Introduction to Electronic Systems

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Part 2

Dynamic Circuit Analysis

(DC+Dynamic/Resistive Element+Topology)

Topology Constraint + Element Constraint = Circuit Behavior

Topology Constraint → KCL, KVL

Element Constraint → Ohm's Law, IVS, ICS, …

The VCR constraint for non-resistive element?

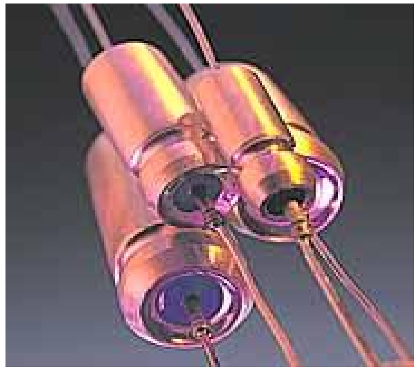
Part 2: Dynamic Circuit Analysis

5. **Capacitors and Inductors**
6. Response of First-order RC and RL Circuits
7. Response of Second-order RLC Circuits*

Chapter 5: Capacitors and Inductors

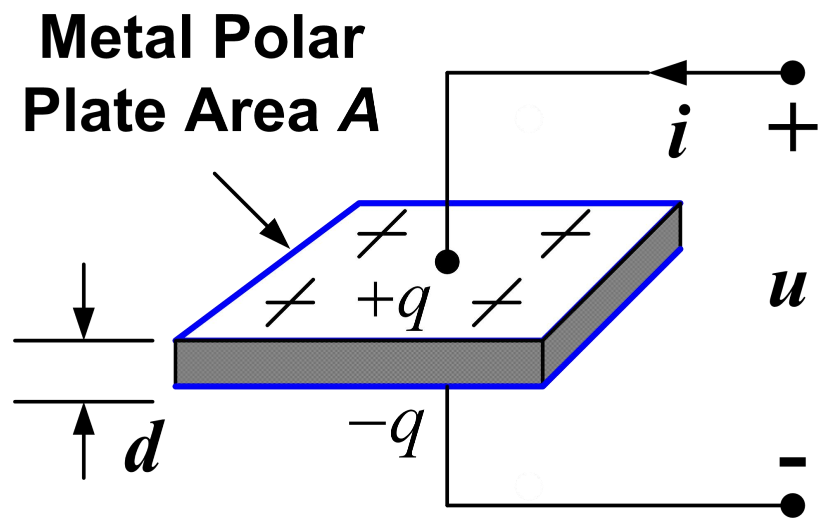
- **Capacitor and Capacitance**
- **Inductor and Inductance**
- **Dynamic Element and Circuit**

5-1 Capacitor and Capacitance



Electrolytic Capacitor

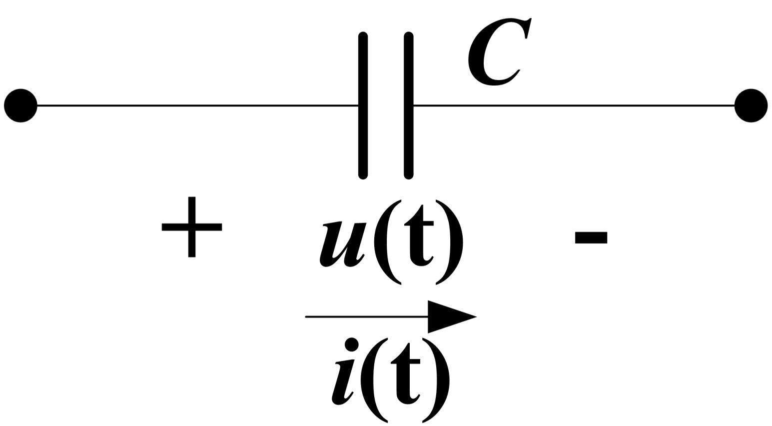
Ceramic Capacitor



Capacitor and Capacitance

- Capacitor is a charge storing device;
- CVR of ideal capacitor:

$$q(t) = Cu(t)$$



- Reference voltage and current defined with passive sign convention

VCR of Capacitors (1)

$$q(t) = Cu(t)$$

$$i(t) = \frac{dq(t)}{dt} = \frac{dCu(t)}{dt} = C\frac{du(t)}{dt}$$

$$\boxed{i(t) = C\frac{du(t)}{dt}}$$

VCR of Capacitors (2)

i **(t) is measured in amperes**

u **(t) in volts**

$$i(t) = C\frac{du(t)}{dt}$$

C **in farads**

t in seconds

VCR of Capacitors (3)

$$i(t) = C\frac{du(t)}{dt}$$

$$u(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau = \frac{1}{C}\int_{-\infty}^{t_0} i(\tau)d\tau + \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau$$

$$= u(t_0) + \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau$$

$$\boxed{u(t) = u(t_0) + \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau \qquad (t \geq t_0)}$$

VCR of Capacitors (4)

$$\boxed{i(t) = C\frac{du(t)}{dt}}$$

- *u*(t) is continuous if *i*(t) is limited;
- For DC, *i*(t) = 0, i.e., capacitor is open.

The voltage across a capacitor can not change abruptly in condition that *current* is limited.

VCR of Capacitors (5)

$$\boxed{u(t)=\frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau}$$

- *u*(t) is related to all “historical” current;
- Capacitor is a “Memorial” element;
- Dynamical element

Power and Energy

- **Power:** $p(t) = u(t)i(t) = Cu(t)\dfrac{du(t)}{dt}$
- **Energy:**

$$w(t) = \int_{-\infty}^{t} p(\lambda)d\lambda$$

$$= \int_{-\infty}^{t} Cu(\lambda)\frac{du(\lambda)}{d\lambda}d\lambda = \int_{-\infty}^{t} Cu(\lambda)du(\lambda)$$

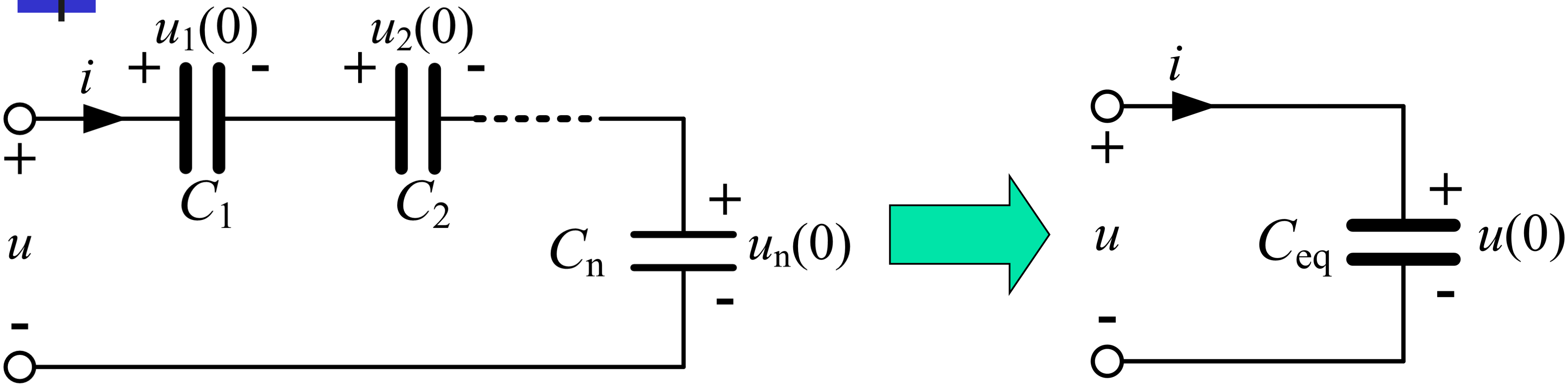
$$= \frac{1}{2}Cu^2(t)\Big|_{-\infty}^{t} = \frac{1}{2}Cu^2(t) \geq 0$$

Power and Energy

$$w(t) = \frac{1}{2} C u^2(t) \geq 0$$

- **Capacitor is a passive element.**

Capacitors in Series



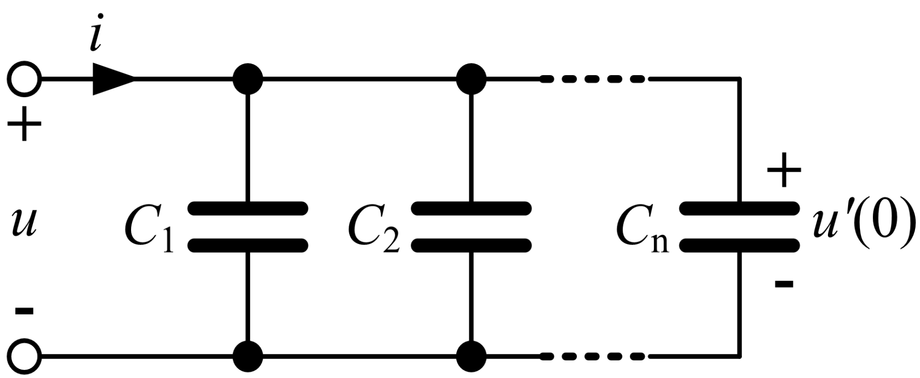
$$u(t)=\sum_{k=1}^{n}u_k(t)$$

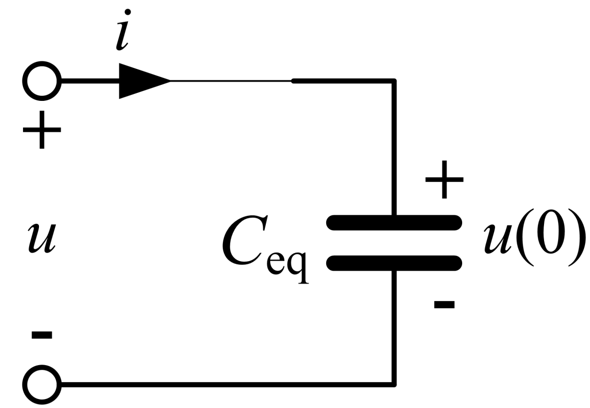
$$=\sum_{k=1}^{n}u_k(0)+\int_0^t\left(\sum_{k=1}^{n}\frac{1}{C_k}\right)i(\tau)\,d\tau$$

$$=u(0)+\int_0^t\frac{1}{C_{eq}}i(\tau)\,d\tau$$

$$\Rightarrow\begin{cases}\dfrac{1}{C_{eq}}=\displaystyle\sum_{k=1}^{n}\left(\frac{1}{C_k}\right)\\[2ex] u(0)=\displaystyle\sum_{k=1}^{n}u_k(0)\end{cases}$$

Capacitors in Parallel





$$i(t) = \sum_{k=1}^{n} i_k(t)$$

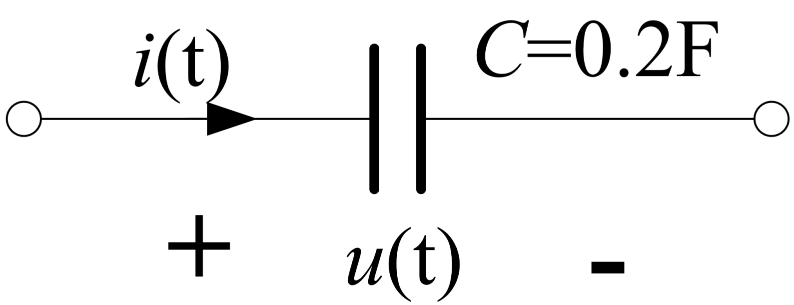
$$= \left(\sum_{k=1}^{n} C_k\right) \frac{du(t)}{dt}$$

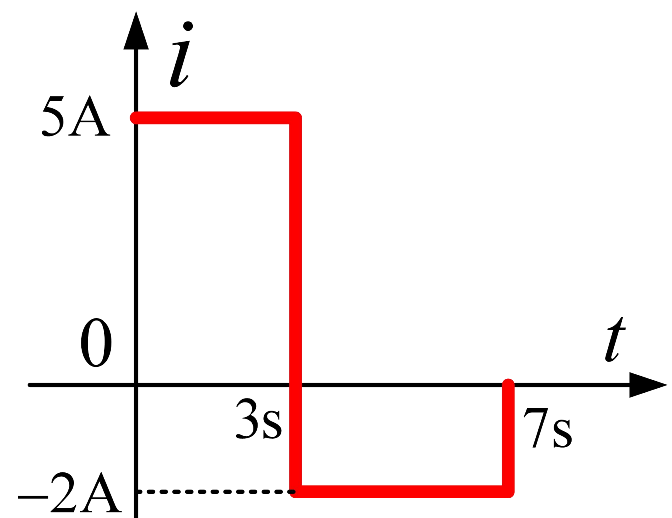
$$= C_{eq} \frac{du(t)}{dt}$$

$$\begin{cases} C_{eq} = \sum_{k=1}^{n} C_k \\ u(0) = u_k(0) \end{cases}$$

Example

A capacitor current has a waveform as shown. The initial voltage u(0)=30V. Find the voltage on the capacitor.





(a)

Solution:

(1) $0 \le t < 3\text{s}$：$i = 5\text{A} > 0$

$$u = u(0) + \frac{1}{C}\int_0^t i(\tau)\,\mathrm{d}\tau$$

$$= 30 + \frac{1}{0.2}\int_0^t 5\,\mathrm{d}\tau = \left(30 + 25t\right)\text{V}$$

$$u(t = 3\text{s}) = 30 + 25 \times 3 = 105\text{V}$$

(2) $3\text{s} \le t < 7\text{s}$： $i = -2\text{A} < 0$

$$u = u(3) + \frac{1}{C}\int_3^t i(\tau)\mathrm{d}\tau$$

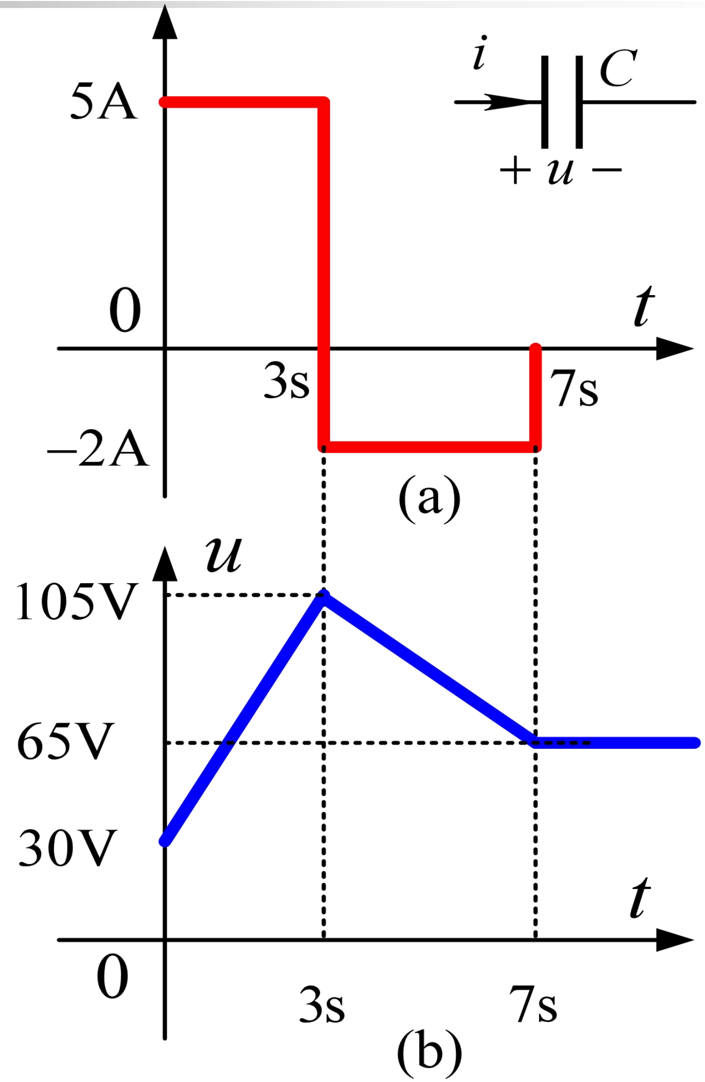
$$= 105 + \frac{1}{0.2}\int_3^t (-2)\mathrm{d}\tau = (135 - 10t)\text{V}$$

$$u(t = 7\text{s}) = 65\text{V}$$

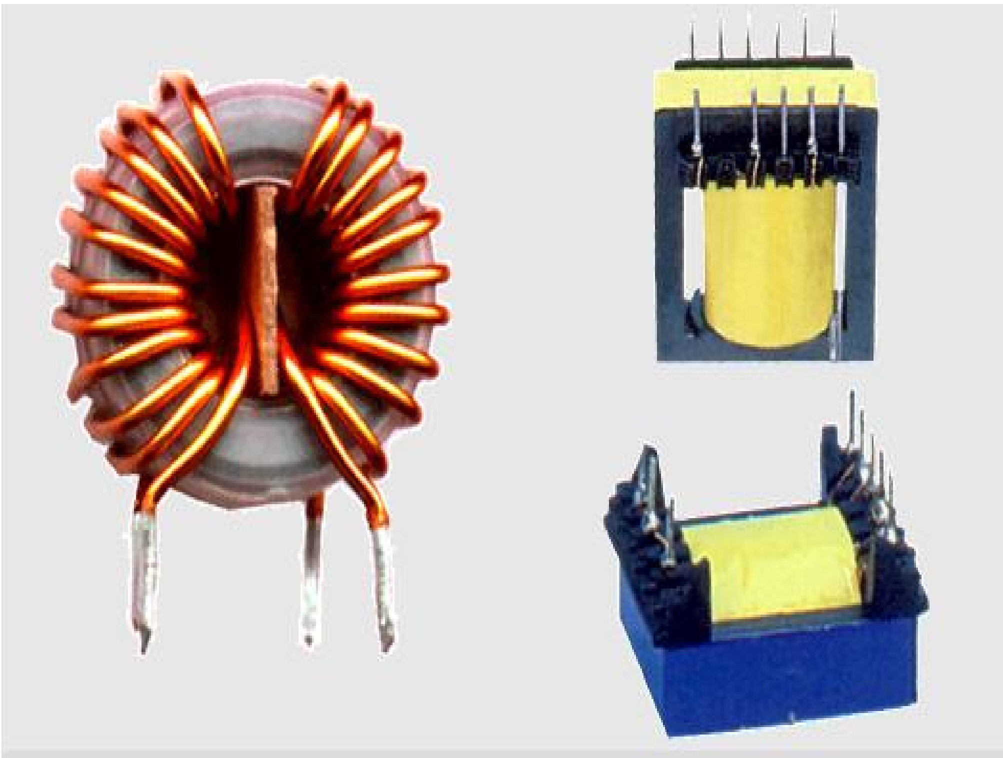
(3) $t \geq 7\text{s}$：$i = 0$

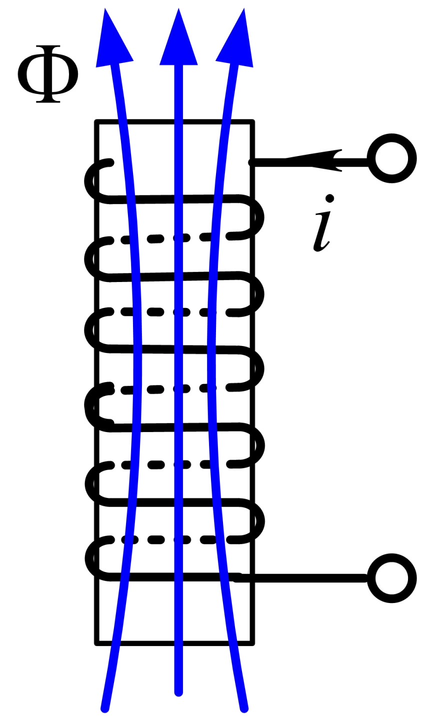
$$u(t) = u(7\text{s})$$

$$= 65\text{V}$$



5-2 Inductor and Inductance



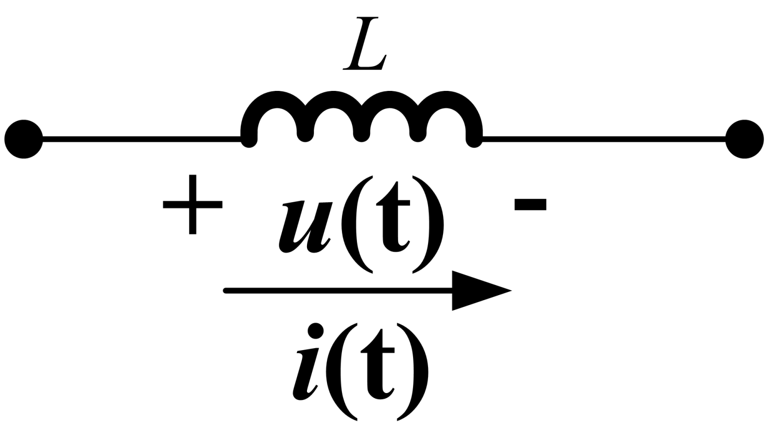


Examples of Real Inductors

Inductor and Inductance

- Inductor is a energy storing device;
- WCR of ideal Inductor:

$$\Psi(t) = Li(t)$$



- Reference voltage and current defined with passive sign convention

VCR of Inductors (1)

$$\Psi(t) = Li(t)$$

$$u(t) = \frac{d\Psi(t)}{dt} = \frac{dLi(t)}{dt} = L\frac{di(t)}{dt}$$

$$\boxed{u(t) = L\frac{di(t)}{dt}}$$

VCR of Inductors (2)

u(t) is measured in volts

i(t) in amperes

$$u(t) = L\frac{di(t)}{dt}$$

L in henrys

t in seconds

VCR of Inductors (3)

$$u(t) = L\frac{di(t)}{dt}$$

$$\Downarrow$$

$$i(t) = \frac{1}{L}\int_{-\infty}^{t} u(\tau)\,d\tau = \frac{1}{L}\int_{-\infty}^{t_0} u(\tau)\,d\tau + \frac{1}{L}\int_{t_0}^{t} u(\tau)\,d\tau$$

$$= i(t_0) + \frac{1}{L}\int_{t_0}^{t} u(\tau)\,d\tau$$

$$\Downarrow$$

$$\boxed{i(t) = i(t_0) + \frac{1}{L}\int_{t_0}^{t} u(\tau)\,d\tau \qquad (t \ge t_0)}$$

VCR of Inductors (4)

$$u(t) = L\frac{di(t)}{dt}$$

- *i*(t) is continuous if *u*(t) is limited;
- For DC, *u*(t) = 0, i.e., inductor is short.

The current through an inductor can not change abruptly in condition that *voltage* is limited.

VCR of Inductors (5)

$$\boxed{i(t) = \frac{1}{L}\int_{-\infty}^{t} u(\xi)d\xi}$$

- *i*(t) is related to all “historical” voltage;
- Inductor is a “Memorial” element;
- Dynamical element

Power and Energy

- **Power:** $p(t) = u(t)i(t) = Li(t)\dfrac{di(t)}{dt}$
- **Energy:**

$$w(t) = \int_{-\infty}^{t} p(\lambda)d\lambda = \int_{-\infty}^{t} Li(\lambda)\frac{di(\lambda)}{d\lambda}d\lambda$$

$$= \int_{-\infty}^{t} Li(\lambda)di(\lambda) = \frac{1}{2}Li^2(t) - \frac{1}{2}Li^2(-\infty)$$

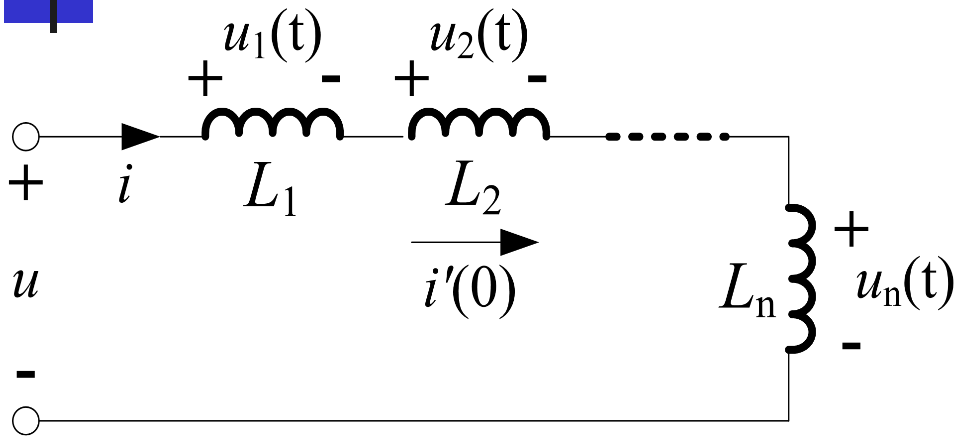
$$= \frac{1}{2}Li^2(t) \geq 0$$

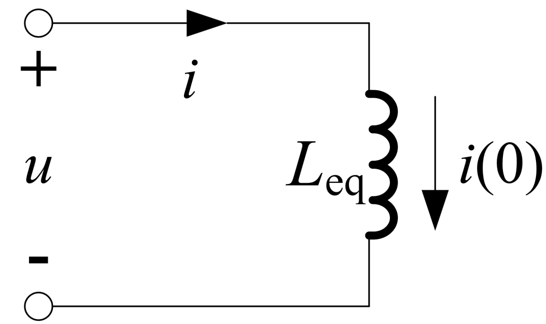
Power and Energy

$$w(t) = \frac{1}{2} L i^2(t) \geq 0$$

- **Inductor is a passive element.**

Inductors in Series





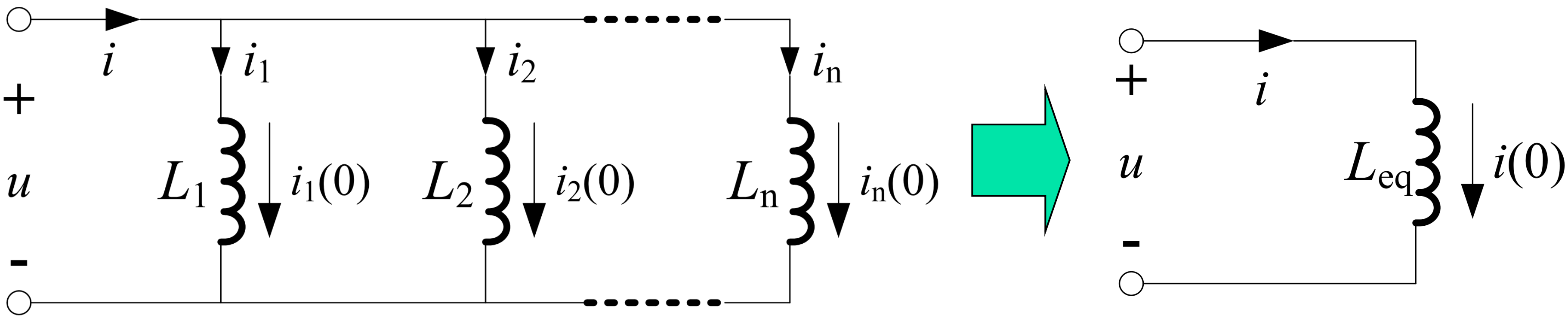
$$u(t)=\sum_{k=1}^{n}u_k(t)$$

$$=\left(\sum_{k=1}^{n}L_k\right)\frac{di(t)}{dt}$$

$$=L_{eq}\frac{di(t)}{dt}$$

$$\begin{cases} L_{eq}=\sum_{k=1}^{n}L_k \\ i(0)=i'(0) \end{cases}$$

Inductors in Parallel



$$i(t) = \sum_{k=1}^{n} i_k(t)$$

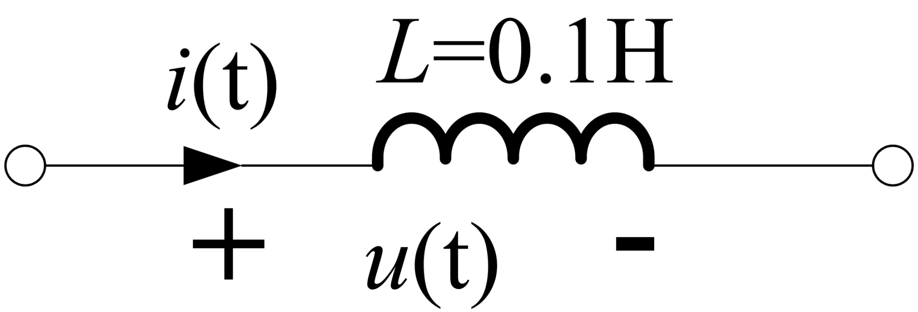
$$= \sum_{k=1}^{n} i_k(0) + \int_0^t \left(\sum_{k=1}^{n} \frac{1}{L_k} \right) u(\xi)\, d\xi$$

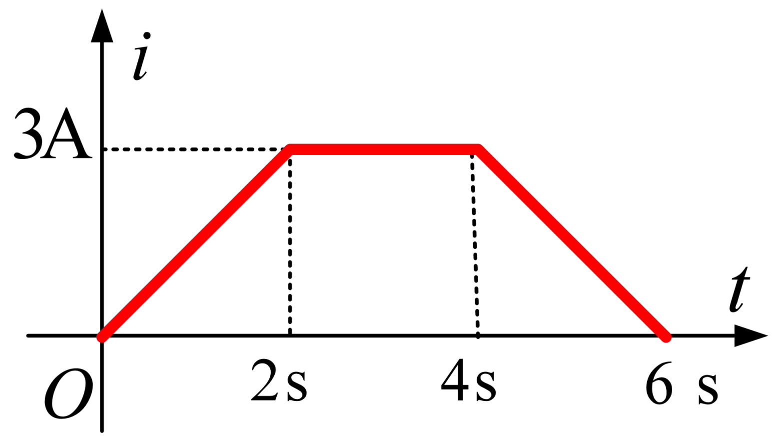
$$= i(0) + \int_0^t \frac{1}{L_{eq}} u(\xi)\, d\xi$$

$$\Rightarrow \begin{cases} \dfrac{1}{L_{eq}} = \displaystyle\sum_{k=1}^{n} \left(\frac{1}{L_k} \right) \\ i(0) = \displaystyle\sum_{k=1}^{n} i_k(0) \end{cases}$$

Example

An inductor current has a waveform as shown. Find the voltage, power, and energy for the inductor.





Solution:

(1) $0 < t < 2\text{s}$： $i = 1.5t\ \text{A}$

$$u = L\frac{di}{dt} = (0.1 \times 1.5)\text{V} = 0.15\text{V}$$

$$p = ui = 0.225t\ \text{W}$$

$$w_{\text{m}} = \frac{1}{2}Li^2 = 0.1125t^2\ \ \text{J}$$

(2) $2\text{s} < t < 4\text{s}$: $i = 3\ \text{A}$

$$u = L\frac{di}{dt} = 0$$

$$p = ui = 0$$

$$w_{\text{m}} = \frac{1}{2}Li^2 = 0.45\ \text{J}$$

(3) $4\text{s} < t < 6\text{s}$**：** $i = -1.5t + 9\ \text{A}$

$$u = L\frac{\mathrm{d}i}{\mathrm{d}t} = -0.1\times 1.5\text{V} = -0.15\text{V}$$

$$p = ui = (0.225t - 1.35)\text{W}$$

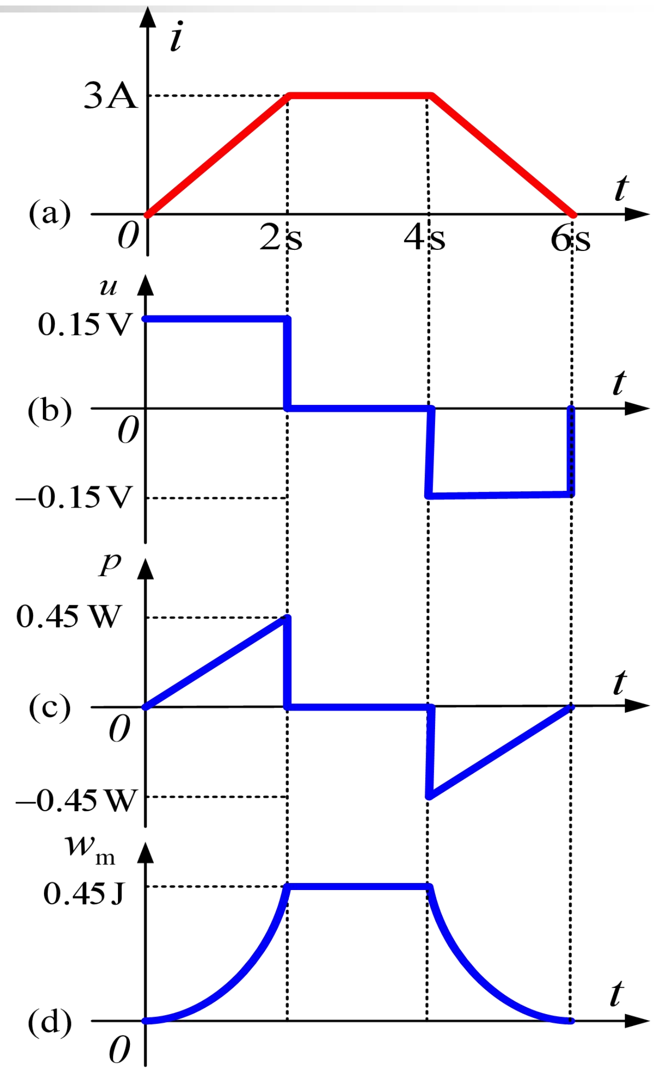
$$w_{\text{m}} = \frac{1}{2}Li^2 = (0.1125t^2 - 1.35t + 4.05)\text{J}$$

(3) $t > 6\text{s}$： $i = 0\,\text{A}$

$$u = L\frac{\mathrm{d}i}{\mathrm{d}t} = 0$$

$$p = ui = 0$$

$$w_m = \frac{1}{2}Li^2 = 0$$



Summary of VCR for Passive Elements

Resistor: $u(t) = Ri(t)$

Capacitor: $\begin{cases} i(t) = C\dfrac{du(t)}{dt} \\ u(t) = u(t_0) + \dfrac{1}{C}\displaystyle\int_{t_0}^{t} i(\xi)d\xi \end{cases}$

Inductor: $\begin{cases} u(t) = L\dfrac{di(t)}{dt} \\ i(t) = i(t_0) + \dfrac{1}{L}\displaystyle\int_{t_0}^{t} u(\xi)d\xi \end{cases}$

Memorize!!

Summary of Equations for Series

Resistor: $$R_{eq} = \sum_{k=1}^{N} R_k$$

Capacitor: $$\begin{cases} \dfrac{1}{C_{eq}} = \displaystyle\sum_{k=1}^{n} \left(\dfrac{1}{C_k} \right) \\ u(0) = \displaystyle\sum_{k=1}^{n} u_k(0) \end{cases}$$

Inductor: $$\begin{cases} L_{eq} = \displaystyle\sum_{k=1}^{n} L_k \\ i(0) = i'(0) \end{cases}$$

Memorize!!

Summary of Equations for Parallel

$$\left.\begin{array}{ll}
\textbf{Resistor:} & \dfrac{1}{R_{eq}} = \displaystyle\sum_{k=1}^{N} \dfrac{1}{R_k} \\[3em]
\textbf{Capacitor:} & \begin{cases} C_{eq} = \displaystyle\sum_{k=1}^{n} C_k \\ u(0) = u'(0) \end{cases} \\[3em]
\textbf{Inductor:} & \begin{cases} \dfrac{1}{L_{eq}} = \displaystyle\sum_{k=1}^{n} \left(\dfrac{1}{L_k}\right) \\ i(0) = \displaystyle\sum_{k=1}^{n} i_k(0) \end{cases}
\end{array}\right\} \textbf{Memorize!!}$$

5-3 Dynamic Element and Circuit

- Dynamic element
- Dynamic circuit

Dynamic Element

- **Resistor:**
  - VCR of terminals is a linear algebraic equation;
  - VCR is “instantaneous” or “memoryless”;
  - Static element.

Dynamic Element

- **Capacitor and Inductor:**
  - **VCR is a differential or integral equation;**
  - **VCR is “historical” or “memorial”;**
  - **Dynamic elements.**

Dynamic Circuit

- **Dynamic circuits**:
  - **contain at least one dynamic element;**
  - **are described by differential equations;**
  - **are “historical” or “memorial”.**

Summary of Chapter 5

- VCR of capacitor
- VCR of inductor
- Power/energy for capacitors/inductors
- Capacitors/inductors in series and parallel
- Conception of dynamic element and circuit