Introduction to Electronic Systems

Zheng Feng

Part 2: Dynamic Circuit Analysis

5. Capacitors and Inductors

6. **Response of First-order RC and RL Circuits**

7. Response of Second-order RLC Circuits*

Chapter 6

- RC and RL Circuits
- Initial Values
- Natural Response of RC/RL Circuits
- Step Response of RC/RL Circuits
- General Solution Method
- Sequential Switching
- Integrating Amplifier

Mathematical Fundamentals

- First-order Differential Equation
- Complex Number

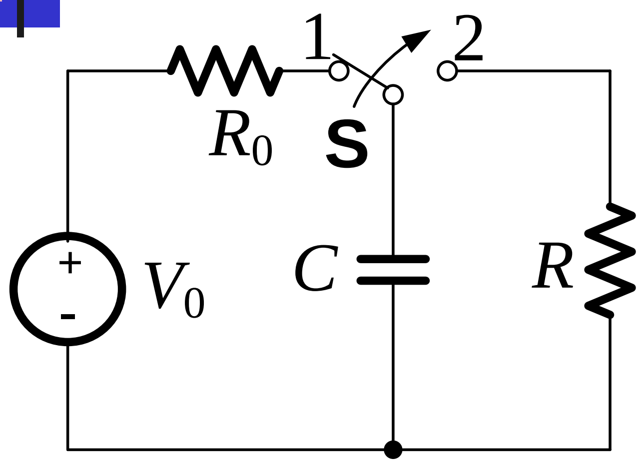
6-1 RC and RL Circuits

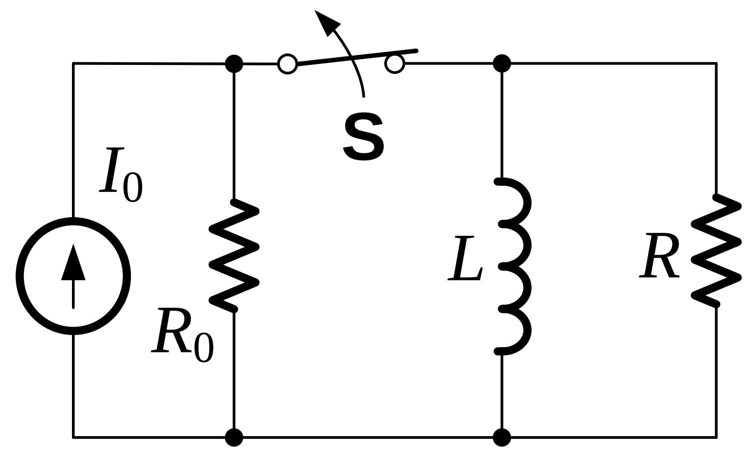
- What is RC and RL Circuits?
- What is Natural Response?
- What is Step Response?

RC and RL Circuits

- Circuit that consists of sources, resistors, and either (but not both) inductor or capacitor is called first-order RC or RL circuit;
- RC: Resistor-Capacitor
- RL: Resistor-Inductor

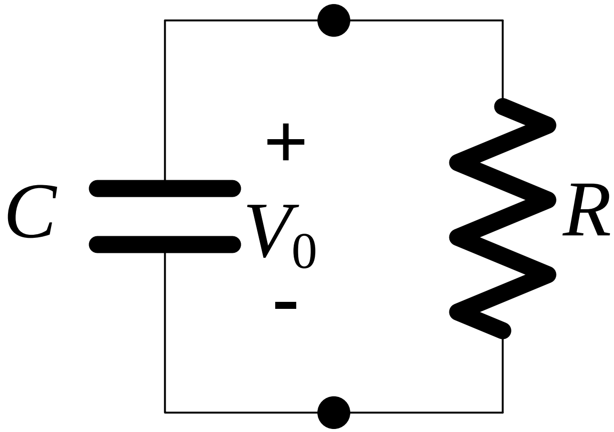
Natural Response

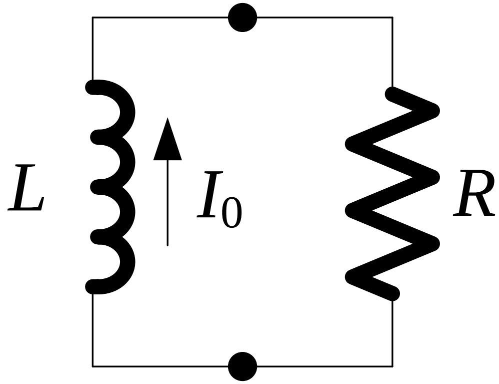




- The capacitor/inductor is abruptly disconnected from its source;
- Energy stored in the capacitor/inductor is released to a resistive network.

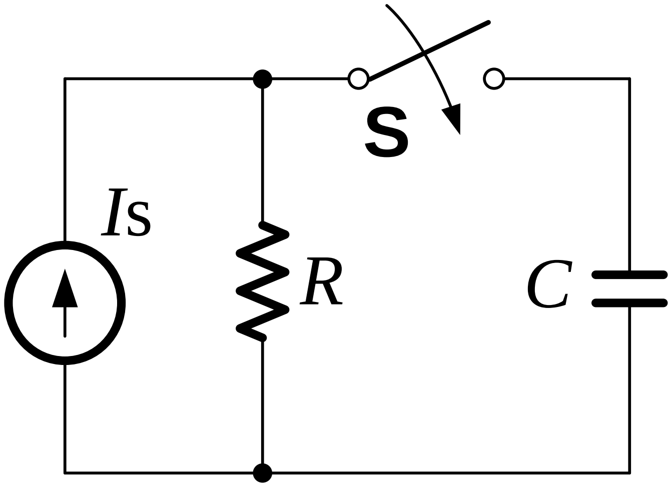
Natural Response

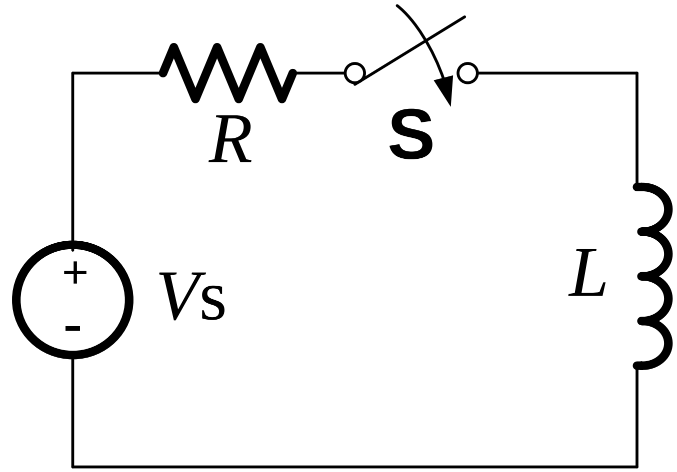




- The voltages and currents arising in the circuit are referred to as the **Natural Response**;
- The circuit behavior is determined by the **circuit itself**, but not the external source excitation.

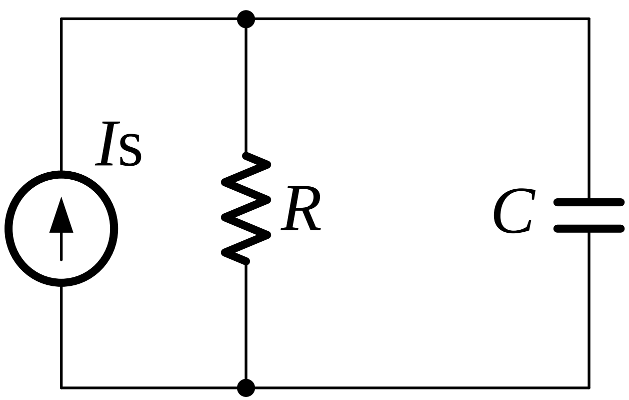
Step Response

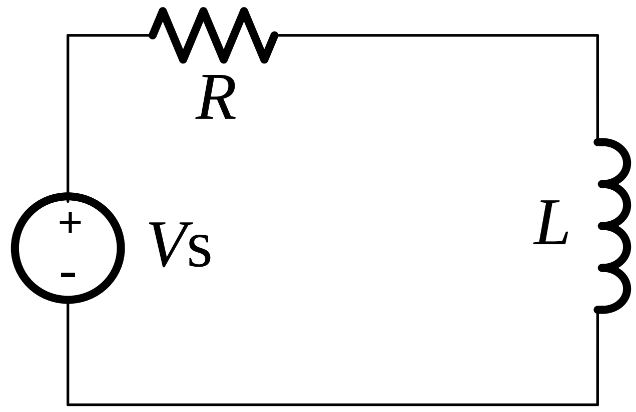




- The voltage or current source is suddenly applied to the capacitor/inductor;
- Energy is acquired by the capacitor/inductor.

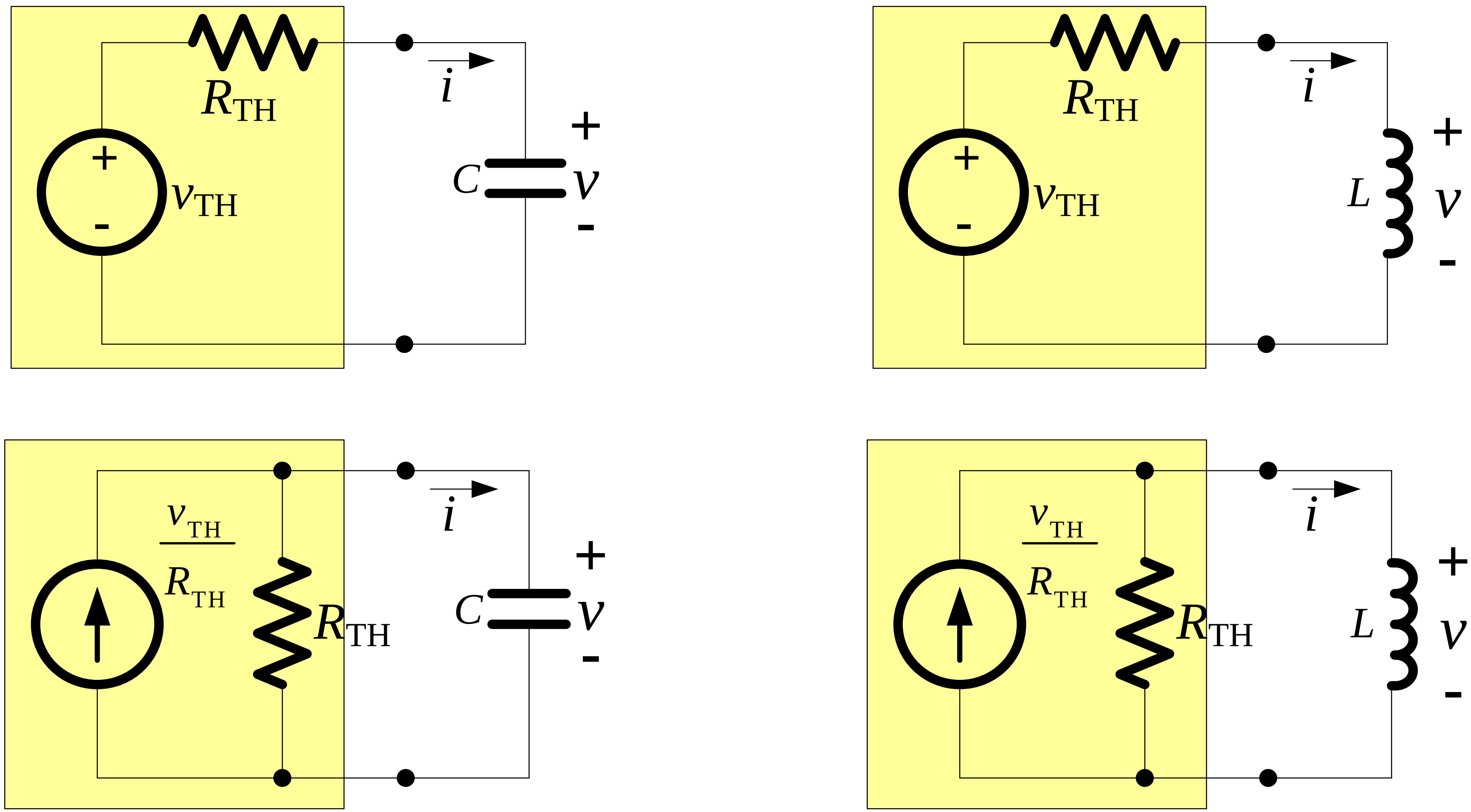
Step Response





- The voltages and currents arising in the circuit are referred to as the **Step Response**;
- The circuit behavior is determined **both by the circuit itself, and the external source excitation**.

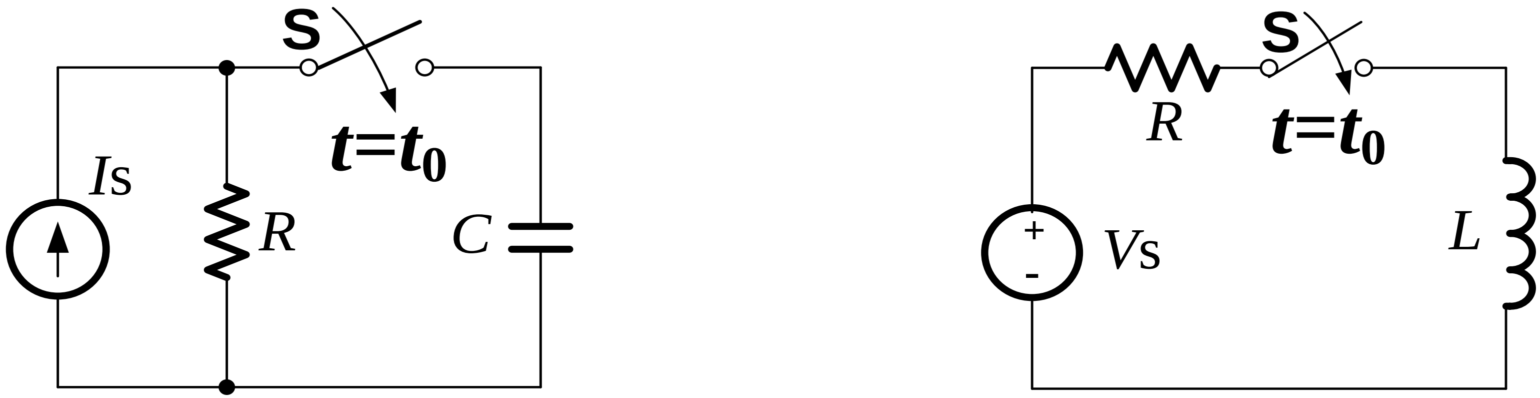
RC and RL Circuits with Source



6-2 Initial Values

- Switching Time
- Switching Theorem
- Method for Initial Values

Switching Time



- Circuit is switched at the time of t_0;
- t_0^- denote the time just prior to switching;
- t_0^+ denote the time immediately following switching.

Switching Time

$$u_C(t_0^-),\quad i_C(t_0^-),\quad u_L(t_0^-),\quad i_L(t_0^-)$$

- **denote the voltages and currents at the time just prior to switching (at the time t_0^-);**

Switching Time

$$u_C(t_0^+),\quad i_C(t_0^+),\quad u_L(t_0^+),\quad i_L(t_0^+)$$

- **denote the voltages and currents at the time immediately following switching (at the time t_0^+);**
- **are referred to as the initial values (initial voltage or initial current) of the circuits.**

Switching Theorem

For a capacitor, $i(t) = C\dfrac{du(t)}{dt}$

If $i(t)$ is limited, then $u(t)$ is **continuous**.

$$\boxed{u_C(t_0^-) = u_C(t_0^+)}$$

- **Specially, if t_0=0,** $u_C(0^+) = u_C(0^-)$

Switching Theorem

For a inductor, $u(t) = L\frac{di(t)}{dt}$

If $u(t)$ is limited, then $i(t)$ is **continuous**.

$$\boxed{i_L(t_0^-) = i_L(t_0^+)}$$

- **Specially, if t_0=0,** $i_L(0^+) = i_L(0^-)$

Method for Initial Values

- **Initial values:**
  - **Independent initial values:**

$$u_C\left(0^+\right),\quad i_L\left(0^+\right)$$

  - **Other dependent initial values:**

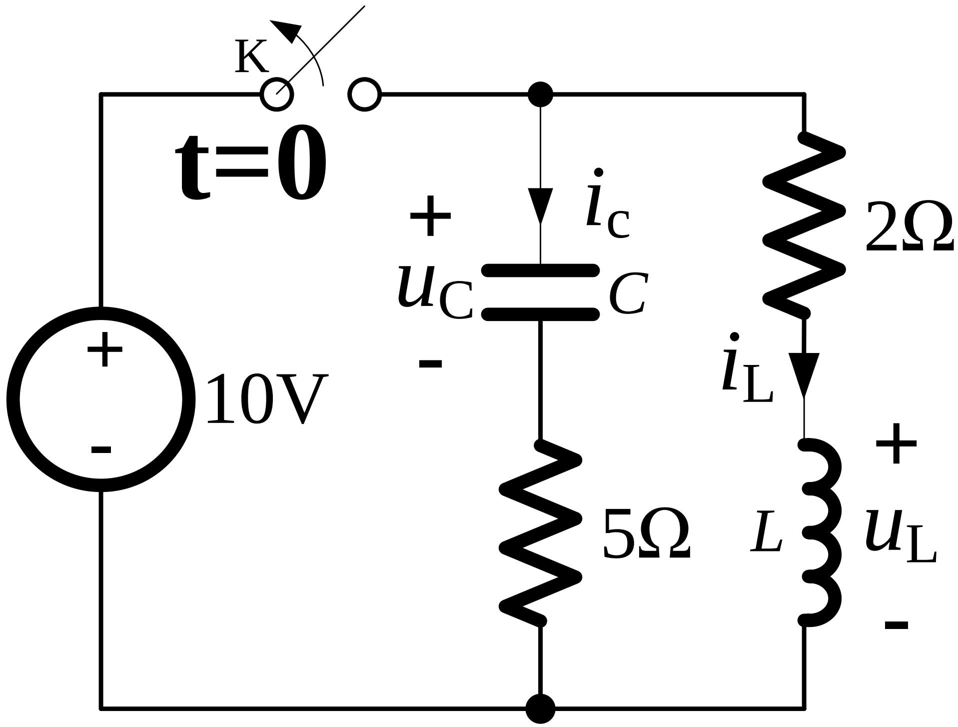
$$i_C\left(0^+\right),\quad u_L\left(0^+\right)\cdots\cdots$$

Method for Initial Values

- **Steps for initial values:**

  1. **Find the values of** $u_C\left(0^-\right),\quad i_L\left(0^-\right)$
  2. **Find independent initial values by switching theorem:** $u_C\left(0^+\right),\quad i_L\left(0^+\right)$
  3. **Redraw the circuit for t=0^+;**
  4. **Find other dependent initial values by KCL and KVL.**

Example



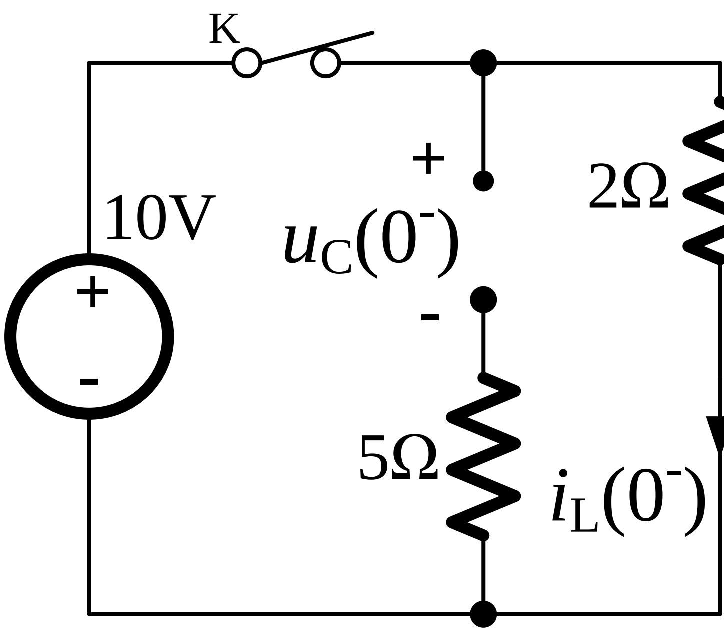
The switch has been closed for a long time, and is opened at t = 0. Find $u_C(0^+)$ and $i_L(0^+)$.

Analysis:

- $u_C(t)$ and $i_L(t)$ are continuous;
- The circuit is switched at t=0;
- $u_C(0^+) = u_C(0^-)$ and $i_L(0^+) = i_L(0^-)$;
- At the time of $t=0^-$, the switch has been closed for a long time;
- For DC source, capacitor is **OPEN**, and inductor is **SHORT**.

Solution:

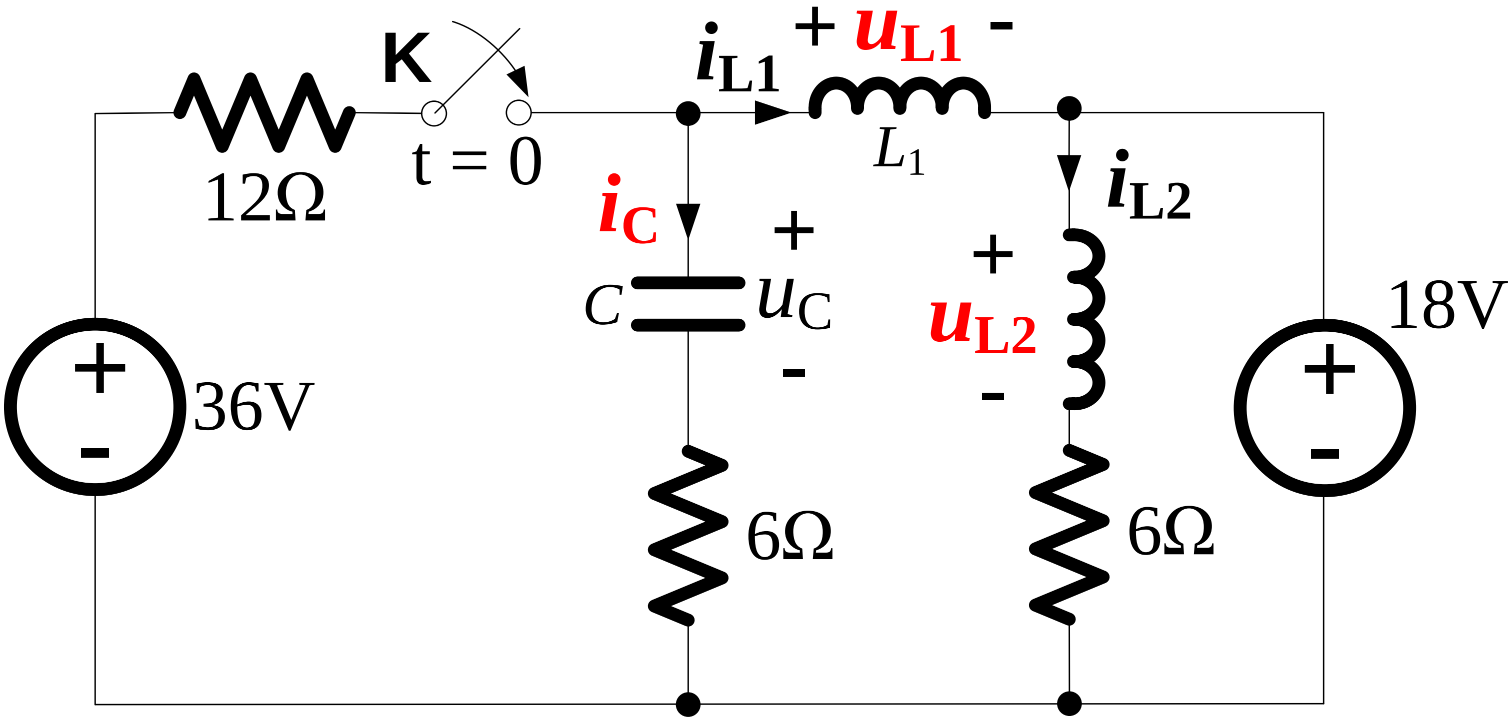
At the time of t=0$^-$, the equivalent circuit is:



$$\begin{cases} u_C(0^-) = 10\text{V} \\ i_L(0^-) = 5\text{A} \end{cases}$$

$$\begin{cases} u_C(0^+) = u_C(0^-) = 10\text{V} \\ i_L(0^+) = i_L(0^-) = 5\text{A} \end{cases}$$

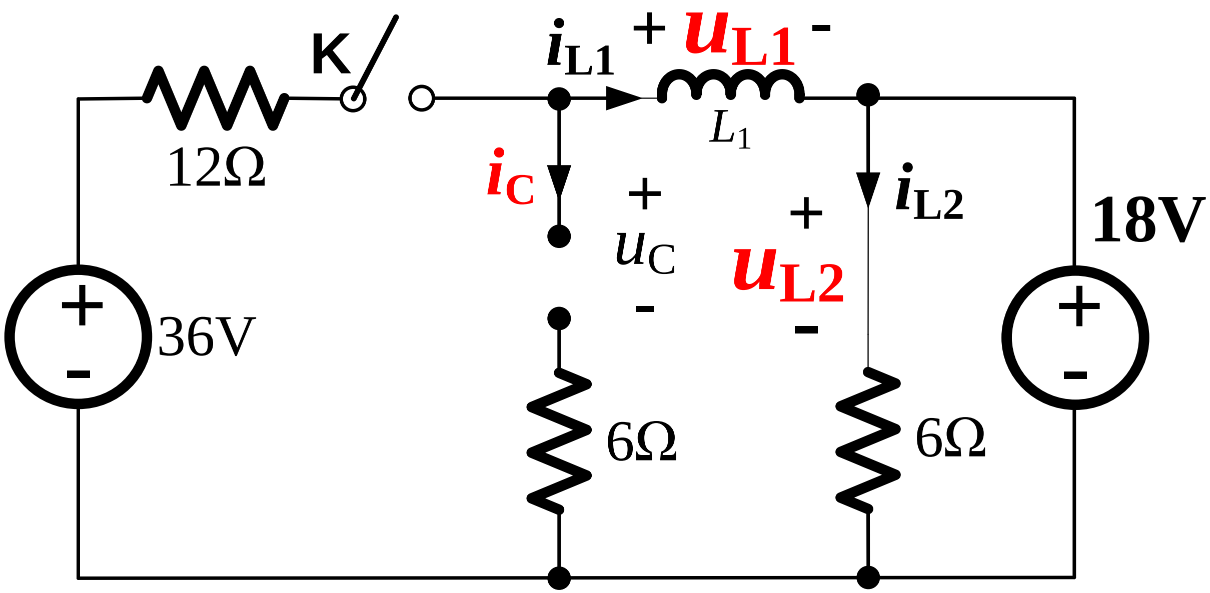
Example



The switch K has been opened for a long time, and is closed at t=0. Find $i_C(0^+)$, $u_{L1}(0^+)$, and $u_{L2}(0^+)$.

Solution:

At the time of $t=0^-$, the equivalent circuit is:

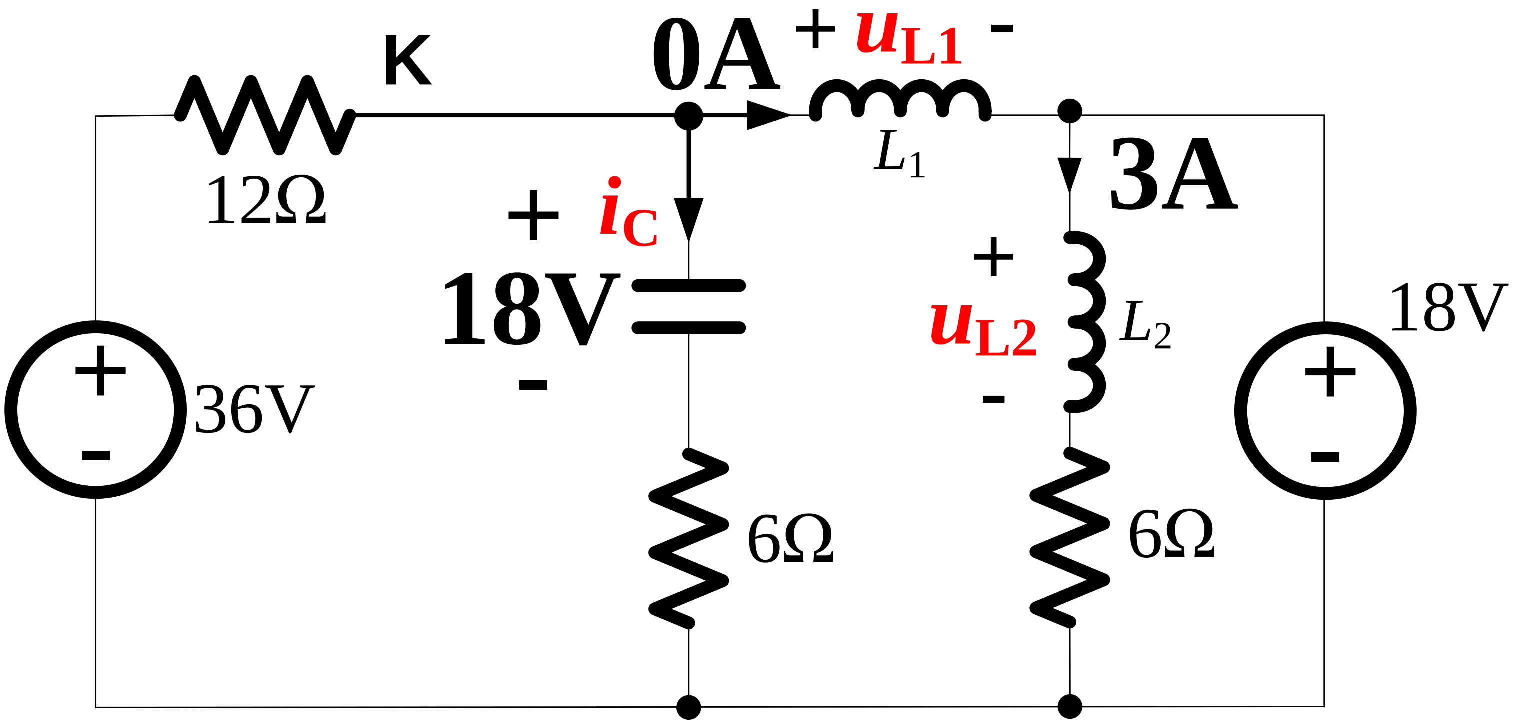


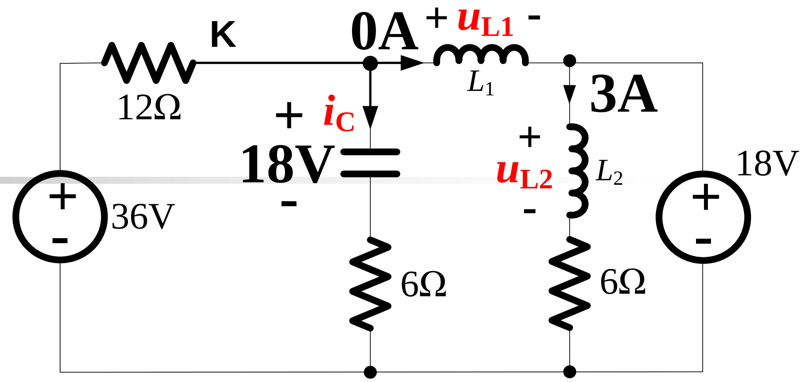
$$
\begin{cases}
i_{L1}\left(0^-\right) = 0\text{A} \\
i_{L2}\left(0^-\right) = 3\text{A} \\
u_C\left(0^-\right) = 18\text{V}
\end{cases}
$$

At t=0$^+$, by switching theorem, we have:

$$\begin{cases} i_{L1}\left(0^+\right) = i_{L1}\left(0^-\right) = 0\text{A} \\ i_{L2}\left(0^+\right) = i_{L2}\left(0^-\right) = 3\text{A} \\ u_C\left(0^+\right) = u_C\left(0^-\right) = 18\text{V} \end{cases}$$

Then, the circuit can be redrawn for t=0$^+$:



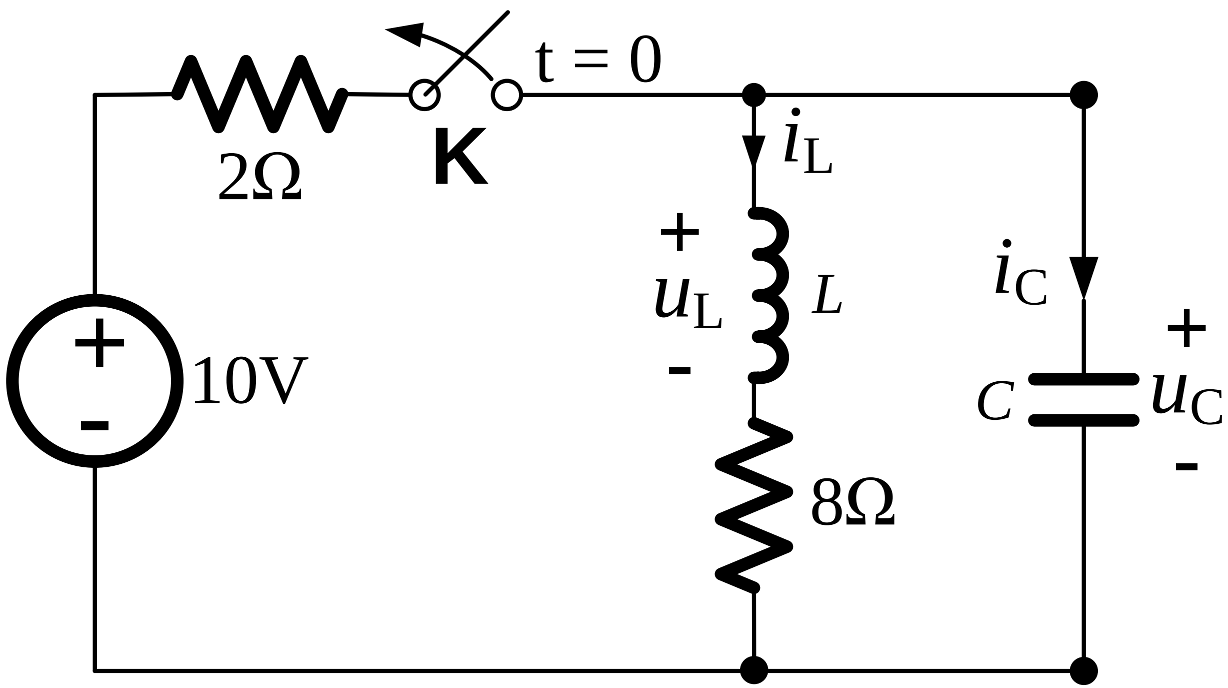


At t=0$^+$, by KVL we have:

$$\begin{cases} (12+6)\,i_C\left(0^+\right)+18-36=0 \\ 12i_C\left(0^+\right)+u_{L1}\left(0^+\right)+18-36=0 \\ u_{L2}\left(0^+\right)+18=18 \end{cases}$$

$$\Rightarrow \begin{cases} i_C\left(0^+\right)=1\text{A} \\ u_{L1}\left(0^+\right)=6\text{V} \\ u_{L2}\left(0^+\right)=0\text{V} \end{cases}$$

Exercise



ANS:

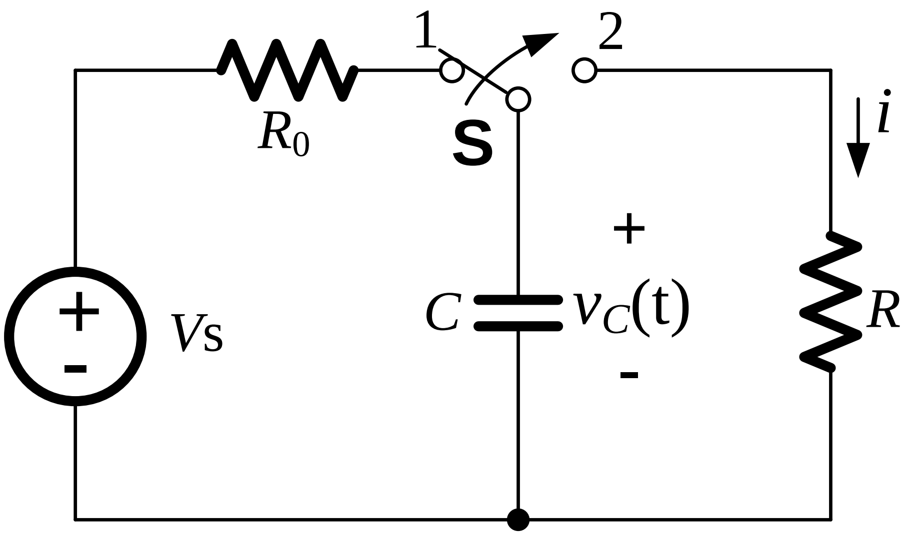
$$\begin{cases} i_L(0^+) = 1\text{A} \\ u_C(0^+) = 8\text{V} \\ i_C(0^+) = -1\text{A} \\ u_L(0^+) = 0\text{V} \\ i_R(0^+) = 1\text{A} \\ u_R(0^+) = 8\text{V} \end{cases}$$

The switch K has been closed for a long time, and is opened at t=0. Find initial value for all elements.

6-3 Natural Response

- Natural Response of RC Circuits
- Natural Response of RL Circuits

Natural Response of RC Circuits



$$\begin{cases} v_C\left(0^-\right) = V_s \\ i_C\left(0^-\right) = 0 \end{cases}$$

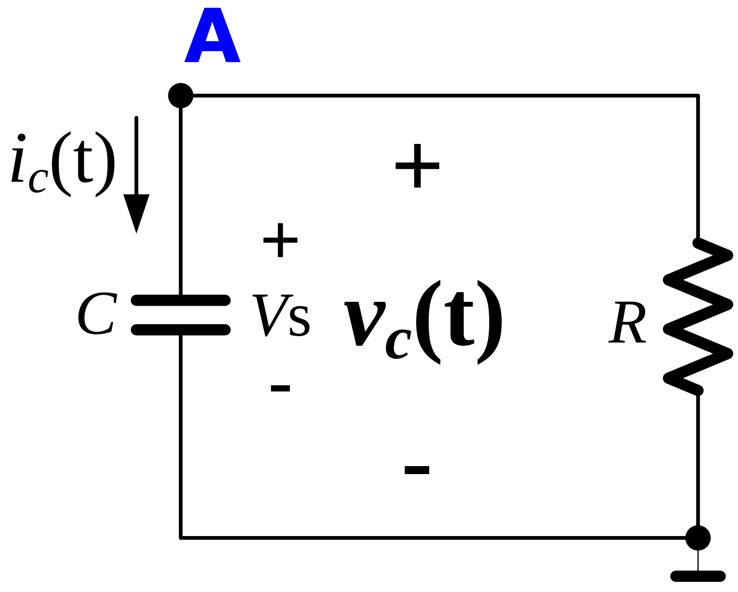
- **The switch has been in the position 1 for a long time;**
- **The switch is moved to 2 at t=0;**
- **Energy stored in *capacitor* is suddenly released to *R*.**

Natural Response of RC Circuits

- By Node-Voltage method:

$$C\frac{dv_C}{dt}+\frac{v_C}{R}=0$$

$$\frac{dv_C}{v_C}=-\frac{1}{RC}dt$$



$$\Rightarrow\quad v_C\left(t\right)=Ke^{-t/RC},\quad t\geq 0$$

Natural Response of RC Circuits

$$v_C(t) = Ke^{-t/RC}, \quad t \geq 0$$

$$v_C(0^+) = v_C(0^-) = V_s$$

$$v_C(t) = V_s e^{-t/RC}, \quad t \geq 0$$

Natural Response of RC Circuits

- The current through the capacitor:

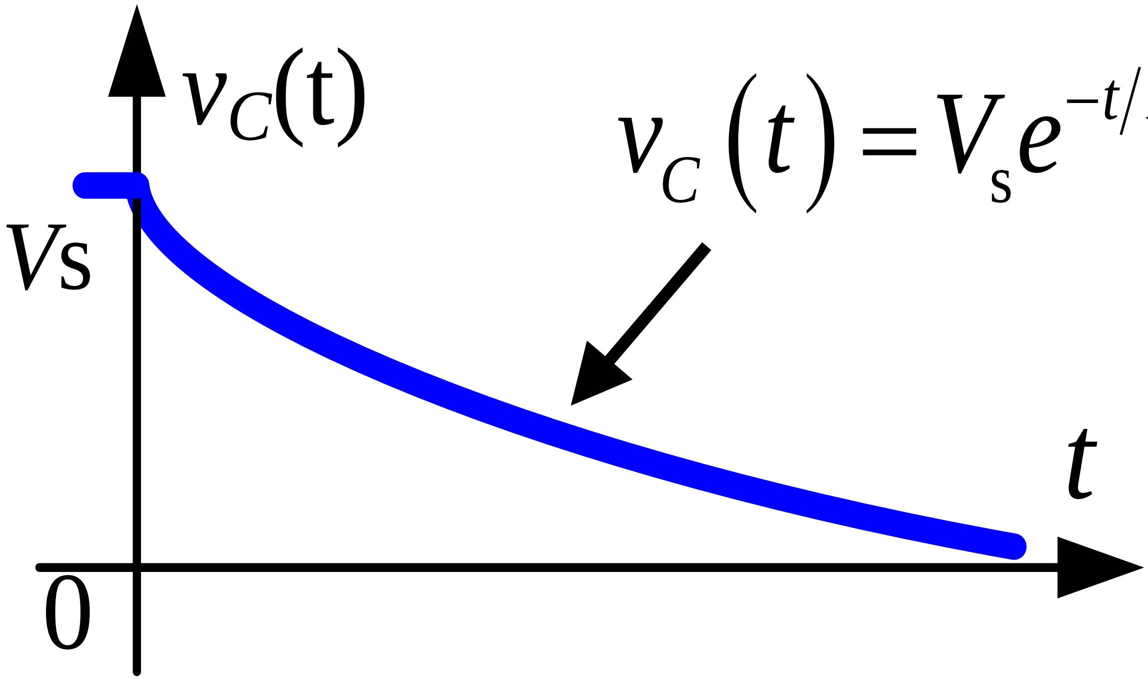
$$i_C(t) = -\frac{v_C(t)}{R} = -\frac{V_s}{R} e^{-t/RC}, \quad t \geq 0^+$$

$$i_C(0^+) = -V_s / R$$

$$i_C(0^-) = 0 \quad \Rightarrow \quad i_C(0^+) \neq i_C(0^-)$$

北京郵電大學
BEIJING UNIVERSITY OF POSTS AND TELECOMMUNICATIONS

Natural Response of RC Circuits

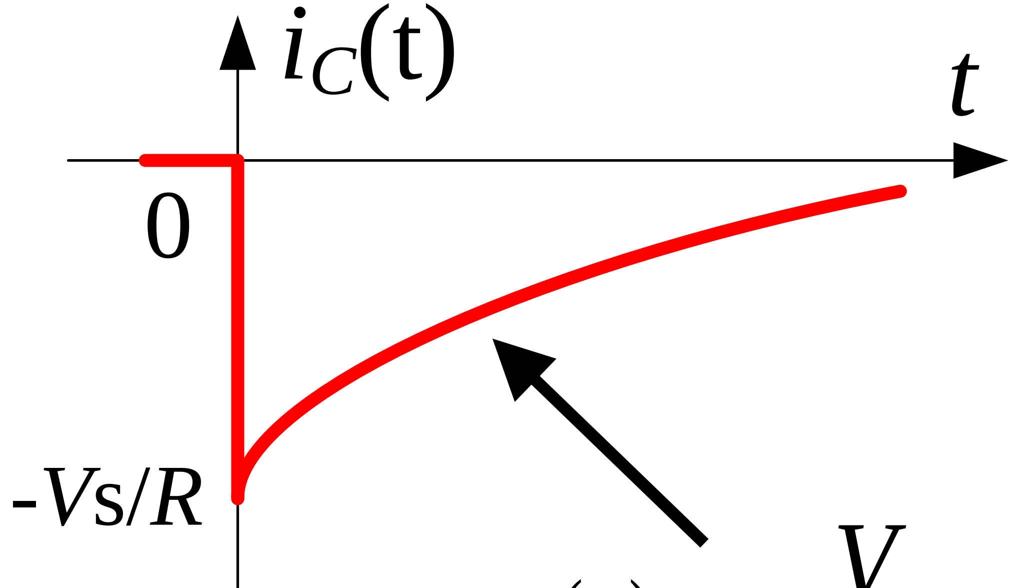


$$v_C(t) = V_s e^{-t/RC}, \quad t \geq 0$$

The voltage across the capacitor

Natural Response of RC Circuits

The current through the capacitor:



$$i_C(t) = -\frac{V_s}{R} e^{-t/RC}, \quad t \geq 0^+$$

Voltage across the capacitor:

$$v_C(t) = V_s e^{-t/RC}, \quad t \geq 0$$

Current through the capacitor:

$$i_C(t) = -(V_s/R)e^{-t/RC}, \quad t \geq 0^+$$

Power delivered to the capacitor:

$$p_C(t) = v_C i_C = -\left(V_s^2 / R\right) e^{-2t/RC}, \quad t \geq 0^+$$

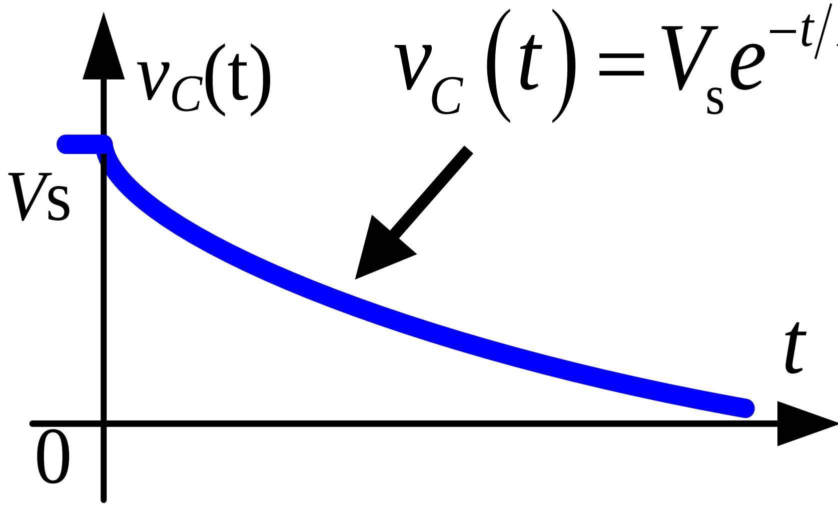
Energy stored in the capacitor:

$$w_C(t) = \int_0^t p_C(t) dt = \frac{1}{2} C V_s^2 \left(e^{-2t/RC} - 1\right), \quad t \geq 0$$

$$w_C(t) = -\frac{1}{2} C V_s^2, \quad t \to \infty$$

北京邮电大学 BEIJING UNIVERSITY OF POSTS AND TELECOMMUNICATIONS

Time Constant



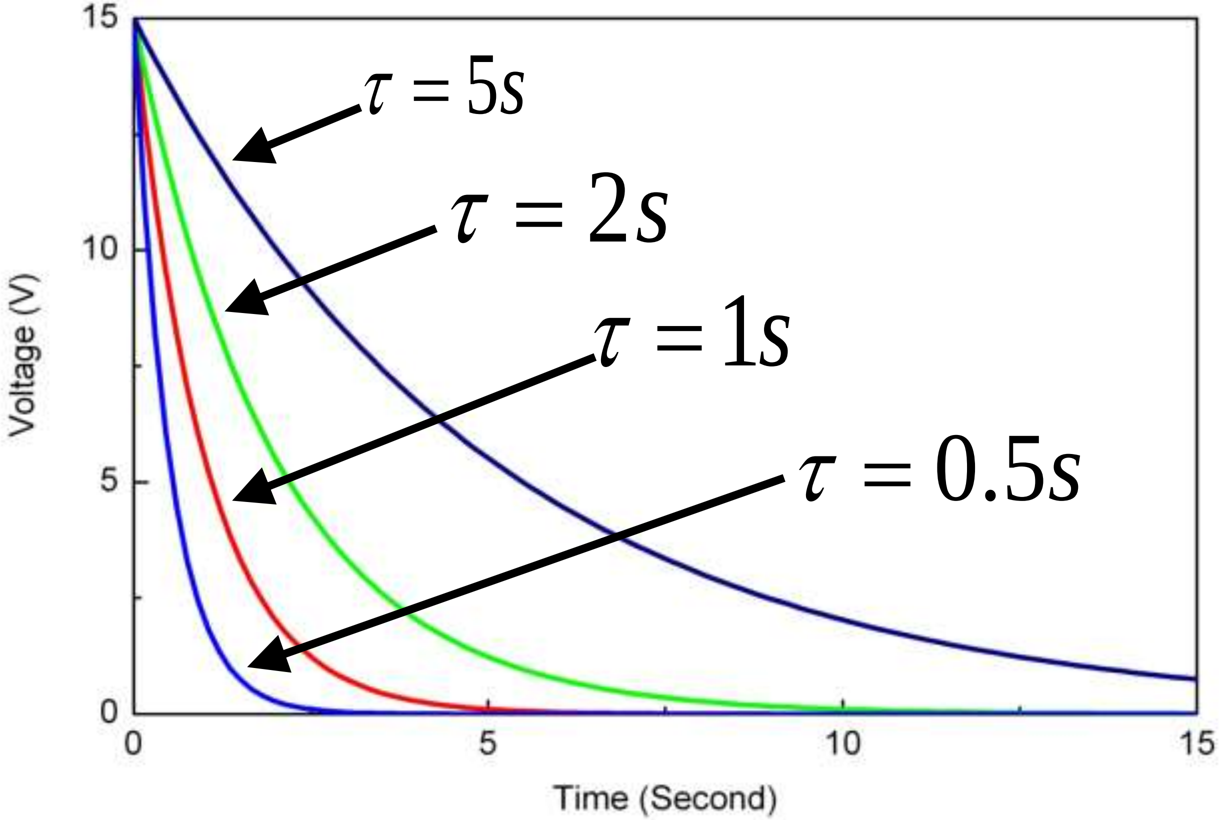
$$v_C(t) = V_s e^{-t/RC}, \quad t \geq 0$$

Time Constant: $\tau = RC$

- **Time Constant: Rate at which the capacitive voltage approaches zero.**

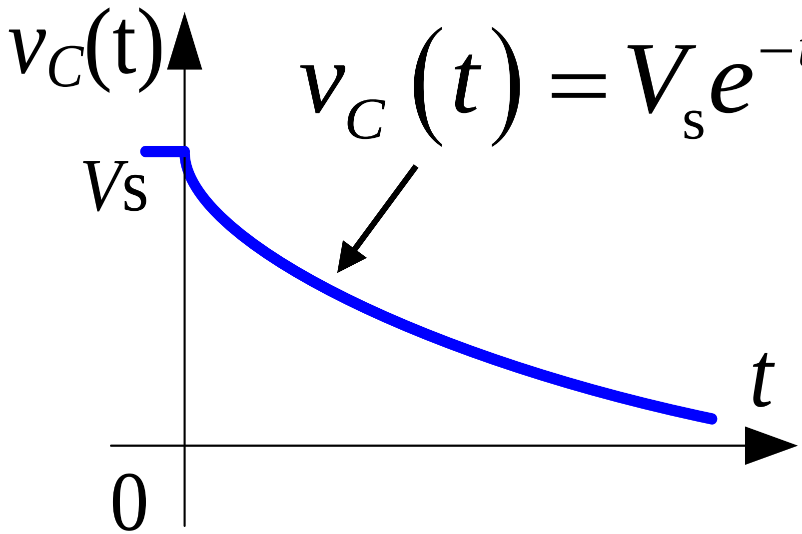
Time Constant

$$v_C(t) = V_s e^{-t/\tau}, \quad t \geq 0$$



Time Constant

$$\tau = RC$$



$$v_C(t) = V_s e^{-t/\tau}$$

- Transient response
- Steady-state response

$t=\tau$: $v = 0.368V_s$

$t=2\tau$: $v = 0.135V_s$

$t=3\tau$: $v = 0.050V_s$

$t=4\tau$: $v = 0.018V_s$

$t=5\tau$: $v = 0.007V_s$

……

$t=\infty$: $v = 0$

Time Constant

Voltage: $$v_C(t) = V_s e^{-t/\tau}, \quad t \geq 0$$

Current: $$i_C(t) = -(V_s / R) e^{-t/\tau}, \quad t \geq 0^+$$

Power: $$p_C(t) = -(V_s^2 / R) e^{-2t/\tau}, \quad t \geq 0^+$$

Energy: $$w_C(t) = \frac{1}{2} C V_s^2 \left(e^{-2t/\tau} - 1\right), \quad t \geq 0$$

北京邮电大学 BEIJING UNIVERSITY OF POSTS AND TELECOMMUNICATIONS

Steps for find natural response of RC circuit

1. Find the initial voltage across the capacitor;

2. Find the time constant of the RC circuit:

$$\tau = R_{TH} C_{eq}$$

3. Use the following equation to get the natural response of RC circuit:

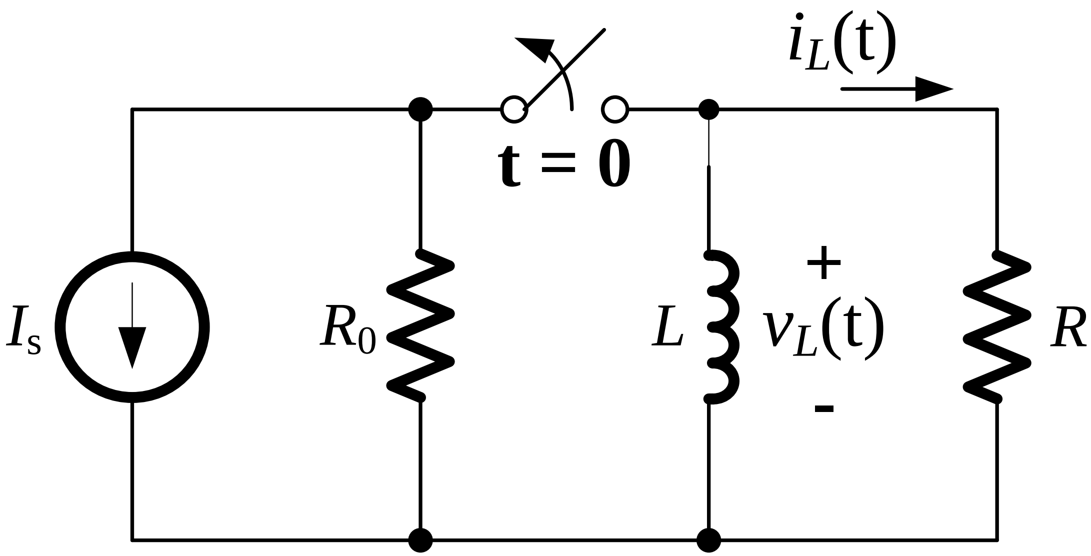
$$v_C(t) = v\left(0^+\right)e^{-t/\tau}, \quad t \geq 0$$

First-Order RC Circuit

$$\tau = R_{TH} C_{eq}$$

- R_{TH} is the Thévenin Equivalent resistance between the two terminals of the capacitor;
- C_{eq} is the equivalent capacitance (If multiple capacitors exist, they must be interconnected in such a way that they can be replaced by a single equivalent capacitor).

Natural Response of RL Circuits



$$\begin{cases} i_L(0^-) = I_s \\ v_L(0^-) = 0 \end{cases}$$

- The switch has been closed for a long time;
- The switch is opened at the instant of t = 0;
- Energy stored in *L* is suddenly released to *R*.

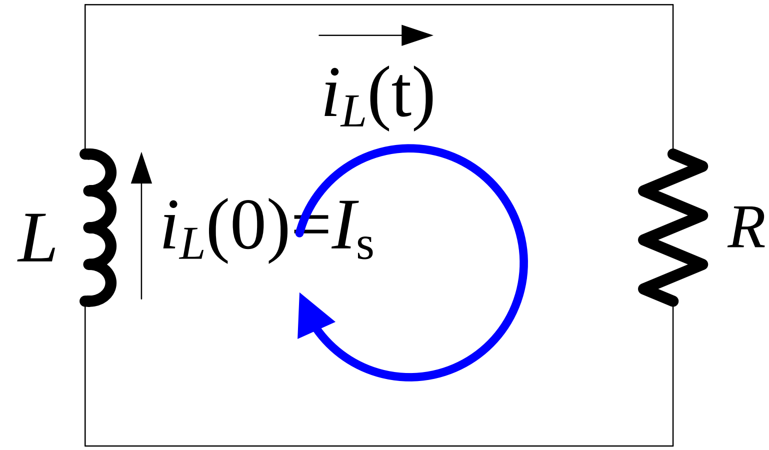
Natural Response of RL Circuits

- **By KVL for the loop:**

$$L\frac{di_L}{dt}+Ri_L=0$$

$$\frac{di_L}{i_L}=-\frac{R}{L}dt$$

$$i_L(t)=Ke^{-(R/L)t},\quad t\geq 0$$



Natural Response of RL Circuits

$$i_L(t) = Ke^{-(R/L)t}, \quad t \geq 0$$

$$i_L(0^+) = i_L(0^-) = I_s$$

$$i_L(t) = I_s e^{-(R/L)t}, \quad t \geq 0$$

Natural Response of RL Circuits

- **The voltage across the inductor:**

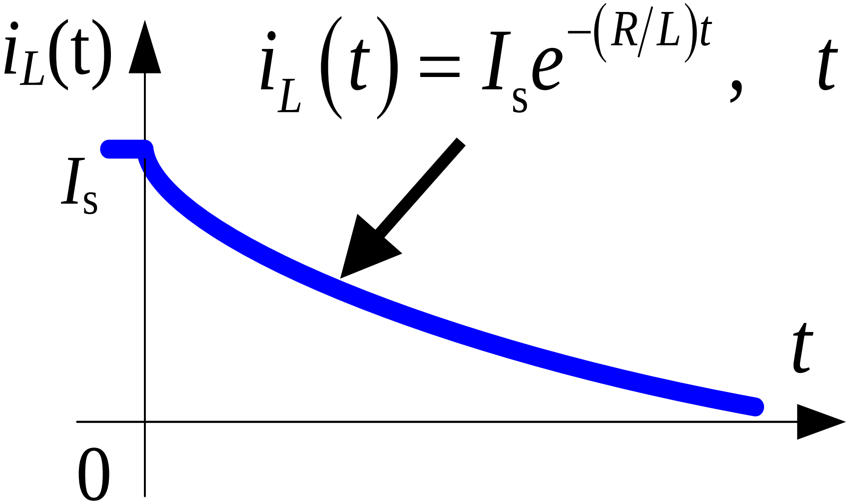
$$v_L(t) = -Ri_L(t) = -RI_s e^{-(R/L)t}, \quad t \geq 0^+$$

$$v_L(0^+) = -RI_s$$

$$v_L(0^-) = 0 \quad \Rightarrow \quad v_L(0^+) \neq v_L(0^-)$$

Natural Response of RL Circuits

$$i_L(t) = I_s e^{-(R/L)t}, \quad t \geq 0$$



The current through the inductor

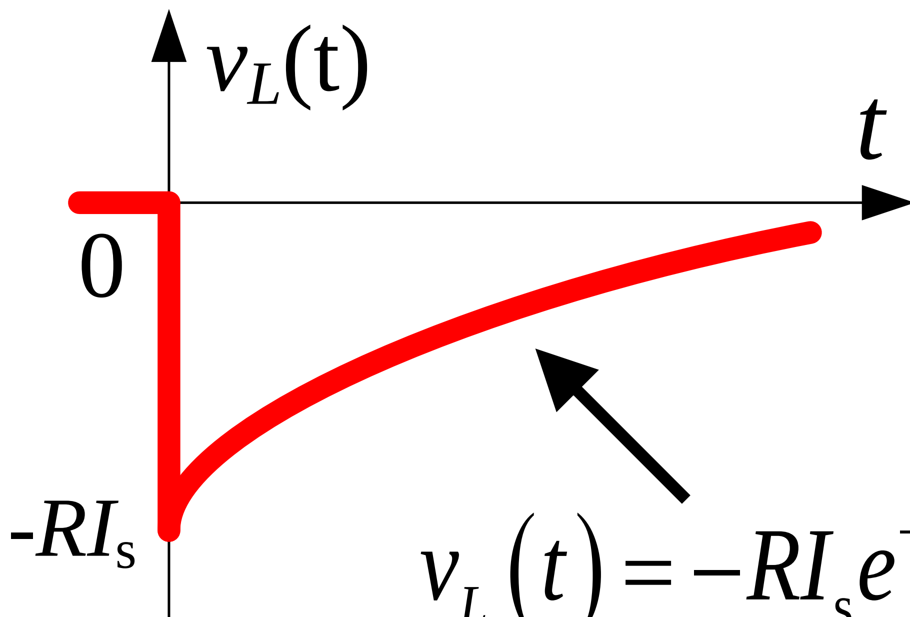
Time Constant :

$$\tau = \frac{L}{R}$$

- **Time Constant: Rate at which the inducitive current approaches zero.**

Natural Response of RL Circuits

The voltage across the inductor:



$$v_L(t) = -RI_s e^{-(R/L)t}, \quad t \geq 0^{+}$$

Time Constant

Current: $$i_L(t) = I_s e^{-t/\tau}, \quad t \geq 0$$

Voltage: $$v_L(t) = -RI_s e^{-t/\tau}, \quad t \geq 0^+$$

Power: $$p_L(t) = RI_s^{\,2} e^{-2t/\tau}, \quad t \geq 0^+$$

Energy: $$w_L(t) = \frac{1}{2} LI_s^2 \left(e^{-2t/\tau} - 1\right), \quad t \geq 0$$

Steps for find natural response of RL circuit

1. Find the initial current through the inductor;

2. Find the time constant of the RL circuit:

$$\tau = L_{eq} / R_{TH}$$

3. Use the following equation to get the natural response of RL circuit:

$$i_L(t) = i(0^+)e^{-t/\tau}, \quad t \geq 0$$

First-Order RL Circuit

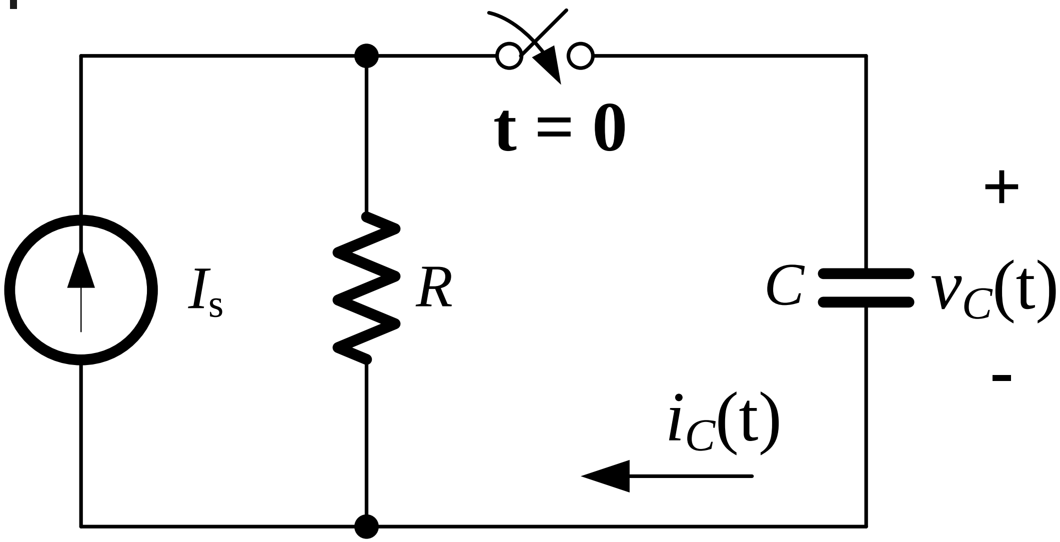
$$\tau = L_{eq} / R_{TH}$$

- R_{TH} is the Thévenin Equivalent resistance between the two terminals of the inductor;
- L_{eq} is the equivalent inductance (If multiple inductors exist, they must be interconnected so that they must be replaced by a single equivalent inductor).

6-4 Step Response

- Step Response of RC Circuits
- Step Response of RL Circuits

Step Response of RC Circuits

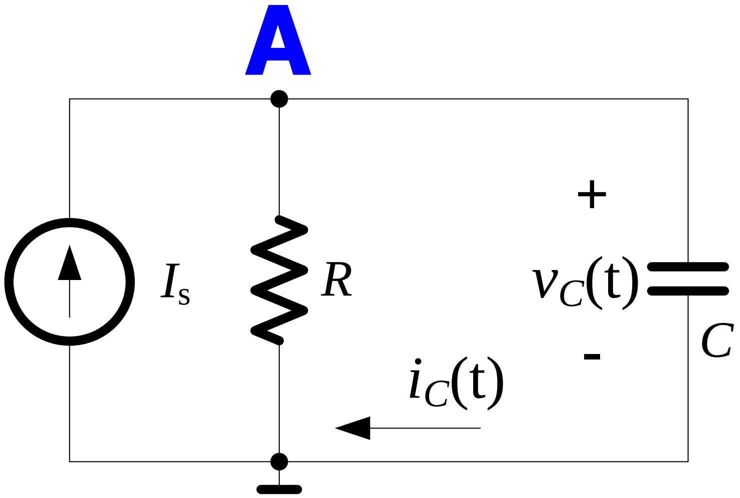


$$v_C\left(0^-\right)=V_0$$

- The capacitor has a initial voltage V_0;
- The switch is closed at t = 0;
- Energy is being stored in the capacitor.

Step Response of RC Circuits

- By KCL at node A:



$$C\frac{dv_C}{dt}+\frac{v_C}{R}=I_s$$

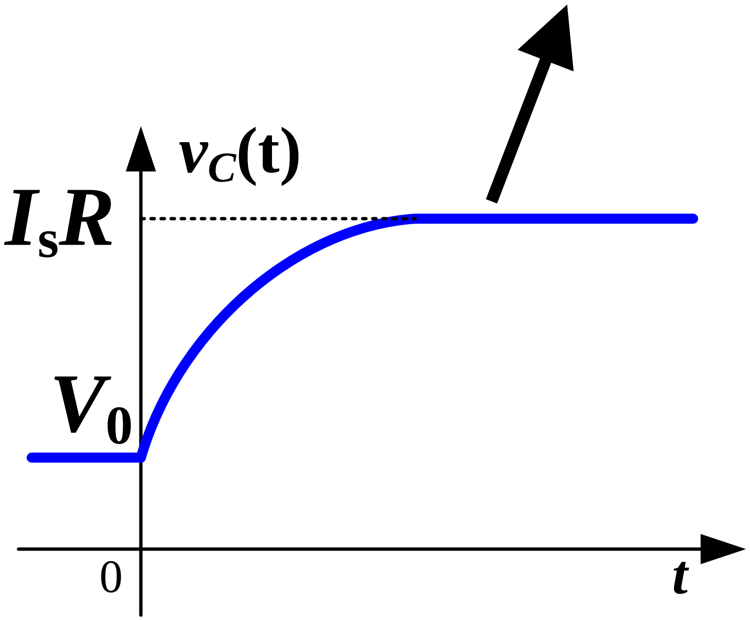
$$\frac{dv_C}{v_C-RI_s}=-\frac{1}{RC}dt$$

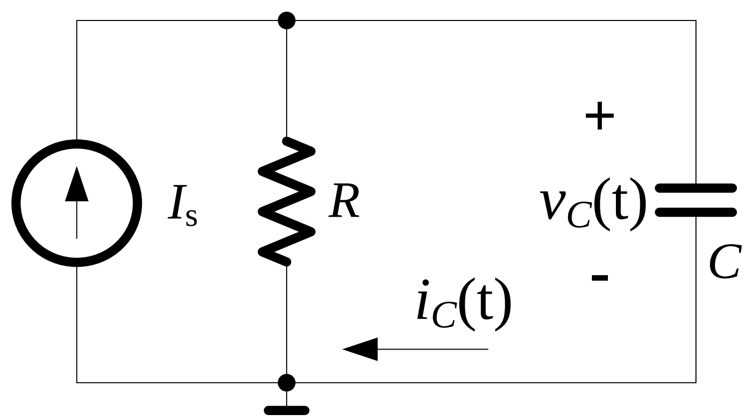
$$v_C\left(0^-\right)=V_0$$

$$v_C\left(t\right)=I_sR+\left(V_0-I_sR\right)e^{-t/RC},\quad t\geq 0$$

Voltage across the capacitor:

$$v_C(t) = I_s R + (V_0 - I_s R)e^{-t/RC}, \quad t \geq 0$$



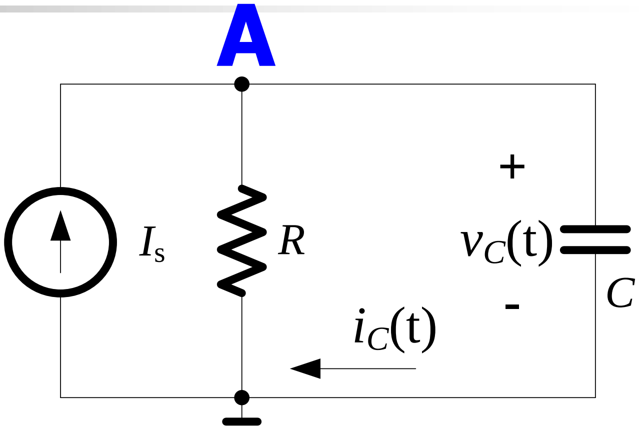


$$\tau = RC$$

$$v_C(t) = I_s R + (V_0 - I_s R)e^{-t/\tau}, \quad t \geq 0$$

Another Method to Find $i_c(t)$

- By KCL at node A:



$$i_C(t)+\frac{v_C(t)}{R}=I_s$$

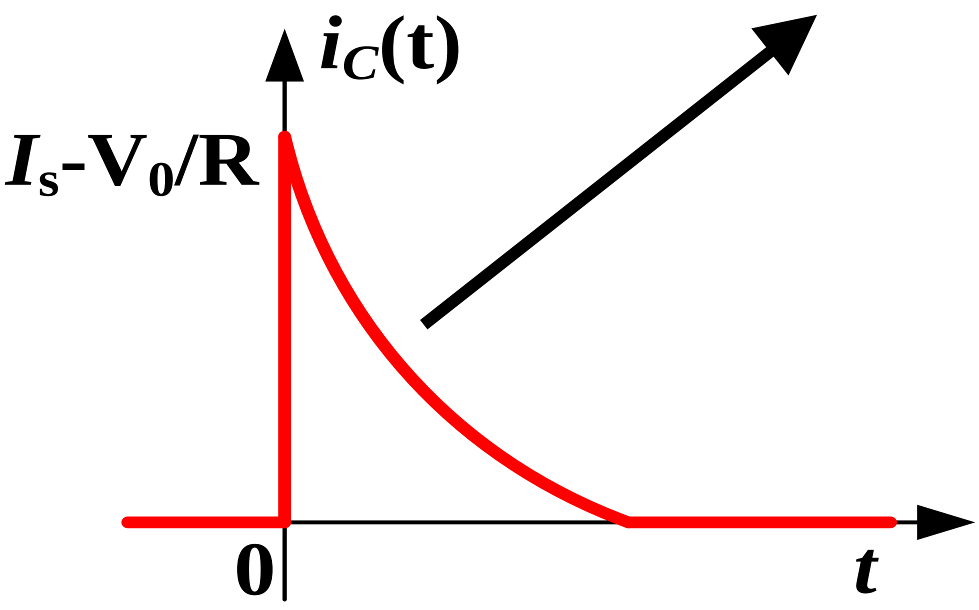
$$\frac{di_C(t)}{dt}=-\frac{1}{RC}i_C(t)$$

$$\begin{cases} v_C(0^+)=v_C(0^-)=V_0 \\ i_C(0^+)=I_s-V_0/R \end{cases}$$

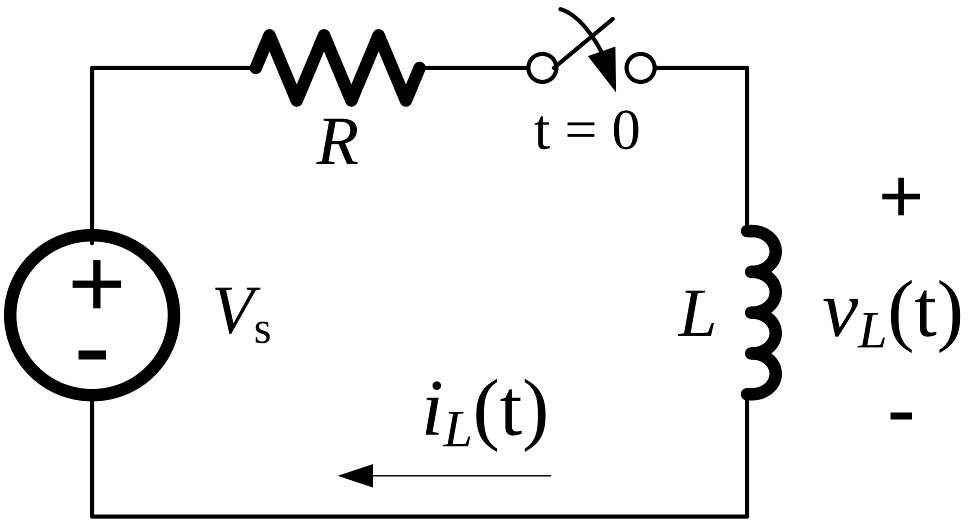
$$i_C(t)=(I_s-V_0/R)e^{-t/RC},\quad t\geq 0^+$$

Current through the capacitor:

$$i_C(t) = \left(I_s - \frac{V_0}{R} \right) e^{-t/RC}, \quad t \geq 0^{+}$$



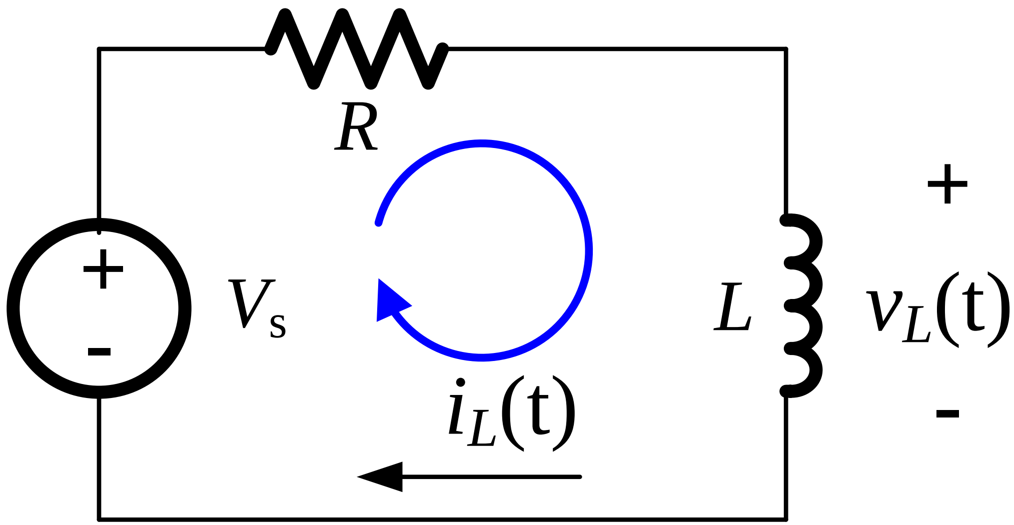
Step Response of RL Circuits



$$i_L\left(0^-\right) = I_0$$

- The switch is closed at t = 0;
- The inductor has a initial current I_0;
- Energy is being stored in the inductor.

Step Response of RL Circuits



- By KVL for the loop: $L\dfrac{di_L}{dt} + Ri_L = V_s$

Step Response of RL Circuits

$$i_L(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R} \right) e^{-(R/L)t}, \quad t \geq 0$$

$$v_L(t) = L\frac{di_L(t)}{dt} = (V_s - RI_0)e^{-(R/L)t}, \quad t \geq 0^+$$

Time Constant: $\tau = \dfrac{L}{R}$

6-5 General Solution Method

- General Solution Method for Natural and Step Response
- Sequential Switching

General Solution Method

$$C\frac{dv_C}{dt}+\frac{v_C}{R}=0 \quad \Rightarrow \quad \frac{dv_C}{dt}+\frac{v_C}{\tau}=0$$

$$\tau = RC$$

$$L\frac{di_L}{dt}+Ri_L=0 \quad \Rightarrow \quad \frac{di_L}{dt}+\frac{i_L}{\tau}=0$$

$$\tau = L/R$$

General Solution Method

$$C\frac{dv_C}{dt}+\frac{v_C}{R}=I_s \quad \Longrightarrow \quad \frac{dv_C}{dt}+\frac{v_C}{\tau}=\frac{I_s}{C}$$

$$\tau = RC$$

$$L\frac{di_L}{dt}+Ri_L=V_s \quad \Longrightarrow \quad \frac{di_L}{dt}+\frac{i_L}{\tau}=\frac{V_s}{L}$$

$$\tau = L/R$$

General Solution Method

Natural Response:

$$v_C(t) = v(0)e^{-t/\tau}, \quad t \ge 0$$

$$i_L(t) = i(0)e^{-t/\tau}, \quad t \ge 0$$

Step Response:

$$v_C(t) = I_s R + (V_0 - I_s R)e^{-t/\tau}, \quad t \ge 0$$

$$i_L(t) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right)e^{-t/\tau}, \quad t \ge 0$$

General Solution Method

- Generally, response of first-order RC and RL circuit can be written as:

$$y(t) = y(+\infty) + \left[y(t_0^{+}) - y(+\infty) \right] e^{-(t-t_0)/\tau}$$

t_0: Switching time

General Solution

$$y(t) = y(+\infty) + \left[y(t_0^{+}) - y(+\infty) \right] e^{-(t-t_0)/\tau}$$

$y(t)$ **:Response of *voltage or current***

$y(t_0^{+})$ **: Initial value of *voltage or current***

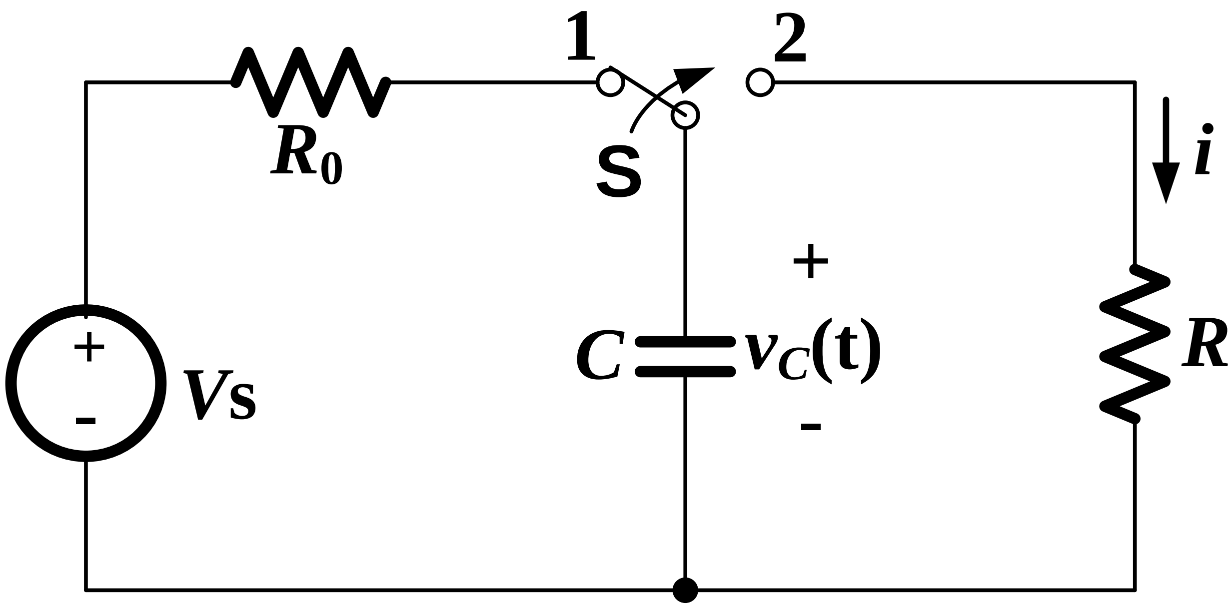
$y(+\infty)$ **: Final value of *voltage or current***

General Solution Method

$y(+\infty)$: **Final value of *voltage or current*, which is the value as t $\rightarrow$ +∞.**

$$\tau = \begin{cases} R_{TH} C_{eq} & \text{: Time constant for RC circuits} \\ L_{eq} / R_{TH} & \text{: Time constant for RL circuits} \end{cases}$$

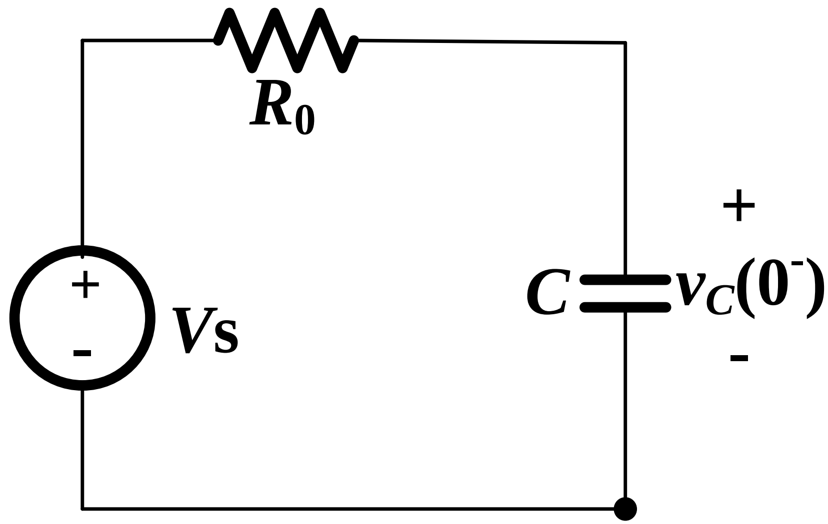
Example



The switch has been in the position 1 for a long time, and is thrown to position 2 at t = 0. Find the voltage across the capacitor.

Solution:

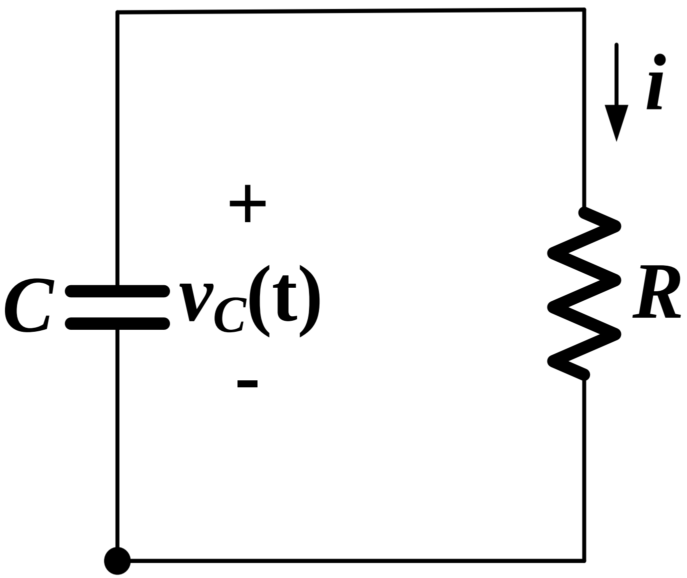
At t=0-, the equivalent circuit is redrawn as:



$$v_C\left(0^-\right) = V_s$$

$$v_C\left(0^+\right) = v_C\left(0^-\right) = V_s$$

At t>0$^+$, the equivalent circuit is redrawn as:



$$v_C(+\infty) = 0$$

$$\tau = R_{TH}C = RC$$

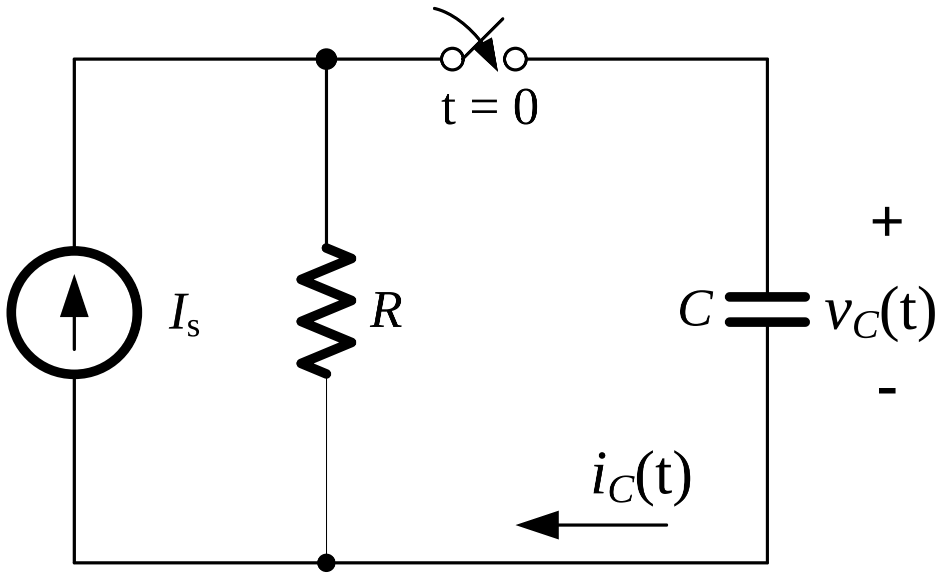
The voltage across the capacitor is:

$$v_C(t) = v_C(+\infty) + \left[v_C\left(0^+\right) - v_C(+\infty)\right] e^{-t/\tau}$$

$$= 0 + [V_s - 0] e^{-t/RC}$$

$$= V_s e^{-t/RC}, \quad t \geq 0$$

Example



The capacitor has a initial voltage V_0, the switch is closed at t = 0. Find the voltage across the capacitor.

Solution:

$$v_C\left(0^{+}\right)=v_C\left(0^{-}\right)=V_0\,,\quad v_C\left(+\infty\right)=RI_s$$

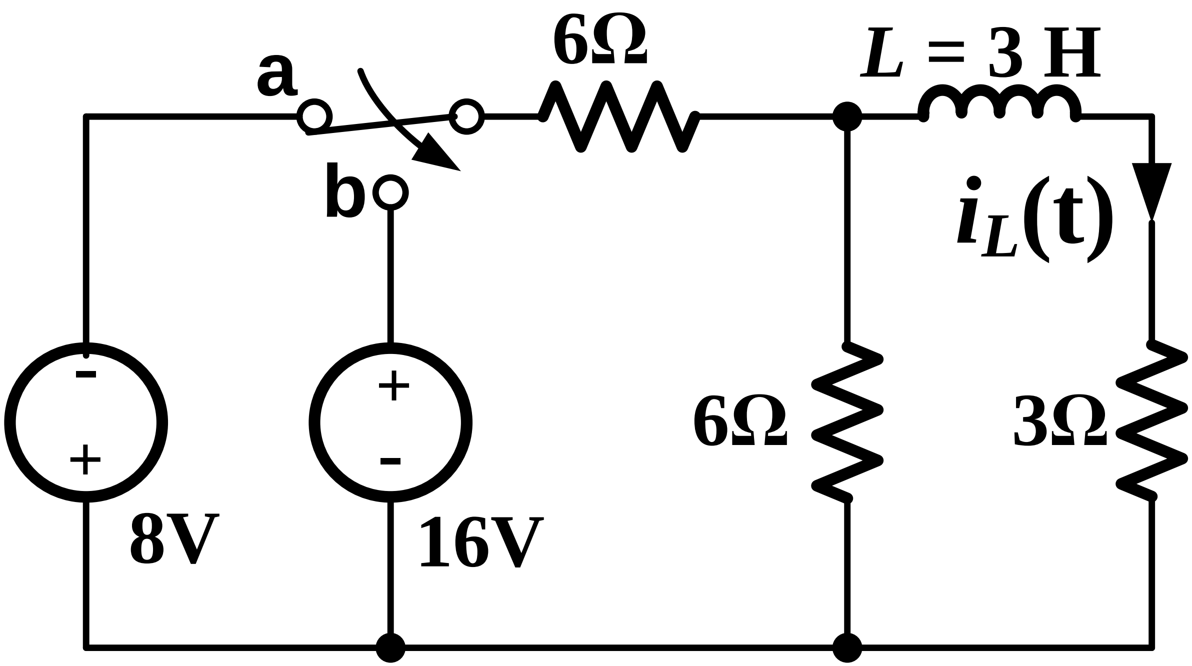
$$\tau=RC\,,\quad t_0=0$$

The voltage across the capacitor is:

$$v_C\left(t\right)=v_C\left(+\infty\right)+\left[v_C\left(0^{+}\right)-v_C\left(+\infty\right)\right]e^{-t/\tau}$$

$$=RI_s+\left(V_0-RI_s\right)e^{-t/RC}\,,\quad t\geq 0$$

Example

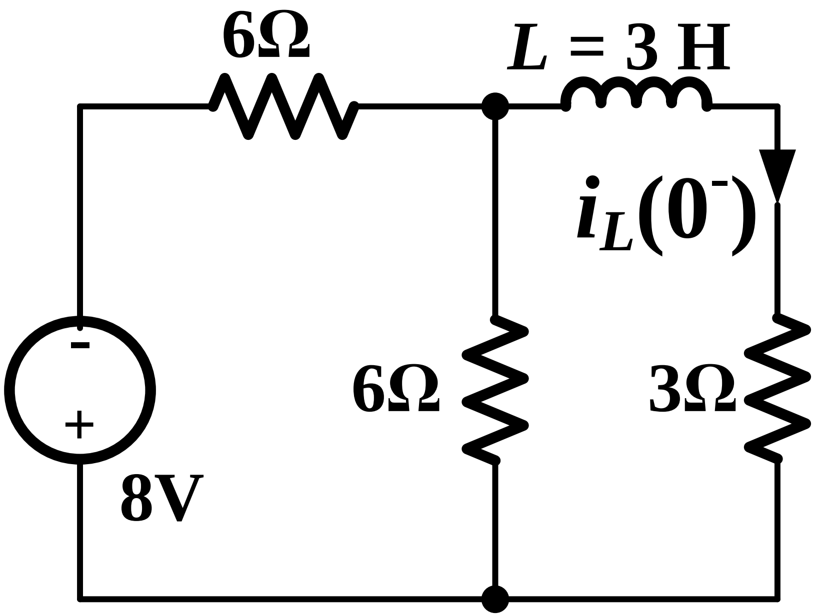


The switch is thrown from a to b at t = 0.

Find i_L(t).

Solution:

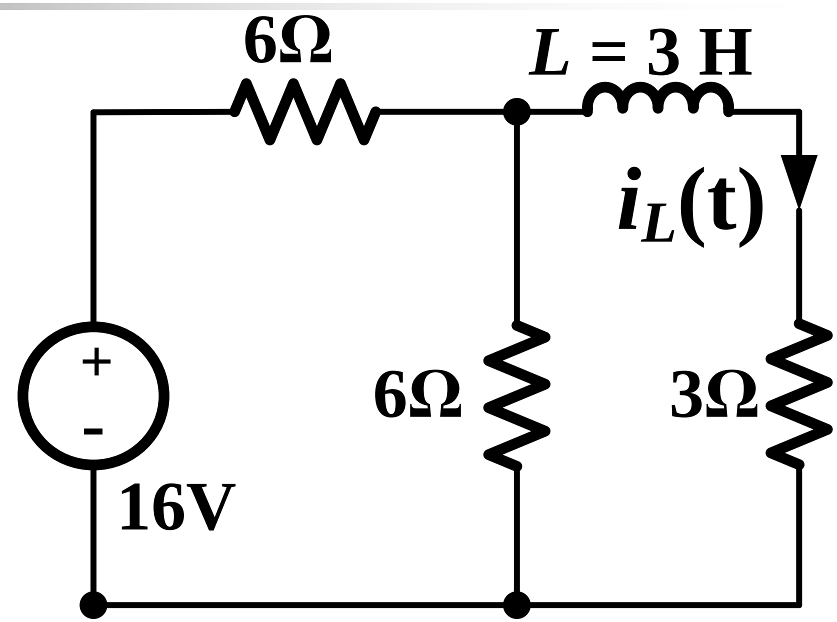
At t=0-, the equivalent circuit is redrawn as:



$$i_L\left(0^-\right) = -\frac{2}{3}\text{A}$$

$$i_L\left(0^+\right) = i_L\left(0^-\right) = -\frac{2}{3}\text{A}$$

At t>0+, the equivalent circuit is redrawn as:



$$i_L(+\infty) = \frac{4}{3}\mathrm{A}$$

$$R_{TH} = (6\Omega \parallel 6\Omega) + 3\Omega = 6\Omega$$

$$\tau = L/R_{TH} = 3/6 = 0.5s$$

The current through the inductor is:

$$i_L(t) = i_L(+\infty) + \left[i_L(0^+) - i_L(+\infty)\right]e^{-t/\tau}$$

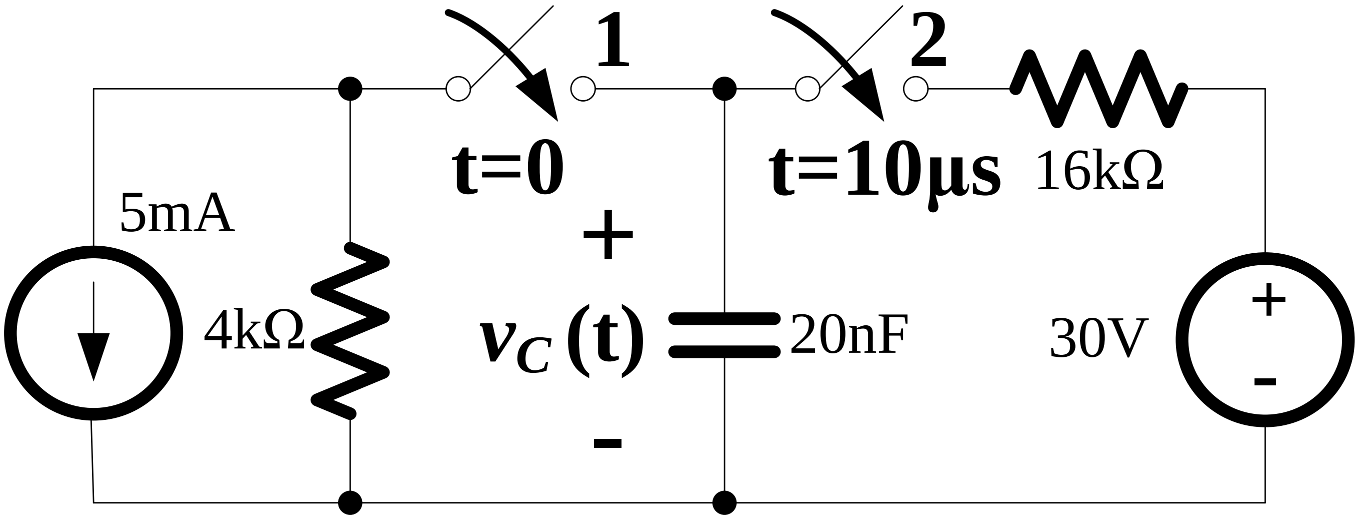
$$= \frac{4}{3} + \left(-\frac{2}{3} - \frac{4}{3}\right)e^{-t/0.5}$$

$$= \frac{4}{3} - 2e^{-2t}\ \mathrm{A}, \quad t \geq 0$$

Sequential Switching

- Switching occurs more than once in a circuit;
- A premium is placed on obtaining the initial values;
- Every time the circuit is switched, initial values should be determined for the switched circuit.

Example



No energy stored in the capacitor when switch 1 is closed at t = 0. 10µs later, switch 2 is closed. Find $v_C(t)$ for $t \geq 0$.

Analysis:

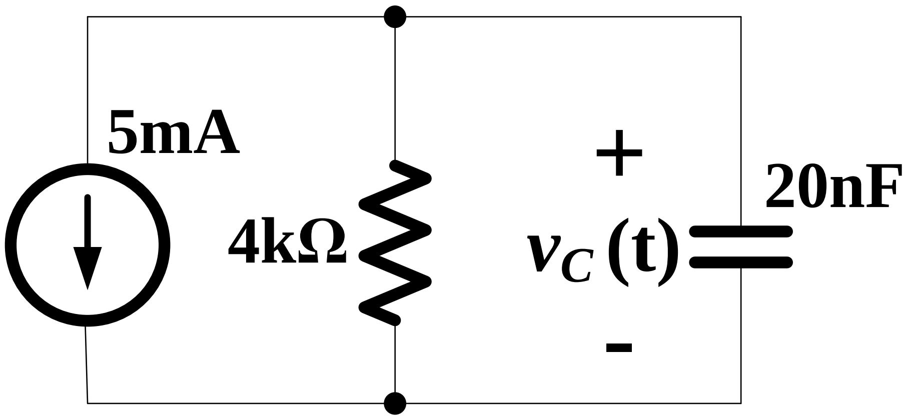
- The circuit is switched two times: one is at the time t=0; the other is at the time t=10μs;
- At the time t=0, switch 1 is closed, and we obtain a circuit containing the capacitor;
- At the time t=10μs, switch 2 is closed, and we obtain a new circuit.

Solution:

At t=0-, no energy is stored in the capacitor:

$$v_C\left(0^-\right)=0 \text{ ,hence: } v_C\left(0^+\right)=v_C\left(0^-\right)=0$$

At t=0, the switch 1 is closed. Then for the time $0^+<t<10\mu s$, the circuit is redrawn as:



For the capacitive voltage in such a circuit:

$$v_C(+\infty) = -20\text{V}, \quad R_{TH1} = 4\text{k}\Omega$$

$$\tau_1 = R_{TH1}C = 4\text{k}\Omega \times 20\text{nF} = 80\mu\text{s}$$

$$\begin{aligned} v_C(t) &= v_C(+\infty) + \left[v_C(0^+) - v_C(+\infty)\right]e^{-t/\tau} \\ &= -20 + \left[0 - (-20)\right]e^{-t/80} \\ &= 20\left(e^{-t/80} - 1\right)\text{V}, \quad (0 \le t \le 10\mu\text{s}) \end{aligned}$$

At the time t=10⁻μs, the capacitive voltage is:

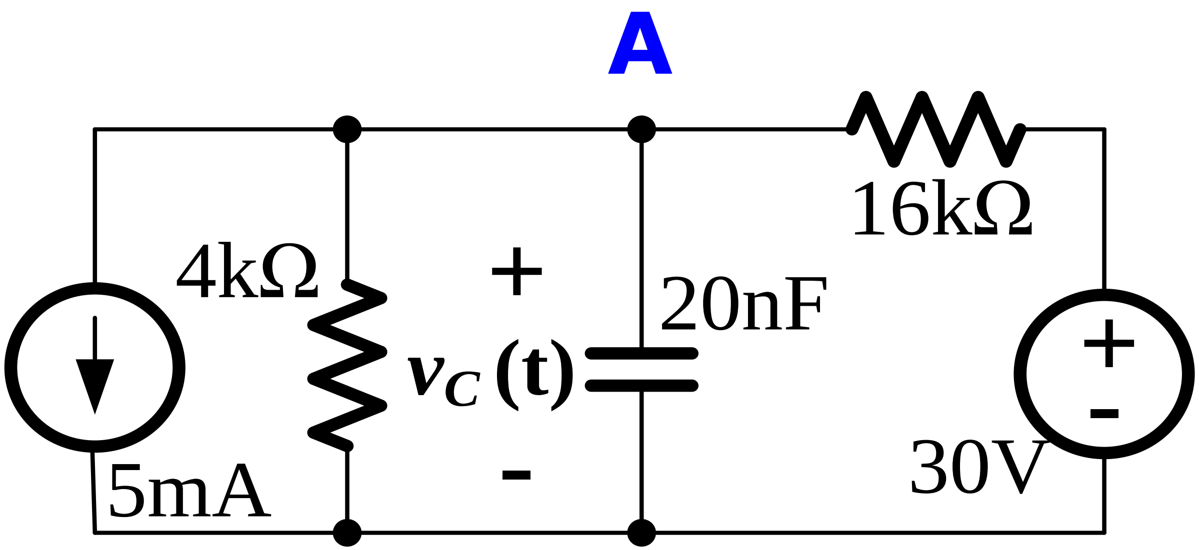
$$v_C\left(t=10^{-}\,\mu\text{s}\right)=20\left(e^{-1/8}-1\right)\text{V}$$

At t=10μs, the switch 2 is closed. Hence,

$$v_C\left(t=10^{+}\,\mu\text{s}\right)=v_C\left(t=10^{-}\,\mu\text{s}\right)$$

$$=20\left(e^{-1/8}-1\right)\text{V}$$

Then for the time $t > 10^{+}\mu s$, the circuit is redrawn as:



For the capacitive voltage in such a circuit:

$$v_C(+\infty) = -10\text{V}$$

$$R_{TH2} = \frac{16}{5}\text{k}\Omega$$

$$\tau_2 = R_{TH2}C = \frac{16}{5}\text{k}\Omega \times 20\text{nF} = 64\mu\text{s}$$

$$v_C(t) = v_C(+\infty) + \left[v_C(10^+) - v_C(+\infty) \right] e^{-(t-10)/\tau}$$

$$= -10 + \left[20e^{-1/8} - 20 - (-10) \right] e^{-(t-10)/64}$$

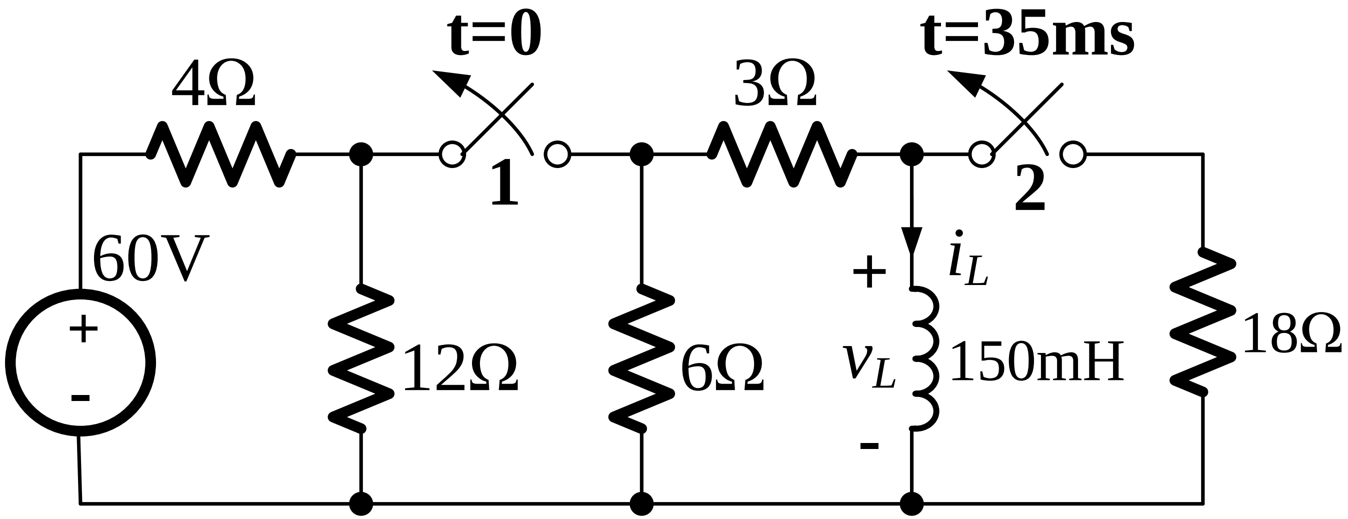
$$= -10 + \left(20e^{-1/8} - 10 \right) e^{-(t-10)/64} \mathrm{V}$$

$$\left(\mathrm{t} \ge 10\mu s \right)$$

Then, for t ≥ 0, v_C (t) can be expressed as:

$$v_C(t)=\begin{cases}20\left(e^{-t/80}-1\right)\mathrm{V}, & 0\le t\le 10\mu\mathrm{s}\\ -10+\left(20e^{-1/8}-10\right)e^{-(t-10)/64}\mathrm{V}, & \mathrm{t}\ge 10\mu\mathrm{s}\end{cases}$$

Example



The two switches have been closed for a long time. At t=0, switch 1 is opened. Then 35ms later, switch 2 is opened.

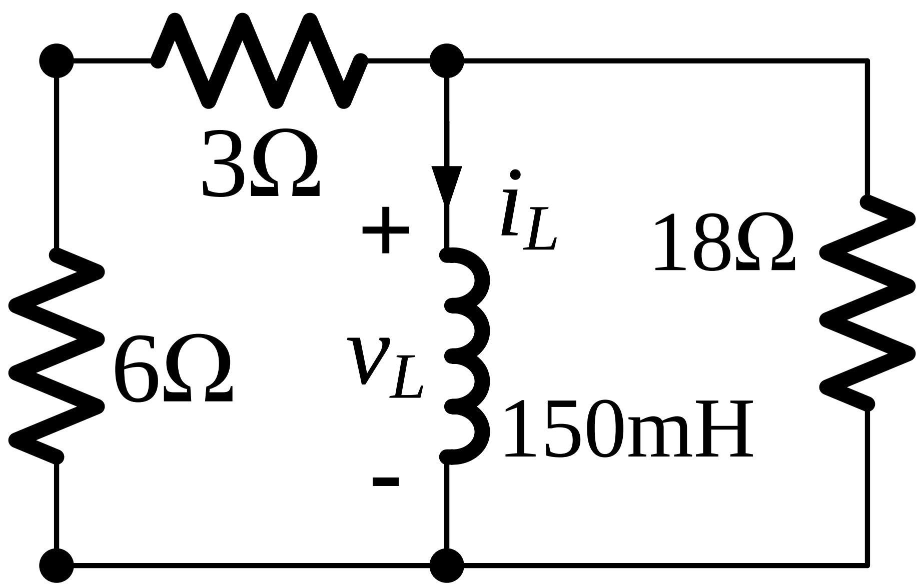
1. Find $i_L(t)$ for $t \geq 0$;

2. What percentage of the initial energy stored in the inductor is dissipated in the 18Ω resistor?

Solution:

1. $$i_L\left(0^{+}\right)=i_L\left(0^{-}\right)=6\text{A}$$

For the time $0^{+}<t<35$ms, the circuit is redrawn as:



$$i_L(+\infty) = 0, \quad R_{TH1} = 6\Omega$$

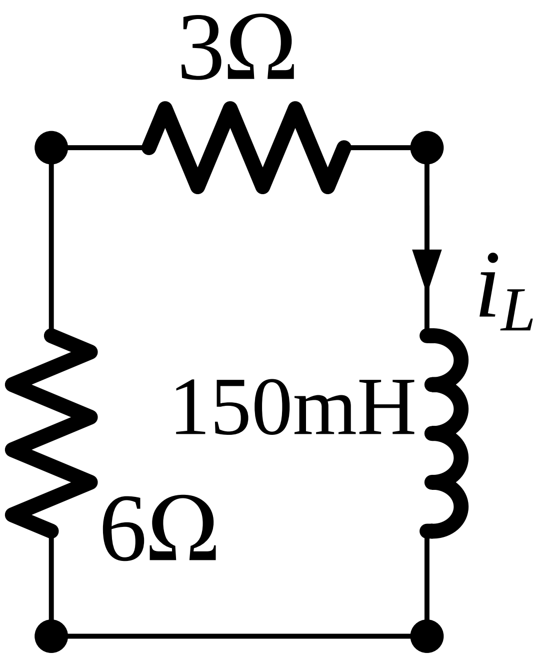
$$\tau_1 = \frac{L}{R_{TH1}} = \frac{150\text{mH}}{6\Omega} = 25\text{ms}$$

$$i_L(t) = 6e^{-t/25\text{ms}}$$

$$= 6e^{-40t}\text{A}, \quad (0 \le t \le 35\text{ms})$$

$$i_L\left(t=35\text{ms}\right)=6e^{-1.4}=1.48\text{A}$$

For the time t > 35ms, the circuit is redrawn as:



$$i_L\left(+\infty\right)=0,\quad R_{TH2}=9\Omega$$

$$\tau_2=\frac{L}{R_{TH2}}=\frac{150\text{mH}}{9\Omega}=\frac{50}{3}\text{ms}$$

$$i_L(t) = 1.48e^{-60(t-0.035)}\text{A}, \quad (t \geq 35\text{ms})$$

Hence,

$$i_L(t) = \begin{cases} 6e^{-40t}\text{A}, & (0 \leq t \leq 35\text{ms}) \\ 1.48e^{-60(t-0.035)}\text{A}, & (t \geq 35\text{ms}) \end{cases}$$

2. The 18Ω resistor is in the circuit only during the first 35ms. During this interval, the voltage across the resistor is:

$$v_L(t) = L\frac{dv_L(t)}{dt} = 0.15\frac{d6e^{-40t}}{dt}$$

$$= -36e^{-40t}\mathrm{V}, \quad (0 \le \mathrm{t} \le 35\mathrm{ms})$$

北京邮電大學 BEIJING UNIVERSITY OF POSTS AND TELECOMMUNICATIONS

The power dissipated in 18Ω resistor is:

$$p = \frac{v_L^2}{18} = 72e^{-80t}\,\mathrm{W}, \quad \left(0 \le \mathrm{t} \le 35\mathrm{ms}\right)$$

The energy dissipated in 18Ω resistor is:

$$w = \int_0^{0.035} 72e^{-80t}\,dt = -\frac{72}{80}e^{-80t}\Big|_0^{0.035} = 845.27\mathrm{mJ}$$

The initial energy stored in the inductor is:

$$w_i = \frac{1}{2} L i_L^2 \left(0^-\right) = \frac{1}{2} \times 0.15 \times 6^2 = 2.7\mathrm{J}$$

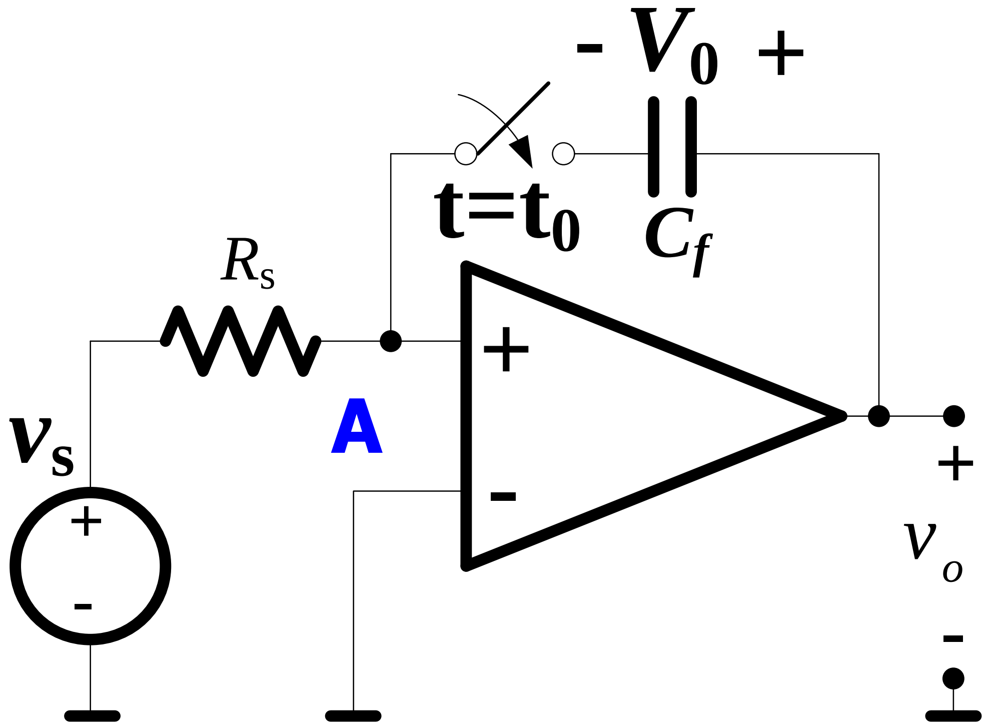
$$\frac{w}{w_i} = \frac{845.27\mathrm{mJ}}{2.7\mathrm{J}} \approx 31.31\%$$

Therefore, 31.31% of the initial energy stored in the inductor is dissipated in the 18Ω resistor.

6-6 Integrating Amplifier

- Integrating amplifier can generate an output voltage proportional to the integral of the input voltage;
- Generally, it contains a capacitor and an operational amplifier.

Example



The initial voltage across the capacitor is V_0.

The switch is closed at t = t_0. Express v_o by v_s.

Solution:

For node A, by KCL:

$$C_f \frac{dv_o}{dt} + \frac{v_s}{R_s} = 0 \qquad v_o\left(t_0^{+}\right) = v_o\left(t_0^{-}\right) = V_0$$

$$v_o(t) = -\frac{1}{R_s C_f}\int_{t_0}^{t} v_s dt + V_0, \quad t \geq t_0$$

- Specially, if $t_0 = 0$, and $V_0 = 0$, we have:

$$v_o(t) = -\frac{1}{R_s C_f}\int_0^t v_s dt$$

- Output voltage is an inverted, scaled replica of the integral of the input voltage.

Summary of Chapter 6

- Conception of First-order RC and RL circuits
- Initial values of circuits
- Natural response and step response
- Conception of time constant
- General solution method for natural and step responses for first-order RC and RL circuits
- Responses for sequential switching circuits