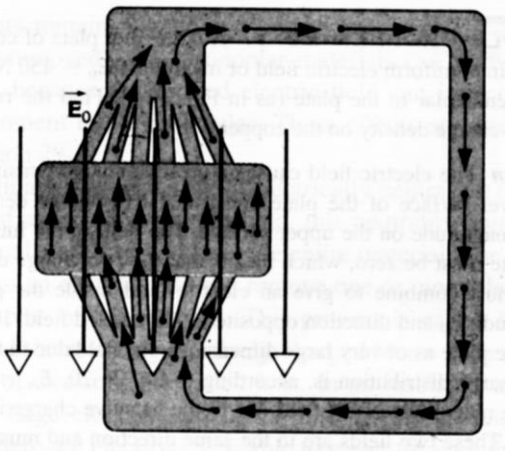
Chapter 23 Current Behaviors in Metallic Conductors

§ 1 Current Density and Drift Speed (p556-559)

- Electric Current

  The net amount of charge per unit time that pass through the full cross section at any point in the conductor.

$$I = \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$



Direction: the positive charges would move

- Current density
  - Current per unit area $j = \dfrac{I}{A}$
  - The current density is a vector. The direction of $\vec{j}$ is defined to be the direction of the flow of positive charge.
  - The relationship between $\vec{j}$ and I :

$$I = \iint_S \vec{j} \cdot d\vec{A}$$

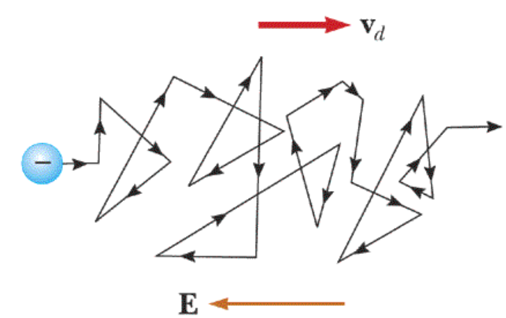
Drift speed

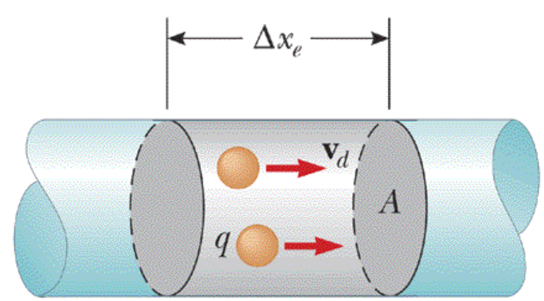
- The electrons collide with the ions of the lattice. On the average, electrons can be described as moving with a constant *drift speed* v_d in a direction opposite to the electric field.
- The relationship between $\vec{j}$ and v_d: In the interval Δt, the magnitude of net charge passing though the surface A is $\Delta Q = n(Av_d\Delta t)q$

$$j = \frac{\Delta Q}{A\Delta t} = nqv_d = -nev_d$$

***n*: the number of carrier per unit volume.**

$$\boxed{\vec{j} = -en\vec{v}_d}$$





Drift speed in a copper wire.

Example: A copper wire of cross-sectional area $3.00\times10^{-6}\text{m}^2$ carries a current of 10.0A. Find the drift speed of the conduction electrons in this copper wire. The density of copper is 8.95g/cm^3. the molar mass of copper is 63.5g/mol.

Solution: $$v_d = \frac{I}{nqA}$$

Evaluate the number of electrons per unit volume: *n*

In copper, there is nearly one conduction electron per atom on average. The number of atoms per unit volume:

$$\frac{\text{atoms/m}^3}{\text{atoms/mol}} = \frac{\text{mass/m}^3}{\text{mass/mol}} \Longleftrightarrow \frac{n}{N_A} = \frac{\rho_m}{M}$$

$$n = \frac{N_A \rho_m}{M} = \frac{(6.02\times10^{23}\text{electrons/mol})(8.95\times10^{3}\text{kg/m}^3)}{63.5\times10^{-3}\text{kg/mol}}$$

$$= 8.48\times10^{28}\text{electrons/m}^3$$

$$v_d = \frac{10.0\text{ C/s}}{(8.48\times10^{28}\text{m}^{-3})(1.60\times10^{-19}\text{C})(3.00\times10^{-6}\text{m}^2)} = 2.46\times10^{-4}\text{m/s}$$

§ 2 The Microscopic View of Ohm's Law

(p550-554)

Ohm's Law in macroscopic view

- A potential difference ΔV is applied across a wire conductor of length l and cross-section area A.

$$\boxed{I = \frac{\Delta V}{R}}$$

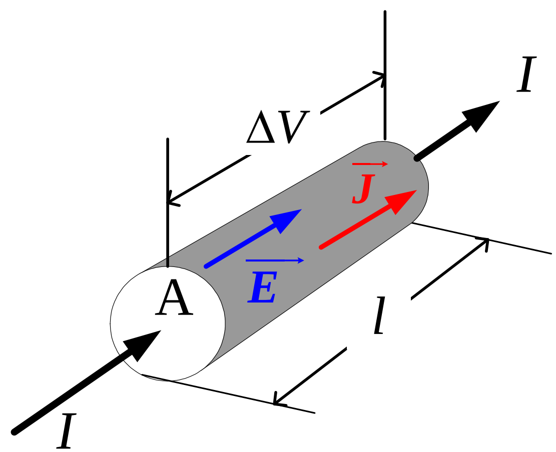
- R is the resistance of the wire. For ohmic materials:

$$R = \rho \frac{l}{A} \qquad dR = \rho \frac{dl}{dA}$$

ρ is resistivity which has the unit ohm-meter ($\Omega \cdot$m).

Ohm's Law in microscopic view

$$j = \frac{dI}{dA} = \frac{dU}{dAdR} = \frac{Edl}{dA\rho \frac{dl}{dA}} = \frac{E}{\rho} = \sigma E$$



$$\boxed{\vec{j} = \sigma \vec{E} = \frac{\vec{E}}{\rho}}$$

σ is conductivity

Example

Example: A hollow cylinder has length *L* and inner and outer radii *a* and *b*. It is made of a material with resistivity ρ. A potential difference is set up between the inner and outer surface of the cylinder so that current flows radially through the cylinder. What is the resistance to this radial current flow?

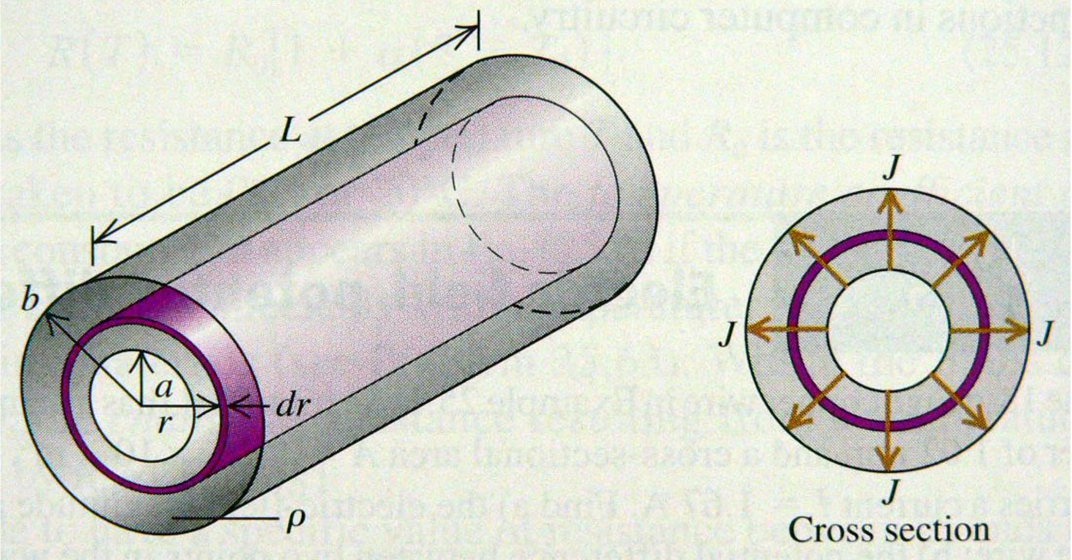
Solution: We consider a thin cylindrical shell of inner radius *r* and thickness *dr*. The resistance *dR* of this shell is that of a conductor with length *dr* and area $2\pi rL$, that is:

$$dR = \frac{\rho dr}{2\pi rL}$$

The total resistance:

$$R = \int dR = \frac{\rho}{2\pi L}\int_a^b \frac{dr}{r}$$

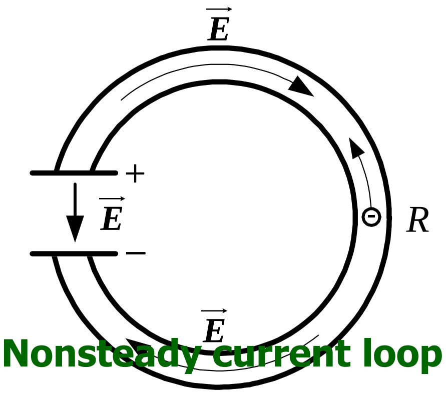
$$= \frac{\rho}{2\pi L}\ln\frac{b}{a}$$



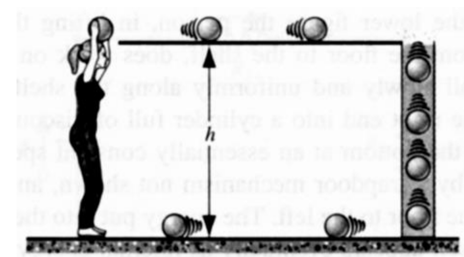
§ 3 The Electromotive Force

- How to maintain a steady current?
  - A steady current can only take place in a closed loop conducting material or complete circuit.
  - In a close loop, $\oint \vec{E} \cdot d\vec{l} = 0$. There is always a decrease in potential energy when a positive charge moves through an ordinary conducting material with resistance.

    Only electrostatic field in the complete circuit can not maintain a steady current.
  - In an complete circuit there must be a device somewhere in the loop that acts like the water pump in a water fountain. In this device there must be a non-electrostatic force F_n that move a positive charge "uphill", from lower to higher potential.



Nonsteady current loop

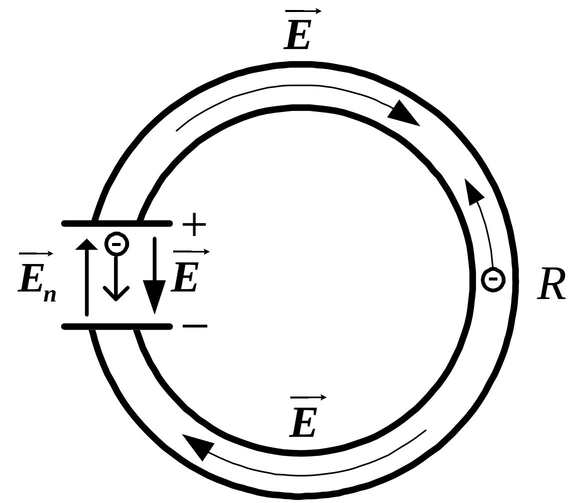


Electromotive force —— emf

- The influence that “pump” positive charges from lower to higher potential is called electromotive force.

Definition of emf

A emf is defined as the work per unit charge done by a non-electrostatic force *Fn*, when a positive charge moves inside the source of emf from “-” terminal to “+” terminal. *En=Fn / q* is called non-electrostatic field.



Steady current loop

$$W_n = \int_{-}^{+} \vec{F}_n \cdot d\vec{l} = \int_{-}^{+} q\vec{E}_n \cdot d\vec{l}$$

$$\boxed{\varepsilon = \frac{W_n}{q} = \int_{-}^{+} \vec{E}_n \cdot d\vec{l}}$$

If the non-electrostatic field E_n exists everywhere in the current loop, then

$$\boxed{\varepsilon = \oint \vec{E}_n \cdot d\vec{l}}$$

- F_n might be a force of chemical or magnetic origin, but it is not associate with an electrostatic field.

- What is the process inside the source of emf?
  - If a positive charge q is moved from ***b*** to ***a*** inside the source, the non-electrostatic force F_n does a positive work $W_n = q\mathcal{E}$ on the charge. This displacement is opposite to the electric force F_e, so the potential energy associate with the charge increases by amount equal to qV_{ab}. For the ideal source of emf, F_e and F_n are equal in magnitude but opposite in direction, so the total work done on the charge q is zero.

$$q\varepsilon = qV_{ab} \longrightarrow V_{ab} = \varepsilon$$