1

Circuit Variables and Circuit Elements

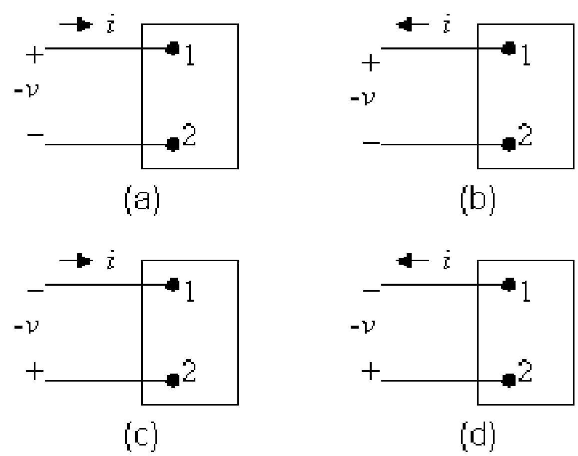
Drill Exercises

DE 1.1 $q = \int_0^\infty 20e^{-5000t}\,dt = 4000\,\mu\text{C}$

DE 1.2 $i = \dfrac{dq}{dt} = te^{-\alpha t}, \qquad \dfrac{di}{dt} = (1-\alpha t)e^{-\alpha t}, \qquad \dfrac{di}{dt} = 0 \qquad \text{when } t = \dfrac{1}{\alpha};$

Therefore $i_{\max} = \dfrac{1}{\alpha e} = \dfrac{1}{0.03679e} \cong 10\ \text{A}$

DE 1.3 [**a**]



Therefore

(a) $v = -20\,\text{V}, \qquad i = -4\,\text{A}$; (b) $v = -20\,\text{V}, \qquad i = 4\,\text{A}$

(c) $v = 20\,\text{V}, \qquad i = -4\,\text{A}$; (d) $v = 20\,\text{V}, \qquad i = 4\,\text{A}$

[**b**] Using the reference system in Fig. 1.3(a), $p = vi = (-20)(-4) = 80\,\text{W}$, so the box is absorbing power.

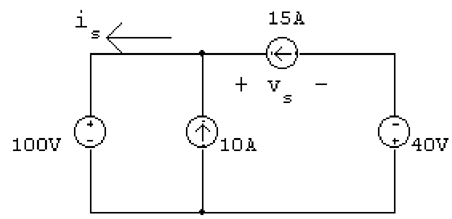
[**c**] The box is absorbing 80 W.

DE 1.4 $p = vi = 20 \times 10^4 e^{-10{,}000t}$ W; $\qquad w = \int_0^\infty 20 \times 10^4 e^{-10{,}000t}\,dt = 20$ J

DE 1.5 $p = 800 \times 10^3 \times 1.8 \times 10^3 = 1440 \times 10^6 = 1440$ MW

from Oregon to California

DE 1.6



The interconnection is valid:

$i_s = 10 + 15 = 25$ A

$p_{100\text{V}} = 100 i_s = 2500$ W (absorbing)

$p_{10\text{A}} = -100(10) = -1000$ W (generating)

$-100 + v_s - 40 = 0 \qquad$ so $v_s = 140$ V

$p_{15\text{A}} = -15(140) = -2100$ W (generating)

$p_{40\text{V}} = 15(40) = 600$ W (absorbing)

$\sum p_{\text{dev}} = p_{10\text{A}} + p_{15\text{A}} = 3100$ W

$\sum p_{\text{abs}} = p_{100\text{V}} + p_{40\text{V}} = 3100$ W

$\sum p_{\text{dev}} = \sum p_{\text{abs}} = 3100$ W

DE 1.7 **[a]** $v_l - v_{\text{c}} + v_1 - v_s = 0, \qquad i_l R_l - i_{\text{c}} R_{\text{c}} + i_1 R_1 - v_s = 0$

$i_s R_l + i_s R_{\text{c}} + i_s R_1 - v_s = 0$

[b] $i_s = v_s/(R_l + R_{\text{c}} + R_1)$

DE 1.8 **[a]** $24 = v_2 + v_5 - v_1 = 3i_5 + 7i_5 - (-2i_5) = 12i_5$

Therefore $i_5 = 24/12 = 2$ A

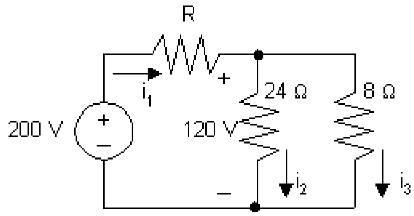
[b] $v_1 = -2i_5 = -4$ V

[c] $v_2 = 3i_5 = 6$ V

[d] $v_5 = 7i_5 = 14$ V

[e] $p_{24} = -(24)(2) = -48$ W; therefore 24 V source is delivering 48 W.

DE 1.9



$i_2 = 120/24 = 5$ A

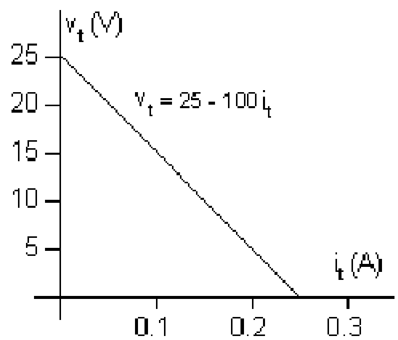
$i_3 = 120/8 = 15$ A

$i_1 = i_2 + i_3 = 20$ A

$-200 + 20R + 120 = 0$

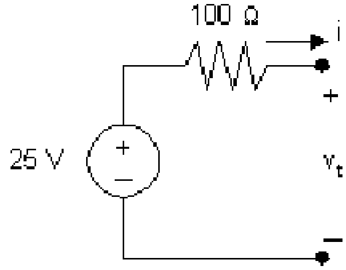
$R = 80/20 = 4\,\Omega$

DE 1.10 **[a]** Plotting a graph of v_t versus i_t gives

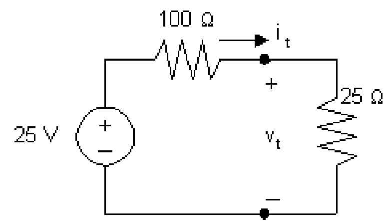


Note that when $i_t = 0$, $v_t = 25$ V; therefore the voltage source must be 25 V. When v_t is zero, $i_t = 0.25$ A, hence the resistor must be 25/0.25 or 100Ω.

A circuit model having the same $v - i$ characteristic is a 25 V source in series with a 100Ω resistor.

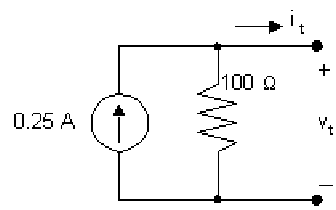


[b]

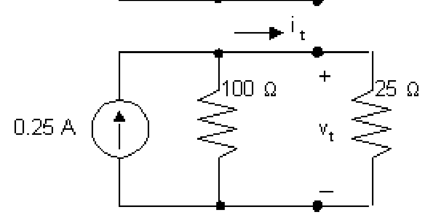


$$i_t = \frac{25}{125} = 0.2 \text{ A}; \qquad p = (0.2)^2(25) = 1 \text{ W}.$$

DE 1.11 **[a]** Since we are constructing the model from two elements, we have two choices on interconnecting them—series or parallel. From the $v - i$ characteristic we require $v_t = 25$ V when $i_t = 0$. The only way we can satisfy this requirement is with a parallel connection. The constraint that $v_t = 0$ when $i_t = 0.25$ A tells us the ideal current source must produce 0.25 A. Therefore the parallel resistor must be 25/0.25 or 100Ω.



[b]



$$-0.25 + \frac{v_t}{100} + \frac{v_t}{25} = 0, \qquad 5v_t = 25, \qquad v_t = 5 \text{ V}$$

$$p = \frac{v_t^2}{25} = 1 \text{ W}.$$

Problems

P 1.1 $\quad i = \dfrac{dq}{dt} = 24\cos 4000t$

Therefore, $dq = 24\cos 4000t\,dt$

$$\int_{q(0)}^{q(t)} dx = 24\int_0^t \cos 4000y\, dy$$

$$q(t) - q(0) = 24\frac{\sin 4000y}{4000}\Big|_0^t$$

But $q(0) = 0$ by hypothesis, i.e., the current passes through its maximum value at $t = 0$, so $q(t) = 6 \times 10^{-3} \sin 4000t\,\mathrm{C} = 6 \sin 4000t\,\mathrm{mC}$

P 1.2 $p = (6)(100) \times 10^{-3} = 0.6$ W; $w = (0.6)(3)(60)(60) = 6480$ J

P 1.3 Assume we are standing at box A looking toward box B, then $p = vi$.

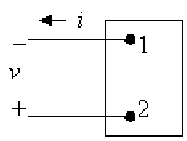
[a] $p = (120)(5) = 600$ W from A to B

[b] $p = (250)(-8) = -2000$ W from B to A

[c] $p = (-150)(16) = -2400$ W from B to A

[d] $p = (-480)(-10) = 4800$ W from A to B

P 1.4 [a]



$p = vi = (40)(-10) = -400$ W
Power is being delivered by the box.

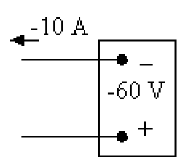
[b] Entering

[c] Gain

P 1.5 [a] $p = vi = (-60)(-10) = 600$ W, so power is being absorbed by the box.

[b] Entering

[c] Lose



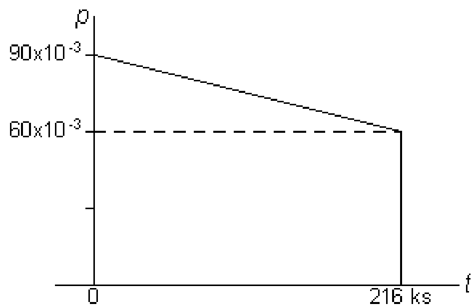
P 1.6 [a] Looking from A to B the current i is in the direction of the voltage rise across the 12 V battery, therefore $p = vi = -12(30) = -360$ W. Thus the power flow is from B to A, and Car A has the “dead” battery.

[b] $w = \int_0^t p\,dx = \int_0^t 360\,dx$

$w = 360t = 360(1 \times 60) = 21.6$ kJ

P 1.7 $p = vi; \qquad w = \int_0^t p\,dx$

Since the energy is the area under the power vs. time plot, let us plot p vs. t.



$$p(0) = (6)(15 \times 10^{-3}) = 90 \times 10^{-3} \text{ W}$$

$$p(216 \text{ ks}) = (4)(15 \times 10^{-3}) = 60 \times 10^{-3} \text{ W}$$

$$w = (60 \times 10^{-3})(216 \times 10^3) + \frac{1}{2}(216)(30) = 16.2 \text{ kJ}$$

Note: 60 hr $\equiv$ 216,000 s = 216 ks

P 1.8 **[a]** $p = vi = 30e^{-500t} - 30e^{-1500t} - 40e^{-1000t} + 50e^{-2000t} - 10e^{-3000t}$

$p(1 \text{ ms}) = 3.1 \text{ mW}$

[b]

$$\begin{aligned} w(t) &= \int_0^t (30e^{-500x} - 30e^{-1500x} - 40e^{-1000x} + \\ &\qquad 50e^{-2000x} - 10e^{-3000x})dx \\ &= 21.67 - 60e^{-500t} + 20e^{-1500t} + 40e^{-1000t} - \\ &\qquad 25e^{-2000t} + 3.33e^{-3000t} \mu\text{J} \\ w(1 \text{ ms}) &= 1.24\mu\text{J} \end{aligned}$$

[c] $w_{\text{total}} = 21.67\mu\text{J}$

P 1.9 **[a]** $v(20 \text{ ms}) = 100e^{-1}\sin 3 = 5.19 \text{ V}$

$i(20 \text{ ms}) = 20e^{-1}\sin 3 = 1.04 \text{ A}$

$p(20 \text{ ms}) = vi = 5.39 \text{ W}$

[b]

$$
\begin{aligned}
p &= vi = 2000e^{-100t}\sin^2 150t \\
&= 2000e^{-100t}\left[\frac{1}{2} - \frac{1}{2}\cos 300t\right] \\
&= 1000e^{-100t} - 1000e^{-100t}\cos 300t \\
w &= \int_0^\infty 1000e^{-100t}\,dt - \int_0^\infty 1000e^{-100t}\cos 300t\,dt \\
&= 1000\left.\frac{e^{-100t}}{-100}\right|_0^\infty - 1000\left.\left\{\frac{e^{-100t}}{(100)^2+(300)^2}\left[-100\cos 300t + 300\sin 300t\right]\right\}\right|_0^\infty \\
&= 10 - 1000\left[\frac{100}{1\times 10^4 + 9\times 10^4}\right] = 10 - 1 \\
\mathrm{w} &= 9 \;\; \mathrm{J}
\end{aligned}
$$

P 1.10 **[a]** $0 \le t \le 10$ ms:

$v = 1000t$ V; $\quad i = 0.6$ mA; $\quad p = 0.6t$ mW

$10 \le t \le 25$ ms:

$v = 10$ V; $\quad i = 0.6$ mA; $\quad p = 6$ mW

$25 \le t \le 35$ ms:

$v = 75 - 2500t$ V; $\quad i = 0$ mA; $\quad p = 0$ mW

$35 \le t \le 60$ ms:

$v = -50 + 1000t$ V; $\quad i = -0.4$ mA; $\quad p = 20 - 400t$ mW

$60 \le t \le 70$ ms:

$v = -50 + 1000t$ V; $\quad i = 0$ mA; $\quad p = 0$ mW

$70 \le t \le 80$ ms:

$v = 20$ V; $\quad i = -0.5$ mA; $\quad p = -10$ mW

$80 \le t \le 90$ ms:

$v = 180 - 2000t$ V; $\quad i = 0$ mA; $\quad p = 0$ mW

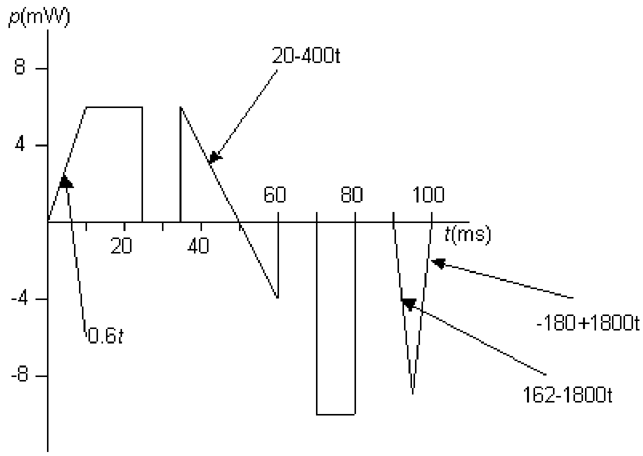
$90 \le t \le 95$ ms:

$v = 180 - 2000t$ V; $\quad i = 0.9$ mA; $\quad p = 162 - 1800t$ mW

$95 \le t \le 100$ ms:

$v = -200 + 2000t$ V; $\quad i = 0.9$ mA; $\quad p = -180 + 1800t$ mW

[**b**]
$$\begin{aligned} w(25) &= \tfrac{1}{2}(6)(10) + (6)(15) = 120\,\mu\text{J} \\ w(60) &= 120 + \tfrac{1}{2}(15)(6) - \tfrac{1}{2}(10)(4) = 145\,\mu\text{J} \\ w(90) &= 145 - (10)(10) = 45\,\mu\text{J} \\ w(100) &= 45 - \tfrac{1}{2}(10)(9) = 0\,\mu\text{J} \end{aligned}$$



P 1.11 [**a**] $p = vi = (2e^{-500t} - 2e^{-1000t})$ W

$$\frac{dp}{dt} = -1000e^{-500t} + 2000e^{-1000t} = 0 \text{ at t} = 1.4 \text{ ms}$$

$p_{\text{max}} = p(1.4 \text{ ms}) = 0.5$ W

[**b**] $$w = \int_0^\infty [2e^{-500t} - 2e^{-1000t}]\,dt = \left[\frac{2}{-500}e^{-500t} - \frac{2}{-1000}e^{-1000t}\Big|_0^\infty\right]$$

$= 2$ mJ

P 1.12 [**a**] $p = vi = 900\sin(200\pi t)\cos(200\pi t) = 450\sin(400\pi t)$ W

Therefore, $p_{\text{max}} = 450$ W

[**b**] $p_{\text{max}}(\text{extracting}) = 450$ W

[**c**]
$$\begin{aligned} p_{\text{avg}} &= 200\int_0^{5\times10^{-3}} 450\sin(400\pi t)\,dt \\ &= 9\times10^4\left[\frac{-\cos 400\pi t}{400\pi}\Big|_0^{2.5\times10^{-3}}\right] = \frac{225}{\pi}[1-\cos 2\pi] = 0 \end{aligned}$$

[**d**] $p_{\text{avg}} = \dfrac{180}{\pi}[1-\cos 2.5\pi] = \dfrac{180}{\pi} = 57.3$ W

P 1.13 [**a**]
$$\begin{aligned} q &= \text{area under } i \text{ vs. } t \text{ plot} \\ &= \left[\tfrac{1}{2}(5)(4) + (10)(4) + \tfrac{1}{2}(8)(4) + (8)(6) + \tfrac{1}{2}(3)(6)\right]\times 10^3 \\ &= [10+40+16+48+9]10^3 = 123{,}000 \text{ C} \end{aligned}$$

[b]

$$\begin{aligned} w &= \int p dt = \int vi\,dt \\ v &= 0.2\times 10^{-3}t + 9 \qquad 0 \le t \le 15 \text{ ks} \end{aligned}$$

$0 \le t \le 4000s$

$$\begin{aligned} i &= 15 - 1.25\times 10^{-3}t \\ p &= 135 - 8.25\times 10^{-3}t - 0.25\times 10^{-6}t^2 \\ w_1 &= \int_0^{4000} (135 - 8.25\times 10^{-3}t - 0.25\times 10^{-6}t^2)\,dt \\ &= (540 - 66 - 5.3333)10^3 = 468.667 \text{ kJ} \end{aligned}$$

$4000 \le t \le 12{,}000$

$$\begin{aligned} i &= 12 - 0.5\times 10^{-3}t \\ p &= 108 - 2.1\times 10^{-3}t - 0.1\times 10^{-6}t^2 \\ w_2 &= \int_{4000}^{12{,}000} (108 - 2.1\times 10^{-3}t - 0.1\times 10^{-6}t^2)\,dt \\ &= (864 - 134.4 - 55.467)10^3 = 674.133 \text{ kJ} \end{aligned}$$

$12{,}000 \le t \le 15{,}000$

$$\begin{aligned} i &= 30 - 2\times 10^{-3}t \\ p &= 270 - 12\times 10^{-3}t - 0.4\times 10^{-6}t^2 \\ w_3 &= \int_{12{,}000}^{15{,}000} (270 - 12\times 10^{-3}t - 0.4\times 10^{-6}t^2)\,dt \\ &= (810 - 486 - 219.6)10^3 = 104.4 \text{ kJ} \\ w_T &= w_1 + w_2 + w_3 = 468.667 + 674.133 + 104.4 = 1247.2 \text{ kJ} \end{aligned}$$

P 1.14 **[a]**

$$\begin{aligned} p &= vi \\ &= 400\times 10^3 t^2 e^{-800t} + 700te^{-800t} + 0.25e^{-800t} \\ &= e^{-800t}[400{,}000t^2 + 700t + 0.25] \\ \frac{dp}{dt} &= \{e^{-800t}[800\times 10^3 t + 700] - 800e^{-800t}[400{,}000t^2 + 700t + 0.25]\} \\ &= [-3{,}200{,}000t^2 + 2400t + 5]100e^{-800t} \end{aligned}$$

Therefore, $\frac{dp}{dt} = 0$ when $3{,}200{,}000t^2 - 2400t - 5 = 0$

so $p_{\max}$ occurs at $t = 1.68$ ms.

[b]

$$\begin{aligned} p_{\max} &= [400{,}000(.00168)^2 + 700(.00168) + 0.25]e^{-800(.00168)} \\ &= 666 \text{ mW} \end{aligned}$$

[c]

$$
\begin{aligned}
w &= \int_0^t p dx \\
w &= \int_0^t 400{,}000x^2e^{-800x}\,dx + \int_0^t 700xe^{-800x}\,dx + \int_0^t 0.25e^{-800x}\,dx \\
&= \frac{400{,}000e^{-800x}}{-512\times 10^6}[64\times 10^4x^2 + 1600x + 2]\Big|_0^t + \\
&\quad \frac{700e^{-800x}}{64\times 10^4}(-800x-1)\Big|_0^t + 0.25\frac{e^{-800x}}{-800}\Big|_0^t
\end{aligned}
$$

When $t = \infty$ all the upper limits evaluate to zero, hence

$$
w = \frac{(400{,}000)(2)}{512\times 10^6} + \frac{700}{64\times 10^4} + \frac{0.25}{800} = 2.97 \text{ mJ}.
$$

P 1.15 **[a]**

$$
\begin{aligned}
p &= 0 \quad t < 0, \qquad p = 0 \quad t > 3 \text{ s} \\
p &= vi = t(3-t)(6-4t) = 18t - 18t^2 + 4t^3 \text{ mW} \qquad 0 \le t \le 3 \text{ s} \\
\frac{dp}{dt} &= 18 - 36t + 12t^2 = 12(t^2 - 3t + 1.5) \\
\frac{dp}{dt} &= 0 \qquad \text{when } t^2 - 3t + 1.5 = 0 \\
t &= \frac{3\pm\sqrt{9-6}}{2} = \frac{3\pm\sqrt{3}}{2} \\
t_1 &= 3/2 - \sqrt{3}/2 = 0.634 \text{ s}; \qquad t_2 = 3/2 + \sqrt{3}/2 = 2.366 \text{ s} \\
p(t_1) &= 18(0.634) - 18(0.634)^2 + 4(0.634)^3 = 5.196 \text{ mW} \\
p(t_2) &= 18(2.366) - 18(2.366)^2 + 4(2.366)^3 = -5.196 \text{ mW}
\end{aligned}
$$

Therefore, maximum power is being delivered at $t = 0.634$ s.

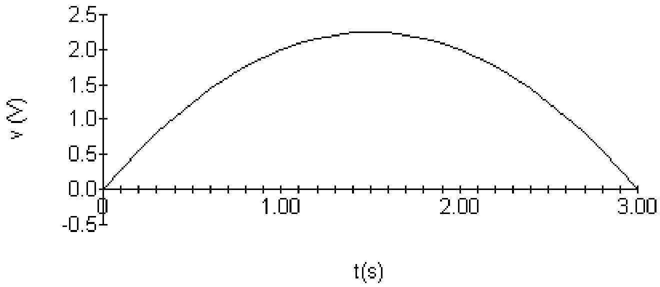
[b] $p_{\max} = 5.196$ mW (delivered)

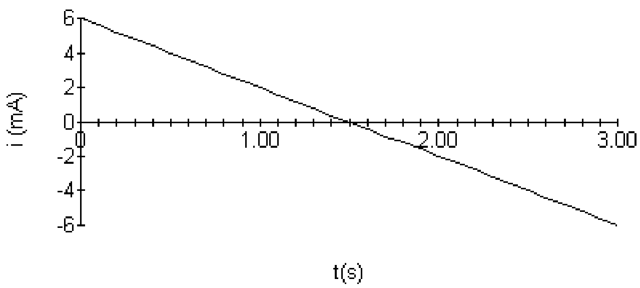
[c] Maximum power is being extracted at $t = 2.366$ s.

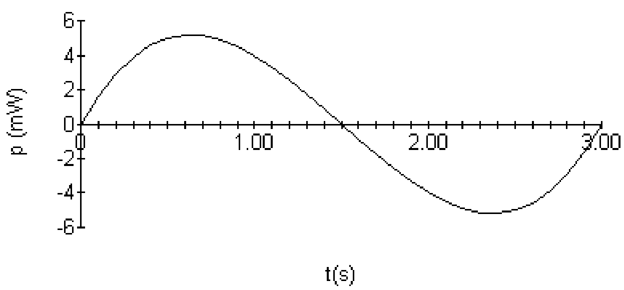
[d] $p_{\max} = 5.196$ mW (extracted)

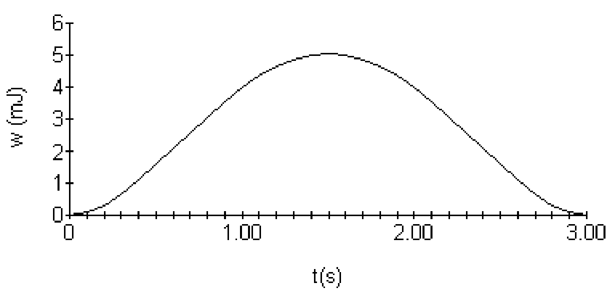
[e] $w = \int_0^t p dx = \int_0^t (18x - 18x^2 + 4x^3)dx = 9t^2 - 6t^3 + t^4$

$$
\begin{aligned}
w(0) &= 0 \text{ mJ} \qquad & w(2) &= 4 \text{ mJ} \\
w(1) &= 4 \text{ mJ} \qquad & w(3) &= 0 \text{ mJ}
\end{aligned}
$$









P 1.16 **[a]**

$$\begin{aligned} p &= vi = 12 \times 10^5 t^2 e^{-1000t} \text{ W} \\ \frac{dp}{dt} &= 12 \times 10^5 [t^2(-1000)e^{-1000t} + e^{-1000t}(2t)] \\ &= 12 \times 10^5 t e^{-1000t}[t(2 - 1000t)] \\ \frac{dp}{dt} &= 0 \text{ at } t = 0, \qquad t = 2 \text{ ms} \end{aligned}$$

We know p is a minimum at $t = 0$ since v and i are zero at $t = 0$.

[b] $p_{\text{max}} = 12 \times 10^5 (2 \times 10^{-3})^2 e^{-2} = 649.61$ mW

[c]
$$\begin{aligned} w &= 12\times 10^5\int_0^\infty t^2e^{-1000t}\,dt \\ &= 12\times 10^5\left\{\frac{e^{-1000t}}{(-1000)^3}[10^6t^2+2{,}000t+2]\Big|_0^\infty\right\} = 2.4\text{ mJ}\end{aligned}$$

P 1.17 [a] From the diagram and the table we have

$$\begin{aligned}
p_{\rm a} &= -v_{\rm a}i_{\rm a} = -(46.16)(6) = -276.96\text{ W} \qquad \text{(del)}\\
p_{\rm b} &= v_{\rm b}i_{\rm b} = (14.16)(4.72) = 66.8352\text{ W} \qquad \text{(abs)}\\
p_{\rm c} &= v_{\rm c}i_{\rm c} = (-32)(-6.4) = 204.80\text{ W} \qquad \text{(abs)}\\
p_{\rm d} &= -v_{\rm d}i_{\rm d} = -(22)(1.28) = -28.16\text{ W} \qquad \text{(del)}\\
p_{\rm e} &= -v_{\rm e}i_{\rm e} = -(33.60)(1.68) = -56.448\text{ W} \qquad \text{(del)}\\
p_{\rm f} &= v_{\rm f}i_{\rm f} = (66)(-0.4) = -26.40\text{ W} \qquad \text{(del)}\\
p_{\rm g} &= v_{\rm g}i_{\rm g} = (2.56)(1.28) = 3.2768\text{ W} \qquad \text{(abs)}\\
p_{\rm h} &= -v_{\rm h}i_{\rm h} = -(-0.4)(0.4) = 0.16\text{ W} \qquad \text{(abs)}\\
\sum P_{\rm del} &= 276.96 + 28.16 + 56.448 + 26.40 = 387.9680\text{ W}\\
\sum P_{\rm abs} &= 66.8352 + 204.80 + 3.2768 + 0.16 = 275.072\text{ W}
\end{aligned}$$

Therefore, $\sum P_{\rm del} \neq \sum P_{\rm abs}$ and the subordinate engineer is correct.

[b] We can also check the data using Kirchhoff's laws.

From Fig. P1.17 the following equations should be satisfied:

$i_{\rm a} - i_{\rm b} - i_{\rm d} = 0$ (ok)

$i_{\rm b} + i_{\rm c} - i_{\rm e} = 0$ (no)

$i_{\rm f} - i_{\rm a} - i_{\rm c} = 0$ (ok)

$i_{\rm d} = i_{\rm g}$ (ok)

$i_{\rm g} + i_{\rm e} + i_{\rm h} = 0$ (no)

$i_{\rm h} = -i_{\rm f}$ (ok)

Using Kirchhoff's current law, it appears $i_{\rm e}$ is in error.

From Kirchhoff's voltage law we have

$v_{\rm b} - v_{\rm a} - v_{\rm c} = 0$ (ok)

$-v_{\rm d} - v_{\rm b} + v_{\rm e} + v_{\rm g} = 0$ (ok)

$-v_{\rm e} + v_{\rm c} + v_{\rm f} + v_{\rm h} = 0$ (ok)

Therefore all the voltages are consistent with Kirchhoff's voltage law.

Assume $i_{\rm e}$ is in error. Therefore,

$i_{\rm e} = i_{\rm b} + i_{\rm c} = -i_{\rm g} - i_{\rm h} = 4.72 - 6.40 = -1.28 - 0.4 = -1.68$ A

So the error is in the sign of $i_{\rm e}$; $i_{\rm e}$ equals minus 1.68 A.

Correcting $i_{\rm e}$ leads to

$\sum P_{\rm del} = \sum P_{\rm abs} = 331.52$ W

P 1.18

$$\begin{aligned}
p_{\rm a} &= v_{\rm a}i_{\rm a} = (48)(12) = 576\ \text{W} \qquad \text{(abs)}\\
p_{\rm b} &= v_{\rm b}i_{\rm b} = (18)(-4) = -72\ \text{W} \qquad \text{(del)}\\
p_{\rm c} &= -v_{\rm c}i_{\rm c} = -(30)(-10) = 300\ \text{W} \qquad \text{(abs)}\\
p_{\rm d} &= v_{\rm d}i_{\rm d} = (36)(16) = 576\ \text{W} \qquad \text{(abs)}\\
p_{\rm e} &= -v_{\rm e}i_{\rm e} = -(36)(8) = -288\ \text{W} \qquad \text{(del)}\\
p_{\rm f} &= -v_{\rm f}i_{\rm f} = -(-54)(14) = 756\ \text{W} \qquad \text{(abs)}\\
p_{\rm g} &= -v_{\rm g}i_{\rm g} = -(84)(22) = -1848\ \text{W} \qquad \text{(del)}
\end{aligned}$$

$\sum P_{\rm del} = 72 + 288 + 1848 = 2208$ W

$\sum P_{\rm abs} = 576 + 300 + 576 + 756 = 2208$ W

Therefore, $\sum P_{\rm del} = \sum P_{\rm abs} = 2208$ W

P 1.19 **[a]** From an examination of reference polarities, the following elements employ the passive convention: a, c, e, and f.

[b]

$$\begin{aligned}
p_{\rm a} &= -56\ \text{W} \qquad \text{(del)}\\
p_{\rm b} &= -14\ \text{W} \qquad \text{(del)}\\
p_{\rm c} &= 150\ \text{W} \qquad \text{(abs)}\\
p_{\rm d} &= -50\ \text{W} \qquad \text{(del)}\\
p_{\rm e} &= -18\ \text{W} \qquad \text{(del)}\\
p_{\rm f} &= -12\ \text{W} \qquad \text{(del)}
\end{aligned}$$

$\sum P_{\rm abs} = 150$ W; $\qquad \sum P_{\rm del} = 56 + 14 + 50 + 18 + 12 = 150$ W.

P 1.20 (a) 9 (b) 7 (c) 4 (d) $v_{\rm a}$–$R_{\rm a}$, $v_{\rm b}$–$R_{\rm b}$, $v_{\rm c}$–$R_{\rm c}$ (e) 6

(f) (1) $v_{\rm a} - R_{\rm a} - R_{\rm d} - R_{\rm b} - v_{\rm b}$

(2) $R_{\rm d} - R_{\rm f} - R_{\rm e}$

(3) $v_{\rm b} - R_{\rm b} - R_{\rm d} - R_{\rm f} - R_{\rm c} - v_{\rm c}$

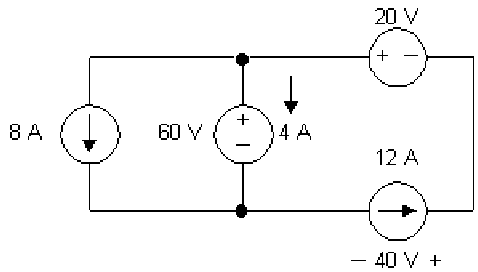
(4) $v_{\rm c} - R_{\rm c} - R_{\rm f} - R_{\rm a} - v_{\rm a}$

(5) $v_{\rm a} - R_{\rm a} - R_{\rm f} - R_{\rm e} - R_{\rm b} - v_{\rm b}$

(6) $v_{\rm a} - R_{\rm a} - R_{\rm d} - R_{\rm e} - R_{\rm c} - v_{\rm c}$

(7) $v_{\rm b} - R_{\rm b} - R_{\rm e} - R_{\rm c} - v_{\rm c}$

P 1.21 The interconnect is valid since it does not violate Kirchhoff's laws.



$-60 + 20 + 40 = 0 \qquad \text{(KVL)}$

$8 + 4 - 12 = 0 \qquad \text{(KCL)}$

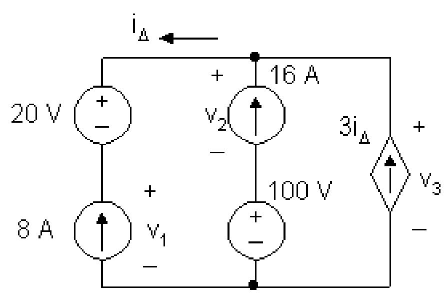
$\sum P_{\text{dev}} = 4(60) + 8(60) = 720 \text{ W}$

$\sum P_{\text{abs}} = 12(20) + 12(40) = 720 \text{ W}$

$\sum P_{\text{dev}} = \sum P_{\text{abs}} = 720 \text{ W}$

P 1.22 **[a]** Yes, Kirchhoff's laws are not violated.

[b] No, because the voltages across the independent and dependent current sources are indeterminate. For example, define v_1, v_2, and v_3 as shown:



Kirchhoff's voltage law requires

$v_1 + 20 = v_3$

$v_2 + 100 = v_3$

Conservation of energy requires

$8(20) + 8v_1 + 16v_2 + 1600 - 24v_3 = 0$

or

$v_1 + 2v_2 - 3v_3 = -220$

Now arbitrarily select a value of v_3 and show the conservation of energy will be satisfied. Examples:
If $v_3 = 200$ V then $v_1 = 180$ V and $v_2 = 100$ V. Then

$180 + 200 - 600 = -220$ (CHECKS)

If $v_3 = -100$ V, then $v_1 = -120$ V and $v_2 = -200$ V. Then

$-120 - 400 + 300 = -220$ (CHECKS)

P 1.23 **[a]** Yes, independent voltage sources can carry whatever current is required by the connection; independent current source can support any voltage required by the connection.

[b] 30 V source: absorbing

10 V source: delivering

8 A source: delivering

[c]
$$\begin{aligned} P_{30\text{V}} &= (30)(8) = 240 \text{ W} \quad \text{(abs)} \\ P_{10\text{V}} &= -10(8) = -80 \text{ W} \quad \text{(del)} \\ P_{8\text{A}} &= -20(8) = -160 \text{ W} \quad \text{(del)} \end{aligned}$$
$\sum P_{\text{abs}} = \sum P_{\text{del}} = 240$ W

[d] Yes, 30 V source is delivering, the 10 V source is delivering, and the 8 A source is absorbing
$$\begin{aligned} P_{30\text{V}} &= -30(8) = -240 \text{ W} \quad \text{(del)} \\ P_{10\text{V}} &= -10(8) = -80 \text{ W} \quad \text{(del)} \\ P_{8\text{A}} &= +40(8) = 320 \text{ W} \quad \text{(abs)} \end{aligned}$$

P 1.24 The interconnection is valid because it does not violate Kirchhoff's laws.

$i_\Delta = -25$ A; $\quad 6i_\Delta = -150$ V

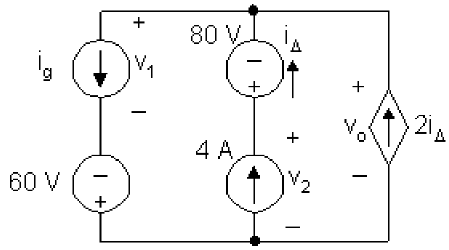
$-200 + 50 - (-150) = 0$

But the power developed in the circuit cannot be determined, as the currents in the 200 V, 50 V, and $6i_\Delta$ sources are unspecified.

P 1.25 The interconnection is not valid because it violates Kirchhoff's current law:

$3 \text{ A} + (-5 \text{ A}) \neq 8 \text{ A}.$

P 1.26



$i_\Delta = 4$ A so $i_g = 12$ A

$v_o = 100$ V

$-60 + v_1 = 100,$ so $v_1 = 160$ V

$v_2 - 80 = 100,$ so $v_2 = 180$ V

$\sum P_{\text{dev}} = 180(4) + 100(8) + 60(12) = 2240$ W

$$\begin{aligned}\text{CHECK:}\quad \sum P_{\text{diss}} &= 160(12) + 80(4) = 1920 + 320\\ &= 2240 \text{ W — CHECKS}\end{aligned}$$

P 1.27 The interconnection is valid because it does not violate Kirchhoff's laws:

$p_{\text{V-sources}} = -(100 - 60)(5) = -200$ W.

P 1.28 First there is no violation of Kirchhoff's laws, hence the interconnection is valid.
Kirchhoff's voltage law requires

$v_1 + v_2 = 150 - 50 = 100$ V

The conservation of energy law requires

$20v_1 - 10v_1 + 10v_2 + 500 - 1500 = 0$

or

$v_1 + v_2 = 100$

Hence any combination of v_1 and v_2 that adds to 100 is a valid solution. For example if $v_1 = 80$ V and $v_2 = 20$ V

$P_{\text{abs}} = 80(20) + 10(20) + 50(10) = 2300$ W

$P_{\text{dev}} = 1500 + 80(10) = 2300$ W

If $v_1 = 60$ V and $v_2 = 40$ V

$P_{\text{abs}} = 60(20) + 10(40) + 500 = 2100$ W

$P_{\text{dev}} = 60(10) + 1500 = 2100$ W

If $v_1 = -100$ V and $v_2 = 200$ V

$P_{\text{abs}} = 10(100) + 10(200) + 10(50) = 3500$ W

$P_{\text{dev}} = 20(100) + 10(150) = 3500$ W

P 1.29 **[a]**

$$\begin{aligned} 1.6 &= i_{\text{g}} - i_{\text{a}} \\ 80i_{\text{a}} &= 1.6(30+90) = 192 \qquad \text{therefore, } i_{\text{a}} = 2.4 \text{ A} \\ i_g &= i_{\text{a}} + 1.6 = 2.4 + 1.6 = 4 \text{ A} \end{aligned}$$

[b] $v_g = 90(1.6) = 144$ V

[c]

$$\begin{aligned} \sum P_{\text{dis}} &= 2.4^2(80) + 1.6^2(120) = 768 \text{ W} \\ \sum P_{\text{dev}} &= (4)(192) = 768 \text{ W} \end{aligned}$$

Therefore, $\sum P_{\text{dis}} = \sum P_{\text{dev}} = 768$ W

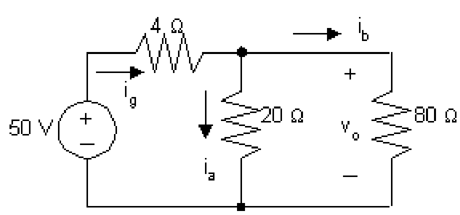
P 1.30 **[a]**

$$\begin{aligned} v_o &= 8i_{\text{a}} + 14i_{\text{a}} + 18i_{\text{a}} = 40(20) = 800 \text{ V} \\ 800 &= 10i_o \\ i_o &= 800/10 = 80 \text{ A} \end{aligned}$$

[b] $i_g = i_{\text{a}} + i_o = 20 + 80 = 100$ A

[c] $p_g(\text{delivered}) = (100)(800) = 80{,}000 \text{ W} = 80$ kW

P 1.31 **[a]**



$$\begin{aligned} 20i_{\text{a}} &= 80i_{\text{b}} \qquad i_g = i_{\text{a}} + i_{\text{b}} = 5i_{\text{b}} \\ i_{\text{a}} &= 4i_{\text{b}} \\ 50 &= 4i_g + 80i_{\text{b}} = 20i_{\text{b}} + 80i_{\text{b}} = 100i_{\text{b}} \\ i_{\text{b}} &= 0.5 \text{ A, therefore, } i_{\text{a}} = 2 \text{ A} \quad \text{and} \quad i_{\text{g}} = 2.5 \text{ A} \end{aligned}$$

[b] $i_b = 0.5$ A

[c] $v_o = 80i_b = 40$ V

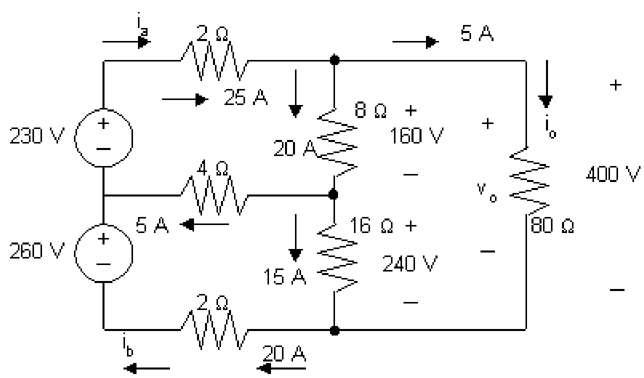
[d]
$$\begin{aligned} p_{4\Omega} &= i_g^2(4) = 6.25(4) = 25 \text{ W} \\ p_{20\Omega} &= i_a^2(20) = (4)(20) = 80 \text{ W} \\ p_{80\Omega} &= i_b^2(80) = 0.25(80) = 20 \text{ W} \end{aligned}$$

[e] p_{5V} (delivered) $= 5i_g = 125$ W
Check:

$$\sum P_{dis} = 25 + 80 + 20 = 125 \text{ W}$$

$$\sum P_{del} = 125 \text{ W}$$

P 1.32 **[a]**



$$\begin{aligned} v_o &= 20(8) + 16(15) = 400 \text{ V} \\ i_o &= 400/80 = 5 \text{ A} \\ i_a &= 25 \text{ A} \end{aligned}$$

P_{230} (supplied) $= (230)(25) = 5750$ W

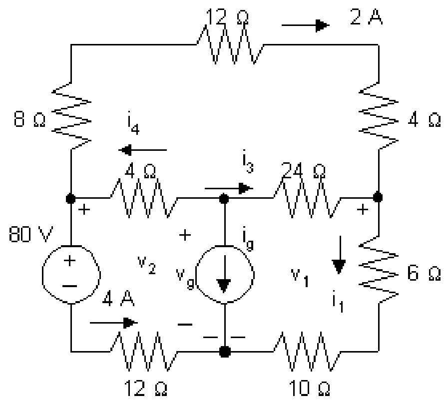
$i_b = 5 + 15 = 20$ A

P_{260} (supplied) $= (260)(20) = 5200$ W

[b]
$$\begin{aligned} \sum P_{dis} &= (25)^2(2) + (20)^2(8) + (5)^2(4) + (15)^2 16 + (20)^2 2 + (5)^2(80) \\ &= 1250 + 3200 + 100 + 3600 + 800 + 2000 = 10{,}950 \text{ W} \\ \sum P_{sup} &= 5750 + 5200 = 10{,}950 \text{ W} \end{aligned}$$

Therefore, $\sum P_{dis} = \sum P_{sup} = 10{,}950$ W

P 1.33 **[a]**



$$
\begin{aligned}
v_2 &= 80 + 4(12) = 128 \text{ V} \\
v_1 &= 128 - 24(2) = 80 \text{ V} \\
i_1 &= \frac{v_1}{16} = \frac{80}{16} = 5 \text{ A} \\
i_3 &= i_1 - 2 = 5 - 2 = 3 \text{ A} \\
v_g &= v_1 + 24i_3 = 80 + 72 = 152 \text{ V} \\
v_g - 4i_4 &= v_2 \\
4i_4 &= v_g - v_2 = 152 - 128 = 24 \text{ V} \\
i_4 &= 24/4 = 6 \text{ A} \\
i_g &= -(i_3 + i_4) = -(3 + 6) = -9 \text{ A}
\end{aligned}
$$

[b]

$$
\begin{aligned}
p_{8\Omega} &= (2)^2(8) = 32 \text{ W} & p_{4\Omega} &= (6)^2(4) = 144 \text{ W} \\
p_{12\Omega} &= (2)^2(12) = 48 \text{ W} & p_{6\Omega} &= (5)^2(6) = 150 \text{ W} \\
p_{4\Omega} &= (2)^2(4) = 16 \text{ W} & p_{10\Omega} &= (5)^2(10) = 250 \text{ W} \\
p_{24\Omega} &= (3)^2(24) = 216 \text{ W} & p_{12\Omega} &= (4)^2(12) = 192 \text{ W}
\end{aligned}
$$

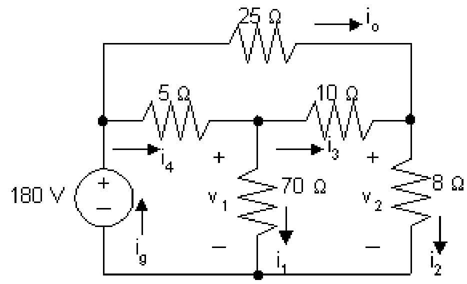
[c] $v_g = 152$ V

[d]

$$
\begin{aligned}
\sum P_{\text{dis}} &= 32 + 48 + 16 + 216 + 144 + 150 + 250 + 192 + 80(4) = 1368 \text{ W} \\
\sum P_{\text{del}} &= (152)(9) = 1368 \text{ W}
\end{aligned}
$$

Therefore, $\sum P_{\text{dis}} = \sum P_{\text{del}}$

P 1.34 **[a]**



$$v_2 = 180 - 100 = 80 \text{ V}$$

$$i_2 = \frac{v_2}{8} = 10 \text{ A}$$

$$i_3 + 4 = i_2, \qquad i_3 = 10 - 4 = 6 \text{ A}$$

$$v_1 = v_2 + v_3 = 80 + 6(10) = 140 \text{ V}$$

$$i_1 = \frac{v_1}{70} = \frac{140}{70} = 2 \text{ A}$$

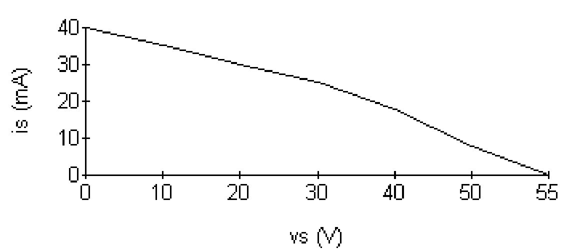
[b]

$$\begin{aligned} p_{5\Omega} &= 8^2(5) = 320 \text{ W} \\ p_{25\Omega} &= (4)^2(25) = 400 \text{ W} \\ p_{70\Omega} &= 2^2(70) = 280 \text{ W} \\ p_{10\Omega} &= 6^2(10) = 360 \text{ W} \\ p_{8\Omega} &= 10^2(8) = 800 \text{ W} \end{aligned}$$

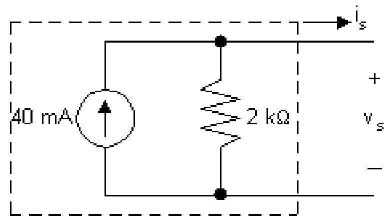
[c] $\sum P_{\text{dis}} = 320 + 400 + 280 + 360 + 800 = 2160 \text{ W}$

$$P_{\text{dev}} = 180 i_g = 180(12) = 2160 \text{ W}$$

P 1.35 **[a]**

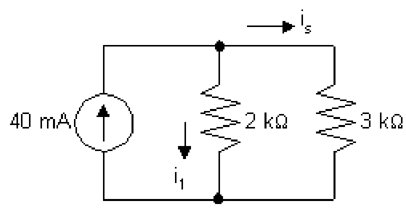


[b] $\Delta v = 20$ V; $\quad \Delta i = 10$ mA; $\quad R = \dfrac{\Delta v}{\Delta i} = 2$ kΩ



[c] $2i_1 = 3i_s, \qquad i_1 = 1.5i_s$

$40 = i_1 + i_s = 2.5i_s, \qquad i_s = 16$ mA

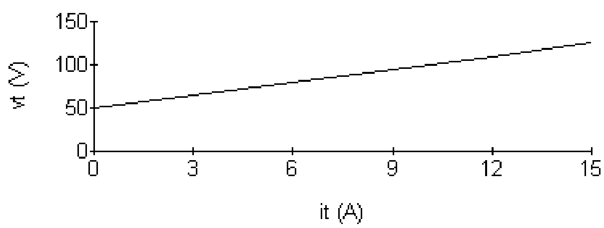


[d] v_s(open circuit) $= (40 \times 10^{-3})(2 \times 10^3) = 80$ V

[e] v_s(open circuit) $= 55$ V

[f] Linear model cannot predict the nonlinear behavior of the practical current source.

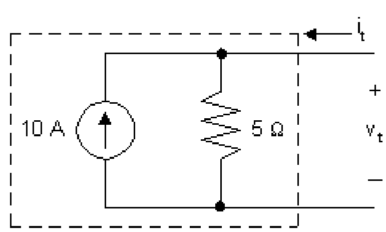
P 1.36 **[a]** Plot the $v - i$ characteristic



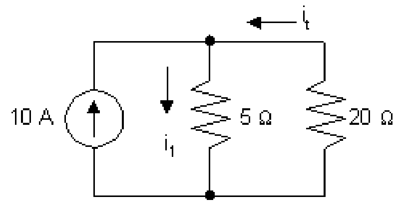
From the plot:

$$R = \frac{\Delta v}{\Delta i} = \frac{(125 - 50)}{(15 - 0)} = 5\,\Omega$$

When $i_t = 0$, $v_t = 50$ V; therefore the ideal current source has a current of 10 A



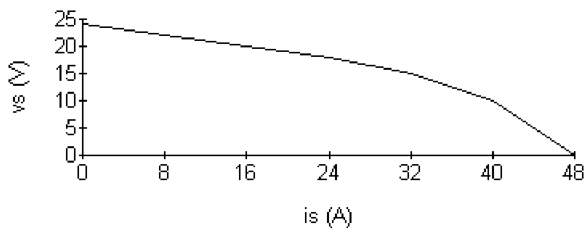
[b]



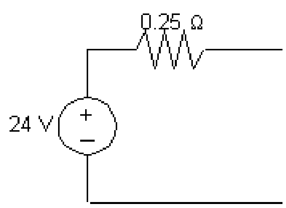
$$10 + i_t = i_1 \qquad \text{and} \qquad 5i_1 = -20i_t$$

Therefore, $10 + i_t = -4i_t$ so $i_t = -2$ A

P 1.37 **[a]**



[b] $R = \dfrac{24 - 18}{24 - 0} = \dfrac{6}{24} = 0.25\,\Omega$



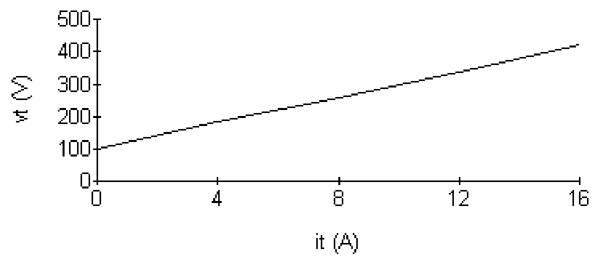
[c] $i = \dfrac{24}{1.25} = 19.2$ A, $\qquad v = 24 - 19.2(0.25) = 19.2$ V

[d] $i_{sc} = \dfrac{24}{0.25} = 96$ A

[e] $i_{sc} = 48$ A (from graph)

[f] Linear model cannot predict nonlinear behavior of voltage source.

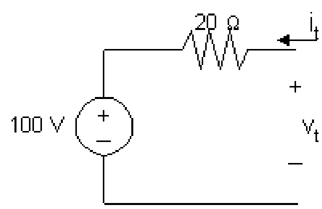
P 1.38 **[a]** Plot the v—i characteristic:



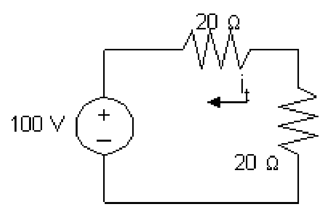
From the plot:

$$R = \frac{\Delta v}{\Delta i} = \frac{(420 - 100)}{(16 - 0)} = 20\,\Omega$$

When $i_t = 0$, $v_t = 100$ V; therefore the ideal voltage source has a voltage of 100 V

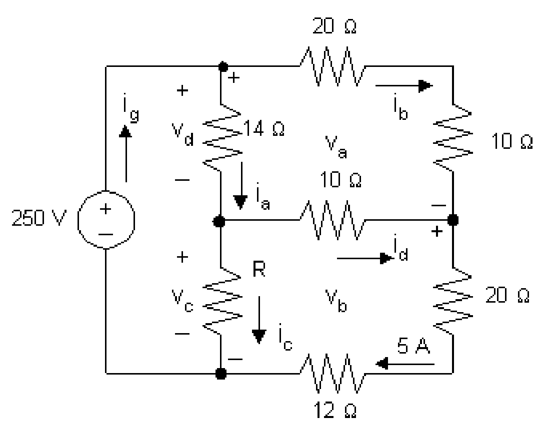


[b]



$i_t = -100/(20 + 20) = -2.5$ A; Therefore, $p_{20\Omega} = (-2.5)^2(20) = 125$ W

P 1.39 **[a]**



$v_b = 5(20 + 12) = 160$ V

$v_b + v_a = 250$ V, so $v_a = 90$ V

$i_b = 90/(20 + 10) = 3$ A

$i_{\rm d} = 5 - i_{\rm b} = 2$ A

$v_{\rm c} = v_{\rm b} + 10(i_{\rm d}) = 180$ V

$v_{\rm d} = 250 - v_{\rm c} = 70$ V $= 14(i_{\rm a})$; therefore, $i_{\rm a} = 5$ A

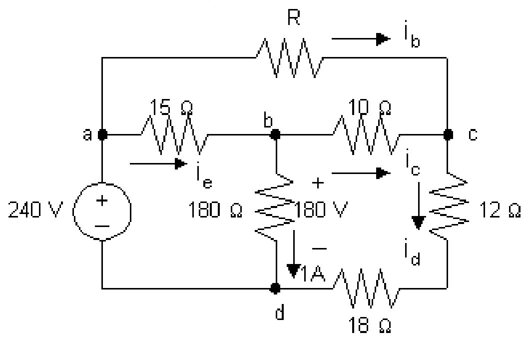
$i_{\rm c} = i_{\rm a} - i_{\rm d} = 5 - 2 = 3$ A

$R = v_{\rm c}/i_{\rm c} = 180/3 = 60\,\Omega$

[b] $i_g = 5 + 3 = 8$ A

p_g (supplied) $= (250)(8) = 2000$ W

P 1.40



$v_{\rm ab} = 240 - 180 = 60$ V; therefore, $i_{\rm e} = 60/15 = 4$ A

$i_{\rm c} = i_{\rm e} - 1 = 4 - 1 = 3$ A; therefore, $v_{\rm bc} = 10 i_{\rm c} = 30$ V

$v_{\rm cd} = 180 - v_{\rm bc} = 180 - 30 = 150$ V;

therefore, $i_{\rm d} = v_{\rm cd}/(12 + 18) = 150/30 = 5$ A

$i_{\rm b} = i_{\rm d} - i_{\rm c} = 5 - 3 = 2$ A

$v_{\rm ac} = v_{\rm ab} + v_{\rm bc} = 60 + 30 = 90$ V

$R = v_{\rm ac}/i_{\rm b} = 90/2 = 45\,\Omega$

CHECK: $i_g = i_{\rm b} + i_{\rm e} = 2 + 4 = 6$ A

$p_{\rm dev} = (240)(6) = 1440$ W

$$\sum P_{\rm dis} = 1(180) + 4(45) + 9(10) + 25(12) + 25(18) + 16(15) = 1440 \text{ W (CHECKS)}$$

P 1.41 **[a]** $15.2 = 10{,}000i_\beta - 0.80 + (200)30i_\beta$

$16 = (16{,}000)i_\beta$

$i_\beta = 1$ mA

$200(30i_\beta) + v_y + 500(29i_\beta) - 25 = 0$

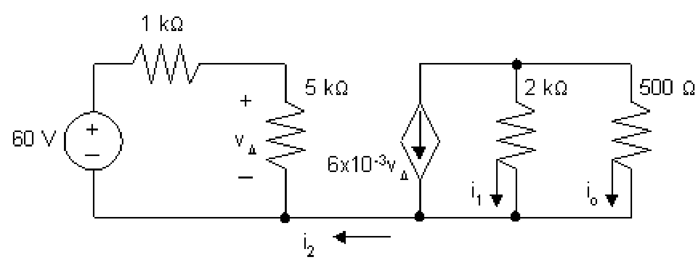
$v_y = 25 - 6000i_\beta - 14{,}500i_\beta$

Therefore, $v_y = 4.5$ V

[b]
$$\begin{aligned}\sum P_{\text{gen}} &= 15.2i_\beta + 25(29)i_\beta + 0.8i_\beta = 741i_\beta = 741 \text{ mW}\\ \sum P_{\text{dis}} &= 10^4(i_\beta)^2 + 200(30i_\beta)^2 + 29i_\beta(4.5) + 500(29i_\beta)^2\\ &= 741 \text{ mW.}\end{aligned}$$

P 1.42 **[a]** $i_2 = 0$ because no current can exist in a single conductor connecting two parts of a circuit.

[b]



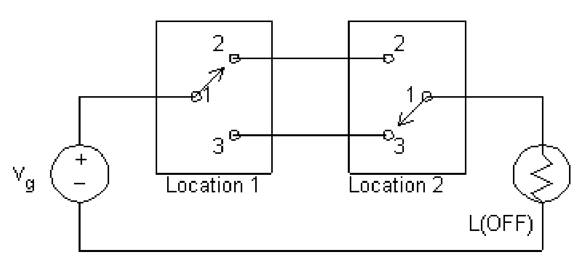
$60 = 6000i_g \qquad i_g = 10$ mA

$v_\Delta = 5000i_g = 50$ V $\qquad 6 \times 10^{-3}v_\Delta = 300$ mA

$2000i_1 = 500i_o$, so $i_1 + 4i_1 = -300$ mA; therefore, $i_1 = -60$ mA

[c] $300 - 60 + i_2 = 0$, so $i_o = -240$ mA.

P 1.43 **[a]**



[b]