1

Circuit Variables and Circuit Elements

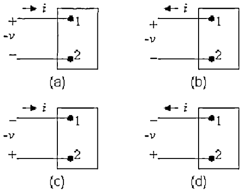
Drill Exercises

DE 1.1 $q = \int_0^\infty 20e^{-5000t}\,dt = 4000\,\mu\text{C}$

DE 1.2 $i = \frac{dq}{dt} = te^{-\alpha t}, \qquad \frac{di}{dt} = (1-\alpha t)e^{-\alpha t}, \qquad \frac{di}{dt} = 0 \qquad \text{when } t = \frac{1}{\alpha};$

Therefore $i_{\max} = \frac{1}{\alpha e} = \frac{1}{0.03679e} \cong 10$ A

DE 1.3 [a]



Therefore

(a) $v = -20\,\text{V}, \quad i = -4\,\text{A}$; (b) $v = -20\,\text{V}, \quad i = 4\,\text{A}$

(c) $v = 20\,\text{V}, \quad i = -4\,\text{A}$; (d) $v = 20\,\text{V}, \quad i = 4\,\text{A}$

[b] Using the reference system in Fig. 1.3(a), $p = vi = (-20)(-4) = 80\,\text{W}$, so the box is absorbing power.

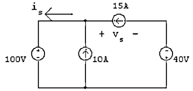
[c] The box is absorbing 80 W.

DE 1.4 $p = vi = 20 \times 10^4 e^{-10{,}000t}$ W; $\quad w = \int_0^\infty 20 \times 10^4 e^{-10{,}000t}\,dt = 20$ J

DE 1.5 $p = 800 \times 10^3 \times 1.8 \times 10^3 = 1440 \times 10^6 = 1440$ MW

from Oregon to California

DE 1.6



The interconnection is valid:

$i_s = 10 + 15 = 25$ A

$p_{100\text{V}} = 100 i_s = 2500$ W (absorbing)

$p_{10\text{A}} = -100(10) = -1000$ W (generating)

$-100 + v_s - 40 = 0 \qquad$ so $v_s = 140$ V

$p_{15\text{A}} = -15(140) = -2100$ W (generating)

$p_{40\text{V}} = 15(40) = 600$ W (absorbing)

$\sum p_{\text{dev}} = p_{10\text{A}} + p_{15\text{A}} = 3100$ W

$\sum p_{\text{abs}} = p_{100\text{V}} + p_{40\text{V}} = 3100$ W

$\sum p_{\text{dev}} = \sum p_{\text{abs}} = 3100$ W

DE 1.7 [a] $v_l - v_c + v_1 - v_s = 0, \qquad i_l R_l - i_c R_c + i_1 R_1 - v_s = 0$

$i_s R_l + i_s R_c + i_s R_1 - v_s = 0$

[b] $i_s = v_s/(R_l + R_c + R_1)$

DE 1.8 [a] $24 = v_2 + v_5 - v_1 = 3i_5 + 7i_5 - (-2i_5) = 12 i_5$

Therefore $i_5 = 24/12 = 2$ A

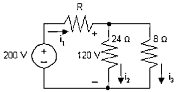
[b] $v_1 = -2i_5 = -4$ V

[c] $v_2 = 3i_5 = 6$ V

[d] $v_5 = 7i_5 = 14$ V

[e] $p_{24} = -(24)(2) = -48$ W; therefore 24 V source is delivering 48 W.

DE 1.9



$i_2 = 120/24 = 5$ A

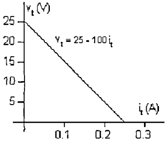
$i_3 = 120/8 = 15$ A

$i_1 = i_2 + i_3 = 20$ A

$-200 + 20R + 120 = 0$

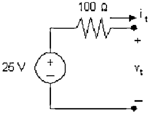
$R = 80/20 = 4\,\Omega$

DE 1.10 [a] Plotting a graph of v_t versus i_t gives

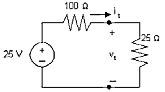


Note that when $i_t = 0$, $v_t = 25$ V; therefore the voltage source must be 25 V. When v_t is zero, $i_t = 0.25$ A, hence the resistor must be 25/0.25 or 100Ω.

A circuit model having the same $v - i$ characteristic is a 25 V source in series with a 100Ω resistor.

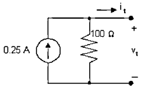


[b]

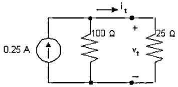


$$i_t = \frac{25}{125} = 0.2 \text{ A}; \qquad p = (0.2)^2(25) = 1 \text{ W}.$$

DE 1.11 [a] Since we are constructing the model from two elements, we have two choices on interconnecting them—series or parallel. From the $v - i$ characteristic we require $v_t = 25$ V when $i_t = 0$. The only way we can satisfy this requirement is with a parallel connection. The constraint that $v_t = 0$ when $i_t = 0.25$ A tells us the ideal current source must produce 0.25 A. Therefore the parallel resistor must be 25/0.25 or 100Ω.



[b]



$$-0.25 + \frac{v_t}{100} + \frac{v_t}{25} = 0, \qquad 5v_t = 25, \qquad v_t = 5 \text{ V}$$

$$p = \frac{v_t^2}{25} = 1 \text{ W}.$$

Problems

P 1.1 $i = \frac{dq}{dt} = 24\cos 4000t$

Therefore, $dq = 24\cos 4000t\,dt$

$$\int_{q(0)}^{q(t)} dx = 24 \int_0^t \cos 4000y \, dy$$

$$q(t) - q(0) = 24 \frac{\sin 4000y}{4000} \bigg|_0^t$$

But $q(0) = 0$ by hypothesis, i.e., the current passes through its maximum value at $t = 0$, so $q(t) = 6 \times 10^{-3} \sin 4000t$ C $= 6 \sin 4000t$ mC

P 1.2 $p = (6)(100) \times 10^{-3} = 0.6$ W; $w = (0.6)(3)(60)(60) = 6480$ J

P 1.3 Assume we are standing at box A looking toward box B, then $p = vi$.

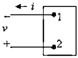
[a] $p = (120)(5) = 600$ W from A to B

[b] $p = (250)(-8) = -2000$ W from B to A

[c] $p = (-150)(16) = -2400$ W from B to A

[d] $p = (-480)(-10) = 4800$ W from A to B

P 1.4 [a]



$p = vi = (40)(-10) = -400$ W
Power is being delivered by the box.

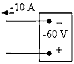
[b] Entering

[c] Gain

P 1.5 [a] $p = vi = (-60)(-10) = 600$ W, so power is being absorbed by the box.

[b] Entering

[c] Lose



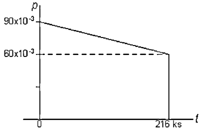
P 1.6 [a] Looking from A to B the current i is in the direction of the voltage rise across the 12 V battery, therefore $p = vi = -12(30) = -360$ W. Thus the power flow is from B to A, and Car A has the "dead" battery.

[b] $w = \int_0^t p \, dx = \int_0^t 360 \, dx$

$w = 360t = 360(1 \times 60) = 21.6$ kJ

P 1.7 $p = vi; \qquad w = \int_0^t p\,dx$

Since the energy is the area under the power vs. time plot, let us plot p vs. t.



$p(0) = (6)(15 \times 10^{-3}) = 90 \times 10^{-3}$ W

$p(216 \text{ ks}) = (4)(15 \times 10^{-3}) = 60 \times 10^{-3}$ W

$w = (60 \times 10^{-3})(216 \times 10^3) + \frac{1}{2}(216)(30) = 16.2$ kJ

Note: 60 hr $\equiv$ 216,000 s = 216 ks

P 1.8 [a] $p = vi = 30e^{-500t} - 30e^{-1500t} - 40e^{-1000t} + 50e^{-2000t} - 10e^{-3000t}$
$p(1 \text{ ms}) = 3.1$ mW

[b]
$$\begin{aligned} w(t) &= \int_0^t (30e^{-500x} - 30e^{-1500x} - 40e^{-1000x} + \\ &\qquad 50e^{-2000x} - 10e^{-3000x})\,dx \\ &= 21.67 - 60e^{-500t} + 20e^{-1500t} + 40e^{-1000t} - \\ &\qquad 25e^{-2000t} + 3.33e^{-3000t}\,\mu\text{J} \\ w(1\text{ ms}) &= 1.24\mu\text{J} \end{aligned}$$

[c] $w_{\text{total}} = 21.67\mu$J

P 1.9 [a] $v(20 \text{ ms}) = 100e^{-1}\sin 3 = 5.19$ V
$i(20 \text{ ms}) = 20e^{-1}\sin 3 = 1.04$ A
$p(20 \text{ ms}) = vi = 5.39$ W

[b]

$$\begin{aligned}
p &= vi = 2000e^{-100t}\sin^2 150t \\
&= 2000e^{-100t}\left[\frac{1}{2} - \frac{1}{2}\cos 300t\right] \\
&= 1000e^{-100t} - 1000e^{-100t}\cos 300t \\
w &= \int_0^\infty 1000e^{-100t}\,dt - \int_0^\infty 1000e^{-100t}\cos 300t\,dt \\
&= 1000\,\frac{e^{-100t}}{-100}\bigg|_0^\infty - 1000\left\{\frac{e^{-100t}}{(100)^2+(300)^2}\left[-100\cos 300t + 300\sin 300t\right]\right\}\bigg|_0^\infty \\
&= 10 - 1000\left[\frac{100}{1\times 10^4 + 9\times 10^4}\right] = 10 - 1 \\
\mathrm{w} &= 9 \ \ \mathrm{J}
\end{aligned}$$

P 1.10 [a] $0 \le t \le 10$ ms:

$v = 1000t$ V; $i = 0.6$ mA; $p = 0.6t$ mW

$10 \le t \le 25$ ms:

$v = 10$ V; $i = 0.6$ mA; $p = 6$ mW

$25 \le t \le 35$ ms:

$v = 75 - 2500t$ V; $i = 0$ mA; $p = 0$ mW

$35 \le t \le 60$ ms:

$v = -50 + 1000t$ V; $i = -0.4$ mA; $p = 20 - 400t$ mW

$60 \le t \le 70$ ms:

$v = -50 + 1000t$ V; $i = 0$ mA; $p = 0$ mW

$70 \le t \le 80$ ms:

$v = 20$ V; $i = -0.5$ mA; $p = -10$ mW

$80 \le t \le 90$ ms:

$v = 180 - 2000t$ V; $i = 0$ mA; $p = 0$ mW

$90 \le t \le 95$ ms:

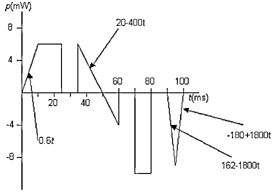
$v = 180 - 2000t$ V; $i = 0.9$ mA; $p = 162 - 1800t$ mW

$95 \le t \le 100$ ms:

$v = -200 + 2000t$ V; $i = 0.9$ mA; $p = -180 + 1800t$ mW

[b]

$$\begin{aligned} w(25) &= \tfrac{1}{2}(6)(10) + (6)(15) = 120\,\mu\text{J} \\ w(60) &= 120 + \tfrac{1}{2}(15)(6) - \tfrac{1}{2}(10)(4) = 145\,\mu\text{J} \\ w(90) &= 145 - (10)(10) = 45\,\mu\text{J} \\ w(100) &= 45 - \tfrac{1}{2}(10)(9) = 0\,\mu\text{J} \end{aligned}$$



P 1.11 [a] $p = vi = (2e^{-500t} - 2e^{-1000t})$ W

$$\frac{dp}{dt} = -1000e^{-500t} + 2000e^{-1000t} = 0 \text{ at t} = 1.4 \text{ ms}$$

$p_{\max} = p(1.4 \text{ ms}) = 0.5$ W

[b] $$w = \int_0^\infty [2e^{-500t} - 2e^{-1000t}]\,dt = \left[\frac{2}{-500}e^{-500t} - \frac{2}{-1000}e^{-1000t}\Big|_0^\infty\right]$$

$= 2$ mJ

P 1.12 [a] $p = vi = 900\sin(200\pi t)\cos(200\pi t) = 450\sin(400\pi t)$ W

Therefore, $p_{\max} = 450$ W

[b] $p_{\max}(\text{extracting}) = 450$ W

[c]

$$\begin{aligned} p_{\text{avg}} &= 200\int_0^{5\times10^{-3}} 450\sin(400\pi t)\,dt \\ &= 9\times10^4\left[\frac{-\cos 400\pi t}{400\pi}\Big|_0^{2.5\times10^{-3}}\right] = \frac{225}{\pi}[1-\cos 2\pi] = 0 \end{aligned}$$

[d] $$p_{\text{avg}} = \frac{180}{\pi}[1-\cos 2.5\pi] = \frac{180}{\pi} = 57.3 \text{ W}$$

P 1.13 [a]

$$\begin{aligned} q &= \text{area under } i \text{ vs. } t \text{ plot} \\ &= \left[\tfrac{1}{2}(5)(4) + (10)(4) + \tfrac{1}{2}(8)(4) + (8)(6) + \tfrac{1}{2}(3)(6)\right]\times10^3 \\ &= [10+40+16+48+9]10^3 = 123{,}000 \text{ C} \end{aligned}$$

$$[b]\quad w = \int p\,dt = \int vi\,dt$$

$$v = 0.2 \times 10^{-3}t + 9 \qquad 0 \le t \le 15 \text{ ks}$$

$0 \le t \le 4000s$

$$i = 15 - 1.25 \times 10^{-3}t$$

$$p = 135 - 8.25 \times 10^{-3}t - 0.25 \times 10^{-6}t^2$$

$$w_1 = \int_0^{4000} (135 - 8.25 \times 10^{-3}t - 0.25 \times 10^{-6}t^2)\,dt$$

$$= (540 - 66 - 5.3333)10^3 = 468.667 \text{ kJ}$$

$4000 \le t \le 12{,}000$

$$i = 12 - 0.5 \times 10^{-3}t$$

$$p = 108 - 2.1 \times 10^{-3}t - 0.1 \times 10^{-6}t^2$$

$$w_2 = \int_{4000}^{12{,}000} (108 - 2.1 \times 10^{-3}t - 0.1 \times 10^{-6}t^2)\,dt$$

$$= (864 - 134.4 - 55.467)10^3 = 674.133 \text{ kJ}$$

$12{,}000 \le t \le 15{,}000$

$$i = 30 - 2 \times 10^{-3}t$$

$$p = 270 - 12 \times 10^{-3}t - 0.4 \times 10^{-6}t^2$$

$$w_3 = \int_{12{,}000}^{15{,}000} (270 - 12 \times 10^{-3}t - 0.4 \times 10^{-6}t^2)\,dt$$

$$= (810 - 486 - 219.6)10^3 = 104.4 \text{ kJ}$$

$$w_T = w_1 + w_2 + w_3 = 468.667 + 674.133 + 104.4 = 1247.2 \text{ kJ}$$

P 1.14 [a]

$$p = vi$$

$$= 400 \times 10^3 t^2 e^{-800t} + 700te^{-800t} + 0.25e^{-800t}$$

$$= e^{-800t}[400{,}000t^2 + 700t + 0.25]$$

$$\frac{dp}{dt} = \{e^{-800t}[800 \times 10^3 t + 700] - 800e^{-800t}[400{,}000t^2 + 700t + 0.25]\}$$

$$= [-3{,}200{,}000t^2 + 2400t + 5]100e^{-800t}$$

Therefore, $\frac{dp}{dt} = 0$ when $3{,}200{,}000t^2 - 2400t - 5 = 0$

so $p_{\max}$ occurs at $t = 1.68$ ms.

[b]

$$p_{\max} = [400{,}000(.00168)^2 + 700(.00168) + 0.25]e^{-800(.00168)}$$

$$= 666 \text{ mW}$$

[c]
$$\begin{aligned} w &= \int_0^t p\,dx \\ w &= \int_0^t 400{,}000x^2e^{-800x}\,dx + \int_0^t 700xe^{-800x}\,dx + \int_0^t 0.25e^{-800x}\,dx \\ &= \frac{400{,}000e^{-800x}}{-512\times 10^6}[64\times 10^4x^2 + 1600x + 2]\Big|_0^t + \\ &\quad \frac{700e^{-800x}}{64\times 10^4}(-800x-1)\Big|_0^t + 0.25\frac{e^{-800x}}{-800}\Big|_0^t \end{aligned}$$

When $t = \infty$ all the upper limits evaluate to zero, hence

$$w = \frac{(400{,}000)(2)}{512\times 10^6} + \frac{700}{64\times 10^4} + \frac{0.25}{800} = 2.97 \text{ mJ}.$$

P 1.15 [a]
$$\begin{aligned} p &= 0 \quad t<0, \qquad p = 0 \quad t > 3\text{ s} \\ p &= vi = t(3-t)(6-4t) = 18t - 18t^2 + 4t^3 \text{ mW} \qquad 0 \le t \le 3\text{ s} \\ \frac{dp}{dt} &= 18 - 36t + 12t^2 = 12(t^2 - 3t + 1.5) \\ \frac{dp}{dt} &= 0 \qquad \text{when } t^2 - 3t + 1.5 = 0 \\ t &= \frac{3\pm\sqrt{9-6}}{2} = \frac{3\pm\sqrt{3}}{2} \\ t_1 &= 3/2 - \sqrt{3}/2 = 0.634\text{ s}; \qquad t_2 = 3/2+\sqrt{3}/2 = 2.366\text{ s} \\ p(t_1) &= 18(0.634) - 18(0.634)^2 + 4(0.634)^3 = 5.196\text{ mW} \\ p(t_2) &= 18(2.366) - 18(2.366)^2 + 4(2.366)^3 = -5.196\text{ mW} \end{aligned}$$

Therefore, maximum power is being delivered at $t = 0.634$ s.

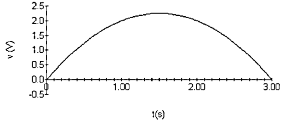
[b] $p_{\max} = 5.196$ mW (delivered)

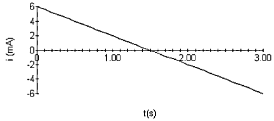
[c] Maximum power is being extracted at $t = 2.366$ s.

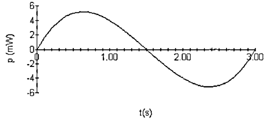
[d] $p_{\max} = 5.196$ mW (extracted)

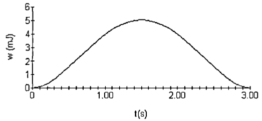
[e] $w = \int_0^t p\,dx = \int_0^t (18x - 18x^2 + 4x^3)dx = 9t^2 - 6t^3 + t^4$

$$\begin{aligned} w(0) &= 0 \text{ mJ} \qquad & w(2) &= 4\text{ mJ} \\ w(1) &= 4 \text{ mJ} \qquad & w(3) &= 0\text{ mJ} \end{aligned}$$









P 1.16 [a]

$$p = vi = 12 \times 10^5 t^2 e^{-1000t} \text{ W}$$

$$\frac{dp}{dt} = 12 \times 10^5[t^2(-1000)e^{-1000t} + e^{-1000t}(2t)]$$

$$= 12 \times 10^5 t e^{-1000t}[t(2 - 1000t)]$$

$$\frac{dp}{dt} = 0 \text{ at } t = 0, \qquad t = 2 \text{ ms}$$

We know p is a minimum at $t = 0$ since v and i are zero at $t = 0$.

[b] $p_{\max} = 12 \times 10^5(2 \times 10^{-3})^2 e^{-2} = 649.61$ mW

[c]
$$\begin{aligned} w &= 12 \times 10^5 \int_0^\infty t^2 e^{-1000t}\,dt \\ &= 12 \times 10^5 \left\{ \frac{e^{-1000t}}{(-1000)^3}[10^6 t^2 + 2{,}000t + 2] \Bigg|_0^\infty \right\} = 2.4 \text{ mJ} \end{aligned}$$

P 1.17 [a] From the diagram and the table we have

$$\begin{aligned} p_\text{a} &= -v_\text{a} i_\text{a} = -(46.16)(6) = -276.96 \text{ W} \qquad \text{(del)} \\ p_\text{b} &= v_\text{b} i_\text{b} = (14.16)(4.72) = 66.8352 \text{ W} \qquad \text{(abs)} \\ p_\text{c} &= v_\text{c} i_\text{c} = (-32)(-6.4) = 204.80 \text{ W} \qquad \text{(abs)} \\ p_\text{d} &= -v_\text{d} i_\text{d} = -(22)(1.28) = -28.16 \text{ W} \qquad \text{(del)} \\ p_\text{e} &= -v_\text{e} i_\text{e} = -(33.60)(1.68) = -56.448 \text{ W} \qquad \text{(del)} \\ p_\text{f} &= v_\text{f} i_\text{f} = (66)(-0.4) = -26.40 \text{ W} \qquad \text{(del)} \\ p_\text{g} &= v_\text{g} i_\text{g} = (2.56)(1.28) = 3.2768 \text{ W} \qquad \text{(abs)} \\ p_\text{h} &= -v_\text{h} i_\text{h} = -(-0.4)(0.4) = 0.16 \text{ W} \qquad \text{(abs)} \\ \sum P_\text{del} &= 276.96 + 28.16 + 56.448 + 26.40 = 387.9680 \text{ W} \\ \sum P_\text{abs} &= 66.8352 + 204.80 + 3.2768 + 0.16 = 275.072 \text{ W} \end{aligned}$$

Therefore, $\sum P_\text{del} \neq \sum P_\text{abs}$ and the subordinate engineer is correct.

[b] We can also check the data using Kirchhoff's laws.

From Fig. P1.17 the following equations should be satisfied:

$i_\text{a} - i_\text{b} - i_\text{d} = 0$ (ok)
$i_\text{b} + i_\text{c} - i_\text{e} = 0$ (no)
$i_\text{f} - i_\text{a} - i_\text{c} = 0$ (ok)
$i_\text{d} = i_\text{g}$ (ok)
$i_\text{g} + i_\text{e} + i_\text{h} = 0$ (no)
$i_\text{h} = -i_\text{f}$ (ok)

Using Kirchhoff's current law, it appears i_e is in error.

From Kirchhoff's voltage law we have

$v_\text{b} - v_\text{a} - v_\text{c} = 0$ (ok)
$-v_\text{d} - v_\text{b} + v_\text{e} + v_\text{g} = 0$ (ok)
$-v_\text{e} + v_\text{c} + v_\text{f} + v_\text{h} = 0$ (ok)

Therefore all the voltages are consistent with Kirchhoff's voltage law.

Assume i_e is in error. Therefore,

$i_\text{e} = i_\text{b} + i_\text{c} = -i_\text{g} - i_\text{h} = 4.72 - 6.40 = -1.28 - 0.4 = -1.68$ A

So the error is in the sign of i_e; i_e equals minus 1.68 A.

Correcting i_e leads to

$\sum P_\text{del} = \sum P_\text{abs} = 331.52$ W

P 1.18
$$\begin{aligned}
p_a &= v_a i_a = (48)(12) = 576\ \text{W} \qquad \text{(abs)}\\
p_b &= v_b i_b = (18)(-4) = -72\ \text{W} \qquad \text{(del)}\\
p_c &= -v_c i_c = -(30)(-10) = 300\ \text{W} \qquad \text{(abs)}\\
p_d &= v_d i_d = (36)(16) = 576\ \text{W} \qquad \text{(abs)}\\
p_e &= -v_e i_e = -(36)(8) = -288\ \text{W} \qquad \text{(del)}\\
p_f &= -v_f i_f = -(-54)(14) = 756\ \text{W} \qquad \text{(abs)}\\
p_g &= -v_g i_g = -(84)(22) = -1848\ \text{W} \qquad \text{(del)}
\end{aligned}$$

$\sum P_{del} = 72 + 288 + 1848 = 2208$ W

$\sum P_{abs} = 576 + 300 + 576 + 756 = 2208$ W

Therefore, $\sum P_{del} = \sum P_{abs} = 2208$ W

P 1.19 [a] From an examination of reference polarities, the following elements employ the passive convention: a, c, e, and f.

[b]
$$\begin{aligned}
p_a &= -56\ \text{W} \qquad \text{(del)}\\
p_b &= -14\ \text{W} \qquad \text{(del)}\\
p_c &= 150\ \text{W} \qquad \text{(abs)}\\
p_d &= -50\ \text{W} \qquad \text{(del)}\\
p_e &= -18\ \text{W} \qquad \text{(del)}\\
p_f &= -12\ \text{W} \qquad \text{(del)}
\end{aligned}$$

$\sum P_{abs} = 150$ W; $\quad \sum P_{del} = 56 + 14 + 50 + 18 + 12 = 150$ W.

P 1.20 (a) 9 (b) 7 (c) 4 (d) v_a–R_a, v_b–R_b, v_c–R_c (e) 6

(f) (1) $v_a - R_a - R_d - R_b - v_b$

(2) $R_d - R_f - R_e$

(3) $v_b - R_b - R_d - R_f - R_c - v_c$

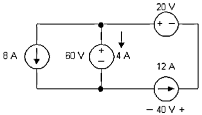
(4) $v_c - R_c - R_f - R_a - v_a$

(5) $v_a - R_a - R_f - R_e - R_b - v_b$

(6) $v_a - R_a - R_d - R_e - R_c - v_c$

(7) $v_b - R_b - R_e - R_c - v_c$

P 1.21 The interconnect is valid since it does not violate Kirchhoff's laws.



$-60 + 20 + 40 = 0 \qquad \text{(KVL)}$

$8 + 4 - 12 = 0 \qquad \text{(KCL)}$

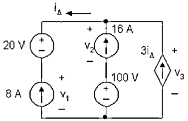
$\sum P_{\text{dev}} = 4(60) + 8(60) = 720 \text{ W}$

$\sum P_{\text{abs}} = 12(20) + 12(40) = 720 \text{ W}$

$\sum P_{\text{dev}} = \sum P_{\text{abs}} = 720 \text{ W}$

P 1.22 [a] Yes, Kirchhoff's laws are not violated.

[b] No, because the voltages across the independent and dependent current sources are indeterminate. For example, define v_1, v_2, and v_3 as shown:



Kirchhoff's voltage law requires

$v_1 + 20 = v_3$

$v_2 + 100 = v_3$

Conservation of energy requires

$8(20) + 8v_1 + 16v_2 + 1600 - 24v_3 = 0$

or

$v_1 + 2v_2 - 3v_3 = -220$

Now arbitrarily select a value of v_3 and show the conservation of energy will be satisfied. Examples:
If $v_3 = 200$ V then $v_1 = 180$ V and $v_2 = 100$ V. Then

$180 + 200 - 600 = -220$ (CHECKS)

If $v_3 = -100$ V, then $v_1 = -120$ V and $v_2 = -200$ V. Then

$-120 - 400 + 300 = -220$ (CHECKS)

P 1.23 [a] Yes, independent voltage sources can carry whatever current is required by the connection; independent current source can support any voltage required by the connection.

[b] 30 V source: absorbing

10 V source: delivering

8 A source: delivering

[c]
$$\begin{aligned} P_{30V} &= (30)(8) = 240 \text{ W} \quad \text{(abs)} \\ P_{10V} &= -10(8) = -80 \text{ W} \quad \text{(del)} \\ P_{8A} &= -20(8) = -160 \text{ W} \quad \text{(del)} \end{aligned}$$
$\sum P_{abs} = \sum P_{del} = 240$ W

[d] Yes, 30 V source is delivering, the 10 V source is delivering, and the 8 A source is absorbing
$$\begin{aligned} P_{30V} &= -30(8) = -240 \text{ W} \quad \text{(del)} \\ P_{10V} &= -10(8) = -80 \text{ W} \quad \text{(del)} \\ P_{8A} &= +40(8) = 320 \text{ W} \quad \text{(abs)} \end{aligned}$$

P 1.24 The interconnection is valid because it does not violate Kirchhoff's laws.

$i_\Delta = -25$ A; $6i_\Delta = -150$ V

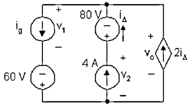
$-200 + 50 - (-150) = 0$

But the power developed in the circuit cannot be determined, as the currents in the 200 V, 50 V, and $6i_\Delta$ sources are unspecified.

P 1.25 The interconnection is not valid because it violates Kirchhoff's current law:

$3 \text{ A} + (-5 \text{ A}) \neq 8 \text{ A}.$

P 1.26



$i_\Delta = 4$ A so $i_g = 12$ A

$v_o = 100$ V

$-60 + v_1 = 100, \text{ so } v_1 = 160$ V

$v_2 - 80 = 100, \text{ so } v_2 = 180$ V

$\sum P_{\text{dev}} = 180(4) + 100(8) + 60(12) = 2240$ W

$$\text{CHECK: } \sum P_{\text{diss}} = 160(12) + 80(4) = 1920 + 320 = 2240 \text{ W — CHECKS}$$

P 1.27 The interconnection is valid because it does not violate Kirchhoff's laws:

$p_{\text{V-sources}} = -(100 - 60)(5) = -200$ W.

P 1.28 First there is no violation of Kirchhoff's laws, hence the interconnection is valid.
Kirchhoff's voltage law requires

$v_1 + v_2 = 150 - 50 = 100$ V

The conservation of energy law requires

$20v_1 - 10v_1 + 10v_2 + 500 - 1500 = 0$

or

$v_1 + v_2 = 100$

Hence any combination of v_1 and v_2 that adds to 100 is a valid solution. For example if $v_1 = 80$ V and $v_2 = 20$ V

$P_{\text{abs}} = 80(20) + 10(20) + 50(10) = 2300$ W

$P_{\text{dev}} = 1500 + 80(10) = 2300$ W

If $v_1 = 60$ V and $v_2 = 40$ V

$P_{\text{abs}} = 60(20) + 10(40) + 500 = 2100$ W

$P_{\text{dev}} = 60(10) + 1500 = 2100$ W

If $v_1 = -100$ V and $v_2 = 200$ V

$P_{\text{abs}} = 10(100) + 10(200) + 10(50) = 3500$ W

$P_{\text{dev}} = 20(100) + 10(150) = 3500$ W

P 1.29 [a]
$$\begin{aligned} 1.6 &= i_g - i_a \\ 80i_a &= 1.6(30+90) = 192 \quad \text{therefore, } i_a = 2.4 \text{ A} \\ i_g &= i_a + 1.6 = 2.4 + 1.6 = 4 \text{ A} \end{aligned}$$

[b] $v_g = 90(1.6) = 144$ V

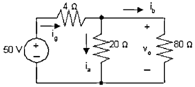
[c]
$$\begin{aligned} \sum P_{\text{dis}} &= 2.4^2(80) + 1.6^2(120) = 768 \text{ W} \\ \sum P_{\text{dev}} &= (4)(192) = 768 \text{ W} \end{aligned}$$
Therefore, $\sum P_{\text{dis}} = \sum P_{\text{dev}} = 768$ W

P 1.30 [a]
$$\begin{aligned} v_o &= 8i_a + 14i_a + 18i_a = 40(20) = 800 \text{ V} \\ 800 &= 10i_o \\ i_o &= 800/10 = 80 \text{ A} \end{aligned}$$

[b] $i_g = i_a + i_o = 20 + 80 = 100$ A

[c] $p_g(\text{delivered}) = (100)(800) = 80{,}000$ W $= 80$ kW

P 1.31 [a]



$$\begin{aligned} 20i_a &= 80i_b \qquad i_g = i_a + i_b = 5i_b \\ i_a &= 4i_b \\ 50 &= 4i_g + 80i_b = 20i_b + 80i_b = 100i_b \\ i_b &= 0.5 \text{ A, therefore, } i_a = 2 \text{ A} \quad \text{and} \quad i_g = 2.5 \text{ A} \end{aligned}$$

[b] $i_b = 0.5$ A

[c] $v_o = 80i_b = 40$ V

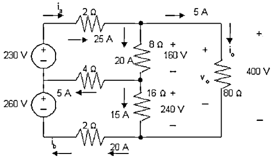
[d]
$$\begin{aligned} p_{4\Omega} &= i_g^2(4) = 6.25(4) = 25 \text{ W} \\ p_{20\Omega} &= i_a^2(20) = (4)(20) = 80 \text{ W} \\ p_{80\Omega} &= i_b^2(80) = 0.25(80) = 20 \text{ W} \end{aligned}$$

[e] p_{5V} (delivered) $= 5i_g = 125$ W
Check:

$$\sum P_{\text{dis}} = 25 + 80 + 20 = 125 \text{ W}$$

$$\sum P_{\text{del}} = 125 \text{ W}$$

P 1.32 [a]



$$\begin{aligned} v_o &= 20(8) + 16(15) = 400 \text{ V} \\ i_o &= 400/80 = 5 \text{ A} \\ i_a &= 25 \text{ A} \end{aligned}$$

P_{230} (supplied) $= (230)(25) = 5750$ W

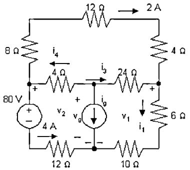
$i_b = 5 + 15 = 20$ A

P_{260} (supplied) $= (260)(20) = 5200$ W

[b]
$$\begin{aligned} \sum P_{\text{dis}} &= (25)^2(2) + (20)^2(8) + (5)^2(4) + (15)^2 16 + (20)^2 2 + (5)^2(80) \\ &= 1250 + 3200 + 100 + 3600 + 800 + 2000 = 10{,}950 \text{ W} \\ \sum P_{\text{sup}} &= 5750 + 5200 = 10{,}950 \text{ W} \end{aligned}$$

Therefore, $\sum P_{\text{dis}} = \sum P_{\text{sup}} = 10{,}950$ W

P 1.33 [a]



$$\begin{aligned}
v_2 &= 80 + 4(12) = 128 \text{ V} \\
v_1 &= 128 - 24(2) = 80 \text{ V} \\
i_1 &= \frac{v_1}{16} = \frac{80}{16} = 5 \text{ A} \\
i_3 &= i_1 - 2 = 5 - 2 = 3 \text{ A} \\
v_g &= v_1 + 24i_3 = 80 + 72 = 152 \text{ V} \\
v_g - 4i_4 &= v_2 \\
4i_4 &= v_g - v_2 = 152 - 128 = 24 \text{ V} \\
i_4 &= 24/4 = 6 \text{ A} \\
i_g &= -(i_3 + i_4) = -(3 + 6) = -9 \text{ A}
\end{aligned}$$

[b]

$$\begin{aligned}
p_{8\Omega} &= (2)^2(8) = 32 \text{ W} & p_{4\Omega} &= (6)^2(4) = 144 \text{ W} \\
p_{12\Omega} &= (2)^2(12) = 48 \text{ W} & p_{6\Omega} &= (5)^2(6) = 150 \text{ W} \\
p_{4\Omega} &= (2)^2(4) = 16 \text{ W} & p_{10\Omega} &= (5)^2(10) = 250 \text{ W} \\
p_{24\Omega} &= (3)^2(24) = 216 \text{ W} & p_{12\Omega} &= (4)^2(12) = 192 \text{ W}
\end{aligned}$$

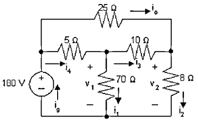
[c] $v_g = 152$ V

[d]

$$\begin{aligned}
\sum P_{\text{dis}} &= 32 + 48 + 16 + 216 + 144 + 150 + 250 + 192 + 80(4) = 1368 \text{ W} \\
\sum P_{\text{del}} &= (152)(9) = 1368 \text{ W}
\end{aligned}$$

Therefore, $\sum P_{\text{dis}} = \sum P_{\text{del}}$

P 1.34 [a]



$$v_2 = 180 - 100 = 80 \text{ V}$$

$$i_2 = \frac{v_2}{8} = 10 \text{ A}$$

$$i_3 + 4 = i_2, \qquad i_3 = 10 - 4 = 6 \text{ A}$$

$$v_1 = v_2 + v_3 = 80 + 6(10) = 140 \text{ V}$$

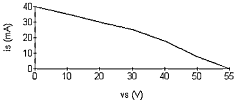
$$i_1 = \frac{v_1}{70} = \frac{140}{70} = 2 \text{ A}$$

[b]
$$\begin{aligned} p_{5\Omega} &= 8^2(5) = 320 \text{ W} \\ p_{25\Omega} &= (4)^2(25) = 400 \text{ W} \\ p_{70\Omega} &= 2^2(70) = 280 \text{ W} \\ p_{10\Omega} &= 6^2(10) = 360 \text{ W} \\ p_{8\Omega} &= 10^2(8) = 800 \text{ W} \end{aligned}$$

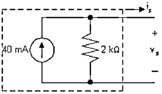
[c] $\sum P_{\text{dis}} = 320 + 400 + 280 + 360 + 800 = 2160$ W

$$P_{\text{dev}} = 180 i_g = 180(12) = 2160 \text{ W}$$

P 1.35 [a]

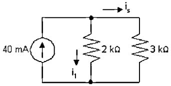


[b] $\Delta v = 20$ V; $\Delta i = 10$ mA; $R = \dfrac{\Delta v}{\Delta i} = 2\ \text{k}\Omega$



[c] $2i_1 = 3i_s$, $i_1 = 1.5i_s$

$40 = i_1 + i_s = 2.5i_s$, $i_s = 16$ mA

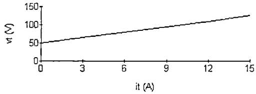


[d] $v_s(\text{open circuit}) = (40 \times 10^{-3})(2 \times 10^{3}) = 80$ V

[e] $v_s(\text{open circuit}) = 55$ V

[f] Linear model cannot predict the nonlinear behavior of the practical current source.

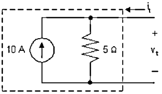
P 1.36 [a] Plot the $v - i$ characteristic



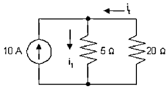
From the plot:

$$R = \frac{\Delta v}{\Delta i} = \frac{(125 - 50)}{(15 - 0)} = 5\,\Omega$$

When $i_t = 0$, $v_t = 50$ V; therefore the ideal current source has a current of 10 A



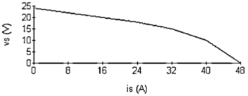
[b]



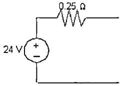
$10 + i_t = i_1$ and $5i_1 = -20i_t$

Therefore, $10 + i_t = -4i_t$ so $i_t = -2$ A

P 1.37 [a]



[b] $R = \dfrac{24 - 18}{24 - 0} = \dfrac{6}{24} = 0.25\,\Omega$



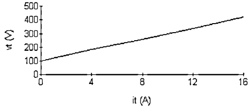
[c] $i = \dfrac{24}{1.25} = 19.2$ A, $\quad v = 24 - 19.2(0.25) = 19.2$ V

[d] $i_{sc} = \dfrac{24}{0.25} = 96$ A

[e] $i_{sc} = 48$ A (from graph)

[f] Linear model cannot predict nonlinear behavior of voltage source.

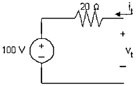
P 1.38 [a] Plot the v—i characteristic:



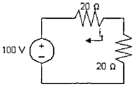
From the plot:

$$R = \frac{\Delta v}{\Delta i} = \frac{(420-100)}{(16-0)} = 20\,\Omega$$

When $i_t = 0$, $v_t = 100$ V; therefore the ideal voltage source has a voltage of 100 V

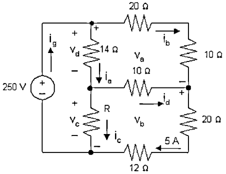


[b]



$i_t = -100/(20+20) = -2.5$ A; Therefore, $p_{20\Omega} = (-2.5)^2(20) = 125$ W

P 1.39 [a]



$v_b = 5(20+12) = 160$ V

$v_b + v_a = 250$ V, so $v_a = 90$ V

$i_b = 90/(20+10) = 3$ A

$i_d = 5 - i_b = 2$ A

$v_c = v_b + 10(i_d) = 180$ V

$v_d = 250 - v_c = 70$ V $= 14(i_a)$; therefore, $i_a = 5$ A

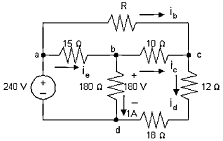
$i_c = i_a - i_d = 5 - 2 = 3$ A

$R = v_c/i_c = 180/3 = 60\,\Omega$

[b] $i_g = 5 + 3 = 8$ A

p_g (supplied) $= (250)(8) = 2000$ W

P 1.40



$v_{ab} = 240 - 180 = 60$ V; therefore, $i_e = 60/15 = 4$ A

$i_c = i_e - 1 = 4 - 1 = 3$ A; therefore, $v_{bc} = 10i_c = 30$ V

$v_{cd} = 180 - v_{bc} = 180 - 30 = 150$ V;

therefore, $i_d = v_{cd}/(12 + 18) = 150/30 = 5$ A

$i_b = i_d - i_c = 5 - 3 = 2$ A

$v_{ac} = v_{ab} + v_{bc} = 60 + 30 = 90$ V

$R = v_{ac}/i_b = 90/2 = 45\,\Omega$

CHECK: $i_g = i_b + i_e = 2 + 4 = 6$ A

$p_{dev} = (240)(6) = 1440$ W

$$\sum P_{dis} = 1(180) + 4(45) + 9(10) + 25(12) + 25(18) + 16(15) = 1440 \text{ W (CHECKS)}$$

P 1.41 [a] $15.2 = 10{,}000i_\beta - 0.80 + (200)30i_\beta$

$16 = (16{,}000)i_\beta$

$i_\beta = 1$ mA

$200(30i_\beta) + v_y + 500(29i_\beta) - 25 = 0$

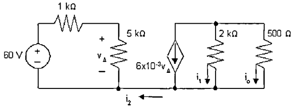
$v_y = 25 - 6000i_\beta - 14{,}500i_\beta$

Therefore, $v_y = 4.5$ V

[b]
$$\begin{aligned}\sum P_{\text{gen}} &= 15.2i_\beta + 25(29)i_\beta + 0.8i_\beta = 741i_\beta = 741 \text{ mW}\\ \sum P_{\text{dis}} &= 10^4(i_\beta)^2 + 200(30i_\beta)^2 + 29i_\beta(4.5) + 500(29i_\beta)^2\\ &= 741 \text{ mW}.\end{aligned}$$

P 1.42 [a] $i_2 = 0$ because no current can exist in a single conductor connecting two parts of a circuit.

[b]



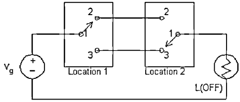
$60 = 6000i_g \qquad i_g = 10$ mA

$v_\Delta = 5000i_g = 50$ V $\qquad 6 \times 10^{-3}v_\Delta = 300$ mA

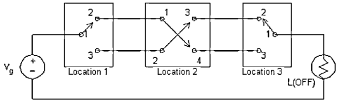
$2000i_1 = 500i_o$, so $i_1 + 4i_1 = -300$ mA; therefore, $i_1 = -60$ mA

[c] $300 - 60 + i_2 = 0$, so $i_o = -240$ mA.

P 1.43 [a]



[b]

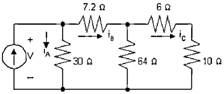


2

Some Circuit Simplification Techniques

Drill Exercises

DE 2.1



$$16\|64 = 12.8\,\Omega, \qquad 12.8 + 7.2 = 20\,\Omega, \qquad 20\|30 = 12\,\Omega$$

[a] $v = 5(12) = 60$ V

[b] $p_{5\text{A}}(\text{del}) = (5)(60) = 300$ W

[c]
$$i_{\text{A}} = 60/30 = 2 \text{ A} \qquad i_{\text{C}} = 3(64)/(80) = 2.4 \text{ A}$$
$$i_{\text{B}} = 5 - 2 = 3 \text{ A} \qquad p_{10\Omega} = (2.4)^2 10 = 57.6 \text{ W}$$

DE 2.2 [a] $v_o(\text{no load}) = 200(75)/100 = 150$ V

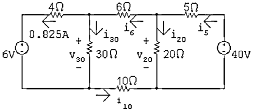
[b] $75\|150 = 50$ kΩ, therefore $v_o = 200(50)/75 = 133.3$ V

[c] $i = 200/25{,}000 = 8$ mA, $p_{25k} = (8 \times 10^{-3})^2(25{,}000) = 1.6$ W

[d] Maximum dissipation at no load since v_o is maximum

$$p = \frac{v_o^2}{75{,}000} = 0.3 \text{ W}$$

DE 2.3



$v_{30} = 6 + 4(0.825) = 9.3$ V; $\qquad i_{30} = \dfrac{v_{30}}{30} = 0.31$ A

$i_6 = i_{30} + 0.825 = 1.135$ A; $\qquad i_{10} = 0.825 + 0.31 = 1.135$ A

$-v_{30} - 6i_b + v_{20} - 10i_{10} = 0$

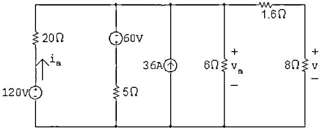
$\therefore\ \ v_{20} = 9.3 + 16(1.135) = 27.46$ V

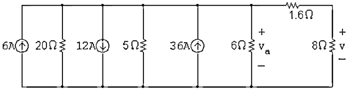
$i_{20} = \dfrac{27.46}{20} = 1.373$ A; $\qquad i_5 = i_6 + i_{20} = 2.508$ A

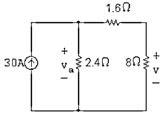
$i_{30} = 0.31$ A; $\quad i_6 = 1.135$ A; $\quad i_{10} = 1.135$ A;

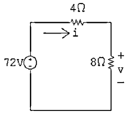
$i_{20} = 1.373$ A; $\quad$ and $\quad i_5 = 2.508$ A

DE 2.4







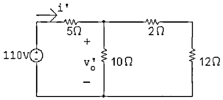


$$i = \frac{72}{12} = 6 \text{ A}$$

[a] $v = \frac{72}{12}(8) = 48$ V, $\qquad i_{120\text{V}} = \frac{120 - 57.6}{20} = 3.12$ A

[b] $v_a = 6(9.6) = 57.6$ V, $\qquad p_{120V}(\text{del}) = 120 i_a = 374.40$ W

DE 2.5 [a] 110 V source acting alone:

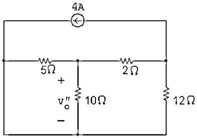


$$R_e = \frac{10(14)}{24} = \frac{35}{6}\,\Omega$$

$$i' = \frac{110}{5 + 35/6} = \frac{132}{13} \text{ A}$$

$$v_o' = \left(\frac{35}{6}\right)\left(\frac{132}{13}\right) = \frac{770}{13} \text{ V}$$

4 A source acting alone:

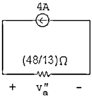


$5\,\Omega \| 10\,\Omega = 50/15 = 10/3\,\Omega$

$10/3 + 2 = 16/3\,\Omega$

$16/3\|12 = 48/13\,\Omega$

Hence our circuit reduces to:



It follows that

$$v_a'' = 4(48/13) = (192/13)\text{ V}$$

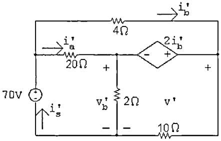
and

$$v_o'' = \frac{-v_a''}{(16/3)}(10/3) = -\frac{5}{8}v_a'' = -(120/13)\text{ V}$$

$$\therefore \quad v_o = v_o' + v_o'' = \frac{770}{13} - \frac{120}{13} = 50\text{ V}$$

[b] $p = \dfrac{v_o^2}{10} = 250\text{ W}$

DE 2.6 70-V source acting alone:



$$v' = 70 - 4i_b'$$

$$i_s' = \frac{v_b'}{2} + \frac{v'}{10} = i_a' + i_b'$$

$$70 = 20i_a' + v_b'$$

$$i_a' = \frac{70 - v_b'}{20}$$

$$\therefore \quad i_b' = \frac{v_b'}{2} + \frac{v'}{10} - \frac{70 - v_b'}{20} = \frac{11}{20}v_b' + \frac{v'}{10} - 3.5$$

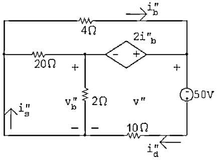
$v' = v_b' + 2i_b'$

$\therefore \; v_b' = v' - 2i_b'$

$$\therefore \; i_b' = \frac{11}{20}(v' - 2i_b') + \frac{v'}{10} - 3.5 \quad \text{or} \quad i_b' = \frac{13}{42}v' - \frac{70}{42}$$

$$\therefore \; v' = 70 - 4\left(\frac{13}{42}v' - \frac{70}{42}\right) \quad \text{or} \quad v' = \frac{3220}{94} = \frac{1610}{47} \text{ V}$$

50-V source acting alone:



$v'' = -4i_b''$

$v'' = v_b'' + 2i_b''$

$v'' = -50 + 10i_d''$

$$\therefore \; i_d'' = \frac{v'' + 50}{10}$$

$$i_s'' = \frac{v_b''}{2} + \frac{v'' + 50}{10}$$

$$i_b'' = \frac{v_b''}{20} + i_s'' = \frac{v_b''}{20} + \frac{v_b''}{2} + \frac{v'' + 50}{10} = \frac{11}{20}v_b'' + \frac{v'' + 50}{10}$$

$v_b'' = v'' - 2i_b''$

$$\therefore \; i_b'' = \frac{11}{20}(v'' - 2i_b'') + \frac{v'' + 50}{10} \quad \text{or} \quad i_b'' = \frac{13}{42}v'' + \frac{100}{42}$$

$$\text{Thus,} \quad v'' = -4\left(\frac{13}{42}v'' + \frac{100}{42}\right) \quad \text{or} \quad v'' = -\frac{200}{47} \text{ V}$$

$$\text{Hence,} \quad v = v' + v'' = \frac{1610}{47} - \frac{200}{47} = \frac{1410}{47} = 30 \text{ V}$$

Problems

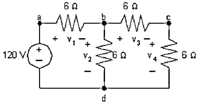
P 2.1 [a] $p_{4\Omega} = i_s^2 4 = (12)^2 4 = 576$ W $\qquad p_{18\Omega} = (4)^2 18 = 288$ W

$p_{3\Omega} = (8)^2 3 = 192$ W $\qquad p_{6\Omega} = (8)^2 6 = 384$ W

[b] $p_{120\text{V}}(\text{delivered}) = 120 i_s = 120(12) = 1440$ W

[c] $p_{\text{diss}} = 576 + 288 + 192 + 384 = 1440$ W

P 2.2 [a] From Ex. 3-1: $i_1 = 4$ A, $\quad i_2 = 8$ A, $\quad i_s = 12$ A
at node x: $-12 + 4 + 8 = 0$, $\quad$ at node y: $12 - 4 - 8 = 0$



[b] $v_1 = 4i_s = 48$ V $\qquad v_3 = 3i_2 = 24$ V

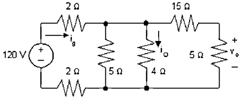
$v_2 = 18i_1 = 72$ V $\qquad v_4 = 6i_2 = 48$ V
loop abda: $-120 + 48 + 72 = 0$,
loop bcdb: $-72 + 24 + 48 = 0$,
loop abcda: $-120 + 48 + 24 + 48 = 0$

P 2.3 $\dfrac{1}{R_{eq}} = \dfrac{1}{6} + \dfrac{1}{10} + \dfrac{1}{15} = \dfrac{10}{30} = \dfrac{1}{3}$; $\qquad R_{eq} = 3\,\Omega$

$v_{(2+8+5)\Omega} = (20)(3) = 60$ V, $\qquad i_{(2+8+5)\Omega} = 60/15 = 4$ A

$P_{5\Omega} = (4)^2(5) = 80$ W

P 2.4 [a]



$R_{eq} = 2 + 2 + (1/4 + 1/5 + 1/20)^{-1} = 6\,\Omega$

$i_g = 120/6 = 20$ A

$v_{4\Omega} = 120 - (2+2)20 = 40$ V

$i_o = 40/4 = 10$ A

$$i_{(15+5)\Omega} = 40/(15+5) = 2 \text{ A}$$

$$v_o = (5)(2) = 10 \text{ V}$$

[b] $i_{15\Omega} = 2$ A; $\quad P_{15\Omega} = (2)^2(15) = 60$ W

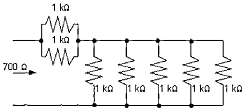
[c] $P_{120\text{V}} = (120)(20) = 2.4$ kW

P 2.5 [a] $R_{\text{eq}} = R \| R = \dfrac{R^2}{2R} = \dfrac{R}{2}$

[b]
$$\begin{aligned} R_{\text{eq}} &= R\|R\|R\| \cdots \|R \qquad (n\ R\text{'s}) \\ &= R\|\frac{R}{n-1} \\ &= \frac{R^2/(n-1)}{R + R/(n-1)} = \frac{R^2}{nR} = \frac{R}{n} \end{aligned}$$

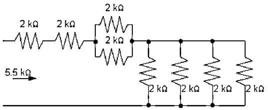
[c] One solution:

$$\begin{aligned} 700\,\Omega &= 200\,\Omega + 500\,\Omega \\ &= 1000/5 + 1000/2 \\ &= 1\text{ k}\Omega\|1\text{ k}\Omega\|1\text{ k}\Omega\|1\text{ k}\Omega\|1\text{ k}\Omega + 1\text{ k}\Omega\|1\text{ k}\Omega \end{aligned}$$



[d] One solution:

$$\begin{aligned} 5.5\text{ k}\Omega &= 5\text{ k}\Omega + 0.5\text{ k}\Omega \\ &= 2\text{ k}\Omega + 2\text{ k}\Omega + 1\text{ k}\Omega + 0.5\text{ k}\Omega \\ &= 2\text{ k}\Omega + 2\text{ k}\Omega + \frac{2\text{ k}\Omega}{2} + \frac{2\text{ k}\Omega}{4} \\ &= 2\text{ k}\Omega + 2\text{ k}\Omega + 2\text{ k}\Omega\|2\text{ k}\Omega + 2\text{ k}\Omega\|2\text{ k}\Omega\|2\text{ k}\Omega\|2\text{ k}\Omega \end{aligned}$$



P 2.6 [a] $12\,\Omega \| 24\,\Omega = 8\,\Omega$ Therefore, $R_{\rm ab} = 8 + 2 + 6 = 16\,\Omega$

[b] $$\frac{1}{R_{\rm eq}} = \frac{1}{24\ \mathrm{k}\Omega} + \frac{1}{30\ \mathrm{k}\Omega} + \frac{1}{20\ \mathrm{k}\Omega} = \frac{15}{120\ \mathrm{k}\Omega} = \frac{1}{8\ \mathrm{k}\Omega}$$

$R_{\rm eq} = 8\ \mathrm{k}\Omega;$ $R_{\rm eq} + 7 = 15\ \mathrm{k}\Omega$

$$\frac{1}{R_{\rm ab}} = \frac{1}{15\ \mathrm{k}\Omega} + \frac{1}{30\ \mathrm{k}\Omega} + \frac{1}{15\ \mathrm{k}\Omega} = \frac{5}{30\ \mathrm{k}\Omega} = \frac{1}{6\ \mathrm{k}\Omega}$$

$R_{\rm ab} = 6\ \mathrm{k}\Omega$

P 2.7 [a] For circuit (a)

$R_{\rm ab} = 15\|(18 + 48\|16) = 10\,\Omega$

For circuit (b)

$$\frac{1}{R_e} = \frac{1}{20} + \frac{1}{15} + \frac{1}{20} + \frac{1}{4} + \frac{1}{12} = \frac{30}{60} = \frac{1}{2}$$

$R_e = 2\,\Omega$

$R_e + 16 = 18\,\Omega$

$18\|18 = 9\,\Omega$

$R_{\rm ab} = 10 + 8 + 9 = 27\,\Omega$

For circuit (c)

$48\|16 = 12\,\Omega$

$12 + 8 = 20\,\Omega$

$20\|30 = 12\,\Omega$

$12 + 18 = 30\,\Omega$

$30\|15 = 10\,\Omega$

$10 + 10 + 20 = 40\,\Omega$

$R_{\rm ab} = 40\|60 = 24\,\Omega$

[b] $$P_a = \frac{20^2}{10} = 40\ \mathrm{W}$$

$$P_b = \frac{144^2}{27} = 768\ \mathrm{W}$$

$P_c = 6^2(24) = 864\ \mathrm{W}$

P 2.8 [a] $5\|20 = 100/25 = 4\,\Omega$ $\qquad$ $5\|20 + 9\|18 + 10 = 20\,\Omega$

$9\|18 = 162/27 = 6\,\Omega$ $\qquad$ $20\|30 = 600/50 = 12\,\Omega$

$R_{\text{ab}} = 5 + 12 + 3 = 20\,\Omega$

[b] $5 + 15 = 20\,\Omega$ $\qquad$ $30\|20 = 600/50 = 12\,\Omega$

$20\|60 = 1200/80 = 15\,\Omega$ $\qquad$ $3\|6 = 18/9 = 2\,\Omega$

$15 + 10 = 25\,\Omega$ $\qquad$ $3\|6 + 30\|20 = 2 + 12 = 14\,\Omega$

$25\|75 = 1875/100 = 18.75\,\Omega$ $\qquad$ $26\|14 = 364/40 = 9.1\,\Omega$

$18.75 + 11.25 = 30\,\Omega$ $\qquad$ $R_{\text{ab}} = 2.5 + 9.1 + 3.4 = 15\,\Omega$

[c] $3 + 5 = 8\,\Omega$ $\qquad$ $60\|40 = 2400/100 = 24\,\Omega$

$8\|12 = 96/20 = 4.8\,\Omega$ $\qquad$ $24 + 6 = 30\,\Omega$

$4.8 + 5.2 = 10\,\Omega$ $\qquad$ $30\|10 = 300/40 = 7.5\,\Omega$

$45 + 15 = 60\,\Omega$ $\qquad$ $R_{\text{ab}} = 1.5 + 7.5 + 1.0 = 10\,\Omega$

P 2.9 [a] $R_{\text{cond}} = 845(0.0397) = 33.5465\,\Omega$

$R_{\text{total}} = 2(1/2)R_{\text{cond}} = 33.5465\,\Omega$

$P_{\text{loss}} = (2000)^2(33.5465) = 134.186$ MW

$P_{\text{calif}} = 800(2) - 134.186 = 1465.814$ MW

Efficiency $= (1465.814/1600) \times 100 = 91.61\%$

[b] $P_{\text{calif}} = 2000 - 134.86 = 1865.814$ MW

Efficiency $= 93.29\%$

[c] $P_{\text{loss}} = (3000)^2 \cdot 2 \cdot (1/3) \cdot 845 \cdot (0.0397) = 201.279$ MW

$P_{\text{oregon}} = 3000$ MW, $\qquad P_{\text{calif}} = 3000 - 201.279 = 2798.7$ MW

Efficiency $= (2798.70/3000) \times 100 = 93.29\%$

P 2.10 $i_{10\text{k}} = \dfrac{(18)(15)}{40} = 6.75$ mA

$v_{15\text{k}} = -(6.75)(15) = -101.25$ V

$i_{3\text{k}} = 18 - 6.75 = 11.25$ mA

$v_{12\text{k}} = -(12)(11.25) = -135$ V

$v_o = -101.25 - (-135) = 33.75$ V

P 2.11 [a] $v_{1k} = \frac{1}{1+5}(30) = 5$ V

$$v_{15k} = \frac{15}{15+60}(30) = 6 \text{ V}$$

$$v_x = v_{15k} - v_{1k} = 6 - 5 = 1 \text{ V}$$

[b] $v_{1k} = \frac{v_s}{6}(1) = v_s/6$

$$v_{15k} = \frac{v_s}{75}(15) = v_s/5$$

$$v_x = (v_s/5) - (v_s/6) = v_s/30$$

P 2.12 $60\|30 = 20\,\Omega$

$$i_{30\Omega} = \frac{(25)(75)}{125} = 15 \text{ A}$$

$$v_o = (15)(20) = 300 \text{ V}$$

$$v_o + 30i_{30} = 750 \text{ V}$$

$$v_g - 12(25) = 750$$

$$v_g = 1050 \text{ V}$$

P 2.13 $5\,\Omega\|20\,\Omega = 4\,\Omega;\qquad 4\,\Omega + 6\,\Omega = 10\,\Omega;\qquad 10\|40 = 8\,\Omega;$

Therefore, $i_g = \frac{125}{8+2} = 12.5$ A

$$i_{6\Omega} = \frac{(40)(12.5)}{50} = 10 \text{ A};\quad i_o = \frac{(5)(10)}{25} = 2 \text{ A}$$

P 2.14 [a] $40\|10 = 8\,\Omega \qquad i_{75\text{V}} = \frac{75}{10} = 7.5$ A

$$8 + 7 = 15\,\Omega \qquad i_{4+3\Omega} = 7.5\left(\frac{30}{45}\right) = 5 \text{ A}$$

$$15\|30 = 10\,\Omega \qquad i_o = -5\left(\frac{10}{50}\right) = -1 \text{ A}$$

[b] $i_{10\Omega} = i_{4+3\Omega} + i_o = 5 - 1 = 4$ A

$$P_{10\Omega} = (4)^2(10) = 160 \text{ W}$$

P 2.15 [a] $v_{9\Omega} = (1)(9) = 9$ V

$i_{2\Omega} = 9/(2+1) = 3$ A

$i_{4\Omega} = 1 + 3 = 4$ A;

$v_{25\Omega} = (4)(4) + 9 = 25$ V

$i_{25\Omega} = 25/25 = 1$ A;

$i_{3\Omega} = i_{25\Omega} + i_{9\Omega} + i_{2\Omega} = 1 + 1 + 3 = 5$ A;

$v_{40\Omega} = v_{25\Omega} - v_{3\Omega} = 25 - (-5)(3) = 40$ V

$i_{40\Omega} = 40/40 = 1$ A

$i_{5\|20\Omega} = i_{40\Omega} + i_{25\Omega} + i_{4\Omega} = 1 + 1 + 4 = 6$ A

$v_{5\|20\Omega} = (4)(6) = 24$ V

$v_{32\Omega} = v_{40\Omega} + v_{5\|20\Omega} = 40 + 24 = 64$ V

$i_{32\Omega} = 64/32 = 2$ A;

$i_{10\Omega} = i_{32\Omega} + i_{5\|20\Omega} = 2 + 6 = 8$ A

$v_g = 10(8) + v_{32\Omega} = 80 + 64 = 144$ V.

[b] $P_{20\Omega} = \dfrac{(v_{5\|20\Omega})^2}{20} = \dfrac{24^2}{20} = 28.8$ W

P 2.16 [a] Let i_s be the current oriented down through the resistors. Then,

$$i_s = \frac{V_s}{R_1 + R_2 + \cdots + R_k + \cdots + R_n}$$

and

$$v_k = R_k i_s = \frac{R_k}{R_1 + R_2 + \cdots + R_k + \cdots + R_n} V_s$$

[b] $i_s = \dfrac{200}{5 + 15 + 30 + 10 + 40} = 2$ A

$v_1 = 2(5) = 10$ V

$v_2 = 2(15) = 30$ V

$v_3 = 2(30) = 60$ V

$v_4 = 2(10) = 20$ V

$v_5 = 2(40) = 80$ V

P 2.17 [a] $v_o = \frac{25}{25}(20) = 20$ V

[b] $v_o = \frac{25}{5 + R_e}R_e$

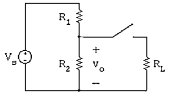
$$R_e = \frac{(20)(12)}{32} = 7.5 \text{ k}\Omega$$

$$v_o = \frac{25}{12.5}(7.5) = 15 \text{ V}$$

[c] $\frac{v_o}{25} = \frac{20}{25} = 0.80$

[d] $\frac{v_o}{25} = \frac{15}{25} = 0.60$

P 2.18 [a]



No load:

$$v_o = \frac{R_2}{R_1 + R_2}V_s = \sigma V_s$$

$$\therefore \quad \sigma = \frac{R_2}{R_1 + R_2}$$

Load:

$$v_o = \frac{R_e}{R_1 + R_e}V_s = \beta V_s$$

$$\therefore \quad \beta = \frac{R_e}{R_e + R_1} \qquad R_e = \frac{R_2 R_L}{R_2 + R_L}$$

$$\therefore \quad \beta = \frac{R_2 R_L}{R_1 R_2 + R_L(R_1 + R_2)}$$

But $\quad R_1 + R_2 = \frac{R_2}{\sigma} \qquad \therefore \quad R_1 = \frac{R_2}{\sigma} - R_2$

$$\therefore \quad \beta = \frac{R_2 R_L}{R_2\left(\frac{R_2}{\sigma} - R_2\right) + \frac{R_L R_2}{\sigma}}$$

$$\beta = \frac{R_L}{R_2\left(\frac{1}{\sigma} - 1\right) + \frac{R_L}{\sigma}}$$

or

$$\beta R_2\left(\frac{1}{\sigma} - 1\right) + \frac{\beta R_L}{\sigma} = R_L$$

$$\beta R_2\left(\frac{1}{\sigma} - 1\right) = R_L\left(1 - \frac{\beta}{\sigma}\right)$$

$$\therefore\ R_2 = \frac{(\sigma - \beta)}{\beta(1 - \sigma)}R_L$$

$$R_1 = \frac{(1 - \sigma)}{\sigma}R_2 = \left(\frac{\sigma - \beta}{\sigma\beta}\right)R_L$$

[b] $R_1 = \dfrac{(0.9 - 0.7)}{0.63}(126)\ \text{k}\Omega = 40\ \text{k}\Omega$

$$R_2 = \frac{(0.9 - 0.7)}{(0.7)(0.1)}(126)\ \text{k}\Omega = 360\ \text{k}\Omega$$

P 2.19 [a] Let v_o be the voltage across the parallel branches, positive at the upper terminal, then

$$i_g = v_oG_1 + v_oG_2 + \cdots + v_oG_N = v_o(G_1 + G_2 + \cdots + G_N)$$

It follows that $\quad v_o = \dfrac{i_g}{(G_1 + G_2 + \cdots + G_N)}$

The current in the k^{th} branch is $\quad i_k = v_oG_k;\qquad$ Thus,

$$i_k = \frac{i_gG_k}{[G_1 + G_2 + \cdot + G_N]}$$

[b] $i_{6.25} = \dfrac{1142(0.16)}{[4 + 0.4 + 1 + 0.16 + 0.1 + 0.05]} = 32\ \text{mA}$

P 2.20 $R_e = \dfrac{4}{8} \times 10^3 = 500\,\Omega$

$$\therefore\ \sum G = \frac{1}{500} = 2\ \text{mS}$$

$$i_1 = 2i_2 = 2(10i_3) = 20i_4$$

$$i_2 = 10i_3 = 10i_4$$

$$i_3 = i_4$$

$$8 = 20i_4 + 10i_4 + i_4 + i_4 = 32i_4$$

$$\therefore \quad i_4 = \frac{8}{32} = 0.25 \text{ mA}$$

$$R_4 = \frac{v_g}{i_4} = \frac{4}{0.25 \times 10^{-3}} = 16 \text{ k}\Omega$$

$$i_3 = i_4 = 0.25 \text{ mA}$$

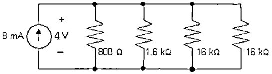
$$\therefore \quad R_3 = 16 \text{ k}\Omega$$

$$i_2 = 10i_4 = 2.5 \text{ mA}$$

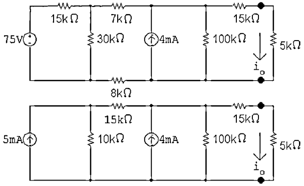
$$R_2 = \frac{v_g}{i_2} = \frac{4}{2.5 \times 10^{-3}} = 1.6 \text{ k}\Omega$$

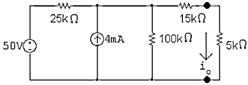
$$i_1 = 20i_4 = 5 \text{ mA}$$

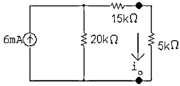
$$R_1 = \frac{v_g}{i_1} = \frac{4}{5 \times 10^{-3}} = 800\,\Omega$$

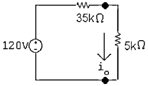


P 2.21 [a]



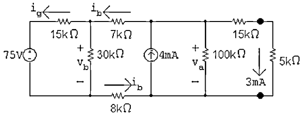






$i_o = 120/40\ \text{k}\Omega = 3\ \text{mA}$

[b]



$v_a = (3)(20) = 60\ \text{V}$

$i_a = \dfrac{v_a}{100} = 0.6\ \text{mA}$

$i_b = 4 - 3.6 = 0.4\ \text{mA}$

$v_b = 60 - (0.4)(15) = 54\ \text{V}$

$i_g = 0.4 - 54/30 = -1.4\ \text{mA}$

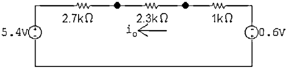
$p_{75\text{V}}$ (developed) $= (75)(1.4) = 105\ \text{mW}$

Check:

$p_{4\text{mA}}$ (developed) $= (60)(4) = 240\ \text{mW}$

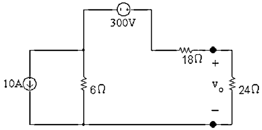
$$\begin{aligned}
\sum P_{\text{dev}} &= 105 + 240 = 345\ \text{mW} \\
\sum P_{\text{dis}} &= (-1.4)^2(15) + (1.8)^2(30) + (0.4)^2(15) + (0.6)^2(100) + \\
&\quad (3)^2(20) \\
&= 345\ \text{mW}
\end{aligned}$$

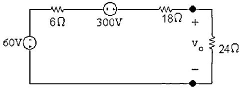
P 2.22 Apply source transformations to both current sources to get

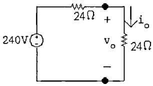


$$i_o = \frac{-6}{6} = -1 \text{ mA}$$

P 2.23 [a]

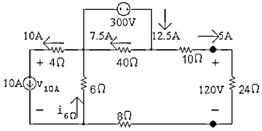






$$\therefore \quad v_o = \frac{1}{2}(240) = 120 \text{ V}; \qquad i_o = 120/24 = 5 \text{ A}$$

[b]



$$p_{300\text{V}} = -12.5(300) = -3750 \text{ W}$$

Therefore, the 300 V source is developing 3.75 kW.

[c] $-10 + i_{6\Omega} + 7.5 - 12.5 = 0; \qquad \therefore\ i_{6\Omega} = 15$ A

$v_{10A} + 4(10) + 6(15) = 0; \qquad \therefore\ v_{10A} = -130$ V

$p_{10A} = 10v_{10A} = -1300$ W

Therefore the 10 A source is developing 1300 W.

[d] $\sum p_{dev} = 3750 + 1300 = 5050$ W

$p_{4\Omega} = 100(4) = 400$ W

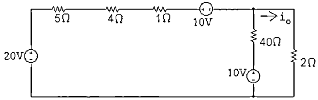
$p_{40\Omega} = (7.5)^2(40) = 2250$ W

$p_{6\Omega} = (15)^2(6) = 1350$ W

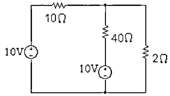
$p_{42\Omega} = (5)^2(42) = 1050$ W

$\sum p_{diss} = 400 + 1350 + 2250 + 1050 = 5050$ W (CHECKS)

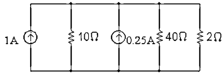
P 2.24 Applying a source transformation to each current source yields



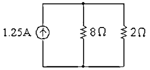
Now combine the 20 V and 10 V sources into a single voltage source and the 5 Ω. 4 Ω and 1 Ω resistors into a single resistor to get



Now use a source transformation on each voltage source, thus

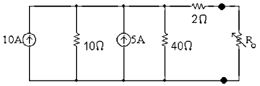


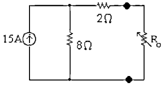
which can be reduced to

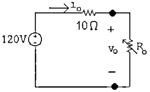


$$\therefore \quad i_o = \frac{(1.25)(8)}{10} = 1 \text{ A}$$

P 2.25 First, find the Thévenin equivalent with respect to R_o.

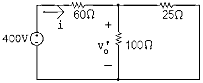






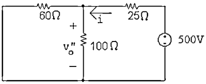
R_o	i_o	v_o	R_o	i_o	v_o
0	12	0	20	4	80
2	10	20	30	3	90
6	7.5	45	40	2.4	96
10	6	60	50	2	100
15	4.8	72	70	1.5	105

P 2.26



$100\,\Omega \| 25\,\Omega = 20\,\Omega \qquad \therefore \quad i = \dfrac{400}{60 + 20} = 5 \text{ A}$

$v_o' = 20i = 100$ V



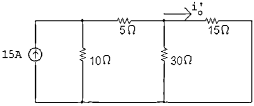
$100\,\Omega\|60\,\Omega = 37.5\,\Omega$

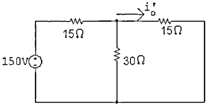
$$i = \frac{500}{25 + 37.5} = 8 \text{ A}$$

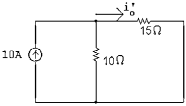
$v_o'' = 37.5i = 300$ V

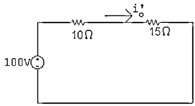
$v_o = v_o' + v_o'' = 100 + 300 = 400$ V

P 2.27

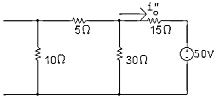




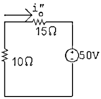




$$i_o' = \frac{100}{25} = 4 \text{ A}$$



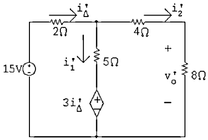
$15\,\Omega \| 30\,\Omega = 10\,\Omega$



$$i_o'' = \frac{-50}{25} = -2 \text{ A}$$

$$\therefore \quad i_o = i_o' + i_o'' = 4 - 2 = 2 \text{ A}$$

P 2.28

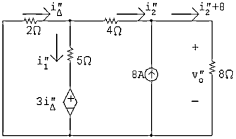


$15 = 2i_\Delta' + 5'i_1 + 3i_\Delta'$

$15 = 2i'_\Delta + 12i'_2$

$i'_\Delta = i'_1 + i'_2, \qquad i'_1 = 27/26 \text{ A}; \qquad i'_\Delta = 51/26 \text{ A}$

$\therefore \quad i'_2 = \dfrac{12}{13} \text{ A}; \qquad v'_o = \dfrac{96}{13} \text{ V}$



$-2i''_\Delta = 5i''_1 + 3i''_\Delta \qquad \therefore \quad i''_\Delta = -i''_1$

$i''_2 = i''_\Delta - i''_1 = 2i''_\Delta$

$4i''_2 + (8 + i''_2)8 = -2i''_\Delta$

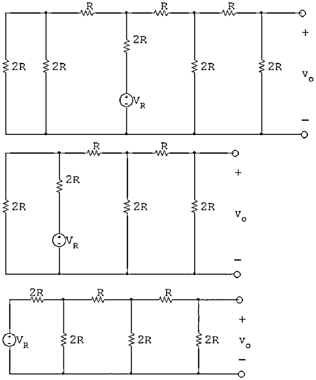
$\therefore \quad i''_2 = -\dfrac{64}{13} \text{ A}; \qquad i''_1 = \dfrac{32}{13} \text{ A}; \qquad i''_\Delta = -\dfrac{32}{13} \text{ A}$

$\therefore \quad 8 + i''_2 = \dfrac{40}{13} \text{ A}$

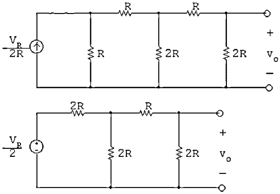
$\therefore \quad v''_o = 8\left(\dfrac{40}{13}\right) = \dfrac{320}{13} \text{ V}$

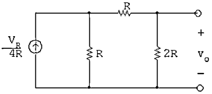
$\therefore \quad v_o = v'_o + v''_o = \dfrac{96}{13} + \dfrac{320}{13} = 32 \text{ V}$

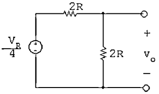
P 2.29 [a] The evolution of the circuit shown in Fig. P2.29 is illustrated in the following steps:



[b] Starting at the left end of the circuit and working toward the right end, a series of source transformations yields:

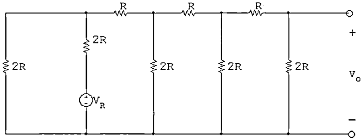


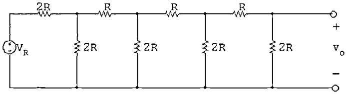




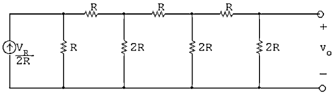
$$\frac{V_R/4}{4R}(2R) = \frac{V_R}{8}$$

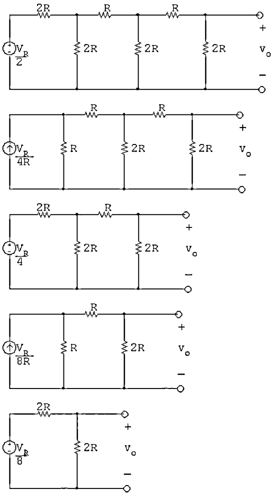
P 2.30 [a] The evolution of the circuit in Fig. P2.30 can be shown in two steps, thus:





[b] Moving from left to right, a series of source transformations yields:





$$v_o = \frac{V_R/8}{4R}(2R) = \frac{V_R}{16}$$

P 2.31

Eq.(2.34)	$v_o = \frac{1}{2}V_R$	(Switch 1)
Eq.(2.35)	$v_o = \frac{1}{4}V_R$	(Switch 2)
Eq.(2.36)	$v_o = \frac{1}{8}V_R$	(Switch 3)
Eq.(2.37)	$v_o = \frac{1}{16}V_R$	(Switch 4)

Given $V_R = 16$ V:

Switch Position				v_o
1	2	3	4	
0	0	0	0	$v_o = 0$ V
0	0	0	V_R	$v_o = \frac{1}{16}V_R = 1$ V
0	0	V_R	0	$v_o = \frac{1}{8}V_R = 2$ V
0	0	V_R	V_R	$v_o = \frac{1}{16}V_R + \frac{1}{8}V_R = 3$ V
0	V_R	0	0	$v_o = \frac{1}{4}V_R = 4$ V
0	V_R	0	V_R	$v_o = \frac{1}{4}V_R + \frac{1}{16}V_R = 5$ V
0	V_R	V_R	0	$v_o = \frac{1}{4}V_R + \frac{1}{8}V_R = 6$ V
0	V_R	V_R	V_R	$v_o = \frac{1}{4}V_R + \frac{1}{8}V_R + \frac{1}{16}V_R = 7$ V
V_R	0	0	0	$v_o = \frac{1}{2}V_R = 8$ V
V_R	0	0	V_R	$v_o = \frac{1}{2}V_R + \frac{1}{16}V_R = 9$ V
V_R	0	V_R	0	$v_o = \frac{1}{2}V_R + \frac{1}{8}V_R = 10$ V
V_R	0	V_R	V_R	$v_o = \frac{1}{2}V_R + \frac{1}{8}V_R + \frac{1}{16}V_R = 11$ V
V_R	V_R	0	0	$v_o = \frac{1}{2}V_R + \frac{1}{4}V_R = 12$ V
V_R	V_R	0	V_R	$v_o = \frac{1}{2}V_R + \frac{1}{4}V_R + \frac{1}{16}V_R = 13$ V
V_R	V_R	V_R	0	$v_o = \frac{1}{2}V_R + \frac{1}{4}V_R + \frac{1}{8}V_R = 14$ V
V_R	V_R	V_R	V_R	$v_o = \frac{1}{2}V_R + \frac{1}{4}V_R + \frac{1}{8}V_R + \frac{1}{16}V_R = 15$ V

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3
Techniques of Circuit Analysis

Drill Exercises

DE 3.1 [a] 11,8 resistors, 2 independent sources, 1 dependent source

[b] 9

[c] 9, $R_4 - R_5$ forms an essential branch as does $R_8 - 10$ V. The remaining seven branches contain a single element.

[d] 7 [e] 6 [f] 4 [g] 6

DE 3.2 Solution given in text.

DE 3.3 Solution given in text.

DE 3.4 Solution given in text.

DE 3.5 [a] The two node voltage equations are

$$-15 + \frac{v_1}{60} + \frac{v_1}{15} + \frac{v_1 - v_2}{5} = 0$$

$$5 + \frac{v_2}{2} + \frac{v_2 - v_1}{5} = 0$$

Solving, $v_1 = 60$ V and $v_2 = 10$ V;
Therefore, $i_1 = (v_1 - v_2)/5 = 10$ A

[b] $p_{15\text{A}}$ (del) $= (15)(60) = 900$ W

[c] $p_{5\text{A}} = -5(10) = -50$ W

DE 3.6 Use the lower node as the reference node. Let $v_1 =$ node voltage across 1 Ω resistor and $v_2 =$ node voltage across 12 Ω resistor. Then

$$\frac{v_1}{1} + \frac{v_1 - v_2}{8} = 4.5$$

$$\frac{v_2}{12} + \frac{v_2 - v_1}{8} + \frac{v_2 - 30}{4} = 0$$

Solving, $v_1 = 6$ V $\quad v_2 = 18$ V Thus, $i = (v_1 - v_2)/8 = -1.5$ A
$v = v_2 + 2i = 15$ V

DE 3.7 Use the lower node as the reference node. Let v_1 = node voltage across the 8 Ω resistor, let v_2 = node voltage across the 4 Ω resistor. Then

$$\frac{v_1 - 50}{6} + \frac{v_1}{8} + \frac{v_1 - v_2}{2} - 3i_1 = 0$$

$$-5 + \frac{v_2}{4} + \frac{v_2 - v_1}{2} + 3i_1 = 0$$

$$i_1 = \frac{50 - v_1}{6}$$

Solving, $v_1 = 32$ V; $\quad v_2 = 16$ V; $\quad i_1 = 3$ A $p_{50V} = -50i_1 = -150$ W (delivering)
$p_{5A} = -5(v_2) = -80$ W (delivering)
$p_{3i_1} = 3i_1(v_2 - v_1) = -144$ W (delivering)

DE 3.8 Use the lower node as the reference node. Let v_1 = node voltage across the 7.5 Ω resistor and v_2 = node voltage across the 2.5 Ω resistor. Place the dependent voltage source inside a supernode between the node voltages v and v_2. The node voltage equations are

node 1: $$\frac{v_1}{7.5} + \frac{v_1 - v}{2.5} = 4.8$$

supernode: $$\frac{v - v_1}{2.5} + \frac{v}{10} + \frac{v_2}{2.5} + \frac{v_2 - 12}{1} = 0$$

We also have: $v + i_x = v_2$ and $i_x = v_1/7.5$. Solving this set of equations for v gives $v = 8$ V

DE 3.9 $$\frac{v_1 - 60}{2} + \frac{v_1}{24} + \frac{v_1 - (60 + 6i_\phi)}{3} = 0, \qquad i_\phi = \frac{60 + 6i_\phi - v_1}{3}$$
Therefore $v_1 = 48$ V

DE 3.10
$$\frac{v_o}{40} + \frac{v_o - 10}{10} + \frac{v_o + 20i_\Delta}{20} = 0, \qquad i_\Delta = \frac{10 - v_o}{10} + \frac{10 + 20i_\Delta}{30}$$
Therefore $v_o = 24$ V

DE 3.11 Define three clockwise mesh currents i_1, i_2, and i_3 in the lower left, upper, and lower right windows. The three mesh-current equations are

$$\begin{aligned} 80 &= 31i_1 - 5i_2 - 26i_3 \\ 0 &= -5i_1 + 125i_2 - 90i_3 \\ 0 &= -26i_1 - 90i_2 + 124i_3 \end{aligned}$$

[a] Solving, $i_1 = 5$ A; therefore the 80 V source is delivering 400 W to the circuit.

[b] Solving, $i_3 = 2.5$ A; therefore $p_{8\Omega} = (6.25)(8) = 50$ W

DE 3.12 [a] $b = 8, \quad n = 6, \quad b - n + 1 = 3$

[b] Define three clockwise mesh currents i_1, i_2, and i_3 in the upper, lower left, and lower right windows. The three mesh-current equations are

$$-(-3v_\phi) + 19i_1 - 2i_2 - 3i_3 = 0$$

$$25 - 10 = -2i_1 + 7i_2 - 5i_3$$

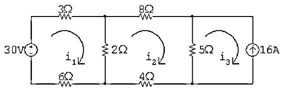
$$10 = -3i_1 - 5i_2 + 9i_3$$

We also have $v_\phi = 3(i_3 - i_1)$
Solving for i_1 and i_3 gives $i_1 = -1$ A, $i_3 = 3$ A Therefore $v_\phi = 12$ V and $p_{3v_\phi} = -(-3v_\phi)i_1 = -36$ W

DE 3.13 Let i_a = lower left mesh current cw, let i_b = upper mesh current cw, and i_c = lower right mesh current cw. Then

$$\begin{aligned} 25 &= 14i_a - 6i_b - 8i_c \\ 0 &= -6i_a + 16i_b - 8i_c \\ 0 &= -8i_a - 8i_b + 16i_c + 5i_\phi \\ i_\phi &= i_a, \qquad i_a = 4 \text{ A}, \qquad i_c = 2A \\ v_o &= 8(i_a - i_c) = 16 \text{ V} \end{aligned}$$

DE 3.14



Mesh 1: $30 = 11i_1 - 2i_2$

Mesh 2: $0 = -2i_1 + 19i_2 - 5i_3$

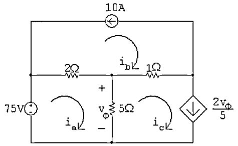
Current source: $i_3 = -16$ A

Solution gives $i_1 = 2$ A, $i_2 = -4$ A, $i_3 = -16$ A

The current in the $2\,\Omega$ resistor is $i_1 - i_2 = 6$ A

$\therefore \quad P_{2\,\Omega} = (6)^2(2) = 72$ W

DE 3.15



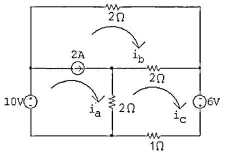
Mesh a: $7i_a - 2i_b - 5i_c = 75$

Current sources: $i_b = -10$ A; $\quad i_c = \dfrac{2v_\phi}{5}$

Dependent variable: $v_\phi = 5(i_a - i_c)$

Solution: $i_a = 15$ A; $\quad i_b = -10$ A; $\quad i_c = 10$ A; $\quad v_\phi = 25$ V

DE 3.16



Supermesh a,b: $2i_a + 4i_b - 4i_c = 10$

Mesh c: $-2i_a - 2i_b + 5i_c = 6$

Current source: $i_a - i_b = 2$ A

Solution: $i_a = 7$ A; $\quad i_b = 5$ A; $\quad i_c = 6$ A

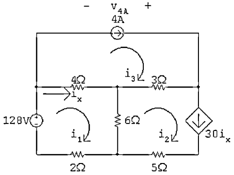
$\therefore \quad p_{1\,\Omega} = i_c^2(1) = (6)^2(1) = 36$ W

DE 3.17 Let v_1 denote the voltage across the 2 A source. Let v_1 be a voltage rise in the direction of the 2 A current.

$$\frac{v_1 - 20}{15} - 2 + \frac{v_1 - 25}{10} = 0, \qquad v_1 = 35 \text{ V}$$

$$p_{2A} = -35(2) = -70 \text{ W} \qquad p_{2A}(\text{del}) = 70 \text{ W}$$

DE 3.18



Mesh 1: $12i_1 - 6i_2 - 4i_3 = 128$

Mesh 2: $-6i_1 + 14i_2 - 3i_3 + 30i_x = 0$

Current source: $i_3 = 4$ A

Dependent variable: $i_x = i_1 - i_3$

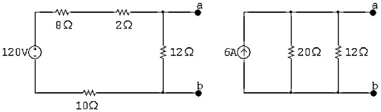
Solution: $i_1 = 9$ A; $i_2 = -6$ A; $i_3 = 4$ A; $i_x = 5$ A

$\therefore \; v_{4A} = 3(i_3 - i_2) - 4i_x = 10$ V

The power delivered by the 4A source is $p_{4A} = (10)(4) = 40$ W

DE 3.19 To find the Thévenin resistance, deactivate the independent voltage source and note that $R_{\text{Th}} = [5\|20 + 8]\|12 = 6\,\Omega$. With the terminals a, b open, the current delivered by the 72 V source is 72/24 or 3 A. The current (left-to-right) in the 5 Ω resistor is $(20/25)(3) = 2.4$ A, and the current (left-to-right) in the 12 Ω resistor is $(5/25)3$ or 0.6 A. The Thévenin voltage $v_{\text{Th}} = v_{ab}$ is the drop across the 8 Ω resistor plus the drop across the 20 Ω resistor. Thus $v_{\text{Th}} = (8)(0.6) + (20)(3) = 64.8$ V.

DE 3.20 After one source transformation, the circuit becomes



Therefore $I_N = 6$ A, $R_N = 20\|12 = 7.5\,\Omega$

DE 3.21 Find the Thévenin equivalent with respect to A, B.

$$\frac{V_{\rm Th} + 36}{12{,}000} + \frac{V_{\rm Th}}{60{,}000} - 0.018 = 0, \qquad V_{\rm Th} = 150 \text{ V}$$

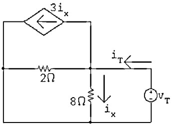
$$R_{\rm Th} = 15{,}000 + \frac{(60{,}000)(12{,}000)}{72{,}000} = 25 \text{ k}\Omega;$$

Therefore, $v_{\rm meas} = 150\left(\dfrac{100{,}000}{125{,}000}\right) = 120$ V

DE 3.22 Summing the currents away from node a, where $v_{\rm Th} = v_{ab}$ We have

$$\frac{v_{\rm Th}}{8} + 4 + 3i_x + \frac{v_{\rm Th} - 24}{2} = 0, \qquad i_x = \frac{v_{\rm Th}}{8}$$

Solving for $v_{\rm Th}$ yields $v_{\rm Th} = 8$ V



$$i_{\rm T} = 4i_x + v_{\rm T}/2, \qquad i_x = v_{\rm T}/8$$

Therefore $i_{\rm T} = v_{\rm T}$ and $R_{\rm Th} = v_{\rm T}/i_{\rm T} = 1\,\Omega$

DE 3.23 Use the bottom node as the reference. Let v_1 be the node voltage across the 60 Ω resistor. Then

$$\frac{v_1}{60}+\frac{v_1-(v_{\rm Th}+160i_\Delta)}{20}-4=0,$$

$$\frac{v_{\rm Th}}{40}+\frac{v_{\rm Th}}{80}+\frac{v_{\rm Th}+160i_\Delta-v_1}{20}=0$$

$$i_\Delta=\frac{v_{\rm Th}}{40},\quad \text{therefore}\quad v_{\rm Th}=30\text{ V}$$

Let $i_{\rm T}$ be the test current into terminal a:

$$i_{\rm T}=\frac{v_{\rm T}}{80}+\frac{v_{\rm T}}{40}+\frac{v_{\rm T}+160(v_{\rm T}/40)}{80},\qquad \frac{i_{\rm T}}{v_{\rm T}}=\frac{1}{10}$$

Therefore, $R_{\rm Th}=10\,\Omega$

DE 3.24 First find the Thévenin equivalent circuit. To find $v_{\rm Th}$, use the bottom node as the reference. Let $v_{\rm Th}=v_{ab}$ and v_1 = node voltage across the 20 V − 4 Ω branch. The two node Voltage equations are

$$\frac{v_{\rm Th}-100-v_\phi}{4}+\frac{v_{\rm Th}-v_1}{4}=0,\qquad (v_\phi=v_1-20)$$

$$\frac{v_1-100}{4}+\frac{v_1-20}{4}+\frac{v_1-v_{\rm Th}}{4}=0$$

Solving for $v_{\rm Th}$ gives $v_{\rm Th}=120$ V. To find $R_{\rm Th}$, deactivate the two independent sources and apply a test voltage source across a, b. Let $v_{\rm T}$ be positive at a and $i_{\rm T}$ directed into a. Then the two node Voltage equations are

$$\frac{v_{\rm T}-v_\phi}{4}+\frac{v_{\rm Th}-v_\phi}{4}=i_{\rm T},\qquad \frac{v_\phi}{4}+\frac{v_\phi}{4}+\frac{v_\phi-v_{\rm T}}{4}=0$$

Therefore $\quad v_\phi=v_{\rm T}/3\quad$ and $\quad 12i_{\rm T}=4v_{\rm T}$
So $\quad R_{\rm Th}=v_{\rm T}/i_{\rm T}=3\,\Omega$

[a] For maximum power transfer, $R_{\rm L}=R_{\rm Th}=3\,\Omega$

[b] $p_{\rm max}=(120/6)^2(3)=1200$ W

DE 3.25 When $R_L = 3\,\Omega$, the voltage across R_L is 60 V. As before, let v_1 be the node voltage across the 20 V—4 Ω branch, then $v_\phi = v_1 - 20$ and

$$\frac{60}{3} + \frac{60 - v_1}{4} + \frac{60 - 100 - v_\phi}{4} = 0$$

Therefore $v_1 = 60$ V and $v_\phi = 40$ V. The current out of the plus terminal of the 100 V source is

$$i_1 = \frac{100 - 60}{4} + \frac{100 + 40 - 60}{4} = 10 + 20 = 30 \text{ A}$$

[a] Therefore 100 V is delivering 3000 W to the circuit.

[b] The current out of the plus terminal of the dependent source is 20 A. Therefore the dependent source is delivering 800 W to the circuit.

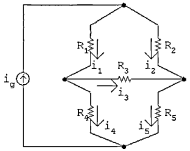
[c] The load power is (1200/3800)100 or 31.58% of this generated power.

Problems

P 3.1 [a] Five

[b] Three

[c]



Sum the currents at any three of the four essential nodes a, b, c, and d. Using nodes a, b, and c we get

$$-i_g + i_1 + i_2 = 0$$

$$-i_1 + i_4 + i_3 = 0$$

$$i_5 - i_2 - i_3 = 0$$

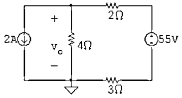
[d] Two.

[e] Sum the voltages around two independent closed paths, avoiding a path that contains the independent current source since the voltage across the current source is not known. Using the upper and lower meshes formed by the five resistors gives

$$R_1 i_1 + R_3 i_3 - R_2 i_2 = 0$$

$$R_3 i_3 + R_5 i_5 - R_4 i_4 = 0$$

P 3.2



$$2 + \frac{v_o}{4} + \frac{v_o - 55}{5} = 0$$

$$v_o = 20 \text{ V}$$

$$p_{2\text{A}} = (20)(2) = 40 \text{ W} \quad \text{(absorbing)}$$

P 3.3 Let v_2 be the node voltage across the 80 Ω resistor, positive at the upper terminal.

Then $\quad -4 + \frac{v_1}{20} + \frac{v_2}{80} + \frac{v_2}{40} = 0$

(Note we have created a super node in writing this expression.)

$$v_1 + 60 = v_2$$

$$\therefore \quad v_1 = 20 \text{ V}$$

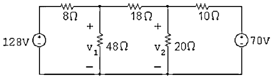
$$\therefore \quad v_2 = 80 \text{ V}$$

$p_{\text{del}} = 60 i_g$where i_g is the current out of the positive terminal

$$4 = i_g + \frac{v_1}{20}; \qquad i_g = 3 \text{ A}$$

$$\therefore \quad p_{\text{del}} = 60(3) = 180 \text{ W}$$

P 3.4 [a]



$$\frac{v_1}{48} + \frac{v_1 - 128}{8} + \frac{v_1 - v_2}{18} = 0$$

$$\frac{v_2}{20} + \frac{v_2 - v_1}{18} + \frac{v_2 - 70}{10} = 0$$

Solving, $v_1 = 96$ V; $\quad v_2 = 60$ V

$$i_a = \frac{128 - 96}{8} = 4 \text{ A}$$

$$i_b = \frac{96}{48} = 2 \text{ A}$$

$$i_c = \frac{96 - 60}{18} = 2 \text{ A}$$

$$i_d = \frac{60}{20} = 3 \text{ A}$$

$$i_e = \frac{60 - 70}{10} = -1 \text{ A}$$

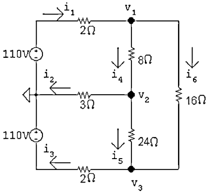
[b] $p_{\text{dev}} = 128(4) + 70(1) = 582$ W

P 3.5 Use the lower terminal of the 5 Ω resistor as the reference node.

$$\frac{v_o - 60}{10} + \frac{v_o}{5} + 3 = 0$$

Solving, $\quad v_o = 10$ V

P 3.6 [a]



$$\frac{v_1 - 110}{2} + \frac{v_1 - v_2}{8} + \frac{v_1 - v_3}{16} = 0$$

$$\frac{v_2 - v_1}{8} + \frac{v_2}{3} + \frac{v_2 - v_3}{24} = 0$$

$$\frac{v_3 + 110}{2} + \frac{v_3 - v_2}{24} + \frac{v_3 - v_1}{16} = 0$$

Solving, $v_1 = 74.64$ V; $v_2 = 11.79$ V; $v_3 = -82.5$ V

Thus, $i_1 = \frac{110 - v_1}{2} = 17.68$ A $\quad i_4 = \frac{v_1 - v_2}{8} = 7.86$ A

$i_2 = \frac{v_2}{3} = 3.93$ A $\quad i_5 = \frac{v_2 - v_3}{24} = 3.93$ A

$i_3 = \frac{v_3 + 110}{2} = 13.75$ A $\quad i_6 = \frac{v_1 - v_3}{16} = 9.82$ A

[b] $\sum P_{\text{dev}} = 110 i_1 + 110 i_3 = 3457.14$ W

$$\sum P_{\text{dis}} = i_1^2(2) + i_2^2(3) + i_3^2(2) + i_4^2(8) + i_5^2(24) + i_6^2(16) = 3457.14 \text{ W}$$

P 3.7 $2.4 + \frac{v_1}{125} + \frac{v_1 - v_2}{25} = 0$

$$\frac{v_2 - v_1}{25} + \frac{v_2}{250} + \frac{v_2}{375} - 3.2 = 0$$

Solving, $v_1 = 25$ V; $\quad v_2 = 90$ V

CHECK:

$$p_{125\Omega} = \frac{(25)^2}{125} = 5 \text{ W}$$

$$p_{25\Omega} = \frac{(90 - 25)^2}{25} = 169 \text{ W}$$

$$p_{250\Omega} = \frac{(90)^2}{250} = 32.4 \text{ W}$$

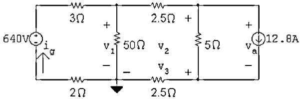
$$p_{375\Omega} = \frac{(90)^2}{375} = 21.6 \text{ W}$$

$$p_{2.4\text{A}} = (25)(2.4) = 60 \text{ W}$$

$$\sum p_{\text{abs}} = 5 + 169 + 32.4 + 21.6 + 60 = 288 \text{ W}$$

$$\sum p_{\text{dev}} = (90)(3.2) = 288 \text{ W} \quad \text{(CHECKS)}$$

P 3.8 [a]



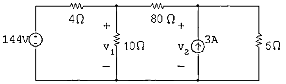
$$\frac{v_1}{50} + \frac{v_1 - 640}{5} + \frac{v_1 - v_2}{2.5} = 0$$
$$\frac{v_2 - v_1}{2.5} + \frac{v_2 - v_3}{5} + 12.8 = 0$$
$$\frac{v_3}{2.5} + \frac{v_3 - v_2}{5} - 12.8 = 0$$

Solving, $v_1 = 380\ V$; $v_2 = 269$ V; $v_3 = 111$ V,

[b] $i_g = \dfrac{640 - 380}{5} = 52$ A

$p_g(\text{del}) = (640)(52) = 33{,}280$ W

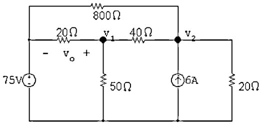
P 3.9



$$\frac{v_1 - 144}{4} + \frac{v_1}{10} + \frac{v_1 - v_2}{80} = 0 \qquad \text{so} \qquad 29v_1 - v_2 = 2880$$
$$-3 + \frac{v_2 - v_1}{80} + \frac{v_2}{5} = 0 \qquad \text{so} \qquad -v_1 + 17v_2 = 240$$

Solving, $v_1 = 100$ V; $v_2 = 20$ V

P 3.10



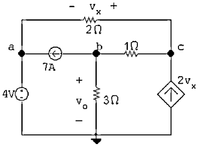
$$\frac{v_1 - 75}{20} + \frac{v_1}{50} + \frac{v_1 - v_2}{40} = 0$$

$$\frac{v_2 - v_1}{40} + \frac{v_2 - 75}{800} - 6 + \frac{v_2}{200} = 0$$

Solving, $v_1 = 115$ V; $\qquad v_2 = 287$ V

$\therefore \quad v_o = 115 - 75 = 40$ V

P 3.11



$v_a = 4$ V

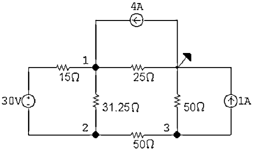
$$7 + \frac{v_b}{3} + \frac{v_b - v_c}{1} = 0$$

$$-2v_x + \frac{v_c - v_b}{1} + \frac{v_c - v_a}{2} = 0$$

$v_x = v_c - v_a = v_c - 4$

Solving, $v_o = v_b = 1.5$ V

P 3.12



$$\frac{v_1 - (v_2 + 30)}{15} + \frac{v_1 - v_2}{31.25} + \frac{v_1}{25} - 4 = 0$$

$$-\left[\frac{v_1 - (v_2 + 30)}{15}\right] + \frac{v_2 - v_3}{50} + \frac{v_2 - v_1}{31.25} = 0$$

$$\frac{v_3 - v_2}{50} + \frac{v_3}{50} + 1 = 0$$

Solving, $v_1 = 76$ V; $v_2 = 46$ V; $v_3 = -2$ V; $i_{30\text{V}} = 0$ A

$p_{4\text{A}} = -4v_1 = -4(76) = -304$ W (del)

$p_{1\text{A}} = (1)(-2) = -2$ W (del)

$p_{30\text{V}} = (30)(0) = 0$ W

$p_{15\Omega} = (0)^2(15) = 0$ W

$$p_{25\Omega} = \frac{v_1^2}{25} = \frac{76^2}{25} = 231.04 \text{ W}$$

$$p_{31.25\Omega} = \frac{(v_1 - v_2)^2}{31.25} = \frac{30^2}{31.25} = 28.8 \text{ W}$$

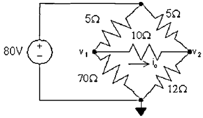
$$p_{50\Omega}(\text{lower}) = \frac{(v_2 - v_3)^2}{50} = \frac{48^2}{50} = 46.08 \text{ W}$$

$$p_{50\Omega}(\text{right}) = \frac{v_3^2}{50} = \frac{4}{50} = 0.08 \text{ W}$$

$\sum p_{\text{diss}} = 0 + 231.04 + 28.8 + 46.8 + 0.08 = 306$ W

$\sum p_{\text{dev}} = 304 + 2 = 306$ W (CHECKS)

P 3.13



$$\frac{v_1}{70} + \frac{v_1 - v_2}{10} + \frac{v_1 - 80}{5} = 0$$
$$\frac{v_2}{12} + \frac{v_2 - v_1}{10} + \frac{v_2 - 80}{5} = 0$$

Solving, $v_1 = 70$ V; $v_2 = 60$ V

Thus, $i_o = \dfrac{v_1 - v_2}{10} = 1$ A

P 3.14 [a] $\frac{v_0 - 60}{10} + \frac{v_o}{5} + 3 = 0; \quad v_o = 10$ V

[b] Let v_x = voltage drop across 3 A source

$v_x = v_o - (100)(3) = -290$ V

$p_{3\text{A}}$ (developed) $= (3)(290) = 870$ W

[c] Let i_g = current into positive terminal of 60 V source

$i_g = (10 - 60)/10 = -5$ A

$p_{60\text{V}}$ (developed) $= (5)(60) = 300$ W

[d] $\sum p_{\text{dis}} = (5)^2(10) + (3)^2(100) + (10)^2/5 = 1170$ W

$\sum p_{\text{dis}} = 300 + 870 = 1170$ W

[e] v_o is independent of any finite resistance connected in series with the 3 A current source

P 3.15 [a] From the solution to Problem 3.5 we know $v_o = 10$ V, therefore

$p_{3\text{A}} = 3v_o = 30$ W

$\therefore \; p_{3\text{A}}$ (developed) $= -30$ W

[b] The current into the negative terminal of the 60 V source is

$$i_g = \frac{60 - 10}{10} = 5 \text{ A}$$

$p_{60\text{V}} = -60(5) = -300$ W

$\therefore \; p_{60\text{V}}$ (developed) $= 300$ W

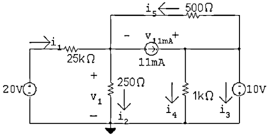
[c] $p_{10\Omega} = (5)^2(10) = 250$ W

$p_{5\Omega} = (10)^2/5 = 20$ W

$\sum p_{\text{dev}} = 300$ W

$\sum p_{\text{dis}} = 250 + 20 + 30 = 300$ W

P 3.16 [a]



$$\frac{v_1 - 20}{25 \times 10^3} + \frac{v_1}{0.25 \times 10^3} + 11 \times 10^{-3} + \frac{v_1 + 10}{0.5 \times 10^3} = 0$$

$$v_1 = -5 \text{ V}$$

$$i_1 = \frac{20+5}{25{,}000} = 1 \text{ mA}$$

$$i_2 = \frac{v_1}{250} = \frac{-5}{250} = -20 \text{ mA}$$

$$i_5 = \frac{-10+5}{500} = -10 \text{ mA}$$

$$i_4 = \frac{-10}{1000} = -10 \text{ mA}$$

$$i_4 + i_3 - 11 + i_5 = 0$$

$$\therefore \quad i_3 = 11 - i_4 - i_5 = 11 + 10 + 10 = 31 \text{ mA}$$

[b] $p_{20\text{V}} = 20i_1 = 20(1 \times 10^{-3}) = 20$ mW

$$p_{10\text{V}} = 10i_3 = 10(31 \times 10^{-3}) = 310 \text{ mW}$$

$$v_{11\text{mA}} + v_1 = -10, \quad v_{11\text{mA}} = -10 + 5 = -5 \text{ V}$$

$$p_{11\text{mA}} = -11v_{11\text{mA}} = -55 \text{ mW} \qquad \text{(del)}$$

$$\sum p_{\text{dev}} = 20 + 310 = 330 \text{ mW}$$

$$p_{25\text{k}} = 25 \times 10^3 i_1^2 = 25 \text{ mW}$$

$$p_{0.25\text{k}} = 0.25 \times 10^3 i_2^2 = 100 \text{ mW}$$

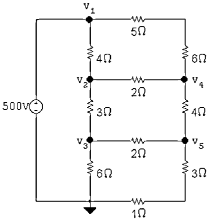
$$p_{0.5\text{k}} = 0.5 \times 10^3 i_5^2 = 50 \text{ mW}$$

$$p_{1\text{k}} = 1 \times 10^3 i_4^2 = 100 \text{ mW}$$

$$\sum p_{\text{diss}} = 25 + 100 + 50 + 100 + 55 = 330 \text{ mW}$$

$$\sum p_{\text{diss}} = \sum p_{\text{dev}} = 330 \text{ mW}$$

P 3.17 [a]



$$\frac{v_2 - 500}{4} + \frac{v_2 - v_4}{2} + \frac{v_2 - v_3}{3} = 0$$

$$\frac{v_3 - v_2}{3} + \frac{v_3}{6} + \frac{v_3 - v_5}{2} = 0$$

$$\frac{v_4 - v_2}{2} + \frac{v_4 - 500}{11} + \frac{v_4 - v_5}{4} = 0$$

$$\frac{v_5 - v_3}{2} + \frac{v_5}{4} + \frac{v_5 - v_4}{4} = 0$$

Solving, $v_2 = 300$ V; $v_3 = 180$ V; $v_4 = 280$ V; $v_5 = 160$ V

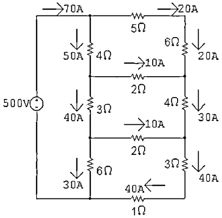
$$i_{5\Omega} = \frac{500 - v_4}{11} = \frac{500 - 280}{11} = 20 \text{ A}$$

$$p_{5\Omega} = (20)^2(5) = 2000 \text{ W}$$

[b]
$$\begin{aligned} i_{500\text{V}} &= \frac{v_1 - v_2}{4} + \frac{v_1 - v_4}{11} \\ &= \frac{500 - 300}{4} + \frac{500 - 280}{11} = 50 + 20 = 70 \text{ A} \end{aligned}$$

$$p_{500\text{V}} = 35{,}000 \text{ W}$$

Check:



$$\begin{aligned} \sum P_{\text{dis}} &= (50)^2(4) + (40)^2(3) + (30)^2(6) + (20)^2(11) + (10)^2(2) \\ &\quad +(30)^2(4) + (10)^2(2) + (40)^2(4) = 35{,}000 \text{ W} \end{aligned}$$

P 3.18 [a] $\dfrac{v_o - v_1}{R} + \dfrac{v_o - v_2}{R} + \dfrac{v_o - v_3}{R} + \cdots + \dfrac{v_o - v_n}{R} = 0$

$\therefore \quad nv_o = v_1 + v_2 + v_3 + \cdots + v_n$

$\therefore \quad v_o = \dfrac{1}{n}[v_1 + v_2 + v_3 + \cdots + v_n] = \dfrac{1}{n}\sum_{k=1}^{n} v_k$

[b] $v_o = \dfrac{1}{3}(150 + 200 - 50) = 100$ V

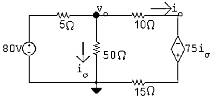
P 3.19 Place $v_\Delta/5$ inside a supernode and use the lower node as a reference. Then

$$\frac{v_1 - 50}{10} + \frac{v_1}{30} + \frac{v_1 - v_\Delta/5}{39} + \frac{v_1 - v_\Delta/5}{78} = 0$$

$134v_1 - 6v_\Delta = 3900; \qquad v_\Delta = 50 - v_1$

Solving, $v_1 = 30$ V; $\quad v_\Delta = 20$ V; $\quad v_o = 30 - v_\Delta/5 = 30 - 4 = 26$ V

P 3.20



$$\frac{v_o - 80}{5} + \frac{v_o}{50} + \frac{v_o + 75i_\sigma}{25} = 0; \quad i_\sigma = \frac{v_o}{50}$$

Solving, $v_o = 50$ V; $\qquad i_\sigma = 1$ A

$$i_o = \frac{50 - (-75)(1)}{25} = 5 \text{ A}$$

$p_{75i_\sigma} = 75i_\sigma i_o = -375$ W

$\therefore$ The dependent voltage source delivers 375 W to the circuit.

P 3.21 $-3 + \dfrac{v_o}{200} + \dfrac{v_o + 5i_\Delta}{10} + \dfrac{v_o - 80}{20} = 0; \quad i_\Delta = \dfrac{v_o - 80}{20}$

[a] Solving, $v_o = 50$ V

[b] $i_{\text{ds}} = \dfrac{v_o + 5i_\Delta}{10}$

$i_\Delta = (50 - 80)/20 = -1.5$ A

$\therefore \quad i_{\text{ds}} = 4.25$ A; $\quad 5i_\Delta = -7.5$ V : $\quad p_{\text{ds}} = (-5i_\Delta)(i_{\text{ds}}) = 31.875$ W

[c] $p_{3\text{A}} = -3v_o = -3(50) = -150$ W $\quad$ (del)

$p_{80\text{V}} = 80i_\Delta = 80(-1.5) = -120$ W $\quad$ (del)

$\sum p_{\text{del}} = 150 + 120 = 270$ W

CHECK:

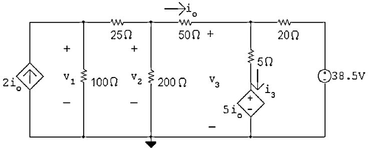
$p_{200\Omega} = 2500/200 = 12.5$ W

$p_{20\Omega} = (80 - 50)^2/20 = 900/20 = 45$ W

$p_{10\Omega} = (4.25)^2/10 = 180.625$ W

$\sum p_{\text{diss}} = 31.875 + 180.625 + 12.5 + 45 = 270$ W

P 3.22 [a]



$$i_o = \frac{v_2 - v_3}{50}$$

$$-2i_o + \frac{v_1}{100} + \frac{v_1 - v_2}{25} = 0$$

$$\frac{v_2 - v_1}{25} + \frac{v_2}{200} + \frac{v_2 - v_3}{50}$$

$$\frac{v_3 - v_2}{50} + \frac{v_3 - 5i_o}{5} + \frac{v_3 - 38.5}{20} = 0$$

Solving, $v_1 = -50$ V; $v_2 = -30$ V; $v_3 = 2.5$ V

[b] $i_o = \dfrac{v_2 - v_3}{50} = \dfrac{-30 - 2.5}{50} = -0.65$ A

$$i_3 = \frac{v_3 - 5i_o}{5} = \frac{2.5 - 5(-0.65)}{5} = 1.15 \text{ A}$$

$$i_g = \frac{38.5 - 2.5}{20} = 1.8 \text{ A}$$

$\sum p_{\text{dis}} = \sum p_{\text{dev}}$

Calculate $\sum p_{\text{dev}}$ because we don't know if the dependent sources are developing or absorbing power. Likewise for the independent source.

$p_{2i_o} = -2i_o v_1 = -2(-0.65)(-50) = -65$ W(dev)

$p_{5i_o} = 5i_o i_3 = 5(-0.65)(1.15) = -3.7375$ W(dev)

$p_g = -38.5(1.8) = -69.30$ W(dev)

$\sum p_{\text{dev}} = 69.3 + 65 + 3.7375 = 138.0375$ W

CHECK

$$\sum p_{\text{dis}} = \frac{2500}{100} + \frac{900}{200} + \frac{400}{25} + (0.65)^2(50) + (1.15)^2 5 + (1.8)^2(20)$$
$$= 138.0375 \text{ W}$$

$$\therefore \quad \sum p_{\text{dev}} = \sum p_{\text{dis}} = 138.0375 \text{ W}$$

P 3.23 [a] $-5 + \dfrac{v_1}{15} + \dfrac{v_1 - v_2}{5} = 0$

$$\frac{v_2 - v_1}{5} + \frac{v_2}{30} + \frac{v_2}{10} + \frac{v_2 + 5i_\Delta}{30} = 0$$

$$i_\Delta = \frac{v_1 - v_2}{5}$$

Solving, $v_1 = 30$ V; $\quad v_2 = 15$ V; $\quad i_\Delta = 3$ A; $\quad i_o = \dfrac{15 + 15}{30} = 1$ A

$$p_{5i_\Delta} = (-15)(1) = -15 \text{ W(del)}$$

$$p_{5\text{A}} = -5(30) = -150 \text{ W(del)}$$

$$\therefore \quad p_{\text{dev}} = 165 \text{ W}$$

[b] $\sum p_{\text{abs}} = \dfrac{(30)^2}{15} + \dfrac{(15)^2}{30} + \dfrac{(15)^2}{10} + (3)^2(5) + (1)^2(30) = 165$ W

$$\therefore \quad \sum p_{\text{dev}} = \sum p_{\text{abs}} = 165 \text{ W}$$

P 3.24 $i_\phi = \dfrac{v_3 - v_4}{4} = \dfrac{235 - 222}{4} = 3.25$ A

$$30i_\phi = 30(3.25) = 97.5 \text{ V}$$

$$v_1 + 30i_\phi = v_4$$

$$v_1 = v_4 - 30i_\phi = 222 - 97.5 = 124.5 \text{ V}$$

$$v_3 + v_\Delta = 250$$

$$\therefore \quad v_\Delta = 250 - 235 = 15 \text{ V}$$

$$3.2v_\Delta = (3.2)(15) = 48 \text{ A}$$

$$i_g = \frac{250 - 124.5}{2} + \frac{250 - 235}{1} = 77.75 \text{ A}$$

$$p_{250\text{V}} = -250i_g = -250(77.75) = -19{,}437.5 \text{ W(del)}$$

$$i_{30i_\phi} - i_\phi + v_4/40 + 48 = 0$$

$$i_{30i_\phi} = i_\phi - 222/40 - 48 = 3.25 - 5.55 - 48 = -50.3 \text{ A}$$

$$p_{30i_\phi} = (30i_\phi)i_{30i_\phi} = (97.5)(-50.3) = -4904.25 \text{ W(dev)}$$

$$p_{3.2v_\Delta} = (3.2v_\Delta)(v_4) = (48)(22) = 10{,}656 \text{ W(abs)}$$

$$\therefore \quad \sum p_{\text{dev}} = 19{,}437.5 + 4904.25 = 24{,}341.75 \text{ W}$$

$$p_{10\Omega} = \frac{v_1^2}{10} = \frac{(124.5)^2}{10} = 1550.025 \text{ W}$$

$$p_{2\Omega} = \frac{(250 - 124.5)^2}{2} = 7875.125 \text{ W}$$

$$p_{1\Omega} = \frac{(250 - 235)^2}{1} = 225 \text{ W}$$

$$p_{20\Omega} = \frac{(235)^2}{20} = 2761.25 \text{ W}$$

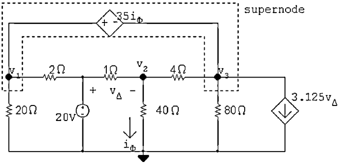
$$p_{4\Omega} = (3.25)^2(4) = 42.25 \text{ W}$$

$$p_{40\Omega} = \frac{(222)^2}{40} = 1232.10 \text{ W}$$

$$\begin{aligned} \therefore \quad \sum p_{\text{diss}} &= 10{,}656 + 1550.025 + 7875.125 + 225 + \\ &\ 2761.250 + 42.25 + 1232.1 = 24{,}341.75 \text{ W} \end{aligned}$$

Thus, $\sum p_{\text{dev}} = \sum p_{\text{diss}}$; Agree with analyst

P 3.25



Node equations:

$$\frac{v_1}{20} + \frac{v_1 - 20}{2} + \frac{v_3 - v_2}{4} + \frac{v_3}{80} + 3.125v_\Delta = 0$$

$$\frac{v_2}{40} + \frac{v_2 - v_3}{4} + \frac{v_2 - 20}{1} = 0$$

Constraint equations:

$$v_\Delta = 20 - v_2$$

$$v_1 - 35i_\phi = v_3$$

$$i_\phi = v_2/40$$

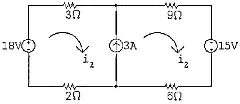
Solving, $v_1 = -20.25$ V; $\quad v_2 = 10$ V; $\quad v_3 = -29$ V

Let i_g be the current delivered by the 20 V source, then

$$i_g = \frac{20 - (20.25)}{2} + \frac{20 - 10}{1} = 30.125 \text{ A}$$

p_g (delivered) $= 20(30.125) = 602.5$ W

P 3.26



$$-18 + 3i_1 + 9i_2 - 15 + 6i_2 + 2i_1 = 0; \quad i_2 - i_1 = 3$$

Solving, $i_1 = -0.6$ A; $\quad i_2 = 2.4$ A

$$p_{18V} = -18i_1 = 10.8 \text{ W (diss)}$$

$$p_{3\Omega} = (-0.6)^2(3) = 1.08 \text{ W}$$

$$p_{2\Omega} = (-0.6)^2(2) = 0.72 \text{ W}$$

$$p_{9\Omega} = (2.4)^2(9) = 51.84 \text{ W}$$

$p_{6\,\Omega} = (2.4)^2(6) = 34.56 \text{ W}$

$\sum p_{\text{diss}} = 99 \text{ W}$

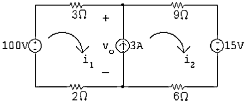
$v_o = 15i_2 - 15 = 36 - 15 = 21 \text{ V}$

$p_{3A} = -3v_o = -63 \text{ W (dev)}$

$p_{15V} = -15i_2 = -36 \text{ W (dev)}$

$\sum p_{\text{dev}} = 99 \text{ W} = \sum p_{\text{diss}}$

P 3.27



$-100 + 5i_1 + 15i_2 - 15 = 0$

$5i_1 + 15i_2 = 115$

$i_2 - i_1 = 3; \qquad i_2 = i_1 + 3; \qquad 15i_2 = 15i_1 + 45$

$\therefore \quad 20i_1 = 70$

$i_1 = 3.5 \text{ A}; \qquad i_2 = 6.5 \text{ A}$

$v_o = 15i_2 - 15 = 97.5 - 15 = 82.5 \text{ V}$

$p_{100\text{V}} = -100i_1 = -350 \text{ W(dev)}$

$p_{3\text{A}} = -3v_o = -247.5 \text{ W(dev)}$

$p_{15\text{V}} = -15i_2 = -97.5 \text{ W(dev)}$

$\sum p_{\text{dev}} = \sum p_{\text{dis}} = 695 \text{ W}$

$\sum p_{\text{dis}} = (3.5)^2(5) + (6.5)^2(15) = 695 \text{ W}$

P 3.28 [a] Summing around the supermesh used in the solution to Problem 3.27 gives

$$-(-10) + 5i_1 + 15i_2 - 15 = 0$$

$$i_2 = i_1 + 3$$

$$\therefore\ i_1 = -2\text{ A}; \qquad i_2 = 1\text{ A}$$

$$p_{10\text{V}} = 10(-2) = -20\text{ W (del)}$$

$$v_o = 15i_2 - 15 = 0\text{ V}$$

$$p_{3\text{A}} = 3v_o = 0\text{ W}$$

$$p_{15\text{V}} = -15i_2 = -15\text{ W (del)}$$

$$\sum p_{\text{diss}} = (-2)^2(5) + (1)^2(15) = 35\text{ W}$$

$$\sum p_{\text{dev}} = 35\text{ W} = \sum p_{\text{diss}}$$

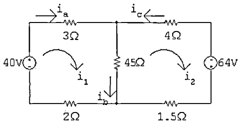
[b] With 3 A current source replaced with a short circuit

$$i_1 = -2\text{ A}, \qquad i_2 = 1\text{ A}$$

$$\therefore\ \sum P_{\text{diss}} = (-2)^2(5) + (1)^2(15) = 35\text{ W}$$

[c] A 3 A source with zero terminal voltage is equivalent to a short circuit carrying 3 A

P 3.29 [a]



$$40 = 50i_1 - 45i_2$$

$$64 = -45i_1 + 50.5i_2$$

Solving, $i_1 = 9.8$ A; $i_2 = 10$ A

$i_a = i_1 = 9.8$ A; $i_b = i_1 - i_2 = -0.2$ A; $i_c = -i_2 = -10$ A

[b] If the polarity of the 64 V source is reversed, we have

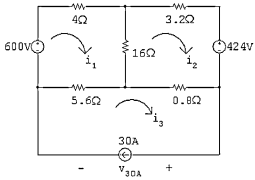
$$40 = 50i_1 - 45i_2$$

$$-64 = -45i_1 + 50.5i_2$$

$i_1 = -1.72$ A and $i_2 = -2.8$ A

$i_a = i_1 = -1.72$ A; $i_b = i_1 - i_2 = 1.08$ A; $i_c = -i_2 = 2.8$ A

P 3.30



$600 = 25.6i_1 - 16i_2 - 5.6i_3$

$-424 = -16i_1 + 20i_2 - 0.8i_3$

$30 = i_3$

Solving, $i_1 = 35$ A; $\quad i_2 = 8$ A; $\quad i_3 = 30$ A

[a] $v_{30\text{A}} = 0.8(i_2 - i_3) + 5.6(i_1 - i_3) = 0.8(8 - 30) + 5.6(35 - 30) = 10.4$ V

$p_{30\text{A}} = 30v_{30\text{A}} = 30(10.4) = 312$ W (abs)

Therefore, the 30 A source delivers -312 W.

[b] $p_{600\text{V}} = -600(35) = -21{,}000$ W(del)

$p_{424\text{V}} = 424(8) = 3392$ W(abs)

Therefore, the total power delivered is 21,000 W

[c] $p_{4\Omega} = (35)^2(4) = 4900$ W

$p_{3.2\Omega} = (8)^2(3.2) = 204.8$ W

$p_{16\Omega} = (35 - 8)^2(16) = 11{,}664$ W

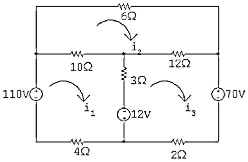
$p_{5.6\Omega} = (35 - 30)^2(5.6) = 140$ W

$p_{0.8\Omega} = (-30 + 8)^2(0.8) = 387.2$ W

$\sum p_{\text{resistors}} = 17{,}296$ W

$\sum p_{\text{abs}} = 17{,}296 + 312 + 3392 = 21{,}000$ W (CHECKS)

P 3.31 [a]



$110 + 12 = 17i_1 - 10i_2 - 3i_3$

$0 = -10i_1 + 28i_2 - 12i_3$

$-12 - 70 = -3i_1 - 12i_2 + 17i_3$

Solving, $i_1 = 8$ A; $i_2 = 2$ A; $i_3 = -2$ A

$p_{110} = -110i_1 = -880$ W(del)

$p_{12} = -12(i_1 - i_3) = -120$ W(del)

$p_{70} = 70i_3 = -140$ W(del)

$\therefore \sum p_{\text{dev}} = 1140$ W

[b] $p_{4\Omega} = (8)^2(4) = 256$ W

$p_{10\Omega} = (6)^2(10) = 360$ W

$p_{12\Omega} = (-4)^2(12) = 192$ W

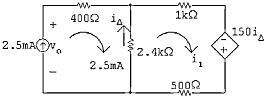
$p_{2\Omega} = (-2)^2(2) = 8$ W

$p_{6\Omega} = (2)^2(6) = 24$ W

$p_{3\Omega} = (10)^2(3) = 300$ W

$\therefore \sum p_{\text{abs}} = 1140$ W

P 3.32 [a]



$2400(i_1 - 0.0025) + 1500i_1 - 150(i_1 - 0.0025) = 0$

$\therefore \quad i_1 = 1.5 \text{ mA}$

$i_\Delta = i_1 - 2.5 = -1.0 \text{ mA}$

[b] $v_o = (0.0025)(400) + (0.001)(2400) = 3.4 \text{ V}$

$p_{2.5\text{mA}} = -3.4(2.5) = -8.5 \text{ mW}$

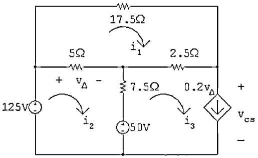
$\therefore \quad p_{2.5\text{mA}} \text{ (deliver)} = 8.5 \text{ mW}$

[c] $150 i_\Delta = 150(-1.0 \times 10^{-3}) = -0.15 \text{ V}$

$p_{\text{dep source}} = 150 i_\Delta i_1 = (-0.15)(0.0015) = -0.225 \text{ mW}$

$p_{\text{dep source}} \text{ (absorbed)} = 0.225 \text{ mW}$

P 3.33



Mesh equations:

$25i_1 - 5i_2 - 2.5i_3 = 0$

$75 = -5i_1 + 12.5i_2 - 7.5i_3$

Constraint equations:

$i_3 = 0.2v_\Delta$

$v_\Delta = 5(i_2 - i_1)$

Solving, $i_1 = 3.6$ A; $\quad i_2 = 13.2$ A; $\quad i_3 = 9.6$ A; $\quad v_\Delta = 48$ V

$v_{\text{cs}} = 125 - v_\Delta - 2.5(i_3 - i_1) = 125 - 48 - 2.5(9.6 - 3.6) = 62 \text{ V}$

$p_{\text{vc}} = (62)(9.6) = 595.2 \text{ W (abs)}$

$p_{50\text{V}} = 50(i_2 - i_3) = 50(13.2 - 9.6) = 180 \text{ W (abs)}$

$$p_{125\text{V}} = -125i_2 = -125(13.2) = -1650 \text{ W(del)}$$

Thus, the total power developed is 1650 W.
CHECK:

$$p_{17.5\Omega} = (3.6)^2(17.5) = 226.8 \text{ W}$$

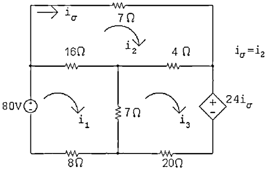
$$p_{5\Omega} = (13.2 - 3.6)^2(5) = 460.8 \text{ W}$$

$$p_{2.5\Omega} = (9.6 - 3.6)^2(2.5) = 90 \text{ W}$$

$$p_{7.5\Omega} = (13.2 - 9.6)^2(7.5) = 97.2 \text{ W}$$

$$\therefore \quad \sum p_{\text{abs}} = 226.8 + 460.8 + 90 + 97.2 + 180 + 595.2 = 1650 \text{ W}$$

P 3.34



$$-80 + 31i_1 - 16i_2 - 7i_3 = 0$$

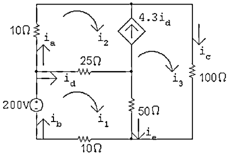
$$-16i_1 + 27i_2 - 4i_3 = 0$$

$$-7i_1 - 4i_2 + 31i_3 + 24i_2 = 0$$

Solving, $i_1 = 3.5$ A

$$p_{8\Omega} = (3.5)^2(8) = 98 \text{ W}$$

P 3.35 [a]



$200 = 85i_1 - 25i_2 - 50i_3$

$0 = -75i_1 + 35i_2 + 150i_3$ (super mesh)

$i_3 - i_2 = 4.3(i_1 - i_2)$

Solving, $i_1 = 4.6$ A; $i_2 = 5.7$ A; $i_3 = 0.97$ A

$i_a = i_2 = 5.7$ A; $i_b = i_1 = 4.6$ A

$i_c = i_3 = 0.97$ A; $i_d = i_1 - i_2 = -1.1$ A

$i_e = i_1 - i_3 = 3.63$ A

[b] $10i_2 + v_o + 25(i_2 - i_1) = 0$

$\therefore \quad v_o = -57 - 27.5 = -84.5$ V

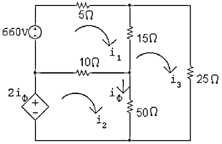
$p_{4.3i_d} = -v_o(4.3i_d) = -(-84.5)(4.3)(-1.1) = -399.685$ W(dev)

$p_{200V} = -200(4.6) = -920$ W(dev)

$$\begin{aligned}\sum P_{\text{dev}} &= 1319.685 \text{ W} \\ \sum P_{\text{dis}} &= (5.7)^2 10 + (1.1)^2(25) + (0.97)^2 100 + (4.6)^2(10) + \\ &\quad (3.63)^2(50) \\ &= 1319.685 \text{ W}\end{aligned}$$

$\therefore \quad \sum P_{\text{dev}} = \sum P_{\text{dis}} = 1319.685$ W

P 3.36



$660 = 30i_1 - 10i_2 - 15i_3$

$20i_\phi = -10i_1 + 60i_2 - 50i_3$

$0 = -15i_1 - 50i_2 + 90i_3$

$i_\phi = i_2 - i_3$

Solving, $i_1 = 42$ A; $i_2 = 27$ A; $i_3 = 22$ A; $i_\phi = 5$ A

$20i_\phi = 100$ V

$p_{20i_\phi} = -100i_2 = -100(27) = -2700$ W

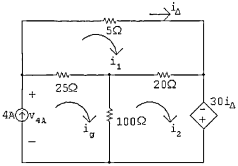
$\therefore\ p_{20i_\phi}$ (developed) $= 2700$ W

CHECK:

$p_{660\text{V}} = -660(42) = -27{,}720$ W (dev)

$$\begin{aligned}\therefore\ \sum P_{\text{dev}} &= 27{,}720 + 2700 = 30{,}420 \text{ W}\\ \sum P_{\text{dis}} &= (42)^2(5) + (22)^2(25) + (20)^2(15) + (5)^2(50) +\\ &\quad (15)^2(10)\\ &= 30{,}420 \text{ W}\end{aligned}$$

P 3.37



Mesh equations:

$50i_1 - 20i_2 - 25i_g = 0$

$-20i_1 + 120i_2 - 30i_\Delta - 100i_g = 0$

Constraint equations:

$i_g = 4$; $i_\Delta = i_1$

Solving, $i_1 = 4$ A; $\qquad i_2 = 5$ A

$i_{25\Omega} = 4 - i_1 = 0$ A

$i_{20\Omega} = i_2 - i_1 = 1$ A

$i_{100\Omega} = 4 - i_2 = -1$ A

$i_{5\Omega} = i_1 = 4$ A

$v_{4A} = 100(4 - i_2) = -100$ V

$p_{4A} = -v_{4A}i_g = -(-100)(4) = 400$ W (abs)

$v_{30i_\Delta} = 30i_\Delta = 30i_1 = 120$ V

$p_{30i_\Delta} = -30i_\Delta i_2 = -120(5) = -600$ W

Therefore, the dependent source is developing 600 W, all other elements are absorbing power, and the total power developed is thus 600 W.
CHECK:

$p_{5\Omega} = 16(5) = 80$ W

$p_{25\Omega} = 0$ W

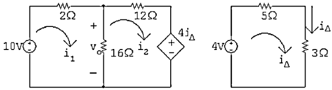
$p_{20\Omega} = 1(20) = 20$ W

$p_{100\Omega} = 1(100) = 100$ W

$p_{4A} = 400$ W

$\sum p_{\rm abs} = 80 + 0 + 20 + 100 + 400 = 600$ W (CHECKS)

P 3.38 [a]



$10 = 18i_1 - 16i_2$

$0 = -16i_1 + 28i_2 + 4i_\Delta$

$4 = 8i_\Delta$

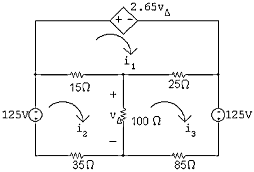
Solving, $i_1 = 1$ A; $\qquad i_2 = 0.5$ A; $\qquad i_\Delta = 0.5$ A

$v_0 = 16(i_1 - i_2) = 16(0.5) = 8$ V

[b] $p_{4i_\Delta} = 4i_\Delta i_2 = (4)(0.5)(0.5) = 1$ W (abs)

$\therefore\ p_{4i_\Delta}$ (deliver) $= -1$ W

P 3.39



Mesh equations:

$2.65v_\Delta + 40i_1 - 15i_2 - 25i_3 = 0$

$-15i_1 + 150i_2 - 100i_3 = -125$

$-25i_1 - 100i_2 - 210i_3 = 125$

Constraint equations:

$v_\Delta = 100(i_2 - i_3)$

Solving, $i_1 = 7$ A; $\quad i_2 = 1.2$ A; $\quad i_3 = 2$ A

$v_\Delta = 100(i_2 - i_3) = 100(1.2 - 2) = -80$ V

$p_{2.65v_\Delta} = 2.65v_\Delta i_1 = -1484$ W

Therefore, the dependent source is developing 1484 W.
CHECK:

$p_{125\text{V}} = 125i_2 = 150$ W (left source)

$p_{125\text{V}} = -125i_3 = -250$ W (right source)

$\sum p_{\text{dev}} = 1484 + 250 = 1734$ W

$p_{35\Omega} = (1.2)^2(35) = 50.4$ W

$p_{85\Omega} = (2)^2(85) = 340$ W

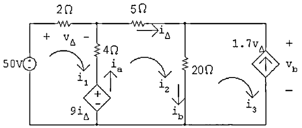
$p_{15\Omega} = (7 - 1.2)^2(15) = 504.6$ W

$p_{25\Omega} = (7 - 2)^2(25) = 625$ W

$p_{100\Omega} = (1.2 - 2)^2(100) = 64$ W

$\sum p_{\text{diss}} = 50.4 + 340 + 504.6 + 625 + 64 + 150 = 1734$ W

P 3.40 [a]



Mesh equations:

$-50 + 6i_1 - 4i_2 + 9i_\Delta = 0$

$-9i_\Delta - 4i_1 + 29i_2 - 20i_3 = 0$

Constraint equations:

$i_\Delta = i_2; \qquad i_3 = -1.7v_\Delta; \qquad v_\Delta = 2i_1$

Solving, $i_1 = -5$ A; $\quad i_2 = 16$ A; $\quad i_3 = 17$ A; $\quad v_\Delta = -10$ V

$9i_\Delta = 9(16) = 144$ V

$i_a = i_2 - i_1 = 21$ A

$i_b = i_2 - i_3 = -1$ A

$v_b = 20i_b = -20$ V

$p_{50V} = -50i_1 = 250$ W (absorbing)

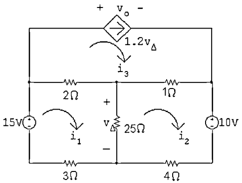
$p_{9i_\Delta} = -i_a(9i_\Delta) = -(21)(144) = -3024$ W (delivering)

$p_{1.7V} = -1.7v_\Delta v_b = i_3 v_b = (17)(-20) = -340$ W (delivering)

[b] $\sum P_{\text{dev}} = 3024 + 340 = 3364$ W

$\sum P_{\text{dis}} = 250 + (-5)^2(2) + (21)^2(4) + (16)^2(5) + (-1)^2(20) = 3364$ W

P 3.41 [a]



Mesh equations:

$15 = 30i_1 - 25i_2 - 2i_3$

$-10 = -25i_1 + 30i_2 - i_3$

Constraint equations:

$i_3 = 1.2v_\Delta; \qquad v_\Delta = 25(i_1 - i_2)$

Solving, $i_1 = 10$ A; $\qquad i_2 = 9$ A; $\qquad i_3 = 30$ A; $\qquad v_\Delta = 25$ V

$i_{2\Omega} = i_1 - i_3 = 9 - 30 = -20$ A

$p_{2\Omega} = (-20)^2(2) = 800$ W

[b] $p_{15\text{V}} = -15(10) = -150$ W(dev)

$p_{10\text{V}} = 10i_2 = 10(9) = 90$ W (abs)

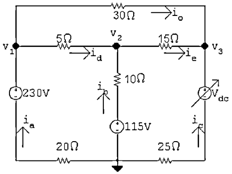
$v_o = (i_1 - i_3)2 + (i_2 - i_3)1 = -40 - 21 = -61$ V

$p_{1.2v_\Delta} = i_3 v_o = (30)(-61) = -1830$ W (dev)

$\sum P_{\text{dev}} = 1830 + 150 = 1980$ W

$$\%\text{ delivered to }2\Omega = \frac{800}{1980} \times 100 = 40.4\%$$

P 3.42 [a]



If $i_o = 0$ then $v_1 = v_3$; therefore,

$$\frac{v_1 - v_2}{5} + \frac{v_1 - 230}{20} = 0$$

$$\frac{v_2 - v_1}{5} + \frac{v_2 - v_3}{15} + \frac{v_2 - 115}{10} = 0$$

Solving, $v_1 = 170\text{ V} = v_3$; $\qquad v_2 = 155\text{ V}$

$$\therefore \quad \frac{170 - 155}{15} + \frac{170 - v_{\text{dc}}}{25} = 0$$

Solving, $v_{\text{dc}} = 195\text{ V}$

[b] $i_a = \dfrac{230 - 170}{20} = 3\text{ A}$

$$i_b = \frac{115 - 155}{10} = -4\text{ A}$$

$$i_c = \frac{195 - 170}{25} = 1\text{ A}$$

$$i_d = \frac{170 - 155}{5} = 3\text{ A}$$

$$i_e = \frac{155 - 170}{15} = -1\text{ A}$$

$p_{230\text{V}} = -230 i_a = -690\text{ W (dev)}$

$p_{115\text{V}} = -115 i_b = 460\text{ W (abs)}$

$p_{v_{\text{dc}}} = -v_{\text{dc}} i_c = -195\text{ W (dev)}$

$p_{20\Omega} = i_a^2(20) = 180\text{ W}$

$p_{5\Omega} = i_d^2(5) = 45\text{ W}$

$p_{10\Omega} = i_b^2(10) = 160\text{ W}$

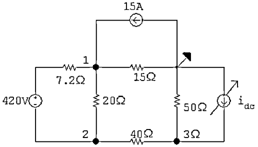
$p_{15\Omega} = i_e^2(15) = 15\text{ W}$

$p_{25\Omega} = i_c^2(25) = 25\text{ W}$

$\sum p_{\text{diss}} = 460 + 180 + 45 + 160 + 15 + 25 = 885\text{ W}$

$\sum p_{\text{dev}} = 690 + 195 = 885\text{ W (CHECKS)}$

P 3.43 Choose the reference node so that a node voltage is identical to the voltage across the 15 A source; thus:



Since the 15 A source is developing 3750 W, v_1 must be 250 V.

Since v_1 is known, we can sum the currents away from node 1 to find v_2; thus:

$$\frac{250-(420+v_2)}{7.2}+\frac{250-v_2}{20}+\frac{250}{15}-15=0$$

$\therefore\quad v_2=-50$ V

Now that we know v_2 we sum the currents away from node 2 to find v_3; thus:

$$\frac{v_2+420-250}{7.2}+\frac{v_2-250}{20}+\frac{v_2-v_3}{40}=0$$

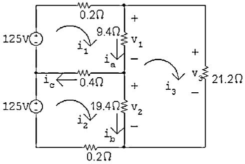
$\therefore\quad v_3=50/3$ V

Now that we know v_3 we sum the currents away from node 3 to find i_{dc}; thus:

$$\frac{v_3}{50}+\frac{v_3+50}{40}=i_{dc}$$

$\therefore\quad i_{dc}=2$ A

P 3.44 [a]



$$125=10i_1-0.4i_2-9.4i_3$$

$125 = -0.4i_1 + 20i_2 - 19.4i_3$

$0 = -9.4i_1 - 19.4i_2 + 50i_3$

Solving, $i_1 = 23.93$ A; $\quad i_2 = 17.79$ A; $\quad i_3 = 11.40$ A

$v_1 = 9.4(i_1 - i_3) = 117.76$ V

$v_2 = 19.4(i_2 - i_3) = 123.90$ V

$v_3 = 21.2i_3 = 241.66$ V

[b] $P_{R1} = (i_1 - i_3)^2(9.4) = 1475.22$ W

$P_{R2} = (i_2 - i_3)^2(19.4) = 791.29$ W

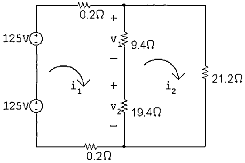
$P_{R3} = i_3^2(21.2) = 2754.64$ W

[c] $\sum p_{\text{dev}} = 125(i_1 + i_2) = 5213.99$ W

$\sum p_{\text{load}} = 5021.15$ W

$$\% \text{ delivered } = \frac{5021.15}{5213.99} \times 100 = 96.3\%$$

[d]



$250 = 29.2i_1 - 28.8i_2$

$0 = -28.8i_1 + 50i_2$

Solving, $i_1 = 19.82$ A; $\quad i_2 = 11.42$ A

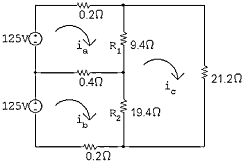
$i_1 - i_2 = 8.41$ A

$v_1 = (8.41)(9.4) = 79.01$ V

$v_2 = 8.41(19.4) = 163.06$ V

Note v_1 is low and v_2 is high. Therefore, loads designed for 125 V would not function properly, and could be damaged.

P 3.45



$$125 = (R_1 + 0.6)i_a - 0.4i_b - R_1 i_c$$

$$125 = -0.4i_a + (R_2 + 0.6)i_b - R_2 i_c$$

$$0 = -R_1 i_a - R_2 i_b + (R_1 + R_2 + 21.2)i_c$$

$$\Delta = \begin{vmatrix} (R_1 + 0.6) & -0.4 & -R_1 \\ -0.4 & (R_2 + 0.6) & -R_2 \\ -R_1 & -R_2 & (R_1 + R_2 + 21.2) \end{vmatrix}$$

When $R_1 = R_2$, Δ reduces to

$$\Delta = 21.6R_1^2 + 25.84R_1 + 4.24.$$

$$\begin{aligned} N_a &= \begin{vmatrix} 125 & -0.4 & -R_1 \\ 125 & (R_2 + 0.6) & -R_2 \\ 0 & -R_2 & (R_1 + R_2 + 21.2) \end{vmatrix} \\ &= 125\,[2R_1R_2 + R_1 + 22.2R_2 + 21.2] \end{aligned}$$

$$\begin{aligned} N_b &= \begin{vmatrix} (R_1 + 0.6) & 125 & -R_1 \\ -0.4 & 125 & -R_2 \\ -R_1 & 0 & (R_1 + R_2 + 21.2) \end{vmatrix} \\ &= 125\,[2R_1R_2 + 22.2R_1 + R_2 + 21.2] \end{aligned}$$

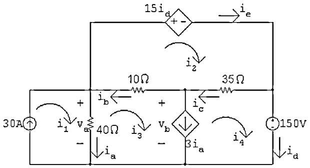
$$i_a = \frac{N_a}{\Delta}, \qquad i_b = \frac{N_b}{\Delta}$$

$$i_{\text{neutral}} = i_a - i_b = \frac{N_a - N_b}{\Delta} = \frac{125[(R_1 - R_2) + 22.2(R_2 - R_1)]}{\Delta}$$

Now note that when $R_1 = R_2$, i_{neutral} reduces to

$$i_{\text{neutral}} = \frac{0}{\Delta} = 0$$

P 3.46 [a]



$40(i_3 - i_1) + 10(i_3 - i_2) + 35(i_4 - i_2) + 150 = 0$

$35(i_2 - i_4) + 10(i_2 - i_3) + 15i_d = 0$

$i_d = i_4;\qquad i_1 = 30$ A

Solving, $i_1 = 30$ A; $\quad i_2 = 8$ A; $\quad i_3 = 24$ A; $\quad i_4 = 6$ A

$i_a = 30 - 24 = 6$ A; $\quad i_b = 8 - 24 = -16$ A; $\quad i_c = 8 - 6 = 2$ A;

$i_d = 6$ A; $\qquad i_e = i_c + i_d = 6 + 2 = 8$ A

[b] $v_a = 40i_a = 240$ V; $\qquad v_b = 150 - 35i_c = 80$ V

$p_{30A} = -30v_a = -30(240) = -7200$ W (gen)

$p_{15i_d} = 15i_di_e = 15(6)(8) = 720$ W (diss)

$p_{3i_a} = 3i_av_b = 3(6)(80) = 1440$ W (diss)

$p_{150V} = 150i_d = 150(6) = 900$ W (diss)

$p_{40\Omega} = (6)^2(40) = 1440$ W (diss)

$p_{10\Omega} = (-16)^2(10) = 2560$ W (diss)

$p_{35\Omega} = (2)^2(35) = 140$ W (diss)

$\sum P_{\text{gen}} = 7200$ W

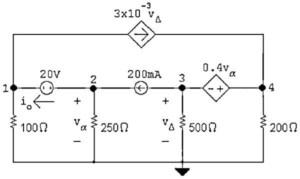
$\sum P_{\text{diss}} = 720 + 1440 + 900 + 1440 + 2560 + 140 = 7200$ W

P 3.47 [a] The node voltage method requires summing the currents at two supernodes in terms of four node voltages and using two constraint equations to reduce the system of equations to two unknowns. If the connection at the bottom of the circuit is used as the reference node, then the voltages controlling the dependent sources are node voltages. This makes it easy to formulate the constraint equations. The current in the 20 V source is obtained by summing the currents at either terminal of the source.

The mesh current method requires summing the voltages around the two meshes not containing current sources in terms of four mesh currents. In addition the voltages controlling the dependent sources must be expressed in terms of the mesh currents. Thus the constraint equations are more complicated, and the reduction to two equations and two unknowns involves more algebraic manipulation. The current in the 20 V source is found by subtracting two mesh currents.

Because the constraint equations are easier to formulate in the node Voltage method, it is the preferred approach.

[b]



Node voltage equations:

$$\frac{v_1}{100} + \frac{v_2}{250} - 0.2 + 3 \times 10^{-3} v_3 = 0$$

$$\frac{v_3}{500} + \frac{v_4}{200} - 3 \times 10^{-3} v_3 + 0.2 = 0$$

Constraints:

$v_2 - v_1 = 20; \qquad v_4 - v_3 = 0.4v_\alpha; v_\alpha = v_2$

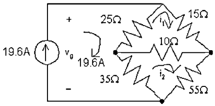
Solving, $v_2 = 44$ V

$i_o = 0.2 - 44/250 = 24$ mA

$p_{20\text{V}} = 20i_o = 480$ mW (abs)

P 3.48 [a] There are three unknown node voltages and only two unknown mesh currents. Use the mesh current method to minimize the number of simultaneous equations.

[b]



$$25(i_1 - 19.6) + 15i_1 + 10(i_1 - i_2) = 0$$

$$35(i_2 - 19.6) + 10(i_2 - i_1) + 55i_2 = 0$$

Solving, $i_1 = 11.4$ A; $\quad i_2 = 8$ A

$$i_{10\Omega} = i_1 - i_2 = 3.4 \text{ A}$$

$$p_{10\Omega} = (3.4)^2(10) = 115.6 \text{ W}$$

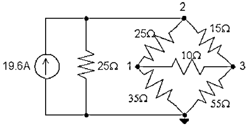
[c] No, the voltage across the 19.6 A current source is readily available from the mesh currents, and solving two simultaneous mesh-current equations is less work than solving three node voltage equations.

[d] $v_g = (19.6 - 11.4)(25) + (19.6 - 8)(35) = 611$ V

$$p_{19.6\text{A}} \text{ (developed) } = 19.6(611) = 11{,}975.6 \text{ W}$$

P 3.49 [a] There are three unknown node voltages and three unknown mesh currents, so the number of simultaneous equations required are the same for both methods. The node voltage method has the advantage of having to solve the three simultaneous equations for one unknown voltage provided the connection at either the top or bottom of the circuit is used as the reference node. Therefore recommend the node voltage method.

[b]



$$\frac{v_1}{35} + \frac{v_1 - v_2}{25} + \frac{v_1 - v_3}{10} = 0$$

$$\frac{v_2}{25} - 19.6 + \frac{v_2 - v_1}{25} + \frac{v_2 - v_3}{15} = 0$$

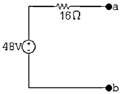
$$\frac{v_3 - v_1}{10} + \frac{v_3 - v_2}{15} + \frac{v_3}{55} = 0$$

Solving, $v_2 = 271.9255$ V

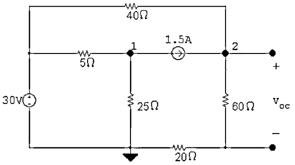
$p_{19.6\text{A}} = -(19.6)(271.9255) = -5329.74$ W(dev)

$\therefore$ The 19.6 A source is developing 5329.74 W

P 3.50 $v_{\text{Th}} = \dfrac{60}{50}(40) = 48$ V $\qquad R_{\text{Th}} = 8 + \dfrac{(40)(10)}{50} = 16\,\Omega$



P 3.51 [a] Open circuit:

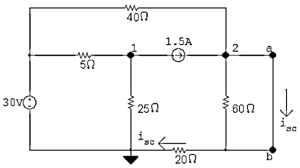


$$\frac{v_2}{80} + \frac{v_2 - 30}{40} - 1.5 = 0$$

$\therefore$ $v_2 = 60$ V

$$v_{\text{oc}} = \frac{60}{80}v_2 = 45 \text{ V} = v_{\text{Th}}$$

Short circuit:

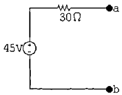


$$\frac{v_2 - 30}{40} - 1.5 + \frac{v_2}{20} = 0$$

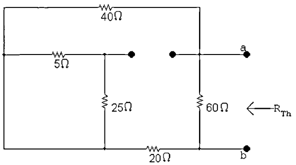
$\therefore \quad v_2 = 30$ V

$$i_{sc} = \frac{v_2}{20} = 1.5 \text{ A}$$

Therefore, $R_{Th} = 45/1.5 = 30\,\Omega$

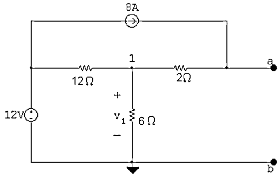


[b]



$R_{Th} = 60\|(40 + 20) = 30\,\Omega$ (CHECKS)

P 3.52

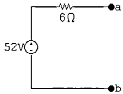


$$\frac{v_1 - 12}{12} + \frac{v_1}{6} - 8 = 0$$

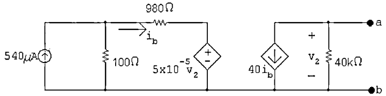
$v_1 = 36$ V

$v_{\rm Th} = v_1 + (2)(8) = 52$ V

$$R_{\rm Th} = 2 + \frac{(12)(6)}{18} = 6\,\Omega$$



P 3.53



OPEN CIRCUIT

$v_2 = -40i_b\ 40 \times 10^3 = -16 \times 10^5 i_b$

$5 \times 10^{-5} v_2 = -80 i_b$

$980 i_b + 5 \times 10^{-5} v_2 = 900 i_b$

$100(540 \times 10^{-6}) = 54$ mV

$\therefore \quad i_b = \dfrac{54 \times 10^{-3}}{1000} = 54\,\mu\text{A}$

$v_{\text{Th}} = -16 \times 10^5(54 \times 10^{-6}) = -86.40$ V

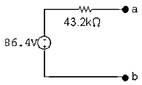
SHORT CIRCUIT

$v_2 = 0; \qquad i_{sc} = -40 i_b$

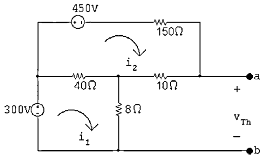
$i_b = \dfrac{54 \times 10^{-3}}{1080} = \dfrac{54}{1.08} \times 10^{-6} = 50\,\mu\text{A}$

$i_{sc} = -40(50) = -2000\,\mu\text{A} = -2$ mA

$R_{\text{Th}} = \dfrac{-86.4}{-2} \times 10^3 = 43.2$ kΩ



P 3.54 After making a source transformation the circuit becomes



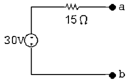
$300 = 48 i_1 - 40 i_2$

$-450 = -40 i_1 + 200 i_2$

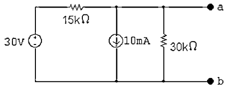
$\therefore \quad i_1 = 5.25$ A and $i_2 = -1.2$ A

$v_{\text{Th}} = 8 i_1 + 10 i_2 = 30$ V

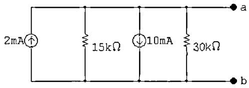
$R_{\text{Th}} = (40\|8 + 10)\|50 = 15\,\Omega$



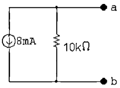
P 3.55 First we make the observation that the 8-mA current source and the 20 kΩ resistor will have no influence on the behavior of the circuit with respect to the terminals a,b. This follows because they are in parallel with an ideal voltage source. Hence our circuit can be simplified to



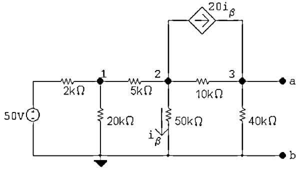
or



Therefore the Norton equivalent is



P 3.56 Open circuit voltage:

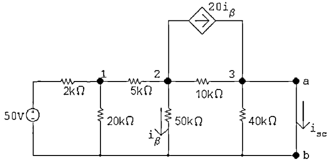


$$\frac{v_1 - 50}{2} + \frac{v_1}{20} + \frac{v_1 - v_2}{5} = 0$$

$$\frac{v_2 - v_1}{5} + \frac{v_2}{50} + \frac{v_2 - v_3}{10} + 20\frac{v_2}{50} = 0$$

$$\frac{v_3}{40} + \frac{v_3 - v_2}{10} - 20\frac{v_2}{50} = 0$$

Solving, $v_3 = 100$ V $= v_{\text{Th}}$
Short circuit current:



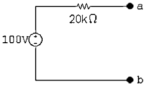
$$\frac{v_1}{20} + \frac{v_1 - 50}{2} + \frac{v_1 - v_2}{5} = 0$$

$$\frac{v_2 - v_1}{5} + \frac{v_2}{50} + \frac{v_2}{10} + 20\frac{v_2}{50} = 0$$

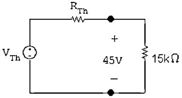
Solving, $v_1 = 36$ V; $\qquad v_2 = 10$ V

$$i_{sc} = \frac{20(10)}{50{,}000} + \frac{10}{10{,}000} = 0.004 + 0.001 = 5 \text{ mA}$$

$$\therefore \quad R_{\text{Th}} = \frac{v_{\text{Th}}}{i_{\text{sc}}} = 100/0.005 = 20\ \text{k}\Omega$$

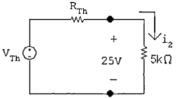


P 3.57



$i_1 = 45/15{,}000 = 3$ mA

$45 = v_{\text{Th}} - 0.003R_{\text{Th}}, \qquad v_{\text{Th}} = 45 + 0.003R_{\text{Th}}$

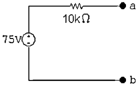


$i_2 = 25/5000 = 5$ mA

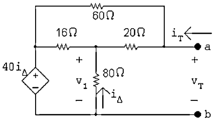
$25 = v_{\text{Th}} - 0.005R_{\text{Th}}, \qquad v_{\text{Th}} = 25 + 0.005R_{\text{Th}}$

$\therefore \quad 45 + 0.003R_{\text{Th}} = 25 + 0.005R_{\text{Th}} \qquad \text{so} \qquad R_{\text{Th}} = 10\ \text{k}\Omega$

$v_{\text{Th}} = 45 + 30 = 75$ V



P 3.58 $V_{\text{Th}} = 0$, since circuit contains no independent sources.



$$i_T = \frac{v_T - v_1}{20} + \frac{v_T - 40i_\Delta}{60}$$

$$\frac{v_1 - 40i_\Delta}{16} + \frac{v_1}{80} + \frac{v_1 - v_T}{20} = 0$$

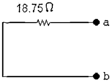
$$\therefore \quad 10v_1 - 200i_\Delta = 4v_T \qquad i_\Delta = \frac{-v_1}{80}, \qquad 200i_\Delta = -2.5v_1$$

$$\therefore \quad 12.5v_1 = 4v_T; \qquad v_1 = 0.32v_T$$

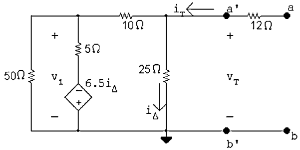
$$60i_T = 4v_T - 2.5v_1 = 3.2v_T$$

$$\therefore \quad \frac{v_T}{i_T} = \frac{60}{3.2} = 18.75\,\Omega$$

$$R_{\text{Th}} = 18.75\,\Omega$$



P 3.59 $V_{\text{Th}} = 0$ since there are no independent sources in the circuit. To find R_{Th} we first find $R_{a'b'}$.



$$i_T = \frac{v_T}{25} + \frac{v_T - v_1}{10}$$

$$\frac{v_1}{50} + \frac{v_1 + 6.5i_\Delta}{5} + \frac{v_1 - v_T}{10} = 0 \text{ so } 16v_1 + 65i_\Delta = 5v_T$$

$$i_\Delta = \frac{v_T}{25}, \qquad 65i_\Delta = 2.6v_T$$

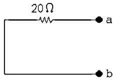
$$16v_1 + 2.6v_T = 5v_T$$

$$\therefore \quad v_1 = 0.15v_T$$

$$i_T = \frac{v_T}{25} + \frac{v_T - 0.15v_T}{10} = \frac{6.25}{50}v_T$$

$$\frac{v_T}{i_T} = 50/6.25 = 8\,\Omega = R_{a'b'}$$

$$\therefore \quad R_{Th} = 12 + 8 = 20\,\Omega$$



P 3.60 [a] Since $0 \le R_o \le \infty$ maximum power will be delivered to the 6 Ω resistor when $R_o = 0$.

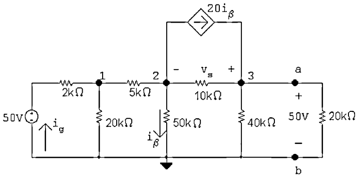
[b] $P = \dfrac{30^2}{6} = 150$ W

P 3.61 [a] From the solution of Problem 3.56 we have $R_{\rm Th} = 20$ kΩ and $v_{\rm Th} = 100$ V. Therefore

$$R_o = R_{\rm Th} = 20 \text{ k}\Omega$$

[b] $p = \dfrac{(50)^2}{20{,}000} = 125$ mW

[c]



$$\frac{v_1}{20{,}000} + \frac{v_1 - 50}{2000} + \frac{v_1 - v_2}{5000} = 0$$

$$\frac{v_2}{50{,}000} + \frac{v_2 - v_1}{5000} + \frac{v_2 - 50}{10{,}000} + 20\left(\frac{v_2}{50{,}000}\right) = 0$$

Solving, $v_1 = 38$ V; $\qquad v_2 = 17.5$ V

$$i_g = \frac{50 - 38}{2000} = 6 \text{ mA}$$

$p_{50\text{V}}$ (delivered) $= (50)(0.006) = 300$ mW

$v_2 + v_s = 50$ V

$v_s = 50 - (17.5) = 32.5$ V

$$i_\beta = \frac{v_2}{50{,}000} = 0.35 \text{ mA}$$

$20i_\beta = 7$ mA

p_{20i_β} (delivered) $= (32.5)(0.007) = 227.5$ mW

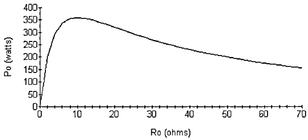
$\sum p_{\rm dev} = 300 + 227.5 = 527.5$ mW

$$\% \text{ delivered } = \frac{125}{527.5} \times 100 = 23.7\%$$

P 3.62 [a] From the solution to Problem 2.25 we have

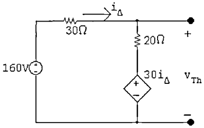
$R_o(\Omega)$	P_o(W)	$R_o(\Omega)$	P_o(W)
0	0	20	320.00
2	200.00	30	270.00
6	337.50	40	230.40
10	360.00	50	200.00
15	345.60	70	157.50

[b]



[c] $R_o = 10\,\Omega$, $\quad P_o$ (max) $= 360$ W

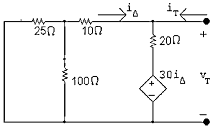
P 3.63 We begin by finding the Thévenin equivalent with respect to R_o. After making a couple of source transformations the circuit simplifies to



$$i_\Delta = \frac{160 - 30i_\Delta}{50}; \qquad i_\Delta = 2 \text{ A}$$

$$v_{\text{Th}} = 20i_\Delta + 30i_\Delta = 50i_\Delta = 100 \text{ V}$$

Using the test-source method to find the Thévenin resistance gives

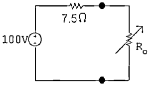


$$i_T = \frac{v_T}{30} + \frac{v_T - 30(-v_T/30)}{20}$$

$$\frac{i_T}{v_T} = \frac{1}{30} + \frac{1}{10} = \frac{4}{30} = \frac{2}{15}$$

$$R_{Th} = \frac{v_T}{i_T} = \frac{15}{2} = 7.5\,\Omega$$

Thus our problem is reduced to analyzing the circuit shown below.



$$p = \left(\frac{100}{7.5 + R_o}\right)^2 R_o = 250$$

$$\frac{10^4}{R_o^2 + 15R_o + 56.25} R_o = 250$$

$$\frac{10^4 R_o}{250} = R_o^2 + 15R_o + 56.25$$

$$40R_o = R_o^2 + 15R_o + 56.25$$

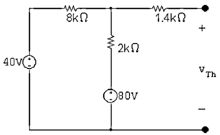
$$R_o^2 - 25R_o + 56.25 = 0$$

$$R_o = 12.5 \pm \sqrt{156.25 - 56.25} = 12.5 \pm 10$$

$$R_o = 22.5\,\Omega$$

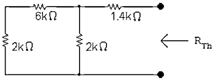
$$R_o = 2.5\,\Omega$$

P 3.64 [a]



$$\frac{v_{Th} - 40}{8000} + \frac{v_{Th} - 80}{2000} = 0$$

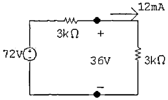
$\therefore\ v_{Th} = 72$ V



$R_{Th} = 1400 + (2000)(8000)/1000 = 3$ kΩ

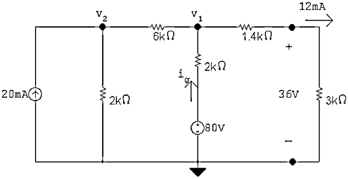
$R_o = R_{Th} = 3$ kΩ

[b]



$$p_{max} = \frac{(36)^2}{3} \times 10^{-3} = 432 \text{ mW}$$

P 3.65



$$v_1 = (12 \times 10^{-3})(1.4 + 3) \times 10^3 = 12(4.4) = 52.8 \text{ V}$$

$$i_g = \frac{80 - 52.8}{2000} = 13.6 \text{ mA}$$

$$p_{80\text{V}} \text{ (dev) } = (80)(0.0136) = 1088 \text{ mW}$$

$$-0.02 + \frac{v_2}{2000} + \frac{v_2 - 52.8}{6000} = 0$$

$$\therefore \quad v_2 = 43.2 \text{ V}$$

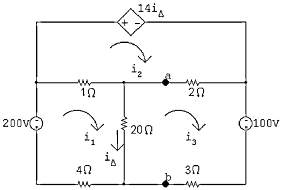
$$p_{20\text{mA}} \text{ (dev) } = (0.02)(43.2) = 864 \text{ mW}$$

$$\sum p_{\text{dev}} = 1088 + 864 = 1952 \text{ mW}$$

$$\% \text{ delivered to } R_o = \frac{432}{1952} \times 100 = 22.13\%$$

P 3.66 [a] We begin by finding the Thévenin equivalent with respect to the terminals of R_o.

Open circuit voltage



$$-200 = 25i_1 - 1i_2 - 20i_3$$

$$0 = -i_1 + 3i_2 - 2i_3 + 14i_\Delta$$

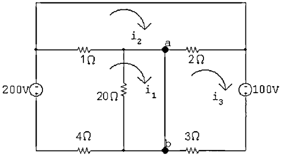
$$100 = -20i_1 - 2i_2 + 25i_3$$

$$i_\Delta = i_1 - i_3$$

Solving, $i_1 = -2.5$ A; $\quad i_2 = 37.5$ A; $\quad i_3 = 5$ A; $\quad i_\Delta = -7.5$ A

$v_{\text{Th}} = 20(i_1 - i_3) = 20(-7.5) = -150$ V

Now find the short-circuit current.



Note with the short circuit from a to b that i_Δ is zero, hence $14i_\Delta$ is also zero.

$$-200 = 5i_1 - 1i_2 + 0i_3$$

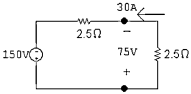
$$0 = -1i_1 + 3i_2 - 2i_3$$

$$100 = 0i_1 - 2i_2 + 5i_3$$

Solving, $i_1 = -40$ A; $\quad i_2 = 0$ A; $\quad i_3 = 20$ A

$i_{\text{sc}} = i_1 - i_3 = -60$ A

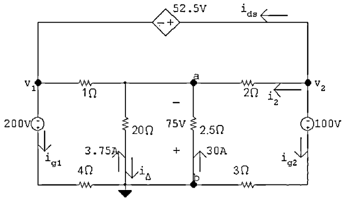
$R_{\text{Th}} = (-150)/(-60) = 2.5\,\Omega$



For maximum power transfer $\quad R_o = R_{\text{Th}} = 2.5\,\Omega$

[b] $p_{\max} = \dfrac{75^2}{2.5} = 2250$ W

P 3.67 From the solution of Problem 3.66 we know that when R_o is 2.5 Ω, the voltage across R_o is 75 V, positive at the lower terminal. Therefore our problem reduces to the analysis of the following circuit. In constructing the circuit we have used the fact that i_Δ is −3.75 A, and hence $14i_\Delta$ is −52.5 V.



Using the node Voltage method to find v_1 and v_2 yields

$$-33.75 + \frac{-75 - v_1}{1} + \frac{-75 - v_2}{2} = 0$$

$$v_1 + 52.5 = v_2$$

Solving, $v_1 = -115$ V; $\quad v_2 = -62.5$ V.It follows that

$$i_{g_1} = \frac{-115 + 200}{4} = 21.25 \text{ A}$$

$$i_{g_2} = \frac{-62.5 + 100}{3} = 12.5 \text{ A}$$

$$i_2 = \frac{-62.5 + 75}{2} = 6.25 \text{ A}$$

$$i_{\text{ds}} = -6.25 - 12.5 = -18.75 \text{ A}$$

$$p_{200\text{V}} = -200 i_{g_1} = -4250 \text{ W(dev)}$$

$$p_{100\text{V}} = -100 i_{g_2} = -1250 \text{ W(dev)}$$

$$p_{\text{ds}} = 52.5 i_{\text{ds}} = -984.375 \text{ W(dev)}$$

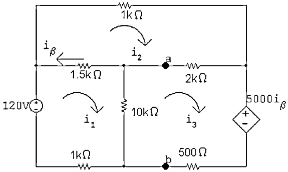
$$\therefore \quad \sum p_{\text{dev}} = 4250 + 1250 + 984.375 = 6484.375 \text{ W}$$

$$\therefore \quad \%\text{ delivered } = \frac{2250}{6484.375}(100) = 34.7\%$$

$\therefore$ 34.7% of developed power is delivered to load

P 3.68 [a] Find the Thévenin equivalent with respect to the terminals of R_L.

Open circuit voltage:



$$120 = 12{,}500i_1 - 1500i_2 - 10{,}000i_3$$

$$0 = -1500i_1 + 4500i_2 - 2000i_3$$

$$0 = -10{,}000i_1 - 2000i_2 + 12{,}500i_3 + 5000i_\beta$$

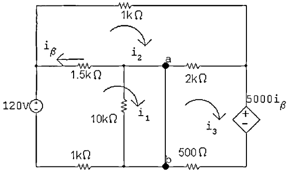
$$i_\beta = i_2 - i_1$$

Solving,

$i_1 = 99.6$ mA; $i_2 = 78$ mA; $i_3 = 100.8$ mA; $i_\beta = -21.6$ mA

$$v_{\text{Th}} = v_{\text{ab}} = 10 \times 10^3(i_1 - i_3) = -12 \text{ V}$$

Short-circuit current:



$$120 = 2500i_1 - 1500i_2 + 0i_3$$

$$0 = -1500i_1 + 4500i_2 - 2000i_3$$

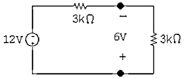
$$0 = 0i_1 - 2000i_2 + 2500i_3 + 5000i_\beta$$

$$i_\beta = i_2 - i_1$$

Solving,

$i_1 = 92$ mA; $\quad i_2 = 73.33$ mA; $\quad i_3 = 96$ mA; $\quad i_\beta = -18.67$ mA

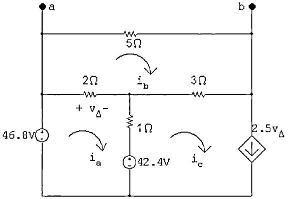
$i_{sc} = i_1 - i_3 = -4$ mA; $\qquad R_{Th} = \dfrac{v_{Th}}{i_{sc}} = \dfrac{-12}{-4 \times 10^{-3}} = 3$ kΩ



$R_L = R_{Th} = 3$ kΩ

[b] $p_{max} = \dfrac{6^2}{3 \times 10^3} = 12$ mW

P 3.69 Find the Thévenin equivalent with respect to the terminals of R_o. Open circuit voltage:



$(46.8 - 42.4) = 3i_a - 2i_b - i_c$

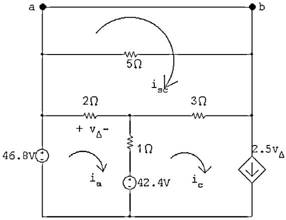
$0 = -2i_a + 10i_b - 3i_c$

$i_c = 2.5v_\Delta; \qquad v_\Delta = 2(i_a - i_b)$

Solving, $i_b = 74.8$ A

$\therefore \; v_{Th} = 5i_b = 374$ V

Short circuit current:



$46.8 - 42.4 = 3i_a - 2i_{\text{sc}} - i_c$

$0 = -2i_a + 5i_{\text{sc}} - 3i_c$

$i_c = 2.5v_\Delta; \qquad v_\Delta = 2(i_a - i_{\text{sc}})$

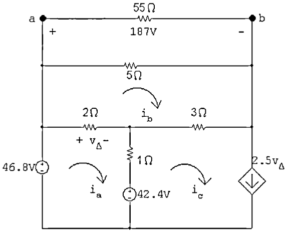
Solving,

$i_{\text{sc}} = 6.8$ A; $\qquad i_a = 8$ A; $\qquad i_c = 6$ A; $\qquad v_\Delta = 2.4$ V

$R_{\text{Th}} = v_{\text{Th}}/i_{\text{sc}} = 374/6.8 = 55\,\Omega$

$R_o = 55\,\Omega$

with R_o equal to $55\,\Omega$ the circuit becomes



$46.8 - 42.4 = 3i_a - 2i_b - 2.5(2)(i_a - i_b)$

$i_c = 2.5v_\Delta$

$v_\Delta = 2(i_a - i_b)$

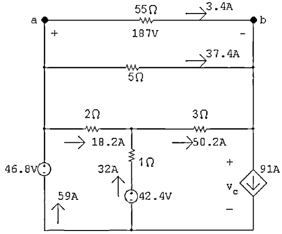
$187 + 3i_b - 3(2.5)(2)(i_a - i_b) + 2i_b - 2i_a = 0$

Solving, $i_a = 59$ A; $\qquad i_b = 40.8$ A

$v_\Delta = 2(59 - 40.80) = 36.4$ V

$i_c = 91$ A

Thus we have



$v_c = 42.4 - 32 - 150.6 = -140.20$ V

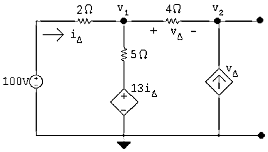
$\sum P_{\text{dev}} = 46.8(59) + 42.4(32) + 140.20(91) = 16{,}876.20$ W

CHECK:

$$\sum P_{\text{dis}} = (18.2)^2(2) + (50.2)^2(3) + (32)^2(1) + 187(3.4) + 187(37.4)$$
$$= 16{,}876.20 \text{ W}$$

$$\% \text{ delivered } = \frac{(55)(3.4)^2(100)}{16{,}876.2} = 3.77\%$$

P 3.70 [a] Open circuit voltage



Node voltage equation:

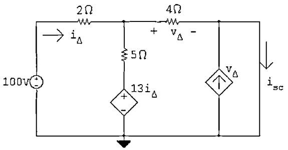
$$\frac{v_1 - 100}{2} + \frac{v_1 - 13i_\Delta}{5} + \frac{v_1 - v_2}{4} = 0$$

Constraint equations:

$$i_\Delta = \frac{100 - v_1}{2}; \qquad \frac{v_2 - v_1}{4} - v_\Delta = 0; \qquad v_\Delta = v_1 - v_2$$

Solving, $v_2 = 90$ V $= v_{\rm Th}$; $\quad v_1 = 0$ V; $\quad v_\Delta = 0$ V; $\quad i_\Delta = 5$ A

Short circuit current:



$$\frac{v_1 - 100}{2} + \frac{v_1 - 13i_\Delta}{5} + \frac{v_1}{4} = 0$$

$$i_\Delta = \frac{100 - v_1}{2}$$

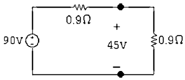
Solving, $v_1 = 80$ V $= v_\Delta$; $\quad i_\Delta = 10$ A

$$i_{\rm sc} = \frac{v_1}{4} + v_\Delta = 20 + 80 = 100 \text{ A}$$

$$R_{\rm Th} = \frac{v_{\rm Th}}{i_{\rm sc}} = \frac{90}{100} = 0.9\,\Omega$$

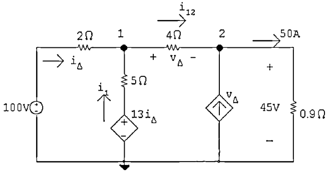
$$\therefore \quad R_o = R_{\rm Th} = 0.9\,\Omega$$

[b]



$$p_{\rm max} = \frac{(45)^2}{0.9} = 2250 \text{ W}$$

[c]



$$\frac{v_1 - 100}{2} + \frac{v_1 - 13i_\Delta}{5} + \frac{v_1 - 45}{4} = 0$$

$$i_\Delta = \frac{100 - v_1}{2}$$

Solving, $v_1 = 85$ V; $\quad i_\Delta = 7.5$ A; $\quad v_\Delta = v_1 - v_2 = 85 - 45 = 40$ V

$i_{100\text{V}} = i_\Delta = 7.5$ A

$p_{100\text{V}}$ (dev) $= 100(7.5) = 750$ W

$i_{12} = v_\Delta/4 = 40/4 = 10$ A

$i_1 = i_{12} - i_\Delta - 10 - 7.5 = 2.5$ A

p_{13i_Δ} (dev) $= (97.5)(2.5) = 243.75$ W

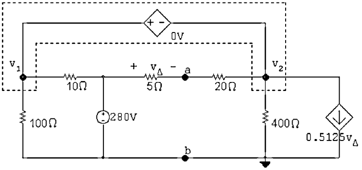
p_{v_Δ} (dev) $= (45)(40) = 1800$ W

$\sum p_{\rm dev} = 750 + 243.75 + 1800 = 2793.75$ W

$$\% \text{ delivered } = \frac{2250}{2793.75} \times 100 = 80.54\%$$

P 3.71 [a] First find the Thévenin equivalent with respect to R_o.

Open circuit voltage: $i_\phi = 0$; $50i_\phi = 0$



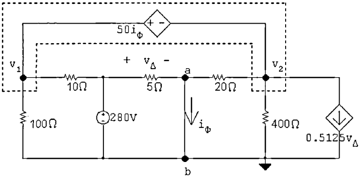
$$\frac{v_1}{100} + \frac{v_1 - 280}{10} + \frac{v_1 - 280}{25} + \frac{v_1}{400} + 0.5125v_\Delta = 0$$

$$v_\Delta = \frac{(280 - v_1)}{25}5 = 56 - 0.2v_1$$

$v_1 = 210$ V; $\qquad v_\Delta = 14$ V

$v_{\mathrm{Th}} = 280 - v_\Delta = 280 - 56 + 0.2(210) = 266$ V

Short circuit current



$$\frac{v_1}{100} + \frac{v_1 - 280}{10} + \frac{v_2}{20} + \frac{v_2}{400} + 0.5125(280) = 0$$

$v_\Delta = 280$ V

$v_2 + 50i_\phi = v_1$

$$i_\phi = \frac{280}{5} + \frac{v_2}{20} = 56 + 0.05v_2$$

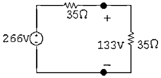
$v_2 = -968$ V; $\qquad v_1 = -588$ V

$i_\phi = i_{sc} = 56 + 0.05(-968) = 7.6$ A

$R_{Th} = v_{Th}/i_{sc} = 266/7.6 = 35\,\Omega$

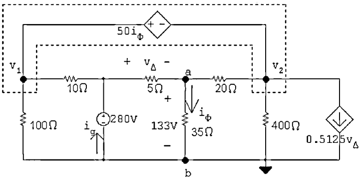
$\therefore \quad R_o = 35\,\Omega$

[b]



$p_{max} = (133)^2/35 = 505.4$ W

[c]



$$\frac{v_1}{100} + \frac{v_1 - 280}{10} + \frac{v_2 - 133}{20} + \frac{v_2}{400} + 0.5125(280 - 133) = 0$$

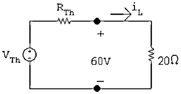
$v_2 + 50i_\phi = v_1; \qquad i_\phi = 133/35 = 3.8$ A

Therefore, $v_1 = -189$ V and $v_2 = -379$ V; thus,

$$i_g = \frac{280 - 133}{5} + \frac{280 + 189}{10} = 76.30 \text{ A}$$

$p_{280\text{V}}$ (dev) $= (280)(76.3) = 21{,}364$ W

P 3.72 [a]



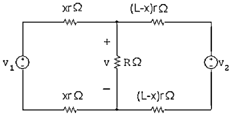
$v_{oc} = V_{Th} = 75$ V; $\qquad i_L = \dfrac{60}{20} = 3$ A; $\qquad i_L = \dfrac{75 - 60}{R_{Th}} = \dfrac{15}{R_{Th}}$

Therefore $\quad R_{\text{Th}} = \dfrac{15}{3} = 5\,\Omega$

[b] $i_L = \dfrac{v_o}{R_L} = \dfrac{V_{\text{Th}} - v_o}{R_{\text{Th}}}$

Therefore $\quad R_{\text{Th}} = \dfrac{V_{\text{Th}} - v_o}{v_o/R_L} = \left(\dfrac{V_{\text{Th}}}{v_o} - 1\right) R_L$

P 3.73 [a]



$$\frac{v - v_1}{2xr} + \frac{v}{R} + \frac{v - v_2}{2r(\ell - x)} = 0$$

$$v\left[\frac{1}{2xr} + \frac{1}{R} + \frac{1}{2r(L - x)}\right] = \frac{v_1}{2xr} + \frac{v_2}{2r(\ell - x)}$$

$$v = \frac{v_1 RL + xR(v_2 - v_1)}{RL + 2rLx - 2rx^2}$$

[b] Let $D = RL + 2rLx - 2rx^2$

$$\frac{dv}{dx} = \frac{(RL + 2rLx - 2rx^2)R(v_2 - v_1) - [v_1 RL + xR(v_2 - v_1)]2rL - 2x)}{D^2}$$

$\dfrac{dv}{dx} = 0 \quad$ when numerator is zero.

The numerator simplifies to

$$x^2 + \frac{2L - v_1}{(v_2 - v_1)}x + \frac{RL(v_2 - v_1) - 2rv_1 L^2}{2r(v_2 - v_1)} = 0$$

Solving for the roots of the quadratic yields

$$x = \frac{L}{v_2 - v_1}\left\{-v_1 \pm \sqrt{v_1 v_2 - \frac{R}{2rL}(v_2 - v_1)^2}\right\}$$

[c] $x = \dfrac{L}{v_2 - v_1}\left\{-v_1 \pm \sqrt{v_1 v_2 - \dfrac{R}{2rL}(v_1 - v_2)^2}\right\}$

$v_2 = 1200$ V, $\qquad v_1 = 1000$ V, $\qquad L = 16$ km

$r = 5 \times 10^{-5}\,\Omega/m; \qquad R = 3.9\,\Omega$

$$\frac{L}{v_2 - v_1} = \frac{16{,}000}{1200 - 1000} = 80; \qquad v_1 v_2 = 1.2 \times 10^6$$

$$\frac{R}{2rL}(v_1 - v_2)^2 = \frac{3.9(-200)^2}{(10 \times 10^{-5})(16 \times 10^3)} = 0.975 \times 10^5$$

$$x = 80\{-1000 \pm \sqrt{1.2 \times 10^6 - 0.0975 \times 10^6}\}$$

$$= 80\{-1000 \pm 1050\} = 80(50) = 4000 \text{ m}$$

[d]

$$\begin{aligned} v_{\min} &= \frac{v_1 RL + R(v_2 - v_1)x}{RL + 2rLx - 2rx^2} \\ &= \frac{(1000)(3.9)(16 \times 10^3) + 3.9(200)(4000)}{(3.9)(16{,}000) + 10 \times 10^{-5}(16{,}000)(4000) - 10 \times 10^{-5}(16 \times 10^6)} \\ &= 975 \text{ V} \end{aligned}$$

P 3.74 $$\frac{dv_1}{dI_{g1}} = \frac{-R_1[R_2(R_3 + R_4) + R_3R_4]}{(R_1 + R_2)(R_3 + R_4) + R_3R_4}$$

$$\frac{dv_1}{dI_{g2}} = \frac{R_1R_3R_4}{(R_1 + R_2)(R_3 + R_4) + R_3R_4}$$

$$\frac{dv_2}{dI_{g1}} + \frac{-R_1R_3R_4}{(R_1 + R_2)(R_3 + R_4) + R_3R_4}$$

$$\frac{dv_2}{dI_{g2}} = \frac{R_3R_4(R_1 + R_2)}{(R_1 + R_2)(R_3 + R_4) + R_3R_4}$$

P 3.75 From the solution to Problem 3.74 we have

$$\frac{dv_1}{dI_{g1}} = \frac{-25[5(125) + 3750]}{30(125) + 3750} = -\frac{175}{12} \text{ V/A}$$

and

$$\frac{dv_2}{dI_{g1}} = \frac{-(25)(50)(75)}{30(125) + 3750} = -12.5 \text{ V/A}$$

By hypothesis, $\Delta I_{g1} = 11 - 12 = -1$ A

$$\therefore \quad \Delta v_1 = (-\frac{175}{12})(-1) = \frac{175}{12} = 14.5833 \text{ V}$$

Thus, $v_1 = 25 + 14.5833 = 39.5833$ V

Also,

$\Delta v_2 = (-12.5)(-1) = 12.5$ V

Thus, $v_2 = 90 + 12.5 = 102.5$ V
The PSpice solution is

$v_1 = 39.5830$ V

and

$v_2 = 102.5000$ V

These values are in agreement with our predicted values.

P 3.76 From the solution to Problem 3.74 we have

$$\frac{dv_1}{dI_{g2}} = \frac{(25)(50)(75)}{30(125) + 3750} = 12.5 \text{ V/A}$$

and

$$\frac{dv_2}{dI_{g2}} = \frac{(50)(75)(30)}{30(125) + 3750} = 15 \text{ V/A}$$

By hypothesis, $\Delta I_{g2} = 17 - 16 = 1$ A

$\therefore \quad \Delta v_1 = (12.5)(1) = 12.5$ V

Thus, $v_1 = 25 + 12.5 = 37.5$ V
Also,

$\Delta v_2 = (15)(1) = 15$ V

Thus, $v_2 = 90 + 15 = 105$ V
The PSpice solution is

$v_1 = 37.5$ V

and

$v_2 = 105$ V

These values are in agreement with our predicted values.

P 3.77 From the solutions to Problems 3.74 — 3.76 we have

$$\frac{dv_1}{dI_{g1}} = -\frac{175}{12}\ \text{V/A}; \qquad \frac{dv_1}{dI_{g2}} = 12.5\ \text{V/A}$$

$$\frac{dv_2}{dI_{g1}} = -12.5\ \text{V/A}; \qquad \frac{dv_2}{dI_{g2}} = 15\ \text{V/A}$$

By hypothesis,

$$\Delta I_{g1} = 11 - 12 = -1\ \text{A}$$

$$\Delta I_{g2} = 17 - 16 = 1\ \text{A}$$

Therefore,

$$\Delta v_1 = \frac{175}{12} + 12.5 = 27.0833\ \text{V}$$

$$\Delta v_2 = 12.5 + 15 = 27.5\ \text{V}$$

Hence

$$v_1 = 25 + 27.0833 = 52.0833\ \text{V}$$

$$v_2 = 90 + 27.5 = 117.5\ \text{V}$$

The PSpice solution is

$$v_1 = 52.0830\ \text{V}$$

and

$$v_2 = 117.5\ \text{V}$$

These values are in agreement with our predicted values.

P 3.78 By hypothesis,

$$\Delta R_1 = 27.5 - 25 = 2.5\,\Omega$$

$$\Delta R_2 = 4.5 - 5 = -0.5\,\Omega$$

$$\Delta R_3 = 55 - 50 = 5\,\Omega$$

$$\Delta R_4 = 67.5 - 75 = -7.5\,\Omega$$

So

$\Delta v_1 = 0.5833(2.5) - 5.417(-0.5) + 0.45(5) + 0.2(-7.5) = 4.9168$ V

$\therefore \quad v_1 = 25 + 4.9168 = 29.9168$ V

$\Delta v_2 = 0.5(2.5) + 6.5(-0.5) + 0.54(5) + 0.24(-7.5) = -1.1$ V

$\therefore \quad v_2 = 90 - 1.1 = 88.9$ V

The PSpice solution is

$v_1 = 29.6710$ V

and

$v_2 = 88.5260$ V

Note our predicted values are within a fraction of a volt of the actual values.

4 The Operational Amplifier

Drill Exercises

DE 4.1 [a] $v_o = (-80/16)v_s, \qquad v_o = -5v_s$

v_s(V)	0.4	2.0	3.5	−0.6	−1.6	−2.4
v_o(V)	−2.0	−10.0	−15.0	3.0	8.0	10.0

[b] $-15 = -5v_s, \quad v_s = 3\ \text{V}; \qquad 10 = -5v_s, \qquad v_s = -2\ \text{V}$

Therefore $\quad -2 \leq v_s \leq 3\ \text{V}$

DE 4.2 $v_o = (-R_x/16)v_s = (0.64R_x/16) = 10\ \text{V}$

Therefore $\quad R_x = \dfrac{160}{0.64} = 250\,\text{k}\Omega, \qquad 0 \leq R_x \leq 250\,\text{k}\Omega$

DE 4.3 [a] $v_o = -\dfrac{250}{5}v_\text{a} - \dfrac{250}{25}v_\text{b} = -50(0.1) - 10(0.25) = -5 - 2.5 = -7.5\ \text{V}$

[b] $v_o = -50v_\text{a} - 2.5 = -10\ \text{V}; \qquad$ therefore $\quad 50v_\text{a} = 7.5, \quad v_\text{a} = 0.15\ \text{V}$

[c] $v_o = -5 - 10v_\text{b} = -10\ \text{V}; \qquad 10v_\text{b} = 5, \quad v_\text{b} = 0.5\ \text{V}$

[d] $v_o = -50v_\text{a} + 10v_\text{b} = -5 + 2.5 = -2.5\ \text{V}$

$v_o = -50v_\text{a} + 2.5 = -10\ \text{V};$

$50v_\text{a} = 12.5, \quad v_\text{a} = 0.25\ \text{V}$

$v_o = -5 + 10v_\text{b} = 15\ \text{V}; \qquad 10v_\text{b} = 20; \qquad v_\text{b} = 2.0\ \text{V}$

DE 4.4 [a] $\dfrac{v_n}{4500} + \dfrac{v_n - v_o}{63{,}000} = 0, \qquad$ therefore $\quad v_o = 15v_n, \quad v_n = v_p$

Thus $\quad v_o = 15v_p, \qquad v_p = \dfrac{0.4R_x}{15{,}000 + R_x}$

So when $\quad R_x = 60\,\text{k}\Omega, \qquad v_p = 0.32\ \text{V}, \qquad v_o = 4.8\ \text{V}$

[b] $\dfrac{15(0.4R_x)}{15{,}000 + R_x} = 5, \qquad R_x = 75\,\text{k}\Omega$

DE 4.5 [a] Assume v_a is acting along. Replacing v_b with a short circuit yields $v_p = 0$, therefore $v_n = 0$ and we have

$$\frac{0 - v_\text{a}}{R_\text{a}} + \frac{0 - v_o'}{R_\text{b}} + i_n = 0, \qquad i_n = 0$$

Therefore

$$\frac{v_o'}{R_\text{b}} = -\frac{v_\text{a}}{R_\text{a}}, \qquad v_o' = \frac{R_\text{b}}{R_\text{a}} v_\text{a}$$

Assume v_b is acting alone. Replace v_a with a short circuit. Now

$$v_p = v_n = \frac{v_\text{b} R_\text{d}}{R_\text{c} + R_\text{d}}$$

$$\frac{v_n}{R_\text{a}} + \frac{v_n - v_o''}{R_\text{b}} + i_n = 0, \qquad i_n = 0$$

$$\left(\frac{1}{R_\text{a}} + \frac{1}{R_\text{b}}\right)\left(\frac{R_\text{d}}{R_\text{c} + R_\text{d}}\right) v_\text{b} - \frac{v_o''}{R_\text{b}} = 0$$

$$v_o'' = \left(\frac{R_\text{b}}{R_\text{a}} + 1\right)\left(\frac{R_\text{d}}{R_\text{c} + R_\text{d}}\right) v_\text{b} = \frac{R_\text{d}}{R_\text{a}}\left(\frac{R_\text{a} + R_\text{b}}{R_\text{c} + R_\text{d}}\right) v_\text{b}$$

$$v_o = v_o' + v_o'' = \frac{R_\text{d}}{R_\text{a}}\left(\frac{R_\text{a} + R_\text{b}}{R_\text{c} + R_\text{d}}\right) v_\text{b} - \frac{R_\text{b}}{R_\text{a}} v_\text{a}$$

[b] $\dfrac{R_\text{d}}{R_\text{a}}\left(\dfrac{R_\text{a} + R_\text{b}}{R_\text{c} + R_\text{d}}\right) = \dfrac{R_\text{b}}{R_\text{a}}, \qquad$ therefore $\quad R_\text{d}(R_\text{a} + R_\text{b}) = R_\text{b}(R_\text{c} + R_\text{d})$

$R_\text{d} R_\text{a} = R_\text{b} R_\text{c}, \qquad$ therefore $\quad \dfrac{R_\text{a}}{R_\text{b}} = \dfrac{R_\text{c}}{R_\text{d}}$

When $\dfrac{R_\text{d}}{R_\text{a}}\left(\dfrac{R_\text{a} + R_\text{b}}{R_\text{c} + R_\text{d}}\right) = \dfrac{R_\text{b}}{R_\text{a}}$

Eq. (4.22) reduces to $\quad v_o = \dfrac{R_\text{b}}{R_\text{a}} v_\text{b} - \dfrac{R_\text{b}}{R_\text{a}} v_\text{a} = \dfrac{R_\text{b}}{R_\text{a}}(v_\text{b} - v_\text{a})$.

DE 4.6 [a] $v_o = \dfrac{20(60)}{10(24)} v_\text{b} - \dfrac{50}{10} v_\text{a} = 5(v_\text{b} - v_\text{a}) = 20 - 5v_\text{a}$

$20 - 5v_\text{a} = \pm 10$ V

$5v_\text{a} = 20 \mp 10, \qquad v_\text{a} = 2$ V, $\qquad v_\text{a} = 6$ V

Therefore $\quad 2 \le v_\text{a} \le 6$ V

[b] $v_o = \dfrac{8(60)}{10(12)}v_b - 5v_a = 4v_b - 5v_a$

$4v_b - 5v_a = 16 - 5v_a = \pm 10\ \text{V}$

$16 \mp 10 = 5v_a, \qquad v_a = 1.2\ \text{V}, \qquad v_a = 5.2\ \text{V}$

Therefore $1.2 \leq v_a \leq 5.2\ \text{V}$

DE 4.7 [a] $A_{\text{dm}} = \dfrac{(24)(26) + (25)(25)}{(2)(1)(25)} = 24.98$

[b] $A_{\text{cm}} = \dfrac{(1)(24) - 25(1)}{1(25)} = -0.04$

[c] $\text{CMRR} = \left|\dfrac{24.98}{0.04}\right| = 624.50$

DE 4.8 $A_{\text{cm}} = \dfrac{(20)(50) - (50)R_x}{20(50 + R_x)}$

$$A_{\text{dm}} = \frac{50(20 + 50) + 50(50 + R_x)}{2(20)(50 + R_x)}$$

$$\frac{A_{\text{dm}}}{A_{\text{cm}}} = \frac{R_x + 120}{2(20 - R_x)}$$

$$\therefore \quad \frac{R_x + 120}{2(20 - R_x)} = \pm 1000$$

If we use $+1000 \quad R_x = 19.93\,\text{k}\Omega$

If we use $-1000 \quad R_x = 20.07\,\text{k}\Omega$

Problems

P 4.1 Since the current into the inverting input terminal of an ideal op-amp is zero, the voltage across the $3.3\,\mathrm{M}\Omega$ resistor is (2.5)(3.3) or 8.25 V. Therefore the voltmeter reads 8.25 V.

P 4.2 $v_p = \dfrac{18}{24}(12) = 9\ \mathrm{V} = v_n$

$$\frac{v_n - 24}{30} + \frac{v_n - v_o}{20} = 0$$

$$v_o = (45 - 48)/3 = -1.0\ \mathrm{V}$$

$$i_{\mathrm{L}} = \frac{v_o}{5} \times 10^{-3} = -\frac{1}{5} \times 10^{-3} = -200 \times 10^{-6}$$

$$i_{\mathrm{L}} = -200\,\mu\mathrm{A}$$

P 4.3 $\dfrac{v_{\mathrm{b}} - v_{\mathrm{a}}}{20} + \dfrac{v_{\mathrm{b}} - v_o}{160} = 0,\qquad$ therefore $\quad v_o = 9v_{\mathrm{b}} - 8v_{\mathrm{a}}$

[a] $v_{\mathrm{a}} = 1.5\ \mathrm{V},\quad v_{\mathrm{b}} = 0\ \mathrm{V},\quad v_o = -12\ \mathrm{V}$

[b] $v_{\mathrm{a}} = 3.0\ \mathrm{V},\quad v_{\mathrm{b}} = 0\ \mathrm{V},\quad v_o = -18\ \mathrm{V}$ (sat)

[c] $v_{\mathrm{a}} = 1.0\ \mathrm{V},\quad v_{\mathrm{b}} = 2\ \mathrm{V},\quad v_o = 10\ \mathrm{V}$

[d] $v_{\mathrm{a}} = 4.0\ \mathrm{V},\quad v_{\mathrm{b}} = 2\ \mathrm{V},\quad v_o = -14\ \mathrm{V}$

[e] $v_{\mathrm{a}} = 6.0\ \mathrm{V},\quad v_{\mathrm{b}} = 8\ \mathrm{V},\quad v_o = 18\ \mathrm{V}$ (sat)

[f] If $v_{\mathrm{b}} = 4.5\ \mathrm{V},\quad v_o = 40.5 - 8v_{\mathrm{a}} = \pm 18$

$$\therefore\ 2.8125 \le v_{\mathrm{a}} \le 7.3125\ \mathrm{V}$$

P 4.4 [a] $i_{\mathrm{a}} = \dfrac{120}{6} \times 10^{-6} = 20\,\mu\mathrm{A}$

$$v_{\mathrm{a}} = -20 \times 10^3 i_{\mathrm{a}} = -400\,\mathrm{mV}$$

[b] $\dfrac{v_{\mathrm{a}}}{60{,}000} + \dfrac{v_{\mathrm{a}}}{20{,}000} + \dfrac{v_{\mathrm{a}} - v_o}{240{,}000} = 0$

$$\therefore\ v_o = 17v_{\mathrm{a}} = -6.8\ \mathrm{V}$$

[c] $i_{\mathrm{a}} = 20\,\mu\mathrm{A}$

[d] $i_o = \dfrac{-v_o}{80{,}000} + \dfrac{v_a - v_o}{240{,}000} = 111.67\,\mu\ \mathrm{A}$

P 4.5 $v_o = (1)(9) = 9$ V; $\quad i_{15k\Omega} = \dfrac{9}{15{,}000} = 0.6\,\text{mA};$

$$i_{6k\Omega} = \frac{9}{6000} = 1.5\,\text{mA}; \qquad i_{9k\Omega} = \frac{9}{9000} = 1\,\text{mA}$$

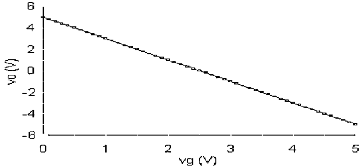
$$\therefore \quad i_o = -0.6 - 1.5 - 1 = -3.1\,\text{mA}$$

P 4.6 [a] First, note that $v_n = v_p = 2.5$ V

Let v_{o1} equal the voltage output of the op-amp. Then

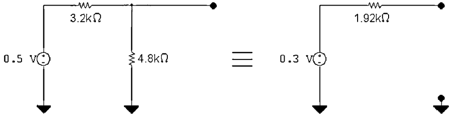
$$\frac{2.5 - v_g}{5000} + \frac{2.5 - v_{o1}}{10{,}000} = 0, \qquad \therefore \quad v_{o1} = 7.5 - 2v_g$$

Also note that $v_{o1} - 2.5 = v_o$, $\qquad \therefore \quad v_o = 5 - 2v_g$



[b] Yes, the circuit designer is correct!

P 4.7 [a] Replace the combination of v_g, 3.2 kΩ, and the 4.8 kΩ resistors with its Thévenin equivalent.



Then $\quad v_o = \dfrac{-[30 + \sigma 170]}{1.92}(0.30)$

At saturation $\quad v_o = -10$ V; $\qquad$ therefore

$$-\left(\frac{30 + \sigma 170}{1.92}\right)(0.3) = -10, \quad \text{or} \quad \sigma = 0.2$$

Thus for $0 \le \sigma < 0.20$ the operational amplifier will not saturate.

[b] When $\sigma = 0.12, \qquad v_o = \dfrac{-(30+20.4)}{1.92}(0.30) = -7.875 \text{ V}$

Also $\dfrac{v_o}{180} + \dfrac{v_o}{50.4} + i_o = 0$

$$\therefore \quad i_o = -\frac{v_o}{180} - \frac{v_o}{50.4} = \frac{7.875}{180} + \frac{7.875}{50.4}\,\text{mA} = 200\,\mu\text{A}$$

P 4.8 [a] Let v_Δ be the voltage from the potentiometer contact to ground. Then

$$\frac{0 - v_g}{5} + \frac{0 - v_\Delta}{15} = 0$$

$$-3v_g - v_\Delta = 0, \qquad \therefore \quad v_\Delta = -150\,\text{mV}$$

$$\frac{v_\Delta}{\alpha R_\Delta} + \frac{v_\Delta - 0}{15{,}000} + \frac{v_\Delta - v_o}{(1-\alpha)R_\Delta} = 0$$

$$\frac{v_\Delta}{\alpha} + 10v_\Delta + \frac{v_\Delta - v_o}{1-\alpha} = 0$$

$$v_\Delta\left(\frac{1}{\alpha} + 10 + \frac{1}{1-\alpha}\right) = \frac{v_o}{1-\alpha}$$

$$\therefore \quad v_o = -0.15\left[1 + 10(1-\alpha) + \frac{(1-\alpha)}{\alpha}\right]$$

When $\alpha = 0.3, \qquad v_o = -0.15(1 + 7 + 7/3) = -1.55 \text{ V}$

When $\alpha = 0.75, \qquad v_o = -0.15(1 + 2.5 + 1/3) = -0.575 \text{ V}$

$$\therefore \quad -1.55 \text{ V} \le v_o \le -0.575 \text{ V}$$

[b] $-0.15\left[1 + 10(1-\alpha) + \dfrac{(1-\alpha)}{\alpha}\right] = -6$

$$\alpha + 10\alpha(1-\alpha) + (1-\alpha) = 40\alpha$$

$$\alpha + 10\alpha - 10\alpha^2 + 1 - \alpha = 40\alpha$$

$$\therefore \quad 10\alpha^2 + 30\alpha - 1 = 0 \quad \text{so} \quad \alpha \cong 0.033$$

P 4.9 [a] $\dfrac{v_d - v_a}{72} + \dfrac{v_d - v_b}{120} + \dfrac{v_d - v_c}{450} + \dfrac{v_d}{600} + \dfrac{v_d - v_o}{180} = 0$

$$v_o = 180\left(-\frac{10}{72{,}000} + \frac{2}{120{,}000} + \frac{23}{450{,}000} + \frac{8}{600{,}000} + \frac{8}{180{,}000}\right) = -2.4 \text{ V}$$

[b] $v_o = -8.4 - 0.4v_c$

$$-8.4 - 0.4v_c = -16; \qquad v_c = 19\ \text{V}$$

$$-8.4 - 0.4v_c = 16; \qquad v_c = -61\ \text{V}$$

$$-61\ \text{V} \le v_c \le 19\ \text{V}$$

P 4.10 [a] $\dfrac{v_d - v_a}{72{,}000} + \dfrac{v_d - v_b}{120{,}000} + \dfrac{v_d - v_c}{450{,}000} + \dfrac{v_d}{600{,}000} + \dfrac{v_d - v_o}{R_f} = 0$

$$(25/3)v_d - (25/3)v_a + 5v_d - 5v_b + (4/3)v_d - (4/3)v_c + v_d +$$

$$\frac{600}{R_f}v_d = \frac{600}{R_f}v_o$$

$$(47/3)v_d + \frac{600}{R_f}v_d - (25/3)v_a - 5v_b - (4/3)v_c = \frac{600}{R_f}v_o$$

$$(376/3) + \frac{4800}{R_f} - 150 - 30 + 20 = \frac{600}{R_f}v_o$$

$$14400 - 104R_f = 1800v_o \quad \text{or} \quad 104R_f = 14400 - 1800v_o$$

$$v_o = \pm 16\ \text{V}, \qquad \text{but} \quad R_f > 0$$

$$\therefore\ 104R_f = 14400 - 1800(-16) \quad \text{or} \quad R_f = 415.38\,\text{k}\Omega$$

[b] $i_f = \dfrac{8 - (-16)}{415.38 \times 10^3} = 57.78\,\mu\text{A}$

$$i_{27\,\text{k}\Omega} = \frac{v_o}{0.027 \times 10^6} = -592.59\,\mu\text{A}$$

$$i_o - i_f + i_{27\,\text{k}\Omega} = 0$$

$$i_o = 57.78 - (-592.59) = 650.37\,\mu\text{A}$$

P 4.11 [a] $v_o = -\dfrac{220}{33}v_a - \dfrac{220}{22}v_b - \dfrac{220}{80}v_c = -8 + 15 - 11 = -4\ \text{V}$

[b] $v_o = -19 - 10v_b = \pm 6$

$$\therefore\ v_b = -1.3\ \text{V} \quad \text{when} \quad v_o = -6\ \text{V};$$

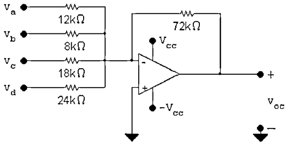
$$v_b = -2.5\ \text{V} \quad \text{when} \quad v_o = 6\ \text{V}$$

$$\therefore\ -2.5\ \text{V} \le v_b \le -1.3\ \text{V}$$

P 4.12 $v_o = -\left[\dfrac{R_f}{3000}(0.15) + \dfrac{R_f}{5000}(0.1) + \dfrac{R_f}{25{,}000}(0.25)\right]$

$$-6 = -8 \times 10^{-5}R_f; \qquad R_f = 75\,\text{k}\Omega; \qquad \therefore\ 0 \le R_f \le 75\,\text{k}\Omega$$

P 4.13



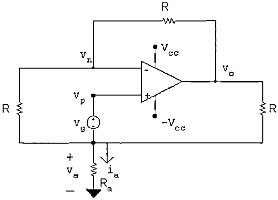
$$v_o = -(6v_a + 9v_b + 4v_c + 3v_d)$$

$$v_o = -\left[\frac{72}{R_a}v_a + \frac{72}{R_b}v_b + \frac{72}{R_c}v_c + \frac{72}{R_d}v_d\right]$$

$\therefore$ $R_a = 72{,}000/6 = 12\,\text{k}\Omega$ $R_c = 72{,}000/4 = 18\,\text{k}\Omega$

$R_b = 72{,}000/9 = 8\,\text{k}\Omega$ $R_d = 72{,}000/3 = 24\,\text{k}\Omega$

P 4.14 [a]



$$\frac{v_n - v_a}{R} + \frac{v_n - v_o}{R} = 0$$

$$2v_n - v_a = v_o$$

$$\frac{v_a}{R_a} + \frac{v_a - v_n}{R} + \frac{v_a - v_o}{R} = 0$$

$$v_a\left[\frac{1}{R_a} + \frac{2}{R}\right] - \frac{v_n}{R} = \frac{v_o}{R}$$

$$v_a\left(2 + \frac{R}{R_a}\right) - v_n = v_o$$

$$v_n = v_p = v_a + v_g$$

$\therefore \quad 2v_n - v_a = 2v_a + 2v_g - v_a = v_a + 2v_g$

$\therefore \quad v_a - v_o = -2v_g \qquad (1)$

$2v_a + v_a\left(\frac{R}{R_a}\right) - v_a - v_g = v_o$

$\therefore \quad v_a\left(1 + \frac{R}{R_a}\right) - v_o = v_g \qquad (2)$

Now combining equations (1) and (2) yields

$-v_a\frac{R}{R_a} = -3v_g$

or $\quad v_a = 3v_g\frac{R_a}{R}$

Hence $\quad i_a = \frac{v_a}{R_a} = \frac{3v_g}{R} \qquad$ Q.E.D.

[b] At saturation $\quad V_o = \pm\, V_{cc}$

$\therefore \quad v_a = \pm\, V_{cc} - 2v_g \qquad (3)$

and

$\therefore \quad v_a\left(1 + \frac{R}{R_a}\right) = \pm\, V_{cc} + v_g \qquad (4)$

Dividing Eq (4) by Eq (3) gives

$1 + \frac{R}{R_a} = \frac{\pm\, V_{cc} + v_g}{\pm\, V_{cc} - 2v_g}$

$\therefore \quad \frac{R}{R_a} = \frac{\pm\, V_{cc} + v_g}{\pm\, V_{cc} - 2v_g} - 1 = \frac{3v_g}{\pm\, V_{cc} - 2v_g}$

or $\quad R_a = \frac{(\pm\, V_{cc} - 2v_g)}{3v_g}R \qquad$ Q.E.D.

P 4.15 [a] Assume the op-amp is operating within its linear range, then

$i_L = \frac{3}{1.5} = 2\,\text{mA}$

For $R_L = 2.5\,\text{k}\Omega \qquad v_o = (2.5 + 1.5)(2) = 8\ \text{V}$

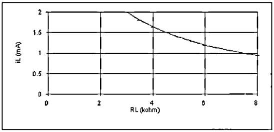
Now since $v_o < 9$ V our assumption of linear operation is correct, therefore

$i_L = 2\,\text{mA}$

[b] $9 = 2(1.5 + R_L); \qquad R_L = 3\,\text{k}\Omega$

[c] As long as the op-amp is operating in its linear region i_L is independent of R_L. From (b) we found the op-amp is operating in its linear region as long as $R_L \leq 3\,\text{k}\Omega$. Therefore when $R_L = 6.5\,\text{k}\Omega$ the op-amp is saturated. We can estimate the value of i_L by assuming $i_p = i_n \ll i_L$. Then $i_L = 9/(1.5 + 6.5) = 1.125\,\text{mA}$. To justify neglecting the current into the op-amp assume the drop across the 47 kΩ resistor is negligible, and the input resistance to the op-amp is at least $500\,\text{k}\Omega$. Then $i_p = i_n = (3 - 1.5)/500 \times 10^{-3} = 3\,\mu\text{A}$. But $3\,\mu\text{A} \ll 1.125\,\text{mA}$, hence our assumption is reasonable.

[d]



P 4.16 [a] The output voltage of the first op-amp is $v_{o1} = -(80/20)v_g = -4v_g$
The output voltage of the second op-amp is $v_{o2} = -1.6v_{o1} = 6.4v_g$
When v_g has its largest value, i.e., 1.2 V,

$$v_{o1} = -4.8 \text{ V} \quad \text{and} \quad v_{o2} = 7.68 \text{ V}$$

Therefore neither op-amp saturates. The expression for i_g is

$$i_g = \frac{v_g}{20{,}000} + \frac{v_g - 6.4v_g}{R_o} = v_g\left[\frac{1}{20{,}000} - \frac{5.4}{R_o}\right]$$

$$i_g = 0 \quad \text{when} \quad \left(\frac{1}{20{,}000} - \frac{5.4}{R_o}\right) = 0, \quad \text{or} \quad R_o = 108\,\text{k}\Omega$$

[b] $$i_{R_o} = \frac{6.4v_g - v_g}{R_o} = \frac{5.4v_g}{R_o} = 50v_g\,\mu\text{A} = 50\,\mu\text{A}$$

$$p_{R_o} = (50 \times 10^{-6})^2(108 \times 10^3) = 270\,\mu\text{W}$$

P 4.17 Let v_{o1} be the output voltage of the first operational amplifier and v_{o2} the output voltage of the second operational amplifier. Then

$$\frac{0-1}{12{,}000} + \frac{0 - v_{o1}}{48{,}000} + \frac{0 - v_{o2}}{100{,}000} = 0$$

$$-50 - 12.5v_{o1} - 6v_{o2} = 0$$

$$\frac{v_{o1}}{30{,}000} + \frac{v_{o1} - v_{o2}}{6000} = 0$$

$$\therefore \quad 6v_{o1} = 5v_{o2}$$

$$\therefore \quad -50 - 12.5[(5/6)v_{o2}] - 6v_{o2} = 0 \quad \text{so} \quad v_{o2} = -3.05 \text{ V}$$

$$i_a = \frac{v_{o2}}{36{,}000} = -0.0846\,\text{mA}$$

$$i_a = -84.6\,\mu\text{A}$$

P 4.18 [a] Let v_{o1} = output voltage of the amplifier on the left. Let v_{o2} = output voltage of the amplifier on the right. Then

$$v_{o1} = \frac{-47}{10}(1) = -4.7 \text{ V}; \qquad v_{o2} = \frac{-220}{33}(-0.15) = 1.0 \text{ V}$$

$$i_a = \frac{v_{o2} - v_{o1}}{1000} = 5.7\,\text{mA}$$

[b] $i_a = 0$ when $v_{o1} = v_{02}$ so from (a) $v_{o2} = 1$ V

Thus

$$\frac{-47}{10}(v_L) = 1$$

$$v_L = -\frac{10}{47} = -212.77 \text{ mV}$$

P 4.19 [a] $p_{16\,\text{k}\Omega} = \dfrac{(320 \times 10^{-3})^2}{(16 \times 10^3)} = 6.4\,\mu\text{W}$

[b] $v_{16\,\text{k}\Omega} = \left(\dfrac{16}{64}\right)(320) = 80\,\text{mV}$

$$p_{16\,\text{k}\Omega} = \frac{(80 \times 10^{-3})^2}{(16 \times 10^3)} = 0.4\,\mu\text{W}$$

[c] $\dfrac{p_a}{p_b} = \dfrac{6.4}{0.4} = 16$

[d] Yes, the operational amplifier serves several useful purposes:

- First, it enables the source to control 16 times as much power delivered to the load resistor. When a small amount of power controls a larger amount of power, we refer to it as *power amplification.*
- Second, it allows the full source voltage to appear across the load resistor, no matter what the source resistance. This is the *voltage follower* function of the operational amplifier.

- Third, it allows the load resistor voltage (and thus its current) to be set without drawing any current from the input voltage source. This is the *current amplification* function of the circuit.

P 4.20 [a] $v_p = v_s, \qquad v_n = \dfrac{R_1 v_o}{R_1 + R_2}, \qquad v_n = v_p$

$$\text{Therefore} \quad v_o = \left(\frac{R_1 + R_2}{R_1}\right) v_s = \left(1 + \frac{R_2}{R_1}\right) v_s$$

[b] $v_o = v_s$

[c] Because $v_o = v_s$, thus the output voltage follows the signal voltage.

P 4.21 [a] $v_p = v_n = \dfrac{45}{75} v_g = 0.6 v_g$

$$\therefore \quad \frac{0.6 v_g}{15} + \frac{0.6 v_g - v_o}{48} = 0$$

$$\therefore \quad v_o = 2.52 v_g = 2.52(3), \qquad v_o = 7.56 \text{ V}$$

[b] $v_o = 2.52 v_g = \pm 10$

$$v_g = \pm 3.97 \text{ V}, \qquad -3.97 \leq v_g \leq 3.97 \text{ V}$$

[c] $\dfrac{0.6 v_g}{15} + \dfrac{0.6 v_g - v_o}{R_f} = 0$

$$\left(\frac{0.6 R_f}{15} + 0.6\right) v_g = v_o = \pm 10$$

$$\therefore \quad 3R_f + 45 = \pm 150; \qquad 3R_f = 150 - 45; \qquad R_f = 35\,\text{k}\Omega$$

P 4.22 [a] $\dfrac{v_p - v_a}{R_a} + \dfrac{v_p - v_b}{R_b} + \dfrac{v_p - v_c}{R_c} + \dfrac{v_p}{R_g} = 0$

$$\therefore \quad v_p = \frac{R_b R_c R_g}{D} v_a + \frac{R_a R_c R_g}{D} v_b + \frac{R_a R_b R_g}{D} v_c$$

where $D = R_b R_c R_g + R_a R_c R_g + R_a R_b R_g + R_a R_b R_c$

$$\frac{v_n}{R_s} + \frac{v_n - v_o}{R_f} = 0$$

$$v_n \left(\frac{1}{R_s} + \frac{1}{R_f}\right) = \frac{v_o}{R_f}$$

$$\therefore \quad v_o = \left(1 + \frac{R_f}{R_s}\right) v_n = k v_n$$

where $k = \left(1 + \dfrac{R_f}{R_s}\right)$

$$v_p = v_n$$

$$\therefore \quad v_o = kv_p$$

or

$$v_o = \frac{kR_gR_bR_c}{D}v_a + \frac{kR_gR_aR_c}{D}v_b + \frac{kR_gR_aR_b}{D}v_c$$

$$\frac{kR_gR_bR_c}{D} = 3 \qquad \therefore \quad \frac{R_b}{R_a} = 1.5$$

$$\frac{kR_gR_aR_c}{D} = 2 \qquad \therefore \quad \frac{R_c}{R_b} = 2$$

$$\frac{kR_gR_aR_b}{D} = 1 \qquad \therefore \quad \frac{R_c}{R_a} = 3$$

Since $\quad R_a = 2\,\text{k}\Omega \qquad R_b = 3\,\text{k}\Omega \qquad R_c = 6\,\text{k}\Omega$

$$\therefore \quad D = [(3)(6)(4) + (2)(6)(4) + (2)(3)(4) + (2)(3)(6)] \times 10^9 = 180 \times 10^9$$

$$\frac{k(4)(3)(6) \times 10^9}{180 \times 10^9} = 3$$

$$k = \frac{540 \times 10^9}{72 \times 10^9} = 7.5$$

$$\therefore \quad 7.5 = 1 + \frac{R_f}{R_s}$$

$$\frac{R_f}{R_s} = 6.5$$

$$R_f = (6.5)(12{,}000) = 78\,\text{k}\Omega$$

[b] $v_o = 3(0.8) + 2(1.5) + 2.10 = 7.5\ \text{V}$

$$v_n = v_p = \frac{7.5}{7.5} = 1.0\ \text{V}$$

$$i_a = \frac{0.8 - 1}{2000} = \frac{-0.2}{2000} = -0.1\,\text{mA} = -100\,\mu\text{A}$$

$$i_b = \frac{1.5 - 1.0}{3000} = \frac{0.5}{3000} = 166.67\,\mu\text{A}$$

$$i_c = \frac{2.10 - 1.0}{6000} = \frac{1.1}{6000} = 183.33\,\mu\text{A}$$

$$i_g = \frac{1}{4000} = 250\,\mu\text{A}$$

$$i_s = \frac{v_n}{12{,}000} = \frac{1}{12{,}000} = 83.33\,\mu\text{A}$$

P 4.23 [a] $\frac{v_p - v_a}{80 \times 10^3} + \frac{v_p - v_b}{64 \times 10^3} = 0$

$\therefore \quad 9v_p = 4v_a + 5v_b$

$$\frac{v_n}{18{,}000} + \frac{v_n - v_o}{72{,}000} = 0$$

$\therefore \quad v_o = 5v_n = 5v_p = (20/9)v_a + (25/9)v_b = 4.44 \text{ V}$

[b] $v_p = v_n = \frac{v_o}{5} = 0.889 \text{ V}$

$$i_a = \frac{v_a - v_p}{80 \times 10^3} = -4.86\,\mu\text{A}$$

$$i_b = \frac{v_b - v_p}{64 \times 10^3} = 4.86\,\mu\text{A}$$

[c] (20/9) for v_a

(25/9) for v_b

P 4.24 [a] $\frac{v_p - v_a}{R_a} + \frac{v_p - v_b}{R_b} + \frac{v_p - v_c}{R_c} = 0$

$$\therefore \quad v_p = \frac{R_bR_c}{D}v_a + \frac{R_aR_c}{D}v_b + \frac{R_aR_b}{D}v_c$$

where $D = R_bR_c + R_aR_c + R_aR_b$

$$\frac{v_n}{10{,}000} + \frac{v_n - v_o}{R_f} = 0$$

$$\left(\frac{R_f}{10{,}000} + 1\right) v_n = v_o$$

Let $\frac{R_f}{10{,}000} + 1 = k$

$v_o = kv_n = kv_p$

$$\therefore \quad v_o = \frac{kR_bR_c}{D}v_a + \frac{kR_aR_c}{D}v_b + \frac{kR_aR_b}{D}v_c$$

$\therefore \quad \frac{kR_bR_c}{D} = 5 \qquad \therefore \quad \frac{R_c}{R_a} = 5$

$\frac{kR_aR_c}{D} = 4$

$\frac{kR_aR_b}{D} = 1 \qquad \therefore \quad \frac{R_c}{R_b} = 4$

$\therefore \quad R_c = 5R_a = 5\,\text{k}\Omega$

$R_b = R_c/4 = 1.25\,\text{k}\Omega$

$\therefore \quad D = (1.25)(5) + (1)(5) + (1.25)(1) = 12.5 \times 10^6$

$$\therefore \quad k = \frac{5D}{R_b R_c} = \frac{(5)(12.5) \times 10^6}{(1.25)(5) \times 10^6} = 10$$

$$\therefore \quad \frac{R_f}{10{,}000} + 1 = 10, \qquad R_f = 90\,\text{k}\Omega$$

[b] $v_o = 5(0.5) + 4(1) + 1.5 = 8\ \text{V}$

$v_n = v_o/10 = 0.8\ \text{V} = v_p$

$$i_a = \frac{v_a - v_p}{1000} = \frac{0.5 - 0.8}{1000} = -300\,\mu\text{A}$$

$$i_b = \frac{v_b - v_p}{1250} = \frac{1 - 0.8}{1250} = 160\,\mu\text{A}$$

$$i_c = \frac{v_c - v_p}{5000} = \frac{1.5 - 0.8}{5000} = 140\,\mu\text{A}$$

P 4.25 [a] $v_o = \dfrac{R_d(R_a + R_b)}{R_a(R_c + R_d)}v_b - \dfrac{R_b}{R_a}v_a = \dfrac{33(100)}{20(80)}(0.90) - 4(0.45)$

$v_o = 1.8563 - 1.8 = 56.25\ \text{mV}$

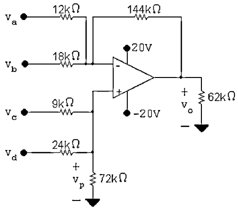
[b] $v_n = v_p = \dfrac{(0.90)(33)}{80} = 371.25\,\text{mV}$

$$i_a = \frac{(450 - 371.25)10^{-3}}{20 \times 10^3} = 3.9375\,\mu\text{A}$$

$$R_a = \frac{v_a}{i_a} = \frac{450 \times 10^{-3}}{3.9375 \times 10^{-6}} = 114.3\,\text{k}\Omega$$

[c] $R_{\text{in b}} = R_c + R_d = 80\,\text{k}\Omega$

P 4.26 [a]



$$\frac{v_p}{72{,}000} + \frac{v_p - v_c}{9{,}000} + \frac{v_p - v_d}{24{,}000} = 0$$

$$\therefore \quad v_p = (2/3)v_c + 0.25v_d = v_n$$

$$\frac{v_n - v_a}{12{,}000} + \frac{v_n - v_b}{18{,}000} + \frac{v_n - v_o}{144{,}000} = 0$$

$$\begin{aligned}\therefore \quad v_o &= 21v_n - 12v_a - 8v_b \\ &= 21[(2/3)v_c + 0.25v_d] - 12v_a - 8v_b \\ &= 21(0.4 + 0.2) - 12(0.5) - 8(0.3) = 4.2 \text{ V}\end{aligned}$$

[b] $v_o = 14v_c + 4.2 - 6 - 2.4$

$$\pm 15 = 14v_c - 4.2$$

$$\therefore \quad 14v_c = \pm 15 + 4.2$$

$$\therefore \quad v_c = 1.371 \text{ V} \qquad \text{and} \qquad v_c = -0.771 \text{ V}$$

$$\therefore \quad -771 \le v_c \le 1371\,\text{mV}$$

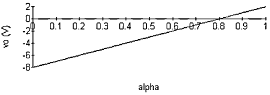
P 4.27 [a] $v_n = v_p = \alpha v_g$

$$\frac{v_n - v_g}{R_1} + \frac{v_n - v_o}{R_f} = 0$$

$$(v_n - v_g)\frac{R_f}{R_1} + v_n - v_o = 0$$

$$\begin{aligned}v_o &= (\alpha v_g - v_g)4 + \alpha v_g \\ &= [(\alpha - 1)4 + \alpha]v_g \\ &= (5\alpha - 4)v_g \\ &= (5\alpha - 4)(2) = 10\alpha - 8\end{aligned}$$

α	v_o	α	v_o	α	v_o
0.0	-8 V	0.4	-4 V	0.8	0 V
0.1	-7 V	0.5	-3 V	0.9	1 V
0.2	-6 V	0.6	-2 V	1.0	2 V
0.3	-5 V	0.7	-1 V		



[b] Rearranging the equation for v_o from (a) gives

$$v_o = \left(\frac{R_f}{R_1} + 1\right) v_g \alpha - \left(\frac{R_f}{R_1}\right) v_g$$

Therefore,

$$\text{slope } = \left(\frac{R_f}{R_1} + 1\right) v_g; \qquad \text{intercept } = -\left(\frac{R_f}{R_1}\right) v_g$$

[c] Using the equations from (b),

$$-6 = \left(\frac{R_f}{R_1} + 1\right) v_g; \qquad 4 = -\left(\frac{R_f}{R_1}\right) v_g$$

Solving,

$$v_g = -2 \text{ V}; \qquad \frac{R_f}{R_1} = 2$$

P 4.28 $v_p = v_n = R_b i_b$

$$\frac{R_b i_b - 3000 i_a}{3000} + \frac{R_b i_b - v_o}{R_f} = 0$$

$$\left(\frac{R_b}{3000} + \frac{R_b}{R_f}\right) i_b - i_a = \frac{v_o}{R_f}$$

$$v_o = \left[\frac{R_b R_f}{3000} + R_b\right] i_b - R_f i_a$$

$$\therefore \quad R_f = 2000\,\Omega$$

$$(2/3)R_b + R_b = 2000$$

$$\therefore \quad R_b = 1200\,\Omega$$

P 4.29 $$v_p = \frac{v_b R_b}{R_a + R_b} = v_n$$

$$\frac{v_n - v_a}{4700} + \frac{v_n - v_o}{R_f} = 0$$

$$v_n\left(\frac{R_f}{4700} + 1\right) - \frac{v_a R_f}{4700} = v_o$$

$$\therefore \quad \left(\frac{R_f}{4700} + 1\right)\frac{R_b}{R_a + R_b}v_b - \frac{R_f}{4700}v_a = v_o$$

$$\therefore \quad \frac{R_f}{4700} = 10; \qquad R_f = 47\,\text{k}\Omega$$

$$\therefore \quad \frac{R_f}{4700} + 1 = 11$$

$$\therefore \quad 11\left(\frac{R_b}{R_a + R_b}\right) = 10$$

$$11R_b = 10R_b + 10R_a \qquad R_b = 10R_a$$

$$R_a + R_b = 220\,\text{k}\Omega$$

$$11R_a = 220\,\text{k}\Omega$$

$$R_a = 20\,\text{k}\Omega$$

$$R_b = 220 - 20 = 200\,\text{k}\Omega$$

P 4.30 $$v_o = \frac{R_d(R_a + R_b)}{R_a(R_c + R_d)}v_b - \frac{R_b}{R_a}v_a$$

By hypothesis: $R_b/R_a = 5$; $R_c + R_d = 600\,\text{k}\Omega$; $\dfrac{R_d(R_a + R_b)}{R_a(R_c + R_d)} = 2$

$$\therefore \quad \frac{R_d}{R_a}\frac{(R_a + 5R_a)}{600{,}000} = 2 \quad \text{so} \quad R_d = 200\,\text{k}\Omega; \quad R_c = 400\,\text{k}\Omega$$

Also, when $v_o = 0$ we have

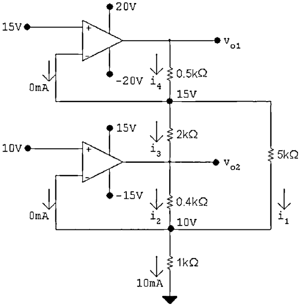
$$\frac{v_n - v_a}{R_a} + \frac{v_n}{R_b} = 0$$

$$\therefore \quad v_n\left(1 + \frac{R_a}{R_b}\right) = v_a; \qquad v_n = (5/6)v_a$$

$$i_a = \frac{v_a - (5/6)v_a}{R_a} = \frac{1}{6}\frac{v_a}{R_a}; \qquad R_{in} = \frac{v_a}{i_a} = 6R_a = 18\,\text{k}\Omega$$

$$\therefore \quad R_a = 3\,\text{k}\Omega; \qquad R_b = 15\,\text{k}\Omega$$

P 4.31



$$i_1 = \frac{15 - 10}{5000} = 1\,\text{mA}$$

$$i_2 + i_1 + 0 = 10\,\text{mA}; \qquad i_2 = 9\,\text{mA}$$

$$v_{o2} = 10 + (400)(9) \times 10^{-3} = 13.6\ \text{V}$$

$$i_3 = \frac{15 - 13.6}{2000} = 0.7\,\text{mA}$$

$$i_4 = i_3 + i_1 = 1.7\,\text{mA}$$

$$v_{o1} = 15 + 1.7(0.5) = 15.85\ \text{V}$$

P 4.32 Let v_{o1} be the output voltage of the first op-amp. Then

$$\frac{0-1.1}{3000}+\frac{0-v_{o1}}{18{,}000}+\frac{0-v_o}{24{,}000}=0$$

$$-26.4-4v_{o1}-3v_o=0$$

$$\text{But}\quad v_{o1}=\frac{v_o}{30}(27)=0.9v_o$$

$$\therefore\quad -3.6v_o-3v_o=26.4 \quad \text{or} \quad v_o=-4\ \text{V}$$

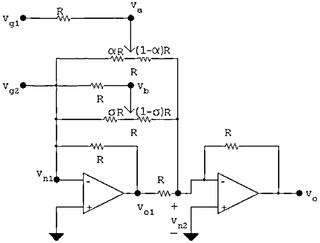
$$i_{24\,\text{k}\Omega}=\frac{0-(-4)}{24}=(1/6)\,\text{mA}$$

$$i_{3\,\text{k}\Omega}=\frac{-4}{30}=(2/15)\,\text{mA}$$

$$i_{4.5\,\text{k}\Omega}=\frac{-4}{4.5}=(8/9)\,\text{mA}$$

$$\frac{1}{6000}=i_o-\frac{2}{15{,}000}-\frac{8}{9000};\qquad i_o=1.1889\,\text{mA}$$

P 4.33 [a] The circuit of Fig. P4.33 is redrawn with intermediate voltages defined to facilitate the analysis.



$$v_{n1}=v_{n2}=0$$

$$\frac{0-v_{o1}}{R}+\frac{0-v_b}{\sigma R}+\frac{0-v_a}{\alpha R}=0$$

therefore $\quad v_{o1} = -\frac{v_a}{\alpha} - \frac{v_b}{\sigma}$

$$\frac{0 - v_b}{(1-\sigma)R} + \frac{0 - v_a}{(1-\alpha)R} + \frac{0 - v_{o1}}{R} + \frac{0 - v_o}{R} = 0$$

therefore $\quad v_o = -v_{o1} - \frac{v_a}{1-\alpha} - \frac{v_b}{1-\sigma}$

$$v_o = \frac{v_a}{\alpha} + \frac{v_b}{\sigma} - \frac{v_a}{1-\alpha} - \frac{v_b}{1-\sigma} = \frac{v_a(1-2\alpha)}{\alpha(1-\alpha)} + v_b\frac{(1-2\sigma)}{\sigma(1-\sigma)}$$

$$\frac{v_a - v_{g1}}{R} + \frac{v_a - 0}{\alpha R} + \frac{v_a - 0}{(1-\alpha)R} = 0$$

$$v_a + \frac{v_a}{\alpha} + \frac{v_a}{1-\alpha} = v_{g1}$$

$$v_a\left(\frac{\alpha(1-\alpha) + (1-\alpha) + \alpha}{\alpha(1-\alpha)}\right) = v_{g1}$$

$$v_a = \frac{v_{g1}\alpha(1-\alpha)}{(\alpha - \alpha^2 + 1)}$$

By symmetry $\quad v_b = \frac{v_{g2}\sigma(1-\sigma)}{\sigma - \sigma^2 + 1)}$

therefore $\quad v_o = \frac{(1-2\alpha)}{(\alpha - \alpha^2 + 1)}v_{g1} + \frac{(1-2\sigma)}{(\sigma - \sigma^2 + 1)}v_{g2}$

[b] $\alpha = \sigma = 1:$

$v_o = -v_{g1} - v_{g2} = -(v_{g1} + v_{g2});$ inverted summing amplifier

[c] $\alpha = \sigma = 0:$

$v_o = v_{g1} + v_{g2};$ noninverted summing amplifier

P 4.34 $\quad v_p = \frac{5.6}{8.0}v_g = 0.7v_g = 7\sin(\pi/3)t$ V

$$\frac{v_n}{15{,}000} + \frac{v_n - v_o}{75{,}000} = 0$$

$6v_n = v_o; \qquad v_n = v_p$

$\therefore\ v_o = 42\sin(\pi/3)t$ V $\qquad 0 \le t \le \infty$

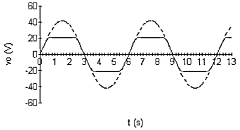
$v_o = 0 \qquad t \le 0$

At saturation

$$42\sin\left(\frac{\pi}{3}\right)t = \pm 21; \qquad \sin\frac{\pi}{3}t = \pm 0.5$$

$$\therefore \quad \frac{\pi}{3}t = \frac{\pi}{6}, \quad \frac{5\pi}{6}, \quad \frac{7\pi}{6}, \quad \frac{11\pi}{6}, \quad \text{etc.}$$

$$t = 0.50\,\text{s}, \quad 2.50\,\text{s}, \quad 3.50\,\text{s}, \quad 5.50\,\text{s}, \quad \text{etc.}$$



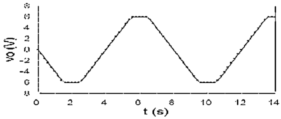
P 4.35 It follows directly from the circuit that $v_o = -16v_g$
From the plot of v_g we have $v_g = 0, \quad t < 0$

$$\begin{aligned}
v_g &= (1/4)t & 0 \le t \le 2 \\
v_g &= -(1/4)t + 1 & 2 \le t \le 6 \\
v_g &= (1/4)t - 2 & 6 \le t \le 10 \\
v_g &= -(1/4)t + 3 & 10 \le t \le 14 \\
v_g &= (1/4)t - 4 & 14 \le t \le 18, \quad \text{etc.}
\end{aligned}$$

Therefore

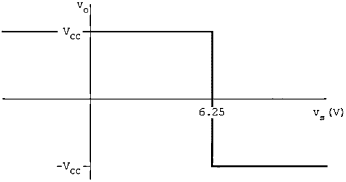
$$\begin{aligned}
v_o &= -4t & 0 \le t \le 2 \\
v_o &= 4t - 16 & 2 \le t \le 6 \\
v_o &= -4t + 32 & 6 \le t \le 10 \\
v_o &= 4t - 48 & 10 \le t \le 14 \\
v_o &= -4t + 64 & 14 \le t \le 18, \quad \text{etc.}
\end{aligned}$$

These expressions for v_o are valid as long as the op amp is not saturated. Since the peak values of v_o are ± 6, the output is clipped at ± 6. The plot is shown below.



P 4.36 [a] v_o will equal V_{CC} when $v_n < v_{\text{ref}}$. Thus

$$v_s\left(\frac{40}{50}\right) < v_{\text{ref}} \qquad \text{or} \qquad v_s < 6.25 \text{ V}$$

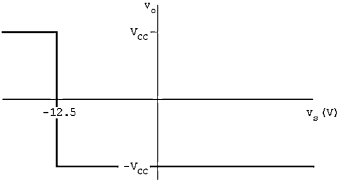


[b] v_o will equal V_{CC} when $v_n < v_{\text{ref}}$. Hence

$v_s < -12.5$ V

v_o will equal $-V_{CC}$ when $v_n > v_{\text{ref}}$. Thus

$v_s < -12.5$ V



[c] Observe that in Problem 4.36 the inputs to the comparator are interchanged with thode in Example 4.2. Hence the v_o versus v_s plots in Problem 4.36 are interchanged with those in Example 4.2. For example, when $v_{\text{ref}} = 5$ V

$$\begin{aligned} v_o = V_{\text{CC}} \quad &\text{when}\ \ v_s > 6.25 \ \text{(Example 4.2)} \\ &\phantom{\text{when}\ \ } v_s < 6.25 \ \text{(Problem 4.36)} \\ v_o = -V_{\text{CC}} \ &\text{when}\ \ v_s < 6.25 \ \text{(Example 4.2)} \\ &\phantom{\text{when}\ \ } v_s > 6.25 \ \text{(Problem 4.36)} \end{aligned}$$

when $v_{\text{ref}} = -10$ V

$$\begin{aligned} v_o = V_{\text{CC}} \quad &\text{when}\ \ v_s > -12.5 \ \text{(Example 4.2)} \\ &\phantom{\text{when}\ \ } v_s < -12.5 \ \text{(Problem 4.36)} \\ v_o = -V_{\text{CC}} \ &\text{when}\ \ v_s < -12.5 \ \text{(Example 4.2)} \\ &\phantom{\text{when}\ \ } v_s > -12.5 \ \text{(Problem 4.36)} \end{aligned}$$

P 4.37 [a] The output of the comparator will be zero when $v_n = 0$. Summing the currents away from the inverting input terminal yields

$$\frac{0 - v_s}{R_1} + \frac{0 - v_{\text{ref}}}{R_2} = 0$$

Solving for v_s gives

$$v_s = -\frac{R_1}{R_2} v_{\text{ref}}$$

[b] The threshold value of v_s is

$$v_s = -\frac{10}{20}(-10) = 5 \text{ V}$$

Assume v_s is slightly less than 5 V, say

$$v_s = (5 - \epsilon) \text{ V}$$

Then

$$\frac{v_n - (5 - \epsilon)}{10} \, \frac{v_n + 10}{20} = 0$$

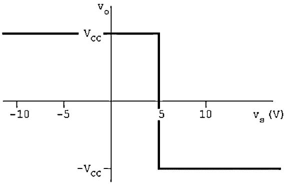
or

$$v_n = -\frac{2}{3}\epsilon$$

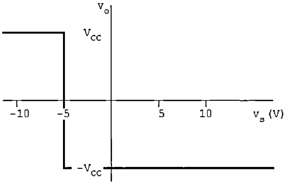
With v_n slightly negative $v_o = V_{\text{CC}}$
If $v_s = (5 + \epsilon)$ then

$$v_n = \frac{2}{3}\epsilon$$

Therefore, when v_s is slightly larger thanthe threshold value v_n goes positive and $v_o = -V_{CC}$. Thus, the v_o versus v_s sketch is



[c] When $v_{ref} = 10$ V, the thrshold value of v_s is -5 V and the sketch of v_o versus v_s is



P 4.38 The voltages at the inverting input terminal of the comparators, starting with the lower comparator, are: 0.875 V, 1.75 V, 2.625 V, 3.5 V, 4.375 V, 5.25 V, and 6.125 V. When $v_s = 1$ V, all comparator output voltages are low except the lowest one. Therefore, the thermometer code is 0 0 0 0 0 0 1.

When $v_s = 3$ V, the comparator output voltages of the three lowest comparators are high, hence the code is 0 0 0 0 1 1 1.

For $v_s = 5$ V the code is 0 0 1 1 1 1 1 and for $v_s = 7$ V the code is 1 1 1 1 1 1 1.

The results are summarized in the following table:

v_s (V)	Thermometer Code						
1	0	0	0	0	0	0	1
3	0	0	0	0	1	1	1
5	0	0	1	1	1	1	1
7	1	1	1	1	1	1	1

P 4.39 Since the current into the terminals of the ideal comparators is zero the current oriented down through the string of resistors is

$$i = \frac{v_{\text{ref}} - (-v_{\text{ref}})}{8R} = \frac{v_{\text{ref}}}{4R}$$

It follows that

$$v_1 = -v_{\text{ref}} + \frac{v_{\text{ref}}}{4R}(R) = -\frac{3}{4}v_{\text{ref}}$$

$$v_2 = -v_{\text{ref}} + \frac{v_{\text{ref}}}{4R}(2R) = -\frac{1}{2}v_{\text{ref}}$$

$$v_3 = -v_{\text{ref}} + \frac{v_{\text{ref}}}{4R}(3R) = -\frac{1}{4}v_{\text{ref}}$$

$$v_4 = -v_{\text{ref}} + \frac{v_{\text{ref}}}{4R}(4R) = 0$$

$$v_5 = -v_{\text{ref}} + \frac{v_{\text{ref}}}{4R}(5R) = \frac{1}{4}v_{\text{ref}}$$

$$v_6 = -v_{\text{ref}} + \frac{v_{\text{ref}}}{4R}(6R) = \frac{1}{2}v_{\text{ref}}$$

$$v_7 = -v_{\text{ref}} + \frac{v_{\text{ref}}}{4R}(7R) = \frac{3}{4}v_{\text{ref}}$$

P 4.40 From the solution to Problem 4.39 we have

$$v_1 = (-3/4)(7) = -5.25 \text{ V}$$

$$v_2 = (-1/2)(7) = -3.5 \text{ V}$$

$$v_3 = (-1/4)(7) = -1.75 \text{ V}$$

$$v_4 = 0$$

$v_5 = (1/4)(7) = 1.75$ V

$v_6 = (1/2)(7) = 3.5$ V

$v_7 = (3/4)(7) = 5.25$ V

When $v_s = -7$ V all the comparator output voltages will be low, thus the thermometer code is 0 0 0 0 0 0 0.

When $v_s = -5$ V, all except the first comparator (counting from the bottom up) output voltage will be low, thus the code is 0 0 0 0 0 0 1.

When $v_s = -3$ V, all except the first two comparator output voltages will be low, hence the code is 0 0 0 0 0 1 1.

When $v_s = -1$ V, the output voltages of the first three comparators will be high, thus the thermometer code is 0 0 0 0 1 1 1.

Then $v_s = 1$ V the output voltages of the first four comparators will be high (0 0 0 1 1 1 1); when $v_s = 3$ V the first five comparators will be high (0 0 1 1 1 1 1); when $v_s = 5$ V the first six comparators will be high (0 1 1 1 1 1 1); and when $v_s = 7$ V the output voltages of all seven comparators will be high (1 1 1 1 1 1 1). Our results are summarized in the following table:

v_s (V)	Thermometer Code						
-7	0	0	0	0	0	0	0
-5	0	0	0	0	0	0	1
-3	0	0	0	0	0	1	1
-1	0	0	0	0	1	1	1
1	0	0	0	1	1	1	1
3	0	0	1	1	1	1	1
5	0	1	1	1	1	1	1
7	1	1	1	1	1	1	1

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5 The Natural and Step Response of RL and RC Circuits

Drill Exercises

DE 5.1 [a] $i_g = 8e^{-300t} - 8e^{-1200t}\text{A}$

$$v = L\frac{di_g}{dt} = -9.6e^{-300t} + 38.4e^{-1200t}\text{V}, \qquad t > 0^+$$

$$v(0^+) = -9.6 + 38.4 = 28.8\,\text{V}$$

[b] $v = 0$ when $38.4e^{-1200t} = 9.6e^{-300t}$ or $t = (\ln 4)/900 = 1.54\,\text{ms}$

[c] $p = vi = 384e^{-1500t} - 76.8e^{-600t} - 307.2e^{-2400t}\,\text{W}$

[d] $\dfrac{dp}{dt} = 0$ when $e^{1800t} - 12.5e^{900t} + 16 = 0$

Let $x = e^{900t}$ and solve the quadratic $x^2 - 12.5x + 16 = 0$

$$x = 1.45, \quad x = 11.05, \quad t = \frac{\ln 1.45}{900} = 411.05\,\mu\text{s}, \quad t = \frac{\ln 11.05}{900} = 2.67\,\text{ms}$$

p is maximum at $t = 411.05\,\mu\text{s}$

[e] $p_{\text{max}} = 384e^{-1.5(0.41105)} - 76.8e^{-0.6(0.41105)} - 307.2e^{-2.4(0.41105)} = 32.72\,\text{W}$

[f] $i_{\text{max}} = 8[e^{-0.3(1.54)} - e^{-1.2(1.54)}] = 3.78\,\text{A}$

$$w_{\text{max}} = (1/2)(4 \times 10^{-3})(3.78)^2 = 28.6\,\text{mJ}$$

[g] W is max when i is max, i is max when di/dt is zero.

When $di/dt = 0$, $v = 0$, therefore $t = 1.54\,\text{ms}$.

DE 5.2 [a] $i = C\dfrac{dv}{dt} = 24 \times 10^{-6}\dfrac{d}{dt}[e^{-15{,}000t}\sin 30{,}000t]$

$$= [0.72\cos 30{,}000t - 0.36\sin 30{,}000t]e^{-15{,}000t}\,\text{A}, \qquad i(0^+) = 0.72\,\text{A}$$

[b] $i\left(\frac{\pi}{80}\,\text{ms}\right) = -31.66\,\text{mA}, \quad v\left(\frac{\pi}{80}\,\text{ms}\right) = 20.505\,\text{V},$

$p = vi = -649.23\,\text{mW}$

[c] $w = \left(\frac{1}{2}\right) Cv^2 = 126.13\,\mu\text{J}$

DE 5.3 [a] $v = \left(\frac{1}{C}\right)\int_{0^-}^{t} i\,dx + v(0^-)$

$$= \frac{1}{0.6\times 10^{-6}}\int_{0^-}^{t} 3\cos 50{,}000x\,dx = 100\sin 50{,}000t\,\text{V}$$

[b] $p(t) = vi = [300\cos 50{,}000t]\sin 50{,}000t$

$$= 150\sin 100{,}000t\,\text{W}, \qquad p_{(\max)} = 150\,\text{W}$$

[c] $w_{(\max)} = \left(\frac{1}{2}\right) Cv_{\max}^2 = 0.30(100)^2 = 3000\,\mu\text{J} = 3\,\text{mJ}$

DE 5.4 [a] $L_{\text{eq}} = \frac{60(240)}{300} = 48\,\text{mH}$

[b] $i(0^+) = 3 + -5 = -2\,\text{A}$

[c] $i = \frac{125}{6}\int_{0^+}^{t}(-0.03e^{-5x})\,dx - 2 = 0.125e^{-5t} - 2.125\,\text{A}$

[d] $i_1 = \frac{50}{3}\int_{0^+}^{t}(-0.03e^{-5x})\,dx + 3 = 0.1e^{-5t} + 2.9\,\text{A}$

$$i_2 = \frac{25}{6}\int_{0^+}^{t}(-0.03e^{-5x})\,dx - 5 = 0.025e^{-5t} - 5.025\,\text{A}$$

$i_1 + i_2 = i$

DE 5.5 $v_1 = 0.5\times 10^6\int_{0^+}^{t} 240\times 10^{-6}e^{-10x}\,dx - 10 = -12e^{-10t} + 2\,\text{V}$

$$v_2 = 0.125\times 10^6\int_{0^+}^{t} 240\times 10^{-6}e^{-10x}\,dx - 5 = -3e^{-10t} - 2\,\text{V}$$

$v_1(\infty) = 2\,\text{V}, \qquad v_2(\infty) = -2\,\text{V}$

$$W = \left[\frac{1}{2}(2)(4) + \frac{1}{2}(8)(4)\right]\times 10^{-6} = 20\,\mu\text{J}$$

DE 5.6 [a] $i = \left(\frac{120}{3+5}\right)\left(\frac{-30}{36}\right) = -12.5\,\text{A}$

[b] $w = 0.5(8\times 10^{-3})(12.5)^2 = 625\,\text{mJ}$

[c] $\tau = \frac{L}{R} = \frac{8\times 10^{-3}}{2} = 4\,\text{ms}$

[d] $i = -12.5e^{-250t}\,\text{A}, \qquad t \geq 0$

[e] $i(5\,\text{ms}) = -3.58\,\text{A}, \qquad w\,(5\,\text{ms}) = (0.5)(8) \times 10^{-3}(3.58)^2 = 51.3\,\text{mJ}$

$$w\,(\text{dis}) = 625 - 51.3 = 573.7\,\text{mJ}$$

$$\%\ \text{dissipated} = \left(\frac{573.7}{625}\right) 100 = 91.8\%$$

DE 5.7 [a] $i_L(0^-) = 6.4\left(\frac{10}{16}\right) = 4\,\text{A} = i_L(0^+), \qquad t > 0$

$$R_{\text{eq}} = \frac{(4)(16)}{20} = 3.2\,\Omega, \qquad \tau = \frac{0.32}{3.2} = 0.1\,\text{s}$$

Therefore $\frac{1}{\tau} = 10, \qquad i_L = 4e^{-10t}\,\text{A}$

Let i_1 equal the current in the $10\,\Omega$ resistor. Let the reference direction for i_1 be up. Then

$$i_1 = \left(\frac{4}{20}\right) i_L = 0.8e^{-10t}\,\text{A}, \qquad v_o = -10i_1 = -8e^{-10t}\,\text{V}, \qquad t \geq 0^+$$

[b] $v_{4\Omega} = L\frac{di_L}{dt} = 0.32(-40)e^{-10t} = -12.8e^{-10t}\,\text{V}, \qquad t \geq 0^+$

$$p_{4\Omega} = \frac{v_{4\Omega}^2}{4} = 40.96e^{-20t}\,\text{W}, \qquad t \geq 0^+$$

$$w_{4\Omega} = \int_0^\infty 40.96e^{-20t}dt = 2.048\,\text{J}$$

$$w_i = \frac{1}{2}Li^2 = \frac{1}{2}(0.32)(16) = 2.56\,\text{J}$$

$$\%\ \text{dissipated} = \left(\frac{2.048}{2.56}\right) 100 = 80\%$$

DE 5.8 [a] $v(0) = \left[\frac{7.5(80)}{150}\right] 50 = 200\,\text{V}$

[b] $\tau = RC = (50 \times 10^3)(0.4 \times 10^{-6}) = 20\,\text{ms}$

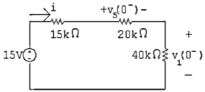
[c] $v = 200e^{-50t}\,\text{V}$

[d] $w(0) = 0.5(0.4 \times 10^{-6})(200)^2 = 8\,\text{mJ}$

[e] $w(t) = 0.5(0.4 \times 10^{-6})(4 \times 10^4)e^{-100t} = 8e^{-100t}\,\text{mJ}$

$$8e^{-100t} = 2, \qquad t = (\ln 4)/100 = 13.86\,\text{ms}$$

DE 5.9 [a] For $t < 0$:



$$i = \frac{15}{75{,}000} = \frac{1}{5}\,\text{mA}, \qquad v_5(0^-) = 4\,\text{V}, \qquad v_1(0^-) = 8\,\text{V}$$

$$\tau_5 = (20 \times 10^3)(5 \times 10^{-6}) = 100\,\text{ms}, \qquad 1/\tau_5 = 10$$

$$\tau_1 = (40 \times 10^3)(1 \times 10^{-6}) = 40\,\text{ms}, \qquad 1/\tau_1 = 25$$

Therefore $v_5 = 4e^{-10t}\,\text{V}, \quad t \geq 0; \qquad v_1 = 8e^{-25t}\,\text{V}, \quad t \geq 0;$

$$v_o = v_1 + v_5 = [8e^{-25t} + 4e^{-10t}]\,\text{V}, \qquad t \geq 0$$

[b] $v_1(60\,\text{ms}) \cong 1.79\,\text{V}, \qquad v_5(60\,\text{ms}) \cong 2.20\,\text{V}$

$$w_1(60\,\text{ms}) = (1/2)(1)(1.79)^2 \cong 1.59\,\mu\text{J}$$

$$w_5(60\,\text{ms}) = (1/2)(5)(2.20)^2 \cong 12.05\,\mu\text{J}$$

$$w_1(0) = \frac{1}{2}(10^{-6})(64) + \frac{1}{2}(5 \times 10^{-6})(16) = 72\,\mu\text{J}$$

$$w_{\text{diss}} = 72 - 13.64 = 58.36\,\mu\text{J}$$

$$\%\ \text{dissipated} = (58.36/72)(100) = 81.05\,\%$$

DE 5.10 [a] $i(0^+) = 24/2 = 12\,\text{A}$

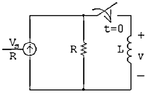
[b] $v(0^+) = -10(8 + 12) = -200\,\text{V}$

[c] $\tau = L/R = (200/10) \times 10^{-3} = 20\,\text{ms}$

[d] $i = -8 + [12 - (-8)]e^{-50t} = [-8 + 20e^{-50t}]\,\text{A}, \qquad t \geq 0^+$

[e] $v = 0 + [-200 - 0]e^{-50t}\,\text{V} = -200e^{-50t}\,\text{V}, \qquad t \geq 0^+$

DE 5.11 [a]



$$\frac{v}{R} + \frac{1}{L}\int_0^t v\,dx = \frac{V_s}{R}$$

$$\frac{1}{R}\frac{dv}{dt} + \frac{v}{L} = 0$$

$$\frac{dv}{dt} + \frac{R}{L}v = 0$$

[b] $$\frac{dv}{dt} = -\frac{R}{L}v$$

$$\frac{dv}{dt}\,dt = -\frac{R}{L}v\,dt$$

$$\therefore \quad \frac{dv}{v} = -\frac{R}{L}\,dt$$

$$\int_{v(0^+)}^{v(t)} \frac{dy}{y} = -\frac{R}{L}\int_{0^+}^{t} dx$$

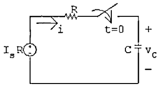
$$\ln y \Big|_{v(0^+)}^{v(t)} = -\left(\frac{R}{L}\right)t$$

$$\ln\left[\frac{v(t)}{v(0^+)}\right] = -\left(\frac{R}{L}\right)t$$

$$v(t) = v(0^+)e^{-(R/L)t}; \qquad v(0^+) = \left(\frac{V_s}{R} - I_o\right)R = V_s - I_oR$$

$$\therefore \quad v(t) = (V_s - I_oR)e^{-(R/L)t}$$

DE 5.12 [a]



$$I_sR = Ri + \frac{1}{C}\int_{0^+}^{t} i\,dx + V_o$$

$$0 = R\frac{di}{dt} + \frac{i}{C} + 0$$

$$\therefore \quad \frac{di}{dt} + \frac{i}{RC} = 0$$

[b] $$\frac{di}{dt} = -\frac{i}{RC}; \qquad \frac{di}{i} = -\frac{dt}{RC}$$

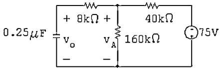
$$\int_{i(0^+)}^{i(t)} \frac{dy}{y} = -\frac{1}{RC}\int_{0^+}^{t} dx$$

$$\ln\frac{i(t)}{i(0^+)} = \frac{-t}{RC}$$

$$i(t) = i(0^+)e^{-t/RC}; \qquad i(0^+) = \frac{I_sR - V_o}{R} = \left(I_s - \frac{V_o}{R}\right)$$

$$\therefore \quad i(t) = \left(I_s - \frac{V_o}{R}\right)e^{-t/RC}$$

DE 5.13 [a]



$$v_o = -60 + 90e^{-100t}\,\text{V}$$

$$\frac{v_A - v_o}{8000} + \frac{v_A}{160{,}000} + \frac{v_A + 75}{40{,}000} = 0$$

$$20v_A - 20v_o + v_A + 4v_A + 300 = 0$$

$$25v_A = 20v_o - 300$$

$$v_A = 0.8v_o - 12$$

$$v_A = -48 + 72e^{-100t} - 12 = -60 + 72e^{-100t}\,\text{V}, \qquad t \geq 0^+$$

[b] $t \geq 0^+$

DE 5.14 [a] $v_c(0^+) = 50\,\text{V}$

[b] $v_c(\infty) = \left(-\frac{30}{25}\right)20 = -24\,\text{V}$

[c] Find the Thévenin equivalent with respect to the terminals of the capacitor:

$v_{\text{Th}} = -24\,\text{V}, \qquad R_{\text{Th}} = 20\|5 = 4\,\Omega,$

Therefore $\tau = 4(25 \times 10^{-9}) = 0.1\,\mu\text{s}$

[d] $i(0^+) = -\frac{50 + 24}{4} = -18.5\,\text{A}$

[e] $v_c = -24 + [50 - (-24)]e^{-t/\tau} = -24 + 74e^{-10^7t}\,\text{V}, \qquad t \geq 0$

[f] $i = -18.5e^{-t/\tau} = -18.5e^{-10^7t}\,\text{A}, \qquad t \geq 0^+$

DE 5.15 [a] $v_c(0^+) = (9/12)(120) = 90\,\text{V}$

[b] $v_c(\infty) = -1.5(40) = -60\,\text{V}$

[c] Find the Thévenin equivalent with respect to the terminals of the capacitor:

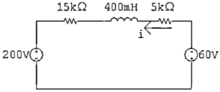
$$v_{\mathrm{Th}} = -60\,\mathrm{V}, \qquad R_{\mathrm{Th}} = 50\,\mathrm{k}\Omega$$

$$\tau = R_{\mathrm{Th}}C = 1\,\mathrm{ms} = 1000\,\mu\mathrm{s}$$

[d] $v_c = -60 + (90 + 60)e^{-1000t} = -60 + 150e^{-1000t}\,\mathrm{V}, \quad t \geq 0$

$$\text{Therefore} \quad t = \frac{\ln(150/60)}{1000} = 916.3\,\mu\mathrm{s}$$

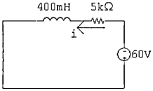
DE 5.16 [a] For $t < 0$, calculate the Thévenin equivalent for the circuit to the left and right of the 400-mH inductor. We get



$$i(0^-) = -260/20 = -13\,\mathrm{mA}$$

$$i(0^-) = i(0^+) = -13\,\mathrm{mA}$$

[b] For $t > 0$, the circuit reduces to



$$\text{Therefore} \quad i(\infty) = -60/5 = -12\,\mathrm{mA}$$

[c] $\tau = (400/5) \times 10^{-6} = 80\,\mu\mathrm{s}$

[d] $i(t) = -12 + [-13 + 12]e^{-12{,}500t} = -12 - e^{-12{,}500t}\,\mathrm{mA}, \qquad t \geq 0$

Problems

P 5.1 $p = vi = 40t[e^{-10t} - 10te^{-20t} - e^{-20t}]$

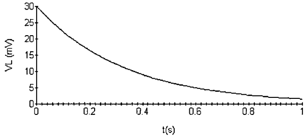
$$W = \int_0^\infty p\,dx = \int_0^\infty 40x[e^{-10x} - 10xe^{-20x} - e^{-20x}]\,dx = 0.2\,\mathrm{J}$$

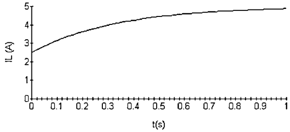
This is energy stored in the inductor at $t = \infty$.

P 5.2 $0 \leq t < \infty$

$$i_L = \frac{10^3}{4}\int_0^t 30 \times 10^{-3}e^{-3x}\,dx + 2.5 = 7.5\frac{e^{-3x}}{-3}\Big|_0^t + 2.5$$

$$= 5 - 2.5e^{-3t}\,\mathrm{A}, \qquad 0 \leq t \leq \infty$$





P 5.3 [a] $v = L\frac{di}{dt}$

$$\frac{di}{dt} = 50[t(-10e^{-10t}) + e^{-10t}] = 50e^{-10t}(1 - 10t)$$

$$v = (2 \times 10^{-3})(50)e^{-10t}(1 - 10t)$$

$$= 100e^{-10t}(1 - 10t)\,\text{mV}, \quad t > 0$$

[b] $p = vi$

$$v(200\,\text{ms}) = 100e^{-2}(1 - 2) = -13.53\,\text{mV}$$

$$i(200\,\text{ms}) = 50(0.2)e^{-2} = 1.35\,\text{A}$$

$$p(200\,\text{ms}) = -13.53 \times 10^{-3}(1.35) = -18.32\,\text{mW}$$

[c] delivering

[d] $w = \frac{1}{2}Li^2 = \frac{1}{2}(2 \times 10^{-3})(1.35)^2 = 1.83\,\text{mJ}$

[e] $\frac{di}{dt} = 0 \quad \text{when} \quad t = \frac{1}{10}\,\text{s} = 100\,\text{ms}$

$$i_{\text{max}} = 50(0.1)e^{-1} = 1.84\,\text{A}$$

$$w_{\text{max}} = \frac{1}{2}(2 \times 10^{-3})(1.84)^2 = 3.38\,\text{mJ}$$

P 5.4 [a] $0 \leq t \leq 1\,\text{ms}:$

$$i = \frac{1}{L}\int_0^t v_s\,dx + i(0) = \frac{10^6}{300}\int_0^t 6 \times 10^{-3}\,dx + 0$$

$$= 20x\Big|_0^t = 20t\,\text{A}$$

$1\,\text{ms} \leq t \leq 2\,\text{ms}:$

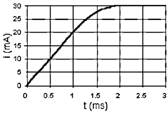
$$i = \frac{10^6}{300}\int_{10^{-3}}^t (12 \times 10^{-3} - 6x)\,dx + 20 \times 10^{-3}$$

$$\therefore \quad i = 40t - 10{,}000t^2 - 10 \times 10^{-3}\,\text{A}$$

$2\,\text{ms} \leq t \leq \infty:$

$$i = \frac{10^6}{300}\int_{2\times10^{-3}}^t (0)\,dx + 30 \times 10^{-3} = 30\,\text{mA}$$

[b]



P 5.5 [a]

$$\begin{aligned}
i &= 0 && t < 0 \\
i &= 16t\,\text{A} && 0 \le t \le 25\,\text{ms} \\
i &= 0.8 - 16t\,\text{A} && 25 \le t \le 50\,\text{ms} \\
i &= 0 && 50\,\text{ms} < t
\end{aligned}$$

[b] $v = L\dfrac{di}{dt} = 375 \times 10^{-3}(16) = 6\,\text{V} \qquad 0 \le t \le 25\,\text{ms}$

$v = 375 \times 10^{-3}(-16) = -6\,\text{V} \qquad 25 \le t \le 50\,\text{ms}$

$$\begin{aligned}
v &= 0 && t < 0 \\
v &= 6\,\text{V} && 0 < t < 25\,\text{ms} \\
v &= -6\,\text{V} && 25 < t < 50\,\text{ms} \\
v &= 0 && 50\,\text{ms} < t
\end{aligned}$$

$p = vi$

$$\begin{aligned}
p &= 0 && t < 0 \\
p &= (16t)(6) = 96t\,\text{W} && 0 < t < 25\,\text{ms} \\
p &= (0.8 - 16t)(-6) = 96t - 4.8\,\text{W} && 25 < t < 50\,\text{ms} \\
p &= 0 && 50\,\text{ms} < t
\end{aligned}$$

$$\begin{aligned}
w &= 0 && t < 0 \\
w &= \int_0^t (16x)6\,dx = 96\frac{x^2}{2}\Big|_0^t = 48t^2\,\text{J} && 0 < t < 25\,\text{ms} \\
w &= \int_{0.025}^t (96x - 4.8)\,dx + 0.03 \\
&= \int_{0.025}^t 96x\,dx - \int_{0.025}^t 4.8\,dx + 0.03 \\
&= 96\frac{x^2}{2}\Big|_{0.025}^t - 4.8x\Big|_{0.025}^t + 0.03 \\
&= 48t^2 - 4.8t + 0.12\,\text{J} && 25 < t < 50\,\text{ms} \\
w &= 0 && 50\,\text{ms} < t
\end{aligned}$$

P 5.6 [a] $0 \le t \le 1\,\text{s}:$

$v = -100t$

$i = \dfrac{1}{5}\displaystyle\int_0^t -100x\,dx + 0 = -20\frac{x^2}{2}\Big|_0^t$

$i = -10t^2\,\text{A}$

$1\,\text{s} \leq t \leq 3\,\text{s}:$

$v = -200 + 100t$

$i(1) = -10\,\text{A}$

$$\begin{aligned}\therefore\ i &= \frac{1}{5}\int_1^t (100x - 200)\,dx - 10\\ &= 20\int_1^t x\,dx - 40\int_1^t dx - 10\\ &= 10(t^2 - 1) - 40(t - 1) - 10\\ &= 10t^2 - 40t + 20\,\text{A}\end{aligned}$$

$3\,\text{s} \leq t \leq 5\,\text{s}:$

$v = 100$

$i(3) = 10(9) - 120 + 20 = -10\,\text{A}$

$$\begin{aligned}i &= \frac{1}{5}\int_3^t 100\,dx - 10\\ &= 20t - 60 - 10 = 20t - 70\,\text{A}\end{aligned}$$

$5\,\text{s} \leq t \leq 6\,\text{s}:$

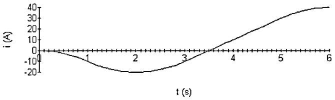
$v = -100t + 600$

$i(5) = 100 - 70 = 30$

$$\begin{aligned}i &= \frac{1}{5}\int_5^t (-100x + 600)\,dx + 30\\ &= -20\int_5^t x\,dx + 120\int_5^t dx + 30\\ &= -10(t^2 - 25) + 120(t - 5) + 30\\ &= -10t^2 + 120t - 320\,\text{A}\end{aligned}$$

[b] $i(6) = -10(36) + 120(6) - 320 = 720 - 680 = 40\,\text{A}, \qquad 6 \leq t \leq \infty$

[c]



P 5.7 [a] $i(0) = A_1 + A_2 = 0.05$

$$\frac{di}{dt} = -2500A_1e^{-2500t} - 7500A_2e^{-7500t}$$

$$v = -50A_1e^{-2500t} - 150A_2e^{-7500t}\,\text{V}$$

$$v(0) = -50A_1 - 150A_2 = 10$$

$$\therefore \quad -5A_1 - 15A_2 = 1$$

But from the equation for $i(0)$, $5A_1 + 5A_2 = 0.25$

Solving, $A_1 = 0.175$ and $A_2 = -0.125$

Thus,

$$i = 0.175e^{-2500t} - 0.125e^{-7500t}\,\text{A}, \qquad t \geq 0$$

$$v = -8.75e^{-2500t} + 18.75e^{-7500t}\,\text{V}, \qquad t \geq 0$$

[b] $p = vi = 4.375e^{-10{,}000t} - 1.53125e^{-5000t} - 2.34375e^{-15{,}000t}\text{W}$

$p = 0$ when $4.375e^{-10{,}000t} - 1.53125e^{-5000t} - 2.34375e^{-15{,}000t} = 0$

Let $x = e^{5000t}$, then

$$4.375x - 1.53125x^2 - 2.34375 = 0$$

Solving,

$$x = 0.7143, \quad x = 2.143$$

If $x < 1$, t must be negative hence the solution for $t > 0$ must be $x = 2.143$

$e^{5000t} = 2.143$ so $t = 152.43\,\mu\text{s}$

P 5.8 [a] From Prob. 5.7 we have

$$i = A_1e^{-2500t} + A_2e^{-7500t}\,\text{A}$$

$$v = -50A_1e^{-2500t} - 150A_2e^{-7500t}\,\text{V}$$

$$i(0) = A_1 + A_2 = 0.05$$

$$v(0) = -50A_1 - 150A_2 = -100$$

$$\therefore\;\; A_1 + A_2 = 0.05 \quad \text{and} \quad A_1 + 3A_2 = 2$$

$$\therefore\;\; A_2 = 0.975\text{A}, \qquad A_1 = -0.925\text{A}$$

Thus,

$$i = -0.925e^{-2500t} + 0.975e^{-7500t}\,\text{A} \qquad t \geq 0$$

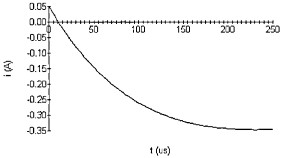
$$v = 46.25e^{-2500t} - 146.25e^{-7500t}\,\text{V} \qquad t \geq 0$$

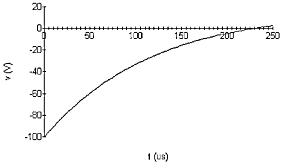
[b] $i = 0$ when $0.975e^{-7500t} = 0.925e^{-2500t}$

$$\therefore\;\; e^{5000t} = 1.0541$$

$$t = (\ln 1.054)/5000 = 10.53\,\mu\text{s}$$

$v = 0$ when $46.25e^{-2500t} = 146.25e^{-7500t}$





$\therefore$ Energy is being stored between $10.53\,\mu$s and $230.25\,\mu$s; energy is being extracted between 0 and $10.53\,\mu$s and between $230.25\,\mu$s and infinity.

[c] $p = vi = 180.375e^{-10{,}000t} - 42.78125e^{-5000t} - 142.59375e^{-15{,}000t}$ W

$$\therefore \quad \mathrm{W}_{\text{stored}} = \int_{t_1}^{t_2} p\,dt + w(0)$$

$$\begin{aligned}
\mathrm{W}_{\text{stored}} &= 10^{-3}\left\{-18.0375e^{-10{,}000t}\Big|_{t_1}^{t_2} + 8.55625e^{-5000t}\Big|_{t_1}^{t_2} + \right. \\
&\quad \left. 9.50625e^{-15{,}000t}\Big|_{t_1}^{t_2}\right\} + 25\times10^{-6} \\
&= 8.55625e^{-5000t_2} + 9.50625e^{-15{,}000t_2} - 18.0375e^{-10{,}000t_2} \\
&\quad - 8.55625e^{-5000t_1} - 9.50625e^{-15{,}000t_1} + 18.0375e^{-10{,}000t_1} \\
&\quad + 0.025\,\mathrm{mW}
\end{aligned}$$

where $t_1 = 10.52\,\mu$s, $t_2 = 230.11\,\mu$s

$\mathrm{W}_{\text{stored}} = 1.23$ mJ.

$$\begin{aligned}
\mathrm{W}_{\text{extracted}} &= \int_0^{t_1} p\,dt + \int_{t_2}^{\infty} p\,dt \\
&= \int_0^{t_1}(180.375e^{-10^4t} - 42.78125e^{-5000t} \\
&\qquad - 142.59375e^{-15{,}000t})\,dt \\
&\quad + \int_{t_2}^{\infty}(180.375e^{-10^4t} - 42.78125e^{-5000t} \\
&\quad - 142.59375e^{-15{,}000t})\,dt \\
&= 10^{-3}\left\{-18.0375e^{-10{,}000t}\Big|_0^{t_1} + 8.55625e^{-5000t}\Big|_0^{t_1}\right. \\
&\quad + 9.50625e^{-15{,}000t}\Big|_0^{t_1} - 18.0375e^{-10{,}000t}\Big|_{t_2}^{\infty} \\
&\quad \left. + 8.55625e^{-5000t}\Big|_{t_2}^{\infty} + 9.50625e^{-15{,}000t}\Big|_{t_2}^{\infty}\right\} \\
&= \{18.0375e^{-10{,}000t_2} - 8.55625e^{-5000t_2} - 9.50625e^{-15{,}000t_2} \\
&\quad + 8.55625e^{-5000t_1} + 9.50625e^{-15{,}000t_1} - 18.0375e^{-10{,}000t_1} \\
&\quad - 0.025\}\,\mathrm{mJ}
\end{aligned}$$

$\mathrm{W}_{\text{ext.}} = -1.23$ mJ $\qquad \therefore\ \mathrm{W}_{\text{stored}} = \mathrm{W}_{\text{extracted}}$

P 5.9 [a] $v_L = L\frac{di}{dt} = [125\sin 400t]e^{-200t}\,\text{V}$

$\therefore\quad \frac{dv_L}{dt} = 25{,}000(2\cos 400t - \sin 400t)e^{-200t}\,\text{V/s}$

$\frac{dv_L}{dt} = 0 \quad \text{when} \quad \tan 400t = 2$

$\therefore\quad t = 2.77\,\text{ms}$

Also $\quad 400t = 1.107 + \pi \qquad$ etc.

Because of the decaying exponential v_L will be maximum the first time the derivative is zero.

[b] $v_L(\text{max}) = [125\sin 1.107]e^{-0.554} = 64.27\,\text{V}$

$v_L\,\text{max} = 64.27\,\text{V}$

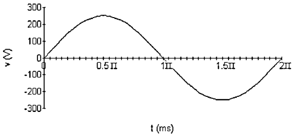
Note: When $\quad t = (1.107 + \pi)/400; \qquad v_L = -13.36\,\text{V}$

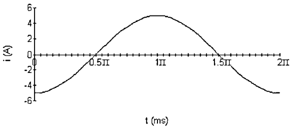
P 5.10 [a]

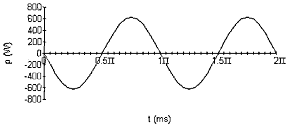
$$\begin{aligned} i &= \frac{1000}{50}\int_0^t 250\sin 1000x\,dx - 5 \\ &= 5000\int_0^t \sin 1000x\,dx - 5 \\ &= 5000\left[\frac{-\cos 1000x}{1000}\right|_0^t - 5 \\ &= 5(1 - \cos 1000t) - 5 \\ i &= -5\cos 1000t\,\text{A} \end{aligned}$$

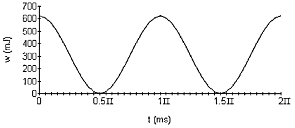
[b]

$$\begin{aligned} p &= vi = (250\sin 1000t)(-5\cos 1000t) \\ &= -1250\sin 1000t\cos 1000t \\ p &= -625\sin 2000t\,\text{W} \\ w &= \frac{1}{2}Li^2 \\ &= \frac{1}{2}(50\times 10^{-3})25\cos^2 1000t \\ &= 625\cos^2 1000t\,\text{mJ} \\ w &= [312.5 + 312.5\cos 2000t]\,\text{mJ}. \end{aligned}$$


300
200
100
0
-100
-200
-300
v (V)
0
0.5Π
1Π
1.5Π
2Π
t (ms)


6
4
2
0
-2
-4
-6
i (A)
0
0.5Π
1Π
1.5Π
2Π
t (ms)


800
600
400
200
0
-200
-400
-600
-800
p (W)
0
0.5Π
1Π
1.5Π
2Π
t (ms)


700
600
500
400
300
200
100
0
w (mJ)
0
0.5Π
1Π
1.5Π
2Π
t (ms)

[c] Absorbing power: Delivering power:

Absorbing power:	Delivering power:
$0.5\pi \leq t \leq \pi$ ms	$0 \leq t \leq 0.5\pi$ ms
$1.5\pi \leq t \leq 2\pi$ ms	$\pi \leq t \leq 1.5\pi$ ms

P 5.11 $i = (B_1 \cos 5t + B_2 \sin 5t)e^{-t}$

$i(0) = B_1 = 25\,\text{A}$

$$\frac{di}{dt} = (B_1 \cos 5t + B_2 \sin 5t)(-e^{-t}) + e^{-t}(-5B_1 \sin 5t + 5B_2 \cos 5t)$$

$$= [(5B_2 - B_1)\cos 5t - (5B_1 + B_2)\sin 5t]e^{-t}$$

$$v = 2\frac{di}{dt} = [(10B_2 - 2B_1)\cos 5t - (10B_1 + 2B_2)\sin 5t]e^{-t}$$

$v(0) = 100 = 10B_2 - 2B_1 = 10B_2 - 50 \quad \therefore \quad B_2 = 150/10 = 15\,\text{A}$

Thus,

$i = (25\cos 5t + 15\sin 5t)e^{-t}\,\text{A}, \qquad t \geq 0$

$v = (100\cos 5t - 280\sin 5t)e^{-t}\,\text{V}, \qquad t \geq 0$

$i(0.5) = -6.70\,\text{A}; \qquad v(0.5) = -150.23\,\text{V}$

$p(0.5) = (-6.70)(-150.23) = 1007.00\,\text{W}$ absorbing

P 5.12 [a]

$$\begin{aligned} v(20\,\mu\text{s}) &= 12.5 \times 10^9(20 \times 10^{-6})^2 = 5\,\text{V} \quad \text{(end of first interval)} \\ v(20\,\mu\text{s}) &= 10^6(20 \times 10^{-6}) - (12.5)(400) \times 10^{-3} - 10 \\ &= 5\,\text{V (start of second interval)} \\ v(40\,\mu\text{s}) &= 10^6(40 \times 10^{-6}) - (12.5)(1600) \times 10^{-3} - 10 \\ &= 10\,\text{V (end of second interval)} \end{aligned}$$

[b] $p(10\mu\text{s}) = 62.5 \times 10^{12}(10^{-5})^3 = 62.5\,\text{mW}, \qquad v(10\,\mu\text{s}) = 1.25\,\text{V},$

$i(10\mu\text{s}) = 50\,\text{mA}, \qquad p(10\,\mu\text{s}) = vi = 62.5\,\text{mW},$

$p(30\,\mu\text{s}) = 437.50\,\text{mW}, \qquad v(30\,\mu\text{s}) = 8.75\,\text{V}, \qquad i(30\,\mu\text{s}) = 0.05\,\text{A}$

[c] $w(10\,\mu\text{s}) = 15.625 \times 10^{12}(10 \times 10^{-6})^4 = 0.15625\,\mu\text{J}$

$w = 0.5Cv^2 = 0.5(0.2 \times 10^{-6})(1.25)^2 = 0.15625\,\mu\text{J}$

$w(30\,\mu\text{s}) = 7.65625\,\mu\text{J}$

$w(30\,\mu\text{s}) = 0.5(0.2 \times 10^{-6})(8.75)^2 = 7.65625\,\mu\text{J}$

P 5.13 [a] $0 \leq t \leq 50\,\mu\text{s}$

$C = 0.5\,\mu\text{F} \qquad \dfrac{1}{C} = 2 \times 10^6$

$$v = 2 \times 10^6 \int_0^t 20 \times 10^{-3}\,dx + 20$$

$v = 40 \times 10^3 t + 20\,\text{V} \qquad 0 \leq t \leq 50\,\mu\text{s}$

$v(50\,\mu\text{s}) = 2 + 20 = 22\,\text{V}$

[b] $50\,\mu\text{s} \leq t \leq 200\,\mu\text{s}$

$$v = 2 \times 10^6 \int_{50\times 10^{-6}}^t -40 \times 10^{-3}\,dx + 22 = -8 \times 10^4 t + 4 + 22$$

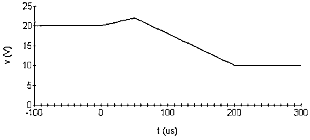
$v = -8 \times 10^4 t + 26\text{V} \qquad 50 \leq t \leq 200\,\mu\text{s}$

$v(200\,\mu\text{s}) = -8 \times 10^4(200 \times 10^{-6}) + 26 = 10\,\text{V}$

[c] $200 \leq t \leq \infty$

$$v = 2 \times 10^6 \int_{200\times 10^{-6}}^t 0\,dx + 10 = 10\,\text{V} \quad 200\,\mu\text{s} \leq t \leq \infty$$

[d]



P 5.14 $i_C = C(dv/dt)$

$0 < t < 1$:

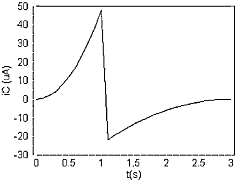
$v_c = 20t^3\,\text{V}$

$i_C = 0.8 \times 10^{-6}(60)t^2 = 48t^2\,\mu\text{A}$

$1 < t < 3$:

$v_c = 2.5(3-t)^3\,\text{V}$

$i_C = 0.8\times10^{-6}(7.5)(3-t)^2(-1) = -6(3-t)^2\,\mu\text{A}$



P 5.15 [a]

$$v = 5\times10^6\int_0^{250\times10^{-6}} 100\times10^{-3}e^{-1000t}\,dt - 60.6$$

$$= 500\times10^3\frac{e^{-1000t}}{-1000}\Big|_0^{250\times10^{-6}} - 60.6$$

$$= 500(1-e^{-0.25}) - 60.6 = 50\,\text{V}$$

$$w = \tfrac{1}{2}Cv^2 = \tfrac{1}{2}(0.2)(10^{-6})(50)^2 = 250\,\mu\text{J}$$

[b] $v = 500 - 60.6 = 439.40\,\text{V}$

$$w = \frac{1}{2}(0.2)\times10^{-6}(439.40)^2 = 19.31\,\text{mJ} = 19{,}307.24\,\mu\text{J}$$

P 5.16 [a] $w(0) = \dfrac{1}{2}C[v(0)]^2 = \dfrac{1}{2}(0.40)\times10^{-6}(25)^2 = 125\,\mu\text{J}$

[b] $v = (A_1t + A_2)e^{-1500t} \qquad v(0) = A_2 = 25\,\text{V}$

$$\frac{dv}{dt} = -1500e^{-1500t}(A_1t + A_2) + e^{-1500t}(A_1)$$

$$= (-1500A_1t - 1500A_2 + A_1)e^{-1500t}$$

$$\frac{dv}{dt}(0) = A_1 - 1500A_2, \qquad i = C\frac{dv}{dt}, \qquad i(0) = C\frac{dv(0)}{dt}$$

$$\therefore\quad \frac{dv(0)}{dt} = \frac{i(0)}{C} = \frac{90\times10^{-3}}{0.40\times10^{-6}} = 225\times10^3$$

$$\therefore\quad 225\times10^3 = A_1 - 1500(25)$$

Thus, $A_1 = 2.25\times10^5 + 3.75\times10^4 = 262{,}500\,\dfrac{\text{V}}{\text{s}}$

[c] $v = (262{,}500t + 25)e^{-1500t}$

$$i = C\frac{dv}{dt} = 0.40 \times 10^{-6}\frac{d}{dt}(262{,}500t + 25)e^{-1500t}$$

$$\begin{aligned} i &= \frac{d}{dt}[(0.105t + 10 \times 10^{-6})e^{-1500t}] \\ &= (0.105t + 10 \times 10^{-6})(-1500)e^{-1500t} + e^{-1500t}(0.105) \\ &= (-157.5t - 15 \times 10^{-3} + 0.105)e^{-1500t} \\ &= (0.09 - 157.5t)e^{-1500t}\,\text{A}, \qquad t \geq 0 \\ &= (90 - 157{,}500t)e^{-1500t}\,\text{mA}, \qquad t \geq 0 \end{aligned}$$

P 5.17 [a] $i = \dfrac{50 \times 10^{-3}}{10 \times 10^{-6}}t = 5 \times 10^3 t \qquad 0 \leq t \leq 10\,\mu\text{s}$

$i = 50 \times 10^{-3} \qquad 10 \leq t \leq 30\,\mu\text{s}$

$$\begin{aligned} q &= \int_0^{10\times10^{-6}} 5 \times 10^3 t\,dt + \int_{10\times10^{-6}}^{30\times10^{-6}} 50 \times 10^{-3}\,dt \\ &= 5 \times 10^3\frac{t^2}{2}\Big|_0^{10\times10^{-6}} + 50 \times 10^{-3}(20 \times 10^{-6}) \\ &= 5 \times 10^3(\tfrac{1}{2})(100 \times 10^{-12}) + 1000 \times 10^{-3} \times 10^{-6} \\ &= 1.25\,\mu\text{C} \end{aligned}$$

[b] $i = 200 \times 10^{-3} - 5 \times 10^{-3}t \qquad 30\,\mu\text{s} \leq t \leq 50\,\mu\text{s}$

$$\begin{aligned} q &= 1.25 \times 10^{-6} + \int_{30\times10^{-6}}^{50\times10^{-6}}[200 \times 10^{-3} - 5 \times 10^3 t]\,dt \\ &= 1.25 \times 10^{-6} + 200 \times 10^{-3}(20 \times 10^{-6}) - 5 \times 10^3\frac{t^2}{2}\Big|_{30\times10^{-6}}^{50\times10^{-6}} \\ &= 1.25 \times 10^{-6} + 4000 \times 10^{-9} - 5 \times 10^3\left[\frac{2500 - 900}{2}\right]10^{-12} \\ &= 1.25\,\mu\text{C} \end{aligned}$$

Since $q = vC, \quad \therefore \quad v = 1.25/0.25 = 5\,\text{V}.$

[c] $i = -300 \times 10^{-3} + 5 \times 10^{-3}t \qquad 50\,\mu\text{s} \leq t \leq 60\,\mu\text{s}$

$$
\begin{aligned}
q &= 1.25 \times 10^{-6} + \int_{50\times10^{-6}}^{60\times10^{-6}} [-300 \times 10^{-3} + 5 \times 10^{3}t]\,dt \\
&= 1.25 \times 10^{-6} - 300 \times 10^{-3}(10 \times 10^{-6}) + 5 \times 10^{3}\left[\frac{3600 - 2500}{2}\right] 10^{-12} \\
&= 1\,\mu\text{C}
\end{aligned}
$$

$$v = \frac{1 \times 10^{-6}}{0.25 \times 10^{-6}} = 4\,\text{V}$$

$$w = \frac{C}{2}v^2 = \frac{1}{2}(0.25) \times 10^{-6}(16) = 2\,\mu\text{J}$$

P 5.18 $v = -60\,\text{V}, \quad t \leq 0; \qquad C = 0.4\,\mu\text{F}$

$$v = 15 - 15e^{-500t}(5\cos 2000t + \sin 2000t)\text{V}, \quad t \geq 0$$

[a] $i = 0, \quad t < 0$

[b]
$$
\begin{aligned}
\frac{dv}{dt} &= -15[(5\cos 2000t + \sin 2000t)(-500e^{-500t}) + \\
&\qquad (e^{-500t})(-10{,}000\sin 2000t + 2000\cos 2000t)] \\
&= 15e^{-500t}(500\cos 2000t + 10{,}500\sin 2000t) \\
i &= C\frac{dv}{dt} = 0.4 \times 10^{-6}(7500)e^{-500t}(\cos 2000t + 21\sin 2000t) \\
&= 3e^{-500t}(\cos 2000t + 21\sin 2000t)\,\text{mA}, \quad t > 0
\end{aligned}
$$

[c] no

[d] yes, from 0 to 3 mA

[e] $v(\infty) = 15\,\text{V}$

$$w = \frac{1}{2}Cv^2 = \frac{1}{2}(0.4)225 \times 10^{-6} = 45\,\mu\text{J}$$

P 5.19 $10\|(15 + 25) = 8\,\text{H}$

$8\|12 = 4.8\,\text{H}$

$44\|(1.2 + 4.8) = 5.28\,\text{H}$

$21\|4 = 3.36\,\text{H}$

$5.28 + 3.36 = 8.64\,\text{H}$

P 5.20 $6\|14 = 4.2\,\text{H}$

$15.8 + 4.2 = 20\,\text{H}$

$20\|60 = 15\,\text{H}$

$15 + 5 = 20\,\text{H}$

$20\|80 = 16\,\text{H}$

$16 + 24 = 40\,\text{H}$

$40\|10 = 8\,\text{H}$

$L_{\text{ab}} = 12 + 8 = 20\,\text{H}$

P 5.21 From Figure 5.17(a) we have

$$v = \frac{1}{C_1}\int_0^t i + v_1(0) + \frac{1}{C_2}\int_0^t i\,dx + v_2(0) + \cdots$$

$$v = \left[\frac{1}{C_1} + \frac{1}{C_2} + \cdots\right]\int_0^t i\,dx + v_1(0) + v_2(0) + \cdots$$

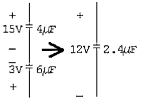
Therefore $\dfrac{1}{C_{\text{eq}}} = \left[\dfrac{1}{C_1} + \dfrac{1}{C_2} + \cdots\right]$, $\quad v_{\text{eq}}(0) = v_1(0) + v_2(0) + \cdots$

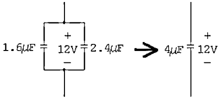
P 5.22 From Fig. 5.18(a)

$$i = C_1\frac{dv}{dt} + C_2\frac{dv}{dt} + \cdots = [C_1 + C_2 + \cdots]\frac{dv}{dt}$$

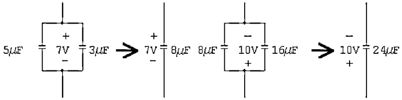
Therefore $C_{\text{eq}} = C_1 + C_2 + \cdots$. Because the capacitors are in parallel, the initial voltage on every capacitor must be the same. This initial voltage would appear on C_{eq}.

P 5.23 $\dfrac{1}{4} + \dfrac{1}{6} = \dfrac{5}{12} \quad \therefore \quad C_{\text{eq}} = 2.4\,\mu\text{F}$

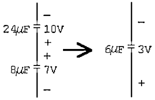




$\dfrac{1}{4} + \dfrac{1}{12} = \dfrac{4}{12} \quad \therefore \quad C_{\text{eq}} = 3\,\mu\text{F}$

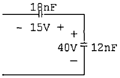


$$\frac{1}{24} + \frac{1}{8} = \frac{4}{24} \qquad \therefore \quad C_{eq} = 6\,\mu\text{F}$$



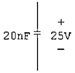
P 5.24 $\dfrac{1}{C_1} = \dfrac{1}{8} + \dfrac{1}{32} = \dfrac{5}{32}; \qquad C_1 = 6.4\,\text{nF}$

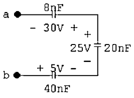
$C_2 = 5.6 + 6.4 = 12\,\text{nF}$



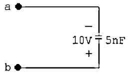
$$\frac{1}{C_3} = \frac{1}{18} + \frac{1}{12} = \frac{10}{72}; \qquad C_3 = 7.2\,\text{nF}$$

$C_4 = 12.8 + 7.2 = 20\,\text{nF}$



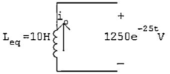


$$\frac{1}{C_5} = \frac{1}{8} + \frac{1}{20} + \frac{1}{40} = \frac{1}{5}; \qquad C_5 = 5\,\text{nF}$$



P 5.25 [a] $i_o(0) = i_1(0) + i_2(0) = 5\,\text{A}$

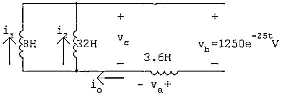
[b]



$$i_o = -\frac{1}{10}\int_0^t 1250e^{-25x}\,dx + 5 = -125\left[\frac{e^{-25x}}{-25}\bigg|_0^t\right] + 5$$

$$= 5(e^{-25t} - 1) + 5 = 5e^{-25t}\,\text{A}, \qquad t \geq 0$$

[c]



$$v_a = 3.6\frac{d}{dt}(5e^{-25t}) = -450e^{-25t}\,\text{V}$$

$$v_c = v_a + v_b = -450e^{-25t} + 1250e^{-25t}$$

$$= 800e^{-25t}\,\text{V}$$

$$i_1 = -\frac{1}{8}\int_0^t 800e^{-25x}\,dx + 10$$

$$= 4e^{-25t} - 4 + 10$$

$$i_1 = 4e^{-25t} + 6\,\text{A} \qquad t \geq 0$$

[d]
$$i_2 = -\frac{1}{32}\int_0^t 800e^{-25x}\,dx - 5$$

$$= e^{-25t} - 1 - 5$$

$$i_2 = e^{-25t} - 6\,\text{A}, \qquad t \geq 0$$

[e] $w(0) = \frac{1}{2}(8)(100) + \frac{1}{2}(32)(25) + \frac{1}{2}(3.6)(25) = 845\,\text{J}$

[f] $w_{\text{del}} = \frac{1}{2}(10)(25) = 125\,\text{J}$

[g] $w_{\text{trapped}} = 845 - 125 = 720\,\text{J}$

P 5.26 $v_b = 1250e^{-25t}\,\text{V}$

$i_o = 5e^{-25t}\,\text{A}$

$p = 6250e^{-50t}\,\text{W}$

$$w = \int_0^t 6250e^{-50x}\,dx = 6250\frac{e^{-50x}}{-50}\Big|_0^t = 125(1 - e^{-50t})\,\text{W}$$

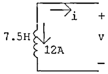
$w_{\text{total}} = 125\,\text{J}$

$80\%w_{\text{total}} = 100\,\text{J}$

Thus,

$125 - 125e^{-50t} = 100; \qquad e^{50t} = 5; \qquad \therefore\ t = 32.19\,\text{ms}$

P 5.27 [a]



$$\begin{aligned} i(t) &= -\frac{1}{7.5}\int_0^t -1800e^{-20x}\,dx - 12 \\ &= 240\frac{e^{-20x}}{-20}\Big|_0^t - 12 \\ &= -12(e^{-20t} - 1) - 12 \\ i(t) &= -12e^{-20t}\,\text{A} \end{aligned}$$

[b]
$$\begin{aligned} i_1(t) &= -\frac{1}{10}\int_0^t -1800e^{-20x}\,dx + 4 \\ &= 180\frac{e^{-20x}}{-20}\Big|_0^t + 4 \\ &= -9(e^{-20t} - 1) + 4 \\ i_1(t) &= -9e^{-20t} + 13\,\text{A} \end{aligned}$$

[c]
$$\begin{aligned} i_2(t) &= -\frac{1}{30}\int_0^t -1800e^{-20x}\,dx - 16 \\ &= 60\frac{e^{-20x}}{-20}\Big|_0^t -16 \\ &= -3(e^{-20t}-1) - 16 \\ i_2(t) &= -3e^{-20t} - 13\,\text{A} \end{aligned}$$

[d] $p = vi = (-1800e^{-20t})(-12e^{-20t}) = 21{,}600e^{-40t}\,\text{W}$

$$\begin{aligned} w &= \int_0^\infty p\,dt = \int_0^\infty 21{,}600e^{-40t}\,dt \\ &= 21{,}600\frac{e^{-40t}}{-40}\Big|_0^\infty \\ &= 540\,\text{J} \end{aligned}$$

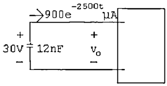
[e] $w = \frac{1}{2}(10)(16) + \frac{1}{2}(30)(256) = 3920\,\text{J}$

[f] $w_{\text{trapped}} = \frac{1}{2}(10)(13)^2 + \frac{1}{2}(30)(13)^2 = 3380\,\text{J}$

$w_{\text{trapped}} = 3920 - 540 = 3380\,\text{J}$ checks

[g] Yes, they agree.

P 5.28 [a]



$$\begin{aligned} v_o &= -\frac{10^9}{12}\int_0^t 900\times 10^{-6}e^{-2500x}\,dx + 30 \\ &= -75{,}000\frac{e^{-2500x}}{-2500}\Big|_0^t +30 \\ &= 30e^{-2500t}\,\text{V}, \qquad t \geq 0 \end{aligned}$$

[b]
$$\begin{aligned} v_1 &= -\frac{10^9}{20}(900\times 10^{-6})\frac{e^{-2500x}}{-2500}\Big|_0^t +45 \\ &= 18e^{-2500t} + 27\,\text{V}, \qquad t \geq 0 \end{aligned}$$

[c]
$$\begin{aligned} v_2 &= -\frac{10^9}{30}(900\times 10^{-6})\frac{e^{-2500x}}{-2500}\Big|_0^t -15 \\ &= 12e^{-2500t} - 27\,\text{V}, \qquad t \geq 0 \end{aligned}$$

[d] $p = vi = (30e^{-2500t})(900 \times 10^{-6})e^{-2500t}$

$$= 27 \times 10^{-3}e^{-5000t}$$

$$w = \int_0^\infty 27 \times 10^{-3}e^{-5000t}\,dt$$

$$= 27 \times 10^{-3}\frac{e^{-5000t}}{-5000}\Big|_0^\infty$$

$$= -5.4 \times 10^{-6}(0-1) = 5.4\,\mu\text{J}$$

[e] $w = \frac{1}{2}(20 \times 10^{-9})(45)^2 + \frac{1}{2}(30 \times 10^{-9})(15)^2$

$$= 20.25 \times 10^{-6} + 3.375 \times 10^{-6}$$

$$= 23.625\,\mu\text{J}$$

[f] $w_{\text{trapped}} = \frac{1}{2}(20 \times 10^{-9})(27)^2 + \frac{1}{2}(30 \times 10^{-9})(27)^2$

$$= (10+15)(27)^2 \times 10^{-9}$$

$$= 18.225\,\mu\text{J}$$

CHECK: $18.225 + 5.4 = 23.625\,\mu\text{J}$

[g] Yes, they agree.

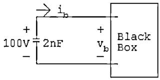
P 5.29 $C_1 = 1 + 1.5 = 2.5\,\text{nF}$

$$\frac{1}{C_2} = \frac{1}{2.5} + \frac{1}{12.5} + \frac{1}{50} = \frac{1}{2}$$

$\therefore\; C_2 = 2\,\text{nF}$

$v_d(0) + v_a(0) - v_c(0) = 40 + 15 + 45 = 100\,\text{V}$

[a]



$$v_b = -\frac{10^9}{2}\int_0^t 50 \times 10^{-6}e^{-250x}\,dx + 100$$

$$= -25{,}000\frac{e^{-250x}}{-250}\Big|_0^t + 100$$

$$= 100(e^{-250t} - 1) + 100$$

$$= 100e^{-250t}\,\text{V}, \quad t \geq 0$$

[b]
$$\begin{aligned} v_{\rm a} &= -\frac{10^9}{12.5}\int_0^t 50\times10^{-6}e^{-250x}\,dx + 15\\ &= -4000\frac{e^{-250x}}{-250}\Big|_0^t + 15\\ &= 16(e^{-250t}-1)+15\\ &= 16e^{-250t} - 1\,\text{V}\end{aligned}$$

[c]
$$\begin{aligned} v_{\rm c} &= \frac{10^9}{50}\int_0^t 50\times10^{-6}e^{-250x}\,dx - 45\\ &= 1000\frac{e^{-250x}}{-250}\Big|_0^t - 45\\ &= -4(e^{-250t}-1)-45\\ &= -4e^{-250t} - 41\,\text{V}, \quad t\geq 0\end{aligned}$$

[d]
$$\begin{aligned} v_{\rm d} &= -\frac{10^9}{2.5}\int_0^t 50\times10^{-6}e^{-250x}\,dx + 40\\ &= -20{,}000\frac{e^{-250x}}{-250}\Big|_0^t + 40\\ &= 80(e^{-250t}-1)+40\\ &= 80e^{-250t} - 40\,\text{V}, \quad t\geq 0\end{aligned}$$

CHECK:
$$\begin{aligned} v_{\rm b} &= v_{\rm d}+v_{\rm a}-v_{\rm c}\\ &= 80e^{-250t}-40+16e^{-250t}-1+4e^{-250t}+41\\ &= 100e^{-250t}\,\text{V} \quad \text{(checks)}\end{aligned}$$

[e]
$$\begin{aligned} i_1 &= -10^{-9}\frac{d}{dt}[80e^{-250t}-40]\\ &= -10^{-9}(-20{,}000e^{-250t})\\ &= 20e^{-250t}\,\mu\text{A}, \quad t\geq 0\end{aligned}$$

[f]
$$\begin{aligned} i_2 &= -1.5\times10^{-9}\frac{d}{dt}[80e^{-250t}-40]\\ &= -1.5\times10^{-9}(-20{,}000e^{-250t})\\ &= 30e^{-250t}\,\mu\text{A}, \quad t\geq 0\end{aligned}$$

CHECK: $i_1+i_2 = 50e^{-250t}\,\mu\text{A} = i_{\rm b}$

P 5.30 [a] $w(0) = [\frac{1}{2}(2.5)(40)^2 + \frac{1}{2}(12.5)(15)^2 + \frac{1}{2}(50)(45)^2] \times 10^{-9}$

$= 54{,}031.25\,\text{nJ}$

[b] $v_a(\infty) = -1\,\text{V}$

$v_c(\infty) = -41\,\text{V}$

$v_d(\infty) = -40\,\text{V}$

$w(\infty) = [\frac{1}{2}(2.5)(40)^2 + \frac{1}{2}(12.5)(1)^2 + \frac{1}{2}(50)(41)^2] \times 10^{-9}$

$= 44{,}031.25\,\text{nJ}$

[c] $w = \int_0^\infty (100e^{-250t})(50e^{-250t}) \times 10^{-6}\,dt = 10{,}000\,\text{nJ}$

CHECK: $54{,}031.25 - 44{,}031.25 = 10{,}000$

[d] $\%$ delivered $= \dfrac{10{,}000}{54{,}031.25} \times 100 = 18.51\%$

[e] $w = 5 \times 10^{-3} \int_0^t e^{-500x}\,dx$

$= 10^4(1 - e^{-500t})\,\text{nJ}$

$\therefore\ 10^4(1 - e^{-500t}) = 5000; \qquad e^{-500t} = 0.5$

Thus, $t = (\ln 2)/500 = 1.39\,\text{ms}$.

P 5.31 [a] $\dfrac{v}{i} = R = \dfrac{100e^{-80t}}{4e^{-80t}} = 25\,\Omega$

[b] $\tau = \dfrac{1}{80} = 12.5\,\text{ms}$

[c] $\tau = \dfrac{L}{R} = 12.5 \times 10^{-3}$

$L = (12.5)(25) \times 10^{-3} = 312.5\,\text{mH}$

[d] $w(0) = \dfrac{1}{2}L[i(0)]^2 = \dfrac{1}{2}(0.3125)(16) = 2.5\,\text{J}$

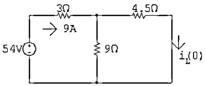
[e] $w_{\text{diss}} = \int_0^t 400e^{-160x}\,dx = 2.5 - 2.5e^{-160t}$

$0.8w(0) = (0.8)(2.5) = 2\,\text{J}$

$2.5 - 2.5e^{-160t} = 2 \qquad \therefore\ e^{160t} = 5$

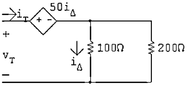
Solving, $t = 10.06\,\text{ms}$.

P 5.32 [a] $t < 0:$



$$\frac{(9)(4.5)}{13.5} = 3\,\Omega; \qquad i_L(0) = 9\frac{9}{13.5} = 6\,\text{A}$$

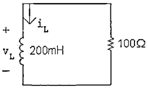
$t > 0$:



$$i_\Delta = \frac{i_T(200)}{300} = \frac{2}{3}i_T$$

$$v_T = 50i_\Delta + i_T\frac{(100)(200)}{300} = 50i_T\frac{2}{3} + \frac{200}{3}i_T$$

$$\frac{v_T}{i_T} = R_{\text{Th}} = \frac{100}{3} + \frac{200}{3} = 100\,\Omega$$

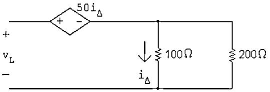


$$\tau = \frac{L}{R} = \frac{200}{100} \times 10^{-3} \qquad \frac{1}{\tau} = 500$$

$$i_L = 6e^{-500t}\,\text{A}, \qquad t \geq 0$$

[b] $v_L = 200 \times 10^{-3}(-3000e^{-500t}) = -600e^{-500t}\,\text{V}, \quad t \geq 0^+$

[c]



$$v_L = 50i_\Delta + 100i_\Delta = 150i_\Delta$$

$$i_\Delta = \frac{v_L}{150} = -4e^{-500t}\,\text{A} \qquad t \geq 0^+$$

P 5.33 $w(0) = \frac{1}{2}(200 \times 10^{-3})(36) = 3.6\,\text{J}$

$$p_{50i_\Delta} = -50 i_\Delta i_L = -50(-4e^{-500t})(6e^{-500t}) = 1200e^{-1000t}\,\text{W}$$

$$w_{50i_\Delta} = \int_0^\infty 1200e^{-1000t}\,dt = 1200\frac{e^{-1000t}}{-1000}\Big|_0^\infty = 1.2\,\text{J}$$

$$\%\text{ dissipated } = \frac{1.2}{3.6}(100) = 33.33\%$$

P 5.34 [a] $i(0) = 125/25 = 5\,\text{A}$

[b] $\tau = \frac{L}{R} = \frac{4}{100} = 40\,\text{ms}$

[c] $i = 5e^{-25t}\,\text{A}, \qquad t \geq 0$

$$v_1 = L\frac{di_1}{dt} = 4(-125e^{-25t}) = -500e^{-25t}\,\text{V} \qquad t \geq 0^+$$

$$v_2 = -80i = -400e^{-25t}\,\text{V} \qquad t \geq 0$$

[d] $p_{\text{diss}} = i^2(20) = 25e^{-50t}(20) = 500e^{-50t}\,\text{W}$

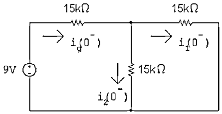
$$w_{\text{diss}} = \int_0^t 500e^{-50x}\,dx = 500\frac{e^{-50x}}{-50}\Big|_0^t = 10 - 10e^{-50t}\,\text{J}$$

$$w_{\text{diss}}(12\,\text{ms}) = 10 - 10e^{-0.6} = 4.51\,\text{J}$$

$$w(0) = \frac{1}{2}(4)(25) = 50\,\text{J}$$

$$\%\text{ dissipated } = \frac{4.51}{50}(100) = 9.02\%$$

P 5.35 [a] $t < 0$



$$15\,\text{k}\Omega \| 15\,\text{k}\Omega = 7.5k\Omega$$

$$i_g(0^-) = \frac{9}{(15+7.5)\times 10^3} = 0.4\,\text{mA}$$

$$i_1(0^-) = i_2(0^-) = (0.4 \times 10^{-3})\frac{(15)}{(30)} = 0.2\,\text{mA}$$

[b] $i_1(0^+) = i_1(0^-) = 0.2\,\text{mA}$

$i_2(0^+) = -i_1(0^+) = -0.2\,\text{mA}$ (when switch is open)

[c] $\tau = \frac{L}{R} = \frac{30 \times 10^{-3}}{30 \times 10^3} = 10^{-6};\qquad \frac{1}{\tau} = 10^6$

$i_1(t) = i_1(0^+)e^{-t/\tau}$

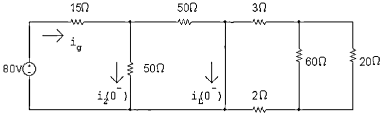
$i_1(t) = 0.2e^{-10^6 t}\,\text{mA},\qquad t \geq 0$

[d] $i_2(t) = -i_1(t)$ when $t \geq 0^+$

$\therefore\ i_2(t) = -0.2e^{-10^6 t}\,\text{mA},\qquad t \geq 0^+$

[e] The current in a resistor can change instantaneously. The switching operation forces $i_2(0^-)$ to equal 0.2 mA and $i_2(0^+) = -0.2\,\text{mA}$.

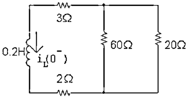
P 5.36 [a] For $t < 0$



$i_g = \frac{80}{40} = 2\,\text{A}$

$i_L(0^-) = \frac{2(50)}{(100)} = 1\,\text{A} = i_L(0^+)$

For $t > 0$



$i_L(t) = i_L(0^+)e^{-t/\tau}\,\text{A},\qquad t \geq 0$

$\tau = \frac{L}{R} = \frac{0.20}{5 + 15} = \frac{1}{100} = 0.01\,\text{s}$

$i_L(0^+) = 1\,\text{A}$

$$i_L(t) = e^{-100t}\,\text{A}, \qquad t \geq 0$$
$$v_o(t) = -15 i_L(t)$$
$$v_o(t) = -15e^{-100t}\,\text{V}, \qquad t \geq 0^+$$

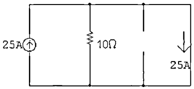
P 5.37 $P_{20\Omega} = \dfrac{v_o^2}{20} = 11.25e^{-200t}\text{W}$

$$w_{\text{diss}} = \int_0^{0.01} 11.25e^{-200t}\,dt$$
$$= \frac{11.25}{-200}e^{-200t}\Big|_0^{0.01}$$
$$= 56.25 \times 10^{-3}(1 - e^{-2}) = 48.64\,\text{mJ}$$

$$w_{\text{stored}} = \frac{1}{2}(0.2)(1)^2 = 100\,\text{mJ}.$$

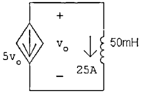
$$\%\ \text{diss} = \frac{48.64}{100} \times 100 = 48.64\%$$

P 5.38 $t < 0$

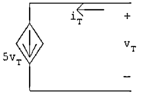


$$i_L(0^-) = i_L(0^+) = 25\,\text{A}$$

$t > 0$

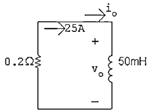


Find Thévenin resistance seen by inductor



$$i_T = 5v_T; \qquad \frac{v_T}{i_T} = R_{\text{Th}} = \frac{1}{5} = 0.2\,\Omega$$

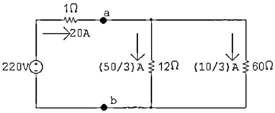
$$\tau = \frac{L}{R} = \frac{50 \times 10^{-3}}{0.2} = 250\,\text{ms}; \qquad 1/\tau = 4$$



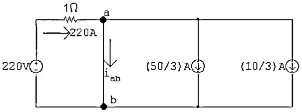
$$i_o = 25e^{-4t}\,\text{A}, \qquad t \geq 0$$

$$v_o = L\frac{di_o}{dt} = (50 \times 10^{-3})(-100e^{-4t}) = -5e^{-4t}\,\text{V}, \quad t \geq 0^+$$

P 5.39 [a] $t < 0$:

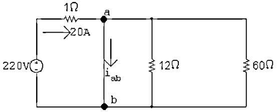


$t = 0^+$:

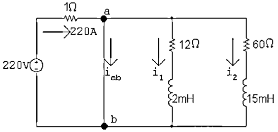


$$220 = i_{\text{ab}} + (50/3) + (10/3), \qquad i_{\text{ab}} = 200\,\text{A}, \quad t = 0^+$$

[b] At $t = \infty$:



$i_{ab} = 220/1 = 220\,\text{A}, \quad t = \infty$



[c] $i_1(0) = 50/3, \qquad \tau_1 = \frac{2}{12} \times 10^{-3} = 0.167\,\text{ms}$

$i_2(0) = 10/3, \qquad \tau_2 = \frac{15}{60} \times 10^{-3} = 0.25\,\text{ms}$

$i_1(t) = (50/3)e^{-6000t}\,\text{A}, \quad t \geq 0$

$i_2(t) = (10/3)e^{-4000t}\,\text{A}, \quad t \geq 0$

$i_{ab} = 220 - (50/3)e^{-6000t} - (10/3)e^{-4000t}\,\text{A}, \quad t \geq 0$

$220 - (50/3)e^{-6000t} - (10/3)e^{-4000t} = 210$

$30 = 50e^{-6000t} + 10e^{-4000t}$

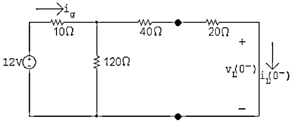
$3 = 5e^{-6000t} + e^{-4000t}$

By trial and error

$t = 123.1\,\mu\text{s}$

P 5.40 [a] $i_o(0^-) = 0$ since the switch is open for $t < 0$.

[b] For $t = 0^-$ the circuit is:

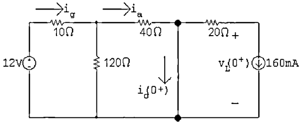


$$120\,\Omega//60\,\Omega = 40\,\Omega$$

$$\therefore \quad i_g = \frac{12}{10+40} = 0.24\,\text{A} = 240\,\text{mA}$$

$$i_L(0^-) = \left(\frac{120}{180}\right) i_g = 160\,\text{mA}$$

[c] For $t = 0^+$ the circuit is:



$$120\,\Omega//40\,\Omega = 30\,\Omega$$

$$\therefore \quad i_g = \frac{12}{10+30} = 0.30\,\text{A} = 300\,\text{mA}$$

$$i_a = \left(\frac{120}{160}\right) 300 = 225\,\text{mA}$$

$$\therefore \quad i_o(0^+) = 225 - 160 = 65\,\text{mA}$$

[d] $i_L(0^+) = i_L(0^-) = 160\,\text{mA}$

[e] $i_o(\infty) = i_a = 225\,\text{mA}$

[f] $i_L(\infty) = 0$, since the switch short circuits the branch containing the 20 Ω resistor and the 100 mH inductor.

[g] $\tau = \dfrac{L}{R} = \dfrac{100 \times 10^{-3}}{20} = 5\,\text{ms}; \qquad \dfrac{1}{\tau} = 200$

$$\therefore \quad i_L = 0 + (160 - 0)e^{-200t} = 160e^{-200t}\,\text{mA}, \qquad t \geq 0$$

[h] $v_L(0^-) = 0$ since for $t < 0$ the current in the inductor is constant

[i] Refer to the circuit at $t = 0^+$ and note:

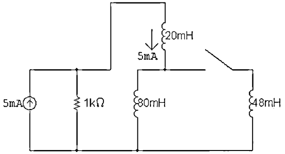
$$20(0.16) + v_L(0^+) = 0; \qquad \therefore \; v_L(0^+) = -3.2\,\text{V}$$

[j] $v_L(\infty) = 0$, since the current in the inductor is a constant at $t = \infty$.

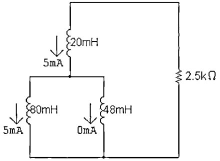
[k] $v_L(t) = 0 + (-3.2 - 0)e^{-200t} = -3.2e^{-200t}\,\text{V}, \qquad t \geq 0^+$

[l] $i_o = i_a - i_L = 225 - 160e^{-200t}\,\text{mA}, \qquad t \geq 0^+$

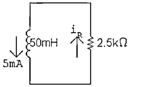
P 5.41 [a] $t < 0$:



$t = 0^+$:

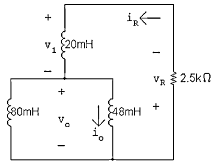


$t > 0$:



$$i_R = 5e^{t/\tau}\,\text{mA}; \qquad \tau = \frac{L}{R} = 20 \times 10^{-6}$$

$$i_R = 5e^{-50{,}000t}\,\text{mA}$$



$$v_R = (2.5 \times 10^3)(5 \times 10^{-3})e^{-50{,}000t} = 12.5e^{-50{,}000t}\,\text{V}$$

$$v_1 = 20 \times 10^{-3}[5 \times 10^{-3}(-50{,}000)e^{-50{,}000t}] = -5e^{-50{,}000t}\,\text{V}$$

$$v_o = -v_1 - v_R = -7.5e^{-50{,}000t}\,\text{V}$$

[b] $i_o = \dfrac{10^3}{48}\displaystyle\int_0^t -7.5e^{-50{,}000x}\,dx + 0 = 3.125e^{-50{,}000t} - 3.125\,\text{mA}$

P 5.42 [a] From the solution to Problem 5.41,

$$i_R = 5 \times 10^{-3}e^{-50{,}000t}\,\text{A}$$

$$p_R = (25 \times 10^{-6}e^{-100{,}000t})(2.5 \times 10^3) = 62.5 \times 10^{-3}e^{-100{,}000t}\,\text{W}$$

$$w_{\text{diss}} = \int_0^\infty 62.5 \times 10^{-3}e^{-100{,}000t}\,dt$$

$$= 62.5 \times 10^{-3}\frac{e^{-100{,}000t}}{-10^5}\Big|_0^\infty = 625\,\text{nJ}$$

[b] $w_{\text{trapped}} = \dfrac{1}{2}L_{\text{eq}}i_{\text{R}}^2(0) = \dfrac{1}{2}(50 \times 10^{-3})(5 \times 10^{-3})^2 = 625\,\text{nJ}$

CHECK:

$w(0) = \frac{1}{2}(20)(25 \times 10^{-6}) \times 10^{-3} + \frac{1}{2}(80)(25 \times 10^{-6}) \times 10^{-3} = 1250\,\text{nJ}$

$\therefore\ \ w(0) = w_{\text{diss}} + w_{\text{trapped}}$

P 5.43 [a] $i_L(0) = \dfrac{80}{40} = 2\,\text{A}$

$$i_o(0^+) = \frac{80}{20} - 2 = 4 - 2 = 2\,\text{A}$$

$$i_o(\infty) = \frac{80}{20} = 4\,\text{A}$$

[b] $i_L = 2e^{-t/\tau}; \qquad \tau = \frac{L}{R} = \frac{20}{20} \times 10^{-3} = 1\,\text{ms}$

$i_L = 2e^{-1000t}\,\text{A}$

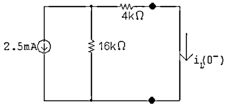
$i_o = 4 - i_L = 4 - 2e^{-1000t}\,\text{A}, \qquad t \geq 0^+$

[c] $4 - 2e^{-1000t} = 3.8$

$0.2 = 2e^{-1000t}$

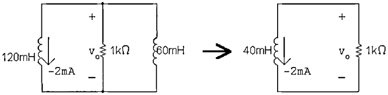
$e^{1000t} = 10 \qquad \therefore \; t = 2.30\,\text{ms}$

P 5.44 [a] $t < 0$



$$i_L(0^-) = \frac{-2.5(16)}{(20)} = -2\,\text{mA}$$

$t \geq 0$



$$\tau = \frac{40 \times 10^{-3}}{10^3} = 40 \times 10^{-6}; \qquad 1/\tau = 25{,}000$$

$$v_o = -1000(-2 \times 10^{-3})e^{-25{,}000t} = 2e^{-25{,}000t}\,\text{V}, \qquad t \geq 0^+$$

[b] $w_{\text{del}} = \frac{1}{2}(40 \times 10^{-3})(4 \times 10^{-6}) = 80\,\text{nJ}$

[c] $0.95 w_{\text{del}} = 76\,\text{nJ}$

$$\therefore \; 76 \times 10^{-9} = \int_0^{t_o} \frac{4e^{-50{,}000t}}{1000}\,dt$$

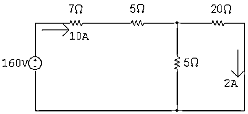
$$\therefore \; 76 \times 10^{-9} = 80 \times 10^{-9} e^{-50{,}000t} \Big|_0^{t_o} = 80 \times 10^{-9}(1 - e^{-50{,}000t_o})$$

$$\therefore \; e^{-50{,}000t_o} = 0.05$$

$50{,}000t_o = \ln 20 \quad \text{so} \quad t_o = 59.9\,\mu\text{s}$

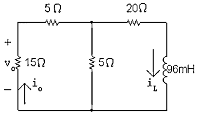
$$\therefore \; \frac{t_o}{\tau} = \frac{59.9}{40} = 1.498 \quad \text{so} \quad t_o \approx 1.5\tau$$

P 5.45 $t < 0$:



$i_L(0^+) = 2\,\text{A}$

$t > 0$:



$$R_e = \frac{(20)(5)}{25} + 20 = 24\,\Omega$$

$$\tau = \frac{L}{R_e} = \frac{96}{24} \times 10^{-3} = 4\,\text{ms}; \qquad \frac{1}{\tau} = 250$$

$$\therefore \quad i_L = 2e^{-250t}\,\text{A}$$

$$\therefore \quad i_o = \frac{5}{25} i_L = 0.4e^{-250t}\,\text{A}$$

$$v_o = -15i_o = -6e^{-250t}\,\text{V}, \quad t \geq 0^+$$

P 5.46 $p_{20\Omega} = 20i_L^2 = 20(4)(e^{-250t})^2 = 80e^{-500t}\,\text{W}$

$$w_{20\Omega} = \int_0^\infty 80e^{-500t}\,dt = 80\frac{e^{-500t}}{-500}\Big|_0^\infty = 160\,\text{mJ}$$

$$w(0) = \frac{1}{2}(96)(10^{-3})(4) = 192\,\text{mJ}$$

$$\%\ \text{diss} = \frac{160}{192}(100) = 83.33\%$$

P 5.47 $w(0) = \frac{1}{2}(20 \times 10^{-3})(10^2) = 1\,\text{J}$

$0.5w(0) = 0.5\,\text{J}$

$i_R = 10e^{-t/\tau}$

$p_{\text{diss}} = i_R^2 R = 100Re^{-2t/\tau}$

$$w_{\text{diss}} = \int_0^t R(100)e^{-2x/\tau}\,dx$$

$$w_{\text{diss}} = 100R\frac{e^{-2x/\tau}}{-2/\tau}\Big|_0^{t_o} = -50\tau R(e^{-2t_o/\tau} - 1) = 50L(1 - e^{-2t_o/\tau})$$

$50L = (50)(20) \times 10^{-3} = 1; \qquad t_o = 10\,\mu\text{s}$

$1 - e^{-2t_o/\tau} = 0.5$

$$e^{2t_o/\tau} = 2; \qquad \frac{2t_o}{\tau} = \frac{2t_oR}{L} = \ln 2$$

$$R = \frac{L\ln 2}{2t_o} = \frac{20 \times 10^{-3}\ln 2}{20 \times 10^{-6}} = 693.15\,\Omega$$

P 5.48 [a] $w(0) = \frac{1}{2}LI_g^2$

$$w_{\text{diss}} = \int_0^{t_o} I_g^2Re^{-2t/\tau}\,dt = I_g^2R\frac{e^{-2t/\tau}}{(-2/\tau)}\Big|_0^{t_o}$$
$$= \frac{1}{2}I_g^2R\tau(1 - e^{-2t_o/\tau}) = \frac{1}{2}I_g^2L(1 - e^{-2t_o/\tau})$$

$w_{\text{diss}} = \sigma w(0)$

$$\therefore \quad \frac{1}{2}LI_g^2(1 - e^{-2t_o/\tau}) = \tau\left(\frac{1}{2}LI_g^2\right)$$

$$1 - e^{-2t_o/\tau} = \sigma; \qquad e^{2t_o/\tau} = \frac{1}{(1-\sigma)}$$

$$\frac{2t_o}{\tau} = \ln\left[\frac{1}{(1-\sigma)}\right]; \qquad \frac{R(2t_o)}{L} = \ln[1/(1-\sigma)]$$

$$R = \frac{L\,\ln[1/(1-\sigma)]}{2t_o}$$

[b] $R = \dfrac{(20 \times 10^{-3})\ln[1/0.5]}{20 \times 10^{-6}}$

$R = 693.15\,\Omega$

P 5.49 [a] $v_o(t) = v_o(0^+)e^{-t/\tau}$

$\therefore\quad v_o(0^+)e^{-5\times10^{-3}/\tau} = 0.25v_o(0^+)$

$\therefore\quad e^{5\times10^{-3}/\tau} = 4$

$\therefore\quad \tau = \dfrac{L}{R} = \dfrac{5\times10^{-3}}{\ln 4}$

$\therefore\quad L = \dfrac{250\times10^{-3}}{\ln 4} = 180.34\,\text{mH}$

[b] $i_L(0^-) = 60\left(\dfrac{1}{6}\right) = 10\,\text{mA} = i_L(0^+)$

$w_{\text{stored}} = \dfrac{1}{2}Li_L(0^+)^2 = \dfrac{1}{2}(R\tau)(100\times10^{-6}) = 2500\tau\,\mu\text{J}.$

$i_L(t) = 10e^{-t/\tau}\,\text{mA}$

$p_{50\Omega} = i_L^2(50) = 5000\times10^{-6}e^{-2t/\tau}$

$$w_{\text{diss}} = \int_0^{5\times10^{-3}} 5000\times10^{-6}e^{-2t/\tau}\,dt$$
$$= 5000\times10^{-6}\frac{e^{-2t/\tau}}{(-2/\tau)}\Bigg|_0^{5\times10^{-3}}$$
$$= 2500\times10^{-6}\tau\left[1 - e^{\frac{-10\times10^{-3}}{\tau}}\right]$$

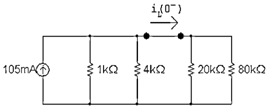
$e^{\frac{-10\times10^{-3}}{\tau}} = e^{-2\ln 4} = 0.0625$

$w_{\text{diss}} = 2500\times10^{-6}\tau(0.9375)$

$\%\ \text{diss} = \dfrac{2500\times10^{-6}\tau(0.9375)}{2500\times10^{-6}\tau}\times 100$

$w_{\text{diss}} = 93.75\%$

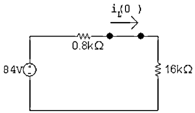
P 5.50 [a] $t < 0$



$1\,\text{k}\Omega \| 4\,\text{k}\Omega = 0.8\,\text{k}\Omega$

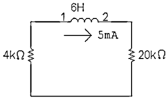
$20\,\text{k}\Omega \| 80\,\text{k}\Omega = 16\,\text{k}\Omega$

$(105)(0.8) = 84\,\text{V}$



$$i_L(0^-) = \frac{84}{16.8} = 5\,\text{mA}$$

$t > 0$



$$\tau = \frac{L}{R} = \frac{6}{24} \times 10^{-3} = 250\,\mu\text{s}; \qquad \frac{1}{\tau} = 4000$$

$$i_L(t) = 5e^{-4000t}\,\text{mA}, \qquad t \geq 0$$

$$p_{4k} = 25 \times 10^{-6} e^{-8000t}(4000) = 0.10e^{-8000t}\,\text{W}$$

$$w_{\text{diss}} = \int_0^t 0.10e^{-8000x}\,dx = 12.5 \times 10^{-6}[1 - e^{-8000t}]\,\text{J}$$

$$w(0) = \frac{1}{2}(6)(25 \times 10^{-6}) = 75\,\mu\text{J}$$

$$0.10w(0) = 7.5\,\mu\text{J}$$

$$12.5(1 - e^{-8000t}) = 7.5; \qquad \therefore \quad e^{8000t} = 2.5$$

$$t = \frac{\ln 2.5}{8000} = 114.54\,\mu\text{s}$$

[b] $w_{\text{diss}}(\text{total}) = 75(1 - e^{-8000t})\,\mu\text{J}$

$w_{\text{diss}}(114.54\,\mu\text{s}) = 45\,\mu\text{J}$

$\% = (45/75)(100) = 60\%$

P 5.51 [a] $R = \dfrac{v}{i} = 20\,\text{k}\Omega$

[b] $\frac{1}{\tau} = \frac{1}{RC} = 1000; \qquad C = \frac{1}{(10^3)(20 \times 10^3)} = 0.05\,\mu\text{F}$

[c] $\tau = \frac{1}{1000} = 1\,\text{ms}$

[d] $w(0) = \frac{1}{2}(0.05 \times 10^{-6})(10^4) = 250\,\mu\text{J}$

[e]

$$W_{\text{diss}} = \int_0^{t_o} \frac{v^2}{R}dt = \int_0^{t_o} \frac{(10^4)e^{-2000t}}{(20 \times 10^3)}dt$$

$$= 0.5\frac{e^{-2000t}}{-2000}\bigg|_0^{t_o} = 250(1 - e^{-2000t_o})\,\mu\text{J}$$

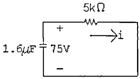
$$200 = 250(1 - e^{-2000t_o})$$

$$\therefore \quad e^{-2000t_o} = 0.2; \qquad e^{2000t_o} = 5$$

$$t_o = \frac{1}{2000}\ln 5; \qquad t_o \cong 804.72\,\mu\text{s}$$

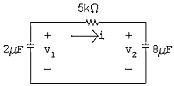
P 5.52 [a] $v_1(0^-) = v_1(0^+) = 75\,\text{V} \qquad v_2(0^+) = 0$

$C_{\text{eq}} = 2 \times 8/10 = 1.6\,\mu\text{F}$



$\tau = (5)(1.6) \times 10^{-3} = 8\text{ms}; \qquad \frac{1}{\tau} = 125$

$i = \frac{75}{5} \times 10^{-3}e^{-125t} = 15e^{-125t}\,\text{mA}, \qquad t \geq 0^+$



$$v_1 = \frac{-10^6}{2}\int_0^t 15 \times 10^{-3}e^{-125x}\,dx + 75 = 60e^{-125t} + 15\,\text{V}, \qquad t \geq 0$$

$$v_2 = \frac{10^6}{8}\int_0^t 15 \times 10^{-3}e^{-125x}\,dx + 0 = -15e^{-125t} + 15\,\text{V}, \qquad t \geq 0$$

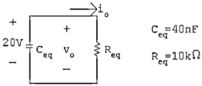
[b] $w(0) = \frac{1}{2}(2 \times 10^{-6})(5625) = 5625\,\mu\text{J}$

[c] $w_{\text{trapped}} = \frac{1}{2}(2 \times 10^{-6})(225) + \frac{1}{2}(8 \times 10^{-6})225 = 1125\,\mu\text{J}.$

$w_{\text{diss}} = \frac{1}{2}(1.6 \times 10^{-6})(5625) = 4500\,\mu\text{J}.$

Check: $w_{\text{trapped}} + w_{\text{diss}} = 1125 + 4500 = 5625\,\mu\text{J}; \qquad w(0) = 5625\,\mu\text{J}.$

P 5.53 [a] The equivalent circuit for $t > 0$:



$\tau = 0.4\,\text{ms}; \qquad 1/\tau = 2500$

$v_o = 20e^{-2500t}\,\text{V}, \qquad t \geq 0$

$i_o = 2e^{-2500t}\,\text{mA}, \qquad t \geq 0^+$

$i_{25k\Omega} = 2e^{-2500t}\left(\frac{15}{40}\right) = 0.75e^{-2500t}\,\text{mA}, \qquad t \geq 0^+$

$p_{25k\Omega} = (0.5625 \times 10^{-6}e^{-5000t})(25{,}000) = 14{,}062.5 \times 10^{-6}e^{-5000t}\,\text{W}$

$w_{25k\Omega} = \int_0^{\infty} 14{,}062.5 \times 10^{-6}e^{-5000t}\,dt = -2.8125 \times 10^{-6}(0 - 1) = 2.8125\,\mu\text{J}$

$w(0) = \frac{1}{2}(0.2 \times 10^{-6})(100) + \frac{1}{2}(0.05 \times 10^{-6})(900) = 32.5\,\mu\text{J}$

$\%\ \text{diss}\,(25\,\text{k}\Omega) = \frac{2.8125}{32.5} \times 100 = 8.65\%$

[b] $p_{625\Omega} = 625(2 \times 10^{-3}e^{-2500t})^2 = 2.5 \times 10^{-3}e^{-5000t}$

$w_{625\Omega} = \int_0^{\infty} p_{625}\,dt = 0.50\,\mu\text{J}$

$\%\ \text{diss}\,(625\Omega) = \frac{0.5}{32.5} \times 100 = 1.54\%$

$i_{15k\Omega} = 2e^{-2500t}\left(\frac{25}{40}\right) = 1.25e^{-2500t}\,\text{mA}, \quad t \geq 0^+$

$p_{15k\Omega} = (1.25 \times 10^{-3}e^{-2500t})^2(15{,}000) = 23.4375 \times 10^{-3}e^{-5000t}\,\text{W}$

$w_{15k\Omega} = \int_0^{\infty} 23.4375 \times 10^{-3}e^{-5000t}\,dt = 4.6875\,\mu\text{J}$

$\%\ \text{diss}\,(15\text{k}\Omega) = 14.42\%$

[c] $\sum w_{\text{diss}} = 2.8125 + 0.50 + 4.6875 = 8\,\mu\text{J}$

$$w_{\text{trapped}} = w(0) - \sum w_{\text{diss}} = 32.5 - 8 = 24.5\,\mu\text{J}$$

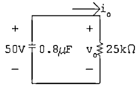
$$\%\text{ trapped} = \frac{24.5}{32.5} \times 100 = 75.38\%$$

Check: $8.65 + 1.54 + 14.42 + 75.38 = 99.99 \approx 100\%$

P 5.54 [a] $\frac{1}{C_e} = 1 + \frac{1}{4} = 1.25$

$$\therefore\ C_e = 0.8\,\mu\text{F}; \qquad v_o(0) = 60 - 10 = 50\,\text{V}$$

$$\tau = (0.8)(25) \times 10^{-3} = 20\,\text{ms}; \qquad \frac{1}{\tau} = 50$$



$$v_o = 50e^{-50t}\,\text{V}, \qquad t > 0^+$$

[b] $w_o = \frac{1}{2}(1 \times 10^{-6})(3600) + \frac{1}{2}(4 \times 10^{-6})(100) = 2\,\text{mJ}$

$$w_{\text{diss}} = \frac{1}{2}(0.8 \times 10^{-6})(2500) = 1\,\text{mJ}$$

$$\%\text{ diss} = \frac{1}{2} \times 100 = 50\%$$

[c] $i_o = \frac{v_o}{25} \times 10^{-3} - 2e^{-50t}\,\text{mA}$

$$v_1 = -\frac{10^6}{4}\int_0^t 2 \times 10^{-3} e^{-50x}\,dx - 10 = -500\int_0^t e^{-50x}\,dx - 10$$
$$= -500\frac{e^{-50x}}{-50}\Big|_0^t -10 = 10e^{-50t} - 20\,\text{V} \qquad t \geq 0$$

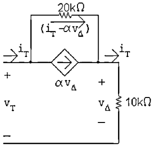
[d] $v_1 + v_2 = v_o$

$$v_2 = v_o - v_1 = 50e^{-50t} - 10e^{-50t} + 20 = 40e^{-50t} + 20\,\text{V} \quad t \geq 0$$

[e] $w_{\text{trapped}} = \frac{1}{2}(4 \times 10^{-6})(400) + \frac{1}{2}(1 \times 10^{-6})(400) = 1\,\text{mJ}$

$$w_{\text{diss}} + w_{\text{trapped}} = 2\,\text{mJ} \qquad \text{(check)}$$

P 5.55 [a] $\tau = RC = R_{\text{Th}}(0.2) \times 10^{-6} = 10^{-3}; \qquad \therefore \quad R_{\text{Th}} = \dfrac{1000}{0.2} = 5\,\text{k}\Omega$



$$v_T = 20 \times 10^3(i_T - \alpha v_\Delta) + 10 \times 10^3 i_T$$

$$v_\Delta = 10 \times 10^3 i_T$$

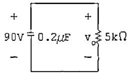
$$v_T = 30 \times 10^3 i_T - 20 \times 10^3 \alpha 10 \times 10^3 i_T$$

$$\frac{v_T}{i_T} = 30 \times 10^3 - 200 \times 10^6 \alpha = 5 \times 10^3$$

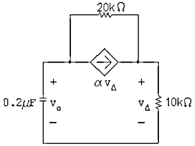
$$\therefore \quad 30 - 200{,}000\alpha = 5; \qquad \alpha = 125 \times 10^{-6}\ \text{A/V}$$

[b] $v_o(0) = (0.018)(5000) = 90\,\text{V} \qquad t < 0$

$t > 0$:



$$v_o = 90e^{-1000t}\,\text{V}, \quad t \geq 0$$

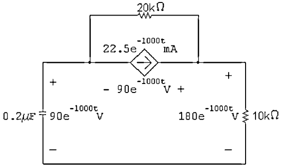


$$\frac{v_\Delta}{10 \times 10^3} + \frac{v_\Delta - v_o}{20{,}000} - 125 \times 10^{-6} v_\Delta = 0$$

$$2v_\Delta + v_\Delta - v_o - 2500 \times 10^{-3} v_\Delta = 0$$

$$\therefore \quad v_\Delta = 2v_o = 180e^{-1000t}\,\text{V}$$

P 5.56 [a]



$$p_{ds} = (-90e^{-1000t})(22.5 \times 10^{-3}e^{-1000t}) = -2025 \times 10^{-3}e^{-2000t}\,\text{W}$$

$$w_{ds} = \int_0^\infty p_{ds}\,dt = -1012.5\,\mu\text{J}.$$

$\therefore$ dependent source is delivering $1012.5\,\mu$J

[b] $$p_{10k} = \frac{(180)^2e^{-2000t}}{10 \times 10^3}$$

$$w_{10k} = \int_0^\infty p_{10k}\,dt = 1620\,\mu\text{J}$$

$$p_{20k} = \frac{(90)^2e^{-2000t}}{20 \times 10^3}$$

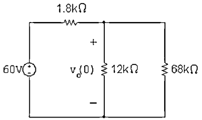
$$w_{20k} = \int_0^\infty p_{20k}\,dt = 202.5\,\mu\text{J}$$

$$w_c(0) = \frac{1}{2}(0.2) \times 10^{-6}(90)^2 = 810\,\mu\text{J}$$

$$\sum w_{\text{dev}} = 810 + 1012.5 = 1822.5\,\mu\text{J}$$

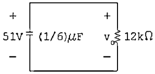
$$\sum w_{\text{diss}} = 202.5 + 1620 = 1822.5\,\mu\text{J}.$$

P 5.57 [a] $t < 0$:



$$v_o(0) = \frac{(60)(10.2)}{12} = 51\,\text{V}$$

$t > 0$:



$$\tau = \frac{1}{6}(12) \times 10^{-3} = 2\,\text{ms}; \qquad \frac{1}{\tau} = 500$$

$$v_o = 51e^{-500t}\,\text{V}, \quad t \geq 0$$

$$p = \frac{v_o^2}{12} \times 10^{-3} = 216.75 \times 10^{-3}e^{-1000t}\,\text{W}$$

$$w_{\text{diss}} = \int_0^{2\times10^{-3}} 216.75 \times 10^{-3}e^{-1000t}\,dt = 216.75 \times 10^{-6}(1 - e^{-2})$$
$$= 187.42\,\mu\text{J}$$

[b] $w(0) = \left(\frac{1}{2}\right)\left(\frac{1}{6}\right)(51)^2 \times 10^{-6} = 216.75\,\mu\text{J}$

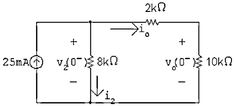
$$0.95w(0) = 205.9125\,\mu\text{J}$$

$$\int_0^{t_o} 216.75 \times 10^{-3}e^{-1000x}\,dx = 205.9125 \times 10^{-6}$$

$$\int_0^{t_o} e^{-1000x}\,dx = 0.95 \times 10^{-3}$$

$$\therefore \quad 1 - e^{-1000t_o} = 0.95; \qquad e^{1000t_o} = 20; \quad \text{so} \quad t_o = 3\,ms$$

P 5.58 [a] $t < 0$:



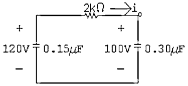
$$i_o(0^-) = \frac{(25)(8)}{(20)} = 10\,\text{mA}$$

$$v_o(0^-) = (10)(10) = 100\,\text{V}$$

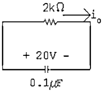
$$i_2(0^-) = 25 - 10 = 15\,\text{mA}$$

$$v_2(0^-) = 15(8) = 120\,\text{V}$$

$t > 0$

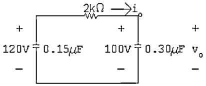


$$\tau = RC = 0.2\,\text{ms} = 200\,\mu\text{s}; \qquad \frac{1}{\tau} = 5000$$



$$i_o(t) = \frac{20}{2 \times 10^3} e^{-t/\tau} = 10e^{-5000t}\,\text{mA}, \qquad t \geq 0^+$$

[b]



$$\begin{aligned} v_o &= \frac{10^6}{0.3}\int_0^t 10 \times 10^{-3} e^{-5000x}\,dx + 100 \\ &= \frac{10^5}{3}\frac{e^{-5000x}}{-5000}\Big|_0^t + 100 \\ &= -(20/3)e^{-5000t} + (20/3) + 100 \\ v_o &= [-(20/3)e^{-5000t} + (320/3)]\,\text{V}, \qquad t \geq 0 \end{aligned}$$

[c] $w_{\text{trapped}} = (1/2)(0.15) \times 10^{-6}(320/3)^2 + (1/2)(0.3) \times 10^{-6}(320/3)^2$

$w_{\text{trapped}} = 2560\,\mu\text{J}.$

Check:

$$w_{\text{diss}} = \frac{1}{2}(0.1 \times 10^{-6})(20)^2 = 20\,\mu\text{J}$$

$$w(0) = \frac{1}{2}(0.15) \times 10^{-6}(120)^2 + \frac{1}{2}(0.3 \times 10^{-6})(100)^2 = 2580\,\mu\text{J}.$$

$$w_{\text{trapped}} + w_{\text{diss}} = w(0)$$

$2560 + 20 = 2580 \qquad$ OK.

P 5.59 [a] $v(0) = \dfrac{(8)(27)(33)}{60} = 118.80\,\text{V}$

$$R_e = \frac{(3)(6)}{9} = 2\,\text{k}\Omega$$

$$\tau = R_eC = (2000)(0.25) \times 10^{-6} = 500\,\mu\text{s}; \qquad \frac{1}{\tau} = 2000$$

$$v = 118.80e^{-2000t}\,\text{V} \qquad t \geq 0$$

$$i_o = \frac{v}{3000} = 39.6e^{-2000t}\,\text{mA}, \quad t \geq 0^+$$

[b] $w(0) = \dfrac{1}{2}(0.25)(118.80)^2 = 1764.18\,\mu\text{J}$

$$i_{4k} = \frac{118.80e^{-2000t}}{6} = 19.8e^{-2000t}\,\text{mA}$$

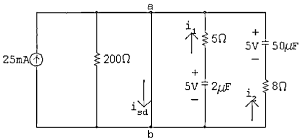
$$p_{4k} = [(19.8)e^{-2000t}]^2(4000) \times 10^{-6} = 1568.16 \times 10^{-3}e^{-4000t}$$

$$w_{4k} = 1568.16 \times 10^{-3}\frac{e^{-4000x}}{-4000}\Bigg|_0^{250\times10^{-6}} = 392.04(1 - e^{-1})\,\mu\text{J}$$

$$= 247.82\,\mu\text{J}$$

$$\% = \frac{247.82}{1764.18} \times 100 = 14.05\%$$

P 5.60 [a] At $t = 0^-$ the voltage on each capacitor will be 5 V$(25 \times 10^{-3} \times 200)$, positive at the upper terminal. Hence at $t \geq 0^+$ we have



$$\therefore \quad i_{sd}(0^+) = 0.025 + \frac{5}{5} + \frac{5}{8} = 1.65\,\text{A}$$

At $t = \infty$, both capacitors will have completely discharged.

$$\therefore \quad i_{sd}(\infty) = 25\,\text{mA}$$

[b] $i_{sd}(t) = 0.025 + i_1(t) + i_2(t)$

$$\tau_1 = (5)(2) \times 10^{-6} = 10\,\mu\text{s}$$

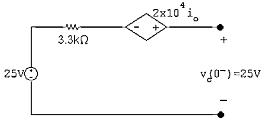
$$\tau_2 = (8)(50 \times 10^{-6}) = 400\,\mu\text{s}$$

$$\therefore \quad i_1(t) = e^{-10^5 t}\,\text{A}, \qquad t \geq 0^+$$

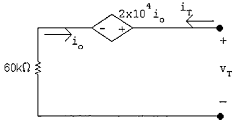
$$i_2(t) = 0.625e^{-2500t}\,\text{A}, \qquad t \geq 0$$

$$\therefore \quad i_{sd} = 25 + 1000e^{-100{,}000t} + 625e^{-2500t}\,\text{mA}, \qquad t \geq 0^+$$

P 5.61 $t < 0$

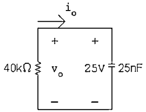


$t > 0$



$$\begin{aligned} v_T &= 2 \times 10^4 i_o + 60{,}000 i_T \\ &= 20{,}000(-i_T) + 60{,}000 i_T = 40{,}000 i_T \end{aligned}$$

$$\therefore \quad \frac{v_T}{i_T} = R_{\text{Th}} = 40\,\text{k}\Omega$$

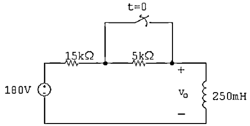


$$\tau = RC = 1\,\text{ms}; \qquad \frac{1}{\tau} = 1000$$

$$v_o = 25e^{-1000t}\,\text{V}, \qquad t \geq 0$$

$$i_o = 25 \times 10^{-9}\frac{d}{dt}[25e^{-1000t}] = -625e^{-1000t}\,\mu\text{A}, \qquad t \geq 0^+$$

P 5.62 After making a Thévenin equivalent we have



$$I_o = 180/15 = 12\,\text{mA}$$

$$\tau = (0.25/20) \times 10^{-3} = 0.125 \times 10^{-4}; \qquad \frac{1}{\tau} = 80{,}000$$

$$\frac{V_s}{R} = \frac{180}{20} = 9\,\text{mA}$$

$$i_o = 9 + (12 - 9)e^{-80{,}000t} = 9 + 3e^{-80{,}000t}\,\text{mA}$$

$$v = [180 - 12(20)]e^{-80{,}000t} = -60e^{-80{,}000t}\,\text{V}$$

P 5.63 [a] $v_o(0^+) = -I_gR_2; \qquad \tau = \dfrac{L}{R_1 + R_2}$

$v_o(\infty) = 0$

$v_o(t) = -I_gR_2e^{-[(R_1+R_2)/L]t}\,\text{V}, \qquad t \geq 0$

[b] $v_o = -(12 \times 10^{-3})(5 \times 10^3)e^{-[\frac{15{,}000+5000}{0.25}]t} = -60e^{-80{,}000t}\,\text{V}, \qquad t \geq 0$

[c] $v_o(0^+) \to \infty$, and the duration of $v_o(t) \to 0$

[d] $v_{\text{sw}} = R_2i_o; \qquad \tau = \dfrac{L}{R_1 + R_2}$

$i_o(0^+) = I_g; \qquad i_o(\infty) = I_g\dfrac{R_1}{R_1 + R_2}$

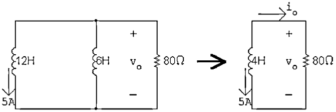
Therefore $\quad i_o(t) = \dfrac{I_gR_1}{R_1 + R_2} + \left[I_g - \dfrac{I_gR_1}{R_1 + R_2}\right]e^{-[(R_1+R_2)/L]t}$

$$i_o(t) = \frac{R_1I_g}{(R_1 + R_2)} + \frac{R_2I_g}{(R_1 + R_2)}e^{-[(R_1+R_2)/L]t}$$

Therefore $\quad v_{\text{sw}} = \dfrac{R_1I_g}{(1 + R_1/R_2)} + \dfrac{R_2I_g}{(1 + R_1/R_2)}e^{-[(R_1+R_2)/L]t}, \qquad t \geq 0$

[e] $|v_{\rm sw}(0^+)| \to \infty$; duration $\to 0$

P 5.64 $t > 0$



$$\tau = \frac{4}{80} = \frac{1}{20}$$

$$i_o = -5e^{-20t}\,\text{A}, \qquad t \geq 0$$

$$v_o = 80i_o = -400e^{-20t}\,\text{V}, \qquad t > 0^+$$

$$-400e^{-20t} = -80; \qquad e^{20t} = 5$$

$$\therefore \quad t = \frac{1}{20}\ln 5 = 80.47\,\text{ms}$$

P 5.65 [a] $w_{\rm diss} = \frac{1}{2}L_e i^2(0) = \frac{1}{2}(4)(25) = 50\,\text{J}$

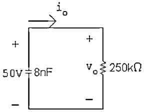
[b]

$$i_{12H} = \frac{1}{12}\int_0^t (-400)e^{-20x}\,dx + 5$$
$$= \frac{-100}{3}\frac{e^{-20x}}{-20}\Big|_0^t + 5 = \frac{5}{3}e^{-20t} + \frac{10}{3}\,\text{A}$$
$$i_{6H} = \frac{1}{6}\int_0^t (-400)e^{-20x}\,dx + 0$$
$$= \frac{-200}{3}\frac{e^{-20x}}{-20}\Big|_0^t + 0 = \frac{10}{3}e^{-20t} - \frac{10}{3}\,\text{A}$$

$$w_{\rm trapped} = \frac{1}{2}(18)(100/9) = 100\,\text{J}$$

[c] $w(0) = \frac{1}{2}(12)(25) = 150\,\text{J}$

P 5.66 [a] For $t > 0$:



$$\tau = RC = 250 \times 10^3 \times 8 \times 10^{-9} = 2\,\text{ms}; \qquad \frac{1}{\tau} = 500$$

$$v_o = 50e^{-500t}\,\text{V}, \qquad t \geq 0^+$$

[b] $i_o = \dfrac{v_o}{250} \times 10^{-3} = \dfrac{50e^{-500t}}{250} \times 10^{-3} = 200e^{-500t}\,\mu\text{A}$

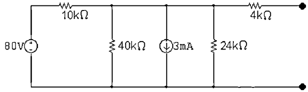
$$v_1 = \frac{-10^9}{40} \times 200 \times 10^{-6} \int_0^t e^{-500x}\,dx + 50 = 10e^{-500t} + 40\,\text{V}, \quad t \geq 0$$

P 5.67 [a] $w = \frac{1}{2}C_e v_e^2 = \frac{1}{2}(8 \times 10^{-9})(2500) = 10\,\mu\text{J}$

[b] $w_{\text{trapped}} = \frac{1}{2}(40)^2(50) \times 10^{-9} = 40\,\mu\text{J}$

[c] $w(0) = \frac{1}{2}(40 \times 10^{-9})(2500) = 50\,\mu\text{J}$

P 5.68 For $t < 0$

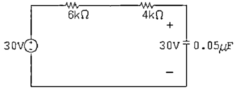


$80/10{,}000 = 8\,\text{mA}, \qquad 10\,\text{k}\Omega \| 40\,\text{k}\Omega \| 24\,\text{k}\Omega = 6\,\text{k}\Omega$

$8\,\text{mA} - 3\,\text{mA} = 5\,\text{mA}$

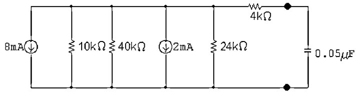
$5\,\text{mA} \times 6\,\text{k}\Omega = 30\,\text{V}$

$t < 0$

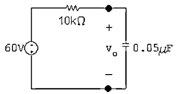


$\therefore \quad v_o(0^-) = v_o(0^+) = 30\,\text{V}$

$t > 0$



$v_o(\infty) = -10 \times 10^{-3}(6 \times 10^3) = -60\,\text{V}$

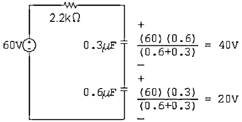


$$\tau = 0.5\,\text{ms}; \qquad \frac{1}{\tau} = 2000$$

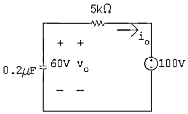
$$v_o = -60 + (30 - (-60))e^{-2000t}$$

$$v_o = -60 + 90e^{-2000t}\,\text{V} \qquad t \geq 0$$

P 5.69 [a] $t < 0$



$t > 0$



$$v_o(0^-) = v_o(0^+) = 60\,\text{V}$$

$$v_o(\infty) = 100\,\text{V}$$

$$\tau = (0.2)(5) \times 10^{-3} = 1\,\text{ms}; \qquad 1/\tau = 1000$$

$$v_o = 100 - 40e^{-1000t}\,\text{V}, \qquad t \geq 0$$

[b] $i_o = -C\dfrac{dv_o}{dt} = -0.2 \times 10^{-6}[40{,}000e^{-1000t}]$

$$= -8e^{-1000t}\,\text{mA}; \qquad t \geq 0^+$$

[c] $v_1 = \frac{-10^6}{0.3}\int_0^t -8\times 10^{-3}e^{-1000x}\,dx + 40$

$= 66.67 - 26.67e^{-1000t}\,\text{V}, \qquad t \geq 0$

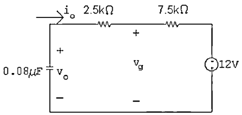
[d] $v_2 = \frac{-10^6}{0.6}\int_0^t -8\times 10^{-3}e^{-1000x}\,dx + 20$

$= 33.33 - 13.33e^{-1000t}\,\text{V}, \qquad t \geq 0$

[e] $w_{\text{trapped}} = \frac{1}{2}(0.3)10^{-6}(66.67)^2 + \frac{1}{2}(0.6)10^{-6}(33.33)^2$

$= 666.67 + 333.33 = 1000\,\mu\text{J}.$

P 5.70 [a] $v_o(0^-) = v_o(0^+) = 48\,\text{V}$



$v_o(\infty) = -12\,\text{V}; \qquad \tau = 0.8\,\text{ms}; \qquad \frac{1}{\tau} = 1250$

$v_o = -12 + (48 - (-12))e^{-1250t}$

$v_o = -12 + 60e^{-1250t}\,\text{V}, \qquad t \geq 0$

[b] $i_o = -0.08\times 10^{-6}[-75{,}000e^{-1250t}]$

$i_o = 6e^{-1250t}\,\text{mA}, \qquad t \geq 0^+$

[c] $v_g = v_o - 2.5\times 10^3 i_o$

$v_g = -12 + 45e^{-1250t}\,\text{V}$

[d] $v_g(0^+) = -12 + 45 = 33\,\text{V}$

Checks:

$v_g(0^+) = i_o(0^+)7.5\times 10^3 - 12 = 45 - 12 = 33\,\text{V}$

$i_{10\text{k}} = \frac{v_g}{10\text{k}} = -1.2 + 4.5e^{-1250t}\,\text{mA}$

$i_{30\text{k}} = \frac{v_g}{30\text{k}} = -0.4 + 1.5e^{-1250t}\,\text{mA}$

$-i_o + i_{10} + i_{30} + 1.6 = 0 \qquad \text{(ok)}$

P 5.71 [a] $0 \le t \le 1\,\text{ms}$:

$v_c(0^+) = 0; \qquad v_c(\infty) = 50\,\text{V};$

$RC = 400 \times 10^3(0.01 \times 10^{-6}) = 4\,\text{ms}; \qquad 1/RC = 250$

$v_c = 50 - 50e^{-250t}$

$v_o = 50 - 50 + 50e^{-250t} = 50e^{-250t}\,\text{V}, \qquad 0 \le t \le 1\,\text{ms}$

$1\,\text{ms} \le t \le \infty$:

$v_c(1\,\text{ms}) = 50 - 50e^{-0.25} = 11.06\,\text{V}$

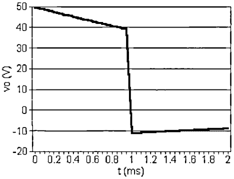
$v_c(\infty) = 0\,\text{V}$

$\tau = 4\,\text{ms}; \qquad 1/\tau = 250$

$v_c = 11.06e^{-250(t-0.001)}\,\text{V}$

$v_o = -v_c = -11.06e^{-250(t-0.001)}\,\text{V}, \qquad 1\,\text{ms} \le t \le \infty$

[b]



P 5.72 [a] $t < 0; \qquad v_o = 0$

$0 \le t \le 10\,\text{ms}$:

$\tau = (50)(0.4) \times 10^{-3} = 20\,\text{ms}; \qquad 1/\tau = 50$

$v_o = 40 - 40e^{-50t}\,\text{V}, \qquad 0 \le t \le 10\,\text{ms}$

$v_o(10\,\text{ms}) = 40(1 - e^{-0.5}) = 15.74\,\text{V}$

$10\,\text{ms} \le t \le 20\,\text{ms}$:

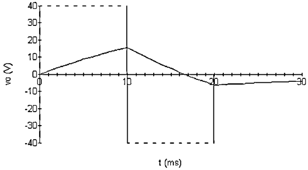
$v_o = -40 + 55.74e^{-50(t-0.01)}\,\text{V}$

$v_o(20\,\text{ms}) = -40 + 55.74e^{-0.5} = -6.19\,\text{V}$

$20\,\text{ms} \le t \le \infty$:

$v_o = -6.19e^{-50(t-0.02)}\,\text{V}$

[b]



[c] $t \leq 0: \qquad v_o = 0$

$0 \leq t \leq 10\,\text{ms}$:

$\tau = 10(0.4 \times 10^{-3}) = 4\,\text{ms}$

$v_o = 40 - 40e^{-250t}\,\text{V}, \qquad 0 \leq t \leq 10\,\text{ms}$

$v_o(10\,\text{ms}) = 40 - 40e^{-2.5} = 36.72\,\text{V}$

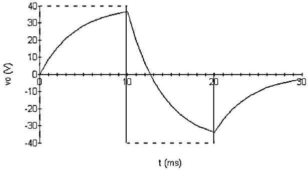
$10\,\text{ms} \leq t \leq 20\,\text{ms}$:

$v_o = -40 + 76.72e^{-250(t-0.01)}\,\text{V}, \qquad 10\,\text{ms} \leq t \leq 20\,\text{ms}$

$v_o(20\,\text{ms}) = -40 + 76.72e^{-2.5} = -33.7\,\text{V}$

$20\,\text{ms} \leq t \leq \infty$:

$v_o = -33.7e^{-250(t-0.02)}\,\text{V}, \qquad 20\,\text{ms} \leq t \leq \infty$



P 5.73 $\quad \dfrac{1}{R_s C_f} = \dfrac{10^6}{50 \times 10^3(0.05)} = 400$

Therefore,

$$
\begin{aligned}
v_o &= -400\int_0^t 75\cos 5000x\,dx + 0 \\
&= -30{,}000\left[\frac{1}{5000}\sin 5000x\,\Big|_0^t\right] \\
&= -6\sin 5000t\text{V}
\end{aligned}
$$

P 5.74 [a] For $0 \leq t \leq 25$ ms:

$$v_s = \frac{600}{25}t = 24t$$

$$\frac{1}{R_sC_f} = \frac{(10^6)(10^{-3})}{(7.5)(0.16)} = \frac{1000}{1.2}$$

$$
\begin{aligned}
\therefore\quad v_o &= -\frac{1000}{1.2}\int_0^t 24x\,dx + 0 \\
&= -20{,}000\left[\frac{x^2}{2}\right]_0^t \\
&= -10^4t^2\text{V} \qquad 0 \leq t \leq 25 \text{ ms}
\end{aligned}
$$

[b] For 25 ms $\leq t \leq 75$ ms:

$$v_s = 1.2 - 24t$$

$$v_o(25 \text{ ms}) = -10^4(625\times 10^{-6}) = -6.25\text{V}$$

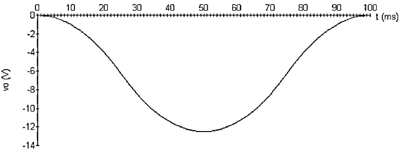
$$
\begin{aligned}
\therefore\quad v_o &= -\frac{1000}{1.2}\int_{25\times10^{-3}}^t (1.2 - 24x)\,dx - 6.25 \\
&= -\frac{1000}{1.2}\left[1.2x - \frac{24x^2}{2}\right]_{25\times10^{-3}}^t - 6.25 \\
&= 10^4t^2 - 10^3t + 12.5\text{V} \qquad 25 \text{ ms } \leq t \leq 75 \text{ ms}
\end{aligned}
$$

$$v_o(75 \text{ ms}) = -10^4(5625\times10^{-6}) - 75 + 12.5 = -6.25\text{V}$$

[c] For 75 ms $\leq t \leq 100$ ms:

$$
\begin{aligned}
v_o &= -\frac{1000}{1.2}\int_{75\times10^{-3}}^t (-2.4 + 24x)\,dx - 6.25 \\
&= -10^4t^2 + 2000t - 100\text{V} \qquad 75 \text{ ms } \leq t \leq 100 \text{ ms}
\end{aligned}
$$

[d]



P 5.75 [a] $v_o(t_1) = \dfrac{4 \times 10^6}{0.8R}(0.25) = 10$

$$\therefore \quad R = \frac{10^6}{8} = 125\,\text{k}\Omega$$

[b] $t_2 - t_1 = \dfrac{4}{10}(250) = 100$ ms

P 5.76 [a] $t_2 - t_1 = \dfrac{3.6}{10}(250) = 90$ ms

$$N_2 = \frac{90}{1000}(10^5) = 9000 \text{ pulses}$$

[b] From (a) we have 9000/3.6 or 2500 pulses/volt.

$\therefore$ 7000 pulses corresponds to 7000/2500 = 2.8V

$\therefore$ $v_a = 2.8$V

P 5.77 Summing the currents at the inverting input terminal yields

$$\frac{0 - v_{\text{ref}}}{R_{\text{ref}}} + \frac{0 - v_x}{R_x} = 0$$

Solving for v_x gives

$$v_x = -\left(\frac{V_{\text{ref}}}{R_{\text{ref}}}\right) R_x$$

Since $(V_{\text{ref}}/R_{\text{ref}})$ is a constant fixed by the circuit designer we see that v_x is directly proportional to the unknown resistance R_x.

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6 Natural and Step Responses of *RLC* Circuits

Drill Exercises

DE 6.1 [a] $\dfrac{1}{(2RC)^2} = \dfrac{1}{LC}, \qquad \text{therefore} \quad C = 500\,\text{nF}$

[b] $\alpha = 5000 = \dfrac{1}{2RC}, \qquad \text{therefore} \quad C = 1\,\mu\text{F}$

$$s_{1,2} = -5000 \pm \sqrt{25 \times 10^6 - \frac{(10^3)(10^6)}{20}} = (-5000 \pm j5000)\ \text{rad/s}$$

[c] $\dfrac{1}{\sqrt{LC}} = 20{,}000, \qquad \text{therefore} \quad C = 125\,\text{nF}$

$$s_{1,2} = \left[-40 \pm \sqrt{(40)^2 - 20^2}\right] 10^3,$$

$$s_1 = -5.36\,\text{krad/s}, \qquad s_2 = -74.64\,\text{krad/s}$$

DE 6.2

$$\begin{aligned}
i_L &= \frac{1000}{50}\int_0^t [-14e^{-5000x} + 26e^{-20{,}000x}]\,dx + 30 \times 10^{-3} \\
&= 20\left\{\frac{-14e^{-5000x}}{-5000}\bigg|_0^t + \frac{26e^{-20{,}000t}}{-20{,}000}\bigg|_0^t\right\} + 30 \times 10^{-3} \\
&= 56 \times 10^{-3}(e^{-5000t} - 1) - 26 \times 10^{-3}(e^{-20{,}000t} - 1) + 30 \times 10^{-3} \\
&= [56e^{-5000t} - 56 - 26e^{-20{,}000t} + 26 + 30]\,\text{mA} \\
&= 56e^{-5000t} - 26e^{-20{,}000t}\,\text{mA}, \qquad t \geq 0
\end{aligned}$$

DE 6.3 From the given values of R, L, and C, $s_1 = -10\,\text{krad/s}$ and $s_2 = -40$ krad/s.

[a] $v(0^-) = v(0^+) = 0, \qquad \text{therefore} \quad i_R(0^+) = 0$

[b] $i_C(0^+) = 4\,\text{A}$

[c] $C\dfrac{dv_c(0^+)}{dt} = 4, \qquad \text{therefore} \quad \dfrac{dv_c(0^+)}{dt} = 4 \times 10^8\,\text{V/s}$

[d] $v = [A_1e^{-10{,}000t} + A_2e^{-40{,}000t}]\,\text{V}, \qquad t \geq 0^+$

$$v(0^+) = A_1 + A_2, \qquad \frac{dv(0^+)}{dt} = -10{,}000A_1 - 40{,}000A_2$$

Therefore $A_1 + A_2 = 0, \quad -A_1 - 4A_2 = 40{,}000, \quad A_1 = 40{,}000/3$

[e] $A_2 = -40{,}000/3$

[f] $v = [40{,}000/3][e^{-10{,}000t} - e^{-40{,}000t}]\,\text{V}, \qquad t \geq 0^+$

DE 6.4 [a] $\dfrac{1}{2RC} = 8000, \qquad \text{therefore} \quad R = 62.5\,\Omega$

[b] $i_R(0^+) = \dfrac{10}{62.5} = 160\,\text{mA}$

$$i_C(0^+) = -80 - 160 = -240\,\text{mA}, \qquad i_C(0^+) = C\frac{dv(0^+)}{dt}$$

Therefore $\dfrac{dv(0^+)}{dt} = -240\,\text{kV/s}$

[c] $B_1 = v(0^+) = 10\,\text{V}, \qquad \dfrac{dv_c(0^+)}{dt} = \omega_d B_2 - \alpha B_1$

Therefore $6000B_2 - 8000B_1 = -240{,}000, \qquad B_2 = (-80/3)\,\text{V}$

[d] $i_L = -(i_R + i_C); \qquad i_R = v/R; \qquad i_C = C\dfrac{dv}{dt}$

$$v = e^{-8000t}[10\cos 6000t - \frac{80}{3}\sin 6000t]\,\text{V}$$

Therefore $i_R = e^{-8000t}[160\cos 6000t - \dfrac{1280}{3}\sin 6000t]\,\text{mA}$

$$i_C = e^{-8000t}[-240\cos 6000t + \frac{460}{3}\sin 6000t]\,\text{mA}$$

$$i_L = 10e^{-8000t}[8\cos 6000t + \frac{82}{3}\sin 6000t]\,\text{mA}, \qquad t \geq 0$$

DE 6.5 [a] $\left(\dfrac{1}{2RC}\right)^2 = \dfrac{1}{LC} = \dfrac{10^6}{4}, \qquad \text{therefore} \quad \dfrac{1}{2RC} = 500, \quad R = 100\,\Omega$

[b] $0.5CV_0^2 = 12.5 \times 10^{-3}, \qquad \text{therefore} \quad V_0 = 50\,\text{V}$

[c] $0.5LI_0^2 = 12.5 \times 10^{-3}, \qquad I_0 = 250\,\text{mA}$

[d] $D_2 = v(0^+) = 50, \qquad \dfrac{dv(0^+)}{dt} = D_1 - \alpha D_2$

$i_R(0^+) = \dfrac{50}{100} = 500\,\text{mA}$

Therefore $i_C(0^+) = -(500 + 250) = -750\,\text{mA}$

Therefore $\dfrac{dv(0^+)}{dt} = -750 \times \dfrac{10^{-3}}{C} = -75{,}000\,\text{V/s}$

Therefore $D_1 - \alpha D_2 = -75{,}000;$

$\alpha = \dfrac{1}{2RC} = 500, \quad D_1 = -50{,}000\ \text{V/s}$

[e] $v = [50e^{-500t} - 50{,}000te^{-500t}]\,\text{V}$

$i_R = \dfrac{v}{R} = [0.5e^{-500t} - 500te^{-500t}]\,\text{A}, \qquad t \geq 0^+$

DE 6.6 [a] $i_R(0^+) = \dfrac{V_0}{R} = \dfrac{40}{500} = 0.08\,\text{A}$

[b] $i_C(0^+) = I - i_R(0^+) - i_L(0^+) = -1 - 0.08 - 0.5 = -1.58\,\text{A}$

[c] $\dfrac{di_L(0^+)}{dt} = \dfrac{40}{0.64} = 62.5\,\text{A/s}$

[d] $\alpha = \dfrac{1}{2RC} = 1000; \qquad \dfrac{1}{LC} = 1{,}562{,}500;$

$s_{1,2} = -1000 \pm j750\ \text{rad/s}$

[e] $i_L = i_f + B_1'e^{-\alpha t}\cos\omega_d t + B_2'e^{-\alpha t}\sin\omega_d t, \qquad i_f = -1\,\text{A}$

$i_L(0^+) = 0.5 = i_f + B_1', \qquad \text{therefore} \quad B_1' = 1.5\,\text{A}$

$\dfrac{di_L(0^+)}{dt} = 62.5 = -\alpha B_1' + \omega_d B_2', \qquad \text{therefore} \quad B_2' = (25/12)\,\text{A}$

$\therefore \quad i_L(t) = -1 + e^{-1000t}[1.5\cos 750t + (25/12)\sin 750t]\,\text{A}, \qquad t \geq 0^+$

[f] $v(t) = \dfrac{\text{L}di_L}{dt} = 40e^{-1000t}[\cos 750t - (154/3)\sin 750t]V \qquad t \geq 0$

DE 6.7 [a] $i(0^+) = 0$

[b] $v_c(0^+) = v_C(0^-) = \left(\dfrac{80}{24}\right)(15) = 50\,\text{V}$

[c] $50 + L\dfrac{di(0^+)}{dt} = 100, \qquad \dfrac{di(0^+)}{dt} = 10{,}000\,\text{A/s}$

[d] $\alpha = 8000; \qquad \dfrac{1}{LC} = 100 \times 10^6; \qquad s_{1,2} = -8000 \pm j6000\ \text{rad/s}$

[e] $i = i_f + e^{-\alpha t}[B_1' \cos \omega_d t + B_2' \sin \omega_d t]; \qquad i_f = 0, \quad i(0^+) = 0$

$$\text{Therefore} \quad B_1' = 0; \qquad \frac{di(0^+)}{dt} = 10{,}000 = -\alpha B_1' + \omega_d B_2'$$

Therefore $B_2' = 1.67\,\text{A}; \qquad i = 1.67e^{-8000t} \sin 6000t\,\text{A}, \quad t \geq 0$

DE 6.8 $v_c(t) = v_f + e^{-\alpha t}[B_1' \cos \omega_d t + B_2' \sin \omega_d t], \qquad v_f = 100\,\text{V}$

$$v_c(0^+) = 50\,\text{V}; \qquad \frac{dv_c(0^+)}{dt} = 0; \qquad \text{therefore} \quad 50 = 100 + B_1'$$

$B_1' = -50\,\text{V}; \qquad 0 = -\alpha B_1' + \omega_d B_2'$

$$\text{Therefore} \quad B_2' = \frac{\alpha}{\omega_d} B_1' = \left(\frac{8000}{6000}\right)(-50) = -66.67\,\text{V}$$

Therefore $v_c(t) = 100 - e^{-8000t}[50 \cos 6000t + 66.67 \sin 6000t]\,\text{V}, \quad t \geq 0$

Problems

P 6.1 [a] $\alpha = \dfrac{1}{2RC} = 1$ rad/s

$$\omega_o^2 = \frac{1}{LC} = 10$$

$\omega_d = \sqrt{10 - 1} = 3$ rad/s

$\therefore \quad v = B_1 e^{-t} \cos 3t + B_2 e^{-t} \sin t$

$v(0) = B_1 = 0; \qquad v = B_2 e^{-t} \sin 3t$

$$i_R(0^+) = 0\,\text{A}; \qquad i_C(0^+) = 3\,\text{A}; \qquad \frac{dv}{dt}(0^+) = \frac{3}{0.25} = 12 \text{ V/s}$$

$12 = -\alpha B_1 + \omega_d B_2 = -1(0) + 3B_2$

$\therefore \quad B_2 = 4$

$\therefore \quad v = 4e^{-t} \sin 3t\,\text{V}, \qquad t \geq 0$

[b] $\dfrac{dv}{dt} = 4e^{-t}(3\cos 3t - \sin 3t)$

$\dfrac{dv}{dt} = 0 \quad \text{when} \quad 3\cos 3t = \sin 3t \quad \text{or} \quad \tan 3t = 3$

$\therefore \quad 3t_1 = 1.25, \qquad t_1 = 416.35\,\text{ms}$

$3t_2 = 1.25 + \pi, \qquad t_2 = 1463.55\,\text{ms}$

$3t_3 = 1.25 + 2\pi, \qquad t_3 = 2510.74\,\text{ms}$

[c] $t_3 - t_1 = 2094.40\,\text{ms}; \qquad T_d = \dfrac{2\pi}{\omega_d} = \dfrac{2\pi}{3} = 2094.40\,\text{ms}$

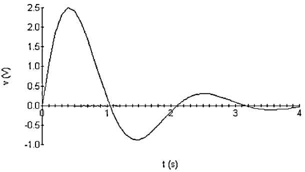
[d] $t_2 - t_1 = 1047.20\,\text{ms}; \qquad \dfrac{T_d}{2} = \dfrac{2094.40}{2} = 1047.20\,\text{ms}$

[e] $v(t_1) = 4e^{-(0.41635)}\sin 3(0.41635) = 2.50\,\text{V}$

$v(t_2) = 4e^{-(1.46355)}\sin 3(1.46355) = -0.88\,\text{V}$

$v(t_3) = 4e^{-(2.51074)}\sin 3(2.51074) = 0.31\,\text{V}$

[f]



P 6.2 [a] $\alpha = 0; \qquad \omega_d = \omega_o = \sqrt{10} = 3.16\,\text{rad/s}$

$v = B_1\cos\omega_o t + B_2\sin\omega_o t; \qquad v(0) = B_1 = 0; \qquad v = B_2\sin\omega_o t$

$C\dfrac{dv}{dt}(0) = -i_{\text{L}}(0) = 3$

$12 = -\alpha B_1 + \omega_d B_2 = -0 + \sqrt{10}B_2$

$\therefore \quad B_2 = 12/\sqrt{10} = 3.79\,\text{V}$

$v = 3.79\sin 3.16t\,\text{V}, \qquad t \geq 0$

[b] $2\pi f = 3.16; \qquad f = \dfrac{3.16}{2\pi} \cong 0.50\,\text{Hz}$

[c] 3.79 V

P 6.3 [a] $\alpha = 4000; \qquad \omega_d = 3000$

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2}$$

$$\therefore \quad \omega_o^2 = \omega_d^2 + \alpha^2 = 9 \times 10^6 + 16 \times 10^6 = 25 \times 10^6$$

$$\frac{1}{LC} = 25 \times 10^6$$

$$L = \frac{1}{(25 \times 10^6)(50 \times 10^{-9})} = 0.8\,\text{H} = 800\,\text{mH}$$

[b] $\alpha = \dfrac{1}{2RC}$

$$\therefore \quad R = \frac{1}{2\alpha C} = \frac{10^9}{(8000)(50)} = 2500\,\Omega$$

[c] $V_o = v(0) = 125\,\text{V}$

[d] $I_o = i_\text{L}(0) = -i_\text{R}(0) - i_\text{C}(0)$

$$i_\text{R}(0) = \frac{V_o}{R} = \frac{125}{2.5} \times 10^{-3} = 50\,\text{mA}$$

$$i_\text{C}(0) = C\frac{dv}{dt}(0)$$

$$\frac{dv}{dt} = 125\{e^{-4000t}[-3000\sin 3000t - 6000\cos 3000t] -$$

$$4000e^{-4000t}[\cos 3000t - 2\sin 3000t]$$

$$\frac{dv}{dt}(0) = 125\{1(-6000) - 4000\} = -125 \times 10^4$$

$$C\frac{dv}{dt}(0) = -125 \times 10^4(40 \times 10^{-9}) = -6250 \times 10^{-5} = -62.5\,\text{mA}$$

$$\therefore \quad I_o = -50 + 62.5 = 12.5\,\text{mA}$$

[e]
$$\begin{aligned}\frac{dv}{dt} &= 125e^{-4000t}[5000\sin 3000t - 10{,}000\cos 3000t] \\ &= 625 \times 10^3 e^{-4000t}[\sin 3000t - 2\cos 3000t]\end{aligned}$$

$$C\frac{dv}{dt} = 31{,}250 \times 10^{-6}e^{-4000t}(\sin 3000t - 2\cos 3000t)$$

$$i_C(t) = 31.25e^{-4000t}(\sin 3000t - 2\cos 3000t)\,\text{mA}$$

$$i_R(t) = 50e^{-4000t}(\cos 3000t - 2\sin 3000t)\,\text{mA}$$

$$i_L(t) = -i_R(t) - i_C(t)$$

$$= e^{-4000t}(12.5\cos 3000t + 68.75\sin 3000t)\,\text{mA}, \quad t \geq 0$$

CHECK:

$$\begin{aligned} \frac{di_L}{dt} &= \{-4000e^{-4000t}[12.5\cos 3000t + 68.75\sin 3000t] \\ &\quad + e^{-4000t}[-37.5\times 10^3\sin 3000t \\ &\quad + 206.25\times 10^3\cos 3000t]\times 10^{-3} \\ &= e^{-4000t}[156.25\cos 3000t - 312.5\sin 3000t] \\ L\frac{di_L}{dt} &= e^{-4000t}[125\cos 3000t - 250\sin 3000t] \\ &= 125e^{-4000t}[\cos 3000t - 2\sin 3000t]\,\text{V} \end{aligned}$$

P 6.4 [a] $\left(\frac{1}{2RC}\right)^2 = \frac{1}{LC} = (4000)^2$

$$\therefore \quad C = \frac{1}{(16\times 10^6)(5)} = 12.5\,\text{nF}$$

$$\frac{1}{2RC} = 4000$$

$$\therefore \quad R = \frac{10^9}{(8000)(12.5)} = 10\,\text{k}\Omega$$

$$v(0) = D_2 = 25\,\text{V}$$

$$i_R(0) = \frac{25}{10} = 2.5\text{mA}$$

$$i_C(0) = -2.5 - 5 = -7.5\,\text{mA}$$

$$\frac{dv}{dt}(0) = D_1 - 4000D_2 = \frac{-7.5\times 10^{-3}}{12.5\times 10^{-9}} = -6\times 10^5$$

$$\therefore \quad D_1 = -6\times 10^5 + 4000(25) = -5\times 10^5 \text{ V/s}$$

[b] $v = -5\times 10^5 te^{-4000t} + 25e^{-4000t}$

$$\frac{dv}{dt} = [20\times 10^8 t - 6\times 10^5]e^{-4000t}$$

$$i_C = C\frac{dv}{dt} = 12.5\times 10^{-9}[20\times 10^8 t - 6\times 10^5]e^{-4000t}$$

$$= (25{,}000t - 7.5)e^{-4000t}\,\text{mA}, \qquad t > 0$$

P 6.5 [a] $-\alpha + \sqrt{\alpha^2 - \omega_o^2} = -5000$

$$-\alpha - \sqrt{\alpha^2 - \omega_o^2} = -20{,}000$$

$$\therefore \quad -2\alpha = -25{,}000$$

$$\alpha = 12{,}500\,\text{rad/s}$$

$$\frac{1}{2RC} = \frac{10^6}{2R(0.05)} = 12{,}500$$

$$R = 800\,\Omega$$

$$2\sqrt{\alpha^2 - \omega_o^2} = 15{,}000$$

$$4(\alpha^2 - \omega_o^2) = 225 \times 10^6$$

$$\therefore \quad \omega_o = 10{,}000\,\text{rad/s}$$

$$\omega_o^2 = 10^8 = \frac{1}{LC}$$

$$\therefore \quad L = \frac{1}{10^8 C} = 200\,\text{mH}$$

[b] $i_{\text{R}} = \dfrac{v(t)}{R} = -6.25e^{-5000t} + 25e^{-20{,}000t}\,\text{mA}, \qquad t \geq 0^+$

$$i_{\text{C}} = C\frac{dv(t)}{dt} = 1.25e^{-5000t} - 20e^{-20{,}000t}\,\text{mA}, \qquad t \geq 0^+$$

$$i_{\text{L}} = -(i_{\text{R}} + i_{\text{C}}) = 5e^{-5000t} - 5e^{-20{,}000t}\,\text{mA}, \qquad t \geq 0^+$$

P 6.6 [a] $\omega_o^2 = \dfrac{1}{LC} = \dfrac{10^9}{40} = 25 \times 10^6$

$$\omega_o = 5000\ \text{rad/s}$$

$$\frac{1}{2RC} = 5000; \qquad R = \frac{1}{10{,}000C}$$

$$R = \frac{10^9}{8 \times 10^4} = 12.5\,\text{k}\Omega$$

[b] $v(t) = D_1 t e^{-5000t} + D_2 e^{-5000t}$

$$v(0) = -25\,\text{V} = D_2$$

$$\frac{dv}{dt} = (D_1 t - 25)(-5000e^{-5000t}) + D_1 e^{-5000t}$$

$$\frac{dv}{dt}(0) = 125 \times 10^3 + D_1 = \frac{i_{\text{C}}(0)}{C}$$

$$i_C(0) = -i_R(0) - i_L(0)$$

$$i_R(0) = \frac{-25}{12.5} = -2\,\text{mA}$$

$$\therefore \quad i_C(0) = 2 - (-1) = 3\,\text{mA}$$

$$\therefore \quad \frac{dv}{dt}(0) = \frac{3 \times 10^{-3}}{8 \times 10^{-9}} = 0.375 \times 10^6 = 3.75 \times 10^5$$

$$\therefore \quad 1.25 \times 10^5 + D_1 = 3.75 \times 10^5$$

$$D_1 = 2.5 \times 10^5 = 25 \times 10^4 \text{ V/s}$$

$$\therefore \quad v(t) = (25 \times 10^4 t - 25)e^{-5000t}\,\text{V}, \qquad t \geq 0$$

[c] $i_C(t) = 0$ when $\dfrac{dv}{dt}(t) = 0$

$$\frac{dv}{dt} = (25 \times 10^4 t - 25)(-5000)e^{-5000t} + e^{-5000t}(25 \times 10^4)$$

$$= (375{,}000 - 125 \times 10^7 t)e^{-5000t}$$

$$\frac{dv}{dt} = 0 \text{ when } 125 \times 10^7 t_1 = 375{,}000; \qquad \therefore \quad t_1 = 300\,\mu\text{s}$$

$$v(300\mu\text{s}) = 50e^{-1.5} = 11.16\,\text{V}$$

[d] $i_L(300\mu\text{s}) = -i_R(300\mu\text{s}) = \dfrac{11.16}{12.5} = 0.89\,\text{mA}$

$$\omega_C(300\mu\text{s}) = 4 \times 10^{-9}(11.16)^2 = 497.87\,\text{nJ}$$

$$\omega_L(300\mu\text{s}) = (2.5)(0.89)^2 \times 10^{-6} = 1991.48\,\text{nJ}$$

$$\omega(300\mu\text{s}) = \omega_C + \omega_L = 2489.35\,\text{nJ}$$

$$\omega(0) = 4 \times 10^{-9}(625) + 2.5(10^{-6}) = 5000\,\text{nJ}$$

$$\%\text{ remaining } = \frac{2489.35}{5000}(100) = 49.79\%$$

P 6.7 [a] $i_R(0) = \dfrac{90}{2000} = 45\text{mA}$

$$i_L(0) = -30\text{mA}$$

$$i_C(0) = -i_L(0) - i_R(0) = 30 - 45 = -15\,\text{mA}$$

[b] $\alpha = \dfrac{1}{2RC} = \dfrac{10^9}{(4000)(10)} = 25{,}000$

$$\omega_o^2 = \frac{1}{LC} = \frac{(10^3)(10^9)}{(250)(10)} = 4 \times 10^8$$

$$s_{1,2} = -25{,}000 \pm \sqrt{6.25 \times 10^8 - 10^8(4)} = -25{,}000 \pm 15{,}000$$

$$s_1 = -10{,}000 \text{ rad/s}; \qquad s_2 = -40{,}000 \text{ rad/s}$$

$$v = A_1 e^{-10{,}000t} + A_2 e^{-40{,}000t}$$

$$v(0) = A_1 + A_2 = 90$$

$$\frac{dv}{dt}(0) = -10^4 A_1 - 4A_2 \times 10^4 = \frac{-15 \times 10^{-3}}{10 \times 10^{-9}} = -1.5 \times 10^6 \text{ V/s}$$

$$-A_1 - 4A_2 = -150$$

$$\therefore \quad -3A_2 = -60; \qquad A_2 = 20; \qquad A_1 = 70$$

$$v = 70e^{-10{,}000t} + 20e^{-40{,}000t} \text{ V}, \qquad t \geq 0$$

[c]
$$\begin{aligned} i_C &= C\frac{dv}{dt} \\ &= 10 \times 10^{-9}[-70 \times 10^4 e^{-10{,}000t} - 80 \times 10^4 e^{-40{,}000t}] \\ &= -7e^{-10{,}000t} - 8e^{-40{,}000t} \text{ mA} \end{aligned}$$

$$i_R = 35e^{-10{,}000t} + 10e^{-40{,}000t} \text{ mA}$$

$$i_L = -i_C - i_R = -28e^{-10{,}000t} - 2e^{-40{,}000t} \text{ mA}, \quad t \geq 0$$

P 6.8 $\dfrac{1}{2RC} = \dfrac{3 \times 10^9}{(25{,}000)(10)} = 12{,}000$

$$\frac{1}{LC} = 4 \times 10^8$$

$$s_{1,2} = -12{,}000 \pm j16{,}000 \text{ rad/s}$$

$\therefore$ response is underdamped

$$v(t) = B_1 e^{-12{,}000t} \cos 16{,}000t + B_2 e^{-12{,}000t} \sin 16{,}000t$$

$$v(0^+) = 90\,\text{V} = B_1; \qquad i_R(0^+) = \frac{90}{(12{,}500/3)} = 21.6\,\text{mA}$$

$i_C(0^+) = [-i_L(0^+) + i_R(0^+)] = -[-30 + 21.6] = 8.4\,\text{mA}$

$$\frac{dv(0^+)}{dt} = \frac{8.4 \times 10^{-3}}{10 \times 10^{-9}} = 840{,}000\,\text{V/s}$$

$$\frac{dv(0)}{dt} = -12{,}000B_1 + 16{,}000B_2 = 840{,}000$$

or $-3B_1 + 4B_2 = 210; \qquad \therefore \quad B_2 = 120\,\text{V}$

$v(t) = 90e^{-12{,}000t}\cos 16{,}000t + 120e^{-12{,}000t}\sin 16{,}000t\,\text{V}, \qquad t \geq 0$

P 6.9 $\alpha = \dfrac{1}{2RC} = \dfrac{10^9}{(5000)(10)} = 2 \times 10^4$

$\alpha^2 = 4 \times 10^8; \qquad \therefore \quad \alpha^2 = \omega_o^2$

Critical damping:

$v = D_1te^{-\alpha t} + D_2e^{-\alpha t}$

$$i_R(0^+) = \frac{90}{2500} = 36\,\text{mA}$$

$i_C(0^+) = -[i_L(0^+) + i_R(0^+)] = -[-30 + 36] = -6\,\text{mA}$

$v(0) = D_2 = 90$

$$\frac{dv}{dt} = D_1[t(-\alpha e^{-\alpha t}) + e^{-\alpha t}] - \alpha D_2e^{-\alpha t}$$

$$\frac{dv}{dt}(0) = D_1 - \alpha D_2 = \frac{i_C(0)}{C} = \frac{-6 \times 10^{-3}}{10 \times 10^{-9}} = -6 \times 10^5$$

$D_1 = \alpha D_2 - 6 \times 10^5 = (2 \times 10^4)(90) - 6 \times 10^5 = 120 \times 10^4$

$v = (120 \times 10^4t + 90)e^{-20{,}000t}\,\text{V}, \qquad t \geq 0$

P 6.10 [a] $\alpha = \dfrac{1}{2RC} = \dfrac{10^9}{(10{,}000)(8)} = 12{,}500$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^9}{(1.25)(8)} = 10^8$$

$s_{1,2} = -12{,}500 \pm \sqrt{(1.5625 - 1)10^8} = -12{,}500 \pm 7500$

$s_1 = -5000\,\text{rad/s}$

$s_2 = -20{,}000\,\text{rad/s}$

[b] overdamped

[c] $\omega_d = \sqrt{\omega_o^2 - \alpha^2}$

$$\therefore \quad \alpha^2 = \omega_o^2 - \omega_d^2 = 10^8 - 36 \times 10^6 = 0.64 \times 10^8$$

$$\alpha = 0.8 \times 10^4 = 8000$$

$$\frac{1}{2RC} = 8000; \qquad \therefore \quad R = \frac{10^9}{(16{,}000)(8)} = 7812.5\,\Omega$$

[d] $s_1 = -8000 + j6000$ rad/s; $\qquad s_2 = -8000 - j6000$ rad/s

[e] $\alpha = 10^4 = \dfrac{1}{2RC}; \qquad \therefore \quad R = \dfrac{1}{2C(10^4)} = 6250\,\Omega$

P 6.11 $\alpha = 2000/2 = 1000$

$$R = \frac{1}{2\alpha C} = \frac{10^6}{(2000)(18)} = 27.78\,\Omega$$

$$v(0^+) = -24\,\text{V}$$

$$i_R(0^+) = \frac{-24}{27.78} = -864\,\text{mA}$$

$$\frac{dv}{dt} = 2400e^{-200t} + 21{,}600e^{-1800t}$$

$$\frac{dv(0^+)}{dt} = 2400 + 21{,}600 = 24{,}000\,\text{V/s}$$

$$i_C(0^+) = 18 \times 10^{-6}(24{,}000) = 432\,\text{mA}$$

$$i_L(0^+) = -[i_R(0^+) + i_C(0^+)] = -[-864 + 432] = 432\,\text{mA}$$

P 6.12 [a] $2\alpha = 200; \qquad \alpha = 100\,\text{rad/s}$

$$2\sqrt{\alpha^2 - \omega_o^2} = 120; \qquad \omega_o = 80\,\text{rad/s}$$

$$C = \frac{1}{2\alpha R} = \frac{1}{200(200)} = 25\,\mu F$$

$$L = \frac{1}{\omega_o^2 C} = \frac{10^6}{(80)^2(25)} = 6.25\,\text{H}$$

$$i_C(0^+) = A_1 + A_2 = 15\,\text{mA}$$

$$\frac{di_C}{dt} + \frac{di_L}{dt} + \frac{di_R}{dt} = 0$$

$$\frac{di_C(0)}{dt} = -\frac{di_L(0)}{dt} - \frac{di_R(0)}{dt}$$

$$\frac{di_L(0)}{dt} = \frac{0}{6.25} = 0\,\text{A/s}$$

$$\frac{di_R(0)}{dt} = \frac{1}{R}\frac{dv(0)}{dt} = \frac{1}{R}\frac{i_C(0)}{C} = \frac{15 \times 10^{-3}}{(200)(25 \times 10^{-6})} = 3\,\text{A/s}$$

$$\therefore \quad \frac{di_C(0)}{dt} = -3\,\text{A/s}$$

$$\therefore \quad 40A_1 + 160A_2 = 3$$

$$A_1 + 4A_2 = 75 \times 10^{-3}; \qquad \therefore \quad A_1 = -5\,\text{mA}; \qquad A_2 = 20\,\text{mA}$$

$$\therefore \quad i_C = 20e^{-160t} - 5e^{-40t}\,\text{mA}, \qquad t \geq 0$$

[b] By hypothesis

$$v = A_3e^{-160t} + A_4e^{-40t}, \qquad t \geq 0$$

$$v(0) = A_3 + A_4 = 0$$

$$\frac{dv(0)}{dt} = \frac{15 \times 10^{-3}}{25 \times 10^{-6}} = 600\,\text{V/s}$$

$$-160A_3 - 40A_4 = 600; \qquad \therefore \quad A_3 = -5\,\text{V}; \qquad A_4 = 5\,\text{V}$$

$$v = -5e^{-160t} + 5e^{-40t}\,\text{V}, \qquad t \geq 0$$

[c] $i_R(t) = \dfrac{v}{200} = -25e^{-160t} + 25e^{-40t}\,\text{mA}, \qquad t \geq 0^+$

[d] $i_L = -i_R - i_C$

$$i_L = 5e^{-160t} - 20e^{-40t}\,\text{mA}, \qquad t \geq 0$$

P 6.13 From the form of the solution we have

$$v(0) = A_1 + A_2$$

$$\frac{dv(0^+)}{dt} = -\alpha(A_1 + A_2) + j\omega_d(A_1 - A_2)$$

We know both $v(0)$ and $dv(0^+)/dt$ will be real numbers. To facilitate the algebra we let these numbers be K_1 and K_2, respectively. Then our two simultaneous equations are

$$K_1 = A_1 + A_2$$

$K_2 = (-\alpha + j\omega_d)A_1 + (-\alpha - j\omega_d)A_2$

The characteristic determinate is

$$\Delta = \begin{vmatrix} 1 & 1 \\ (-\alpha + j\omega_d) & (-\alpha - j\omega_d) \end{vmatrix} = -j2\omega_d$$

The numerator determinates are

$$N_1 = \begin{vmatrix} K_1 & 1 \\ K_2 & (-\alpha - j\omega_d) \end{vmatrix} = -(\alpha + j\omega_d)K_1 - K_2$$

and $N_2 = \begin{vmatrix} 1 & K_1 \\ (-\alpha + j\omega_d) & K_2 \end{vmatrix} = K_2 + (\alpha - j\omega_d)K_1$

It follows that $A_1 = \dfrac{N_1}{\Delta} = \dfrac{\omega_d K_1 - j(\alpha K_1 + K_2)}{2\omega_d}$

and $A_2 = \dfrac{N_2}{\Delta} = \dfrac{\omega_d K_1 + j(\alpha K_1 + K_2)}{2\omega_d}$

We see from these expressions that $A_1 = A_2^*$

P 6.14 By definition, $B_1 = A_1 + A_2$. From the solution to Problem 6.13 we have

$$A_1 + A_2 = \frac{2\omega_d K_1}{2\omega_d} = K_1$$

But K_1 is $v(0)$, therefore, $B_1 = v(0)$, which is identical to Eq. (6.30).
By definition, $B_2 = j(A_1 - A_2)$. From Problem 6.13 we have

$$B_2 = j(A_1 - A_2) = \frac{j[-2j(\alpha K_1 + K_2)]}{2\omega_d} = \frac{\alpha K_1 + K_2}{\omega_d}$$

It follows that

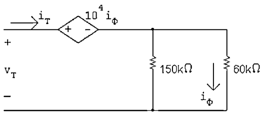
$K_2 = -\alpha K_1 + \omega_d B_2$, but $K_2 = \dfrac{dv(0^+)}{dt}$ and $K_1 = B_1$

Thus we have

$$\frac{dv}{dt}(0^+) = -\alpha B_1 + \omega_d B_2,$$

which is identical to Eq. (6.31).

P 6.15



$$v_T = 10^4 \frac{i_T(150\times 10^3)}{210\times 10^3} + \frac{(150)(60)10^6}{210\times 10^3} i_T$$

$$\frac{v_T}{i_T} = \frac{1500\times 10^3}{210} + \frac{9000\times 10^3}{210} = \frac{10{,}500}{210}\times 10^3 = 50\,\text{k}\Omega$$

$$V_o = \frac{75}{10}(6) = 45\,\text{V}; \qquad I_o = 0$$

$$i_C(0) = -i_R(0) - i_L(0) = -\frac{45}{50{,}000} = -0.9\,\text{mA}$$

$$\frac{i_C(0)}{C} = \frac{-0.9}{1.25}\times 10^6 = -720\times 10^3$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^9}{(8)(1.25)} = 10^8; \qquad \omega_o = 10^4 \text{ rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{10^9}{(2)(50)(1.25)\times 10^3} = 8000 \text{ rad/s}$$

$$\omega_d = \sqrt{(100-64)\times 10^6} = 6000 \text{ rad/s}$$

$$v_o = B_1 e^{-8000t}\cos 6000t + B_2 e^{-8000t}\sin 6000t$$

$$v_o(0) = B_1 = 45\,\text{V}$$

$$\frac{dv_o}{dt}(0) = 6000B_2 - 8000B_1 = -720\times 10^3$$

$$\therefore\ 6000B_2 = 8000(45) - 720\times 10^3; \qquad \therefore\ B_2 = -60\,\text{V}$$

$$v_o = 45e^{-8000t}\cos 6000t - 60e^{-8000t}\sin 6000t\,\text{V}, \qquad t\geq 0$$

P 6.16 [a] $\alpha = \dfrac{1}{2RC} = 1250, \qquad \omega_o = 10^3, \qquad$ therefore overdamped

$s_1 = -500, \qquad s_2 = -2000$

therefore $\quad v = A_1e^{-500t} + A_2e^{-2000t}$

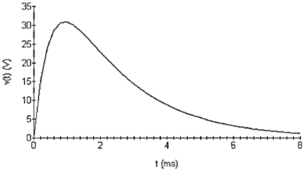
$v(0^+) = 0 = A_1 + A_2; \qquad \left[\dfrac{dv(0^+)}{dt}\right] = \dfrac{i_C(0^+)}{C} = 98{,}000\,\text{V/s}$

Therefore $\quad -500A_1 - 2000A_2 = 98{,}000$

$A_1 = \dfrac{+980}{15}, \quad A_2 = \dfrac{-980}{15}$

$v(t) = \left[\dfrac{980}{15}\right][e^{-500t} - e^{-2000t}]\,\text{V}, \qquad t \geq 0$

[b]



Example 6.4: $\quad v_{\max} \cong 74\,\text{V} \quad$ at 1.4 ms

Example 6.5: $\quad v_{\max} \cong 36.1\,\text{V} \quad$ at 1.0 ms

Problem 6.16: $\quad v_{\max} \cong 30.9 \quad$ at 0.92 ms

P 6.17 [a] $v = L\left(\dfrac{di_L}{dt}\right) = 16[e^{-20{,}000t} - e^{-80{,}000t}]\,\text{V}, \qquad t \geq 0$

[b] $i_R = \dfrac{v}{R} = 40[e^{-20{,}000t} - e^{-80{,}000t}]\,\text{mA}, \qquad t \geq 0^+$

[c] $i_C = I - i_L - i_R = [-8e^{-20{,}000t} + 32e^{-80{,}000t}]\,\text{mA}, \qquad t \geq 0^+$

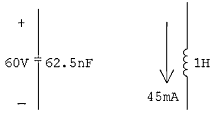
P 6.18 [a] $v = L\left(\dfrac{di_L}{dt}\right) = 40e^{-32{,}000t}\sin 24{,}000t\,\text{V}, \qquad t \geq 0$

[b] $i_C(t) = I - i_R - i_L = 24 \times 10^{-3} - \dfrac{v}{625} - i_L$

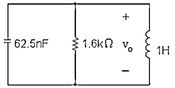
$= [24e^{-32{,}000t}\cos 24{,}000t - 32e^{-32{,}000t}\sin 24{,}000t]\,\text{mA}, \qquad t \geq 0^+$

P 6.19 $v = L\left(\frac{di_L}{dt}\right) = 960{,}000te^{-40{,}000t}\,\text{V}, \qquad t \geq 0$

P 6.20 $t < 0: \qquad V_o = 60\,\text{V}, \qquad I_o = 45\,\text{mA}$



$t > 0$:



$$i_R(0) = \frac{60}{1600} = 37.5\,\text{mA}; \qquad i_L(0) = 45\,\text{mA}$$

$$i_C(0) = -37.5 - 45 = -82.5\,\text{mA}$$

$$\alpha = \frac{1}{2RC} = \frac{10^9}{3200(62.5)} = 5000 \text{ rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^9}{62.5} = 16 \times 10^6$$

$$s_{1,2} = -5000 \pm \sqrt{25 \times 10^6 - 16 \times 10^6} = -5000 \pm 3000$$

$$s_1 = -2000 \text{ rad/s}; \qquad s_2 = -8000 \text{ rad/s}$$

$$\therefore \quad v_o = A_1e^{-2000t} + A_2e^{-8000t}$$

$$A_1 + A_2 = v_o(0) = 60$$

$$\frac{dv_o}{dt}(0) = -2000A_1 - 8000A_2 = \frac{-82.5 \times 10^{-3}}{62.5 \times 10^{-9}} = -1320 \times 10^3$$

Solving, $\qquad A_1 = -140\,\text{V}, \qquad A_2 = 200\,\text{V}$

$$\therefore \quad v_o = -140e^{-2000t} + 200e^{-8000t}\,\text{V}, \qquad t \geq 0$$

P 6.21 $\omega_o^2 = \frac{1}{LC} = \frac{16 \times 10^6}{0.64} = 25 \times 10^6$

$\alpha = \frac{1}{2RC} = \frac{16 \times 10^6}{4000} = 4000$ nepers; $\qquad \alpha^2 = 16 \times 10^3$

$\omega_d = \sqrt{(25 - 16) \times 10^6} = 3000$ rad/s

$s_{1,2} = -4000 \pm j3000$rad/s

$v_o(t) = B_1e^{-4000t}\cos 3000t + B_2e^{-4000t}\sin 3000t$

$v_o(0) = B_1 = 60\,\text{V}$

$i_R(0) = \frac{60}{2000} = 30\,\text{mA}$

$i_\text{L}(0) = 45\,\text{mA}$

$i_\text{C}(0) = -i_R(0) - i_\text{L}(0) = -75\,\text{mA}$

$\frac{i_\text{C}(0)}{C} = (-75 \times 10^{-3})(16 \times 10^6) = -12 \times 10^5$

$\frac{dv_o}{dt}(0) = -4000B_1 + 3000B_2 = -12 \times 10^5$

$\therefore \quad 3B_2 = 4B_1 - 1200 = 240 - 1200 = -960; \quad \therefore \quad B_2 = -320\,\text{V}$

$v_o(t) = 60e^{-4000t}\cos 3000t - 320e^{-4000t}\sin 3000t\,\text{V}, \qquad t \geq 0$

P 6.22 $\omega_o^2 = \frac{1}{LC} = \frac{16 \times 10^6}{0.16} = 10^8; \qquad \omega_o = 10^4$

$\alpha = \frac{1}{2RC} = \frac{16 \times 10^6}{1600} = 10^4$

$\therefore \quad \alpha^2 = \omega_o^2$ (critical damping)

$v_o(t) = D_1te^{-10{,}000t} + D_2e^{-10{,}000t}$

$v_o(0) = D_2 = 60\,\text{V}$

$$i_R(0) = \frac{60}{800} = 75\,\text{mA}$$

$$i_{\text{L}}(0) = 45\,\text{mA}$$

$$i_{\text{C}}(0) = -120\,\text{mA}$$

$$\frac{dv_o}{dt}(0) = -10{,}000D_2 + D_1$$

$$\frac{i_{\text{C}}(0)}{C} = (-120 \times 10^{-3})(16 \times 10^6) = -1920 \times 10^3$$

$$D_1 - 10{,}000D_2 = -1920 \times 10^3; \qquad D_1 = -1320 \times 10^3\text{V/s}$$

$$v_o(t) = (60 - 132 \times 10^4 t)e^{-10{,}000t}\,\text{V}, \qquad t > 0$$

P 6.23 [a] $2\alpha = 5000; \quad \alpha = 2500\,\text{rad/s}$

$$\sqrt{\alpha^2 - \omega_o^2} = 1500; \qquad \omega_o^2 = 4 \times 10^6; \qquad \omega_o = 2000\,\text{rad/s}$$

$$\alpha = \frac{R}{2L} = 2500; \qquad R = 5000L$$

$$\omega_o^2 = \frac{1}{LC} = 4 \times 10^6; \qquad L = \frac{10^9}{4 \times 10^6(50)} = 5\text{H}$$

$$R = 25{,}000\,\Omega$$

[b] $i(0) = 0$

$$L\frac{di(0)}{dt} = v_c(0); \qquad \frac{1}{2}(50) \times 10^{-9}v_c^2(0) = 90 \times 10^{-6}$$

$$\therefore \quad v_c^2(0) = 3600; \qquad v_c(0) = 60\,\text{V}$$

$$\frac{di(0)}{dt} = \frac{60}{5} = 12\,\text{A/s}$$

[c] $i(t) = A_1e^{-1000t} + A_2e^{-4000t}$

$$i(0) = A_1 + A_2 = 0$$

$$\frac{di(0)}{dt} = -1000A_1 - 4000A_2 = 12$$

$$\therefore \quad A_1 + 4A_2 = -12 \times 10^{-3}$$

$$\therefore \quad A_2 = -4\,\text{mA}; \qquad A_1 = +4\,\text{mA}$$

$$i(t) = +4e^{-1000t} - 4e^{-4000t}\,\text{mA} \qquad t \geq 0$$

[d] $\dfrac{di(t)}{dt} = -4e^{-1000t} + 16e^{-4000t}$

$\dfrac{di}{dt} = 0$ when $16e^{-4000t} = 4e^{-1000t}$

or $e^{3000t} = 4$

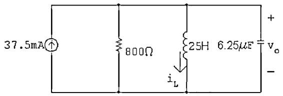
$\therefore \quad t = \dfrac{\ln 4}{3000}\mu\text{s} = 462.10\,\mu\text{s}$

[e] $i_{\max} = 4e^{-0.4621} - 4e^{-1.8484} = 1.89\,\text{mA}$

[f] $v_L(t) = 5\dfrac{di}{dt} = [-20e^{-1000t} + 80e^{-4000t}]\,\text{V}, \quad t \geq 0^+$

P 6.24 $i_L(0^-) = i_L(0^+) = 37.5\,\text{mA}$

For $t > 0$



$i_L(0^-) = i_L(0^+) = 37.5\,\text{mA}$

$\alpha = \dfrac{1}{2RC} = 100\,\text{rad/s}; \qquad \omega_o^2 = \dfrac{1}{LC} = 6400$

$s_1 = -40\,\text{rad/s} \qquad s_2 = -160\,\text{rad/s}$

$v_o(\infty) = 0 = V_f$

$v_o = A_1'e^{-40t} + A_2'e^{-160t}$

$i_C(0^+) = -37.5 + 37.5 + 0 = 0$

$\therefore \quad \dfrac{dv_o}{dt} = 0$

$\dfrac{dv_o}{dt}(0) = -40A_1' - 160A_2'$

$\therefore \quad A_1' + 4A_2' = 0; \qquad A_1' + A_2' = 0$

$\therefore \quad A_1' = 0; \qquad A_2' = 0$

$\therefore \quad v_o = 0$ for $t \geq 0$

Note: $\quad v_o(0) = 0; \qquad v_o(\infty) = 0; \qquad \dfrac{dv_o(0)}{dt} = 0$

Hence the 37.5 mA current circulates between the current source and the ideal inductor in the equivalent circuit. In the original circuit the 30 V source sustains a current of 37.5 mA in the inductor. This is an example of a circuit going directly into steady state when the switch is closed. There is no transient period, or interval.

P 6.25 $\quad i_C(0) = 0; \qquad v_o(0) = 200\,\text{V}$

$$\alpha = \frac{R}{2L} = \frac{4}{2(0.04)} = 50\,\text{rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^3}{0.4} = 2500$$

$\therefore \quad \alpha^2 = \omega_o^2; \qquad$ critical damping

$$v_o(t) = V_f + D_1' te^{-50t} + D_2' e^{-50t}$$

$$V_f = 100\,\text{V}$$

$$v_o(0) = 100 + D_2' = 200; \qquad D_2' = 100\,\text{V}$$

$$\frac{dv_o}{dt}(0) = -50D_2' + D_1' = 0$$

$$D_1' = 50D_2' = 5000\ \text{V/s}$$

$$v_o = 100 + 5000te^{-50t} + 100e^{-50t}\,\text{V}, \quad t \geq 0$$

P 6.26 $\quad \alpha = 800\,\text{rad/s}; \qquad \omega_d = 600\,\text{rad/s}$

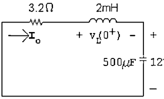
$$\omega_o^2 - \alpha^2 = 36 \times 10^4; \qquad \omega_o^2 = 100 \times 10^4; \quad w_o = 1000\,\text{rad/s}$$

$$\alpha = \frac{R}{2L} = 800; \qquad R = 1600L$$

$$\frac{1}{LC} = 100 \times 10^4; \qquad L = \frac{10^6}{(100 \times 10^4)(500)} = 2\,\text{mH}$$

$$\therefore \quad R = 3.2\,\Omega$$

$i(0^+) = B_1 = 0\,\text{A}; \qquad$ at $t = 0^+$



$$12 + 0 + v_L(0^+) = 0; \qquad v_L(0^+) = -12\,\text{V}$$

$$\frac{di(0^+)}{dt} = \frac{-12}{0.002} = -6000\,\text{A/s}$$

$$\therefore \quad \frac{di(0^+)}{dt} = 600B_2 - 800B_1 = -6000$$

$$\therefore \quad 600B_2 = 800B_1 - 6000; \qquad \therefore \quad B_2 = -10\,\text{A}$$

$$\therefore \quad i = -10e^{-800t}\sin 600t\,\text{A}, \quad t \geq 0$$

P 6.27 From Prob. 6.26 we know v_c will be of the form

$$v_c = B_3e^{-800t}\cos 600t + B_4e^{-800t}\sin 600t$$

From Prob. 6.26 we have

$$v_c(0) = -12\,\text{V} = B_3$$

and

$$\frac{dv_c(0)}{dt} = \frac{i_C(0)}{C} = 0$$

$$\frac{dv_c(0)}{dt} = 600B_4 - 800B_3$$

$$\therefore \quad 600B_4 = 800B_3 + 0; \qquad B_4 = -16\,\text{V}$$

$$v_c(t) = -12e^{-800t}\cos 600t - 16e^{-800t}\sin 600t\,\text{V} \qquad t \geq 0$$

P 6.28 $v_C(0^+) = \frac{1}{2}(240) = 120\,\text{V}$

$i_L(0^+) = 60\,\text{mA}; \qquad i_L(\infty) = \frac{240}{5} \times 10^{-3} = 48\,\text{mA}$

$\alpha = \frac{1}{2RC} = \frac{10^6}{2(2500)(5)} = 40$

$\omega_o^2 = \frac{1}{LC} = \frac{10^6}{400} = 2500$

$\alpha^2 = 1600; \qquad \alpha^2 < \omega_o^2; \qquad \therefore \quad \text{underdamped}$

$s_{1,2} = -40 \pm j\sqrt{2500 - 1600} = -40 \pm j30 \text{ rad/s}$

$$\begin{aligned} i_L &= I_f + B_1' e^{-\alpha t}\cos\omega_d t + B_2' e^{-\alpha t}\sin\omega_d t \\ &= 48 + B_1' e^{-40t}\cos 30t + B_2' e^{-40t}\sin 30t \end{aligned}$$

$i_L(0) = 48 + B_1'; \qquad B_1' = 60 - 48 = 12\,\text{mA}$

$\frac{di_L}{dt}(0) = 30B_2' - 40B_1' = \frac{120}{80} = 1.5 = 1500 \times 10^{-3}$

$\therefore \quad 30B_2 = 40(12) \times 10^{-3} + 1500 \times 10^{-3}; \qquad B_2' = 66\,\text{mA}$

$\therefore \quad i_L = 46 + 12e^{-40t}\cos 30t + 66e^{-40t}\sin 30t\,\text{mA}, \qquad t \geq 0$

P 6.29 $\alpha = \frac{R}{2L} = 5000\,\text{rad/s}$

$\omega_o^2 = \frac{1}{LC} = \frac{10^9}{200} = 50 \times 10^6$

$s_{1,2} = -5000 \pm \sqrt{25 \times 10^6 - 50 \times 10^6} = -5000 \pm j5000\,\text{rad/s}$

$v_o = V_f + B_1' e^{-5000t}\cos 5000t + B_2' e^{-5000t}\sin 5000t$

$v_o(0) = 0 = V_f + B_1'$

$v_o(\infty) = 40\,\text{V}; \qquad \therefore \quad B_1' = -40\,\text{V}$

$\frac{dv_o(0)}{dt} = 0 = 5000B_2' - 5000B_1'$

$\therefore \quad B_2' = B_1' = -40\,\text{V}$

$v_o = 40 - 40e^{-5000t}\cos 5000t - 40e^{-5000t}\sin 5000t\,\text{V}, \quad t \geq 0$

P 6.30 $\alpha = \dfrac{1}{2RC} = \dfrac{10^6}{(1600)(6.25)} = 100; \qquad \alpha^2 = 10^4$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^6}{(25)(6.25)} = 6400$$

$$s_{1,2} = -200 \pm \sqrt{10^4 - 6400} = -100 \pm 60$$

$$s_1 = -40 \text{ rad/s}; \qquad s_2 = -160 \text{ rad/s}$$

$$v_o(\infty) = 0 = V_f$$

$$\therefore \quad v_o = A_1' e^{-40t} + A_2' e^{-160t}$$

$$v_o(0) = 30 = A_1' + A_2'$$

Note: $\qquad i_C(0^+) = 0$

$$\therefore \quad \frac{dv_o}{dt}(0) = 0 = -40A_1' - 160A_2'$$

Solving, $\qquad A_1' = 40\,\text{V}, \qquad A_2' = -10\,\text{V}$

$$v_o(t) = 40e^{-40t} - 10e^{-160t}\,\text{V}, \qquad t > 0^+$$

P 6.31 [a] $i_o = I_f + A_1' e^{-40t} + A_2' e^{-160t}$

$$I_f = \frac{30}{800} = 37.5\text{mA}; \qquad i_o(0) = 0$$

$$0 = 37.5 \times 10^{-3} + A_1' + A_2', \qquad \therefore \quad A_1' + A_2' = -37.5 \times 10^{-3}$$

$$\frac{di_o}{dt}(0) = \frac{30}{25} = -40A_1' - 160A_2'$$

Solving, $\qquad A_1' = -40\,\text{mA}; \qquad A_2' = 2.5\,\text{mA}$

$$i_o = 37.5 - 40e^{-40t} + 2.5e^{-160t}\,\text{mA}, \quad t \geq 0$$

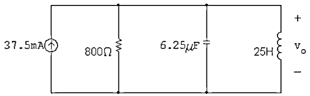
[b] $\dfrac{di_o}{dt} = [1600e^{-40t} - 400e^{-160t}] \times 10^{-3}$

$$L\frac{di_o}{dt} = 25(1.6)e^{-40t} - 25(0.4)e^{-160t}$$

$$\therefore \quad v_o = 40e^{-40t} - 10e^{-160t}\,\text{V}, \quad t \geq 0$$

This agrees with the solution to Problem 6.30.

P 6.32 For $t > 0$



$$\alpha = \frac{1}{2RC} = 100; \qquad \frac{1}{LC} = 6400$$

$$s_{1,2} = -100 \pm 60$$

$$s_1 = -40\,\text{rad/s}; \qquad s_2 = -160\,\text{rad/s}$$

$$v_o = V_f + A_1'e^{-40t} + A_2'e^{-160t}$$

$$V_f = 0; \qquad v_o(0^+) = 0; \qquad i_C(0^+) = 37.5\,\text{mA}$$

$$\therefore \quad A_1' + A_2' = 0$$

$$\frac{dv_o(0^+)}{dt} = \frac{i_C(0^+)}{6.25 \times 10^{-6}} = 6000\,\text{V/s}$$

$$\frac{dv_o(0^+)}{dt} = -40A_1' - 160A_2'$$

$$-40A_1' - 160A_2' = 6000$$

$$A_1' + 4A_2' = -150$$

$$A_1' + A_2' = 0$$

$$\therefore \quad A_1' = 50\,\text{V}; \qquad A_2' = -50\,\text{V}$$

$$v_o = 50e^{-40t} - 50e^{-160t}\,\text{V}, \qquad t \geq 0$$

P 6.33 [a] From the solution to Prob. 6.32 $s_1 = -40\,\text{rad/s}$ and $s_2 = -160\,\text{rad/s}$, therefore

$$i_o = I_f + A_1' e^{-40t} + A_2' e^{-160t}$$

$$I_f = 37.5\,\text{mA}; \qquad i_o(0^+) = 0; \qquad \frac{di_o(0^+)}{dt} = 0$$

$$\therefore \quad 0 = 37.5 + A_1' + A_2'; \qquad -40A_1' - 160A_2' = 0$$

It follows that

$$A_1' = -50\,\text{mA}; \qquad A_2' = 12.5\,\text{mA}$$

$$\therefore \quad i_o = 37.5 - 50e^{-40t} + 12.5e^{-160t}\,\text{mA}, \qquad t \geq 0$$

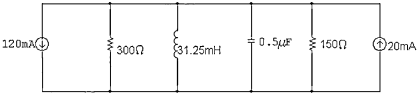
[b] $$\frac{di_o}{dt} = 2e^{-40t} - 2e^{-160t}$$

$$v_o = L\frac{di_o}{dt} = 25[2e^{-40t} - 2e^{-160t}]$$

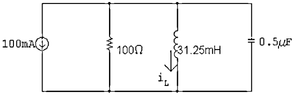
$$v_o = 50e^{-40t} - 50e^{-160t}\,\text{V}, \qquad t \geq 0$$

This agrees with the solution to Problem 6.32

P 6.34 $t < 0:$ $\quad i_L = 3/150 = 20\,\text{mA}$
$t > 0:$



$300\|150 = 100\,\Omega$



$$i_L(0) = 20\,\text{mA}, \qquad i_L(\infty) = -100\,\text{mA}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^9}{(31.25)(0.5)} = 64 \times 10^6; \qquad \omega_o = 8000\ \text{rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{10^6}{(200)(0.5)} = 10^4; \qquad \alpha^2 = 100 \times 10^6$$

$$\alpha^2 - \omega_o^2 = (100 - 64)10^6 = 36 \times 10^6$$

$$s_{1,2} = -10{,}000 \pm 6000$$

$$s_1 = -4000 \text{ rad/s}; \qquad s_2 = -16{,}000 \text{ rad/s}$$

$$i_{\text{L}} = I_f + A_1' e^{-4000t} + A_2' e^{-16{,}000t}$$

$$i_{\text{L}}(\infty) = I_f = -100\text{mA}$$

$$i_{\text{L}}(0) = A_1' + A_2' + I_f = 20\,\text{mA}$$

$$\therefore \quad A_1' + A_2' - 100 = 20 \quad \text{so} \quad A_1' + A_2' = 120\,\text{mA}$$

$$\frac{di_{\text{L}}}{dt}(0) = 0 = -4000A_1 - 16{,}000A_2'$$

Solving, $\quad A_1' = 160\,\text{mA}, \qquad A_2' = -40\,\text{mA}$

$$i_{\text{L}} = -100 + 160e^{-4000t} - 40e^{-16{,}000t}\,\text{mA}, \qquad t \geq 0$$

P 6.35 $\quad \omega_o^2 = \dfrac{1}{LC} = \dfrac{10^6}{200} = 5000$

$$\alpha = \frac{R}{2L} = \frac{400}{40} = 10; \qquad \alpha^2 = 100$$

$$\alpha^2 < \omega_o^2 \quad \therefore \qquad \text{underdamped}$$

$$s_{1,2} = -10 \pm j\sqrt{4900} = -10 \pm j70 \text{ rad/s}$$

$$i = B_1 e^{-10t}\cos 70t + B_2 e^{-10t}\sin 70t$$

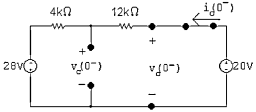
$$i(0) = B_1 = 147/420 = 350\,\text{mA}$$

$$\frac{di}{dt}(0) = 70B_2 - 10B_1 = 0$$

$$\therefore \quad B_2 = 50\,\text{mA}$$

$$i = 50e^{-10t}(7\cos 70t + \sin 70t)\,\text{mA}, \qquad t \geq 0^+$$

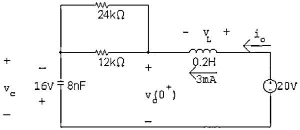
P 6.36 [a] $t < 0$:



$$i_o(0^-) = \frac{48}{16{,}000} = 3\,\text{mA}$$

$$v_C(0^-) = 20 - (12{,}000)(0.003) = -16\,\text{V}$$

$t = 0^+$:



$$12\,\text{k}\Omega \| 24\,\text{k}\Omega = 8\,\text{k}\Omega$$

$$\therefore\quad v_o(0^+) = (0.003)(8000) - 16 = 24 - 16 = 8\,\text{V}$$

$$\text{and } v_L(0^+) = 20 - 8 = 12\,\text{V}$$

[b] $v_o(t) = 8000i_o + v_C$

$$\frac{dv_o}{dt}(t) = 8000\frac{di_o}{dt} + \frac{dv_C}{dt}$$

$$\frac{dv_o}{dt}(0^+) = 8000\frac{di_o}{dt}(0^+) + \frac{dv_C}{dt}(0^+)$$

$$20 = L\frac{di_o}{dt} + 8000i_o + v_C$$

$$20 = 0.2\frac{di_o}{dt}(0^+) + 24 - 16$$

$$\therefore\quad 0.2\frac{di_o}{dt}(0^+) = 20 - 8 = 12$$

$$\frac{di_o}{dt}(0^+) = \frac{12}{0.2} = 60\ \text{A/s}$$

$$C\frac{dv_c}{dt}(0^+) = i_o(0^+)$$

$$\therefore \quad \frac{dv_c}{dt}(0^+) = \frac{3 \times 10^{-3}}{8 \times 10^{-9}} = 375{,}000$$

$$\therefore \quad \frac{dv_o}{dt}(0^+) = 8000(60) + 375{,}000 = 855{,}000 \text{ V/s}$$

[c] $\omega_o^2 = \dfrac{1}{LC} = \dfrac{10^9}{1.6} = 625 \times 10^6; \qquad \omega_o = 25{,}000$ rad/s

$$\alpha = \frac{R}{2L} = \frac{8000}{0.4} = 20{,}000 \text{ rad/s}; \qquad \alpha^2 = 400 \times 10^6$$

$\alpha^2 < \omega_o^2$ underdamped

$$s_{1,2} = -20{,}000 \pm j15{,}000 \text{ rad/s}$$

$$v_o(t) = V_f + B_1' e^{-20{,}000t} \cos 15{,}000t + B_2' e^{-20{,}000t} \sin 15{,}000t$$

$$V_f = v_o(\infty) = 20 \text{ V}$$

$$8 = 20 + B_1'; \qquad B_1' = -12 \text{ V}$$

$$-20{,}000B_1' + 15{,}000B_2' = 855{,}000$$

Solving, $\quad B_2' = 41$ V

$$\therefore \quad v_o(t) = 20 - 12e^{-20{,}000t} \cos 15{,}000t + 41e^{-20{,}000t} \sin 15{,}000t \text{ V}, \qquad t \geq 0^+$$

P 6.37 [a] $t < 0$:

$$i_o = \frac{120}{8000} = 15 \text{ mA}; \qquad v_o = (5000)(0.015) = 75 \text{ V}$$

$t > 0$:

$$\alpha = \frac{R}{2L} = \frac{5000}{2(1)} = 2500 \text{ rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^9}{(1)(250)} = 4 \times 10^6 = 400 \times 10^4$$

$$\alpha^2 - \omega_o^2 = 625 \times 10^4 - 400 \times 10^4 = 225 \times 10^4$$

$$\therefore \quad s_{1,2} = -2500 \pm 1500$$

$$s_1 = -1000 \text{ rad/s} \qquad s_2 = -4000 \text{ rad/s}$$

$$\therefore \quad i_o(t) = A_1 e^{-1000t} + A_2 e^{-4000t}$$

$$i_o(0) = A_1 + A_2 = 15 \times 10^{-3}$$

$$\frac{di_o}{dt}(0) = -1000A_1 - 4000A_2 = 0$$

Solving, $\quad A_1 = 20\,\text{mA}; \qquad A_2 = -5\,\text{mA}$

$$i_o(t) = 20e^{-1000t} - 5e^{-4000t}\,\text{mA}, \qquad t \geq 0^+$$

[b] $v_o(t) = A_1e^{-1000t} + A_2e^{-4000t}$

$$v_o(0) = A_1 + A_2 = 75$$

$$\frac{dv_o}{dt}(0) = -1000A_1 - 4000A_2 = \frac{-15 \times 10^{-3}}{250 \times 10^{-9}}$$

Solving, $\quad A_1 = 80\,\text{V}; \qquad A_2 = -5\,\text{V}$

$$v_o(t) = 80e^{-1000t} - 5e^{-4000t}\,\text{V}, \qquad t \geq 0^+$$

Check:

$$5000i_o + 1\frac{di_o}{dt} = v_o$$

$$5000i_o = 100e^{-1000t} - 25e^{-4000t}$$

$$\frac{di_o}{dt} = -20e^{-1000t} + 20e^{-4000t}$$

$$\therefore \quad 5000i_o + \frac{di_o}{dt} = 80e^{-1000t} - 5e^{-4000t}\,\text{V} \qquad \text{(checks)}$$

P 6.38 $\omega_o^2 = \dfrac{1}{LC} = \dfrac{10^6}{(20)(5)} = 10^4; \qquad \omega_o = 100\text{ rad/s}$

$$\alpha = \frac{1}{2RC} = \frac{10^6}{(1600)(5)} = \frac{10^4}{80} = 125\text{ rad/s}$$

$$s_{1,2} = -125 \pm \sqrt{(125)^2 - 10^4} = -125 \pm 75$$

$$s_1 = -50\text{ rad/s}; \qquad s_2 = -200\text{ rad/s}$$

$$I_f = 15\,\text{mA}$$

$$i_L = 15 + A_1'e^{-50t} + A_2'e^{-200t}$$

$$\therefore \quad -30 = 15 + A_1' + A_2'; \qquad A_1' + A_2' = -45 \times 10^{-3}$$

$$\frac{di_L}{dt} = -50A_1' - 200A_2' = \frac{60}{20} = 3$$

Solving, $\quad A_1' = -40\,\text{mA}; \qquad A_2' = -5\,\text{mA}$

$$i_L = 15 - 40e^{-50t} - 5e^{-200t}\,\text{mA}, \quad t \geq 0$$

P 6.39 $\quad \alpha = \dfrac{1}{2RC} = \dfrac{10^6}{(2500)(5)} = 80; \qquad \alpha^2 = 6400$

$$\omega_o^2 = 10^4$$

$$s_{1,2} = -80 \pm j\sqrt{10^4 - 6400} = -80 \pm j60 \text{ rad/s}$$

$$i_L = 15 + B_1'e^{-80t}\cos 60t + B_2'e^{-80t}\sin 60t$$

$$-30 = 15 + B_1' \qquad \therefore \quad B_1' = -45\,\text{mA}$$

$$\frac{di_L}{dt}(0) = -80B_1' + 60B_2' = 3$$

$$\therefore \quad B_2' = -10\,\text{mA}$$

$$i_L = 15 - 45e^{-80t}\cos 60t - 10e^{-80t}\sin 60t\,\text{mA}, \quad t \geq 0$$

P 6.40 $\quad \alpha = \dfrac{1}{2RC} = \dfrac{10^6}{(2000)(5)} = 100$

$$\alpha^2 = 10^4 = \omega_o^2 \qquad \text{critical damping}$$

$$i_L = I_f + D_1'te^{-100t} + D_2'e^{-100t} = 15 + D_1'te^{-100t} + D_2'e^{-100t}$$

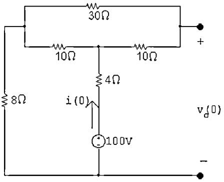
$$i_L(0) = -30 = 15 + D_2'; \qquad \therefore \quad D_2' = -45\,\text{mA}$$

$$\frac{di_L}{dt}(0) = -100D_2' + D_1' = 3000 \times 10^{-3}$$

$$\therefore \quad D_1' = 3000 \times 10^{-3} + 100(-45 \times 10^{-3}) = -1500 \times 10^{-3}$$

$$i_L = 15 - 1500te^{-100t} - 45e^{-100t}\,\text{mA}, \quad t \geq 0$$

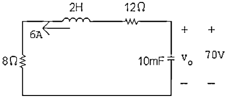
P 6.41 $t < 0$:



$$i(0) = \frac{100}{4+8+8} = \frac{100}{20} = 5\,\text{A}$$

$$v_o(0) = 100 - 5(4) - 10(5)\left(\frac{10}{50}\right) = 70\,\text{V}$$

$t > 0$:



$$\alpha = \frac{R}{2L} = \frac{20}{4} = 5, \qquad \alpha^2 = 25$$

$$\omega_o^2 = \frac{1}{LC} = \frac{100}{2} = 50$$

$\omega_o^2 > \alpha^2$ underdamped

$$v_o = B_1 e^{-\alpha t}\cos\omega_d t + B_2 e^{-\alpha t}\sin\omega_d t; \qquad \omega_d = \sqrt{50-25} = 5$$

$$v_o = B_1 e^{-5t}\cos 5t + B_2 e^{-5t}\sin 5t$$

$$v_o(0) = B_1 = 70\,\text{V}$$

$$C\frac{dv_o}{dt}(0) = -5, \qquad \frac{dv_o}{dt} = \frac{-5}{10}\times 10^3 = -500\,\text{V/s}$$

$$\frac{dv_o}{dt}(0) = -5B_1 + 5B_2 = -500$$

$$5B_2 = -500 + 5B_1 = -500 + 350; \qquad B_2 = -150/5 = -30\,\text{V}$$

$$\therefore\ v_o = 70e^{-5t}\cos 5t - 30e^{-5t}\sin 5t\,\text{V}, \qquad t \geq 0$$

P 6.42 [a] Let i be the current in the direction of the voltage drop $v_o(t)$. Then by hypothesis

$$i = i_f + B_1'e^{-\alpha t}\cos\omega_d t + B_2'e^{-\alpha t}\sin\omega_d t$$

$$i_f = i(\infty) = 0, \qquad i(0) = \frac{V_g}{R} = B_1'$$

Therefore $\quad i = B_1'e^{-\alpha t}\cos\omega_d t + B_2'e^{-\alpha t}\sin\omega_d t$

$$L\frac{di(0)}{dt} = 0, \qquad \text{therefore} \quad \frac{di(0)}{dt} = 0$$

$$\frac{di}{dt} = \left[(\omega_d B_2' - \alpha B_1')\cos\omega_d t - (\alpha B_2' + \omega_d B_1')\sin\omega_d t\right]e^{-\alpha t}$$

Therefore $\quad \omega_d B_2' - \alpha B_1' = 0; \qquad B_2' = \frac{\alpha}{\omega_d}B_1' = \frac{\alpha}{\omega_d}\frac{V_g}{R}$

Therefore

$$v_o = L\frac{di}{dt} = -\left\{L\left(\frac{\alpha^2 V_g}{\omega_d R} + \frac{\omega_d V_g}{R}\right)\sin\omega_d t\right\}e^{-\alpha t}$$

$$= -\left\{\frac{LV_g}{R}\left(\frac{\alpha^2}{\omega_d} + \omega_d\right)\sin\omega_d t\right\}e^{-\alpha t}$$

$$= -\frac{V_g L}{R}\left(\frac{\alpha^2 + \omega_d^2}{\omega_d}\right)e^{-\alpha t}\sin\omega_d t$$

$$v_o = -\frac{V_g}{RC\omega_d}e^{-\alpha t}\sin\omega_d t\,\text{V}, \quad t \geq 0^+$$

[b] $\dfrac{dv_o}{dt} = -\dfrac{V_g}{\omega_d RC}\{\omega_d\cos\omega_d t - \alpha\sin\omega_d t\}e^{-\alpha t}$

$$\frac{dv_o}{dt} = 0 \quad \text{when} \quad \tan\omega_d t = \frac{\omega_d}{\alpha}$$

Therefore $\quad \omega_d t = \tan^{-1}(\omega_d/\alpha) \quad$ (smallest t)

$$t = \frac{1}{\omega_d}\tan^{-1}\left(\frac{\omega_d}{\alpha}\right)$$

P 6.43 [a] From Problem 6.42 we have

$$v_o = \frac{-V_g}{RC\omega_d}e^{-\alpha t}\sin\omega_d t$$

$$\alpha = \frac{R}{2L} = \frac{120}{0.01} = 12{,}000\,\text{rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^{12}}{2500} = 400 \times 10^6$$

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = 16\,\text{krad/s}$$

$$\frac{-V_g}{RC\omega_d} = \frac{-(-600)10^9}{(120)(500)(16)\times 10^3} = 625$$

$$\therefore \quad v_o = 625e^{-12{,}000t}\sin 16{,}000t\,\text{V}$$

[b] From Problem 6.42

$$t_d = \frac{1}{\omega_d}\tan^{-1}\left(\frac{\omega_d}{\alpha}\right) = \frac{1}{16{,}000}\tan^{-1}\left(\frac{16{,}000}{12{,}000}\right)$$

$$t_d = 57.96\,\mu\text{s}$$

[c] $v_{\max} = 625e^{-0.012(57.96)}\sin[(0.016)(57.96)] = 249.42\,\text{V}$

[d] $R = 12\,\Omega;\qquad \alpha = 1200\,\text{rad/s}$

$$\omega_d = 19{,}963.97\,\text{rad/s}$$

$$v_o = 5009.02e^{-1200t}\sin 19{,}963.97t\,\text{V}, \quad t \geq 0$$

$$t_d = 75.67\,\mu\text{s}$$

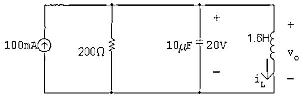
$$v_{\max} = 4565.96\,\text{V}$$

P 6.44 $t < 0$:

$$v_o(0^-) = v_o(0^+) = \frac{1000}{1250}(25) = 20\,\text{V}$$

$$i_L(0^-) = i_L(0^+) = 0$$

$t > 0$



$$-100 + \frac{20}{0.2} + i_C(0^+) + 0 = 0; \qquad \therefore \quad i_C(0^+) = 0$$

$$\frac{1}{2RC} = \frac{10^6}{(400)(10)} = 250\,\text{rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^9}{10(1.6)} = 62{,}500$$

$$\therefore \quad \alpha^2 = \omega_o^2 \quad \text{critically damped}$$

[a] $v_o = V_f + D_1'te^{-250t} + D_2'e^{-250t}$

$V_f = 0$

$\frac{dv_o(0)}{dt} = -250D_2' + D_1' = 0$

$v_o(0^+) = 20 = D_2'$

$D_1' = 250D_2' = 5000\,\text{V/s}$

$\therefore \quad v_o = 5000te^{-250t} + 20e^{-250t}\,\text{V}, \quad t \geq 0^+$

[b] $i_{\text{L}} = I_f + D_3'te^{-250t} + D_4'e^{-250t}$

$i_{\text{L}}(0^+) = 0; \qquad I_f = 100\,\text{mA}; \qquad \frac{di_{\text{L}}(0^+)}{dt} = \frac{20}{1.6} = 12.5\ \text{A/s}$

$\therefore \quad 0 = 100 + D_4'; \qquad D_4' = -100\,\text{mA};$

$-250D_4' + D_3' = 12.5; \qquad D_3' = -12.5\,\text{A/s}$

$\therefore \quad i_{\text{L}} = 100 - 12{,}500te^{-250t} - 100e^{-250t}\,\text{mA} \qquad t \geq 0$

P 6.45 [a] $w_{\text{L}} = \int_0^\infty p\,dt = \int_0^\infty v_o i_{\text{L}}\,dt$

$v_o = 5000te^{-250t} + 20e^{-250t}\,\text{V}$

$i_{\text{L}} = 0.1 - 12.5te^{-250t} - 0.1e^{-250t}\,\text{A}$

$p = 2e^{-250t} + 500te^{-250t} - 750te^{-500t} - 62{,}500t^2e^{-500t} - 2e^{-500t}\,\text{W}$

$$\frac{w_{\text{L}}}{2} = \int_0^\infty e^{-250t}\,dt + 250\int_0^\infty te^{-250t}\,dt - 375\int_0^\infty te^{-500t} -$$

$$31{,}250\int_0^\infty t^2e^{-500t}\,dt - \int_0^\infty e^{-500t}\,dt$$

$$= \left.\frac{e^{-250t}}{-250}\right|_0^\infty + \left.\frac{250}{(250)^2}e^{-250t}(-250t-1)\right|_0^\infty -$$

$$\left.\frac{375}{(500)^2}e^{-500t}(-500t-1)\right|_0^\infty -$$

$$\left.\frac{31{,}250}{(-500)^3}e^{-500t}(500^2t^2 + 1000t + 2)\right|_0^\infty -$$

$$\left.\frac{e^{-500t}}{(-500)}\right|_0^\infty$$

All the upper limits evaluate to zero hence

$$\frac{w_L}{2} = \frac{1}{250} + \frac{250}{62{,}500} - \frac{375}{25 \times 10^4} - \frac{(31{,}250)(2)}{(5)^3 10^6} - \frac{1}{500}$$

$$w_L = 8 + 8 - 3 - 1 - 4 = 8\,\text{mJ}$$

Note this value corresponds to the final energy stored in the inductor, i.e.

$$w_L(\infty) = \frac{1}{2}(1.6)(0.1)^2 = 8\,\text{mJ}.$$

[b] $v = 5000te^{-250t} + 20e^{-250t}\,\text{V}$

$$i_R = \frac{v}{200} = 25te^{-250t} + 0.1e^{-250t}\,\text{A}$$

$$p_R = vi_R = 2e^{-500t}[62{,}500t^2 + 500t + 1]$$

$$w_R = \int_0^\infty p_R\,dt$$

$$\frac{w_R}{2} = 62{,}500\int_0^\infty t^2e^{-500t}\,dt + 500\int_0^\infty te^{-500t}\,dt + \int_0^\infty e^{-500t}\,dt$$

$$= \frac{62{,}500e^{-500t}}{-125 \times 10^6}[25 \times 10^4t^2 + 1000t + 2]\Big|_0^\infty +$$

$$\frac{500e^{-500t}}{25 \times 10^4}(-500t - 1)\Big|_0^\infty + \frac{e^{-500t}}{(-500)}\Big|_0^\infty$$

Since all the upper limits evaluate to zero we have

$$\frac{w_R}{2} = \frac{62{,}500(2)}{125 \times 10^6} + \frac{500}{25 \times 10^4} + \frac{1}{500}$$

$$w_R = 2 + 4 + 4 = 10\,\text{mJ}$$

[c] $100 = i_R + i_C + i_L \quad (\text{mA})$

$$i_R + i_L = 25{,}000te^{-250t} + 100e^{-250t} + 100$$

$$-12{,}500te^{-250t} - 100e^{-250t}\,\text{mA}$$

$$= 100 + 12{,}500te^{-250t}\,\text{mA}$$

$$\therefore\; i_C = 100 - (i_R + i_L) = -12{,}500te^{-250t}\,\text{mA} = -12.5te^{-250t}\,\text{A}$$

$$p_C = vi_C = [5000te^{-250t} + 20e^{-250t}][-12.5te^{-250t}]$$

$$= -250[250t^2e^{-500t} + te^{-500t}]$$

$$\frac{w_C}{-250} = 250\int_0^\infty t^2e^{-500t}\,dt + \int_0^\infty te^{-500t}\,dt$$

$$\frac{w_C}{-250} = \frac{250e^{-500t}}{-125\times 10^6}[25\times 10^4t^2+1000t+2]\Big|_0^\infty + \frac{e^{-500t}}{25\times 10^4}(-500t-1)\Big|_0^\infty$$

Since all upper limits evaluate to zero we have

$$w_C = \frac{-250(250)(2)}{125\times 10^6} - \frac{250(1)}{25\times 10^4} = -1000\times 10^{-6} - 10\times 10^{-4} = -2\,\text{mJ}$$

Note this 2 mJ corresponds to the initial energy stored in the capacitor, i.e.,

$$w_C(0) = \frac{1}{2}(10\times 10^{-6})(20)^2 = 2\,\text{mJ}.$$

Thus $w_C(\infty) = 0$ mJ which agrees with the final value of $v = 0$.

[d] $i_s = 100$ mA

$$p_s(\text{del}) = 100v_o\,\text{mW}$$
$$= 0.1[5000te^{-250t} + 20e^{-250t}]$$
$$= 2e^{-250t} + 500te^{-250t}\,\text{W}$$
$$\frac{w_s}{2} = \int_0^\infty e^{-250t}\,dt + \int_0^\infty 250te^{-250t}\,dt$$
$$= \frac{e^{-250t}}{-250}\Big|_0^\infty + \frac{250e^{-250t}}{62{,}500}(-250t-1)\Big|_0^\infty$$
$$= \frac{1}{250} + \frac{1}{250}$$
$$w_s = \frac{2(2)}{250} = \frac{4}{250} = 16\,\text{mJ}$$

[e] $w_L = 8$ mJ (absorbed)

$w_R = 10$ mJ (absorbed)

$w_C = 2$ mJ (delivered)

$w_S = 16$ mJ (delivered)

$\sum w_{\text{del}} = w_{\text{abs}} = 18$ mJ.

P 6.46 [a] $\omega_o^2 = \dfrac{1}{LC} = \dfrac{10^9}{(0.25)(160)} = \dfrac{10^8}{4} = 25\times 10^6$

$$\alpha = \frac{R}{2L} = \omega_o = 5000\text{ rad/s}$$

$\therefore\quad R = (5000)(2)L = 2500\,\Omega$

[b] $i(0) = i_L(0) = 24\,\text{mA}$

$$v_L(0) = 90 - (0.024)(2500) = 30\,\text{V}$$

$$\frac{di}{dt}(0) = \frac{30}{0.25} = 120 \text{ A/s}$$

[c] $v_C = D_1te^{-5000t} + D_2e^{-5000t}$

$$v_C(0) = D_2 = 90\,\text{V}$$

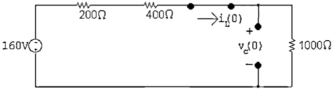
$$\frac{dv_C}{dt}(0) = D_1 - 5000D_2 = \frac{i_C(0)}{C} = \frac{-i_L(0)}{C}$$

$$D_1 - 450{,}000 = -\frac{24 \times 10^{-3}}{160 \times 10^{-9}} = -150{,}000$$

$$\therefore \quad D_1 = 300{,}000 \text{ V/s}$$

$$v_C = 300{,}000te^{-5000t} + 90e^{-5000t}\,\text{V}, \qquad t \geq 0^+$$

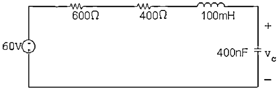
P 6.47 $t < 0$:



$$i_L(0) = \frac{-160}{1600} = -100\,\text{mA}$$

$$v_C(0) = 1000i_L(0) = -100\,\text{V}$$

$t > 0$:



$$\alpha = \frac{R}{2L} = \frac{1000}{200} \times 10^3 = 5000 \text{ rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{(10^9)(10^3)}{(100)(400)} = \frac{10^8}{4} = 25 \times 10^6$$

$\omega_o = 5000$ rad/s $\quad \therefore \quad$ critical damping

$$v_C(t) = V_f + D_1'te^{-5000t} + D_2'e^{-5000t}$$

$$v_C(0) = -100\,\text{V}; \qquad V_f = -60\,\text{V}$$

$$\therefore \quad -100 = -60 + D_2'; \qquad D_2' = -40\,\text{V}$$

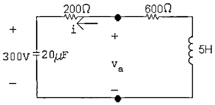
$$C\frac{dv_C}{dt}(0) = i_L(0) = -100 \times 10^{-3}$$

$$\frac{dv_C}{dt}(0) = \frac{-100 \times 10^{-3}}{400 \times 10^{-9}} = -250{,}000 \text{ V/s}$$

$$\therefore \quad D_1' = 5000(-40) - 250{,}000 = -450{,}000$$

$$v_C(t) = -60 - 450{,}000te^{-5000t} - 40e^{-5000t}\,\text{V}, \qquad t \geq 0$$

P 6.48 [a] For $t > 0$:



Since $i(0^-) = i(0^+) = 0$

$v_a(0^+) = 300\,\text{V}$

[b] $v_a = 200i + 5 \times 10^4 \int_0^t i\,dx + 300$

$$\frac{dv_a}{dt} = 200\frac{di}{dt} + 5 \times 10^4 i$$

$$\frac{dv_a(0^+)}{dt} = 200\frac{di(0^+)}{dt} + 5 \times 10^4 i(0^+) = 200\frac{di(0^+)}{dt}$$

$$-L\frac{di(0^+)}{dt} = 300$$

$$\frac{di(0^+)}{dt} = -0.2(300) = -60\,\text{A/s}$$

$$\therefore \quad \frac{dv_a(0^+)}{dt} = -12{,}000\,\text{V/s}$$

[c] $\alpha = \dfrac{R}{2L} = \dfrac{800}{10} = 80\,\text{rad/s}$

$$\omega_o^2 = \frac{1}{LC} = \frac{10^6}{(5)(20)} = 10^4$$

$$s_{1,2} = -80 \pm \sqrt{6400 - 10^4} = -80 \pm j60\,\text{rad/s}$$

Underdamped:

$$v_a = B_1 e^{-80t}\cos 60t + B_2 e^{-80t}\sin 60t$$

$$v_a(0) = B_1 = 300\,\text{V}$$

$$\frac{dv_a(0)}{dt} = -80B_1 + 60B_2 = -12{,}000; \qquad \therefore\ B_2 = 200$$

$$v_a = 300e^{-80t}\cos 60t + 200e^{-80t}\sin 60t\,\text{V}, \quad t \geq 0^+$$

P 6.49 [a] When $L = 1.6$ nH,

$$s_{1,2} = -\frac{100}{3.2\times 10^{-9}} \pm \sqrt{\left(\frac{100}{3.2}\times 10^9\right)^2 - \frac{10^{12}}{1.6\times 10^{-9}}}$$
$$= -3.125\times 10^{10} \pm 1.875\times 10^{10}$$

$$s_1 = -12.5\times 10^9\,\text{rad/s} \qquad s_2 = -50\times 10^9\,\text{rad/s}$$

$$\therefore\ v_o = V_f + A_1' e^{-12.5\times 10^9 t} + A_2' e^{-50\times 10^9 t}$$

$$V_f = 5\text{V}$$

$$v_o(0) = 1\text{V} = A_1' + A_2' + 5$$

$$\frac{dv_o(0)}{dt} = 0 = -12.5\times 10^9 A_1' - 50\times 10^9 A_2'$$

$$\therefore\ A_1' + A_2' = -4; \qquad A_1' = -4A_2'$$

$$\therefore\ A_1' = -\frac{16}{3}\text{V}; \qquad A_2' = \frac{4}{3}\text{V}$$

$$\therefore\ v_o = 5 - \frac{16}{3}e^{-12.5\times 10^9 t} + \frac{4}{3}e^{-50\times 10^9 t}\text{V} \qquad t \geq 0$$

[b] When $L = 2.5$ nH,

$$\frac{R}{2L} = 2\times 10^{10}; \qquad \left(\frac{R}{2L}\right)^2 = 4\times 10^{20}$$

$$\frac{1}{LC} = \frac{10^{12}}{2.5\times 10^{-9}} = 4\times 10^{20}$$

$$\therefore \quad \left(\frac{R}{2L}\right)^2 = \frac{1}{LC}; \qquad s_{1,2} = -2 \times 10^{10} \text{ rad/s}$$

$$\therefore \quad v_o = V_f + D_1' te^{-2\times 10^{10}t} + D_2' e^{-2\times 10^{10}t}$$

$$V_f = 5\text{V}$$

$$v_o(0) = 5 + D_2' = 1; \qquad D_2' = -4\text{V}$$

$$\frac{dv_o(0)}{dt} = 0 = D_1' - 2 \times 10^{10} D_2'$$

$$\therefore \quad D_1' = -8 \times 10^{10} \text{ V/s}$$

$$\therefore \quad v_o = 5 - 8 \times 10^{10} te^{-2\times 10^{10}t} - 4e^{-2\times 10^{10}t}\text{V}, \quad t \geq 0$$

[c] When $L = 5$ nH,

$$\frac{R}{2L} = \frac{50}{5} \times 10^9 = 10^{10}$$

$$\frac{1}{LC} = \frac{10^{12} \times 10^9}{5} = 2 \times 10^{20}$$

$$s_{1,2} = -10^{10} \pm \sqrt{10^{20} - 2 \times 10^{20}} = -10^{10} \pm j10^{10} \text{ rad/s}$$

$$v_o = 5 + B_1' e^{-10^{10}t} \cos 10^{10}t + B_2' e^{10^{10}t} \sin 10^{10}t$$

$$v_o(0) = 5 + B_1' = 1; \qquad B_1' = -4\text{V}$$

$$\frac{dv_o(0)}{dt} = -10^{10} B_1' + 10^{10} B_2' = 0; \qquad B_1' = B_2' = -4\text{V}$$

$$v_o = 5 - 4e^{-10^{10}t}(\cos 10^{10}t + \sin 10^{10}t)\text{V}, \quad t \geq 0$$

[d] When $L = 25$ nH,

$$\frac{R}{2L} = \frac{50}{25} \times 10^9 = 2 \times 10^9 \left(\frac{R}{2L}\right)^2 = 4 \times 10^{18}$$

$$\frac{1}{LC} = \frac{10^{12} \times 10^9}{25} = 40 \times 10^{18}$$

$$s_{1,2} = -2 \times 10^9 \pm j6 \times 10^9 \text{ rad/s}$$

$$v_o = 5 + B_1' e^{-2\times 10^9 t} \cos 6 \times 10^9 t + B_2' e^{-2\times 10^9 t} \sin 6 \times 10^9 t$$

$$v_o(0) = 1 = 5 + B_1'; \qquad B_1' = -4\text{V}$$

$$\frac{dv_o(0)}{dt} = -2 \times 10^9 B_1' + 6 \times 10^9 B_2' = 0; \qquad B_2' = -\frac{4}{3}\text{V}$$

$$v_o = 5 - 4e^{-2\times 10^9 t}(\cos 6 \times 10^9 t + (1/3) \sin 6 \times 10^9 t)\text{V}, \quad t \geq 0$$

P 6.50 Use the $L = 0$ value of t_x as a first estimate. Then by successive approximations find that:

$t_x = 133.79$ ps when $L = 1.6$ nH

$t_x = 134.64$ ps when $L = 2.5$ nH

$t_x = 147.41$ ps when $L = 5$ nH

$t_x = 268.64$ ps when $L = 25$ nH

7 Sinusoidal Steady State Analysis

Drill Exercises

DE 7.1 [a] $\omega = 2\pi f = 3769.91\,\text{rad/s}, \qquad f = 600\,\text{Hz}$

[b] $T = 1/f = 1.67\,\text{ms}$

[c] $V_m = 10\,\text{V}$

[d] $v(0) = 10(0.6) = 6\,\text{V}$

[e] $\phi = -53.13^\circ; \qquad \phi = \dfrac{-53.13^\circ(2\pi)}{360^\circ} = -0.9273\ \text{rad}$

[f] $3769.91t = 143.13/57.3 = 2.498\,\text{rad}, \qquad t = 662.62\,\mu\text{s}$

[g] $(dv/dt) = (-10)3769.91\sin(3769.91t - 53.13^\circ)$

$(dv/dt) = 0 \quad \text{when} \quad 3769.91t - 53.13^\circ = 0^\circ$

or $\qquad 3769.91t = 0.9273\,\text{rad}$

Therefore $\quad t = 245.97\,\mu\text{s}$

DE 7.2 $V_{\text{rms}} = \sqrt{\dfrac{1}{T}\displaystyle\int_0^{T/2} V_m^2 \sin^2 \frac{2\pi}{T} t\, dt}$

$$\int_0^{T/2} V_m^2 \sin^2\left(\frac{2\pi}{T}\right) t\, dt = \frac{V_m^2}{2}\int_0^{T/2}\left(1 - \cos\frac{4\pi}{T}t\right)\, dt = \frac{V_m^2 T}{4}$$

Therefore $\quad V_{\text{rms}} = \sqrt{\dfrac{1}{T}\dfrac{V_m^2 T}{4}} = \dfrac{V_m}{2}$

DE 7.3 [a] The numerical values of the terms in Eq. 7.9 are

$V_m = 20, \qquad R/L = 1066.67, \qquad \omega L = 60$

$\sqrt{R^2 + \omega^2 L^2} = 100$

$\phi = 25^\circ, \qquad \theta = \tan^{-1} 60/80, \qquad \theta = 36.87^\circ$

$i = \left[-195.72e^{-1066.67t} + 200\cos(800t - 11.87^\circ)\right]\ \text{mA}, \quad t \geq 0$

[b] Transient component $= -195.72e^{-1066.67t}$ mA
Steady-state component $= 200\cos(800t - 11.87°)$ mA

[c] By direct substitution into Eq 7.9, $i(1.875\,\text{ms}) = 28.39\,\text{mA}$

[d] 0.2 A, 800 rad/s, −11.87°

[e] The current lags the voltage by 36.87°.

DE 7.4 [a] $\mathbf{V} = 170\underline{/-40°}\,\text{V}$

[b] $\mathbf{I} = 10\underline{/-70°}\,\text{A}$

[c] $\mathbf{I} = 5\underline{/36.87°} + 10\underline{/-53.13°}$

$= 4 + j3 + 6 - j8 = 10 - j5 = 11.18\underline{/-26.57°}\,\text{A}$

[d] $\mathbf{V} = 300\underline{/45°} - 100\underline{/-60°} = 212.13 + j212.13 - (50 - j86.60)$

$= 162.13 + j298.73 = 339.90\underline{/61.51°}\,\text{mV}$

DE 7.5 [a] $v = 18.6\cos(\omega t - 54°)\,\text{V}$

[b] $\mathbf{I} = 20\underline{/45°} - 50\underline{/-30°} = 14.14 + j14.14 - 43.3 + j25$

$= -29.16 + j39.14 = 48.81\underline{/126.68°}$

Therefore $i = 48.81\cos(\omega t + 126.68°)\,\text{mA}$

[c] $\mathbf{V} = 20 + j80 - 30\underline{/15°} = 20 + j80 - 28.98 - j7.76$

$= -8.98 + j72.24 = 72.79\underline{/97.08°}$

$v = 72.79\cos(\omega t + 97.08°)\,\text{V}$

DE 7.6 [a] $\omega L = (10^4)(20 \times 10^{-3}) = 200\,\Omega$

[b] $Z_L = j200\,\Omega$

[c] $\mathbf{V}_L = \mathbf{I}Z_L = (10\underline{/30°})(200\underline{/90°}) \times 10^{-3} = 2\underline{/120°}\,\text{V}$

[d] $v_L = 2\cos(10{,}000t + 120°)\,\text{V}$

DE 7.7 [a] $X_C = \dfrac{-1}{\omega C} = -\dfrac{10^6}{4000(5)} = -50\,\Omega$

[b] $Z_C = jX_C = -j50\,\Omega$

[c] $\mathbf{I} = \dfrac{30\underline{/25°}}{50\underline{/-90°}} = 0.6\underline{/115°}\,\text{A}$

[d] $i = 0.6\cos(4000t + 115°)\,\text{A}$

DE 7.8 $\mathbf{I}_1 = 100\underline{/25^\circ} = 90.63 + j42.26$

$\mathbf{I}_2 = 100\underline{/145^\circ} = -81.92 + j57.36$

$\mathbf{I}_3 = 100\underline{/-95^\circ} = -8.71 - j99.62$

$\mathbf{I}_4 = -(\mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3) = (0 + j0)\,\text{A}, \qquad \text{therefore} \quad i_4 = 0\,\text{A}$

DE 7.9 [a] $\mathbf{I} = \dfrac{125\underline{/-60^\circ}}{|Z|\underline{/\theta_z}} = \dfrac{125}{|Z|}\underline{/(-60 - \theta_Z)^\circ}$

But $-60 - \theta_Z = -105^\circ \qquad \therefore \ \theta_Z = 45^\circ$

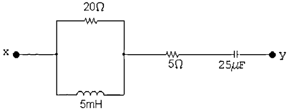
$Z = 90 + j160 + jX_C$

$\therefore \ X_C = -70\,\Omega; \qquad -\dfrac{1}{\omega C} = -70$

$\therefore \ C = \dfrac{1}{(70)(5000)} = 2.86\,\mu\text{F}$

[b] $\mathbf{I} = \dfrac{125\underline{/-60^\circ}}{(90 + j90)} = 0.982\underline{/-105^\circ}A; \qquad \therefore \ |\mathbf{I}| = 0.982\,\text{A}$

DE 7.10 [a]



$\omega = 2000\,\text{rad/s}$

$\omega L = 10\,\Omega, \qquad \dfrac{-1}{\omega C} = -20\,\Omega$

$$Z_{xy} = \frac{20(j10)}{(20 + j10)} + 5 - j20 = 4 + j8 + 5 - j20 = (9 - j12)\,\Omega$$

[b] $\omega L = 40\,\Omega, \qquad \dfrac{-1}{\omega C} = -5\,\Omega$

$$Z_{xy} = 5 - j5 + \left[\frac{(20)(j40)}{20 + j40}\right] = 5 - j5 + 16 + j8 = (21 + j3)\,\Omega$$

[c] $Z_{xy} = \left[\frac{20(j\omega L)}{20 + j\omega L}\right] + \left(5 - \frac{j10^6}{25\omega}\right)$

$= \frac{20\omega^2 L^2}{400 + \omega^2 L^2} + \frac{j400\omega L}{400 + \omega^2 L^2} + 5 - \frac{j10^6}{25\omega}$

The impedance will be purely resistive when the j terms cancel, i.e.,

$\frac{400\omega L}{400 + \omega^2 L^2} = \frac{10^6}{25\omega}$

Solving for ω yields $\omega = 4000\,\text{rad/s}$.

[d] $Z_{xy} = \frac{20\omega^2 L^2}{400 + \omega^2 L^2} + 5 = 10 + 5 = 15\,\Omega$

DE 7.11

$\mathbf{V} = 150\underline{/0^\circ}, \qquad \mathbf{I}_s = \frac{150\underline{/0^\circ}}{15} = 10\underline{/0^\circ}\,\text{A}$

$\mathbf{I}_L = \frac{10(20)}{20 + j20} = 5 - j5 = 7.07\underline{/-45^\circ}\,\text{A}$

$i_L = 7.07\cos(4000t - 45^\circ)\,\text{A}, \qquad I_m = 7.07\,\text{A}$

DE 7.12 [a] $Y = \frac{1}{3 + j4} + \frac{1}{16 - j12} + \frac{1}{-j4}$

$= 0.12 - j0.16 + 0.04 + j0.03 + j0.25$

$= 0.16 + j0.12 = 200\underline{/36.87^\circ}\,\text{mS}$

[b] $G = 160\,\text{mS}$

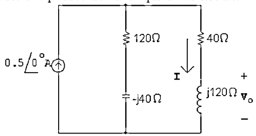
[c] $B = 120\,\text{mS}$

[d] $\mathbf{I} = 8\underline{/0^\circ}\,\text{A}, \qquad \mathbf{V} = \frac{\mathbf{I}}{Y} = \frac{8}{0.2\underline{/36.87^\circ}} = 40\underline{/-36.87^\circ}\,\text{V}$

$\mathbf{I}_C = \frac{\mathbf{V}}{Z_C} = \frac{40\underline{/-36.87^\circ}}{4\underline{/-90^\circ}} = 10\underline{/53.13^\circ}\,\text{A}$

$i_C = 10\cos(\omega t + 53.13^\circ)\,\text{A}, \qquad I_m = 10\,\text{A}$

DE 7.13 Construct the phasor domain equivalent circuit:



$\mathbf{I} = \frac{0.5(120 - j40)}{160 + j80} = 0.25 - j0.25\,\text{A}$

$\mathbf{V}_o = j120\mathbf{I} = 30 + j30 = 42.43\underline{/45^\circ}$

$v_o = 42.43\cos(2000t + 45^\circ)\,\text{V}$

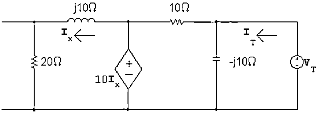
DE 7.14 Use the lower node as the reference node. Let $\mathbf{V}_1$ = node voltage across the $20\,\Omega$ resistor and $\mathbf{V}_{\text{Th}}$ = node voltage across the capacitor. Writing the node voltage equations gives us

$$\frac{\mathbf{V}_1}{20} - 2\underline{/45^\circ} + \frac{\mathbf{V}_1 - 10\mathbf{I}_x}{j10} = 0 \quad \text{and} \quad \mathbf{V}_{\text{Th}} = \frac{-j10}{10 - j10}(10\mathbf{I}_x)$$

We also have

$$\mathbf{I}_x = \frac{\mathbf{V}_1}{20}$$

Solving these equations for $\mathbf{V}_{\text{Th}}$ gives $\mathbf{V}_{\text{Th}} = 10\underline{/45^\circ}V$. To find the Thévenin impedance, we remove the independent current source and apply a test voltage source at the terminals a, b. Thus



It follows from the circuit that

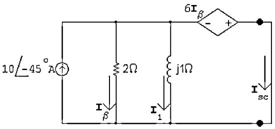
$$10\mathbf{I}_x = (20 + j10)\mathbf{I}_x$$

Therefore

$$\mathbf{I}_x = 0 \quad \text{and} \quad \mathbf{I}_T = \frac{\mathbf{V}_T}{-j10} + \frac{\mathbf{V}_T}{10}$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T}, \quad \text{therefore} \quad Z_{\text{Th}} = (5 - j5)\,\Omega$$

DE 7.15 Short circuit current



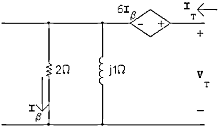
With the short circuit

$$\mathbf{I}_\beta = \frac{-6\mathbf{I}_\beta}{2}$$

$$2\mathbf{I}_\beta = -6\mathbf{I}_\beta; \qquad \therefore \quad \mathbf{I}_\beta = 0$$

$$\mathbf{I}_1 = 0; \qquad \therefore \quad \mathbf{I}_{sc} = 10\underline{/-45^\circ}\,\text{A} = \mathbf{I}_N$$

The Norton impedance is the same as the Thévenin impedance. Thus



$$\mathbf{V}_T = 6\mathbf{I}_\beta + 2\mathbf{I}_\beta = 8\mathbf{I}_\beta, \qquad \mathbf{I}_\beta = \frac{j1}{2+j1}\mathbf{I}_T$$

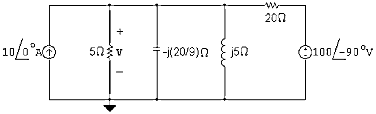
$$Z_{\text{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T} = \frac{8\mathbf{I}_\beta}{[(2+j1)/j1]\mathbf{I}_\beta} = \frac{j8}{2+j1} = 1.6 + j3.2\,\Omega$$

DE 7.16 The phasor domain circuit is as shown in the following diagram. The node voltage equation is

$$-10 + \frac{\mathbf{V}}{5} + \frac{9\mathbf{V}}{-j20} + \frac{\mathbf{V}}{j5} + \frac{\mathbf{V} - 100\underline{/-90^\circ}}{20} = 0$$

Therefore $\mathbf{V} = 10 - j30 = 31.62\underline{/-71.57^\circ}$

Therefore $v = 31.62\cos(50{,}000t - 71.57^\circ)\,\text{V}$



DE 7.17 Let $\mathbf{I}_a$, $\mathbf{I}_b$, and $\mathbf{I}_c$ be the three clockwise mesh currents going from left to right. Summing the voltages around meshes a and b gives

$$33.8 = (1 + j2)\mathbf{I}_a + (3 - j5)(\mathbf{I}_a - \mathbf{I}_b)$$

and

$$0 = (3 - j5)(\mathbf{I}_b - \mathbf{I}_a) + 2(\mathbf{I}_b - \mathbf{I}_c).$$

But

$$\mathbf{V}_x = -j5(\mathbf{I}_a - \mathbf{I}_b),$$

therefore

$$\mathbf{I}_c = -0.75[-j5(\mathbf{I}_a - \mathbf{I}_b)].$$

Solving for $\mathbf{I} = \mathbf{I}_a = 29 + j2 = 29.07\underline{/3.95^\circ}$ A.

DE 7.18 [a] $\mathbf{V} = 100\underline{/-45^\circ}$ V, $\quad \mathbf{I} = 20\underline{/15^\circ}$ A

Therefore

$$P = \frac{1}{2}(100)(20)\cos[-45 - (15)] = 500\,\text{W}, \qquad \text{A} \rightarrow \text{B}$$

$$Q = 1000 \sin -60^\circ = -866.03\ \text{VAR}, \qquad \text{B} \rightarrow \text{A}$$

[b] $\mathbf{V} = 100\underline{/-45^\circ}$, $\quad \mathbf{I} = 20\underline{/165^\circ}$

$$P = 1000\cos(-210^\circ) = -866.03\,\text{W}, \qquad \text{B} \rightarrow \text{A}$$

$$Q = 1000\sin(-210^\circ) = 500\ \text{VAR}, \qquad \text{A} \rightarrow \text{B}$$

[c] $\mathbf{V} = 106\underline{/-45^\circ}$, $\quad \mathbf{I} = 20\underline{/-105^\circ}$

$$P = 1000\cos(60^\circ) = 500\,\text{W}, \qquad \text{A} \rightarrow \text{B}$$

$$Q = 1000\sin(60^\circ) = 866.03\ \text{VAR}, \qquad \text{A} \rightarrow \text{B}$$

[d] $P = 1000\cos(-120^\circ) = -500\,\text{W}, \qquad \text{B} \rightarrow \text{A}$

$$Q = 1000\sin(-120^\circ) = -866.03\ \text{VAR}, \qquad \text{B} \rightarrow \text{A}$$

DE 7.19

$$p_f = \cos(\theta_v - \theta_i) = \cos[15 - (75)] = \cos -60^\circ = 0.5 \text{ leading}$$

$$r_f = \sin(\theta_v - \theta_i) = \sin -60^\circ = -0.866$$

DE 7.20 From Example 7.4,

$$I_{\text{eff}} = \frac{0.18}{\sqrt{3}}$$

$$\begin{aligned} P &= I_{\text{eff}}^2 R \\ &= \left(\frac{0.0324}{3}\right)(5000) \\ &= 54\,\text{W} \end{aligned}$$

DE 7.21 [a] $Z = (39 + j26)\|(-j52) = 48 - j20 = 52\underline{/-22.62^\circ}\,\Omega$

$$\text{Therefore} \quad \mathbf{I}_\ell = \frac{250\underline{/0^\circ}}{48 - j20 + 1 + j4} = 4.85\underline{/18.08^\circ}\,\text{A(rms)}$$

$$\mathbf{V}_\text{L} = Z\mathbf{I}_\ell = (52\underline{/-22.62^\circ})(4.85\underline{/18.08^\circ}) = 252.20\underline{/-4.54^\circ}\,\text{V(rms)}$$

$$\mathbf{I}_\text{L} = \frac{\mathbf{V}_\text{L}}{39 + j26} = 5.38\underline{/-38.23^\circ}\,\text{A(rms)}$$

[b] $S_\text{L} = (252.20\underline{/-4.54^\circ})(5.38\underline{/+38.23^\circ}) = 1357\underline{/33.69^\circ}$

$= (1129.09 + j752.73)\,\text{VA}$

$P_\text{L} = 1129.09\,\text{W}; \qquad Q_\text{L} = 752.73\,\text{VAR}$

[c] $P_\ell = |\mathbf{I}_\ell|^2 1 = (4.85)^2 \cdot 1 = 23.52\,\text{W}; \qquad Q_\ell = |\mathbf{I}_\ell|^2 4 = 94.09\,\text{VAR}$

[d] $S_g(\text{delivering}) = 250\mathbf{I}_\ell^* = (1152.62 - j376.36)\,\text{VA}$

Therefore the source is delivering 1152.62 W and absorbing 376.36 magnetizing VAR.

[e] $Q_\text{cap} = \dfrac{|\mathbf{V}_\text{L}|^2}{-52} = \dfrac{(252.20)^2}{-52} = -1223.18\,\text{VAR}$

Therefore the capacitor is delivering 1223.18 magnetizing VAR.

Check: $94.09 + 752.73 + 376.36 = 1223.18\,\text{VAR}$ and

$1129.09 + 23.52 = 1152.62\,\text{W}$

DE 7.22 Series circuit derivation:

$$250\mathbf{I}^* = (40{,}000 - j30{,}000)$$

$$\text{Therefore} \quad \mathbf{I}^* = 160 - j120 = 200\underline{/-36.87^\circ}\,\text{A(rms)}$$

$$\mathbf{I} = 200\underline{/36.87^\circ}\,\text{A(rms)}$$

$$Z = \frac{250}{200\underline{/36.87^\circ}} = 1.25\underline{/-36.87^\circ} = (1 - j0.75)\,\Omega$$

Therefore $\quad R = 1\,\Omega, \qquad X_C = -0.75\,\Omega$

Parallel circuit derivation:

$$P = \frac{(250)^2}{R}; \qquad \text{therefore} \quad R = \frac{(250)^2}{40{,}000} = 1.5625\,\Omega$$

$$Q = \frac{(250)^2}{X_C}; \qquad \text{therefore} \quad X_C = \frac{(250)^2}{-30{,}000} = -2.083\,\Omega$$

DE 7.23

$$S_1 = 15{,}000(0.6) + j15{,}000(0.8) = 9000 + j12{,}000\,\text{VA}$$

$$S_2 = 6000(0.8) + j6000(0.6) = 4800 - j3600\,\text{VA}$$

$$S_T = S_1 + S_2 = 13{,}800 + j8400\,\text{VA}$$

$$S_T = 200\mathbf{I}^*; \qquad \text{therefore} \quad \mathbf{I}^* = 69 + j42 \qquad \mathbf{I} = 69 - j42\,\text{A}$$

$$\mathbf{V}_s = 200 + j\mathbf{I} = 200 + j69 + 42 = 242 + j69 = 251.64\underline{/15.91^\circ}\,\text{V(rms)}$$

Problems

P 7.1 [a] By hypothesis

$$i = 10\cos(\omega t + \theta)$$

$$\frac{di}{dt} = -10\omega\sin(\omega t + \theta)$$

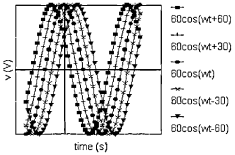
$$\therefore \quad 10\omega = 20{,}000\pi; \qquad \omega = 2000\pi \text{ rad/s}$$

[b] $f = \dfrac{\omega}{2\pi} = 1000$ Hz; $\qquad T = \dfrac{1}{f} = 1$ ms $= 1000\,\mu$s

$$\frac{150}{1000} = \frac{3}{20}, \qquad \therefore \quad \theta = -90 - \frac{3}{20}(360) = -144^\circ$$

$$\therefore \quad i = 10\cos(2000\pi t - 144^\circ)\text{ A}$$

P 7.2



[a] Left as ϕ becomes more positive

[b] Right

P 7.3 [a] 170 V

[b] $2\pi f = 120\pi; \qquad f = 60\text{Hz}$

[c] $\omega = 120\pi = 376.99$ rad/s

[d] $\theta(\text{rad}) = \dfrac{-\pi}{180}(60) = \dfrac{-\pi}{3} = -1.05$ rad

[e] $\theta = -60^\circ$

[f] $T = \dfrac{1}{f} = \dfrac{1}{60} = 16.67\,\text{ms}$

[g] $120\pi t - \dfrac{\pi}{3} = 0; \qquad \therefore \quad t = \dfrac{1}{360} = 2.78\,\text{ms}$

[h] $v = 170\cos\left[120\pi\left(t + \frac{0.125}{18}\right) - \frac{\pi}{3}\right]$

$= 170\cos[120\pi t + (15\pi/18) - (\pi/3)]$

$= 170\cos[120\pi t + (\pi/2)]$

$= -170\sin 120\pi t\,\text{V}$

[i] $120\pi(t - t_o) - (\pi/3) = 120\pi t - (\pi/2)$

$\therefore\quad 120\pi t_o = \frac{\pi}{6}; \qquad t_o = \frac{25}{18}\,\text{ms}$

[j] $120\pi(t - t_o) - (\pi/3) = 120\pi t$

$\therefore\quad 120\pi t_o = \frac{\pi}{3}; \qquad t_o = \frac{25}{9}\,\text{ms}$

P 7.4 [a] $\frac{T}{2} = \frac{1250}{6} + \frac{250}{6} = 250\,\mu\text{s}; \qquad T = 500\,\mu\text{s}$

$f = \frac{1}{T} = \frac{10^6}{500} = 2000\text{Hz}$

[b] $v = V_m\sin(\omega t + \theta)$

$\omega = 2\pi f = 4000\pi$ rad/s

$4000\pi\left(\frac{-250}{6} \times 10^{-6}\right) + \theta = 0; \qquad \therefore\quad \theta = \frac{\pi}{6}\text{ rad} = 30^\circ$

$v = V_m\sin[4000\pi t + 30^\circ]$

$75 = V_m\sin 30^\circ; \qquad V_m = 150\,\text{V}$

$v = 150\sin[4000\pi t + 30^\circ] = 150\cos[4000\pi t - 60^\circ]\,\text{V}$

P 7.5 [a] From Eq. 7.9 we have

$$L\frac{di}{dt} = \frac{V_m R\cos(\phi - \theta)}{\sqrt{R^2 + \omega^2 L^2}}e^{-(R/L)t} - \frac{\omega L V_m\sin(\omega t + \phi - \theta)}{\sqrt{R^2 + \omega^2 L^2}}$$

$$Ri = \frac{-V_m R\cos(\phi - \theta)e^{-(R/L)t}}{\sqrt{R^2 + \omega^2 L^2}} + \frac{V_m R\cos(\omega t + \phi - \theta)}{\sqrt{R^2 + \omega^2 L^2}}$$

$$L\frac{di}{dt} + Ri = V_m\left[\frac{R\cos(\omega t + \phi - \theta) - \omega L\sin(\omega t + \phi - \theta)}{\sqrt{R^2 + \omega^2 L^2}}\right]$$

But

$$\frac{R}{\sqrt{R^2 + \omega^2 L^2}} = \cos\theta \quad\text{and}\quad \frac{\omega L}{\sqrt{R^2 + \omega^2 L^2}} = \sin\theta$$

Therefore the right-hand side reduces to

$V_m \cos(\omega t + \phi)$

At $t = 0$, Eq. 7.9 reduces to

$$i(0) = \frac{-V_m \cos(\phi - \theta)}{\sqrt{R^2 - \omega^2 L^2}} + \frac{V_m \cos(\phi - \theta)}{\sqrt{R^2 + \omega^2 L^2}} = 0$$

[b] $i_{ss} = \dfrac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t + \phi - \theta)$

Therefore

$$L\frac{di_{ss}}{dt} = \frac{-\omega L V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin(\omega t + \phi - \theta)$$

and

$$Ri_{ss} = \frac{V_m R}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t + \phi - \theta)$$

$$L\frac{di_{ss}}{dt} + Ri_{ss} = V_m \left[\frac{R\cos(\omega t + \phi - \theta) - \omega L \sin(\omega t + \phi - \theta)}{\sqrt{R^2 + \omega^2 L^2}}\right]$$

$$= V_m \cos(\omega t + \phi)$$

P 7.6 [a] $\mathbf{Y} = 100\underline{/45^\circ} + 500\underline{/-60^\circ} = 483.86\underline{/-48.48^\circ}$

$y = 483.86 \cos(300t - 48.48^\circ)$

[b] $\mathbf{Y} = 250\underline{/30^\circ} - 150\underline{/50^\circ} = 120.51\underline{/4.8^\circ}$

$y = 120.51 \cos(377t + 4.8^\circ)$

[c] $\mathbf{Y} = 60\underline{/60^\circ} - 120\underline{/-215^\circ} + 100\underline{/90^\circ} = 152.88\underline{/32.94^\circ}$

$y = 152.88 \cos(100t + 32.94^\circ)$

[d] $\mathbf{Y} = 100\underline{/40^\circ} + 100\underline{/160^\circ} + 100\underline{/-80^\circ} = 0$

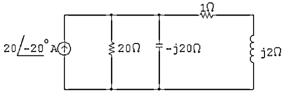
$y = 0$

P 7.7
$$u = \int_{t_o}^{t_o+T} V_m^2 \cos^2(\omega t + \phi)\, dt$$
$$= V_m^2 \int_{t_o}^{t_o+T} \frac{1}{2} + \frac{1}{2}\cos(2\omega t + 2\phi)\, dt$$
$$= \frac{V_m^2}{2}\left\{\int_{t_o}^{t_o+T} dt + \int_{t_o}^{t_o+T} \cos(2\omega t + 2\phi)\, dt\right\}$$
$$= \frac{V_m^2}{2}\left\{T + \frac{1}{2\omega}\left[\sin(2\omega t + 2\phi)\,\Big|_{t_o}^{t_o+T}\right]\right\}$$
$$= \frac{V_m^2}{2}\left\{T + \frac{1}{2\omega}\left[\sin(2\omega t_o + 4\pi + 2\phi) - \sin(2\omega t_o + 2\phi)\right]\right\}$$
$$= V_m^2\left(\frac{T}{2}\right) + \frac{1}{2\omega}(0) = V_m^2\left(\frac{T}{2}\right)$$

P 7.8 $V_m = \sqrt{2}V_{\rm rms} = \sqrt{2}(120) = 169.71\,\text{V}$

P 7.9 [a] $j\omega L = j(5\times 10^4)(40\times 10^{-6}) = j2\,\Omega$

$$\frac{1}{j\omega C} = -j\frac{10^6}{5\times 10^4} = -j20\,\Omega; \qquad \mathbf{I}_g = 20\angle{-20^\circ}\,\text{A}$$



[b] $\mathbf{V}_o = 20\angle{-20^\circ}Z_e$

$$Z_e = \frac{1}{Y_e}; \qquad Y_e = \frac{1}{20} + j\frac{1}{20} + \frac{1}{1+j2}$$

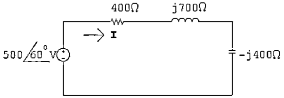
$$Y_e = 0.05 + j0.05 + 0.20 - j0.40 = 0.25 - j0.35\,\text{S}$$

$$Z_e = \frac{1}{0.25 - j0.35} = 2.32\angle{54.46^\circ}\,\Omega$$

$$\mathbf{V}_o = (20\angle{-20^\circ})(2.32\angle{54.46^\circ}) = 46.4\angle{34.46^\circ}\,\text{V}$$

[c] $v_o = 46.4\cos(5\times 10^4 t + 34.46^\circ)\,\text{V}$

P 7.10 [a]



[b] $\mathbf{I} = \dfrac{500\angle{60^\circ}}{400 + j700 - j400} = 1\angle{23.13^\circ}\,\text{A}$

[c] $i = 1\cos(8000t + 23.13^\circ)\,\text{A}$

P 7.11 [a] 50Hz

[b] $\theta_v = 0^\circ$

[c] $\mathbf{I} = \dfrac{340\angle{0^\circ}}{j\omega L} = \dfrac{340}{\omega L}\angle{-90^\circ} = 8.5\angle{-90^\circ}; \qquad \theta_i = -90^\circ$

[d] $\dfrac{340}{\omega L} = 8.5; \qquad \omega L = 40\,\Omega$

[e] $L = \dfrac{40}{100\pi} = \dfrac{400}{\pi}\,\text{mH} = 127.32\,\text{mH}$

[f] $Z_L = j\omega L = j40\,\Omega$

P 7.12 [a] $\omega = 2\pi f = 80\pi \times 10^3 = 251.33\,\text{krad/s} = 251{,}327.41\,\text{rad/s}$

[b] $\mathbf{I} = \dfrac{2.5 \times 10^{-3}\underline{/0^\circ}}{1/j\omega C} = j\omega C(2.5 \times 10^{-3})\underline{/0^\circ} = 2.5 \times 10^{-3}\omega C\underline{/90^\circ}$

$\therefore \quad \theta_i = 90^\circ$

[c] $125.66 \times 10^{-6} = 2.5 \times 10^{-3}\,\omega C$

$$\frac{1}{\omega C} = \frac{2.5 \times 10^{-3}}{125.66 \times 10^{-6}} = 19.89\,\Omega, \quad \therefore \; X_C = -19.89\,\Omega$$

[d] $C = \dfrac{1}{19.89(\omega)} = \dfrac{1}{(19.89)(80\pi \times 10^3)}$

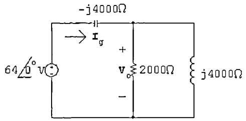
$C = 0.2 \times 10^{-6} = 0.2\,\mu\text{F}$

[e] $Z_c = j\left(\dfrac{-1}{\omega C}\right) = -j19.89\,\Omega$

P 7.13 $\dfrac{1}{j\omega C} = \dfrac{10^9}{(31.25)(8000)} = -j4000\,\Omega$

$j\omega L = j8000(500)10^{-3} = j4000\,\Omega$

$\mathbf{V}_g = 64\underline{/0^\circ}\,\text{V}$



$$Z_e = \frac{(2000)(j4000)}{2000 + j4000} = 1600 + j800\,\Omega$$

$$Z_T = 1600 + j800 - j4000 = 1600 - j3200\,\Omega$$

$$\mathbf{I}_g = \frac{64\underline{/0^\circ}}{1600 - j3200} = 8 + j16\,\text{mA}$$

$$\mathbf{V}_o = Z_e\mathbf{I}_g = (1600 + j800)(0.008 + j0.016) = j32 = 32\underline{/90^\circ}\,\text{V}$$

$$v_o = 32\cos(8000t + 90^\circ)\,\text{V}$$

P 7.14 $Z = 400 + j(5)(40) - j\dfrac{1000}{(5)(0.4)} = 500\underline{/-36.87^\circ}\,\Omega$

$$\mathbf{I}_o = \frac{750\underline{/0^\circ} \times 10^{-3}}{500\underline{/-36.87^\circ}} = 1.5\underline{/36.87^\circ}\,\text{mA}$$

$$i_o(t) = 1.5\cos(5000t + 36.87^\circ)\,\text{mA}$$

P 7.15 [a] $Z_p = \dfrac{\dfrac{R}{j\omega C}}{R + (1/j\omega C)} = \dfrac{R}{1 + j\omega RC}$

$$= \frac{12{,}500}{1 + j(1000)(12{,}500)C} = \frac{12{,}500}{1 + j12.5 \times 10^6 C}$$

$$= \frac{12{,}500(1 - j12.5 \times 10^6 C)}{1 + 156.25 \times 10^{12} C^2}$$

$$= \frac{12{,}500}{1 + 156.25 \times 10^{12} C^2} - j\frac{156.25 \times 10^9 C}{1 + 156.25 \times 10^{12} C^2}$$

$$j\omega L = j1000(5) = j5000$$

$$\therefore\quad 5000 = \frac{156.25 \times 10^9 C}{1 + 156.25 \times 10^{12} C^2}$$

$$\therefore\quad 781.25 \times 10^{15} C^2 - 156.25 \times 10^9 C + 5000 = 0$$

$$\therefore\quad C^2 - 20 \times 10^{-8} C + 64 \times 10^{-16} = 0$$

$$\therefore\quad C_{1,2} = 10 \times 10^{-8} \pm \sqrt{100 \times 10^{-16} - 64 \times 10^{-16}}$$

$$C_1 = 10 \times 10^{-8} + 6 \times 10^{-8} = 16 \times 10^{-8} = 160\,\text{nF} = 0.16\,\mu\text{F}$$

$$C_2 = 10 \times 10^{-8} - 6 \times 10^{-8} = 4 \times 10^{-8} = 40\,\text{nF} = 0.04\,\mu\text{F}$$

[b] $R_e = \dfrac{12{,}500}{1 + 156.25 \times 10^{12} C^2}$

When $C = 160\,\text{nF}$ $R_e = 2500\,\Omega$;

$$\mathbf{I}_g = \frac{250\underline{/0^\circ}}{2500} = 0.1\underline{/0^\circ}\,\text{A}; \qquad i_g = 100\cos 1000t\,\text{mA}$$

When $C = 40\,\text{nF}$ $R_e = 10{,}000\,\Omega$;

$$\mathbf{I}_g = \frac{250\underline{/0^\circ}}{10{,}000} = 0.025\underline{/0^\circ}\,\text{A}; \qquad i_g = 25\cos 1000t\,\text{mA}$$

P 7.16 [a] $Y_p = \dfrac{1}{10 + j2\omega} + j4 \times 10^{-3}\omega$

$$= \frac{10 - j2\omega}{100 + 4\omega^2} + j4 \times 10^{-3}\omega$$

$$= \frac{10}{100 + 4\omega^2} - \frac{j2\omega}{100 + 4\omega^2} + j4 \times 10^{-3}\omega$$

Y_p is real when

$$4 \times 10^{-3}\omega = \frac{2\omega}{100 + 4\omega^2}$$

or $\quad \omega^2 = 100; \qquad \omega = 10 \text{ rad/s}; \qquad f = 5/\pi = 1.59\text{Hz}$

[b] $Y_p(10\,\text{rad/s}) = \dfrac{10}{500} = 20\,\text{mS}$

$$Z_p(10\,\text{rad/s}) = \frac{10^3}{20} = 50\,\Omega$$

$$Z(10\,\text{rad/s}) = 50 + 150 = 200\,\Omega$$

$$\mathbf{I}_o = \frac{\mathbf{V}_g}{200}\,\text{A} = \frac{10\underline{/0^\circ}}{200} = 50\underline{/0^\circ}\,\text{mA}$$

$$i_o = 50\cos 10t\,\text{mA}$$

P 7.17 $\mathbf{V}_g = 50\underline{/-45^\circ}\,\text{V}; \qquad \mathbf{I}_g = 100\underline{/-8.13^\circ}\,\text{mA}$

$$Z = \frac{\mathbf{V}_g}{\mathbf{I}_g} = 500\underline{/-36.87^\circ}\,\Omega = 400 - j300\,\Omega$$

$$Z = 400 + j\left(0.04\omega - \frac{2.5 \times 10^6}{\omega}\right)$$

$$\therefore \quad 0.04\omega - \frac{2.5 \times 10^6}{\omega} = -300$$

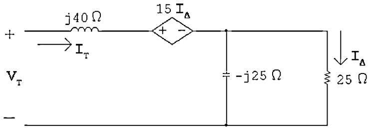
$$\therefore \quad \omega^2 + 7500\omega - 62.5 \times 10^6 = 0$$

$$\therefore \quad \omega = -3750 \pm \sqrt{(3750)^2 + 62.5 \times 10^6} = -3750 \pm 8750$$

$$\omega > 0, \qquad \therefore \quad \omega = 5000\,\text{rad/s}$$

P 7.18 $j\omega L = j1.6 \times 10^6(25 \times 10^{-6}) = j40\,\Omega$

$$\frac{1}{j\omega C} = \frac{10^{-6} \times 10^9}{j1.6(25)} = -j25\,\Omega$$



$$\mathbf{V}_T = j40\mathbf{I}_T + 15\mathbf{I}_\Delta + 25\mathbf{I}_\Delta$$

$$\mathbf{I}_\Delta = \frac{\mathbf{I}_T(-j25)}{25 - j25} = \frac{-j\mathbf{I}_T}{1 - j1}$$

$$\mathbf{V}_T = j40\mathbf{I}_T + 40\frac{(-j\mathbf{I}_T)}{1 - j1}$$

$$\frac{\mathbf{V}_T}{\mathbf{I}_T} = Z_{\text{ab}} = j40 + 20(-j)(1 + j) = 20 + j20\,\Omega = 28.28\underline{/45^\circ}\,\Omega$$

P 7.19 First find the admittance of the parallel branches

$$Y_p = \frac{1}{6 - j2} + \frac{1}{4 + j12} + \frac{1}{5} + \frac{1}{j10} = 0.375 - j0.125\,\text{S}$$

$$Z_p = \frac{1}{Y_p} = \frac{1}{0.375 - j0.125} = 2.4 + j0.8\,\Omega$$

$$Z_{\text{ab}} = -j12.8 + 2.4 + j0.8 + 13.6 = 16 - j12\,\Omega$$

$$Y_{\text{ab}} = \frac{1}{Z_{\text{ab}}} = \frac{1}{16 - j12} = 0.04 + j0.03\,\text{S}$$

$$= 40 + j30\,\text{mS} = 50\underline{/36.87^\circ}\,\text{mS}$$

P 7.20 [a] $Z_g = 4000 - j\dfrac{10^9}{25\omega} + \dfrac{10^4(j2\omega)}{10^4 + j2\omega}$

$$= 4000 - j\frac{10^9}{25\omega} + \frac{2 \times 10^4 j\omega(10^4 - j2\omega)}{10^8 + 4\omega^2}$$

$$= 4000 - j\frac{10^9}{25\omega} + \frac{4 \times 10^4\omega^2}{10^8 + 4\omega^2} + j\frac{2 \times 10^8\omega}{10^8 + 4\omega^2}$$

$$\therefore \quad \frac{10^9}{25\omega} = \frac{0.2 \times 10^9\omega}{10^8 + 4\omega^2}$$

$$10^8 + 4\omega^2 = 5\omega^2$$

$$\omega^2 = 10^8; \qquad \omega = 10{,}000\,\text{rad/s}$$

[b] When $\omega = 10{,}000$ rad/s

$$Z_g = 4000 + \frac{4 \times 10^4(10^4)^2}{10^8 + 4(10^4)^2} = 12{,}000\,\Omega$$

$$\therefore \quad \mathbf{I}_g = \frac{45\underline{/0^\circ}}{12{,}000} = 3.75\underline{/0^\circ}\,\text{mA}$$

$$\mathbf{V}_o = \mathbf{V}_g - \mathbf{I}_g Z_1$$

$$Z_1 = 4000 - j\frac{10^9}{25 \times 10^4} = 4000 - j4000\,\Omega$$

$$\mathbf{V}_o = 45\underline{/0^\circ} - (3.75 \times 10^{-3})(4000 - j4000) = 45 - (15 - j15)$$

$$= 30 + j15 = 33.54\underline{/26.57^\circ}\,\text{V}$$

$$v_o = 33.54\cos(10{,}000t + 26.57^\circ)\,\text{V}$$

P 7.21 [a] $Z_1 = 1600 - j\dfrac{10^9}{10^4(62.5)} = 1600 - j1600\,\Omega$

$$Z_1 = \frac{4000(j10^4L)}{4000 + j10^4L} = \frac{4 \times 10^5L^2 + j16 \times 10^4L}{16 + 100L^2}$$

$$Z_T = Z_1 + Z_2 = 1600 + \frac{4 \times 10^5L^2}{16 + 100L^2} - j1600 + j\frac{16 \times 10^4L}{16 + 100L^2}$$

Z_T is resistive when

$$\frac{16 \times 10^4L}{16 + 100L^2} = 1600 \qquad \text{or}$$

$$L^2 - L + 0.16 = 0$$

Solving, $L_1 = 0.8$ H and $L_2 = 0.2$ H.

[b] When $L = 0.8$ H:

$$Z_T = 1600 + \frac{4 \times 10^5(0.64)}{16 + 64} = 4800\,\Omega$$

$$\mathbf{I}_g = \frac{96\underline{/0^\circ}}{4.8} \times 10^{-3} = 20\underline{/0^\circ}\,\text{mA}$$

$$i_g = 20\cos 10{,}000t\,\text{mA}$$

When $L = 0.2$ H:

$$Z_T = 1600 + \frac{4 \times 10^5(0.04)}{16 + 4} = 2400\,\Omega$$

$$i_g = 40\cos 10{,}000t\,\text{mA}$$

P 7.22 [a] $Z_{\text{ab}} = j5\omega + \dfrac{(4000)(10^9/j\omega 625)}{4000 + (10^9/j625\omega)}$

$$= j5\omega + \frac{4 \times 10^{12}}{2500 \times 10^3 j\omega + 10^9}$$

$$= j5\omega + \frac{4 \times 10^7}{10^4 + j25\omega}$$

$$= j5\omega + \frac{4 \times 10^{11}}{10^8 + 625\omega^2} - j\frac{100 \times 10^7\omega}{10^8 + 625\omega^2}$$

$$\therefore \quad 5 = \frac{10^9}{10^8 + 625\omega^2}$$

$$5 \times 10^8 + 3125\omega^2 = 10^9$$

$$\omega = 4 \times 10^2 = 400\,\text{rad/s}$$

[b] $Z_{\text{ab}}(400) = j2000 + \dfrac{(4000)(-j4000)}{4000 - j4000} = 2\,\text{k}\Omega$

P 7.23 $Z_1 = 10 - j40\,\Omega$

$$Z_2 = \frac{(5 - j10)(10 + j30)}{15 + j20} = 10 - j10\,\Omega$$

$$Z_3 = \frac{20(j20)}{20 + j20} = 10 + j10\,\Omega$$

$$\therefore \quad Z_{\text{ab}} = Z_1 + Z_2 + Z_3 = 30 - j40\,\Omega = 50\underline{/-53.13^\circ}\,\Omega$$

P 7.24 [a] $Y_1 = \dfrac{1}{5000} = 0.2 \times 10^{-3}\,\text{S}$

$$Y_2 = \frac{1}{1200 + j0.2\omega}$$

$$= \frac{1200}{1.44 \times 10^6 + 0.04\omega^2} - j\frac{0.2\omega}{1.44 \times 10^6 + 0.04\omega^2}$$

$$Y_3 = j\omega 50 \times 10^{-9}$$

$$Y_T = Y_1 + Y_2 + Y_3$$

For i_g and v_o to be in phase the j component of Y_T must be zero; thus,

$$\omega 50 \times 10^{-9} = \frac{0.2\omega}{1.44 \times 10^6 + 0.04\omega^2}$$

or

$$0.04\omega^2 + 1.44 \times 10^6 = \frac{0.2 \times 10^9}{50} = 4 \times 10^6$$

$$\therefore\quad 0.04\omega^2 = 2.56 \times 10^6 \qquad \therefore\quad \omega = 8000\,\text{rad/s} = 8\,\text{krad/s}$$

[b] $Y_T = 0.2 \times 10^{-3} + \dfrac{1200}{1.44 \times 10^6 + 0.04(64) \times 10^6} = 0.5 \times 10^{-3}\,\text{S}$

$$\therefore\quad Z_T = 2000\,\Omega$$

$$\mathbf{V}_o = (2.5 \times 10^{-3}\underline{/0^\circ})(2000) = 5\underline{/0^\circ}$$

$$v_o = 5\cos 8000t\,\text{V}$$

P 7.25 [a] $Z_1 = R_1 + j\omega L_1$

$$Z_2 = \frac{R_2(j\omega L_2)}{R_2 + j\omega L_2} = \frac{\omega^2 L_2^2 R_2 + j\omega L_2 R_2^2}{R^2 + \omega^2 L_2^2}$$

$$Z_1 = Z_2 \quad \text{when} \quad R_1 = \frac{\omega^2 L_2^2 R_2}{R_2^2 + \omega^2 L_2^2} \quad \text{and} \quad L_1 = \frac{R_2^2 L_2}{R_2^2 + \omega^2 L_2^2}$$

[b] $R_1 = \dfrac{(4 \times 10^8)(6.25)(5 \times 10^4)}{25 \times 10^8 + (4 \times 10^8)(6.25)} = 2.5 \times 10^4$

$$\therefore\quad R_1 = 25\,\text{k}\Omega$$

$$L_1 = \frac{(25 \times 10^8)2.5}{50 \times 10^8} = 1.25\,\text{H}$$

P 7.26 [a] $Y_2 = \frac{1}{R_2} - \frac{j}{\omega L_2}$

$$Y_1 = \frac{1}{R_1 + j\omega L_1} = \frac{R_1 - j\omega L_1}{R_1^2 + \omega^2 L_1^2}$$

Therefore $\quad Y_2 = Y_1 \quad$ when

$$R_2 = \frac{R_1^2 + \omega^2 L_1^2}{R_1} \quad \text{and} \quad L_2 = \frac{R_1^2 + \omega^2 L_1^2}{\omega^2 L_1}$$

[b] $R_2 = \frac{25 \times 10^6 + 10^8(0.25)}{5 \times 10^3} = 10 \times 10^3$

$\therefore \quad R_2 = 10\,\text{k}\Omega$

$$L_2 = \frac{50 \times 10^6}{10^8(0.5)} = 1\,\text{H}$$

P 7.27 [a] $Z_1 = R_1 - j\frac{1}{\omega C_1}$

$$Z_2 = \frac{R_2/j\omega C_2}{R_2 + (1/j\omega C_2)} = \frac{R_2}{1 + j\omega R_2 C_2} = \frac{R_2 - j\omega R_2^2 C_2}{1 + \omega^2 R_2^2 C_2^2}$$

$$Z_1 = Z_2 \quad \text{when} \quad R_1 = \frac{R_2}{1 + \omega^2 R_2^2 C_2^2} \quad \text{and}$$

$$\frac{1}{\omega C_1} = \frac{\omega R_2^2 C_2}{1 + \omega^2 R_2^2 C_2^2} \quad \text{or} \quad C_1 = \frac{1 + \omega^2 R_2^2 C_2^2}{\omega^2 R_2^2 C_2}$$

[b] $R_1 = \frac{500}{1 + (64 \times 10^8)(25 \times 10^4)(625 \times 10^{-18})} = 250\,\Omega$

$$C_1 = \frac{2}{(64 \times 10^8)(25 \times 10^4)(25 \times 10^{-9})} = 50\,\text{nF}$$

P 7.28 [a] $Y_2 = \frac{1}{R_2} + j\omega C_2$

$$Y_1 = \frac{1}{R_1 + (1/j\omega C_1)} = \frac{j\omega C_1}{1 + j\omega R_1 C_1} = \frac{\omega^2 R_1 C_1^2 + j\omega C_1}{1 + \omega^2 R_1^2 C_1^2}$$

Therefore $\quad Y_1 = Y_2 \quad$ when

$$R_2 = \frac{1 + \omega^2 R_1^2 C_1^2}{\omega^2 R_1 C_1^2} \quad \text{and} \quad C_2 = \frac{C_1}{1 + \omega^2 R_1^2 C_1^2}$$

[b] $R_2 = \frac{1 + (4 \times 10^8)(4 \times 10^6)(2500 \times 10^{-18})}{(4 \times 10^8)(2 \times 10^3)(2500 \times 10^{-18})} = 2500 = 2.5\text{k}\Omega$

$$C_2 = \frac{50 \times 10^{-9}}{5} = 10\,\text{nF}$$

P 7.29 [a] $\mathbf{V}_g = 150\angle 20^\circ; \qquad \mathbf{I}_g = 30\angle -52^\circ$

$$\therefore \quad Z = \frac{\mathbf{V}_g}{\mathbf{I}_g} = 5\angle 72^\circ\,\Omega$$

[b] i_g lags v_g by 72°:

$$2\pi f = 8000\pi; \qquad f = 4000\,\text{Hz}; \qquad T = 1/f = 250\,\mu\text{s}$$

$$\therefore \quad i_g \text{ lags } v_g \text{ by } \frac{72}{360}(250) = 50\,\mu\text{s}$$

P 7.30 $\dfrac{1}{j\omega C} = -j\dfrac{10^6}{10^4} = -j100\,\Omega$

$j\omega L = j(500)(1) = j500\,\Omega$

Let $Z_1 = 50 - j100\,\Omega; \qquad Z_2 = 250 + j500\,\Omega$

$\mathbf{I}_g = 125\angle 0^\circ\,\text{mA}$

$$\mathbf{I}_o = \frac{\mathbf{I}_g Z_1}{Z_1 + Z_2} = \frac{125\angle 0^\circ(50 - j100)}{(300 + j400)}$$

$$= -12.5 - j25\,\text{mA} = 27.95\angle -116.57^\circ\,\text{mA}$$

$i_o = 27.95\cos(500t - 116.57^\circ)\,\text{mA}$

P 7.31 $Z_o = 600 - j\dfrac{10^6}{(5000)(0.25)} = 600 - j800\,\Omega$

$Z_T = 300 + j2000 + 600 - j800 = 900 + j1200\,\Omega = 1500\angle 53.13^\circ\,\Omega$

$$\mathbf{V}_o = \mathbf{V}_g\frac{Z_o}{Z_T} = \frac{(75\angle 0^\circ)(1000\angle -53.13^\circ)}{1500\angle 53.13^\circ} = 50\angle -106.26^\circ\,\text{V}$$

$v_o = 50\cos(5000t - 106.26^\circ)\,\text{V}$

P 7.32 $\mathbf{V}_1 = 240\angle 53.13^\circ = 144 + j192\,\text{V}$

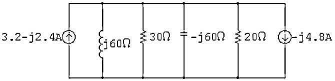
$\mathbf{V}_2 = 96\angle -90^\circ = -j96\,\text{V}$

$j\omega L = j(4000)(15 \times 10^{-3}) = j60\,\Omega$

$$\frac{1}{j\omega C} = -j\frac{6\times 10^6}{(4000)(25)} = -j60\,\Omega$$

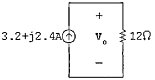
$$\frac{\mathbf{V}_1}{j60} = \frac{144+j192}{j60} = 3.2 - j2.4\,\text{A}$$

$$\frac{\mathbf{V}_2}{20} = -j\frac{96}{20} = -j4.8\,\text{A}$$



$$Y = \frac{1}{j60} + \frac{1}{30} + \frac{1}{-j60} + \frac{1}{20} = \frac{j5}{j60} = \frac{1}{12}$$

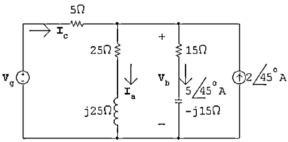
$$Z = \frac{1}{Y} = 12\,\Omega$$



$$\mathbf{V}_o = 12(3.2+j2.4) = 38.4 + j28.8\,\text{V} = 48\underline{/36.87^\circ}\,\text{V}$$

$$v_o = 48\cos(4000t + 36.87^\circ)\,\text{V}$$

P 7.33 [a]



$$\mathbf{V}_b = (15 - j15)5\underline{/45^\circ} = 75\sqrt{2}\underline{/0^\circ}\,\text{V}$$

$$\mathbf{I}_a = \frac{75\sqrt{2}}{25 + j25} = 3\underline{/-45^\circ}\,\text{A}$$

$$\mathbf{I}_c = \mathbf{I}_a + 5\underline{/45^\circ} - 2\underline{/45^\circ} = 3\sqrt{2}\,\text{A}$$

$$\mathbf{V}_g = 5\mathbf{I}_c + \mathbf{V}_b = 15\sqrt{2} + 75\sqrt{2} = 90\sqrt{2}\,\text{V} = 127.28\underline{/0^\circ}\,\text{V}$$

[b] $i_a = 3\cos(800t - 45^\circ)$ A

$i_c = 4.24\cos 800t$ A

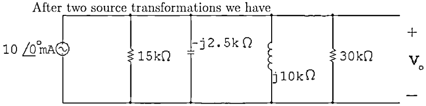
$v_g = 127.28\cos 800t$ V

P 7.34 $\mathbf{I}_s = 15\underline{/0^\circ}$ mA

$$\frac{1}{j\omega C} = \frac{10^6}{j0.05(8000)} = -j2500\,\Omega$$

$$j\omega L = j8000(1.25) = j10{,}000\,\Omega$$

After two source transformations we have



$15\,\text{k}\Omega \| 30\,\text{k}\Omega = 10\,\text{k}\Omega$

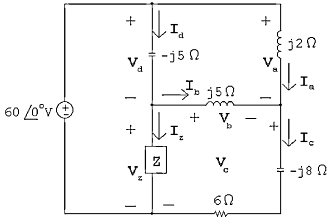
$$Y_o = \frac{10^{-3}}{10} + \frac{1}{-j2500} + \frac{1}{j10^4} = 10^{-4}(1 + j3)$$

$$Z_o = \frac{10^4}{1 + j3} = (1 - j3)\,\text{k}\Omega$$

$$\mathbf{V}_o = \mathbf{I}_g Z_o = (10)(1 - j3) = 10 - j30 = 31.62\underline{/-71.57^\circ}\,\text{V}$$

$$v_o = 31.62\cos(8000t - 71.57^\circ)\,\text{V}$$

P 7.35



$$\mathbf{V}_a = j2\mathbf{I}_a = j2(-j5) = 10\underline{/0^\circ}\,\text{V}$$

$$\mathbf{V}_c = 60\underline{/0^\circ} - \mathbf{V}_a = 50\underline{/0^\circ}\,\text{V}$$

$$\mathbf{I}_c = \frac{\mathbf{V}_c}{6 - j8} = \frac{50\underline{/0^\circ}}{10\underline{/-53.13^\circ}} = 5\underline{/53.13^\circ} = 3 + j4\,\text{A}$$

$$\mathbf{I}_b = \mathbf{I}_c - \mathbf{I}_a = 3 + j4 - (-j5) = 3 + j9\,\text{A} = 9.49\underline{/71.57^\circ}\,\text{A}$$

$$\mathbf{V}_b = \mathbf{I}_b(j5) = (3 + j9)(j5) = -45 + j15\,\text{V}$$

$$\mathbf{V}_z = \mathbf{V}_b + \mathbf{V}_c = -45 + j15 + 50 + j0 = 5 + j15\,\text{V}$$

$$\mathbf{V}_d + \mathbf{V}_z = 60\underline{/0^\circ}; \qquad \therefore\ \mathbf{V}_d = 60 - 5 - j15 = 55 - j15\,\text{V}$$

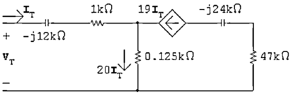
$$\mathbf{I}_d = \frac{\mathbf{V}_d}{-j5} = 3 + j11\,\text{A}$$

$$\mathbf{I}_z = \mathbf{I}_d - \mathbf{I}_b = 3 + j11 - 3 - j9 = j2\,\text{A}$$

$$Z = \frac{\mathbf{V}_z}{\mathbf{I}_z} = \frac{5 + j15}{j2} = 7.5 - j2.5\,\Omega$$

P 7.36 $\dfrac{1}{\omega C_1} = \dfrac{(10^{-3})(10^9)}{25(10/3)} = 12\,\text{k}\Omega$

$$\frac{1}{\omega C_2} = \frac{(10^{-3})(10^9)}{25(5/3)} = 24\,\text{k}\Omega$$

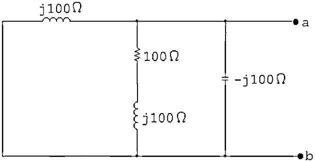


$$\mathbf{V}_T = (1 - j12)\mathbf{I}_T + 20\mathbf{I}_T(0.125)$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T} = 3.5 - j12\,\text{k}\Omega$$

P 7.37 [a] $j\omega L = j(1000)(100) \times 10^{-3} = j100\,\Omega$

$$\frac{1}{j\omega C} = -j\frac{10^6}{(1000)(10)} = -j100\,\Omega$$



$$Y_{\text{ab}} = \frac{1}{j100} + \frac{1}{100 + j100} + \frac{1}{-j100}$$

$$= \frac{1}{100}\left[\frac{1}{j} + \frac{1}{1 + j1} + \frac{j}{1}\right]$$

$$Y_{\text{ab}} = \frac{1}{100}\left[-j + \frac{1 - j1}{2} + j\right]$$

$$= \frac{1 - j1}{200}; \qquad Z_{\text{ab}} = \frac{200}{1 - j1} = 100(1 + j1)$$

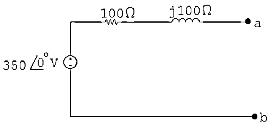
$$\therefore\ \mathbf{V}_{\text{ab}} = 100(1 + j1)\left[\frac{247.49\underline{/45^\circ}}{j100}\right]$$

$$= \sqrt{2}\underline{/45^\circ} \cdot 1\underline{/-90^\circ} \cdot 247.49\underline{/45^\circ}$$

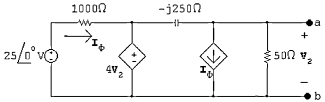
$$\mathbf{V}_{\text{Th}} = 350\underline{/0^\circ}\,\text{V}$$

[b] $Z_{\rm Th} = Z_{\rm ab} = 100 + j100\,\Omega$

[c]



P 7.38



$$\frac{\mathbf{V}_2}{50} + \frac{25 - 4\mathbf{V}_2}{1000} + \frac{\mathbf{V}_2 - 4\mathbf{V}_2}{-j250} = 0$$

Solving,

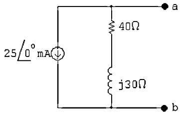
$$\mathbf{V}_2 = -10 - j0.75\,\text{V} = 1.25\underline{/216.87^\circ}\,\text{V}$$

$$\mathbf{I}_{sc} = -\mathbf{I}_\phi = \frac{-25\underline{/0^\circ}}{1000} = -25\underline{/0^\circ}\,\text{mA}$$

$$Z_{\rm Th} = \frac{1.25\underline{/216.87^\circ}}{-25 \times 10^{-3}\underline{/0^\circ}} = 50\underline{/36.87^\circ}\,\Omega = 40 + j30\,\Omega$$

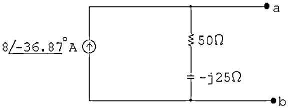
$$\mathbf{I}_N = \mathbf{I}_{sc} = -25\underline{/0^\circ}\,\text{mA}$$

$$Z_N = Z_{\rm Th} = 50\underline{/36.87^\circ} = 40 + j30\,\Omega$$

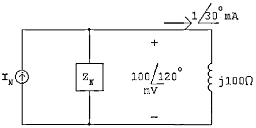


P 7.39 $\mathbf{I}_N = \mathbf{I}_{sc} = \dfrac{(16\underline{/0^\circ})(25)}{25 + 15 + j30} = 6.4 - j4.8\,\text{A} \;= 8\underline{/-36.87^\circ}\,\text{A}$

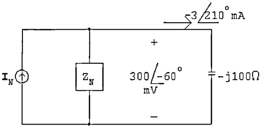
$$Z_N = \frac{(-j50)(40 + j30)}{40 + j30 - j50} = 50 - j25\,\Omega$$



P 7.40



$$\mathbf{I}_N = \frac{0.1\underline{/120^\circ}}{Z_N} + 1\underline{/30^\circ}\,\text{mA}, \quad Z_N \text{ in k}\Omega$$



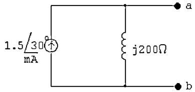
$$\mathbf{I}_N = \frac{0.3\underline{/-60^\circ}}{Z_N} + (-3\underline{/210^\circ})\,\text{mA}, \quad Z_N \text{ in k}\Omega$$

$$\frac{0.1\underline{/120^\circ}}{Z_N} + 1\underline{/30^\circ} = \frac{0.3\underline{/-60^\circ}}{Z_N} + (-3\underline{/210^\circ})$$

$$\frac{0.3\underline{/-60^\circ} - 0.1\underline{/120^\circ}}{Z_N} = 1\underline{/30^\circ} + 3\underline{/210^\circ}$$

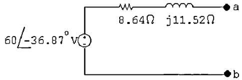
$$Z_N = \frac{0.3\angle{-60^\circ} - 0.1\angle{120^\circ}}{1\angle{30^\circ} + 3\angle{210^\circ}} = 0.2\angle{90^\circ} = j0.2\,\text{k}\Omega$$

$$\mathbf{I}_N = \frac{0.1\angle{120^\circ}}{0.2\angle{90^\circ}} + 1\angle{30^\circ} = 1.5\angle{30^\circ}\,\text{mA}$$

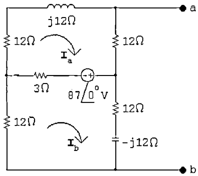


P 7.41 $\mathbf{V}_{\text{Th}} = \dfrac{75(24)}{24 + j18} = 60\angle{-36.87^\circ}\,\text{V}$

$$Z_{\text{Th}} = \frac{(24)(j18)}{24 + j18} = 8.64 + j11.52\,\Omega$$



P 7.42



$$(27 + j12)\mathbf{I}_a - 3\mathbf{I}_b = -87\angle{0^\circ}$$

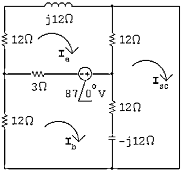
$$-3\mathbf{I}_a + (27 - j12)\mathbf{I}_b = 87\angle{0^\circ}$$

Solving,

$\mathbf{I}_a = -2.4167 + j1.21; \qquad \mathbf{I}_b = 2.4167 + j1.21$

$\mathbf{V}_{\mathrm{Th}} = 12\mathbf{I}_{\mathrm{a}} + (12 - j12)\mathbf{I}_{\mathrm{b}} = 14.5\underline{/0^\circ}\,\mathrm{V}$

Short Circuit Test:



$(27 + j12)\mathbf{I}_{\mathrm{a}} - 3\mathbf{I}_{\mathrm{b}} - 12\mathbf{I}_{\mathrm{sc}} = -87$

$-3\mathbf{I}_{\mathrm{a}} + (27 - j12)\mathbf{I}_{\mathrm{b}} - (12 - j12)\mathbf{I}_{\mathrm{sc}} = 87$

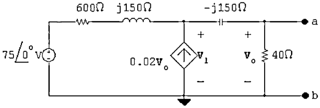
$-12\mathbf{I}_{\mathrm{a}} - (12 - j12)\mathbf{I}_{\mathrm{b}} + (24 - j12)\mathbf{I}_{\mathrm{sc}} = 0$

Solving,

$\mathbf{I}_{\mathrm{sc}} = 1\underline{/0^\circ}$

$$Z_{\mathrm{Th}} = \frac{\mathbf{V}_{\mathrm{Th}}}{\mathbf{I}_{\mathrm{sc}}} = \frac{14.5\underline{/0^\circ}}{1\underline{/0^\circ}} = 14.5\,\Omega$$

P 7.43



$$\frac{\mathbf{V}_1 - 75}{150(4 + j1)} - \frac{0.02\mathbf{V}_1(40)}{40 - j150} + \frac{\mathbf{V}_1}{40 - j150} = 0$$

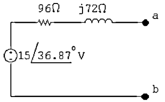
$$\therefore \quad \mathbf{V}_1 = \frac{75(4 - j15)}{16 - j12}$$

$$\mathbf{V}_{\text{Th}} = \frac{40\mathbf{V}_1}{40 - j150} = \frac{4}{4 - j15} \cdot \frac{75(4 - j15)}{16 - j12}$$

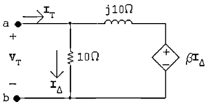
$$= \frac{75}{4 - j3} = 15\underline{/36.87^\circ}\ \text{V}$$

$$\mathbf{I}_{\text{sc}} = \frac{75}{600} = \frac{1}{8}\ \text{A}$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = 120\underline{/36.87^\circ} = 96 + j72\ \Omega$$



P 7.44 [a]



$$\mathbf{I}_T = \frac{\mathbf{V}_T}{10} + \frac{\mathbf{V}_T + \beta\mathbf{V}_T/10}{j10}$$

$$\frac{\mathbf{I}_T}{\mathbf{V}_T} = \frac{1}{10} + \frac{(1 - \beta/10)}{j10} = \frac{(10 - \beta) + j10}{j100}$$

$$\therefore\quad Z_{\text{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T} = \frac{1000 + j100(10 - \beta)}{(10 - \beta)^2 + 100}$$

Z_{Th} is real when $\beta = 10$.

[b] $Z_{\text{Th}} = \dfrac{1000}{100} = 10\ \Omega$

[c] $Z_{\text{Th}} = 5 + j5$

$$\frac{1000}{(10 - \beta)^2 + 100} = 5; \qquad (10 - \beta)^2 = 100$$

$$\therefore\quad 10 - \beta = \pm 10; \qquad \beta = 10 \mp 10$$

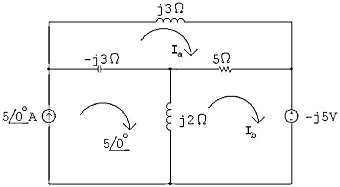
$$\beta = 0; \qquad \beta = 20$$

But the j term can only equal the real term with $\beta = 0$. Thus, $\beta = 0$.

[d] Z_{Th} will be capacitive when $\beta > 10$:

$\therefore \quad 10 < \beta \leq 50$

P 7.45



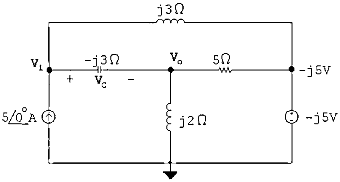
$j3\mathbf{I}_a + 5(\mathbf{I}_a - \mathbf{I}_b) - j3(\mathbf{I}_a - 5) = 0$

$j2(\mathbf{I}_b - 5) + 5(\mathbf{I}_b - \mathbf{I}_a) - j5 = 0$

Solving,

$\mathbf{I}_a = -j3; \qquad \mathbf{I}_g = -j3 = 3\underline{/-90^\circ}\,\text{A}$

P 7.46



$$\frac{\mathbf{V}_o}{j2} + \frac{\mathbf{V}_o + j5}{5} + \frac{\mathbf{V}_o - \mathbf{V}_1}{-j3} = 0$$

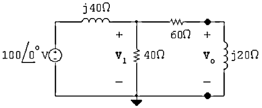
$(5 + j6)\mathbf{V}_o + 10\mathbf{V}_1 = 30$

$$-5 + \frac{\mathbf{V}_1 - \mathbf{V}_o}{-j3} + \frac{\mathbf{V}_1 + j5}{j3} = 0$$

$\mathbf{V}_o = j10; \qquad \mathbf{V}_1 = 9 - j5$

$\mathbf{V}_c = \mathbf{V}_1 - \mathbf{V}_o = 9 - j5 - j10 = 9 - j15 = 17.49\underline{/-59.04^\circ}\,\text{V}$

P 7.47



$$\frac{\mathbf{V}_1 - 100}{j40} + \frac{\mathbf{V}_1}{40} + \frac{\mathbf{V}_1}{60 + j20} = 0$$

Solving for $\mathbf{V}_1$ yields

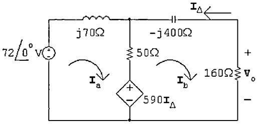
$$\mathbf{V}_1 = 30 - j40\,\text{V}$$

$$\mathbf{V}_o = \frac{\mathbf{V}_1}{60 + j20}(j20) = \left(\frac{j}{3+j}\right)\mathbf{V}_1$$

$$\mathbf{V}_o = 15 + j5\,\text{V} = 15.81\underline{/18.43^\circ}\,\text{V}$$

P 7.48 $j\omega L = j5000(14 \times 10^{-3}) = j70\,\Omega$

$$\frac{1}{j\omega C} = \frac{-j}{(5000)(0.5 \times 10^{-6})} = -j400\,\Omega$$



$$72\underline{/0^\circ} = (50 + j70)\mathbf{I}_a - 50\mathbf{I}_b + 590(-\mathbf{I}_b)$$

$$0 = -50\mathbf{I}_a - 590(-\mathbf{I}_b) + (210 - j400)\mathbf{I}_b$$

Solving,

$$\mathbf{I}_b = (50 - j50)\,\text{mA}$$

$$\mathbf{V}_o = 160\mathbf{I}_b = 8 - j8 = 11.31\underline{/-45^\circ}$$

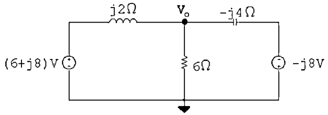
$$v_o = 11.31\cos(5000t - 45^\circ)\,\text{V}$$

P 7.49 $j\omega L = j(5000)(0.4 \times 10^{-3}) = j2\,\Omega$

$$\frac{1}{j\omega C} = -j\frac{10^6}{(5000)(50)} = -j4\,\Omega$$

$\mathbf{V}_{g1} = 10\underline{/53.13^\circ} = 6 + j8\,\text{V}$

$\mathbf{V}_{g2} = 8\underline{/-90^\circ} = -j8\,\text{V}$



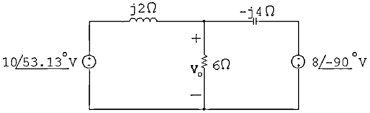
$$\frac{\mathbf{V}_o - 6 - j8}{j2} + \frac{\mathbf{V}_o}{6} + \frac{\mathbf{V}_o + (-j8)}{-j4} = 0$$

Solving,

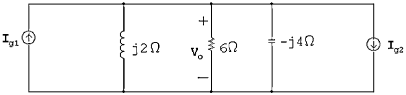
$\mathbf{V}_o = 12\underline{/0^\circ}$

$v_o(t) = 12\cos 5000t\,\text{V}$

P 7.50 From the solution to Problem 7.49 the phasor-domain circuit is



Making two source transformations yields



$$\mathbf{I}_{g1} = \frac{10\underline{/53.13^\circ}}{j2} = 5\underline{/-36.87^\circ} = 4 - j3\,\text{A}$$

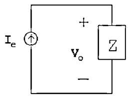
$$\mathbf{I}_{g2} = \frac{8/\!-90^\circ}{-j4} = 2/\!0^\circ = 2\,\text{A}$$

$$Y = \frac{1}{j2} + \frac{1}{6} + \frac{1}{-j4}\,\text{S}$$

$$Z = \frac{1}{Y} = \frac{1}{(1/6) - j(1/4)} = 1.85 + j2.77\,\Omega$$

$$\mathbf{I}_e = \mathbf{I}_{g1} - \mathbf{I}_{g2} = 4 - j3 - 2 = 2 - j3\,\text{A}$$

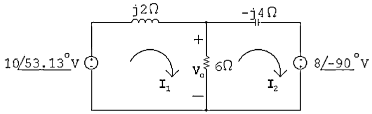
Hence the circuit reduces to



$$\mathbf{V}_o = Z\mathbf{I}_e = (1.85 + j2.77)(2 - j3) = 12/\!0^\circ$$

$$\therefore\quad v_o(t) = 12\cos 5000t\,\text{V}$$

P 7.51 From the solution to Problem 7.49 the phasor-domain circuit is



$$10/\!53.13^\circ = (6 + j2)\mathbf{I}_1 - 6\mathbf{I}_2$$

$$8/\!-90^\circ = -6\mathbf{I}_1 + (6 - j4)\mathbf{I}_2$$

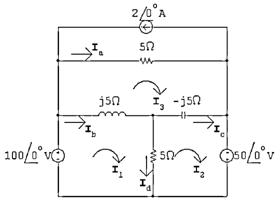
$$\mathbf{V}_o = (\mathbf{I}_1 - \mathbf{I}_2)6$$

Solving,

$$\mathbf{V}_o = 12/\!0^\circ\,\text{V}$$

$$v_o(t) = 12\cos 5000t\,\text{V}$$

P 7.52



$100\underline{/0^\circ} = (5 + j5)\mathbf{I}_1 - 5\mathbf{I}_2 - j5\mathbf{I}_3$

$50\underline{/0^\circ} = -5\mathbf{I}_1 + (5 - j5)\mathbf{I}_2 + j5\mathbf{I}_3$

$-10\underline{/0^\circ} = -j5\mathbf{I}_1 + j5\mathbf{I}_2 + 5\mathbf{I}_3$

Solving,

$\mathbf{I}_1 = 58 - j20\,\text{A}; \qquad \mathbf{I}_2 = 58 + j10\,\text{A}; \qquad \mathbf{I}_3 = 28 + j0\,\text{A}$

$\mathbf{I}_a = \mathbf{I}_3 + 2 = 30 + j0\,\text{A}$

$\mathbf{I}_b = \mathbf{I}_1 - \mathbf{I}_3 = 58 - j20 - 28 = 30 - j20\,\text{A}$

$\mathbf{I}_c = \mathbf{I}_2 - \mathbf{I}_3 = 58 + j10 - 28 = 30 + j10\,\text{A}$

$\mathbf{I}_d = \mathbf{I}_1 - \mathbf{I}_2 = 58 - j20 - 58 - j10 = -j30\,\text{A}$

P 7.53 $\mathbf{V}_2$ is the voltage across the $-j10\,\Omega$ impedance.

$$\frac{\mathbf{V}_1 - \mathbf{V}_g}{20} + \frac{\mathbf{V}_1}{j5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{Z} = 0$$

$$\frac{(40 + j30) - (100 - j50)}{20} + \frac{40 + j30}{j5} + \frac{(40 + j30) - \mathbf{V}_2}{Z} = 0$$

$$\therefore \quad \mathbf{V}_2 = 40 + j30 + (3 - j4)Z$$

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{Z} + \frac{\mathbf{V}_L}{-j10} - \mathbf{I}_g + \frac{\mathbf{V}_2 - \mathbf{V}_g}{3 + j1} = 0$$

$$\frac{\mathbf{V}_2 - (40 + j30)}{Z} + \frac{\mathbf{V}_2}{-j10} - (20 + j30) + \frac{\mathbf{V}_2 - (100 - j50)}{3 + j1} = 0$$

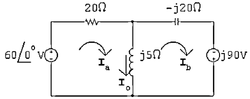
Substituting the expression for $\mathbf{V}_2$ found at the start and simplifying yields

$Z = 12 + j16\,\Omega$

P 7.54 $\mathbf{V}_a = 60\underline{/0^\circ}\,\text{V}; \qquad \mathbf{V}_b = 90\underline{/90^\circ}\,\text{V}$

$$j\omega L = j(4\times 10^4)(125\times 10^{-6}) = j5\Omega$$

$$\frac{-j}{\omega C} = \frac{-j10^6}{40{,}000(1.25)} = -j20\,\Omega$$



$$60 = (20+j5)\mathbf{I}_a - j5\mathbf{I}_b$$

$$j90 = -j5\mathbf{I}_a - j15\mathbf{I}_b$$

Solving,

$$\mathbf{I}_a = 2.25 - j2.25\,\text{A}; \qquad \mathbf{I}_b = -6.75 + j0.75\,\text{A}$$

$$\mathbf{I}_o = \mathbf{I}_a - \mathbf{I}_b = 9 - j3 = 9.49\underline{/-18.43^\circ}\,\text{A}$$

$$i_o(t) = 9.49\cos(40{,}000t - 18.43^\circ)\,\text{A}$$

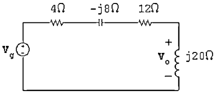
P 7.55 [a] $\dfrac{1}{j\omega C} = \dfrac{10^9}{j8\times 10^5(125)} = -j10\,\Omega$

$$j\omega L = j8\times 10^5(25\times 10^{-6}) = j20\,\Omega$$

$$Z_e = \frac{(-j10)(20)}{20-j10} = 4 - j8\,\Omega$$

$$\mathbf{I}_g = 5\underline{/0^\circ}$$

$$\mathbf{V}_g = \mathbf{I}_g Z_e = 5(4-j8) = 20 - j40\,\text{V}$$



$$\mathbf{V}_o = \frac{(20-j40)(j20)}{(16+j12)} = 44 - j8 = 44.72\underline{/-10.30^\circ}\,\text{V}$$

$$v_o = 44.72\cos(8\times 10^5 t - 10.30^\circ)\,\text{V}$$

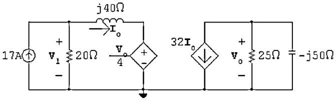
[b] $\omega = 2\pi f = 8 \times 10^5; \qquad f = \dfrac{4 \times 10^5}{\pi}$

$$T = \frac{1}{f} = \frac{\pi}{4 \times 10^5} = 2.5\pi\,\mu\text{s}$$

$$\therefore \quad \frac{10.30}{360}(2.5\pi) = 224.82\,\text{ns}$$

$\therefore \quad v_o$ lags i_g by 224.82 ns

P 7.56



$$\frac{\mathbf{V}_o}{25} + \frac{\mathbf{V}_o}{-j50} + 32\mathbf{I}_o = 0$$

$$(2 + j)\mathbf{V}_o = -1600\mathbf{I}_o$$

$$\mathbf{V}_o = (-640 + j320)\mathbf{I}_o$$

$$\mathbf{I}_o = \frac{\mathbf{V}_1 - (\mathbf{V}_o/4)}{j40}$$

$$\therefore \quad \mathbf{V}_1 = (-160 + j120)\mathbf{I}_o$$

$$17 = \frac{\mathbf{V}_1}{20} + \mathbf{I}_o = (-8 + j6)\mathbf{I}_o + \mathbf{I}_o = (-7 + j6)\mathbf{I}_o$$

$$\therefore \quad \mathbf{I}_o = \frac{17}{(-7 + j6)} = -1.4 - j1.2\,\text{A} = 1.84\underline{/-139.40^\circ}\,\text{A}$$

$$\mathbf{V}_o = (-640 + j320)\mathbf{I}_o = 1280 + j320 = 1319.39\underline{/14.04^\circ}\,\text{V}$$

P 7.57 $\quad -15\underline{/0^\circ} + \dfrac{\mathbf{V}_o}{8} + \dfrac{\mathbf{V}_o - 2.5\mathbf{I}_\Delta}{j5} + \dfrac{\mathbf{V}_o}{-j10} = 0$

$$\mathbf{I}_\Delta = \frac{\mathbf{V}_o}{-j10}$$

Solving,

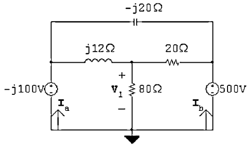
$$\mathbf{V}_o = 72 + j96 = 120\underline{/53.13^\circ}\,\text{V}$$

P 7.58 $j\omega L = j10^4(1.2 \times 10^{-3}) = j12\,\Omega$

$$\frac{1}{j\omega C} = \frac{-j10^6}{5 \times 10^4} = -j20\,\Omega$$

$$\mathbf{V}_a = 100\underline{/-90^\circ} = -j100\,\text{V}$$

$$\mathbf{V}_b = 500\underline{/0^\circ} = 500\,\text{V}$$



$$\frac{\mathbf{V}_1}{80} + \frac{\mathbf{V}_1 - 500}{20} + \frac{\mathbf{V}_1 + j100}{j12} = 0$$

Solving,

$$\mathbf{V}_1 = 160\underline{/53.13^\circ}\,\text{V} = 96 + j128\,\text{V}$$

$$\mathbf{I}_a = \frac{-j100 - 96 - j128}{j12} + \frac{-j100 - 500}{-j20}$$

$$= 14 - j17 = 22.02\underline{/-129.47^\circ}\,\text{A}$$

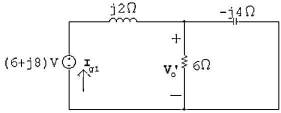
$$i_a = 22.02\cos(10{,}000t - 129.47^\circ)\,\text{A}$$

$$\mathbf{I}_b = \frac{500 - 96 - j128}{20} + \frac{500 + j100}{-j20}$$

$$= 15.2 + j18.6 = 24.02\underline{/50.74^\circ}\,\text{A}$$

$$i_b = 24.02\cos(10{,}000t + 50.74^\circ)\,\text{A}$$

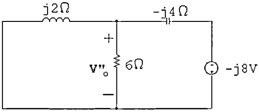
P 7.59 From the solution to Problem 7.49 the phasor-domain circuit with the right-hand source removed is



$$Z_{e1} = \frac{6(-j4)}{(6-j4)} = \frac{-j24}{6-j4}\,\Omega$$

$$\mathbf{V}_o' = \frac{Z_{e1}}{Z_{e1}+j2}(6+j8) = \frac{192-j144}{8-j12}\,\mathrm{V}$$

With the left hand source removed



$$Z_{e2} = \frac{6(j2)}{6+j2} = \frac{j12}{6+j2}\,\Omega$$

$$\mathbf{V}_o'' = \frac{Z_{e2}}{-j4+Z_{e2}}(j8) = \frac{-96}{8-j12}\,\mathrm{V}$$

$$\mathbf{V}_o = \mathbf{V}_o' + \mathbf{V}_o'' = \frac{192-j144-96}{8-j12} = 12+j0\,\mathrm{V}$$

$$v_o(t) = 12\cos 5000t\,\mathrm{V}$$

P 7.60 [a] $P = \frac{1}{2}(340)(20)\cos(60-15) = 2400\cos 45^\circ = 2404.16\,\mathrm{W}$ (abs)

$Q = 2400\sin 45^\circ = 2404.16\,\mathrm{VAR}$ (abs)

[b] $P = \frac{1}{2}(16)(75)\cos(-15 - 60) = 600\cos(-75^\circ) = 155.29\,\text{W} \quad \text{(abs)}$

$Q = 600\sin(-75^\circ) = -579.56\ \text{VAR} \quad \text{(del)}$

[c] $P = \frac{1}{2}(625)(4)\cos(40 - 150) = 1250\cos(-110^\circ) = -427.53\,\text{W} \quad \text{(del)}$

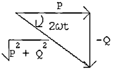
$Q = 1250\sin(-110^\circ) = -1174.62\ \text{VAR} \quad \text{(del)}$

[d] $P = \frac{1}{2}(180)(10)\cos(130 - 20) = 900\cos(110^\circ) = -307.82\,\text{W} \quad \text{(del)}$

$Q = 900\sin(110^\circ) = 845.72\ \text{VAR} \quad \text{(abs)}$

P 7.61 $p = P + P\cos 2\omega t - Q\sin 2\omega t; \qquad \frac{dp}{dt} = -2\omega P\sin 2\omega t - 2\omega Q\cos 2\omega t$

$$\frac{dp}{dt} = 0 \quad \text{when} \quad -2\omega P\sin 2\omega t = 2\omega Q\cos 2\omega t \quad \text{or} \quad \tan 2\omega t = -\frac{Q}{P}$$



$$\cos 2\omega t = \frac{P}{\sqrt{P^2+Q^2}}; \qquad \sin 2\omega t = -\frac{Q}{\sqrt{P^2+Q^2}}$$

Let $\theta = \tan^{-1}(-Q/P)$, then p is maximum when $2\omega t = \theta$ and p is minimum when $2\omega t = (\theta + \pi)$.

Therefore $p_{\max} = P + P\cdot\frac{P}{\sqrt{P^2+Q^2}} - \frac{Q(-Q)}{\sqrt{P^2+Q^2}} = P + \sqrt{P^2+Q^2}$

and $p_{\min} = P - P\cdot\frac{P}{\sqrt{P^2+Q^2}} - Q\cdot\frac{Q}{\sqrt{P^2+Q^2}} = P - \sqrt{P^2+Q^2}$

P 7.62 $W_{\text{dc}} = \frac{V_{\text{dc}}^2}{R}T; \qquad W_s = \int_{t_o}^{t_o+T}\frac{v_s^2}{R}\,dt$

$$\therefore \quad \frac{V_{\text{dc}}^2}{R}T = \int_{t_o}^{t_o+T}\frac{v_s^2}{R}\,dt$$

$$V_{\text{dc}}^2 = \frac{1}{T}\int_{t_o}^{t_o+T} v_s^2\,dt$$

$$V_{\text{dc}} = \sqrt{\frac{1}{T}\int_{t_o}^{t_o+T} v_s^2\,dt} = V_{\text{rms}} = V_{\text{eff}}$$

P 7.63 [a] Area under on cycle of v_g^2:

$$A = (400)(4)(20 \times 10^{-6}) + 10{,}000(2)(20 \times 10^{-6})$$
$$= 21{,}600(20 \times 10^{-6})$$

Mean value of v_g^2:

$$\text{M.V.} = \frac{A}{120 \times 10^{-6}} = \frac{21{,}600(20 \times 10^{-6})}{120 \times 10^{-6}} = 3600$$

$$\therefore \quad V_{\text{rms}} = \sqrt{3600} = 60\,\text{V(rms)}$$

[b] $P = \dfrac{V_{\text{rms}}^2}{R} = \dfrac{3600}{12} = 300\,\text{W}$

P 7.64 $i(t) = \dfrac{30}{40} \times 10^3 t = 750t \qquad 0 \leq t \leq 40\,\text{ms}$

$$i(t) = M - \frac{30}{10} \times 10^3 t \qquad 40\,\text{ms} \leq t \leq 50\,\text{ms}$$

$i(t) = 0$ when $t = 50\,\text{ms}$

$$\therefore \quad M = 3000(50 \times 10^{-3}) = 150$$

$$i(t) = 150 - 3000t \qquad 40\,\text{ms} \leq t \leq 50\,\text{ms}$$

$$\therefore \quad I_{\text{rms}} = \sqrt{\frac{1000}{50}\left\{\int_0^{0.04} (750)^2 t^2\,dt + \int_{0.04}^{0.05} (150 - 3000t)^2\,dt\right\}}$$

$$\int_0^{0.04} (750)^2 t^2\,dt = (750)^2 \frac{t^3}{3}\Big|_0^{0.04} = 12$$

$$(150 - 3000t)^2 = 22{,}500 - 9 \times 10^5 t + 9 \times 10^6 t^2$$

$$\int_{0.04}^{0.05} 22{,}500\,dt = 225$$

$$\int_{0.04}^{0.05} 9 \times 10^5 t\,dt = 45 \times 10^4 t^2 \Big|_{0.04}^{0.05} = 405$$

$$9 \times 10^6 \int_{0.04}^{0.05} t^2\,dt = 3 \times 10^6 t^3 \Big|_{0.04}^{0.05} = 183$$

$$\therefore \quad I_{\text{rms}} = \sqrt{20\{12 + (225 - 405 + 183)\}} = \sqrt{300} = 17.32\,\text{A}$$

P 7.65 $P = I_{\rm rms}^2 R \qquad \therefore \quad R = \dfrac{24 \times 10^3}{300} = 80\,\Omega$

P 7.66 $\dfrac{1}{\omega C} = \dfrac{10^9}{(5000)(80)} = 2500\,\Omega$

$$Z_{\rm f} = \frac{-j2500(7500)}{7500 - j2500} = 750 - j2250\,\Omega$$

$$Z_{\rm i} = 1500\,\Omega$$

$$\therefore \quad \frac{Z_{\rm f}}{Z_{\rm i}} = \frac{750 - j2250}{1500} = 0.5 - j1.5$$

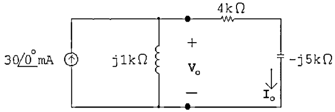
$$\mathbf{V}_o = -\frac{Z_{\rm f}}{Z_{\rm i}}\mathbf{V}_g; \qquad \mathbf{V}_g = 4\underline{/0^\circ}\,\text{V}$$

$$\mathbf{V}_o = (-0.5 + j1.5)(4) = -2 + j6 = 6.32\underline{/108.43^\circ}\,\text{V}$$

$$P = \frac{1}{2}\frac{V_m^2}{R} = \frac{1}{2}\frac{(4)(10)}{1000} = 20 \times 10^{-3} = 20\,\text{mW}$$

P 7.67 $\mathbf{I}_g = 30\underline{/0^\circ}\,\text{mA}$

$$j\omega L = j(100)(10) = j1000\,\Omega; \qquad \frac{1}{j\omega C} = \frac{10^6}{j(100)(2)} = -j5000\,\Omega$$



$$\mathbf{I}_o = \frac{30\underline{/0^\circ}(j1000)}{4000 - j4000} = 3.75\sqrt{2}\underline{/135^\circ}\,\text{mA}$$

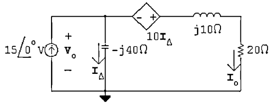
$$P = |\mathbf{I}_o|_{\rm rms}^2(4000) = (3.75)^2(4000) = 56.25\,\text{mW}$$

$$Q = |\mathbf{I}_o|_{\rm rms}^2(-5000) = -70.3125\,\text{mVAR}$$

$$S = P + jQ = 56.25 - j70.3125\,\text{mVA}$$

$$|S| = 90.044\,\text{mVA}$$

P 7.68 $j\omega L = j10{,}000(10^{-3}) = j10\,\Omega;\qquad \frac{1}{j\omega C} = \frac{10^6}{j10{,}000(2.5)} = -j40\,\Omega$



$$-15 + \frac{\mathbf{V}_o}{-j40} + \frac{\mathbf{V}_o + 10(\mathbf{V}_o/-j40)}{20+j10} = 0$$

$$\therefore\quad \mathbf{V}_o\left[\frac{1}{-j40} + \frac{1+j0.25}{20+j10}\right] = 15$$

$$\therefore\quad \mathbf{V}_o = 300 - j100\,\text{V}$$

$$\therefore\quad \mathbf{I}_\Delta = \frac{\mathbf{V}_o}{-j40} = 2.5 + j7.5\,\text{A}$$

$$\mathbf{I}_o = 15\underline{/0^\circ} - \mathbf{I}_\Delta = 15 - 2.5 - j7.5 = 12.5 - j7.5 = 14.58\underline{/-30.9^\circ}\,\text{A}$$

$$P_{20\Omega} = \frac{1}{2}|\mathbf{I}_o|^2 20 = 2125\,\text{W}$$

P 7.69 [a] $Z_1 = 240 + j70 = 250\underline{/16.26^\circ}\,\Omega$

pf $= \cos(16.26^\circ) = 0.96$ lagging

rf $= \sin(16.26^\circ) = 0.28$

$Z_2 = 160 - j120 = 200\underline{/-36.87^\circ}\,\Omega$

pf $= \cos(-36.87^\circ) = 0.80$ leading

rf $= \sin(-36.87^\circ) = -0.60$

$Z_3 = 30 - j40 = 50\underline{/-53.13^\circ}\,\Omega$

pf $= \cos(-53.13^\circ) = 0.6$ leading

rf $= \sin(-53.13^\circ) = -0.8$

[b] $Y = Y_1 + Y_2 + Y_3$

$$Y_1 = \frac{1}{250\angle 16.26^\circ}; \qquad Y_2 = \frac{1}{200\angle -36.87^\circ}; \qquad Y_3 = \frac{1}{50\angle -53.13^\circ}$$

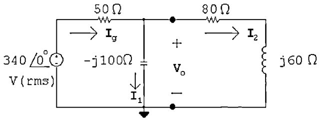
$$Y = 19.84 + j17.88\,\text{mS}$$

$$Z = \frac{1}{Y} = 37.44\angle -42.03^\circ\,\Omega$$

$$\text{pf} = \cos(-42.03^\circ) = 0.74 \text{ leading}$$

$$\text{rf} = \sin(-42.03^\circ) = -0.67$$

P 7.70 [a]



$$\frac{\mathbf{V}_o}{-j100} + \frac{\mathbf{V}_o - 340}{50} + \frac{\mathbf{V}_o}{80 + j60} = 0$$

$$\therefore \quad \mathbf{V}_o = 238 - j34\,\text{V}$$

$$\mathbf{I}_g = \frac{340 - 238 + j34}{50} = 2.04 + j0.68\,\text{A}$$

$$S_g = \mathbf{V}_g\mathbf{I}_g^* = (340)(2.04 - j0.68)$$
$$= 693.6 - j231.2\,\text{VA}$$

[b] Source is delivering 693.6 W.

[c] Source is absorbing 231.2 magnetizing VAR.

[d] $\mathbf{I}_1 = \dfrac{\mathbf{V}_o}{-j100} = 0.34 + j2.38\,\text{A}$

$$S_1 = \mathbf{V}_o\mathbf{I}_1^* = (238 - j34)(0.34 - j2.38)$$
$$= 0 - j578\,\text{VA}$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_o}{80 + j60} = \frac{238 - j34}{80 + j60} = 1.7 - j1.7\,\text{A}$$

$$S_2 = \mathbf{V}_o\mathbf{I}_2^* = (238 - j34)(1.7 + j1.7)$$
$$= 462.4 + j346.8\,\text{VA}$$

$$S_{50\Omega} = |\mathbf{I}_g|^2(50) + j0 = (2.15)^2(50) = 231.2\,\text{W}$$

[e] $\sum P_{\text{del}} = 693.6\,\text{W}$

$\sum P_{\text{diss}} = 462.4 + 231.2 = 693.6\,\text{W}$

$\therefore \quad \sum P_{\text{del}} = \sum P_{\text{diss}} = 693.6\,\text{W}$

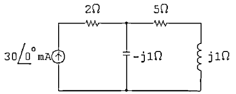
[f] $\sum Q_{\text{abs}} = 231.2 + 346.8 = 578\,\text{VAR}$

$\sum Q_{\text{dev}} = 578\,\text{VAR}$

$\therefore \quad \sum \text{ mag VAR dev } = \sum \text{ mag VAR abs } = 578$

P 7.71 $\mathbf{I}_g = 30\underline{/0^\circ}\,\text{mA}; \qquad \dfrac{1}{j\omega C} = \dfrac{10^6}{j(25\times10^3)(40)} = -j1\,\Omega$

$j\omega L = j(25\times10^3)(40)\times10^{-6} = j1\,\Omega$



$Z_1 = j1\|(5+j1) = 0.2 - j1\,\Omega$

$Z_{\text{eq}} = 2 + Z_1 = 2.2 - j1\,\Omega$

$P_g = |I_{\text{rms}}|^2\text{Re}\{Z_{\text{eq}}\} = \left(\dfrac{30}{\sqrt{2}}\times10^{-3}\right)^2(2.2) = 990\,\mu\text{W}$

P 7.72 [a] $P = \dfrac{1}{2}\dfrac{(240)^2}{480} = 60\,\text{W}$

$-\dfrac{1}{\omega C} = \dfrac{-9\times10^6}{(5000)(5)} = -360\,\Omega$

$Q = \dfrac{1}{2}\dfrac{(240)^2}{(-360)} = -80\,\text{VAR}$

$p_{\max} = P + \sqrt{P^2+Q^2} = 60 + \sqrt{(60)^2+(80)^2} = 160\,\text{W(del)}$

[b] $p_{\min} = 60 - 100 = -40\,\text{W(abs)}$

[c] $P = 60\,\text{W}$

[d] $Q = -80\,\text{VAR}$

[e] generate

[f] pf $= \cos(\theta_v - \theta_i)$

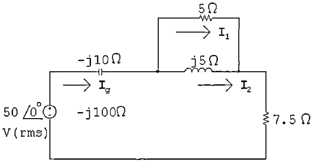
$$\mathbf{I} = \frac{240}{480} + \frac{240}{-j360} = 0.5 + j0.67 = 0.83\underline{/53.13^\circ}\,\text{A}$$

$$\therefore \quad \text{pf} = \cos(0 - 53.13^\circ) = 0.6 \text{ leading}$$

[g] rf $= \sin(-53.13^\circ) = -0.8$

P 7.73 [a] $\dfrac{1}{j\omega C} = \dfrac{10^6}{j10^5} = -j10\,\Omega$

$$j\omega L = j10^5(50 \times 10^{-6}) = j5\,\Omega$$



$$Z = -j10 + \frac{(5)(j5)}{5 + j5} + 7.5 = 10 - j7.5\,\Omega$$

$$\mathbf{I}_g = \frac{50\underline{/0^\circ}}{10 - j7.5} = 3.2 + j2.4\,\text{A}$$

$$S_g = \frac{1}{2}\mathbf{V}_g\mathbf{I}_g^* = 25(3.2 - j2.4) = 80 - j60\,\text{VA}$$

$$P = 80\,\text{W(del)}; \qquad Q = 60\,\text{VAR(abs)}$$

$$|S| = |S_g| = 100\,\text{VA}$$

[b] $\mathbf{I}_1 = \dfrac{\mathbf{I}_g(j5)}{5 + j5} = \dfrac{1}{2}(3.2 + j2.4)(1 + j1) = 0.4 + j2.8\,\text{A}$

$$P_{5\Omega} = \frac{1}{2}|\mathbf{I}_1|^2(5) = 20\,\text{W}$$

$$P_{7.5\Omega} = \frac{1}{2}|\mathbf{I}_g|^2(7.5) = 60\,\text{W}$$

$$\sum P_{\text{diss}} = 20 + 60 = 80\,\text{W} = \sum P_{\text{dev}}$$

[c] $\mathbf{I}_{j5} = \dfrac{\mathbf{I}_g 5}{5+j5} = \dfrac{1}{2}(3.2+j2.4)(1-j1) = 2.8 - j0.4\,\text{A}$

$$Q_{j5\Omega} = \frac{1}{2}|\mathbf{I}_{j5}|^2(5) = 20\,\text{VAR(abs)}$$

$$Q_{-j10\Omega} = \frac{1}{2}|\mathbf{I}_g|^2(-10) = -80\,\text{VAR(dev)}$$

$$\sum Q_{\text{abs}} = 20 + 60 = 80\,\text{VAR} = \sum Q_{\text{dev}}$$

P 7.74 [a] $S_1 = 24{,}960 + j47{,}040\,\text{VA}$

$$S_2 = \frac{|\mathbf{V}_\text{L}|^2}{Z_2^*} = \frac{(480)^2}{5+j5} = 23{,}040 - j23{,}040\,\text{VA}$$

$$S_1 + S_2 = 48{,}000 + j24{,}000\,\text{VA}$$

$$480\mathbf{I}_\text{L}^* = 48{,}000 + j24{,}000; \qquad \therefore\quad \mathbf{I}_\text{L} = 100 - j50\,\text{A(rms)}$$

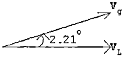
$$\mathbf{V}_g = \mathbf{V}_\text{L} + \mathbf{I}_\text{L}(0.02 + j0.20) = 480 + (100 - j50)(0.02 + j0.20)$$
$$= 492 + j19 = 492.37\underline{/2.21^\circ}\,\text{Vrms}$$

$$|\mathbf{V}_g| = 492.37\,\text{Vrms}$$

[b] $T = \dfrac{1}{f} = \dfrac{1}{60} = 16.67\,\text{ms}$

$$\frac{2.21^\circ}{360^\circ} = \frac{t}{16.67\ \text{ms}}; \qquad \therefore\quad t = 102.39\,\mu\text{s}$$

[c] $\mathbf{V}_\text{L}$ lags $\mathbf{V}_g$ by 2.21° or 102.31 μs



P 7.75 [a] $S_1 = 18 + j24\,\text{kVA}; \qquad S_2 = 36 - j48\,\text{kVA}; \qquad S_3 = 18 + j0\,\text{kVA}$

$$S_\text{T} = S_1 + S_2 + S_3 = 72 - j24\,\text{kVA}$$

$$2400\mathbf{I}^* = (72 - j24)\times 10^3; \qquad \therefore\quad \mathbf{I} = 30 + j10\,\text{A}$$

$$Z = \frac{2400}{30 + j10} = 72 - j24\,\Omega = 75.89\underline{/-18.43^\circ}\,\Omega$$

[b] pf $= \cos(-18.43^\circ) = 0.9487$ leading

P 7.76 [a] From the solution to Problem 7.75 we have

$\mathbf{I}_L = 30 + j10\,\text{A(rms)}$

$\therefore\quad \mathbf{V}_s = 2400\underline{/0^\circ} + (30 + j10)(0.2 + j1.6) = 2390 + j50$

$= 2390.52\underline{/1.20^\circ}\,\text{V(rms)}$

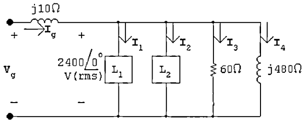
[b] $|\mathbf{I}_L| = \sqrt{1000}$

$P_\ell = (1000)(0.2) = 200\,\text{W} \qquad Q_\ell = (1000)(1.6) = 1600\,\text{VAR}$

[c] $P_s = 72{,}000 + 200 = 72.2\,\text{kW} \qquad Q_s = -24{,}000 + 1600 = -22.4\,\text{kVAR}$

[d] $\eta = \dfrac{72}{72.2}(100) = 99.72\%$

P 7.77



$2400\mathbf{I}_1^* = 24{,}000 + j18{,}000$

$\mathbf{I}_1^* = 10 + j7.5; \qquad \therefore\quad \mathbf{I}_1 = 10 - j7.5\,\text{A(rms)}$

$2400\mathbf{I}_2^* = 48{,}000 - j30{,}000$

$\mathbf{I}_2^* = 20 - j12.5; \qquad \therefore\quad \mathbf{I}_2 = 20 + j12.5\,\text{A(rms)}$

$\mathbf{I}_3 = \dfrac{2400\underline{/0^\circ}}{60} = 40 + j0\,\text{A}; \qquad \mathbf{I}_4 = \dfrac{2400\underline{/0^\circ}}{j480} = 0 - j5\,\text{A}$

$\mathbf{I}_g = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 + \mathbf{I}_4 = 70\,\text{A}$

$\mathbf{V}_g = 2400 + (70)(j10) = 2400 + j700 = 2500\underline{/16.26^\circ}\,\text{V(rms)}$

P 7.78 $S_T = 52{,}800 - j\dfrac{52{,}800}{0.8}(0.6) = 52{,}800 - j39{,}600\,\text{VA}$

$S_1 = 40{,}000(0.96 + j0.28) = 38{,}400 + j11{,}200\,\text{VA}$

$S_2 = S_T - S_1 = 14{,}400 - j50{,}800 = 52{,}801.52\underline{/-74.17^\circ}\,\text{VA}$

pf $= \cos(-74.17^\circ) = 0.2727$ leading

P 7.79 [a] $\mathbf{I} = \dfrac{7200\underline{/0^\circ}}{140 + j480} = 14.4\underline{/-73.74^\circ}\,\text{A(rms)}$

$P = (14.4)^2(2) = 414.72\,\text{W}$

[b] $Y_\text{L} = \dfrac{1}{138 + j460} = \dfrac{138 - j460}{230{,}644}$

$\therefore\quad -j\omega C = -j\dfrac{460}{230{,}644} \qquad \therefore\quad X_\text{C} = \dfrac{-230{,}644}{460} = -501.40\,\Omega$

[c] $Z_\text{L} = \dfrac{230{,}644}{138} = 1671.33\,\Omega$

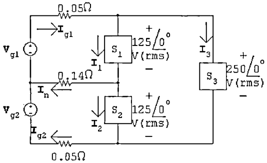
[d] $\mathbf{I} = \dfrac{7200}{1673.33 + j20} = 4.30\underline{/-0.68^\circ}\,\text{A}$

$P = (4.30)^2(2) = 37.02\,\text{W}$

[e] $\% = \dfrac{37.02}{414.72}(100) = 8.93\%$

Thus the power loss after the capacitor is added is 8.93% of the power loss before the capacitor is added.

P 7.80 [a]



$\mathbf{I}_1 = \dfrac{5000 - j2000}{125} = 40 - j16\,\text{A (rms)}$

$\mathbf{I}_2 = \dfrac{3750 - j1500}{125} = 30 - j12\,\text{A (rms)}$

$\mathbf{I}_3 = \dfrac{8000 + j0}{250} = 32 + j0\,\text{A (rms)}$

$\therefore\quad \mathbf{I}_{g1} = 72 - j16\,\text{A (rms)}$

$\mathbf{I}_n = \mathbf{I}_1 - \mathbf{I}_2 = 10 - j4\,\text{A (rms)}$

$\mathbf{I}_{g2} = 62 - j12\,\text{A}$

$\mathbf{V}_{g1} = 0.05\mathbf{I}_{g1} + 125 + j0 + 0.14\mathbf{I}_n = 130 - j1.36\,\text{V(rms)}$

$\mathbf{V}_{g2} = -0.14\mathbf{I}_n + 125 + j0 + 0.05\mathbf{I}_{g2} = 126.7 - j0.04\,\text{V(rms)}$

$S_{g1} = [(130 - j1.36)(72 + j16)] = [9381.76 + j1982.08]\,\text{VA}$

$S_{g2} = [(126.7 - j0.04)(62 + j12)] = [7855.88 + j1517.92]\,\text{VA}$

Note: Both sources are delivering average power and magnetizing VAR to the circuit.

[b] $P_{0.05} = |\mathbf{I}_{g1}|^2(0.05) = 272\,\text{W}$

$P_{0.14} = |\mathbf{I}_n|^2(0.14) = 16.24\,\text{W}$

$P_{0.05} = |\mathbf{I}_{g2}|^2(0.05) = 199.4\,\text{W}$

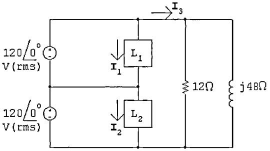
$\sum P_{\text{dis}} = 272 + 16.24 + 199.4 + 5000 + 3750 + 8000 = 17{,}237.64\,\text{W}$

$\sum P_{\text{dev}} = 9381.76 + 7855.88 = 17{,}237.64\,\text{W} = \sum P_{\text{dis}}$

$\sum Q_{\text{abs}} = 2000 + 1500 = 2500\,\text{VAR}$

$\sum Q_{\text{del}} = 1982.08 + 1517.92 = 3500\,\text{VAR} = \sum Q_{\text{abs}}$

P 7.81 [a]



$120\mathbf{I}_1^* = 1800 + j600; \qquad \therefore \quad \mathbf{I}_1 = 15 - j5\,\text{A(rms)}$

$120\mathbf{I}_2^* = 1200 - j900; \qquad \therefore \quad \mathbf{I}_2 = 10 + j7.5\,\text{A(rms)}$

$$\mathbf{I}_3 = \frac{240}{12} + \frac{240}{j48} = 20 - j5\,\text{A(rms)}$$

$\mathbf{I}_{g1} = \mathbf{I}_1 + \mathbf{I}_3 = 35 - j10\,\text{A}$

$S_{g1} = 120(35 + j10) = 4200 + j1200\,\text{VA}$

Thus the $\mathbf{V}_{g1}$ source is delivering 4200 W and 1200 magnetizing vars.

$\mathbf{I}_{g2} = \mathbf{I}_2 + \mathbf{I}_3 = 30 + j2.5\,\text{A(rms)}$

$S_{g2} = 120(30 - j2.5) = 3600 - j300\,\text{VA}$

Thus the $\mathbf{V}_{g2}$ source is delivering 3600 W and absorbing 300 magnetizing vars.

[b] $\sum P_{\text{gen}} = 4200 + 3600 = 7800\,\text{W}$

$$\sum P_{\text{abs}} = 1800 + 1200 + \frac{(240)^2}{12} = 7800\,\text{W} = \sum P_{\text{gen}}$$

$$\sum Q_{\text{del}} = 1200 + 900 = 2100\,\text{VAR}$$

$$\sum Q_{\text{abs}} = 300 + 600 + \frac{(240)^2}{48} = 2100\,\text{VAR} = \sum Q_{\text{del}}$$

P 7.82 [a] $S_{\text{L}} = 24 + j7\,\text{kVA}$

$$125\mathbf{I}_{\text{L}}^* = (24 + j7) \times 10^3; \quad \mathbf{I}_{\text{L}}^* = 192 + j56\,\text{A(rms)}$$

$$\therefore \quad \mathbf{I}_{\text{L}} = 192 - j56\,\text{A(rms)}$$

$$\mathbf{V}_s = 125 + (192 - j56)(0.006 + j0.048) = 128.84 + j8.88$$
$$= 129.15\underline{/3.94^\circ}\,\text{V(rms)}$$

$$|\mathbf{V}_s| = 129.15\,\text{V(rms)}$$

[b] $P_\ell = |\mathbf{I}_\ell|^2(0.006) = (200)^2(0.006) = 240\,\text{W}$

[c] $\dfrac{(125)^2}{X_{\text{C}}} = -7000; \qquad X_{\text{C}} = -2.23\,\Omega$

$$-\frac{1}{\omega C} = -2.23; \qquad C = \frac{1}{(2.23)(120\pi)} = 1188.36\,\mu\text{F}$$

[d] $\mathbf{I}_\ell = 192 + j0\,\text{A(rms)}$

$$\mathbf{V}_s = 125 + 192(0.006 + j0.048) = 126.152 + j9.216$$
$$= 126.49\underline{/4.18^\circ}\,\text{V(rms)}$$

$$|\mathbf{V}_s| = 126.49\,\text{V(rms)}$$

[e] $P_\ell = (192)^2(0.006) = 221.184\,\text{W}$

P 7.83 [a] $\Delta = R_aR_bR_c - R_1^2R_b - R_2^2R_a - R_n(2R_1R_2 + R_nR_c)$

$$R_a = R_1 + R_n + R_l = 30 + 1 + 0.5 = 31.5\,\Omega$$

$$R_b = R_2 + R_n + R_l = 300 + 1 + 0.5 = 301.5\,\Omega$$

$$R_c = R_1 + R_2 + R_3 = 30 + 300 + 15 = 345\,\Omega$$

$$\Delta = (31.5)(301.5)(345) - 900(301.5) - 9 \times 10^4(31.5)$$
$$-1[2(30)(300) + 1(345)]$$
$$= 151{,}856.25$$

$$N_a = \mathbf{V}_{g1}[(R_bR_c - R_2^2) + R_nR_c + R_1R_2]$$
$$= 120[(301.5)(345) - 9 \times 10^4 + 345 + 30(300)]$$
$$= 2{,}803{,}500$$
$$N_b = \mathbf{V}_{g1}[R_nR_c + R_1R_2 + R_aR_c - R_1^2]$$
$$= 120[345 + (30)(300) + 31.5(345) - 900]$$
$$= 2{,}317{,}500$$

$$\mathbf{I}_a = \frac{N_a}{\Delta}; \qquad \mathbf{I}_b = \frac{N_b}{\Delta}$$

$$\mathbf{I}_n = \mathbf{I}_a - \mathbf{I}_b = \frac{N_a - N_b}{\Delta} = 3.2\underline{/0^\circ}\text{A(rms)}$$

[b]

$$N_c = \mathbf{V}_{g1}[R_2R_n + R_1R_b + R_2R_a + R_1R_n]$$
$$= 120[300 + 30(301.5) + 31.5(300) + 30]$$
$$= 2{,}259{,}000$$

$$\mathbf{I}_{L1} = \frac{N_a - N_c}{\Delta}$$

$$\mathbf{V}_1 = 30\mathbf{I}_{L1} = \frac{30(N_a - N_c)}{\Delta} = 107.57\underline{/0^\circ}\text{V(rms)}$$

[c] $\mathbf{I}_{L2} = \dfrac{N_b - N_c}{\Delta}$

$$\mathbf{V}_2 = 300\mathbf{I}_{L2} = \frac{300(N_b - N_c)}{\Delta} = 115.57\underline{/0^\circ}\text{V(rms)}$$

[d] $\mathbf{V}_3 = 15\mathbf{I}_c = 15\dfrac{N_c}{\Delta} = 233.14\underline{/0^\circ}\text{V(rms)}$

CHECK:

$$\mathbf{V}_3 = \mathbf{V}_1 + \mathbf{V}_2 = 107.57\underline{/0^\circ} + 115.57\underline{/0^\circ} = 233.14\underline{/0^\circ}\text{V(rms)}$$

[e] $P_1 = \dfrac{|\mathbf{V}_1|^2}{R_1} = \dfrac{(107.57)^2}{30} = 385.70\text{ W}$

$$P_2 = \frac{|\mathbf{V}_2|^2}{R_2} = \frac{(115.57)^2}{300} = 44.52\text{ W}$$

$$P_3 = \frac{|\mathbf{V}_3|^2}{R_3} = \frac{(233.14)^2}{15} = 3319.39\text{ W}$$

[f] $\mathbf{I}_a = \dfrac{N_a}{\Delta} = 18.46\underline{/0^\circ}\text{A(rms)}$

$\mathbf{I}_b = \dfrac{N_b}{\Delta} = 15.26\underline{/0^\circ}\text{A(rms)}$

$P_a = (120)(18.46)\cos 0^\circ$

$P_b = (120)(15.26)\cos 0^\circ$

$\sum P_{\text{gen}} = 120(18.46 + 15.26) = 4046.72 \text{ W}$

[g] $P_a = |\mathbf{I}_a|^2(0.5) = 170.41 \text{ W}$

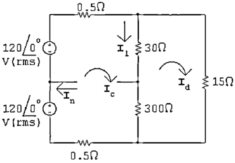
$P_n = |\mathbf{I}_n|^2(1) = 10.24 \text{ W}$

$P_b = |\mathbf{I}_b|^2(0.5) = 116.45 \text{ W}$

$$\begin{aligned}\sum P_{\text{diss}} &= 170.41 + 10.24 + 116.45 + 385.70 \\ &\quad +44.52 + 3319.39 \\ &= 4046.72 \text{ W}\end{aligned}$$

P 7.84 [a] $\mathbf{I}_n = 0$ by hypothesis.

[b] With the neutral conductor open the circuit becomes:



The two mesh current equations are

$240\underline{/0^\circ} = 331\mathbf{I}_c - 330\mathbf{I}_d$

$0 = -330\mathbf{I}_c + 345\mathbf{I}_d$

$\therefore \quad \mathbf{I}_c = \dfrac{82{,}800}{5295} + 15.64\underline{/0^\circ}\text{A(rms)}$

$\mathbf{I}_d = \dfrac{79{,}200}{5295} + 14.96\underline{/0^\circ}\text{A(rms)}$

$\mathbf{I}_1 = \mathbf{I}_c - \mathbf{I}_d = 0.68\underline{/0^\circ}\text{A(rms)}$

$\mathbf{V}_1 = 30\mathbf{I}_1 = 20.40\underline{/0^\circ}\text{V(rms)}$

[c] $\mathbf{V}_2 = 300\mathbf{I}_1 = 203.97\underline{/0^\circ}\mathrm{V(rms)}$

[d] $\mathbf{V}_3 = 15\mathbf{I}_d = 224.36\underline{/0^\circ}\mathrm{V(rms)}$

[e] $P_{R_1} = (20.40)^2/30 = 13.87$ W

$$P_{R_2} = (203.97)^2/300 = 138.67 \text{ W}$$

$$P_{R_3} = (224.36)^2/15 = 3355.91 \text{ W}$$

[f] $\sum P_{\text{gen}} = 240|\mathbf{I}_c|\cos 0^\circ = (240)(15.64)(1) = 3752.97$ W

[g] $\sum P_{\text{diss}} = (15.64)^2(1) + 13.87 + 138.67 + 3355.91 = 3752.97$ W