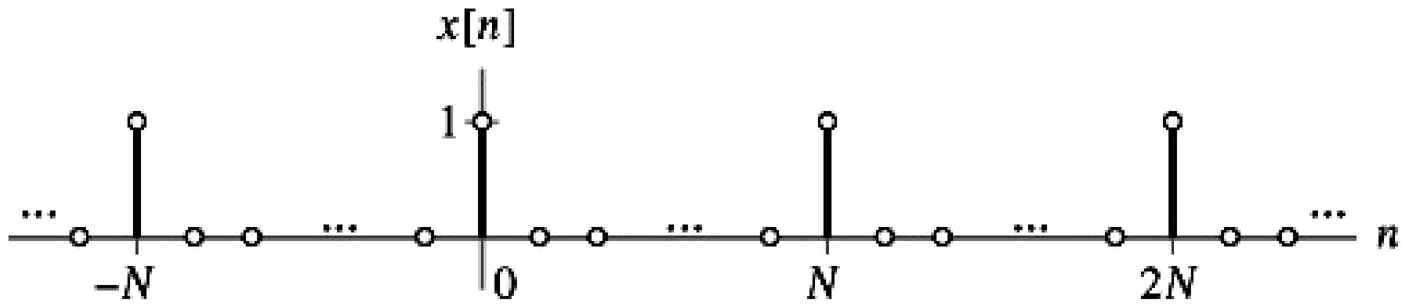
$$\Rightarrow \quad X[k] = \begin{cases} e^{-j\phi}/2, & k = -1 \\ e^{j\phi}/2, & k = 1 \\ 0, & k = 0, \pm 2, 3 \end{cases}$$

DT Periodic Signals: The DT Fourier Series (DTFS)

$$X[k] = \begin{cases} e^{-j\phi}/2, & k=-1 \\ e^{j\phi}/2, & k=1 \\ 0, & k=0,\pm 2,3 \end{cases}$$



DT Periodic Signals: The DT Fourier Series (DTFS)



Example 3.4 ***DTFS Representation of An Impulse Train***

Find the DTFS coefficients of the *N*-periodic impulse train as shown in Fig. 3.9.

$$x[n] = \sum_{l=-\infty}^{\infty} \delta[n - lN]$$

<Sol.>

- **Period:** N ⟹ $\Omega_o = 2\pi/N$

$$X[k] = \frac{1}{N}\sum_{n=0}^{N-1} x[n]e^{-jk\Omega_0 n} = \frac{1}{N}\sum_{n=0}^{N-1}\delta[n]e^{-jkn2\pi/N} = \frac{1}{N}$$ **~ period=1**

In case where some of the values of $x[n]$ are zero, $X[k]$ may be periodic in k with period less than N.

DT Periodic Signals: The DT Fourier Series (DTFS)

$$x[n]=\sum_{k=0}^{N-1}X[k]e^{jk\Omega_0 n}$$

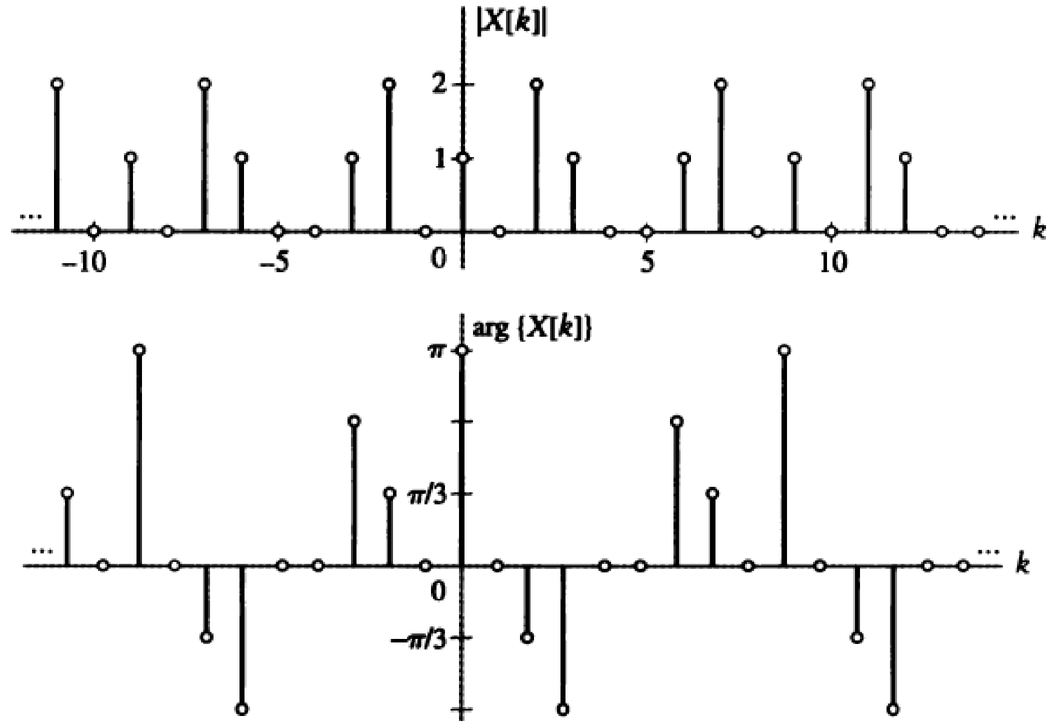
Example 3.5 ***The Inverse DTFS***

Determine the time-domain signal $x[n]$ from the DTFS coefficients depicted in Fig. 3.10.

<Sol.>

- **Period of DTFS coefficients**

$N = 9 \Rightarrow \Omega_o = 2\pi/9$



$$x[n]=\sum_{k=-4}^{4}X[k]e^{jk2\pi n/9}$$

$$=e^{j2\pi/3}e^{-j6\pi n/9}+2e^{j\pi/3}e^{-j4\pi n/9}-1$$

$$+2e^{-j\pi/3}e^{j4\pi n/9}+e^{-j2\pi/3}e^{j6\pi n/9}$$

$$=2\cos(6\pi n/9-2\pi/3)+4\cos(4\pi n/9-\pi/3)-1$$

Summary

- **Three forms of Fourier Series**
  - Trigonometric form for real-valued signals:

$$x(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos(k\omega_0 t) + \sum_{k=1}^{\infty} b_k \sin(k\omega_0 t)$$

  - Cosine-with-phase form: $x(t) = a_0 + \sum_{k=1}^{\infty} A_k \cos(k\omega_0 t + \theta_k)$
  - Exponential form: $x(t) = \sum_{n=-\infty}^{\infty} X_n e^{jn\omega_0 t}, \quad X_n = \frac{1}{T}\int_0^T x(t) e^{-jn\omega_0 t} dt$

- **The Discrete-Time Fourier Series**

$$x[n] = \sum_{k=0}^{N-1} X[k] e^{jk\Omega_0 n}, \quad X[k] = \frac{1}{N}\sum_{n=0}^{N-1} x[n] e^{-jk\Omega_0 n}$$

- Reference in textbook: 3.4, 3.5
- Homework: 3.50(a,b,d,e), 3.51(b,d,e); 3.48(a,c,e), 3.49(b,d,e)