7 Sinusoidal Steady State Analysis

Drill Exercises

DE 7.1 **[a]** $\omega = 2\pi f = 3769.91\,\text{rad/s}, \qquad f = 600\,\text{Hz}$

[b] $T = 1/f = 1.67\,\text{ms}$

[c] $V_m = 10\,\text{V}$

[d] $v(0) = 10(0.6) = 6\,\text{V}$

[e] $\phi = -53.13^\circ; \qquad \phi = \dfrac{-53.13^\circ(2\pi)}{360^\circ} = -0.9273\ \text{rad}$

[f] $3769.91t = 143.13/57.3 = 2.498\,\text{rad}, \qquad t = 662.62\,\mu\text{s}$

[g] $(dv/dt) = (-10)3769.91\sin(3769.91t - 53.13^\circ)$

$(dv/dt) = 0 \quad \text{when} \quad 3769.91t - 53.13^\circ = 0^\circ$

or $\qquad 3769.91t = 0.9273\,\text{rad}$

Therefore $\quad t = 245.97\,\mu\text{s}$

DE 7.2 $V_{\text{rms}} = \sqrt{\dfrac{1}{T}\displaystyle\int_0^{T/2} V_m^2 \sin^2 \dfrac{2\pi}{T} t\,dt}$

$$\int_0^{T/2} V_m^2 \sin^2\left(\frac{2\pi}{T}\right) t\,dt = \frac{V_m^2}{2}\int_0^{T/2}\left(1 - \cos\frac{4\pi}{T}t\right)\,dt = \frac{V_m^2 T}{4}$$

Therefore $\quad V_{\text{rms}} = \sqrt{\dfrac{1}{T}\dfrac{V_m^2 T}{4}} = \dfrac{V_m}{2}$

DE 7.3 **[a]** The numerical values of the terms in Eq. 7.9 are

$V_m = 20, \qquad R/L = 1066.67, \qquad \omega L = 60$

$\sqrt{R^2 + \omega^2 L^2} = 100$

$\phi = 25^\circ, \qquad \theta = \tan^{-1} 60/80, \qquad \theta = 36.87^\circ$

$i = \left[-195.72e^{-1066.67t} + 200\cos(800t - 11.87^\circ)\right]\ \text{mA}, \quad t \geq 0$

[b] Transient component $= -195.72e^{-1066.67t}$ mA
Steady-state component $= 200\cos(800t - 11.87°)$ mA

[c] By direct substitution into Eq 7.9, $i(1.875\,\text{ms}) = 28.39\,\text{mA}$

[d] 0.2 A, 800 rad/s, $-11.87°$

[e] The current lags the voltage by 36.87°.

DE 7.4 [a] $\mathbf{V} = 170\underline{/-40°}\,\text{V}$

[b] $\mathbf{I} = 10\underline{/-70°}\,\text{A}$

[c] $\mathbf{I} = 5\underline{/36.87°} + 10\underline{/-53.13°}$

$$= 4 + j3 + 6 - j8 = 10 - j5 = 11.18\underline{/-26.57°}\,\text{A}$$

[d] $\mathbf{V} = 300\underline{/45°} - 100\underline{/-60°} = 212.13 + j212.13 - (50 - j86.60)$

$$= 162.13 + j298.73 = 339.90\underline{/61.51°}\,\text{mV}$$

DE 7.5 [a] $v = 18.6\cos(\omega t - 54°)\,\text{V}$

[b] $\mathbf{I} = 20\underline{/45°} - 50\underline{/-30°} = 14.14 + j14.14 - 43.3 + j25$

$$= -29.16 + j39.14 = 48.81\underline{/126.68°}$$

Therefore $i = 48.81\cos(\omega t + 126.68°)\,\text{mA}$

[c] $\mathbf{V} = 20 + j80 - 30\underline{/15°} = 20 + j80 - 28.98 - j7.76$

$$= -8.98 + j72.24 = 72.79\underline{/97.08°}$$

$v = 72.79\cos(\omega t + 97.08°)\,\text{V}$

DE 7.6 [a] $\omega L = (10^4)(20 \times 10^{-3}) = 200\,\Omega$

[b] $Z_L = j200\,\Omega$

[c] $\mathbf{V}_L = \mathbf{I}Z_L = (10\underline{/30°})(200\underline{/90°}) \times 10^{-3} = 2\underline{/120°}\,\text{V}$

[d] $v_L = 2\cos(10{,}000t + 120°)\,\text{V}$

DE 7.7 [a] $X_C = \dfrac{-1}{\omega C} = -\dfrac{10^6}{4000(5)} = -50\,\Omega$

[b] $Z_C = jX_C = -j50\,\Omega$

[c] $\mathbf{I} = \dfrac{30\underline{/25°}}{50\underline{/-90°}} = 0.6\underline{/115°}\,\text{A}$

[d] $i = 0.6\cos(4000t + 115°)\,\text{A}$

DE 7.8 $\mathbf{I}_1 = 100\underline{/25^\circ} = 90.63 + j42.26$

$$\mathbf{I}_2 = 100\underline{/145^\circ} = -81.92 + j57.36$$

$$\mathbf{I}_3 = 100\underline{/-95^\circ} = -8.71 - j99.62$$

$$\mathbf{I}_4 = -(\mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3) = (0 + j0)\,\text{A}, \qquad \text{therefore} \quad i_4 = 0\,\text{A}$$

DE 7.9 **[a]** $\mathbf{I} = \dfrac{125\underline{/-60^\circ}}{|Z|\underline{/\theta_z}} = \dfrac{125}{|Z|}\underline{/(-60 - \theta_Z)^\circ}$

But $-60 - \theta_Z = -105^\circ \qquad \therefore \quad \theta_Z = 45^\circ$

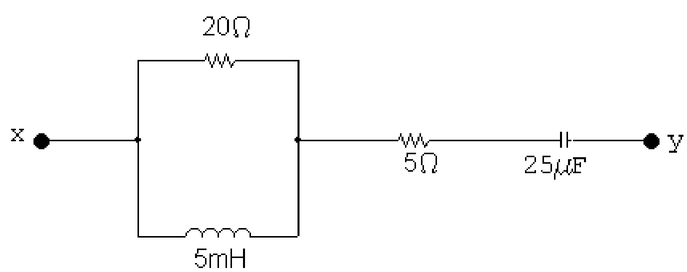
$$Z = 90 + j160 + jX_C$$

$$\therefore \quad X_C = -70\,\Omega; \qquad -\frac{1}{\omega C} = -70$$

$$\therefore \quad C = \frac{1}{(70)(5000)} = 2.86\,\mu\text{F}$$

[b] $\mathbf{I} = \dfrac{125\underline{/-60^\circ}}{(90 + j90)} = 0.982\underline{/-105^\circ}A; \qquad \therefore \quad |\mathbf{I}| = 0.982\,\text{A}$

DE 7.10 **[a]**



$$\omega = 2000\,\text{rad/s}$$

$$\omega L = 10\,\Omega, \qquad \frac{-1}{\omega C} = -20\,\Omega$$

$$Z_{xy} = \frac{20(j10)}{(20 + j10)} + 5 - j20 = 4 + j8 + 5 - j20 = (9 - j12)\,\Omega$$

[b] $\omega L = 40\,\Omega, \qquad \dfrac{-1}{\omega C} = -5\,\Omega$

$$Z_{xy} = 5 - j5 + \left[\frac{(20)(j40)}{20 + j40}\right] = 5 - j5 + 16 + j8 = (21 + j3)\,\Omega$$

[c] $Z_{xy} = \left[\dfrac{20(j\omega L)}{20 + j\omega L}\right] + \left(5 - \dfrac{j10^6}{25\omega}\right)$

$$= \frac{20\omega^2 L^2}{400 + \omega^2 L^2} + \frac{j400\omega L}{400 + \omega^2 L^2} + 5 - \frac{j10^6}{25\omega}$$

The impedance will be purely resistive when the j terms cancel, i.e.,

$$\frac{400\omega L}{400 + \omega^2 L^2} = \frac{10^6}{25\omega}$$

Solving for ω yields $\omega = 4000\,\text{rad/s}$.

[d] $Z_{xy} = \dfrac{20\omega^2 L^2}{400 + \omega^2 L^2} + 5 = 10 + 5 = 15\,\Omega$

DE 7.11

$$\mathbf{V} = 150\underline{/0^\circ}, \qquad \mathbf{I}_s = \frac{150\underline{/0^\circ}}{15} = 10\underline{/0^\circ}\,\text{A}$$

$$\mathbf{I}_L = \frac{10(20)}{20 + j20} = 5 - j5 = 7.07\underline{/-45^\circ}\,\text{A}$$

$$i_L = 7.07\cos(4000t - 45^\circ)\,\text{A}, \qquad I_m = 7.07\,\text{A}$$

DE 7.12 [a] $Y = \dfrac{1}{3 + j4} + \dfrac{1}{16 - j12} + \dfrac{1}{-j4}$

$$= 0.12 - j0.16 + 0.04 + j0.03 + j0.25$$

$$= 0.16 + j0.12 = 200\underline{/36.87^\circ}\,\text{mS}$$

[b] $G = 160\,\text{mS}$

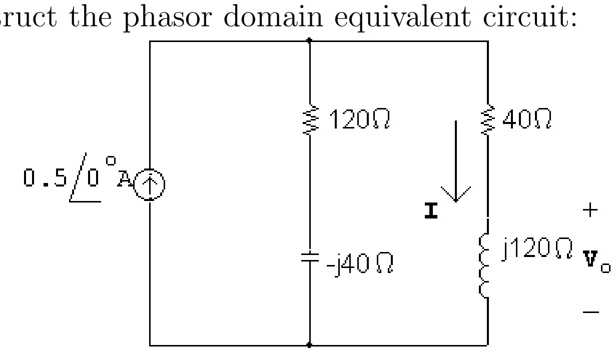
[c] $B = 120\,\text{mS}$

[d] $\mathbf{I} = 8\underline{/0^\circ}\,\text{A}, \qquad \mathbf{V} = \dfrac{\mathbf{I}}{Y} = \dfrac{8}{0.2\underline{/36.87^\circ}} = 40\underline{/-36.87^\circ}\,\text{V}$

$$\mathbf{I}_C = \frac{\mathbf{V}}{Z_C} = \frac{40\underline{/-36.87^\circ}}{4\underline{/-90^\circ}} = 10\underline{/53.13^\circ}\,\text{A}$$

$$i_C = 10\cos(\omega t + 53.13^\circ)\,\text{A}, \qquad I_m = 10\,\text{A}$$

DE 7.13 Construct the phasor domain equivalent circuit:



$$\mathbf{I} = \frac{0.5(120 - j40)}{160 + j80} = 0.25 - j0.25\,\text{A}$$

$$\mathbf{V}_o = j120\mathbf{I} = 30 + j30 = 42.43\angle 45^\circ$$

$$v_o = 42.43\cos(2000t + 45^\circ)\text{ V}$$

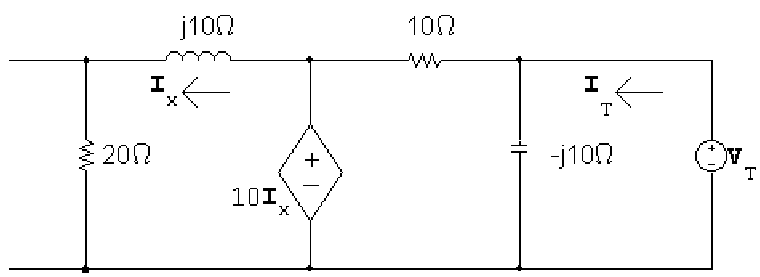
DE 7.14 Use the lower node as the reference node. Let $\mathbf{V}_1$ = node voltage across the $20\,\Omega$ resistor and $\mathbf{V}_{\text{Th}}$ = node voltage across the capacitor. Writing the node voltage equations gives us

$$\frac{\mathbf{V}_1}{20} - 2\angle 45^\circ + \frac{\mathbf{V}_1 - 10\mathbf{I}_x}{j10} = 0 \quad \text{and} \quad \mathbf{V}_{\text{Th}} = \frac{-j10}{10 - j10}(10\mathbf{I}_x)$$

We also have

$$\mathbf{I}_x = \frac{\mathbf{V}_1}{20}$$

Solving these equations for $\mathbf{V}_{\text{Th}}$ gives $\mathbf{V}_{\text{Th}} = 10\angle 45^\circ V$. To find the Thévenin impedance, we remove the independent current source and apply a test voltage source at the terminals a, b. Thus



It follows from the circuit that

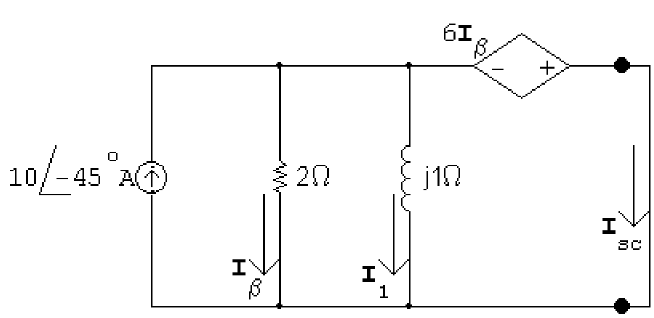
$$10\mathbf{I}_x = (20 + j10)\mathbf{I}_x$$

Therefore

$$\mathbf{I}_x = 0 \quad \text{and} \quad \mathbf{I}_T = \frac{\mathbf{V}_T}{-j10} + \frac{\mathbf{V}_T}{10}$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T}, \quad \text{therefore} \quad Z_{\text{Th}} = (5 - j5)\,\Omega$$

DE 7.15 Short circuit current



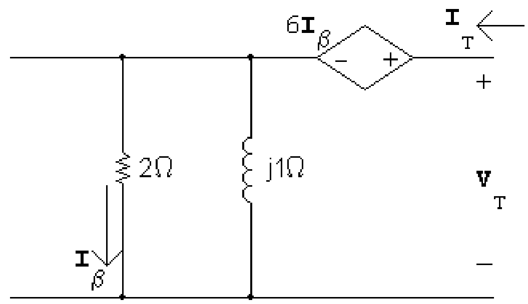
With the short circuit

$$\mathbf{I}_\beta = \frac{-6\mathbf{I}_\beta}{2}$$

$$2\mathbf{I}_\beta = -6\mathbf{I}_\beta; \qquad \therefore \quad \mathbf{I}_\beta = 0$$

$$\mathbf{I}_1 = 0; \qquad \therefore \quad \mathbf{I}_{sc} = 10\underline{/-45^\circ}\,\mathrm{A} = \mathbf{I}_N$$

The Norton impedance is the same as the Thévenin impedance. Thus



$$\mathbf{V}_T = 6\mathbf{I}_\beta + 2\mathbf{I}_\beta = 8\mathbf{I}_\beta, \qquad \mathbf{I}_\beta = \frac{j1}{2+j1}\mathbf{I}_T$$

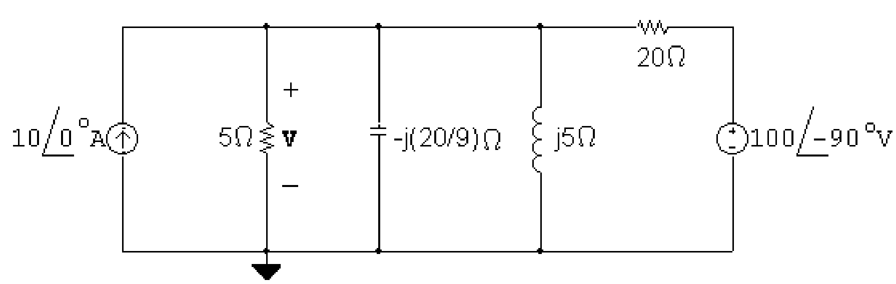
$$Z_{\mathrm{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T} = \frac{8\mathbf{I}_\beta}{[(2+j1)/j1]\mathbf{I}_\beta} = \frac{j8}{2+j1} = 1.6 + j3.2\,\Omega$$

DE 7.16 The phasor domain circuit is as shown in the following diagram. The node voltage equation is

$$-10 + \frac{\mathbf{V}}{5} + \frac{9\mathbf{V}}{-j20} + \frac{\mathbf{V}}{j5} + \frac{\mathbf{V} - 100\underline{/-90^\circ}}{20} = 0$$

Therefore $\mathbf{V} = 10 - j30 = 31.62\underline{/-71.57^\circ}$

Therefore $v = 31.62\cos(50{,}000t - 71.57^\circ)\,\mathrm{V}$



DE 7.17 Let $\mathbf{I}_a$, $\mathbf{I}_b$, and $\mathbf{I}_c$ be the three clockwise mesh currents going from left to right. Summing the voltages around meshes a and b gives

$$33.8 = (1 + j2)\mathbf{I}_a + (3 - j5)(\mathbf{I}_a - \mathbf{I}_b)$$

and

$$0 = (3 - j5)(\mathbf{I}_b - \mathbf{I}_a) + 2(\mathbf{I}_b - \mathbf{I}_c).$$

But

$$\mathbf{V}_x = -j5(\mathbf{I}_a - \mathbf{I}_b),$$

therefore

$$\mathbf{I}_c = -0.75[-j5(\mathbf{I}_a - \mathbf{I}_b)].$$

Solving for $\mathbf{I} = \mathbf{I}_a = 29 + j2 = 29.07\underline{/3.95^\circ}$ A.

DE 7.18 [a] $\mathbf{V} = 100\underline{/-45^\circ}$ V, $\quad \mathbf{I} = 20\underline{/15^\circ}$ A

Therefore

$$P = \frac{1}{2}(100)(20)\cos[-45 - (15)] = 500\,\text{W}, \qquad \text{A} \rightarrow \text{B}$$

$$Q = 1000 \sin -60^\circ = -866.03\ \text{VAR}, \qquad \text{B} \rightarrow \text{A}$$

[b] $\mathbf{V} = 100\underline{/-45^\circ}$, $\quad \mathbf{I} = 20\underline{/165^\circ}$

$$P = 1000\cos(-210^\circ) = -866.03\,\text{W}, \qquad \text{B} \rightarrow \text{A}$$

$$Q = 1000\sin(-210^\circ) = 500\ \text{VAR}, \qquad \text{A} \rightarrow \text{B}$$

[c] $\mathbf{V} = 106\underline{/-45^\circ}$, $\quad \mathbf{I} = 20\underline{/-105^\circ}$

$$P = 1000\cos(60^\circ) = 500\,\text{W}, \qquad \text{A} \rightarrow \text{B}$$

$$Q = 1000\sin(60^\circ) = 866.03\ \text{VAR}, \qquad \text{A} \rightarrow \text{B}$$

[d] $P = 1000\cos(-120^\circ) = -500\,\text{W}, \qquad \text{B} \rightarrow \text{A}$

$$Q = 1000\sin(-120^\circ) = -866.03\ \text{VAR}, \qquad \text{B} \rightarrow \text{A}$$

DE 7.19

$$p_f = \cos(\theta_v - \theta_i) = \cos[15 - (75)] = \cos -60^\circ = 0.5 \text{ leading}$$

$$r_f = \sin(\theta_v - \theta_i) = \sin -60^\circ = -0.866$$

DE 7.20 From Example 7.4,

$$I_{\text{eff}} = \frac{0.18}{\sqrt{3}}$$

$$\begin{aligned} P &= I_{\text{eff}}^2 R \\ &= \left(\frac{0.0324}{3}\right)(5000) \\ &= 54\,\text{W} \end{aligned}$$

DE 7.21 **[a]** $Z = (39 + j26)\|(-j52) = 48 - j20 = 52\underline{/-22.62^\circ}\,\Omega$

$$\text{Therefore} \quad \mathbf{I}_\ell = \frac{250\underline{/0^\circ}}{48 - j20 + 1 + j4} = 4.85\underline{/18.08^\circ}\,\text{A(rms)}$$

$$\mathbf{V}_\text{L} = Z\mathbf{I}_\ell = (52\underline{/-22.62^\circ})(4.85\underline{/18.08^\circ}) = 252.20\underline{/-4.54^\circ}\,\text{V(rms)}$$

$$\mathbf{I}_\text{L} = \frac{\mathbf{V}_\text{L}}{39 + j26} = 5.38\underline{/-38.23^\circ}\,\text{A(rms)}$$

[b] $S_\text{L} = (252.20\underline{/-4.54^\circ})(5.38\underline{/+38.23^\circ}) = 1357\underline{/33.69^\circ}$

$= (1129.09 + j752.73)\,\text{VA}$

$P_\text{L} = 1129.09\,\text{W}; \qquad Q_\text{L} = 752.73\,\text{VAR}$

[c] $P_\ell = |\mathbf{I}_\ell|^2 1 = (4.85)^2 \cdot 1 = 23.52\,\text{W}; \qquad Q_\ell = |\mathbf{I}_\ell|^2 4 = 94.09\,\text{VAR}$

[d] $S_g(\text{delivering}) = 250\mathbf{I}_\ell^* = (1152.62 - j376.36)\,\text{VA}$

Therefore the source is delivering 1152.62 W and absorbing 376.36 magnetizing VAR.

[e] $Q_\text{cap} = \dfrac{|\mathbf{V}_\text{L}|^2}{-52} = \dfrac{(252.20)^2}{-52} = -1223.18\,\text{VAR}$

Therefore the capacitor is delivering 1223.18 magnetizing VAR.

Check: $94.09 + 752.73 + 376.36 = 1223.18\,\text{VAR}$ and

$1129.09 + 23.52 = 1152.62\,\text{W}$

DE 7.22 Series circuit derivation:

$$250\mathbf{I}^* = (40{,}000 - j30{,}000)$$

$$\text{Therefore} \quad \mathbf{I}^* = 160 - j120 = 200\underline{/-36.87^\circ}\,\text{A(rms)}$$

$$\mathbf{I} = 200\underline{/36.87^\circ}\,\text{A(rms)}$$

$$Z = \frac{250}{200\underline{/36.87^\circ}} = 1.25\underline{/-36.87^\circ} = (1 - j0.75)\,\Omega$$

Therefore $\quad R = 1\,\Omega, \qquad X_\mathrm{C} = -0.75\,\Omega$

Parallel circuit derivation:

$$P = \frac{(250)^2}{R}; \qquad \text{therefore} \quad R = \frac{(250)^2}{40{,}000} = 1.5625\,\Omega$$

$$Q = \frac{(250)^2}{X_\mathrm{C}}; \qquad \text{therefore} \quad X_\mathrm{C} = \frac{(250)^2}{-30{,}000} = -2.083\,\Omega$$

DE 7.23

$$S_1 = 15{,}000(0.6) + j15{,}000(0.8) = 9000 + j12{,}000\,\mathrm{VA}$$

$$S_2 = 6000(0.8) + j6000(0.6) = 4800 - j3600\,\mathrm{VA}$$

$$S_T = S_1 + S_2 = 13{,}800 + j8400\,\mathrm{VA}$$

$$S_T = 200\mathbf{I}^*; \qquad \text{therefore} \quad \mathbf{I}^* = 69 + j42 \qquad \mathbf{I} = 69 - j42\,\mathrm{A}$$

$$\mathbf{V}_s = 200 + j\mathbf{I} = 200 + j69 + 42 = 242 + j69 = 251.64\underline{/15.91^\circ}\,\mathrm{V(rms)}$$

Problems

P 7.1 **[a]** By hypothesis

$$i = 10\cos(\omega t + \theta)$$

$$\frac{di}{dt} = -10\omega\sin(\omega t + \theta)$$

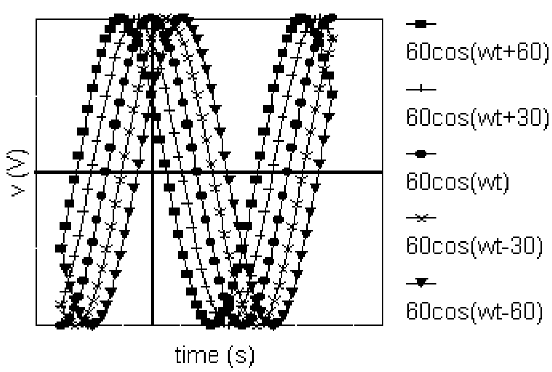
$$\therefore \quad 10\omega = 20{,}000\pi; \qquad \omega = 2000\pi\,\text{rad/s}$$

[b] $f = \dfrac{\omega}{2\pi} = 1000$ Hz; $\qquad T = \dfrac{1}{f} = 1$ ms $= 1000\,\mu$s

$$\frac{150}{1000} = \frac{3}{20}, \qquad \therefore \quad \theta = -90 - \frac{3}{20}(360) = -144^\circ$$

$$\therefore \quad i = 10\cos(2000\pi t - 144^\circ)\,\text{A}$$

P 7.2



[a] Left as ϕ becomes more positive

[b] Right

P 7.3 **[a]** 170 V

[b] $2\pi f = 120\pi; \qquad f = 60$Hz

[c] $\omega = 120\pi = 376.99$ rad/s

[d] $\theta(\text{rad}) = \dfrac{-\pi}{180}(60) = \dfrac{-\pi}{3} = -1.05$ rad

[e] $\theta = -60^\circ$

[f] $T = \dfrac{1}{f} = \dfrac{1}{60} = 16.67\,$ms

[g] $120\pi t - \dfrac{\pi}{3} = 0; \qquad \therefore \quad t = \dfrac{1}{360} = 2.78\,$ms

[h] $v = 170\cos\left[120\pi\left(t + \frac{0.125}{18}\right) - \frac{\pi}{3}\right]$

$= 170\cos[120\pi t + (15\pi/18) - (\pi/3)]$

$= 170\cos[120\pi t + (\pi/2)]$

$= -170\sin 120\pi t\,\text{V}$

[i] $120\pi(t - t_o) - (\pi/3) = 120\pi t - (\pi/2)$

$\therefore \quad 120\pi t_o = \frac{\pi}{6}; \qquad t_o = \frac{25}{18}\,\text{ms}$

[j] $120\pi(t - t_o) - (\pi/3) = 120\pi t$

$\therefore \quad 120\pi t_o = \frac{\pi}{3}; \qquad t_o = \frac{25}{9}\,\text{ms}$

P 7.4 **[a]** $\frac{T}{2} = \frac{1250}{6} + \frac{250}{6} = 250\,\mu\text{s}; \qquad T = 500\,\mu\text{s}$

$f = \frac{1}{T} = \frac{10^6}{500} = 2000\text{Hz}$

[b] $v = V_m\sin(\omega t + \theta)$

$\omega = 2\pi f = 4000\pi \text{ rad/s}$

$4000\pi\left(\frac{-250}{6} \times 10^{-6}\right) + \theta = 0; \qquad \therefore \quad \theta = \frac{\pi}{6} \text{ rad} = 30^\circ$

$v = V_m\sin[4000\pi t + 30^\circ]$

$75 = V_m\sin 30^\circ; \qquad V_m = 150\,\text{V}$

$v = 150\sin[4000\pi t + 30^\circ] = 150\cos[4000\pi t - 60^\circ]\,\text{V}$

P 7.5 **[a]** From Eq. 7.9 we have

$$L\frac{di}{dt} = \frac{V_m R\cos(\phi - \theta)}{\sqrt{R^2 + \omega^2L^2}}e^{-(R/L)t} - \frac{\omega L V_m\sin(\omega t + \phi - \theta)}{\sqrt{R^2 + \omega^2L^2}}$$

$$Ri = \frac{-V_m R\cos(\phi - \theta)e^{-(R/L)t}}{\sqrt{R^2 + \omega^2L^2}} + \frac{V_m R\cos(\omega t + \phi - \theta)}{\sqrt{R^2 + \omega^2L^2}}$$

$$L\frac{di}{dt} + Ri = V_m\left[\frac{R\cos(\omega t + \phi - \theta) - \omega L\sin(\omega t + \phi - \theta)}{\sqrt{R^2 + \omega^2L^2}}\right]$$

But

$$\frac{R}{\sqrt{R^2 + \omega^2L^2}} = \cos\theta \quad \text{and} \quad \frac{\omega L}{\sqrt{R^2 + \omega^2L^2}} = \sin\theta$$

Therefore the right-hand side reduces to

$V_m \cos(\omega t + \phi)$

At $t = 0$, Eq. 7.9 reduces to

$$i(0) = \frac{-V_m \cos(\phi - \theta)}{\sqrt{R^2 - \omega^2 L^2}} + \frac{V_m \cos(\phi - \theta)}{\sqrt{R^2 + \omega^2 L^2}} = 0$$

[b] $i_{ss} = \dfrac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t + \phi - \theta)$

Therefore

$$L\frac{di_{ss}}{dt} = \frac{-\omega L V_m}{\sqrt{R^2 + \omega^2 L^2}} \sin(\omega t + \phi - \theta)$$

and

$$Ri_{ss} = \frac{V_m R}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t + \phi - \theta)$$

$$L\frac{di_{ss}}{dt} + Ri_{ss} = V_m \left[\frac{R\cos(\omega t + \phi - \theta) - \omega L \sin(\omega t + \phi - \theta)}{\sqrt{R^2 + \omega^2 L^2}}\right]$$

$$= V_m \cos(\omega t + \phi)$$

P 7.6 **[a]** $\mathbf{Y} = 100\underline{/45^\circ} + 500\underline{/-60^\circ} = 483.86\underline{/-48.48^\circ}$

$y = 483.86 \cos(300t - 48.48^\circ)$

[b] $\mathbf{Y} = 250\underline{/30^\circ} - 150\underline{/50^\circ} = 120.51\underline{/4.8^\circ}$

$y = 120.51 \cos(377t + 4.8^\circ)$

[c] $\mathbf{Y} = 60\underline{/60^\circ} - 120\underline{/-215^\circ} + 100\underline{/90^\circ} = 152.88\underline{/32.94^\circ}$

$y = 152.88 \cos(100t + 32.94^\circ)$

[d] $\mathbf{Y} = 100\underline{/40^\circ} + 100\underline{/160^\circ} + 100\underline{/-80^\circ} = 0$

$y = 0$

P 7.7

$$u = \int_{t_o}^{t_o+T} V_m^2 \cos^2(\omega t + \phi)\, dt$$

$$= V_m^2 \int_{t_o}^{t_o+T} \frac{1}{2} + \frac{1}{2}\cos(2\omega t + 2\phi)\, dt$$

$$= \frac{V_m^2}{2}\left\{\int_{t_o}^{t_o+T} dt + \int_{t_o}^{t_o+T} \cos(2\omega t + 2\phi)\, dt\right\}$$

$$= \frac{V_m^2}{2}\left\{T + \frac{1}{2\omega}\left[\sin(2\omega t + 2\phi)\,\Big|_{t_o}^{t_o+T}\right]\right\}$$

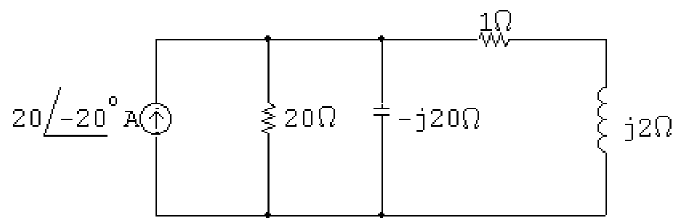
$$= \frac{V_m^2}{2}\left\{T + \frac{1}{2\omega}\left[\sin(2\omega t_o + 4\pi + 2\phi) - \sin(2\omega t_o + 2\phi)\right]\right\}$$

$$= V_m^2\left(\frac{T}{2}\right) + \frac{1}{2\omega}(0) = V_m^2\left(\frac{T}{2}\right)$$

P 7.8 $V_m = \sqrt{2}V_{\text{rms}} = \sqrt{2}(120) = 169.71\,\text{V}$

P 7.9 **[a]** $j\omega L = j(5\times10^4)(40\times10^{-6}) = j2\,\Omega$

$$\frac{1}{j\omega C} = -j\frac{10^6}{5\times10^4} = -j20\,\Omega; \qquad \mathbf{I}_g = 20\underline{/-20^\circ}\,\text{A}$$



[b] $\mathbf{V}_o = 20\underline{/-20^\circ}Z_e$

$$Z_e = \frac{1}{Y_e}; \qquad Y_e = \frac{1}{20} + j\frac{1}{20} + \frac{1}{1+j2}$$

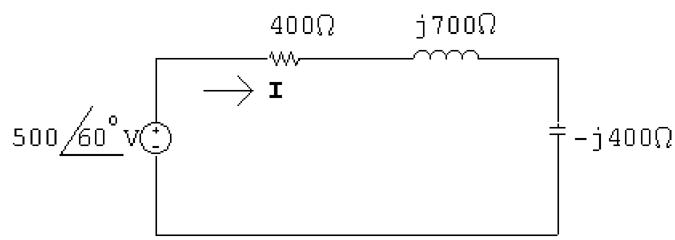
$$Y_e = 0.05 + j0.05 + 0.20 - j0.40 = 0.25 - j0.35\,\text{S}$$

$$Z_e = \frac{1}{0.25 - j0.35} = 2.32\underline{/54.46^\circ}\,\Omega$$

$$\mathbf{V}_o = (20\underline{/-20^\circ})(2.32\underline{/54.46^\circ}) = 46.4\underline{/34.46^\circ}\,\text{V}$$

[c] $v_o = 46.4\cos(5\times10^4 t + 34.46^\circ)\,\text{V}$

P 7.10 **[a]**



[b] $\mathbf{I} = \dfrac{500\underline{/60^\circ}}{400 + j700 - j400} = 1\underline{/23.13^\circ}\,\text{A}$

[c] $i = 1\cos(8000t + 23.13^\circ)\,\text{A}$

P 7.11 **[a]** 50Hz

[b] $\theta_v = 0^\circ$

[c] $\mathbf{I} = \dfrac{340\underline{/0^\circ}}{j\omega L} = \dfrac{340}{\omega L}\underline{/-90^\circ} = 8.5\underline{/-90^\circ}; \qquad \theta_i = -90^\circ$

[d] $\dfrac{340}{\omega L} = 8.5; \qquad \omega L = 40\,\Omega$

[e] $L = \dfrac{40}{100\pi} = \dfrac{400}{\pi}\,\text{mH} = 127.32\,\text{mH}$

[f] $Z_L = j\omega L = j40\,\Omega$

P 7.12 **[a]** $\omega = 2\pi f = 80\pi \times 10^3 = 251.33\,\text{krad/s} = 251{,}327.41\,\text{rad/s}$

[b] $\mathbf{I} = \dfrac{2.5 \times 10^{-3}\underline{/0^\circ}}{1/j\omega C} = j\omega C(2.5 \times 10^{-3})\underline{/0^\circ} = 2.5 \times 10^{-3}\omega C\underline{/90^\circ}$

$\therefore \quad \theta_i = 90^\circ$

[c] $125.66 \times 10^{-6} = 2.5 \times 10^{-3}\,\omega C$

$$\frac{1}{\omega C} = \frac{2.5 \times 10^{-3}}{125.66 \times 10^{-6}} = 19.89\,\Omega, \quad \therefore \quad X_{\text{C}} = -19.89\,\Omega$$

[d] $C = \dfrac{1}{19.89(\omega)} = \dfrac{1}{(19.89)(80\pi \times 10^3)}$

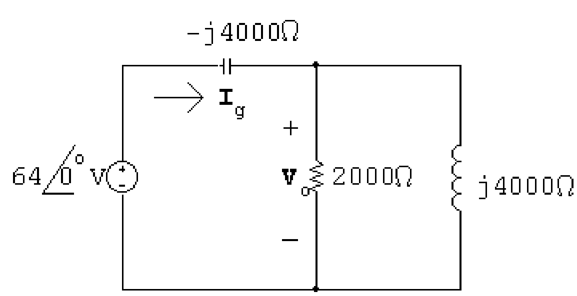
$C = 0.2 \times 10^{-6} = 0.2\,\mu\text{F}$

[e] $Z_c = j\left(\dfrac{-1}{\omega C}\right) = -j19.89\,\Omega$

P 7.13 $\dfrac{1}{j\omega C} = \dfrac{10^9}{(31.25)(8000)} = -j4000\,\Omega$

$j\omega L = j8000(500)10^{-3} = j4000\,\Omega$

$\mathbf{V}_g = 64\underline{/0^\circ}\,\text{V}$



$$Z_e = \frac{(2000)(j4000)}{2000 + j4000} = 1600 + j800\,\Omega$$

$$Z_T = 1600 + j800 - j4000 = 1600 - j3200\,\Omega$$

$$\mathbf{I}_g = \frac{64\underline{/0^\circ}}{1600 - j3200} = 8 + j16\,\text{mA}$$

$$\mathbf{V}_o = Z_e\mathbf{I}_g = (1600 + j800)(0.008 + j0.016) = j32 = 32\underline{/90^\circ}\,\text{V}$$

$$v_o = 32\cos(8000t + 90^\circ)\,\text{V}$$

P 7.14 $Z = 400 + j(5)(40) - j\dfrac{1000}{(5)(0.4)} = 500\underline{/-36.87^\circ}\,\Omega$

$$\mathbf{I}_o = \frac{750\underline{/0^\circ} \times 10^{-3}}{500\underline{/-36.87^\circ}} = 1.5\underline{/36.87^\circ}\,\text{mA}$$

$$i_o(t) = 1.5\cos(5000t + 36.87^\circ)\,\text{mA}$$

P 7.15 **[a]** $Z_p = \dfrac{\dfrac{R}{j\omega C}}{R + (1/j\omega C)} = \dfrac{R}{1 + j\omega RC}$

$$= \frac{12{,}500}{1 + j(1000)(12{,}500)C} = \frac{12{,}500}{1 + j12.5 \times 10^6 C}$$

$$= \frac{12{,}500(1 - j12.5 \times 10^6 C)}{1 + 156.25 \times 10^{12}C^2}$$

$$= \frac{12{,}500}{1 + 156.25 \times 10^{12}C^2} - j\frac{156.25 \times 10^9 C}{1 + 156.25 \times 10^{12}C^2}$$

$$j\omega L = j1000(5) = j5000$$

$$\therefore \quad 5000 = \frac{156.25 \times 10^9 C}{1 + 156.25 \times 10^{12}C^2}$$

$$\therefore \quad 781.25 \times 10^{15}C^2 - 156.25 \times 10^9 C + 5000 = 0$$

$$\therefore \quad C^2 - 20 \times 10^{-8}C + 64 \times 10^{-16} = 0$$

$$\therefore \quad C_{1,2} = 10 \times 10^{-8} \pm \sqrt{100 \times 10^{-16} - 64 \times 10^{-16}}$$

$$C_1 = 10 \times 10^{-8} + 6 \times 10^{-8} = 16 \times 10^{-8} = 160\,\text{nF} = 0.16\,\mu\text{F}$$

$$C_2 = 10 \times 10^{-8} - 6 \times 10^{-8} = 4 \times 10^{-8} = 40\,\text{nF} = 0.04\,\mu\text{F}$$

[b] $R_e = \dfrac{12{,}500}{1 + 156.25 \times 10^{12}C^2}$

When $C = 160\,\text{nF}$ $\qquad R_e = 2500\,\Omega$;

$$\mathbf{I}_g = \frac{250\underline{/0^\circ}}{2500} = 0.1\underline{/0^\circ}\,\text{A}; \qquad i_g = 100\cos 1000t\,\text{mA}$$

When $C = 40\,\text{nF}$ $\qquad R_e = 10{,}000\,\Omega$;

$$\mathbf{I}_g = \frac{250\underline{/0^\circ}}{10{,}000} = 0.025\underline{/0^\circ}\,\text{A}; \qquad i_g = 25\cos 1000t\,\text{mA}$$

P 7.16 **[a]** $Y_p = \dfrac{1}{10 + j2\omega} + j4 \times 10^{-3}\omega$

$$= \frac{10 - j2\omega}{100 + 4\omega^2} + j4 \times 10^{-3}\omega$$

$$= \frac{10}{100 + 4\omega^2} - \frac{j2\omega}{100 + 4\omega^2} + j4 \times 10^{-3}\omega$$

Y_p is real when

$$4 \times 10^{-3}\omega = \frac{2\omega}{100 + 4\omega^2}$$

or $\qquad \omega^2 = 100; \qquad \omega = 10$ rad/s; $\qquad f = 5/\pi = 1.59$Hz

[b] $Y_p(10\,\text{rad/s}) = \dfrac{10}{500} = 20\,\text{mS}$

$$Z_p(10\,\text{rad/s}) = \frac{10^3}{20} = 50\,\Omega$$

$$Z(10\,\text{rad/s}) = 50 + 150 = 200\,\Omega$$

$$\mathbf{I}_o = \frac{\mathbf{V}_g}{200}\,\text{A} = \frac{10\underline{/0^\circ}}{200} = 50\underline{/0^\circ}\,\text{mA}$$

$$i_o = 50\cos 10t\,\text{mA}$$

P 7.17 $\mathbf{V}_g = 50\underline{/-45^\circ}\,\text{V}; \qquad \mathbf{I}_g = 100\underline{/-8.13^\circ}\,\text{mA}$

$$Z = \frac{\mathbf{V}_g}{\mathbf{I}_g} = 500\underline{/-36.87^\circ}\,\Omega = 400 - j300\,\Omega$$

$$Z = 400 + j\left(0.04\omega - \frac{2.5 \times 10^6}{\omega}\right)$$

$$\therefore \quad 0.04\omega - \frac{2.5 \times 10^6}{\omega} = -300$$

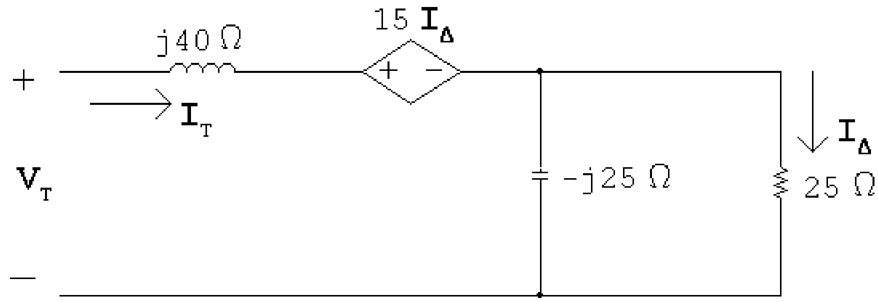
$$\therefore \quad \omega^2 + 7500\omega - 62.5 \times 10^6 = 0$$

$$\therefore \quad \omega = -3750 \pm \sqrt{(3750)^2 + 62.5 \times 10^6} = -3750 \pm 8750$$

$\omega > 0, \qquad \therefore \quad \omega = 5000\,\text{rad/s}$

P 7.18 $j\omega L = j1.6 \times 10^6(25 \times 10^{-6}) = j40\,\Omega$

$$\frac{1}{j\omega C} = \frac{10^{-6} \times 10^9}{j1.6(25)} = -j25\,\Omega$$



$$\mathbf{V}_T = j40\mathbf{I}_T + 15\mathbf{I}_\Delta + 25\mathbf{I}_\Delta$$

$$\mathbf{I}_\Delta = \frac{\mathbf{I}_T(-j25)}{25 - j25} = \frac{-j\mathbf{I}_T}{1 - j1}$$

$$\mathbf{V}_T = j40\mathbf{I}_T + 40\frac{(-j\mathbf{I}_T)}{1 - j1}$$

$$\frac{\mathbf{V}_T}{\mathbf{I}_T} = Z_{\text{ab}} = j40 + 20(-j)(1 + j) = 20 + j20\,\Omega = 28.28\underline{/45^\circ}\,\Omega$$

P 7.19 First find the admittance of the parallel branches

$$Y_p = \frac{1}{6 - j2} + \frac{1}{4 + j12} + \frac{1}{5} + \frac{1}{j10} = 0.375 - j0.125\,\text{S}$$

$$Z_p = \frac{1}{Y_p} = \frac{1}{0.375 - j0.125} = 2.4 + j0.8\,\Omega$$

$$Z_{\text{ab}} = -j12.8 + 2.4 + j0.8 + 13.6 = 16 - j12\,\Omega$$

$$Y_{\text{ab}} = \frac{1}{Z_{\text{ab}}} = \frac{1}{16 - j12} = 0.04 + j0.03\,\text{S}$$

$$= 40 + j30\,\text{mS} = 50\underline{/36.87^\circ}\,\text{mS}$$

P 7.20 **[a]** $Z_g = 4000 - j\dfrac{10^9}{25\omega} + \dfrac{10^4(j2\omega)}{10^4 + j2\omega}$

$$= 4000 - j\frac{10^9}{25\omega} + \frac{2\times 10^4 j\omega(10^4 - j2\omega)}{10^8 + 4\omega^2}$$

$$= 4000 - j\frac{10^9}{25\omega} + \frac{4\times 10^4\omega^2}{10^8 + 4\omega^2} + j\frac{2\times 10^8\omega}{10^8 + 4\omega^2}$$

$$\therefore \quad \frac{10^9}{25\omega} = \frac{0.2\times 10^9\omega}{10^8 + 4\omega^2}$$

$$10^8 + 4\omega^2 = 5\omega^2$$

$$\omega^2 = 10^8; \qquad \omega = 10{,}000\,\text{rad/s}$$

[b] When $\omega = 10{,}000$ rad/s

$$Z_g = 4000 + \frac{4\times 10^4(10^4)^2}{10^8 + 4(10^4)^2} = 12{,}000\,\Omega$$

$$\therefore \quad \mathbf{I}_g = \frac{45\underline{/0^\circ}}{12{,}000} = 3.75\underline{/0^\circ}\,\text{mA}$$

$$\mathbf{V}_o = \mathbf{V}_g - \mathbf{I}_g Z_1$$

$$Z_1 = 4000 - j\frac{10^9}{25\times 10^4} = 4000 - j4000\,\Omega$$

$$\mathbf{V}_o = 45\underline{/0^\circ} - (3.75\times 10^{-3})(4000 - j4000) = 45 - (15 - j15)$$

$$= 30 + j15 = 33.54\underline{/26.57^\circ}\,\text{V}$$

$$v_o = 33.54\cos(10{,}000t + 26.57^\circ)\,\text{V}$$

P 7.21 **[a]** $Z_1 = 1600 - j\dfrac{10^9}{10^4(62.5)} = 1600 - j1600\,\Omega$

$$Z_1 = \frac{4000(j10^4L)}{4000 + j10^4L} = \frac{4\times 10^5L^2 + j16\times 10^4L}{16 + 100L^2}$$

$$Z_T = Z_1 + Z_2 = 1600 + \frac{4\times 10^5L^2}{16 + 100L^2} - j1600 + j\frac{16\times 10^4L}{16 + 100L^2}$$

Z_T is resistive when

$$\frac{16\times 10^4L}{16 + 100L^2} = 1600 \qquad \text{or}$$

$$L^2 - L + 0.16 = 0$$

Solving, $L_1 = 0.8$ H and $L_2 = 0.2$ H.

[b] When $L = 0.8$ H:

$$Z_T = 1600 + \frac{4 \times 10^5(0.64)}{16 + 64} = 4800\,\Omega$$

$$\mathbf{I}_g = \frac{96\underline{/0^\circ}}{4.8} \times 10^{-3} = 20\underline{/0^\circ}\,\text{mA}$$

$$i_g = 20\cos 10{,}000t\,\text{mA}$$

When $L = 0.2$ H:

$$Z_T = 1600 + \frac{4 \times 10^5(0.04)}{16 + 4} = 2400\,\Omega$$

$$i_g = 40\cos 10{,}000t\,\text{mA}$$

P 7.22 **[a]** $$Z_{\text{ab}} = j5\omega + \frac{(4000)(10^9/j\omega 625)}{4000 + (10^9/j625\omega)}$$

$$= j5\omega + \frac{4 \times 10^{12}}{2500 \times 10^3 j\omega + 10^9}$$

$$= j5\omega + \frac{4 \times 10^7}{10^4 + j25\omega}$$

$$= j5\omega + \frac{4 \times 10^{11}}{10^8 + 625\omega^2} - j\frac{100 \times 10^7\omega}{10^8 + 625\omega^2}$$

$$\therefore \quad 5 = \frac{10^9}{10^8 + 625\omega^2}$$

$$5 \times 10^8 + 3125\omega^2 = 10^9$$

$$\omega = 4 \times 10^2 = 400\,\text{rad/s}$$

[b] $$Z_{\text{ab}}(400) = j2000 + \frac{(4000)(-j4000)}{4000 - j4000} = 2\,\text{k}\Omega$$

P 7.23 $$Z_1 = 10 - j40\,\Omega$$

$$Z_2 = \frac{(5 - j10)(10 + j30)}{15 + j20} = 10 - j10\,\Omega$$

$$Z_3 = \frac{20(j20)}{20 + j20} = 10 + j10\,\Omega$$

$$\therefore \quad Z_{\text{ab}} = Z_1 + Z_2 + Z_3 = 30 - j40\,\Omega = 50\underline{/-53.13^\circ}\,\Omega$$

P 7.24 **[a]** $Y_1 = \dfrac{1}{5000} = 0.2 \times 10^{-3}\,\text{S}$

$$Y_2 = \frac{1}{1200 + j0.2\omega}$$

$$= \frac{1200}{1.44 \times 10^6 + 0.04\omega^2} - j\frac{0.2\omega}{1.44 \times 10^6 + 0.04\omega^2}$$

$$Y_3 = j\omega 50 \times 10^{-9}$$

$$Y_T = Y_1 + Y_2 + Y_3$$

For i_g and v_o to be in phase the j component of Y_T must be zero; thus,

$$\omega 50 \times 10^{-9} = \frac{0.2\omega}{1.44 \times 10^6 + 0.04\omega^2}$$

or

$$0.04\omega^2 + 1.44 \times 10^6 = \frac{0.2 \times 10^9}{50} = 4 \times 10^6$$

$$\therefore \quad 0.04\omega^2 = 2.56 \times 10^6 \qquad \therefore \quad \omega = 8000\,\text{rad/s} = 8\,\text{krad/s}$$

[b] $Y_T = 0.2 \times 10^{-3} + \dfrac{1200}{1.44 \times 10^6 + 0.04(64) \times 10^6} = 0.5 \times 10^{-3}\,\text{S}$

$$\therefore \quad Z_T = 2000\,\Omega$$

$$\mathbf{V}_o = (2.5 \times 10^{-3}\underline{/0^\circ})(2000) = 5\underline{/0^\circ}$$

$$v_o = 5\cos 8000t\,\text{V}$$

P 7.25 **[a]** $Z_1 = R_1 + j\omega L_1$

$$Z_2 = \frac{R_2(j\omega L_2)}{R_2 + j\omega L_2} = \frac{\omega^2 L_2^2 R_2 + j\omega L_2 R_2^2}{R^2 + \omega^2 L_2^2}$$

$$Z_1 = Z_2 \quad \text{when} \quad R_1 = \frac{\omega^2 L_2^2 R_2}{R_2^2 + \omega^2 L_2^2} \quad \text{and} \quad L_1 = \frac{R_2^2 L_2}{R_2^2 + \omega^2 L_2^2}$$

[b] $R_1 = \dfrac{(4 \times 10^8)(6.25)(5 \times 10^4)}{25 \times 10^8 + (4 \times 10^8)(6.25)} = 2.5 \times 10^4$

$$\therefore \quad R_1 = 25\,\text{k}\Omega$$

$$L_1 = \frac{(25 \times 10^8)2.5}{50 \times 10^8} = 1.25\,\text{H}$$

P 7.26 **[a]** $Y_2 = \dfrac{1}{R_2} - \dfrac{j}{\omega L_2}$

$$Y_1 = \frac{1}{R_1 + j\omega L_1} = \frac{R_1 - j\omega L_1}{R_1^2 + \omega^2 L_1^2}$$

Therefore $\quad Y_2 = Y_1 \quad$ when

$$R_2 = \frac{R_1^2 + \omega^2 L_1^2}{R_1} \quad \text{and} \quad L_2 = \frac{R_1^2 + \omega^2 L_1^2}{\omega^2 L_1}$$

[b] $R_2 = \dfrac{25 \times 10^6 + 10^8(0.25)}{5 \times 10^3} = 10 \times 10^3$

$$\therefore \quad R_2 = 10\,\text{k}\Omega$$

$$L_2 = \frac{50 \times 10^6}{10^8(0.5)} = 1\,\text{H}$$

P 7.27 **[a]** $Z_1 = R_1 - j\dfrac{1}{\omega C_1}$

$$Z_2 = \frac{R_2/j\omega C_2}{R_2 + (1/j\omega C_2)} = \frac{R_2}{1 + j\omega R_2 C_2} = \frac{R_2 - j\omega R_2^2 C_2}{1 + \omega^2 R_2^2 C_2^2}$$

$$Z_1 = Z_2 \quad \text{when} \quad R_1 = \frac{R_2}{1 + \omega^2 R_2^2 C_2^2} \quad \text{and}$$

$$\frac{1}{\omega C_1} = \frac{\omega R_2^2 C_2}{1 + \omega^2 R_2^2 C_2^2} \quad \text{or} \quad C_1 = \frac{1 + \omega^2 R_2^2 C_2^2}{\omega^2 R_2^2 C_2}$$

[b] $R_1 = \dfrac{500}{1 + (64 \times 10^8)(25 \times 10^4)(625 \times 10^{-18})} = 250\,\Omega$

$$C_1 = \frac{2}{(64 \times 10^8)(25 \times 10^4)(25 \times 10^{-9})} = 50\,\text{nF}$$

P 7.28 **[a]** $Y_2 = \dfrac{1}{R_2} + j\omega C_2$

$$Y_1 = \frac{1}{R_1 + (1/j\omega C_1)} = \frac{j\omega C_1}{1 + j\omega R_1 C_1} = \frac{\omega^2 R_1 C_1^2 + j\omega C_1}{1 + \omega^2 R_1^2 C_1^2}$$

Therefore $\quad Y_1 = Y_2 \quad$ when

$$R_2 = \frac{1 + \omega^2 R_1^2 C_1^2}{\omega^2 R_1 C_1^2} \quad \text{and} \quad C_2 = \frac{C_1}{1 + \omega^2 R_1^2 C_1^2}$$

[b] $R_2 = \dfrac{1 + (4 \times 10^8)(4 \times 10^6)(2500 \times 10^{-18})}{(4 \times 10^8)(2 \times 10^3)(2500 \times 10^{-18})} = 2500 = 2.5\text{k}\Omega$

$$C_2 = \frac{50 \times 10^{-9}}{5} = 10\,\text{nF}$$

P 7.29 [a] $\mathbf{V}_g = 150\underline{/20^\circ}$; $\quad \mathbf{I}_g = 30\underline{/-52^\circ}$

$$\therefore \quad Z = \frac{\mathbf{V}_g}{\mathbf{I}_g} = 5\underline{/72^\circ}\,\Omega$$

[b] i_g lags v_g by 72°:

$$2\pi f = 8000\pi; \qquad f = 4000\,\text{Hz}; \qquad T = 1/f = 250\,\mu\text{s}$$

$$\therefore \quad i_g \text{ lags } v_g \text{ by } \frac{72}{360}(250) = 50\,\mu\text{s}$$

P 7.30 $$\frac{1}{j\omega C} = -j\frac{10^6}{10^4} = -j100\,\Omega$$

$$j\omega L = j(500)(1) = j500\,\Omega$$

Let $Z_1 = 50 - j100\,\Omega$; $\quad Z_2 = 250 + j500\,\Omega$

$$\mathbf{I}_g = 125\underline{/0^\circ}\,\text{mA}$$

$$\mathbf{I}_o = \frac{\mathbf{I}_g Z_1}{Z_1 + Z_2} = \frac{125\underline{/0^\circ}(50 - j100)}{(300 + j400)}$$

$$= -12.5 - j25\,\text{mA} = 27.95\underline{/-116.57^\circ}\,\text{mA}$$

$$i_o = 27.95\cos(500t - 116.57^\circ)\,\text{mA}$$

P 7.31 $$Z_o = 600 - j\frac{10^6}{(5000)(0.25)} = 600 - j800\,\Omega$$

$$Z_T = 300 + j2000 + 600 - j800 = 900 + j1200\,\Omega = 1500\underline{/53.13^\circ}\,\Omega$$

$$\mathbf{V}_o = \mathbf{V}_g\frac{Z_o}{Z_T} = \frac{(75\underline{/0^\circ})(1000\underline{/-53.13^\circ})}{1500\underline{/53.13^\circ}} = 50\underline{/-106.26^\circ}\,\text{V}$$

$$v_o = 50\cos(5000t - 106.26^\circ)\,\text{V}$$

P 7.32 $$\mathbf{V}_1 = 240\underline{/53.13^\circ} = 144 + j192\,\text{V}$$

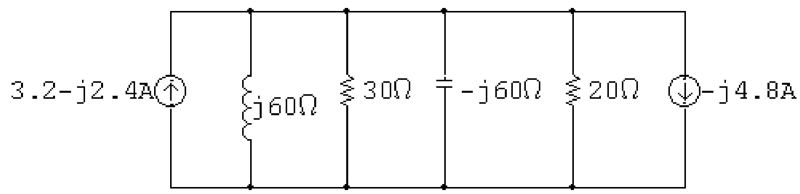
$$\mathbf{V}_2 = 96\underline{/-90^\circ} = -j96\,\text{V}$$

$$j\omega L = j(4000)(15 \times 10^{-3}) = j60\,\Omega$$

$$\frac{1}{j\omega C} = -j\frac{6\times 10^6}{(4000)(25)} = -j60\,\Omega$$

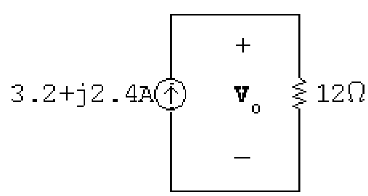
$$\frac{\mathbf{V}_1}{j60} = \frac{144+j192}{j60} = 3.2 - j2.4\,\text{A}$$

$$\frac{\mathbf{V}_2}{20} = -j\frac{96}{20} = -j4.8\,\text{A}$$



$$Y = \frac{1}{j60} + \frac{1}{30} + \frac{1}{-j60} + \frac{1}{20} = \frac{j5}{j60} = \frac{1}{12}$$

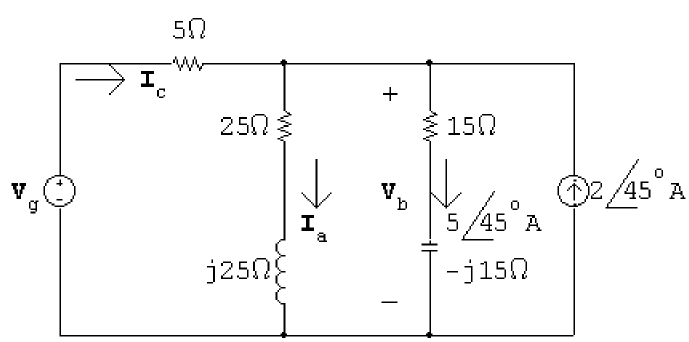
$$Z = \frac{1}{Y} = 12\,\Omega$$



$$\mathbf{V}_o = 12(3.2+j2.4) = 38.4 + j28.8\,\text{V} = 48\underline{/36.87^\circ}\,\text{V}$$

$$v_o = 48\cos(4000t + 36.87^\circ)\,\text{V}$$

P 7.33 **[a]**



$$\mathbf{V}_\text{b} = (15 - j15)5\underline{/45^\circ} = 75\sqrt{2}\underline{/0^\circ}\,\text{V}$$

$$\mathbf{I}_\mathrm{a} = \frac{75\sqrt{2}}{25 + j25} = 3\underline{/-45^\circ}\,\mathrm{A}$$

$$\mathbf{I}_\mathrm{c} = \mathbf{I}_\mathrm{a} + 5\underline{/45^\circ} - 2\underline{/45^\circ} = 3\sqrt{2}\,\mathrm{A}$$

$$\mathbf{V}_g = 5\mathbf{I}_c + \mathbf{V}_b = 15\sqrt{2} + 75\sqrt{2} = 90\sqrt{2}\,\mathrm{V} = 127.28\underline{/0^\circ}\,\mathrm{V}$$

[b] $i_\mathrm{a} = 3\cos(800t - 45^\circ)\,\mathrm{A}$

$i_\mathrm{c} = 4.24\cos 800t\,\mathrm{A}$

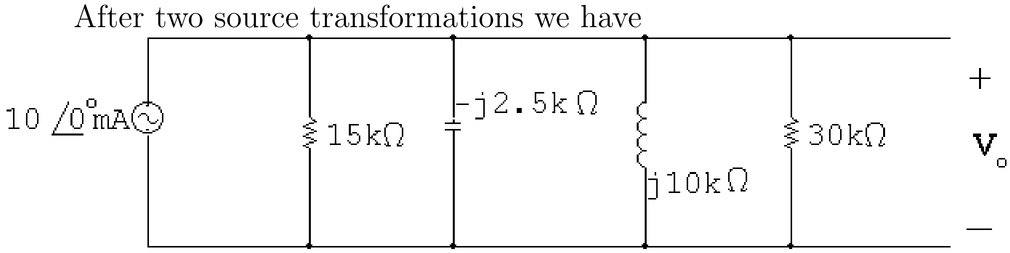
$v_g = 127.28\cos 800t\,\mathrm{V}$

P 7.34 $\mathbf{I}_s = 15\underline{/0^\circ}\,\mathrm{mA}$

$$\frac{1}{j\omega C} = \frac{10^6}{j0.05(8000)} = -j2500\,\Omega$$

$$j\omega L = j8000(1.25) = j10{,}000\,\Omega$$

After two source transformations we have



$$15\,\mathrm{k}\Omega \| 30\,\mathrm{k}\Omega = 10\,\mathrm{k}\Omega$$

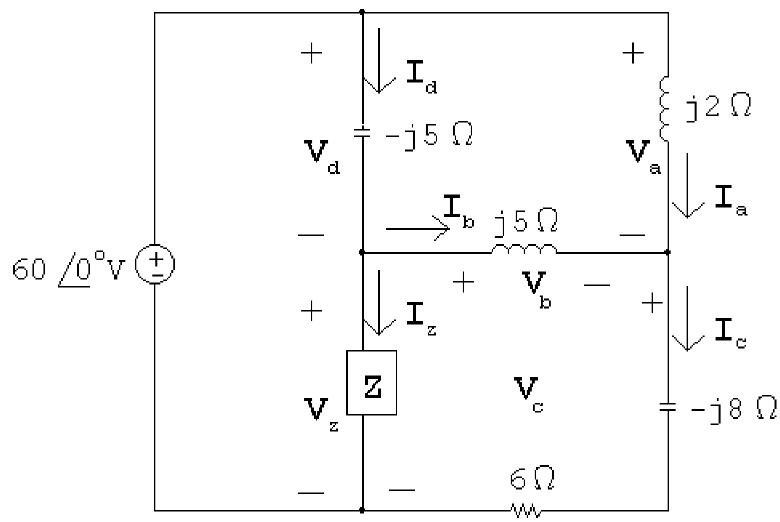
$$Y_o = \frac{10^{-3}}{10} + \frac{1}{-j2500} + \frac{1}{j10^4} = 10^{-4}(1 + j3)$$

$$Z_o = \frac{10^4}{1 + j3} = (1 - j3)\,\mathrm{k}\Omega$$

$$\mathbf{V}_o = \mathbf{I}_g Z_o = (10)(1 - j3) = 10 - j30 = 31.62\underline{/-71.57^\circ}\,\mathrm{V}$$

$$v_o = 31.62\cos(8000t - 71.57^\circ)\,\mathrm{V}$$

P 7.35



$$\mathbf{V}_\mathrm{a} = j2\mathbf{I}_\mathrm{a} = j2(-j5) = 10\underline{/0^\circ}\,\mathrm{V}$$

$$\mathbf{V}_\mathrm{c} = 60\underline{/0^\circ} - \mathbf{V}_\mathrm{a} = 50\underline{/0^\circ}\,\mathrm{V}$$

$$\mathbf{I}_\mathrm{c} = \frac{\mathbf{V}_\mathrm{c}}{6 - j8} = \frac{50\underline{/0^\circ}}{10\underline{/-53.13^\circ}} = 5\underline{/53.13^\circ} = 3 + j4\,\mathrm{A}$$

$$\mathbf{I}_\mathrm{b} = \mathbf{I}_\mathrm{c} - \mathbf{I}_\mathrm{a} = 3 + j4 - (-j5) = 3 + j9\,\mathrm{A} = 9.49\underline{/71.57^\circ}\,\mathrm{A}$$

$$\mathbf{V}_\mathrm{b} = \mathbf{I}_\mathrm{b}(j5) = (3 + j9)(j5) = -45 + j15\,\mathrm{V}$$

$$\mathbf{V}_\mathrm{z} = \mathbf{V}_\mathrm{b} + \mathbf{V}_\mathrm{c} = -45 + j15 + 50 + j0 = 5 + j15\,\mathrm{V}$$

$$\mathbf{V}_\mathrm{d} + \mathbf{V}_\mathrm{z} = 60\underline{/0^\circ}; \qquad \therefore \quad \mathbf{V}_\mathrm{d} = 60 - 5 - j15 = 55 - j15\,\mathrm{V}$$

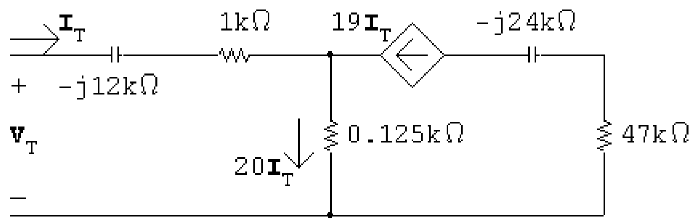
$$\mathbf{I}_\mathrm{d} = \frac{\mathbf{V}_\mathrm{d}}{-j5} = 3 + j11\,\mathrm{A}$$

$$\mathbf{I}_\mathrm{z} = \mathbf{I}_\mathrm{d} - \mathbf{I}_\mathrm{b} = 3 + j11 - 3 - j9 = j2\,\mathrm{A}$$

$$Z = \frac{\mathbf{V}_\mathrm{z}}{\mathbf{I}_\mathrm{z}} = \frac{5 + j15}{j2} = 7.5 - j2.5\,\Omega$$

P 7.36 $$\frac{1}{\omega C_1} = \frac{(10^{-3})(10^9)}{25(10/3)} = 12\,\mathrm{k}\Omega$$

$$\frac{1}{\omega C_2} = \frac{(10^{-3})(10^9)}{25(5/3)} = 24\,\mathrm{k}\Omega$$

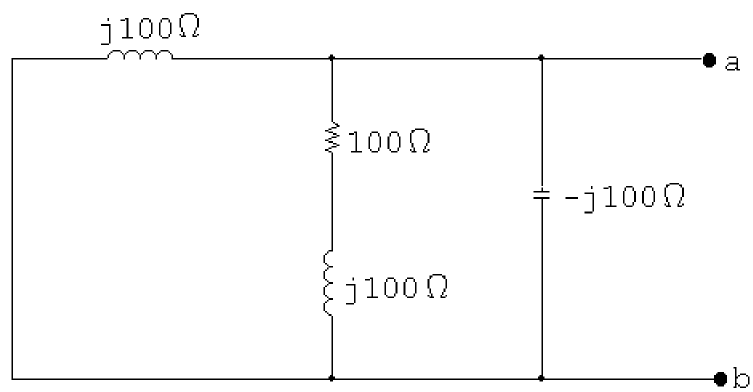


$$\mathbf{V}_T = (1 - j12)\mathbf{I}_T + 20\mathbf{I}_T(0.125)$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T} = 3.5 - j12\,\text{k}\Omega$$

P 7.37 **[a]** $j\omega L = j(1000)(100) \times 10^{-3} = j100\,\Omega$

$$\frac{1}{j\omega C} = -j\frac{10^6}{(1000)(10)} = -j100\,\Omega$$



$$Y_{\text{ab}} = \frac{1}{j100} + \frac{1}{100 + j100} + \frac{1}{-j100}$$

$$= \frac{1}{100}\left[\frac{1}{j} + \frac{1}{1 + j1} + \frac{j}{1}\right]$$

$$Y_{\text{ab}} = \frac{1}{100}\left[-j + \frac{1 - j1}{2} + j\right]$$

$$= \frac{1 - j1}{200}; \qquad Z_{\text{ab}} = \frac{200}{1 - j1} = 100(1 + j1)$$

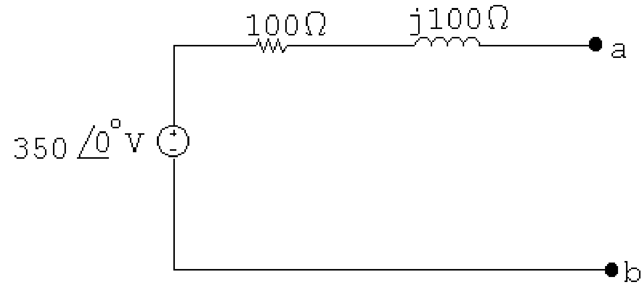
$$\therefore \quad \mathbf{V}_{\text{ab}} = 100(1 + j1)\left[\frac{247.49\underline{/45^\circ}}{j100}\right]$$

$$= \sqrt{2}\underline{/45^\circ} \cdot 1\underline{/-90^\circ} \cdot 247.49\underline{/45^\circ}$$

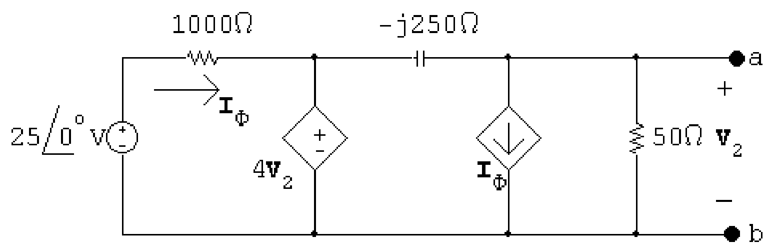
$$\mathbf{V}_{\text{Th}} = 350\underline{/0^\circ}\,\text{V}$$

[b] $Z_{\text{Th}} = Z_{\text{ab}} = 100 + j100\,\Omega$

[c]



P 7.38



$$\frac{\mathbf{V}_2}{50} + \frac{25 - 4\mathbf{V}_2}{1000} + \frac{\mathbf{V}_2 - 4\mathbf{V}_2}{-j250} = 0$$

Solving,

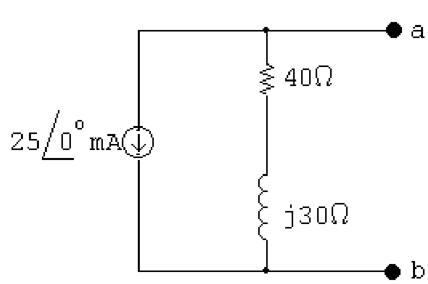
$$\mathbf{V}_2 = -10 - j0.75\,\text{V} = 1.25\underline{/216.87^\circ}\,\text{V}$$

$$\mathbf{I}_{sc} = -\mathbf{I}_\phi = \frac{-25\underline{/0^\circ}}{1000} = -25\underline{/0^\circ}\,\text{mA}$$

$$Z_{\text{Th}} = \frac{1.25\underline{/216.87^\circ}}{-25 \times 10^{-3}\underline{/0^\circ}} = 50\underline{/36.87^\circ}\,\Omega = 40 + j30\,\Omega$$

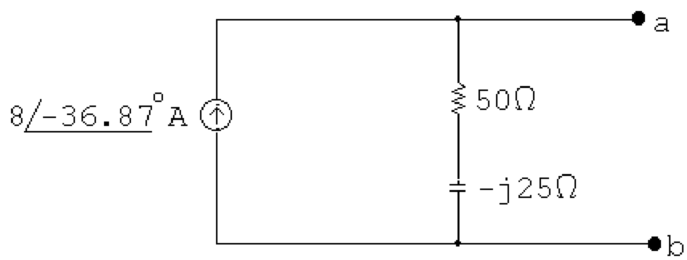
$$\mathbf{I}_N = \mathbf{I}_{sc} = -25\underline{/0^\circ}\,\text{mA}$$

$$Z_N = Z_{\text{Th}} = 50\underline{/36.87^\circ} = 40 + j30\,\Omega$$

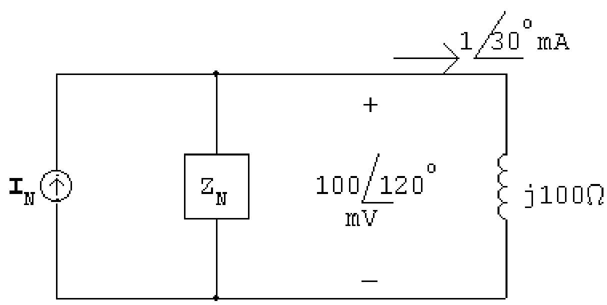


P 7.39 $\mathbf{I}_N = \mathbf{I}_{sc} = \dfrac{(16\angle 0^\circ)(25)}{25 + 15 + j30} = 6.4 - j4.8\,\text{A} \;= 8\angle -36.87^\circ\,\text{A}$

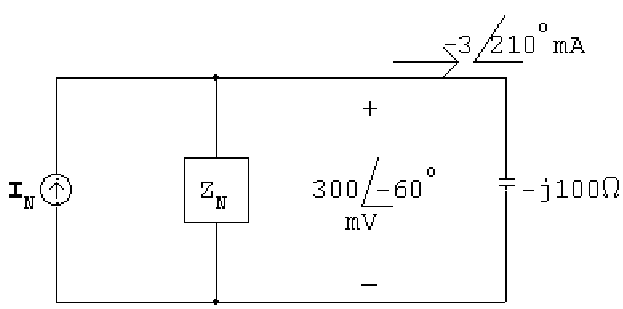
$$Z_N = \frac{(-j50)(40 + j30)}{40 + j30 - j50} = 50 - j25\,\Omega$$



P 7.40



$$\mathbf{I}_N = \frac{0.1\angle 120^\circ}{Z_N} + 1\angle 30^\circ\,\text{mA}, \quad Z_N \text{ in k}\Omega$$



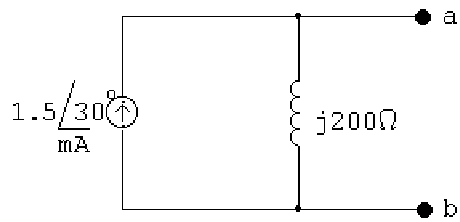
$$\mathbf{I}_N = \frac{0.3\angle -60^\circ}{Z_N} + (-3\angle 210^\circ)\,\text{mA}, \quad Z_N \text{ in k}\Omega$$

$$\frac{0.1\angle 120^\circ}{Z_N} + 1\angle 30^\circ = \frac{0.3\angle -60^\circ}{Z_N} + (-3\angle 210^\circ)$$

$$\frac{0.3\angle -60^\circ - 0.1\angle 120^\circ}{Z_N} = 1\angle 30^\circ + 3\angle 210^\circ$$

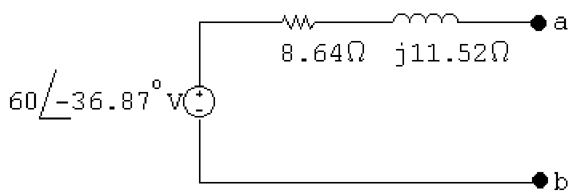
$$Z_N = \frac{0.3\angle -60^\circ - 0.1\angle 120^\circ}{1\angle 30^\circ + 3\angle 210^\circ} = 0.2\angle 90^\circ = j0.2\,\text{k}\Omega$$

$$\mathbf{I}_N = \frac{0.1\angle 120^\circ}{0.2\angle 90^\circ} + 1\angle 30^\circ = 1.5\angle 30^\circ\,\text{mA}$$

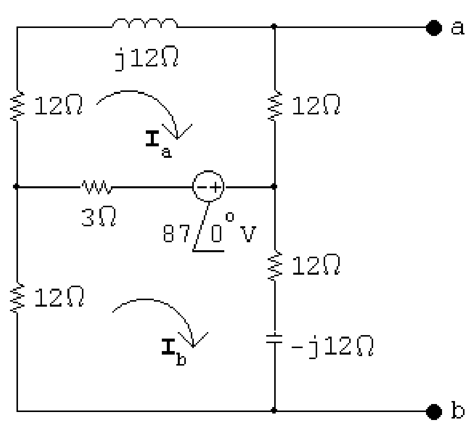


P 7.41 $\mathbf{V}_{\text{Th}} = \frac{75(24)}{24 + j18} = 60\angle -36.87^\circ\,\text{V}$

$$Z_{\text{Th}} = \frac{(24)(j18)}{24 + j18} = 8.64 + j11.52\,\Omega$$



P 7.42



$$(27 + j12)\mathbf{I}_{\text{a}} - 3\mathbf{I}_{\text{b}} = -87\angle 0^\circ$$

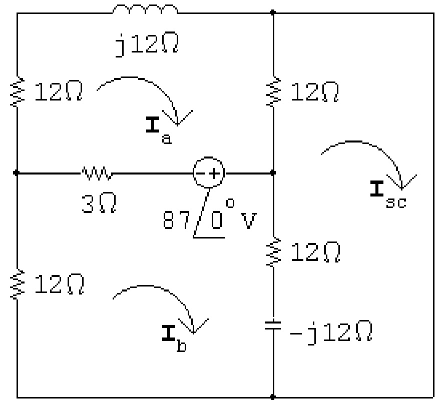
$$-3\mathbf{I}_{\text{a}} + (27 - j12)\mathbf{I}_{\text{b}} = 87\angle 0^\circ$$

Solving,

$\mathbf{I}_{\text{a}} = -2.4167 + j1.21;\qquad \mathbf{I}_{\text{b}} = 2.4167 + j1.21$

$$\mathbf{V}_{\text{Th}} = 12\mathbf{I}_{\text{a}} + (12 - j12)\mathbf{I}_{\text{b}} = 14.5\underline{/0^\circ}\,\text{V}$$

Short Circuit Test:



$$(27 + j12)\mathbf{I}_{\text{a}} - 3\mathbf{I}_{\text{b}} - 12\mathbf{I}_{\text{sc}} = -87$$

$$-3\mathbf{I}_{\text{a}} + (27 - j12)\mathbf{I}_{\text{b}} - (12 - j12)\mathbf{I}_{\text{sc}} = 87$$

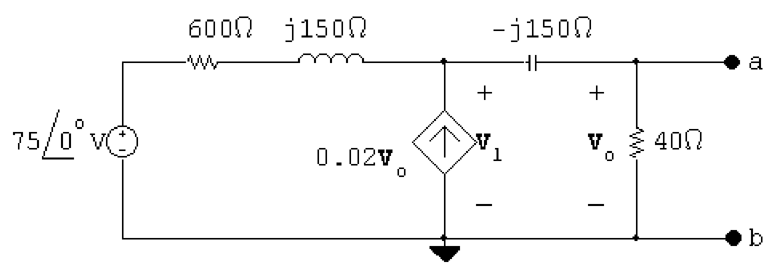
$$-12\mathbf{I}_{\text{a}} - (12 - j12)\mathbf{I}_{\text{b}} + (24 - j12)\mathbf{I}_{\text{sc}} = 0$$

Solving,

$$\mathbf{I}_{\text{sc}} = 1\underline{/0^\circ}$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{14.5\underline{/0^\circ}}{1\underline{/0^\circ}} = 14.5\,\Omega$$

P 7.43



$$\frac{\mathbf{V}_1 - 75}{150(4 + j1)} - \frac{0.02\mathbf{V}_1(40)}{40 - j150} + \frac{\mathbf{V}_1}{40 - j150} = 0$$

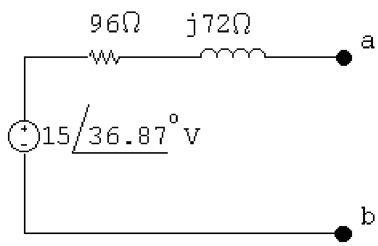
$$\therefore \quad \mathbf{V}_1 = \frac{75(4 - j15)}{16 - j12}$$

$$\mathbf{V}_{\text{Th}} = \frac{40\mathbf{V}_1}{40 - j150} = \frac{4}{4 - j15} \cdot \frac{75(4 - j15)}{16 - j12}$$

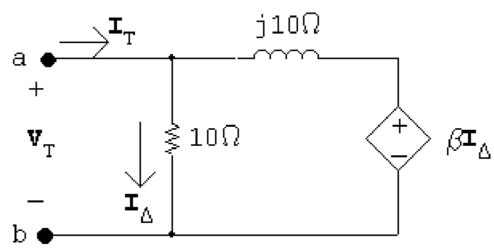
$$= \frac{75}{4 - j3} = 15\underline{/36.87^\circ}\,\text{V}$$

$$\mathbf{I}_{\text{sc}} = \frac{75}{600} = \frac{1}{8}\,\text{A}$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = 120\underline{/36.87^\circ} = 96 + j72\,\Omega$$



P 7.44 [a]



$$\mathbf{I}_T = \frac{\mathbf{V}_T}{10} + \frac{\mathbf{V}_T + \beta\mathbf{V}_T/10}{j10}$$

$$\frac{\mathbf{I}_T}{\mathbf{V}_T} = \frac{1}{10} + \frac{(1 - \beta/10)}{j10} = \frac{(10 - \beta) + j10}{j100}$$

$$\therefore \quad Z_{\text{Th}} = \frac{\mathbf{V}_T}{\mathbf{I}_T} = \frac{1000 + j100(10 - \beta)}{(10 - \beta)^2 + 100}$$

Z_{Th} is real when $\beta = 10$.

[b] $Z_{\text{Th}} = \dfrac{1000}{100} = 10\,\Omega$

[c] $Z_{\text{Th}} = 5 + j5$

$$\frac{1000}{(10 - \beta)^2 + 100} = 5; \qquad (10 - \beta)^2 = 100$$

$$\therefore \quad 10 - \beta = \pm 10; \qquad \beta = 10 \mp 10$$

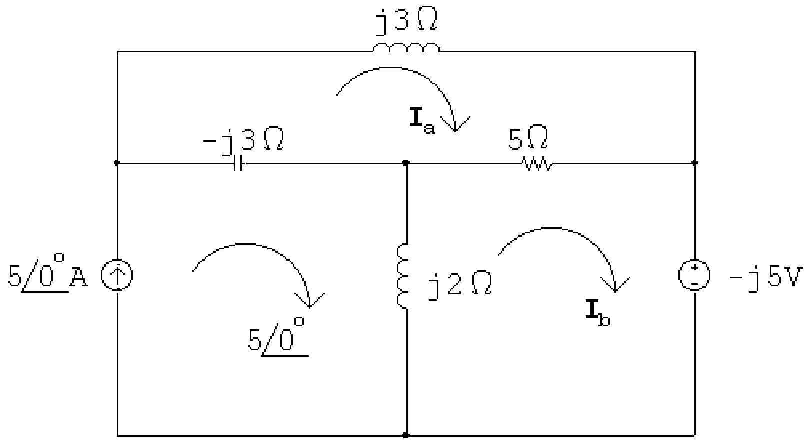
$$\beta = 0; \qquad \beta = 20$$

But the j term can only equal the real term with $\beta = 0$. Thus, $\beta = 0$.

[d] Z_{Th} will be capacitive when $\beta > 10$:

$\therefore \quad 10 < \beta \le 50$

P 7.45



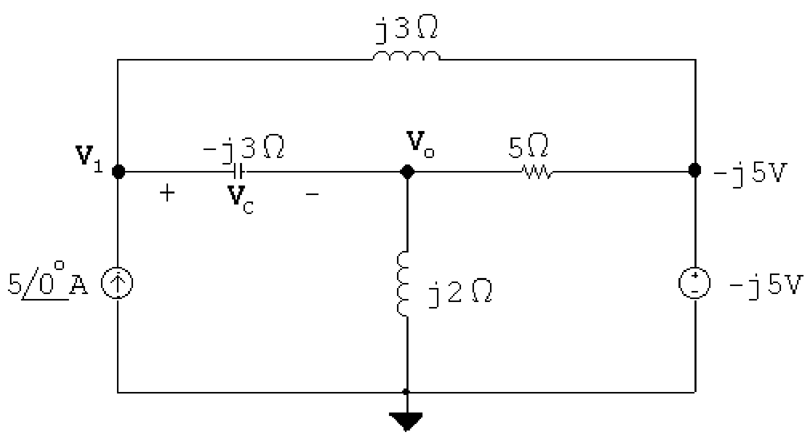
$$j3\mathbf{I}_{\text{a}} + 5(\mathbf{I}_{\text{a}} - \mathbf{I}_{\text{b}}) - j3(\mathbf{I}_{\text{a}} - 5) = 0$$

$$j2(\mathbf{I}_{\text{b}} - 5) + 5(\mathbf{I}_{\text{b}} - \mathbf{I}_{\text{a}}) - j5 = 0$$

Solving,

$\mathbf{I}_{\text{a}} = -j3; \qquad \mathbf{I}_g = -j3 = 3\underline{/-90^\circ}\,\text{A}$

P 7.46



$$\frac{\mathbf{V}_o}{j2} + \frac{\mathbf{V}_o + j5}{5} + \frac{\mathbf{V}_o - \mathbf{V}_1}{-j3} = 0$$

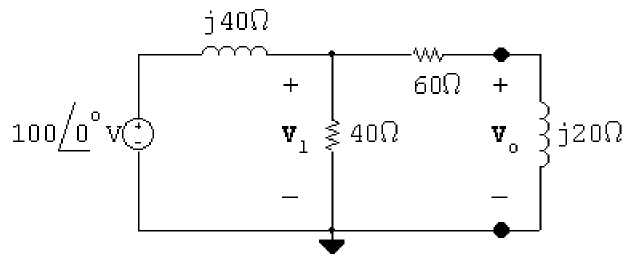
$$(5 + j6)\mathbf{V}_o + 10\mathbf{V}_1 = 30$$

$$-5 + \frac{\mathbf{V}_1 - \mathbf{V}_o}{-j3} + \frac{\mathbf{V}_1 + j5}{j3} = 0$$

$\mathbf{V}_o = j10; \qquad \mathbf{V}_1 = 9 - j5$

$$\mathbf{V}_{\text{c}} = \mathbf{V}_1 - \mathbf{V}_o = 9 - j5 - j10 = 9 - j15 = 17.49\underline{/-59.04^\circ}\,\text{V}$$

P 7.47



$$\frac{\mathbf{V}_1 - 100}{j40} + \frac{\mathbf{V}_1}{40} + \frac{\mathbf{V}_1}{60 + j20} = 0$$

Solving for $\mathbf{V}_1$ yields

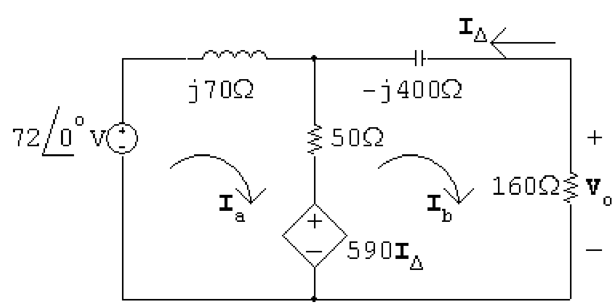
$$\mathbf{V}_1 = 30 - j40\,\mathrm{V}$$

$$\mathbf{V}_o = \frac{\mathbf{V}_1}{60 + j20}(j20) = \left(\frac{j}{3 + j}\right)\mathbf{V}_1$$

$$\mathbf{V}_o = 15 + j5\,\mathrm{V} = 15.81\underline{/18.43^\circ}\,\mathrm{V}$$

P 7.48 $j\omega L = j5000(14 \times 10^{-3}) = j70\,\Omega$

$$\frac{1}{j\omega C} = \frac{-j}{(5000)(0.5 \times 10^{-6})} = -j400\,\Omega$$



$$72\underline{/0^\circ} = (50 + j70)\mathbf{I}_\mathrm{a} - 50\mathbf{I}_\mathrm{b} + 590(-\mathbf{I}_\mathrm{b})$$

$$0 = -50\mathbf{I}_\mathrm{a} - 590(-\mathbf{I}_\mathrm{b}) + (210 - j400)\mathbf{I}_\mathrm{b}$$

Solving,

$$\mathbf{I}_\mathrm{b} = (50 - j50)\,\mathrm{mA}$$

$$\mathbf{V}_o = 160\mathbf{I}_\mathrm{b} = 8 - j8 = 11.31\underline{/-45^\circ}$$

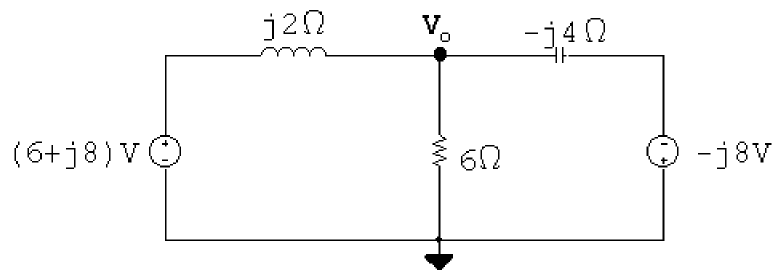
$$v_o = 11.31\cos(5000t - 45^\circ)\,\mathrm{V}$$

P 7.49 $j\omega L = j(5000)(0.4 \times 10^{-3}) = j2\,\Omega$

$$\frac{1}{j\omega C} = -j\frac{10^6}{(5000)(50)} = -j4\,\Omega$$

$\mathbf{V}_{g1} = 10\underline{/53.13^\circ} = 6 + j8\,\mathrm{V}$

$\mathbf{V}_{g2} = 8\underline{/-90^\circ} = -j8\,\mathrm{V}$



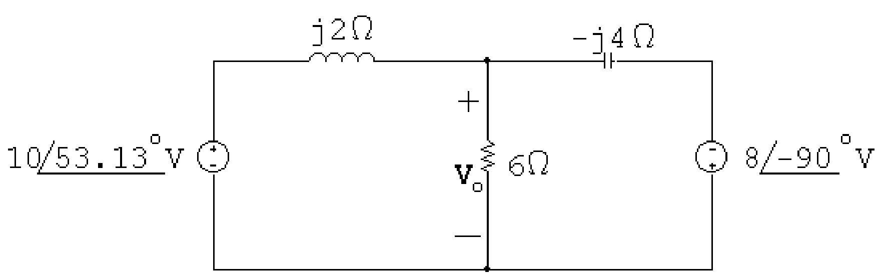
$$\frac{\mathbf{V}_o - 6 - j8}{j2} + \frac{\mathbf{V}_o}{6} + \frac{\mathbf{V}_o + (-j8)}{-j4} = 0$$

Solving,

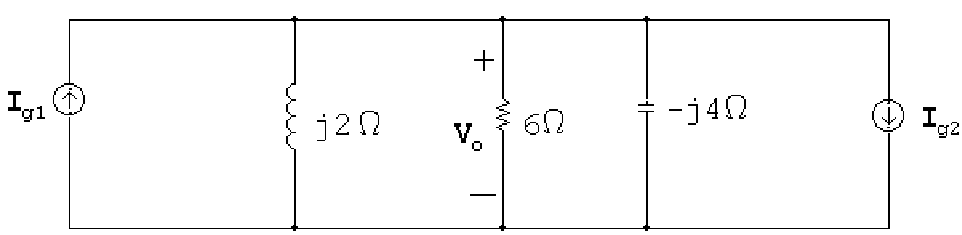
$\mathbf{V}_o = 12\underline{/0^\circ}$

$v_o(t) = 12\cos 5000t\,\mathrm{V}$

P 7.50 From the solution to Problem 7.49 the phasor-domain circuit is



Making two source transformations yields



$$\mathbf{I}_{g1} = \frac{10\underline{/53.13^\circ}}{j2} = 5\underline{/-36.87^\circ} = 4 - j3\,\mathrm{A}$$

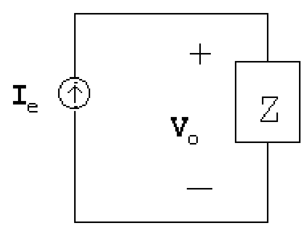
$$\mathbf{I}_{g2} = \frac{8\underline{/-90^\circ}}{-j4} = 2\underline{/0^\circ} = 2\,\text{A}$$

$$Y = \frac{1}{j2} + \frac{1}{6} + \frac{1}{-j4}\,\text{S}$$

$$Z = \frac{1}{Y} = \frac{1}{(1/6) - j(1/4)} = 1.85 + j2.77\,\Omega$$

$$\mathbf{I}_e = \mathbf{I}_{g1} - \mathbf{I}_{g2} = 4 - j3 - 2 = 2 - j3\,\text{A}$$

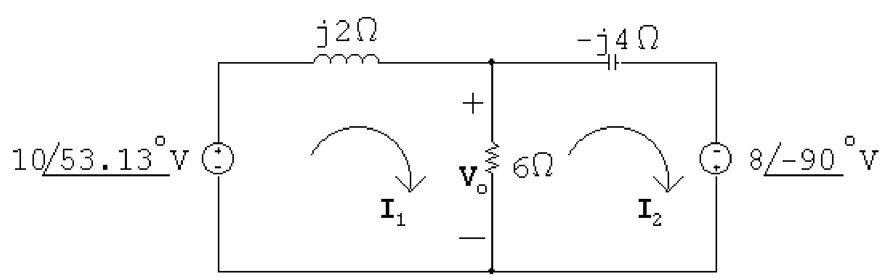
Hence the circuit reduces to



$$\mathbf{V}_o = Z\mathbf{I}_e = (1.85 + j2.77)(2 - j3) = 12\underline{/0^\circ}$$

$$\therefore \quad v_o(t) = 12\cos 5000t\,\text{V}$$

P 7.51 From the solution to Problem 7.49 the phasor-domain circuit is



$$10\underline{/53.13^\circ} = (6 + j2)\mathbf{I}_1 - 6\mathbf{I}_2$$

$$8\underline{/-90^\circ} = -6\mathbf{I}_1 + (6 - j4)\mathbf{I}_2$$

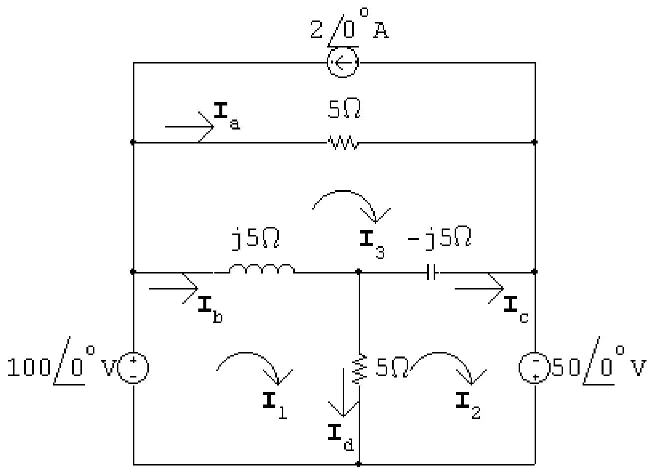
$$\mathbf{V}_o = (\mathbf{I}_1 - \mathbf{I}_2)6$$

Solving,

$$\mathbf{V}_o = 12\underline{/0^\circ}\,\text{V}$$

$$v_o(t) = 12\cos 5000t\,\text{V}$$

P 7.52



$100\underline{/0^\circ} = (5 + j5)\mathbf{I}_1 - 5\mathbf{I}_2 - j5\mathbf{I}_3$

$50\underline{/0^\circ} = -5\mathbf{I}_1 + (5 - j5)\mathbf{I}_2 + j5\mathbf{I}_3$

$-10\underline{/0^\circ} = -j5\mathbf{I}_1 + j5\mathbf{I}_2 + 5\mathbf{I}_3$

Solving,

$\mathbf{I}_1 = 58 - j20\,\text{A}; \qquad \mathbf{I}_2 = 58 + j10\,\text{A}; \qquad \mathbf{I}_3 = 28 + j0\,\text{A}$

$\mathbf{I}_\text{a} = \mathbf{I}_3 + 2 = 30 + j0\,\text{A}$

$\mathbf{I}_\text{b} = \mathbf{I}_1 - \mathbf{I}_3 = 58 - j20 - 28 = 30 - j20\,\text{A}$

$\mathbf{I}_\text{c} = \mathbf{I}_2 - \mathbf{I}_3 = 58 + j10 - 28 = 30 + j10\,\text{A}$

$\mathbf{I}_\text{d} = \mathbf{I}_1 - \mathbf{I}_2 = 58 - j20 - 58 - j10 = -j30\,\text{A}$

P 7.53 $\mathbf{V}_2$ is the voltage across the $-j10\,\Omega$ impedance.

$$\frac{\mathbf{V}_1 - \mathbf{V}_g}{20} + \frac{\mathbf{V}_1}{j5} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{Z} = 0$$

$$\frac{(40 + j30) - (100 - j50)}{20} + \frac{40 + j30}{j5} + \frac{(40 + j30) - \mathbf{V}_2}{Z} = 0$$

$$\therefore \quad \mathbf{V}_2 = 40 + j30 + (3 - j4)Z$$

$$\frac{\mathbf{V}_2 - \mathbf{V}_1}{Z} + \frac{\mathbf{V}_L}{-j10} - \mathbf{I}_g + \frac{\mathbf{V}_2 - \mathbf{V}_g}{3 + j1} = 0$$

$$\frac{\mathbf{V}_2 - (40 + j30)}{Z} + \frac{\mathbf{V}_2}{-j10} - (20 + j30) + \frac{\mathbf{V}_2 - (100 - j50)}{3 + j1} = 0$$

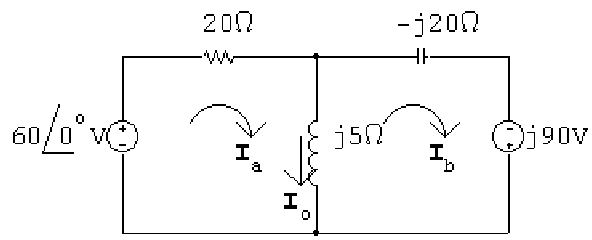
Substituting the expression for $\mathbf{V}_2$ found at the start and simplifying yields

$Z = 12 + j16\,\Omega$

P 7.54 $\mathbf{V}_\mathrm{a} = 60\underline{/0^\circ}\,\mathrm{V}; \qquad \mathbf{V}_\mathrm{b} = 90\underline{/90^\circ}\,\mathrm{V}$

$$j\omega L = j(4\times10^4)(125\times10^{-6}) = j5\Omega$$

$$\frac{-j}{\omega C} = \frac{-j10^6}{40{,}000(1.25)} = -j20\,\Omega$$



$$60 = (20+j5)\mathbf{I}_\mathrm{a} - j5\mathbf{I}_\mathrm{b}$$

$$j90 = -j5\mathbf{I}_\mathrm{a} - j15\mathbf{I}_\mathrm{b}$$

Solving,

$$\mathbf{I}_\mathrm{a} = 2.25 - j2.25\,\mathrm{A}; \qquad \mathbf{I}_\mathrm{b} = -6.75 + j0.75\,\mathrm{A}$$

$$\mathbf{I}_o = \mathbf{I}_\mathrm{a} - \mathbf{I}_\mathrm{b} = 9 - j3 = 9.49\underline{/-18.43^\circ}\,\mathrm{A}$$

$$i_o(t) = 9.49\cos(40{,}000t - 18.43^\circ)\,\mathrm{A}$$

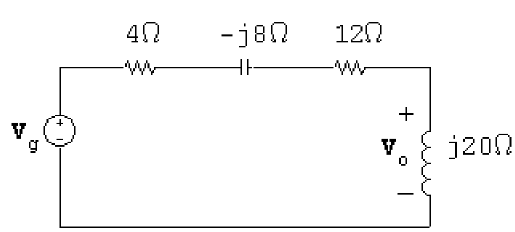
P 7.55 [**a**] $\dfrac{1}{j\omega C} = \dfrac{10^9}{j8\times10^5(125)} = -j10\,\Omega$

$$j\omega L = j8\times10^5(25\times10^{-6}) = j20\,\Omega$$

$$Z_e = \frac{(-j10)(20)}{20-j10} = 4 - j8\,\Omega$$

$$\mathbf{I}_g = 5\underline{/0^\circ}$$

$$\mathbf{V}_g = \mathbf{I}_g Z_e = 5(4-j8) = 20 - j40\,\mathrm{V}$$



$$\mathbf{V}_o = \frac{(20-j40)(j20)}{(16+j12)} = 44 - j8 = 44.72\underline{/-10.30^\circ}\,\mathrm{V}$$

$$v_o = 44.72\cos(8\times10^5t - 10.30^\circ)\,\mathrm{V}$$

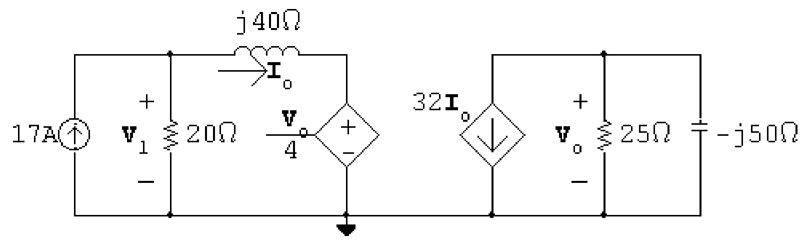
[b] $\omega = 2\pi f = 8 \times 10^5; \qquad f = \dfrac{4 \times 10^5}{\pi}$

$$T = \frac{1}{f} = \frac{\pi}{4 \times 10^5} = 2.5\pi\,\mu\text{s}$$

$$\therefore \quad \frac{10.30}{360}(2.5\pi) = 224.82\,\text{ns}$$

$$\therefore \quad v_o \text{ lags } i_g \text{ by } 224.82\,\text{ns}$$

P 7.56



$$\frac{\mathbf{V}_o}{25} + \frac{\mathbf{V}_o}{-j50} + 32\mathbf{I}_o = 0$$

$$(2 + j)\mathbf{V}_o = -1600\mathbf{I}_o$$

$$\mathbf{V}_o = (-640 + j320)\mathbf{I}_o$$

$$\mathbf{I}_o = \frac{\mathbf{V}_1 - (\mathbf{V}_o/4)}{j40}$$

$$\therefore \quad \mathbf{V}_1 = (-160 + j120)\mathbf{I}_o$$

$$17 = \frac{\mathbf{V}_1}{20} + \mathbf{I}_o = (-8 + j6)\mathbf{I}_o + \mathbf{I}_o = (-7 + j6)\mathbf{I}_o$$

$$\therefore \quad \mathbf{I}_o = \frac{17}{(-7 + j6)} = -1.4 - j1.2\,\text{A} = 1.84\underline{/-139.40^\circ}\,\text{A}$$

$$\mathbf{V}_o = (-640 + j320)\mathbf{I}_o = 1280 + j320 = 1319.39\underline{/14.04^\circ}\,\text{V}$$

P 7.57 $\quad -15\underline{/0^\circ} + \dfrac{\mathbf{V}_o}{8} + \dfrac{\mathbf{V}_o - 2.5\mathbf{I}_\Delta}{j5} + \dfrac{\mathbf{V}_o}{-j10} = 0$

$$\mathbf{I}_\Delta = \frac{\mathbf{V}_o}{-j10}$$

Solving,

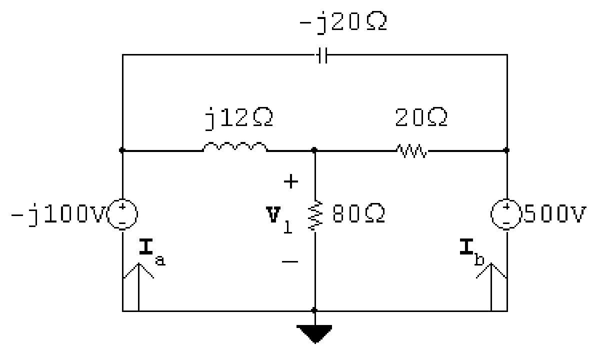
$$\mathbf{V}_o = 72 + j96 = 120\underline{/53.13^\circ}\,\text{V}$$

P 7.58 $j\omega L = j10^4(1.2 \times 10^{-3}) = j12\,\Omega$

$$\frac{1}{j\omega C} = \frac{-j10^6}{5 \times 10^4} = -j20\,\Omega$$

$$\mathbf{V}_{\rm a} = 100\underline{/-90^\circ} = -j100\,\mathrm{V}$$

$$\mathbf{V}_{\rm b} = 500\underline{/0^\circ} = 500\,\mathrm{V}$$



$$\frac{\mathbf{V}_1}{80} + \frac{\mathbf{V}_1 - 500}{20} + \frac{\mathbf{V}_1 + j100}{j12} = 0$$

Solving,

$$\mathbf{V}_1 = 160\underline{/53.13^\circ}\,\mathrm{V} = 96 + j128\,\mathrm{V}$$

$$\mathbf{I}_{\rm a} = \frac{-j100 - 96 - j128}{j12} + \frac{-j100 - 500}{-j20}$$

$$= 14 - j17 = 22.02\underline{/-129.47^\circ}\,\mathrm{A}$$

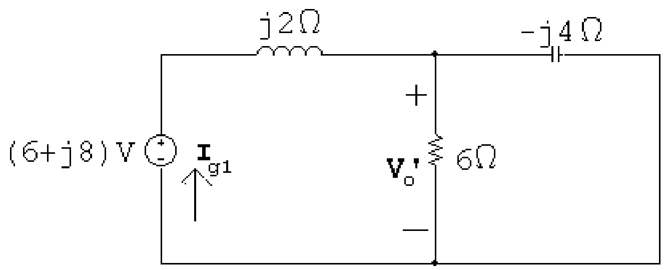
$$i_{\rm a} = 22.02\cos(10{,}000t - 129.47^\circ)\,\mathrm{A}$$

$$\mathbf{I}_{\rm b} = \frac{500 - 96 - j128}{20} + \frac{500 + j100}{-j20}$$

$$= 15.2 + j18.6 = 24.02\underline{/50.74^\circ}\,\mathrm{A}$$

$$i_{\rm b} = 24.02\cos(10{,}000t + 50.74^\circ)\,\mathrm{A}$$

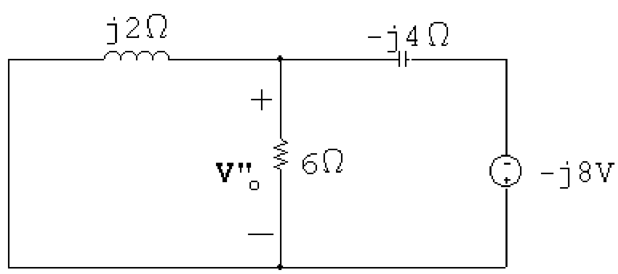
P 7.59 From the solution to Problem 7.49 the phasor-domain circuit with the right-hand source removed is



$$Z_{e1} = \frac{6(-j4)}{(6-j4)} = \frac{-j24}{6-j4}\,\Omega$$

$$\mathbf{V}_o' = \frac{Z_{e1}}{Z_{e1}+j2}(6+j8) = \frac{192-j144}{8-j12}\,\mathrm{V}$$

With the left hand source removed



$$Z_{e2} = \frac{6(j2)}{6+j2} = \frac{j12}{6+j2}\,\Omega$$

$$\mathbf{V}_o'' = \frac{Z_{e2}}{-j4+Z_{e2}}(j8) = \frac{-96}{8-j12}\,\mathrm{V}$$

$$\mathbf{V}_o = \mathbf{V}_o' + \mathbf{V}_o'' = \frac{192-j144-96}{8-j12} = 12+j0\,\mathrm{V}$$

$$v_o(t) = 12\cos 5000t\,\mathrm{V}$$

P 7.60 **[a]** $P = \frac{1}{2}(340)(20)\cos(60-15) = 2400\cos 45^\circ = 2404.16\,\mathrm{W}\quad\text{(abs)}$

$$Q = 2400\sin 45^\circ = 2404.16\ \mathrm{VAR}\quad\text{(abs)}$$

[b] $P = \frac{1}{2}(16)(75)\cos(-15 - 60) = 600\cos(-75^\circ) = 155.29\,\text{W} \quad \text{(abs)}$

$Q = 600\sin(-75^\circ) = -579.56\,\text{VAR} \quad \text{(del)}$

[c] $P = \frac{1}{2}(625)(4)\cos(40 - 150) = 1250\cos(-110^\circ) = -427.53\,\text{W} \quad \text{(del)}$

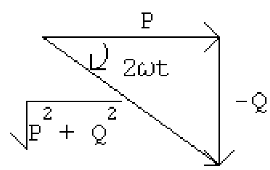
$Q = 1250\sin(-110^\circ) = -1174.62\,\text{VAR} \quad \text{(del)}$

[d] $P = \frac{1}{2}(180)(10)\cos(130 - 20) = 900\cos(110^\circ) = -307.82\,\text{W} \quad \text{(del)}$

$Q = 900\sin(110^\circ) = 845.72\,\text{VAR} \quad \text{(abs)}$

P 7.61 $p = P + P\cos 2\omega t - Q\sin 2\omega t; \qquad \frac{dp}{dt} = -2\omega P\sin 2\omega t - 2\omega Q\cos 2\omega t$

$$\frac{dp}{dt} = 0 \quad \text{when} \quad -2\omega P\sin 2\omega t = 2\omega Q\cos 2\omega t \quad \text{or} \quad \tan 2\omega t = -\frac{Q}{P}$$



$$\cos 2\omega t = \frac{P}{\sqrt{P^2+Q^2}}; \qquad \sin 2\omega t = -\frac{Q}{\sqrt{P^2+Q^2}}$$

Let $\theta = \tan^{-1}(-Q/P)$, then p is maximum when $2\omega t = \theta$ and p is minimum when $2\omega t = (\theta + \pi)$.

Therefore $\quad p_{\max} = P + P\cdot\frac{P}{\sqrt{P^2+Q^2}} - \frac{Q(-Q)}{\sqrt{P^2+Q^2}} = P + \sqrt{P^2+Q^2}$

and $\quad p_{\min} = P - P\cdot\frac{P}{\sqrt{P^2+Q^2}} - Q\cdot\frac{Q}{\sqrt{P^2+Q^2}} = P - \sqrt{P^2+Q^2}$

P 7.62 $W_{\text{dc}} = \frac{V_{\text{dc}}^2}{R}T; \qquad W_s = \int_{t_o}^{t_o+T}\frac{v_s^2}{R}\,dt$

$$\therefore \quad \frac{V_{\text{dc}}^2}{R}T = \int_{t_o}^{t_o+T}\frac{v_s^2}{R}\,dt$$

$$V_{\text{dc}}^2 = \frac{1}{T}\int_{t_o}^{t_o+T} v_s^2\,dt$$

$$V_{\text{dc}} = \sqrt{\frac{1}{T}\int_{t_o}^{t_o+T} v_s^2\,dt} = V_{\text{rms}} = V_{\text{eff}}$$

P 7.63 **[a]** Area under on cycle of v_g^2:

$$A = (400)(4)(20 \times 10^{-6}) + 10{,}000(2)(20 \times 10^{-6})$$
$$= 21{,}600(20 \times 10^{-6})$$

Mean value of v_g^2:

$$\text{M.V.} = \frac{A}{120 \times 10^{-6}} = \frac{21{,}600(20 \times 10^{-6})}{120 \times 10^{-6}} = 3600$$

$$\therefore \quad V_{\text{rms}} = \sqrt{3600} = 60\,\text{V(rms)}$$

[b] $P = \dfrac{V_{\text{rms}}^2}{R} = \dfrac{3600}{12} = 300\,\text{W}$

P 7.64 $i(t) = \dfrac{30}{40} \times 10^3 t = 750t \qquad 0 \le t \le 40\,\text{ms}$

$$i(t) = M - \frac{30}{10} \times 10^3 t \qquad 40\,\text{ms} \le t \le 50\,\text{ms}$$

$$i(t) = 0 \text{ when } t = 50\,\text{ms}$$

$$\therefore \quad M = 3000(50 \times 10^{-3}) = 150$$

$$i(t) = 150 - 3000t \qquad 40\,\text{ms} \le t \le 50\,\text{ms}$$

$$\therefore \quad I_{\text{rms}} = \sqrt{\frac{1000}{50}\left\{\int_0^{0.04} (750)^2 t^2\,dt + \int_{0.04}^{0.05} (150 - 3000t)^2\,dt\right\}}$$

$$\int_0^{0.04} (750)^2 t^2\,dt = (750)^2 \frac{t^3}{3}\Big|_0^{0.04} = 12$$

$$(150 - 3000t)^2 = 22{,}500 - 9 \times 10^5 t + 9 \times 10^6 t^2$$

$$\int_{0.04}^{0.05} 22{,}500\,dt = 225$$

$$\int_{0.04}^{0.05} 9 \times 10^5 t\,dt = 45 \times 10^4 t^2 \Big|_{0.04}^{0.05} = 405$$

$$9 \times 10^6 \int_{0.04}^{0.05} t^2\,dt = 3 \times 10^6 t^3 \Big|_{0.04}^{0.05} = 183$$

$$\therefore \quad I_{\text{rms}} = \sqrt{20\{12 + (225 - 405 + 183)\}} = \sqrt{300} = 17.32\,\text{A}$$

P 7.65 $P = I_{\text{rms}}^2 R \qquad \therefore \quad R = \dfrac{24 \times 10^3}{300} = 80\,\Omega$

P 7.66 $\dfrac{1}{\omega C} = \dfrac{10^9}{(5000)(80)} = 2500\,\Omega$

$$Z_{\text{f}} = \frac{-j2500(7500)}{7500 - j2500} = 750 - j2250\,\Omega$$

$$Z_{\text{i}} = 1500\,\Omega$$

$$\therefore \quad \frac{Z_{\text{f}}}{Z_{\text{i}}} = \frac{750 - j2250}{1500} = 0.5 - j1.5$$

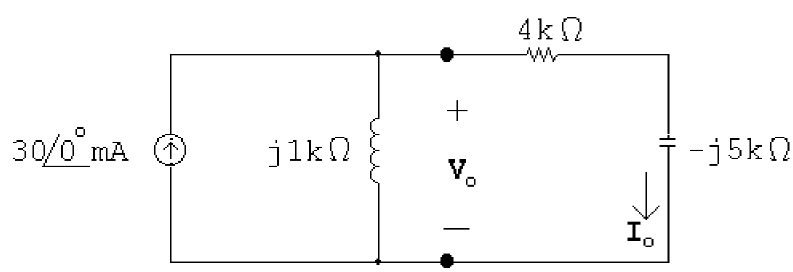
$$\mathbf{V}_o = -\frac{Z_{\text{f}}}{Z_{\text{i}}}\mathbf{V}_g; \qquad \mathbf{V}_g = 4\underline{/0^\circ}\,\text{V}$$

$$\mathbf{V}_o = (-0.5 + j1.5)(4) = -2 + j6 = 6.32\underline{/108.43^\circ}\,\text{V}$$

$$P = \frac{1}{2}\frac{V_m^2}{R} = \frac{1}{2}\frac{(4)(10)}{1000} = 20 \times 10^{-3} = 20\,\text{mW}$$

P 7.67 $\mathbf{I}_g = 30\underline{/0^\circ}\,\text{mA}$

$$j\omega L = j(100)(10) = j1000\,\Omega; \qquad \frac{1}{j\omega C} = \frac{10^6}{j(100)(2)} = -j5000\,\Omega$$



$$\mathbf{I}_o = \frac{30\underline{/0^\circ}(j1000)}{4000 - j4000} = 3.75\sqrt{2}\underline{/135^\circ}\,\text{mA}$$

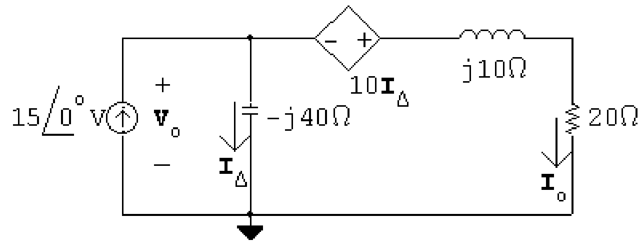
$$P = |\mathbf{I}_o|_{\text{rms}}^2(4000) = (3.75)^2(4000) = 56.25\,\text{mW}$$

$$Q = |\mathbf{I}_o|_{\text{rms}}^2(-5000) = -70.3125\,\text{mVAR}$$

$$S = P + jQ = 56.25 - j70.3125\,\text{mVA}$$

$$|S| = 90.044\,\text{mVA}$$

P 7.68 $j\omega L = j10{,}000(10^{-3}) = j10\,\Omega; \qquad \frac{1}{j\omega C} = \frac{10^6}{j10{,}000(2.5)} = -j40\,\Omega$



$$-15 + \frac{\mathbf{V}_o}{-j40} + \frac{\mathbf{V}_o + 10(\mathbf{V}_o/-j40)}{20+j10} = 0$$

$$\therefore \quad \mathbf{V}_o\left[\frac{1}{-j40} + \frac{1+j0.25}{20+j10}\right] = 15$$

$$\therefore \quad \mathbf{V}_o = 300 - j100\,\mathrm{V}$$

$$\therefore \quad \mathbf{I}_\Delta = \frac{\mathbf{V}_o}{-j40} = 2.5 + j7.5\,\mathrm{A}$$

$$\mathbf{I}_o = 15\underline{/0^\circ} - \mathbf{I}_\Delta = 15 - 2.5 - j7.5 = 12.5 - j7.5 = 14.58\underline{/-30.9^\circ}\,\mathrm{A}$$

$$P_{20\Omega} = \frac{1}{2}|\mathbf{I}_o|^2 20 = 2125\,\mathrm{W}$$

P 7.69 **[a]** $Z_1 = 240 + j70 = 250\underline{/16.26^\circ}\,\Omega$

pf $= \cos(16.26^\circ) = 0.96$ lagging

rf $= \sin(16.26^\circ) = 0.28$

$Z_2 = 160 - j120 = 200\underline{/-36.87^\circ}\,\Omega$

pf $= \cos(-36.87^\circ) = 0.80$ leading

rf $= \sin(-36.87^\circ) = -0.60$

$Z_3 = 30 - j40 = 50\underline{/-53.13^\circ}\,\Omega$

pf $= \cos(-53.13^\circ) = 0.6$ leading

rf $= \sin(-53.13^\circ) = -0.8$

[b] $Y = Y_1 + Y_2 + Y_3$

$$Y_1 = \frac{1}{250\underline{/16.26^\circ}}; \qquad Y_2 = \frac{1}{200\underline{/-36.87^\circ}}; \qquad Y_3 = \frac{1}{50\underline{/-53.13^\circ}}$$

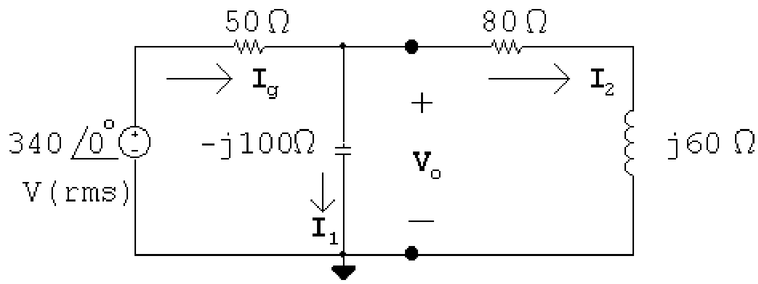
$$Y = 19.84 + j17.88\,\text{mS}$$

$$Z = \frac{1}{Y} = 37.44\underline{/-42.03^\circ}\,\Omega$$

$$\text{pf} = \cos(-42.03^\circ) = 0.74 \text{ leading}$$

$$\text{rf} = \sin(-42.03^\circ) = -0.67$$

P 7.70 **[a]**



$$\frac{\mathbf{V}_o}{-j100} + \frac{\mathbf{V}_o - 340}{50} + \frac{\mathbf{V}_o}{80 + j60} = 0$$

$$\therefore \quad \mathbf{V}_o = 238 - j34\,\text{V}$$

$$\mathbf{I}_g = \frac{340 - 238 + j34}{50} = 2.04 + j0.68\,\text{A}$$

$$S_g = \mathbf{V}_g\mathbf{I}_g^* = (340)(2.04 - j0.68)$$

$$= 693.6 - j231.2\,\text{VA}$$

[b] Source is delivering 693.6 W.

[c] Source is absorbing 231.2 magnetizing VAR.

[d] $\mathbf{I}_1 = \dfrac{\mathbf{V}_o}{-j100} = 0.34 + j2.38\,\text{A}$

$$S_1 = \mathbf{V}_o\mathbf{I}_1^* = (238 - j34)(0.34 - j2.38)$$

$$= 0 - j578\,\text{VA}$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_o}{80 + j60} = \frac{238 - j34}{80 + j60} = 1.7 - j1.7\,\text{A}$$

$$S_2 = \mathbf{V}_o\mathbf{I}_2^* = (238 - j34)(1.7 + j1.7)$$

$$= 462.4 + j346.8\,\text{VA}$$

$$S_{50\Omega} = |\mathbf{I}_g|^2(50) + j0 = (2.15)^2(50) = 231.2\,\text{W}$$

[e] $\sum P_{\text{del}} = 693.6\,\text{W}$

$\sum P_{\text{diss}} = 462.4 + 231.2 = 693.6\,\text{W}$

$\therefore \quad \sum P_{\text{del}} = \sum P_{\text{diss}} = 693.6\,\text{W}$

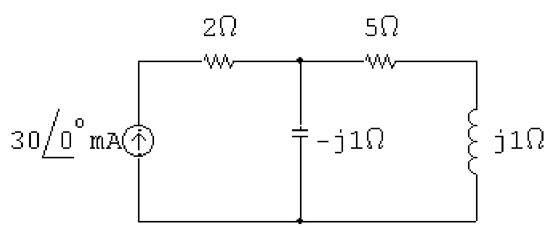
[f] $\sum Q_{\text{abs}} = 231.2 + 346.8 = 578\,\text{VAR}$

$\sum Q_{\text{dev}} = 578\,\text{VAR}$

$\therefore \quad \sum \text{ mag VAR dev } = \sum \text{ mag VAR abs } = 578$

P 7.71 $\mathbf{I}_g = 30\angle 0^\circ\,\text{mA}; \qquad \frac{1}{j\omega C} = \frac{10^6}{j(25\times 10^3)(40)} = -j1\,\Omega$

$j\omega L = j(25\times 10^3)(40)\times 10^{-6} = j1\,\Omega$



$Z_1 = j1\|(5+j1) = 0.2 - j1\,\Omega$

$Z_{\text{eq}} = 2 + Z_1 = 2.2 - j1\,\Omega$

$$P_g = |I_{\text{rms}}|^2\text{Re}\{Z_{\text{eq}}\} = \left(\frac{30}{\sqrt{2}}\times 10^{-3}\right)^2(2.2) = 990\,\mu\text{W}$$

P 7.72 **[a]** $P = \frac{1}{2}\frac{(240)^2}{480} = 60\,\text{W}$

$$-\frac{1}{\omega C} = \frac{-9\times 10^6}{(5000)(5)} = -360\,\Omega$$

$$Q = \frac{1}{2}\frac{(240)^2}{(-360)} = -80\,\text{VAR}$$

$$p_{\max} = P + \sqrt{P^2+Q^2} = 60 + \sqrt{(60)^2+(80)^2} = 160\,\text{W(del)}$$

[b] $p_{\min} = 60 - 100 = -40\,\text{W(abs)}$

[c] $P = 60\,\text{W}$

[d] $Q = -80\,\text{VAR}$

[**e**] generate

[**f**] pf $= \cos(\theta_v - \theta_i)$

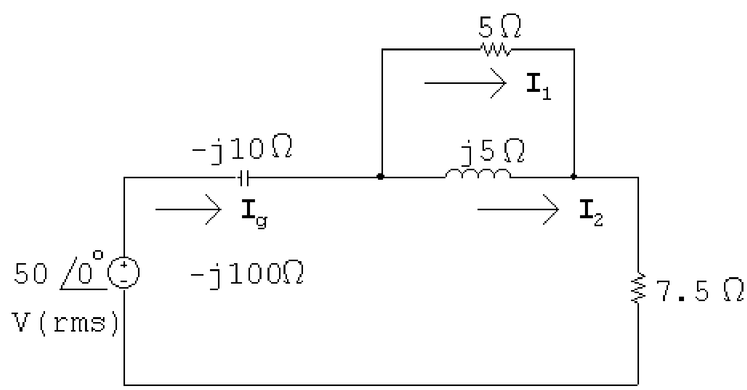
$$\mathbf{I} = \frac{240}{480} + \frac{240}{-j360} = 0.5 + j0.67 = 0.83\underline{/53.13^\circ}\,\text{A}$$

$$\therefore \quad \text{pf} = \cos(0 - 53.13^\circ) = 0.6 \text{ leading}$$

[**g**] rf $= \sin(-53.13^\circ) = -0.8$

P 7.73 [**a**] $\dfrac{1}{j\omega C} = \dfrac{10^6}{j10^5} = -j10\,\Omega$

$$j\omega L = j10^5(50 \times 10^{-6}) = j5\,\Omega$$



$$Z = -j10 + \frac{(5)(j5)}{5+j5} + 7.5 = 10 - j7.5\,\Omega$$

$$\mathbf{I}_g = \frac{50\underline{/0^\circ}}{10 - j7.5} = 3.2 + j2.4\,\text{A}$$

$$S_g = \frac{1}{2}\mathbf{V}_g\mathbf{I}_g^* = 25(3.2 - j2.4) = 80 - j60\,\text{VA}$$

$$P = 80\,\text{W(del)}; \qquad Q = 60\,\text{VAR(abs)}$$

$$|S| = |S_g| = 100\,\text{VA}$$

[**b**] $\mathbf{I}_1 = \dfrac{\mathbf{I}_g(j5)}{5+j5} = \dfrac{1}{2}(3.2 + j2.4)(1 + j1) = 0.4 + j2.8\,\text{A}$

$$P_{5\Omega} = \frac{1}{2}|\mathbf{I}_1|^2(5) = 20\,\text{W}$$

$$P_{7.5\Omega} = \frac{1}{2}|\mathbf{I}_g|^2(7.5) = 60\,\text{W}$$

$$\sum P_{\text{diss}} = 20 + 60 = 80\,\text{W} = \sum P_{\text{dev}}$$

[c] $\mathbf{I}_{j5} = \dfrac{\mathbf{I}_g 5}{5 + j5} = \dfrac{1}{2}(3.2 + j2.4)(1 - j1) = 2.8 - j0.4\,\text{A}$

$$Q_{j5\Omega} = \frac{1}{2}|\mathbf{I}_{j5}|^2(5) = 20\,\text{VAR(abs)}$$

$$Q_{-j10\Omega} = \frac{1}{2}|\mathbf{I}_g|^2(-10) = -80\,\text{VAR(dev)}$$

$$\sum Q_{\text{abs}} = 20 + 60 = 80\,\text{VAR} = \sum Q_{\text{dev}}$$

P 7.74 **[a]** $S_1 = 24{,}960 + j47{,}040\,\text{VA}$

$$S_2 = \frac{|\mathbf{V}_\text{L}|^2}{Z_2^*} = \frac{(480)^2}{5 + j5} = 23{,}040 - j23{,}040\,\text{VA}$$

$$S_1 + S_2 = 48{,}000 + j24{,}000\,\text{VA}$$

$$480\mathbf{I}_\text{L}^* = 48{,}000 + j24{,}000; \qquad \therefore \quad \mathbf{I}_\text{L} = 100 - j50\,\text{A(rms)}$$

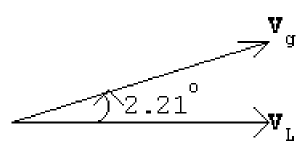
$$\mathbf{V}_g = \mathbf{V}_\text{L} + \mathbf{I}_\text{L}(0.02 + j0.20) = 480 + (100 - j50)(0.02 + j0.20)$$
$$= 492 + j19 = 492.37\underline{/2.21^\circ}\,\text{Vrms}$$

$$|\mathbf{V}_g| = 492.37\,\text{Vrms}$$

[b] $T = \dfrac{1}{f} = \dfrac{1}{60} = 16.67\,\text{ms}$

$$\frac{2.21^\circ}{360^\circ} = \frac{t}{16.67\text{ ms}}; \qquad \therefore \quad t = 102.39\,\mu\text{s}$$

[c] $\mathbf{V}_\text{L}$ lags $\mathbf{V}_g$ by 2.21° or $102.31\,\mu$s



P 7.75 **[a]** $S_1 = 18 + j24\,\text{kVA}; \qquad S_2 = 36 - j48\,\text{kVA}; \qquad S_3 = 18 + j0\,\text{kVA}$

$$S_\text{T} = S_1 + S_2 + S_3 = 72 - j24\,\text{kVA}$$

$$2400\mathbf{I}^* = (72 - j24) \times 10^3; \qquad \therefore \quad \mathbf{I} = 30 + j10\,\text{A}$$

$$Z = \frac{2400}{30 + j10} = 72 - j24\,\Omega = 75.89\underline{/-18.43^\circ}\,\Omega$$

[b] pf $= \cos(-18.43^\circ) = 0.9487$ leading

P 7.76 **[a]** From the solution to Problem 7.75 we have

$$\mathbf{I}_{\rm L} = 30 + j10\,\mathrm{A(rms)}$$

$$\therefore\quad \mathbf{V}_s = 2400\underline{/0^\circ} + (30 + j10)(0.2 + j1.6) = 2390 + j50$$

$$= 2390.52\underline{/1.20^\circ}\,\mathrm{V(rms)}$$

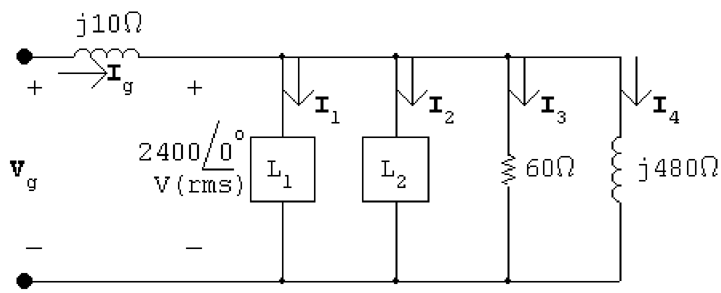
[b] $|\mathbf{I}_{\rm L}| = \sqrt{1000}$

$$P_\ell = (1000)(0.2) = 200\,\mathrm{W} \qquad Q_\ell = (1000)(1.6) = 1600\,\mathrm{VAR}$$

[c] $P_s = 72{,}000 + 200 = 72.2\,\mathrm{kW} \qquad Q_s = -24{,}000 + 1600 = -22.4\,\mathrm{kVAR}$

[d] $\eta = \dfrac{72}{72.2}(100) = 99.72\%$

P 7.77



$$2400\mathbf{I}_1^* = 24{,}000 + j18{,}000$$

$$\mathbf{I}_1^* = 10 + j7.5; \qquad \therefore\quad \mathbf{I}_1 = 10 - j7.5\,\mathrm{A(rms)}$$

$$2400\mathbf{I}_2^* = 48{,}000 - j30{,}000$$

$$\mathbf{I}_2^* = 20 - j12.5; \qquad \therefore\quad \mathbf{I}_2 = 20 + j12.5\,\mathrm{A(rms)}$$

$$\mathbf{I}_3 = \frac{2400\underline{/0^\circ}}{60} = 40 + j0\,\mathrm{A}; \qquad \mathbf{I}_4 = \frac{2400\underline{/0^\circ}}{j480} = 0 - j5\,\mathrm{A}$$

$$\mathbf{I}_g = \mathbf{I}_1 + \mathbf{I}_2 + \mathbf{I}_3 + \mathbf{I}_4 = 70\,\mathrm{A}$$

$$\mathbf{V}_g = 2400 + (70)(j10) = 2400 + j700 = 2500\underline{/16.26^\circ}\,\mathrm{V(rms)}$$

P 7.78 $S_{\rm T} = 52{,}800 - j\dfrac{52{,}800}{0.8}(0.6) = 52{,}800 - j39{,}600\,\mathrm{VA}$

$$S_1 = 40{,}000(0.96 + j0.28) = 38{,}400 + j11{,}200\,\mathrm{VA}$$

$$S_2 = S_{\rm T} - S_1 = 14{,}400 - j50{,}800 = 52{,}801.52\underline{/-74.17^\circ}\,\mathrm{VA}$$

pf $= \cos(-74.17^\circ) = 0.2727$ leading

P 7.79 [**a**] $\mathbf{I} = \dfrac{7200\underline{/0^\circ}}{140 + j480} = 14.4\underline{/-73.74^\circ}\,\text{A(rms)}$

$$P = (14.4)^2(2) = 414.72\,\text{W}$$

[**b**] $Y_\text{L} = \dfrac{1}{138 + j460} = \dfrac{138 - j460}{230{,}644}$

$$\therefore\quad -j\omega C = -j\frac{460}{230{,}644} \qquad \therefore\quad X_\text{C} = \frac{-230{,}644}{460} = -501.40\,\Omega$$

[**c**] $Z_\text{L} = \dfrac{230{,}644}{138} = 1671.33\,\Omega$

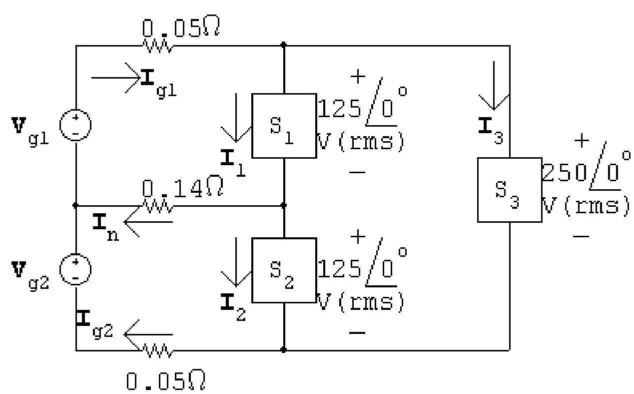
[**d**] $\mathbf{I} = \dfrac{7200}{1673.33 + j20} = 4.30\underline{/-0.68^\circ}\,\text{A}$

$$P = (4.30)^2(2) = 37.02\,\text{W}$$

[**e**] $\% = \dfrac{37.02}{414.72}(100) = 8.93\%$

Thus the power loss after the capacitor is added is 8.93% of the power loss before the capacitor is added.

P 7.80 [**a**]



$$\mathbf{I}_1 = \frac{5000 - j2000}{125} = 40 - j16\,\text{A (rms)}$$

$$\mathbf{I}_2 = \frac{3750 - j1500}{125} = 30 - j12\,\text{A (rms)}$$

$$\mathbf{I}_3 = \frac{8000 + j0}{250} = 32 + j0\,\text{A (rms)}$$

$$\therefore\quad \mathbf{I}_{g1} = 72 - j16\,\text{A (rms)}$$

$$\mathbf{I}_n = \mathbf{I}_1 - \mathbf{I}_2 = 10 - j4\,\text{A (rms)}$$

$$\mathbf{I}_{g2} = 62 - j12\,\text{A}$$

$$\mathbf{V}_{g1} = 0.05\mathbf{I}_{g1} + 125 + j0 + 0.14\mathbf{I}_n = 130 - j1.36\,\text{V(rms)}$$

$$\mathbf{V}_{g2} = -0.14\mathbf{I}_n + 125 + j0 + 0.05\mathbf{I}_{g2} = 126.7 - j0.04\,\text{V(rms)}$$

$$S_{g1} = [(130 - j1.36)(72 + j16)] = [9381.76 + j1982.08]\,\text{VA}$$

$$S_{g2} = [(126.7 - j0.04)(62 + j12)] = [7855.88 + j1517.92]\,\text{VA}$$

Note: Both sources are delivering average power and magnetizing VAR to the circuit.

[b] $P_{0.05} = |\mathbf{I}_{g1}|^2(0.05) = 272\,\text{W}$

$$P_{0.14} = |\mathbf{I}_n|^2(0.14) = 16.24\,\text{W}$$

$$P_{0.05} = |\mathbf{I}_{g2}|^2(0.05) = 199.4\,\text{W}$$

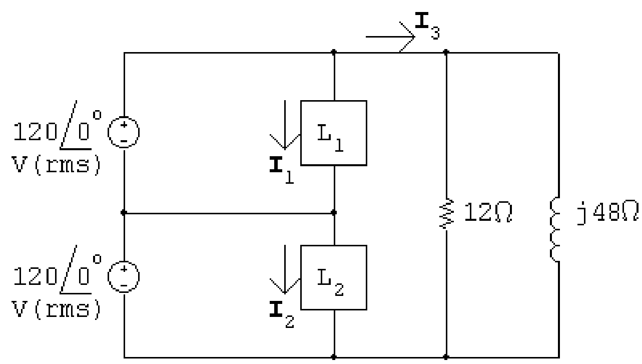
$$\sum P_{\text{dis}} = 272 + 16.24 + 199.4 + 5000 + 3750 + 8000 = 17{,}237.64\,\text{W}$$

$$\sum P_{\text{dev}} = 9381.76 + 7855.88 = 17{,}237.64\,\text{W} = \sum P_{\text{dis}}$$

$$\sum Q_{\text{abs}} = 2000 + 1500 = 2500\,\text{VAR}$$

$$\sum Q_{\text{del}} = 1982.08 + 1517.92 = 3500\,\text{VAR} = \sum Q_{\text{abs}}$$

P 7.81 [a]



$$120\mathbf{I}_1^* = 1800 + j600; \qquad \therefore \quad \mathbf{I}_1 = 15 - j5\,\text{A(rms)}$$

$$120\mathbf{I}_2^* = 1200 - j900; \qquad \therefore \quad \mathbf{I}_2 = 10 + j7.5\,\text{A(rms)}$$

$$\mathbf{I}_3 = \frac{240}{12} + \frac{240}{j48} = 20 - j5\,\text{A(rms)}$$

$$\mathbf{I}_{g1} = \mathbf{I}_1 + \mathbf{I}_3 = 35 - j10\,\text{A}$$

$$S_{g1} = 120(35 + j10) = 4200 + j1200\,\text{VA}$$

Thus the $\mathbf{V}_{g1}$ source is delivering 4200 W and 1200 magnetizing vars.

$$\mathbf{I}_{g2} = \mathbf{I}_2 + \mathbf{I}_3 = 30 + j2.5\,\text{A(rms)}$$

$$S_{g2} = 120(30 - j2.5) = 3600 - j300\,\text{VA}$$

Thus the $\mathbf{V}_{g2}$ source is delivering 3600 W and absorbing 300 magnetizing vars.

[b] $\sum P_{\text{gen}} = 4200 + 3600 = 7800\,\text{W}$

$$\sum P_{\text{abs}} = 1800 + 1200 + \frac{(240)^2}{12} = 7800\,\text{W} = \sum P_{\text{gen}}$$

$$\sum Q_{\text{del}} = 1200 + 900 = 2100\,\text{VAR}$$

$$\sum Q_{\text{abs}} = 300 + 600 + \frac{(240)^2}{48} = 2100\,\text{VAR} = \sum Q_{\text{del}}$$

P 7.82 **[a]** $S_{\text{L}} = 24 + j7\,\text{kVA}$

$$125\mathbf{I}_{\text{L}}^* = (24 + j7) \times 10^3; \quad \mathbf{I}_{\text{L}}^* = 192 + j56\,\text{A(rms)}$$

$$\therefore \quad \mathbf{I}_{\text{L}} = 192 - j56\,\text{A(rms)}$$

$$\mathbf{V}_s = 125 + (192 - j56)(0.006 + j0.048) = 128.84 + j8.88$$

$$= 129.15\underline{/3.94^\circ}\,\text{V(rms)}$$

$$|\mathbf{V}_s| = 129.15\,\text{V(rms)}$$

[b] $P_\ell = |\mathbf{I}_\ell|^2(0.006) = (200)^2(0.006) = 240\,\text{W}$

[c] $\dfrac{(125)^2}{X_{\text{C}}} = -7000; \qquad X_{\text{C}} = -2.23\,\Omega$

$$-\frac{1}{\omega C} = -2.23; \qquad C = \frac{1}{(2.23)(120\pi)} = 1188.36\,\mu\text{F}$$

[d] $\mathbf{I}_\ell = 192 + j0\,\text{A(rms)}$

$$\mathbf{V}_s = 125 + 192(0.006 + j0.048) = 126.152 + j9.216$$

$$= 126.49\underline{/4.18^\circ}\,\text{V(rms)}$$

$$|\mathbf{V}_s| = 126.49\,\text{V(rms)}$$

[e] $P_\ell = (192)^2(0.006) = 221.184\,\text{W}$

P 7.83 **[a]** $\Delta = R_aR_bR_c - R_1^2R_b - R_2^2R_a - R_n(2R_1R_2 + R_nR_c)$

$$R_a = R_1 + R_n + R_l = 30 + 1 + 0.5 = 31.5\,\Omega$$

$$R_b = R_2 + R_n + R_l = 300 + 1 + 0.5 = 301.5\,\Omega$$

$$R_c = R_1 + R_2 + R_3 = 30 + 300 + 15 = 345\,\Omega$$

$$\Delta = (31.5)(301.5)(345) - 900(301.5) - 9 \times 10^4(31.5)$$
$$-1[2(30)(300) + 1(345)]$$
$$= 151{,}856.25$$

$$N_a = \mathbf{V}_{g1}[(R_bR_c - R_2^2) + R_nR_c + R_1R_2]$$
$$= 120[(301.5)(345) - 9\times 10^4 + 345 + 30(300)]$$
$$= 2{,}803{,}500$$
$$N_b = \mathbf{V}_{g1}[R_nR_c + R_1R_2 + R_aR_c - R_1^2]$$
$$= 120[345 + (30)(300) + 31.5(345) - 900]$$
$$= 2{,}317{,}500$$

$$\mathbf{I}_a = \frac{N_a}{\Delta}; \qquad \mathbf{I}_b = \frac{N_b}{\Delta}$$

$$\mathbf{I}_n = \mathbf{I}_a - \mathbf{I}_b = \frac{N_a - N_b}{\Delta} = 3.2\underline{/0^\circ}\mathrm{A(rms)}$$

[b]

$$N_c = \mathbf{V}_{g1}[R_2R_n + R_1R_b + R_2R_a + R_1R_n]$$
$$= 120[300 + 30(301.5) + 31.5(300) + 30]$$
$$= 2{,}259{,}000$$

$$\mathbf{I}_{L1} = \frac{N_a - N_c}{\Delta}$$

$$\mathbf{V}_1 = 30\mathbf{I}_{L1} = \frac{30(N_a - N_c)}{\Delta} = 107.57\underline{/0^\circ}\mathrm{V(rms)}$$

[c] $\mathbf{I}_{L2} = \dfrac{N_b - N_c}{\Delta}$

$$\mathbf{V}_2 = 300\mathbf{I}_{L2} = \frac{300(N_b - N_c)}{\Delta} = 115.57\underline{/0^\circ}\mathrm{V(rms)}$$

[d] $\mathbf{V}_3 = 15\mathbf{I}_c = 15\dfrac{N_c}{\Delta} = 233.14\underline{/0^\circ}\mathrm{V(rms)}$

CHECK:

$$\mathbf{V}_3 = \mathbf{V}_1 + \mathbf{V}_2 = 107.57\underline{/0^\circ} + 115.57\underline{/0^\circ} = 233.14\underline{/0^\circ}\mathrm{V(rms)}$$

[e] $P_1 = \dfrac{|\mathbf{V}_1|^2}{R_1} = \dfrac{(107.57)^2}{30} = 385.70$ W

$$P_2 = \frac{|\mathbf{V}_2|^2}{R_2} = \frac{(115.57)^2}{300} = 44.52 \text{ W}$$

$$P_3 = \frac{|\mathbf{V}_3|^2}{R_3} = \frac{(233.14)^2}{15} = 3319.39 \text{ W}$$

[f] $\mathbf{I}_a = \dfrac{N_a}{\Delta} = 18.46\underline{/0^\circ}\text{A(rms)}$

$\mathbf{I}_b = \dfrac{N_b}{\Delta} = 15.26\underline{/0^\circ}\text{A(rms)}$

$P_a = (120)(18.46)\cos 0^\circ$

$P_b = (120)(15.26)\cos 0^\circ$

$\sum P_{\text{gen}} = 120(18.46 + 15.26) = 4046.72 \text{ W}$

[g] $P_a = |\mathbf{I}_a|^2(0.5) = 170.41 \text{ W}$

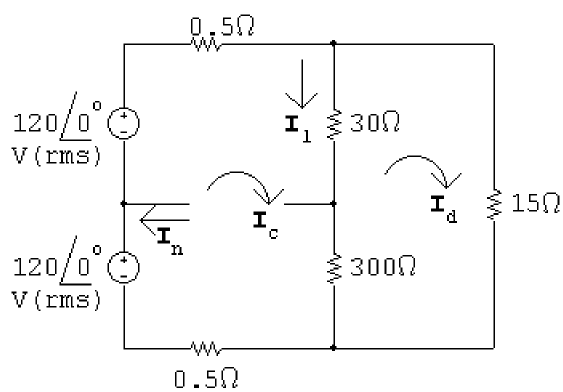
$P_n = |\mathbf{I}_n|^2(1) = 10.24 \text{ W}$

$P_b = |\mathbf{I}_b|^2(0.5) = 116.45 \text{ W}$

$$\begin{aligned}\sum P_{\text{diss}} &= 170.41 + 10.24 + 116.45 + 385.70 \\ &\quad +44.52 + 3319.39 \\ &= 4046.72 \text{ W}\end{aligned}$$

P 7.84 [a] $\mathbf{I}_n = 0$ by hypothesis.

[b] With the neutral conductor open the circuit becomes:



The two mesh current equations are

$$240\underline{/0^\circ} = 331\mathbf{I}_c - 330\mathbf{I}_d$$

$$0 = -330\mathbf{I}_c + 345\mathbf{I}_d$$

$$\therefore \quad \mathbf{I}_c = \frac{82{,}800}{5295} + 15.64\underline{/0^\circ}\text{A(rms)}$$

$$\mathbf{I}_d = \frac{79{,}200}{5295} + 14.96\underline{/0^\circ}\text{A(rms)}$$

$$\mathbf{I}_1 = \mathbf{I}_c - \mathbf{I}_d = 0.68\underline{/0^\circ}\text{A(rms)}$$

$$\mathbf{V}_1 = 30\mathbf{I}_1 = 20.40\underline{/0^\circ}\text{V(rms)}$$

[c] $\mathbf{V}_2 = 300\mathbf{I}_1 = 203.97\underline{/0^\circ}\mathrm{V(rms)}$

[d] $\mathbf{V}_3 = 15\mathbf{I}_d = 224.36\underline{/0^\circ}\mathrm{V(rms)}$

[e] $P_{R_1} = (20.40)^2/30 = 13.87\ \mathrm{W}$

$$P_{R_2} = (203.97)^2/300 = 138.67\ \mathrm{W}$$

$$P_{R_3} = (224.36)^2/15 = 3355.91\ \mathrm{W}$$

[f] $\sum P_{\text{gen}} = 240|\mathbf{I}_c|\cos 0^\circ = (240)(15.64)(1) = 3752.97\ \mathrm{W}$

[g] $\sum P_{\text{diss}} = (15.64)^2(1) + 13.87 + 138.67 + 3355.91 = 3752.97\ \mathrm{W}$

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