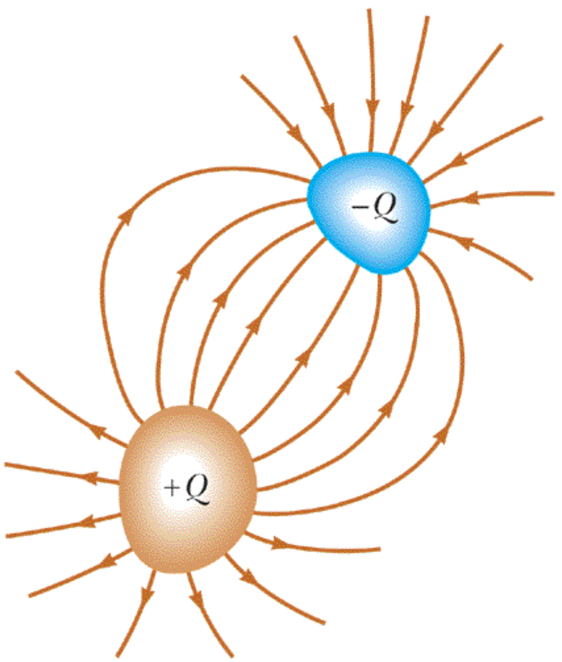
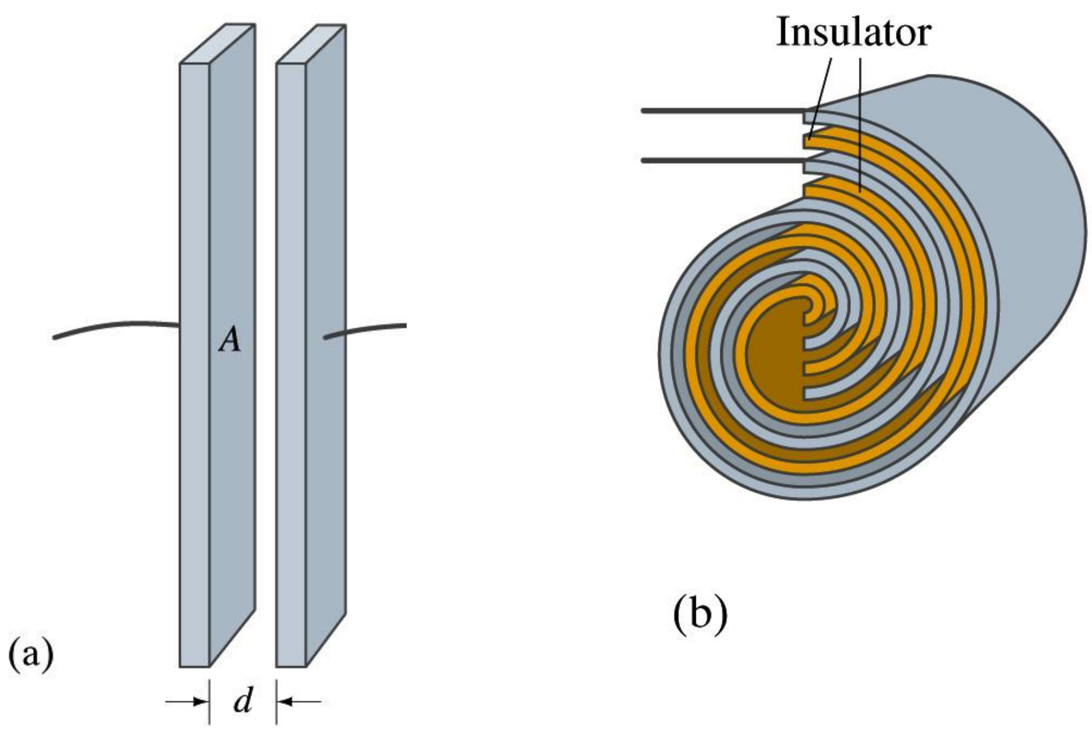
# Chapter 22 Capacitance, Dielectrics, Electric Energy Storage

## § 22-1 Capacitance (P525)

- **Capacitors**
  - **Any two conductors separated by an insulator (or a vacuum) form a capacitor, which can store amount of charge.**



# Capacitance of a capacitor

- **The capacitance $C$ of a capacitor**

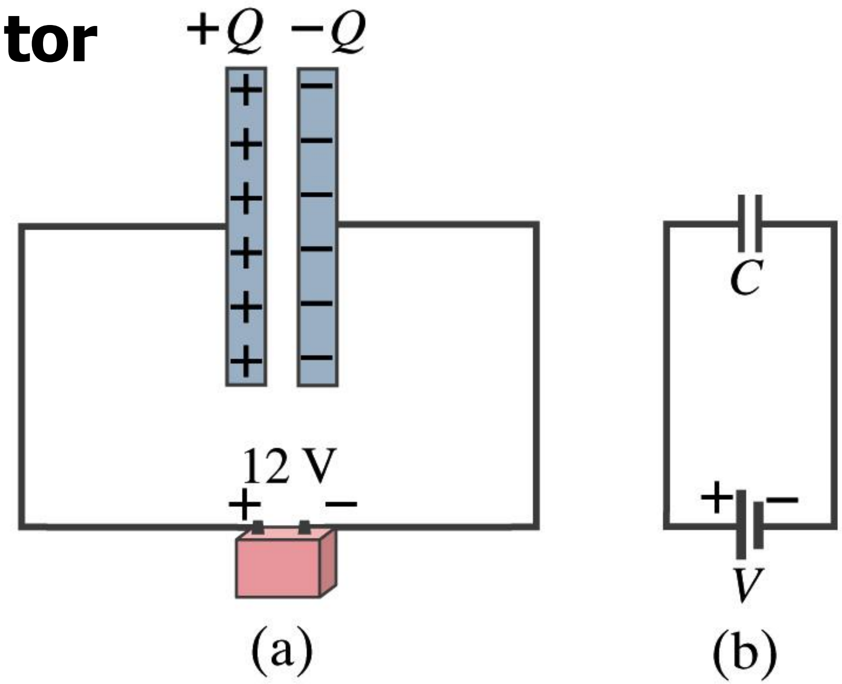
$$Q = C(\Delta V)$$

$$\boxed{C \equiv \frac{Q}{\Delta V}}$$

Farad

$\mu$F($10^{-6}$F)

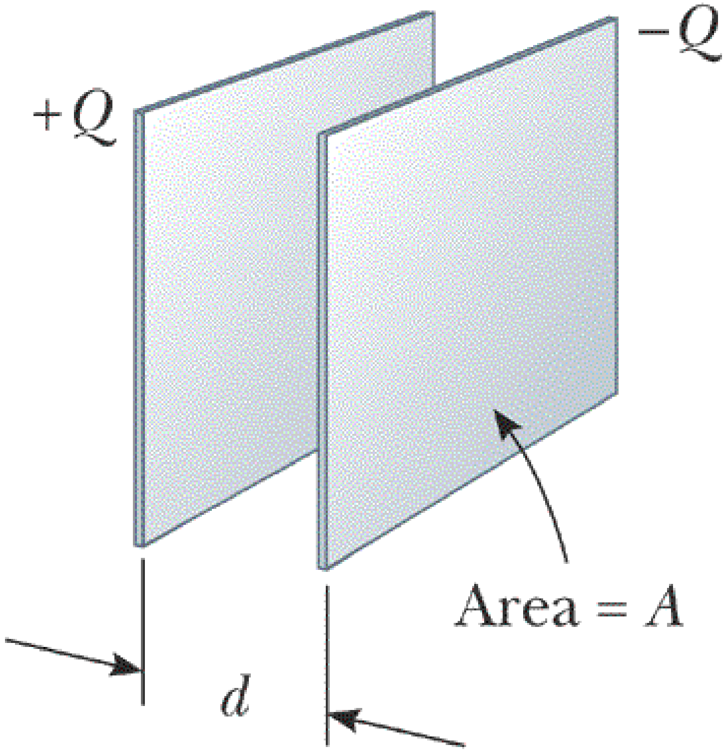
$p$F($10^{-12}$F)




- **The capacitance of a capacitor depends on the geometric arrangement of the conductors, and is independent of the charge $Q$ or the potential difference $\Delta V$. Because the potential difference is proportional to the charge, the ratio $Q/\Delta V$ is constant for a given capacitor.**

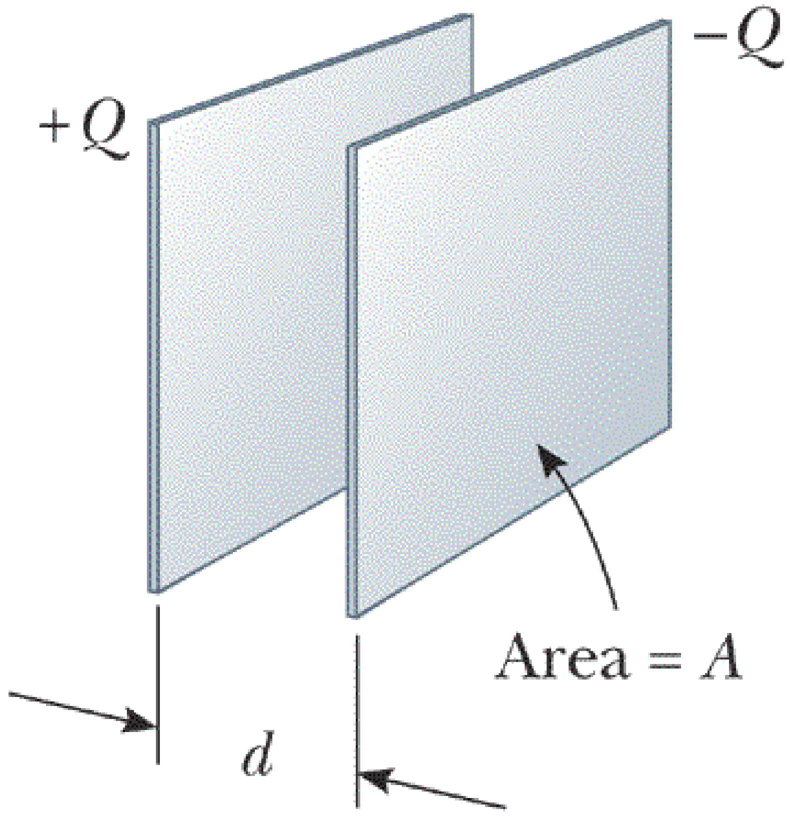
**Problem-Solving Strategy :**

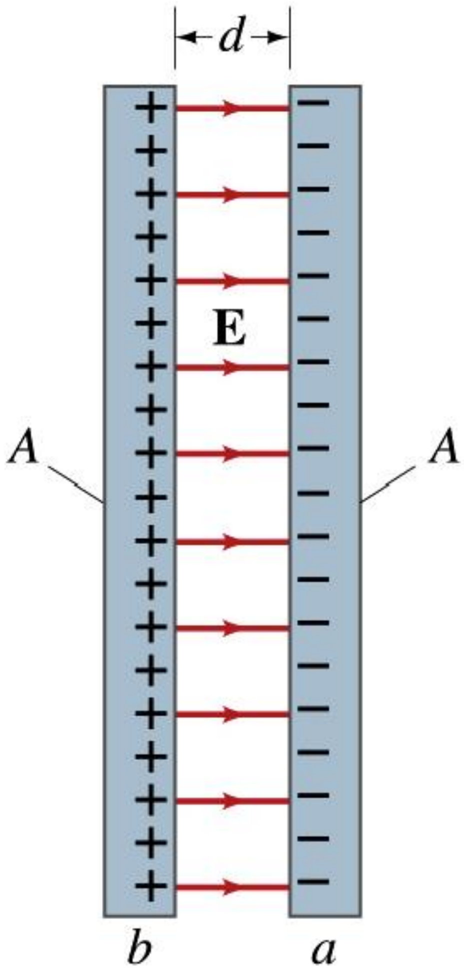
- A convenient charge of magnitude $Q$ is assumed.
- The potential difference $\Delta V$ is calculated.
- Use $C=Q/\Delta V$ to evaluate the capacitance.



# The parallel-plate capacitor (P527)

A parallel-plate capacitor consists of two parallel plates of equal area $A$, separated by a distance $d$. Find the capacitance.

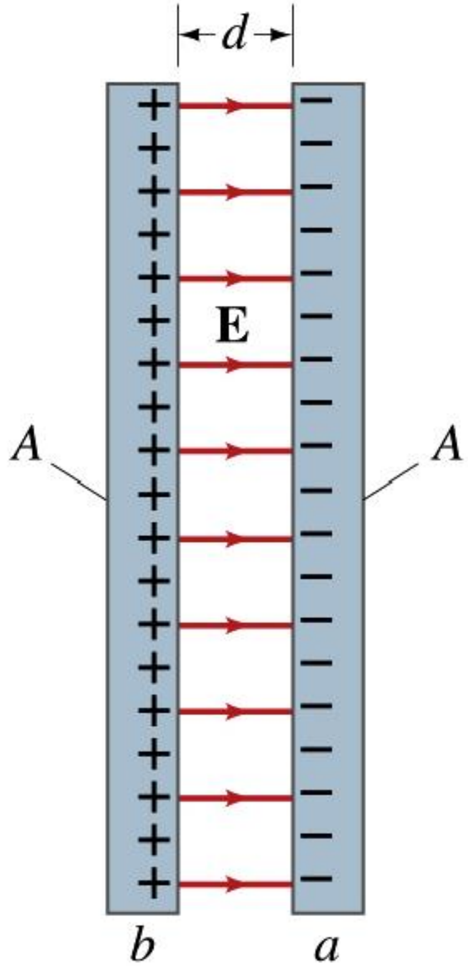


# The parallel-plate capacitor

**Solution: Assume the two plates have opposite charges $+Q$ and $-Q$. An uniform electric field is:**

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$



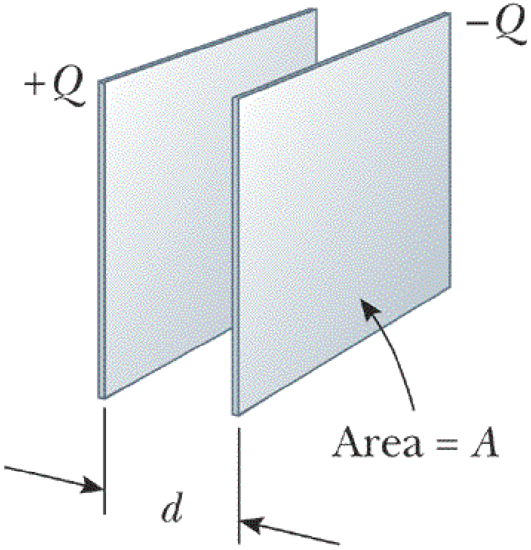

**The potential difference:**

$$\Delta V = \int_{+}^{-} \vec{E} \cdot d\vec{l} = Ed = \frac{Qd}{\varepsilon_0 A}$$

$$\boxed{C = \frac{Q}{\Delta V} = \frac{\varepsilon_0 A}{d}}$$

# The parallel-plate capacitor

$$C = \varepsilon_0 \frac{A}{d}$$

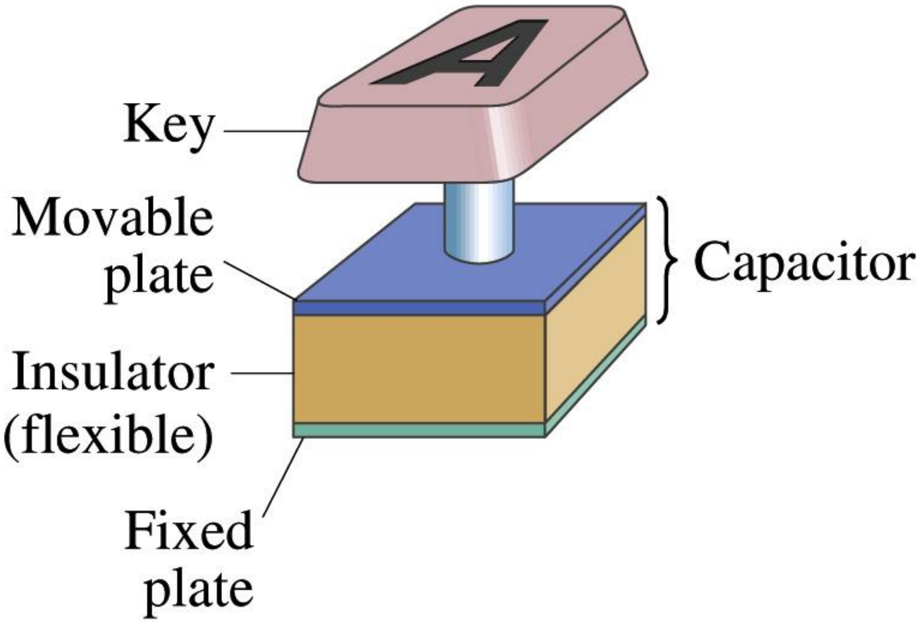



- **The capacitance of a parallel-plate capacitor is proportional to the area of its plates and inversely proportional to the plate separation, which are the geometrical factors.**
- **The capacitance does not depend on the potential difference or the charge carried by the plates.**
- **The capacitance has form of $\varepsilon_0$ times a quantity with the dimension of length ($A/d$), which is essential form for all the capacitors.**

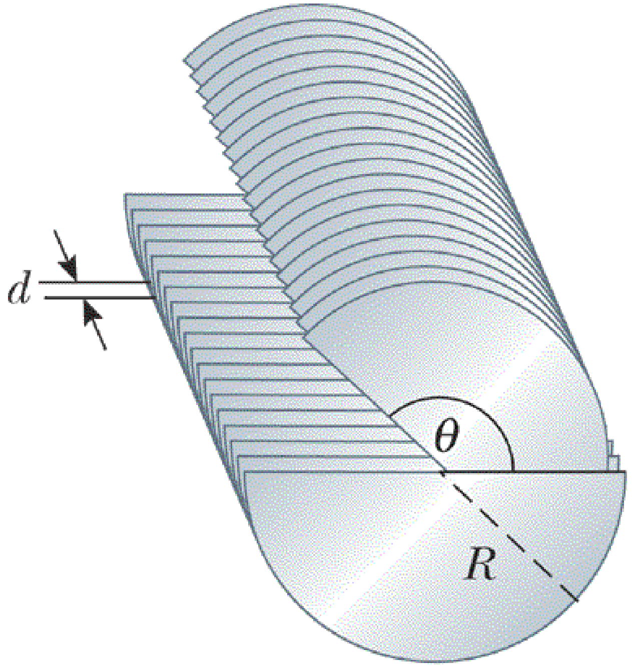
$$\varepsilon_0 = 8.85\times 10^{-12}\,\mathrm{F/m} = 8.85\,\mathrm{pF/m}$$

# The parallel-plate capacitor

$$C = \frac{\varepsilon_0 A}{d}$$



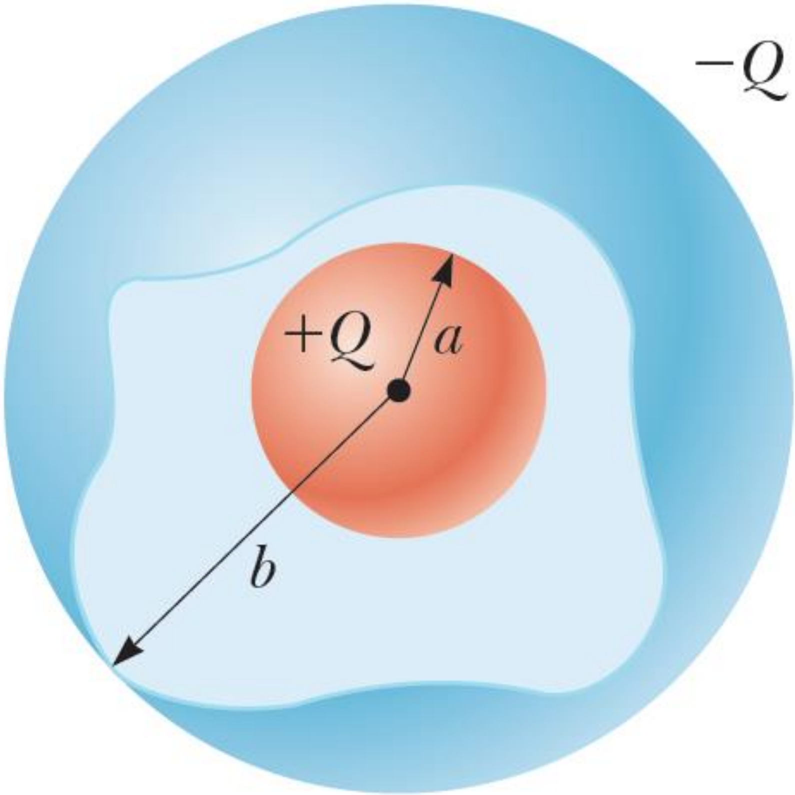

**Key on a computer keyboard**



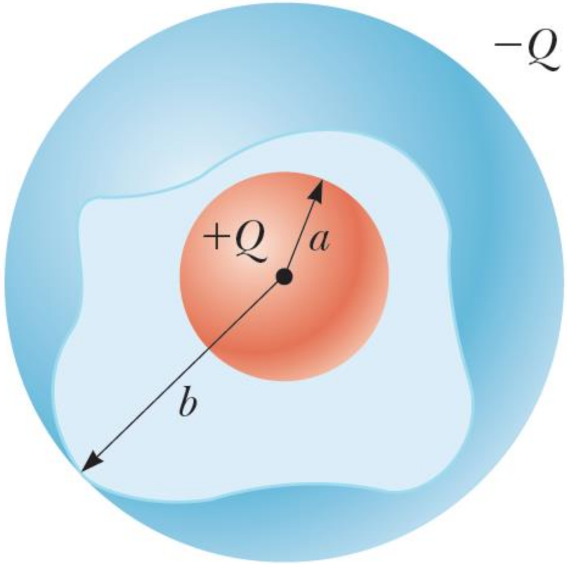

**A radio tuner**

## The Spherical Capacitor (P529 Ex. 22-3)

A spherical capacitor in which the inner conductor is a solid sphere of radius $a$, and outer conductor is a hollow spherical shell of inner radius $b$. Find the capacitance.



**Solution: Assume the inner and outer sphere have opposite charges $+Q$ and $-Q$. In the region $a<r<b$, we can use Gauss' law to determine:**




$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}, \qquad (a < r < b)$$

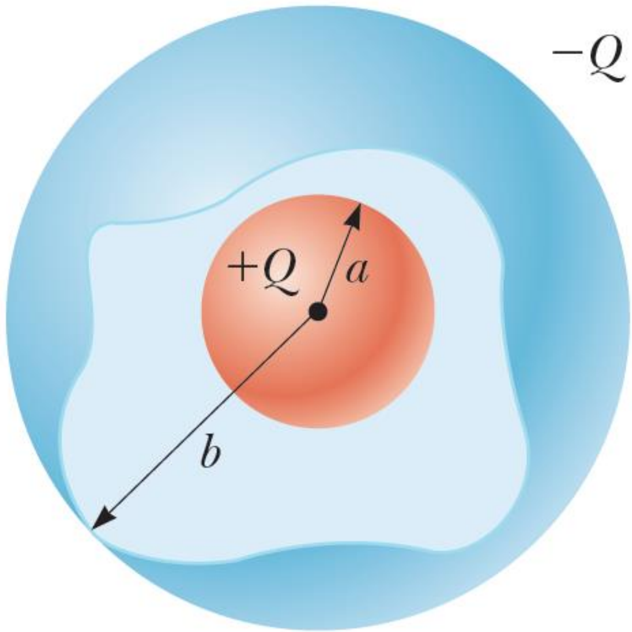
**The potential difference:**

$$\Delta V = \int_+^- \vec{E}\cdot d\vec{s} = \frac{Q}{4\pi\varepsilon_0}\int_{r_a}^{r_b}\frac{dr}{r^2} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right)$$

$$= \frac{Q}{4\pi\varepsilon_0}\frac{b-a}{ab}, \qquad C = \frac{Q}{\Delta V} = 4\pi\varepsilon_0\frac{ab}{b-a}$$

# The Spherical Capacitor

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a}$$




- $\varepsilon_0$ **times a quantity with dimension of length.**

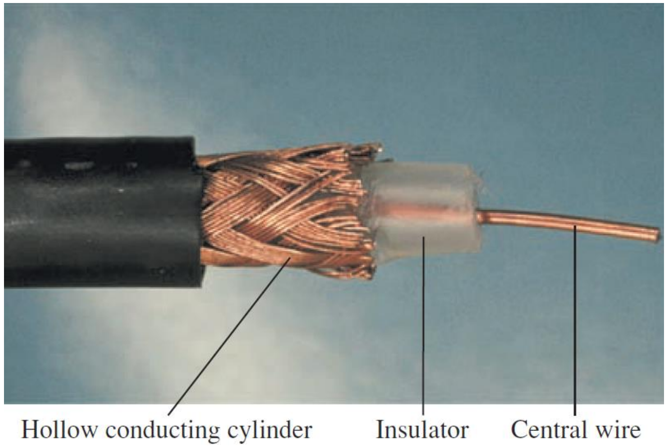
- **When** $b\to\infty$, $C=4\pi\varepsilon_0 a$ **(isolated conducting sphere )**

- **When** $b-a<<a$, $ab\approx a^2$, $d=b-a$, $A=4\pi a^2$, $C=\varepsilon_0 A/d$

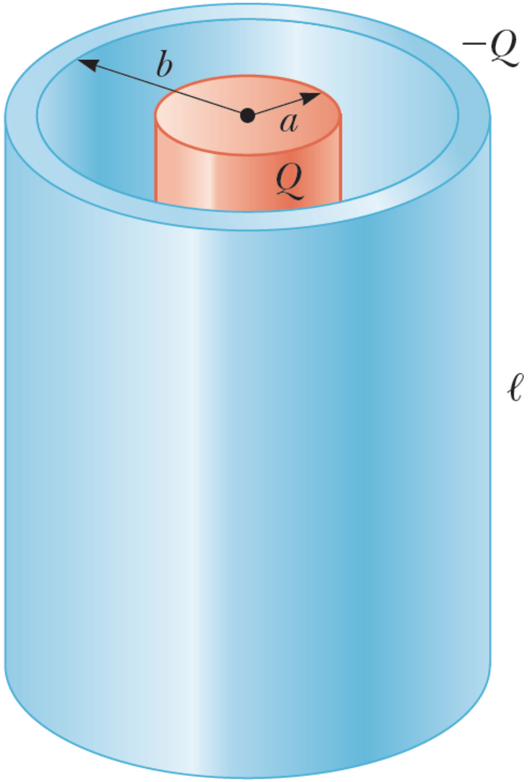
  **(parallel-plate capacitor)**

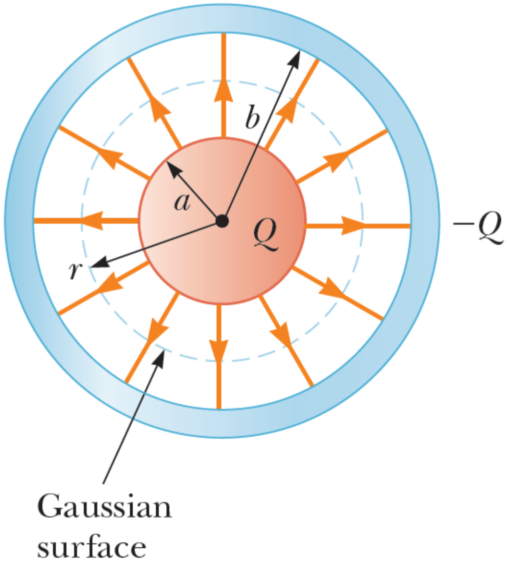
## The Cylindrical Capacitor (P528 Ex.22-2)

A cylindrical capacitor consists of a cylindrical conductor of radius $a$ coaxial with a larger cylindrical shell of radius $b$. Find the **capacitance** of this device if its length is $l$.




coaxial cable

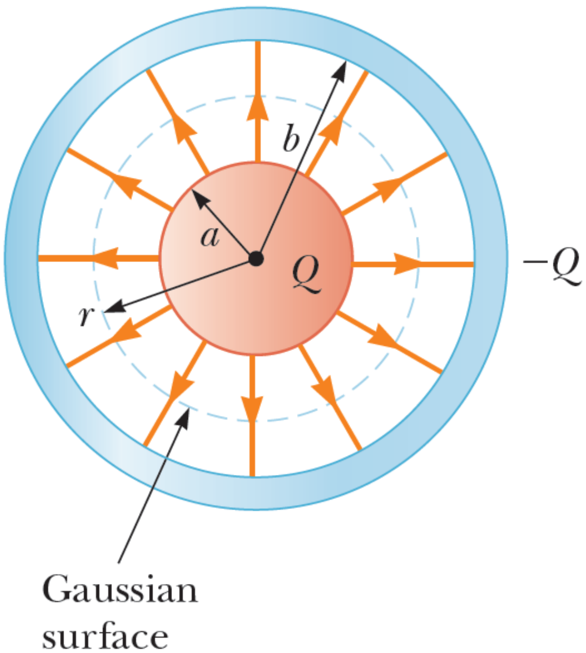


# The Cylindrical Capacitor

**Solution: Assume the inner and outer conductors have opposite charges +*Q* and –*Q*. In the region *a*<*r*<*b*, we can use Gauss' law to determine:**

$$\oiint_S \vec{E}\cdot d\vec{A} = E(2\pi r l) = \frac{\lambda l}{\varepsilon_0}, \qquad E = \frac{\lambda}{2\pi\varepsilon_0 r}$$




**The potential difference:**

$$\Delta V = \int_+^- \vec{E}\cdot d\vec{s} = \int_a^b \frac{\lambda}{2\pi\varepsilon_0}\frac{dr}{r} = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{b}{a}\right)$$

$$Q = \lambda l, \qquad C = \frac{Q}{\Delta V} = \frac{2\pi\varepsilon_0 l}{\ln(b/a)}$$

# The Cylindrical Capacitor

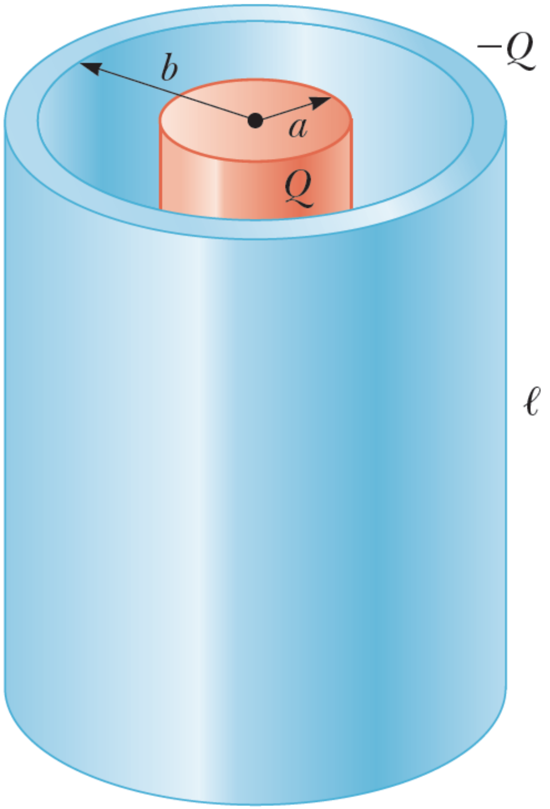
$$C = \frac{2\pi\varepsilon_0 l}{\ln(b/a)}$$

- **The form of $\varepsilon_0$ times a quantity with dimension of length.**
- **When $d=b-a<<a$**

$$\ln\left(\frac{b}{a}\right) = \ln\left(\frac{a+d}{a}\right) = \ln\left(1+\frac{d}{a}\right) \approx \frac{d}{a}$$

$$C = \frac{2\pi\varepsilon_0 la}{d} = \varepsilon_0 \frac{A}{d}, \quad A = (2\pi a)l$$

**(parallel-plate capacitor)**



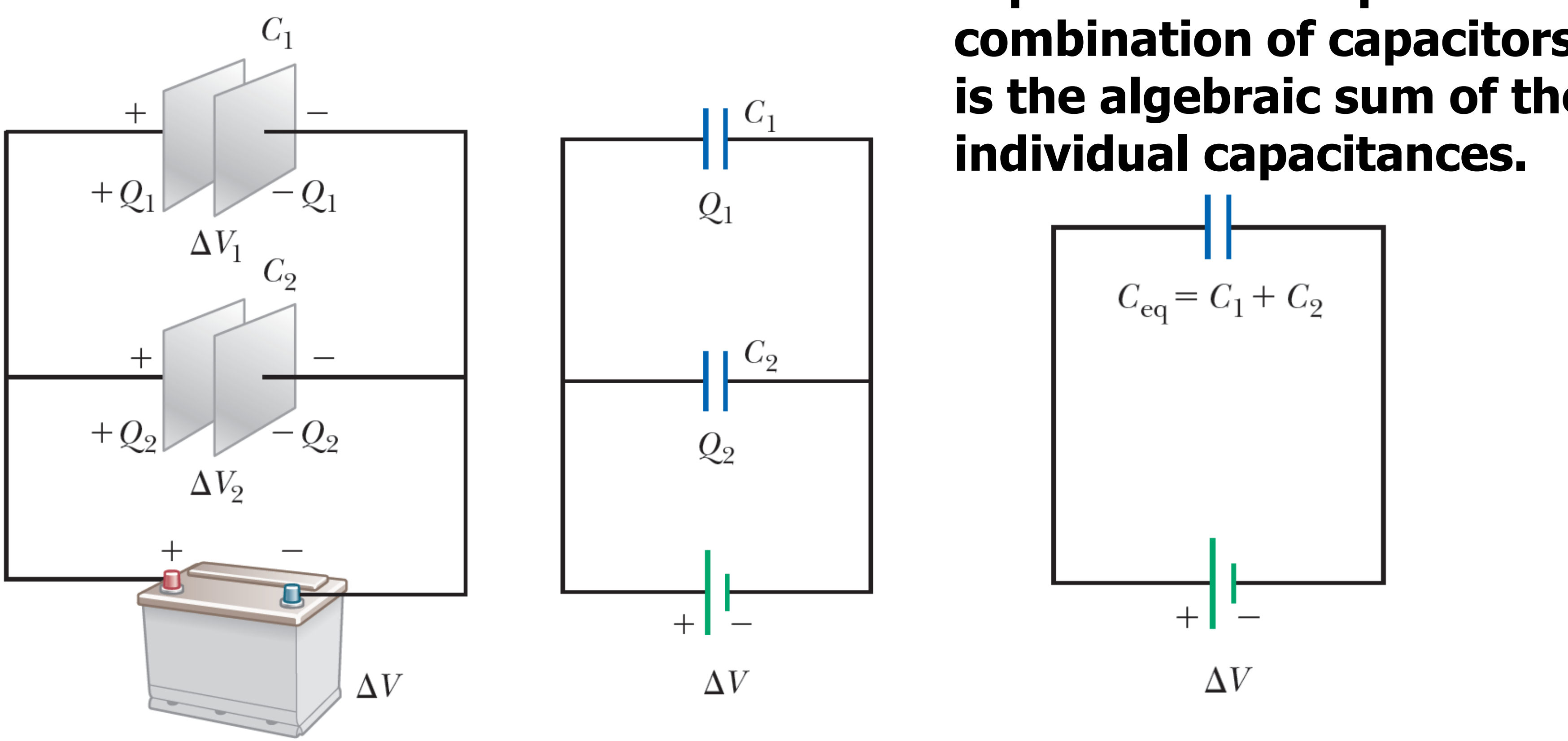
# § 22-3 Combinations of Capacitor (P529)

## Parallel Combination

$$C_1 = \frac{Q_1}{\Delta V_1}, \quad C_2 = \frac{Q_2}{\Delta V_2}, \qquad C_{eq} = \frac{Q}{\Delta V} = \frac{Q_1 + Q_2}{\Delta V} = C_1 + C_2$$

$$Q = Q_1 + Q_2, \quad \Delta V = \Delta V_1 = \Delta V_2$$

- **The equivalent capacitance of a parallel combination of capacitors is the algebraic sum of the individual capacitances.**



# Combinations of Capacitor

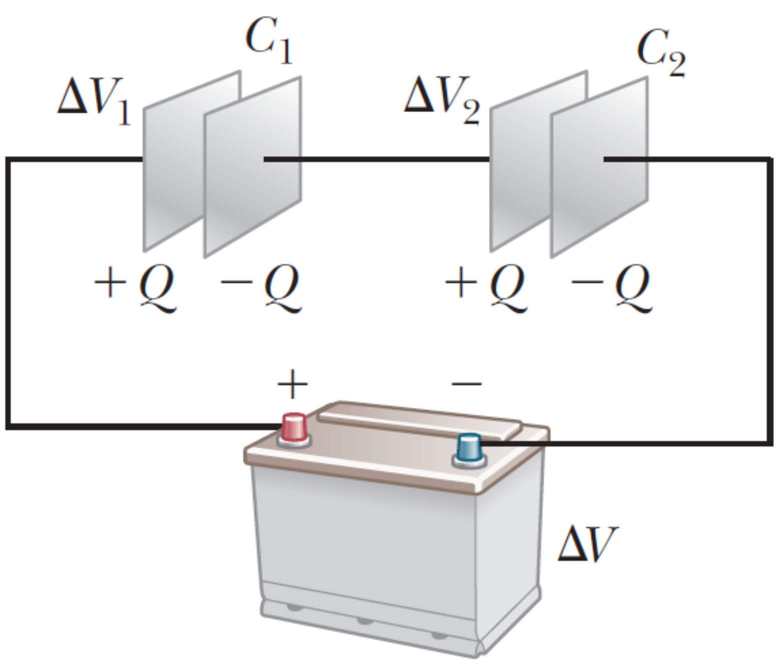
## Series Combination

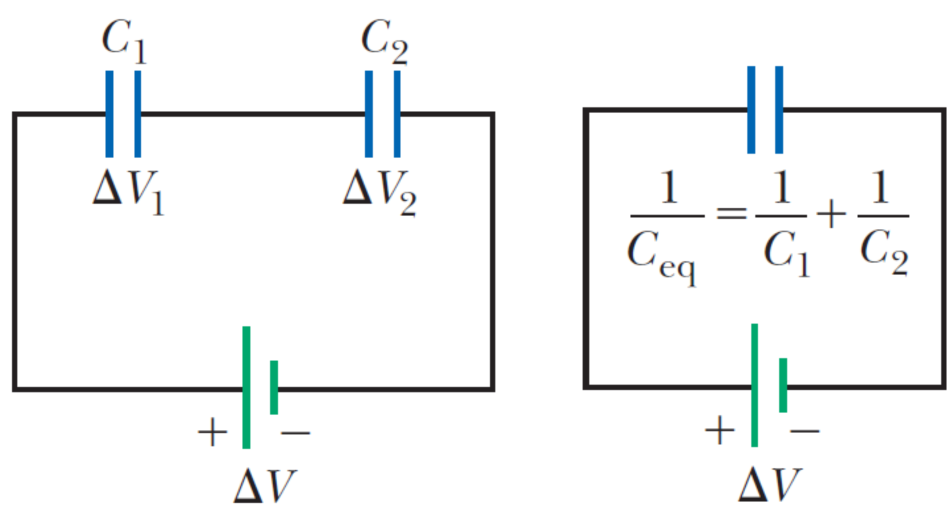
$$C_1 = \frac{Q_1}{\Delta V_1}, \quad C_2 = \frac{Q_2}{\Delta V_2},$$

$$\frac{1}{C_{eq}} = \frac{\Delta V}{Q} = \frac{\Delta V_1 + \Delta V_2}{Q} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$Q = Q_1 = Q_2, \quad \Delta V = \Delta V_1 + \Delta V_2$$

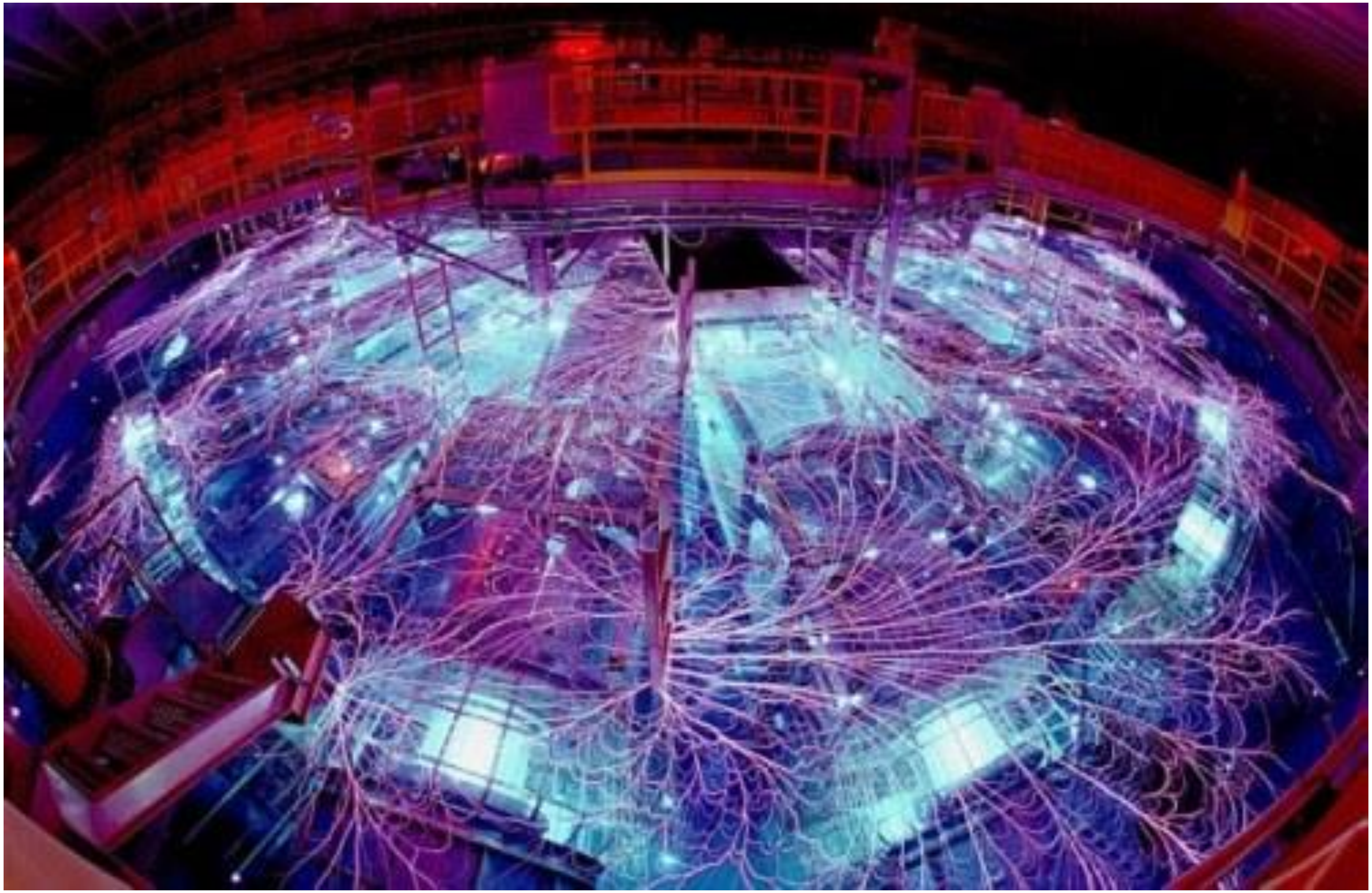
- **The inverse of the equivalent capacitance is the algebraic sum of the inverse of the individual capacitances.**



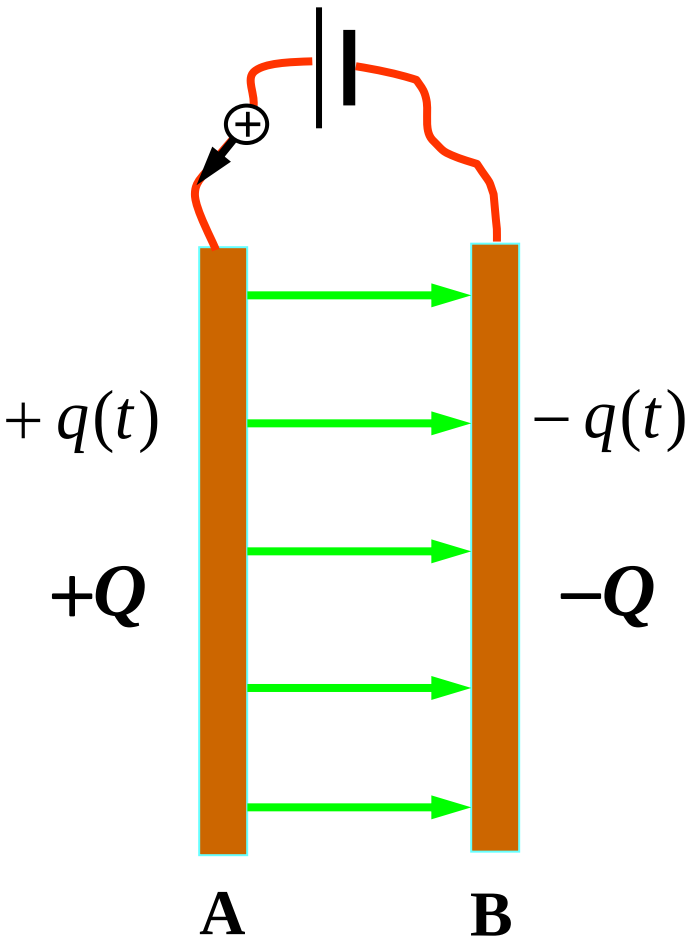
# § 22-4 Electric Energy Storage (P532)

**A capacitor can store charge, and can also store energy!**



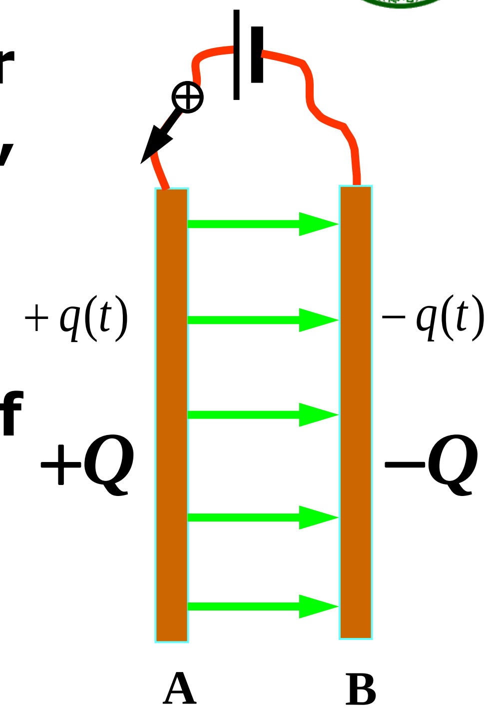
Sandia's Z machine is the world's most powerful and efficient laboratory radiation source.

- **The potential energy of a charged capacitor**
  - **The energy stored in a capacitor will be equal to the work done to charge it.**
  - **We evaluate the work of charging that an external agent continuously pulls charge $dq$ from negative plate to positive plate until the capacitor has the opposite charge of $\pm Q$.**



# The potential energy of a charged capacitor

- **Suppose that $q$ is the charge on the capacitor at some instant during this charging process, the potential difference across the capacitor is $\Delta V=q/C$. Imaging that the external agent transfers an additional increment of charge $dq$ from the plate of charge $-q$ to the plate of charge $q$, the resulting small change $dU$ in the electric potential energy is:**




$$dU = (\Delta V)dq = \frac{q}{C}dq$$

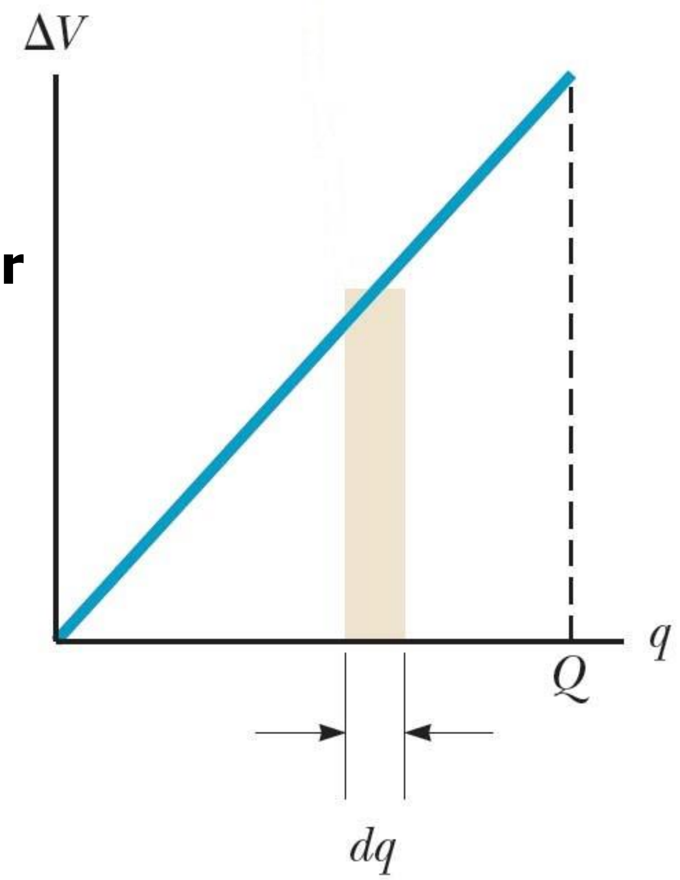
- **If this process is continued until to charge the capacitor from $q=0$ to the final charge $q=Q$, the total potential energy is:**

$$U = \int dU = \int_0^Q \frac{q}{C}dq = \frac{Q^2}{2C}$$

# The potential energy of a charged capacitor

**Using** $C = \dfrac{Q}{\Delta V}$, $\quad U = \dfrac{1}{2}\dfrac{Q^2}{C} = \dfrac{1}{2}C(\Delta V)^2 = \dfrac{1}{2}Q\Delta V$

**Graphical interpretation:**

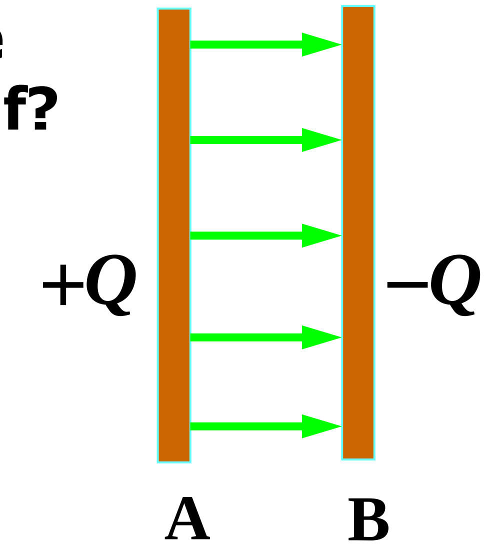



**A plot of potential difference versus charge for a capacitor is a straight line having slope $1/C$.**

**The total area under the curve is the potential energy stored in a charged capacitor.**

# Where does the potential energy reside?

**Question: Which one is the storehouse of the energy? The charges or the electric field itself?**




- **From the equation $U=Q^2/2C$, we conclude that the energy relates to the charging.**

- **Another point of view:**

$$C=\varepsilon_0\frac{A}{d},$$

$$\Delta V=Ed,$$

$$U=\frac{1}{2}C(\Delta V)^2=\left(\frac{1}{2}\varepsilon_0E^2\right)(Ad)$$

$U$ **is proportional to the volume** $Ad$ **between the two plates.**

- **Because the electric field is present in the space between the two plates, the energy is stored in the electric field that is present in this region.**

# Where does the potential energy reside?

$$U = \left(\frac{1}{2}\varepsilon_0 E^2\right)(Ad)$$

- **The energy density:** $$u = \frac{U}{Ad} = \frac{1}{2}\varepsilon_0 E^2$$
- **If an electric field $\vec{E}$ exists at any point in empty space, we can think of that point as the site of stored energy in amount of $\frac{1}{2}\varepsilon_0 E^2$ .**

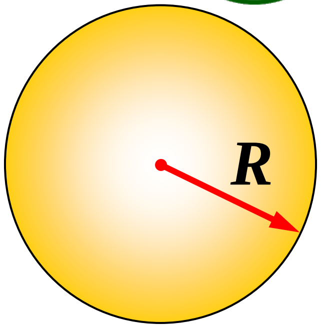
$$U = \int dU = \iiint_V u dV = \iiint_V \left(\frac{1}{2}\varepsilon_0 E^2\right) dV$$

# Where does the potential energy reside?

- In the case of electrostatic field, we can not answer which one is the storehouse of the energy.
  - Because in the case of electrostatic field, the electric field is always accompanied with the charge.
- In the case of time-varying electromagnetism field
  - The electromagnetic wave can exists in the vacuum, whether the charge exists or not.

# Example

**How much electrostatic energy is stored in the electric field of an isolated conducting sphere of radius $R$ and charge $Q$ .**




**Solution (I):**

**The energy stored in the electric field is** $U = \iiint\limits_V \left( \frac{1}{2} \varepsilon_0 E^2 \right) dV$

**The electric field distribution:** $E = \begin{cases} 0 & \text{if} \quad r < R \\ \dfrac{Q}{4\pi\varepsilon_0 r^2} & \text{if} \quad r > R \end{cases}$

**We choose a differential spherical shell of radius $r$ and thickness $\mathrm{d}r$ and integrate the energies in the shells**

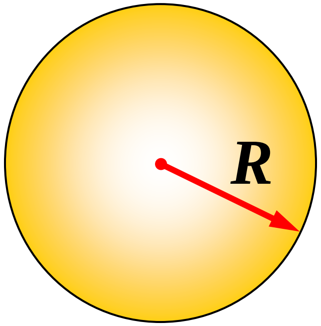
$$U = \iiint \left[ \frac{1}{2} \varepsilon_0 \left( \frac{Q}{4\pi\varepsilon_0 r^2} \right)^2 \right] dV = \int_R^\infty \left[ \frac{1}{2} \varepsilon_0 \left( \frac{Q}{4\pi\varepsilon_0 r^2} \right)^2 \right] \left( 4\pi r^2 dr \right) = \frac{Q^2}{8\pi\varepsilon_0} \int_R^\infty \frac{dr}{r^2} = \frac{Q^2}{8\pi\varepsilon_0 R}$$

## Example

**Solution (II):**

**The energy stored in the spherical capacitor is**

$$U = \frac{Q^2}{2C}, \qquad C = 4\pi\varepsilon_0 R, \qquad U = \frac{Q^2}{8\pi\varepsilon_0 R}$$




**Solution (III):**

**The work required to bring a differential charge d$q$ to the sphere is**

$$dU = Vdq, \qquad V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

$$U = \int_0^Q Vdq = \frac{1}{4\pi\varepsilon_0 R}\int_0^Q qdq = \frac{Q^2}{8\pi\varepsilon_0 R}$$

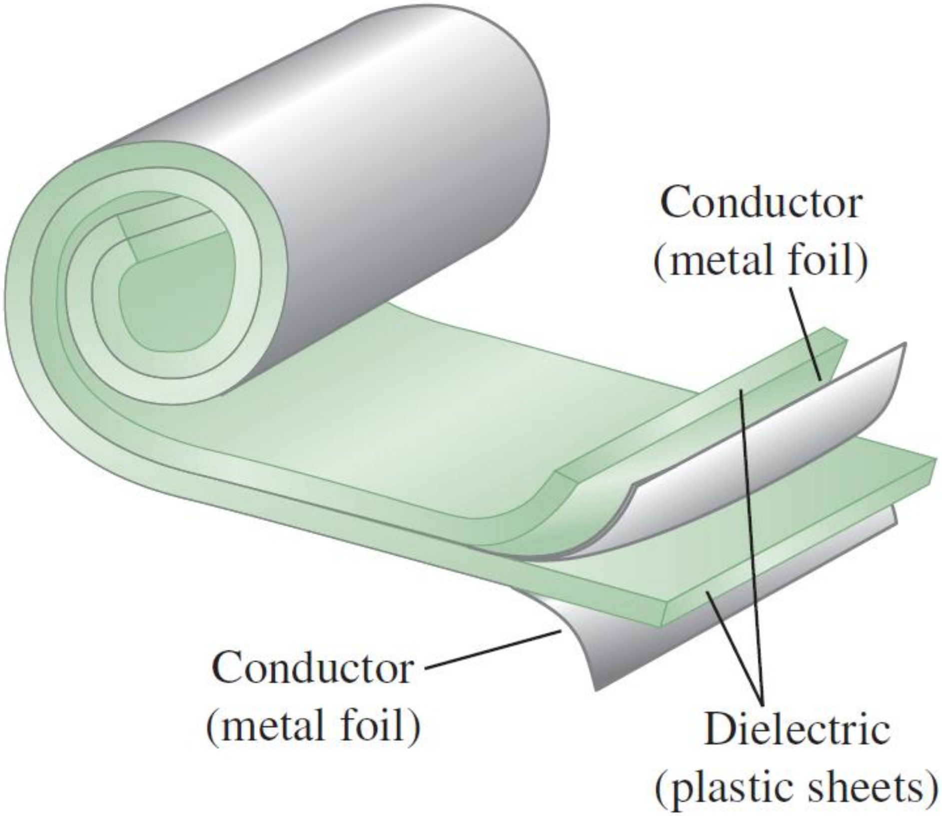
# Problems

## Ch22 Prob. 49, 50, 85 (P543)

## § 22-5, 22-6 Dielectric Materials (P533, P536)

- **Dielectrics vs. conductors**

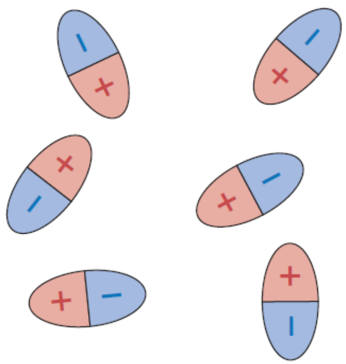
**Most capacitors have a nonconducting material, or dielectric, between their conducting plates.**



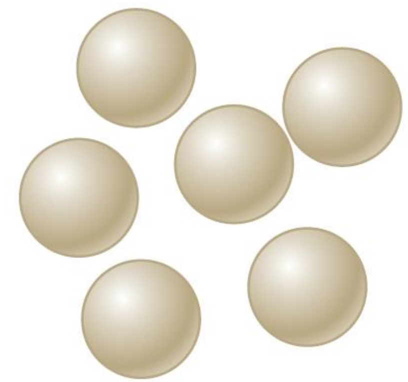

**Why?**

# Polar vs. nonpolar dielectric materials

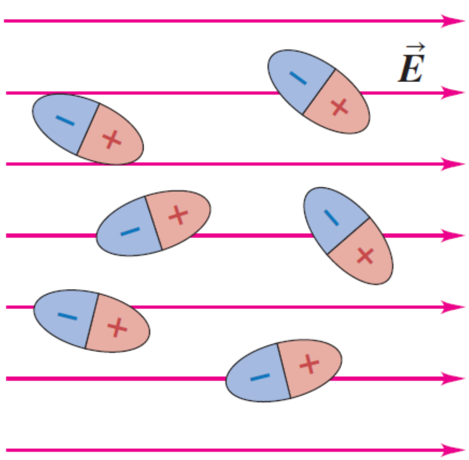
In the absence of an electric field, **Polar** molecules orient randomly.



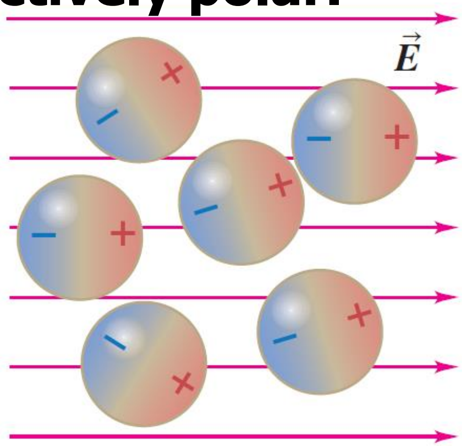
**Nonpolar** molecules are not electric dipoles.



When an electric field is applied, **polar** molecules tend to align with it.



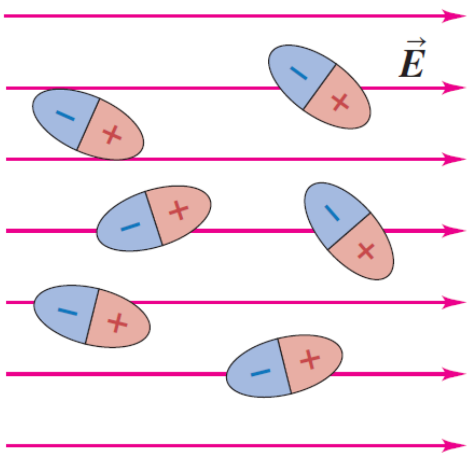

**Nonpolar** molecules are made effectively polar.



# Polar and nonpolar dielectric materials

- **Polar dielectric material —— its molecule has a permanent electric dipole moment, such as water.**

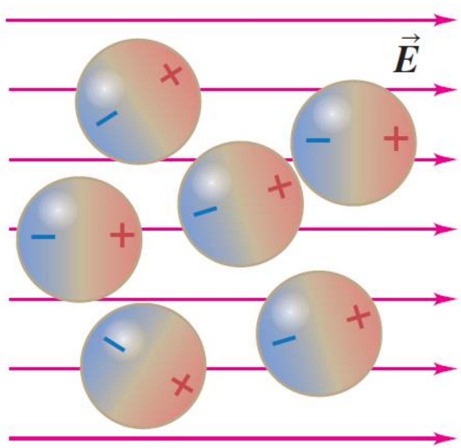
  **The external electric field exerts a torque on the dipole that tries to align it with the field.**



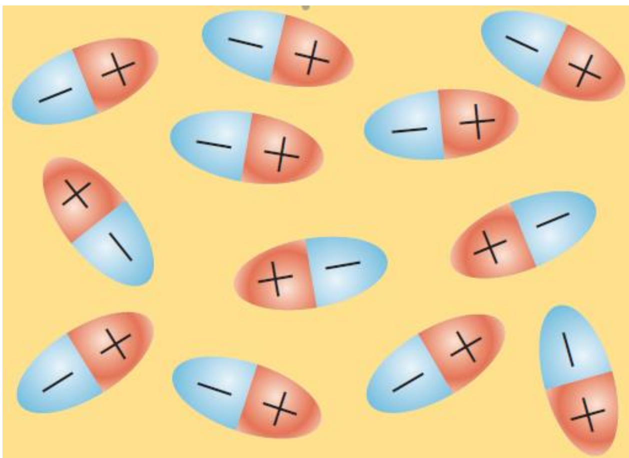

$$\vec{\tau} = \vec{p} \times \vec{E}$$

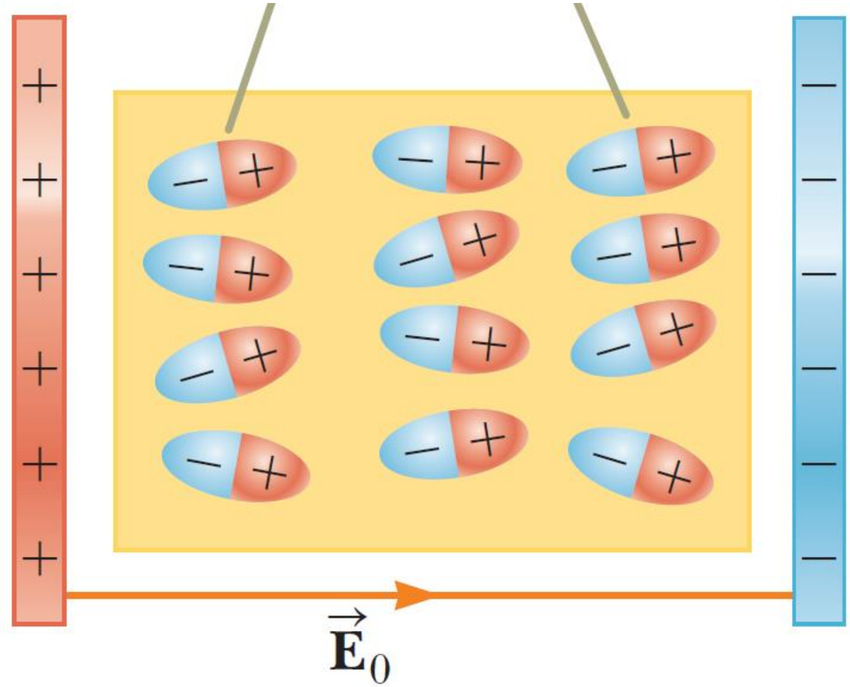
- **Nonpolar dielectric material —— its molecule has no permanent electric dipole.**

  **The atom acquires an induced dipole moment when the atom is placed in an external electric field.**



# Polarization of a dielectric material






**Either polar or nonpolar materials are put in an external field.**

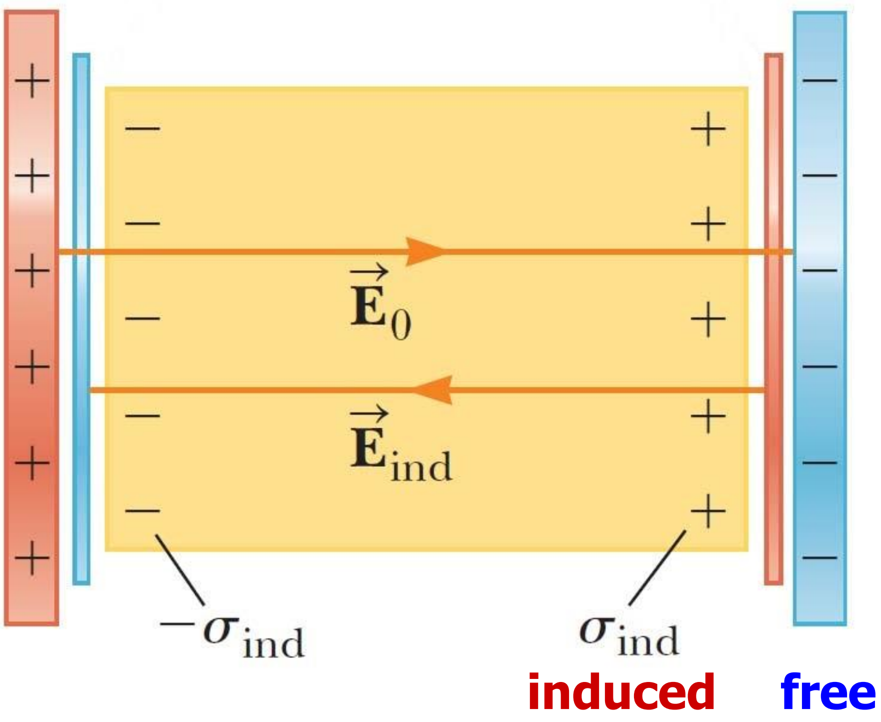
**The induced surface charges arise as a result of redistribution of positive and negative charge within the dielectric material, a phenomenon called polarization.**

# Induced polarization field

- When a dielectric material is placed in an external applied field $E_0$, induced surface charges $q_{ind}$ appear that tend to weaken the original field $E_0$ by a polarization field $E_{ind}$ within the material. For a linear material, the net field inside the material is

$$\vec{E} = \vec{E}_0 + \vec{E}_{ind}$$

- The charge $q_0$, the origin of $E_0$, that resides in the conductors is called free charge, and induced charge $q_{ind}$ that resides in the surface of dielectric materials, that not free to move and bound to a molecule, is called induced bound charge.



# Permittivity

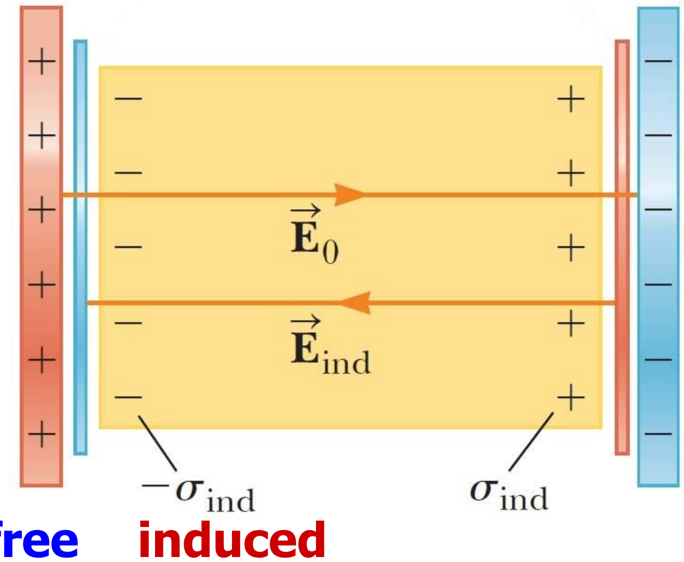
$$\vec{E} = \vec{E}_0 + \vec{E}_{\text{ind}},$$

$$\boxed{E = \frac{E_0}{\kappa}}$$

- $\kappa$ is called the **relavitity** permittivity (dielectric constant)（相对介电常数）, which is greater than 1.

**Absolute** permittivity of the dielectric: $\varepsilon = \kappa\varepsilon_0$

The electric field within the dielectric: $E = \dfrac{E_0}{\kappa} = \dfrac{\sigma_0}{\kappa\varepsilon_0} = \dfrac{\sigma_0}{\varepsilon}$




Induced charge density:

$$E = E_0 - E_{\text{ind}}$$

$$\frac{\sigma_0}{\kappa\varepsilon_0} = \frac{\sigma_0}{\varepsilon_0} - \frac{\sigma_{\text{ind}}}{\varepsilon_0} \Rightarrow \quad \sigma_{\text{ind}} = \left(1 - \frac{1}{\kappa}\right)\sigma_0$$

# The dielectric strength

- **The dielectric strength:** $E_{\text{break}}$

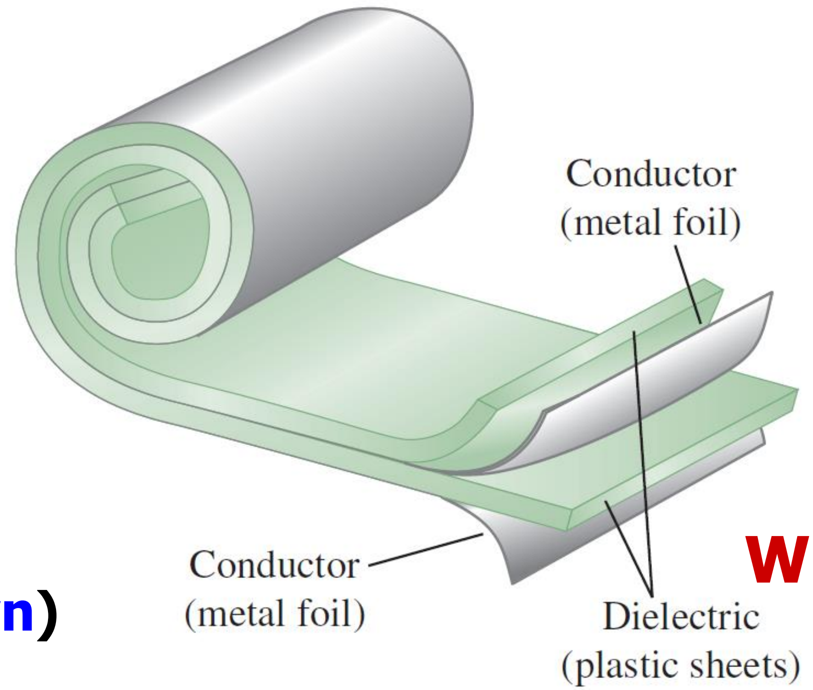
**If we apply a large enough electric field to an insulator, we can ionize atoms or molecules of the insulator and thus create a condition for electric charge to flow, as in a conductor. The field necessary for the breakdown of the insulator is called the dielectric strength.**

| Material | Dielectric Constant $\kappa$ | Dielectric Strength[a] ($10^6$ V/m) |
|---|---|---|
| Air (dry) | 1.000 59 | 3 |
| Bakelite | 4.9 | 24 |
| Fused quartz | 3.78 | 8 |
| Mylar | 3.2 | 7 |
| Neoprene rubber | 6.7 | 12 |
| Nylon | 3.4 | 14 |
| Paper | 3.7 | 16 |
| Paraffin-impregnated paper | 3.5 | 11 |
| Polystyrene | 2.56 | 24 |
| Polyvinyl chloride | 3.4 | 40 |
| Porcelain | 6 | 12 |
| Pyrex glass | 5.6 | 14 |
| Silicone oil | 2.5 | 15 |
| Strontium titanate | 233 | 8 |
| Teflon | 2.1 | 60 |
| Vacuum | 1.000 00 | — |
| Water | 80 | — |

# § 7 Capacitors with Dielectrics (P534)

**Placing a solid dielectric between the plates of a capacitor serves three functions.**

**First, it solves the mechanical problem of maintaining two large metal sheets at a very small separation without actual contact.**




**Why?**

**Second, using a dielectric increases the maximum possible potential difference between the capacitor plates. (without dielectric breakdown)**

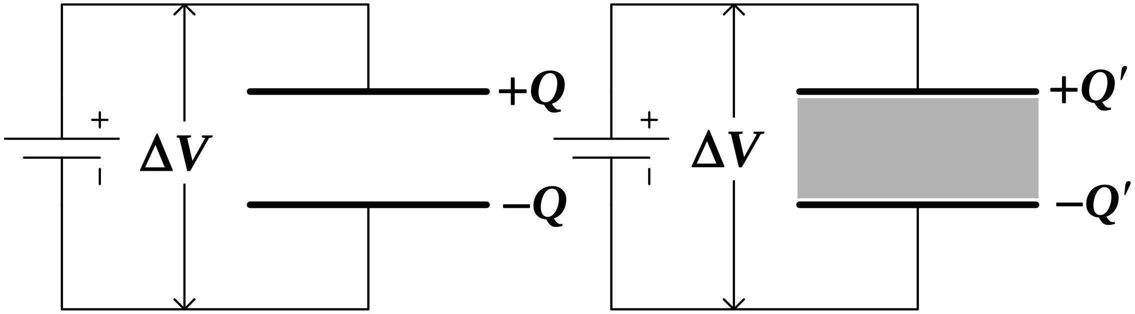
**Third, the capacitance of a capacitor of given dimensions is greater when there is a dielectric material between the plates than when there is vacuum.**

## Capacitors with Dielectrics

**Two identical capacitors, filling one with a dielectric material and leaving the other with air between its plates**

- **When both capacitors are connected to batteries with the same potential difference.**

$$\left.\begin{aligned} &\Delta V = \Delta V' \Rightarrow E = E' \\ &E = \frac{Q}{\varepsilon_0 A}, \quad E' = \frac{1}{\kappa}\frac{Q'}{\varepsilon_0 A} \end{aligned}\right\}$$



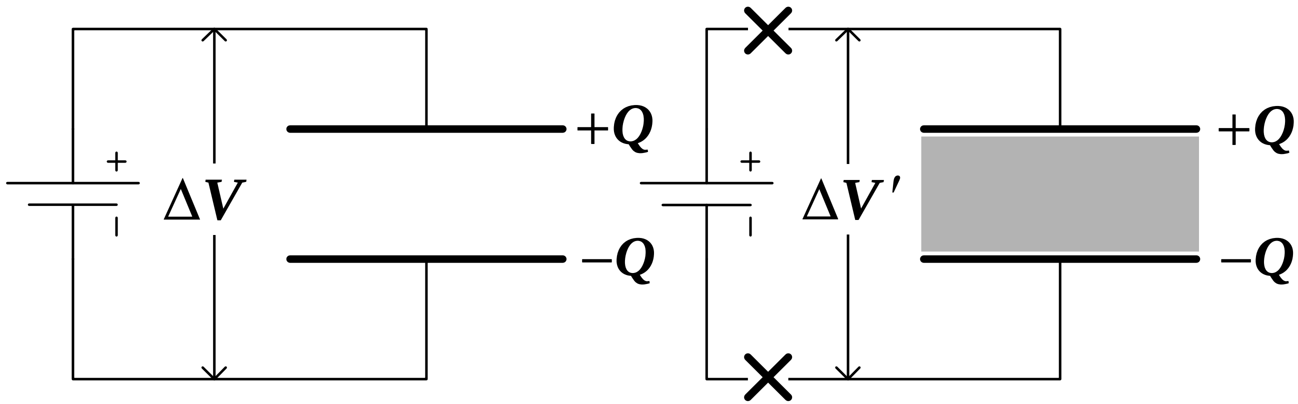

$$Q' = \kappa Q$$

$$C' = \frac{Q'}{\Delta V'} = \frac{\kappa Q}{\Delta V} \Rightarrow C' = \kappa C \Rightarrow C' = \frac{\kappa\varepsilon_0 A}{d} = \frac{\varepsilon A}{d}$$

$\varepsilon = \kappa\varepsilon_0$ **permittivity**

# Capacitors with Dielectrics

- **When both are disconnected the batteries with the same charge.**




$$Q' = Q, \quad E' = \frac{E}{\kappa},$$

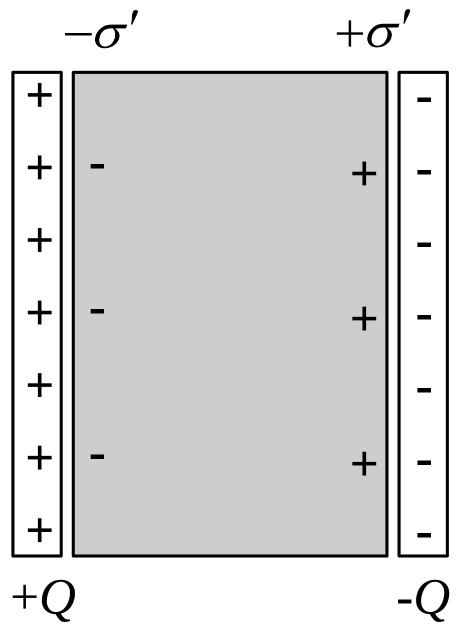
$$\Delta V' = E'd = \frac{Ed}{\kappa} = \frac{\Delta V}{\kappa}$$

$$C' = \frac{Q'}{\Delta V'} = \kappa \frac{Q}{\Delta V} = \kappa C = \frac{\varepsilon A}{d}$$

# The electric field energy stored in a capacitor with dielectric

- **The electric field energy stored in a capacitor with dielectric**

$$U=\frac{Q^2}{2C}=\frac{Q^2 d}{2\kappa\varepsilon_0 A}=\frac{1}{2}\kappa\varepsilon_0\left(\frac{Q}{\kappa\varepsilon_0 A}\right)^2(Ad)$$




$$E=\frac{E_0}{\kappa}=\frac{1}{\kappa}\frac{\sigma}{\varepsilon_0}=\frac{Q}{\kappa\varepsilon_0 A},\qquad \boxed{U=\frac{1}{2}\kappa\varepsilon_0 E^2(Ad)}$$

- **The electric field energy density in dielectric materials**

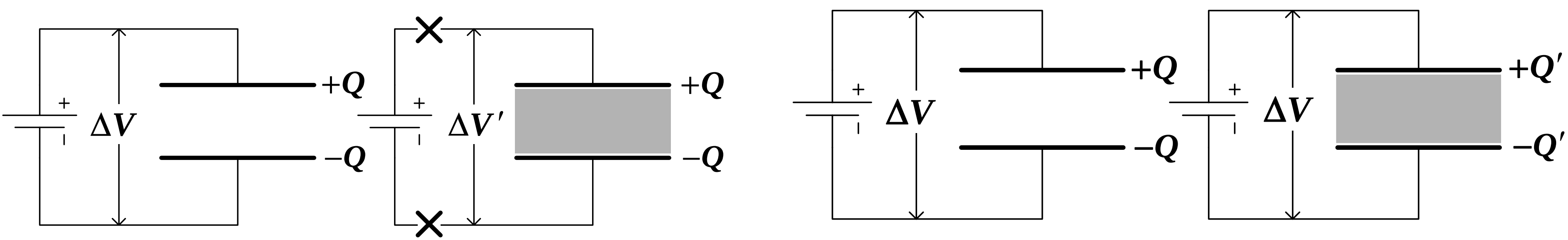
$$\boxed{u=\frac{1}{2}\kappa\varepsilon_0 E^2=\frac{1}{2}\varepsilon E^2}$$

# Example

In following two cases, find the **electric field energy** stored in a parallel-plate capacitor **before** and **after** the dielectric is inserted. The capacitor without dielectric is $C_0$, and dielectric material has dielectric constant $\kappa$.

(1) At beginning, the capacitor, with empty, is connected to the battery of voltage $\Delta V$ . The battery is then removed, and the capacitor is fill with the dielectric material.

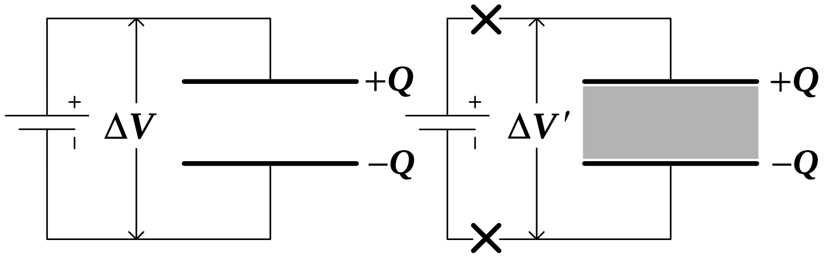
(2) From beginning to end, the capacitor is always connected to the battery of voltage $\Delta V$ ;



# Solution

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}Q\Delta V$$

**(1) Before and after removing the battery, the charges in the capacitor are the same and the capacitance increases.**




**Before inserting the dielectric:** $U_{before} = \frac{1}{2}\frac{Q^2}{C_0}$

**After inserting the dielectric:** $U_{after} = \frac{1}{2}\frac{Q^2}{\kappa C_0} = \frac{U_{before}}{\kappa}$

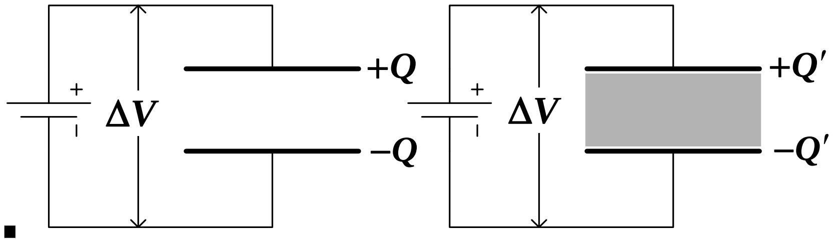
$$\Delta U = U_{after} - U_{before} = (1-\kappa)U_{after} < 0$$

**The dielectric, when inserted, is pulled into the device. To keep the dielectric from accelerating, an external agent must do negative work on the dielectric.**

## Solution

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}Q\Delta V$$

**(2) Before inserting the dielectric:**




$$U_{before} = \frac{1}{2}C_0(\Delta V)^2$$

**After inserting the dielectric:**

$$U_{after} = \frac{1}{2}\kappa C_0(\Delta V)^2 = \kappa U_{before}$$

$$\Delta U = U_{after} - U_{before} = (\kappa - 1)U_{before} > 0$$

# Problems

Ch22 Prob. 65, 87 (P544)