SOLUTIONS

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| Module: | Singles and Systems Theory | | |
| Module Code | BBU4374 | Paper | A |
| Time allowed | 2hrs | Filename | Solutions\_2223\_BBU4374\_A |
| Rubric | Answer ALL FOUR questions | | |
| Examiners | Dr Changchuan Yin, Dr Feng Zheng, Dr Dong Liang, Dr Yang Yang, Dr Shaoshi Yang, Dr Li Li | | |

**Question 1 [18 marks]**

1. Suppose a discrete-time sinusoidal signal is defined as . Determine whether  is periodic; if yes, find its fundamental period.

**(4 marks)**

1. The sequence  is shown in **Figure 1.1**. Please plot the sequence .

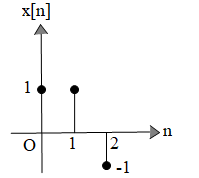
**(4 marks)**

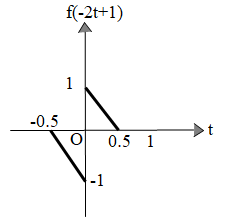
1. The waveform of  is shown in **Figure 1.2**. Please draw the waveform of .

**(4 marks)**

1. A discrete-time sequence is defined as , where  is the unit-impulse sequence. Please plot sequence  and calculate its energy.

**(6 marks)**





**Figure 1.1 Figure 1.2**

**Solutions:**

1.  is periodic (2 marks).

, then the fundamental period for  (2 marks).

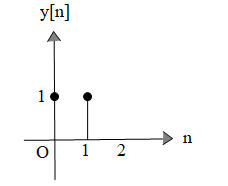
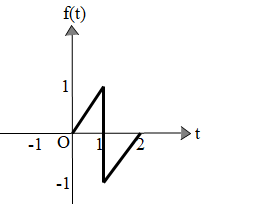
**(4 marks)**

1. is shown in Figure (1.1) (4 marks).

**(4 marks)**

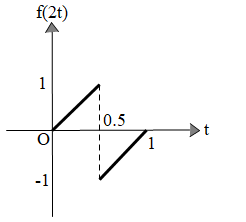
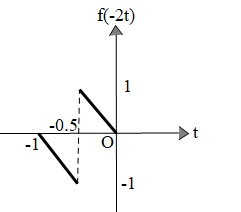
1.  is shown in Figure (1.2) (4 marks).

**(4 marks)**



**Figure (1.1) Figure (1.2)**

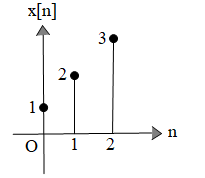
The detailed process of the third question is as follows:



**Figure (1.3)**

1.  is shown in Figure (1.4) (3 marks).

**(6 marks)**



**Figure (1.4)**

The energy of  is (3 marks).

**Question 2**

1. Determine and sketch the convolution of the following two signals:

 h[n]=δ[n+2]+2δ[n+1]

**[10 marks]**

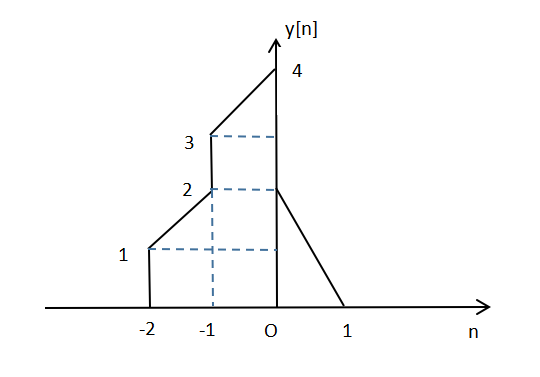
**Solution:**

y[n]=x[n]\*h[n]=x[n]\*δ[n+2]+x[n]\*2δ[n+1]=x[n+2]+2x[n+1] （2marks）

 （2marks）

（4marks）

The waveform of y[n] is shown in the figure below

（2marks）

1. Consider a causal LTI system whose input x[n] and output y[n] are related by the difference equation:y[n]=1/4y[n-1]+x[n].Determine y[n] if x[n]=δ[n-1].

**[10 marks]**

**Solution:**

Because the system is causal,so y[n]=0 while n<1;（2marks）

While n=1 and n>1,we can use iterative method to find y[n].（2marks）

y[1]=1/4y[0]+x[1]=δ[0]=1,

y[2]=1/4y[1]+x[2]=1/4+0=1/4,

y[3]=1/4y[2]+x[3]=1/16=(1/4)2,

y[4]=1/4y[3]+x[4]=(1/4)3,...,y[m]=(1/4)m-1,...（4marks）

So,y[n]=(1/4)n-1u[n-1]（2marks）

**Question 3**

Consider a continuous-time LTI system with the impulse response . If the system with impulse response  is composed of the parallel connection of  and , as shown in **Figure. 2**:

**[15 marks]**



Figure 2: Block diagram of , which is equivalent to the parallel connection of  and .

1. Find the Fourier transform (FT) of the signals , and ;

**(3 marks)**

ii) If the impulse response and , find the frequency response , magnitude spectrum  and phase spectrum ;

**(4 marks)**

iii) Find the output signal  and its FT  with input ;

**(4 marks)**

iv) If the output is , find its corresponding input  and its FT .

**(4 marks)**

**Solution:**

|  |  |  |
| --- | --- | --- |
| i) |  | 1 marks  1 marks  1 marks |
| ii) |  | 1 marks  1 marks  1 marks  1 marks |
| iii) |  | 2 marks  2 marks |
| iiii) |  | 1 marks  1 marks  2 marks |

**Question 4**

a)Consider a discrete-time LTI system, 

**[8 marks]**

1. Determine the frequency response ;

**(2 marks)**

1. Find the Discrete-time Fourier series (DTFS) coefficients of the signal ;

**(3 marks)**

1. **C**alculate the output of the system  with the input signal .

**(3 marks)**

**Solutions:**

1. ； 2 marks
2. *Ω****o* =** *π***/ 8, *N*=2***π***/** *Ω****o* =16**

 1 mark



 2 marks

1.  1 mark



2 marks

b) The input to a discrete-time system is given by , use the DTFT to find the output of the system, , if the impulse response is given by .

**[7 marks]**

**Solution:**

 2 marks



2 marks

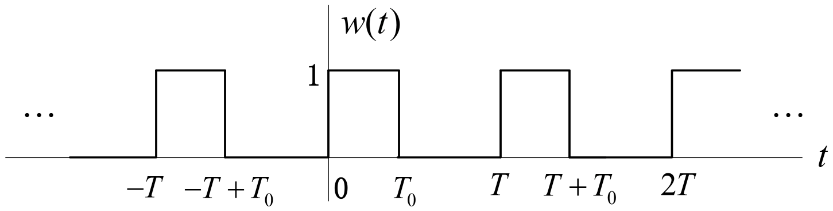
 2 marks

 1 mark

**Question 5**

1. Find the FT representation of  as depicted in **Figure. 3**.

**[5 marks]**



**Figure. 3**

**Solution:**

FT representation of  is given by

 1 mark

where ,



2 marks

 2 marks

1. Use  as depicted in Fig. 1 as the input to an LTI system with the frequency response 

If we have , , determine the system output .

**[4 marks]**

**Solution:**

 1 mark

When  and , we have 



where



Therefore

 2 marks

Its inverse FT is

 1 mark

1. Consider sampling the signal  with sampling interval , determine the bound on , which guarantee that there will be no aliasing.

**[3 marks]**

**Solution:**

The maximum frequency  in  is

 1 mark

According to the sampling theorem, we require that



or . 2 marks

**Question 6**

a) A differential equation of a linear time-invariant causal continuous time system can be described as

The input signal is , the output signal is

.

Find the zero-input response (t), zero-state response (t)

**[10 marks]**

**Solutions:**

Using Laplace transform to process both sides of the differential equation, we obtain

3s2 (2 marks)

Therefore,

(1 mark)

And,

(1 mark)

So,

, (2 marks)

Suppose,

Then we have

. (2 marks)

So

. (2 marks)

b) Find the inverse transform x[n] of under the following three conditions:

(1)

(2)

(3)

**[10 marks]**

**Solution:**

Partial-fraction expansion:

(4 marks)

(1) Convergence region , x[n] is a right-sided sequence

(2 marks)

(2) Convergence region , x[n] is a bilateral sequence

(2 marks)

(3) Convergence region , x[n] is a left-sided sequence

(2 marks)