Chapter 13-14 Mechanical waves

§ 1 Categories of Waves

- Mechanical waves —— require an elastic medium
  - Sound wave, water wave, earthquakes.
- Electromagnetic waves —— do not require any medium
  - Lightwave, radio wave, microwave， x rays, and radar waves.
- Matter waves —— any matter has wave-like and particle-like behaviors
  - Electrons, protons, and other fundamental particles; ultracold atoms.

All types of waves use similar mathematical descriptions. We can therefore learn a great deal about waves in general by making a careful study of one type of wave —— For example, mechanical wave.

§ 2 Conceptual ideas of Waves

- Requirements for mechanical waves

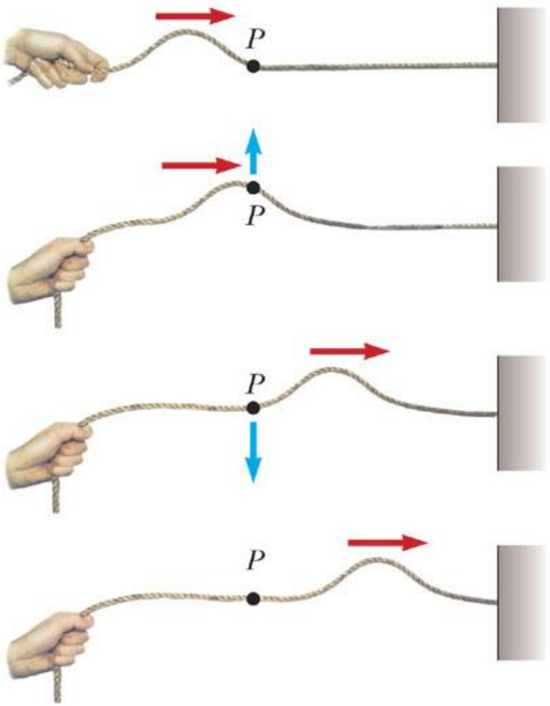
  A mechanical wave is the propagation of a disturbances in a medium.

  - Source of disturbance (origin of wave).
  - Medium through which the wave can propagate.

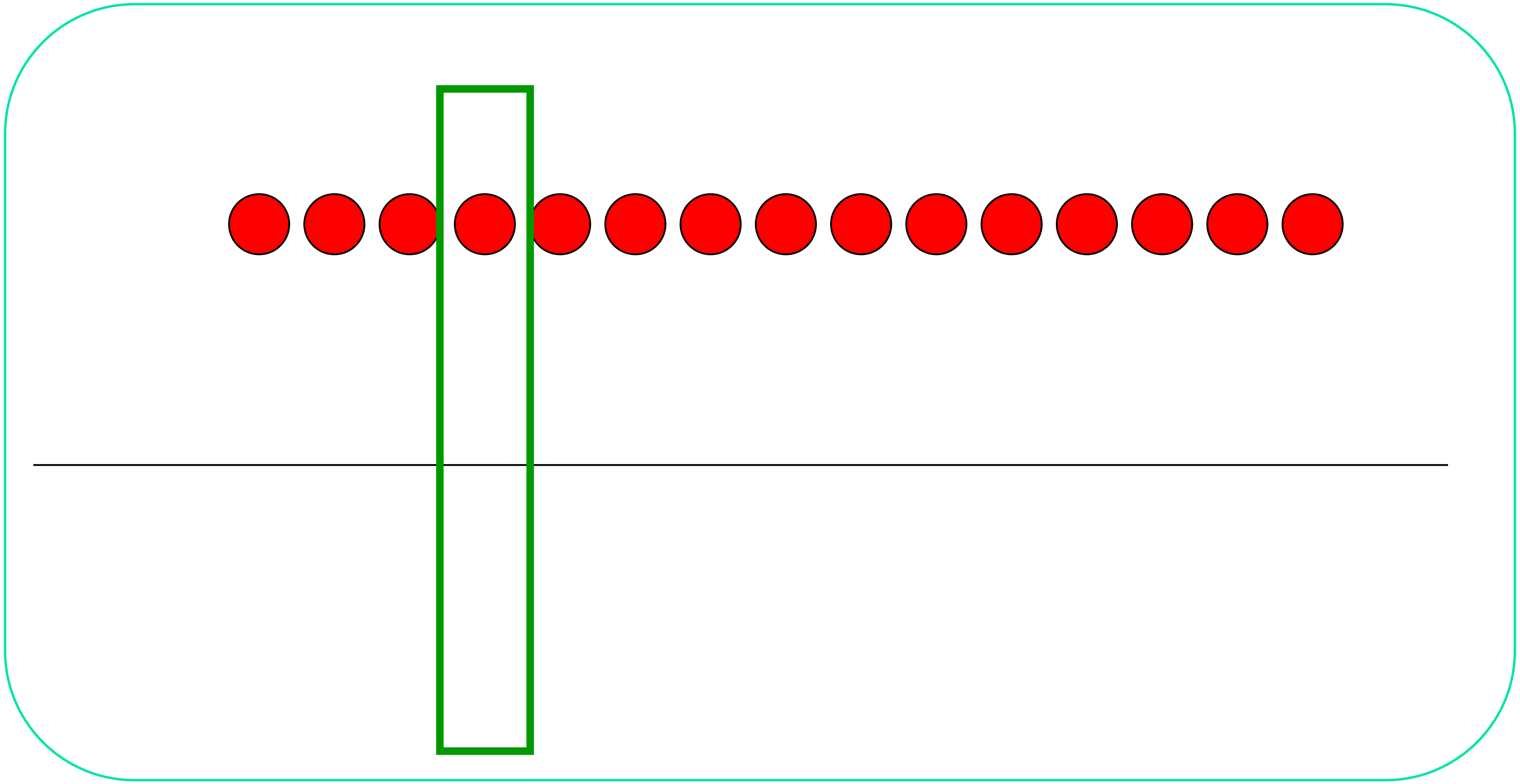
•A wave is a disturbance in a medium that travels outward from its source.

•It travels from one place to another by means of a medium, but the medium itself is not transported.

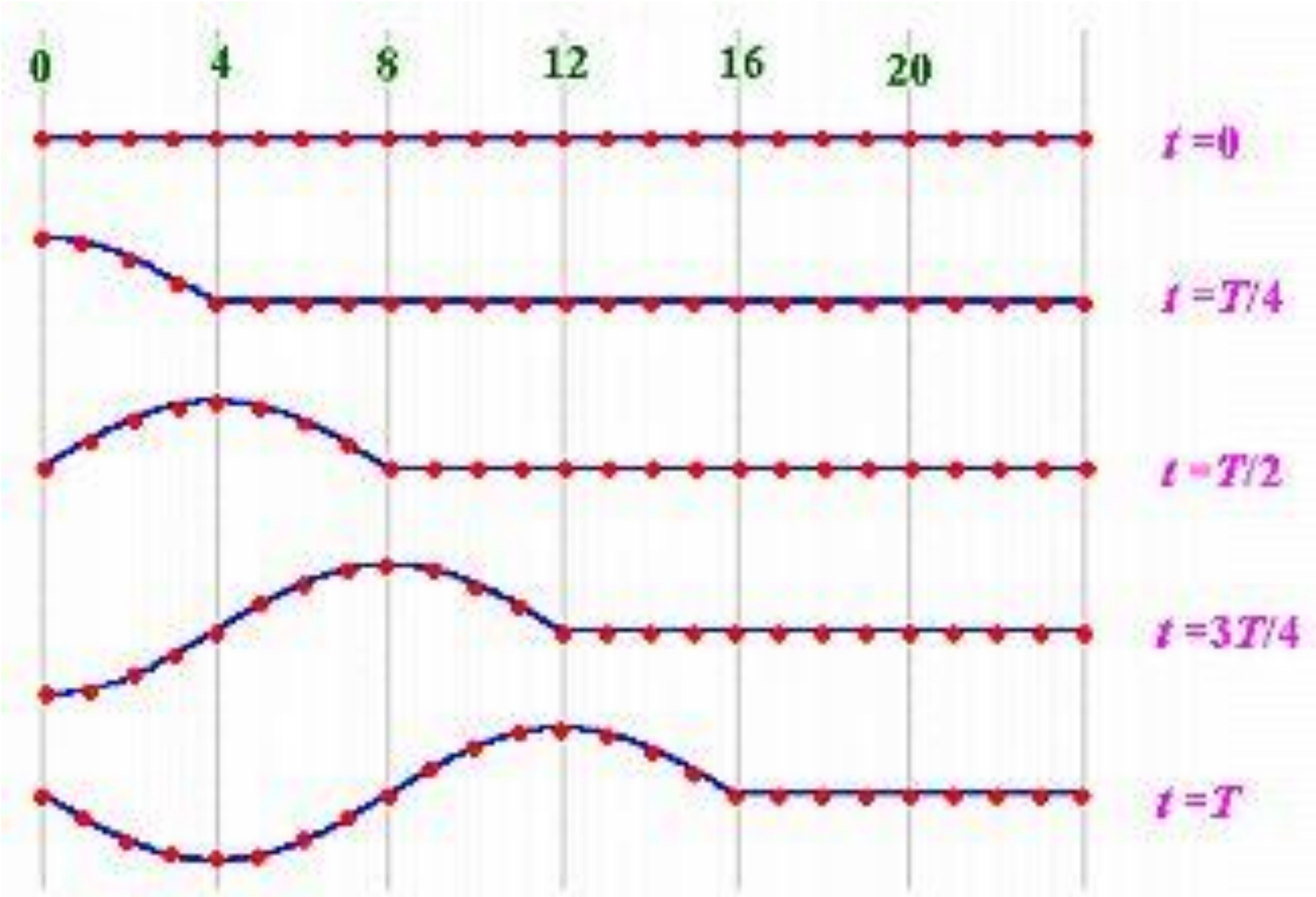
•All material media- solids, liquids, and gases- can carry energy and information by means of waves



2. Waves' creation and propagation



The Feature of wave's propagation:



The essence of wave motion:

- Wave transports the disturbance (also state of motion and energy) through space without the accompanying transfer of matter.
- The particles of the medium do not experience any net displacement as the wave passes, the particles simply move back and forth through small distance about their equilibrium positions.

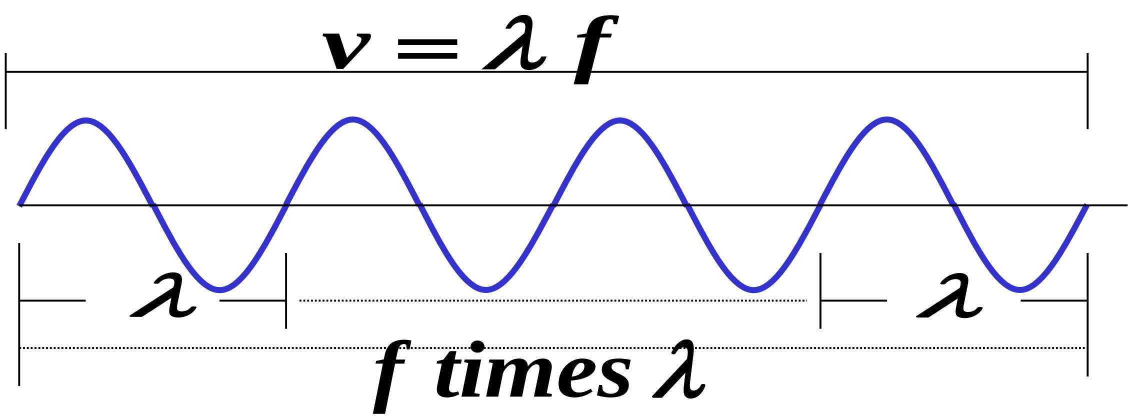
The Characteristic Quantities of wave:

(1) The wavelength λ of a wave is the distance between repetitions of the wave shape , their phase difference is 2π.

(2) The T or f are the T or f of wave source, respectively.

(3) Wave speed is the phase speed, The distance of an oscillatory state propagating in a unit-time, in one T, is the wave moves a distance of λ ——*wavelength*, so

$$\boxed{v = \frac{\lambda}{T}}$$



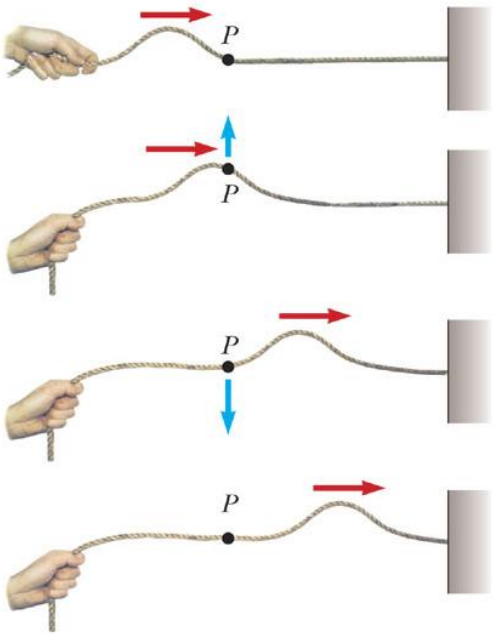
$$f = \frac{1}{T}$$

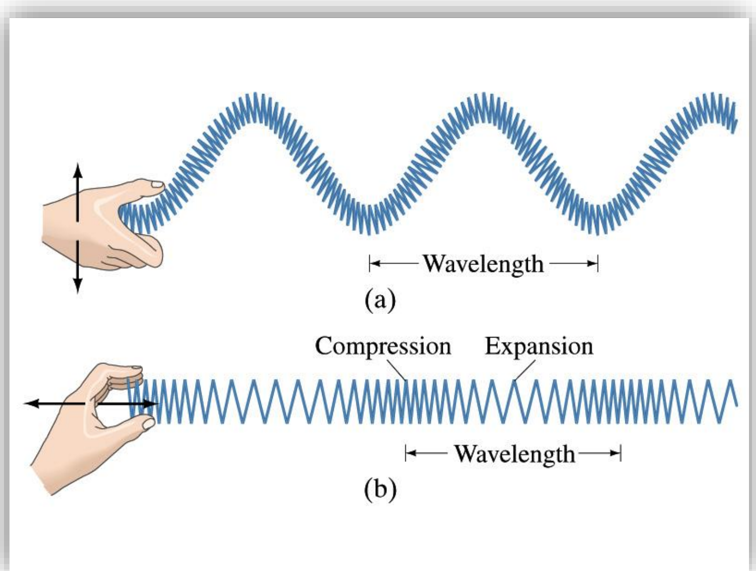
$$v = \lambda f$$

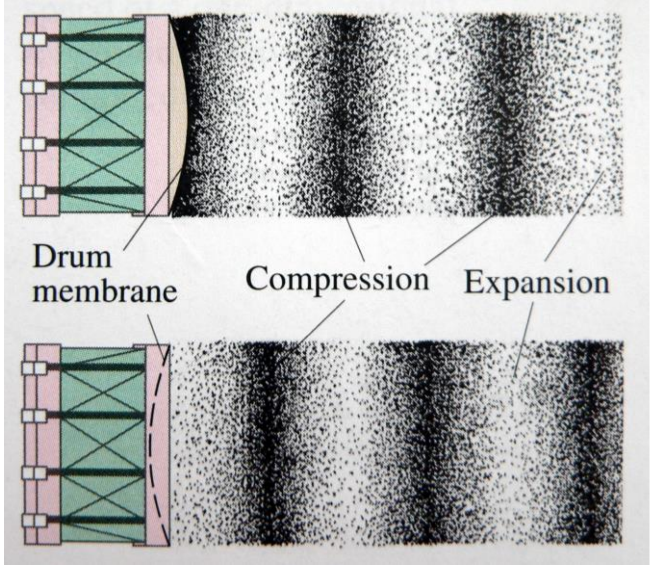
- Transverse and longitudinal waves
  - Transverse wave: the motion of the particles of the medium is perpendicular to direction of propagation.

    string wave, electromagnetic wave.
  - Longitudinal wave: the motion of the particles is back and forth parallel to the direction of propagation.

    sound wave, spring compress and stretch wave.







The speeds of some kinds of waves (p328-329)

- The speed of Transverse waves on string: $v = \sqrt{\dfrac{T}{\mu}}$
- The speed of longitudinal wave in a fluid:

$$v = \sqrt{\frac{B}{\rho}}$$

B: the bulk modulus; ρ: density of medium

- The speed of longitudinal wave in a solid rod:

$$v = \sqrt{\frac{Y}{\rho}}$$

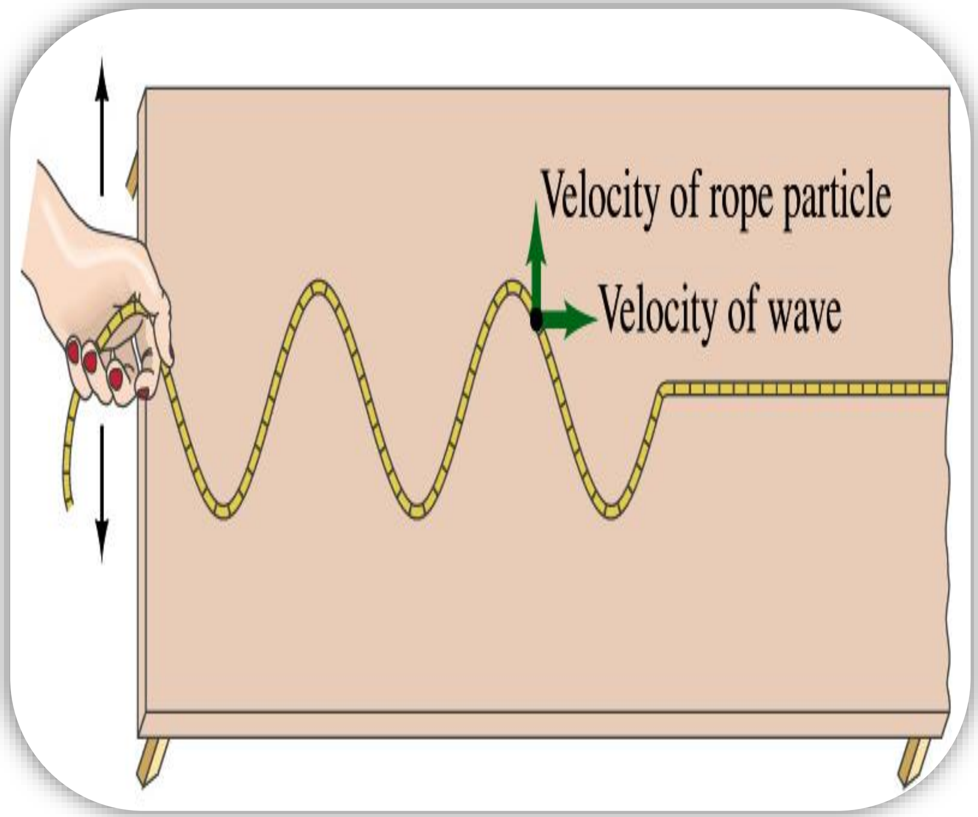
Y: Young's modulus; ρ: density of medium

- The speed of sound in an ideal gas: $v = \sqrt{\dfrac{\gamma RT}{M}}$

$\gamma = C_p/C_v$: dimensionless ratio of heat capacity; R: the gas constant; M: molar mass

Waves vs. Particle Velocity

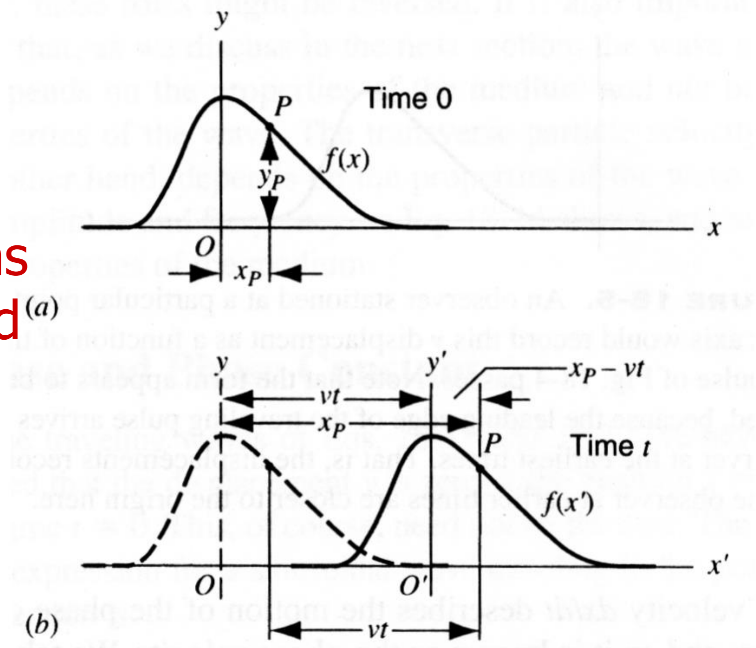
Is the velocity of a wave moving along a cord the same as the velocity of a particle of the cord?



No. The two velocities are different both in magnitude and direction. The wave on the rope moves to the right but each piece of the rope only vibrates up and down.

§ 3 Wave Function for Traveling Wave

- Wave function for the wave travels to the right
  - Wave shape at time ***t*=0** : $y(x,0) = f(x)$
  - The element of the string at ***x*** at time ***t*** has the same ***y*** position as the element located at ***x*-*vt*** had at time ***t*=0**.



$$y(x,t) = y(x-vt,0) = f(x-vt)$$

- With the view of time:
  - The motion of origin at x=0: $y(0,t) = F(t)$
  - The motion of point x at time t is the same as the motion of point x=0 at the earlier time t-x/v.

$$y(x,t) = y\left(0, t-\frac{x}{v}\right) = F\left(t-\frac{x}{v}\right)$$

Wave function

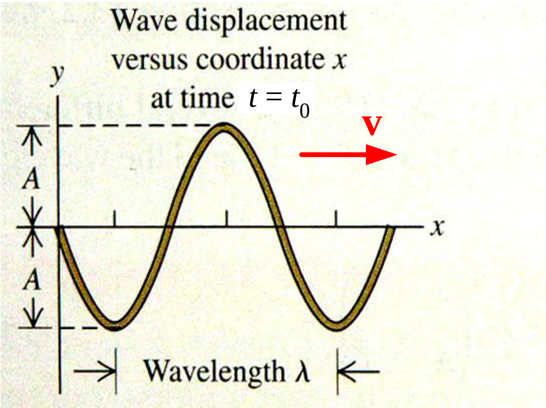
- Wave function for the wave travels to the left

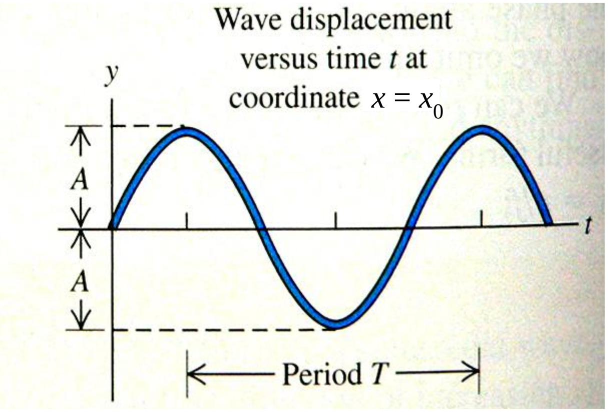
$$y(x,t) = f(x+vt) = F\left(t+\frac{x}{v}\right)$$

- The meaning of wave function

  The wave function y(x,t) represents the y coordinate of any point P located at position x at any time t.

  - If t is fixed ($t=t_0$), the wave function $y=y(x,t_0)$ defines a curve representing the actual geometric shape of wave at that time, called the waveform (the photo of all group of particles in the medium at the same time).
  - If x is fixed ($x=x_0$), the wave function $y=y(x_0,t)$ is actually the kinematics' equation for particles located at $x=x_0$.





§ 4 Harmonic Wave

- Suppose the origin of wave at point x=0 is disturbed with sine or cosine function.

$$y(0,t) = A\cos(\omega t + \phi) = A\cos(2\pi f t + \phi)$$

- Wave function moving in +x-direction

$$y(x,t) = ?$$

How to get a wave function

- Wave function obtained by the view of phase retardation.
  - The phase at point x is retarded with amount of $-2\pi \frac{x}{\lambda}$
  - The wave function: $$y(x,t) = A\cos\left(\omega t - 2\pi \frac{x}{\lambda} + \phi\right)$$ $$= A\cos(\omega t - kx + \phi)$$
  - Wave function obtained by the view of time
  - The motion of point x at time t is the same as the motion of point x=0 at the earlier time t-x/v.
  - The wave function:

$$y(x,t) = y\left(0, t - \frac{x}{v}\right) = A\cos\left[\omega\left(t - \frac{x}{v}\right) + \phi\right]$$

How to get a wave function

- Wave function obtained with reference point not at x=0
- Suppose an oscillation is disturbed at point $x=x_0$ with

$$y(x_0,t)=A\cos(\omega t+\phi)$$

- The phase at point x is retarded with amount of $-2\pi\dfrac{x-x_0}{\lambda}$
- The wave function:

$$y(x,t)=A\cos\left(\omega t-2\pi\frac{x-x_0}{\lambda}+\phi\right)$$

- The motion of point x at time t is the same as the motion of point $x=x_0$ at the earlier time $t-\dfrac{x-x_0}{v}$

$$y(x,t)=y\left(x_0,t-\frac{x-x_0}{v}\right)=A\cos\left[\omega\left(t-\frac{x-x_0}{v}\right)+\phi\right]$$

The speed of wave and the speed of oscillation particle

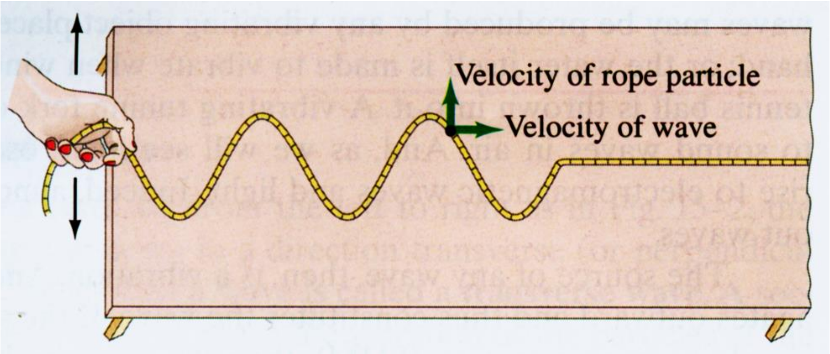
- Difference between the speed of wave and the speed of oscillation particle.
  - The speed of particle: change rate of its displacement with time t.

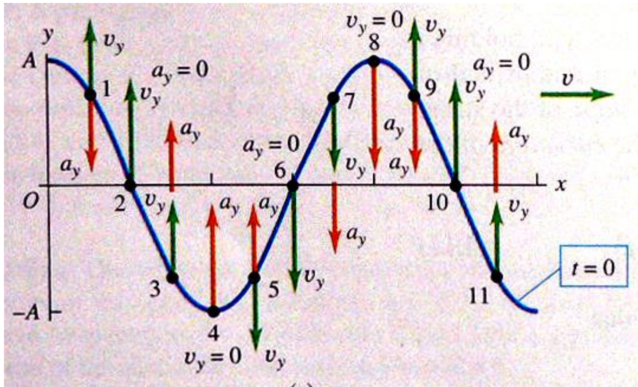
$$v_y(x,t) = \frac{\partial y(x,t)}{\partial t}$$

  - The speed of wave: the speed of the phase propagation (phase velocity) or the transfer speed of the disturbance status.

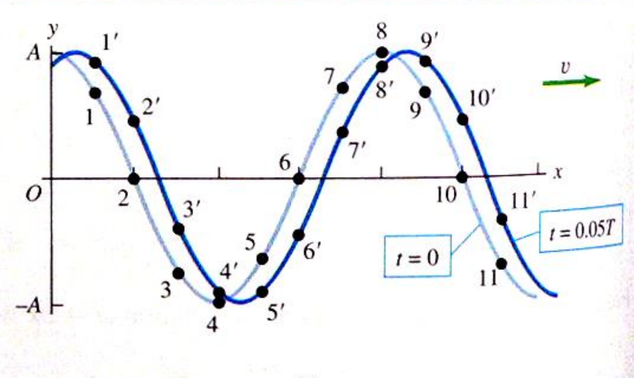
Keeping phase constant:

$$\omega t - kx = \text{constant} \qquad v = \frac{dx}{dt} = \frac{\omega}{k}$$





(a)



(b)

Example

Example: A harmonic wave travels in +x-direction with speed v and wavelength λ. The particle at $x_0=\lambda/4$ oscillates with the function:

$$y(x_0,t)=A\cos\omega t$$

Write the wave function describing the wave.

Solution I: by comparison with the standard wave function

Suppose the wave function has the form:

$$y(x,t)=A\cos(\omega t-\frac{2\pi}{\lambda}x+\phi)$$

At $x=x_0=\lambda/4$ $$y(\frac{\lambda}{4},t)=A\cos(\omega t-\frac{2\pi}{\lambda}\cdot\frac{\lambda}{4}+\phi)$$

$$=A\cos(\omega t-\frac{\pi}{2}+\phi)$$

Compare it with $y(x_0,t)=A\cos\omega t$ We have $-\frac{\pi}{2}+\phi=0$ $\phi=\frac{\pi}{2}$

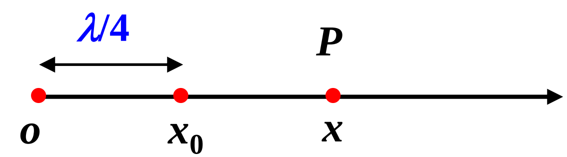
$$\omega=\frac{2\pi}{\lambda}v$$

$$\boxed{y(x,t)=A\cos\left[\frac{2\pi}{\lambda}(vt-x)+\frac{\pi}{2}\right]}$$

Example Cont'd

Solution II: by phase comparison with the reference point x_0

The phase at point x is retarded with respect to x_0



$$\frac{2\pi}{\lambda}(x-x_0)$$

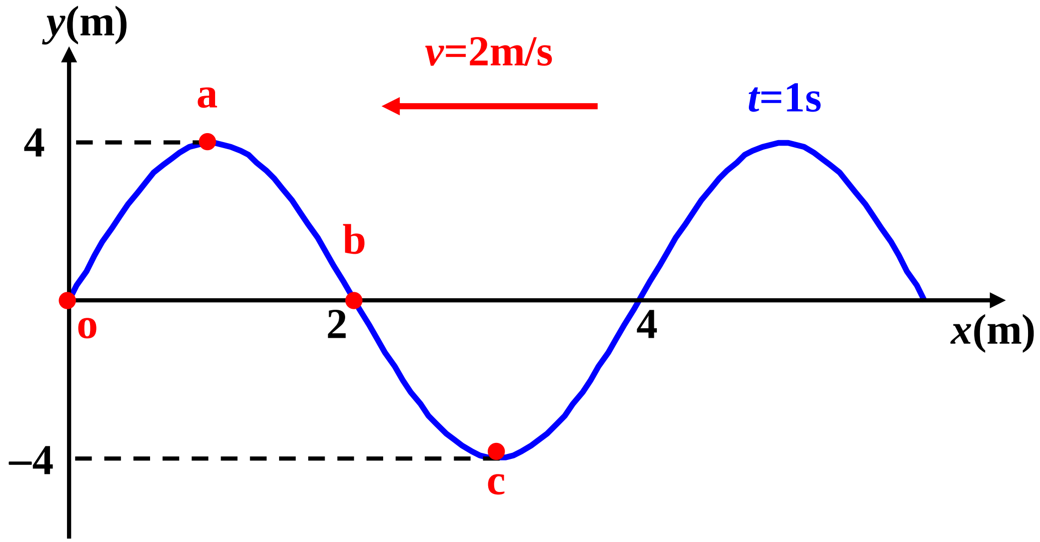
$$y(x,t)=A\cos\left[\omega t-\frac{2\pi}{\lambda}(x-x_0)\right]$$

$$=A\cos\left[\frac{2\pi}{\lambda}vt-\frac{2\pi}{\lambda}(x-\frac{\lambda}{4})\right]$$

$$=A\cos\left[\frac{2\pi}{\lambda}(vt-x)+\frac{\pi}{2}\right]$$

Example

Example: A harmonic wave travels in –x-direction. The waveform at time t=1s is shown in figure.

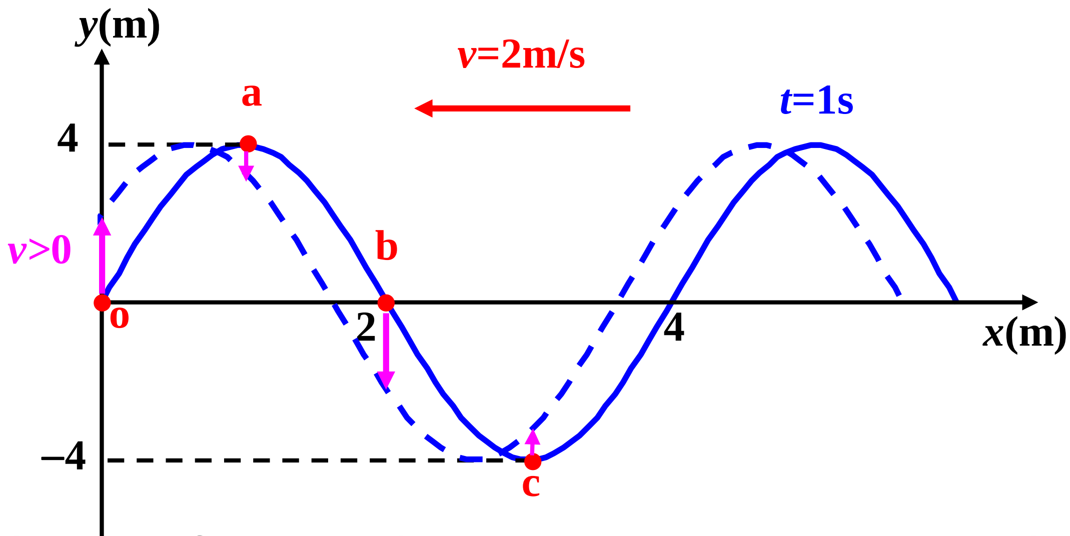


(1) Draw the direction of motion of particle marked with o, a, b, c.

(2) Write the wave function.

(3) Draw the waveform graph at time t=2s.

Example Cont'd



Solution: (2) From the waveform graph, we get A=4m, λ=4m, T=λ/v=2s, ω=2π/T=π rad/s

$$y(x,t) = A\cos(\omega t + \frac{2\pi}{\lambda}x + \phi) = 4\cos(\pi t + \frac{\pi}{2}x + \phi)$$

At x=0, t=1 $\quad y(0,1) = 4\cos(\pi \times 1 + \frac{\pi}{2} \times 0 + \phi)$ $\quad \pi + \phi = \pm\frac{\pi}{2}$

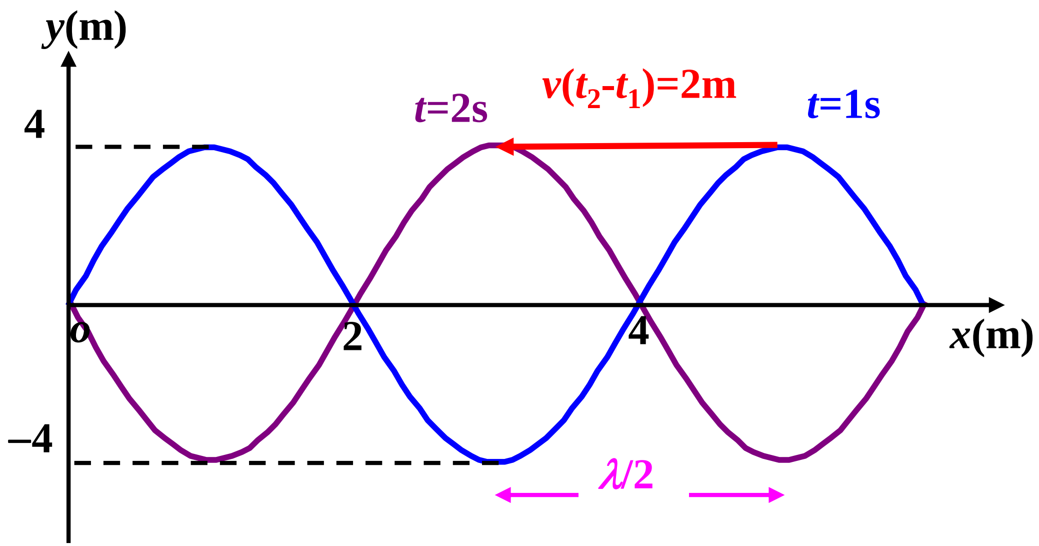
$$= 4\cos(\pi + \phi) = 0$$

v>0

$\pi + \phi = -\frac{\pi}{2}$ $\quad \phi = -\frac{3}{2}\pi$ or $\phi = \frac{\pi}{2}$

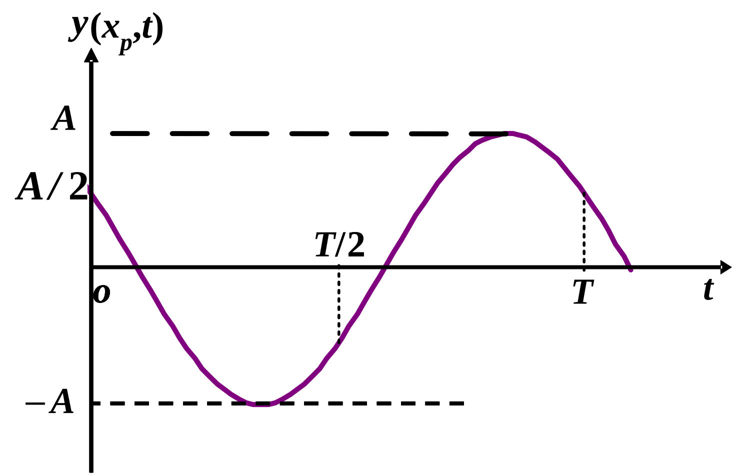
$$\boxed{y(x,t) = 4\cos(\pi t + \frac{\pi}{2}x + \frac{\pi}{2}) \quad \text{(SI)}}$$

Example Cont'd



Example

Example: A harmonic wave with the amplitude of A, the wavelength λ, and the period of T travels in +x-direction. The x-t graph of the particle P at $x_p = \lambda/2$ is shown in the figure. Find (1) the wave function; (2) the waveform at time $t = T/2$.

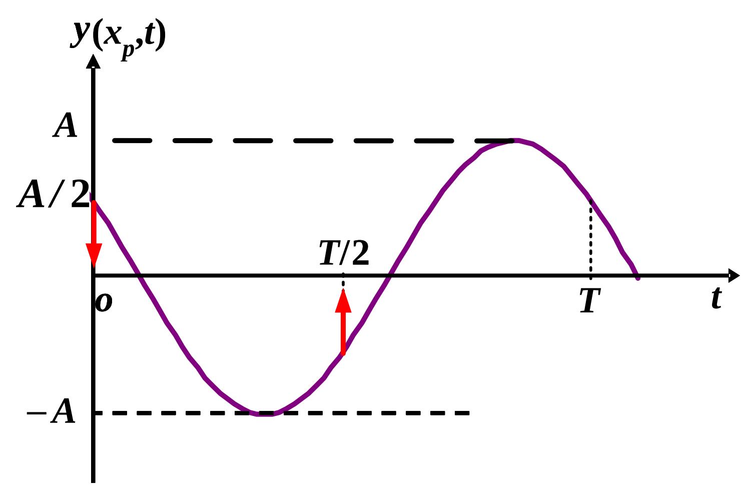


Solution: (1) The phase angle for the oscillation at point P is $\pi/3$

$$y(x_P, t) = A\cos\left(\frac{2\pi}{T}t + \frac{\pi}{3}\right)$$

$$y(x,t) = A\cos\left[\frac{2\pi}{T}t - \frac{2\pi}{\lambda}\left(x - \frac{\lambda}{2}\right) + \frac{\pi}{3}\right]$$

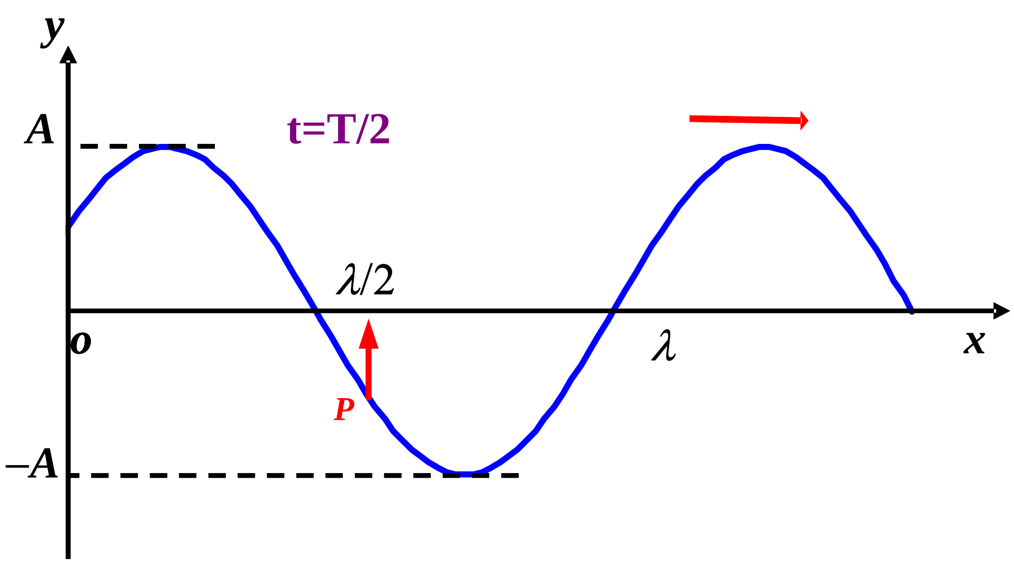
$$= A\cos\left[\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x + \frac{4\pi}{3}\right]$$

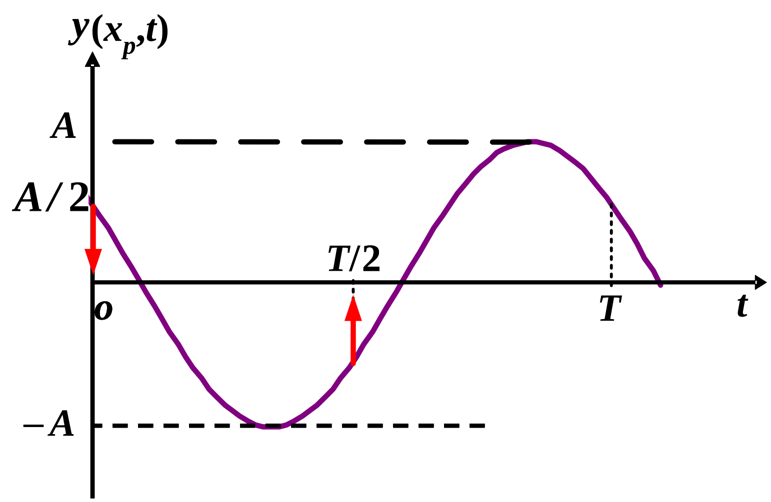


Example Cont'd

$$y(x,t) = A\cos\left[\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x + \frac{4\pi}{3}\right]$$

(2) At t=T/2, the phase angle at x=0: $\frac{2\pi}{T}\cdot\frac{T}{2} + \frac{4\pi}{3} = 2\pi + \frac{\pi}{3} \leftrightarrow \frac{\pi}{3}$





§ 5 The Linear Wave Equation (p335)

$$y(x,t) = A\cos(\omega t - kx)$$

$$v_y = \frac{\partial y}{\partial t} = -\omega A\sin(\omega t - kx) \qquad \frac{\partial y}{\partial x} = kA\sin(\omega t - kx)$$

$$a_y = \frac{\partial^2 y}{\partial t^2} = -\omega^2 A\cos(\omega t - kx) \qquad \frac{\partial^2 y}{\partial x^2} = -k^2 A\cos(\omega t - kx)$$

$$\frac{\partial^2 y}{\partial x^2} = -\frac{k^2}{\omega^2}\omega^2 A\cos(\omega t - kx) = \frac{k^2}{\omega^2}\frac{\partial^2 y}{\partial t^2}$$

$$\boxed{\frac{\partial^2 y}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} = 0}$$

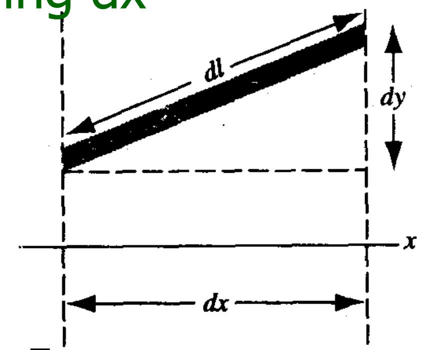
Linear Wave Equation

§ 6 Energy Transfer in Wave Motion (p331)

Waves transport $\left\{ \begin{array}{l} \text{state of motion} \\ \text{energy} \end{array} \right.$

- Total energy density of a wave.

  Consider a wave traveling along a string. A segment of string dx



  ➢ The kinetic energy:

$$dK = \frac{1}{2} dm\, v_y^2 = \frac{1}{2} dm \left(\frac{\partial y}{\partial t} \right)^2 = \frac{1}{2} (\mu\, dx) \omega^2 A^2 \sin^2(\omega t - kx)$$

  ➢ The potential energy:

$$dU = T(dl - dx) = T\left[\sqrt{(dx)^2 + (dy)^2} - dx \right] = T dx \left[\sqrt{1 + \left(\frac{\partial y}{\partial x} \right)^2} - 1 \right] \approx \frac{1}{2} T dx \left(\frac{\partial y}{\partial x} \right)^2$$

$$dU = \frac{1}{2} (\mu dx) \omega^2 A^2 \sin^2(\omega t - kx) \qquad T = \mu v^2 = \mu \left(\frac{\omega}{k} \right)^2$$

  ➢ Energy density: $\boxed{w = \frac{dE}{dx} = \frac{dK + dU}{dx} = \mu \omega^2 A^2 \sin^2(\omega t - kx)}$

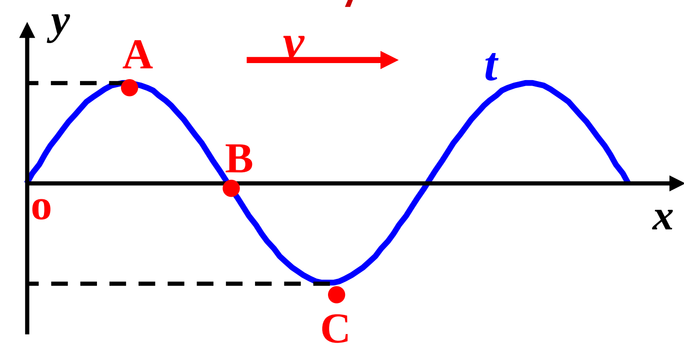
Energy density for volume mass distribution: $$\boxed{w = \frac{dE}{dV} = \rho\omega^2 A^2 \sin^2(\omega t - kx)}$$

- Energy characteristics in wave motion
  - For a particle in medium, the kinetic energy and potential energy are in phase —— They reach their maximum simultaneously.

    At point A, C,

    $$v_y = 0, \quad \frac{\partial y}{\partial x} = 0, \quad dK = dU = 0$$

    At point B, dK and dU reach their maximum.



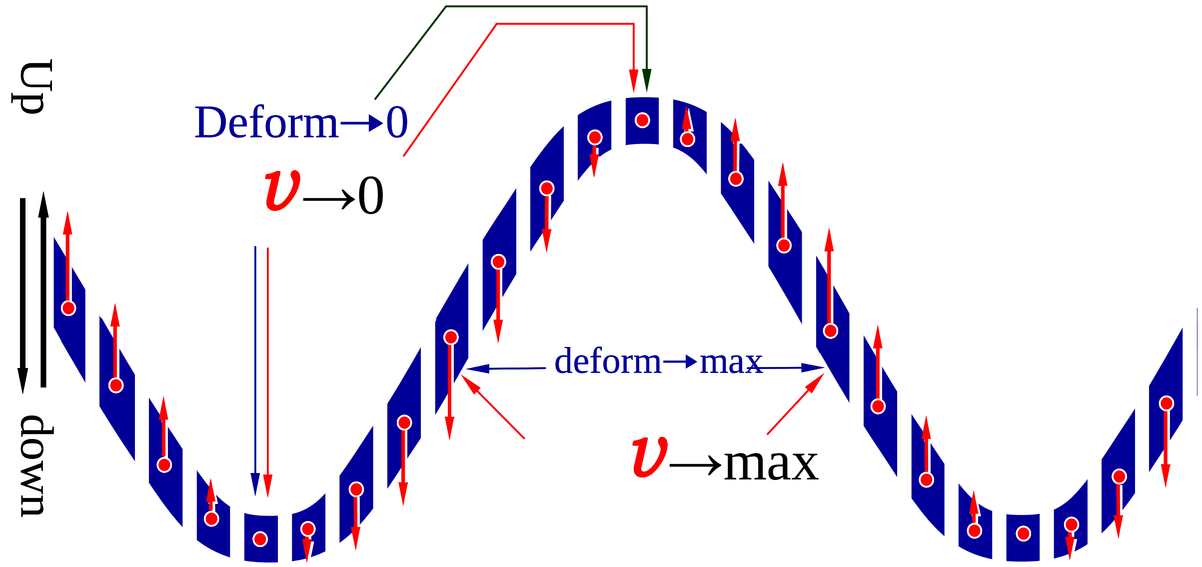
  - The energy in the volume dV is not conserved. Sometime the energy is net input, sometime is net output—energy is transported.
  - In volume dV, the average value of energy in one period is constant

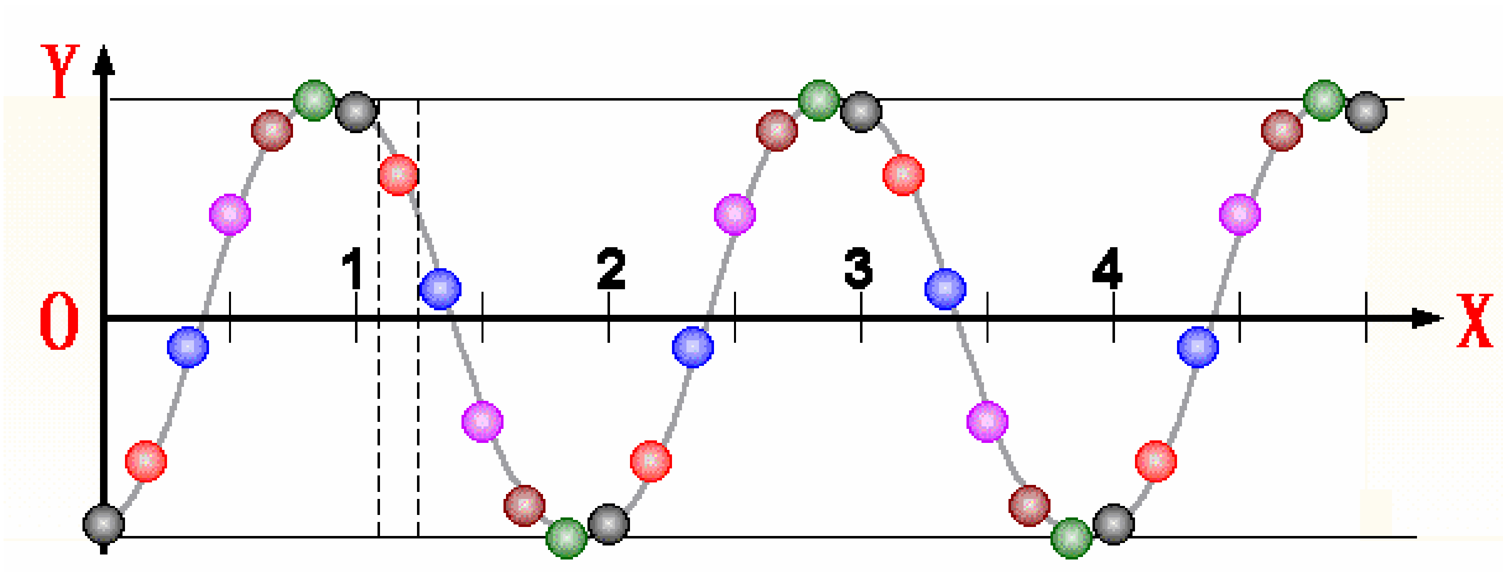
$$\overline{w} = \frac{1}{T}\int_{-T/2}^{T/2} w\,dt$$

$$= \frac{1}{T}\int_{-T/2}^{T/2} \rho\omega^2 A^2 \sin^2(\omega t - kx)\,dt$$

$$= \frac{1}{2}\rho\omega^2 A^2 \propto \begin{cases} \omega^2 \\ A^2 \end{cases}$$

The description of the energy characteristics in wave motion

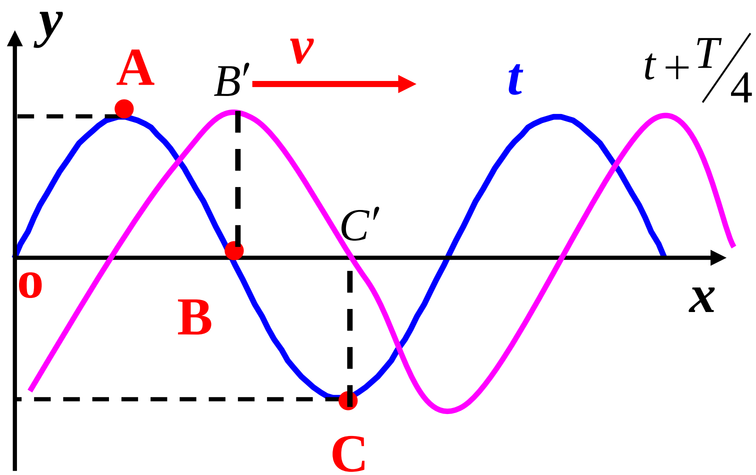




The energy features for wave and SHM

$$w = \frac{dE}{dV} = \rho\omega^2 A^2 \sin^2(\omega t - kx)$$

	t	t+T/4
B	w→max	w→0
C	w→0	w→max

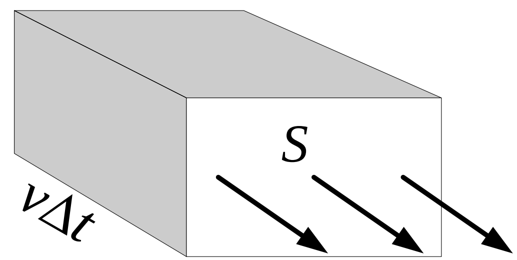


Wave	SHM
For segment, E doesn't conserve	ΔE=0
Transfer energy	Doesn't transfer energy

The energy in volume dV is not conserved. Sometime the energy is net input, sometime is net output — energy is transported.

- Energy current density in wave motion and intensity of wave
  - Energy current: the net energy that flows through a cross-sectional area per unit time interval.

    In time interval **Δt**, the energy that can flow through the surface ***S*** is the energy in the cuboid volume ***$Sv\Delta t$***.



$$\text{energy current} = \frac{wV}{\Delta t} = \frac{wSv\Delta t}{\Delta t} = wvS$$

  - Energy current density: $P = \dfrac{\text{energy current}}{S} = wv$
  - Intensity of wave: time average of energy current density.

$$I = \overline{P} = \overline{w}v = \frac{1}{2}\rho\omega^2 A^2 v$$

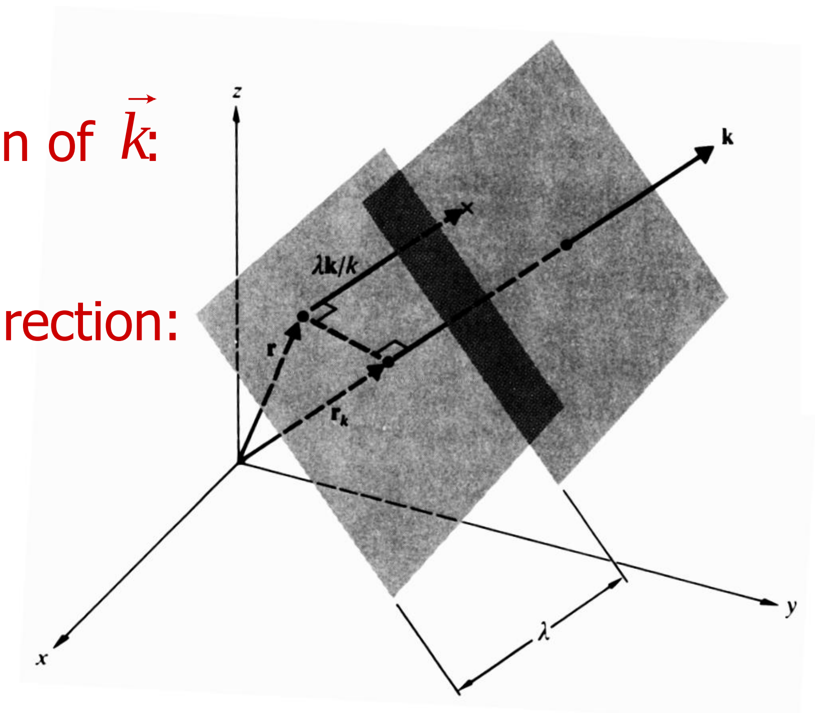
§ 7 Plane Wave and Spherical Wave

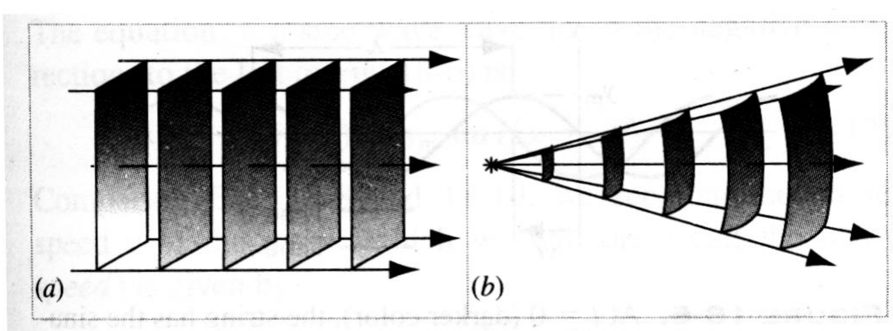
- Wavefronts and Rays
  - Wavefront: the surface composed of all the points having the same state of motion (with equal phase)
  - Ray: A line normal to the wavefronts, indicating the direction of motion of the wave.
- Plane wave and spherical wave
  - A plane wave traveling in the direction of $\vec{k}$:

$$\psi(\vec{r},t) = A\cos(\omega t - \vec{k}\cdot\vec{r})$$

  - A spherical wave traveling in radial direction:

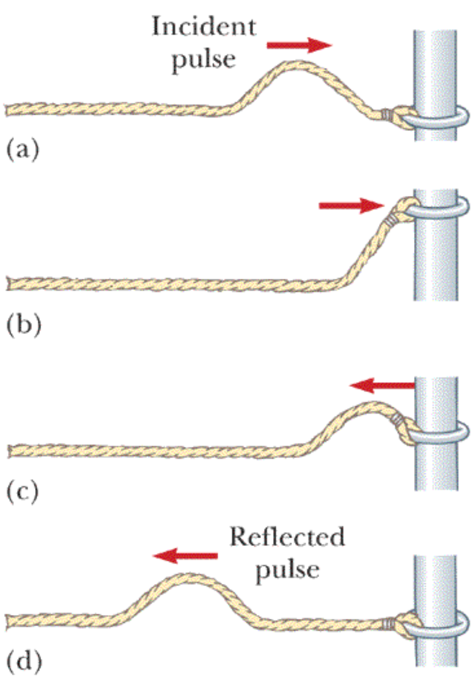
$$\psi(\vec{r},t) = \frac{A}{r}\cos(\omega t - kr)$$

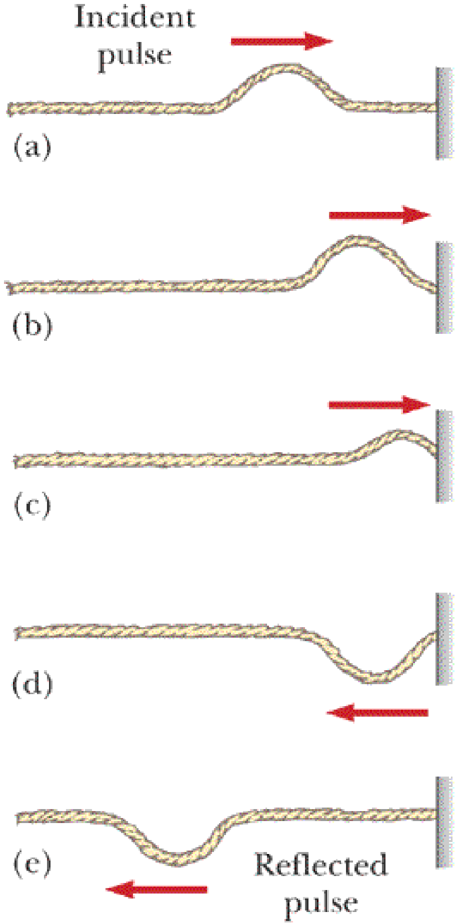




§ 8 Reflection and Transmission of Waves(p338)

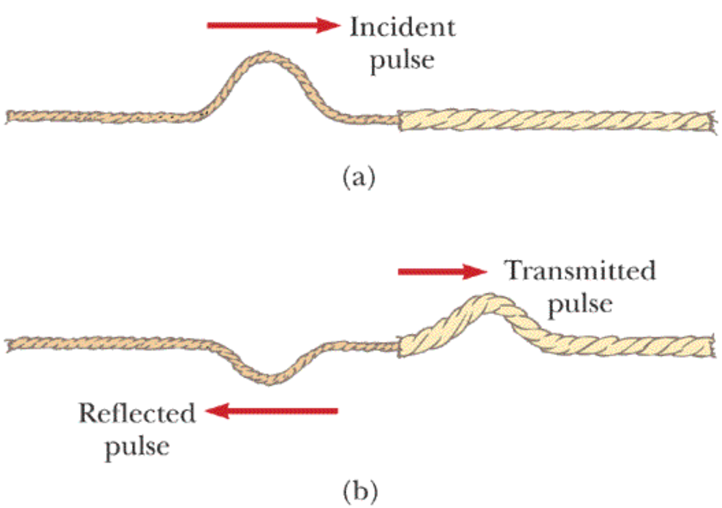
- Reflection through the fixed boundary
  - The reflection at a rigid end causes to invert on reflection.
  - For a sinusoidal wave, the inversion of a wave causes to a π phase shift.
- Reflection through the free boundary
  - The reflected wave is not inverted.

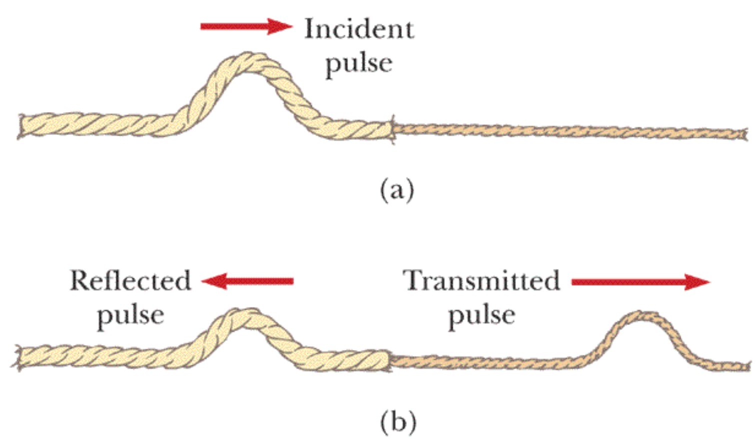




Reflection and Transmission of Waves

- Boundary of light string attached to a heavier (more dense) string
  - The inversion in the reflected wave is similar to the behavior of a wave meeting a fixed boundary, but partially reflected.
  - The transmitted wave has the same shape of the incident wave.
- Boundary of heavy string attached to a lighter (less dense) string
  - The incident wave is partially reflected and partially transmitted. The reflected wave is not inverted.





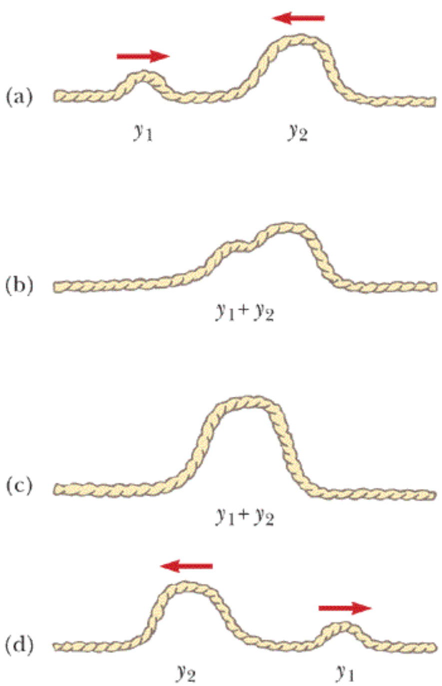
For mechanical wave, the larger is ρv, more dense is the medium.

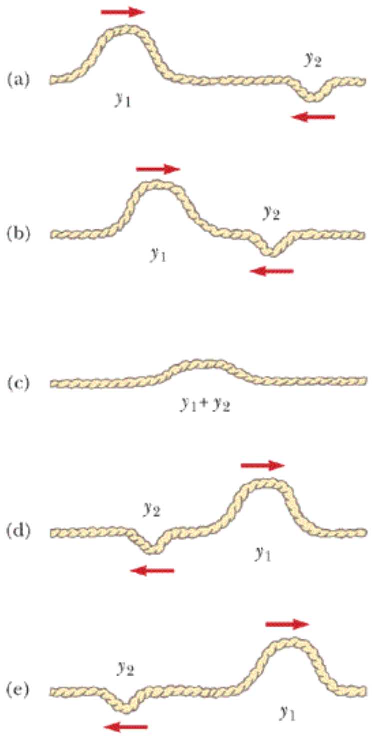
For optical wave, the larger is the index of refraction n, more dense is the medium.

§ 10 The Principle of Superposition (p337)

- The principle of superposition
  - If two or more traveling waves are moving through a medium and combine at a given point, the resultant displacement of the medium at that point is the sum of the displacements of the individual waves.

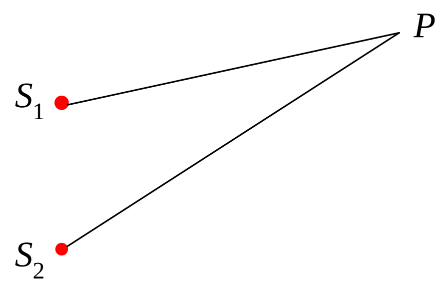
    In the region they overlap: $y(x,t) = y_1(x,t) + y_2(x,t)$
  - Two traveling waves can pass through each other without being destroyed or even altered.





§ 11 Interference of waves (p339)

- The overlapping of waves is called interference
  - If two wave overlap in a region:



$$y_1 = A_1\cos(\omega t - kr_1 + \phi_1)$$
$$y_2 = A_2\cos(\omega t - kr_2 + \phi_2)$$

$$y = y_1 + y_2$$

$$A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\left[k(r_2 - r_1) - (\phi_2 - \phi_1)\right]$$

$$I = I_1 + I_2 + \underbrace{2\sqrt{I_1 I_2}\cos\Delta\varphi}_{\text{Coherent term}}$$

Phase difference: $\Delta\varphi = k(r_2 - r_1) - (\phi_2 - \phi_1)$

Coherent term

- For some point:

$$\Delta\varphi = \pm 2m\pi, \quad m = 0, 1, 2, \ldots$$

Two waves are in phase

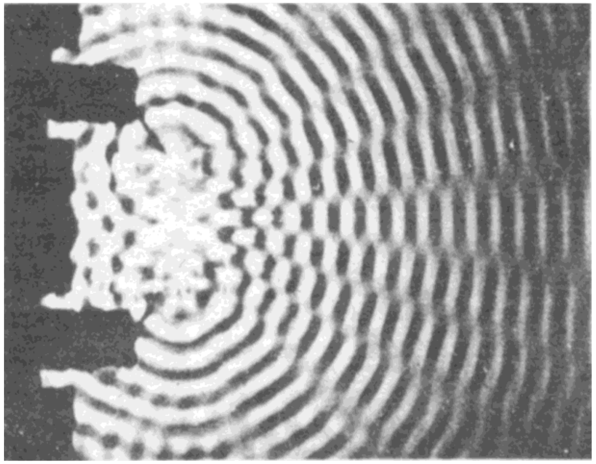
$I = I_{\max}$ —— constructive interference.

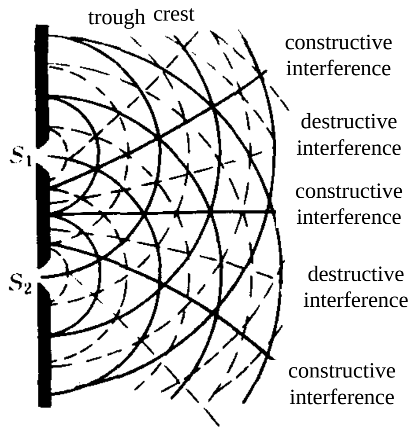
- For some point:

$$\Delta\varphi = \pm(2m+1)\pi, \quad m = 0, 1, 2, \ldots$$

Two waves are out of phase

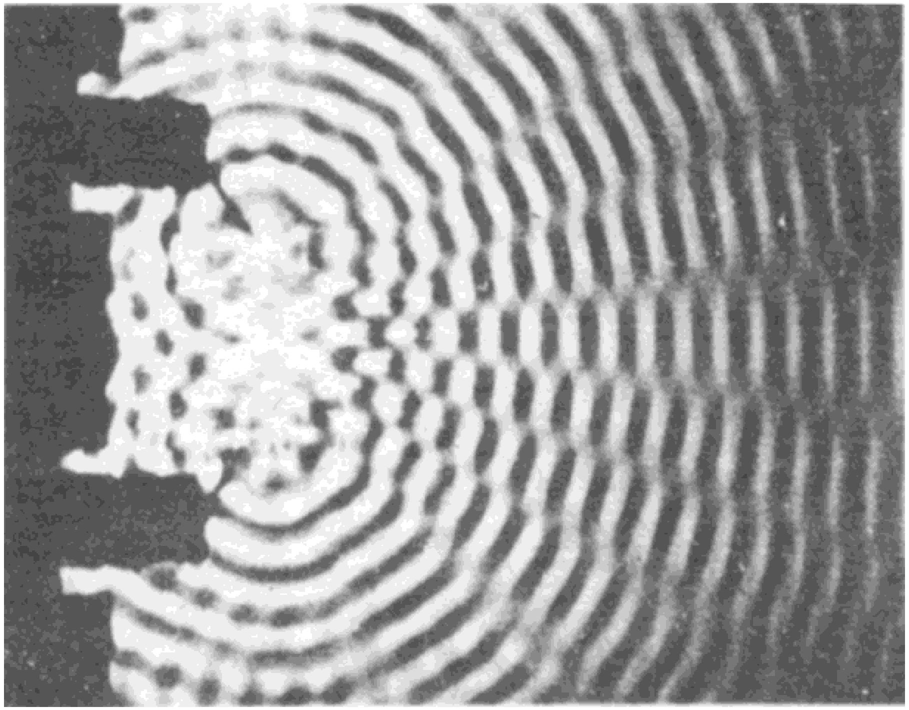
$I = I_{\min}$ —— destructive interference.

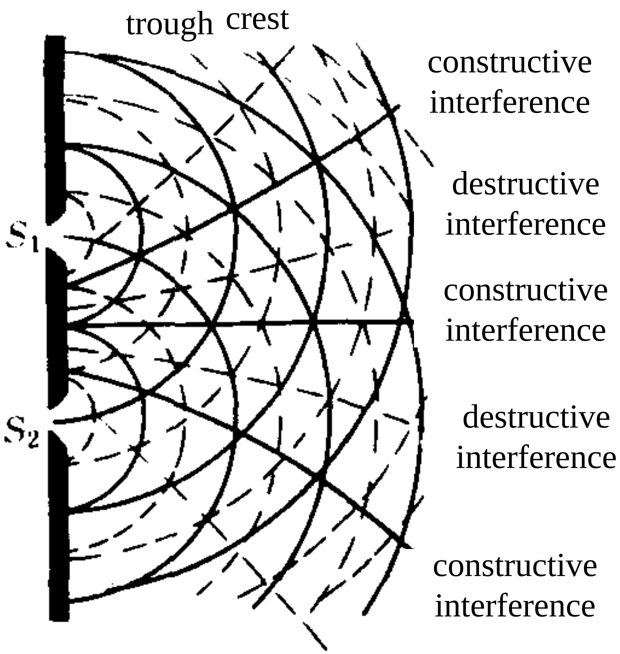




The interference pattern

Interference produces a redistribution of energy, out of the regions where it is destructive into the region where it is constructive.





$$\Delta\varphi = k(r_2 - r_1) - (\phi_2 - \phi_1)$$

$$I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2} \xrightarrow{I_1 = I_2} 4I_1$$

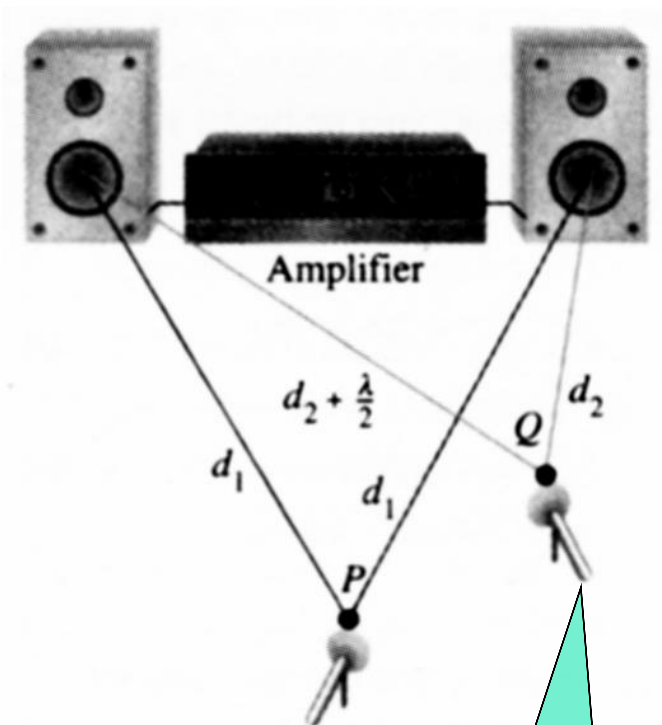
$$I_{\min} = I_1 + I_2 - 2\sqrt{I_1 I_2} \xrightarrow{I_1 = I_2} 0$$

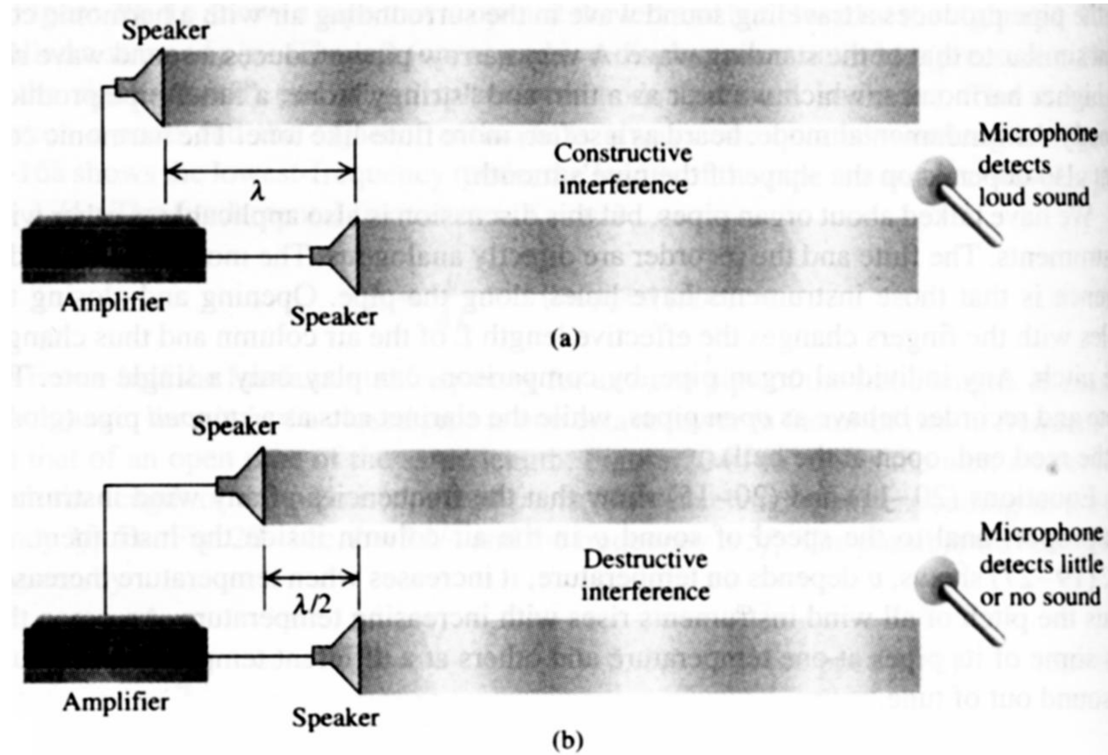
The trajectories of all points for both constructive or destructive are governed by

$$r_2 - r_1 = \text{constant}$$

The surfaces of hyperboloid.

The Examples of Interference





Constructive interference
Loud sound

Destructive interference
Little sound

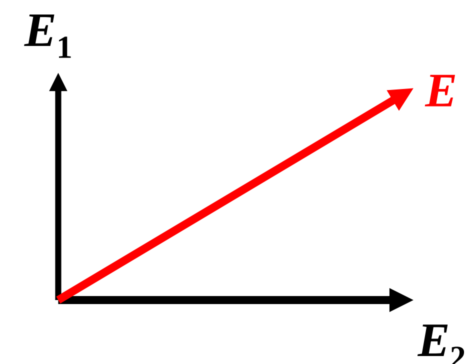
The conditions for coherent interference

- The conditions for coherent interference

  Coherent interference: when $2\sqrt{I_1 I_2}\cos\Delta\varphi \neq 0$

  Incoherent interference: when $2\sqrt{I_1 I_2}\cos\Delta\varphi = 0$

  - Have the same components of vibrations;



$$E_1^2 + E_2^2 = E^2$$

without an interference term

$$I_1 + I_2 = I$$

  - Have the same frequencies;

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\left[\left(\omega_2 - \omega_1\right)t - k(r_2 - r_1) + (\phi_2(t) - \phi_1(t))\right]$$

$\omega_1 \neq \omega_2 \qquad \omega \sim 10^6\,\mathrm{Hz}, \quad \Delta\omega \sim 10^5 - 10^6\,\mathrm{Hz}$

$$\bar{I} = \frac{1}{\tau}\int_0^{\tau} I\,dt = I_1 + I_2$$

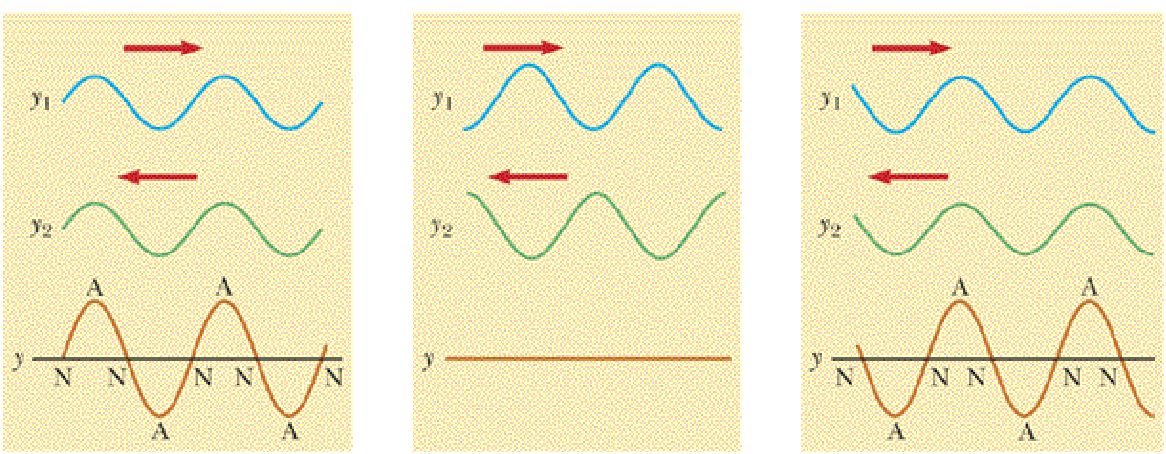
  - Phase difference is unchanged.

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\left[\underbrace{\Delta\phi(t)}_{\text{Varies with time}} - k(r_2 - r_1)\right]$$

$$\bar{I} = \frac{1}{\tau}\int_0^{\tau} I\,dt = I_1 + I_2$$

§ 12 Standing waves (p341)

- Consider two waves that are identical except for traveling in opposite direction $+x:\ y_1 = A\cos(\omega t - kx),\quad -x:\ y_2 = A\cos(\omega t + kx)$
  - Resultant wave: $\boxed{y = y_1 + y_2 = 2A\left(\cos kx\right)\cos\omega t}$
- Features of standing wave
  - x and t appear separately, not in the combination x±vt required for a traveling wave. The equation looks like more a simple harmonic motion than a wave motion.
  - In traveling wave each particle of the string vibrates with the same amplitude. In standing wave, however, the amplitude is not the same for different particles but varies with the location x of the particle.

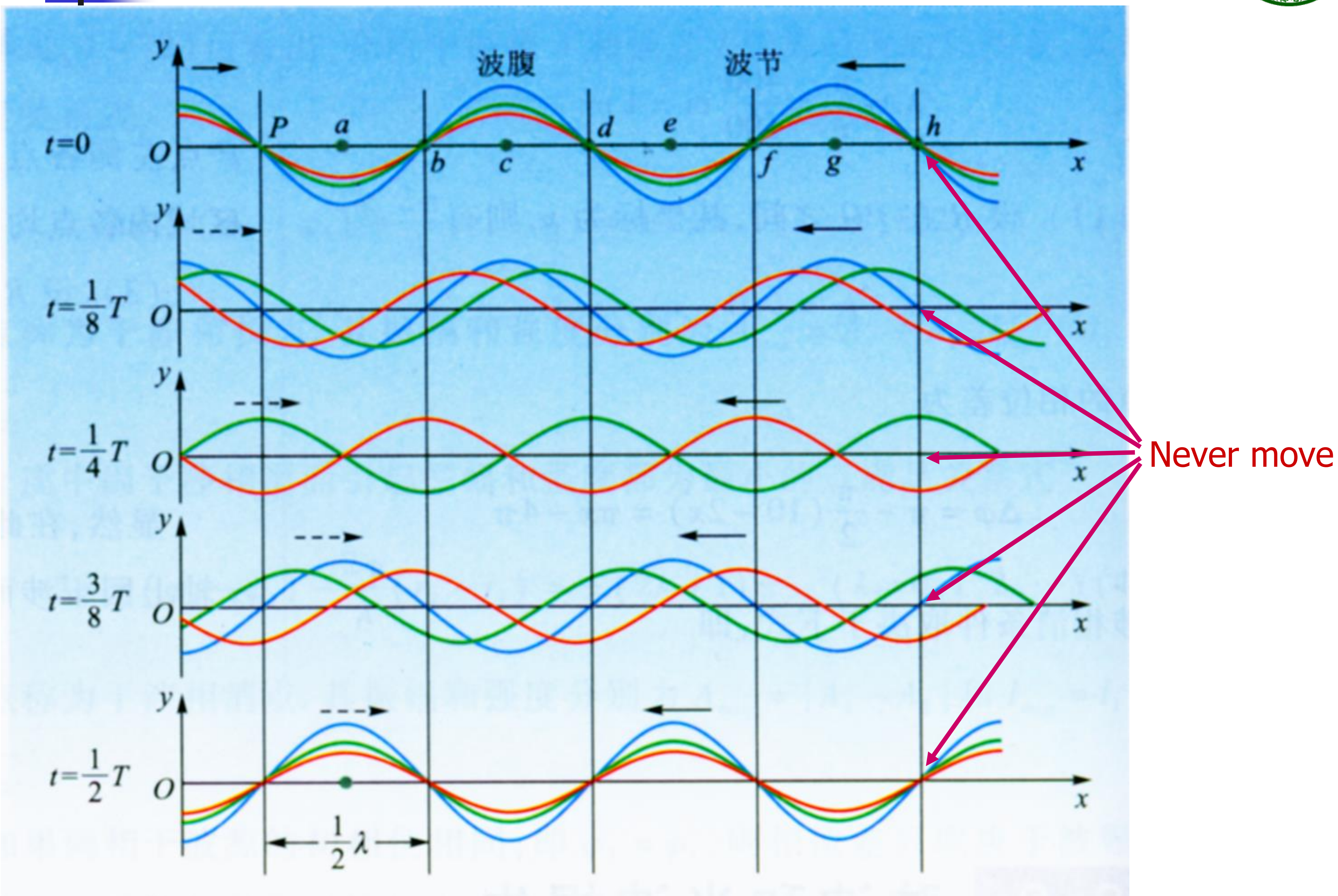


(a) $t = 0$ (b) $t = T/4$ (c) $t = T/2$

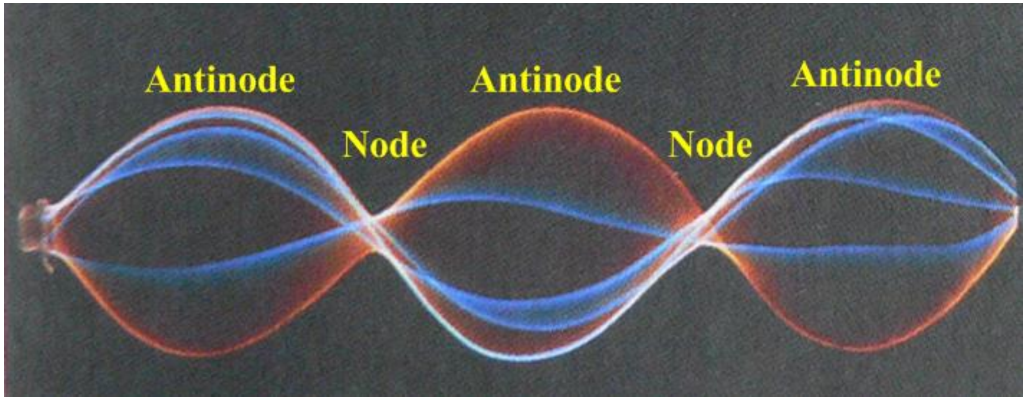
The amplitude for the particle located at x is

$$|2A\cos kx|$$

How is a standing wave produced?



- The nodes and the antinodes in a standing wave
  - Nodes: the amplitude |2Acoskx| has a minimum value of zero at positions where $$kx=\frac{2\pi}{\lambda}x=\pm(2m+1)\frac{\pi}{2},\quad x=\pm(m+\frac{1}{2})\frac{\lambda}{2},\quad m=0,1,2,\ldots$$
    The adjacent nodes are spaced one-half wavelength apart.
  - Antinodes: the amplitude has a maximum value of 2A at positions where $$kx=\frac{2\pi}{\lambda}x=\pm m\pi,\quad x=\pm m\frac{\lambda}{2},\quad m=0,1,2,\ldots$$
    The adjacent antinodes are also spaced one-half wavelength apart.
  - All the particles within two nodes are in phase, The particles at two side of a node are π out of phase.

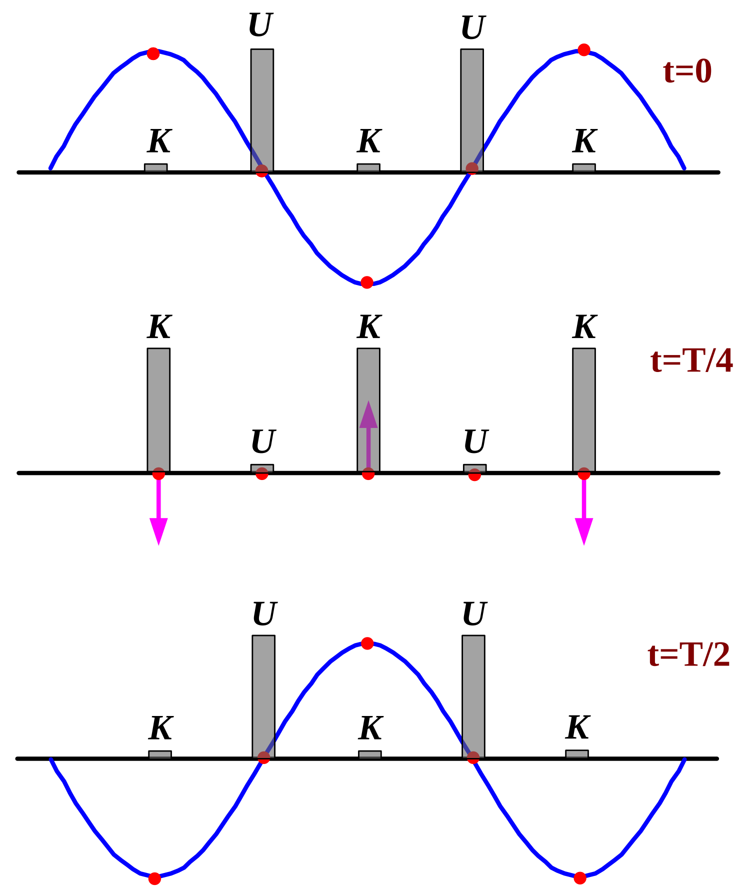


Energy feature of a standing wave

- Energy feature of a standing wave
  - t=0, T/2

    The kinetic energies for all the particles are zero. The potential energies of particles at nodes reach maximum.
  - t=T/4

    The potential energies for all the particles are zero. The kinetic energies of particles at antinodes reach maximum.
  - The energy can only exchange between node and antinode, and cannot be transported along the string to the right or to the left.

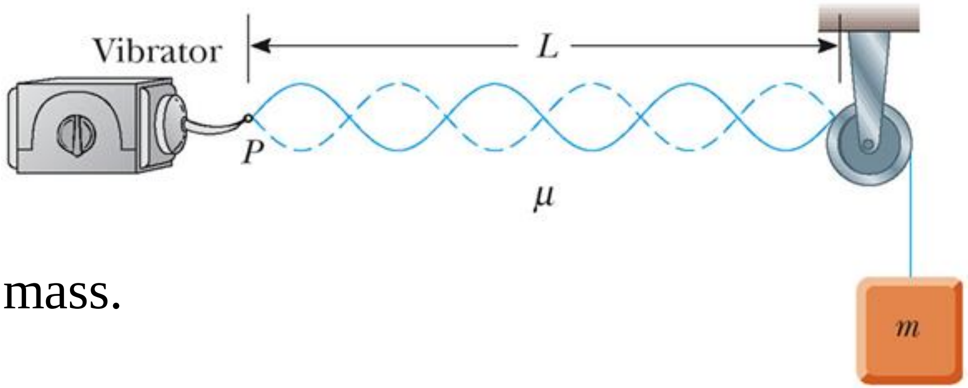


Example: An object is hung from a string (with mass density μ) that pass over a light pulley. The string is connected to a vibrator of length frequency f, and the length of the string between point P and the pulley is L. (1) What should the mass of the object be in order stimulate a clear standing wave in the string? (2) What is the largest mass for which standing waves could be observed?

Solution: In order to generate a clear standing wave, $L = n\frac{\lambda}{2}$

$$\lambda = \frac{v}{f}, \quad v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{mg}{\mu}} \Rightarrow L = \frac{n}{2f}\sqrt{\frac{mg}{\mu}}$$

$$m = \frac{4\mu f^2 L^2}{n^2 g}$$

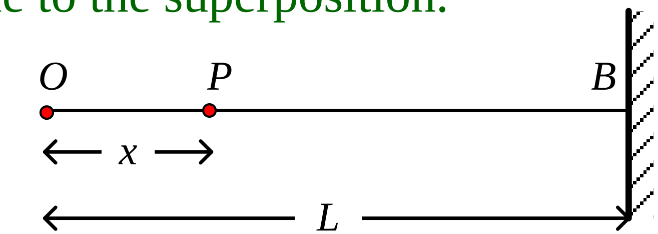


When n=1, we get the maximum value of mass.

$$m_{\max} = \frac{4\mu f^2 L^2}{g}$$

Example

Example: A string of length ***L*** and mass ***m*** is pulled under the tension ***T***. One end is connected to a vibrator of angular frequency ***ω***, amplitude ***A***, and initial phase angle ***ϕ***, while the other end is fixed to a wall. (1) Write the incident wave function; (2) Write the reflected wave function by wall (assume the amplitude is same as the incident wave); (3) Write the resultant wave function due to the superposition.



Solution: Take point O as origin, and positive x-direction to the right.

The wave velocity on the string: $v=\sqrt{T/\mu}=\sqrt{TL/m}$

(1) The wave function for incident wave: $$y_{in}=A\cos\left[\omega\left(t-\frac{x}{\sqrt{TL/m}}\right)+\phi\right]$$

(2) The phase retardation at arbitrary point P due to reflected wave with respect to point O is: $$-k(2L-x)\pm\pi=-\frac{\omega}{\sqrt{TL/m}}(2L-x)\pm\pi$$

The reflected wave:

$$y_{reflect}=A\cos\left[\omega t+\phi-\frac{\omega}{\sqrt{TL/m}}(2L-x)-\pi\right]$$
$$=A\cos\left[\omega t+\frac{\omega x}{\sqrt{TL/m}}-\frac{2\omega L}{\sqrt{TL/m}}+\phi-\pi\right]$$

Example Cont'd

(3) The resultant wave function:

$$y = y_{in} + y_{reflect} = 2A\cos\left[\frac{\omega}{\sqrt{TL/m}}(L-x)+\frac{\pi}{2}\right]\cos\left[\omega t-\left(\frac{\omega}{\sqrt{TL/m}}L-\phi+\frac{\pi}{2}\right)\right]$$

How many nodes exist in the string? Their locations?

§ 13 Standing waves in Strings

- Normal modes (简正模) for standing waves in strings
  - Normal modes: the possible natural patterns of vibration.

$$L = n\frac{\lambda}{2}, n = 1, 2, 3, \ldots$$ for string fixed at both ends

- Fundamental frequency (基频) and harmonic series (谐频系列)

$$\lambda_n = \frac{2L}{n}, \quad f_n = \frac{v}{\lambda_n} = \frac{n}{2L}v = \frac{n}{2L}\sqrt{\frac{T}{\mu}}, \quad n = 1, 2, 3, \ldots$$

  - n=1, fundamental frequency corresponds to fundamental mode.

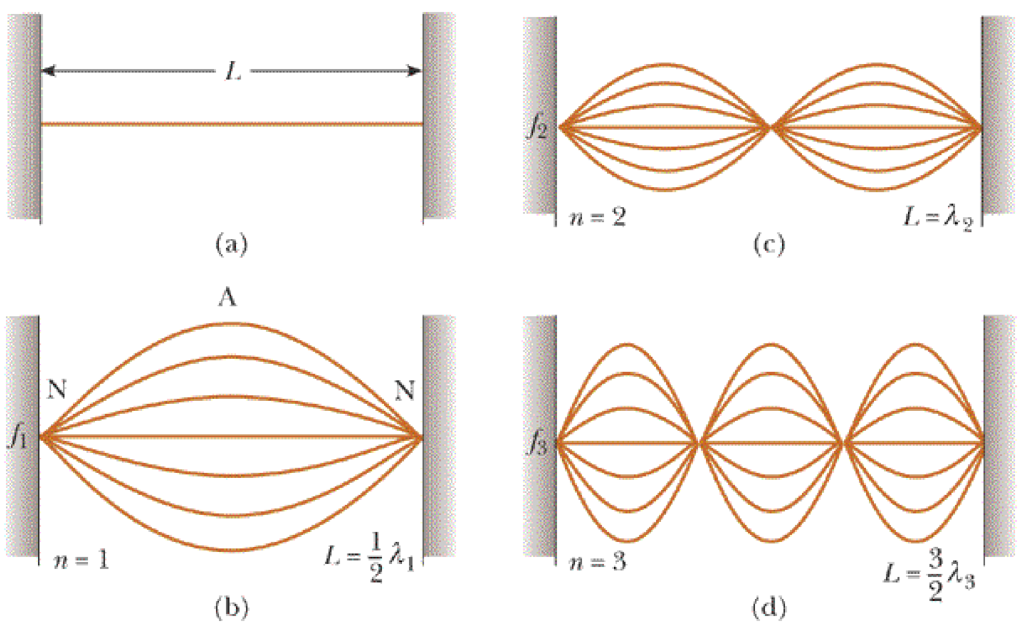
$$f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

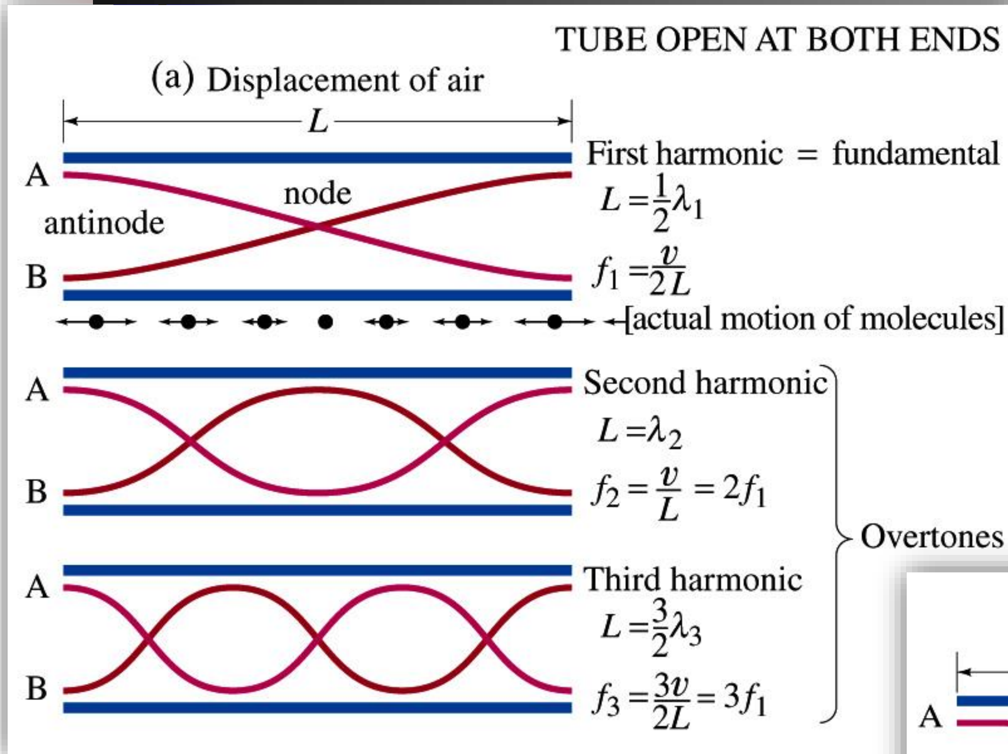
  - For higher modes

$$f_n = nf_1, \quad n = 1, 2, 3, \ldots$$

n-th harmonic frequency

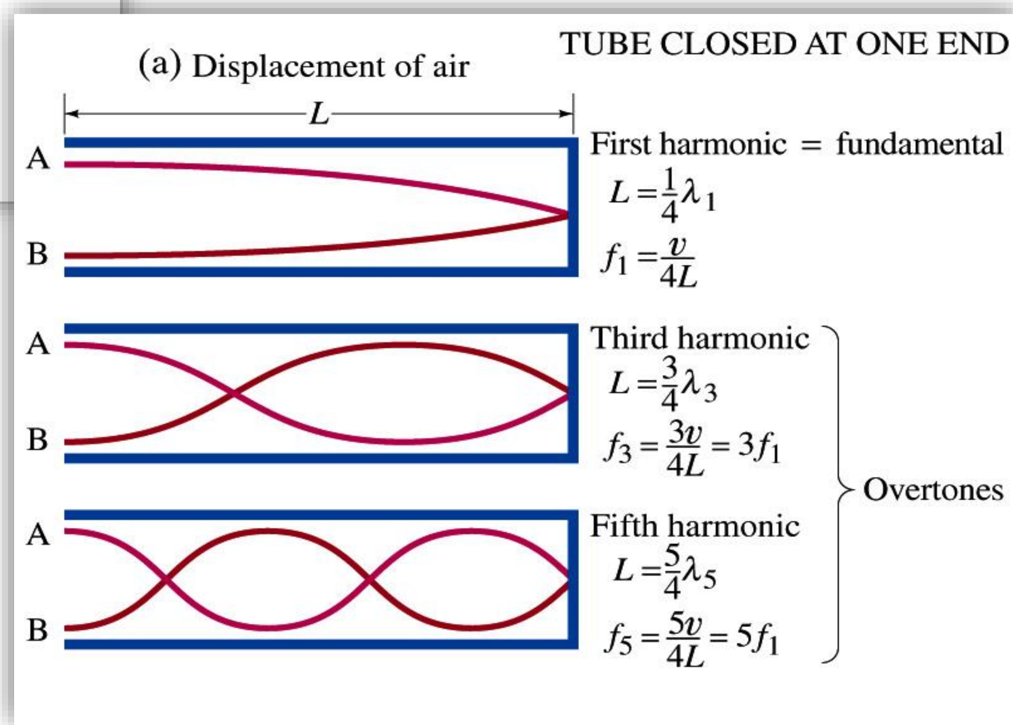
Harmonics other than fundamental are called overtone.





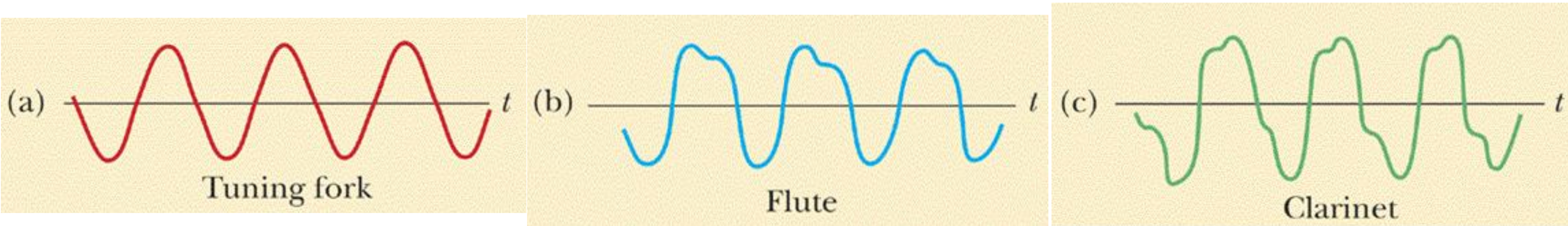
Waves in an Open-Open Pipe

Waves in an Open-Closed Pipe

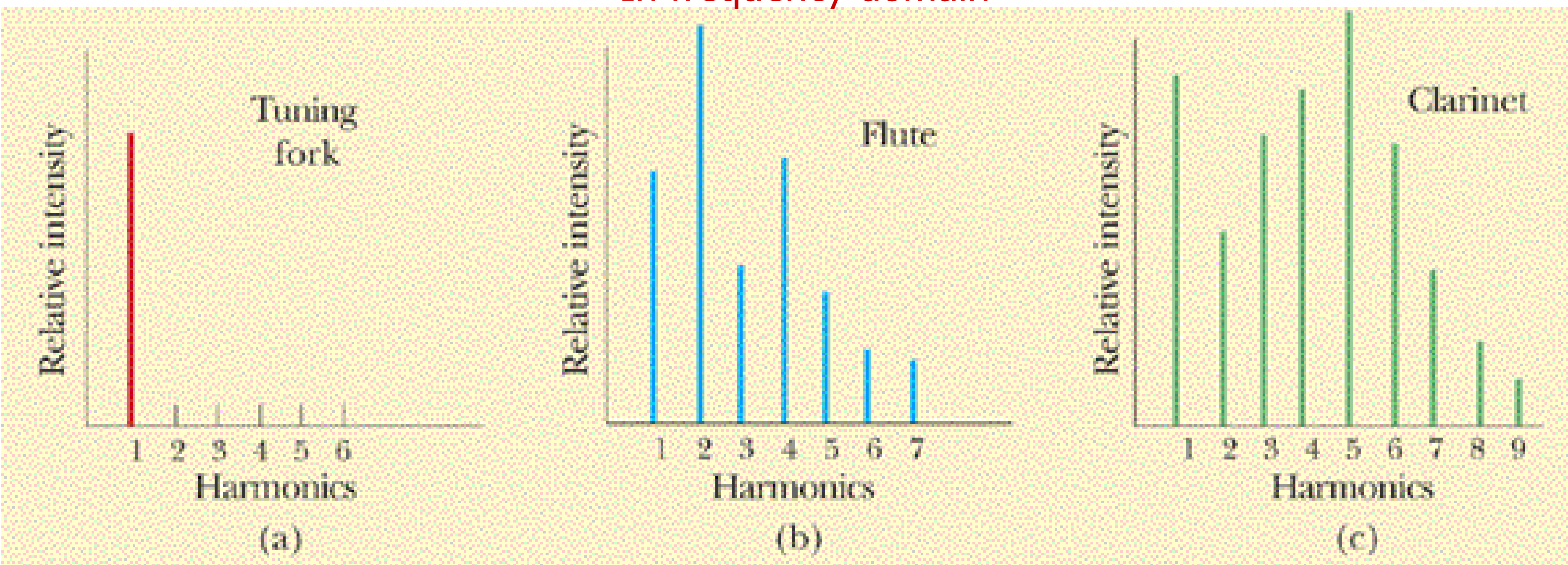


The Waveforms and Harmonics for some musical instruments

In time domain



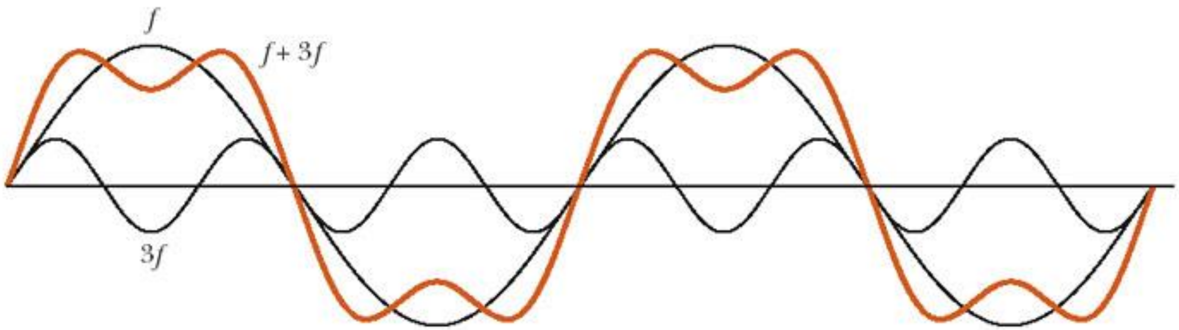
In frequency domain



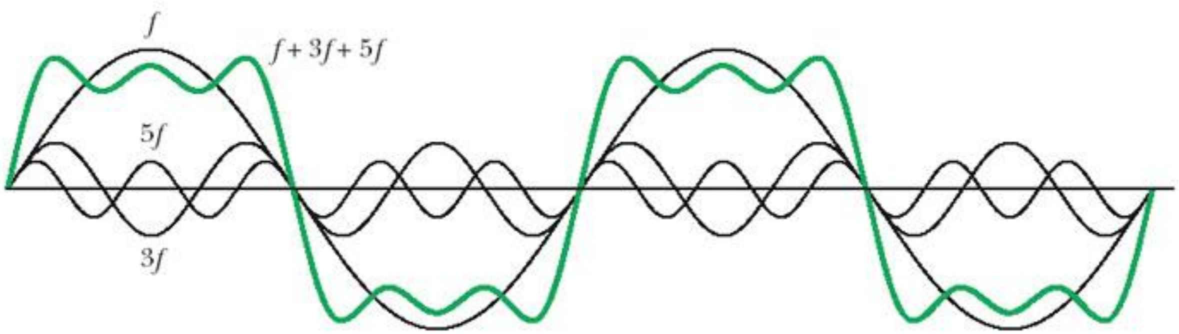
音叉　　　　长笛　　　　单簧管

How to construct a square wave

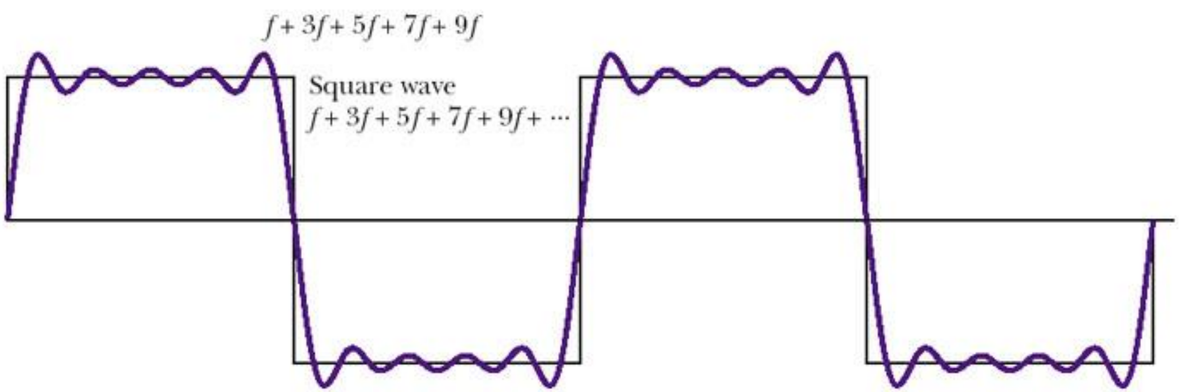
$$y = A\sin\omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t + \cdots$$



(a)



(b)



(c)

Homework:

P349-21,23,26

P350-35,43,44,46

P351-50

PS: Wave function is in the form of cosine.