Introduction to Electronic Systems

Zheng Feng

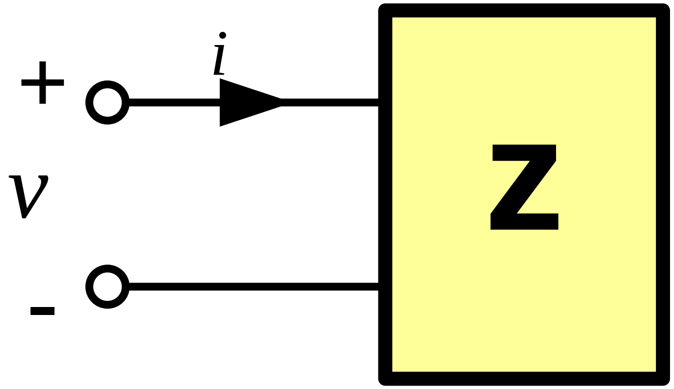
Part 3: Sinusoidal Steady-State Analysis

8. Sinusoidal Steady-State Analysis
9. **Sinusoidal Steady-State Power Calculations**
10. Frequency Selective Circuits *

Chapter 9

- Instantaneous Power
- Average Power
- RMS and Effective Value
- Complex Power and Apparent Power
- Maximum Average Power Transfer

9-1 Instantaneous Power



$$\begin{cases} v = V_m \cos(\omega t + \theta_v - \theta_i) \\ i = I_m \cos(\omega t) \end{cases}$$

- **Instantaneous Power:**

$$p = vi = V_m \cos(\omega t + \theta_v - \theta_i) \cdot I_m \cos(\omega t)$$

$$= V_m I_m \cos(\omega t + \theta_v - \theta_i) \cos(\omega t)$$

Instantaneous Power

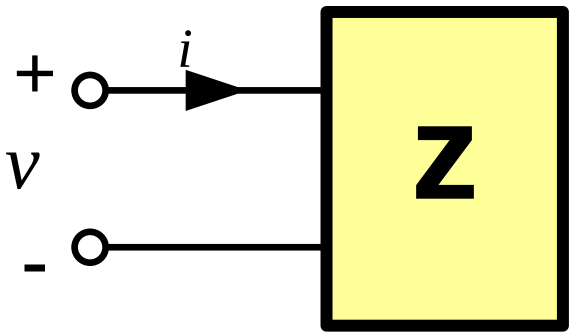
$$p = vi == V_m I_m \cos(\omega t + \theta_v - \theta_i)\cos(\omega t)$$

$$= \frac{V_m I_m}{2}\cos(\theta_v - \theta_i) + \frac{V_m I_m}{2}\cos(2\omega t + \theta_v - \theta_i)$$

$$= \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)$$

$$+ \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)\cos(2\omega t) - \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)\sin(2\omega t)$$

9-2 Average and Reactive Power



$$\begin{cases} v = V_m \cos\left(\omega t + \theta_v - \theta_i\right) \\ i = I_m \cos\left(\omega t\right) \end{cases}$$

$$\begin{aligned} p &= vi \\ &= \frac{V_m I_m}{2}\cos\left(\theta_v - \theta_i\right) \\ &\quad + \frac{V_m I_m}{2}\cos\left(\theta_v - \theta_i\right)\cos\left(2\omega t\right) - \frac{V_m I_m}{2}\sin\left(\theta_v - \theta_i\right)\sin\left(2\omega t\right) \\ &= P + P\cos\left(2\omega t\right) - Q\sin\left(2\omega t\right) \end{aligned}$$

Average Power

$$p = P + P\cos(2\omega t) - Q\sin(2\omega t)$$

$$P = \frac{1}{T}\int_{t_0}^{t_0+T} p\,dt = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)$$

- ***P*** **is defined as Average Power;**
- ***P*** **is also called Real Power, or Active Power;**
- **The unit for Average Power is Watt (W).**

Reactive Power

$$p = P + P\cos(2\omega t) - Q\sin(2\omega t)$$

$$Q = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)$$

- ***Q*** **is defined as Reactive Power;**
- **The unit for Reactive Power is VAR (*Volt-Amp Reactive*).**

Power Factor

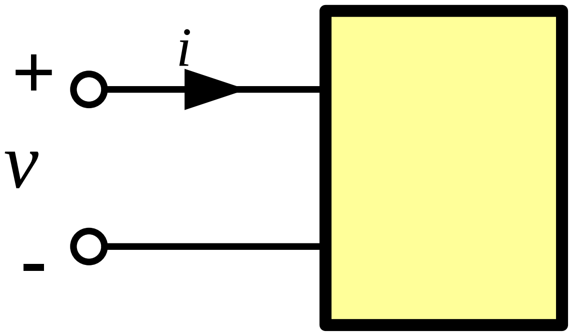
$$P = \frac{V_m I_m}{2}\cos\left(\theta_v - \theta_i\right) = \frac{V_m I_m}{2}\cos\phi$$

$$Q = \frac{V_m I_m}{2}\sin\left(\theta_v - \theta_i\right) = \frac{V_m I_m}{2}\sin\phi$$

$$\phi = \theta_v - \theta_i$$

: is defined as Power Factor Angle.

Example



$$\begin{cases} v = 100\cos\left(\omega t + 15^\circ\right)\text{V} \\ i = 4\sin\left(\omega t - 15^\circ\right)\text{A} \end{cases}$$

Calculate the average power and the reactive power at the terminals of the network shown above.

Solution:

$$\begin{cases} v = 100\cos\left(\omega t + 15^\circ\right)\text{V} \\ i = 4\sin\left(\omega t - 15^\circ\right) = 4\cos\left(\omega t - 105^\circ\right)\text{A} \end{cases}$$

$$P = \frac{V_m I_m}{2}\cos\left(\theta_v - \theta_i\right) = \frac{100\times 4}{2}\times\cos\left[15^\circ - \left(-105^\circ\right)\right] = -100\text{W}$$

$$Q = \frac{V_m I_m}{2}\sin\left(\theta_v - \theta_i\right) = \frac{100\times 4}{2}\sin\left[15^\circ - \left(-105^\circ\right)\right] = 173.21\text{VAR}$$

9-3 rms and Effective Value

rms value:

ROOT of the MEAN value of the SQUARED function

$$v(t) = V_m \cos(\omega t + \phi)$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} v^2 dt} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} V_m^2 \cos^2(\omega t + \phi) dt} = \frac{V_m}{\sqrt{2}}$$

北京郵電大學
BEIJING UNIVERSITY OF POSTS AND TELECOMMUNICATIONS

Effective Value

- For the Sinusoidal voltage/current source, the **rms value** is also referred to as the **Effective Value**.

$$v(t) = V_m \cos(\omega t + \theta_v) \qquad i(t) = I_m \cos(\omega t + \theta_i)$$

$$V_{eff} = \frac{V_m}{\sqrt{2}} = 0.707 V_m \qquad I_{eff} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

Effective Value and Power

- **Average power:**

$$P = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)$$

$$= \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos(\theta_v - \theta_i)$$

$$= V_{eff} I_{eff} \cos(\theta_v - \theta_i)$$

- **Reactive power:**

$$Q = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)$$

$$= \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\sin(\theta_v - \theta_i)$$

$$= V_{eff} I_{eff} \sin(\theta_v - \theta_i)$$

9-4 Complex Power and Apparent Power

$\hat{S} = P + jQ$: is defined as **Complex Power**.

- **The unit for Complex Power is VA (*Volt-Amp*).**
- **The unit for Average Power is Watt;**
- **The unit for Reactive Power is VAR.**

$$\hat{S} = P + jQ$$

Average Power: $$P = \frac{V_m I_m}{2}\cos\phi = \mathrm{Re}\left[\hat{S}\right]$$

Reactive Power: $$Q = \frac{V_m I_m}{2}\sin\phi = \mathrm{Im}\left[\hat{S}\right]$$

$$\left|\hat{S}\right| = \sqrt{P^2 + Q^2}, \quad \phi = \arctan\frac{Q}{P}$$

Complex Power Calculation

$$\hat{S} = P + jQ$$

$$= \frac{V_m I_m}{2}\cos\left(\theta_v - \theta_i\right) + j\frac{V_m I_m}{2}\sin\left(\theta_v - \theta_i\right)$$

$$= \frac{V_m I_m}{2}\left[\cos\left(\theta_v - \theta_i\right) + j\sin\left(\theta_v - \theta_i\right)\right]$$

$$= \frac{V_m I_m}{2}e^{j\left(\theta_v - \theta_i\right)} = \frac{1}{2}V_m e^{j\theta_v} I_m e^{-j\theta_i} = \frac{1}{2}\hat{V}\hat{I}^*$$

北京郵電大學
BEIJING UNIVERSITY OF POSTS AND TELECOMMUNICATIONS

Complex Power Calculation

$$\hat{S} = P + jQ = \frac{1}{2}\hat{V}\hat{I}^*$$

$$\hat{S} = \frac{1}{2}\hat{V}\hat{I}^* = \frac{1}{2}\left(Z\hat{I}\right)\hat{I}^* = \frac{1}{2}Z\left|\hat{I}\right|^2 = \frac{1}{2}ZI_m^2$$

Apparent Power

$$\left|\hat{S}\right| = \sqrt{P^2 + Q^2}$$: is defined as Apparent Power.

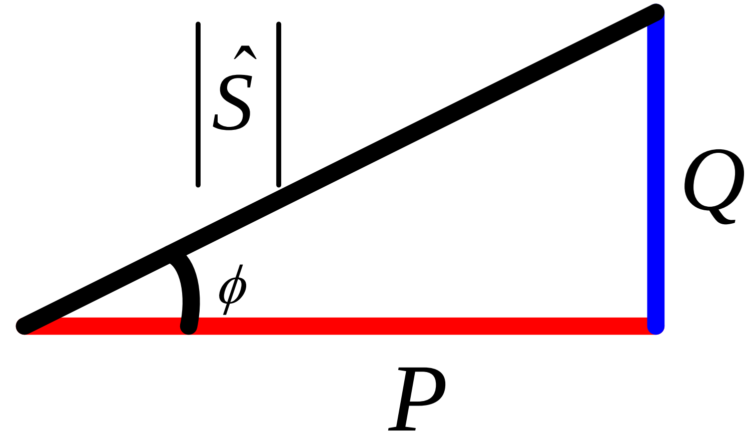
- The unit for Apparent Power is VA (Volt-Amp);
- The unit for Average Power is Watt;
- The unit for Reactive Power is VAR.
- The unit for Complex Power is VA (*Volt-Amp*);

Apparent Power

$$P = \frac{V_m I_m}{2}\cos\phi, \quad Q = \frac{V_m I_m}{2}\sin\phi$$

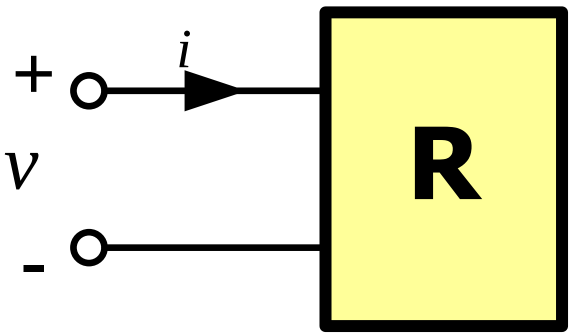
$$\left|\hat{S}\right| = \sqrt{P^2 + Q^2} = \frac{1}{2}V_m I_m$$

$$\phi = \arctan\frac{Q}{P}$$



Power Triangle

Power for Resistive Circuits



$$\begin{cases} \hat{I} = I_m e^{j\theta_i} \\ \hat{V} = R\hat{I} = RI_m e^{j\theta_i} \end{cases}$$

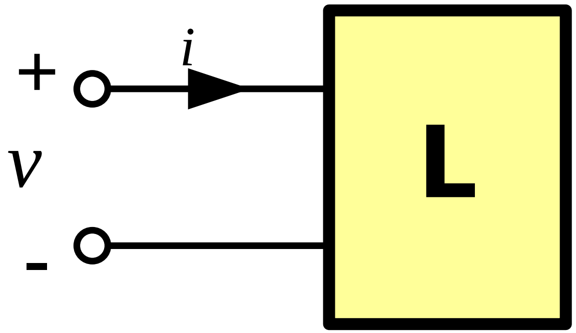
- **Complex Power:**

$$\hat{S} = \frac{1}{2}\hat{V}\hat{I}^* = \frac{1}{2}\hat{V}\left(\frac{\hat{V}^*}{R}\right) = \frac{V_m^2}{2R}$$

$$= \frac{1}{2}R\hat{I}\hat{I}^* = \frac{1}{2}R\left|\hat{I}\right|^2 = \frac{1}{2}RI_m^2$$

$$\Rightarrow \begin{cases} P = \dfrac{V_m^2}{2R} = \dfrac{1}{2}RI_m^2 \\ Q = 0 \\ \phi = 0 \end{cases}$$

Power for Inductive Circuits



$$\begin{cases} \hat{I} = I_m e^{j\theta_i} \\ \hat{V} = Z\hat{I} = j\omega L\hat{I} \end{cases}$$

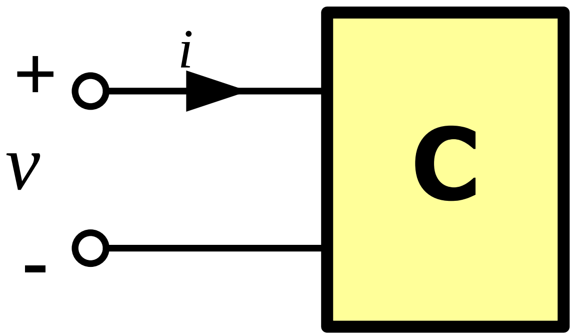
- **Complex Power:**

$$\hat{S} = \frac{1}{2}\hat{V}\hat{I}^* = \frac{1}{2}j\omega L\hat{I}\hat{I}^*$$

$$= j\frac{1}{2}\omega L I_m^2$$

$$\Rightarrow \begin{cases} P = 0 \\ Q = \frac{1}{2}\omega L I_m^2 \\ \phi = 90^0 \end{cases}$$

Power for Capacitive Circuits



$$\begin{cases} \hat{I} = I_m e^{j\theta_i} \\ \hat{V} = Z\hat{I} = \dfrac{1}{j\omega C}\hat{I} \end{cases}$$

- **Complex Power:**

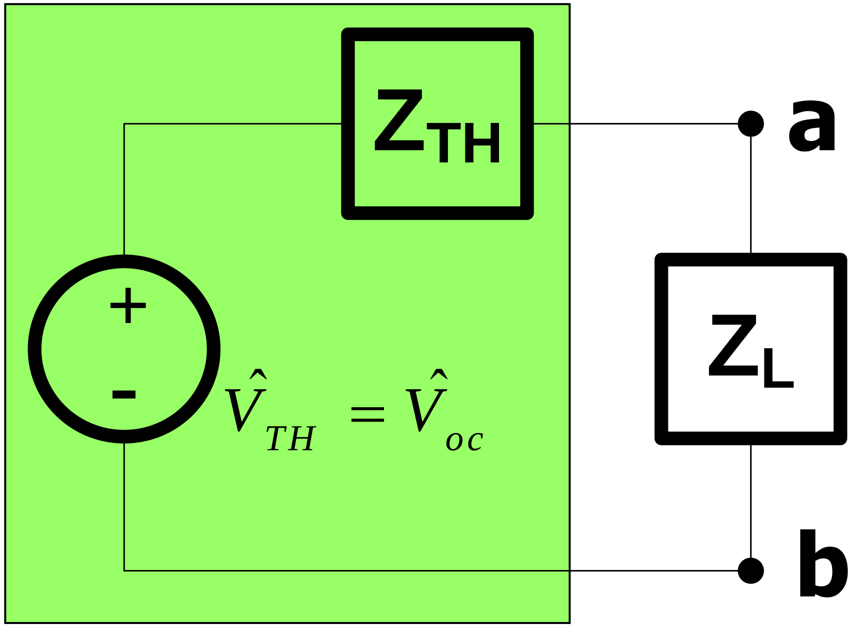
$$\hat{S} = \frac{1}{2}\hat{V}\hat{I}^* = \frac{1}{2}\frac{1}{j\omega C}\hat{I}\hat{I}^*$$

$$= -j\frac{I_m^2}{2\omega C}$$

$$\Rightarrow \begin{cases} P = 0 \\ Q = -\dfrac{I_m^2}{2\omega C} \\ \phi = -90^0 \end{cases}$$

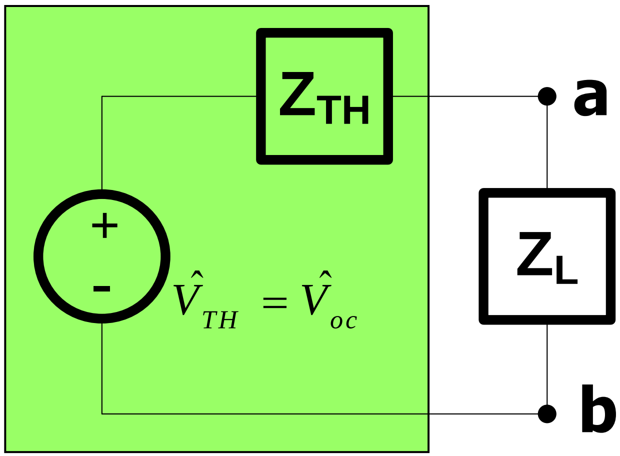
9-5 Maximum Average Power Transfer

- Any linear network may be viewed from the terminals of the load in terms of **Thévenin equivalents**;
- What is the load impedance required to deliver maximum **average power** to the load?
- What is the maximum average power?



$$Z_L = R_L + jX_L$$

Maximum Power Transfer



$$\begin{cases} Z_{TH} = R_{TH} + jX_{TH} \\ Z_L = R_L + jX_L \end{cases}$$

- **Complex Power:**

$$\hat{S} = \frac{1}{2}\hat{V}\hat{I}^* = \frac{1}{2}Z_L\hat{I}\hat{I}^* = \frac{1}{2}Z_L\left|\hat{I}\right|^2$$

$$= \frac{1}{2}\left|\hat{I}\right|^2 (R_L + jX_L)$$

$$\Rightarrow \quad P = \frac{1}{2}\left|\hat{I}\right|^2 R_L$$

$$P=\frac{1}{2}\left|\hat{I}\right|^{2}R_{L}=\frac{1}{2}\left|\frac{\hat{V}_{TH}}{Z_{L}+Z_{TH}}\right|^{2}R_{L}$$

$$=\frac{1}{2}\left|\frac{\hat{V}_{TH}}{\left(R_{L}+R_{TH}\right)+j\left(X_{L}+X_{TH}\right)}\right|^{2}R_{L}$$

$$=\frac{1}{2}\frac{\left|\hat{V}_{TH}\right|^{2}R_{L}}{\left(R_{L}+R_{TH}\right)^{2}+\left(X_{L}+X_{TH}\right)^{2}}$$

Let: $\begin{cases} \dfrac{\partial P}{\partial X_L} = 0 \\ \dfrac{\partial P}{\partial R_L} = 0 \end{cases} \Rightarrow \begin{cases} X_L = -X_{TH} \\ R_L = R_{TH} \end{cases}$

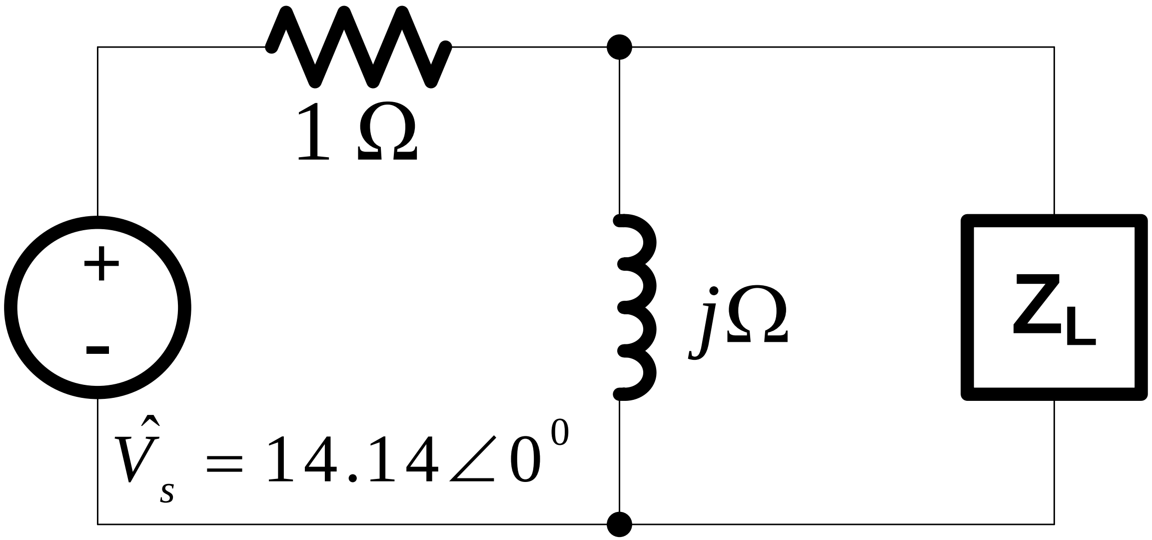
$$\Rightarrow \boxed{Z_L = Z_{TH}^*}$$

$$P_{\max} = \frac{1}{2}\frac{\left|\hat{V}_{TH}\right|^2 R_L}{\left(R_L + R_{TH}\right)^2 + \left(R_{TH} + X_{TH}\right)^2}$$

$$= \frac{1}{2}\frac{\left|\hat{V}_{TH}\right|^2 R_L}{\left(R_L + R_L\right)^2} = \frac{\left|\hat{V}_{TH}\right|^2}{8R_L}$$

$$\Rightarrow \quad P_{L\max} = \frac{\left|\hat{V}_{TH}\right|^2}{8R_L}$$

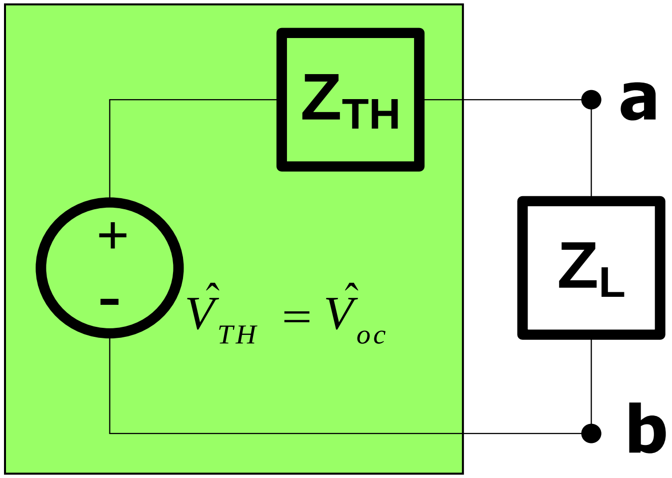
Example



For the circuit shown, what value of Z_L results in maximum average power transferred to Z_L? What is the maximum power?

Solution:

The Thévenin equivalent with respect to the two terminals of the load is:



$$\hat{V}_{Th} = \hat{V}_{oc} = \frac{j}{1+j} 14.14\angle 0^0 \, \mathrm{V} = 10\angle 45^0 \, \mathrm{V}$$

$$Z_{Th} = \frac{j}{1+j} = \frac{1}{\sqrt{2}} \angle 45^0 \, \Omega = (0.5 + j0.5)\, \Omega$$

$$Z_L = Z_{Th}^* = (0.5 - j0.5)\Omega$$

Hence, $Z_L = (0.5 - j0.5)\Omega$ **can result in maximum average power transferred to Z_L. The maximum power is:**

$$P_{L\max} = \frac{\left|\hat{V}_{Th}\right|^2}{8R_L} = \frac{\left|10\angle 45^0\right|^2}{8\times 0.5} = 25\mathrm{W}$$

Summary of Chapter 9

- Definitions of instantaneous powers, average power, complex power, and apparent power
- Sinusoidal steady-state power calculation
- Maximum average power transfer