Introduction to Electronic Systems

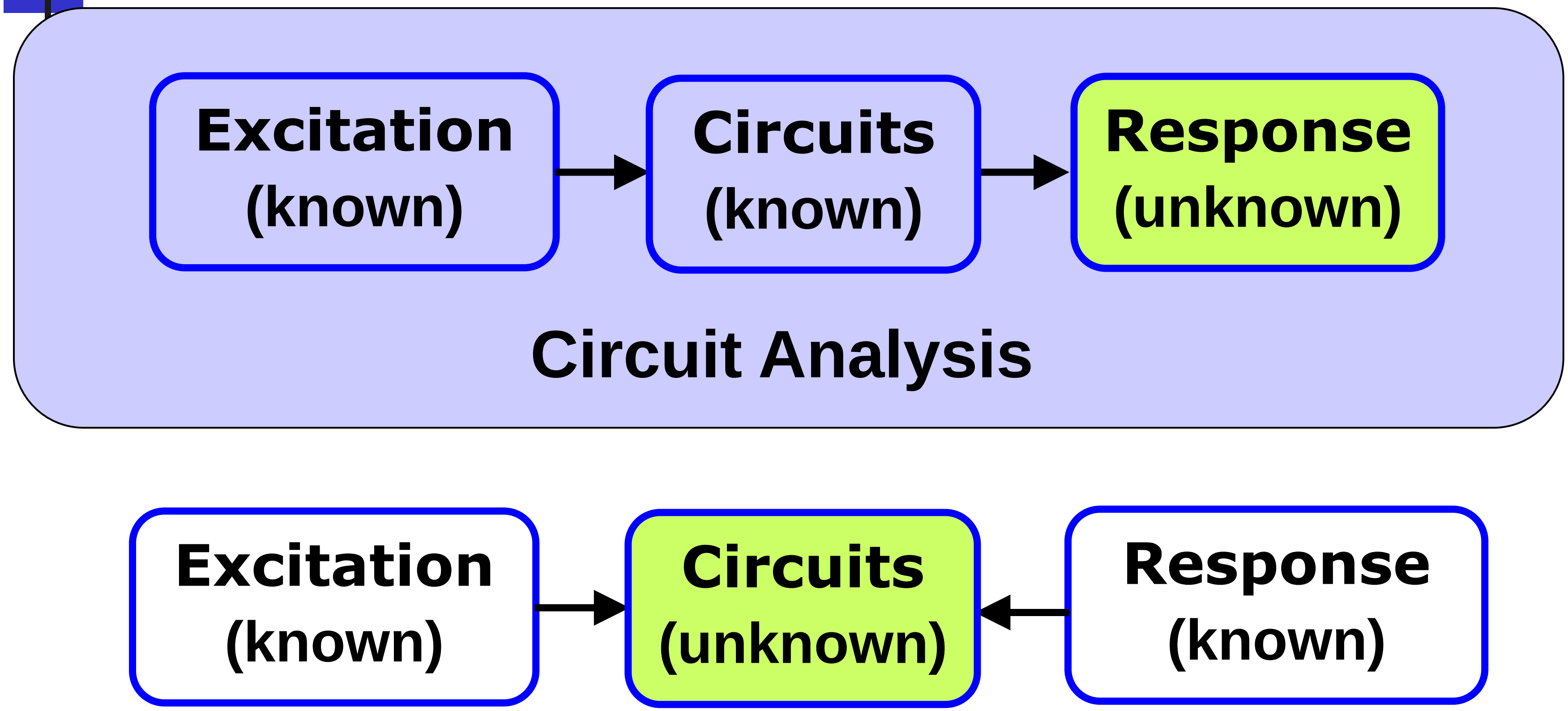
Overview of Circuit Analysis

- What is Circuit Analysis?
- Why do we implement Circuit Analysis?
- How do we implement Circuit Analysis?

What is Circuit Analysis?

Circuit Theory $\begin{cases} \text{Circuit Analysis} \\ \\ \\ \text{Circuit Design} \end{cases}$

What is Circuit Analysis?



Circuit Design

Why do we implement Circuit Analysis?

- **Quantitatively predict or determine the behavior of a circuit:**

  ---- *determine all voltages and currents*

- **Lead to circuit improvements and refinements**
- **Circuit Analysis is vital for circuit design.**

How is circuit behavior determined?

Topology Constraint + **Element Constraint** = **Circuit Behavior**

- Element Constraint
  - Passive Element
    - Resistive Element
    - Dynamic Element
  - Active Element
    - DC Source
    - AC Source

Three Parts of the Lecture

1. **Resistive Circuit Analysis**

   DC + Resistive Element + Topology

2. **Dynamic Circuit Analysis**

   DC + Dynamic/Resistive Element + Topology

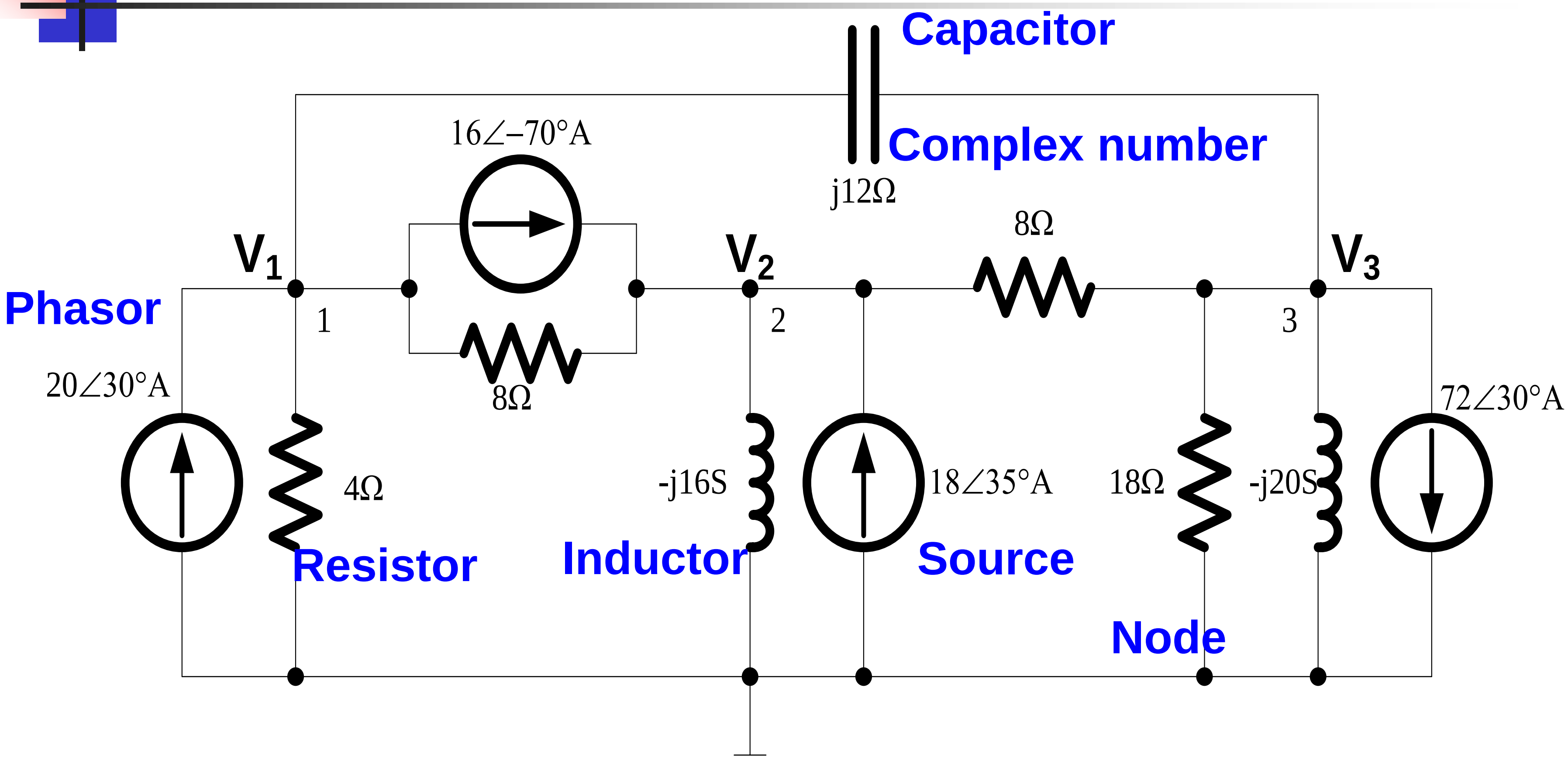
3. **Sinusoidal Steady-State Analysis**

   AC + Dynamic/Resistive Element + Topology

How do we implement Circuit Analysis?

- **Build equations** for a given circuit by using basic laws, theorems, and circuit analysis techniques;
- **Solve equations** to predict or determine circuit behavior (required currents and voltages).

A Typical Problem you should be able to solve after this course:



Find the voltage V_1, V_2 and V_3.

Part 1

Resistive Circuit Analysis

(DC + Resistive Element + Topology)

Part 1: Resistive Circuit Analysis

1. **Circuit Variables and Circuit Elements**
2. Simple Resistive Circuit Analysis
3. Techniques of Circuit Analysis
4. Operational Amplifier

Chapter 1: Circuit Variables and Circuit Elements

- Circuit Variables
- Circuit Elements
- Power and Energy
- Voltage and Current Sources

1-1 Circuit Variables

- Electric Charge
- Current
- Voltage
- Power
- Energy

Electric Charge (1)

- Bipolar: Positive (proton) and Negative (electron) charge;
- The fundamental unit of charge is Coulomb (C);
- A single electron has a charge of -1.602×10^{-19} C, and a single proton has a charge of $+1.602\times10^{-19}$ C;

Electric Charge (2)

- Charge in motion ---- Current;
- Separation of charge ---- voltage;
- Charge conservation law:

  Electrons (or protons) are neither created nor destroyed.

Current (1)

- Motion of charge creates electric fluid (current);
- The fundamental unit of current is Ampere (A);
- The rate of charge flow is defined as current, and expressed as:

$$i(t) = \frac{dq(t)}{dt}$$

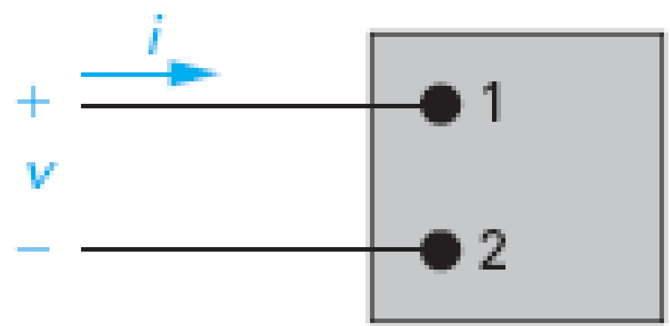
- i : current in A;
- q : charge in C;
- t : time in s.

1.1 The current at the terminals of the element in Fig. 1.2 is

$$i = 0, \qquad t < 0;$$

$$i = 20e^{-5000t}\ \text{A}, \qquad t \geq 0.$$

Calculate the total charge (in microcoulombs) entering the element at its upper terminal.



Solution:

$$Q=\int_{-\infty}^{+\infty} i(t)\,dt=\int_{0}^{+\infty} 20e^{-5000t}\,dt$$

$$=\frac{20}{-5000}e^{-5000t}\Big|_{0}^{+\infty}$$

$$=\frac{20}{-5000}(0-1)=4\times 10^{-3}C$$

$$=4000\mu C$$

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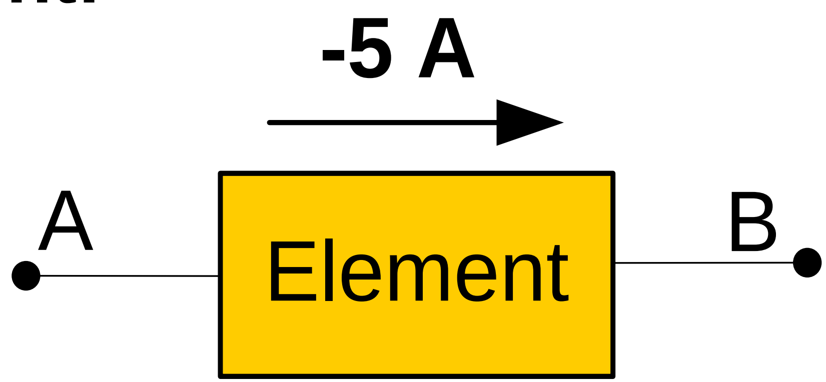
Current (2)

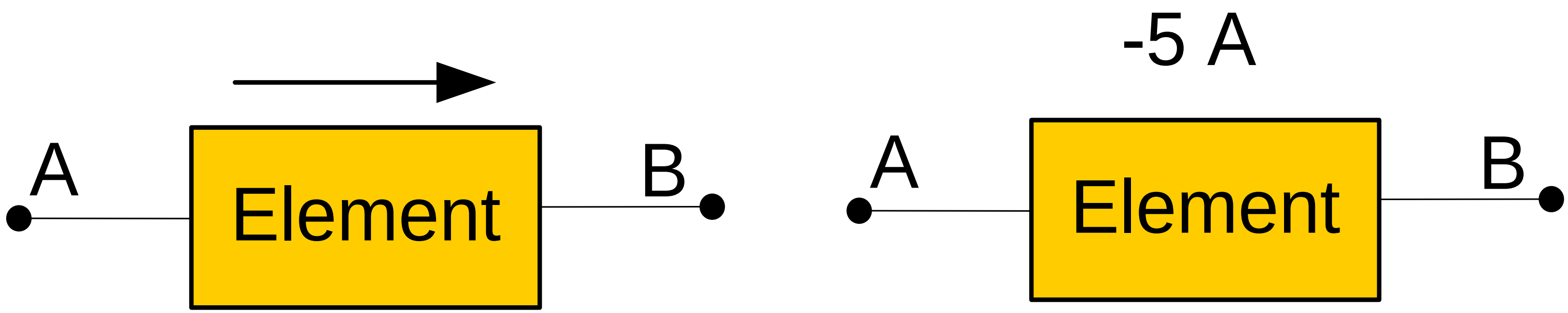
- Current direction
  - Real direction: Moving direction of positive charges (proton)
  - **Reference direction**: Arbitrary assigned direction by an arrow

Current (3)

- A current is completely specified by both its **magnitude** and **reference direction**.
- **Reference direction** is a fundamental part of a current!

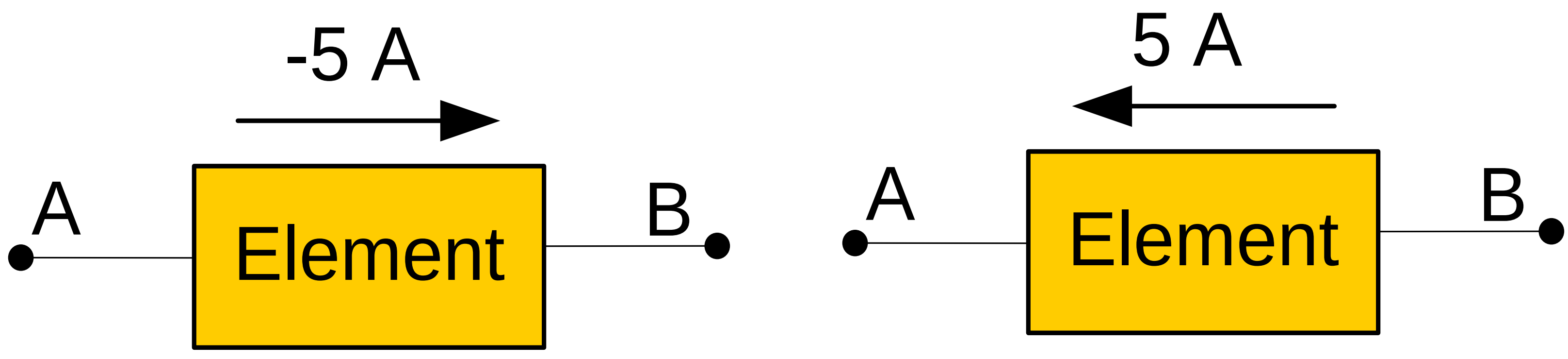


Current (4)



Incorrect definitions of a current

Current (5)



Two correct equivalent representations for the exact same current

Voltage (1)

- Charge separation creates electric force (voltage);
- The fundamental unit of voltage is Volt (V);
- The energy per unit charge created by the separation is defined as voltage:

$$u = \frac{dw}{dq}$$

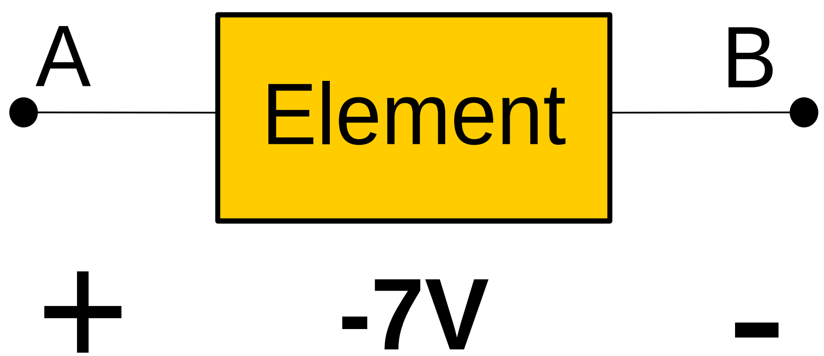
- u : voltage in V;
- w : energy in J;
- q : charge in C.

Voltage (2)

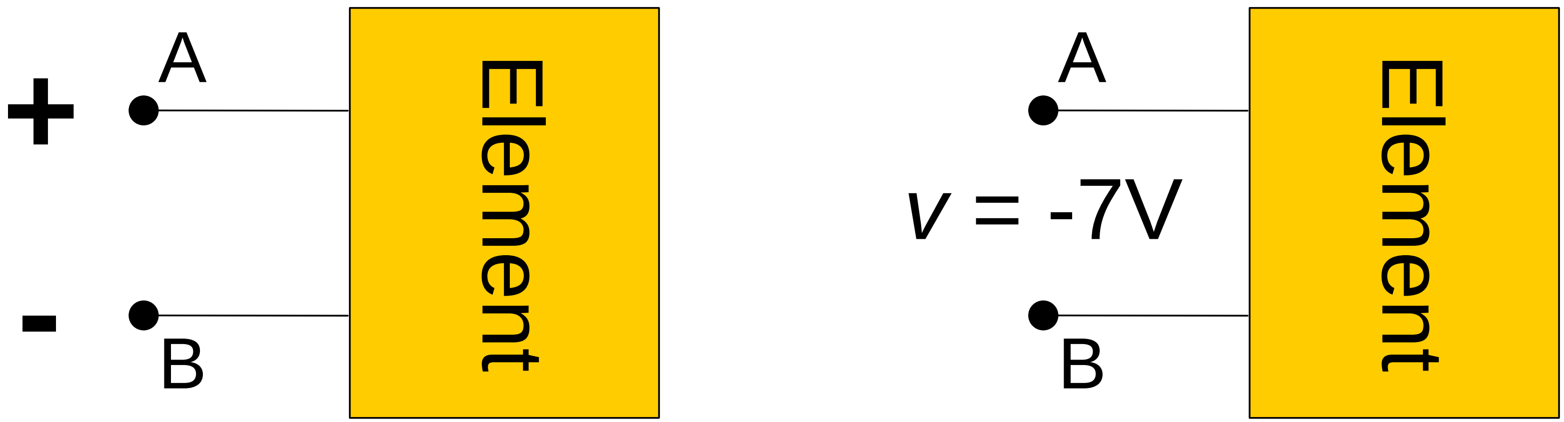
- Voltage polarity
  - Real polarity: From positive terminal to negative terminal
  - Reference polarity: Arbitrary assigned polarity by a plus-minus sign pair

Voltage (3)

- A voltage is completely specified by both its **magnitude** and **reference polarity**.
- The definition of any voltage must include **a plus-minus sign pair**!

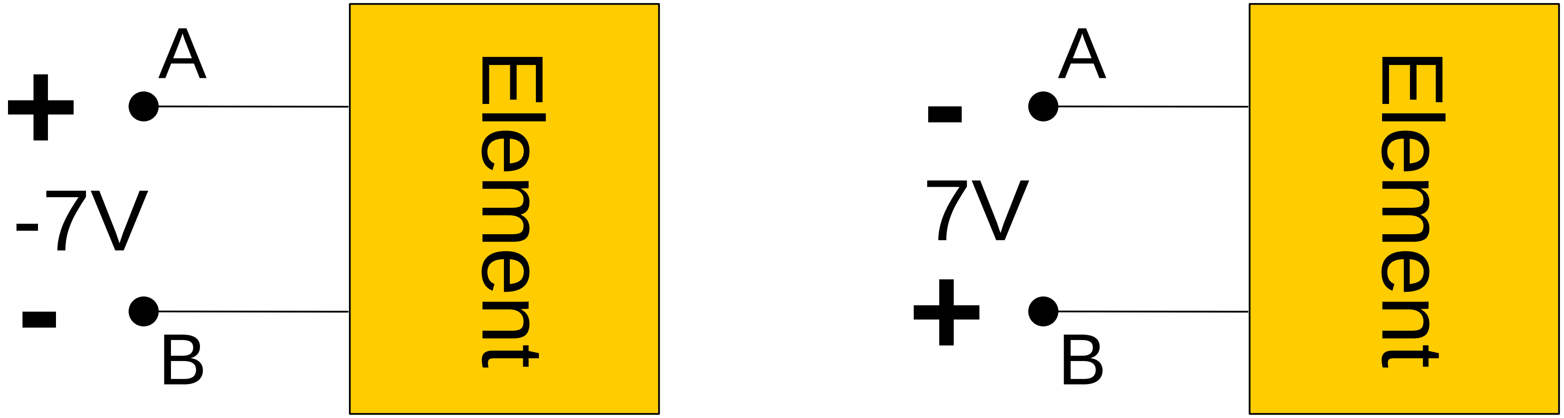


Voltage (4)



Incorrect definitions of a voltage

Voltage (5)



Two correct equivalent representations for the exact same voltage

1-2 Circuit Elements

- Ideal Basic Circuit Element
- Resistor (Ohm's Law)
- Passive Sign Convention
- Voltage and Current Source
- op amp, Capacitor, Inductor
- Diode, Transistor, Transformer ……

Ideal Basic Circuit Element

- Only has two terminals
- Described mathematically in terms of voltage and/or current
- Can not be further reduced or subdivided into other elements
- Resistor, Voltage/Current source, Capacitor, Inductor

Ideal Basic Circuit Element

- **Ideal:**

  Can not be realizable physically

- **Basic:**

  Can not be further reduced or subdivided into other elements

- **Ideal Basic Circuit Element:**

  Resistor, Voltage/Current source, Capacitor, Inductor

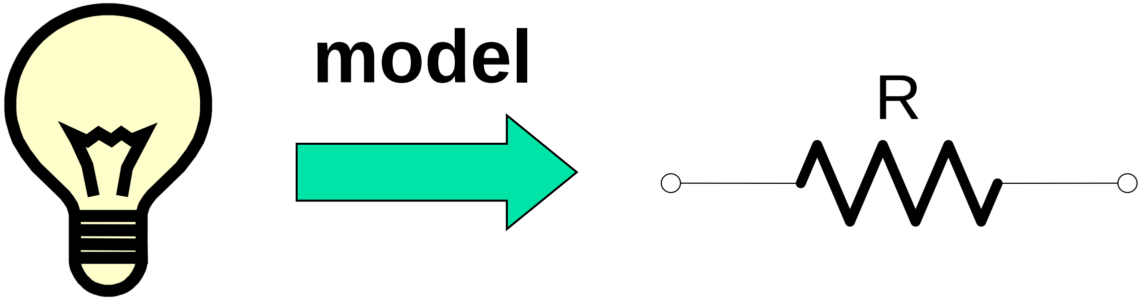
Ideal Circuit Element

- The dimension of element is 1/10th (or smaller) of the operation wavelength.

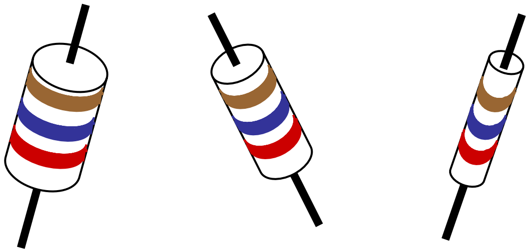
$$\lambda = \frac{C}{f}$$

$C=3\times10^{8}$m/s

f(Hz)	50	25k	500M	30G
λ(m)	6×10^{6}	12k	0.6	0.01

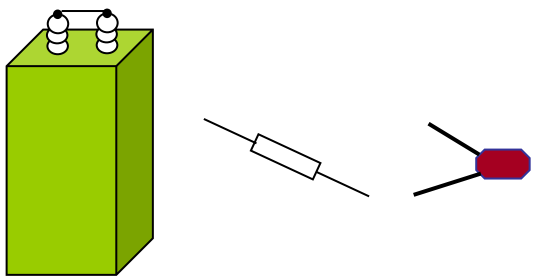


Ideal Basic Circuit Elements



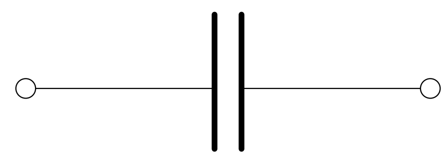
Resistor

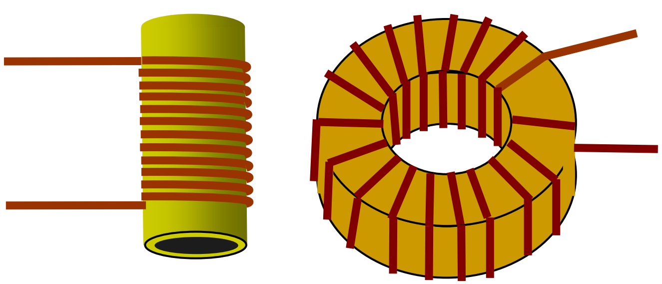
$R_1 = 1k\Omega$



Capacitor

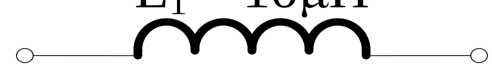
$C_1 = 1nF$



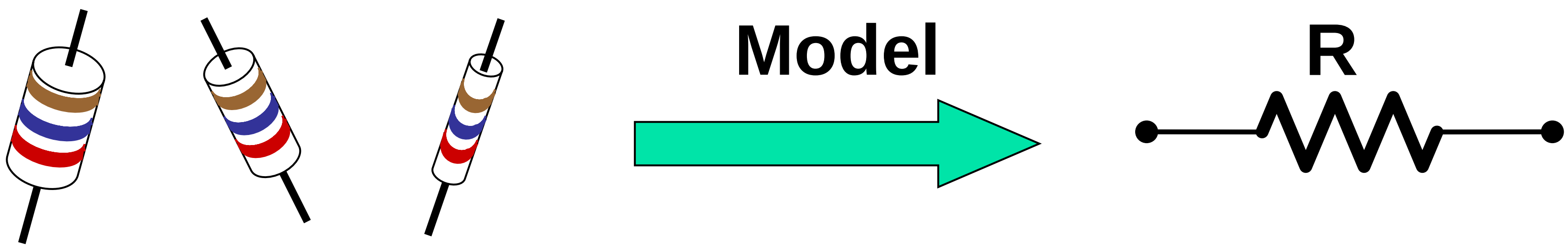


Inductor

$L_1 = 10\mu H$



Resistor

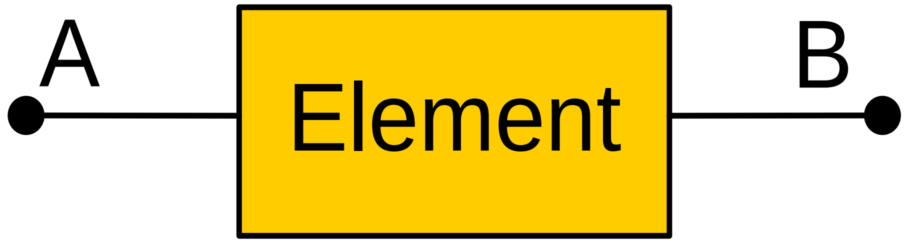


- Resistor is an ideal basic circuit element.

Passive Sign Convention

- Theoretically, the assignments of the reference direction for voltage and current are entirely arbitrary.
- But we must write all subsequent equations to agree with the chosen references.

Example:



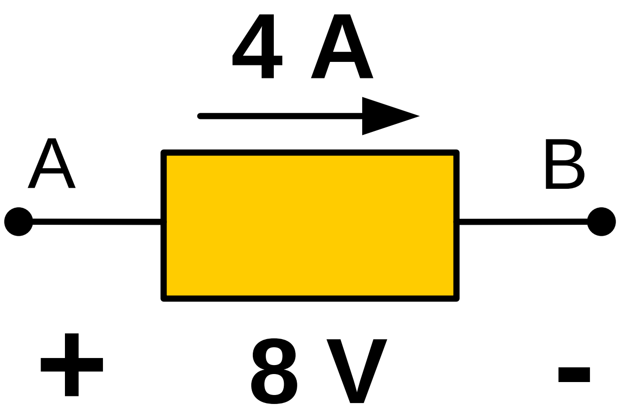
- The **REAL** direction for the current is:

  4A flowing from terminal A to B

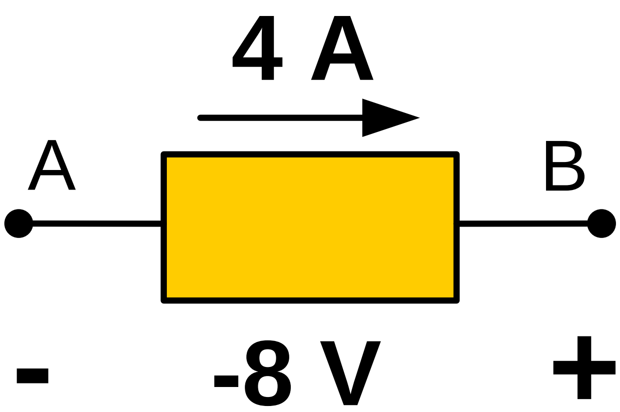
- The **REAL** polarity for the voltage is:

  Terminal A is 8V positive with respective to B

- By using Reference polarity for voltage and Reference direction for current, we have FOUR choices to represent this case :

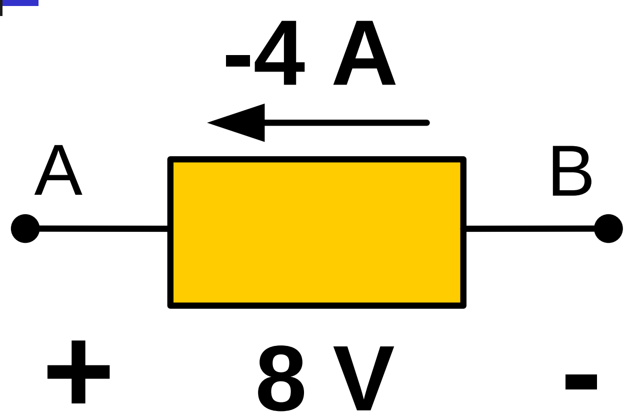


Choice 1

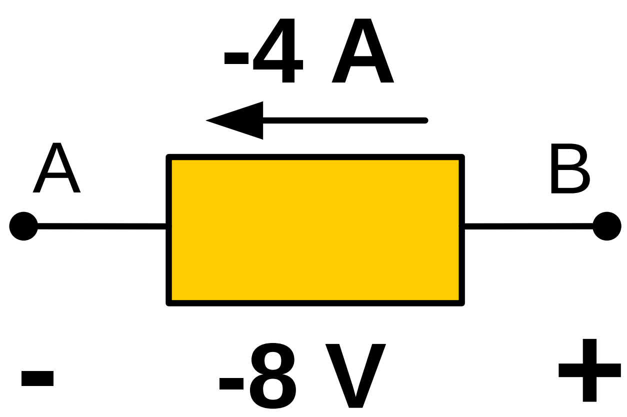


Choice 2

- **If the element is a resistor, how to find the value of the resistance for each case?**



Choice 3



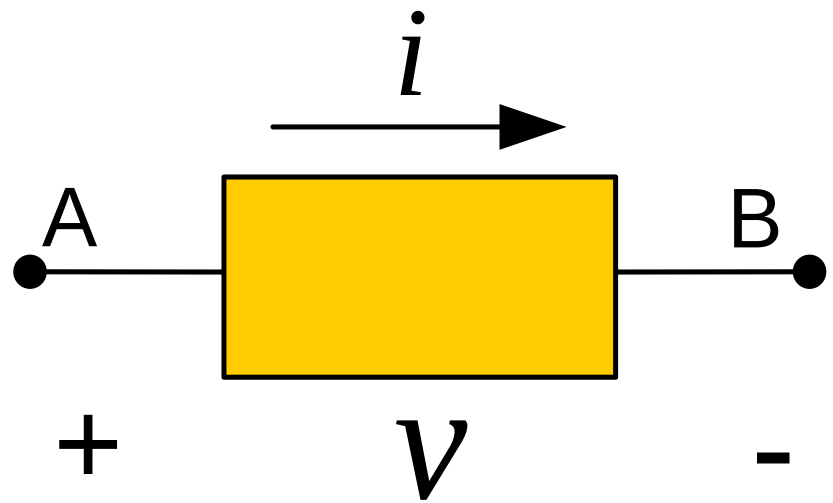
Choice 4

- **If the element is a resistor, how to find the value of the resistance for each case?**

Passive Sign Convention (PSC)

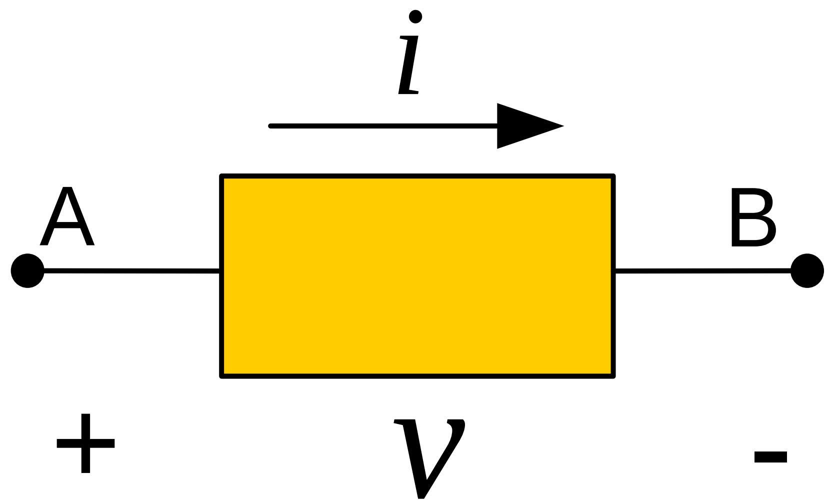
- Whenever the reference direction for the current in an element is in the direction of the reference voltage drop across the element, use a positive sign in any expression that relates the voltage and the current. Otherwise use a negative sign.

Passive Sign Convention



- **The reference direction for the current is assigned in the direction of the reference voltage drop across the element.**

Passive Sign Convention



- Current enters the positive terminal;
- Current leaves the negative terminal.

Passive Sign Convention

无源符号约定

+ − 与 → 同向

- **Passive sign convention is used only for the convenience of circuit analysis.**
- **We apply the passive sign convention in all the lectures that follow.**

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Ohm’s Law

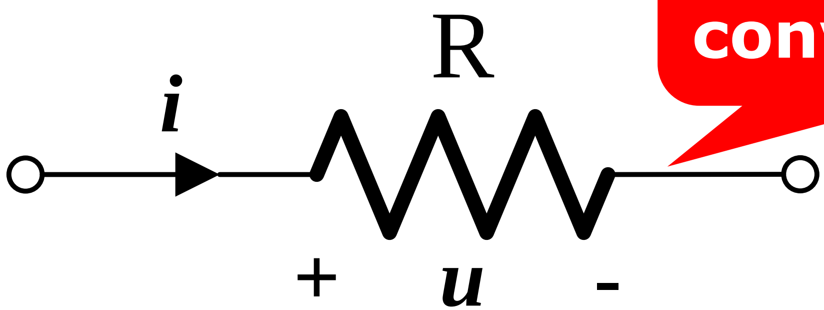
Ohm’s Law with PSC

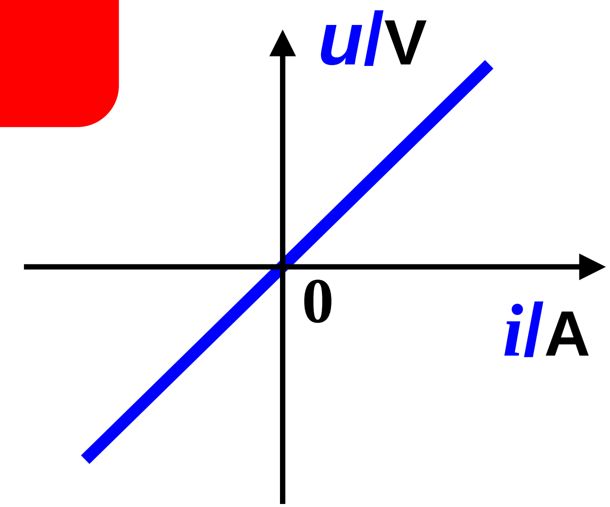
Ohm’s Law w/o PSC

Power and Energy of Resistors

Ohm's Law with PSC

Passive sign convention

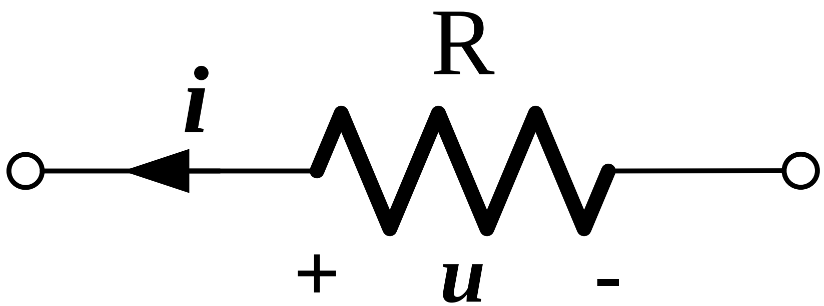


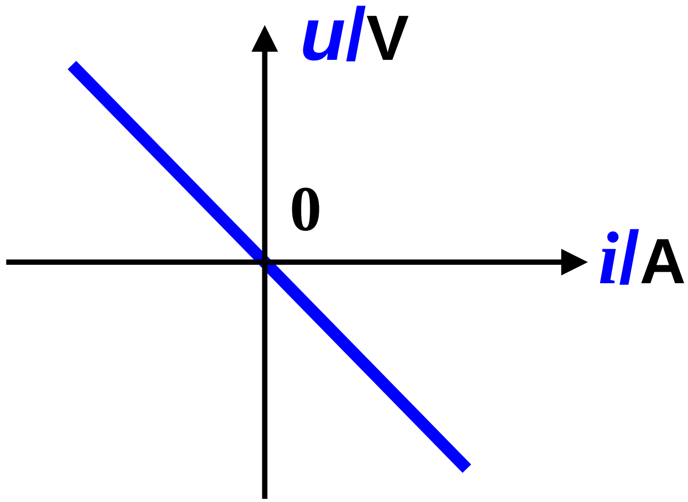


VCR of Resistor (Ohm's law):

$$u = Ri$$

Ohm's Law w/o PSC





Ohm's law (w/o PSC):

$$u = -Ri$$

1-3 Power and Energy

- Power
- Energy
- Passive and Active Element

Resistance and Conductance

In Ohm's law, R is measured in ohm (Ω) :

$$u = Ri \qquad R = u/i$$

R can also be characterized by conductance;

Conductance is measured in Siemens (S); 电导

$$i = Gu \qquad G = \frac{1}{R}$$

Power

- Power is defined as the time rate of expending or absorbing energy, and expressed as:

$$p = \frac{dw}{dt}$$

$$= \left(\frac{dw}{dq}\right)\left(\frac{dq}{dt}\right) = vi$$

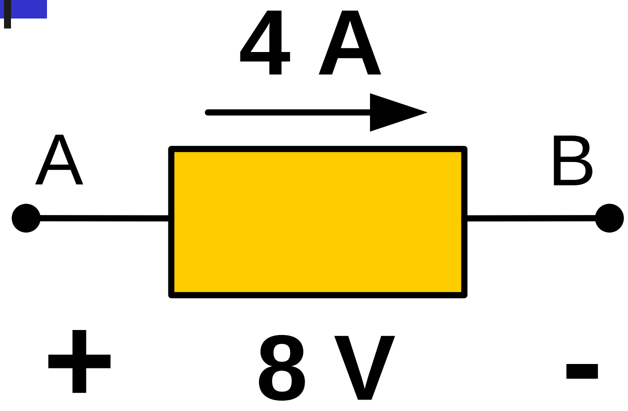
Passive sign convention

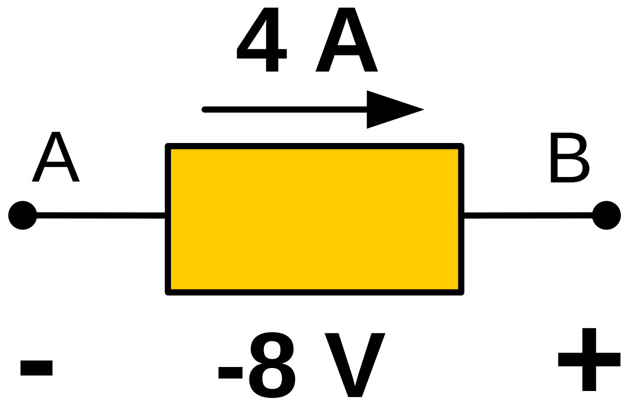
Power

- **If the passive sign convention is not applied, the power is expressed as:**

$$p = -vi$$

Example:





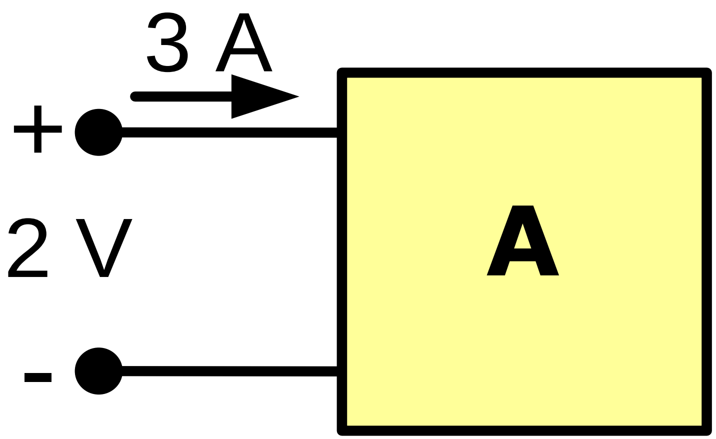
$$
\begin{aligned}
p &= vi \\
&= 8 \times 4 \\
&= 32\text{W}
\end{aligned}
$$

$$
\begin{aligned}
p &= -vi \\
&= -(-8) \times 4 \\
&= 32\text{W}
\end{aligned}
$$

Power

- If p is **positive**, power is absorbed by the element; or, power is being delivered to the element;
- If p is **negative**, the element supplies power; or, power is being extracted from the element.

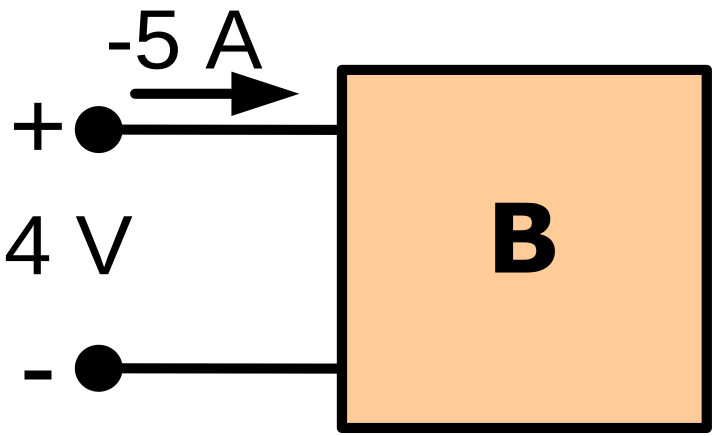
Examples:



$$p = (2\mathrm{V})(3\mathrm{A})$$
$$= 6\mathrm{W}$$

- Power of 6W is **absorbed** by A;
- Or, power of 6W is **being delivered** to A.

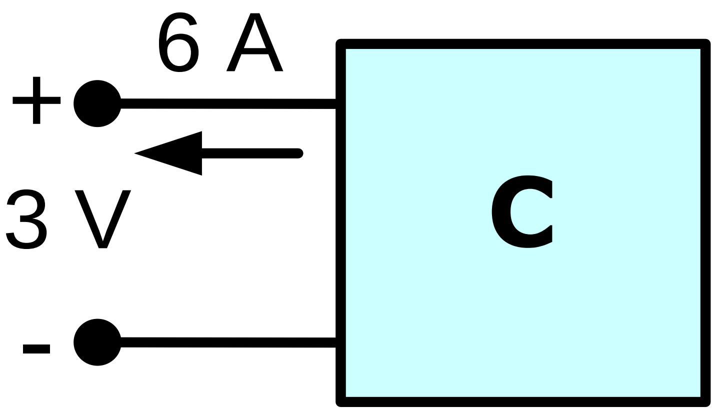
Examples :



$$p = (4\text{V})(\text{-}5\text{A}) = -20\text{W}$$

- Power of 20W is **supplied** by B;
- Or, power of 20W is **being extracted** from B.

Examples:



$$p = -(3\text{V})(6\text{A})$$
$$= -18\text{W}$$

- **Power of 18W is supplied by C;**
- **Or, power of 18W is being extracted from C.**

Energy

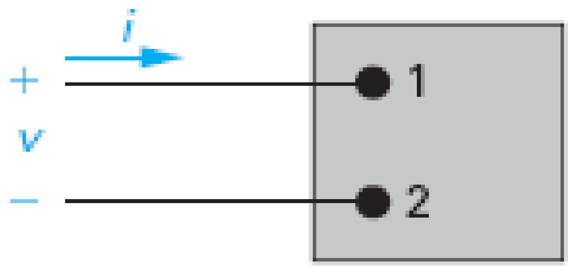
- **Inversely, energy can be calculated as:**

$$w = \int_{-\infty}^{t} p(\tau)\, d\tau$$

1.8. The voltage and current at the terminals of the circuit element in Fig. 1.2 are zero for $t < 0$. For $t \geq 0$ they are

$$v = e^{-500t} - e^{-1500t} \text{ V},$$

$$i = 30 - 40e^{-500t} + 10e^{-1500t} \text{ mA}.$$



(a) Find the power at $t = 1$ ms.

(b) How much energy is delivered to the circuit element between 0 and 1 ms?

(c) Find the total energy delivered to the element.

Solution:

P 1.8 [a] $p = vi = 30e^{-500t} - 30e^{-1500t} - 40e^{-1000t} + 50e^{-2000t} - 10e^{-3000t}$

$$p(1 \text{ ms}) = 3.1 \text{ mW}$$

[b]

$$\begin{aligned} w(t) &= \int_0^t (30e^{-500x} - 30e^{-1500x} - 40e^{-1000x} + \\ &\quad 50e^{-2000x} - 10e^{-3000x})dx \\ &= 21.67 - 60e^{-500t} + 20e^{-1500t} + 40e^{-1000t} - \\ &\quad 25e^{-2000t} + 3.33e^{-3000t} \mu\text{J} \\ w(1 \text{ ms}) &= 1.24 \mu\text{J} \end{aligned}$$

[c] $w_{\text{total}} = 21.67 \mu\text{J}$

Solution:

(c) p=vi

总能量就是对p从0到+∞做积分

$$
\begin{aligned}
E &= \int_0^{+\infty} p\,dt = \int_0^{+\infty} 1000(e^{-500t} - e^{-1500t})(30 - 40e^{-500t} + 10e^{-1500t})dt \\
&= 1000\int_0^{+\infty} (30e^{-500t} - 40e^{-100t} + 10e^{-2000t} - 30e^{-1500t} + 40e^{-2000t} - 10e^{-3000t})dt \\
&= 1000 \times \left(-\frac{30}{500}e^{-500t} + \frac{40}{1000}e^{-1000t} + \frac{30}{1500}e^{-1500t} - \frac{50}{2000}e^{-2000t} + \frac{10}{3000}e^{-3000t}\right)\Big|_0^{+\infty} \\
&= 1000 \times \left(\frac{3}{50} - \frac{1}{25} - \frac{1}{50} + \frac{1}{40} - \frac{1}{300}\right) \\
&= 1000 \times \frac{13}{600} \\
&\approx 21.67
\end{aligned}
$$

Passive and Active Element

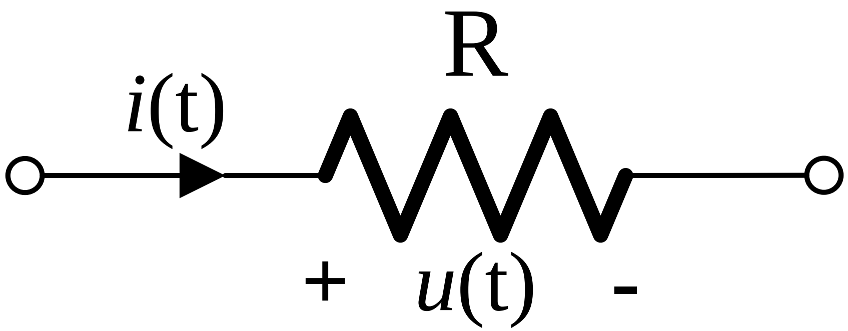
- **Energy:**

$$w=\int_{-\infty}^{t}p(\tau)d\tau=\int_{-\infty}^{t}u(\tau)i(\tau)d\tau$$

- **Passive:** $w \geq 0$

- **Active:** $w < 0$

Power of a Resistor

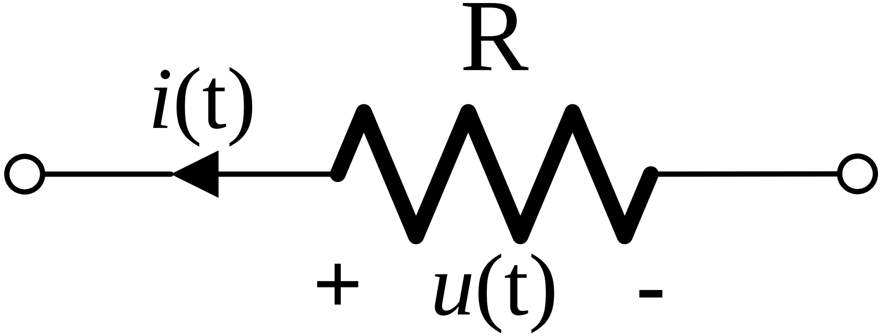


(with PSC)

$$p = vi = (iR)i = i^2 R$$

$$= vi = v\left(\frac{v}{R}\right) = \frac{v^2}{R}$$

Power of a Resistor



(w/o PSC)

$$p = -vi = -\left(-iR\right)i = i^2 R$$

$$= -vi = -v\left(-\frac{v}{R}\right) = \frac{v^2}{R}$$

Power of a Resistor

$$p = i^2 R = \frac{v^2}{R} \geq 0$$

A resistor always absorbs power from the circuit.

Energy of a Resistor

$$w(t) = \int_{-\infty}^{t} p(\tau)d\tau$$

$$= \int_{-\infty}^{t} i^2 R d\tau = \int_{-\infty}^{t} \frac{v^2}{R} d\tau \geqslant 0$$

Resistor is a passive element.

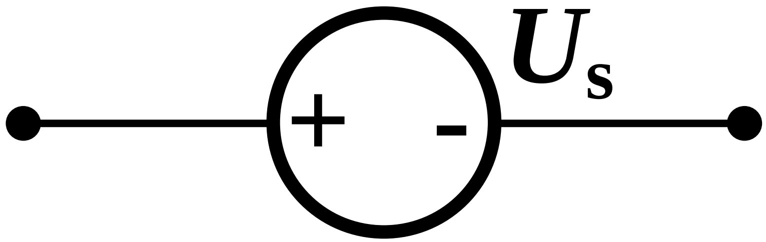
1-4 Voltage and Current Source

- **Independent source**
  - IVS
  - ICS
- **Dependent source (Controlled source)**
  - CCCS
  - VCCS
  - CCVS
  - VCVS

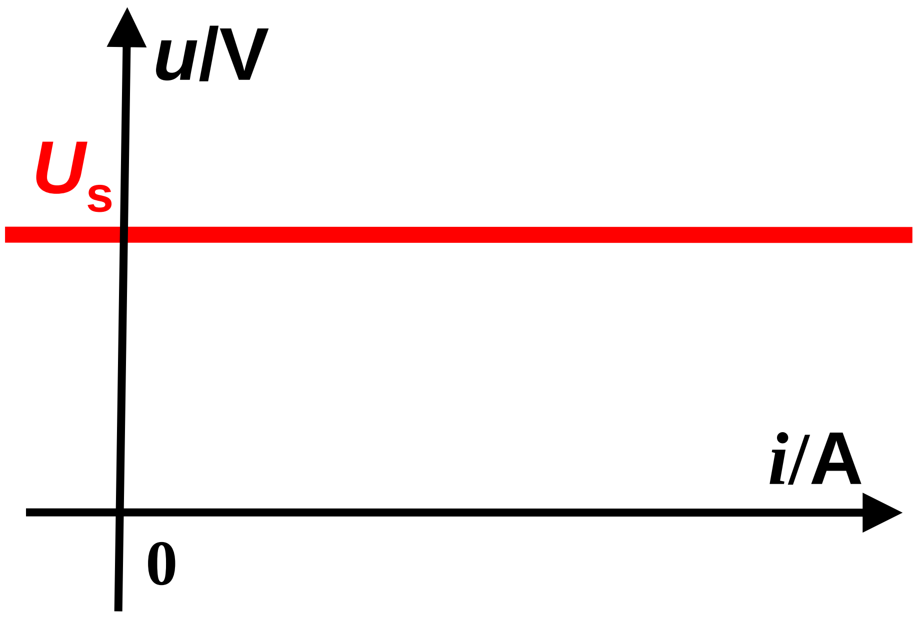
Independent Voltage Source

- **An ideal Independent Voltage Source (IVS) is a circuit element that maintains a prescribed voltage across its terminals regardless of the current flowing in those terminals.**

Independent Voltage Source



IVS

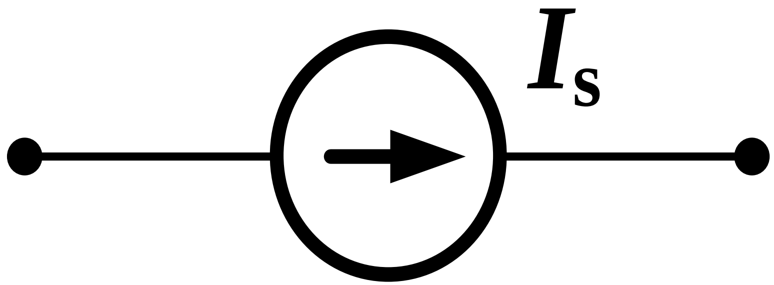


- **Symbol**
  - Reference polarity
  - Magnitude

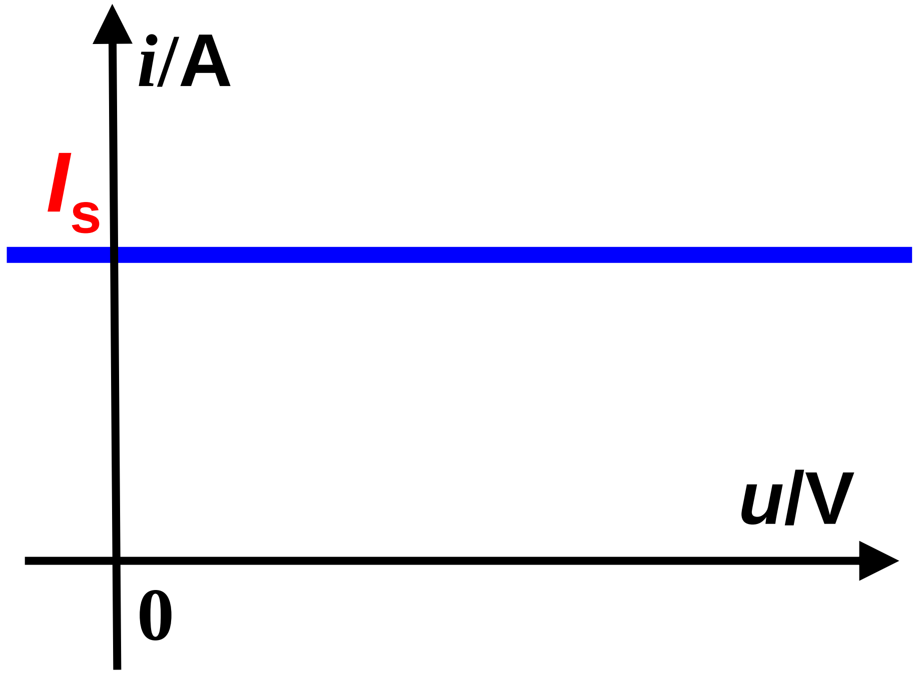
Independent Current Source

- **An ideal Independent Current Source (ICS) is a circuit element that maintains a prescribed current through its terminals regardless of the voltage across those terminals.**

Independent Current Source



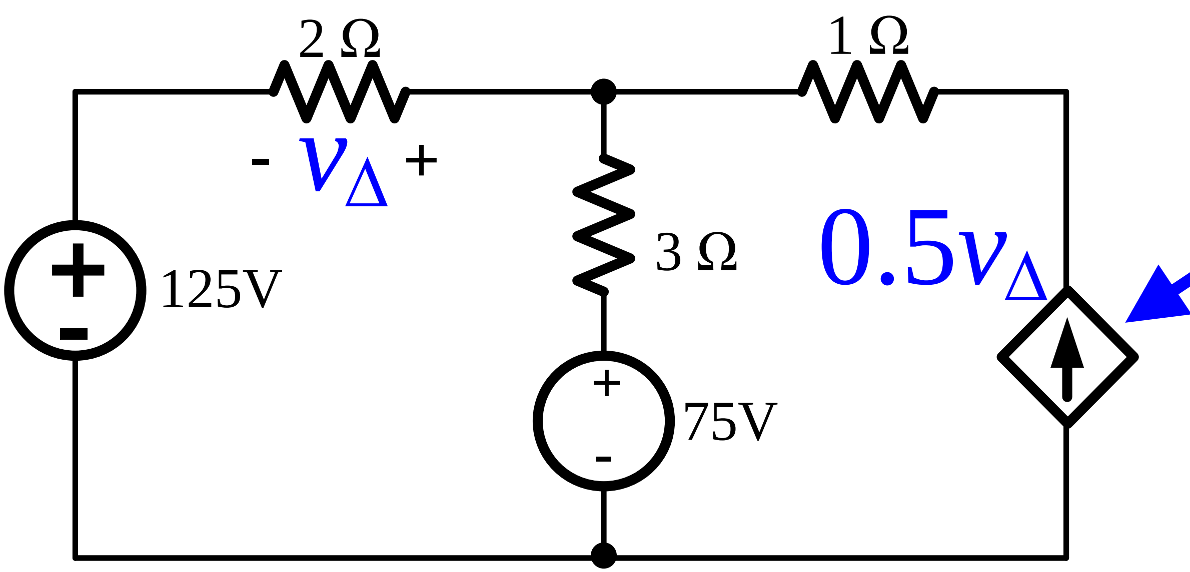
ICS



- **Symbol**
  - **Reference direction**
  - **Magnitude**

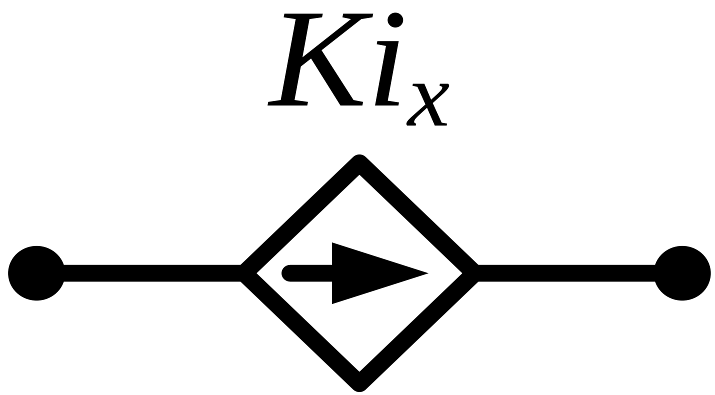
Dependent (Controlled) Sources

- Dependent source is determined by a voltage (or current) existing at some other location in the circuit.



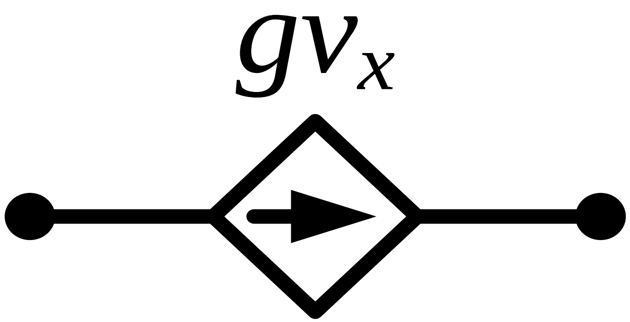
Voltage Controlled Current Source

CCCS



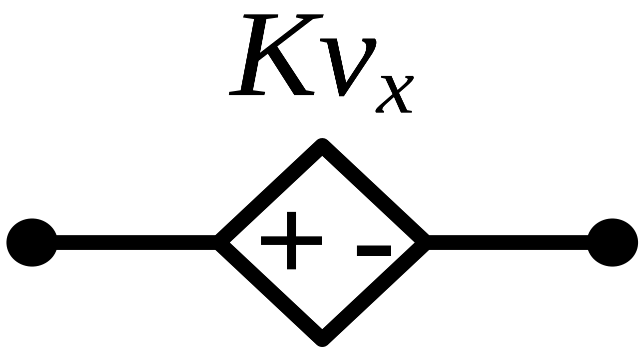
- Current-Controlled Current Source (CCCS)
- *K* is a constant factor

VCCS



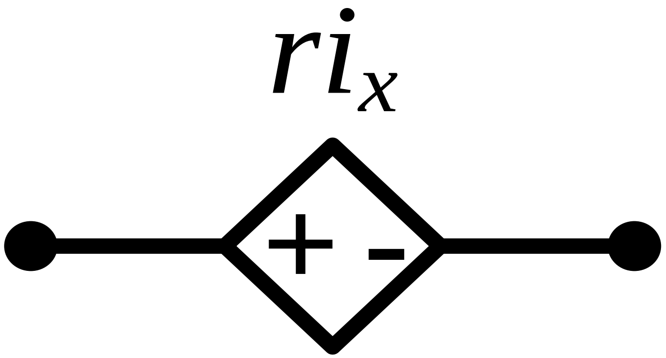
- Voltage-Controlled Current Source (VCCS)
- *g* is a constant factor

VCVS



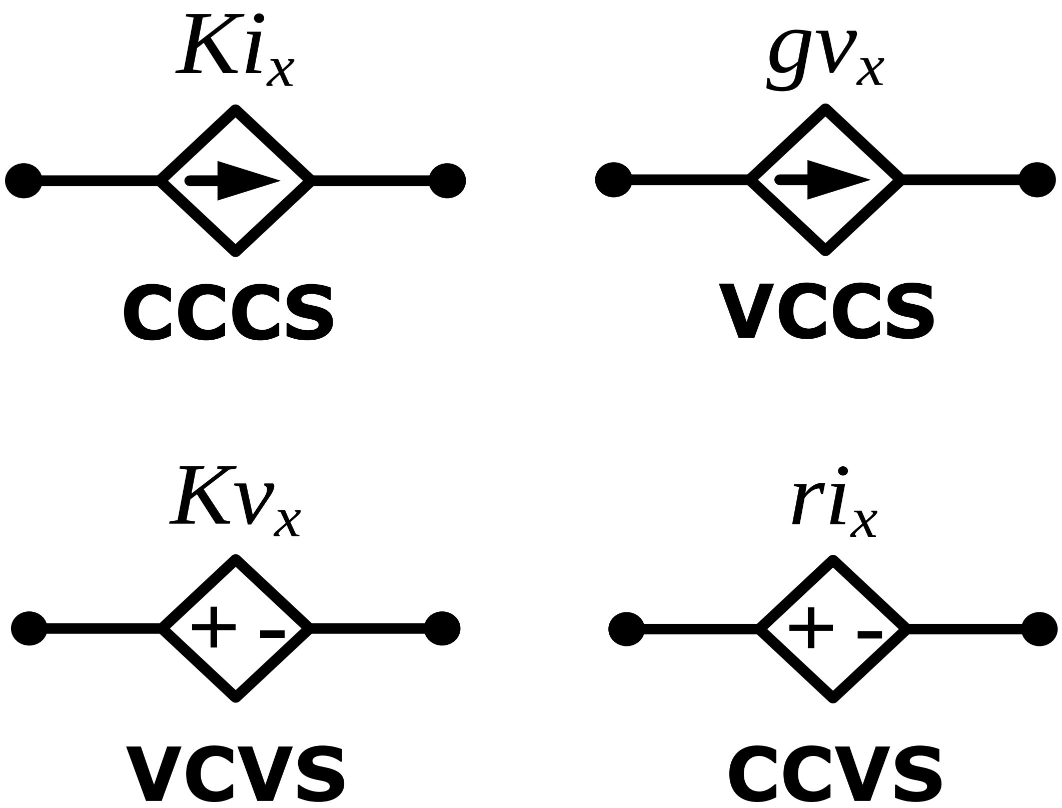
- **Voltage-Controlled Voltage Source (VCVS)**
- ***K* is a constant factor**

CCVS



- Current-Controlled Voltage Source (CCVS)
- *r* is a constant factor

Dependent (Controlled) Sources



- ***K*, *g*, *r* are all const. factors.**

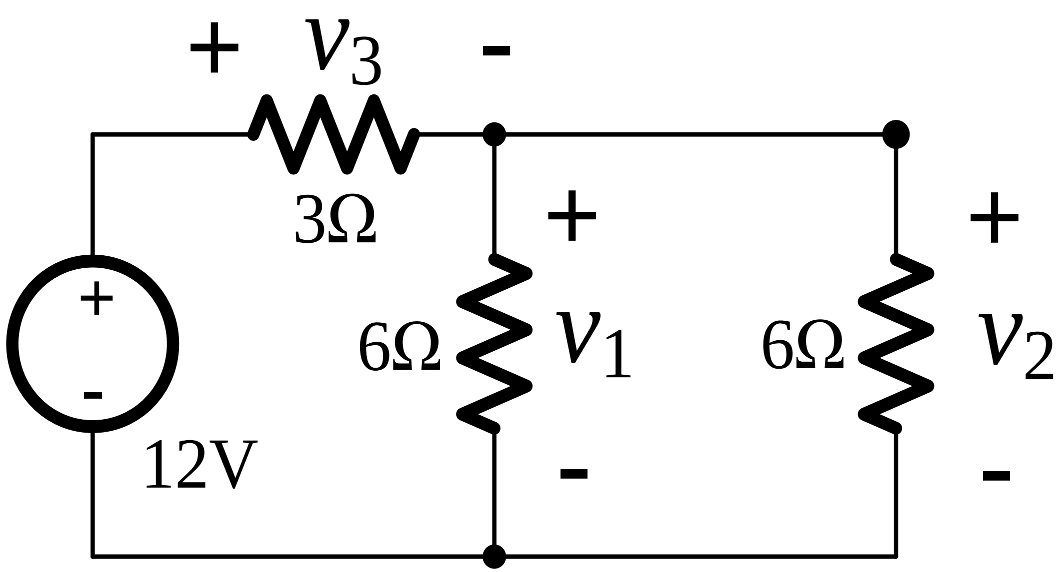
Question 1:

What are the units of the factors of *K*, *g*, and *r* of those controlled sources in last page?

- ***K*** **is a dimensionless scaling factor;**
- ***g*** **is a factor with unit of A/V;**
- ***r*** **is a factor with unit of V/A.**

Question 2:

How to calculate the power of a voltage or current source?



Solution

- 电阻R1与电阻R2并联,根据电阻并联计算公式:

$$R' = \frac{R1 \cdot R2}{R1 + R2} = \frac{6 \times 6}{6 + 6} = 3\Omega$$

则电压源发出功率为:

$$P=UI=\frac{U^2}{R3+R'}=\frac{144}{6}=24W。$$

Summary of Chapter 1

- Basic conceptions of charge, current, voltage, power, and energy
- Reference polarity for voltage and reference direction for current
- Ideal basic circuit element
- Passive sign convention (PSC)
- Power and energy calculation
- IVS, ICS, CCCS, VCCS, CCVS, VCVS