Introduction to Electronic Systems

Zheng Feng

Part 1: Resistive Circuit Analysis

1. Circuit Variables and Circuit Elements
2. Simple Resistive Circuit Analysis
3. **Techniques of Circuit Analysis**
4. Operational Amplifier

Chapter 3: Techniques of Circuit Analysis

- Node-Voltage Analysis Method
- Mesh-Current Analysis Method
- Superposition Theorem
- Source Transformation
- Thévenin and Norton Theorem
- Maximum Power Transfer Theorem

- **What should we do for circuit analysis?**
  - **Build equations** for a given circuit;
  - **Solve equations** to determine currents and voltages for all (or required) circuit elements.

What is the problem now?

- How many, at least, independent equations should we have to solve for a given circuit?
- How can we get the necessary equations ?

Theorem 1

- For a circuit with n nodes, KCL yields (n-1) independent equations for any (n-1) nodes.

Theorem 2

- For a circuit with n nodes and b branches:
  - The circuit must have $m = b-(n-1)$ meshes;
  - KVL yields $m = b-(n-1)$ independent equations for the $m = b-(n-1)$ meshes.

- **Node-Voltage Analysis Method**
- **Mesh-Current Analysis Method**

3-1 Node-Voltage Analysis Method

- What is node voltage?
- How is node voltage analysis method implemented?

What is Node Voltage?

- Node voltage is defined as the **voltage rise** from the reference node;
- Voltages across all branches can be represented by node voltages.

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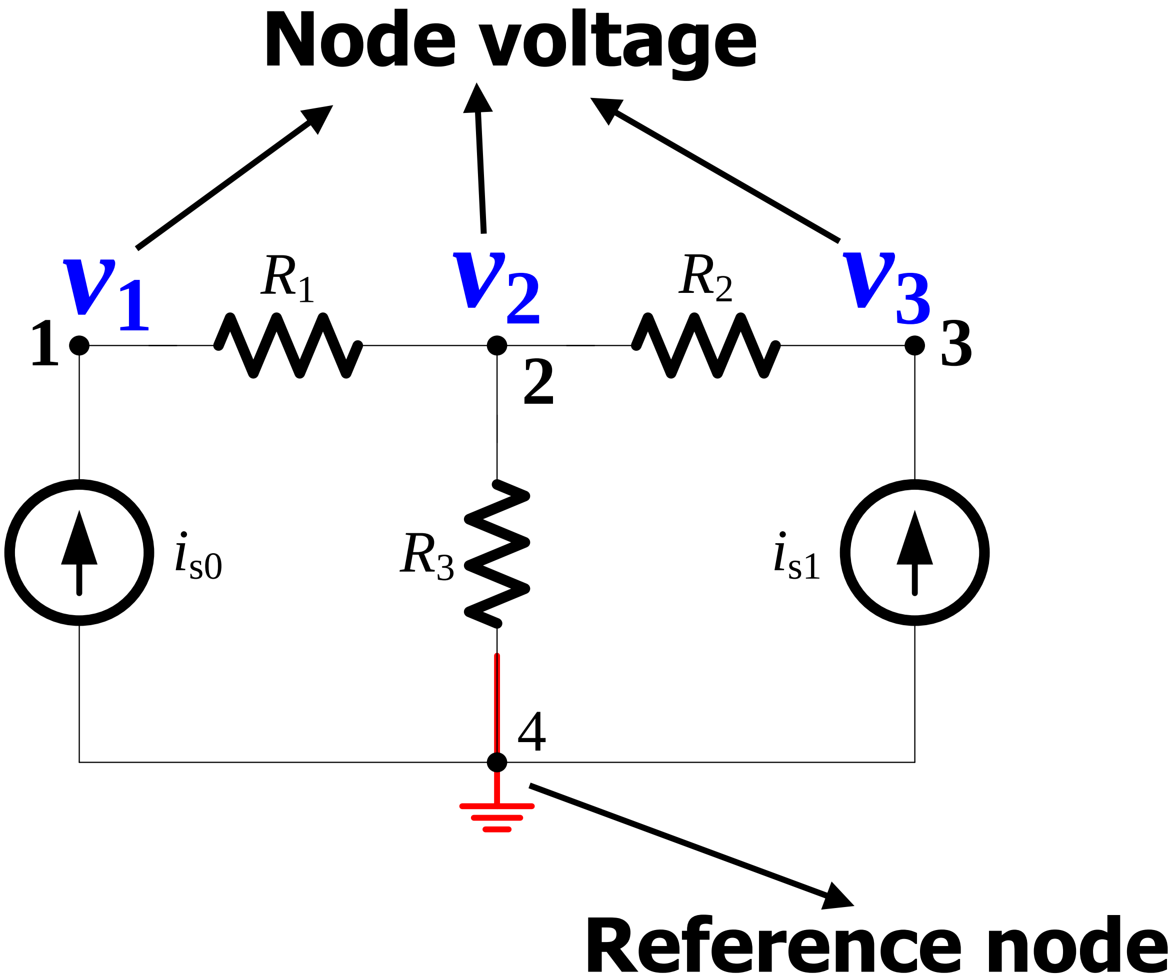
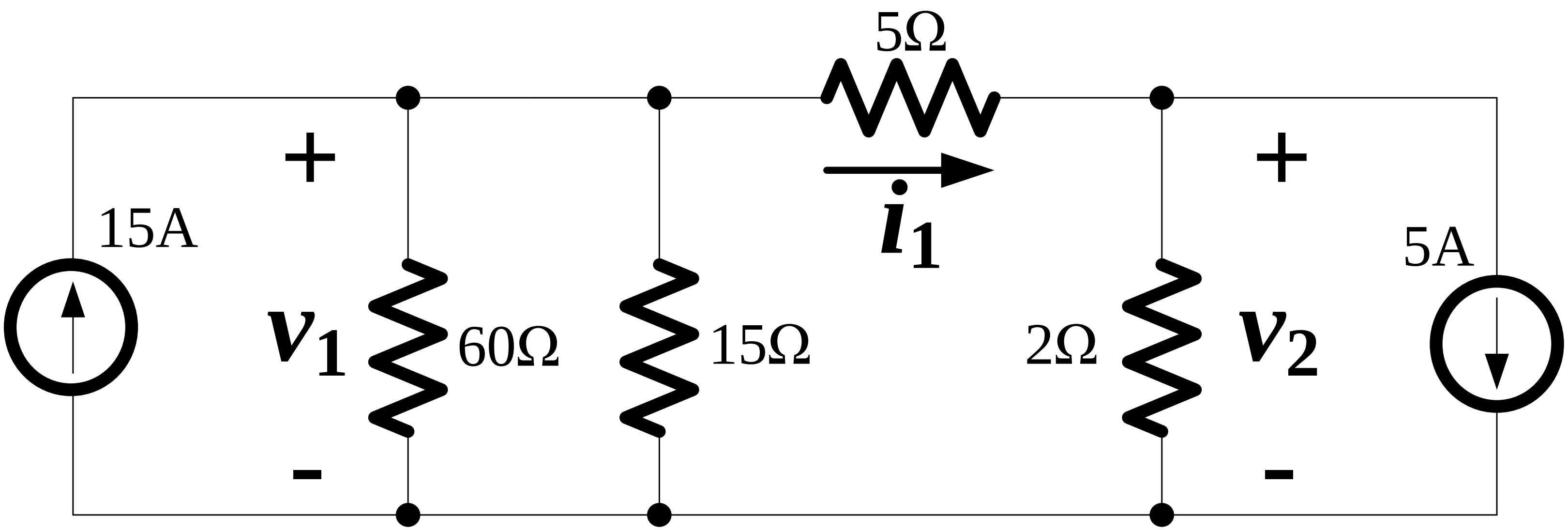
What is Node Voltage?



HOW is node voltage analysis method implemented?

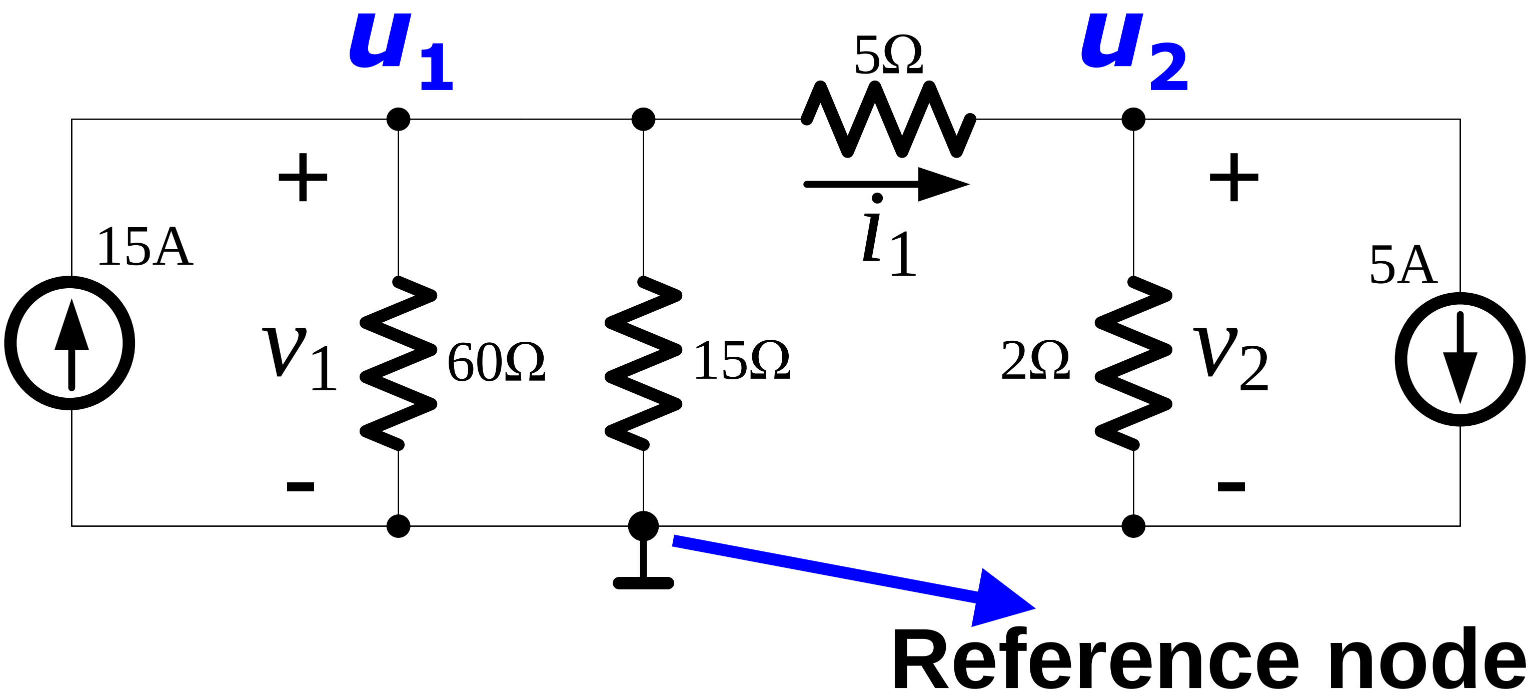
Example



Find v_1, v_2, and i_1.

Solution:

- **Step 1: Select reference node and define node voltages:**



- **Step 2: Build KCL equations for nodes except the reference node**

$$\begin{cases} 15 - \dfrac{u_1}{60} - \dfrac{u_1}{15} - \dfrac{u_1 - u_2}{5} = 0 \\ \dfrac{u_1 - u_2}{5} - \dfrac{u_2}{2} - 5 = 0 \end{cases}$$

- **Step 3: Solve equations to get node voltages:**

$$\begin{cases} u_1 = 60\text{V} \\ u_2 = 10\text{V} \end{cases}$$

- **Step 4: Determine required unknown voltages and currents by solved node voltages:**

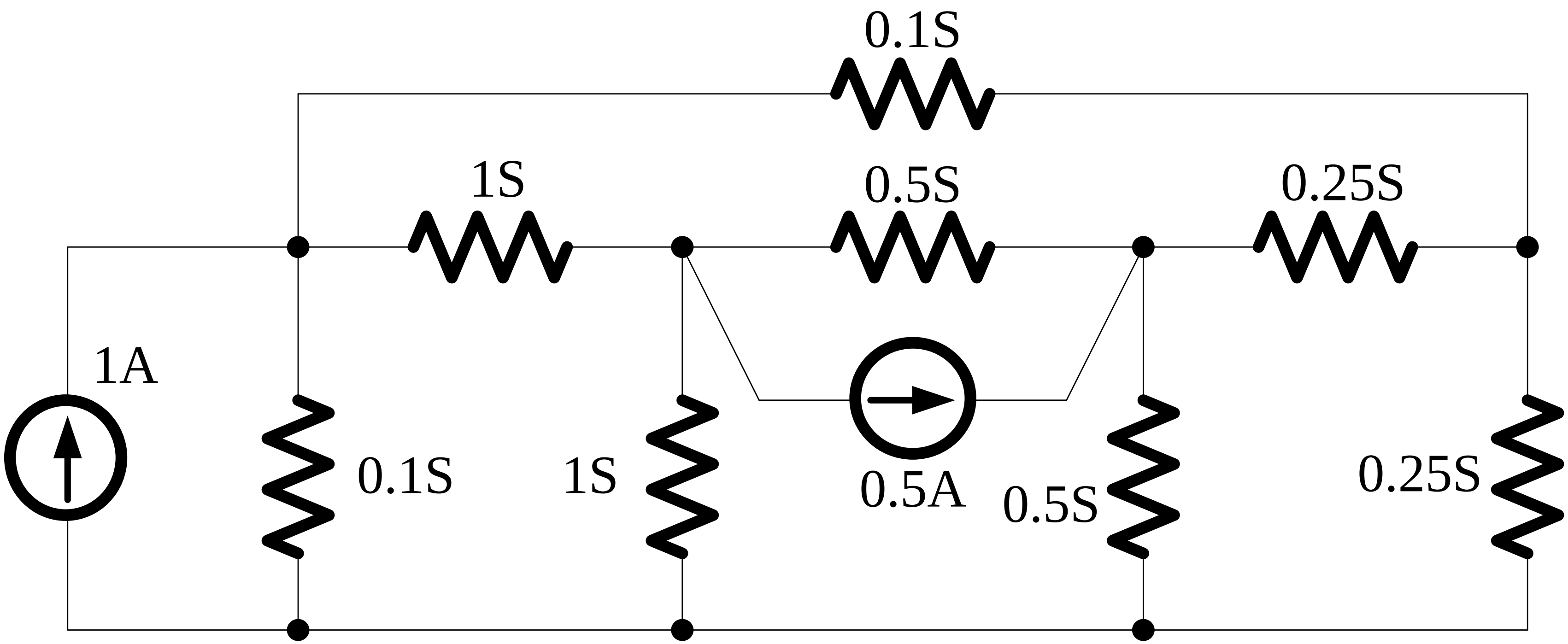
$$\begin{cases} v_1 = u_1 = 60\text{V} \\ v_2 = u_2 = 10\text{V} \\ i_1 = \dfrac{u_1 - u_2}{5} = \dfrac{60-10}{5} = 10\text{A} \end{cases}$$

- Further, if necessary, other unknown voltages, currents, or powers can also be determined, and the circuit is completely solved;
- If necessary, answers can be checked by power balance.

Steps of Node-Voltage Method

1. Select one of the nodes as the reference node, and define node voltages for other nodes;
2. Build KCL equations for nodes except the reference node;
3. Solve equations for the node voltages;
4. Determine required voltages, currents or powers by node voltages.

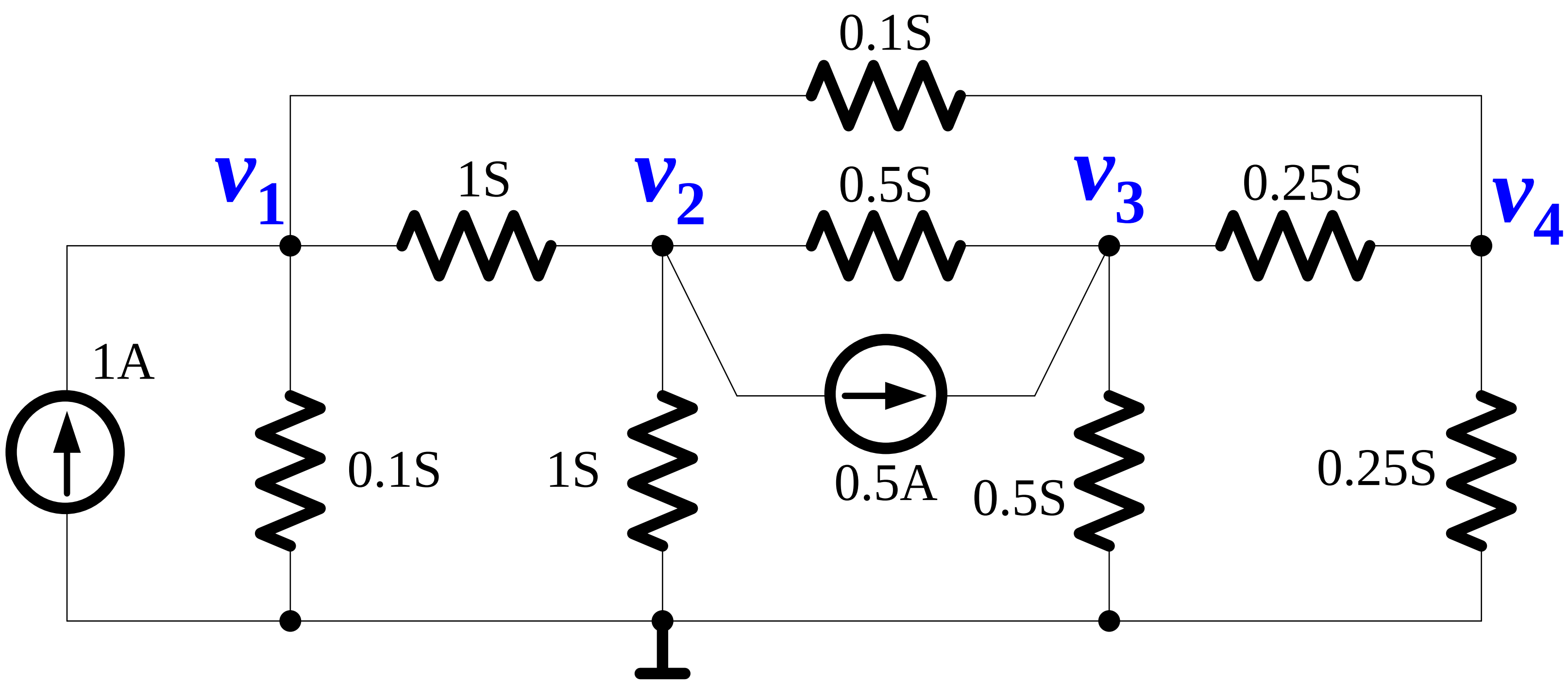
Example



Find the power dissipated by all resistors in the circuit.

Solution:

Select the reference node and define node voltages:



Build KCL equations for nodes except the reference node:

$$\begin{cases} 1-0.1v_1-0.1(v_1-v_4)-(v_1-v_2)=0 \\ (v_1-v_2)-v_2-0.5(v_2-v_3)-0.5=0 \\ 0.5(v_2-v_3)+0.5-0.5v_3-0.25(v_3-v_4)=0 \\ 0.1(v_1-v_4)+0.25(v_3-v_4)-0.25v_4=0 \end{cases}$$

$$\begin{cases} 1.2v_1 - v_2 - 0.1v_4 = 1 \\ v_1 - 2.5v_2 + 0.5v_3 = 0.5 \\ 0.5v_2 - 1.25v_3 + 0.25v_4 = -0.5 \\ 0.1v_1 + 0.25v_3 - 0.6v_4 = 0 \end{cases}$$

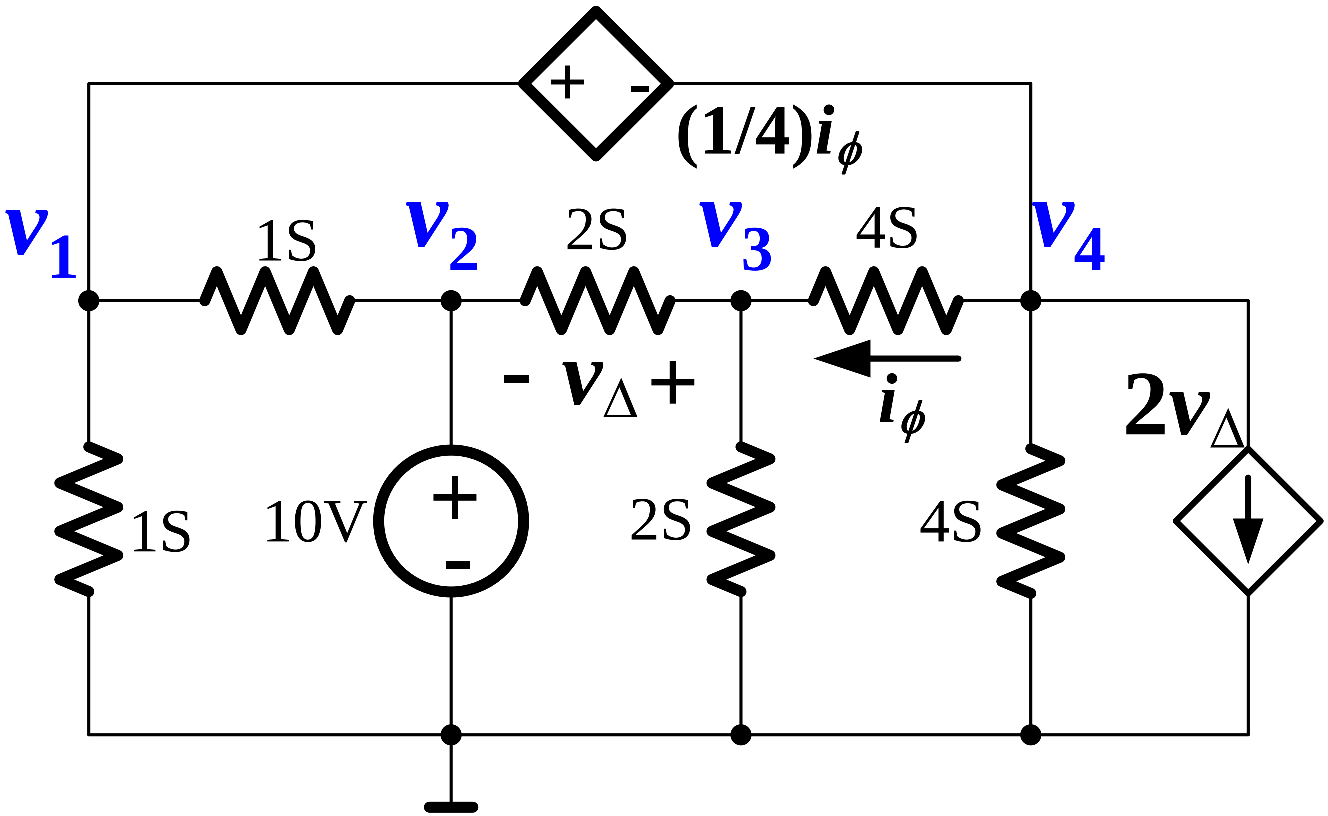
$$\Rightarrow \begin{pmatrix} 1.2 & -1 & 0 & -0.1 \\ 1 & -2.5 & 0.5 & 0 \\ 0 & 0.5 & -1.25 & 0.25 \\ 0.1 & 0 & 0.25 & -0.6 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 0.5 \\ -0.5 \\ 0 \end{pmatrix}$$

Solve equations for node voltages by Cramer's rule:

$$\begin{cases} v_1 = 1.23\text{V} \\ v_2 = 0.42\text{V} \\ v_3 = 0.67\text{V} \\ v_4 = 0.48\text{V} \end{cases}$$

Then, calculate power dissipated by all resistors…

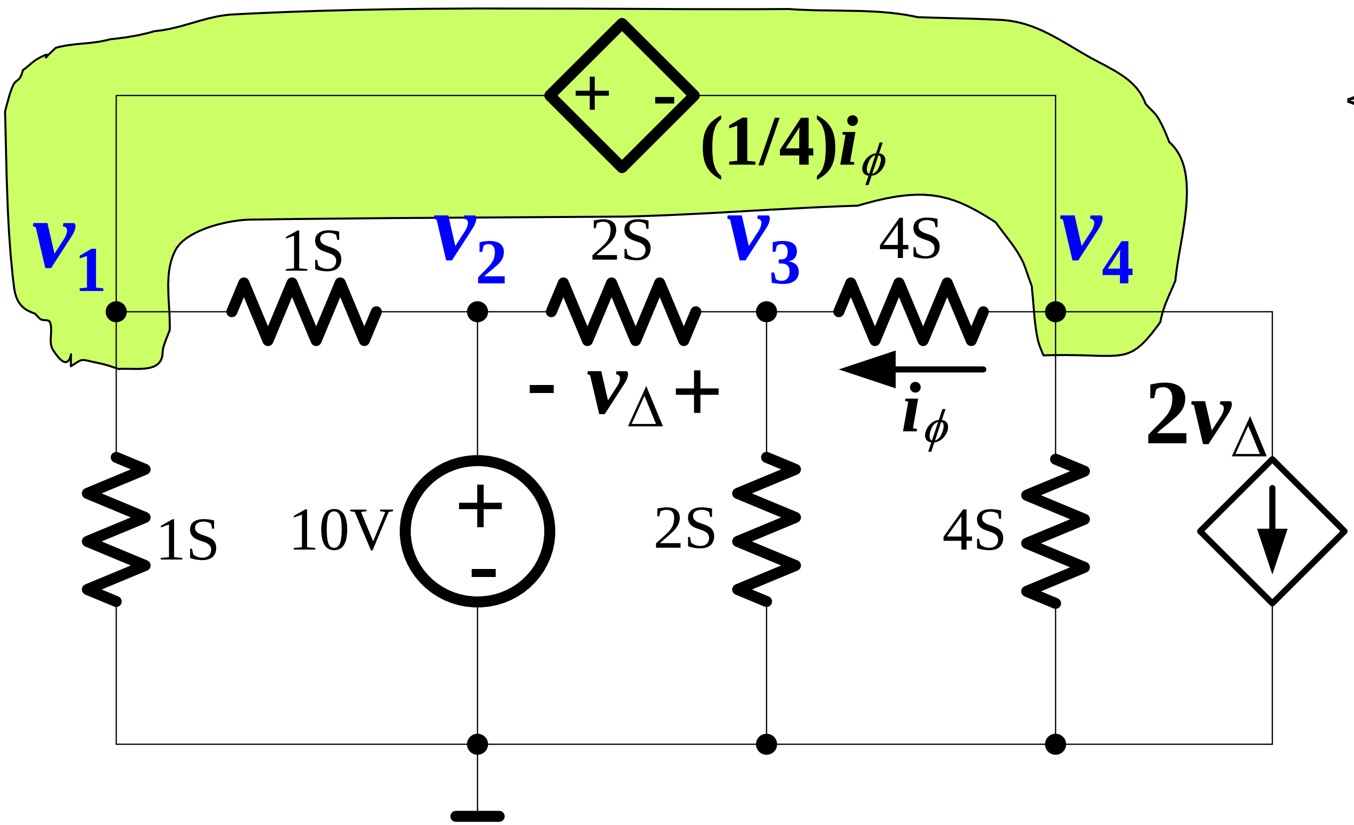
Example with Dependent Source



$$\begin{cases} v_3 = \dfrac{9}{2}\text{V} \\ v_4 = 4\text{V} \end{cases}$$

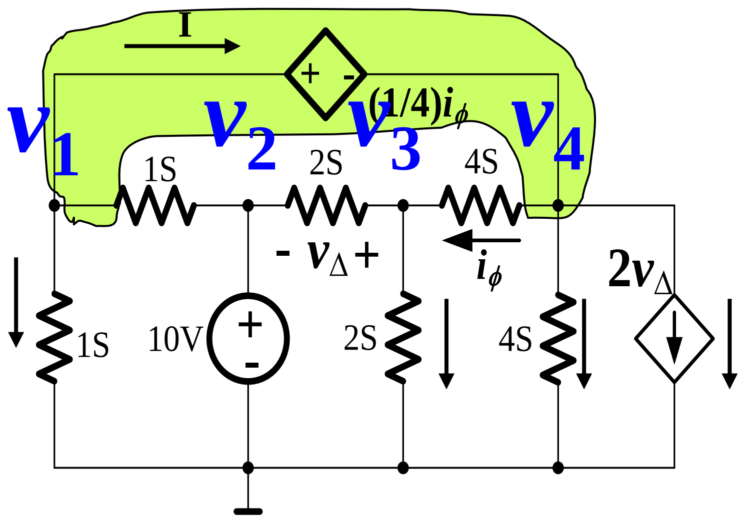
Use node-voltage method to find the value of v_3 and v_4.

Concept of Super Node



$$\begin{cases} v_3 = \dfrac{9}{2}\text{V} \\ v_4 = 4\text{V} \end{cases}$$

Solution:



1. $V_1 - V_4 = V_4 - V_3$
2. $-V_1 - I + V_2 - V_1 = 0$
3. $V_2 = 10V$
4. $-2(V_3 - V_2) + 4(V_4 - V_3) - 2V_3 = 0$
5. $-4(V_4 - V_3) - 2(V_3 - V_2) - 4V_4 + I = 0$

解得：

$V_1=7/2V$；$V_2=10V$；$V_3=9/2V$；

$V_4=4V$；$I=3A$

其中，1式由超节点得到V1 与V4的关系并带入受控方程得出，2-5式分别为V1 ~V4 的节点方程

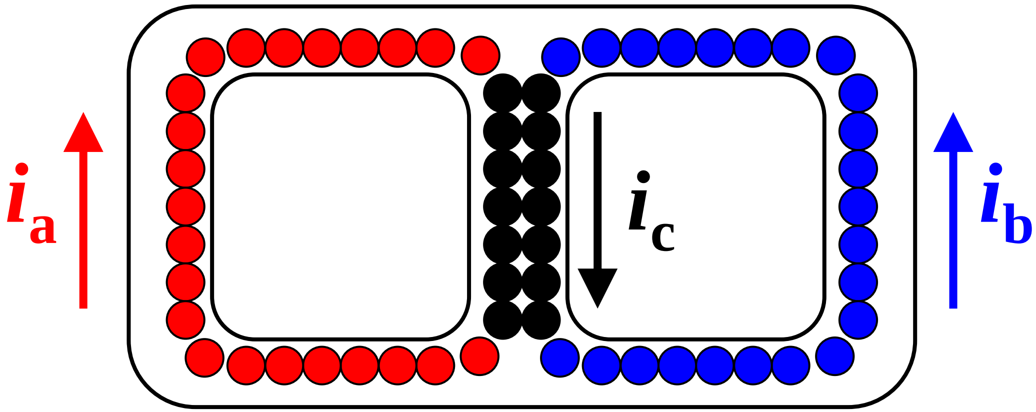
3-2 Mesh-Current Analysis Method

- What is mesh current?
- How is mesh current analysis method implemented?

What is Mesh Current?

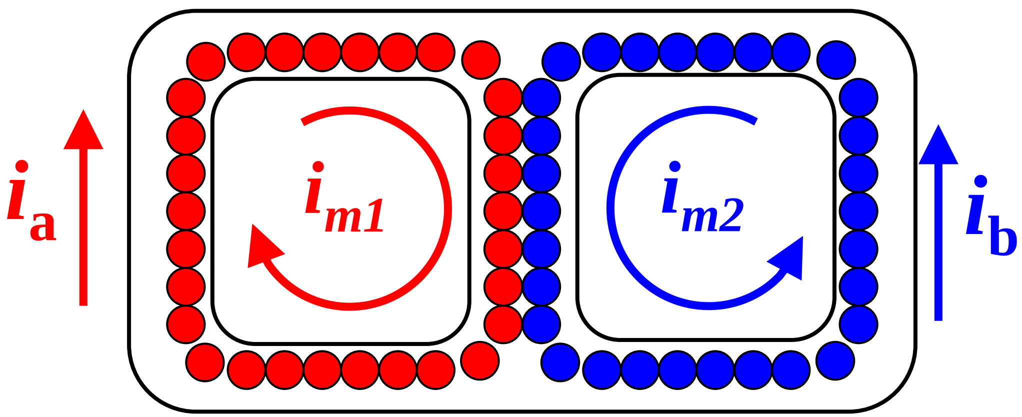
- Mesh current is introduced just as an imaginary quantity;
- Mesh current flows around a mesh;
- All branch currents do not change for introducing mesh currents
- All branch currents can be represented by mesh currents.

What is Mesh Current?



Branch currents:

$$i_c = i_a + i_b$$



Mesh currents:

$$\begin{cases} i_c = i_{m1} + i_{m2} \\ i_a = i_{m1} \\ i_b = i_{m2} \end{cases}$$

HOW is mesh current analysis method implemented?

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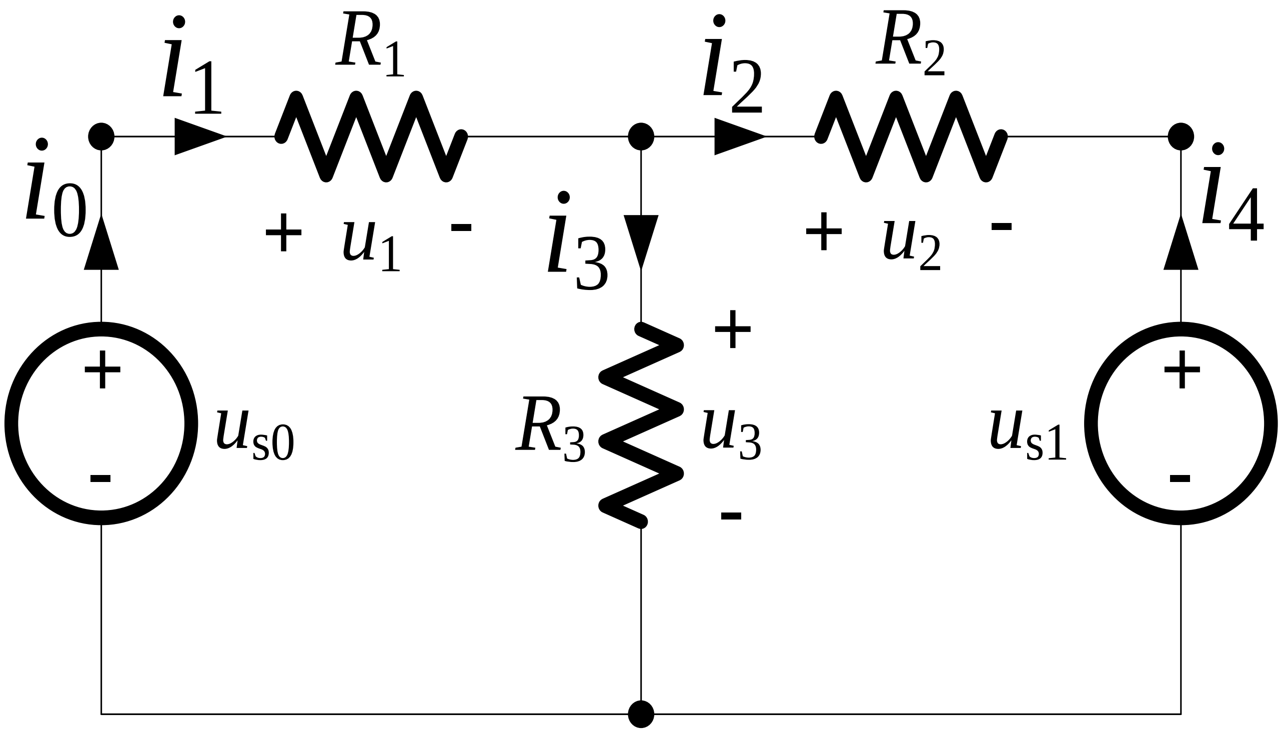
Example:

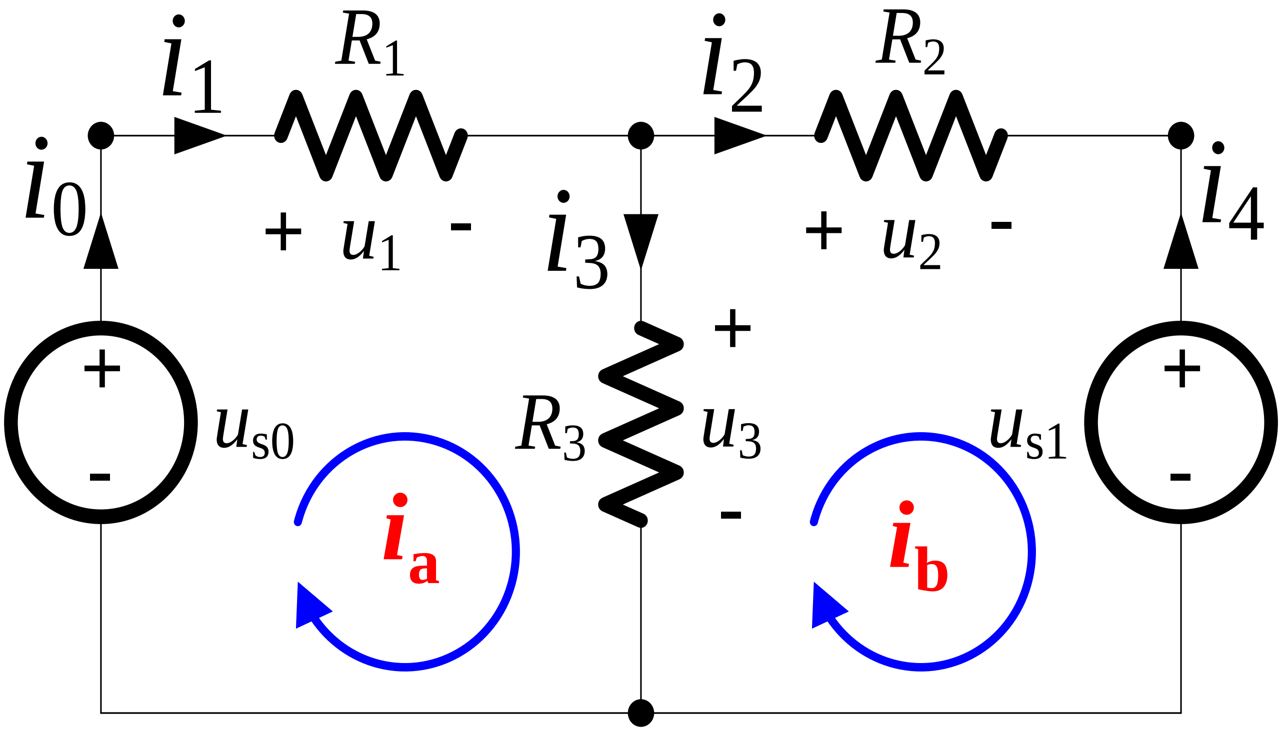


$R_1 = 5\ \Omega$,
$R_2 = 10\ \Omega$,
$R_3 = 20\ \Omega$,
$u_{s0} = 20$ V,
$u_{s1} = 10$ V

Determine the currents of every branch.

Solution:

- **Step 1: Select mesh current i_a and i_b**



- Step 2: Build KVL equations for all meshes

$$\begin{cases} R_1 i_a + R_3\left(i_a - i_b\right) - u_{s0} = 0 \\ R_2 i_b + u_{s1} + R_3\left(i_b - i_a\right) = 0 \end{cases}$$

- Step 3: Solve equations to get mesh current i_a and i_b

$$\begin{cases} 5i_a + 20(i_a - i_b) - 20 = 0 \\ 10i_b + 10 + 20(i_b - i_a) = 0 \end{cases} \Rightarrow \begin{cases} i_a = \dfrac{8}{7}\text{A} \\ i_b = \dfrac{3}{7}\text{A} \end{cases}$$

Step 4: Determine required currents by mesh current i_a and i_b.

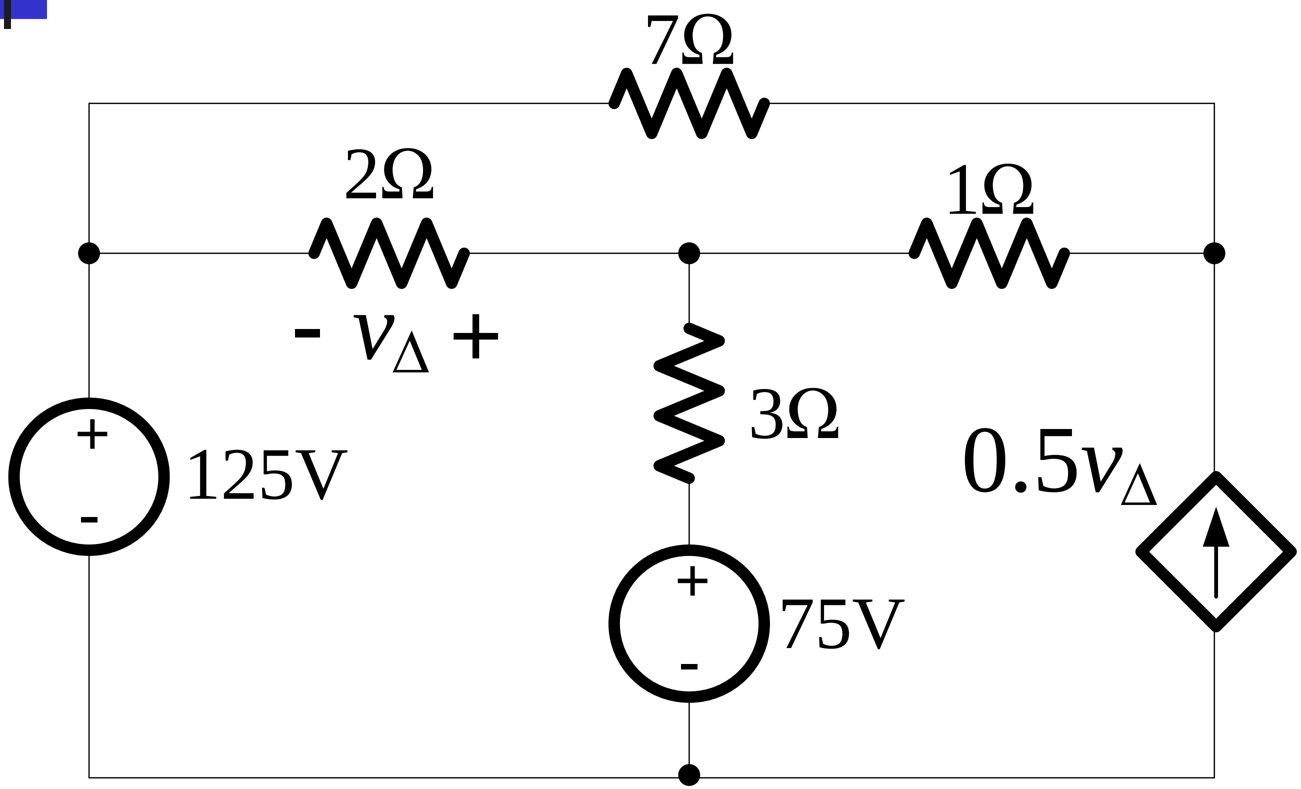
$$\begin{cases} i_0 = i_1 = i_a = \frac{8}{7}\text{A} \\ i_2 = i_b = \frac{3}{7}\text{A} \end{cases} \qquad \begin{cases} i_3 = i_a - i_b = \frac{5}{7}\text{A} \\ i_4 = -i_b = -\frac{3}{7}\text{A} \end{cases}$$

- Further, if necessary, other unknown voltages and currents can also be determined, and the circuit is completely solved;
- If necessary, answers can be checked by power balance.

Steps of Mesh-Current Method

1. Assume mesh current and its reference direction (clockwise) for every mesh;
2. Build KVL equations for every mesh;
3. Solve equations for mesh currents;
4. Determine required unknown currents and voltages by mesh currents.

Example with Dependent Source

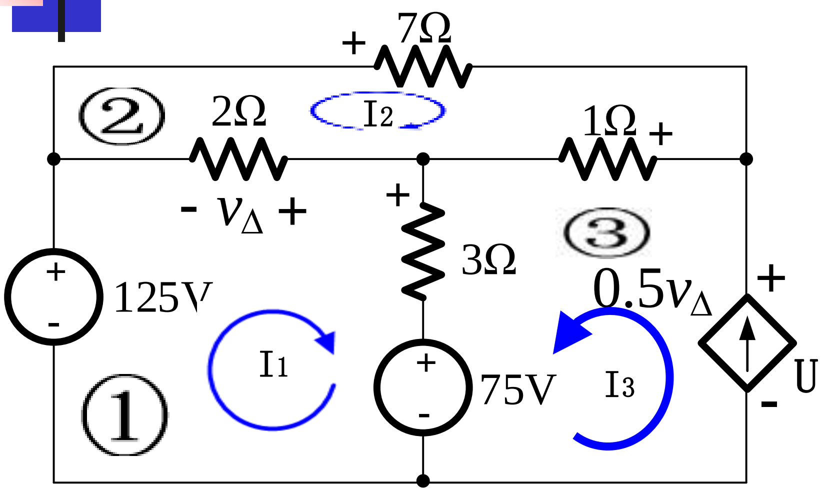


$P_{dissipated}$

$= 2750\text{W}$

Use mesh-current method to find the total power developed in the circuit.

Solution:



对回路1，2列写回路方程：

$$\begin{cases} -125-2(I_2-I_1)+3(I_1+I_2-I_1)+75=0 \\ 7I_2+I_2+I_2-I_1+2(I_2-I_1)=0 \end{cases}$$

解得：

$I_1=22A$；$I_2=6A$；

$I_3=I_2-I_1=-16A$；

对回路3列回路方程：

$$-U-10+6\times3+75=0$$

解得：

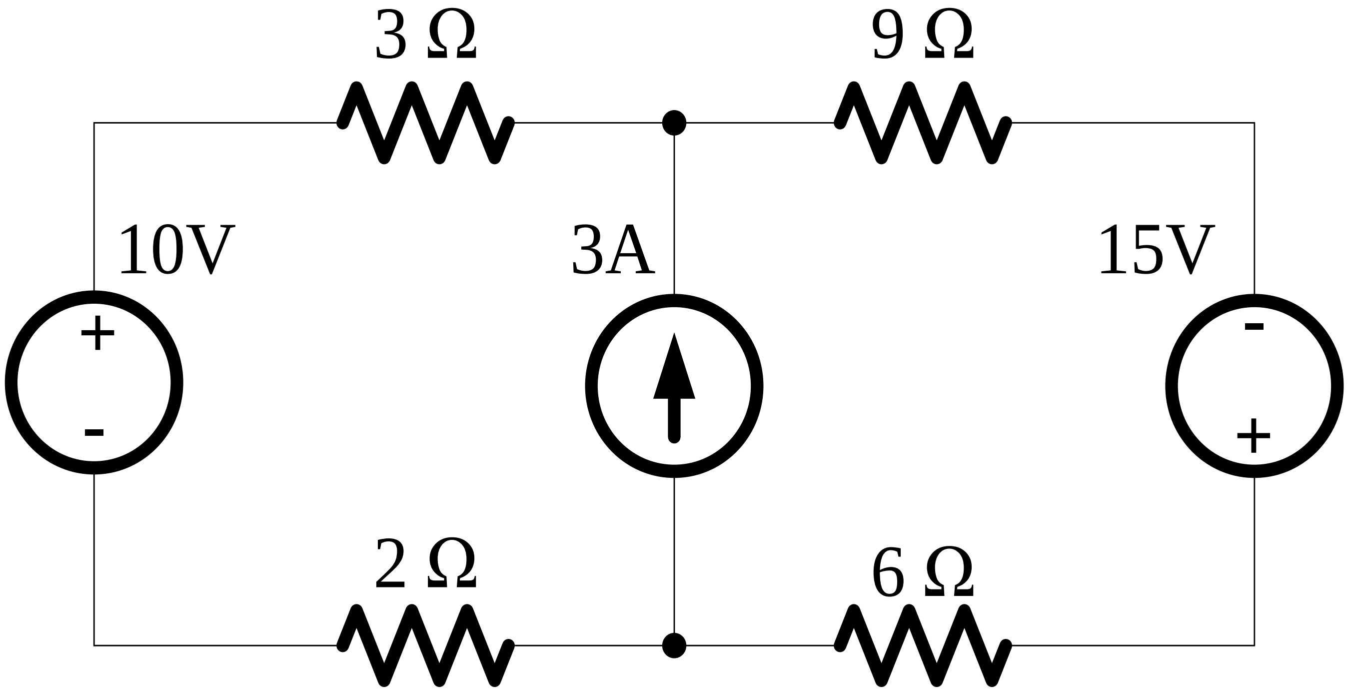
$$U=83V$$

两个电压源和一个电流源由其各自的电压电流关联关系可知：

只有电压为125V的电压源发出功率。

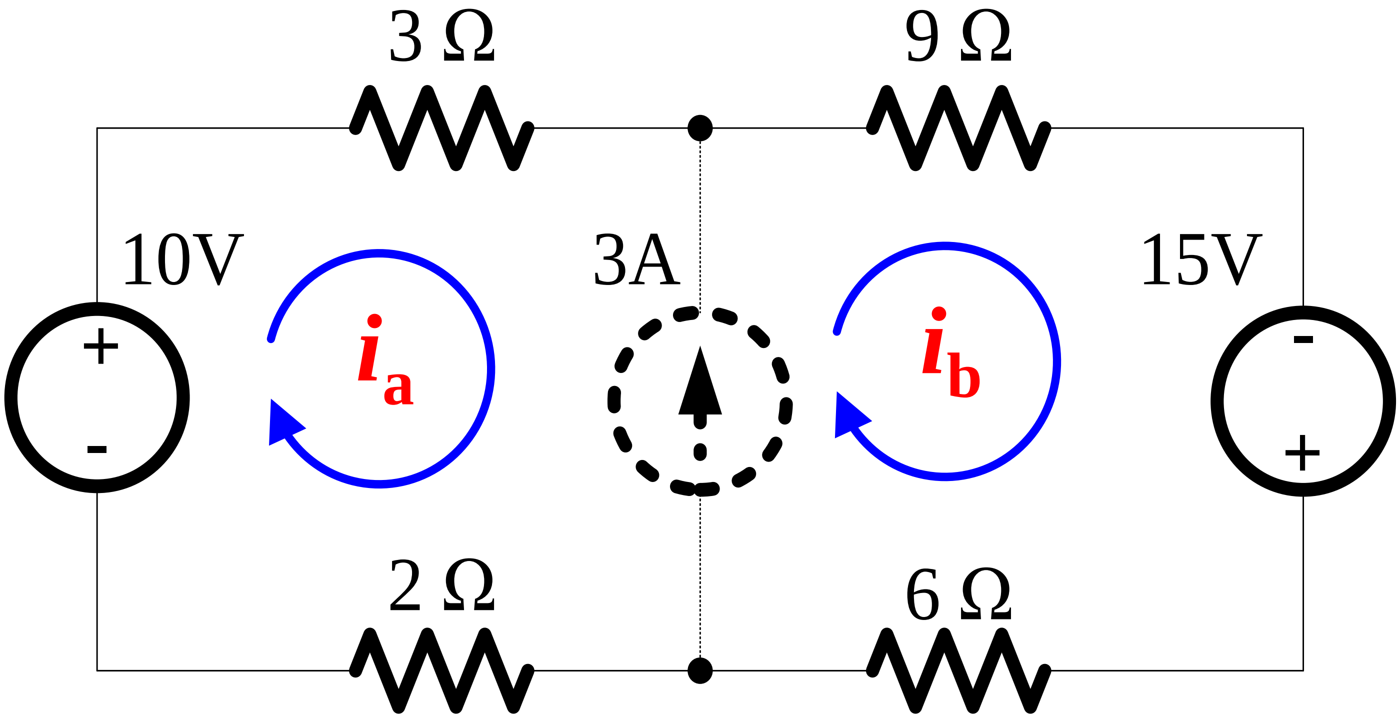
故$P_{吸}=P_{发}=125\times22=2750W$。

Example of Super-Mesh



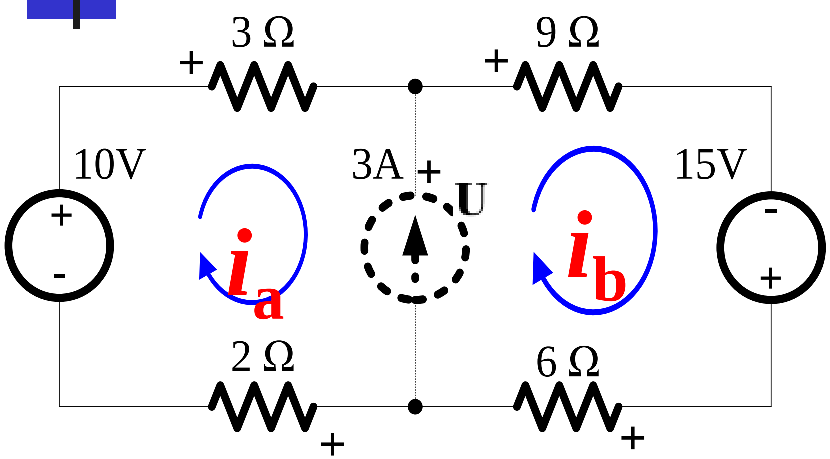
Use mesh-current method to find the total power dissipated in the circuit.

Concept of Super-Mesh



$$P_{dissipated} = 75\text{W}$$

Solution:



解得：

I_a =-1A； I_b =2A；

由其电压电流关联关系可知：
10V电压源吸收功率，15V电压源发出功率；

采用超回路法：

$$\begin{cases} -10+3I_a+9I_b-15+6I_b+2I_a=0 \\ I_a+3=I_b \end{cases}$$

对a回路列回路方程：

$$-10-3+U-2=0$$

解得U=15V；U为受控源两端电压；
由于受控源电流电压非关联，故受控源发出功率；
故总的发出功率为：
P=3×15+2×15=75W

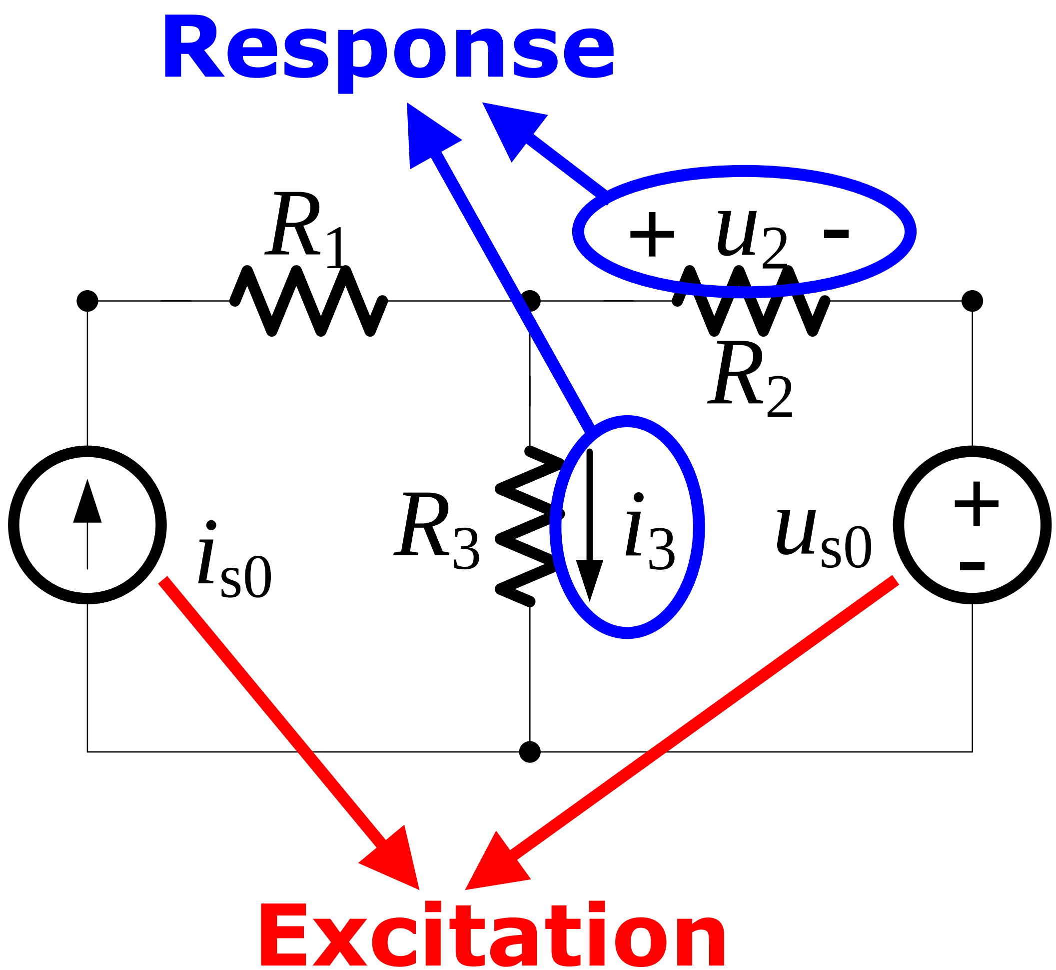
Nodal method versus Mesh method

- **Mesh analysis only applies to planar circuits;**
- **Node-voltage method results in direct calculation of voltage; whereas mesh-current analysis provides currents;**
- **Mesh analysis results in $m=b-(n-1)$ equations; and nodal analysis results in $(n-1)$ equations.**

3-3 Superposition Theorem

- If a linear circuit is excited by more than one **independent source**, the total response is the **algebraic sum** of all the individual responses.
- The individual response is the result of an independent source **acting alone**.

Excitation and Response



Comments on Superposition

- **Act alone:**
  - **Consider each independent source one at a time;**
  - **Other independent sources are “killed”, or “turned off”, or “zeroed out”.**

Comments on Superposition

- **If a voltage source is zeroed out, it is treated as a short circuit;**
- **If a current source is zeroed out, it is treated as an open circuit;**
- **Superposition theorem does not apply to power calculation.**

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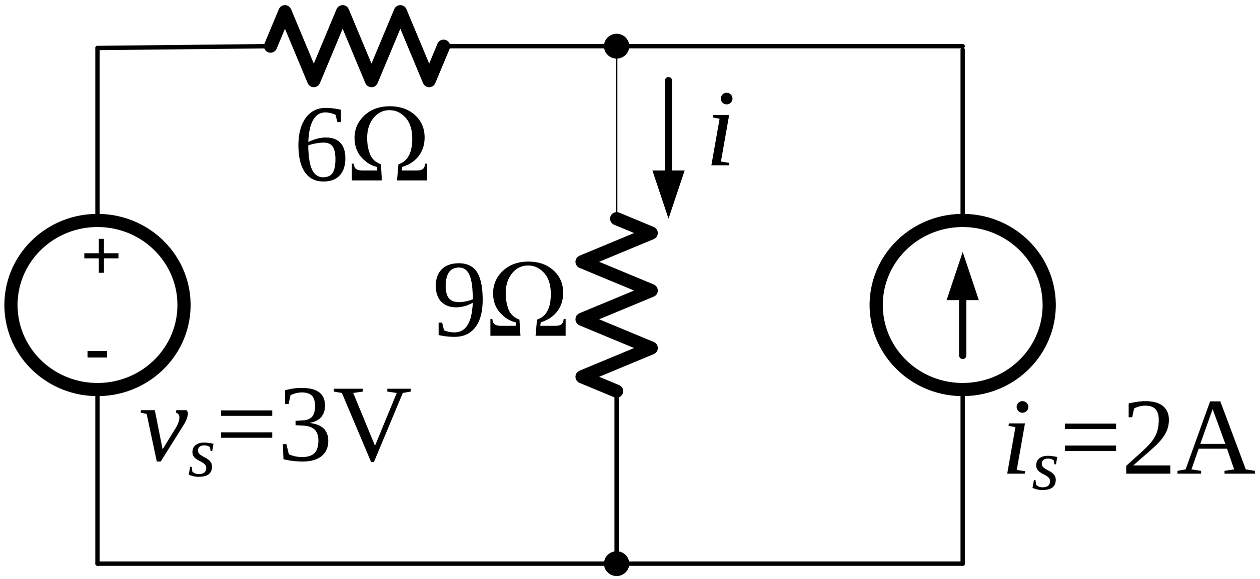
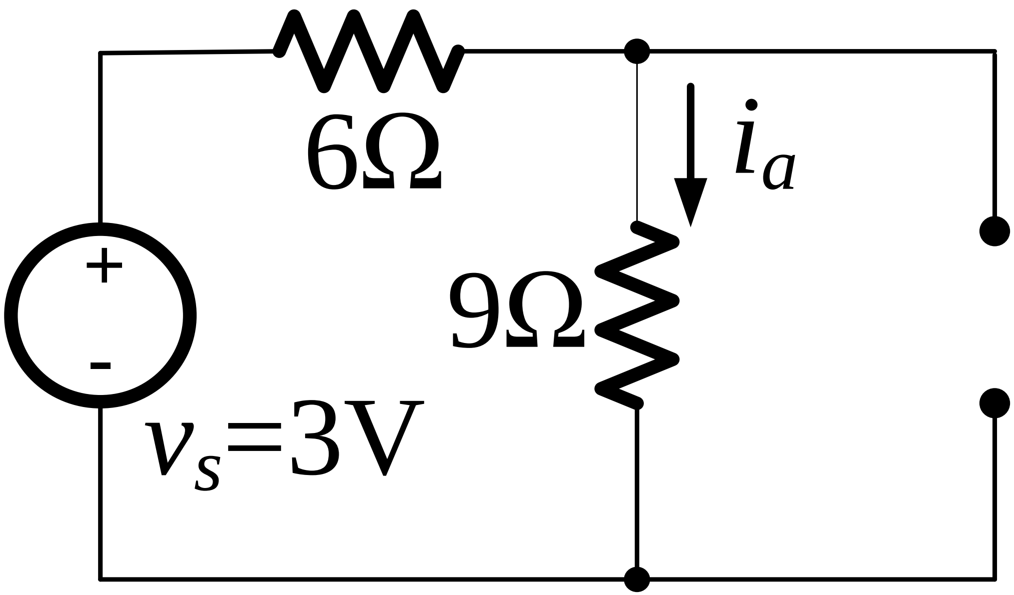
Example



Find the unknown branch current *i*.

Solution:

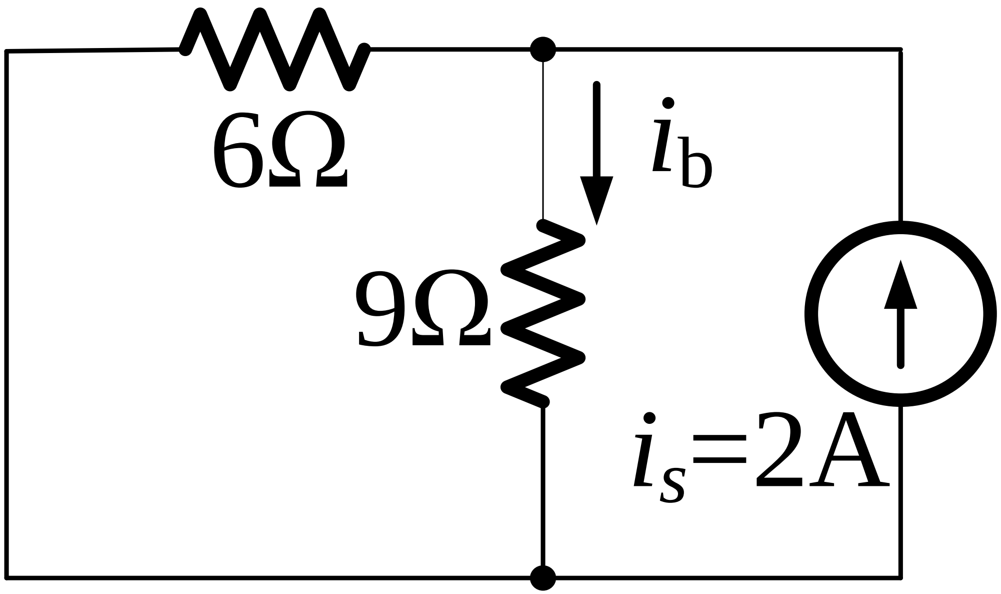
1. Set the current source to be zero:



$$i_a = \frac{3\text{V}}{(6+9)\Omega}$$

$$= 0.2\text{A}$$

2. Set the voltage source to be zero:



$$i_b = \left(\frac{6}{6+9}\right)2\text{A}$$

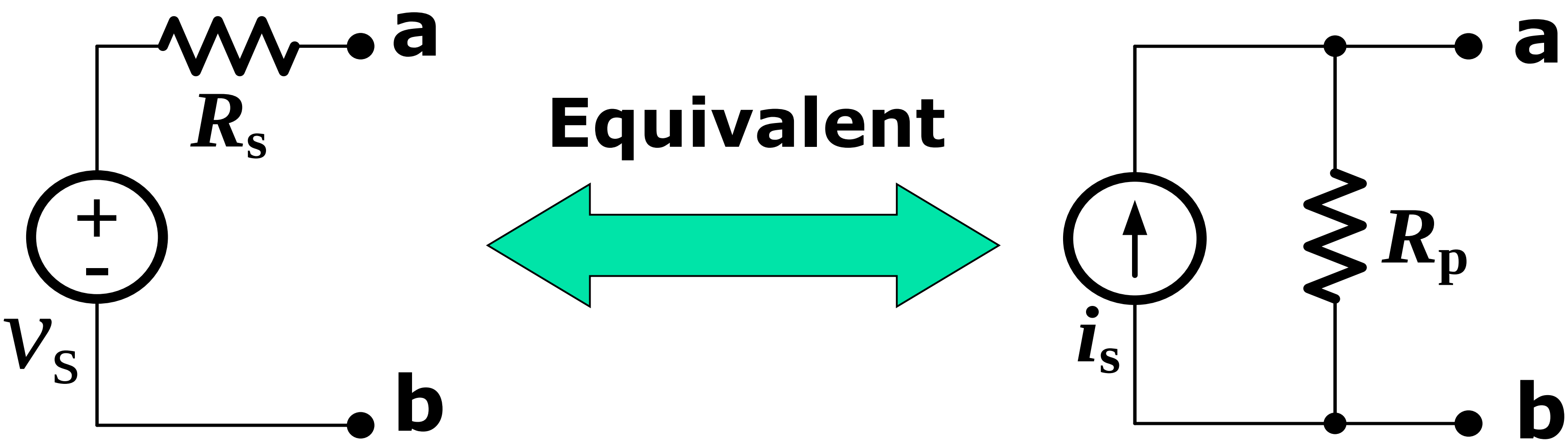
$$= 0.8\text{A}$$

3. By superposition, we have:

$$i = i_a + i_b = 0.2 + 0.8 = 1\text{A}$$

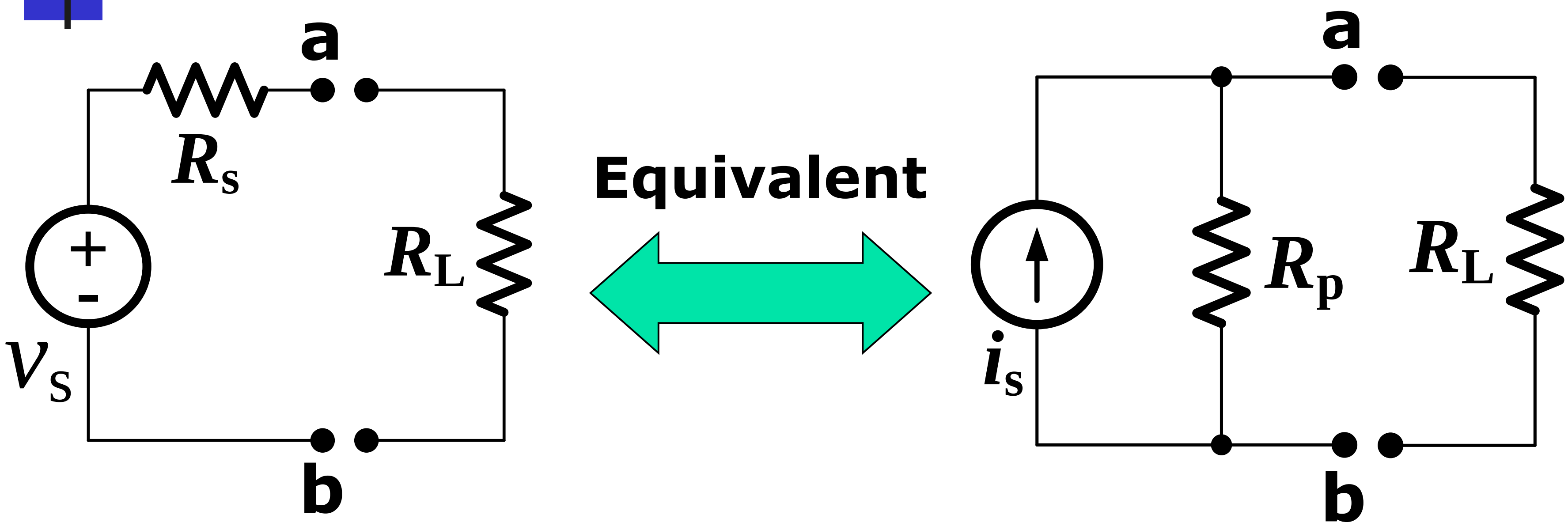
3-4 Source Transformation

Source Transformation



$$\begin{cases} R_s = R_p \\ v_s = i_s R_p \end{cases}$$

Source Transformation



$$v_{Ls} = \frac{v_s}{R_s + R_L} R_L \quad = \quad v_{Lp} = \frac{i_s R_p}{R_p + R_L} R_L$$

Source Transformation

$$v_{Ls} = \frac{v_s}{R_s + R_L} R_L$$

$$\|$$

$$v_{Lp} = \frac{i_s R_p}{R_p + R_L} R_L$$

$$\boxed{\begin{cases} R_s = R_p \\ v_s = i_s R_p \end{cases}}$$

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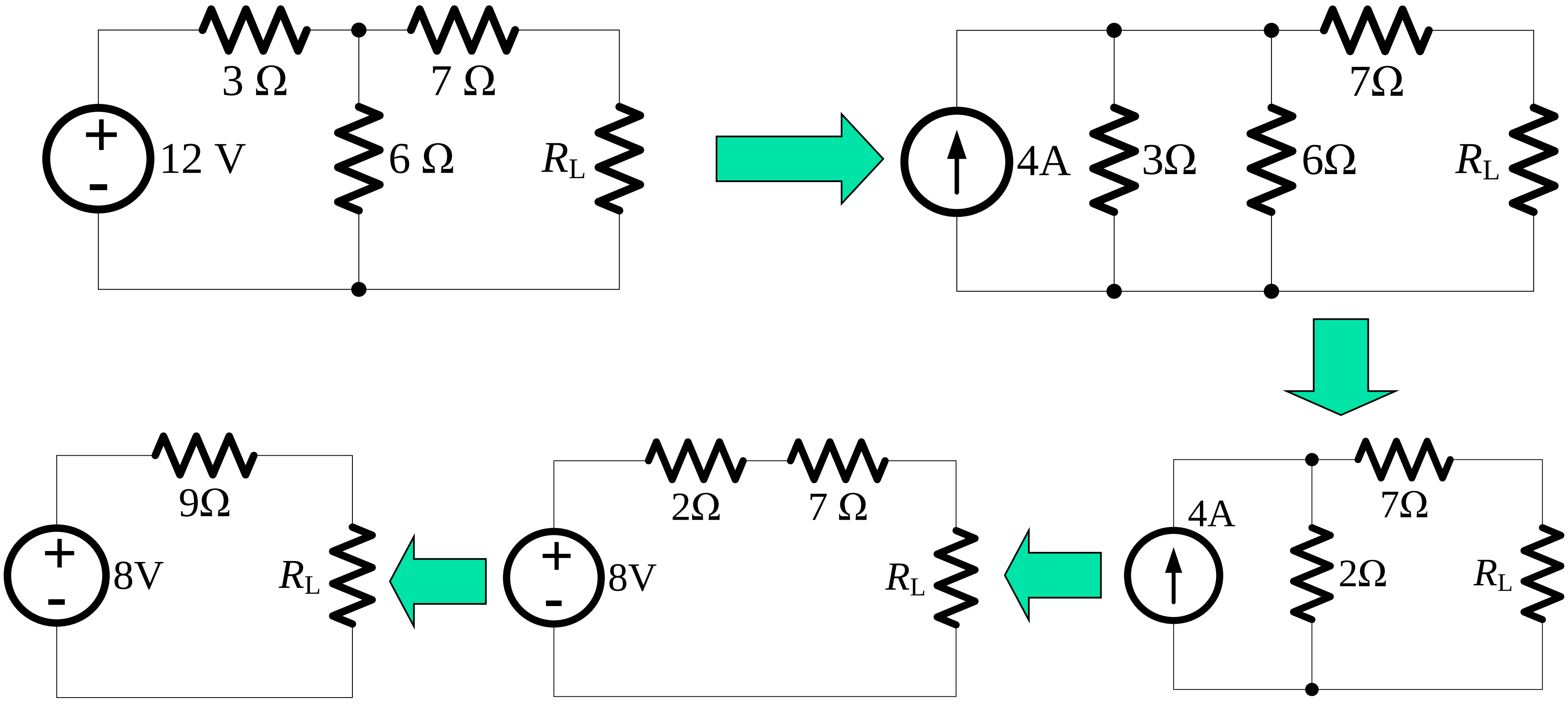
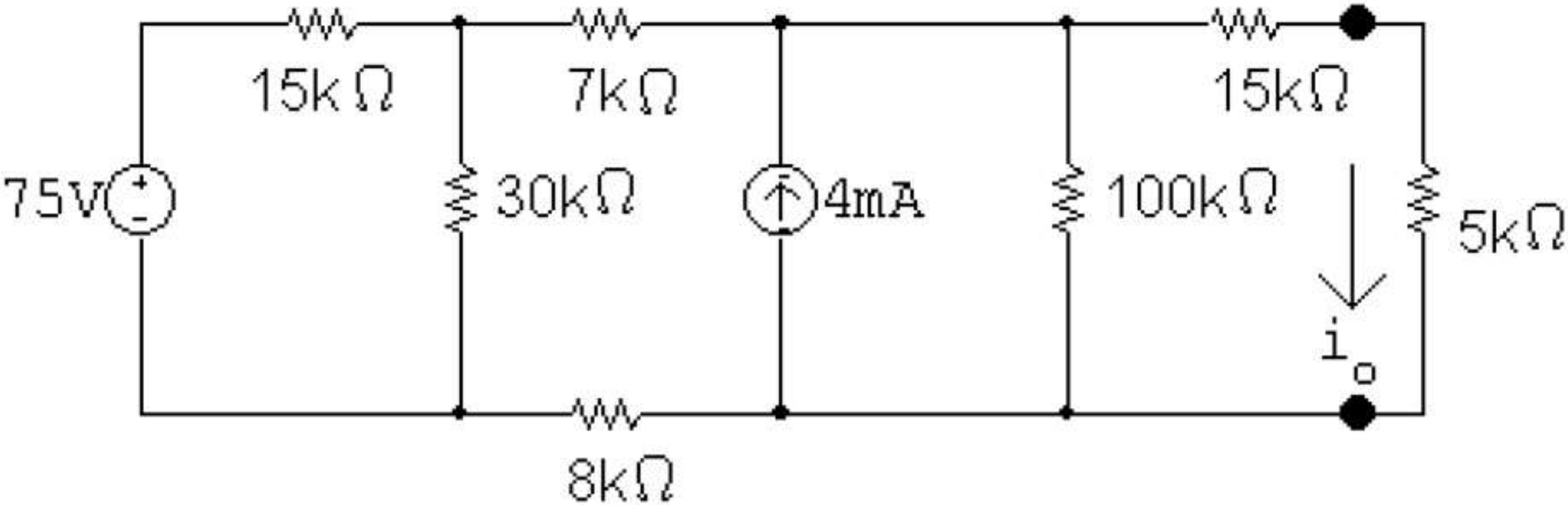
Example



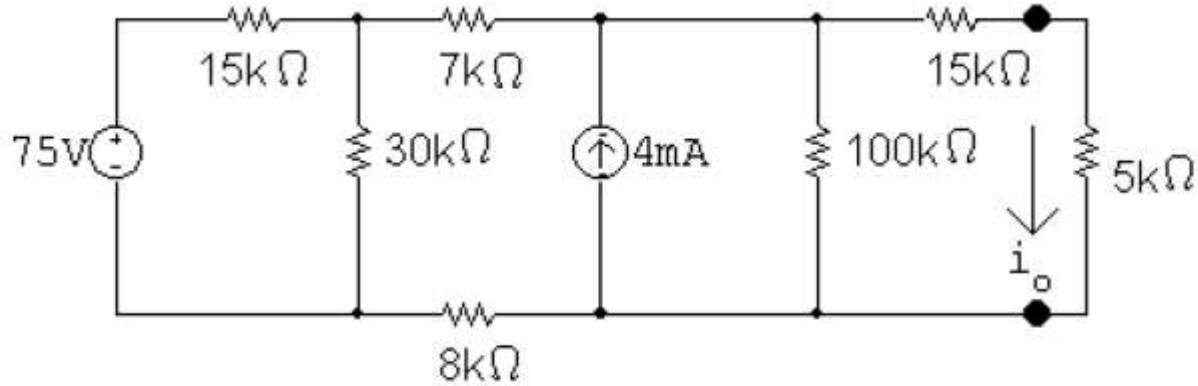
NOTE the reference direction of voltage and current sources.

Example

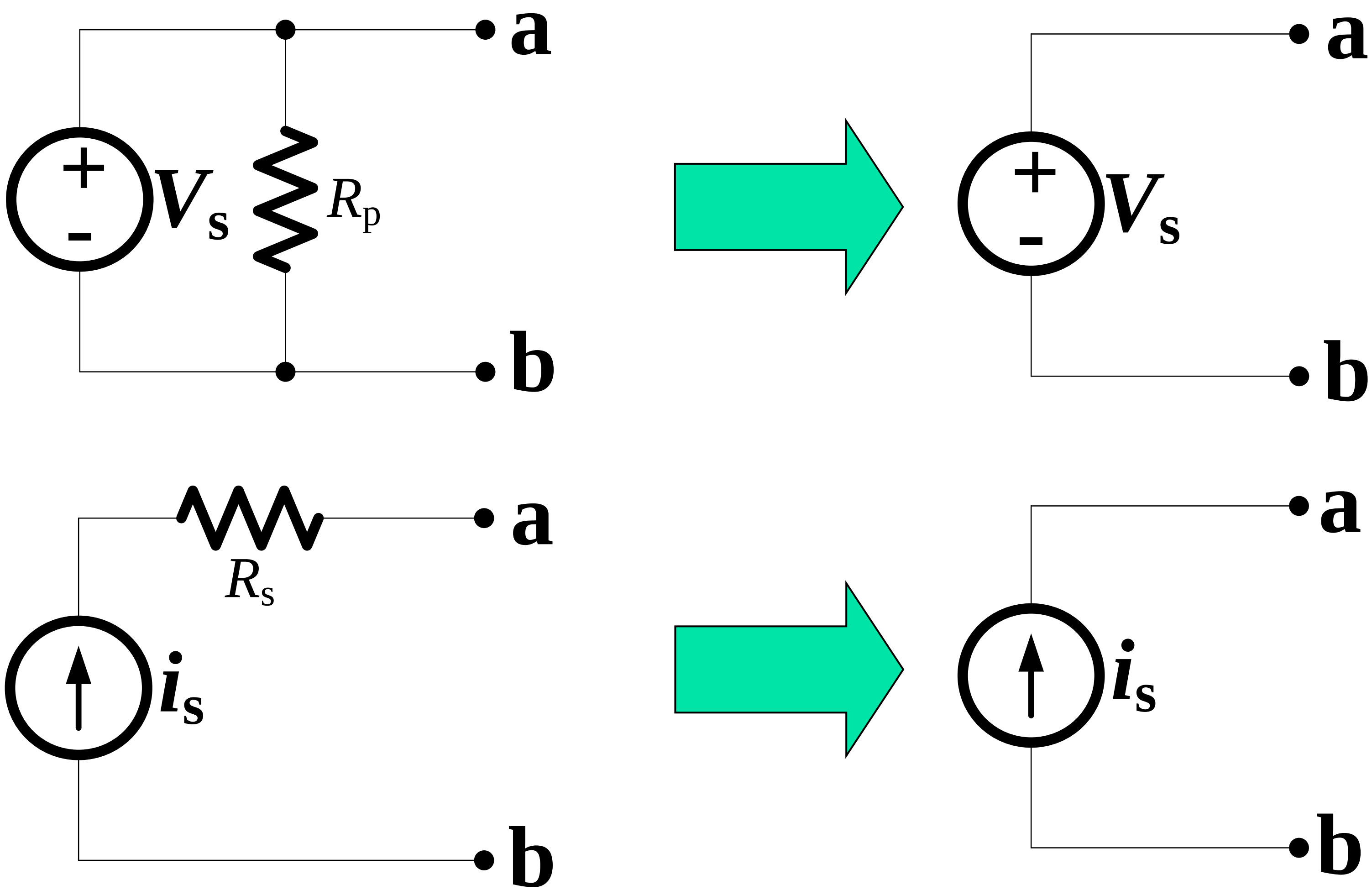


Find the current in the 5 kΩ resistor by source transformations.

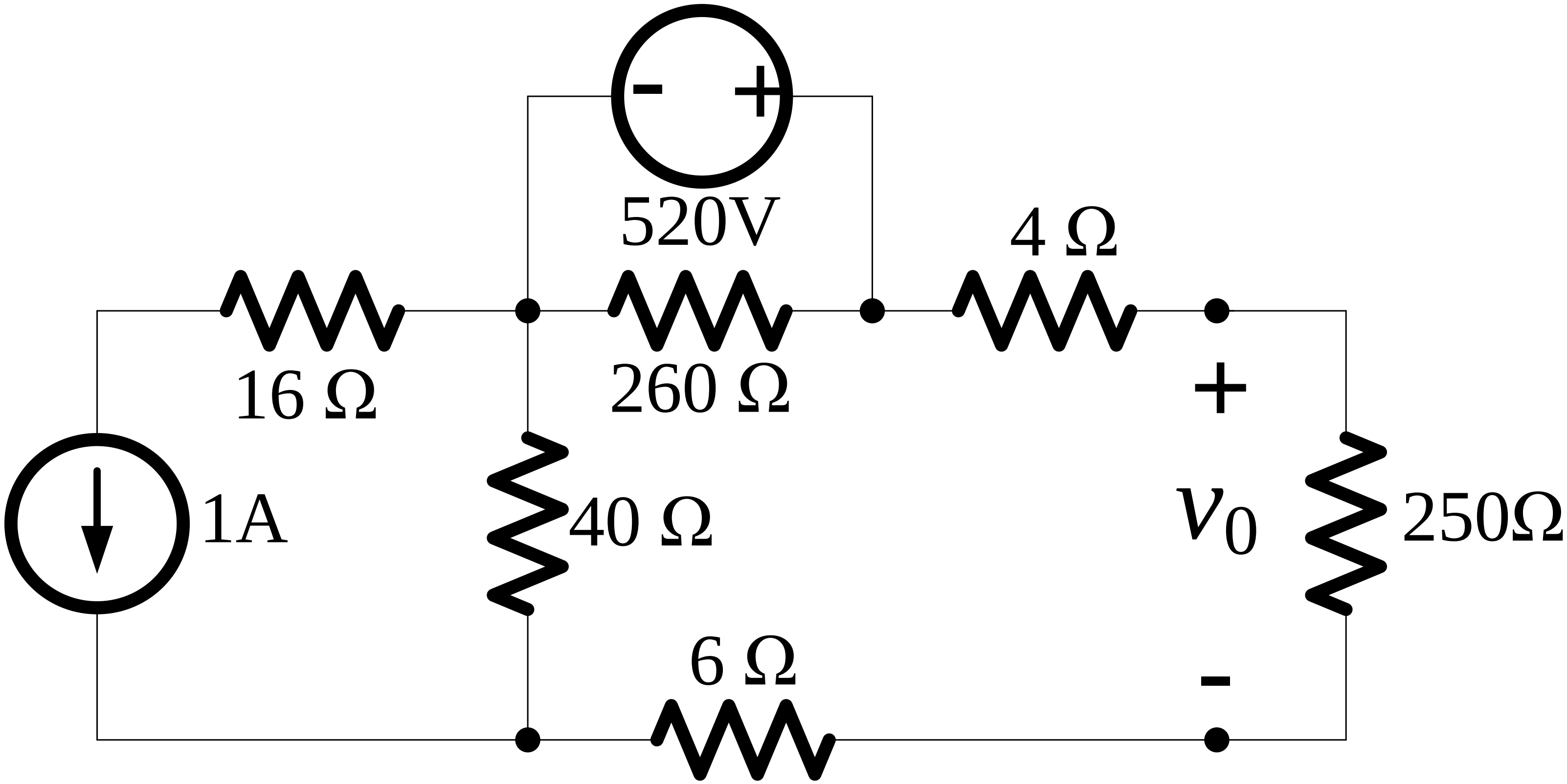
ANS: i_0 = 3mA


15kΩ
7kΩ
15kΩ
75V
30kΩ
4mA
100kΩ
5kΩ
i_o
8kΩ

Special Cases of Source Transformation

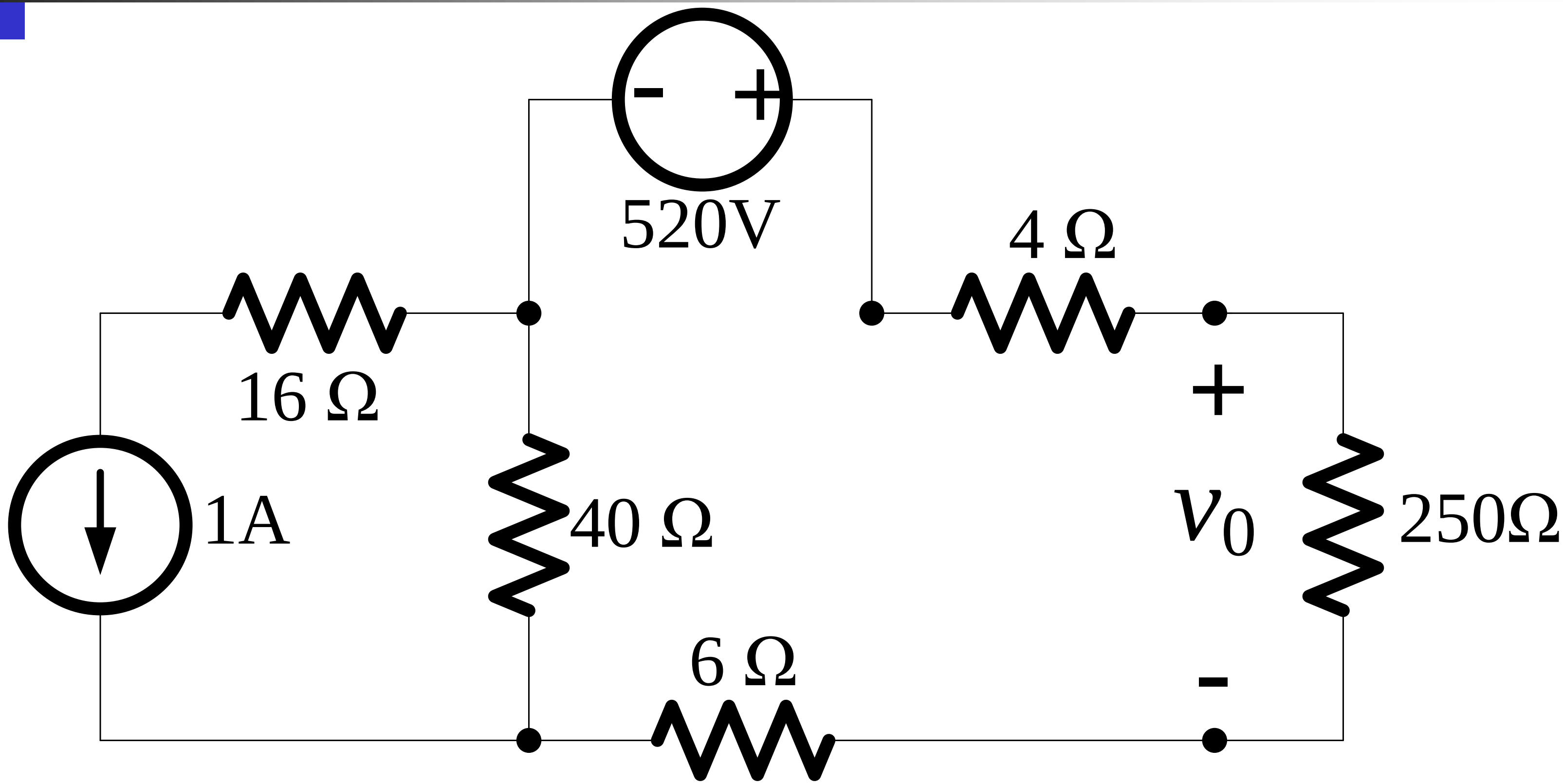


Example



Use source transformation to find v_0.

Solution:

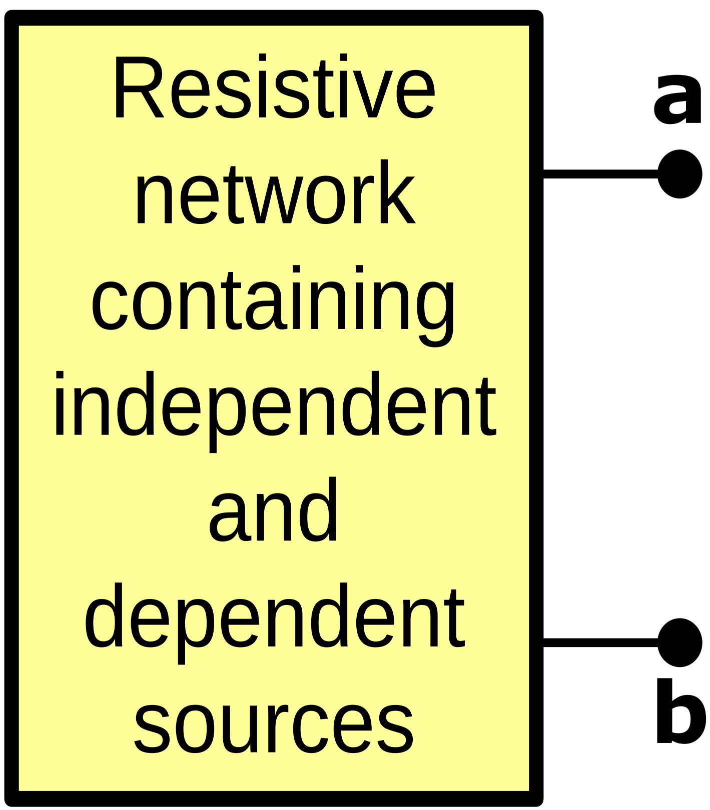


ANS: v_0 = 400V

3-5 Thévenin and Norton Equivalents

- Thévenin Equivalent Circuit
- Norton Equivalent Circuit

Thévenin and Norton Equivalents

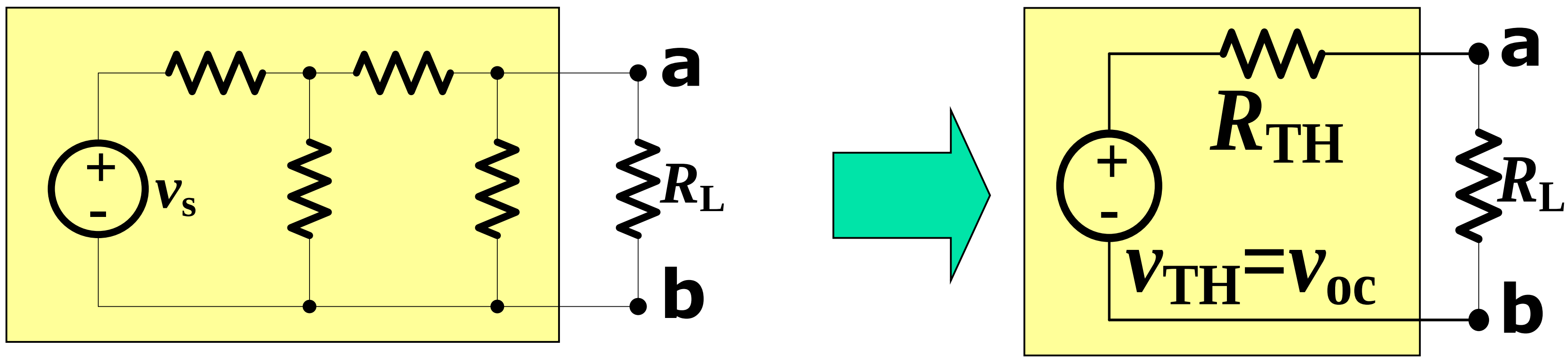


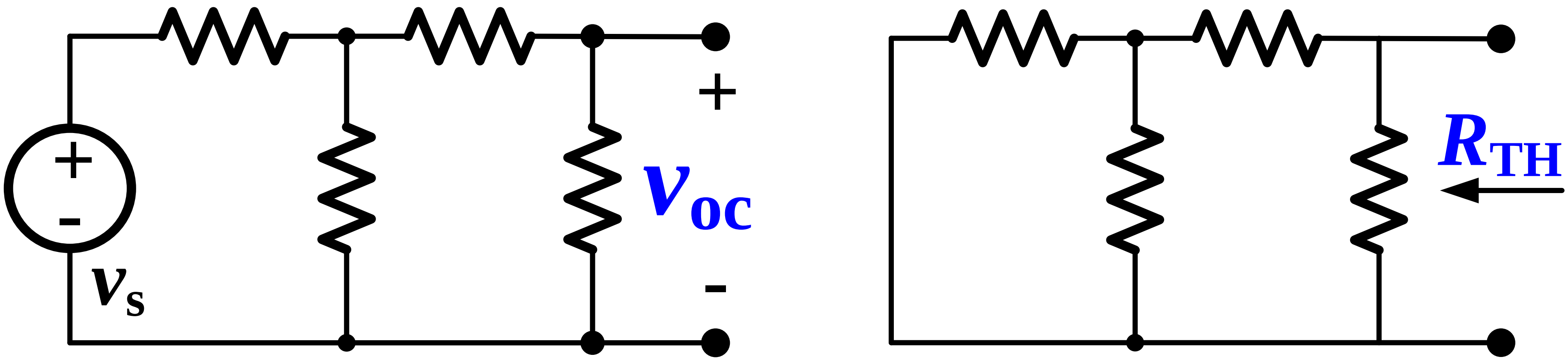
- At times, we only have interest in the behavior at the two terminals;
- Thévenin and Norton Equivalents are circuit simplification techniques that focus on terminal behavior.

Thévenin Equivalent Circuit

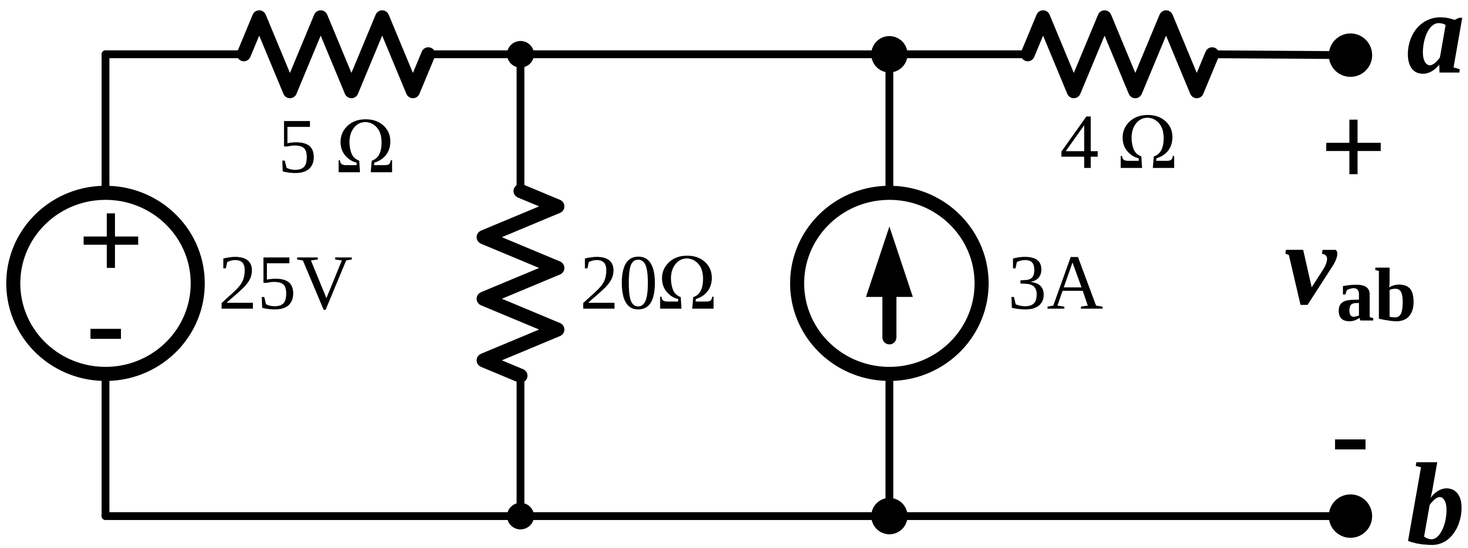
1. A network with two terminals can be equivalent to an independent voltage source v_{TH} in series with a resistor R_{TH};
2. v_{TH} is the open-circuit voltage when the two terminals are disconnected;
3. R_{TH} is the equivalent resistance of the two-terminal network when all independent sources in it are zeroed out (killed).

Thévenin Equivalent Circuit





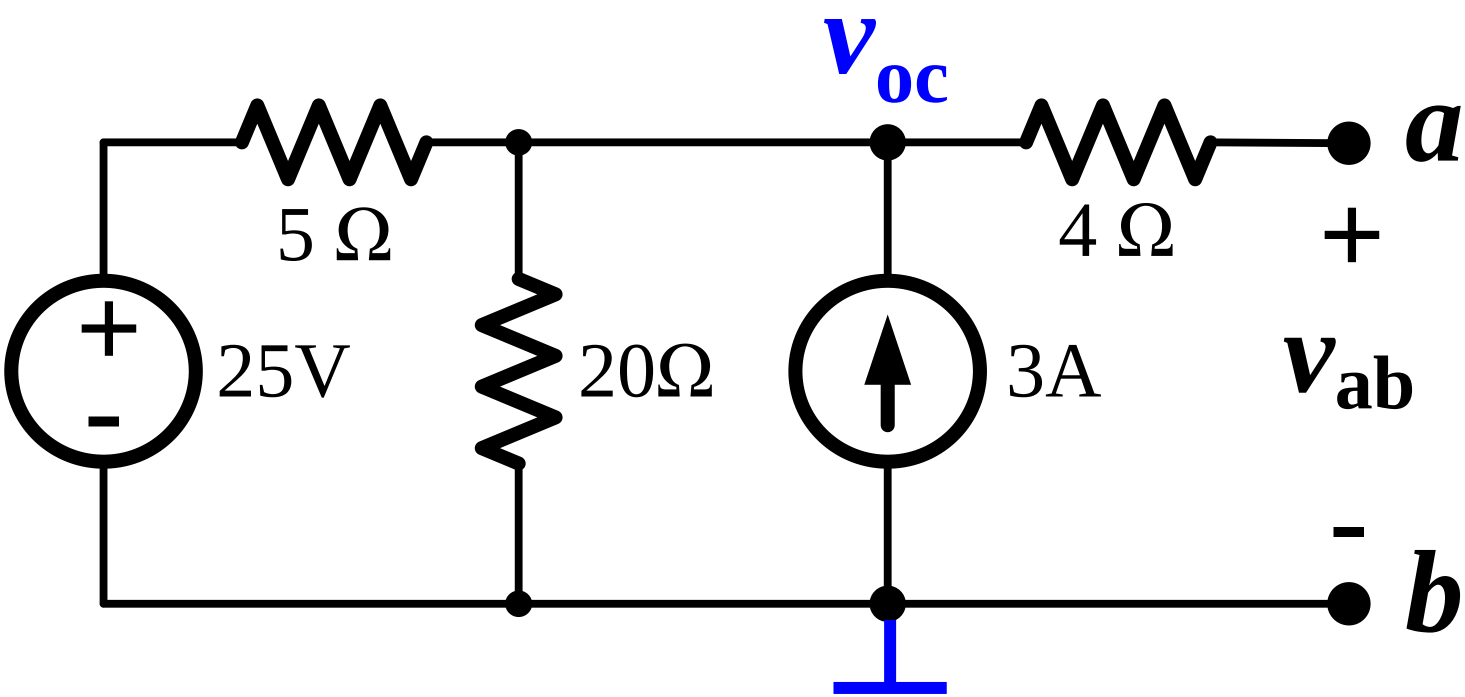
Example



Find the Thévenin equivalent of the circuit between terminals a and b.

Solution:

Find the open circuit voltage between two terminals a and b:



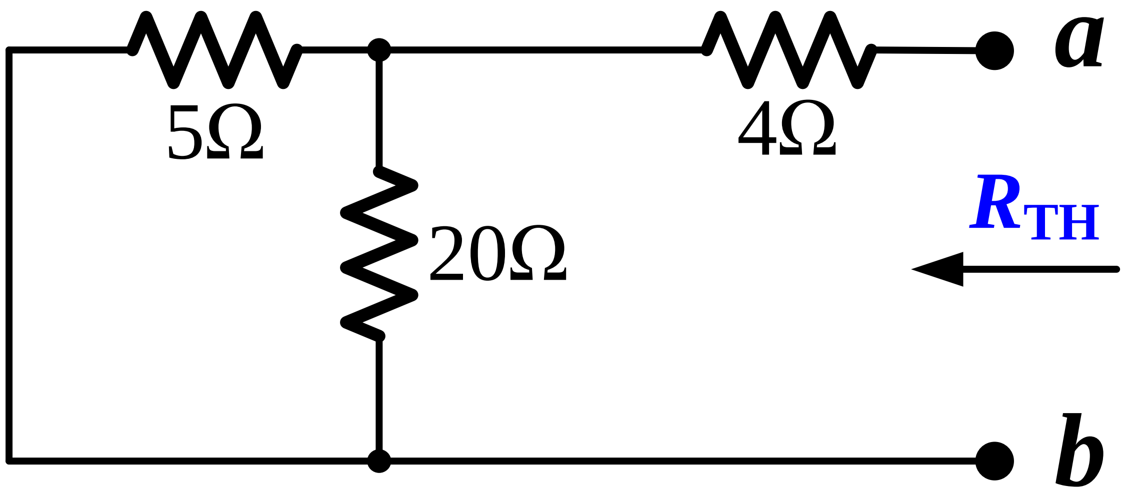
By node-voltage analysis method, we have:

$$\frac{v_{oc}-25}{5}+\frac{v_{oc}}{20}-3=0$$

$$v_{oc}=32\text{V}$$

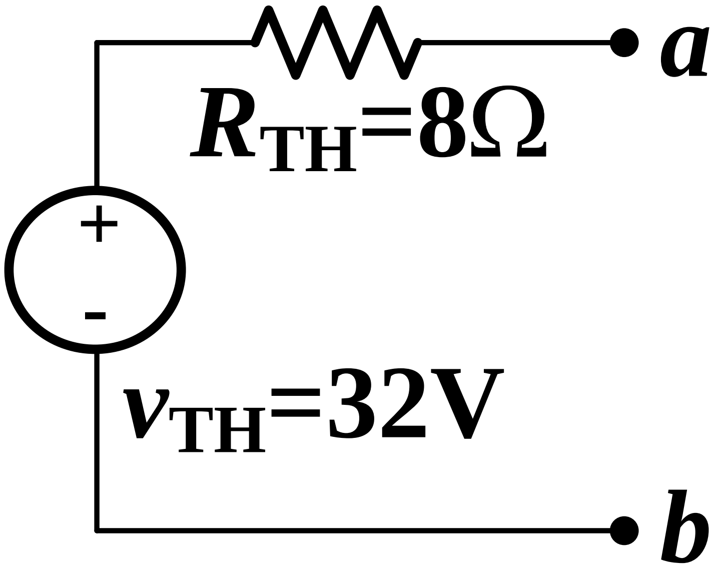
$$v_{TH}=v_{oc}=32\text{V}$$

Find the Thévenin equivalent resistance between two terminals a and b:

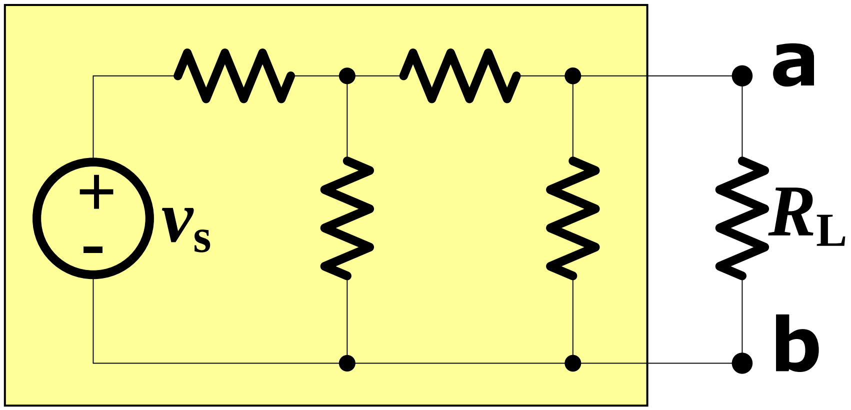


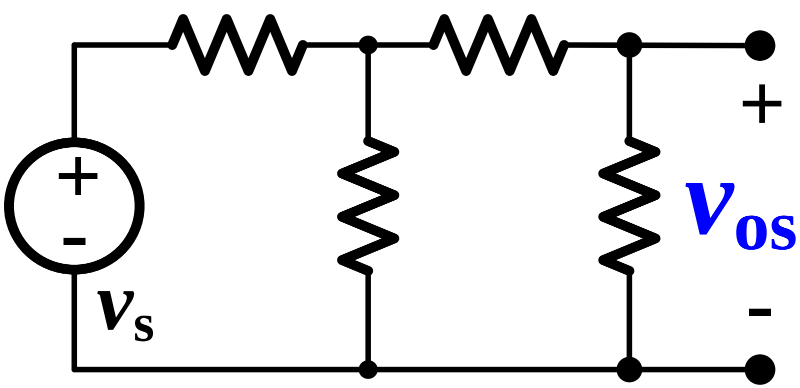
$$R_{TH} = 8\Omega$$

Then, the Thévenin equivalent of the circuit between terminals a and b is:

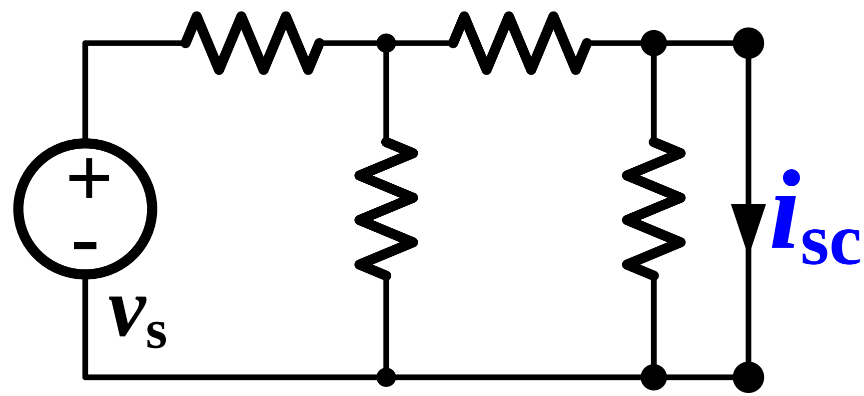


Thévenin Equivalent Circuit





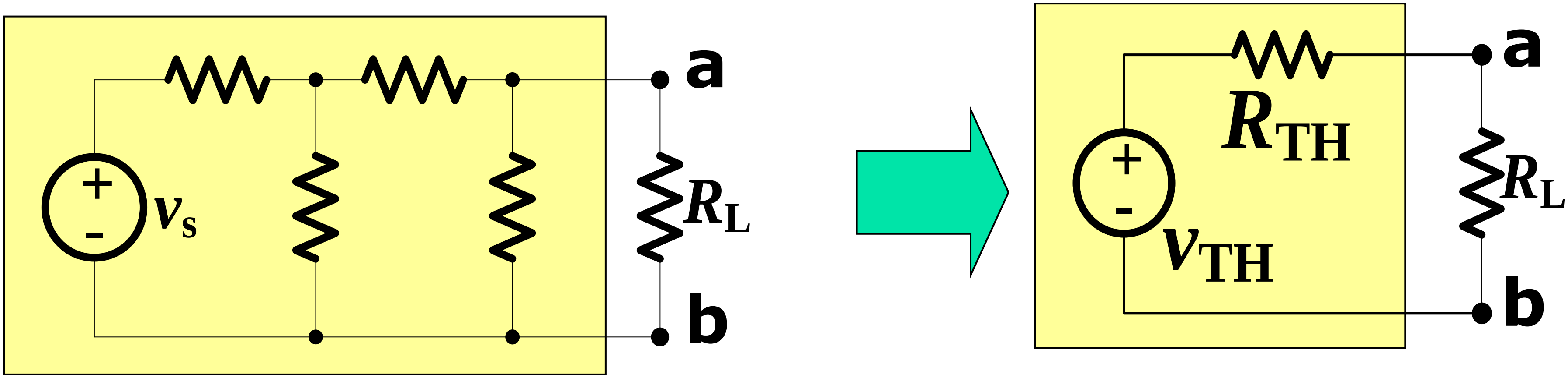
Open-Circuit Voltage



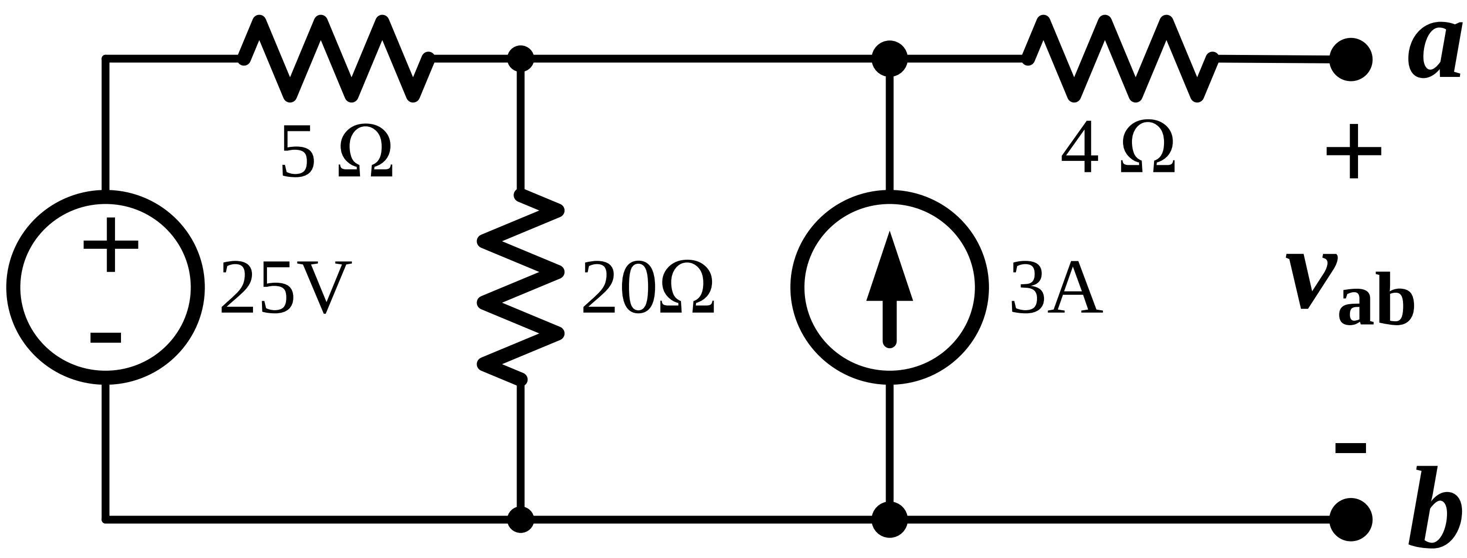
Short-Circuit Current

Thévenin Equivalent Circuit

$$\begin{cases} v_{TH} = v_{oc} \\ R_{TH} = v_{oc} / i_{sc} \end{cases}$$



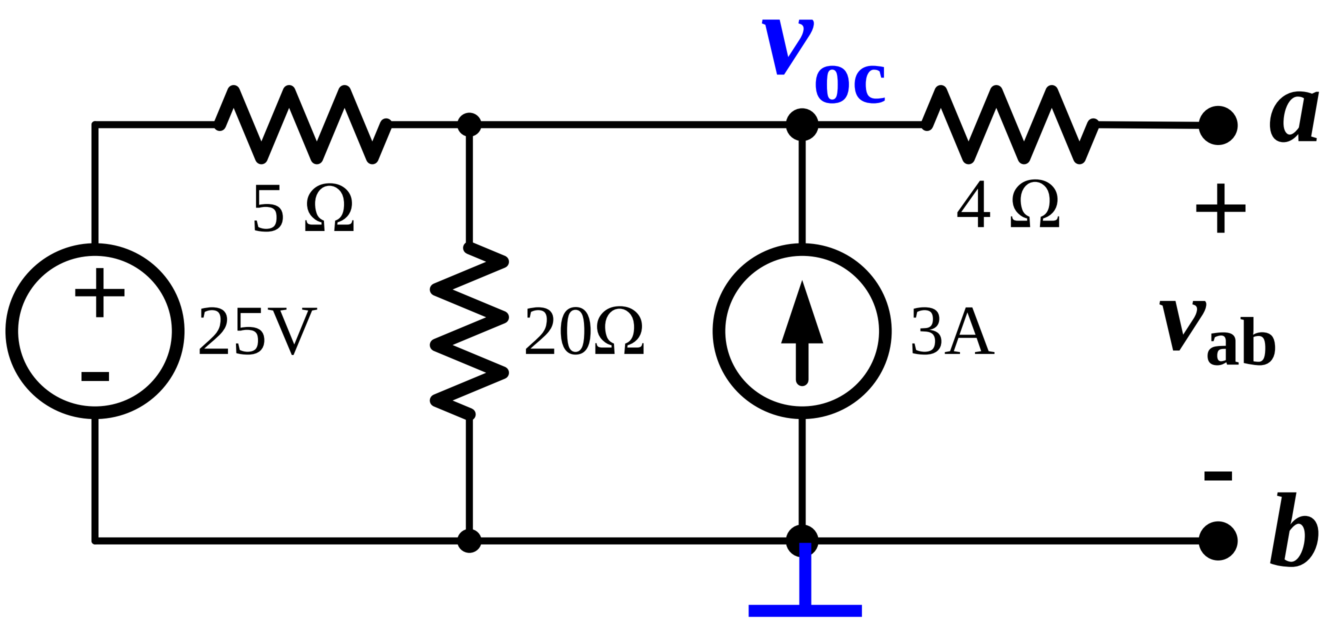
Example



Find the Thévenin equivalent of the circuit between terminals a and b.

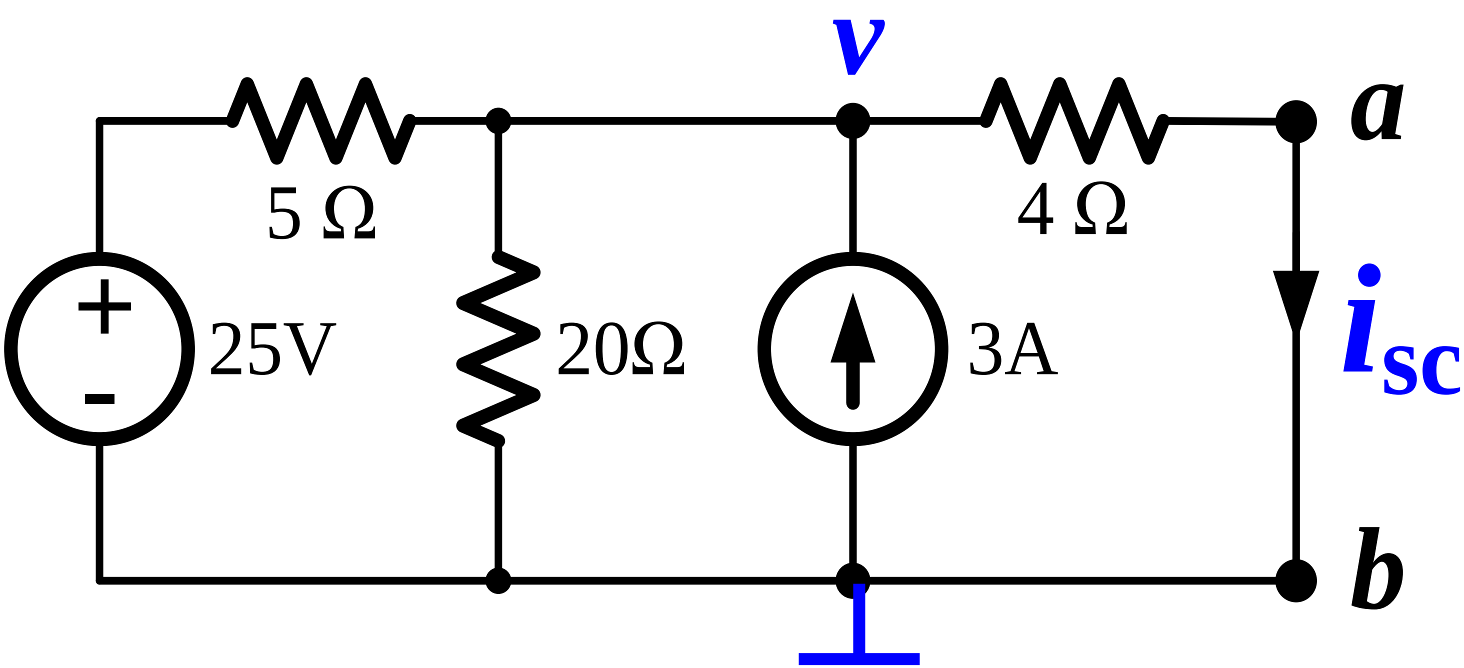
Solution:

Find the open circuit voltage between a and b:



$$v_{oc} = 32\text{V}$$

Find the short circuit current from a to b:



By node-voltage method, we have:

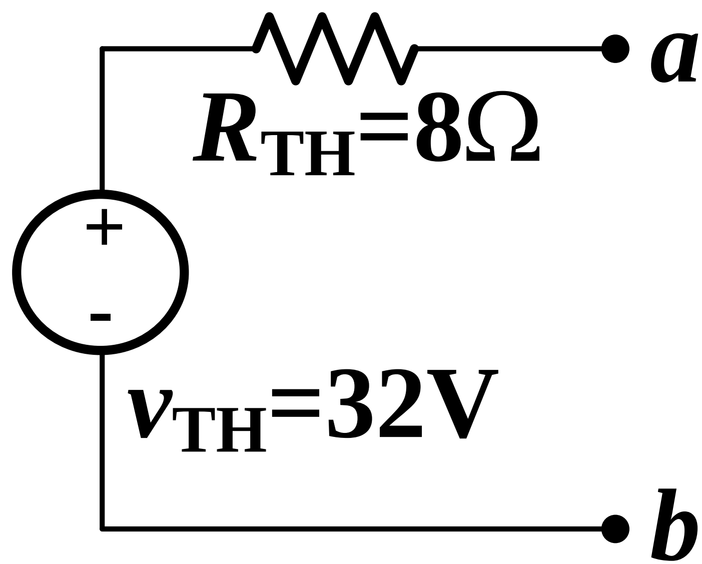
$$\frac{v-25}{5}+\frac{v}{20}-3+\frac{v}{4}=0$$

$$v=16\text{V}$$

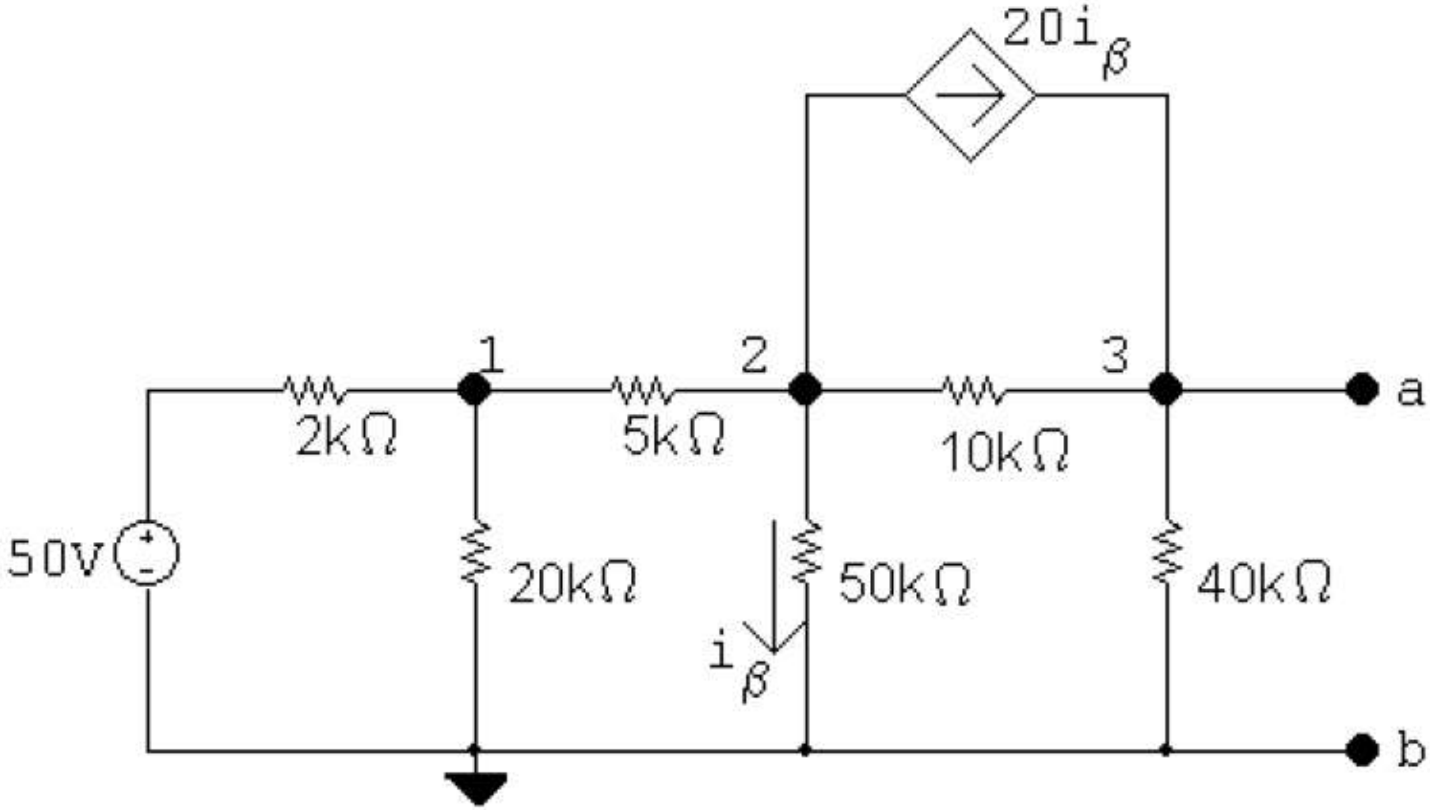
$$i_{sc}=16/4=4\text{A}$$

$$\begin{cases} v_{TH} = v_{oc} = 32\text{V} \\ R_{TH} = v_{oc}/i_{sc} = 32/4 = 8\Omega \end{cases}$$

Then, we have:



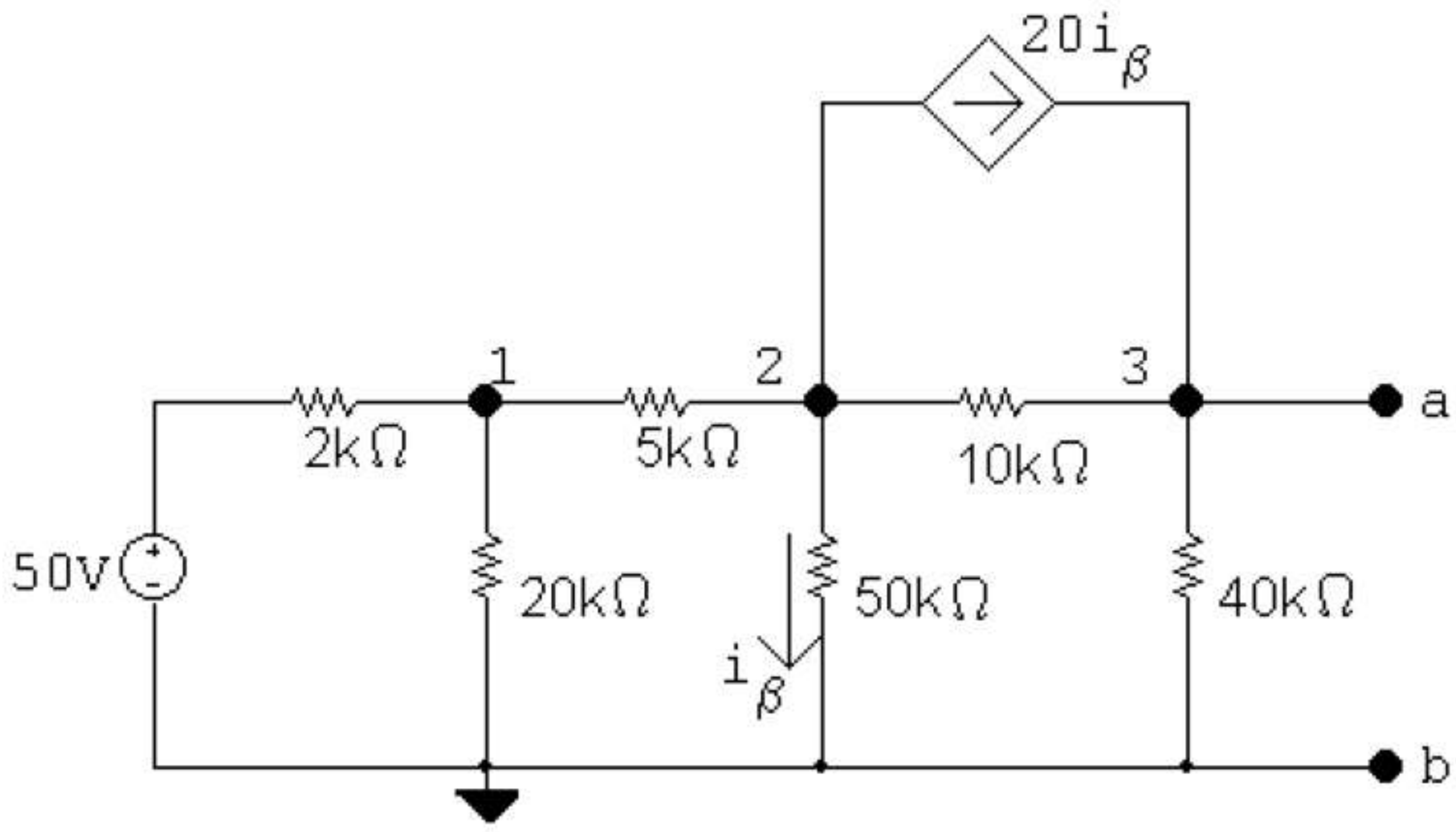
Example



Find the Thévenin equivalent with respect to the terminals of a and b.

Solution:

Find the open circuit voltage:

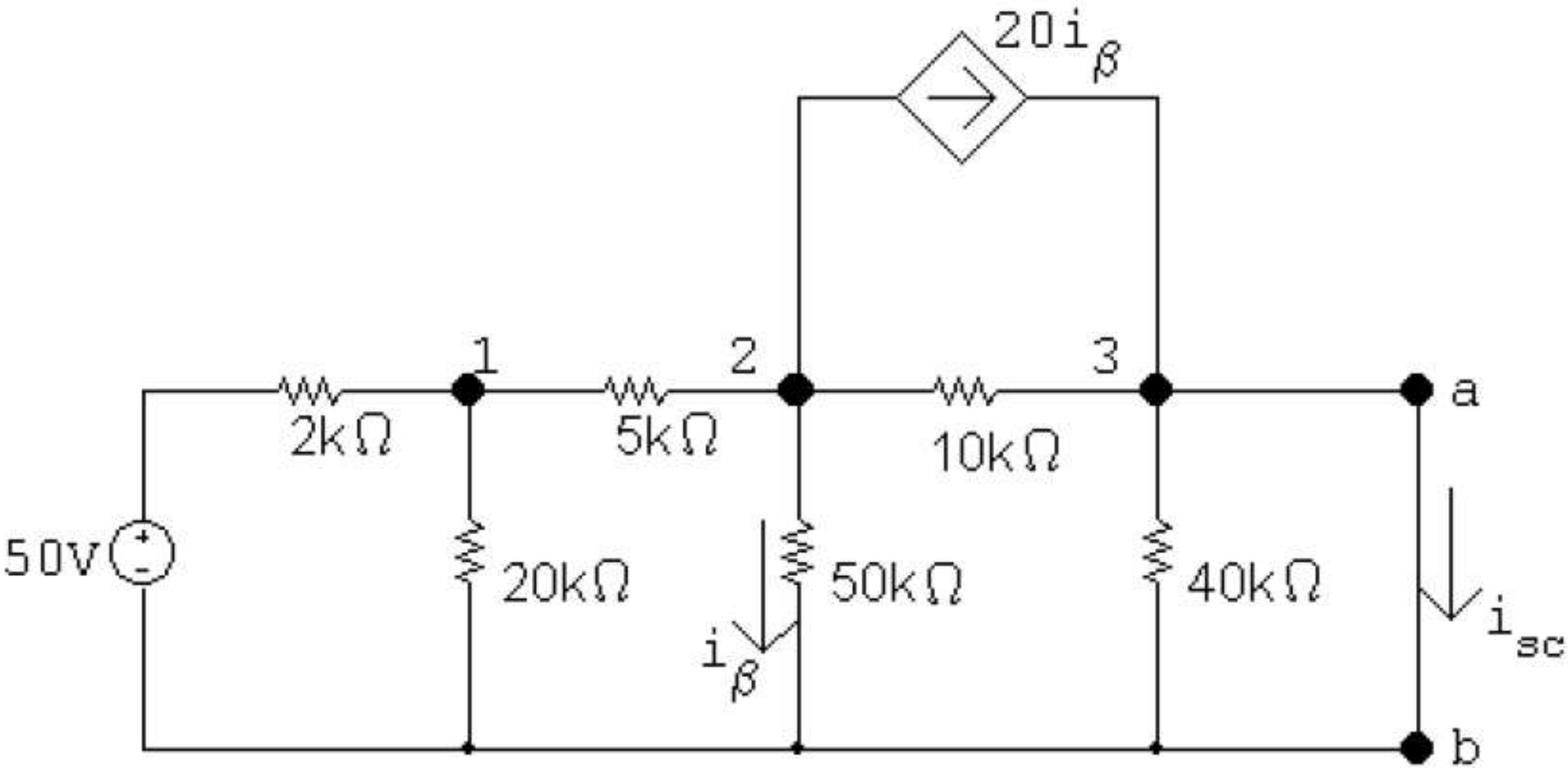


By node-voltage method, we have:

$$\begin{cases} \dfrac{v_1 - 50}{2} + \dfrac{v_1}{20} + \dfrac{v_1 - v_2}{5} = 0 \\ \dfrac{v_2 - v_1}{5} + \dfrac{v_2}{50} + \dfrac{v_2 - v_3}{10} + 20\dfrac{v_2}{50} = 0 \\ \dfrac{v_3}{40} + \dfrac{v_3 - v_2}{10} - 20\dfrac{v_2}{50} = 0 \end{cases}$$

$$\Rightarrow \quad v_3 = 100\text{V} = v_{oc}$$

Find short circuit current from a to b:

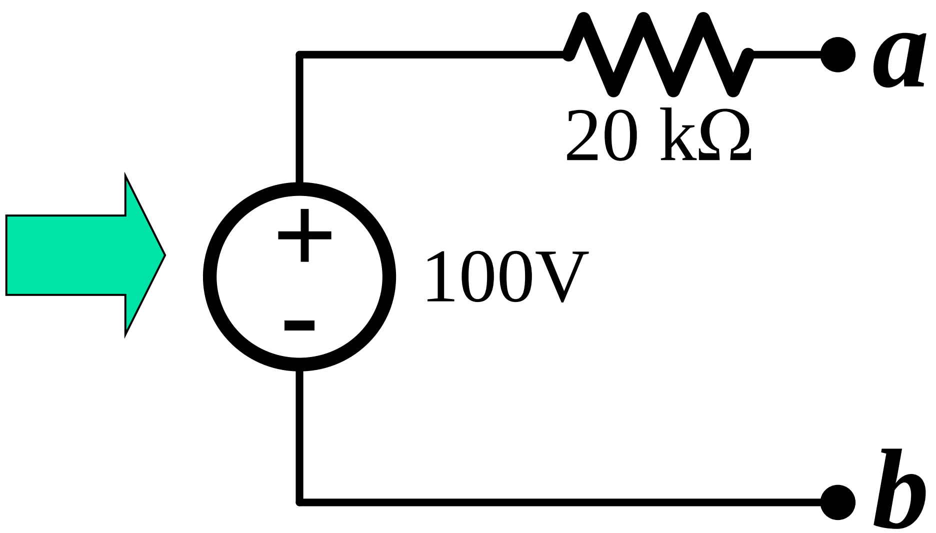


By node-voltage method, we have:

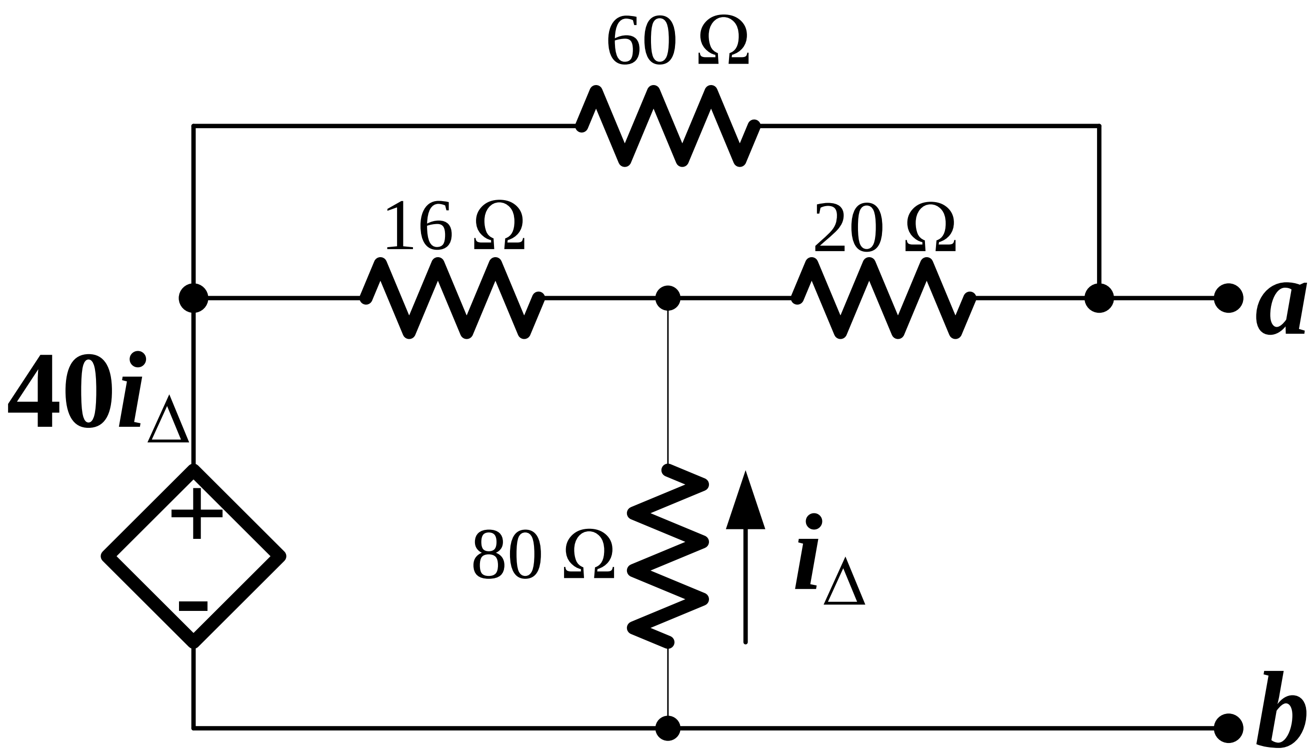
$$\begin{cases} \dfrac{v_1}{20} + \dfrac{v_1 - 50}{2} + \dfrac{v_1 - v_2}{5} = 0 \\ \dfrac{v_2 - v_1}{5} + \dfrac{v_2}{50} + \dfrac{v_2}{10} + 20\dfrac{v_2}{50} = 0 \end{cases} \Rightarrow \begin{cases} v_1 = 36\text{V} \\ v_2 = 10\text{V} \end{cases}$$

$$i_{sc} = \frac{v_2}{10} + 20\frac{v_2}{50} \Rightarrow i_{sc} = 5\text{mA}$$

$$\begin{cases} v_{TH} = v_{oc} = 100\text{V} \\ R_{TH} = \dfrac{v_{oc}}{i_{sc}} = \dfrac{100\text{V}}{5\text{mA}} = 20\text{k}\Omega \end{cases}$$



Example



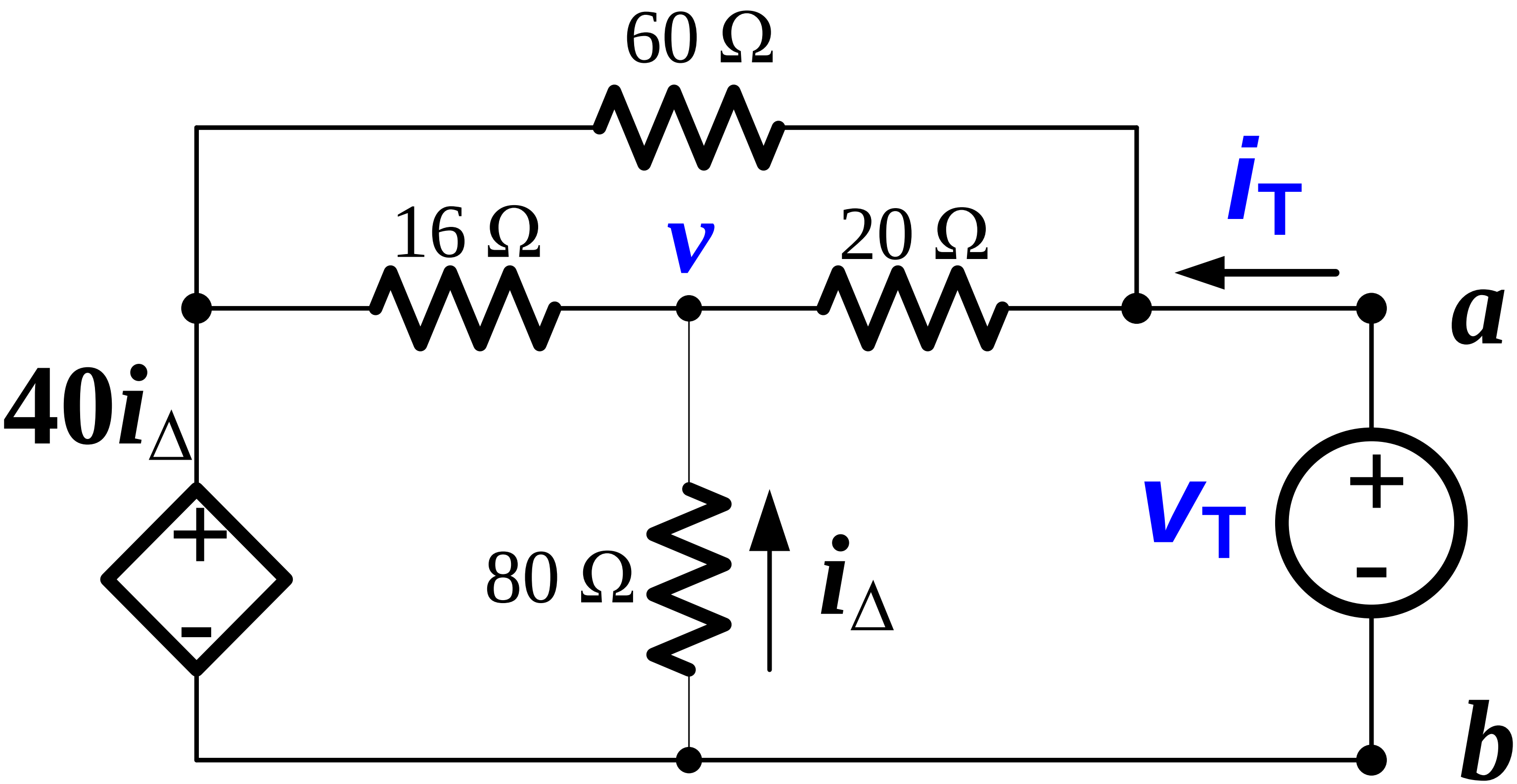
Find the Thévenin equivalent with respect to the terminals a, b.

Solution:

Since the circuit contains no independent sources,

$$\therefore \qquad v_{TH} = v_{oc} = 0\text{V}$$

Apply a test voltage source to terminal a and b:



By node voltage method, we have:

$$\begin{cases} i_T = \dfrac{v_T - v}{20} + \dfrac{v_T - 40i_\Delta}{60} \\ \dfrac{v - 40i_\Delta}{16} + \dfrac{v}{80} + \dfrac{v - v_T}{20} = 0 \\ i_\Delta = \dfrac{-v}{80} \end{cases}$$

$$\Rightarrow \frac{v_T}{i_T} = 18.75\Omega$$

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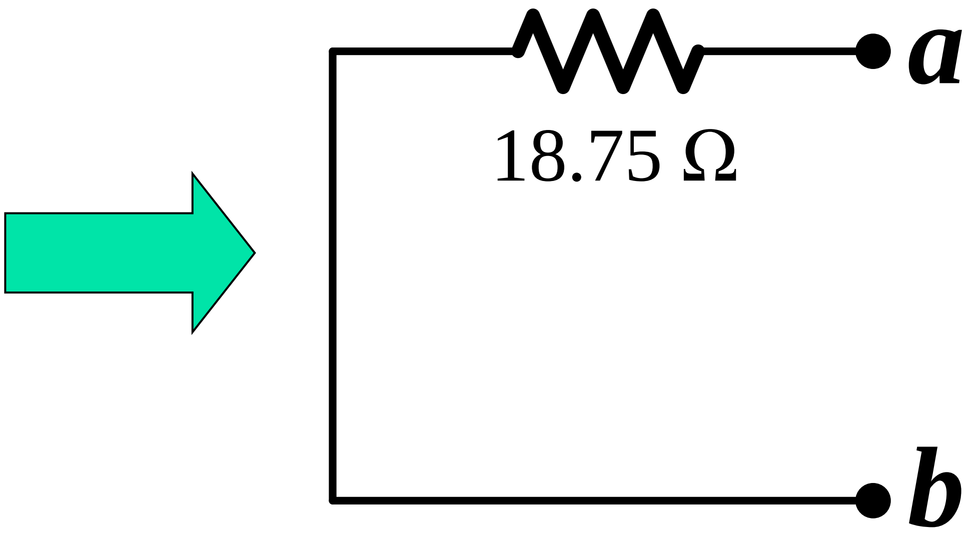
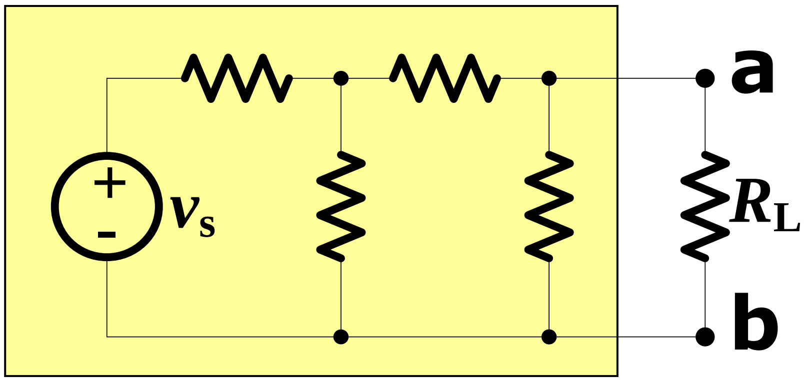
$$\begin{cases} v_{TH} = 0 \\ R_{TH} = \dfrac{v_T}{i_T} = 18.75\Omega \end{cases}$$

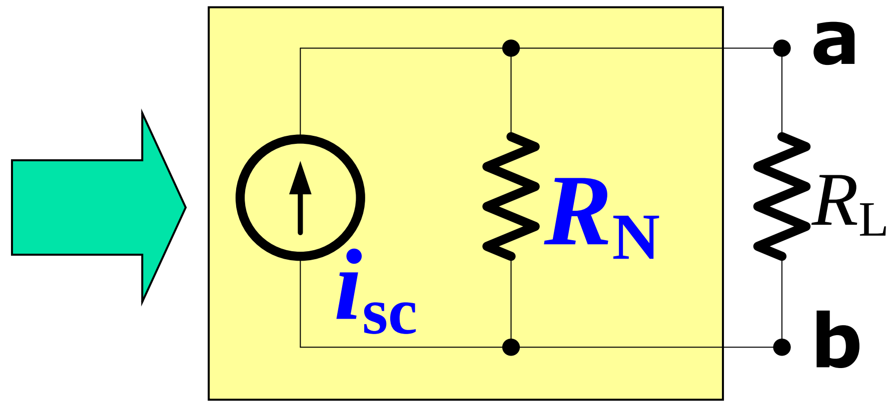


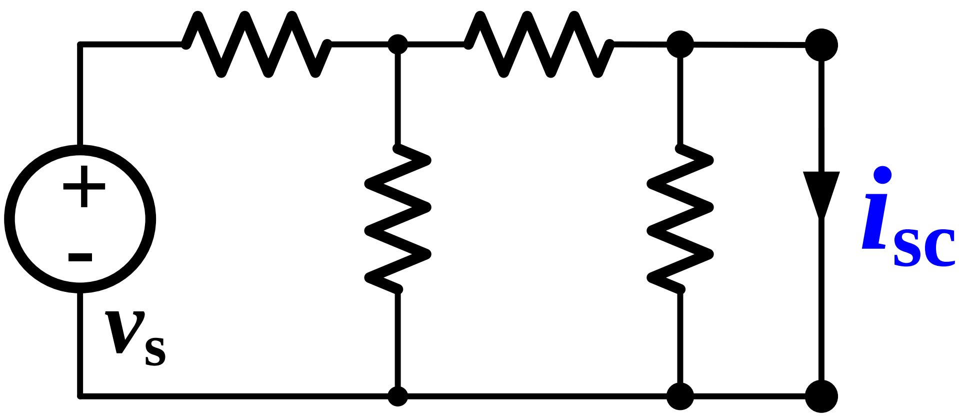
Norton Equivalent Circuit

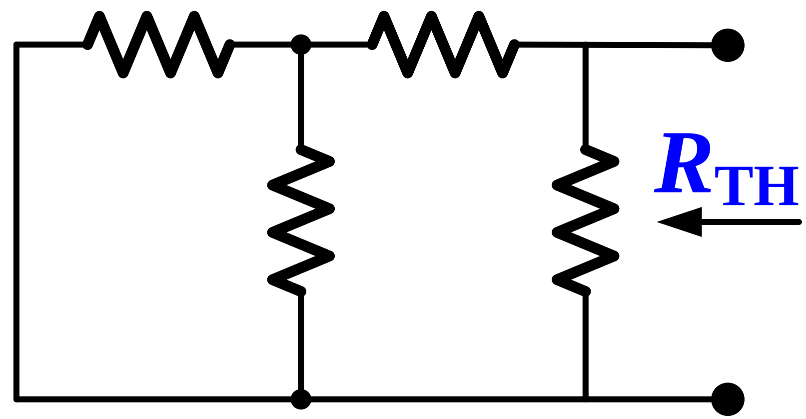
1. A network with two terminals can be equivalent to an independent current source i_{sc} in parallel with a resistor R_N;
2. i_{sc} is the short-circuit current when the two terminals are directly connected;
3. R_N is the equivalent resistance of the two-terminal network when all independent sources in it are zeroed out (killed).

Norton Equivalent Circuit

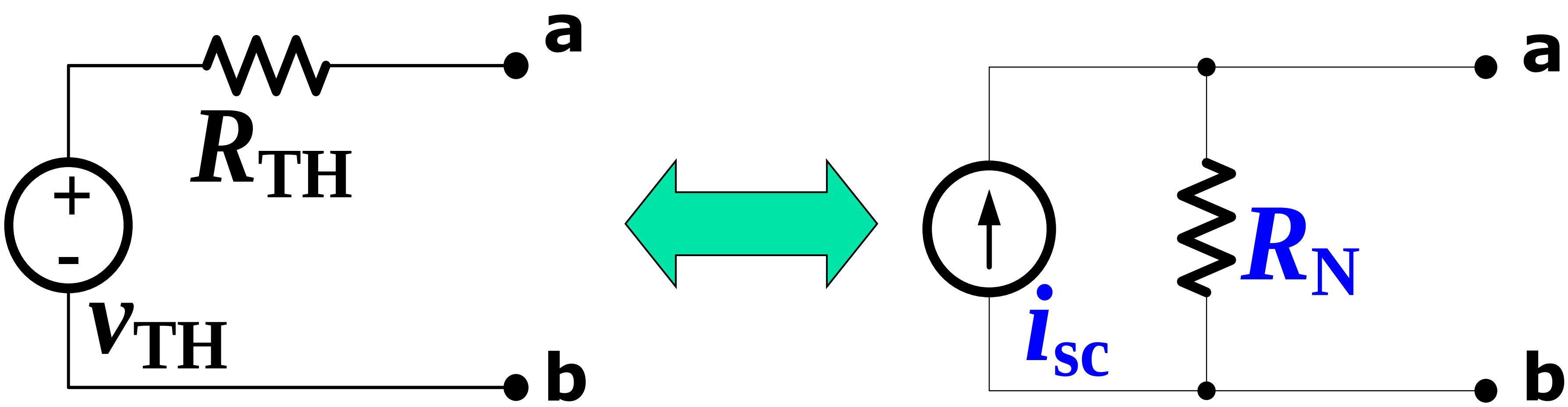








Norton Equivalent Circuit

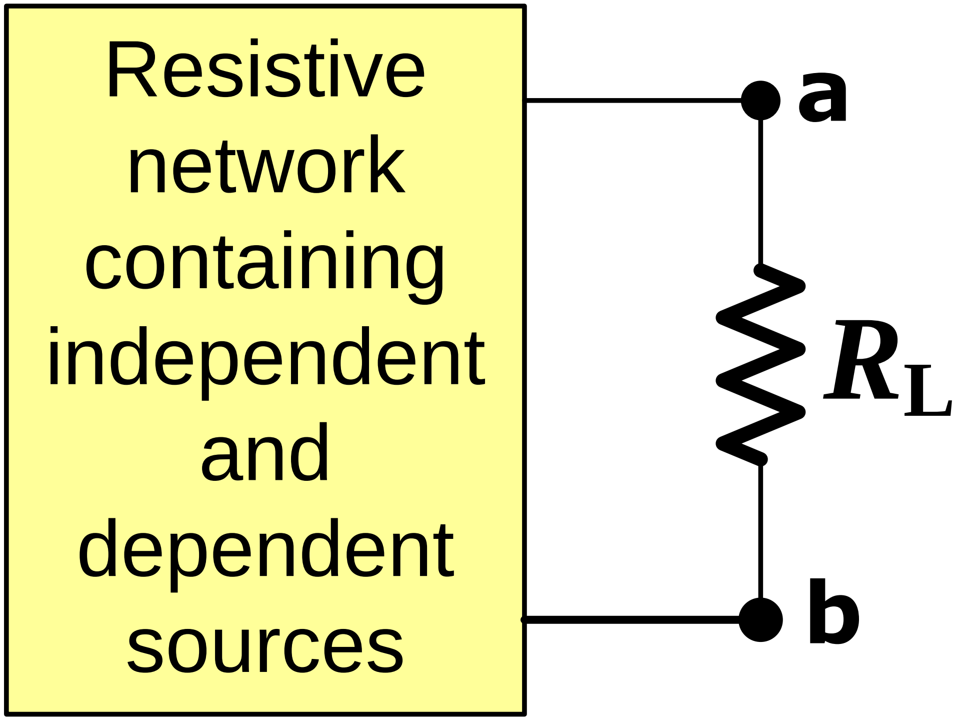


Norton equivalent can be derived from Thévenin equivalent by source transformation; and vice versa.

Thévenin and Norton Equivalents

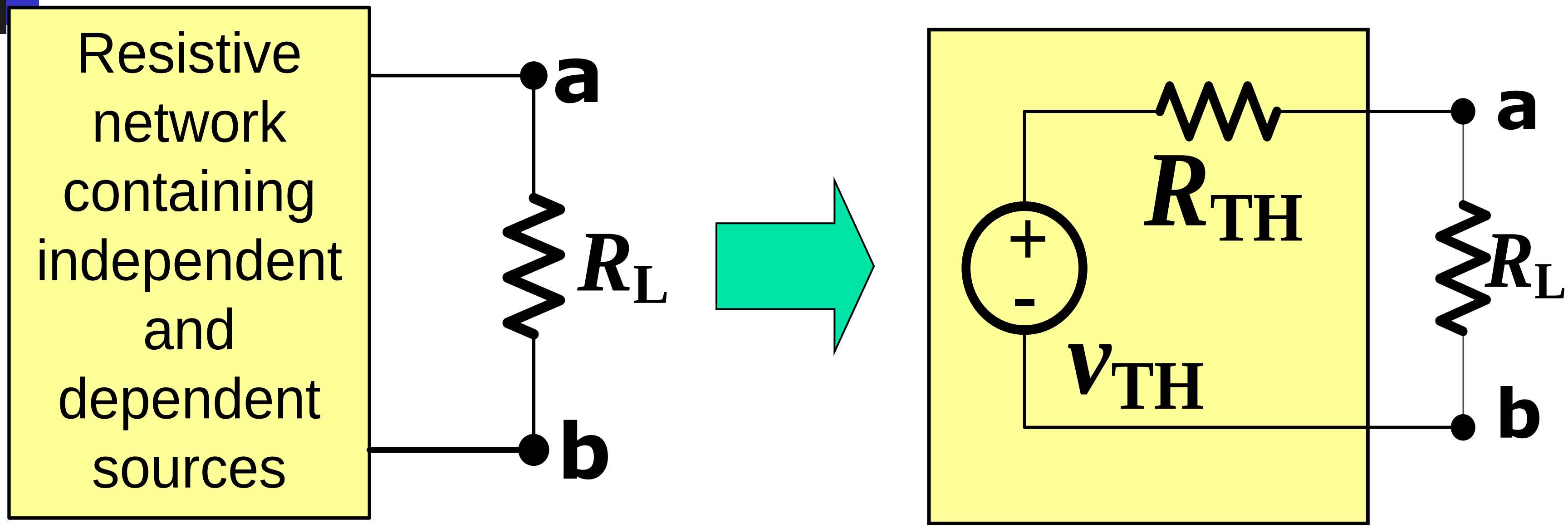
- Both Thévenin and Norton circuits provide an equivalent for a 2-terminal network, and the "load" network (R_L) would not "know" if it was "seeing" the equivalent;
- Thévenin equivalent can be derived from Norton equivalent by source transformation; and vice versa.

3-6 Maximum Power Transfer



Problem:

To determine the value of the load R_L that permits maximum power delivered to R_L.



Power delivered to R_L: $$p = \left(\frac{v_{TH}}{R_{TH} + R_L}\right)^2 R_L$$

The derivative of power *p* with respect to R_L:

$$\frac{dp}{dR_L} = \frac{R_{TH} - R_L}{\left(R_{TH} + R_L\right)^3} v_{TH}^2$$

Let $\frac{dp}{dR_L} = 0$, we get:

Maximum power transfer occurs

$$R_L = R_{TH}$$

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Maximum Power Transfer

- **Condition of maximum power transfer:**

$$R_L = R_{TH}$$

- **The maximum power delivered to R_L is:**

$$p_{max} = i^2 R_L = \left(\frac{v_{TH}}{R_{TH} + R_L} \right)^2 R_L = \frac{v_{TH}^2}{4R_L}$$

Summary of Chapter 3

- Node-voltage analysis method
- Mesh-current analysis method
- Superposition theorem
- Source transformation
- Thévenin and Norton Equivalent
- Maximum power transfer

Primary Goals of Chapter 3

By this chapter, you should be able to:

- **Understand and use node-voltage and mesh-current method for circuit analysis;**
- **Implement superposition and source transformation to solve a circuit;**

- Understand the concept of Thévenin and Norton equivalent circuit;
- Construct the Thévenin and Norton equivalent for a circuit;
- Find the condition for maximum power transfer to a resistive load.