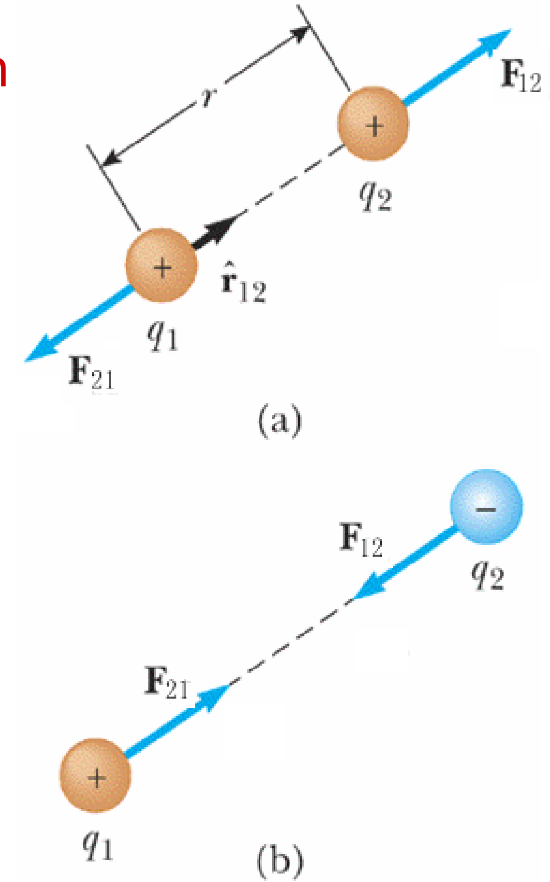
Chapter 19-20 The Electric Field

§ 1 Coulomb's Law (p460)

- Coulomb's Law
  - The electrostatic force exerted by point charge q_1 on q_2, written F_{12}, can be expressed in vector form as (inverse square law):

$$\boxed{\vec{F}_{12} = k_e \frac{q_1 q_2}{r^2}\hat{r}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}_{12}}$$



$k_e = 8.99\times10^{9}\,\mathrm{N\cdot m^2/C^2}$ —— Coulomb constant.

$\varepsilon_0 = 8.8542\times10^{-12}\,\mathrm{C^2/N\cdot m^2}$ —— permittivity of free space (electric constant)

$\hat{r}_{12}$ is a unit vector directed from q_1 toward q_2

  - From Newton's third law, $\vec{F}_{21} = -\vec{F}_{12}$
  - The Coulomb forces between the two charges having the same sign are repulsive, while the two charges with opposite sign result in attractive Coulomb forces.

- It is only suitable for static point charges at the free space.
- If several charges are present, the net force on any one of them will be the vector sum of the forces due to each of the others. It is satisfied the principle of superposition.

Comparison between Coulomb's Law and Newton's law of gravitation

- Comparison between Coulomb's Law and Newton's law of gravitation
  - Both are inverse square laws, and charge q plays the same role in Coulomb's law as that the mass m plays in Newton's law of gravitation.
  - One difference between the two laws is that gravitational forces are always attractive, whereas electrostatic forces can be either repulsive or attractive.
- Comparison of magnitude between electrostatic forces and gravitational forces

  Example: the two forces in the hydrogen atom — the distance between the electron and proton: 5.3×10^{-11}m.

  The electrostatic force: $F_e = k_e \frac{e^2}{r^2} = (8.99\times10^9\,\mathrm{N\cdot m^2/C^2})\frac{(1.60\times10^{-19}\,\mathrm{C})^2}{(5.3\times10^{-11}\,\mathrm{m})^2} = 8.2\times10^{-8}\,\mathrm{N}$

  The gravitational force:

$$F_g = G\frac{m_e m_p}{r^2} = (6.67\times10^{-11}\,\mathrm{N\cdot m^2/kg^2})\frac{(9.11\times10^{-31}\,\mathrm{kg})(1.67\times10^{-27}\,\mathrm{kg})}{(5.3\times10^{-11}\,\mathrm{m})^2} = 3.6\times10^{-47}\,\mathrm{N}$$

  - The gravitational force is weaker than the electrostatic force by factor of about 10^{-39}.

§ 2 Electric fields (p464-471)

- Does Coulomb's law means that the interaction between separated charges is an action-at-a-distance? Is the interaction direct and instantaneous?
  - Historical view: The interaction model for charges is the action-at-a-distance

    charge $\rightleftarrows$ charge

  - The real interaction model for charges——The interaction between two charges is realized through the electric fields established around the charges.

    charge $\rightleftarrows$ field $\rightleftarrows$ charge

    The first charge sets up an electric field, and the second charge interacts with the electric field of the first charge.

- The problem of determining the interaction between the charges is therefore reduced to two separate problem

  ① Determine, by measurement or calculation, the electric field established by the first charge at every point in space.

  ② Calculate the force that the field exerts on the second charge placed at a particular point in space.

- The definition of electric field
  - The definition of the electric field $\vec{E}$ in terms of the electric force $\vec{F}_e$ exerted on a positive test charge q_0 placed at a particular point.

$$\vec{E} \equiv \frac{\vec{F}_e}{q_0}$$

SI unit: N/C or V/m

The direction of $\vec{E}$ is the same as the direction of $\vec{F}_e$.

The test particle q_0 is used only to detect the existence of the field and evaluate its strength. The existence and strength of the electric field is feature of electric field itself, not dependent on the q_0.

- The electric force exerted on a charge.
  - Once the electric field is known at some point, the electric force on any particle with charge q placed at that point can be calculated by

$$\vec{F}_e = q\vec{E}$$

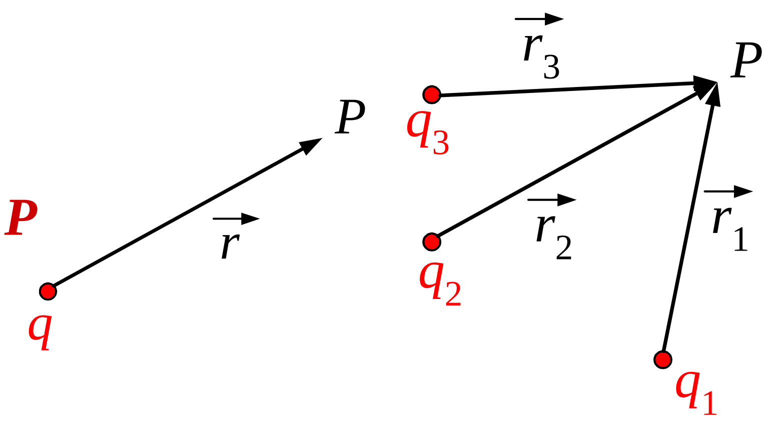
  - Here the electric field $\vec{E}$ is caused by other charges that may be present, not by the charge q.

The electric field of point charge

- The calculation of electric field due to the individual point charges
  - The electric field due to single point charge

    According to Coulomb's law, a test q_0 experience a electric force

$$\vec{F}_e = \frac{1}{4\pi\varepsilon_0}\frac{q\,q_0}{r^2}\hat{r}$$



The electric field created by q at point P

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$$

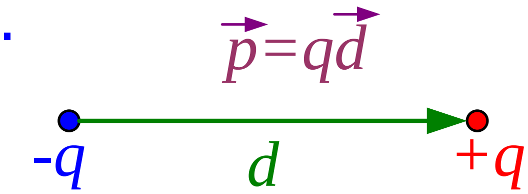
  - The electric field due to a series of point charges distributed in space.

    According to the superposition principle, the total electric field at point P

$$\vec{E} = \sum_i \vec{E}_i = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i^2}\hat{r}_i$$

Example —— The Electric Dipole (p475)

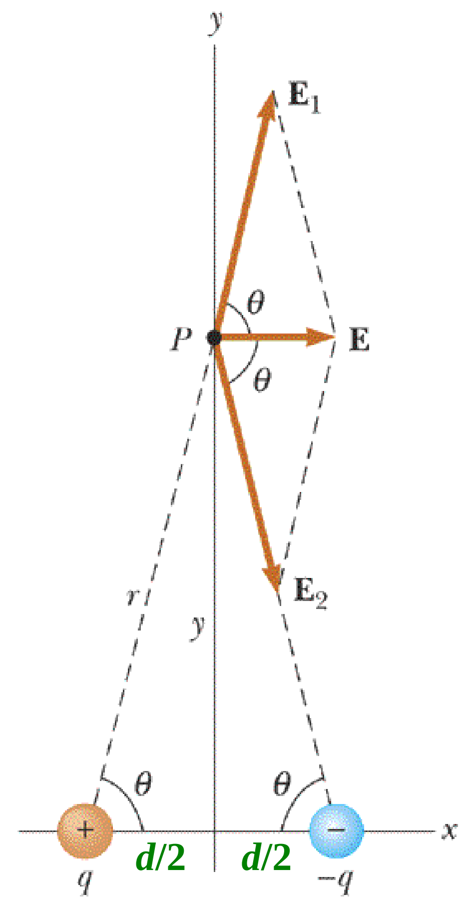
Example: the electric dipole —— consists of equal positive and negative charges $+q$ and $-q$ separated by a fixed distance d.



- Definition of the electric dipole moment: $\vec{p} \equiv q\vec{d}$

(a) Find the electric field E due to the dipole along the y axis at the point P;

(b) Find the electric field E due to the dipole along the x axis at the point P.



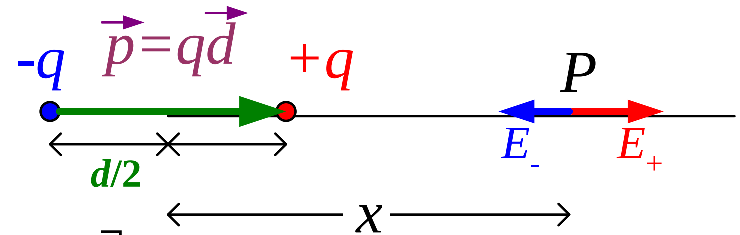
Solution: (a) $\vec{E} = \vec{E}_+ + \vec{E}_-$ $\quad E_+ = E_- = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{y^2+(d/2)^2}$

$$E = 2E_+\cos\theta = \frac{2}{4\pi\varepsilon_0}\frac{q}{y^2+(d/2)^2}\frac{(d/2)}{\sqrt{y^2+(d/2)^2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{qd}{\left[y^2+(d/2)^2\right]^{3/2}} = \frac{1}{4\pi\varepsilon_0}\frac{p}{\left[y^2+(d/2)^2\right]^{3/2}}$$

$$y >> d/2, \qquad \boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{p}{y^3} \propto \frac{p}{y^3}}$$

Example Cont'd



Solution: (b)

$$E = E_+ - E_- = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{\left(x-(d/2)\right)^2} - \frac{1}{\left(x+(d/2)\right)^2}\right]$$

$$x >> d/2 \qquad \frac{1}{\left(x-d/2\right)^2} = \frac{1}{x^2}\left(1-\frac{d}{2x}\right)^{-2} \approx \frac{1}{x^2}\left(1+\frac{d}{x}\right)$$

$$\frac{1}{\left(x+d/2\right)^2} = \frac{1}{x^2}\left(1+\frac{d}{2x}\right)^{-2} \approx \frac{1}{x^2}\left(1-\frac{d}{x}\right)$$

$$\boxed{E = \frac{q}{4\pi\varepsilon_0}\frac{1}{x^2}\frac{2d}{x} = \frac{1}{2\pi\varepsilon_0}\frac{p}{x^3} \propto \frac{p}{x^3}}$$

Example —— The Torque on an Electric Dipole

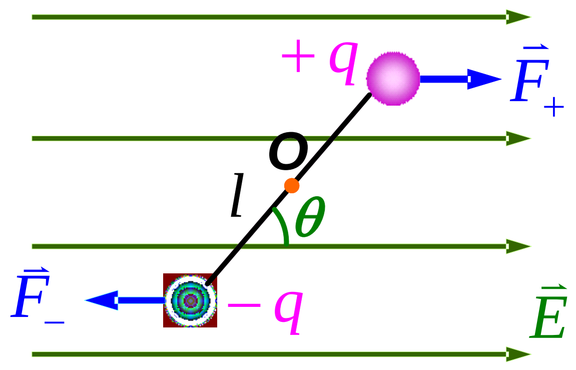
Example: Find the torque exert on an electric dipole.

Solution: to the origin point O

The magnitude:

$$\tau = \frac{l}{2} F_{+} \sin\theta + \frac{l}{2} F_{-} \sin\theta$$

$$= qlE\sin\theta = pE\sin\theta$$



The vector description:

$$\boxed{\vec{\tau} = q\vec{l} \times \vec{E} = \vec{p} \times \vec{E}}$$

The effect of this torque is to try to turn the dipole so $\vec{p}$ is parallel to $\vec{E}$

The electric field due to continuous charge distributions

- The calculation method for electric field due to continuous charge distributions

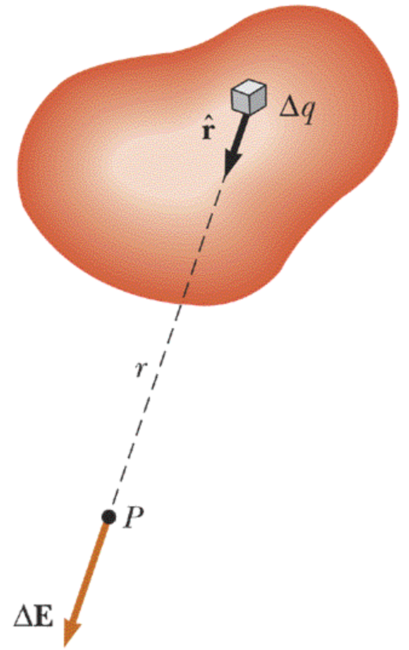
  The procedure:

  - Divide the charge distribution into small elements **dq** ;
  - Model each element as a point charge;

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{r}$$

  - Apply the superposition principle to get the total field at ***P***

$$\vec{E} = \int d\vec{E} = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r^2}\hat{r}$$



- Charge distribution manners

  The linear charge density: $dq = \lambda dl$

  The surface charge density: $dq = \sigma dA$

  The volume charge density: $dq = \rho dV$

The electric field due to a charged rod

Example: A rod of length ***l*** has a uniform linear charge density λ and a total charge ***Q***. Calculate the electric field at a point *P* along the axis of the rod, a distance ***a*** from one end.

Solution: Step 1: Choose the segment dq. $dq = \lambda\, dx = \frac{Q}{l} dx$

Step 2: Write the expression of $\vec{E}$ due to dq.

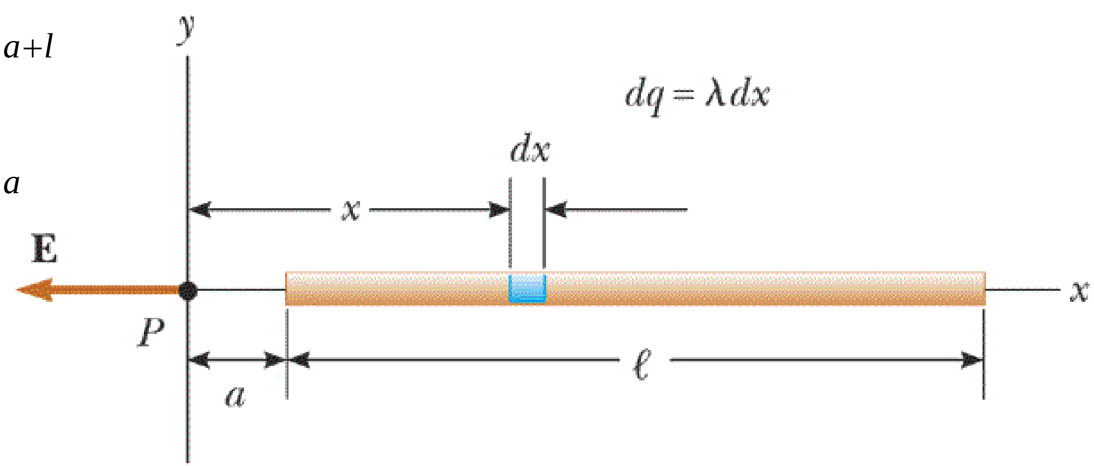
$$d\vec{E} = -\frac{1}{4\pi\varepsilon_0}\frac{dq}{x^2}\hat{i} = -\frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{x^2}\hat{i}$$

Step 3: Obtain the total field $\vec{E}$ by integration.

$$E = -\frac{\lambda}{4\pi\varepsilon_0}\int_a^{a+l}\frac{dx}{x^2} = \frac{\lambda}{4\pi\varepsilon_0}\left[\frac{1}{x}\right]_a^{a+l}$$

$$= -\frac{\lambda}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{a+l}\right)$$

$$= -\frac{Q}{4\pi\varepsilon_0 a(a+l)}$$



The electric field of long line of charge

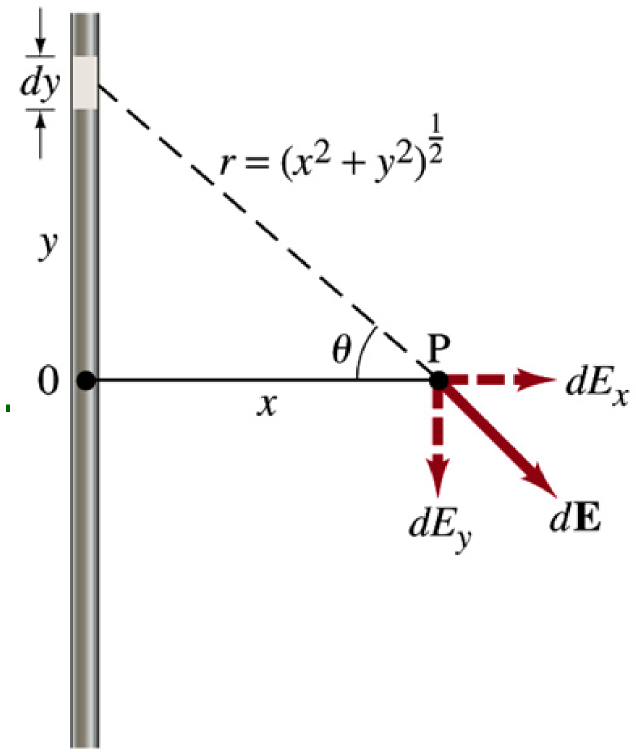
Example: A long wire has a uniform linear charge density λ. Calculate the electric field at a point P a distance x from the wire.

Solution: Step 1: Choose the segment dq. $dQ = \lambda dy$

Step 2: Write the expression of $\vec{E}$ due to dq.

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dQ}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dy}{(x^2+y^2)}$$

$$dE_x = dE\cos\theta, \quad dE_y = dE\sin\theta$$



Step 3: Obtain the total field $\vec{E}$ by integration.

for the symmetry: $E_y = \int dE\sin\theta = 0$

$$E_x = \int dE\cos\theta = \frac{\lambda}{4\pi\varepsilon_0}\int_{-\infty}^{\infty}\frac{\cos\theta dy}{x^2+y^2}$$

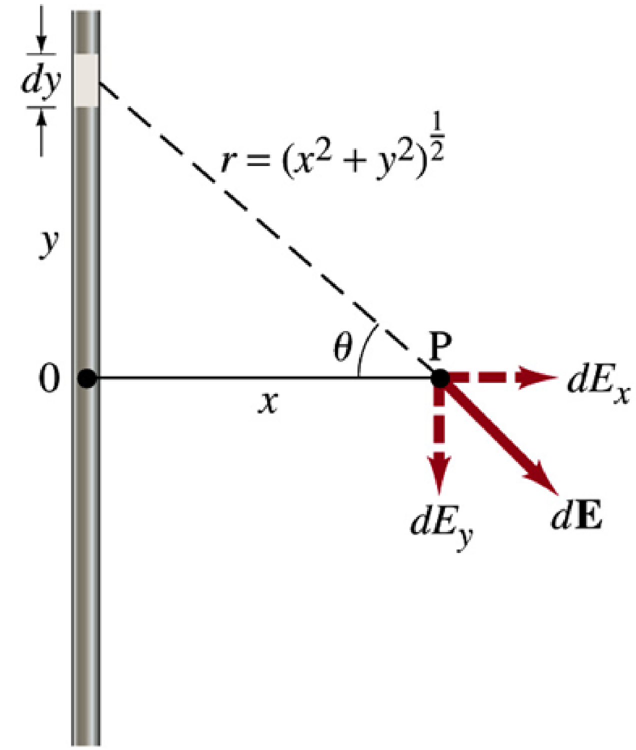
Example cont'd

$$E_x = \int dE\cos\theta = \frac{\lambda}{4\pi\varepsilon_0}\int_{-\infty}^{\infty}\frac{\cos\theta dy}{x^2+y^2}$$

$$y = x\tan\theta \qquad dy = x\,d\theta/\cos^2\theta \qquad (x^2+y^2) = x^2/\cos^2\theta$$

$$E_x = \frac{\lambda}{4\pi\varepsilon_0}\frac{1}{x}\int_{-\pi/2}^{\pi/2}\cos\theta d\theta$$

$$= \frac{\lambda}{4\pi\varepsilon_0 x}\left(\sin\theta\right)\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{x}$$



The electric field of a uniform ring of charge

Example: A ring of radius ***a*** has a uniform positive charge distribution, with a total charge ***Q***. Calculate the electric field at a point *P* on the axis of the ring, at a distance ***x*** from the center of the ring.

Solution: Choose the segment ***dq***.

Write the expression of $d\vec{E}$ due to ***dq***.

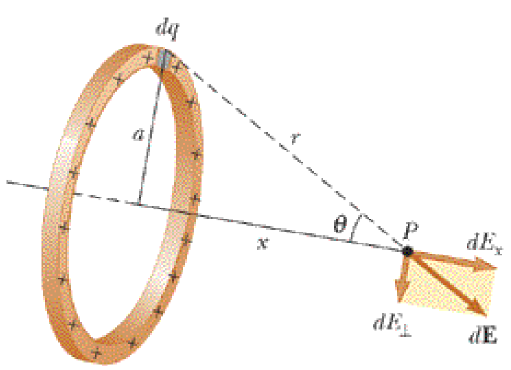
$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}$$

This field include the x component $\mathbf{d}E_x = \mathbf{d}E\cos\theta$, and perpendicular component $\mathbf{d}E_\perp$, which is canceled by another $\mathbf{d}E_\perp$ on the opposite side of the ring.

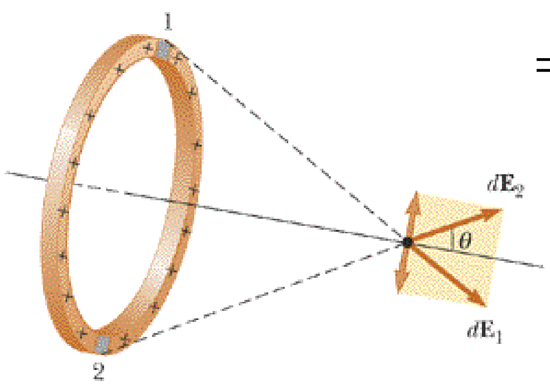
$$dE_x = dE\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\left(\frac{x}{r}\right)$$

$$E_x = \frac{1}{4\pi\varepsilon_0}\frac{x}{r^3}\int dq = \frac{x}{4\pi\varepsilon_0 r^3}Q$$

$$= \frac{x}{4\pi\varepsilon_0\left(x^2+a^2\right)^{3/2}}Q$$



(a)

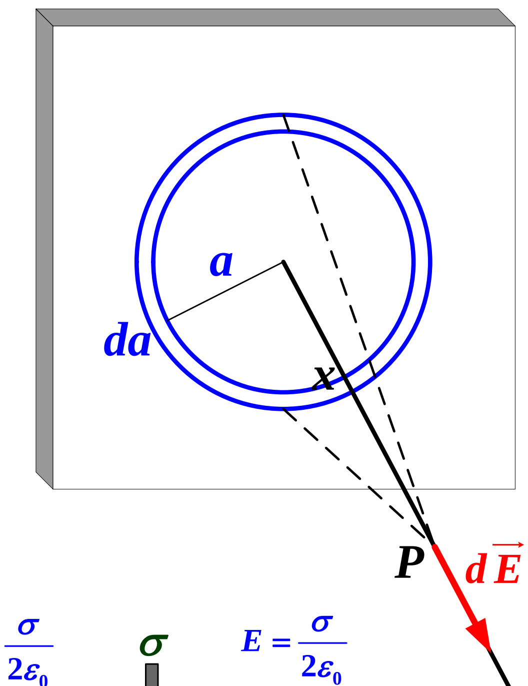


(b)

Example

The electric field of an infinite plane sheet of charge

Example: An infinite plane sheet with uniform surface charge density σ. Calculate the electric field at a point *P* on the axis perpendicular to the plane, at a distance x from the plane.



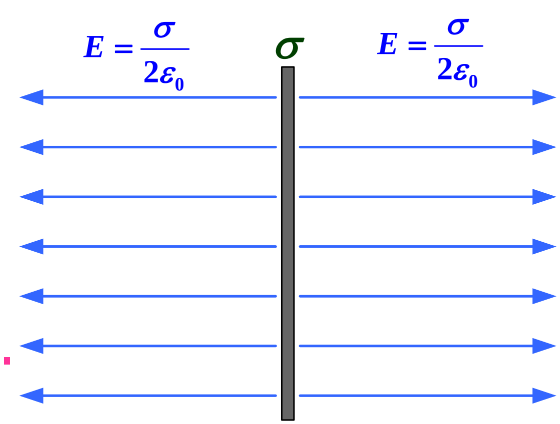
Solution I: Choose the ring with radius a and width da as dq. Starting from:

$$dE = \frac{x}{4\pi\varepsilon_0\left(x^2+a^2\right)^{3/2}}dq$$

$$dq = \sigma dA = \sigma\, 2\pi a\, da$$

$$E = \int_0^{\infty}\frac{\sigma x a da}{2\varepsilon_0\left(x^2+a^2\right)^{3/2}} = \frac{\sigma}{4\varepsilon_0}\int_0^{\infty}\frac{x d\left(x^2+a^2\right)}{\left(x^2+a^2\right)^{3/2}}$$

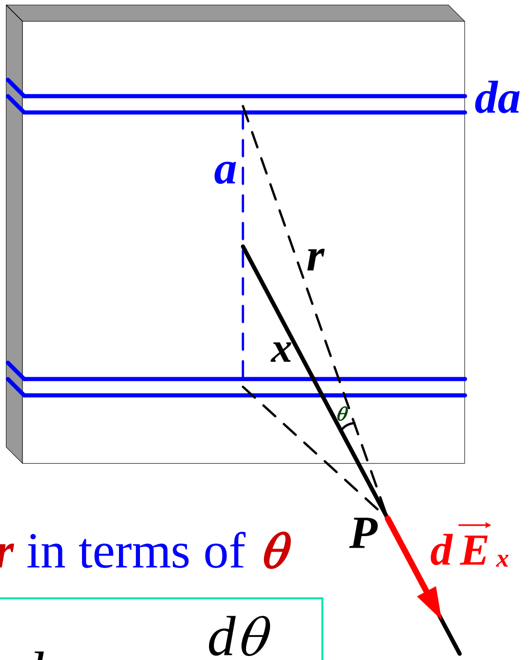
$$= \frac{\sigma}{4\varepsilon_0}\left[-2\frac{x}{\sqrt{x^2+a^2}}\right]_0^{\infty} = \frac{\sigma}{2\varepsilon_0}$$



The electric field keeps constant at any distance from the plane —— the filed is uniform everywhere.

Example cont'd

Solution II: Choose the infinite lengthy rod as ***dq*** , which is at the distance ***a*** from the center, and width ***da***. Using the conclusion of previous example:



$$E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}$$

where y is the distance from the rod along its perpendicular bisector.

$$dq = \sigma\, da\, dl = \lambda\, dl \quad \Rightarrow \quad \lambda = \sigma\, da$$

$$dE = \frac{2k_e\sigma\, da}{r} = \frac{\sigma}{2\pi\varepsilon_0}\frac{da}{r} \quad \Leftrightarrow$$

Find the ***a*** and ***r*** in terms of ***θ***

$$a = x\tan\theta, \quad da = x\frac{d\theta}{\cos^2\theta}$$

$$\frac{1}{r} = \frac{\cos\theta}{x}$$

$$\Downarrow$$

$$dE_x = dE\cos\theta = \frac{\sigma}{2\pi\varepsilon_0}\cos\theta\frac{da}{r}$$

$$= \frac{\sigma}{2\pi\varepsilon_0}d\theta \quad \Rightarrow$$

$$\boxed{E_x = \frac{\sigma}{2\pi\varepsilon_0}\int_{-\pi/2}^{\pi/2} d\theta = \frac{\sigma}{2\varepsilon_0}}$$

Problem-Solving Strategy to Calculating Electric field Due to Continuous Charge Distributions

- Divided the charge distribution into small elements dq.
  - Model each element as a point charge.

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{r}$$

  - Apply the superposition principle to get the total field at P.

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r^2}\hat{r}$$

- Establish a convenient coordinate system to complete the integral.
  - Using component representations to solve the vector integral separately.
- Choose appropriate infinitesimal charge element to simplified the integral.
  - Generally choice: $dq = \lambda\, dl$ for line distribution, $dq = \sigma\, dA$ for surface distribution, and $dq = \rho\, dV$ for volume distribution.
  - Using some symmetry of charge distribution to canceling some field components.

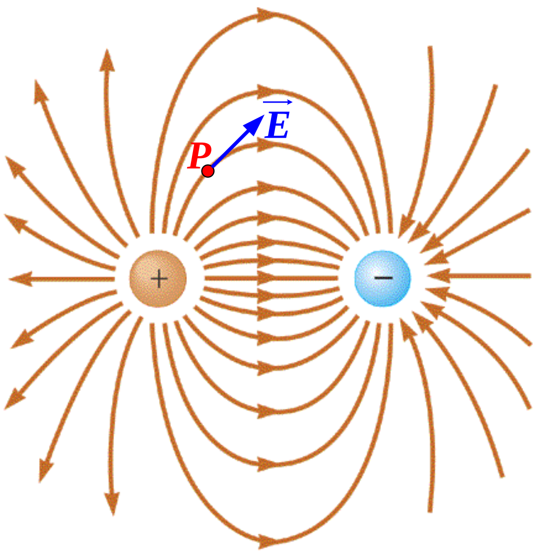
Problem-Solving Strategy to Calculating Electric field Due to Continuous Charge Distributions cont'd

- Using the known low-dimensional results for calculating field due to high-dimensional charge distribution.
  - For example, using the result of line charge distribution in a rod or a ring as the bases for calculation of field in the case of surface charge distribution.
  - Using the symmetry as possible as you can. For a disk of charge, adopt the ring result as the base. For a plane of charge, choose the rod result as the base.

§ 3 Electric Field Lines (p471-473)

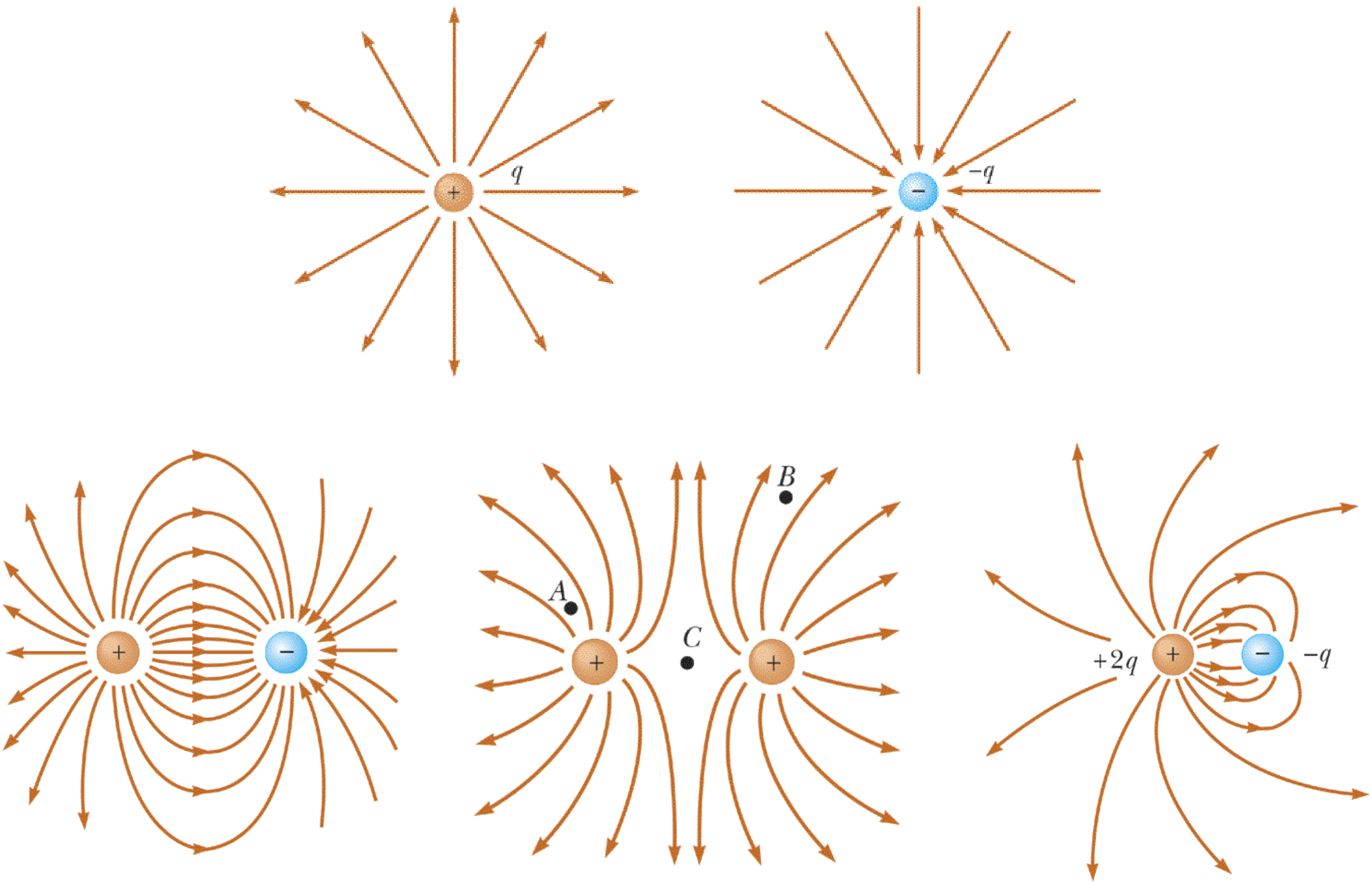
Why introduce electric field lines?

A graphic way for description of electric field
- Visualize the electric field which is not visible;
- Clarified the characteristic of electric field.

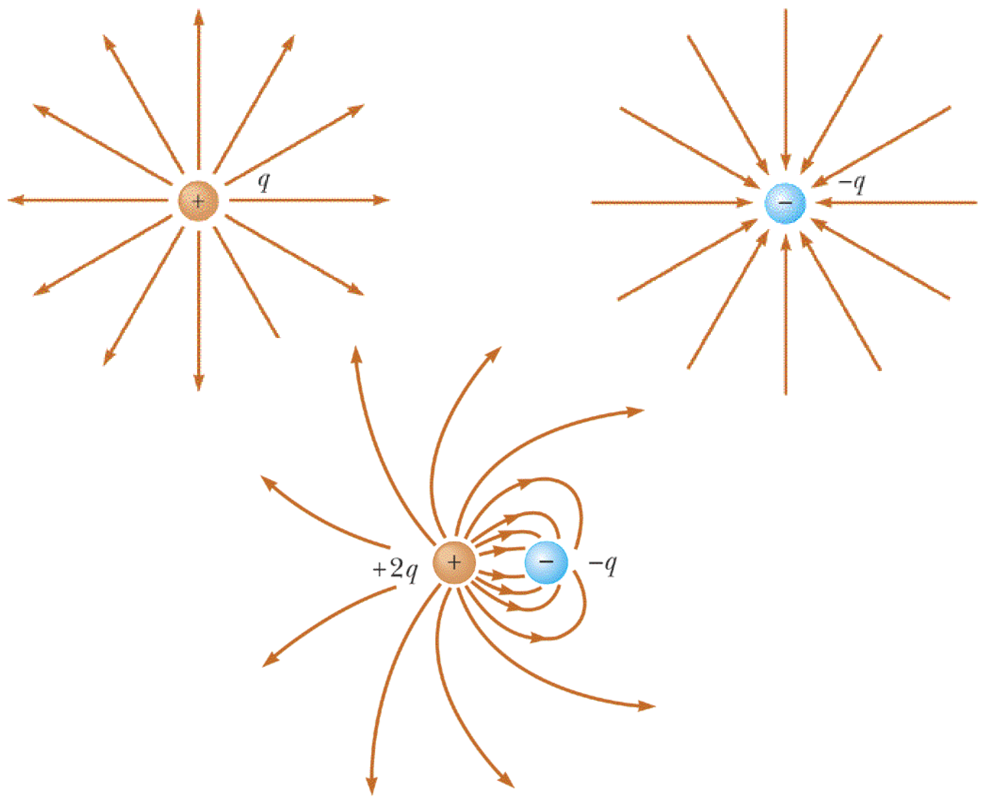


- Electric field lines are related to the electric field in the following manner:
  - Direction —— is tangent to the electric field line at that point.
  - Magnitude —— is proportional to the number of electric field lines per unit area through the cross-sectional surface in that region. E is larger where the field lines are close together and small where they are far apart.

Some typical electric field lines



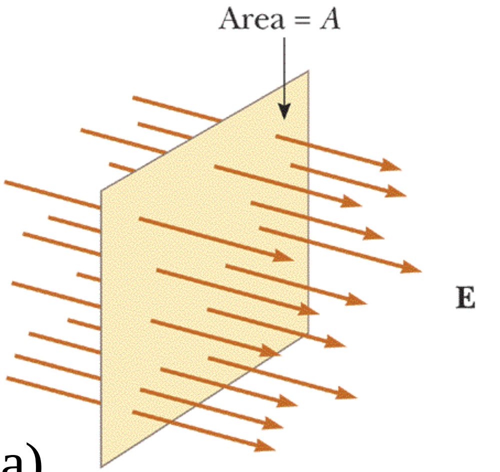
- The fundamental properties for electric field lines :
  - Begin on positive charges (or infinite far away) and end on negative charges (or infinite far away). In the case of an excess of one type of charge, some lines will begin or end infinitely far away.
  - No two field lines can cross each other.



§ 4 Electric Flux (p487-489)

- Introduction of electric flux
  - Electric field:

$$E \propto \frac{\text{the number of electric field lines}}{\text{cross-sectional area perpendicular to the lines}}$$



  - Electric flux Φ_E :

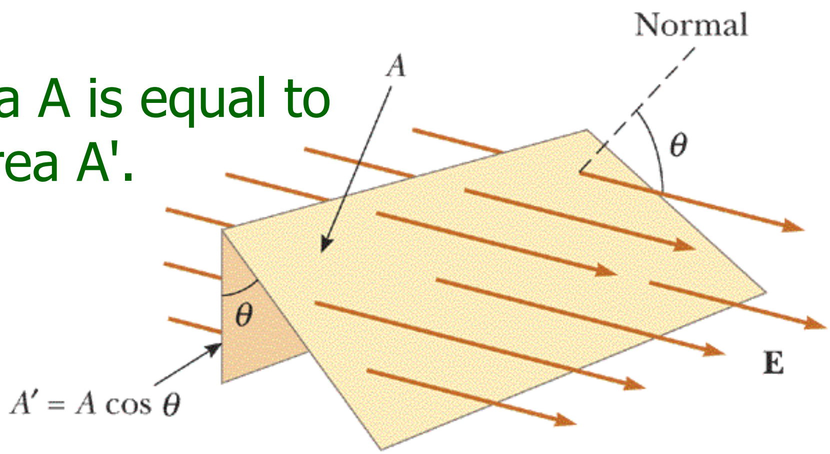
$$\Phi_E \propto \text{the number of field lines} = E \cdot A \text{ (perpendicular area)}$$

  - For uniform electric field, area A is perpendicular to the field $\Phi_E = EA$

  - For uniform electric field, area A is at an angle θ to the field

The number of lines that cross the area A is equal to the number that cross the projected area A'.

$$\Phi_E = EA' = EA\cos\theta = \vec{E} \cdot \vec{A}$$

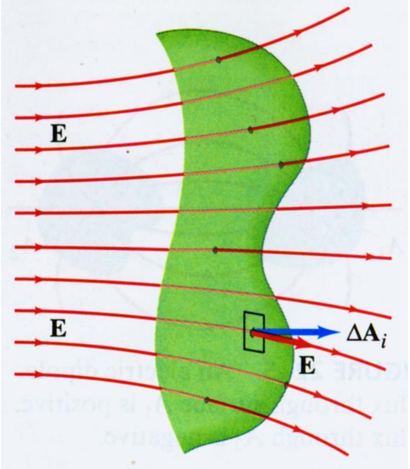


Introduction of electric flux cont'd

- For general electric field that may vary in both magnitude and direction, curved surface —— be divided into a large number of a small element of area: ΔA_i.

  The flux through a small element:

$$\Delta\Phi_E = E_i \Delta A_i \cos\theta_i = \vec{E}_i \cdot \Delta\vec{A}_i$$



  The total electric flux:

$$\Phi_E = \lim_{\Delta A_i \to 0} \sum \vec{E}_i \cdot \Delta\vec{A}_i = \int_{surface} \vec{E} \cdot d\vec{A}$$

- The net electric flux through a closed surface:

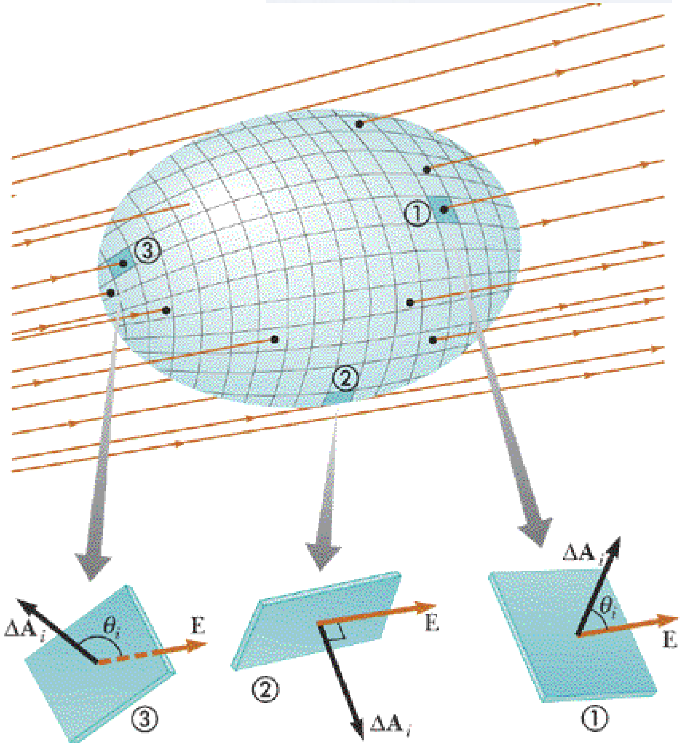
$$\Phi_E = \oint \vec{E} \cdot d\vec{A}$$

  For a closed surface, outward direction is defined to be positive.

  At point ①, θ_i<90°, Φ_E>0.

  At point ②, θ_i=90°, Φ_E=0.

  At point ③, θ_i>90°, Φ_E<0.



Example

Example: Consider a uniform electric field E directed along the +x axis. Find the net electric flux through the surface of a cube of edges ***l*** shown in the figure.

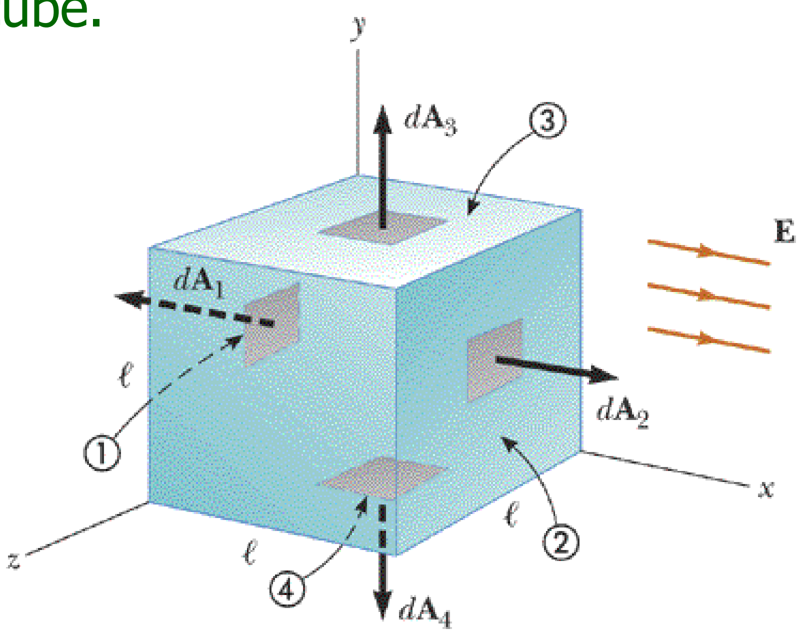
Solution: For the faces labeled ③ and ④, the orientation of $d\vec{A}$ is perpendicular to $\vec{E}$.

The net flux through the surface of cube.

$$\Phi_E = \iint_1 \vec{E}\cdot d\vec{A} + \iint_2 \vec{E}\cdot d\vec{A}$$

$$= \iint_1 EdA\cos 180^\circ + \iint_1 EdA\cos 0^\circ$$

$$= -EA + EA = 0$$

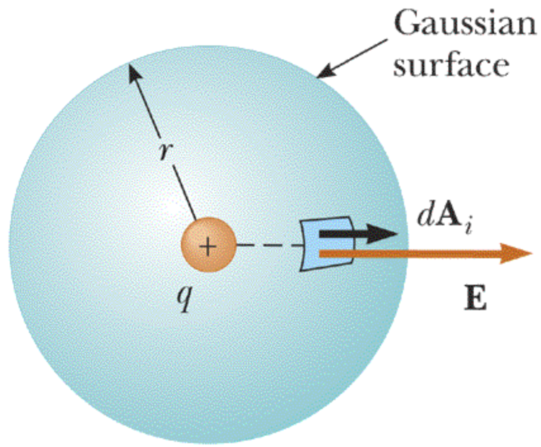


§ 5 Gauss's Law (p489-490)

- A point charge q locates at the center of a spherical surface.

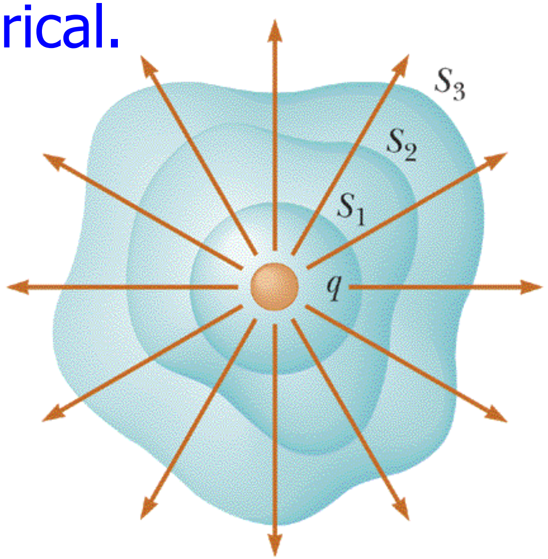
$$\Phi_E = \oint_{\substack{\text{spherical}\\\text{surface}}} \vec{E}\cdot d\vec{A} = \oint_{\substack{\text{spherical}\\\text{surface}}} E_n dA = \oint_{\substack{\text{spherical}\\\text{surface}}} E dA$$

$$= E \oint_{\substack{\text{spherical}\\\text{surface}}} dA = \left(\frac{q}{4\pi\varepsilon_0 r^2}\right)\left(4\pi r^2\right) = \frac{q}{\varepsilon_0}$$



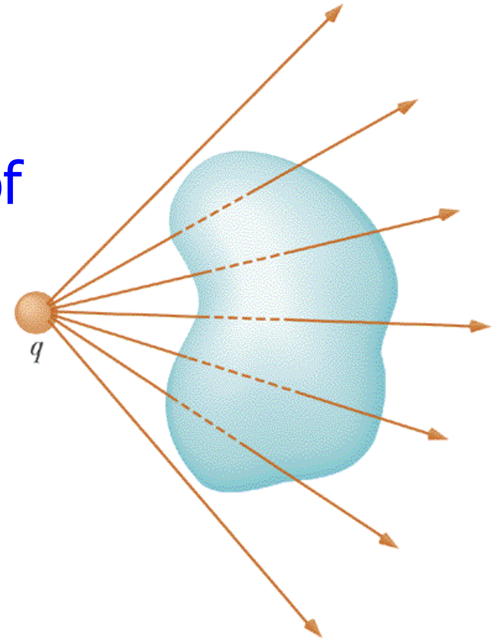
- The net flux is proportional to charge inside the surface;
- The net flux is independent of the radius r —— every field line from the charge must pass through the surface
- The fact that the net flux is independent of the radius is consequence of inverse-square dependence of the electric field according to Coulomb's law.

- The charge q inside, the closed surface not spherical.
  - The flux that passes through spherical surface S_1 has the value q / ε_0.
  - The number of electric field lines through the spherical surface S_1 is equal to the number of electric field lines through the nonspherical surfaces S_2 and S_3.



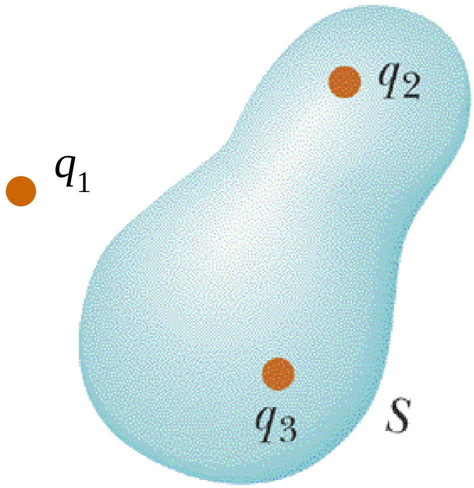
$$\oint_{S_1} \vec{E}\cdot d\vec{A} = \oint_{S_2} \vec{E}\cdot d\vec{A} = \oint_{S_3} \vec{E}\cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

- A point charge locates outside a closed surface of arbitrary shape.
  - The number of electric field lines entering the surface equals the number leaving the surface



$$\Phi_E = \Phi_E^{in} + \Phi_E^{out} = 0$$

- The series of charges, some inside, some outside the closed surface.
  - The total electric field at any point:



$$\vec{E} = \sum_i \vec{E}_i$$

$$\Phi_E = \oint_S \vec{E} \cdot d\vec{A} = \oint_S \sum_i \vec{E}_i \cdot d\vec{A}$$

$$= \sum_i \oint_S \vec{E}_i \cdot d\vec{A} = \sum_i \Phi_{Ei}$$

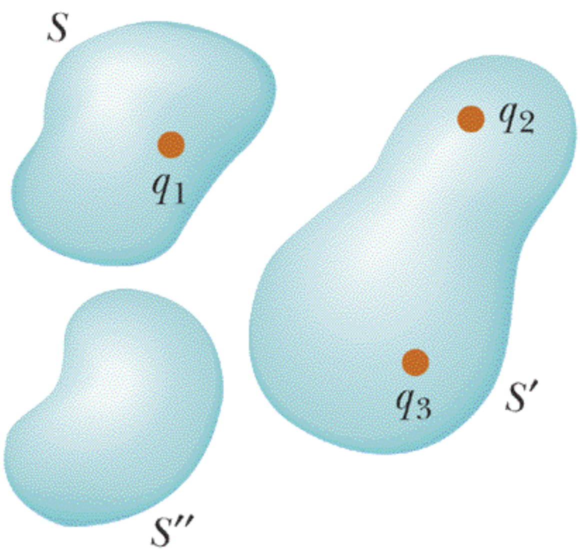
$$\Phi_{Ei} = \begin{cases} \dfrac{q_i}{\varepsilon_0} & \text{if } q_i \text{ inside the } S \\ 0 & \text{if } q_i \text{ outside the } S \end{cases} \Rightarrow \boxed{\Phi_E = \sum_i \Phi_{Ei} = \frac{q_{in}}{\varepsilon_0}}$$

- Gauss's Law: $$\boxed{\Phi_E = \oint_S \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}}$$
  - The net electric flux through any closed surface is equal to the net charge inside the surface divided by ε_0.

$$\oint_S \vec{E}\cdot d\vec{A} = \frac{q_1}{\varepsilon_0},$$

$$\oint_{S'} \vec{E}\cdot d\vec{A} = \frac{q_2+q_3}{\varepsilon_0},$$

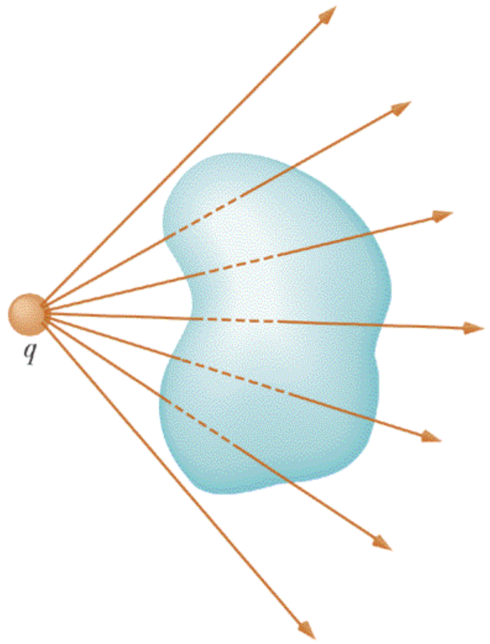
$$\oint_{S''} \vec{E}\cdot d\vec{A} = 0$$



Some comments on Gauss's Law

$$\boxed{\Phi_E = \oint_S \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}}$$

- Some comments on Gauss's Law.
  - The flux only depends on the charges inside (enclosed).
  - *E* on the left side of Guass's law is the *E* in Guassian surface, it is not necessarily due to the charge inside the surface. It is produced by all the charges in the space.
  - Zero flux doesn't mean the zero field.



- Gauss's Law and Coulomb's Law.
  - Gauss's law is deduced from Coulomb's law. Coulomb's law can also be deduced from Gauss's law.

    For an isolated charge q locates inside a spherical surface

$$\oint_S \vec{E}\cdot d\vec{A} = \oint_S E dA = E\oint_S dA = E\left(4\pi r^2\right) = \frac{q}{\varepsilon_0} \Longrightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

  - These two laws can regarded as equivalent in the situation of electrostatics, Gauss's law is found to hold also for electric fields generated by changing magnetic field.
  - Gauss's law is a more general law than Coulomb's law, and so is regarded as a more fundamental equation of electromagnetisms.

§ 6 Application of Gauss's Law to Symmetric Charge Distributions (p491-495)

Generally

The charge distribution is known $\xrightarrow{\text{Coulomb's Law}}$ The electric field

The electric field is known $\xrightarrow{\text{Gauss's Law}}$ The Charge distribution

$$\Phi_E = \oint_S \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$

- For special case where the charge distribution possesses a high degree of symmetry, Gauss's law can be used to evaluate the electric field.
- The Choice of appropriate gaussian surface that allows E to be removed from the integral in Gauss's law is the key problem.
- With the appropriate gaussian surface, the dot product $\vec{E} \cdot d\vec{A}$ should be zero or equal to $E\,dA$, with the magnitude of E being constant.

A spherical symmetric charge distribution

Example: An insulating solid sphere of radius ***a*** has a uniform volume charge density ρ and carries a total positive charge ***Q***.

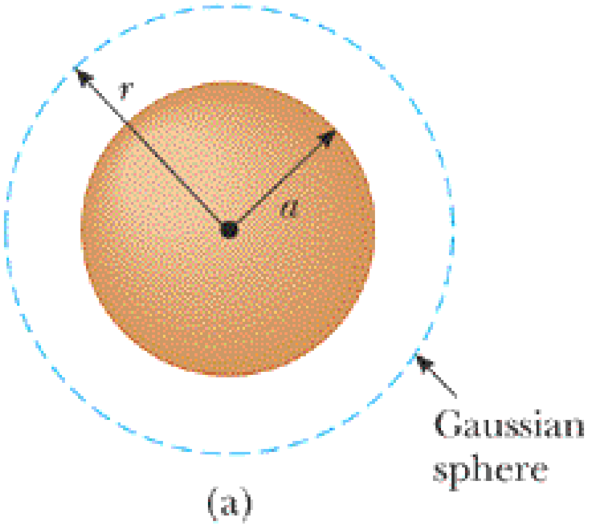
(1) Calculate the magnitude of the electric field at a point outside the sphere.

(2) Find the magnitude of the electric field at a point inside the sphere.

Solution: (1) Select a spherical gaussian surface of radius ***r > a***

$$\oint_S \vec{E} \cdot d\vec{A} = E \oint_S dA = E\left(4\pi r^2\right) = \frac{q_{in}}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} \qquad (\text{for } r > a)}$$



This result is identical to that obtained for a point charge.

Example cont'd

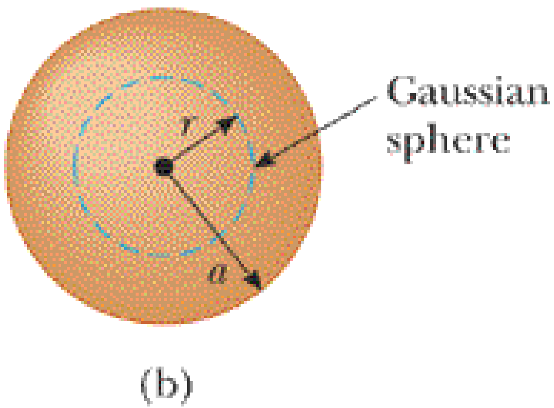
Solution: (2) Select a spherical gaussian surface of radius ***r* < *a***

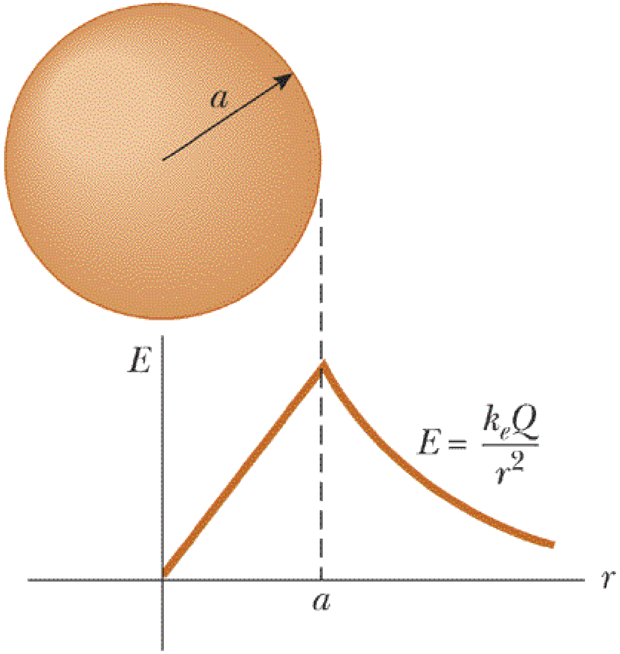
$$\oint_S \vec{E}\cdot d\vec{A} = E\oint_S dA = E\left(4\pi r^2\right) = \frac{q_{in}}{\varepsilon_0}$$

$$= \frac{1}{\varepsilon_0}\rho\left(\frac{4}{3}\pi r^3\right) = \frac{Q}{\varepsilon_0}\frac{r^3}{a^3}$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{a^3}r \quad (\text{for } r < a)}$$

The expressions of electric fields inside and outside the sphere match when r=a.





A cylindrically symmetric charge distribution

Example: Find the electric field a distance ***r*** from a line of positive charge of infinite length and constant charge per unit length λ.

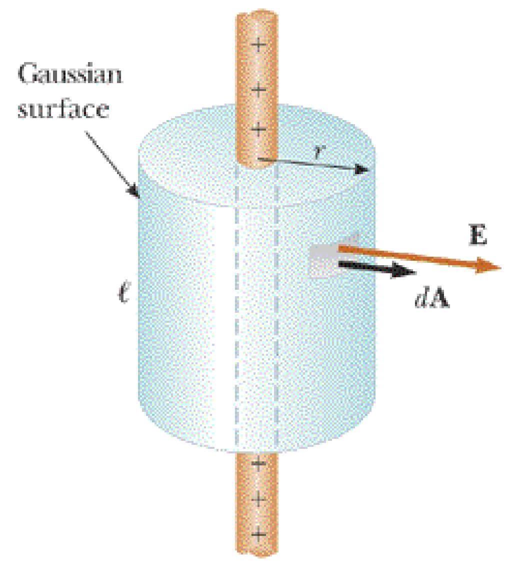
Solution: Select a cylindrical gaussian surface of radius ***r*** and length ***l*** that is coaxial with the line charge.

$$\oint_S \vec{E}\cdot d\vec{A} = \iint_{\text{side surface}} \vec{E}\cdot d\vec{A} + \iint_{\text{top and bottom}} \vec{E}\cdot d\vec{A}$$

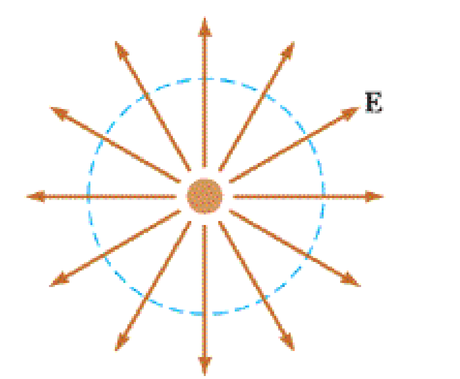
$$= \iint_{\text{side surface}} \vec{E}\cdot d\vec{A} = E(2\pi r l) = \frac{q_{in}}{\varepsilon_0}$$

$$= \frac{\lambda l}{\varepsilon_0}$$

$$\boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}}$$



(a)

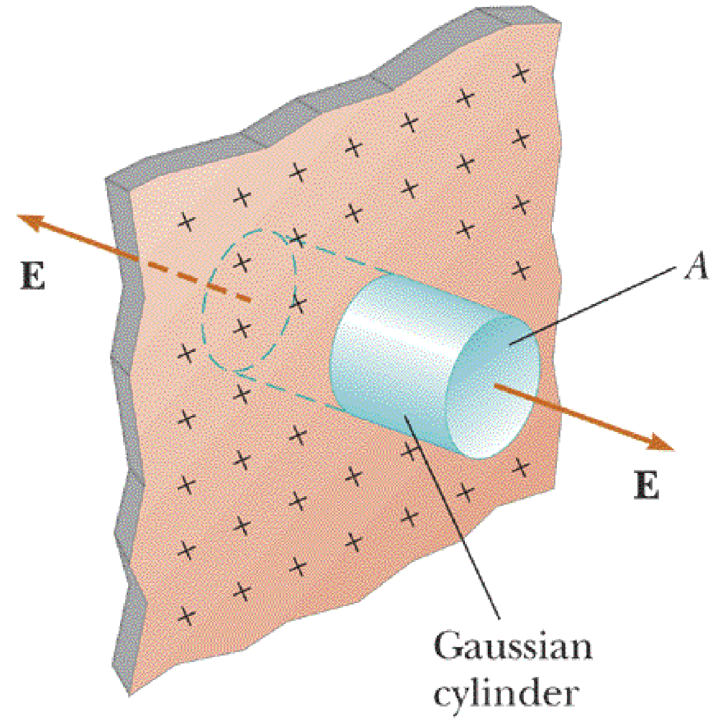


(b)

A Nonconducting plane sheet of charge

Example: Find the electric field due to a nonconducting, infinite plane with uniform surface charge density σ.

Solution: Select the gaussian surface to be a small cylinder whose axis is perpendicular to the plane and whose ends each have an area A and are equidistant from the plane.



$$\oint_S \vec{E}\cdot d\vec{A} = \iint_{\text{side surface}} \vec{E}\cdot d\vec{A} + \iint_{\text{two ends of the cylinder}} \vec{E}\cdot d\vec{A}$$

$$= \iint_{\text{two ends of the cylinder}} \vec{E}\cdot d\vec{A} = E(2A) = \frac{q_{in}}{\varepsilon_0}$$

$$= \frac{\sigma A}{\varepsilon_0}$$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

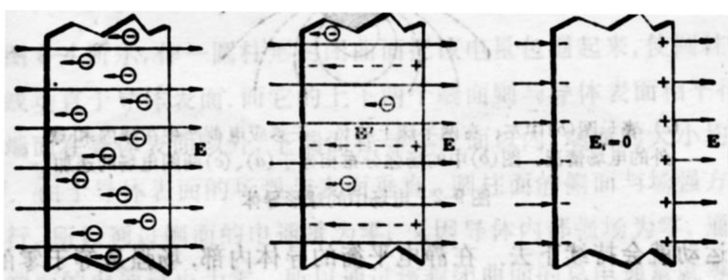
§ 7 Conductors in Electrostatic Equilibrium (p473)

- The characteristics of a electrical conductor.
  - A good electrical conductor contains charges that are not bound to any atom and free to move about within the conductor —— called free charge.
  - When no motion of charge occurs within the conductor, the conductor is in electrostatic equilibrium.
- The properties that an isolated conductor in electrostatic equilibrium.

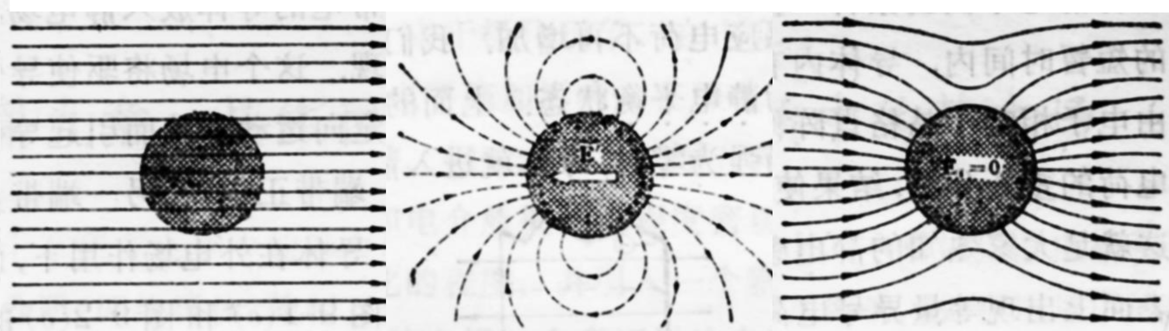
① The electric field is zero everywhere inside the conductor.

If the field were not zero, free charges in the conductor would accelerate under the action of the electric field —— not the case in electrostatic equilibrium

$$\vec{E}_{inside} = \vec{E} + \vec{E}' = 0$$



(a) 导体中自由电子在外电场作用下作宏观的定向运动 (b) 导体的两个相对表面上出现的感应电荷在增加中 (c) 最后，导体处于静电平衡状态，感应电荷不再变化，导体内的总场强 $E_i=0$

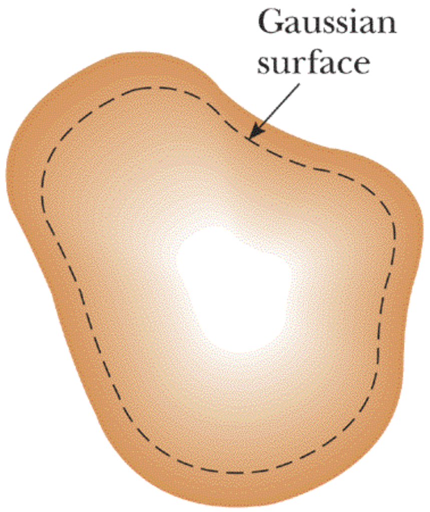


The properties that an isolated conductor in electrostatic equilibrium

- The properties that an isolated conductor in electrostatic equilibrium (cont'd)

  ② If the isolated conductor carries a net charge, the net charge resides entirely on its surface.

$$\oint_S \vec{E}\cdot d\vec{A}=\frac{q_{in}}{\varepsilon_0},\quad \vec{E}=0 \ \text{ inside the conductor,} \ \Rightarrow \ q_{in}=0$$



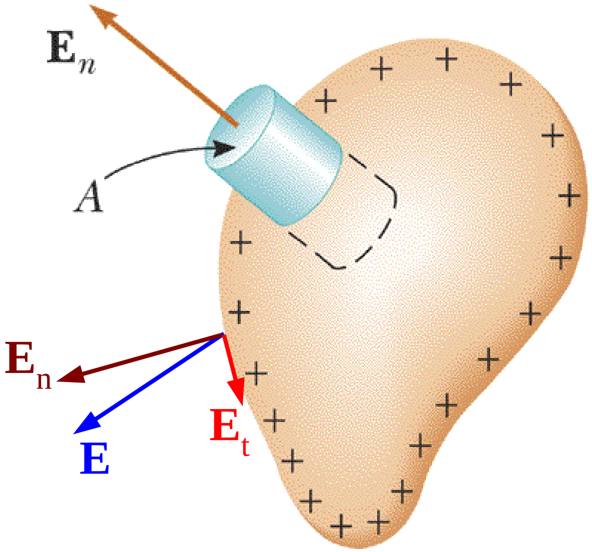
  ③ The electric field just outside the charged conductor is perpendicular to the conductor surface and has a magnitude σ / ε_0, where σ is the surface charge density at that point.

  If E had a component parallel to the surface, the free charges would move along the surface, and so the conductor would not be in equilibrium.

  Draw a small cylinder just containing the surface of the conductor.

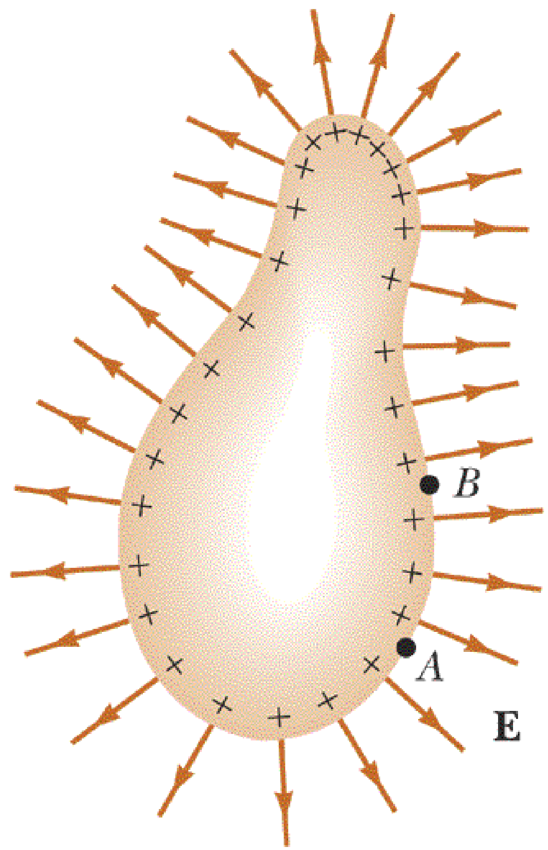
$$\oint_S \vec{E}\cdot d\vec{A}=EA=\frac{q_{in}}{\varepsilon_0}=\frac{\sigma A}{\varepsilon_0}$$

$$\boxed{E=\frac{\sigma}{\varepsilon_0}}$$



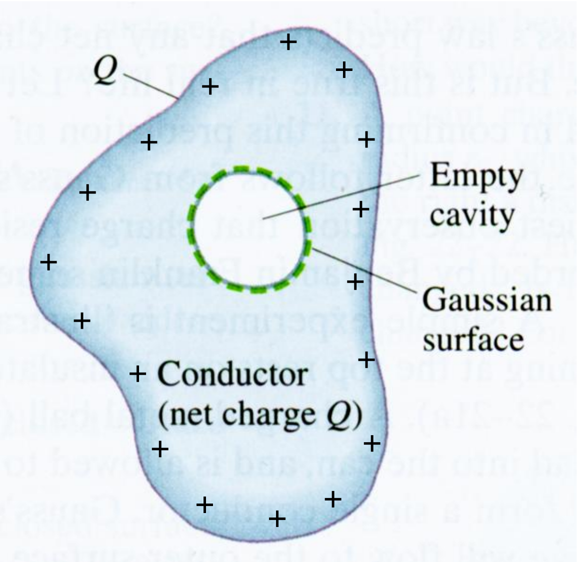
The properties that an isolated conductor in electrostatic equilibrium

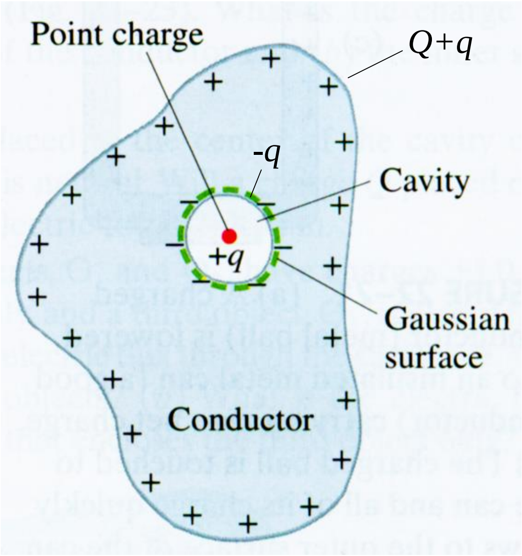
- The properties that an isolated conductor in electrostatic equilibrium (cont'd)
  - ④ On an irregularly shaped conductor, the surface charge density is highest at locations where the radius of curvature of the surface is smallest.



The charge distribution for a conductor cavity

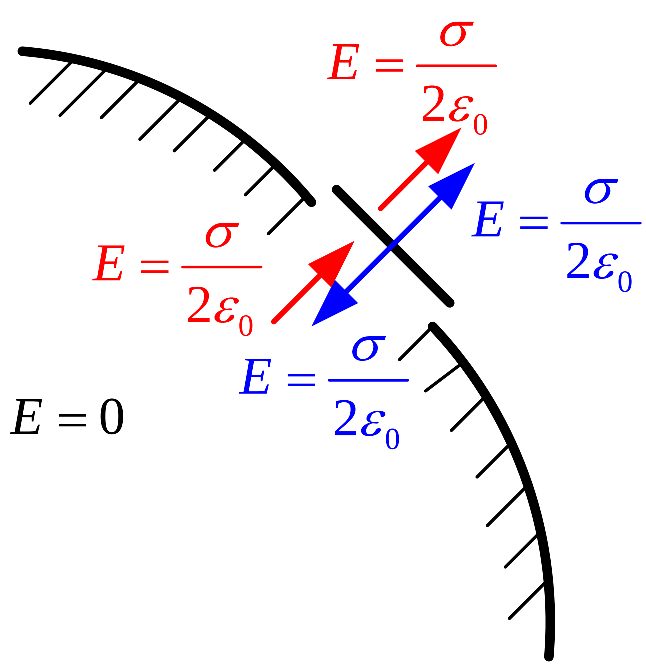
- No charge in the internal cavity of the conductor.
  - There is no charge at the surface of the cavity.
- A point charge +q is place inside the cavity.
  - A charge –q must be attracted to the inner surface of the cavity to keep the net charge zero within the gaussian surface.
  - A charge of Q+q will appear on the outer surface of the cavity, so that the net charge of the conductor does not change.

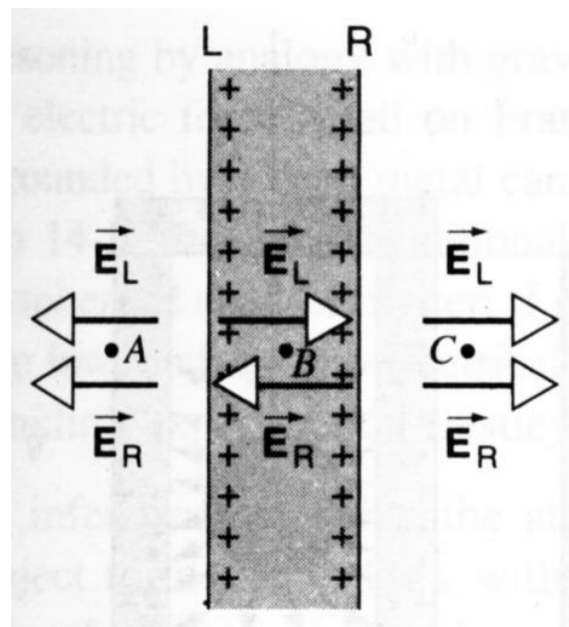




The puzzle

- Why is it that the field outside a large plane nonconductor is $E=\sigma/2\varepsilon_0$ whereas outside a conductor it is $E=\sigma/\varepsilon_0$.
  - Imagine the surface of the conductor to be divided into two sections: the region near where we wish to find the electric field and the remainder of the conductor.





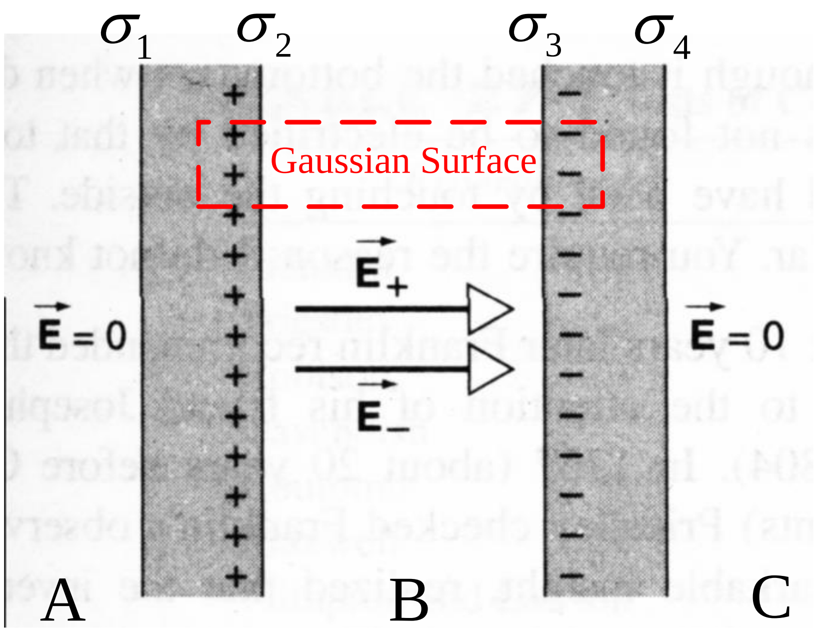
$$E_A=-\frac{\sigma}{2\varepsilon_0}-\frac{\sigma}{2\varepsilon_0}=-\frac{\sigma}{\varepsilon_0}$$

$$E_B=\frac{\sigma}{2\varepsilon_0}-\frac{\sigma}{2\varepsilon_0}=0$$

$$E_C=\frac{\sigma}{2\varepsilon_0}+\frac{\sigma}{2\varepsilon_0}=\frac{\sigma}{\varepsilon_0}$$

Example

Example: Two thin conducting plates carry equal and opposite charges ***+q*** and ***–q***. Find the electric fields between the two plates and at the two sides of the plates.



Solution: Conservation of net charge:

$$(\sigma_1 + \sigma_2)S = +q$$

$$(\sigma_3 + \sigma_4)S = -q$$

Gauss's law:

$$\oint_S \vec{E} \cdot d\vec{A} = 0 \cdot A = (\sigma_2 + \sigma_3)A \quad \Rightarrow \quad \sigma_2 = -\sigma_3$$

The field inside the plate 2 is zero:

$$E_{2in} = \frac{\sigma_1}{2\varepsilon_0} + \frac{\sigma_2}{2\varepsilon_0} + \frac{\sigma_3}{2\varepsilon_0} - \frac{\sigma_4}{2\varepsilon_0} = 0$$

$$\sigma_1 = \sigma_4 = 0$$

$$\sigma_2 = -\sigma_3 = \sigma = \frac{q}{S}$$

$$\boxed{E_A = \frac{\sigma_1}{\varepsilon_0} = 0, \quad E_B = \frac{\sigma_2}{\varepsilon_0} = \frac{q}{S\varepsilon_0}, \quad E_C = \frac{\sigma_4}{\varepsilon_0} = 0}$$