Chapter 4-5 Newton's Laws and Their Applications

§ 1 Newton's First Law — Law of Inertia

P77-79

- In the absence of external forces (or no net force), an object at rest remains at rest, and an object in motion continues in motion with a constant velocity (i.e. with a constant speed in a straight line).
  - An object has a tendency to maintain its original state of motion in the absence of a force. — This tendency is called inertia. The mass is an intrinsic characteristic of a body, it characterizes the inertia of the body.
  - The force is the only reason which makes the states of body change. It indicates the concept of force: An interaction ——can cause an acceleration of a body.

- Newton's first law defines a special set of reference frames called inertial frames — An inertial frame of reference is one in which Newton's first law (also second law) is valid.
- Any reference frame that moves with constant velocity with respect to an inertial frame is itself an inertial frame.
- Reference frames where the law of inertia does not hold, such as the accelerating reference frames are called noninertial frames.

The Earth as a Inertia Frame

- The Earth is not an inertial frame because it is connected with two kinds of motions.
  - Orbital motion about the sun: $a_n \approx 6.0\times10^{-3}\mathrm{m/s^2}$
  - Rotational motion about its own axis: $a_n \approx 3.4\times10^{-2}\mathrm{m/s^2}$
  - Very small compared with $g = 9.8\mathrm{m/s^2}$.
- In most situations, we consider a reference frame connected to the Earth as the approximate inertial frame.

§ 2 Newton's Second Law

P79-82

- The acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass.

$$\sum \vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = m\frac{d^2\vec{r}}{dt^2}$$

- $\vec{F}_{net}$ is the vector sum of all net external forces.

  - An instantaneous relation: once the force acting on an object changes (whether in magnitude or in direction), the acceleration also changes at that moment. Once the force acting on it vanished, the acceleration becomes zero immediately. $\sum \vec{F}(t) = m\vec{a}(t)$

- It is only suitable for the inertial frame, and particles or particle-like bodies. To solve problems with it, a free-body diagram need to be drawn.
  - The component expressions:

$$\sum F_x = ma_x \quad \sum F_y = ma_y \quad \sum F_z = ma_z$$

In Cartesian coordinate

$$\sum F_t = ma_t = m\frac{dv}{dt} \quad \sum F_n = ma_n = m\frac{v^2}{\rho}$$

In natrual coordinate

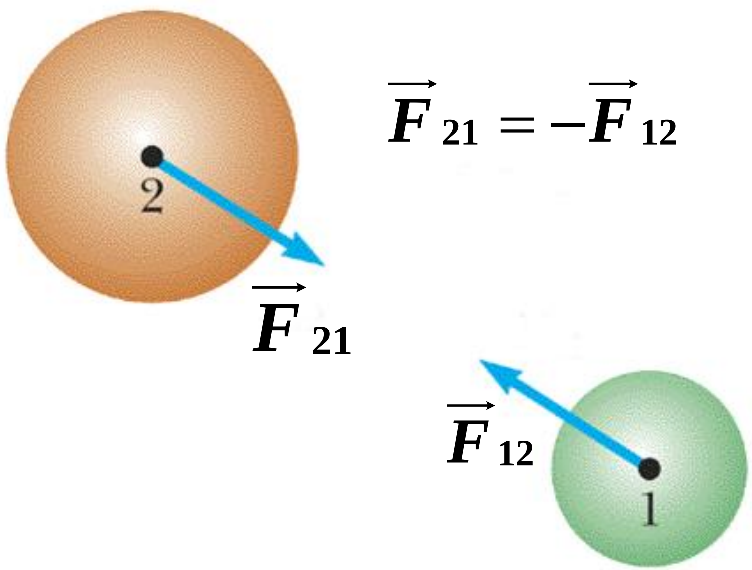
§ 3 Newton's Third Law

P82-85

- If two objects interact, the force $\boldsymbol{F}_{21}$ exerted by object 1 on object 2 is equal in magnitude but opposite in direction to the force $\boldsymbol{F}_{12}$ exerted by object 2 on object 1.

$$\vec{F}_{12} = -\vec{F}_{21}$$

  - $\boldsymbol{F}_{ba}$ means "the force exerted by a on b".



1. Simultaneous;
2. Same type of force;
3. Acting on different bodies, do not cancel each other

§ 4 Solving Problems using Newton's Laws

P88-95, p105-121

Problem-Solving Strategy

- Isolate the object whose motion is being analyzed. Draw a separate free-body diagram for each object.
  - Be sure to include all the forces acting on the object, but be equally careful not to include any force exerted by this object on other object.
  - Never include the quantity $m\vec{a}$ in you free-body diagram. It's not a force.
- Establish a convenient reference frame and an appropriate coordinate system attached to it.

Problem-Solving Strategy (continued)

- For each object, write the equations for Newton's second law in component manner.
  - Generally, the number of unknowns must be equal to number of equations.
  - If number of unknowns < number of equations, there must be equivalent equations.
  - If number of unknowns > number of equations of Newton's second law, find relationship between motions. (this situation mostly occurs to many objects whose motions are dependent.)
- Solve the equations to find unknowns.
- Check the result
  - by introducing particular or extreme cases of quantities, when possible, and compare the results with your intuitive expectations. Ask, "Does this result make sense?"
  - by dimensional analysis.

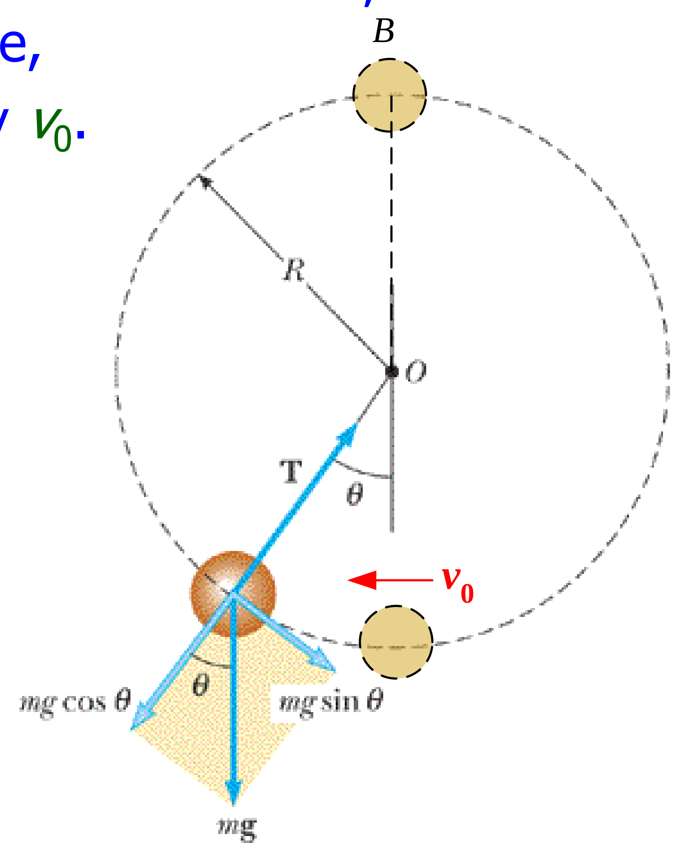
Example

Example: A small ball of mass m is attached to the end of a cord of length R, which rotates under the influence of gravitational force in vertical circle about a fixed point O.

(1) Determine the tension in the cord at any angle θ.

(2) When the ball starts motion in the bottom of the circle, in order to pass point B which is the top of the circle,

find the minimum value of initial velocity v_0.



Solution：

Reference frame: the Earth.

Coordinate system: natural coordinate.

Tangential: $$-mg\sin\theta = m\frac{dv}{dt} \qquad (1)$$

Normal: $$T - mg\cos\theta = m\frac{v^2}{R} \qquad (2)$$

Unknown: θ, T, v. Additional equation to be found

Circular motion:

$$v = R\omega = R\frac{d\theta}{dt} \qquad (3)$$

Example (continued)

Change the independent variable t to θ.

$$(2) \rightarrow \quad -mg\sin\theta = m\frac{dv}{d\theta}\frac{d\theta}{dt} = m\omega\frac{dv}{d\theta} = m\frac{v}{R}\frac{dv}{d\theta}$$

$$\int_{v_0}^{v} mvdv = \int_0^{\theta} -mgR\sin\theta d\theta$$

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv^2 + (mgR - mgR\cos\theta)$$

Equivalent to conservation of mechanical energy.

$$\boxed{T = \frac{mv_0^2 - mgR(2-3\cos\theta)}{R}}$$

At the point B, T≥0, and θ=180°

$$\frac{mv_0^2 - mgR(2+3)}{R} \geq 0$$

$$\boxed{v_0 \geq \sqrt{5gR}}$$

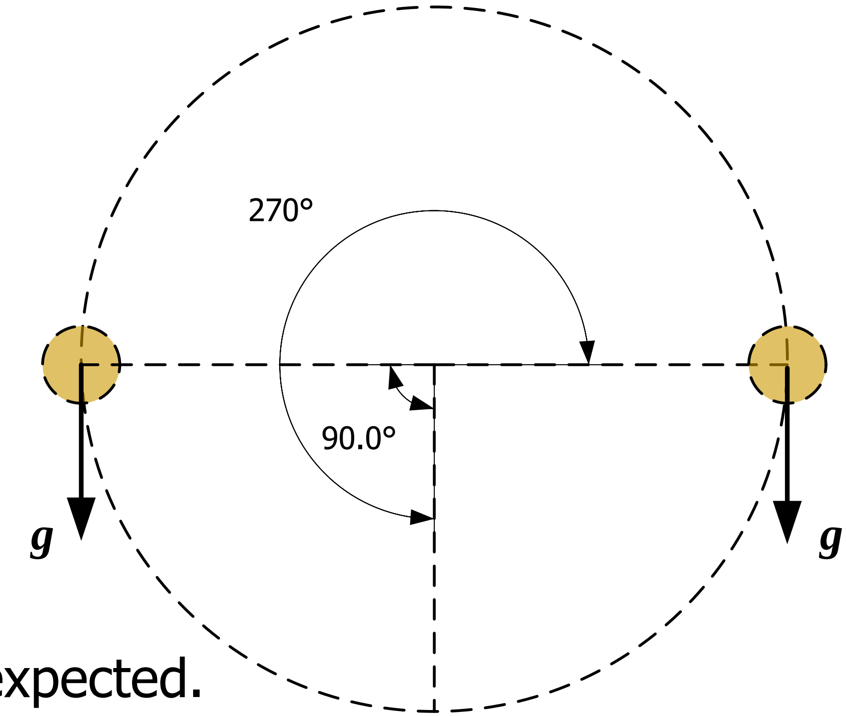
Example (continued)

Check the result: in the case of θ=90°, θ=270°, the tangential acceleration will be $-g$ and g.

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_0^2 - mgR + mgR\cos\theta$$

$$mv\frac{dv}{dt} = -mgR\sin\theta\frac{d\theta}{dt} = -mgR\omega\sin\theta$$

$$\frac{dv}{dt} = -g\sin\theta$$



θ=90° $a_t = -g$

θ=270° $a_t = g$

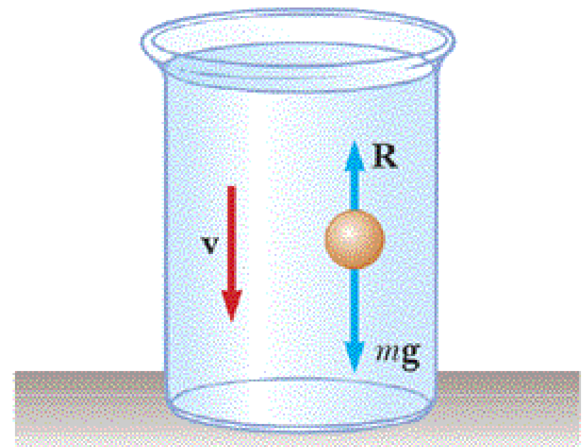
The results are expected.

Example

Example: Motion in the presence of velocity-dependent resistive force

In viscous fluid, moving body experiences a resistive force (or drag force) *R* exerted by the fluid.

For a moving body at low speed: $\vec{R} = -b\vec{v}$



A ball of mass m is released from rest in a liquid.

Choose the downward direction to be positive.

$$mg - bv = m\frac{dv}{dt}$$

$$\int_0^v \frac{dv}{g - \frac{b}{m}v} = \int_0^t dt \qquad \ln\frac{g - \frac{b}{m}v}{g} = -\frac{b}{m}t$$

$$\boxed{v = \frac{mg}{b}\left(1 - e^{-bt/m}\right) = v_T\left(1 - e^{-t/\tau}\right)}$$

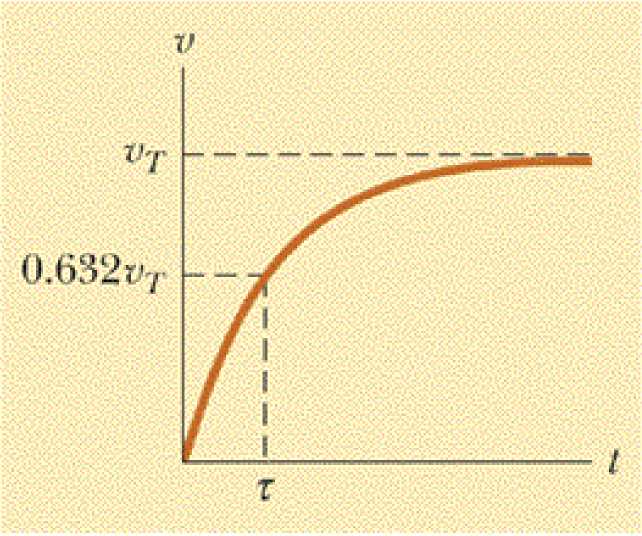
v_T: terminal velocity;

τ: characteristic time.

Example (continued)

When $t = \tau, \quad v = v_T(1 - \frac{1}{e}) \qquad (1 - \frac{1}{e}) = 0.632$

$\tau = m/b$ is the time required for the ball to reach 63.2% of $\boldsymbol{v_T}$, and reflects how fast the ball approaches $\boldsymbol{v_T}$.



Check the result:

(1) Terminal velocity can also be obtained by equation of stationary state of the body:

$$mg - bv = m\frac{dv}{dt} \quad \Delta t \to \infty, \ \ dv/dt = 0, \ \ mg - bv_T = 0 \qquad v_T = \frac{mg}{b}$$

(2) Dimensional analysis: correct

$$\dim R = \mathrm{MLT^{-2}}, \dim v = \mathrm{LT^{-1}}, \dim b = \frac{\dim R}{\dim v} = \mathrm{MT^{-1}}$$

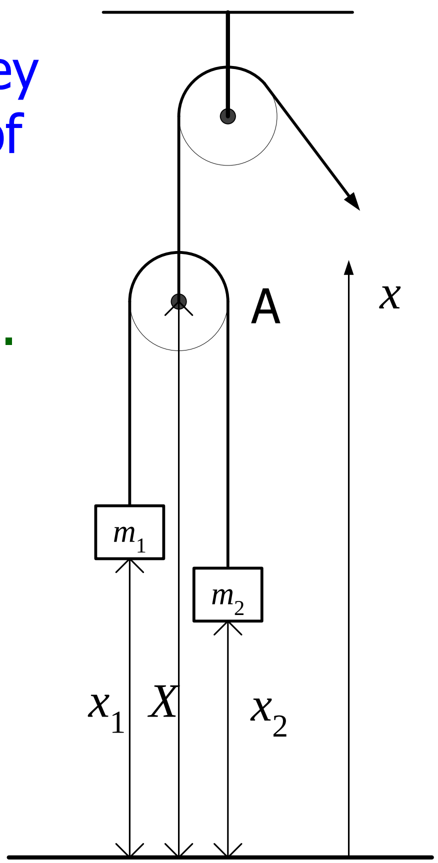
$$\dim v_T = \frac{\dim m \dim g}{\dim b} = \frac{\mathrm{MLT^{-2}}}{\mathrm{MT^{-1}}} = \mathrm{LT^{-1}}$$

$$\dim \tau = \frac{\dim m}{\dim b} = \mathrm{T}$$

Example

Example: A pulley system is shown in Figure. Following quantities are known: (1) ***a*** — the acceleration of pulley A; (2) m_1 — the mass of block 1; (3) m_2 — the mass of block 2. The pulleys are modeled as massless and frictionless.

Determine the accelerations a_1 and a_2 of block 1 and 2.



Choose the upward direction to be positive.

m_1: $$T - m_1 g = m_1 a_1 \qquad (1)$$

m_2: $$T - m_2 g = m_2 a_2 \qquad (2)$$

Unknown: *T*, a_1, a_2.

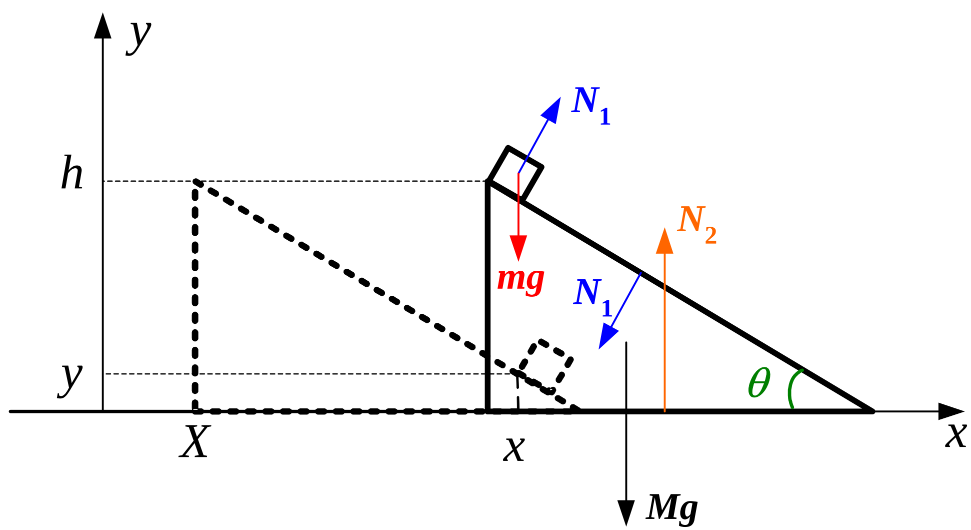
Relationship between m_1 and m_2: the length of rope which passes over pulley A is constant.

$$l = \pi R + (X - x_1) + (X - x_2) = \text{constant}$$

$$2\ddot{X} - \ddot{x}_1 - \ddot{x}_2 = 0 \qquad 2a = a_1 + a_2 \qquad (3)$$

Example

Example: A block of mass m is put on a wedge M, which, in turn, is put on a horizontal table. The incline angle of wedge is θ. All surfaces are frictionless. Determine the accelerations of block m and wedge M.



Example

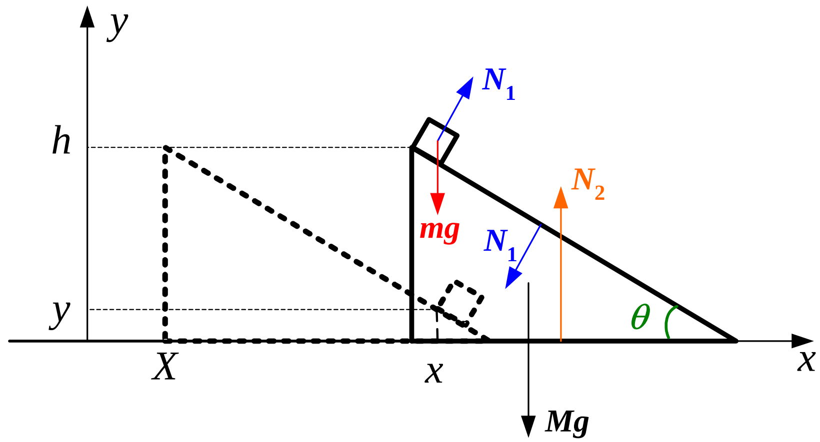
Solution: establish the coordinate system

m: Horizontal: $N_1 \sin\theta = ma_x$ (1)

Vertical: $N_1 \cos\theta - mg = ma_y$ (2)

M: Horizontal: $-N_1 \sin\theta = Ma_0$ (3)

Vertical: $N_2 - N_1 \cos\theta - Mg = 0$ (4)



Unknown: ***N_1, N_2, a_x, a_y, a_0***

$$N_1 = mg\cos\theta$$

$$N_2 = Mg + mg$$

$$\tan\theta = -\frac{a_y}{a_x}$$

Example

Solution: establish the coordinate system

m: Horizontal: $N_1 \sin\theta = ma_x$ (1)

Vertical: $N_1 \cos\theta - mg = ma_y$ (2)

M: Horizontal: $-N_1 \sin\theta = Ma_0$ (3)

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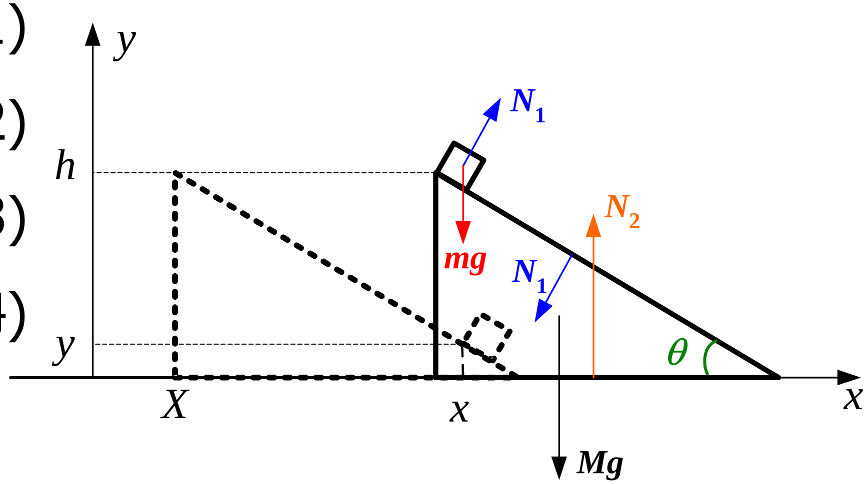


Unknown: $\boldsymbol{N_1, N_2, a_x, a_y, a_0}$

Motion relation: $\vec{a}_{me} = \vec{a}_{mM} + \vec{a}_{Me}$

Horizontal: $a_x = a' \cos\theta + a_0$ (5)

Vertical: $a_y = -a' \sin\theta$ (6)

$$a_x = \frac{g \sin\theta \cos\theta}{1 + \frac{m}{M}\sin^2\theta} \qquad a_y = -\frac{\left(1 + \frac{m}{M}\right) g \sin^2\theta}{1 + \frac{m}{M}\sin^2\theta} \qquad a_0 = -\frac{\frac{m}{M} g \sin\theta \cos\theta}{1 + \frac{m}{M}\sin^2\theta}$$

Example (continued)

Check the results:

(1) by dimensional analysis. ——reasonable.

(2) directions are correct.

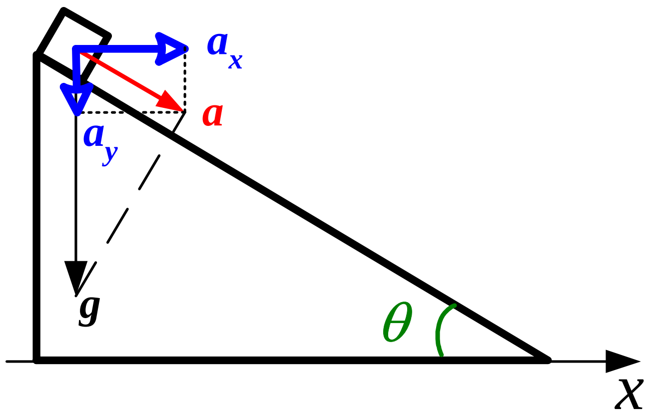
(2) by introducing extreme cases.

If M>>m

$$a_0 \to 0$$

$$a_x \to g\sin\theta\cos\theta$$

$$a_y \to -g\sin^2\theta$$



If m>>M

$$a_x \to 0$$

$$a_y \to -g$$

The results are reasonable.

Example

Example: The Rotor: In many amusement parks we find a device often called the rotor. The rotor is a hollow cylindrical room that can be set rotating about the central vertical axis of the cylinder. A person enters the rotor, closes the door, and stands up against the wall. The rotor gradually increases its rotational speed from rest until, at a predetermined speed, the floor below the person is opened downward, revealing a deep pit. The person does not fall but remains "pinned up" against the wall of the rotor. What minimum rotational speed is necessary to prevent falling?

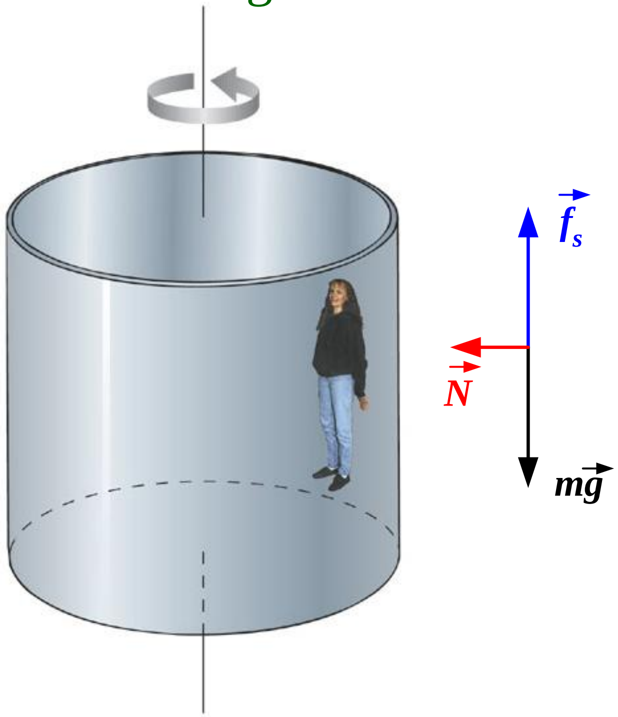
Solution:

Vertical: $N = m\dfrac{v^2}{R}$

Horizontal: $f_s - mg = 0$

$f_s \le \mu_s N$

$$v \ge \sqrt{\frac{gR}{\mu_s}}$$



Take: μ_s=0.4, R=2m.

$$n = \frac{\omega}{2\pi} = \frac{v}{2\pi R} \ge \frac{1}{2\pi}\sqrt{\frac{g}{\mu_s R}} = 0.56 \text{ revolution/s}$$

Example

Example: A device called a capstan (绞盘) is used aboard ships in order to control a rope which is under great tension. The rope is wrapped around a fixed drum, usually for several turns. The load on the rope (end B) pulls it with a force T_B, and the sailor (end A) holds it with a much smaller force T_A. Show that $T_A = T_B \exp(-\mu\theta)$, where μ is the coefficient of friction and θ is the total angle subtended by the rope on the drum.

Example

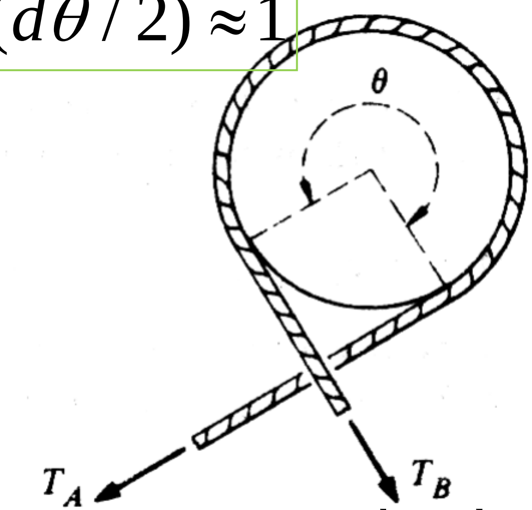
Solution: isolate a element of rope to consider.

Tangential: $(T+dT)\cos(d\theta/2)-T\cos(d\theta/2)-\mu N=0$

Normal: $(T+dT)\sin(d\theta/2)+T\sin(d\theta/2)-N=0$

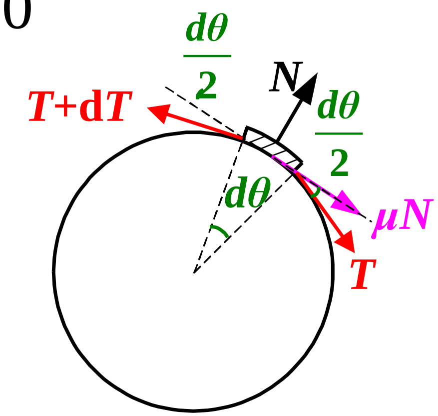
$$\boxed{\sin(d\theta/2)\approx d\theta/2 \qquad \cos(d\theta/2)\approx 1}$$

$$\boxed{\begin{aligned} & dT-\mu N=0 \\ & Td\theta+\frac{1}{2}dTd\theta-N=0 \end{aligned}}$$



The end for sailor holds

The end attached to ship



Neglect the second order infinitesimal ***dTdθ***, we get

$$Td\theta=\frac{dT}{\mu} \qquad \int_{T_A}^{T_B}\frac{dT}{T}=\int_0^{\theta}\mu d\theta \quad \Rightarrow \quad \boxed{T_A=T_B\exp(-\mu\theta)}$$

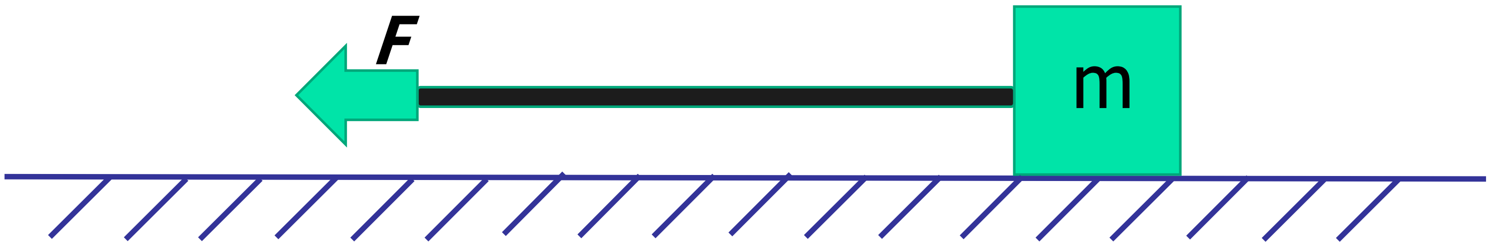
As long as the θ is large enough, we can get $T_A \ll T_B$

设置

The uniform rope with length L and mass M is connected with a body with mass m, which is sit on a surface. The force F action on the other end of the rope. Calculate the tension in the rope?

(1) The surface is frictionless.

(2) The friction coefficient is *u*

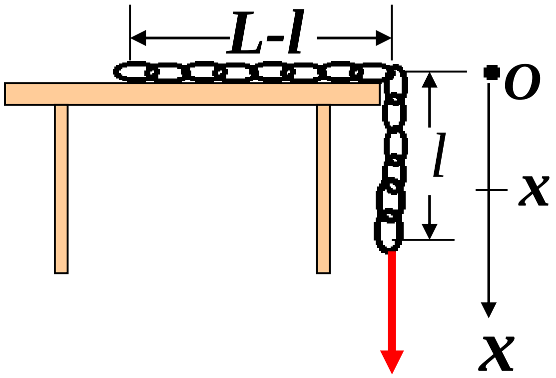


作答

Example

A chain has length *L* and mass *M*, and was put on a frictionless table. At t = 0, the chain is stationary and length *l* is hanged over the edge. (a) what is the velocity at the moment that the whole chain leave the table (整个链条刚离开桌面时的速度) (b) The time covered the whole process (由开始运动到完全离开桌面所经历的时间)?

Solution:



(a) Consider the moment of $l=x$,

$$F = \frac{M}{L} g x$$ **(Variable)**

$$\frac{M}{L} g\, x = Ma; \quad \text{(Variable)} \qquad \frac{M}{L} g\, x = Mv \frac{\mathrm{d}v}{\mathrm{d}x}$$

$$Lv\mathrm{d}v = gx\mathrm{d}x \qquad L\int_0^v v\mathrm{d}v = g\int_l^x x\mathrm{d}x$$

$$v(x) = \sqrt{\frac{g}{L}(x^2 - l^2)}\;; \qquad v(L) = \sqrt{\frac{g}{L}(L^2 - l^2)}$$

(b) $$v(x) = \sqrt{\frac{g}{L}(x^2 - l^2)} = \frac{\mathrm{d}x}{\mathrm{d}t}$$

$$\int_l^L \frac{\mathrm{d}x}{\sqrt{x^2 - l^2}} = \int_0^t \sqrt{\frac{g}{L}}\mathrm{d}t \quad \text{so} \quad t = \sqrt{\frac{L}{g}} \ln \frac{L + \sqrt{L^2 - l^2}}{l}$$

Homework:

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