5

The Natural and Step Response of RL and RC Circuits

Drill Exercises

DE 5.1 **[a]** $i_g = 8e^{-300t} - 8e^{-1200t}\,\text{A}$

$$v = L\frac{di_g}{dt} = -9.6e^{-300t} + 38.4e^{-1200t}\text{V}, \qquad t > 0^+$$

$$v(0^+) = -9.6 + 38.4 = 28.8\,\text{V}$$

[b] $v = 0$ when $38.4e^{-1200t} = 9.6e^{-300t}$ or $t = (\ln 4)/900 = 1.54\,\text{ms}$

[c] $p = vi = 384e^{-1500t} - 76.8e^{-600t} - 307.2e^{-2400t}\,\text{W}$

[d] $\dfrac{dp}{dt} = 0$ when $e^{1800t} - 12.5e^{900t} + 16 = 0$

Let $x = e^{900t}$ and solve the quadratic $x^2 - 12.5x + 16 = 0$

$$x = 1.45, \quad x = 11.05, \quad t = \frac{\ln 1.45}{900} = 411.05\,\mu\text{s}, \quad t = \frac{\ln 11.05}{900} = 2.67\,\text{ms}$$

p is maximum at $t = 411.05\,\mu\text{s}$

[e] $p_{\text{max}} = 384e^{-1.5(0.41105)} - 76.8e^{-0.6(0.41105)} - 307.2e^{-2.4(0.41105)} = 32.72\,\text{W}$

[f] $i_{\text{max}} = 8[e^{-0.3(1.54)} - e^{-1.2(1.54)}] = 3.78\,\text{A}$

$$w_{\text{max}} = (1/2)(4 \times 10^{-3})(3.78)^2 = 28.6\,\text{mJ}$$

[g] W is max when i is max, i is max when di/dt is zero.

When $di/dt = 0$, $v = 0$, therefore $t = 1.54\,\text{ms}$.

DE 5.2 **[a]** $i = C\dfrac{dv}{dt} = 24 \times 10^{-6}\dfrac{d}{dt}[e^{-15,000t}\sin 30,000t]$

$$= [0.72\cos 30,000t - 0.36\sin 30,000t]e^{-15,000t}\,\text{A}, \qquad i(0^+) = 0.72\,\text{A}$$

[b] $i\left(\frac{\pi}{80}\,\text{ms}\right) = -31.66\,\text{mA}, \quad v\left(\frac{\pi}{80}\,\text{ms}\right) = 20.505\,\text{V},$

$p = vi = -649.23\,\text{mW}$

[c] $w = \left(\frac{1}{2}\right)Cv^2 = 126.13\,\mu\text{J}$

DE 5.3 **[a]** $v = \left(\frac{1}{C}\right)\int_{0^-}^{t} i\,dx + v(0^-)$

$$= \frac{1}{0.6\times 10^{-6}}\int_{0^-}^{t} 3\cos 50{,}000x\,dx = 100\sin 50{,}000t\,\text{V}$$

[b] $p(t) = vi = [300\cos 50{,}000t]\sin 50{,}000t$

$$= 150\sin 100{,}000t\,\text{W}, \qquad p_{(\text{max})} = 150\,\text{W}$$

[c] $w_{(\text{max})} = \left(\frac{1}{2}\right)Cv_{\text{max}}^2 = 0.30(100)^2 = 3000\,\mu\text{J} = 3\,\text{mJ}$

DE 5.4 **[a]** $L_{\text{eq}} = \frac{60(240)}{300} = 48\,\text{mH}$

[b] $i(0^+) = 3 + -5 = -2\,\text{A}$

[c] $i = \frac{125}{6}\int_{0^+}^{t}(-0.03e^{-5x})\,dx - 2 = 0.125e^{-5t} - 2.125\,\text{A}$

[d] $i_1 = \frac{50}{3}\int_{0^+}^{t}(-0.03e^{-5x})\,dx + 3 = 0.1e^{-5t} + 2.9\,\text{A}$

$i_2 = \frac{25}{6}\int_{0^+}^{t}(-0.03e^{-5x})\,dx - 5 = 0.025e^{-5t} - 5.025\,\text{A}$

$i_1 + i_2 = i$

DE 5.5 $v_1 = 0.5\times 10^6\int_{0^+}^{t} 240\times 10^{-6}e^{-10x}\,dx - 10 = -12e^{-10t} + 2\,\text{V}$

$v_2 = 0.125\times 10^6\int_{0^+}^{t} 240\times 10^{-6}e^{-10x}\,dx - 5 = -3e^{-10t} - 2\,\text{V}$

$v_1(\infty) = 2\,\text{V}, \qquad v_2(\infty) = -2\,\text{V}$

$W = \left[\frac{1}{2}(2)(4) + \frac{1}{2}(8)(4)\right]\times 10^{-6} = 20\,\mu\text{J}$

DE 5.6 **[a]** $i = \left(\frac{120}{3+5}\right)\left(\frac{-30}{36}\right) = -12.5\,\text{A}$

[b] $w = 0.5(8\times 10^{-3})(12.5)^2 = 625\,\text{mJ}$

[c] $\tau = \frac{L}{R} = \frac{8\times 10^{-3}}{2} = 4\,\text{ms}$

[d] $i = -12.5e^{-250t}\,\text{A}, \qquad t \geq 0$

[e] $i(5\,\text{ms}) = -3.58\,\text{A}, \qquad w\,(5\,\text{ms}) = (0.5)(8) \times 10^{-3}(3.58)^2 = 51.3\,\text{mJ}$

$$w\,(\text{dis}) = 625 - 51.3 = 573.7\,\text{mJ}$$

$$\%\ \text{dissipated} = \left(\frac{573.7}{625}\right)100 = 91.8\%$$

DE 5.7 [a] $i_L(0^-) = 6.4\left(\frac{10}{16}\right) = 4\,\text{A} = i_L(0^+), \qquad t > 0$

$$R_{\text{eq}} = \frac{(4)(16)}{20} = 3.2\,\Omega, \qquad \tau = \frac{0.32}{3.2} = 0.1\,\text{s}$$

$$\text{Therefore} \quad \frac{1}{\tau} = 10, \qquad i_L = 4e^{-10t}\,\text{A}$$

Let i_1 equal the current in the $10\,\Omega$ resistor. Let the reference direction for i_1 be up. Then

$$i_1 = \left(\frac{4}{20}\right)i_L = 0.8e^{-10t}\,\text{A}, \qquad v_o = -10i_1 = -8e^{-10t}\,\text{V}, \qquad t \geq 0^+$$

[b] $v_{4\Omega} = L\frac{di_L}{dt} = 0.32(-40)e^{-10t} = -12.8e^{-10t}\,\text{V}, \qquad t \geq 0^+$

$$p_{4\Omega} = \frac{v_{4\Omega}^2}{4} = 40.96e^{-20t}\,\text{W}, \qquad t \geq 0^+$$

$$w_{4\Omega} = \int_0^\infty 40.96e^{-20t}dt = 2.048\,\text{J}$$

$$w_i = \frac{1}{2}Li^2 = \frac{1}{2}(0.32)(16) = 2.56\,\text{J}$$

$$\%\ \text{dissipated} = \left(\frac{2.048}{2.56}\right)100 = 80\%$$

DE 5.8 [a] $v(0) = \left[\frac{7.5(80)}{150}\right]50 = 200\,\text{V}$

[b] $\tau = RC = (50 \times 10^3)(0.4 \times 10^{-6}) = 20\,\text{ms}$

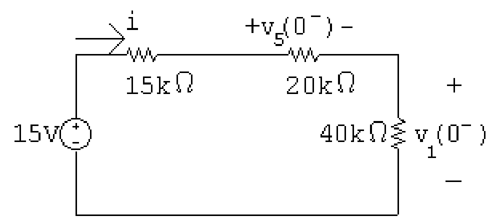
[c] $v = 200e^{-50t}\,\text{V}$

[d] $w(0) = 0.5(0.4 \times 10^{-6})(200)^2 = 8\,\text{mJ}$

[e] $w(t) = 0.5(0.4 \times 10^{-6})(4 \times 10^4)e^{-100t} = 8e^{-100t}\,\text{mJ}$

$$8e^{-100t} = 2, \qquad t = (\ln 4)/100 = 13.86\,\text{ms}$$

DE 5.9 **[a]** For $t < 0$:



$$i = \frac{15}{75{,}000} = \frac{1}{5}\,\text{mA}, \qquad v_5(0^-) = 4\,\text{V}, \qquad v_1(0^-) = 8\,\text{V}$$

$$\tau_5 = (20 \times 10^3)(5 \times 10^{-6}) = 100\,\text{ms}, \qquad 1/\tau_5 = 10$$

$$\tau_1 = (40 \times 10^3)(1 \times 10^{-6}) = 40\,\text{ms}, \qquad 1/\tau_1 = 25$$

Therefore $v_5 = 4e^{-10t}\,\text{V}, \quad t \geq 0; \qquad v_1 = 8e^{-25t}\,\text{V}, \quad t \geq 0;$

$$v_o = v_1 + v_5 = [8e^{-25t} + 4e^{-10t}]\,\text{V}, \qquad t \geq 0$$

[b] $v_1(60\,\text{ms}) \cong 1.79\,\text{V}, \qquad v_5(60\,\text{ms}) \cong 2.20\,\text{V}$

$$w_1(60\,\text{ms}) = (1/2)(1)(1.79)^2 \cong 1.59\,\mu\text{J}$$

$$w_5(60\,\text{ms}) = (1/2)(5)(2.20)^2 \cong 12.05\,\mu\text{J}$$

$$w_1(0) = \frac{1}{2}(10^{-6})(64) + \frac{1}{2}(5 \times 10^{-6})(16) = 72\,\mu\text{J}$$

$$w_{\text{diss}} = 72 - 13.64 = 58.36\,\mu\text{J}$$

% dissipated $= (58.36/72)(100) = 81.05\,\%$

DE 5.10 **[a]** $i(0^+) = 24/2 = 12\,\text{A}$

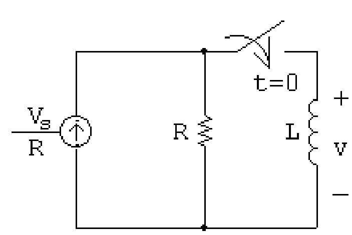
[b] $v(0^+) = -10(8 + 12) = -200\,\text{V}$

[c] $\tau = L/R = (200/10) \times 10^{-3} = 20\,\text{ms}$

[d] $i = -8 + [12 - (-8)]e^{-50t} = [-8 + 20e^{-50t}]\,\text{A}, \qquad t \geq 0^+$

[e] $v = 0 + [-200 - 0]e^{-50t}\,\text{V} = -200e^{-50t}\,\text{V}, \qquad t \geq 0^+$

DE 5.11 **[a]**



$$\frac{v}{R} + \frac{1}{L}\int_0^t v\,dx = \frac{V_s}{R}$$

$$\frac{1}{R}\frac{dv}{dt} + \frac{v}{L} = 0$$

$$\frac{dv}{dt} + \frac{R}{L}v = 0$$

[b] $$\frac{dv}{dt} = -\frac{R}{L}v$$

$$\frac{dv}{dt}\,dt = -\frac{R}{L}v\,dt$$

$$\therefore \quad \frac{dv}{v} = -\frac{R}{L}\,dt$$

$$\int_{v(0^+)}^{v(t)} \frac{dy}{y} = -\frac{R}{L}\int_{0^+}^{t} dx$$

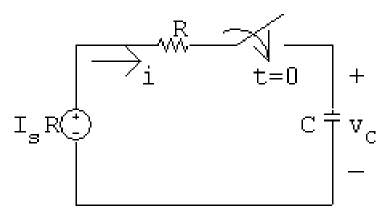
$$\ln y \,\Big|_{v(0^+)}^{v(t)} = -\left(\frac{R}{L}\right)t$$

$$\ln\left[\frac{v(t)}{v(0^+)}\right] = -\left(\frac{R}{L}\right)t$$

$$v(t) = v(0^+)e^{-(R/L)t}; \qquad v(0^+) = \left(\frac{V_s}{R} - I_o\right)R = V_s - I_oR$$

$$\therefore \quad v(t) = (V_s - I_oR)e^{-(R/L)t}$$

DE 5.12 **[a]**



$$I_sR = Ri + \frac{1}{C}\int_{0^+}^{t} i\,dx + V_o$$

$$0 = R\frac{di}{dt} + \frac{i}{C} + 0$$

$$\therefore \quad \frac{di}{dt} + \frac{i}{RC} = 0$$

[b] $$\frac{di}{dt} = -\frac{i}{RC}; \qquad \frac{di}{i} = -\frac{dt}{RC}$$

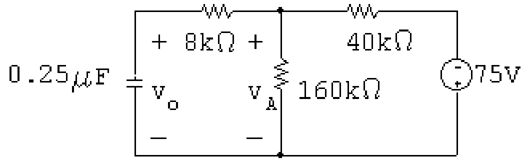
$$\int_{i(0^+)}^{i(t)} \frac{dy}{y} = -\frac{1}{RC}\int_{0^+}^{t} dx$$

$$\ln \frac{i(t)}{i(0^+)} = \frac{-t}{RC}$$

$$i(t) = i(0^+)e^{-t/RC}; \qquad i(0^+) = \frac{I_sR - V_o}{R} = \left(I_s - \frac{V_o}{R}\right)$$

$$\therefore \quad i(t) = \left(I_s - \frac{V_o}{R}\right) e^{-t/RC}$$

DE 5.13 **[a]**



$$v_o = -60 + 90e^{-100t}\,\text{V}$$

$$\frac{v_A - v_o}{8000} + \frac{v_A}{160{,}000} + \frac{v_A + 75}{40{,}000} = 0$$

$$20v_A - 20v_o + v_A + 4v_A + 300 = 0$$

$$25v_A = 20v_o - 300$$

$$v_A = 0.8v_o - 12$$

$$v_A = -48 + 72e^{-100t} - 12 = -60 + 72e^{-100t}\,\text{V}, \qquad t \geq 0^+$$

[b] $t \geq 0^+$

DE 5.14 **[a]** $v_c(0^+) = 50\,\text{V}$

[b] $v_c(\infty) = \left(-\dfrac{30}{25}\right) 20 = -24\,\text{V}$

[c] Find the Thévenin equivalent with respect to the terminals of the capacitor:

$$v_{\text{Th}} = -24\,\text{V}, \qquad R_{\text{Th}} = 20\|5 = 4\,\Omega,$$

Therefore $\quad \tau = 4(25 \times 10^{-9}) = 0.1\,\mu\text{s}$

[d] $i(0^+) = -\dfrac{50 + 24}{4} = -18.5\,\text{A}$

[e] $v_c = -24 + [50 - (-24)]e^{-t/\tau} = -24 + 74e^{-10^7t}\,\text{V}, \qquad t \geq 0$

[f] $i = -18.5e^{-t/\tau} = -18.5e^{-10^7t}\,\text{A}, \qquad t \geq 0^+$

DE 5.15 **[a]** $v_c(0^+) = (9/12)(120) = 90\,\text{V}$

[b] $v_c(\infty) = -1.5(40) = -60\,\text{V}$

[c] Find the Thévenin equivalent with respect to the terminals of the capacitor:

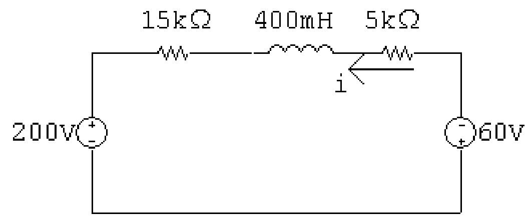
$$v_{\rm Th} = -60\,{\rm V}, \qquad R_{\rm Th} = 50\,{\rm k}\Omega$$

$$\tau = R_{\rm Th}C = 1\,{\rm ms} \;= 1000\,\mu{\rm s}$$

[d] $v_c = -60 + (90+60)e^{-1000t} = -60 + 150e^{-1000t}\,{\rm V}, \quad t \geq 0$

$$\text{Therefore} \quad t = \frac{\ln(150/60)}{1000} = 916.3\,\mu{\rm s}$$

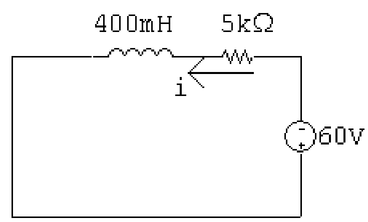
DE 5.16 **[a]** For $t < 0$, calculate the Thévenin equivalent for the circuit to the left and right of the 400-mH inductor. We get



$$i(0^-) = -260/20 = -13\,{\rm mA}$$

$$i(0^-) = i(0^+) = -13\,{\rm mA}$$

[b] For $t > 0$, the circuit reduces to



$$\text{Therefore} \quad i(\infty) = -60/5 = -12\,{\rm mA}$$

[c] $\tau = (400/5) \times 10^{-6} = 80\,\mu{\rm s}$

[d] $i(t) = -12 + [-13+12]e^{-12{,}500t} = -12 - e^{-12{,}500t}\,{\rm mA}, \qquad t \geq 0$

Problems

P 5.1 $p = vi = 40t[e^{-10t} - 10te^{-20t} - e^{-20t}]$

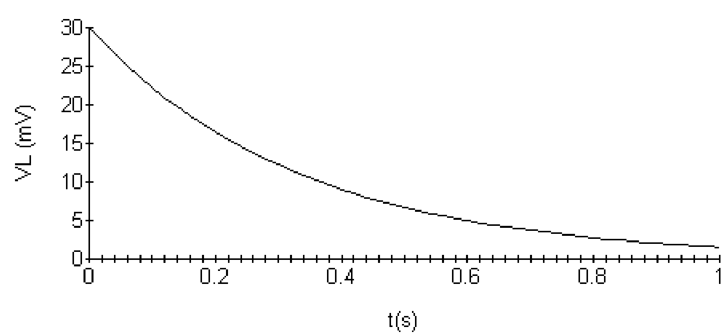
$$W = \int_0^\infty p\,dx = \int_0^\infty 40x[e^{-10x} - 10xe^{-20x} - e^{-20x}]\,dx = 0.2\,\text{J}$$

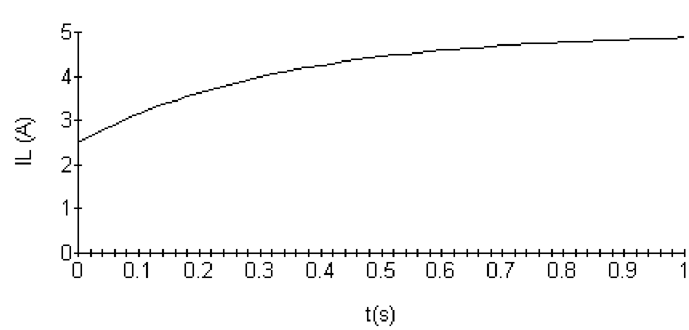
This is energy stored in the inductor at $t = \infty$.

P 5.2 $0 \le t < \infty$

$$i_L = \frac{10^3}{4}\int_0^t 30 \times 10^{-3} e^{-3x}\,dx + 2.5 = 7.5\frac{e^{-3x}}{-3}\Big|_0^t + 2.5$$

$$= 5 - 2.5e^{-3t}\,\text{A}, \qquad 0 \le t \le \infty$$





P 5.3 **[a]** $v = L\dfrac{di}{dt}$

$$\frac{di}{dt} = 50[t(-10e^{-10t}) + e^{-10t}] = 50e^{-10t}(1 - 10t)$$

$$v = (2 \times 10^{-3})(50)e^{-10t}(1 - 10t)$$

$$= 100e^{-10t}(1 - 10t)\,\text{mV}, \quad t > 0$$

[b] $p = vi$

$$v(200\,\text{ms}) = 100e^{-2}(1 - 2) = -13.53\,\text{mV}$$

$$i(200\,\text{ms}) = 50(0.2)e^{-2} = 1.35\,\text{A}$$

$$p(200\,\text{ms}) = -13.53 \times 10^{-3}(1.35) = -18.32\,\text{mW}$$

[c] delivering

[d] $w = \frac{1}{2}Li^2 = \frac{1}{2}(2 \times 10^{-3})(1.35)^2 = 1.83\,\text{mJ}$

[e] $\frac{di}{dt} = 0 \quad \text{when} \quad t = \frac{1}{10}\,\text{s} = 100\,\text{ms}$

$$i_{\max} = 50(0.1)e^{-1} = 1.84\,\text{A}$$

$$w_{\max} = \frac{1}{2}(2 \times 10^{-3})(1.84)^2 = 3.38\,\text{mJ}$$

P 5.4 **[a]** $0 \le t \le 1\,\text{ms}:$

$$i = \frac{1}{L}\int_0^t v_s\,dx + i(0) = \frac{10^6}{300}\int_0^t 6 \times 10^{-3}\,dx + 0$$

$$= 20x \Big|_0^t = 20t\,\text{A}$$

$1\,\text{ms} \le t \le 2\,\text{ms}:$

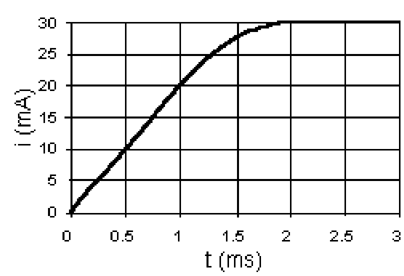
$$i = \frac{10^6}{300}\int_{10^{-3}}^t (12 \times 10^{-3} - 6x)\,dx + 20 \times 10^{-3}$$

$$\therefore \quad i = 40t - 10{,}000t^2 - 10 \times 10^{-3}\,\text{A}$$

$2\,\text{ms} \le t \le \infty:$

$$i = \frac{10^6}{300}\int_{2\times10^{-3}}^t (0)\,dx + 30 \times 10^{-3} = 30\,\text{mA}$$

[b]



P 5.5 **[a]**

$$\begin{aligned} i &= 0 && t < 0 \\ i &= 16t\,\text{A} && 0 \leq t \leq 25\,\text{ms} \\ i &= 0.8 - 16t\,\text{A} && 25 \leq t \leq 50\,\text{ms} \\ i &= 0 && 50\,\text{ms} < t \end{aligned}$$

[b] $v = L\dfrac{di}{dt} = 375 \times 10^{-3}(16) = 6\,\text{V} \qquad 0 \leq t \leq 25\,\text{ms}$

$v = 375 \times 10^{-3}(-16) = -6\,\text{V} \qquad 25 \leq t \leq 50\,\text{ms}$

$$\begin{aligned} v &= 0 && t < 0 \\ v &= 6\,\text{V} && 0 < t < 25\,\text{ms} \\ v &= -6\,\text{V} && 25 < t < 50\,\text{ms} \\ v &= 0 && 50\,\text{ms} < t \end{aligned}$$

$p = vi$

$$\begin{aligned} p &= 0 && t < 0 \\ p &= (16t)(6) = 96t\,\text{W} && 0 < t < 25\,\text{ms} \\ p &= (0.8 - 16t)(-6) = 96t - 4.8\,\text{W} && 25 < t < 50\,\text{ms} \\ p &= 0 && 50\,\text{ms} < t \end{aligned}$$

$$\begin{aligned} w &= 0 && t < 0 \\ w &= \int_0^t (16x)6\,dx = 96\frac{x^2}{2}\Big|_0^t = 48t^2\,\text{J} && 0 < t < 25\,\text{ms} \\ w &= \int_{0.025}^t (96x - 4.8)\,dx + 0.03 \\ &= \int_{0.025}^t 96x\,dx - \int_{0.025}^t 4.8\,dx + 0.03 \\ &= 96\frac{x^2}{2}\Big|_{0.025}^t - 4.8x\Big|_{0.025}^t + 0.03 \\ &= 48t^2 - 4.8t + 0.12\,\text{J} && 25 < t < 50\,\text{ms} \\ w &= 0 && 50\,\text{ms} < t \end{aligned}$$

P 5.6 **[a]** $0 \leq t \leq 1\,\text{s}:$

$v = -100t$

$i = \dfrac{1}{5}\displaystyle\int_0^t -100x\,dx + 0 = -20\frac{x^2}{2}\Big|_0^t$

$i = -10t^2$ A

$1\,\text{s} \le t \le 3\,\text{s}:$

$v = -200 + 100t$

$i(1) = -10$ A

$$\therefore\quad i = \frac{1}{5}\int_1^t (100x - 200)\,dx - 10$$
$$= 20\int_1^t x\,dx - 40\int_1^t dx - 10$$
$$= 10(t^2 - 1) - 40(t - 1) - 10$$
$$= 10t^2 - 40t + 20\ \text{A}$$

$3\,\text{s} \le t \le 5\,\text{s}:$

$v = 100$

$i(3) = 10(9) - 120 + 20 = -10$ A

$$i = \frac{1}{5}\int_3^t 100\,dx - 10$$
$$= 20t - 60 - 10 = 20t - 70\ \text{A}$$

$5\,\text{s} \le t \le 6\,\text{s}:$

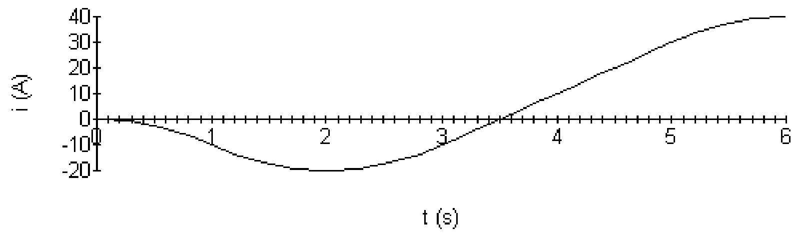
$v = -100t + 600$

$i(5) = 100 - 70 = 30$

$$i = \frac{1}{5}\int_5^t (-100x + 600)\,dx + 30$$
$$= -20\int_5^t x\,dx + 120\int_5^t dx + 30$$
$$= -10(t^2 - 25) + 120(t - 5) + 30$$
$$= -10t^2 + 120t - 320\ \text{A}$$

[b] $i(6) = -10(36) + 120(6) - 320 = 720 - 680 = 40\ \text{A}, \qquad 6 \le t \le \infty$

[c]



P 5.7 [a] $i(0) = A_1 + A_2 = 0.05$

$$\frac{di}{dt} = -2500A_1e^{-2500t} - 7500A_2e^{-7500t}$$

$$v = -50A_1e^{-2500t} - 150A_2e^{-7500t}\,\text{V}$$

$$v(0) = -50A_1 - 150A_2 = 10$$

$$\therefore \quad -5A_1 - 15A_2 = 1$$

But from the equation for $i(0)$, $5A_1 + 5A_2 = 0.25$

Solving, $A_1 = 0.175$ and $A_2 = -0.125$

Thus,

$$i = 0.175e^{-2500t} - 0.125e^{-7500t}\,\text{A}, \qquad t \geq 0$$

$$v = -8.75e^{-2500t} + 18.75e^{-7500t}\,\text{V}, \qquad t \geq 0$$

[b] $p = vi = 4.375e^{-10,000t} - 1.53125e^{-5000t} - 2.34375e^{-15,000t}\text{W}$

$p = 0$ when $4.375e^{-10,000t} - 1.53125e^{-5000t} - 2.34375e^{-15,000t} = 0$

Let $x = e^{5000t}$, then

$$4.375x - 1.53125x^2 - 2.34375 = 0$$

Solving,

$$x = 0.7143, \quad x = 2.143$$

If $x < 1$, t must be negative hence the solution for $t > 0$ must be $x = 2.143$

$e^{5000t} = 2.143$ so $t = 152.43\,\mu\text{s}$

P 5.8 **[a]** From Prob. 5.7 we have

$$i = A_1 e^{-2500t} + A_2 e^{-7500t}\,\text{A}$$

$$v = -50A_1 e^{-2500t} - 150A_2 e^{-7500t}\,\text{V}$$

$$i(0) = A_1 + A_2 = 0.05$$

$$v(0) = -50A_1 - 150A_2 = -100$$

$$\therefore \quad A_1 + A_2 = 0.05 \quad \text{and} \quad A_1 + 3A_2 = 2$$

$$\therefore \quad A_2 = 0.975\text{A}, \qquad A_1 = -0.925\text{A}$$

Thus,

$$i = -0.925e^{-2500t} + 0.975e^{-7500t}\,\text{A} \qquad t \geq 0$$

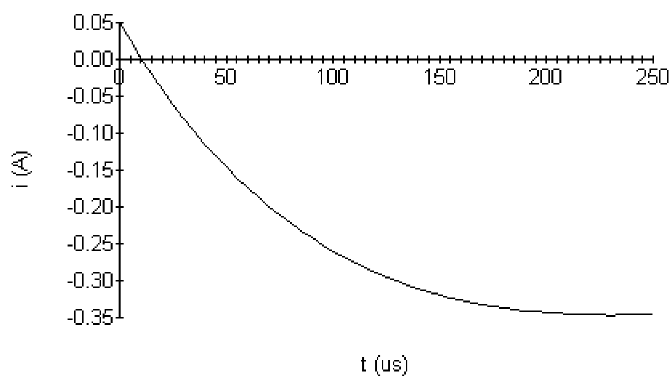
$$v = 46.25e^{-2500t} - 146.25e^{-7500t}\,\text{V} \qquad t \geq 0$$

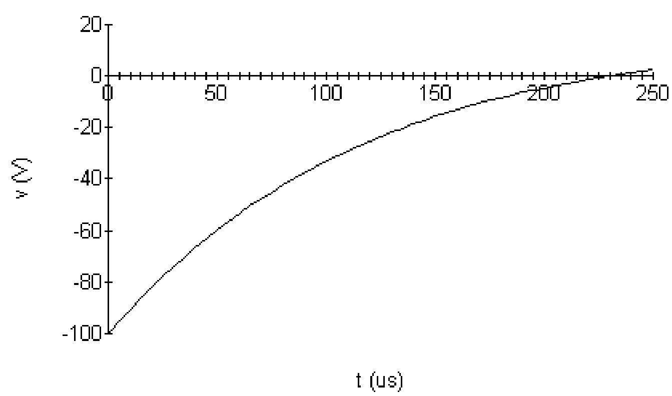
[b] $i = 0 \qquad \text{when} \quad 0.975e^{-7500t} = 0.925e^{-2500t}$

$$\therefore \quad e^{5000t} = 1.0541$$

$$t = (\ln 1.054)/5000 = 10.53\,\mu\text{s}$$

$$v = 0 \qquad \text{when} \quad 46.25e^{-2500t} = 146.25e^{-7500t}$$





$\therefore$ Energy is being stored between 10.53 μs and 230.25 μs; energy is being extracted between 0 and 10.53 μs and between 230.25 μs and infinity.

[c] $p = vi = 180.375e^{-10{,}000t} - 42.78125e^{-5000t} - 142.59375e^{-15{,}000t}\,\text{W}$

$$\therefore \quad \text{W}_{\text{stored}} = \int_{t_1}^{t_2} p\,dt + w(0)$$

$$\begin{aligned}
\text{W}_{\text{stored}} &= 10^{-3}\left\{-18.0375e^{-10{,}000t}\Big|_{t_1}^{t_2} + 8.55625e^{-5000t}\Big|_{t_1}^{t_2} + \right.\\
&\quad \left. 9.50625e^{-15{,}000t}\Big|_{t_1}^{t_2}\right\} + 25\times 10^{-6}\\
&= 8.55625e^{-5000t_2} + 9.50625e^{-15{,}000t_2} - 18.0375e^{-10{,}000t_2}\\
&\quad - 8.55625e^{-5000t_1} - 9.50625e^{-15{,}000t_1} + 18.0375e^{-10{,}000t_1}\\
&\quad + 0.025\,\text{mW}
\end{aligned}$$

where $\quad t_1 = 10.52\,\mu\text{s}, \quad t_2 = 230.11\,\mu\text{s}$

$\text{W}_{\text{stored}} = 1.23\,\text{mJ}.$

$$\begin{aligned}
\text{W}_{\text{extracted}} &= \int_0^{t_1} p\,dt + \int_{t_2}^{\infty} p\,dt\\
&= \int_0^{t_1}(180.375e^{-10^4t} - 42.78125e^{-5000t}\\
&\qquad - 142.59375e^{-15{,}000t})\,dt\\
&\quad + \int_{t_2}^{\infty}(180.375e^{-10^4t} - 42.78125e^{-5000t}\\
&\quad - 142.59375e^{-15{,}000t})\,dt\\
&= 10^{-3}\left\{-18.0375e^{-10{,}000t}\Big|_0^{t_1} + 8.55625e^{-5000t}\Big|_0^{t_1}\right.\\
&\quad + 9.50625e^{-15{,}000t}\Big|_0^{t_1} - 18.0375e^{-10{,}000t}\Big|_{t_2}^{\infty}\\
&\quad \left. + 8.55625e^{-5000t}\Big|_{t_2}^{\infty} + 9.50625e^{-15{,}000t}\Big|_{t_2}^{\infty}\right\}\\
&= \{18.0375e^{-10{,}000t_2} - 8.55625e^{-5000t_2} - 9.50625e^{-15{,}000t_2}\\
&\quad + 8.55625e^{-5000t_1} + 9.50625e^{-15{,}000t_1} - 18.0375e^{-10{,}000t_1}\\
&\quad - 0.025\}\,\text{mJ}
\end{aligned}$$

$\text{W}_{\text{ext.}} = -1.23\,\text{mJ} \qquad \therefore \quad \text{W}_{\text{stored}} = \text{W}_{\text{extracted}}$

P 5.9 **[a]** $v_L = L\dfrac{di}{dt} = [125\sin 400t]e^{-200t}\,\text{V}$

$$\therefore\quad \frac{dv_L}{dt} = 25{,}000(2\cos 400t - \sin 400t)e^{-200t}\,\text{V/s}$$

$$\frac{dv_L}{dt} = 0 \quad \text{when} \quad \tan 400t = 2$$

$$\therefore\quad t = 2.77\,\text{ms}$$

Also $\quad 400t = 1.107 + \pi \qquad$ etc.

Because of the decaying exponential v_L will be maximum the first time the derivative is zero.

[b] $v_L(\text{max}) = [125\sin 1.107]e^{-0.554} = 64.27\,\text{V}$

$v_L\,\text{max} = 64.27\,\text{V}$

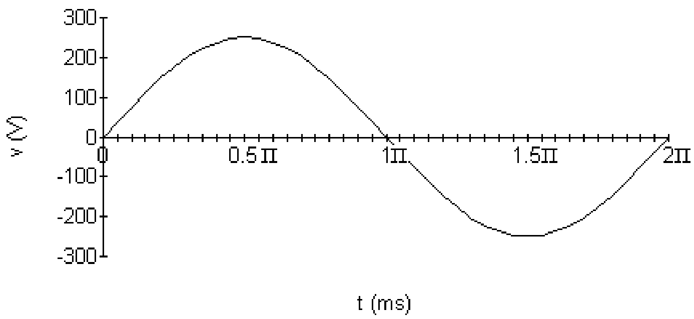
Note: When $\quad t = (1.107 + \pi)/400; \qquad v_L = -13.36\,\text{V}$

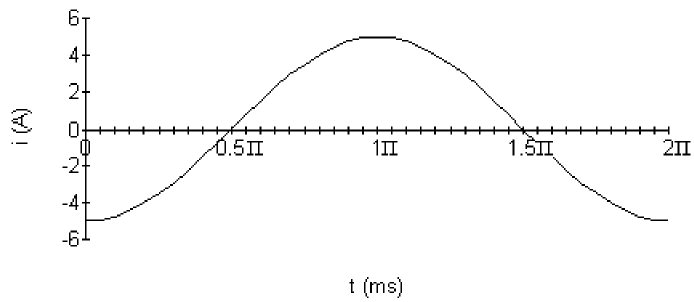
P 5.10 **[a]**

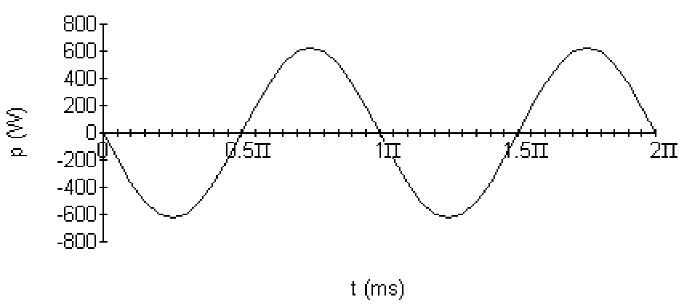
$$\begin{aligned}
i &= \frac{1000}{50}\int_0^t 250\sin 1000x\,dx - 5 \\
&= 5000\int_0^t \sin 1000x\,dx - 5 \\
&= 5000\left[\frac{-\cos 1000x}{1000}\right|_0^t - 5 \\
&= 5(1 - \cos 1000t) - 5 \\
i &= -5\cos 1000t\,\text{A}
\end{aligned}$$

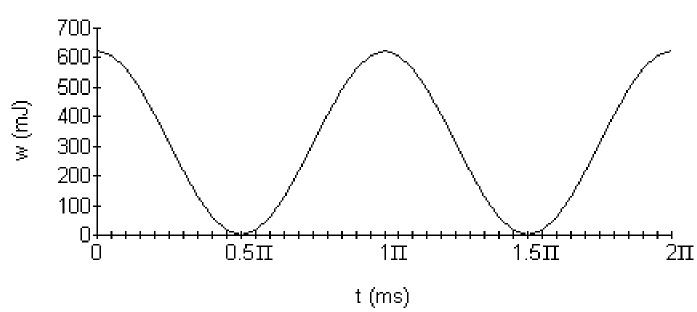
[b]

$$\begin{aligned}
p &= vi = (250\sin 1000t)(-5\cos 1000t) \\
&= -1250\sin 1000t\cos 1000t \\
p &= -625\sin 2000t\,\text{W} \\
w &= \frac{1}{2}Li^2 \\
&= \frac{1}{2}(50\times 10^{-3})25\cos^2 1000t \\
&= 625\cos^2 1000t\,\text{mJ} \\
w &= [312.5 + 312.5\cos 2000t]\,\text{mJ.}
\end{aligned}$$


300
200
100
0
-100
-200
-300
v (V)
0
0.5π
1π
1.5π
2π
t (ms)


6
4
2
0
-2
-4
-6
i (A)
0
0.5π
1π
1.5π
2π
t (ms)


800
600
400
200
0
-200
-400
-600
-800
p (W)
0
0.5π
1π
1.5π
2π
t (ms)


700
600
500
400
300
200
100
0
w (mJ)
0
0.5π
1π
1.5π
2π
t (ms)

[c] Absorbing power: Delivering power:

$0.5\pi \leq t \leq \pi$ ms $\qquad 0 \leq t \leq 0.5\pi$ ms

$1.5\pi \leq t \leq 2\pi$ ms $\qquad \pi \leq t \leq 1.5\pi$ ms

P 5.11 $i = (B_1 \cos 5t + B_2 \sin 5t)e^{-t}$

$$i(0) = B_1 = 25\,\text{A}$$

$$\frac{di}{dt} = (B_1 \cos 5t + B_2 \sin 5t)(-e^{-t}) + e^{-t}(-5B_1 \sin 5t + 5B_2 \cos 5t)$$

$$= [(5B_2 - B_1)\cos 5t - (5B_1 + B_2)\sin 5t]e^{-t}$$

$$v = 2\frac{di}{dt} = [(10B_2 - 2B_1)\cos 5t - (10B_1 + 2B_2)\sin 5t]e^{-t}$$

$$v(0) = 100 = 10B_2 - 2B_1 = 10B_2 - 50 \quad \therefore \quad B_2 = 150/10 = 15\,\text{A}$$

Thus,

$$i = (25\cos 5t + 15\sin 5t)e^{-t}\,\text{A}, \qquad t \geq 0$$

$$v = (100\cos 5t - 280\sin 5t)e^{-t}\,\text{V}, \qquad t \geq 0$$

$$i(0.5) = -6.70\,\text{A}; \qquad v(0.5) = -150.23\,\text{V}$$

$$p(0.5) = (-6.70)(-150.23) = 1007.00\,\text{W absorbing}$$

P 5.12 **[a]**

$$\begin{aligned} v(20\,\mu\text{s}) &= 12.5 \times 10^9(20 \times 10^{-6})^2 = 5\,\text{V} \quad \text{(end of first interval)} \\ v(20\,\mu\text{s}) &= 10^6(20 \times 10^{-6}) - (12.5)(400) \times 10^{-3} - 10 \\ &= 5\,\text{V (start of second interval)} \\ v(40\,\mu\text{s}) &= 10^6(40 \times 10^{-6}) - (12.5)(1600) \times 10^{-3} - 10 \\ &= 10\,\text{V (end of second interval)} \end{aligned}$$

[b] $p(10\mu\text{s}) = 62.5 \times 10^{12}(10^{-5})^3 = 62.5\,\text{mW}, \qquad v(10\,\mu\text{s}) = 1.25\,\text{V},$

$i(10\mu\text{s}) = 50\,\text{mA}, \qquad p(10\,\mu\text{s}) = vi = 62.5\,\text{mW},$

$p(30\,\mu\text{s}) = 437.50\,\text{mW}, \qquad v(30\,\mu\text{s}) = 8.75\,\text{V}, \qquad i(30\,\mu\text{s}) = 0.05\,\text{A}$

[c] $w(10\,\mu\text{s}) = 15.625 \times 10^{12}(10 \times 10^{-6})^4 = 0.15625\,\mu\text{J}$

$$w = 0.5Cv^2 = 0.5(0.2 \times 10^{-6})(1.25)^2 = 0.15625\,\mu\text{J}$$

$$w(30\,\mu\text{s}) = 7.65625\,\mu\text{J}$$

$$w(30\,\mu\text{s}) = 0.5(0.2 \times 10^{-6})(8.75)^2 = 7.65625\,\mu\text{J}$$

P 5.13 **[a]** $0 \le t \le 50\,\mu\text{s}$

$$C = 0.5\,\mu\text{F} \qquad \frac{1}{C} = 2 \times 10^6$$

$$v = 2 \times 10^6 \int_0^t 20 \times 10^{-3}\,dx + 20$$

$$v = 40 \times 10^3 t + 20\,\text{V} \qquad 0 \le t \le 50\,\mu\text{s}$$

$$v(50\,\mu\text{s}) = 2 + 20 = 22\,\text{V}$$

[b] $50\,\mu\text{s} \le t \le 200\,\mu\text{s}$

$$v = 2 \times 10^6 \int_{50\times 10^{-6}}^t -40 \times 10^{-3}\,dx + 22 = -8 \times 10^4 t + 4 + 22$$

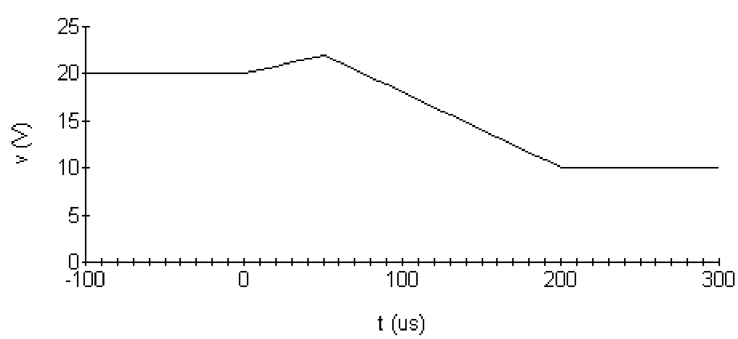
$$v = -8 \times 10^4 t + 26\text{V} \qquad 50 \le t \le 200\,\mu\text{s}$$

$$v(200\,\mu\text{s}) = -8 \times 10^4(200 \times 10^{-6}) + 26 = 10\,\text{V}$$

[c] $200 \le t \le \infty$

$$v = 2 \times 10^6 \int_{200\times 10^{-6}}^t 0\,dx + 10 = 10\,\text{V} \quad 200\,\mu\text{s} \le t \le \infty$$

[d]



P 5.14 $i_C = C(dv/dt)$

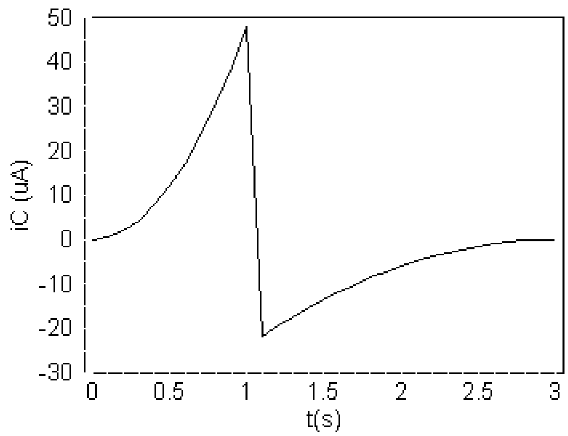
$0 < t < 1$:

$v_c = 20t^3\,\text{V}$

$i_C = 0.8 \times 10^{-6}(60)t^2 = 48t^2\,\mu\text{A}$

$1 < t < 3$:

$v_c = 2.5(3-t)^3\,\text{V}$
$i_C = 0.8\times10^{-6}(7.5)(3-t)^2(-1) = -6(3-t)^2\,\mu\text{A}$



P 5.15 **[a]**

$$\begin{aligned} v &= 5\times10^6\int_0^{250\times10^{-6}} 100\times10^{-3}e^{-1000t}\,dt - 60.6 \\ &= 500\times10^3\frac{e^{-1000t}}{-1000}\bigg|_0^{250\times10^{-6}} -60.6 \\ &= 500(1-e^{-0.25}) - 60.6 = 50\,\text{V} \\ w &= \tfrac{1}{2}Cv^2 = \tfrac{1}{2}(0.2)(10^{-6})(50)^2 = 250\,\mu\text{J} \end{aligned}$$

[b] $v = 500 - 60.6 = 439.40\,\text{V}$

$$w = \frac{1}{2}(0.2)\times10^{-6}(439.40)^2 = 19.31\,\text{mJ} = 19{,}307.24\,\mu\text{J}$$

P 5.16 **[a]** $w(0) = \dfrac{1}{2}C[v(0)]^2 = \dfrac{1}{2}(0.40)\times10^{-6}(25)^2 = 125\,\mu\text{J}$

[b] $v = (A_1t + A_2)e^{-1500t}$ $\qquad v(0) = A_2 = 25\,\text{V}$

$$\begin{aligned} \frac{dv}{dt} &= -1500e^{-1500t}(A_1t + A_2) + e^{-1500t}(A_1) \\ &= (-1500A_1t - 1500A_2 + A_1)e^{-1500t} \end{aligned}$$

$$\frac{dv}{dt}(0) = A_1 - 1500A_2, \qquad i = C\frac{dv}{dt}, \qquad i(0) = C\frac{dv(0)}{dt}$$

$$\therefore\quad \frac{dv(0)}{dt} = \frac{i(0)}{C} = \frac{90\times10^{-3}}{0.40\times10^{-6}} = 225\times10^3$$

$$\therefore\quad 225\times10^3 = A_1 - 1500(25)$$

Thus, $A_1 = 2.25\times10^5 + 3.75\times10^4 = 262{,}500\,\dfrac{\text{V}}{\text{s}}$

[c] $v = (262{,}500t + 25)e^{-1500t}$

$$i = C\frac{dv}{dt} = 0.40 \times 10^{-6}\frac{d}{dt}(262{,}500t + 25)e^{-1500t}$$

$$\begin{aligned}
i &= \frac{d}{dt}[(0.105t + 10 \times 10^{-6})e^{-1500t}] \\
&= (0.105t + 10 \times 10^{-6})(-1500)e^{-1500t} + e^{-1500t}(0.105) \\
&= (-157.5t - 15 \times 10^{-3} + 0.105)e^{-1500t} \\
&= (0.09 - 157.5t)e^{-1500t}\,\text{A}, \qquad t \geq 0 \\
&= (90 - 157{,}500t)e^{-1500t}\,\text{mA}, \qquad t \geq 0
\end{aligned}$$

P 5.17 **[a]** $i = \dfrac{50 \times 10^{-3}}{10 \times 10^{-6}}t = 5 \times 10^3 t \qquad 0 \leq t \leq 10\,\mu\text{s}$

$i = 50 \times 10^{-3} \qquad 10 \leq t \leq 30\,\mu\text{s}$

$$\begin{aligned}
q &= \int_0^{10\times10^{-6}} 5 \times 10^3 t\,dt + \int_{10\times10^{-6}}^{30\times10^{-6}} 50 \times 10^{-3}\,dt \\
&= 5 \times 10^3 \frac{t^2}{2}\Big|_0^{10\times10^{-6}} + 50 \times 10^{-3}(20 \times 10^{-6}) \\
&= 5 \times 10^3(\tfrac{1}{2})(100 \times 10^{-12}) + 1000 \times 10^{-3} \times 10^{-6} \\
&= 1.25\,\mu\text{C}
\end{aligned}$$

[b] $i = 200 \times 10^{-3} - 5 \times 10^{-3}t \qquad 30\,\mu\text{s} \leq t \leq 50\,\mu\text{s}$

$$\begin{aligned}
q &= 1.25 \times 10^{-6} + \int_{30\times10^{-6}}^{50\times10^{-6}} [200 \times 10^{-3} - 5 \times 10^3 t]\,dt \\
&= 1.25 \times 10^{-6} + 200 \times 10^{-3}(20 \times 10^{-6}) - 5 \times 10^3 \frac{t^2}{2}\Big|_{30\times10^{-6}}^{50\times10^{-6}} \\
&= 1.25 \times 10^{-6} + 4000 \times 10^{-9} - 5 \times 10^3\left[\frac{2500 - 900}{2}\right]10^{-12} \\
&= 1.25\,\mu\text{C}
\end{aligned}$$

Since $q = vC, \qquad \therefore \quad v = 1.25/0.25 = 5\,\text{V}.$

[c] $i = -300 \times 10^{-3} + 5 \times 10^{-3}t \qquad 50\,\mu\text{s} \leq t \leq 60\,\mu\text{s}$

$$\begin{aligned} q &= 1.25\times10^{-6} + \int_{50\times10^{-6}}^{60\times10^{-6}} [-300\times10^{-3} + 5\times10^3 t]\,dt \\ &= 1.25\times10^{-6} - 300\times10^{-3}(10\times10^{-6}) + 5\times10^3\left[\frac{3600-2500}{2}\right]10^{-12} \\ &= 1\,\mu\text{C} \end{aligned}$$

$$v = \frac{1\times10^{-6}}{0.25\times10^{-6}} = 4\,\text{V}$$

$$w = \frac{C}{2}v^2 = \frac{1}{2}(0.25)\times10^{-6}(16) = 2\,\mu\text{J}$$

P 5.18 $v = -60\,\text{V}, \quad t \le 0; \qquad C = 0.4\,\mu\text{F}$

$$v = 15 - 15e^{-500t}(5\cos 2000t + \sin 2000t)\text{V}, \quad t \ge 0$$

[a] $i = 0, \quad t < 0$

[b]
$$\begin{aligned} \frac{dv}{dt} &= -15[(5\cos 2000t + \sin 2000t)(-500e^{-500t}) + \\ &\quad (e^{-500t})(-10{,}000\sin 2000t + 2000\cos 2000t)] \\ &= 15e^{-500t}(500\cos 2000t + 10{,}500\sin 2000t) \\ i &= C\frac{dv}{dt} = 0.4\times10^{-6}(7500)e^{-500t}(\cos 2000t + 21\sin 2000t) \\ &= 3e^{-500t}(\cos 2000t + 21\sin 2000t)\,\text{mA}, \quad t > 0 \end{aligned}$$

[c] no

[d] yes, from 0 to 3 mA

[e] $v(\infty) = 15\,\text{V}$

$$w = \frac{1}{2}Cv^2 = \frac{1}{2}(0.4)225\times10^{-6} = 45\,\mu\text{J}$$

P 5.19 $10\|(15+25) = 8\,\text{H}$

$8\|12 = 4.8\,\text{H}$

$44\|(1.2+4.8) = 5.28\,\text{H}$

$21\|4 = 3.36\,\text{H}$

$5.28 + 3.36 = 8.64\,\text{H}$

P 5.20 $6\|14 = 4.2\,\mathrm{H}$

$15.8 + 4.2 = 20\,\mathrm{H}$

$20\|60 = 15\,\mathrm{H}$

$15 + 5 = 20\,\mathrm{H}$

$20\|80 = 16\,\mathrm{H}$

$16 + 24 = 40\,\mathrm{H}$

$40\|10 = 8\,\mathrm{H}$

$L_{\mathrm{ab}} = 12 + 8 = 20\,\mathrm{H}$

P 5.21 From Figure 5.17(a) we have

$$v = \frac{1}{C_1}\int_0^t i + v_1(0) + \frac{1}{C_2}\int_0^t i\,dx + v_2(0) + \cdots$$

$$v = \left[\frac{1}{C_1} + \frac{1}{C_2} + \cdots\right]\int_0^t i\,dx + v_1(0) + v_2(0) + \cdots$$

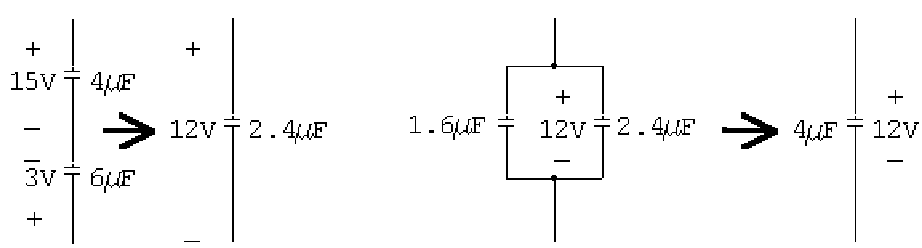
Therefore $\dfrac{1}{C_{\mathrm{eq}}} = \left[\dfrac{1}{C_1} + \dfrac{1}{C_2} + \cdots\right], \qquad v_{\mathrm{eq}}(0) = v_1(0) + v_2(0) + \cdots$

P 5.22 From Fig. 5.18(a)

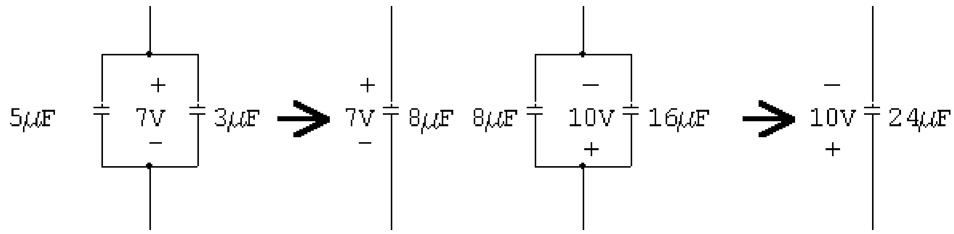
$$i = C_1\frac{dv}{dt} + C_2\frac{dv}{dt} + \cdots = [C_1 + C_2 + \cdots]\frac{dv}{dt}$$

Therefore $C_{\mathrm{eq}} = C_1 + C_2 + \cdots$. Because the capacitors are in parallel, the initial voltage on every capacitor must be the same. This initial voltage would appear on C_{eq}.

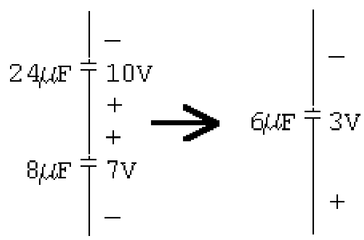
P 5.23 $\dfrac{1}{4} + \dfrac{1}{6} = \dfrac{5}{12} \quad \therefore \quad C_{\mathrm{eq}} = 2.4\,\mu\mathrm{F}$



$\dfrac{1}{4} + \dfrac{1}{12} = \dfrac{4}{12} \quad \therefore \quad C_{\mathrm{eq}} = 3\,\mu\mathrm{F}$

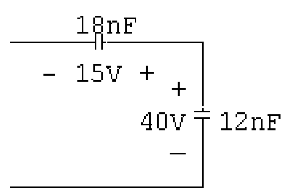


$$\frac{1}{24}+\frac{1}{8}=\frac{4}{24} \qquad \therefore \quad C_{\rm eq}=6\,\mu\text{F}$$



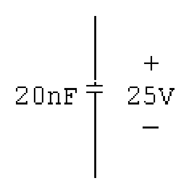
P 5.24 $\quad \dfrac{1}{C_1}=\dfrac{1}{8}+\dfrac{1}{32}=\dfrac{5}{32}; \qquad C_1=6.4\,\text{nF}$

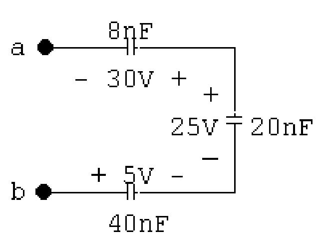
$C_2=5.6+6.4=12\,\text{nF}$



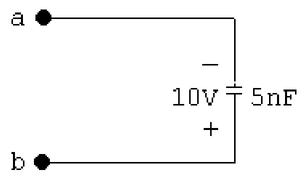
$$\frac{1}{C_3}=\frac{1}{18}+\frac{1}{12}=\frac{10}{72}; \qquad C_3=7.2\,\text{nF}$$

$C_4=12.8+7.2=20\,\text{nF}$



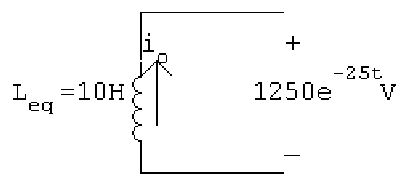


$$\frac{1}{C_5} = \frac{1}{8} + \frac{1}{20} + \frac{1}{40} = \frac{1}{5}; \qquad C_5 = 5\,\text{nF}$$



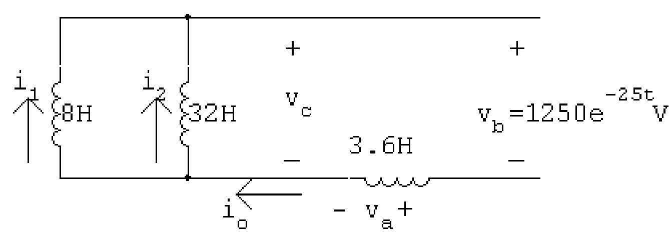
P 5.25 **[a]** $i_o(0) = i_1(0) + i_2(0) = 5\,\text{A}$

[b]



$$\begin{aligned} i_o &= -\frac{1}{10}\int_0^t 1250e^{-25x}\,dx + 5 = -125\left[\frac{e^{-25x}}{-25}\bigg|_0^t\right] + 5 \\ &= 5(e^{-25t} - 1) + 5 = 5e^{-25t}\,\text{A}, \qquad t \geq 0 \end{aligned}$$

[c]



$$\begin{aligned} v_a &= 3.6\frac{d}{dt}(5e^{-25t}) = -450e^{-25t}\,\text{V} \\ v_c &= v_a + v_b = -450e^{-25t} + 1250e^{-25t} \\ &= 800e^{-25t}\,\text{V} \\ i_1 &= -\frac{1}{8}\int_0^t 800e^{-25x}\,dx + 10 \\ &= 4e^{-25t} - 4 + 10 \\ i_1 &= 4e^{-25t} + 6\,\text{A} \qquad t \geq 0 \end{aligned}$$

[d]
$$\begin{aligned} i_2 &= -\frac{1}{32}\int_0^t 800e^{-25x}\,dx - 5 \\ &= e^{-25t} - 1 - 5 \\ i_2 &= e^{-25t} - 6\,\text{A}, \qquad t \geq 0 \end{aligned}$$

[e] $w(0) = \frac{1}{2}(8)(100) + \frac{1}{2}(32)(25) + \frac{1}{2}(3.6)(25) = 845\,\text{J}$

[f] $w_{\text{del}} = \frac{1}{2}(10)(25) = 125\,\text{J}$

[g] $w_{\text{trapped}} = 845 - 125 = 720\,\text{J}$

P 5.26 $v_{\text{b}} = 1250e^{-25t}\,\text{V}$

$i_o = 5e^{-25t}\,\text{A}$

$p = 6250e^{-50t}\,\text{W}$

$$w = \int_0^t 6250e^{-50x}\,dx = 6250\frac{e^{-50x}}{-50}\Big|_0^t = 125(1 - e^{-50t})\,\text{W}$$

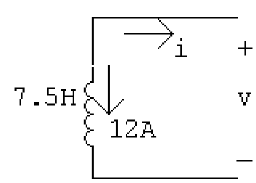
$w_{\text{total}} = 125\,\text{J}$

$80\% w_{\text{total}} = 100\,\text{J}$

Thus,

$125 - 125e^{-50t} = 100; \qquad e^{50t} = 5; \qquad \therefore \quad t = 32.19\,\text{ms}$

P 5.27 **[a]**



$$\begin{aligned} i(t) &= -\frac{1}{7.5}\int_0^t -1800e^{-20x}\,dx - 12 \\ &= 240\frac{e^{-20x}}{-20}\Big|_0^t -12 \\ &= -12(e^{-20t} - 1) - 12 \\ i(t) &= -12e^{-20t}\,\text{A} \end{aligned}$$

[b]

$$\begin{aligned} i_1(t) &= -\frac{1}{10}\int_0^t -1800e^{-20x}\,dx + 4 \\ &= 180\frac{e^{-20x}}{-20}\Big|_0^t +4 \\ &= -9(e^{-20t} - 1) + 4 \\ i_1(t) &= -9e^{-20t} + 13\,\text{A} \end{aligned}$$

[c]
$$\begin{aligned} i_2(t) &= -\frac{1}{30}\int_0^t -1800e^{-20x}\,dx - 16 \\ &= 60\frac{e^{-20x}}{-20}\Big|_0^t -16 \\ &= -3(e^{-20t}-1) - 16 \\ i_2(t) &= -3e^{-20t} - 13\,\text{A} \end{aligned}$$

[d] $p = vi = (-1800e^{-20t})(-12e^{-20t}) = 21{,}600e^{-40t}\,\text{W}$

$$\begin{aligned} w &= \int_0^\infty p\,dt = \int_0^\infty 21{,}600e^{-40t}\,dt \\ &= 21{,}600\frac{e^{-40t}}{-40}\Big|_0^\infty \\ &= 540\,\text{J} \end{aligned}$$

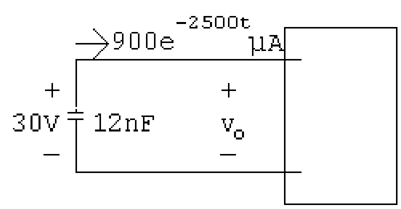
[e] $w = \frac{1}{2}(10)(16) + \frac{1}{2}(30)(256) = 3920\,\text{J}$

[f] $w_{\text{trapped}} = \frac{1}{2}(10)(13)^2 + \frac{1}{2}(30)(13)^2 = 3380\,\text{J}$

$w_{\text{trapped}} = 3920 - 540 = 3380\,\text{J}$ checks

[g] Yes, they agree.

P 5.28 **[a]**



$$\begin{aligned} v_o &= -\frac{10^9}{12}\int_0^t 900\times10^{-6}e^{-2500x}\,dx + 30 \\ &= -75{,}000\frac{e^{-2500x}}{-2500}\Big|_0^t +30 \\ &= 30e^{-2500t}\,\text{V}, \qquad t\geq 0 \end{aligned}$$

[b]
$$\begin{aligned} v_1 &= -\frac{10^9}{20}(900\times10^{-6})\frac{e^{-2500x}}{-2500}\Big|_0^t +45 \\ &= 18e^{-2500t} + 27\,\text{V}, \qquad t\geq 0 \end{aligned}$$

[c]
$$\begin{aligned} v_2 &= -\frac{10^9}{30}(900\times10^{-6})\frac{e^{-2500x}}{-2500}\Big|_0^t -15 \\ &= 12e^{-2500t} - 27\,\text{V}, \qquad t\geq 0 \end{aligned}$$

[d]
$$\begin{aligned} p &= vi = (30e^{-2500t})(900 \times 10^{-6})e^{-2500t} \\ &= 27 \times 10^{-3}e^{-5000t} \\ w &= \int_0^\infty 27 \times 10^{-3}e^{-5000t}\,dt \\ &= 27 \times 10^{-3}\frac{e^{-5000t}}{-5000}\Big|_0^\infty \\ &= -5.4 \times 10^{-6}(0-1) = 5.4\,\mu\text{J} \end{aligned}$$

[e]
$$\begin{aligned} w &= \tfrac{1}{2}(20 \times 10^{-9})(45)^2 + \tfrac{1}{2}(30 \times 10^{-9})(15)^2 \\ &= 20.25 \times 10^{-6} + 3.375 \times 10^{-6} \\ &= 23.625\,\mu\text{J} \end{aligned}$$

[f]
$$\begin{aligned} w_{\text{trapped}} &= \tfrac{1}{2}(20 \times 10^{-9})(27)^2 + \tfrac{1}{2}(30 \times 10^{-9})(27)^2 \\ &= (10+15)(27)^2 \times 10^{-9} \\ &= 18.225\,\mu\text{J} \end{aligned}$$

CHECK: $18.225 + 5.4 = 23.625\,\mu$J

[g] Yes, they agree.

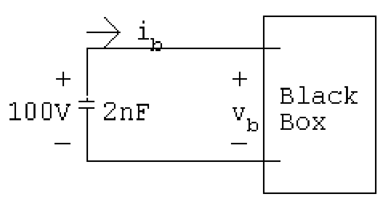
P 5.29 $C_1 = 1 + 1.5 = 2.5\,$nF

$$\frac{1}{C_2} = \frac{1}{2.5} + \frac{1}{12.5} + \frac{1}{50} = \frac{1}{2}$$

$\therefore \quad C_2 = 2\,$nF

$v_{\text{d}}(0) + v_{\text{a}}(0) - v_{\text{c}}(0) = 40 + 15 + 45 = 100\,$V

[a]



$$\begin{aligned} v_{\text{b}} &= -\frac{10^9}{2}\int_0^t 50 \times 10^{-6}e^{-250x}\,dx + 100 \\ &= -25{,}000\frac{e^{-250x}}{-250}\Big|_0^t + 100 \\ &= 100(e^{-250t} - 1) + 100 \\ &= 100e^{-250t}\,\text{V}, \quad t \geq 0 \end{aligned}$$

[b]
$$\begin{aligned} v_{\rm a} &= -\frac{10^9}{12.5}\int_0^t 50\times 10^{-6}e^{-250x}\,dx + 15 \\ &= -4000\frac{e^{-250x}}{-250}\Big|_0^t + 15 \\ &= 16(e^{-250t}-1)+15 \\ &= 16e^{-250t} - 1\,\text{V} \end{aligned}$$

[c]
$$\begin{aligned} v_{\rm c} &= \frac{10^9}{50}\int_0^t 50\times 10^{-6}e^{-250x}\,dx - 45 \\ &= 1000\frac{e^{-250x}}{-250}\Big|_0^t - 45 \\ &= -4(e^{-250t}-1)-45 \\ &= -4e^{-250t} - 41\,\text{V}, \quad t\ge 0 \end{aligned}$$

[d]
$$\begin{aligned} v_{\rm d} &= -\frac{10^9}{2.5}\int_0^t 50\times 10^{-6}e^{-250x}\,dx + 40 \\ &= -20{,}000\frac{e^{-250x}}{-250}\Big|_0^t + 40 \\ &= 80(e^{-250t}-1)+40 \\ &= 80e^{-250t} - 40\,\text{V}, \quad t\ge 0 \end{aligned}$$

CHECK:
$$\begin{aligned} v_{\rm b} &= v_{\rm d}+v_{\rm a}-v_{\rm c} \\ &= 80e^{-250t}-40+16e^{-250t}-1+4e^{-250t}+41 \\ &= 100e^{-250t}\,\text{V} \quad \text{(checks)} \end{aligned}$$

[e]
$$\begin{aligned} i_1 &= -10^{-9}\frac{d}{dt}[80e^{-250t}-40] \\ &= -10^{-9}(-20{,}000e^{-250t}) \\ &= 20e^{-250t}\,\mu\text{A}, \quad t\ge 0 \end{aligned}$$

[f]
$$\begin{aligned} i_2 &= -1.5\times 10^{-9}\frac{d}{dt}[80e^{-250t}-40] \\ &= -1.5\times 10^{-9}(-20{,}000e^{-250t}) \\ &= 30e^{-250t}\,\mu\text{A}, \quad t\ge 0 \end{aligned}$$

CHECK: $i_1+i_2 = 50e^{-250t}\,\mu\text{A} = i_{\rm b}$

P 5.30 **[a]**

$$w(0) = [\tfrac{1}{2}(2.5)(40)^2 + \tfrac{1}{2}(12.5)(15)^2 + \tfrac{1}{2}(50)(45)^2] \times 10^{-9}$$

$$= 54{,}031.25\,\text{nJ}$$

[b]

$$v_\text{a}(\infty) = -1\,\text{V}$$

$$v_\text{c}(\infty) = -41\,\text{V}$$

$$v_\text{d}(\infty) = -40\,\text{V}$$

$$w(\infty) = [\tfrac{1}{2}(2.5)(40)^2 + \tfrac{1}{2}(12.5)(1)^2 + \tfrac{1}{2}(50)(41)^2] \times 10^{-9}$$

$$= 44{,}031.25\,\text{nJ}$$

[c] $w = \int_0^\infty (100e^{-250t})(50e^{-250t}) \times 10^{-6}\,dt = 10{,}000\,\text{nJ}$

CHECK: $54{,}031.25 - 44{,}031.25 = 10{,}000$

[d] $\%\text{ delivered} = \dfrac{10{,}000}{54{,}031.25} \times 100 = 18.51\%$

[e]

$$w = 5 \times 10^{-3}\int_0^t e^{-500x}\,dx$$

$$= 10^4(1 - e^{-500t})\,\text{nJ}$$

$\therefore \quad 10^4(1 - e^{-500t}) = 5000; \qquad e^{-500t} = 0.5$

Thus, $t = (\ln 2)/500 = 1.39\,\text{ms}$.

P 5.31 **[a]** $\dfrac{v}{i} = R = \dfrac{100e^{-80t}}{4e^{-80t}} = 25\,\Omega$

[b] $\tau = \dfrac{1}{80} = 12.5\,\text{ms}$

[c] $\tau = \dfrac{L}{R} = 12.5 \times 10^{-3}$

$L = (12.5)(25) \times 10^{-3} = 312.5\,\text{mH}$

[d] $w(0) = \dfrac{1}{2}L[i(0)]^2 = \dfrac{1}{2}(0.3125)(16) = 2.5\,\text{J}$

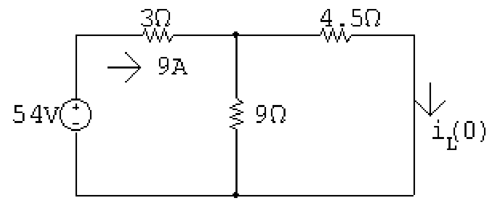
[e] $w_\text{diss} = \int_0^t 400e^{-160x}\,dx = 2.5 - 2.5e^{-160t}$

$0.8w(0) = (0.8)(2.5) = 2\,\text{J}$

$2.5 - 2.5e^{-160t} = 2 \qquad \therefore \quad e^{160t} = 5$

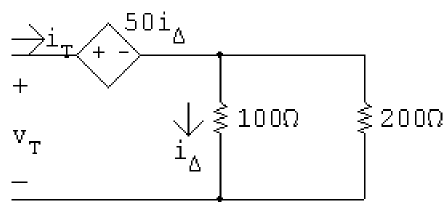
Solving, $t = 10.06$ ms.

P 5.32 **[a]** $t < 0:$



$$\frac{(9)(4.5)}{13.5} = 3\,\Omega; \qquad i_L(0) = 9\frac{9}{13.5} = 6\,\text{A}$$

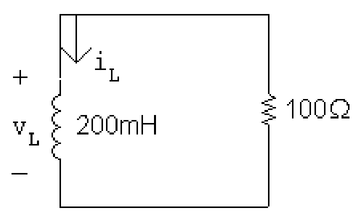
$t > 0$:



$$i_\Delta = \frac{i_T(200)}{300} = \frac{2}{3}i_T$$

$$v_T = 50i_\Delta + i_T\frac{(100)(200)}{300} = 50i_T\frac{2}{3} + \frac{200}{3}i_T$$

$$\frac{v_T}{i_T} = R_{\text{Th}} = \frac{100}{3} + \frac{200}{3} = 100\,\Omega$$

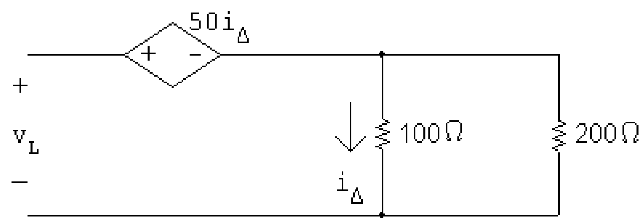


$$\tau = \frac{L}{R} = \frac{200}{100} \times 10^{-3} \qquad \frac{1}{\tau} = 500$$

$$i_L = 6e^{-500t}\,\text{A}, \qquad t \geq 0$$

[b] $v_L = 200 \times 10^{-3}(-3000e^{-500t}) = -600e^{-500t}\,\text{V}, \quad t \geq 0^+$

[c]



$$v_L = 50i_\Delta + 100i_\Delta = 150i_\Delta$$

$$i_\Delta = \frac{v_L}{150} = -4e^{-500t}\,\text{A} \qquad t \geq 0^+$$

P 5.33 $$w(0) = \frac{1}{2}(200\times10^{-3})(36) = 3.6\,\text{J}$$

$$p_{50i_\Delta} = -50i_\Delta i_L = -50(-4e^{-500t})(6e^{-500t}) = 1200e^{-1000t}\,\text{W}$$

$$w_{50i_\Delta} = \int_0^\infty 1200e^{-1000t}\,dt = 1200\frac{e^{-1000t}}{-1000}\Big|_0^\infty = 1.2\,\text{J}$$

$$\%\ \text{dissipated} = \frac{1.2}{3.6}(100) = 33.33\%$$

P 5.34 **[a]** $i(0) = 125/25 = 5\,\text{A}$

[b] $\tau = \dfrac{L}{R} = \dfrac{4}{100} = 40\,\text{ms}$

[c] $i = 5e^{-25t}\,\text{A}, \qquad t \geq 0$

$$v_1 = L\frac{di_1}{dt} = 4(-125e^{-25t}) = -500e^{-25t}\,\text{V} \qquad t \geq 0^+$$

$$v_2 = -80i = -400e^{-25t}\,\text{V} \qquad t \geq 0$$

[d] $p_{\text{diss}} = i^2(20) = 25e^{-50t}(20) = 500e^{-50t}\,\text{W}$

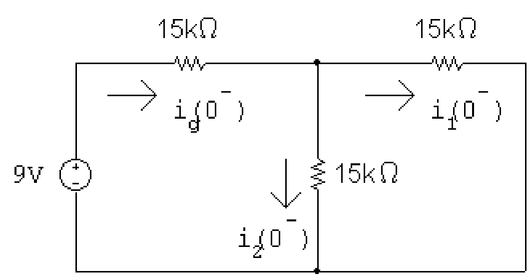
$$w_{\text{diss}} = \int_0^t 500e^{-50x}\,dx = 500\frac{e^{-50x}}{-50}\Big|_0^t = 10 - 10e^{-50t}\,\text{J}$$

$$w_{\text{diss}}(12\,\text{ms}) = 10 - 10e^{-0.6} = 4.51\,\text{J}$$

$$w(0) = \frac{1}{2}(4)(25) = 50\,\text{J}$$

$$\%\ \text{dissipated} = \frac{4.51}{50}(100) = 9.02\%$$

P 5.35 **[a]** $t < 0$



$$15\,\text{k}\Omega \| 15\,\text{k}\Omega = 7.5k\Omega$$

$$i_g(0^-) = \frac{9}{(15+7.5)\times10^3} = 0.4\,\text{mA}$$

$$i_1(0^-) = i_2(0^-) = (0.4\times10^{-3})\frac{(15)}{(30)} = 0.2\,\text{mA}$$

[b] $i_1(0^+) = i_1(0^-) = 0.2\,\text{mA}$

$$i_2(0^+) = -i_1(0^+) = -0.2\,\text{mA} \qquad \text{(when switch is open)}$$

[c] $\tau = \dfrac{L}{R} = \dfrac{30 \times 10^{-3}}{30 \times 10^{3}} = 10^{-6}; \qquad \dfrac{1}{\tau} = 10^{6}$

$$i_1(t) = i_1(0^+)e^{-t/\tau}$$

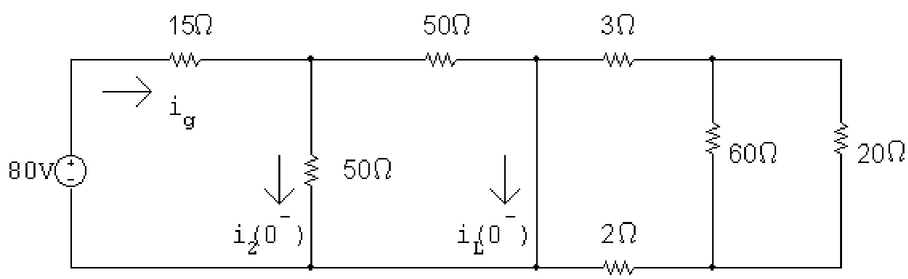
$$i_1(t) = 0.2e^{-10^6 t}\,\text{mA}, \qquad t \geq 0$$

[d] $i_2(t) = -i_1(t) \qquad \text{when} \quad t \geq 0^+$

$$\therefore \quad i_2(t) = -0.2e^{-10^6 t}\,\text{mA}, \qquad t \geq 0^+$$

[e] The current in a resistor can change instantaneously. The switching operation forces $i_2(0^-)$ to equal $0.2\,\text{mA}$ and $i_2(0^+) = -0.2\,\text{mA}$.

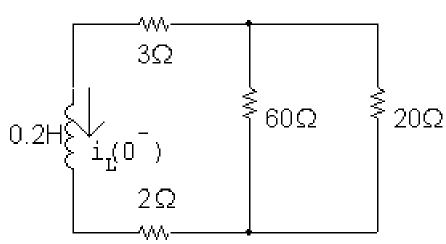
P 5.36 **[a]** For $\quad t < 0$



$$i_g = \frac{80}{40} = 2\,\text{A}$$

$$i_L(0^-) = \frac{2(50)}{(100)} = 1\,\text{A} = i_L(0^+)$$

For $\quad t > 0$



$$i_L(t) = i_L(0^+)e^{-t/\tau}\,\text{A}, \qquad t \geq 0$$

$$\tau = \frac{L}{R} = \frac{0.20}{5 + 15} = \frac{1}{100} = 0.01\,\text{s}$$

$$i_L(0^+) = 1\,\text{A}$$

$$i_L(t) = e^{-100t}\,\text{A}, \qquad t \geq 0$$

$$v_o(t) = -15 i_L(t)$$

$$v_o(t) = -15e^{-100t}\,\text{V}, \qquad t \geq 0^+$$

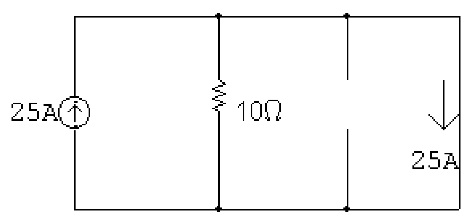
P 5.37 $P_{20\Omega} = \dfrac{v_o^2}{20} = 11.25e^{-200t}\text{W}$

$$w_{\text{diss}} = \int_0^{0.01} 11.25e^{-200t}\,dt$$
$$= \frac{11.25}{-200}e^{-200t}\Big|_0^{0.01}$$
$$= 56.25 \times 10^{-3}(1 - e^{-2}) = 48.64\,\text{mJ}$$

$$w_{\text{stored}} = \frac{1}{2}(0.2)(1)^2 = 100\,\text{mJ}.$$

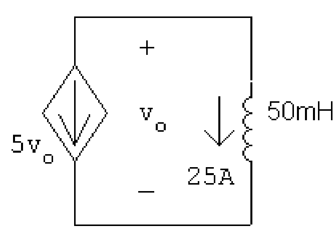
$$\%\ \text{diss} = \frac{48.64}{100} \times 100 = 48.64\%$$

P 5.38 $t < 0$

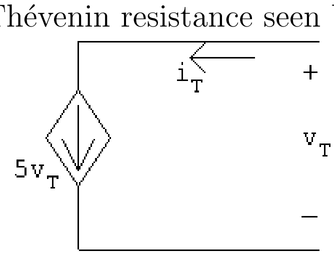


$$i_L(0^-) = i_L(0^+) = 25\,\text{A}$$

$t > 0$

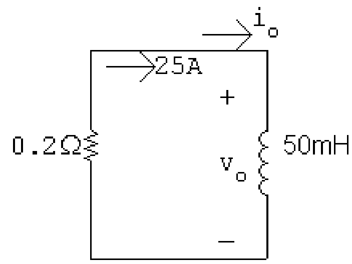


Find Thévenin resistance seen by inductor



$$i_T = 5v_T; \qquad \frac{v_T}{i_T} = R_{\text{Th}} = \frac{1}{5} = 0.2\,\Omega$$

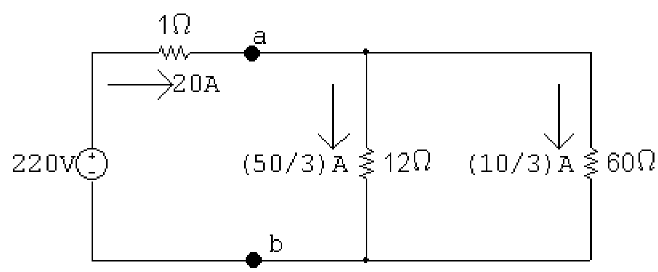
$$\tau = \frac{L}{R} = \frac{50 \times 10^{-3}}{0.2} = 250\,\text{ms}; \qquad 1/\tau = 4$$



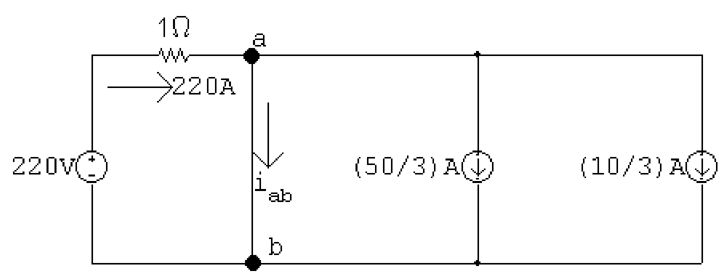
$$i_o = 25e^{-4t}\,\text{A}, \qquad t \geq 0$$

$$v_o = L\frac{di_o}{dt} = (50 \times 10^{-3})(-100e^{-4t}) = -5e^{-4t}\,\text{V}, \quad t \geq 0^+$$

P 5.39 **[a]** $t < 0$:

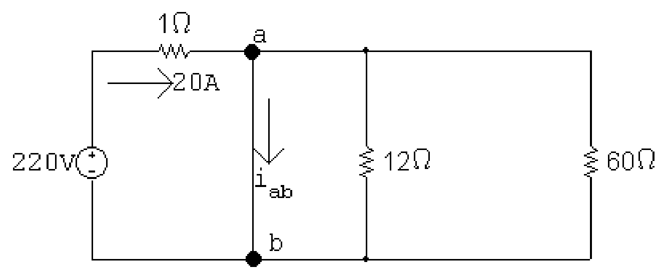


$t = 0^+$:

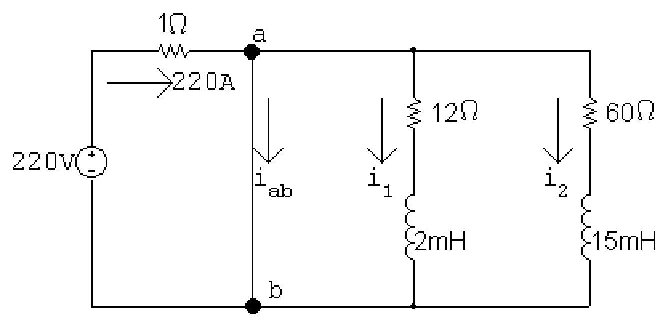


$$220 = i_{\text{ab}} + (50/3) + (10/3), \qquad i_{\text{ab}} = 200\,\text{A}, \quad t = 0^+$$

[b] At $\quad t = \infty$:



$$i_{\rm ab} = 220/1 = 220\,\text{A}, \quad t = \infty$$



[c] $i_1(0) = 50/3, \qquad \tau_1 = \dfrac{2}{12} \times 10^{-3} = 0.167\,\text{ms}$

$$i_2(0) = 10/3, \qquad \tau_2 = \frac{15}{60} \times 10^{-3} = 0.25\,\text{ms}$$

$$i_1(t) = (50/3)e^{-6000t}\,\text{A}, \quad t \geq 0$$

$$i_2(t) = (10/3)e^{-4000t}\,\text{A}, \quad t \geq 0$$

$$i_{\rm ab} = 220 - (50/3)e^{-6000t} - (10/3)e^{-4000t}\,\text{A}, \quad t \geq 0$$

$$220 - (50/3)e^{-6000t} - (10/3)e^{-4000t} = 210$$

$$30 = 50e^{-6000t} + 10e^{-4000t}$$

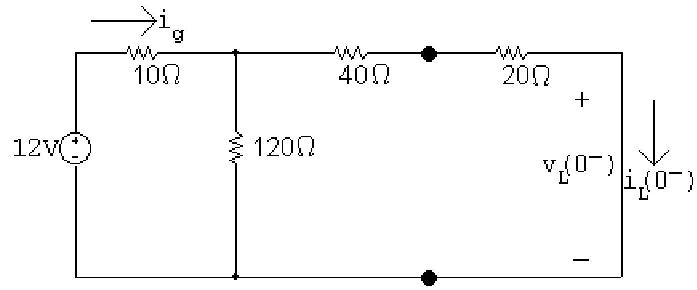
$$3 = 5e^{-6000t} + e^{-4000t}$$

By trial and error

$$t = 123.1\,\mu\text{s}$$

P 5.40 **[a]** $i_o(0^-) = 0 \qquad$ since the switch is open for $t < 0$.

[b] For $t = 0^-$ the circuit is:

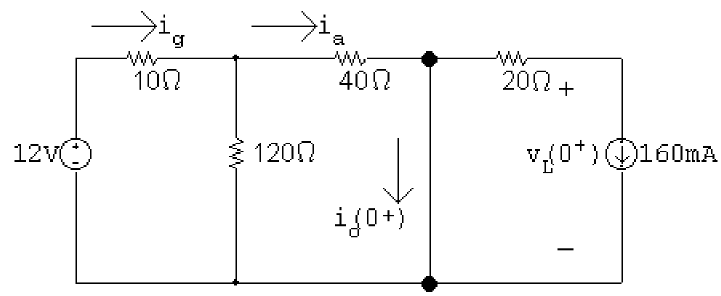


$$120\,\Omega//60\,\Omega = 40\,\Omega$$

$$\therefore \quad i_g = \frac{12}{10+40} = 0.24\,\text{A} = 240\,\text{mA}$$

$$i_L(0^-) = \left(\frac{120}{180}\right) i_g = 160\,\text{mA}$$

[c] For $t = 0^+$ the circuit is:



$$120\,\Omega//40\,\Omega = 30\,\Omega$$

$$\therefore \quad i_g = \frac{12}{10+30} = 0.30\,\text{A} = 300\,\text{mA}$$

$$i_\text{a} = \left(\frac{120}{160}\right) 300 = 225\,\text{mA}$$

$$\therefore \quad i_o(0^+) = 225 - 160 = 65\,\text{mA}$$

[d] $i_L(0^+) = i_L(0^-) = 160\,\text{mA}$

[e] $i_o(\infty) = i_\text{a} = 225\,\text{mA}$

[f] $i_L(\infty) = 0,$ since the switch short circuits the branch containing the 20 Ω resistor and the 100 mH inductor.

[g] $\tau = \dfrac{L}{R} = \dfrac{100 \times 10^{-3}}{20} = 5\,\text{ms}; \qquad \dfrac{1}{\tau} = 200$

$$\therefore \quad i_L = 0 + (160 - 0)e^{-200t} = 160e^{-200t}\,\text{mA}, \qquad t \geq 0$$

[h] $v_L(0^-) = 0$ since for $t < 0$ the current in the inductor is constant [**i**] Refer to the circuit at $t = 0^+$ and note:

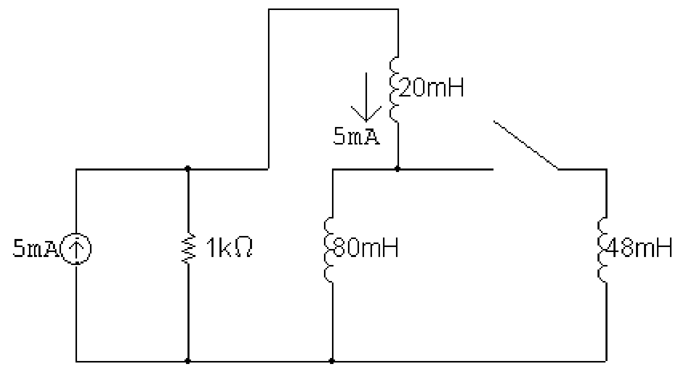
$$20(0.16) + v_L(0^+) = 0; \qquad \therefore \quad v_L(0^+) = -3.2\,\text{V}$$

[**j**] $v_L(\infty) = 0$, since the current in the inductor is a constant at $t = \infty$.

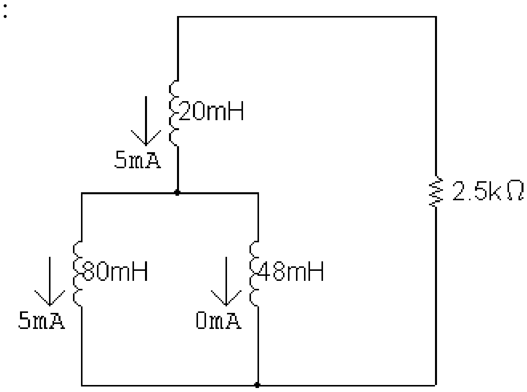
[**k**] $v_L(t) = 0 + (-3.2 - 0)e^{-200t} = -3.2e^{-200t}\,\text{V}, \qquad t \geq 0^+$

[**l**] $i_o = i_\text{a} - i_L = 225 - 160e^{-200t}\,\text{mA}, \qquad t \geq 0^+$

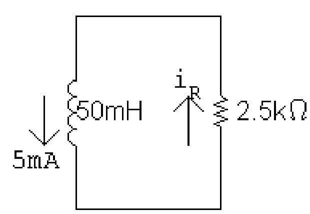
P 5.41 [**a**] $t < 0$:



$t = 0^+$:

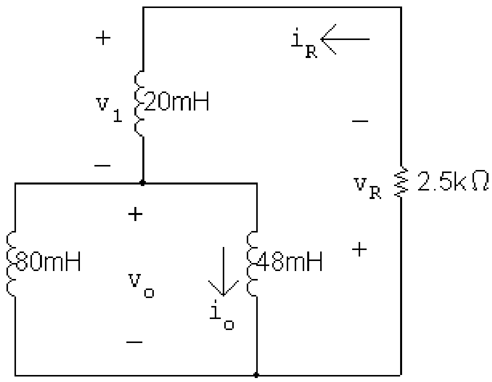


$t > 0$:



$$i_R = 5e^{t/\tau}\,\text{mA}; \qquad \tau = \frac{L}{R} = 20 \times 10^{-6}$$

$$i_R = 5e^{-50{,}000t}\,\text{mA}$$



$$v_R = (2.5 \times 10^3)(5 \times 10^{-3})e^{-50{,}000t} = 12.5e^{-50{,}000t}\,\text{V}$$

$$v_1 = 20 \times 10^{-3}[5 \times 10^{-3}(-50{,}000)e^{-50{,}000t}] = -5e^{-50{,}000t}\,\text{V}$$

$$v_o = -v_1 - v_R = -7.5e^{-50{,}000t}\,\text{V}$$

[b] $i_o = \dfrac{10^3}{48}\displaystyle\int_0^t -7.5e^{-50{,}000x}\,dx + 0 = 3.125e^{-50{,}000t} - 3.125\,\text{mA}$

P 5.42 **[a]** From the solution to Problem 5.41,

$$i_R = 5 \times 10^{-3}e^{-50{,}000t}\,\text{A}$$

$$p_R = (25 \times 10^{-6}e^{-100{,}000t})(2.5 \times 10^3) = 62.5 \times 10^{-3}e^{-100{,}000t}\,\text{W}$$

$$w_{\text{diss}} = \int_0^\infty 62.5 \times 10^{-3}e^{-100{,}000t}\,dt$$

$$= 62.5 \times 10^{-3}\frac{e^{-100{,}000t}}{-10^5}\bigg|_0^\infty = 625\,\text{nJ}$$

[b] $w_{\text{trapped}} = \dfrac{1}{2}L_{\text{eq}}i_{\text{R}}^2(0) = \dfrac{1}{2}(50 \times 10^{-3})(5 \times 10^{-3})^2 = 625\,\text{nJ}$

CHECK:

$w(0) = \frac{1}{2}(20)(25 \times 10^{-6}) \times 10^{-3} + \frac{1}{2}(80)(25 \times 10^{-6}) \times 10^{-3} = 1250\,\text{nJ}$

$\therefore\quad w(0) = w_{\text{diss}} + w_{\text{trapped}}$

P 5.43 **[a]** $i_L(0) = \dfrac{80}{40} = 2\,\text{A}$

$$i_o(0^+) = \frac{80}{20} - 2 = 4 - 2 = 2\,\text{A}$$

$$i_o(\infty) = \frac{80}{20} = 4\,\text{A}$$

[b] $i_L = 2e^{-t/\tau}; \qquad \tau = \frac{L}{R} = \frac{20}{20} \times 10^{-3} = 1\,\text{ms}$

$i_L = 2e^{-1000t}\,\text{A}$

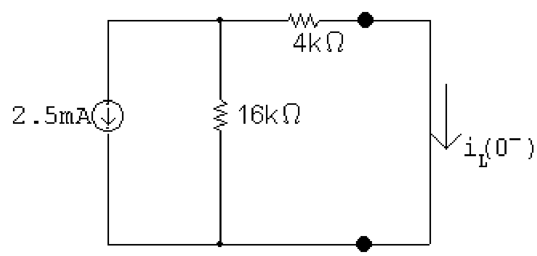
$i_o = 4 - i_L = 4 - 2e^{-1000t}\,\text{A}, \qquad t \geq 0^+$

[c] $4 - 2e^{-1000t} = 3.8$

$0.2 = 2e^{-1000t}$

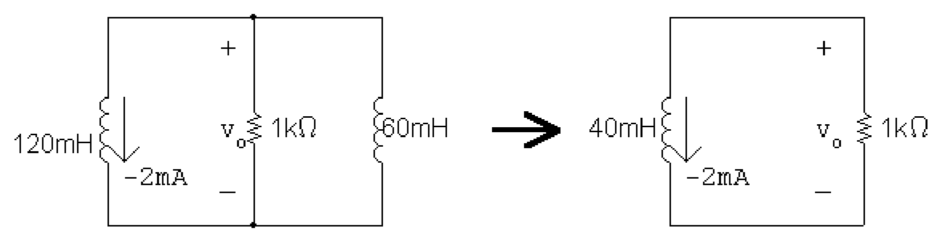
$e^{1000t} = 10 \qquad \therefore \quad t = 2.30\,\text{ms}$

P 5.44 **[a]** $t < 0$



$$i_L(0^-) = \frac{-2.5(16)}{(20)} = -2\,\text{mA}$$

$t \geq 0$



$$\tau = \frac{40 \times 10^{-3}}{10^3} = 40 \times 10^{-6}; \qquad 1/\tau = 25{,}000$$

$v_o = -1000(-2 \times 10^{-3})e^{-25{,}000t} = 2e^{-25{,}000t}\,\text{V}, \qquad t \geq 0^+$

[b] $w_{\text{del}} = \frac{1}{2}(40 \times 10^{-3})(4 \times 10^{-6}) = 80\,\text{nJ}$

[c] $0.95w_{\text{del}} = 76\,\text{nJ}$

$$\therefore \quad 76 \times 10^{-9} = \int_0^{t_o} \frac{4e^{-50{,}000t}}{1000}\,dt$$

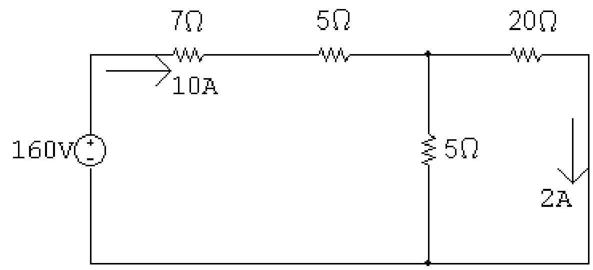
$$\therefore \quad 76 \times 10^{-9} = 80 \times 10^{-9} e^{-50{,}000t}\Big|_0^{t_o} = 80 \times 10^{-9}(1 - e^{-50{,}000t_o})$$

$\therefore \quad e^{-50{,}000t_o} = 0.05$

$50{,}000t_o = \ln 20 \quad \text{so} \quad t_o = 59.9\,\mu\text{s}$

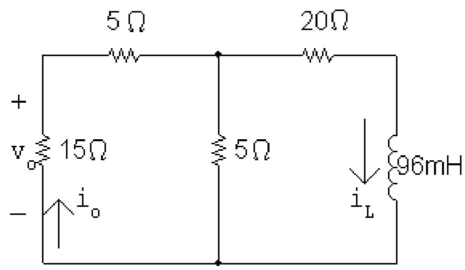
$$\therefore \quad \frac{t_o}{\tau} = \frac{59.9}{40} = 1.498 \quad \text{so} \quad t_o \approx 1.5\tau$$

P 5.45 $t < 0$:



$i_L(0^+) = 2\,\text{A}$

$t > 0$:



$$R_e = \frac{(20)(5)}{25} + 20 = 24\,\Omega$$

$$\tau = \frac{L}{R_e} = \frac{96}{24} \times 10^{-3} = 4\,\text{ms}; \qquad \frac{1}{\tau} = 250$$

$$\therefore \quad i_L = 2e^{-250t}\,\text{A}$$

$$\therefore \quad i_o = \frac{5}{25} i_L = 0.4e^{-250t}\,\text{A}$$

$$v_o = -15 i_o = -6e^{-250t}\,\text{V}, \quad t \geq 0^+$$

P 5.46 $p_{20\Omega} = 20 i_L^2 = 20(4)(e^{-250t})^2 = 80e^{-500t}\,\text{W}$

$$w_{20\Omega} = \int_0^\infty 80e^{-500t}\,dt = 80\frac{e^{-500t}}{-500}\Big|_0^\infty = 160\,\text{mJ}$$

$$w(0) = \frac{1}{2}(96)(10^{-3})(4) = 192\,\text{mJ}$$

$$\%\text{ diss } = \frac{160}{192}(100) = 83.33\%$$

P 5.47 $w(0) = \frac{1}{2}(20 \times 10^{-3})(10^2) = 1\,\text{J}$

$0.5w(0) = 0.5\,\text{J}$

$i_R = 10e^{-t/\tau}$

$p_{\text{diss}} = i_R^2 R = 100Re^{-2t/\tau}$

$$w_{\text{diss}} = \int_0^t R(100)e^{-2x/\tau}\,dx$$

$$w_{\text{diss}} = 100R\frac{e^{-2x/\tau}}{-2/\tau}\Big|_0^{t_o} = -50\tau R(e^{-2t_o/\tau} - 1) = 50L(1 - e^{-2t_o/\tau})$$

$50L = (50)(20) \times 10^{-3} = 1; \qquad t_o = 10\,\mu\text{s}$

$1 - e^{-2t_o/\tau} = 0.5$

$$e^{2t_o/\tau} = 2; \qquad \frac{2t_o}{\tau} = \frac{2t_oR}{L} = \ln 2$$

$$R = \frac{L\ln 2}{2t_o} = \frac{20 \times 10^{-3}\ln 2}{20 \times 10^{-6}} = 693.15\,\Omega$$

P 5.48 **[a]** $w(0) = \frac{1}{2}LI_g^2$

$$w_{\text{diss}} = \int_0^{t_o} I_g^2 Re^{-2t/\tau}\,dt = I_g^2 R\frac{e^{-2t/\tau}}{(-2/\tau)}\Big|_0^{t_o}$$
$$= \frac{1}{2}I_g^2 R\tau(1 - e^{-2t_o/\tau}) = \frac{1}{2}I_g^2 L(1 - e^{-2t_o/\tau})$$

$w_{\text{diss}} = \sigma w(0)$

$$\therefore \quad \frac{1}{2}LI_g^2(1 - e^{-2t_o/\tau}) = \tau\left(\frac{1}{2}LI_g^2\right)$$

$$1 - e^{-2t_o/\tau} = \sigma; \qquad e^{2t_o/\tau} = \frac{1}{(1-\sigma)}$$

$$\frac{2t_o}{\tau} = \ln\left[\frac{1}{(1-\sigma)}\right]; \qquad \frac{R(2t_o)}{L} = \ln[1/(1-\sigma)]$$

$$R = \frac{L\,\ln[1/(1-\sigma)]}{2t_o}$$

[b] $R = \dfrac{(20 \times 10^{-3})\ln[1/0.5]}{20 \times 10^{-6}}$

$R = 693.15\,\Omega$

P 5.49 **[a]** $v_o(t) = v_o(0^+)e^{-t/\tau}$

$\therefore \quad v_o(0^+)e^{-5\times 10^{-3}/\tau} = 0.25 v_o(0^+)$

$\therefore \quad e^{5\times 10^{-3}/\tau} = 4$

$\therefore \quad \tau = \dfrac{L}{R} = \dfrac{5 \times 10^{-3}}{\ln 4}$

$\therefore \quad L = \dfrac{250 \times 10^{-3}}{\ln 4} = 180.34\,\text{mH}$

[b] $i_L(0^-) = 60\left(\dfrac{1}{6}\right) = 10\,\text{mA} = i_L(0^+)$

$w_{\text{stored}} = \dfrac{1}{2}Li_L(0^+)^2 = \dfrac{1}{2}(R\tau)(100 \times 10^{-6}) = 2500\tau\,\mu\text{J}.$

$i_L(t) = 10e^{-t/\tau}\,\text{mA}$

$p_{50\Omega} = i_L^2(50) = 5000 \times 10^{-6}e^{-2t/\tau}$

$$w_{\text{diss}} = \int_0^{5\times 10^{-3}} 5000 \times 10^{-6}e^{-2t/\tau}\,dt$$

$$= 5000 \times 10^{-6}\frac{e^{-2t/\tau}}{(-2/\tau)}\Bigg|_0^{5\times 10^{-3}}$$

$$= 2500 \times 10^{-6}\tau\left[1 - e^{\frac{-10\times 10^{-3}}{\tau}}\right]$$

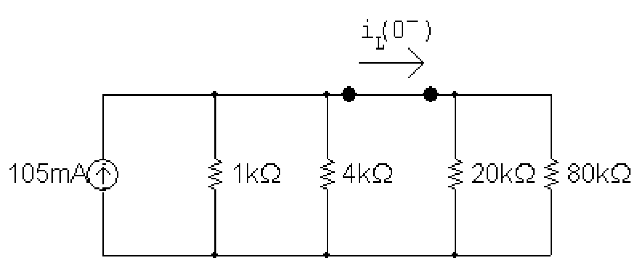
$e^{\frac{-10\times 10^{-3}}{\tau}} = e^{-2\ln 4} = 0.0625$

$w_{\text{diss}} = 2500 \times 10^{-6}\tau(0.9375)$

$\%\text{ diss } = \dfrac{2500 \times 10^{-6}\tau(0.9375)}{2500 \times 10^{-6}\tau} \times 100$

$w_{\text{diss}} = 93.75\%$

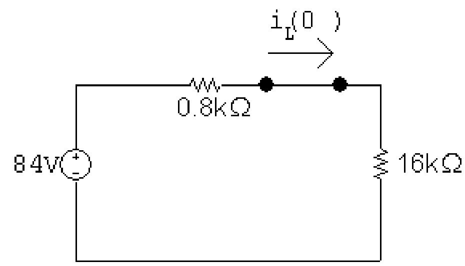
P 5.50 **[a]** $t < 0$



$1\,\text{k}\Omega \| 4\,\text{k}\Omega = 0.8\,\text{k}\Omega$

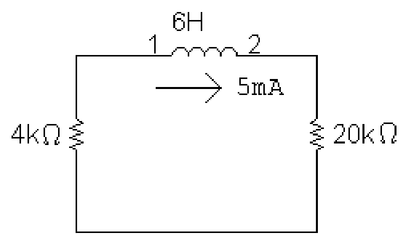
$20\,\mathrm{k}\Omega \| 80\,\mathrm{k}\Omega = 16\,\mathrm{k}\Omega$

$(105)(0.8) = 84\,\mathrm{V}$



$$i_L(0^-) = \frac{84}{16.8} = 5\,\mathrm{mA}$$

$t > 0$



$$\tau = \frac{L}{R} = \frac{6}{24} \times 10^{-3} = 250\,\mu\mathrm{s}; \qquad \frac{1}{\tau} = 4000$$

$$i_L(t) = 5e^{-4000t}\,\mathrm{mA}, \qquad t \geq 0$$

$$p_{4k} = 25 \times 10^{-6} e^{-8000t}(4000) = 0.10e^{-8000t}\,\mathrm{W}$$

$$w_{\mathrm{diss}} = \int_0^t 0.10e^{-8000x}\,dx = 12.5 \times 10^{-6}[1 - e^{-8000t}]\,\mathrm{J}$$

$$w(0) = \frac{1}{2}(6)(25 \times 10^{-6}) = 75\,\mu\mathrm{J}$$

$$0.10w(0) = 7.5\,\mu\mathrm{J}$$

$$12.5(1 - e^{-8000t}) = 7.5; \qquad \therefore \quad e^{8000t} = 2.5$$

$$t = \frac{\ln 2.5}{8000} = 114.54\,\mu\mathrm{s}$$

[b] $w_{\mathrm{diss}}(\mathrm{total}) = 75(1 - e^{-8000t})\,\mu\mathrm{J}$

$w_{\mathrm{diss}}(114.54\,\mu\mathrm{s}) = 45\,\mu\mathrm{J}$

$\% = (45/75)(100) = 60\%$

P 5.51 **[a]** $R = \dfrac{v}{i} = 20\,\mathrm{k}\Omega$

[b] $\dfrac{1}{\tau} = \dfrac{1}{RC} = 1000; \qquad C = \dfrac{1}{(10^3)(20 \times 10^3)} = 0.05\,\mu\text{F}$

[c] $\tau = \dfrac{1}{1000} = 1\,\text{ms}$

[d] $w(0) = \dfrac{1}{2}(0.05 \times 10^{-6})(10^4) = 250\,\mu\text{J}$

[e]

$$W_{\text{diss}} = \int_0^{t_o} \frac{v^2}{R} dt = \int_0^{t_o} \frac{(10^4)e^{-2000t}}{(20 \times 10^3)} dt$$

$$= 0.5\frac{e^{-2000t}}{-2000}\bigg|_0^{t_o} = 250(1 - e^{-2000t_o})\,\mu\text{J}$$

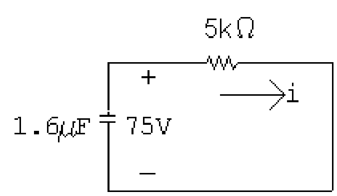
$$200 = 250(1 - e^{-2000t_o})$$

$$\therefore \quad e^{-2000t_o} = 0.2; \qquad e^{2000t_o} = 5$$

$$t_o = \frac{1}{2000}\ln 5; \qquad t_o \cong 804.72\,\mu\text{s}$$

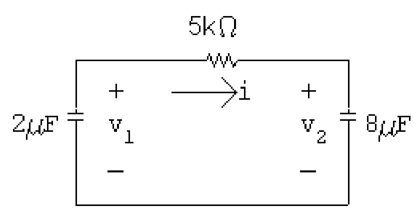
P 5.52 [a] $v_1(0^-) = v_1(0^+) = 75\,\text{V} \qquad\qquad v_2(0^+) = 0$

$$C_{\text{eq}} = 2 \times 8/10 = 1.6\,\mu\text{F}$$



$$\tau = (5)(1.6) \times 10^{-3} = 8\text{ms}; \qquad \frac{1}{\tau} = 125$$

$$i = \frac{75}{5} \times 10^{-3}e^{-125t} = 15e^{-125t}\,\text{mA}, \qquad t \geq 0^+$$



$$v_1 = \frac{-10^6}{2}\int_0^t 15 \times 10^{-3}e^{-125x}\,dx + 75 = 60e^{-125t} + 15\,\text{V}, \qquad t \geq 0$$

$$v_2 = \frac{10^6}{8}\int_0^t 15 \times 10^{-3}e^{-125x}\,dx + 0 = -15e^{-125t} + 15\,\text{V}, \qquad t \geq 0$$

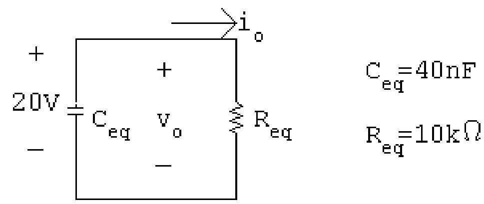
[b] $w(0) = \frac{1}{2}(2 \times 10^{-6})(5625) = 5625\,\mu\text{J}$

[c] $w_{\text{trapped}} = \frac{1}{2}(2 \times 10^{-6})(225) + \frac{1}{2}(8 \times 10^{-6})225 = 1125\,\mu\text{J}.$

$w_{\text{diss}} = \frac{1}{2}(1.6 \times 10^{-6})(5625) = 4500\,\mu\text{J}.$

Check: $w_{\text{trapped}} + w_{\text{diss}} = 1125 + 4500 = 5625\,\mu\text{J}; \qquad w(0) = 5625\,\mu\text{J}.$

P 5.53 **[a]** The equivalent circuit for $t > 0$:



$\tau = 0.4\,\text{ms}; \qquad 1/\tau = 2500$

$v_o = 20e^{-2500t}\,\text{V}, \qquad t \geq 0$

$i_o = 2e^{-2500t}\,\text{mA}, \qquad t \geq 0^+$

$i_{25k\Omega} = 2e^{-2500t}\left(\frac{15}{40}\right) = 0.75e^{-2500t}\,\text{mA}, \qquad t \geq 0^+$

$p_{25k\Omega} = (0.5625 \times 10^{-6}e^{-5000t})(25{,}000) = 14{,}062.5 \times 10^{-6}e^{-5000t}\,\text{W}$

$w_{25k\Omega} = \int_0^\infty 14{,}062.5 \times 10^{-6}e^{-5000t}\,dt = -2.8125 \times 10^{-6}(0 - 1) = 2.8125\,\mu\text{J}$

$w(0) = \frac{1}{2}(0.2 \times 10^{-6})(100) + \frac{1}{2}(0.05 \times 10^{-6})(900) = 32.5\,\mu\text{J}$

$\%\ \text{diss}\,(25\,\text{k}\Omega) = \frac{2.8125}{32.5} \times 100 = 8.65\%$

[b] $p_{625\Omega} = 625(2 \times 10^{-3}e^{-2500t})^2 = 2.5 \times 10^{-3}e^{-5000t}$

$w_{625\Omega} = \int_0^\infty p_{625}\,dt = 0.50\,\mu\text{J}$

$\%\ \text{diss}\,(625\Omega) = \frac{0.5}{32.5} \times 100 = 1.54\%$

$i_{15k\Omega} = 2e^{-2500t}\left(\frac{25}{40}\right) = 1.25e^{-2500t}\,\text{mA}, \quad t \geq 0^+$

$p_{15k\Omega} = (1.25 \times 10^{-3}e^{-2500t})^2(15{,}000) = 23.4375 \times 10^{-3}e^{-5000t}\,\text{W}$

$w_{15k\Omega} = \int_0^\infty 23.4375 \times 10^{-3}e^{-5000t}\,dt = 4.6875\,\mu\text{J}$

$\%\ \text{diss}\,(15\text{k}\Omega) = 14.42\%$

[c] $\sum w_{\text{diss}} = 2.8125 + 0.50 + 4.6875 = 8\,\mu\text{J}$

$$w_{\text{trapped}} = w(0) - \sum w_{\text{diss}} = 32.5 - 8 = 24.5\,\mu\text{J}$$

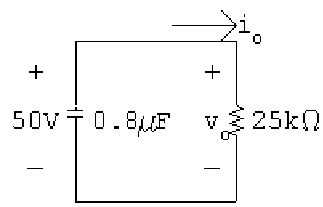
$$\%\ \text{trapped} = \frac{24.5}{32.5} \times 100 = 75.38\%$$

Check: $8.65 + 1.54 + 14.42 + 75.38 = 99.99 \approx 100\%$

P 5.54 **[a]** $\dfrac{1}{C_e} = 1 + \dfrac{1}{4} = 1.25$

$$\therefore \quad C_e = 0.8\,\mu\text{F}; \qquad v_o(0) = 60 - 10 = 50\,\text{V}$$

$$\tau = (0.8)(25) \times 10^{-3} = 20\,\text{ms}; \qquad \frac{1}{\tau} = 50$$



$$v_o = 50e^{-50t}\,\text{V}, \qquad t > 0^+$$

[b] $w_o = \dfrac{1}{2}(1 \times 10^{-6})(3600) + \dfrac{1}{2}(4 \times 10^{-6})(100) = 2\,\text{mJ}$

$$w_{\text{diss}} = \frac{1}{2}(0.8 \times 10^{-6})(2500) = 1\,\text{mJ}$$

$$\%\ \text{diss} = \frac{1}{2} \times 100 = 50\%$$

[c] $i_o = \dfrac{v_o}{25} \times 10^{-3} - 2e^{-50t}\,\text{mA}$

$$v_1 = -\frac{10^6}{4}\int_0^t 2 \times 10^{-3} e^{-50x}\,dx - 10 = -500\int_0^t e^{-50x}\,dx - 10$$

$$= -500\frac{e^{-50x}}{-50}\Big|_0^t - 10 = 10e^{-50t} - 20\,\text{V} \qquad t \geq 0$$

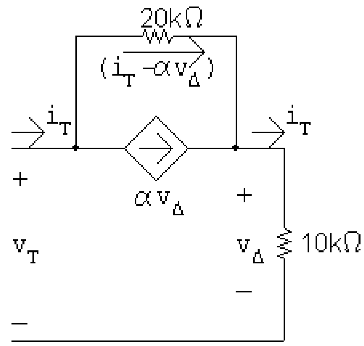
[d] $v_1 + v_2 = v_o$

$$v_2 = v_o - v_1 = 50e^{-50t} - 10e^{-50t} + 20 = 40e^{-50t} + 20\,\text{V} \quad t \geq 0$$

[e] $w_{\text{trapped}} = \dfrac{1}{2}(4 \times 10^{-6})(400) + \dfrac{1}{2}(1 \times 10^{-6})(400) = 1\,\text{mJ}$

$$w_{\text{diss}} + w_{\text{trapped}} = 2\,\text{mJ} \qquad \text{(check)}$$

P 5.55 **[a]** $\tau = RC = R_{\text{Th}}(0.2) \times 10^{-6} = 10^{-3}; \qquad \therefore \quad R_{\text{Th}} = \dfrac{1000}{0.2} = 5\,\text{k}\Omega$



$$v_T = 20 \times 10^3(i_T - \alpha v_\Delta) + 10 \times 10^3 i_T$$

$$v_\Delta = 10 \times 10^3 i_T$$

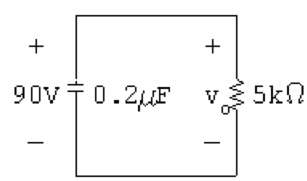
$$v_T = 30 \times 10^3 i_T - 20 \times 10^3 \alpha 10 \times 10^3 i_T$$

$$\frac{v_T}{i_T} = 30 \times 10^3 - 200 \times 10^6 \alpha = 5 \times 10^3$$

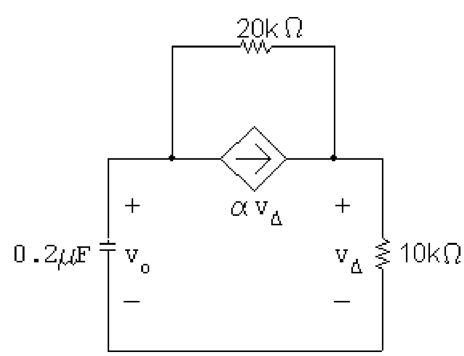
$$\therefore \quad 30 - 200{,}000\alpha = 5; \qquad \alpha = 125 \times 10^{-6} \text{ A/V}$$

[b] $v_o(0) = (0.018)(5000) = 90\,\text{V} \qquad t < 0$

$t > 0$:



$$v_o = 90e^{-1000t}\,\text{V}, \quad t \geq 0$$

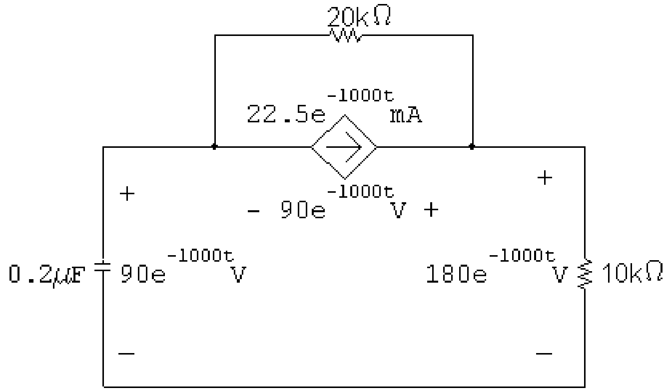


$$\frac{v_\Delta}{10 \times 10^3} + \frac{v_\Delta - v_o}{20{,}000} - 125 \times 10^{-6} v_\Delta = 0$$

$$2v_\Delta + v_\Delta - v_o - 2500 \times 10^{-3} v_\Delta = 0$$

$$\therefore \quad v_\Delta = 2v_o = 180e^{-1000t}\,\text{V}$$

P 5.56 [a]



$$p_{ds} = (-90e^{-1000t})(22.5 \times 10^{-3}e^{-1000t}) = -2025 \times 10^{-3}e^{-2000t}\,\text{W}$$

$$w_{ds} = \int_0^\infty p_{ds}\,dt = -1012.5\,\mu\text{J}.$$

$\therefore$ dependent source is delivering $1012.5\,\mu$J

[b] $$p_{10k} = \frac{(180)^2 e^{-2000t}}{10 \times 10^3}$$

$$w_{10k} = \int_0^\infty p_{10k}\,dt = 1620\,\mu\text{J}$$

$$p_{20k} = \frac{(90)^2 e^{-2000t}}{20 \times 10^3}$$

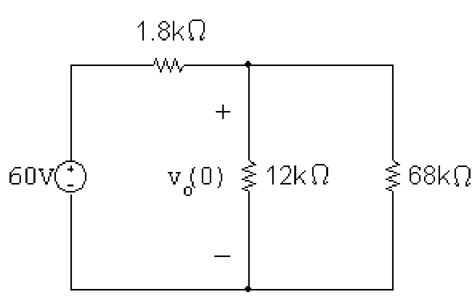
$$w_{20k} = \int_0^\infty p_{20k}\,dt = 202.5\,\mu\text{J}$$

$$w_c(0) = \frac{1}{2}(0.2) \times 10^{-6}(90)^2 = 810\,\mu\text{J}$$

$$\sum w_{\text{dev}} = 810 + 1012.5 = 1822.5\,\mu\text{J}$$

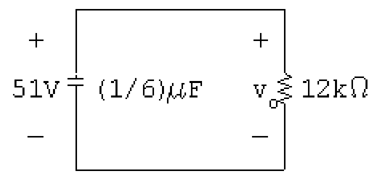
$$\sum w_{\text{diss}} = 202.5 + 1620 = 1822.5\,\mu\text{J}.$$

P 5.57 [a] $t < 0$:



$$v_o(0) = \frac{(60)(10.2)}{12} = 51\,\text{V}$$

$t > 0$:



$$\tau = \frac{1}{6}(12) \times 10^{-3} = 2\,\text{ms}; \qquad \frac{1}{\tau} = 500$$

$$v_o = 51e^{-500t}\,\text{V}, \quad t \geq 0$$

$$p = \frac{v_o^2}{12} \times 10^{-3} = 216.75 \times 10^{-3} e^{-1000t}\,\text{W}$$

$$w_{\text{diss}} = \int_0^{2\times 10^{-3}} 216.75 \times 10^{-3} e^{-1000t}\,dt = 216.75 \times 10^{-6}(1 - e^{-2})$$
$$= 187.42\,\mu\text{J}$$

[b] $w(0) = \left(\frac{1}{2}\right)\left(\frac{1}{6}\right)(51)^2 \times 10^{-6} = 216.75\,\mu\text{J}$

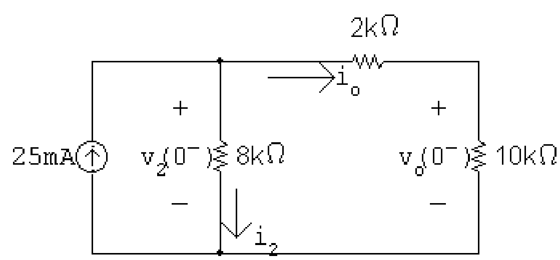
$$0.95w(0) = 205.9125\,\mu\text{J}$$

$$\int_0^{t_o} 216.75 \times 10^{-3} e^{-1000x}\,dx = 205.9125 \times 10^{-6}$$

$$\int_0^{t_o} e^{-1000x}\,dx = 0.95 \times 10^{-3}$$

$$\therefore \quad 1 - e^{-1000t_o} = 0.95; \qquad e^{1000t_o} = 20; \quad \text{so} \quad t_o = 3\,ms$$

P 5.58 **[a]** $t < 0$:



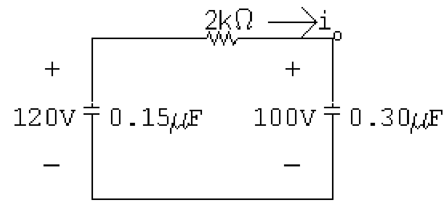
$$i_o(0^-) = \frac{(25)(8)}{(20)} = 10\,\text{mA}$$

$$v_o(0^-) = (10)(10) = 100\,\text{V}$$

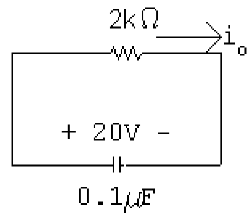
$$i_2(0^-) = 25 - 10 = 15\,\text{mA}$$

$$v_2(0^-) = 15(8) = 120\,\text{V}$$

$t > 0$

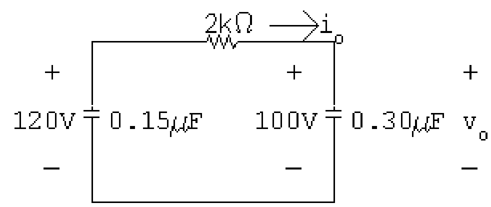


$$\tau = RC = 0.2\,\text{ms} = 200\,\mu\text{s}; \qquad \frac{1}{\tau} = 5000$$



$$i_o(t) = \frac{20}{2 \times 10^3} e^{-t/\tau} = 10e^{-5000t}\,\text{mA}, \qquad t \geq 0^+$$

[b]



$$\begin{aligned} v_o &= \frac{10^6}{0.3}\int_0^t 10 \times 10^{-3} e^{-5000x}\,dx + 100 \\ &= \frac{10^5}{3}\frac{e^{-5000x}}{-5000}\bigg|_0^t + 100 \\ &= -(20/3)e^{-5000t} + (20/3) + 100 \\ v_o &= [-(20/3)e^{-5000t} + (320/3)]\,\text{V}, \qquad t \geq 0 \end{aligned}$$

[c] $w_{\text{trapped}} = (1/2)(0.15) \times 10^{-6}(320/3)^2 + (1/2)(0.3) \times 10^{-6}(320/3)^2$

$w_{\text{trapped}} = 2560\,\mu\text{J}.$

Check:

$$w_{\text{diss}} = \frac{1}{2}(0.1 \times 10^{-6})(20)^2 = 20\,\mu\text{J}$$

$$w(0) = \frac{1}{2}(0.15) \times 10^{-6}(120)^2 + \frac{1}{2}(0.3 \times 10^{-6})(100)^2 = 2580\,\mu\text{J}.$$

$$w_{\text{trapped}} + w_{\text{diss}} = w(0)$$

$2560 + 20 = 2580 \qquad$ OK.

P 5.59 **[a]** $v(0) = \frac{(8)(27)(33)}{60} = 118.80\,\text{V}$

$$R_e = \frac{(3)(6)}{9} = 2\,\text{k}\Omega$$

$$\tau = R_eC = (2000)(0.25)\times 10^{-6} = 500\,\mu\text{s}; \qquad \frac{1}{\tau} = 2000$$

$$v = 118.80e^{-2000t}\,\text{V} \qquad t \geq 0$$

$$i_o = \frac{v}{3000} = 39.6e^{-2000t}\,\text{mA}, \quad t \geq 0^+$$

[b] $w(0) = \frac{1}{2}(0.25)(118.80)^2 = 1764.18\,\mu\text{J}$

$$i_{4k} = \frac{118.80e^{-2000t}}{6} = 19.8e^{-2000t}\,\text{mA}$$

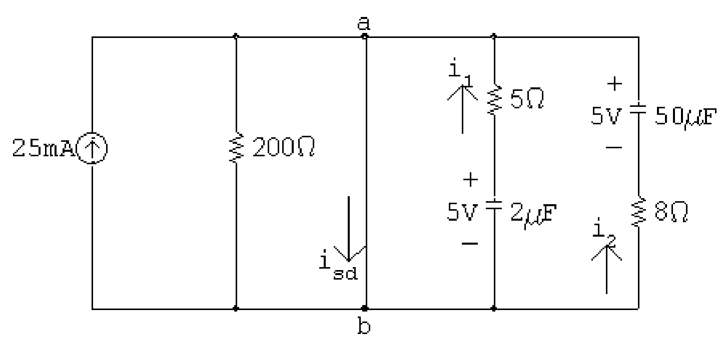
$$p_{4k} = [(19.8)e^{-2000t}]^2(4000)\times 10^{-6} = 1568.16\times 10^{-3}e^{-4000t}$$

$$w_{4k} = 1568.16\times 10^{-3}\frac{e^{-4000x}}{-4000}\Bigg|_0^{250\times 10^{-6}} = 392.04(1-e^{-1})\,\mu\text{J}$$

$$= 247.82\,\mu\text{J}$$

$$\% = \frac{247.82}{1764.18}\times 100 = 14.05\%$$

P 5.60 **[a]** At $t = 0^-$ the voltage on each capacitor will be $5\,\text{V}(25\times 10^{-3}\times 200)$, positive at the upper terminal. Hence at $t \geq 0^+$ we have



$$\therefore \quad i_{sd}(0^+) = 0.025 + \frac{5}{5} + \frac{5}{8} = 1.65\,\text{A}$$

At $t = \infty$, both capacitors will have completely discharged.

$$\therefore \quad i_{sd}(\infty) = 25\,\text{mA}$$

[b] $i_{sd}(t) = 0.025 + i_1(t) + i_2(t)$

$\tau_1 = (5)(2) \times 10^{-6} = 10\,\mu\text{s}$

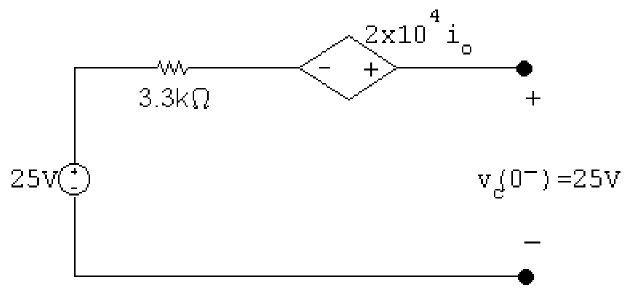
$\tau_2 = (8)(50 \times 10^{-6}) = 400\,\mu\text{s}$

$\therefore \quad i_1(t) = e^{-10^5 t}\,\text{A}, \qquad t \geq 0^+$

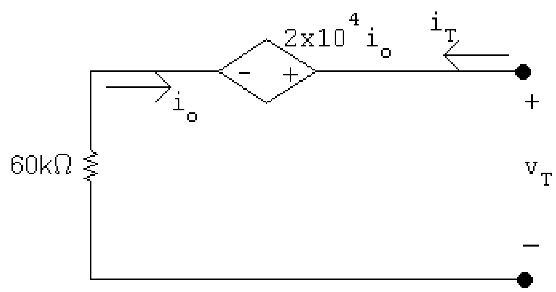
$i_2(t) = 0.625e^{-2500t}\,\text{A}, \qquad t \geq 0$

$\therefore \quad i_{sd} = 25 + 1000e^{-100{,}000t} + 625e^{-2500t}\,\text{mA}, \qquad t \geq 0^+$

P 5.61 $t < 0$

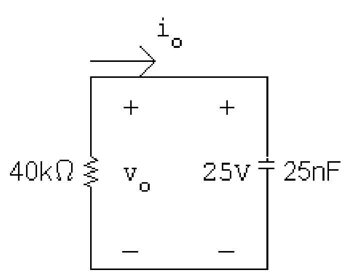


$t > 0$



$$v_T = 2 \times 10^4 i_o + 60{,}000 i_T$$
$$= 20{,}000(-i_T) + 60{,}000 i_T = 40{,}000 i_T$$

$$\therefore \quad \frac{v_T}{i_T} = R_{\text{Th}} = 40\,\text{k}\Omega$$

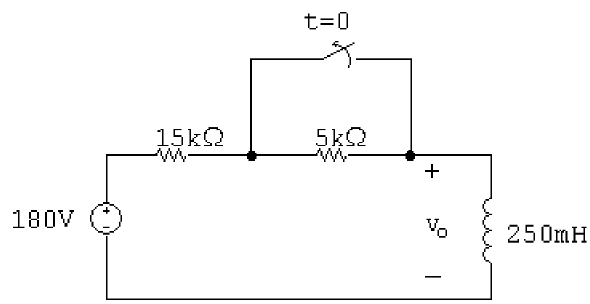


$\tau = RC = 1\,\text{ms}; \qquad \dfrac{1}{\tau} = 1000$

$v_o = 25e^{-1000t}\,\text{V}, \qquad t \geq 0$

$$i_o = 25 \times 10^{-9}\frac{d}{dt}[25e^{-1000t}] = -625e^{-1000t}\,\mu\text{A}, \qquad t \geq 0^+$$

P 5.62 After making a Thévenin equivalent we have



$$I_o = 180/15 = 12\,\text{mA}$$

$$\tau = (0.25/20) \times 10^{-3} = 0.125 \times 10^{-4}; \qquad \frac{1}{\tau} = 80{,}000$$

$$\frac{V_s}{R} = \frac{180}{20} = 9\,\text{mA}$$

$$i_o = 9 + (12 - 9)e^{-80{,}000t} = 9 + 3e^{-80{,}000t}\,\text{mA}$$

$$v = [180 - 12(20)]e^{-80{,}000t} = -60e^{-80{,}000t}\,\text{V}$$

P 5.63 **[a]** $v_o(0^+) = -I_g R_2; \qquad \tau = \dfrac{L}{R_1 + R_2}$

$$v_o(\infty) = 0$$

$$v_o(t) = -I_g R_2 e^{-[(R_1+R_2)/L]t}\,\text{V}, \qquad t \geq 0$$

[b] $v_o = -(12 \times 10^{-3})(5 \times 10^3)e^{-[\frac{15{,}000+5000}{0.25}]t} = -60e^{-80{,}000t}\,\text{V}, \qquad t \geq 0$

[c] $v_o(0^+) \to \infty$, and the duration of $v_o(t) \to 0$

[d] $v_{\text{sw}} = R_2 i_o; \qquad \tau = \dfrac{L}{R_1 + R_2}$

$$i_o(0^+) = I_g; \qquad i_o(\infty) = I_g\frac{R_1}{R_1 + R_2}$$

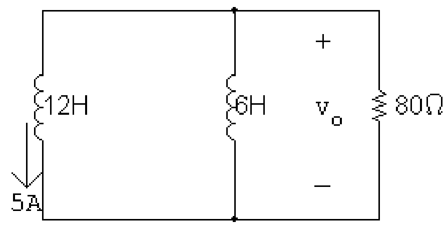
Therefore $\quad i_o(t) = \dfrac{I_g R_1}{R_1 + R_2} + \left[I_g - \dfrac{I_g R_1}{R_1 + R_2}\right]e^{-[(R_1+R_2)/L]t}$

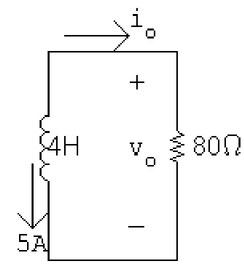
$$i_o(t) = \frac{R_1 I_g}{(R_1 + R_2)} + \frac{R_2 I_g}{(R_1 + R_2)}e^{-[(R_1+R_2)/L]t}$$

Therefore $\quad v_{\text{sw}} = \dfrac{R_1 I_g}{(1 + R_1/R_2)} + \dfrac{R_2 I_g}{(1 + R_1/R_2)}e^{-[(R_1+R_2)/L]t}, \qquad t \geq 0$

[e] $|v_{\rm sw}(0^+)| \to \infty; \qquad \text{duration} \to 0$

P 5.64 $t > 0$





$$\tau = \frac{4}{80} = \frac{1}{20}$$

$$i_o = -5e^{-20t}\,\text{A}, \qquad t \geq 0$$

$$v_o = 80i_o = -400e^{-20t}\,\text{V}, \qquad t > 0^+$$

$$-400e^{-20t} = -80; \qquad e^{20t} = 5$$

$$\therefore \quad t = \frac{1}{20}\ln 5 = 80.47\,\text{ms}$$

P 5.65 **[a]** $w_{\rm diss} = \frac{1}{2}L_e i^2(0) = \frac{1}{2}(4)(25) = 50\,\text{J}$

[b]

$$i_{12H} = \frac{1}{12}\int_0^t (-400)e^{-20x}\,dx + 5$$

$$= \frac{-100}{3}\frac{e^{-20x}}{-20}\Big|_0^t + 5 = \frac{5}{3}e^{-20t} + \frac{10}{3}\,\text{A}$$

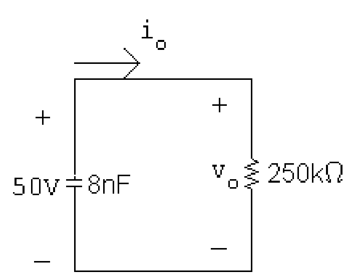
$$i_{6H} = \frac{1}{6}\int_0^t (-400)e^{-20x}\,dx + 0$$

$$= \frac{-200}{3}\frac{e^{-20x}}{-20}\Big|_0^t + 0 = \frac{10}{3}e^{-20t} - \frac{10}{3}\,\text{A}$$

$$w_{\rm trapped} = \frac{1}{2}(18)(100/9) = 100\,\text{J}$$

[c] $w(0) = \frac{1}{2}(12)(25) = 150\,\text{J}$

P 5.66 **[a]** For $t > 0$:



$$\tau = RC = 250 \times 10^3 \times 8 \times 10^{-9} = 2\,\text{ms}; \qquad \frac{1}{\tau} = 500$$

$$v_o = 50e^{-500t}\,\text{V}, \qquad t \geq 0^+$$

[b] $$i_o = \frac{v_o}{250} \times 10^{-3} = \frac{50e^{-500t}}{250} \times 10^{-3} = 200e^{-500t}\,\mu\text{A}$$

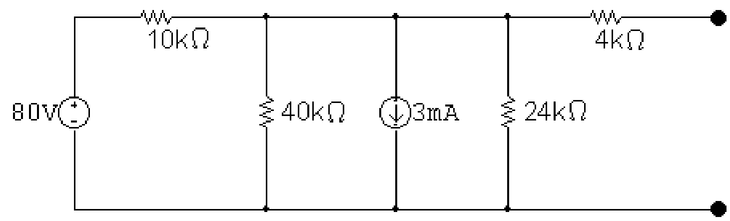
$$v_1 = \frac{-10^9}{40} \times 200 \times 10^{-6} \int_0^t e^{-500x}\,dx + 50 = 10e^{-500t} + 40\,\text{V}, \quad t \geq 0$$

P 5.67 **[a]** $$w = \frac{1}{2}C_e v_e^2 = \frac{1}{2}(8 \times 10^{-9})(2500) = 10\,\mu\text{J}$$

[b] $$w_{\text{trapped}} = \frac{1}{2}(40)^2(50) \times 10^{-9} = 40\,\mu\text{J}$$

[c] $$w(0) = \frac{1}{2}(40 \times 10^{-9})(2500) = 50\,\mu\text{J}$$

P 5.68 For $t < 0$

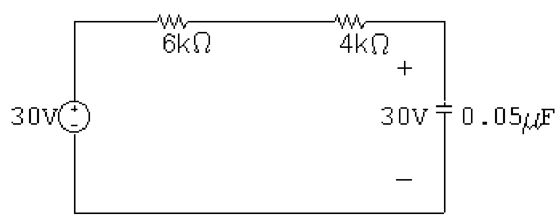


$80/10{,}000 = 8\,\text{mA}, \qquad 10\,\text{k}\Omega \| 40\,\text{k}\Omega \| 24\,\text{k}\Omega = 6\,\text{k}\Omega$

$8\,\text{mA} - 3\,\text{mA} = 5\,\text{mA}$

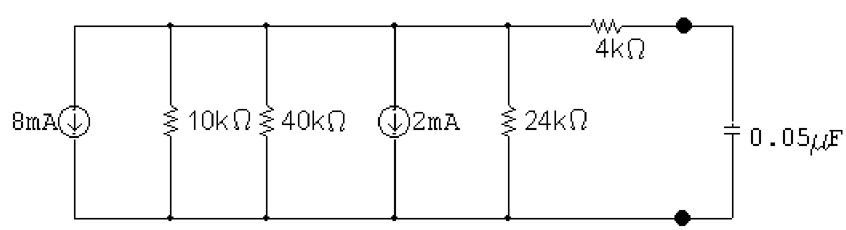
$5\,\text{mA} \times 6\,\text{k}\Omega = 30\,\text{V}$

$t < 0$

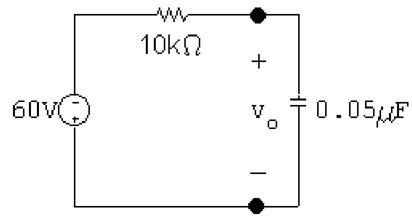


$\therefore \quad v_o(0^-) = v_o(0^+) = 30\,\text{V}$

$t > 0$



$$v_o(\infty) = -10 \times 10^{-3}(6 \times 10^3) = -60\,\text{V}$$

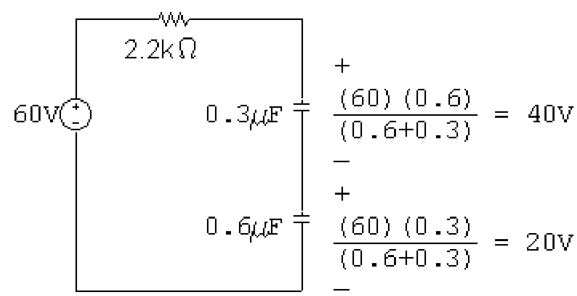


$\tau = 0.5\,\mathrm{ms}; \qquad \frac{1}{\tau} = 2000$

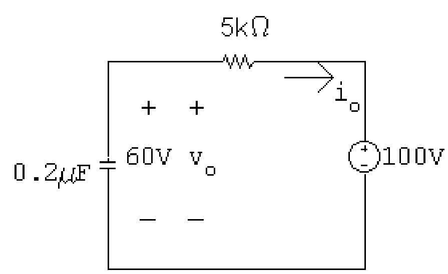
$v_o = -60 + (30 - (-60))e^{-2000t}$

$v_o = -60 + 90e^{-2000t}\,\mathrm{V} \qquad t \geq 0$

P 5.69 **[a]** $t < 0$



$t > 0$



$v_o(0^-) = v_o(0^+) = 60\,\mathrm{V}$

$v_o(\infty) = 100\,\mathrm{V}$

$\tau = (0.2)(5) \times 10^{-3} = 1\,\mathrm{ms}; \qquad 1/\tau = 1000$

$v_o = 100 - 40e^{-1000t}\,\mathrm{V}, \qquad t \geq 0$

[b] $i_o = -C\frac{dv_o}{dt} = -0.2 \times 10^{-6}[40{,}000e^{-1000t}]$

$= -8e^{-1000t}\,\mathrm{mA}; \qquad t \geq 0^+$

[c] $v_1 = \dfrac{-10^6}{0.3}\displaystyle\int_0^t -8\times 10^{-3}e^{-1000x}\,dx + 40$

$= 66.67 - 26.67e^{-1000t}\,\text{V}, \qquad t \geq 0$

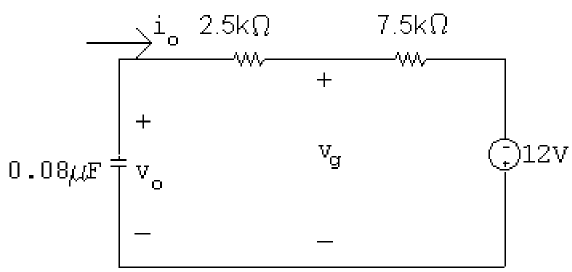
[d] $v_2 = \dfrac{-10^6}{0.6}\displaystyle\int_0^t -8\times 10^{-3}e^{-1000x}\,dx + 20$

$= 33.33 - 13.33e^{-1000t}\,\text{V}, \qquad t \geq 0$

[e] $w_{\text{trapped}} = \frac{1}{2}(0.3)10^{-6}(66.67)^2 + \frac{1}{2}(0.6)10^{-6}(33.33)^2$

$= 666.67 + 333.33 = 1000\,\mu\text{J}.$

P 5.70 **[a]** $v_o(0^-) = v_o(0^+) = 48\,\text{V}$



$v_o(\infty) = -12\,\text{V}; \qquad \tau = 0.8\,\text{ms}; \qquad \dfrac{1}{\tau} = 1250$

$v_o = -12 + (48 - (-12))e^{-1250t}$

$v_o = -12 + 60e^{-1250t}\,\text{V}, \qquad t \geq 0$

[b] $i_o = -0.08\times 10^{-6}[-75{,}000e^{-1250t}]$

$i_o = 6e^{-1250t}\,\text{mA}, \qquad t \geq 0^+$

[c] $v_g = v_o - 2.5\times 10^3 i_o$

$v_g = -12 + 45e^{-1250t}\,\text{V}$

[d] $v_g(0^+) = -12 + 45 = 33\,\text{V}$

Checks:

$v_g(0^+) = i_o(0^+)7.5\times 10^3 - 12 = 45 - 12 = 33\,\text{V}$

$i_{10\text{k}} = \dfrac{v_g}{10\text{k}} = -1.2 + 4.5e^{-1250t}\,\text{mA}$

$i_{30\text{k}} = \dfrac{v_g}{30\text{k}} = -0.4 + 1.5e^{-1250t}\,\text{mA}$

$-i_o + i_{10} + i_{30} + 1.6 = 0 \qquad \text{(ok)}$

P 5.71 **[a]** $0 \leq t \leq 1\,\text{ms}$:

$$v_c(0^+) = 0; \qquad v_c(\infty) = 50\,\text{V};$$

$$RC = 400 \times 10^3(0.01 \times 10^{-6}) = 4\,\text{ms}; \qquad 1/RC = 250$$

$$v_c = 50 - 50e^{-250t}$$

$$v_o = 50 - 50 + 50e^{-250t} = 50e^{-250t}\,\text{V}, \qquad 0 \leq t \leq 1\,\text{ms}$$

$1\,\text{ms} \leq t \leq \infty$:

$$v_c(1\,\text{ms}) = 50 - 50e^{-0.25} = 11.06\,\text{V}$$

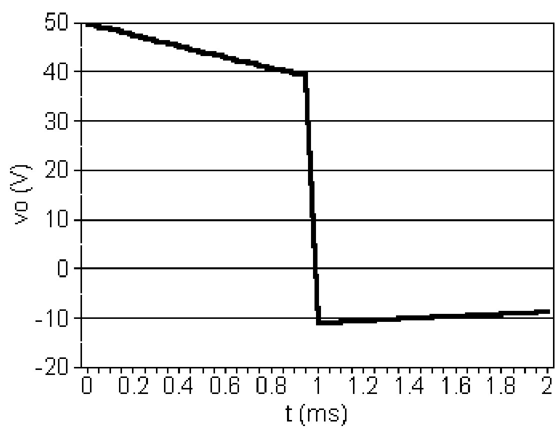
$$v_c(\infty) = 0\,\text{V}$$

$$\tau = 4\,\text{ms}; \qquad 1/\tau = 250$$

$$v_c = 11.06e^{-250(t\,-\,0.001)}\,\text{V}$$

$$v_o = -v_c = -11.06e^{-250(t\,-\,0.001)}\,\text{V}, \qquad 1\,\text{ms} \leq t \leq \infty$$

[b]



P 5.72 **[a]** $t < 0; \qquad v_o = 0$

$0 \leq t \leq 10\,\text{ms}$:

$$\tau = (50)(0.4) \times 10^{-3} = 20\,\text{ms}; \qquad 1/\tau = 50$$

$$v_o = 40 - 40e^{-50t}\,\text{V}, \qquad 0 \leq t \leq 10\,\text{ms}$$

$$v_o(10\,\text{ms}) = 40(1 - e^{-0.5}) = 15.74\,\text{V}$$

$10\,\text{ms} \leq t \leq 20\,\text{ms}$:

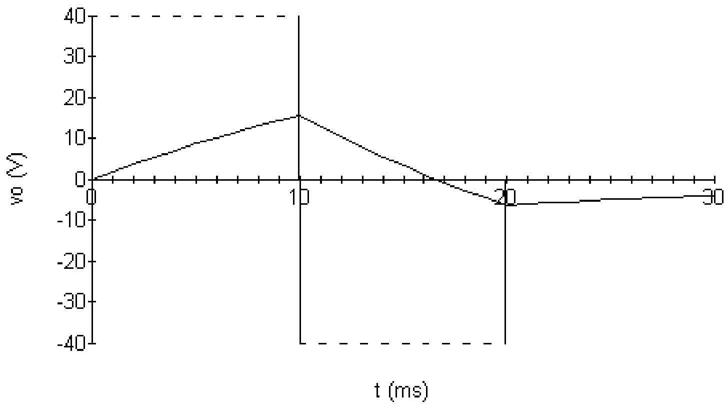
$$v_o = -40 + 55.74e^{-50(t-0.01)}\,\text{V}$$

$$v_o(20\,\text{ms}) = -40 + 55.74e^{-0.5} = -6.19\,\text{V}$$

$20\,\text{ms} \leq t \leq \infty$:

$$v_o = -6.19e^{-50(t-0.02)}\,\text{V}$$

[b]



[c] $t \leq 0: \qquad v_o = 0$

$0 \leq t \leq 10\,\text{ms}$:

$$\tau = 10(0.4 \times 10^{-3}) = 4\,\text{ms}$$

$$v_o = 40 - 40e^{-250t}\,\text{V}, \qquad 0 \leq t \leq 10\,\text{ms}$$

$$v_o(10\,\text{ms}) = 40 - 40e^{-2.5} = 36.72\,\text{V}$$

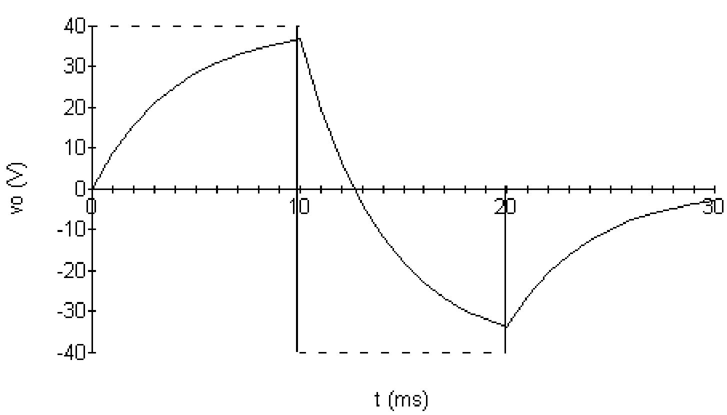
$10\,\text{ms} \leq t \leq 20\,\text{ms}$:

$$v_o = -40 + 76.72e^{-250(t-0.01)}\,\text{V}, \qquad 10\,\text{ms} \leq t \leq 20\,\text{ms}$$

$$v_o(20\,\text{ms}) = -40 + 76.72e^{-2.5} = -33.7\,\text{V}$$

$20\,\text{ms} \leq t \leq \infty$:

$$v_o = -33.7e^{-250(t-0.02)}\,\text{V}, \qquad 20\,\text{ms} \leq t \leq \infty$$



P 5.73 $$\frac{1}{R_sC_f} = \frac{10^6}{50 \times 10^3(0.05)} = 400$$

Therefore,

$$
\begin{aligned}
v_o &= -400\int_0^t 75\cos 5000x\,dx + 0 \\
&= -30{,}000\left[\frac{1}{5000}\sin 5000x\,\Big|_0^t\right] \\
&= -6\sin 5000t\text{V}
\end{aligned}
$$

P 5.74 **[a]** For $0 \leq t \leq 25$ ms:

$$v_s = \frac{600}{25}t = 24t$$

$$\frac{1}{R_sC_f} = \frac{(10^6)(10^{-3})}{(7.5)(0.16)} = \frac{1000}{1.2}$$

$$
\begin{aligned}
\therefore\quad v_o &= -\frac{1000}{1.2}\int_0^t 24x\,dx + 0 \\
&= -20{,}000\left[\frac{x^2}{2}\right]_0^t \\
&= -10^4t^2\text{V} \qquad 0 \leq t \leq 25 \text{ ms}
\end{aligned}
$$

[b] For 25 ms $\leq t \leq 75$ ms:

$$v_s = 1.2 - 24t$$

$$v_o(25\text{ ms}) = -10^4(625\times 10^{-6}) = -6.25\text{V}$$

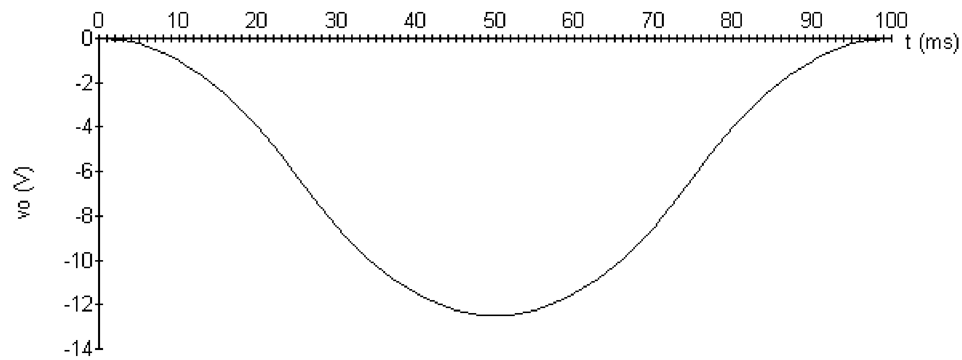
$$
\begin{aligned}
\therefore\quad v_o &= -\frac{1000}{1.2}\int_{25\times 10^{-3}}^t (1.2 - 24x)\,dx - 6.25 \\
&= -\frac{1000}{1.2}\left[1.2x - \frac{24x^2}{2}\right]_{25\times 10^{-3}}^t - 6.25 \\
&= 10^4t^2 - 10^3t + 12.5\text{V} \qquad 25\text{ ms } \leq t \leq 75\text{ ms}
\end{aligned}
$$

$$v_o(75\text{ ms}) = -10^4(5625\times 10^{-6}) - 75 + 12.5 = -6.25\text{V}$$

[c] For 75 ms $\leq t \leq 100$ ms:

$$
\begin{aligned}
v_o &= -\frac{1000}{1.2}\int_{75\times 10^{-3}}^t (-2.4 + 24x)\,dx - 6.25 \\
&= -10^4t^2 + 2000t - 100\text{V} \qquad 75\text{ ms } \leq t \leq 100\text{ ms}
\end{aligned}
$$

[d]



P 5.75 **[a]** $v_o(t_1) = \dfrac{4 \times 10^6}{0.8R}(0.25) = 10$

$$\therefore \quad R = \frac{10^6}{8} = 125\,\text{k}\Omega$$

[b] $t_2 - t_1 = \dfrac{4}{10}(250) = 100$ ms

P 5.76 **[a]** $t_2 - t_1 = \dfrac{3.6}{10}(250) = 90$ ms

$$N_2 = \frac{90}{1000}(10^5) = 9000 \text{ pulses}$$

[b] From (a) we have 9000/3.6 or 2500 pulses/volt.

$\therefore$ 7000 pulses corresponds to 7000/2500 = 2.8V

$\therefore$ $v_a = 2.8$V

P 5.77 Summing the currents at the inverting input terminal yields

$$\frac{0 - v_{\text{ref}}}{R_{\text{ref}}} + \frac{0 - v_x}{R_x} = 0$$

Solving for v_x gives

$$v_x = -\left(\frac{V_{\text{ref}}}{R_{\text{ref}}}\right) R_x$$

Since $(V_{\text{ref}}/R_{\text{ref}})$ is a constant fixed by the circuit designer we see that v_x is directly proportional to the unknown resistance R_x.

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