3
Techniques of Circuit Analysis

Drill Exercises

DE 3.1 **[a]** 11,8 resistors, 2 independent sources, 1 dependent source

[b] 9

[c] 9, $R_4 - R_5$ forms an essential branch as does $R_8 - 10$ V. The remaining seven branches contain a single element.

[d] 7 **[e]** 6 **[f]** 4 **[g]** 6

DE 3.2 Solution given in text.

DE 3.3 Solution given in text.

DE 3.4 Solution given in text.

DE 3.5 **[a]** The two node voltage equations are

$$-15 + \frac{v_1}{60} + \frac{v_1}{15} + \frac{v_1 - v_2}{5} = 0$$

$$5 + \frac{v_2}{2} + \frac{v_2 - v_1}{5} = 0$$

Solving, $v_1 = 60$ V and $v_2 = 10$ V;
Therefore, $i_1 = (v_1 - v_2)/5 = 10$ A

[b] $p_{15\text{A}}$ (del) $= (15)(60) = 900$ W

[c] $p_{5\text{A}} = -5(10) = -50$ W

DE 3.6 Use the lower node as the reference node. Let v_1 = node voltage across 1 Ω resistor and v_2 = node voltage across 12 Ω resistor. Then

$$\frac{v_1}{1} + \frac{v_1 - v_2}{8} = 4.5$$

$$\frac{v_2}{12} + \frac{v_2 - v_1}{8} + \frac{v_2 - 30}{4} = 0$$

Solving, $v_1 = 6$ V $\qquad v_2 = 18$ V Thus, $i = (v_1 - v_2)/8 = -1.5$ A
$v = v_2 + 2i = 15$ V

DE 3.7 Use the lower node as the reference node. Let v_1 = node voltage across the 8 Ω resistor, let v_2 = node voltage across the 4 Ω resistor. Then

$$\frac{v_1 - 50}{6} + \frac{v_1}{8} + \frac{v_1 - v_2}{2} - 3i_1 = 0$$

$$-5 + \frac{v_2}{4} + \frac{v_2 - v_1}{2} + 3i_1 = 0$$

$$i_1 = \frac{50 - v_1}{6}$$

Solving, $v_1 = 32$ V; $\qquad v_2 = 16$ V; $\qquad i_1 = 3$ A $p_{50\text{V}} = -50i_1 = -150$ W (delivering)
$p_{5\text{A}} = -5(v_2) = -80$ W $\quad$ (delivering)
$p_{3i_1} = 3i_1(v_2 - v_1) = -144$ W $\quad$ (delivering)

DE 3.8 Use the lower node as the reference node. Let v_1 = node voltage across the 7.5 Ω resistor and v_2 = node voltage across the 2.5 Ω resistor. Place the dependent voltage source inside a supernode between the node voltages v and v_2. The node voltage equations are

node 1: $\quad \dfrac{v_1}{7.5} + \dfrac{v_1 - v}{2.5} = 4.8$

supernode: $\quad \dfrac{v - v_1}{2.5} + \dfrac{v}{10} + \dfrac{v_2}{2.5} + \dfrac{v_2 - 12}{1} = 0$

We also have: $v + i_x = v_2$ and $i_x = v_1/7.5$. Solving this set of equations for v gives $v = 8$ V

DE 3.9 $\dfrac{v_1 - 60}{2} + \dfrac{v_1}{24} + \dfrac{v_1 - (60 + 6i_\phi)}{3} = 0, \qquad i_\phi = \dfrac{60 + 6i_\phi - v_1}{3}$
Therefore $\quad v_1 = 48$ V

DE 3.10
$\dfrac{v_o}{40} + \dfrac{v_o - 10}{10} + \dfrac{v_o + 20i_\Delta}{20} = 0, \qquad i_\Delta = \dfrac{10 - v_o}{10} + \dfrac{10 + 20i_\Delta}{30}$
Therefore $\quad v_o = 24$ V

DE 3.11 Define three clockwise mesh currents i_1, i_2, and i_3 in the lower left, upper, and lower right windows. The three mesh-current equations are

$$\begin{aligned} 80 &= 31i_1 - 5i_2 - 26i_3 \\ 0 &= -5i_1 + 125i_2 - 90i_3 \\ 0 &= -26i_1 - 90i_2 + 124i_3 \end{aligned}$$

[a] Solving, $i_1 = 5$ A; therefore the 80 V source is delivering 400 W to the circuit.

[b] Solving, $i_3 = 2.5$ A; therefore $p_{8\Omega} = (6.25)(8) = 50$ W

DE 3.12 **[a]** $b = 8, \qquad n = 6, \qquad b - n + 1 = 3$

[b] Define three clockwise mesh currents i_1, i_2, and i_3 in the upper, lower left, and lower right windows. The three mesh-current equations are

$$-(-3v_\phi) + 19i_1 - 2i_2 - 3i_3 = 0$$

$$25 - 10 = -2i_1 + 7i_2 - 5i_3$$

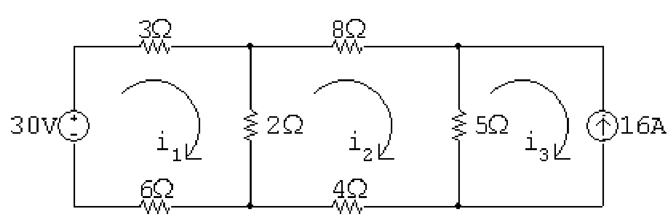
$$10 = -3i_1 - 5i_2 + 9i_3$$

We also have $v_\phi = 3(i_3 - i_1)$
Solving for i_1 and i_3 gives $i_1 = -1$ A, $i_3 = 3$ A Therefore $v_\phi = 12$ V and $p_{3v_\phi} = -(-3v_\phi)i_1 = -36$ W

DE 3.13 Let i_a = lower left mesh current cw, let i_b = upper mesh current cw, and i_c = lower right mesh current cw. Then

$$\begin{aligned} 25 &= 14i_a - 6i_b - 8i_c \\ 0 &= -6i_a + 16i_b - 8i_c \\ 0 &= -8i_a - 8i_b + 16i_c + 5i_\phi \\ i_\phi &= i_a, \qquad i_a = 4\text{ A}, \qquad i_c = 2A \\ v_o &= 8(i_a - i_c) = 16\text{ V} \end{aligned}$$

DE 3.14



Mesh 1: $30 = 11i_1 - 2i_2$

Mesh 2: $0 = -2i_1 + 19i_2 - 5i_3$

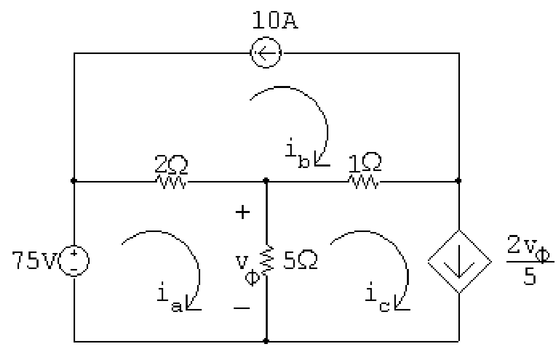
Current source: $i_3 = -16$ A

Solution gives $i_1 = 2$ A, $i_2 = -4$ A, $i_3 = -16$ A

The current in the $2\,\Omega$ resistor is $i_1 - i_2 = 6$ A

$\therefore \quad P_{2\,\Omega} = (6)^2(2) = 72$ W

DE 3.15



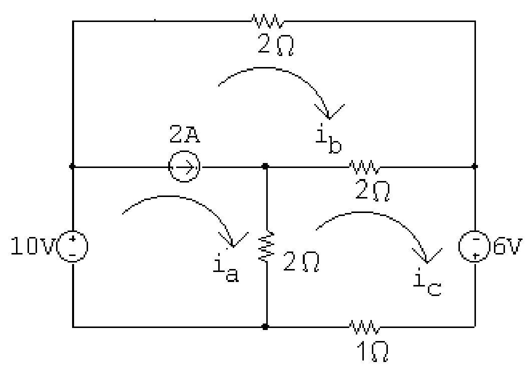
Mesh a: $7i_a - 2i_b - 5i_c = 75$

Current sources: $i_b = -10$ A; $\quad i_c = \dfrac{2v_\phi}{5}$

Dependent variable: $v_\phi = 5(i_a - i_c)$

Solution: $i_a = 15$ A; $\quad i_b = -10$ A; $\quad i_c = 10$ A; $\quad v_\phi = 25$ V

DE 3.16



Supermesh a,b: $2i_a + 4i_b - 4i_c = 10$

Mesh c: $-2i_a - 2i_b + 5i_c = 6$

Current source: $i_a - i_b = 2$ A

Solution: $i_a = 7$ A; $\quad i_b = 5$ A; $\quad i_c = 6$ A

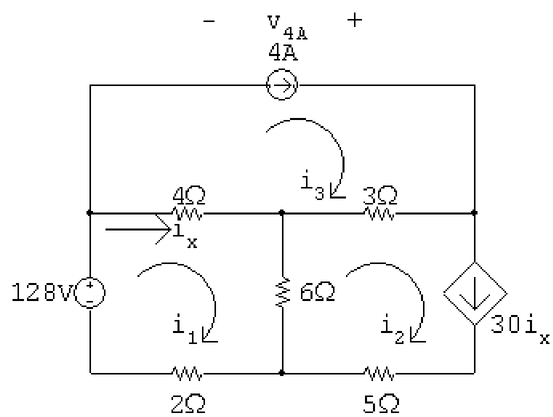
$\therefore \; p_{1\,\Omega} = i_c^2(1) = (6)^2(1) = 36$ W

DE 3.17 Let v_1 denote the voltage across the 2 A source. Let v_1 be a voltage rise in the direction of the 2 A current.

$$\frac{v_1 - 20}{15} - 2 + \frac{v_1 - 25}{10} = 0, \qquad v_1 = 35 \text{ V}$$

$$p_{2A} = -35(2) = -70 \text{ W} \qquad p_{2A}(\text{del}) = 70 \text{ W}$$

DE 3.18



Mesh 1: $\quad 12i_1 - 6i_2 - 4i_3 = 128$

Mesh 2: $\quad -6i_1 + 14i_2 - 3i_3 + 30i_x = 0$

Current source: $\quad i_3 = 4 \text{ A}$

Dependent variable: $\quad i_x = i_1 - i_3$

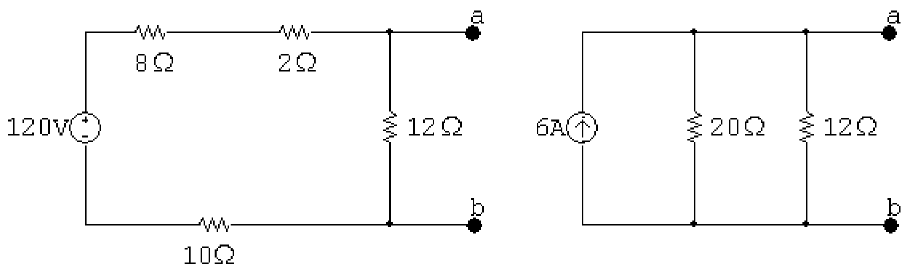
Solution: $\quad i_1 = 9$ A; $\qquad i_2 = -6$ A; $\qquad i_3 = 4$ A; $\qquad i_x = 5$ A

$\therefore \quad v_{4A} = 3(i_3 - i_2) - 4i_x = 10$ V

The power delivered by the 4A source is $\quad p_{4A} = (10)(4) = 40$ W

DE 3.19 To find the Thévenin resistance, deactivate the independent voltage source and note that $R_{\text{Th}} = [5\|20 + 8]\|12 = 6\,\Omega$. With the terminals a, b open, the current delivered by the 72 V source is 72/24 or 3 A. The current (left-to-right) in the 5 Ω resistor is (20/25)(3) = 2.4 A, and the current (left-to-right) in the 12 Ω resistor is (5/25)3 or 0.6 A. The Thévenin voltage $v_{\text{Th}} = v_{ab}$ is the drop across the 8 Ω resistor plus the drop across the 20 Ω resistor. Thus $v_{\text{Th}} = (8)(0.6) + (20)(3) = 64.8$ V.

DE 3.20 After one source transformation, the circuit becomes



Therefore $I_N = 6$ A, $R_N = 20\|12 = 7.5\,\Omega$

DE 3.21 Find the Thévenin equivalent with respect to A, B.

$$\frac{V_{\rm Th} + 36}{12{,}000} + \frac{V_{\rm Th}}{60{,}000} - 0.018 = 0, \qquad V_{\rm Th} = 150 \text{ V}$$

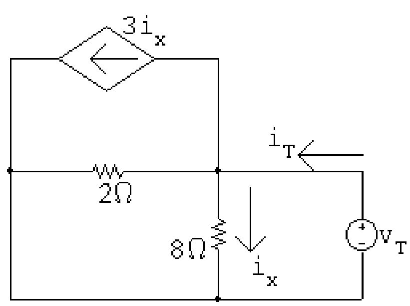
$$R_{\rm Th} = 15{,}000 + \frac{(60{,}000)(12{,}000)}{72{,}000} = 25 \text{ k}\Omega;$$

$$\text{Therefore, } v_{\rm meas} = 150\left(\frac{100{,}000}{125{,}000}\right) = 120 \text{ V}$$

DE 3.22 Summing the currents away from node a, where $v_{\rm Th} = v_{ab}$ We have

$$\frac{v_{\rm Th}}{8} + 4 + 3i_x + \frac{v_{\rm Th} - 24}{2} = 0, \qquad i_x = \frac{v_{\rm Th}}{8}$$

Solving for $v_{\rm Th}$ yields $v_{\rm Th} = 8$ V



$$i_{\rm T} = 4i_x + v_{\rm T}/2, \qquad i_x = v_{\rm T}/8$$

Therefore $i_{\rm T} = v_{\rm T}$ and $R_{\rm Th} = v_{\rm T}/i_{\rm T} = 1\,\Omega$

DE 3.23 Use the bottom node as the reference. Let v_1 be the node voltage across the 60 Ω resistor. Then

$$\frac{v_1}{60} + \frac{v_1 - (v_{\text{Th}} + 160 i_\Delta)}{20} - 4 = 0,$$

$$\frac{v_{\text{Th}}}{40} + \frac{v_{\text{Th}}}{80} + \frac{v_{\text{Th}} + 160 i_\Delta - v_1}{20} = 0$$

$$i_\Delta = \frac{v_{\text{Th}}}{40}, \quad \text{therefore} \quad v_{\text{Th}} = 30 \text{ V}$$

Let i_{T} be the test current into terminal a:

$$i_{\text{T}} = \frac{v_{\text{T}}}{80} + \frac{v_{\text{T}}}{40} + \frac{v_{\text{T}} + 160(v_{\text{T}}/40)}{80}, \qquad \frac{i_{\text{T}}}{v_{\text{T}}} = \frac{1}{10}$$

Therefore, $R_{\text{Th}} = 10\,\Omega$

DE 3.24 First find the Thévenin equivalent circuit. To find v_{Th}, use the bottom node as the reference. Let $v_{\text{Th}} = v_{ab}$ and v_1 = node voltage across the 20 V − 4 Ω branch. The two node Voltage equations are

$$\frac{v_{\text{Th}} - 100 - v_\phi}{4} + \frac{v_{\text{Th}} - v_1}{4} = 0, \qquad (v_\phi = v_1 - 20)$$

$$\frac{v_1 - 100}{4} + \frac{v_1 - 20}{4} + \frac{v_1 - v_{\text{Th}}}{4} = 0$$

Solving for v_{Th} gives $v_{\text{Th}} = 120$ V. To find R_{Th}, deactivate the two independent sources and apply a test voltage source across a, b. Let v_{T} be positive at a and i_{T} directed into a. Then the two node Voltage equations are

$$\frac{v_{\text{T}} - v_\phi}{4} + \frac{v_{\text{Th}} - v_\phi}{4} = i_{\text{T}}, \qquad \frac{v_\phi}{4} + \frac{v_\phi}{4} + \frac{v_\phi - v_{\text{T}}}{4} = 0$$

Therefore $\quad v_\phi = v_{\text{T}}/3 \quad$ and $\quad 12 i_{\text{T}} = 4 v_{\text{T}}$
So $\quad R_{\text{Th}} = v_{\text{T}}/i_{\text{T}} = 3\,\Omega$

[a] For maximum power transfer, $R_{\text{L}} = R_{\text{Th}} = 3\,\Omega$

[b] $p_{\max} = (120/6)^2(3) = 1200$ W

DE 3.25 When $R_L = 3\,\Omega$, the voltage across R_L is 60 V. As before, let v_1 be the node voltage across the 20 V—4 Ω branch, then $v_\phi = v_1 - 20$ and

$$\frac{60}{3} + \frac{60 - v_1}{4} + \frac{60 - 100 - v_\phi}{4} = 0$$

Therefore $v_1 = 60$ V and $v_\phi = 40$ V. The current out of the plus terminal of the 100 V source is

$$i_1 = \frac{100 - 60}{4} + \frac{100 + 40 - 60}{4} = 10 + 20 = 30 \text{ A}$$

[a] Therefore 100 V is delivering 3000 W to the circuit.

[b] The current out of the plus terminal of the dependent source is 20 A. Therefore the dependent source is delivering 800 W to the circuit.

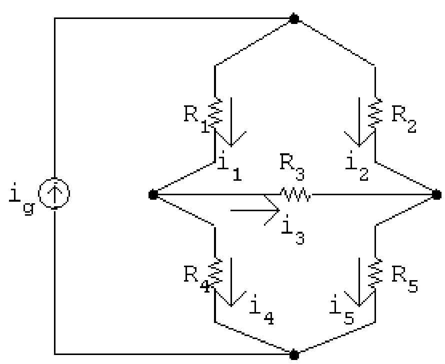
[c] The load power is (1200/3800)100 or 31.58% of this generated power.

Problems

P 3.1 **[a]** Five

[b] Three

[c]



Sum the currents at any three of the four essential nodes a, b, c, and d. Using nodes a, b, and c we get

$-i_g + i_1 + i_2 = 0$

$-i_1 + i_4 + i_3 = 0$

$i_5 - i_2 - i_3 = 0$

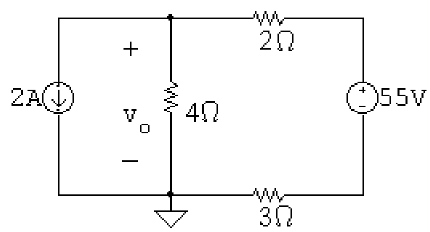
[d] Two.

[e] Sum the voltages around two independent closed paths, avoiding a path that contains the independent current source since the voltage across the current source is not known. Using the upper and lower meshes formed by the five resistors gives

$$R_1 i_1 + R_3 i_3 - R_2 i_2 = 0$$

$$R_3 i_3 + R_5 i_5 - R_4 i_4 = 0$$

P 3.2



$$2 + \frac{v_o}{4} + \frac{v_o - 55}{5} = 0$$

$$v_o = 20 \text{ V}$$

$$p_{2\text{A}} = (20)(2) = 40 \text{ W} \quad \text{(absorbing)}$$

P 3.3 Let v_2 be the node voltage across the 80 Ω resistor, positive at the upper terminal.

Then $\quad -4 + \frac{v_1}{20} + \frac{v_2}{80} + \frac{v_2}{40} = 0$

(Note we have created a super node in writing this expression.)

$$v_1 + 60 = v_2$$

$$\therefore \quad v_1 = 20 \text{ V}$$

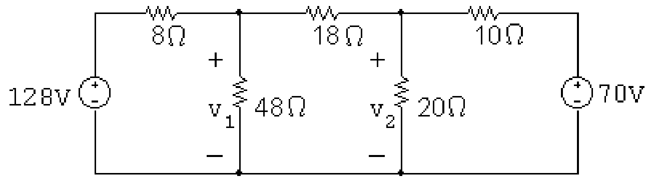
$$\therefore \quad v_2 = 80 \text{ V}$$

$p_{\text{del}} = 60 i_g$where i_g is the current out of the positive terminal

$$4 = i_g + \frac{v_1}{20}; \qquad i_g = 3 \text{ A}$$

$$\therefore \quad p_{\text{del}} = 60(3) = 180 \text{ W}$$

P 3.4 **[a]**



$$\frac{v_1}{48} + \frac{v_1 - 128}{8} + \frac{v_1 - v_2}{18} = 0$$

$$\frac{v_2}{20} + \frac{v_2 - v_1}{18} + \frac{v_2 - 70}{10} = 0$$

Solving, $v_1 = 96$ V; $\qquad v_2 = 60$ V

$$i_\mathrm{a} = \frac{128 - 96}{8} = 4 \text{ A}$$

$$i_\mathrm{b} = \frac{96}{48} = 2 \text{ A}$$

$$i_\mathrm{c} = \frac{96 - 60}{18} = 2 \text{ A}$$

$$i_\mathrm{d} = \frac{60}{20} = 3 \text{ A}$$

$$i_\mathrm{e} = \frac{60 - 70}{10} = -1 \text{ A}$$

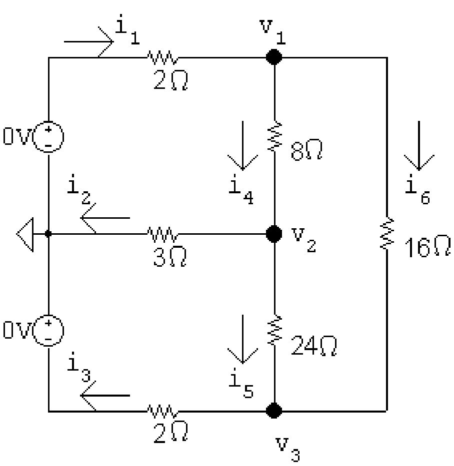
[b] $p_\mathrm{dev} = 128(4) + 70(1) = 582$ W

P 3.5 Use the lower terminal of the 5 Ω resistor as the reference node.

$$\frac{v_o - 60}{10} + \frac{v_o}{5} + 3 = 0$$

Solving, $\qquad v_o = 10$ V

P 3.6 **[a]**



$$\frac{v_1 - 110}{2} + \frac{v_1 - v_2}{8} + \frac{v_1 - v_3}{16} = 0$$
$$\frac{v_2 - v_1}{8} + \frac{v_2}{3} + \frac{v_2 - v_3}{24} = 0$$
$$\frac{v_3 + 110}{2} + \frac{v_3 - v_2}{24} + \frac{v_3 - v_1}{16} = 0$$

Solving, $v_1 = 74.64$ V; $v_2 = 11.79$ V; $v_3 = -82.5$ V

Thus, $i_1 = \frac{110 - v_1}{2} = 17.68$ A $\quad i_4 = \frac{v_1 - v_2}{8} = 7.86$ A

$i_2 = \frac{v_2}{3} = 3.93$ A $\quad i_5 = \frac{v_2 - v_3}{24} = 3.93$ A

$i_3 = \frac{v_3 + 110}{2} = 13.75$ A $\quad i_6 = \frac{v_1 - v_3}{16} = 9.82$ A

[b] $\sum P_{\text{dev}} = 110i_1 + 110i_3 = 3457.14$ W

$$\sum P_{\text{dis}} = i_1^2(2) + i_2^2(3) + i_3^2(2) + i_4^2(8) + i_5^2(24) + i_6^2(16) = 3457.14 \text{ W}$$

P 3.7 $2.4 + \frac{v_1}{125} + \frac{v_1 - v_2}{25} = 0$

$$\frac{v_2 - v_1}{25} + \frac{v_2}{250} + \frac{v_2}{375} - 3.2 = 0$$

Solving, $v_1 = 25$ V; $\quad v_2 = 90$ V
CHECK:

$$p_{125\Omega} = \frac{(25)^2}{125} = 5 \text{ W}$$

$$p_{25\Omega} = \frac{(90 - 25)^2}{25} = 169 \text{ W}$$

$$p_{250\Omega} = \frac{(90)^2}{250} = 32.4 \text{ W}$$

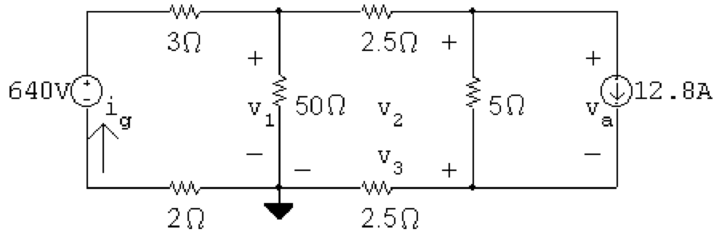
$$p_{375\Omega} = \frac{(90)^2}{375} = 21.6 \text{ W}$$

$$p_{2.4\text{A}} = (25)(2.4) = 60 \text{ W}$$

$$\sum p_{\text{abs}} = 5 + 169 + 32.4 + 21.6 + 60 = 288 \text{ W}$$

$$\sum p_{\text{dev}} = (90)(3.2) = 288 \text{ W} \quad \text{(CHECKS)}$$

P 3.8 **[a]**



$$\frac{v_1}{50} + \frac{v_1 - 640}{5} + \frac{v_1 - v_2}{2.5} = 0$$

$$\frac{v_2 - v_1}{2.5} + \frac{v_2 - v_3}{5} + 12.8 = 0$$

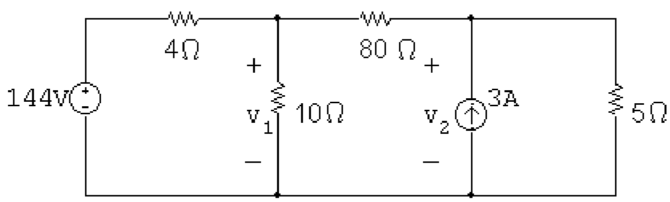
$$\frac{v_3}{2.5} + \frac{v_3 - v_2}{5} - 12.8 = 0$$

Solving,$v_1 = 380\ V$; $v_2 = 269$ V; $v_3 = 111$ V,

[b] $i_g = \dfrac{640 - 380}{5} = 52$ A

$p_g(\text{del}) = (640)(52) = 33{,}280$ W

P 3.9

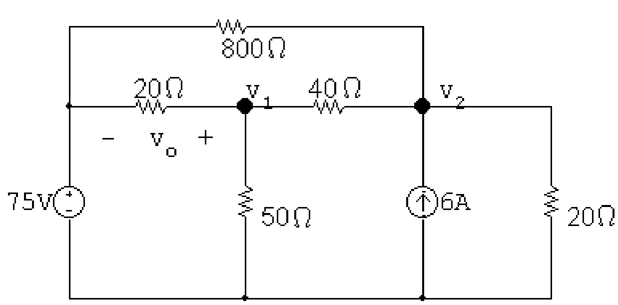


$$\frac{v_1 - 144}{4} + \frac{v_1}{10} + \frac{v_1 - v_2}{80} = 0 \qquad \text{so} \qquad 29v_1 - v_2 = 2880$$

$$-3 + \frac{v_2 - v_1}{80} + \frac{v_2}{5} = 0 \qquad \text{so} \qquad -v_1 + 17v_2 = 240$$

Solving, $v_1 = 100$ V; $v_2 = 20$ V

P 3.10



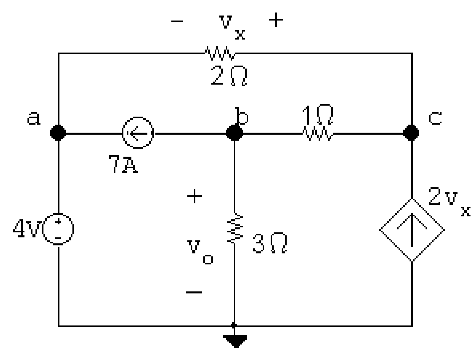
$$\frac{v_1 - 75}{20} + \frac{v_1}{50} + \frac{v_1 - v_2}{40} = 0$$

$$\frac{v_2 - v_1}{40} + \frac{v_2 - 75}{800} - 6 + \frac{v_2}{200} = 0$$

Solving, $v_1 = 115$ V; $\quad v_2 = 287$ V

$\therefore \quad v_o = 115 - 75 = 40$ V

P 3.11



$v_{\rm a} = 4$ V

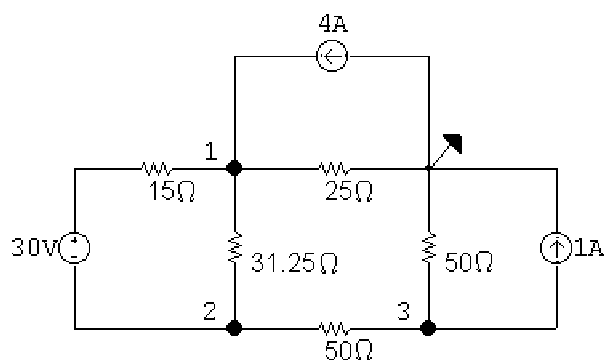
$$7 + \frac{v_{\rm b}}{3} + \frac{v_{\rm b} - v_{\rm c}}{1} = 0$$

$$-2v_x + \frac{v_{\rm c} - v_{\rm b}}{1} + \frac{v_{\rm c} - v_{\rm a}}{2} = 0$$

$v_x = v_{\rm c} - v_{\rm a} = v_{\rm c} - 4$

Solving, $v_o = v_{\rm b} = 1.5$ V

P 3.12



$$\frac{v_1 - (v_2 + 30)}{15} + \frac{v_1 - v_2}{31.25} + \frac{v_1}{25} - 4 = 0$$

$$-\left[\frac{v_1 - (v_2 + 30)}{15}\right] + \frac{v_2 - v_3}{50} + \frac{v_2 - v_1}{31.25} = 0$$

$$\frac{v_3 - v_2}{50} + \frac{v_3}{50} + 1 = 0$$

Solving, $v_1 = 76$ V; $v_2 = 46$ V; $v_3 = -2$ V; $i_{30\text{V}} = 0$ A

$p_{4\text{A}} = -4v_1 = -4(76) = -304$ W (del)

$p_{1\text{A}} = (1)(-2) = -2$ W (del)

$p_{30\text{V}} = (30)(0) = 0$ W

$p_{15\Omega} = (0)^2(15) = 0$ W

$$p_{25\Omega} = \frac{v_1^2}{25} = \frac{76^2}{25} = 231.04 \text{ W}$$

$$p_{31.25\Omega} = \frac{(v_1 - v_2)^2}{31.25} = \frac{30^2}{31.25} = 28.8 \text{ W}$$

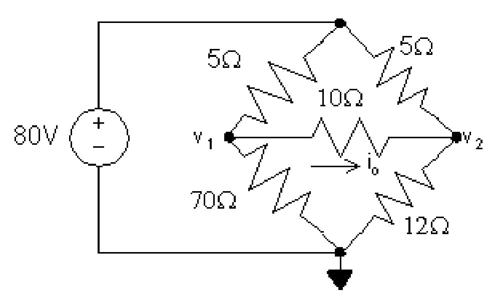
$$p_{50\Omega}(\text{lower}) = \frac{(v_2 - v_3)^2}{50} = \frac{48^2}{50} = 46.08 \text{ W}$$

$$p_{50\Omega}(\text{right}) = \frac{v_3^2}{50} = \frac{4}{50} = 0.08 \text{ W}$$

$$\sum p_{\text{diss}} = 0 + 231.04 + 28.8 + 46.8 + 0.08 = 306 \text{ W}$$

$$\sum p_{\text{dev}} = 304 + 2 = 306 \text{ W} \qquad \text{(CHECKS)}$$

P 3.13



$$\frac{v_1}{70} + \frac{v_1 - v_2}{10} + \frac{v_1 - 80}{5} = 0$$
$$\frac{v_2}{12} + \frac{v_2 - v_1}{10} + \frac{v_2 - 80}{5} = 0$$

Solving, $v_1 = 70$ V; $v_2 = 60$ V

Thus, $i_o = \dfrac{v_1 - v_2}{10} = 1$ A

P 3.14 **[a]** $\dfrac{v_0 - 60}{10} + \dfrac{v_o}{5} + 3 = 0; \quad v_o = 10$ V

[b] Let v_x = voltage drop across 3 A source

$v_x = v_o - (100)(3) = -290$ V

$p_{3\text{A}}$ (developed) $= (3)(290) = 870$ W

[c] Let i_g = current into positive terminal of 60 V source

$i_g = (10 - 60)/10 = -5$ A

$p_{60\text{V}}$ (developed) $= (5)(60) = 300$ W

[d] $\sum p_{\text{dis}} = (5)^2(10) + (3)^2(100) + (10)^2/5 = 1170$ W

$\sum p_{\text{dis}} = 300 + 870 = 1170$ W

[e] v_o is independent of any finite resistance connected in series with the 3 A current source

P 3.15 **[a]** From the solution to Problem 3.5 we know $v_o = 10$ V, therefore

$p_{3\text{A}} = 3v_o = 30$ W

$\therefore \quad p_{3\text{A}}$ (developed) $= -30$ W

[b] The current into the negative terminal of the 60 V source is

$$i_g = \frac{60 - 10}{10} = 5 \text{ A}$$

$p_{60\text{V}} = -60(5) = -300$ W

$\therefore \quad p_{60\text{V}}$ (developed) $= 300$ W

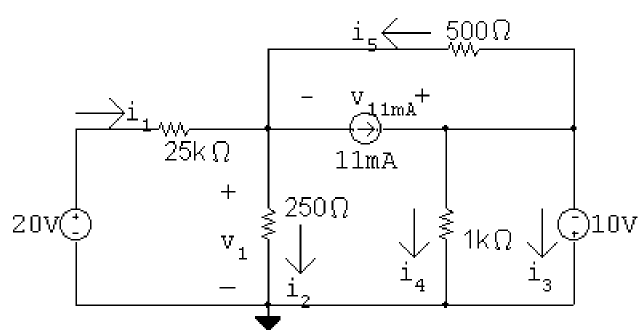
[c] $p_{10\Omega} = (5)^2(10) = 250$ W

$p_{5\Omega} = (10)^2/5 = 20$ W

$\sum p_{\text{dev}} = 300$ W

$\sum p_{\text{dis}} = 250 + 20 + 30 = 300$ W

P 3.16 **[a]**



$$\frac{v_1 - 20}{25 \times 10^3} + \frac{v_1}{0.25 \times 10^3} + 11 \times 10^{-3} + \frac{v_1 + 10}{0.5 \times 10^3} = 0$$

$$v_1 = -5 \text{ V}$$

$$i_1 = \frac{20+5}{25{,}000} = 1 \text{ mA}$$

$$i_2 = \frac{v_1}{250} = \frac{-5}{250} = -20 \text{ mA}$$

$$i_5 = \frac{-10+5}{500} = -10 \text{ mA}$$

$$i_4 = \frac{-10}{1000} = -10 \text{ mA}$$

$$i_4 + i_3 - 11 + i_5 = 0$$

$$\therefore \quad i_3 = 11 - i_4 - i_5 = 11 + 10 + 10 = 31 \text{ mA}$$

[b] $p_{20\text{V}} = 20i_1 = 20(1 \times 10^{-3}) = 20 \text{ mW}$

$$p_{10\text{V}} = 10i_3 = 10(31 \times 10^{-3}) = 310 \text{ mW}$$

$$v_{11\text{mA}} + v_1 = -10, \quad v_{11\text{mA}} = -10 + 5 = -5 \text{ V}$$

$$p_{11\text{mA}} = -11v_{11\text{mA}} = -55 \text{ mW} \qquad \text{(del)}$$

$$\sum p_{\text{dev}} = 20 + 310 = 330 \text{ mW}$$

$$p_{25\text{k}} = 25 \times 10^3 i_1^2 = 25 \text{ mW}$$

$$p_{0.25\text{k}} = 0.25 \times 10^3 i_2^2 = 100 \text{ mW}$$

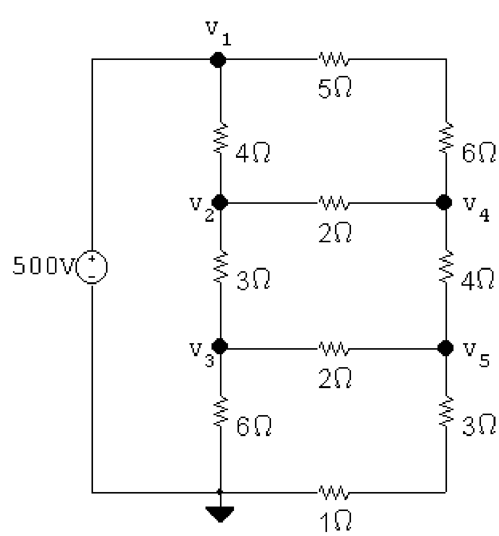
$$p_{0.5\text{k}} = 0.5 \times 10^3 i_5^2 = 50 \text{ mW}$$

$$p_{1\text{k}} = 1 \times 10^3 i_4^2 = 100 \text{ mW}$$

$$\sum p_{\text{diss}} = 25 + 100 + 50 + 100 + 55 = 330 \text{ mW}$$

$$\sum p_{\text{diss}} = \sum p_{\text{dev}} = 330 \text{ mW}$$

P 3.17 **[a]**



$$\frac{v_2 - 500}{4} + \frac{v_2 - v_4}{2} + \frac{v_2 - v_3}{3} = 0$$
$$\frac{v_3 - v_2}{3} + \frac{v_3}{6} + \frac{v_3 - v_5}{2} = 0$$
$$\frac{v_4 - v_2}{2} + \frac{v_4 - 500}{11} + \frac{v_4 - v_5}{4} = 0$$
$$\frac{v_5 - v_3}{2} + \frac{v_5}{4} + \frac{v_5 - v_4}{4} = 0$$

Solving, $v_2 = 300$ V; $v_3 = 180$ V; $v_4 = 280$ V; $v_5 = 160$ V

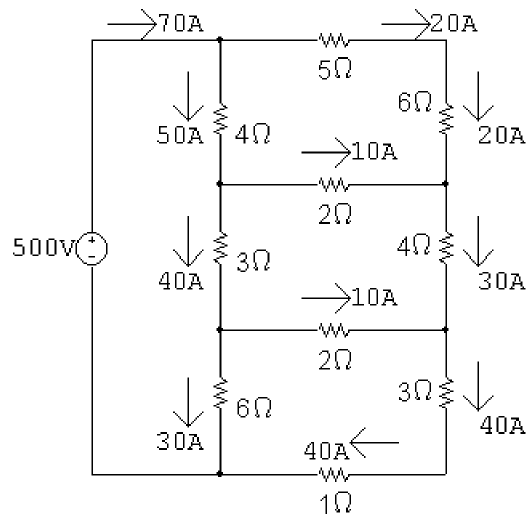
$$i_{5\Omega} = \frac{500 - v_4}{11} = \frac{500 - 280}{11} = 20 \text{ A}$$

$$p_{5\Omega} = (20)^2(5) = 2000 \text{ W}$$

[**b**] $$i_{500\text{V}} = \frac{v_1 - v_2}{4} + \frac{v_1 - v_4}{11} = \frac{500 - 300}{4} + \frac{500 - 280}{11} = 50 + 20 = 70 \text{ A}$$

$$p_{500\text{V}} = 35{,}000 \text{ W}$$

Check:



$$\sum P_{\text{dis}} = (50)^2(4) + (40)^2(3) + (30)^2(6) + (20)^2(11) + (10)^2(2) + (30)^2(4) + (10)^2(2) + (40)^2(4) = 35{,}000 \text{ W}$$

P 3.18 [**a**] $$\frac{v_o - v_1}{R} + \frac{v_o - v_2}{R} + \frac{v_o - v_3}{R} + \cdots + \frac{v_o - v_n}{R} = 0$$

$$\therefore \quad nv_o = v_1 + v_2 + v_3 + \cdots + v_n$$

$$\therefore \quad v_o = \frac{1}{n}[v_1 + v_2 + v_3 + \cdots + v_n] = \frac{1}{n}\sum_{k=1}^{n} v_k$$

[**b**] $v_o = \frac{1}{3}(150 + 200 - 50) = 100$ V

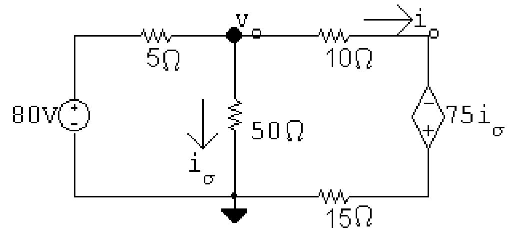
P 3.19 Place $v_\Delta/5$ inside a supernode and use the lower node as a reference. Then

$$\frac{v_1 - 50}{10} + \frac{v_1}{30} + \frac{v_1 - v_\Delta/5}{39} + \frac{v_1 - v_\Delta/5}{78} = 0$$

$$134v_1 - 6v_\Delta = 3900; \qquad v_\Delta = 50 - v_1$$

Solving, $v_1 = 30$ V; $v_\Delta = 20$ V; $v_o = 30 - v_\Delta/5 = 30 - 4 = 26$ V

P 3.20



$$\frac{v_o - 80}{5} + \frac{v_o}{50} + \frac{v_o + 75i_\sigma}{25} = 0; \quad i_\sigma = \frac{v_o}{50}$$

Solving, $v_o = 50$ V; $i_\sigma = 1$ A

$$i_o = \frac{50 - (-75)(1)}{25} = 5 \text{ A}$$

$$p_{75i_\sigma} = 75i_\sigma i_o = -375 \text{ W}$$

$\therefore$ The dependent voltage source delivers 375 W to the circuit.

P 3.21 $-3 + \dfrac{v_o}{200} + \dfrac{v_o + 5i_\Delta}{10} + \dfrac{v_o - 80}{20} = 0; \quad i_\Delta = \dfrac{v_o - 80}{20}$

[a] Solving, $v_o = 50$ V

[b] $i_{\text{ds}} = \dfrac{v_o + 5i_\Delta}{10}$

$i_\Delta = (50 - 80)/20 = -1.5$ A

$\therefore$ $i_{\text{ds}} = 4.25$ A; $5i_\Delta = -7.5$ V : $p_{\text{ds}} = (-5i_\Delta)(i_{\text{ds}}) = 31.875$ W

[c] $p_{3\text{A}} = -3v_o = -3(50) = -150$ W (del)

$p_{80\text{V}} = 80i_\Delta = 80(-1.5) = -120$ W (del)

$\sum p_{\text{del}} = 150 + 120 = 270$ W

CHECK:

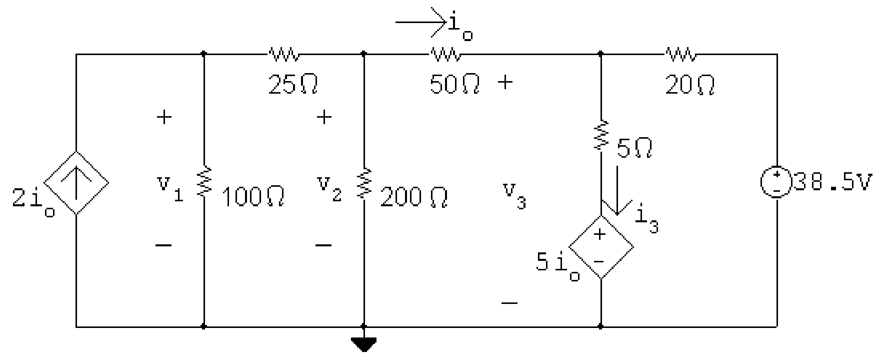
$p_{200\Omega} = 2500/200 = 12.5$ W

$p_{20\Omega} = (80 - 50)^2/20 = 900/20 = 45$ W

$p_{10\Omega} = (4.25)^2/10 = 180.625$ W

$\sum p_{\text{diss}} = 31.875 + 180.625 + 12.5 + 45 = 270$ W

P 3.22 **[a]**



$$i_o = \frac{v_2 - v_3}{50}$$

$$-2i_o + \frac{v_1}{100} + \frac{v_1 - v_2}{25} = 0$$
$$\frac{v_2 - v_1}{25} + \frac{v_2}{200} + \frac{v_2 - v_3}{50}$$
$$\frac{v_3 - v_2}{50} + \frac{v_3 - 5i_o}{5} + \frac{v_3 - 38.5}{20} = 0$$

Solving, $v_1 = -50$ V; $v_2 = -30$ V; $v_3 = 2.5$ V

[b] $i_o = \dfrac{v_2 - v_3}{50} = \dfrac{-30 - 2.5}{50} = -0.65$ A

$$i_3 = \frac{v_3 - 5i_o}{5} = \frac{2.5 - 5(-0.65)}{5} = 1.15 \text{ A}$$

$$i_g = \frac{38.5 - 2.5}{20} = 1.8 \text{ A}$$

$\sum p_{\text{dis}} = \sum p_{\text{dev}}$

Calculate $\sum p_{\text{dev}}$ because we don't know if the dependent sources are developing or absorbing power. Likewise for the independent source.

$p_{2i_o} = -2i_o v_1 = -2(-0.65)(-50) = -65$ W(dev)

$p_{5i_o} = 5i_o i_3 = 5(-0.65)(1.15) = -3.7375$ W(dev)

$p_g = -38.5(1.8) = -69.30$ W(dev)

$\sum p_{\text{dev}} = 69.3 + 65 + 3.7375 = 138.0375$ W

CHECK

$$\sum p_{\rm dis} = \frac{2500}{100} + \frac{900}{200} + \frac{400}{25} + (0.65)^2(50) + (1.15)^2 5 + (1.8)^2(20)$$

$$= 138.0375 \text{ W}$$

$$\therefore \quad \sum p_{\rm dev} = \sum p_{\rm dis} = 138.0375 \text{ W}$$

P 3.23 **[a]** $-5 + \dfrac{v_1}{15} + \dfrac{v_1 - v_2}{5} = 0$

$$\frac{v_2 - v_1}{5} + \frac{v_2}{30} + \frac{v_2}{10} + \frac{v_2 + 5i_\Delta}{30} = 0$$

$$i_\Delta = \frac{v_1 - v_2}{5}$$

Solving, $v_1 = 30$ V; $\quad v_2 = 15$ V; $\quad i_\Delta = 3$ A; $\quad i_o = \dfrac{15 + 15}{30} = 1$ A

$$p_{5i_\Delta} = (-15)(1) = -15 \text{ W(del)}$$

$$p_{5\rm A} = -5(30) = -150 \text{ W(del)}$$

$$\therefore \quad p_{\rm dev} = 165 \text{ W}$$

[b] $\sum p_{\rm abs} = \dfrac{(30)^2}{15} + \dfrac{(15)^2}{30} + \dfrac{(15)^2}{10} + (3)^2(5) + (1)^2(30) = 165$ W

$$\therefore \quad \sum p_{\rm dev} = \sum p_{\rm abs} = 165 \text{ W}$$

P 3.24 $i_\phi = \dfrac{v_3 - v_4}{4} = \dfrac{235 - 222}{4} = 3.25$ A

$$30i_\phi = 30(3.25) = 97.5 \text{ V}$$

$$v_1 + 30i_\phi = v_4$$

$$v_1 = v_4 - 30i_\phi = 222 - 97.5 = 124.5 \text{ V}$$

$$v_3 + v_\Delta = 250$$

$$\therefore \quad v_\Delta = 250 - 235 = 15 \text{ V}$$

$$3.2v_\Delta = (3.2)(15) = 48 \text{ A}$$

$$i_g = \frac{250 - 124.5}{2} + \frac{250 - 235}{1} = 77.75 \text{ A}$$

$p_{250\mathrm{V}} = -250i_g = -250(77.75) = -19{,}437.5\ \mathrm{W(del)}$

$i_{30i_\phi} - i_\phi + v_4/40 + 48 = 0$

$i_{30i_\phi} = i_\phi - 222/40 - 48 = 3.25 - 5.55 - 48 = -50.3\ \mathrm{A}$

$p_{30i_\phi} = (30i_\phi)i_{30i_\phi} = (97.5)(-50.3) = -4904.25\ \mathrm{W(dev)}$

$p_{3.2v_\Delta} = (3.2v_\Delta)(v_4) = (48)(22) = 10{,}656\ \mathrm{W(abs)}$

$\therefore\quad \sum p_{\mathrm{dev}} = 19{,}437.5 + 4904.25 = 24{,}341.75\ \mathrm{W}$

$$p_{10\Omega} = \frac{v_1^2}{10} = \frac{(124.5)^2}{10} = 1550.025\ \mathrm{W}$$

$$p_{2\Omega} = \frac{(250 - 124.5)^2}{2} = 7875.125\ \mathrm{W}$$

$$p_{1\Omega} = \frac{(250 - 235)^2}{1} = 225\ \mathrm{W}$$

$$p_{20\Omega} = \frac{(235)^2}{20} = 2761.25\ \mathrm{W}$$

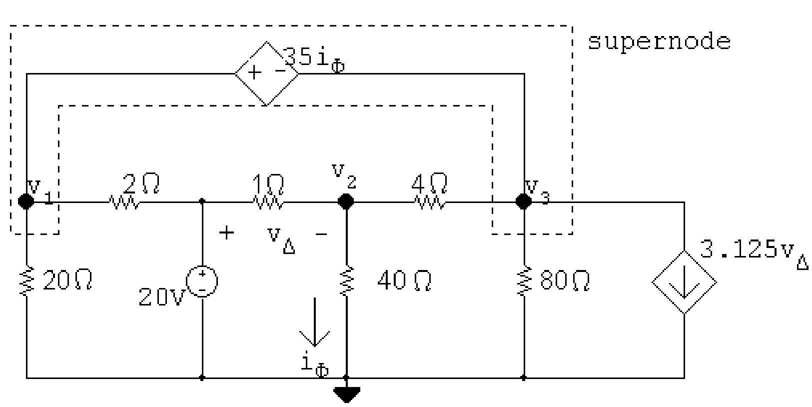
$$p_{4\Omega} = (3.25)^2(4) = 42.25\ \mathrm{W}$$

$$p_{40\Omega} = \frac{(222)^2}{40} = 1232.10\ \mathrm{W}$$

$$\begin{aligned}\therefore\quad \sum p_{\mathrm{diss}} &= 10{,}656 + 1550.025 + 7875.125 + 225 + \\ &\quad 2761.250 + 42.25 + 1232.1 = 24{,}341.75\ \mathrm{W}\end{aligned}$$

Thus, $\sum p_{\mathrm{dev}} = \sum p_{\mathrm{diss}}$; Agree with analyst

P 3.25



Node equations:

$$\frac{v_1}{20} + \frac{v_1 - 20}{2} + \frac{v_3 - v_2}{4} + \frac{v_3}{80} + 3.125v_\Delta = 0$$

$$\frac{v_2}{40} + \frac{v_2 - v_3}{4} + \frac{v_2 - 20}{1} = 0$$

Constraint equations:

$$v_\Delta = 20 - v_2$$

$$v_1 - 35i_\phi = v_3$$

$$i_\phi = v_2/40$$

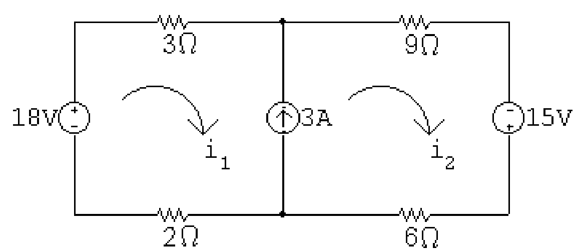
Solving, $v_1 = -20.25$ V; $v_2 = 10$ V; $v_3 = -29$ V

Let i_g be the current delivered by the 20 V source, then

$$i_g = \frac{20 - (20.25)}{2} + \frac{20 - 10}{1} = 30.125 \text{ A}$$

$$p_g \text{ (delivered) } = 20(30.125) = 602.5 \text{ W}$$

P 3.26



$$-18 + 3i_1 + 9i_2 - 15 + 6i_2 + 2i_1 = 0; \quad i_2 - i_1 = 3$$

Solving, $i_1 = -0.6$ A; $i_2 = 2.4$ A

$$p_{18V} = -18i_1 = 10.8 \text{ W (diss)}$$

$$p_{3\,\Omega} = (-0.6)^2(3) = 1.08 \text{ W}$$

$$p_{2\,\Omega} = (-0.6)^2(2) = 0.72 \text{ W}$$

$$p_{9\,\Omega} = (2.4)^2(9) = 51.84 \text{ W}$$

$p_{6\,\Omega} = (2.4)^2(6) = 34.56\ \text{W}$

$\sum p_{\text{diss}} = 99\ \text{W}$

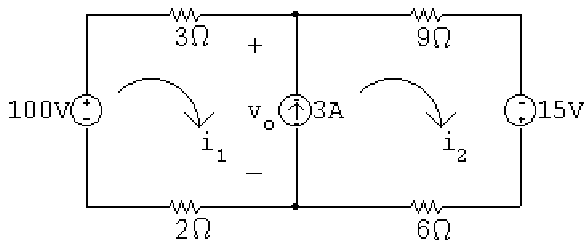
$v_o = 15i_2 - 15 = 36 - 15 = 21\ \text{V}$

$p_{3A} = -3v_o = -63\ \text{W (dev)}$

$p_{15V} = -15i_2 = -36\ \text{W (dev)}$

$\sum p_{\text{dev}} = 99\ \text{W} = \sum p_{\text{diss}}$

P 3.27



$-100 + 5i_1 + 15i_2 - 15 = 0$

$5i_1 + 15i_2 = 115$

$i_2 - i_1 = 3; \qquad i_2 = i_1 + 3; \qquad 15i_2 = 15i_1 + 45$

$\therefore \quad 20i_1 = 70$

$i_1 = 3.5\ \text{A}; \qquad i_2 = 6.5\ \text{A}$

$v_o = 15i_2 - 15 = 97.5 - 15 = 82.5\ \text{V}$

$p_{100\text{V}} = -100i_1 = -350\ \text{W(dev)}$

$p_{3\text{A}} = -3v_o = -247.5\ \text{W(dev)}$

$p_{15\text{V}} = -15i_2 = -97.5\ \text{W(dev)}$

$\sum p_{\text{dev}} = \sum p_{\text{dis}} = 695\ \text{W}$

$\sum p_{\text{dis}} = (3.5)^2(5) + (6.5)^2(15) = 695\ \text{W}$

P 3.28 **[a]** Summing around the supermesh used in the solution to Problem 3.27 gives

$$-(-10) + 5i_1 + 15i_2 - 15 = 0$$

$$i_2 = i_1 + 3$$

$$\therefore \quad i_1 = -2 \text{ A}; \qquad i_2 = 1 \text{ A}$$

$$p_{10\text{V}} = 10(-2) = -20 \text{ W (del)}$$

$$v_o = 15i_2 - 15 = 0 \text{ V}$$

$$p_{3\text{A}} = 3v_o = 0 \text{ W}$$

$$p_{15\text{V}} = -15i_2 = -15 \text{ W (del)}$$

$$\sum p_{\text{diss}} = (-2)^2(5) + (1)^2(15) = 35 \text{ W}$$

$$\sum p_{\text{dev}} = 35 \text{ W} = \sum p_{\text{diss}}$$

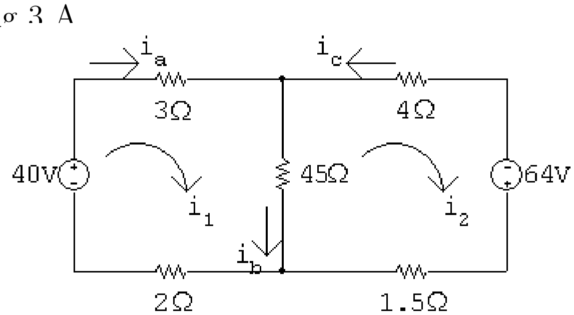
[b] With 3 A current source replaced with a short circuit

$$i_1 = -2 \text{ A}, \qquad i_2 = 1 \text{ A}$$

$$\therefore \quad \sum P_{\text{diss}} = (-2)^2(5) + (1)^2(15) = 35 \text{ W}$$

[c] A 3 A source with zero terminal voltage is equivalent to a short circuit carrying 3 A

P 3.29 **[a]**



$$40 = 50i_1 - 45i_2$$

$$64 = -45i_1 + 50.5i_2$$

Solving, $i_1 = 9.8$ A; $\quad i_2 = 10$ A

$$i_{\text{a}} = i_1 = 9.8 \text{ A}; \quad i_{\text{b}} = i_1 - i_2 = -0.2 \text{ A}; \quad i_{\text{c}} = -i_2 = -10 \text{ A}$$

[b] If the polarity of the 64 V source is reversed, we have

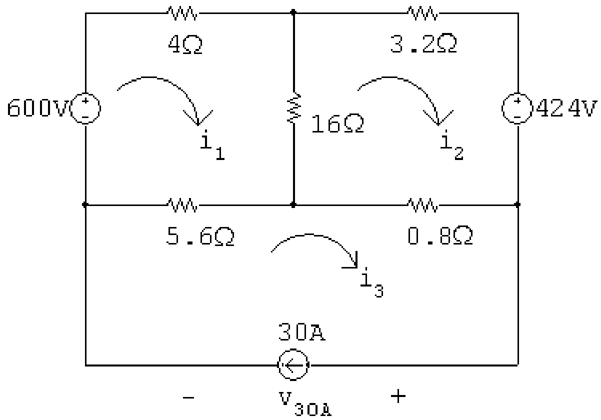
$$40 = 50i_1 - 45i_2$$

$$-64 = -45i_1 + 50.5i_2$$

$$i_1 = -1.72 \text{ A} \quad \text{and} \quad i_2 = -2.8 \text{ A}$$

$$i_{\text{a}} = i_1 = -1.72 \text{ A}; \quad i_{\text{b}} = i_1 - i_2 = 1.08 \text{ A}; \quad i_{\text{c}} = -i_2 = 2.8 \text{ A}$$

P 3.30



$600 = 25.6i_1 - 16i_2 - 5.6i_3$

$-424 = -16i_1 + 20i_2 - 0.8i_3$

$30 = i_3$

Solving, $i_1 = 35$ A; $\quad i_2 = 8$ A; $\quad i_3 = 30$ A

[a] $v_{30\text{A}} = 0.8(i_2 - i_3) + 5.6(i_1 - i_3) = 0.8(8 - 30) + 5.6(35 - 30) = 10.4$ V

$p_{30\text{A}} = 30v_{30\text{A}} = 30(10.4) = 312$ W (abs)

Therefore, the 30 A source delivers -312 W.

[b] $p_{600\text{V}} = -600(35) = -21{,}000$ W(del)

$p_{424\text{V}} = 424(8) = 3392$ W(abs)

Therefore, the total power delivered is 21,000 W

[c] $p_{4\Omega} = (35)^2(4) = 4900$ W

$p_{3.2\Omega} = (8)^2(3.2) = 204.8$ W

$p_{16\Omega} = (35 - 8)^2(16) = 11{,}664$ W

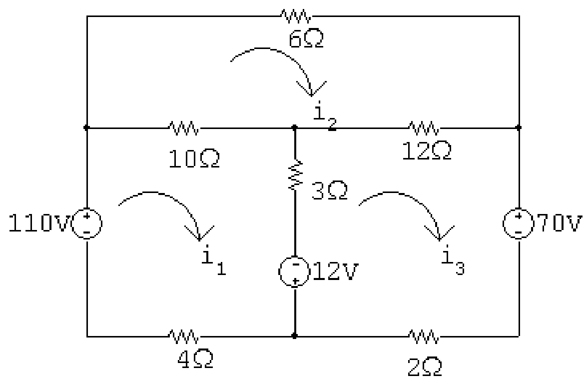
$p_{5.6\Omega} = (35 - 30)^2(5.6) = 140$ W

$p_{0.8\Omega} = (-30 + 8)^2(0.8) = 387.2$ W

$\sum p_{\text{resistors}} = 17{,}296$ W

$\sum p_{\text{abs}} = 17{,}296 + 312 + 3392 = 21{,}000$ W (CHECKS)

P 3.31 **[a]**



$$110 + 12 = 17i_1 - 10i_2 - 3i_3$$

$$0 = -10i_1 + 28i_2 - 12i_3$$

$$-12 - 70 = -3i_1 - 12i_2 + 17i_3$$

Solving, $i_1 = 8$ A; $\quad i_2 = 2$ A; $\quad i_3 = -2$ A

$$p_{110} = -110i_1 = -880 \text{ W(del)}$$

$$p_{12} = -12(i_1 - i_3) = -120 \text{ W(del)}$$

$$p_{70} = 70i_3 = -140 \text{ W(del)}$$

$$\therefore \quad \sum p_{\text{dev}} = 1140 \text{ W}$$

[b] $p_{4\Omega} = (8)^2(4) = 256$ W

$$p_{10\Omega} = (6)^2(10) = 360 \text{ W}$$

$$p_{12\Omega} = (-4)^2(12) = 192 \text{ W}$$

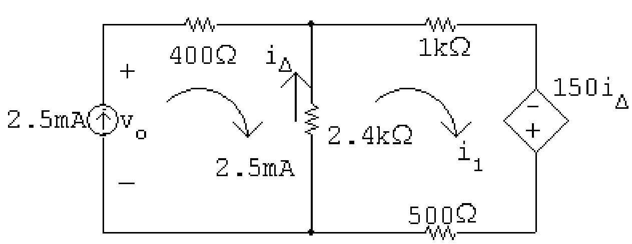
$$p_{2\Omega} = (-2)^2(2) = 8 \text{ W}$$

$$p_{6\Omega} = (2)^2(6) = 24 \text{ W}$$

$$p_{3\Omega} = (10)^2(3) = 300 \text{ W}$$

$$\therefore \quad \sum p_{\text{abs}} = 1140 \text{ W}$$

P 3.32 **[a]**



$$2400(i_1 - 0.0025) + 1500i_1 - 150(i_1 - 0.0025) = 0$$

$\therefore \quad i_1 = 1.5$ mA

$i_\Delta = i_1 - 2.5 = -1.0$ mA

[b] $v_o = (0.0025)(400) + (0.001)(2400) = 3.4$ V

$p_{2.5\text{mA}} = -3.4(2.5) = -8.5$ mW

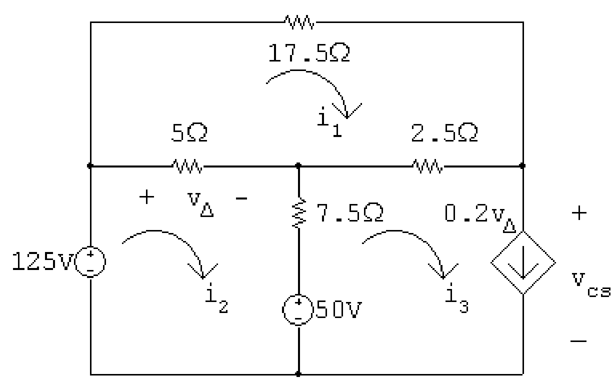
$\therefore \quad p_{2.5\text{mA}}$ (deliver) $= 8.5$ mW

[c] $150i_\Delta = 150(-1.0 \times 10^{-3}) = -0.15$ V

$p_{\text{dep source}} = 150 i_\Delta i_1 = (-0.15)(0.0015) = -0.225$ mW

$p_{\text{dep source}}$ (absorbed) $= 0.225$ mW

P 3.33



Mesh equations:

$$25i_1 - 5i_2 - 2.5i_3 = 0$$

$$75 = -5i_1 + 12.5i_2 - 7.5i_3$$

Constraint equations:

$$i_3 = 0.2v_\Delta$$

$$v_\Delta = 5(i_2 - i_1)$$

Solving, $i_1 = 3.6$ A; $\quad i_2 = 13.2$ A; $\quad i_3 = 9.6$ A; $\quad v_\Delta = 48$ V

$$v_{\text{cs}} = 125 - v_\Delta - 2.5(i_3 - i_1) = 125 - 48 - 2.5(9.6 - 3.6) = 62 \text{ V}$$

$$p_{\text{vc}} = (62)(9.6) = 595.2 \text{ W (abs)}$$

$$p_{50\text{V}} = 50(i_2 - i_3) = 50(13.2 - 9.6) = 180 \text{ W (abs)}$$

$$p_{125\text{V}} = -125i_2 = -125(13.2) = -1650 \text{ W(del)}$$

Thus, the total power developed is 1650 W.
CHECK:

$$p_{17.5\Omega} = (3.6)^2(17.5) = 226.8 \text{ W}$$

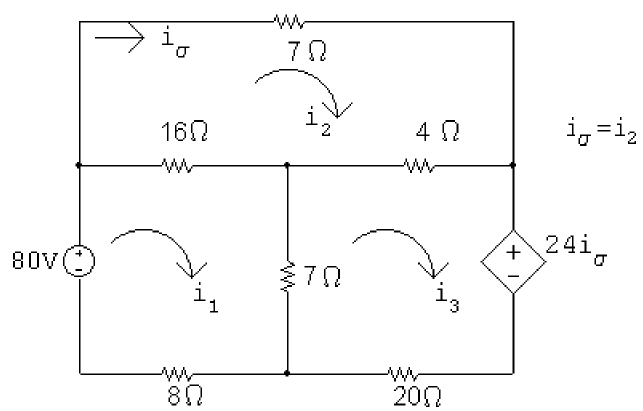
$$p_{5\Omega} = (13.2 - 3.6)^2(5) = 460.8 \text{ W}$$

$$p_{2.5\Omega} = (9.6 - 3.6)^2(2.5) = 90 \text{ W}$$

$$p_{7.5\Omega} = (13.2 - 9.6)^2(7.5) = 97.2 \text{ W}$$

$$\therefore \quad \sum p_{\text{abs}} = 226.8 + 460.8 + 90 + 97.2 + 180 + 595.2 = 1650 \text{ W}$$

P 3.34



$$-80 + 31i_1 - 16i_2 - 7i_3 = 0$$

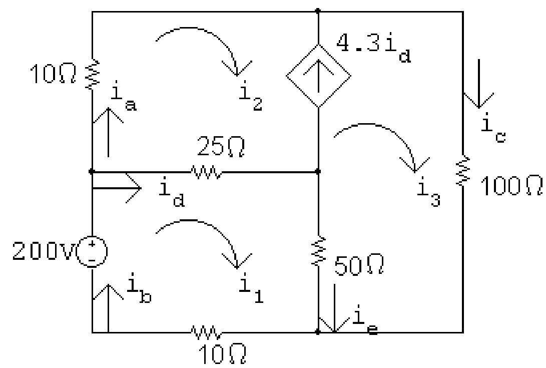
$$-16i_1 + 27i_2 - 4i_3 = 0$$

$$-7i_1 - 4i_2 + 31i_3 + 24i_2 = 0$$

Solving, $i_1 = 3.5$ A

$$p_{8\Omega} = (3.5)^2(8) = 98 \text{ W}$$

P 3.35 **[a]**



$$200 = 85i_1 - 25i_2 - 50i_3$$

$$0 = -75i_1 + 35i_2 + 150i_3 \qquad \text{(super mesh)}$$

$$i_3 - i_2 = 4.3(i_1 - i_2)$$

Solving, $i_1 = 4.6$ A; $\quad i_2 = 5.7$ A; $\quad i_3 = 0.97$ A

$i_\text{a} = i_2 = 5.7$ A; $\quad i_\text{b} = i_1 = 4.6$ A

$i_\text{c} = i_3 = 0.97$ A; $\quad i_\text{d} = i_1 - i_2 = -1.1$ A

$i_\text{e} = i_1 - i_3 = 3.63$ A

[b] $10i_2 + v_o + 25(i_2 - i_1) = 0$

$\therefore \quad v_o = -57 - 27.5 = -84.5$ V

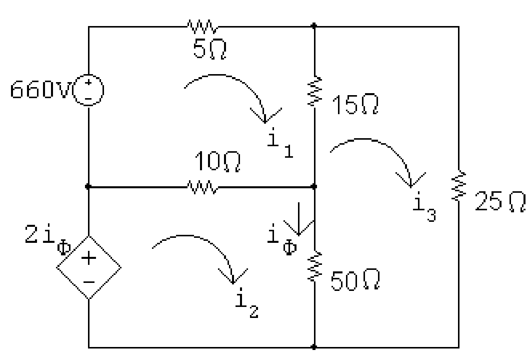
$p_{4.3i_\text{d}} = -v_o(4.3i_\text{d}) = -(-84.5)(4.3)(-1.1) = -399.685$ W(dev)

$p_{200\text{V}} = -200(4.6) = -920$ W(dev)

$$\begin{aligned}\sum P_\text{dev} &= 1319.685 \text{ W}\\ \sum P_\text{dis} &= (5.7)^2 10 + (1.1)^2(25) + (0.97)^2 100 + (4.6)^2(10) +\\ &\quad (3.63)^2(50)\\ &= 1319.685 \text{ W}\end{aligned}$$

$$\therefore \quad \sum P_\text{dev} = \sum P_\text{dis} = 1319.685 \text{ W}$$

P 3.36



$$660 = 30i_1 - 10i_2 - 15i_3$$

$20i_\phi = -10i_1 + 60i_2 - 50i_3$

$0 = -15i_1 - 50i_2 + 90i_3$

$i_\phi = i_2 - i_3$

Solving, $i_1 = 42$ A; $\quad i_2 = 27$ A; $\quad i_3 = 22$ A; $\quad i_\phi = 5$ A

$20i_\phi = 100$ V

$p_{20i_\phi} = -100i_2 = -100(27) = -2700$ W

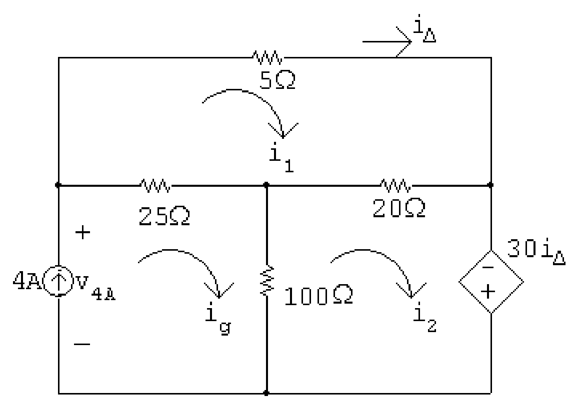
$\therefore \quad p_{20i_\phi}$ (developed) $= 2700$ W

CHECK:

$p_{660\text{V}} = -660(42) = -27{,}720$ W (dev)

$$\begin{aligned} \therefore \quad \sum P_{\text{dev}} &= 27{,}720 + 2700 = 30{,}420 \text{ W} \\ \sum P_{\text{dis}} &= (42)^2(5) + (22)^2(25) + (20)^2(15) + (5)^2(50) + \\ &\quad (15)^2(10) \\ &= 30{,}420 \text{ W} \end{aligned}$$

P 3.37



Mesh equations:

$50i_1 - 20i_2 - 25i_g = 0$

$-20i_1 + 120i_2 - 30i_\Delta - 100i_g = 0$

Constraint equations:

$i_g = 4; \qquad i_\Delta = i_1$

Solving, $i_1 = 4$ A; $\qquad i_2 = 5$ A

$i_{25\Omega} = 4 - i_1 = 0$ A

$i_{20\Omega} = i_2 - i_1 = 1$ A

$i_{100\Omega} = 4 - i_2 = -1$ A

$i_{5\Omega} = i_1 = 4$ A

$v_{4\text{A}} = 100(4 - i_2) = -100$ V

$p_{4\text{A}} = -v_{4\text{A}} i_g = -(-100)(4) = 400$ W (abs)

$v_{30i_\Delta} = 30i_\Delta = 30i_1 = 120$ V

$p_{30i_\Delta} = -30i_\Delta i_2 = -120(5) = -600$ W

Therefore, the dependent source is developing 600 W, all other elements are absorbing power, and the total power developed is thus 600 W.
CHECK:

$p_{5\Omega} = 16(5) = 80$ W

$p_{25\Omega} = 0$ W

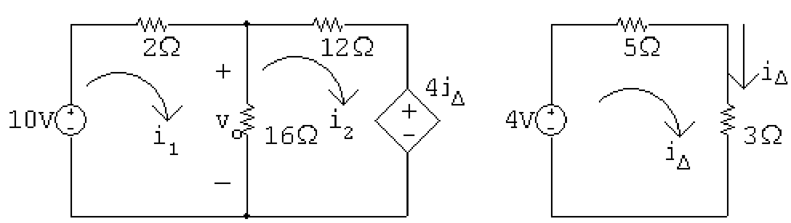
$p_{20\Omega} = 1(20) = 20$ W

$p_{100\Omega} = 1(100) = 100$ W

$p_{4\text{A}} = 400$ W

$\sum p_{\text{abs}} = 80 + 0 + 20 + 100 + 400 = 600$ W (CHECKS)

P 3.38 **[a]**



$10 = 18i_1 - 16i_2$

$0 = -16i_1 + 28i_2 + 4i_\Delta$

$4 = 8i_\Delta$

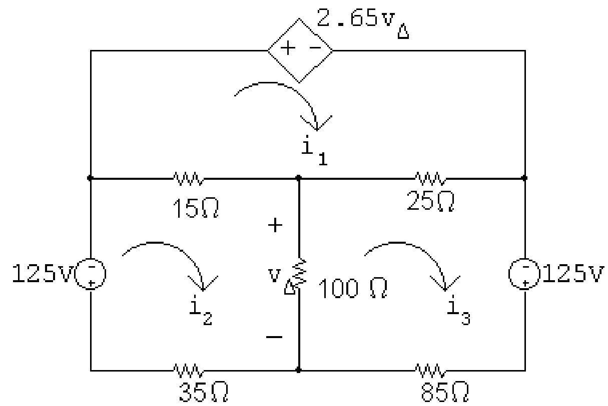
Solving, $i_1 = 1$ A; $\qquad i_2 = 0.5$ A; $\qquad i_\Delta = 0.5$ A

$v_0 = 16(i_1 - i_2) = 16(0.5) = 8$ V

[b] $p_{4i_\Delta} = 4i_\Delta i_2 = (4)(0.5)(0.5) = 1$ W (abs)

$\therefore\ \ p_{4i_\Delta}$ (deliver) $= -1$ W

P 3.39



Mesh equations:

$$2.65v_\Delta + 40i_1 - 15i_2 - 25i_3 = 0$$

$$-15i_1 + 150i_2 - 100i_3 = -125$$

$$-25i_1 - 100i_2 - 210i_3 = 125$$

Constraint equations:

$$v_\Delta = 100(i_2 - i_3)$$

Solving, $i_1 = 7$ A; $\qquad i_2 = 1.2$ A; $\qquad i_3 = 2$ A

$$v_\Delta = 100(i_2 - i_3) = 100(1.2 - 2) = -80 \text{ V}$$

$$p_{2.65v_\Delta} = 2.65v_\Delta i_1 = -1484 \text{ W}$$

Therefore, the dependent source is developing 1484 W.
CHECK:

$p_{125\text{V}} = 125i_2 = 150$ W (left source)

$p_{125\text{V}} = -125i_3 = -250$ W (right source)

$$\sum p_{\text{dev}} = 1484 + 250 = 1734 \text{ W}$$

$$p_{35\Omega} = (1.2)^2(35) = 50.4 \text{ W}$$

$$p_{85\Omega} = (2)^2(85) = 340 \text{ W}$$

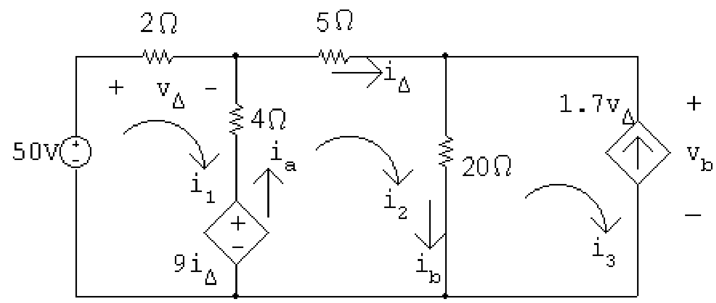
$$p_{15\Omega} = (7 - 1.2)^2(15) = 504.6 \text{ W}$$

$$p_{25\Omega} = (7 - 2)^2(25) = 625 \text{ W}$$

$$p_{100\Omega} = (1.2 - 2)^2(100) = 64 \text{ W}$$

$$\sum p_{\text{diss}} = 50.4 + 340 + 504.6 + 625 + 64 + 150 = 1734 \text{ W}$$

P 3.40 **[a]**



Mesh equations:

$$-50 + 6i_1 - 4i_2 + 9i_\Delta = 0$$

$$-9i_\Delta - 4i_1 + 29i_2 - 20i_3 = 0$$

Constraint equations:

$i_\Delta = i_2; \qquad i_3 = -1.7v_\Delta; \qquad v_\Delta = 2i_1$

Solving, $i_1 = -5$ A; $\quad i_2 = 16$ A; $\quad i_3 = 17$ A; $\quad v_\Delta = -10$ V

$$9i_\Delta = 9(16) = 144 \text{ V}$$

$$i_\text{a} = i_2 - i_1 = 21 \text{ A}$$

$$i_\text{b} = i_2 - i_3 = -1 \text{ A}$$

$$v_\text{b} = 20i_\text{b} = -20 \text{ V}$$

$$p_{50\text{V}} = -50i_1 = 250 \text{ W (absorbing)}$$

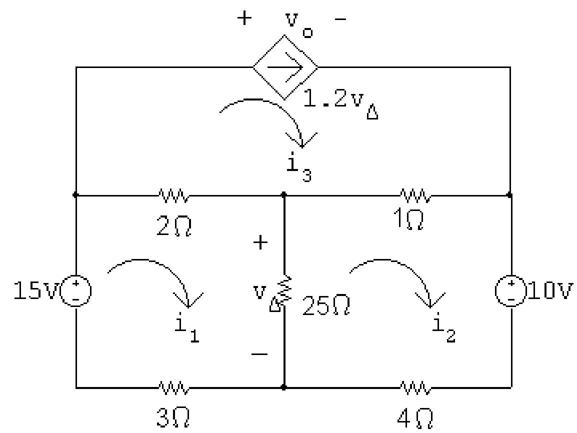
$$p_{9i_\Delta} = -i_\text{a}(9i_\Delta) = -(21)(144) = -3024 \text{ W (delivering)}$$

$$p_{1.7\text{V}} = -1.7v_\Delta v_\text{b} = i_3 v_\text{b} = (17)(-20) = -340 \text{ W (delivering)}$$

[b] $\sum P_{\text{dev}} = 3024 + 340 = 3364$ W

$$\sum P_{\text{dis}} = 250 + (-5)^2(2) + (21)^2(4) + (16)^2(5) + (-1)^2(20) = 3364 \text{ W}$$

P 3.41 [**a**]



Mesh equations:

$15 = 30i_1 - 25i_2 - 2i_3$

$-10 = -25i_1 + 30i_2 - i_3$

Constraint equations:

$i_3 = 1.2v_\Delta; \qquad v_\Delta = 25(i_1 - i_2)$

Solving, $i_1 = 10$ A; $\qquad i_2 = 9$ A; $\qquad i_3 = 30$ A; $\qquad v_\Delta = 25$ V

$i_{2\Omega} = i_1 - i_3 = 9 - 30 = -20$ A

$p_{2\Omega} = (-20)^2(2) = 800$ W

[**b**] $p_{15\text{V}} = -15(10) = -150$ W(dev)

$p_{10\text{V}} = 10i_2 = 10(9) = 90$ W (abs)

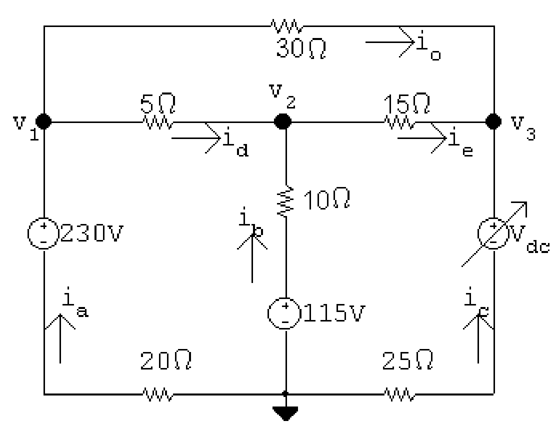
$v_o = (i_1 - i_3)2 + (i_2 - i_3)1 = -40 - 21 = -61$ V

$p_{1.2v_\Delta} = i_3 v_o = (30)(-61) = -1830$ W (dev)

$\sum P_{\text{dev}} = 1830 + 150 = 1980$ W

$\%$ delivered to $2\Omega = \dfrac{800}{1980} \times 100 = 40.4\%$

P 3.42 [**a**]



If $i_o = 0$ then $v_1 = v_3$; therefore,

$$\frac{v_1 - v_2}{5} + \frac{v_1 - 230}{20} = 0$$

$$\frac{v_2 - v_1}{5} + \frac{v_2 - v_3}{15} + \frac{v_2 - 115}{10} = 0$$

Solving, $v_1 = 170$ V $= v_3$; $\qquad v_2 = 155$ V

$$\therefore \quad \frac{170 - 155}{15} + \frac{170 - v_{\text{dc}}}{25} = 0$$

Solving, $v_{\text{dc}} = 195$ V

[b] $i_a = \dfrac{230 - 170}{20} = 3$ A

$$i_b = \frac{115 - 155}{10} = -4 \text{ A}$$

$$i_c = \frac{195 - 170}{25} = 1 \text{ A}$$

$$i_d = \frac{170 - 155}{5} = 3 \text{ A}$$

$$i_e = \frac{155 - 170}{15} = -1 \text{ A}$$

$$p_{230\text{V}} = -230 i_a = -690 \text{ W (dev)}$$

$$p_{115\text{V}} = -115 i_b = 460 \text{ W (abs)}$$

$$p_{v_{\text{dc}}} = -v_{\text{dc}} i_c = -195 \text{ W (dev)}$$

$$p_{20\Omega} = i_a^2(20) = 180 \text{ W}$$

$$p_{5\Omega} = i_d^2(5) = 45 \text{ W}$$

$$p_{10\Omega} = i_b^2(10) = 160 \text{ W}$$

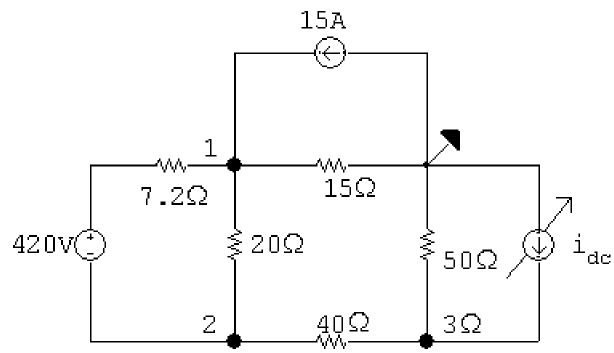
$$p_{15\Omega} = i_e^2(15) = 15 \text{ W}$$

$$p_{25\Omega} = i_c^2(25) = 25 \text{ W}$$

$$\sum p_{\text{diss}} = 460 + 180 + 45 + 160 + 15 + 25 = 885 \text{ W}$$

$$\sum p_{\text{dev}} = 690 + 195 = 885 \text{ W (CHECKS)}$$

P 3.43 Choose the reference node so that a node voltage is identical to the voltage across the 15 A source; thus:



Since the 15 A source is developing 3750 W, v_1 must be 250 V.

Since v_1 is known, we can sum the currents away from node 1 to find v_2; thus:

$$\frac{250-(420+v_2)}{7.2}+\frac{250-v_2}{20}+\frac{250}{15}-15=0$$

$\therefore \quad v_2=-50$ V

Now that we know v_2 we sum the currents away from node 2 to find v_3; thus:

$$\frac{v_2+420-250}{7.2}+\frac{v_2-250}{20}+\frac{v_2-v_3}{40}=0$$

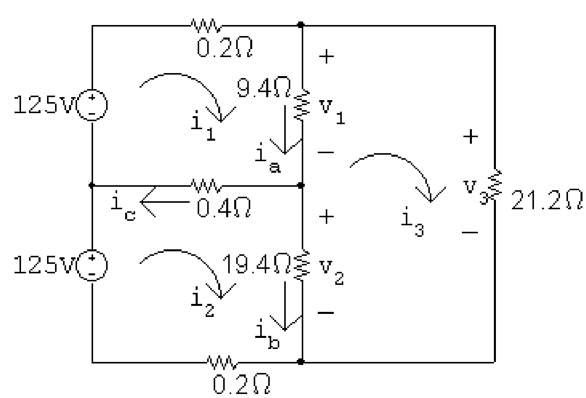
$\therefore \quad v_3=50/3$ V

Now that we know v_3 we sum the currents away from node 3 to find $i_{\rm dc}$; thus:

$$\frac{v_3}{50}+\frac{v_3+50}{40}=i_{\rm dc}$$

$\therefore \quad i_{\rm dc}=2$ A

P 3.44 **[a]**



$$125=10i_1-0.4i_2-9.4i_3$$

$125 = -0.4i_1 + 20i_2 - 19.4i_3$

$0 = -9.4i_1 - 19.4i_2 + 50i_3$

Solving, $i_1 = 23.93$ A; $i_2 = 17.79$ A; $i_3 = 11.40$ A

$v_1 = 9.4(i_1 - i_3) = 117.76$ V

$v_2 = 19.4(i_2 - i_3) = 123.90$ V

$v_3 = 21.2i_3 = 241.66$ V

[b] $P_{R1} = (i_1 - i_3)^2(9.4) = 1475.22$ W

$P_{R2} = (i_2 - i_3)^2(19.4) = 791.29$ W

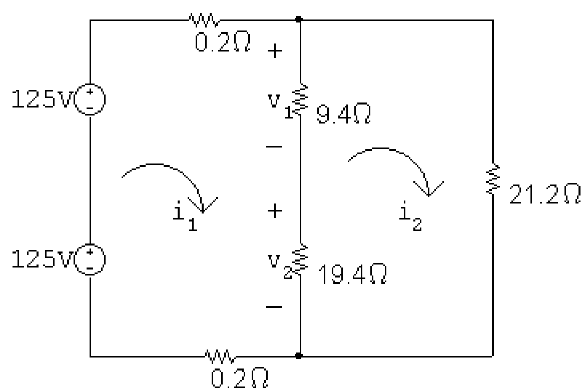
$P_{R3} = i_3^2(21.2) = 2754.64$ W

[c] $\sum p_{\text{dev}} = 125(i_1 + i_2) = 5213.99$ W

$\sum p_{\text{load}} = 5021.15$ W

$$\% \text{ delivered } = \frac{5021.15}{5213.99} \times 100 = 96.3\%$$

[d]



$250 = 29.2i_1 - 28.8i_2$

$0 = -28.8i_1 + 50i_2$

Solving, $i_1 = 19.82$ A; $i_2 = 11.42$ A

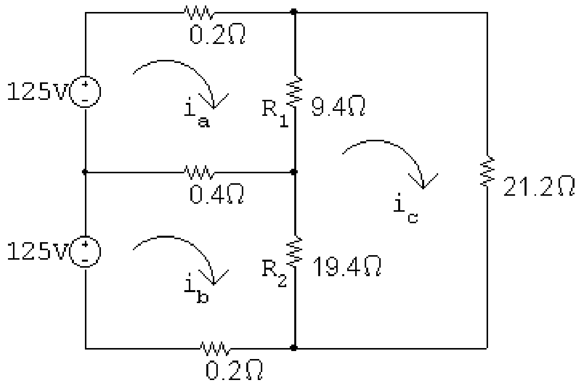
$i_1 - i_2 = 8.41$ A

$v_1 = (8.41)(9.4) = 79.01$ V

$v_2 = 8.41(19.4) = 163.06$ V

Note v_1 is low and v_2 is high. Therefore, loads designed for 125 V would not function properly, and could be damaged.

P 3.45



$$125 = (R_1 + 0.6)i_\mathrm{a} - 0.4i_\mathrm{b} - R_1 i_\mathrm{c}$$

$$125 = -0.4i_\mathrm{a} + (R_2 + 0.6)i_\mathrm{b} - R_2 i_\mathrm{c}$$

$$0 = -R_1 i_\mathrm{a} - R_2 i_\mathrm{b} + (R_1 + R_2 + 21.2)i_\mathrm{c}$$

$$\Delta = \begin{vmatrix} (R_1 + 0.6) & -0.4 & -R_1 \\ -0.4 & (R_2 + 0.6) & -R_2 \\ -R_1 & -R_2 & (R_1 + R_2 + 21.2) \end{vmatrix}$$

When $R_1 = R_2$, Δ reduces to

$$\Delta = 21.6R_1^2 + 25.84R_1 + 4.24.$$

$$\begin{aligned} N_\mathrm{a} &= \begin{vmatrix} 125 & -0.4 & -R_1 \\ 125 & (R_2 + 0.6) & -R_2 \\ 0 & -R_2 & (R_1 + R_2 + 21.2) \end{vmatrix} \\ &= 125\,[2R_1R_2 + R_1 + 22.2R_2 + 21.2] \end{aligned}$$

$$\begin{aligned} N_\mathrm{b} &= \begin{vmatrix} (R_1 + 0.6) & 125 & -R_1 \\ -0.4 & 125 & -R_2 \\ -R_1 & 0 & (R_1 + R_2 + 21.2) \end{vmatrix} \\ &= 125\,[2R_1R_2 + 22.2R_1 + R_2 + 21.2] \end{aligned}$$

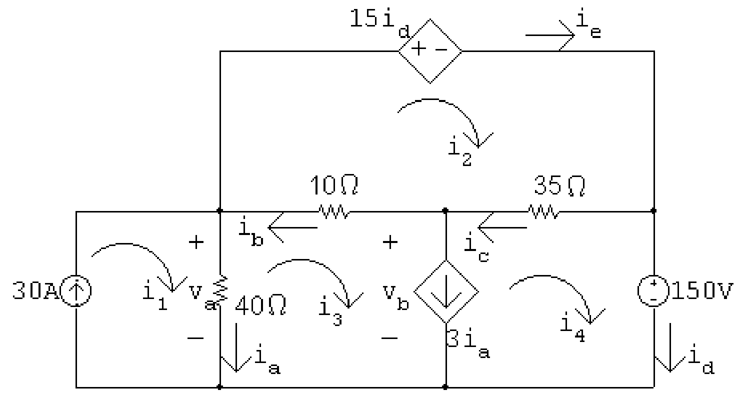
$$i_\mathrm{a} = \frac{N_\mathrm{a}}{\Delta}, \qquad i_\mathrm{b} = \frac{N_\mathrm{b}}{\Delta}$$

$$i_\mathrm{neutral} = i_\mathrm{a} - i_\mathrm{b} = \frac{N_\mathrm{a} - N_\mathrm{b}}{\Delta} = \frac{125[(R_1 - R_2) + 22.2(R_2 - R_1)]}{\Delta}$$

Now note that when $R_1 = R_2$, i_neutral reduces to

$$i_\mathrm{neutral} = \frac{0}{\Delta} = 0$$

P 3.46 **[a]**



$$40(i_3 - i_1) + 10(i_3 - i_2) + 35(i_4 - i_2) + 150 = 0$$

$$35(i_2 - i_4) + 10(i_2 - i_3) + 15i_d = 0$$

$$i_d = i_4; \qquad i_1 = 30 \text{ A}$$

Solving, $i_1 = 30$ A; $\quad i_2 = 8$ A; $\quad i_3 = 24$ A; $\quad i_4 = 6$ A

$i_a = 30 - 24 = 6$ A; $\quad i_b = 8 - 24 = -16$ A; $\quad i_c = 8 - 6 = 2$ A;

$i_d = 6$ A; $\qquad i_e = i_c + i_d = 6 + 2 = 8$ A

[b] $v_{\text{a}} = 40i_{\text{a}} = 240$ V; $\qquad v_{\text{b}} = 150 - 35i_c = 80$ V

$$p_{30\text{A}} = -30v_a = -30(240) = -7200 \text{ W (gen)}$$

$$p_{15i_d} = 15i_d i_e = 15(6)(8) = 720 \text{ W (diss)}$$

$$p_{3i_a} = 3i_a v_b = 3(6)(80) = 1440 \text{ W (diss)}$$

$$p_{150\text{V}} = 150i_d = 150(6) = 900 \text{ W (diss)}$$

$$p_{40\Omega} = (6)^2(40) = 1440 \text{ W (diss)}$$

$$p_{10\Omega} = (-16)^2(10) = 2560 \text{ W (diss)}$$

$$p_{35\Omega} = (2)^2(35) = 140 \text{ W (diss)}$$

$$\sum P_{\text{gen}} = 7200 \text{ W}$$

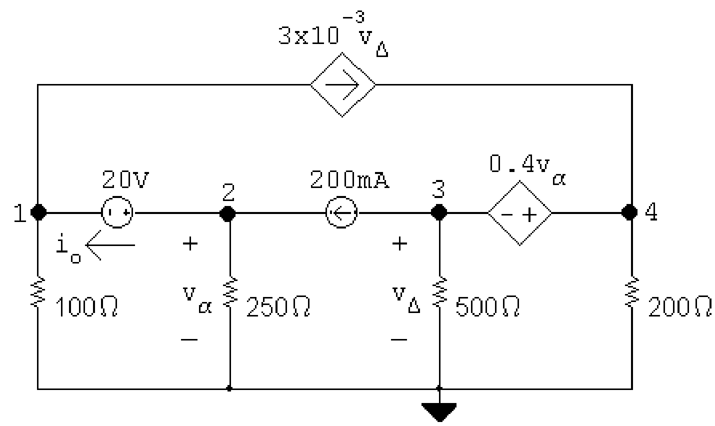
$$\sum P_{\text{diss}} = 720 + 1440 + 900 + 1440 + 2560 + 140 = 7200 \text{ W}$$

P 3.47 **[a]** The node voltage method requires summing the currents at two supernodes in terms of four node voltages and using two constraint equations to reduce the system of equations to two unknowns. If the connection at the bottom of the circuit is used as the reference node, then the voltages controlling the dependent sources are node voltages. This makes it easy to formulate the constraint equations. The current in the 20 V source is obtained by summing the currents at either terminal of the source.

The mesh current method requires summing the voltages around the two meshes not containing current sources in terms of four mesh currents. In addition the voltages controlling the dependent sources must be expressed in terms of the mesh currents. Thus the constraint equations are more complicated, and the reduction to two equations and two unknowns involves more algebraic manipulation. The current in the 20 V source is found by subtracting two mesh currents.

Because the constraint equations are easier to formulate in the node Voltage method, it is the preferred approach.

[b]



Node voltage equations:

$$\frac{v_1}{100} + \frac{v_2}{250} - 0.2 + 3 \times 10^{-3} v_3 = 0$$

$$\frac{v_3}{500} + \frac{v_4}{200} - 3 \times 10^{-3} v_3 + 0.2 = 0$$

Constraints:

$v_2 - v_1 = 20; \qquad v_4 - v_3 = 0.4 v_\alpha; v_\alpha = v_2$

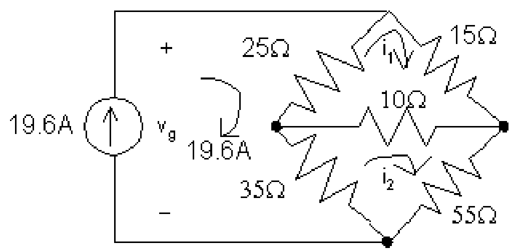
Solving, $v_2 = 44$ V

$i_o = 0.2 - 44/250 = 24$ mA

$p_{20\text{V}} = 20 i_o = 480$ mW (abs)

P 3.48 **[a]** There are three unknown node voltages and only two unknown mesh currents. Use the mesh current method to minimize the number of simultaneous equations.

[b]



$$25(i_1 - 19.6) + 15i_1 + 10(i_1 - i_2) = 0$$

$$35(i_2 - 19.6) + 10(i_2 - i_1) + 55i_2 = 0$$

Solving, $i_1 = 11.4$ A; $\qquad i_2 = 8$ A

$$i_{10\Omega} = i_1 - i_2 = 3.4 \text{ A}$$

$$p_{10\Omega} = (3.4)^2(10) = 115.6 \text{ W}$$

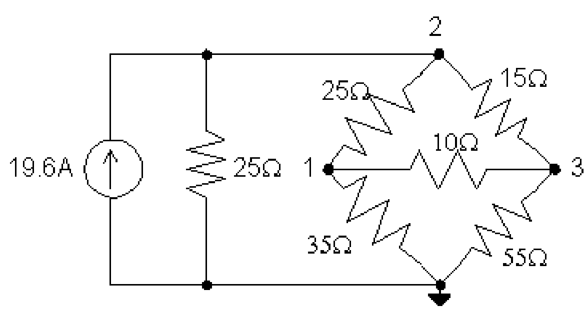
[c] No, the voltage across the 19.6 A current source is readily available from the mesh currents, and solving two simultaneous mesh-current equations is less work than solving three node voltage equations.

[d] $v_g = (19.6 - 11.4)(25) + (19.6 - 8)(35) = 611$ V

$$p_{19.6\text{A}} \text{ (developed) } = 19.6(611) = 11{,}975.6 \text{ W}$$

P 3.49 [a] There are three unknown node voltages and three unknown mesh currents, so the number of simultaneous equations required are the same for both methods. The node voltage method has the advantage of having to solve the three simultaneous equations for one unknown voltage provided the connection at either the top or bottom of the circuit is used as the reference node. Therefore recommend the node voltage method.

[b]



$$\frac{v_1}{35} + \frac{v_1 - v_2}{25} + \frac{v_1 - v_3}{10} = 0$$

$$\frac{v_2}{25} - 19.6 + \frac{v_2 - v_1}{25} + \frac{v_2 - v_3}{15} = 0$$

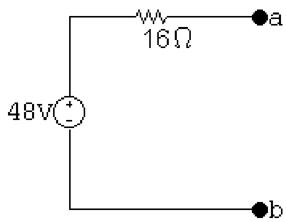
$$\frac{v_3 - v_1}{10} + \frac{v_3 - v_2}{15} + \frac{v_3}{55} = 0$$

Solving, $v_2 = 271.9255$ V

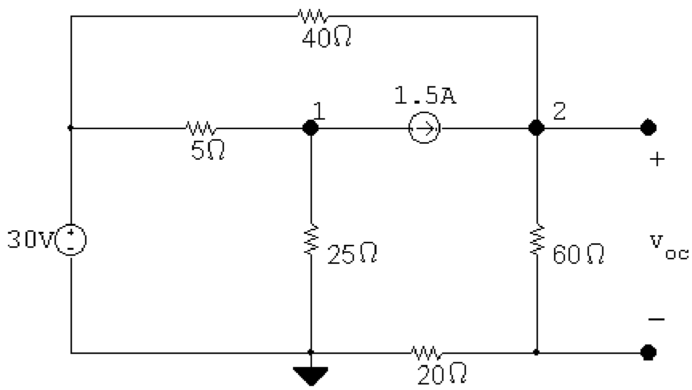
$p_{19.6\text{A}} = -(19.6)(271.9255) = -5329.74$ W(dev)

$\therefore$ The 19.6 A source is developing 5329.74 W

P 3.50 $v_{\text{Th}} = \dfrac{60}{50}(40) = 48$ V $\qquad R_{\text{Th}} = 8 + \dfrac{(40)(10)}{50} = 16\,\Omega$



P 3.51 **[a]** Open circuit:

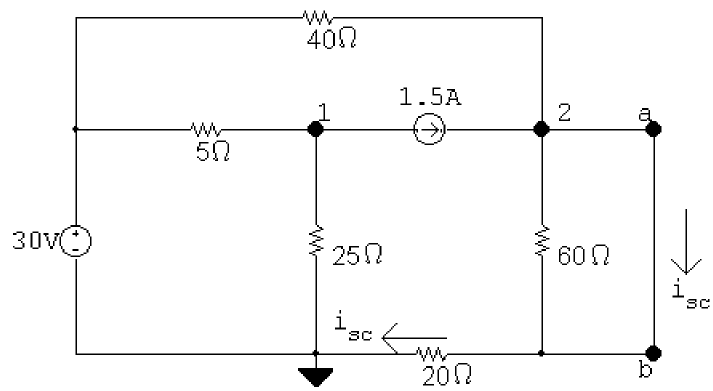


$$\frac{v_2}{80} + \frac{v_2 - 30}{40} - 1.5 = 0$$

$\therefore$ $v_2 = 60$ V

$$v_{\text{oc}} = \frac{60}{80}v_2 = 45 \text{ V} = v_{\text{Th}}$$

Short circuit:

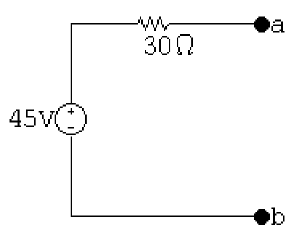


$$\frac{v_2 - 30}{40} - 1.5 + \frac{v_2}{20} = 0$$

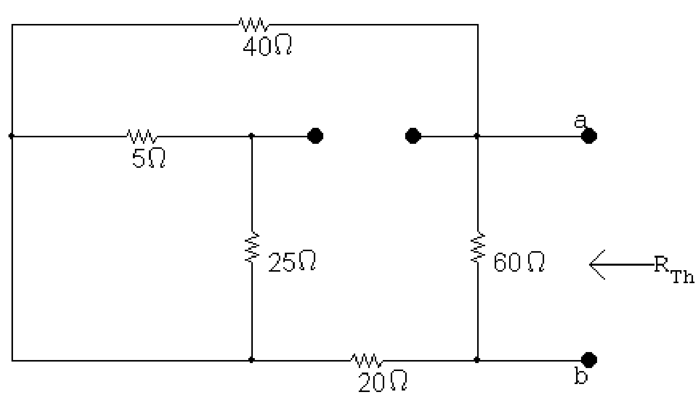
$$\therefore \quad v_2 = 30 \text{ V}$$

$$i_{sc} = \frac{v_2}{20} = 1.5 \text{ A}$$

Therefore, $R_{Th} = 45/1.5 = 30\,\Omega$

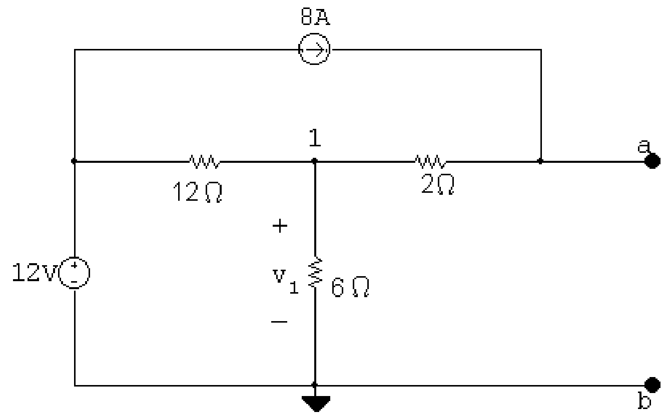


[b]



$R_{Th} = 60\|(40 + 20) = 30\,\Omega$ (CHECKS)

P 3.52

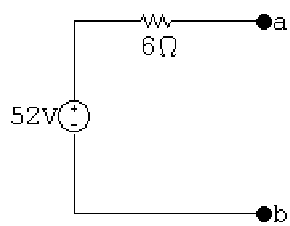


$$\frac{v_1 - 12}{12} + \frac{v_1}{6} - 8 = 0$$

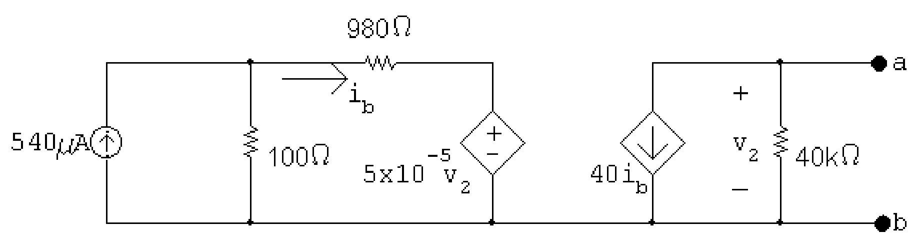
$$v_1 = 36 \text{ V}$$

$$v_{\text{Th}} = v_1 + (2)(8) = 52 \text{ V}$$

$$R_{\text{Th}} = 2 + \frac{(12)(6)}{18} = 6\,\Omega$$



P 3.53



OPEN CIRCUIT

$$v_2 = -40i_b\ 40 \times 10^3 = -16 \times 10^5 i_b$$

$$5 \times 10^{-5} v_2 = -80i_b$$

$$980i_b + 5 \times 10^{-5} v_2 = 900i_b$$

$$100(540 \times 10^{-6}) = 54 \text{ mV}$$

$$\therefore \quad i_b = \frac{54 \times 10^{-3}}{1000} = 54\,\mu\text{A}$$

$$v_{\text{Th}} = -16 \times 10^5 (54 \times 10^{-6}) = -86.40 \text{ V}$$

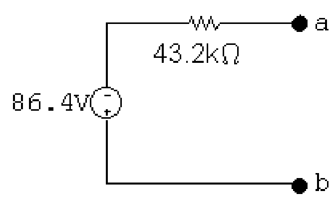
SHORT CIRCUIT

$$v_2 = 0; \qquad i_{\text{sc}} = -40 i_b$$

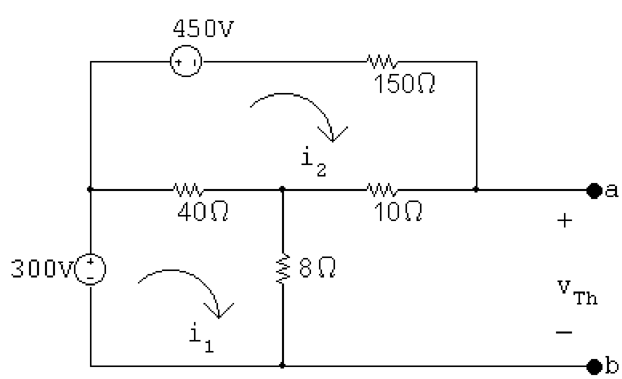
$$i_b = \frac{54 \times 10^{-3}}{1080} = \frac{54}{1.08} \times 10^{-6} = 50\,\mu\text{A}$$

$$i_{\text{sc}} = -40(50) = -2000\,\mu\text{A} = -2 \text{ mA}$$

$$R_{\text{Th}} = \frac{-86.4}{-2} \times 10^3 = 43.2 \text{ k}\Omega$$



P 3.54 After making a source transformation the circuit becomes



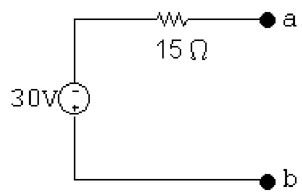
$$300 = 48 i_1 - 40 i_2$$

$$-450 = -40 i_1 + 200 i_2$$

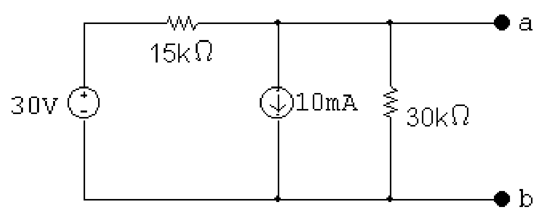
$$\therefore \quad i_1 = 5.25 \text{ A and } i_2 = -1.2 \text{ A}$$

$$v_{\text{Th}} = 8 i_1 + 10 i_2 = 30 \text{ V}$$

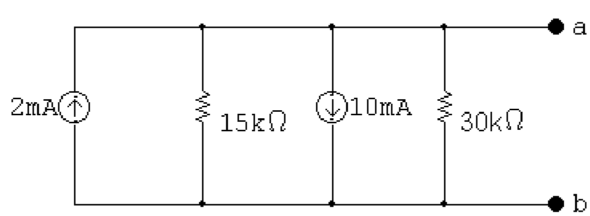
$R_{\text{Th}} = (40\|8 + 10)\|50 = 15\,\Omega$



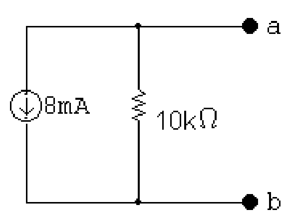
P 3.55 First we make the observation that the 8-mA current source and the 20 kΩ resistor will have no influence on the behavior of the circuit with respect to the terminals a,b. This follows because they are in parallel with an ideal voltage source. Hence our circuit can be simplified to



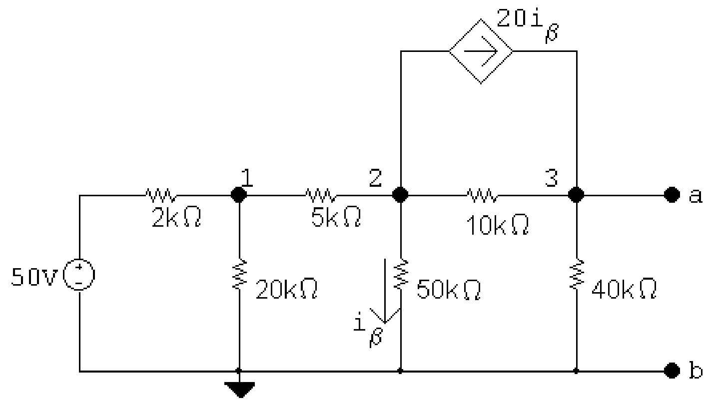
or



Therefore the Norton equivalent is



P 3.56 Open circuit voltage:

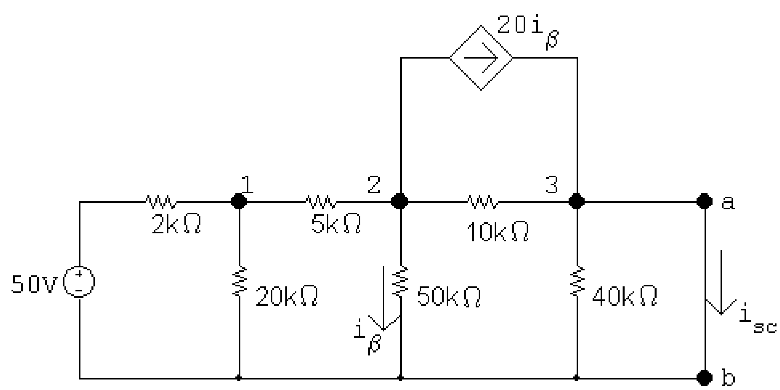


$$\frac{v_1 - 50}{2} + \frac{v_1}{20} + \frac{v_1 - v_2}{5} = 0$$

$$\frac{v_2 - v_1}{5} + \frac{v_2}{50} + \frac{v_2 - v_3}{10} + 20\frac{v_2}{50} = 0$$

$$\frac{v_3}{40} + \frac{v_3 - v_2}{10} - 20\frac{v_2}{50} = 0$$

Solving, $v_3 = 100$ V $= v_{\text{Th}}$
Short circuit current:



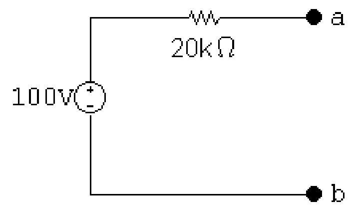
$$\frac{v_1}{20} + \frac{v_1 - 50}{2} + \frac{v_1 - v_2}{5} = 0$$

$$\frac{v_2 - v_1}{5} + \frac{v_2}{50} + \frac{v_2}{10} + 20\frac{v_2}{50} = 0$$

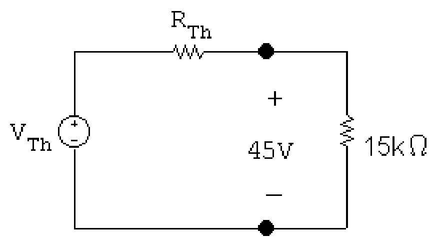
Solving, $v_1 = 36$ V; $\qquad v_2 = 10$ V

$$i_{\text{sc}} = \frac{20(10)}{50{,}000} + \frac{10}{10{,}000} = 0.004 + 0.001 = 5 \text{ mA}$$

$$\therefore \quad R_{\text{Th}} = \frac{v_{\text{Th}}}{i_{\text{sc}}} = 100/0.005 = 20\ \text{k}\Omega$$

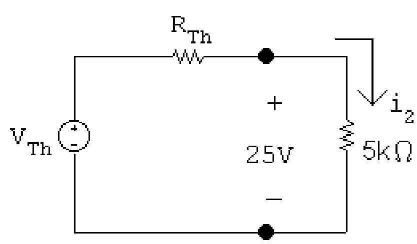


P 3.57



$i_1 = 45/15{,}000 = 3$ mA

$45 = v_{\text{Th}} - 0.003R_{\text{Th}}, \qquad v_{\text{Th}} = 45 + 0.003R_{\text{Th}}$

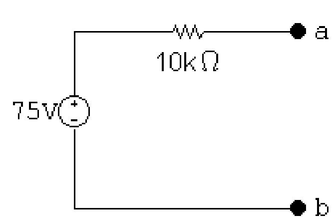


$i_2 = 25/5000 = 5$ mA

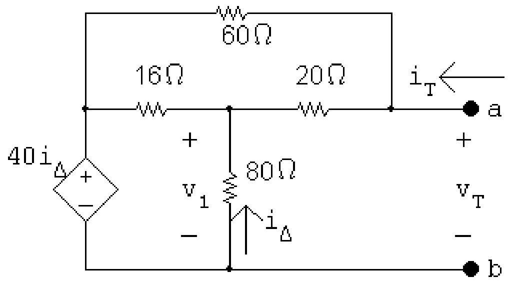
$25 = v_{\text{Th}} - 0.005R_{\text{Th}}, \qquad v_{\text{Th}} = 25 + 0.005R_{\text{Th}}$

$\therefore \quad 45 + 0.003R_{\text{Th}} = 25 + 0.005R_{\text{Th}} \qquad \text{so} \qquad R_{\text{Th}} = 10\ \text{k}\Omega$

$v_{\text{Th}} = 45 + 30 = 75$ V



P 3.58 $V_{\rm Th} = 0$, since circuit contains no independent sources.



$$i_{\rm T} = \frac{v_{\rm T} - v_1}{20} + \frac{v_{\rm T} - 40i_\Delta}{60}$$

$$\frac{v_1 - 40i_\Delta}{16} + \frac{v_1}{80} + \frac{v_1 - v_{\rm T}}{20} = 0$$

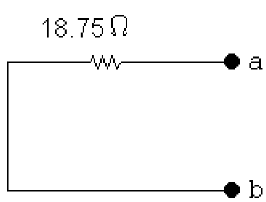
$$\therefore \quad 10v_1 - 200i_\Delta = 4v_{\rm T} \qquad i_\Delta = \frac{-v_1}{80}, \qquad 200i_\Delta = -2.5v_1$$

$$\therefore \quad 12.5v_1 = 4v_{\rm T}; \qquad v_1 = 0.32v_{\rm T}$$

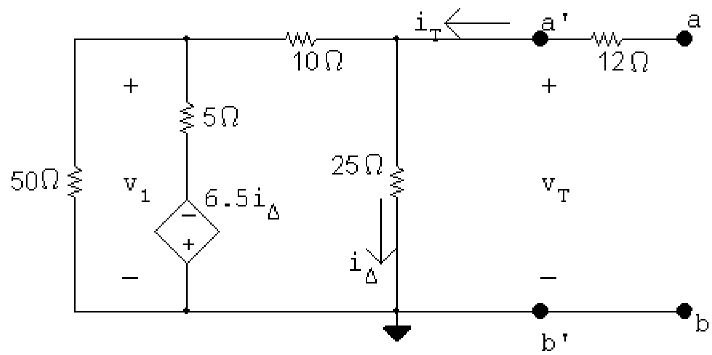
$$60i_{\rm T} = 4v_{\rm T} - 2.5v_1 = 3.2v_{\rm T}$$

$$\therefore \quad \frac{v_{\rm T}}{i_{\rm T}} = \frac{60}{3.2} = 18.75\,\Omega$$

$$R_{\rm Th} = 18.75\,\Omega$$



P 3.59 $V_{\rm Th} = 0$ since there are no independent sources in the circuit. To find $R_{\rm Th}$ we first find $R_{a'b'}$.



$$i_T = \frac{v_T}{25} + \frac{v_T - v_1}{10}$$

$$\frac{v_1}{50} + \frac{v_1 + 6.5i_\Delta}{5} + \frac{v_1 - v_T}{10} = 0 \text{ so } 16v_1 + 65i_\Delta = 5v_T$$

$$i_\Delta = \frac{v_T}{25}, \qquad 65i_\Delta = 2.6v_T$$

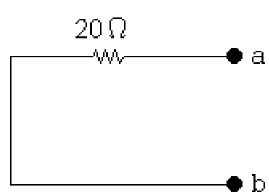
$$16v_1 + 2.6v_T = 5v_T$$

$$\therefore \quad v_1 = 0.15v_T$$

$$i_T = \frac{v_T}{25} + \frac{v_T - 0.15v_T}{10} = \frac{6.25}{50}v_T$$

$$\frac{v_T}{i_T} = 50/6.25 = 8\,\Omega = R_{a'b'}$$

$$\therefore \quad R_{Th} = 12 + 8 = 20\,\Omega$$



P 3.60 **[a]** Since $0 \leq R_o \leq \infty$ maximum power will be delivered to the 6 Ω resistor when $R_o = 0$.

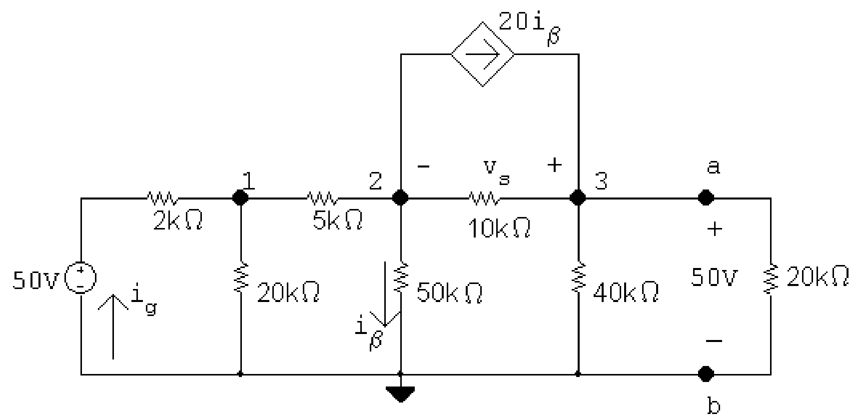
[b] $P = \dfrac{30^2}{6} = 150$ W

P 3.61 **[a]** From the solution of Problem 3.56 we have $R_{\text{Th}} = 20\ \text{k}\Omega$ and $v_{\text{Th}} = 100$ V. Therefore

$$R_o = R_{\text{Th}} = 20\ \text{k}\Omega$$

[b] $p = \dfrac{(50)^2}{20{,}000} = 125$ mW

[c]



$$\frac{v_1}{20{,}000} + \frac{v_1 - 50}{2000} + \frac{v_1 - v_2}{5000} = 0$$

$$\frac{v_2}{50{,}000} + \frac{v_2 - v_1}{5000} + \frac{v_2 - 50}{10{,}000} + 20\left(\frac{v_2}{50{,}000}\right) = 0$$

Solving, $v_1 = 38$ V; $\qquad v_2 = 17.5$ V

$$i_g = \frac{50 - 38}{2000} = 6\ \text{mA}$$

$$p_{50\text{V}}\ (\text{delivered}) = (50)(0.006) = 300\ \text{mW}$$

$$v_2 + v_s = 50\ \text{V}$$

$$v_s = 50 - (17.5) = 32.5\ \text{V}$$

$$i_\beta = \frac{v_2}{50{,}000} = 0.35\ \text{mA}$$

$$20i_\beta = 7\ \text{mA}$$

$$p_{20i_\beta}\ (\text{delivered}) = (32.5)(0.007) = 227.5\ \text{mW}$$

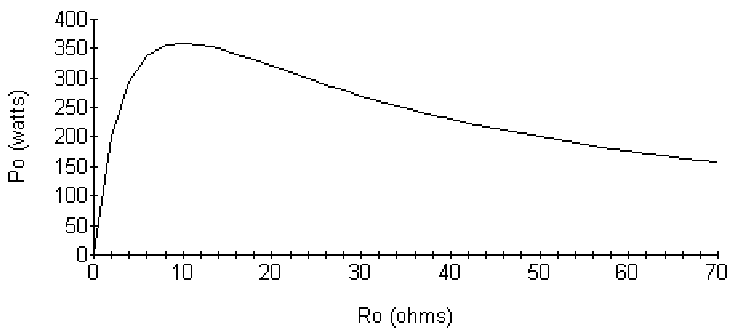
$$\sum p_{\text{dev}} = 300 + 227.5 = 527.5\ \text{mW}$$

$$\%\ \text{delivered} = \frac{125}{527.5} \times 100 = 23.7\%$$

P 3.62 **[a]** From the solution to Problem 2.25 we have

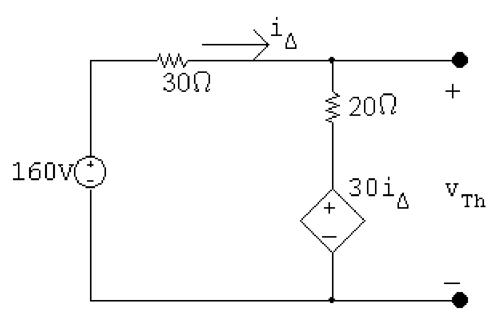
$R_o(\Omega)$	P_o(W)	$R_o(\Omega)$	P_o(W)
0	0	20	320.00
2	200.00	30	270.00
6	337.50	40	230.40
10	360.00	50	200.00
15	345.60	70	157.50

[b]



[c] $R_o = 10\,\Omega, \qquad P_o\ (\text{max}) = 360$ W

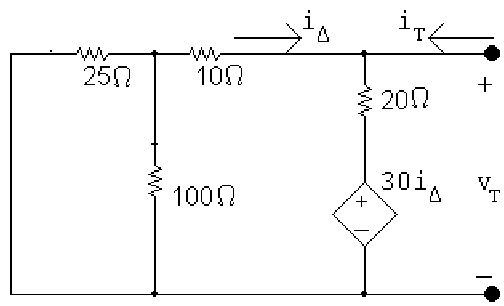
P 3.63 We begin by finding the Thévenin equivalent with respect to R_o. After making a couple of source transformations the circuit simplifies to



$$i_\Delta = \frac{160 - 30i_\Delta}{50}; \qquad i_\Delta = 2\text{ A}$$

$$v_{\text{Th}} = 20i_\Delta + 30i_\Delta = 50i_\Delta = 100\text{ V}$$

Using the test-source method to find the Thévenin resistance gives

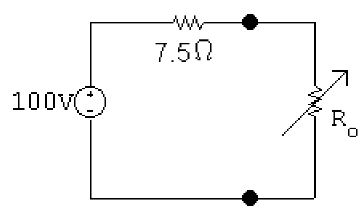


$$i_{\rm T} = \frac{v_{\rm T}}{30} + \frac{v_{\rm T} - 30(-v_{\rm T}/30)}{20}$$

$$\frac{i_{\rm T}}{v_{\rm T}} = \frac{1}{30} + \frac{1}{10} = \frac{4}{30} = \frac{2}{15}$$

$$R_{\rm Th} = \frac{v_{\rm T}}{i_{\rm T}} = \frac{15}{2} = 7.5\,\Omega$$

Thus our problem is reduced to analyzing the circuit shown below.



$$p = \left(\frac{100}{7.5 + R_o}\right)^2 R_o = 250$$

$$\frac{10^4}{R_o^2 + 15R_o + 56.25}R_o = 250$$

$$\frac{10^4 R_o}{250} = R_o^2 + 15R_o + 56.25$$

$$40R_o = R_o^2 + 15R_o + 56.25$$

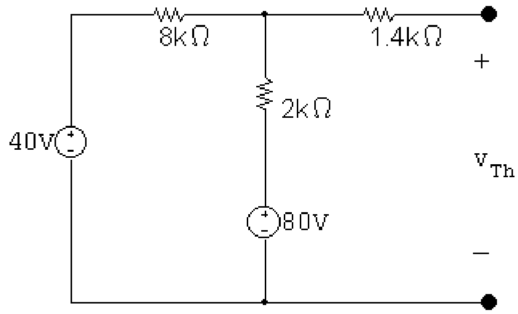
$$R_o^2 - 25R_o + 56.25 = 0$$

$$R_o = 12.5 \pm \sqrt{156.25 - 56.25} = 12.5 \pm 10$$

$$R_o = 22.5\,\Omega$$

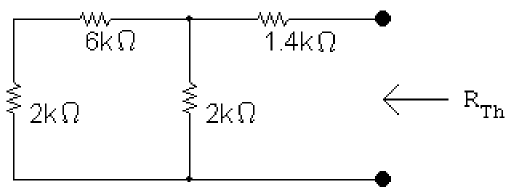
$$R_o = 2.5\,\Omega$$

P 3.64 [**a**]



$$\frac{v_{\rm Th} - 40}{8000} + \frac{v_{\rm Th} - 80}{2000} = 0$$

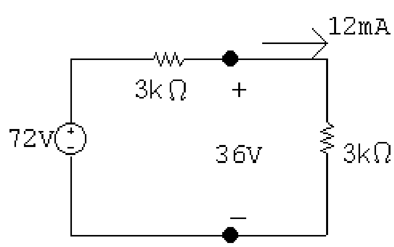
$\therefore \quad v_{\rm Th} = 72$ V



$R_{\rm Th} = 1400 + (2000)(8000)/1000 = 3$ kΩ

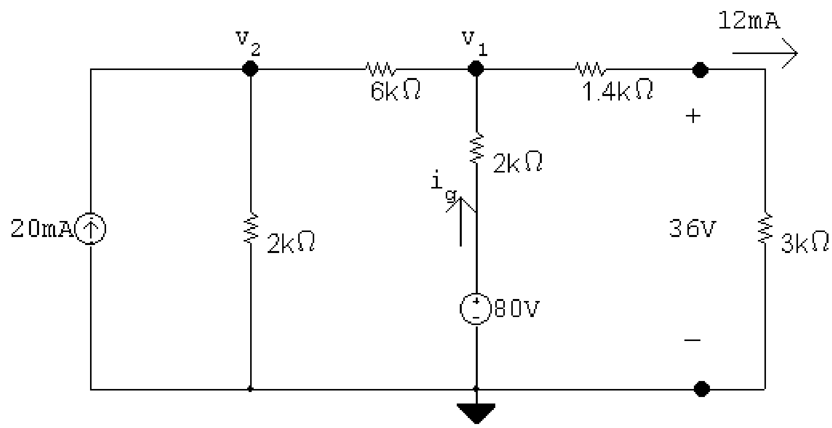
$R_o = R_{\rm Th} = 3$ kΩ

[**b**]



$$p_{\rm max} = \frac{(36)^2}{3} \times 10^{-3} = 432 \text{ mW}$$

P 3.65



$$v_1 = (12 \times 10^{-3})(1.4 + 3) \times 10^3 = 12(4.4) = 52.8 \text{ V}$$

$$i_g = \frac{80 - 52.8}{2000} = 13.6 \text{ mA}$$

$$p_{80\text{V}} \text{ (dev) } = (80)(0.0136) = 1088 \text{ mW}$$

$$-0.02 + \frac{v_2}{2000} + \frac{v_2 - 52.8}{6000} = 0$$

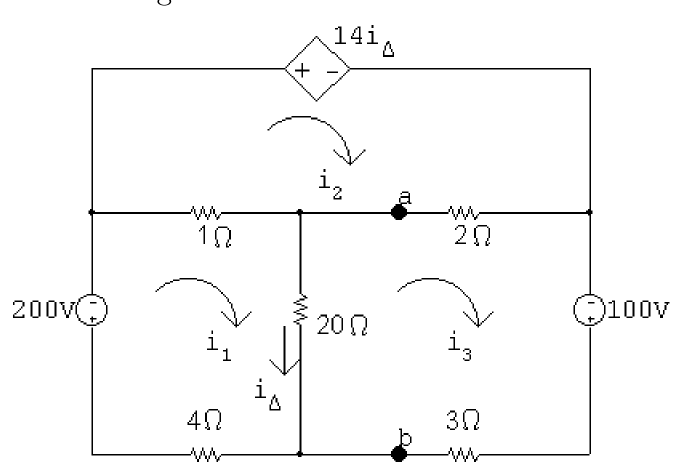
$$\therefore \quad v_2 = 43.2 \text{ V}$$

$$p_{20\text{mA}} \text{ (dev) } = (0.02)(43.2) = 864 \text{ mW}$$

$$\sum p_{\text{dev}} = 1088 + 864 = 1952 \text{ mW}$$

$$\% \text{ delivered to } R_o = \frac{432}{1952} \times 100 = 22.13\%$$

P 3.66 **[a]** We begin by finding the Thévenin equivalent with respect to the terminals of R_o.
Open circuit voltage



$$-200 = 25i_1 - 1i_2 - 20i_3$$

$0 = -i_1 + 3i_2 - 2i_3 + 14i_\Delta$

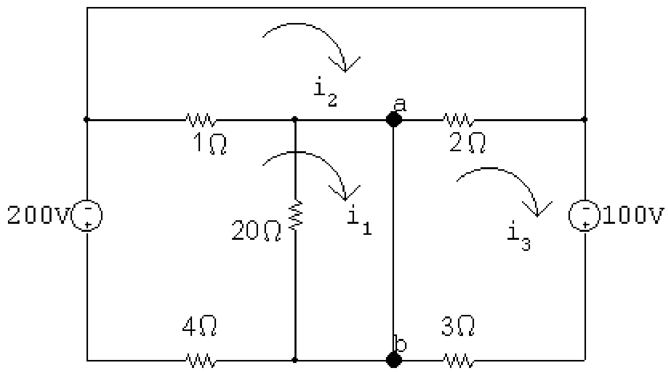
$100 = -20i_1 - 2i_2 + 25i_3$

$i_\Delta = i_1 - i_3$

Solving, $i_1 = -2.5$ A; $i_2 = 37.5$ A; $i_3 = 5$ A; $i_\Delta = -7.5$ A

$v_{\rm Th} = 20(i_1 - i_3) = 20(-7.5) = -150$ V

Now find the short-circuit current.



Note with the short circuit from a to b that i_Δ is zero, hence $14i_\Delta$ is also zero.

$-200 = 5i_1 - 1i_2 + 0i_3$

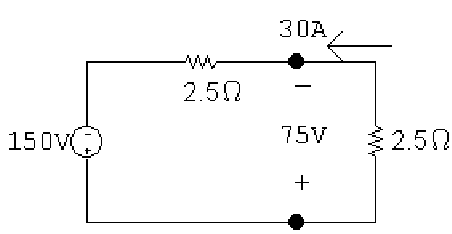
$0 = -1i_1 + 3i_2 - 2i_3$

$100 = 0i_1 - 2i_2 + 5i_3$

Solving, $i_1 = -40$ A; $i_2 = 0$ A; $i_3 = 20$ A

$i_{\rm sc} = i_1 - i_3 = -60$ A

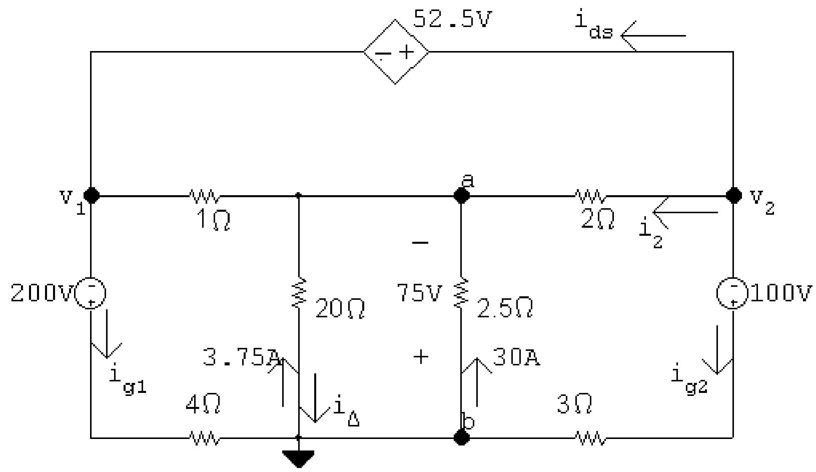
$R_{\rm Th} = (-150)/(-60) = 2.5\,\Omega$



For maximum power transfer $R_o = R_{\rm Th} = 2.5\,\Omega$

[b] $p_{\rm max} = \dfrac{75^2}{2.5} = 2250$ W

P 3.67 From the solution of Problem 3.66 we know that when R_o is $2.5\,\Omega$, the voltage across R_o is 75 V, positive at the lower terminal. Therefore our problem reduces to the analysis of the following circuit. In constructing the circuit we have used the fact that i_Δ is -3.75 A, and hence $14i_\Delta$ is -52.5 V.



Using the node Voltage method to find v_1 and v_2 yields

$$-33.75 + \frac{-75 - v_1}{1} + \frac{-75 - v_2}{2} = 0$$

$$v_1 + 52.5 = v_2$$

Solving, $v_1 = -115$ V; $\qquad v_2 = -62.5$ V.It follows that

$$i_{g_1} = \frac{-115 + 200}{4} = 21.25 \text{ A}$$

$$i_{g_2} = \frac{-62.5 + 100}{3} = 12.5 \text{ A}$$

$$i_2 = \frac{-62.5 + 75}{2} = 6.25 \text{ A}$$

$$i_{\text{ds}} = -6.25 - 12.5 = -18.75 \text{ A}$$

$$p_{200\text{V}} = -200 i_{g_1} = -4250 \text{ W(dev)}$$

$$p_{100\text{V}} = -100 i_{g_2} = -1250 \text{ W(dev)}$$

$$p_{\text{ds}} = 52.5 i_{\text{ds}} = -984.375 \text{ W(dev)}$$

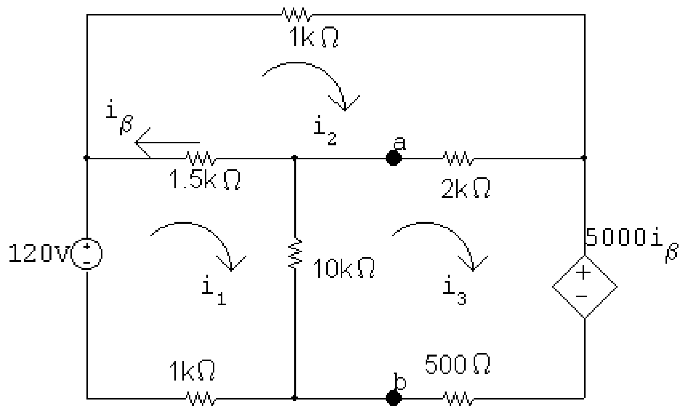
$$\therefore \quad \sum p_{\text{dev}} = 4250 + 1250 + 984.375 = 6484.375 \text{ W}$$

$$\therefore \quad \%\text{ delivered } = \frac{2250}{6484.375}(100) = 34.7\%$$

$\therefore$ 34.7% of developed power is delivered to load

P 3.68 **[a]** Find the Thévenin equivalent with respect to the terminals of R_L.

Open circuit voltage:



$$120 = 12{,}500i_1 - 1500i_2 - 10{,}000i_3$$

$$0 = -1500i_1 + 4500i_2 - 2000i_3$$

$$0 = -10{,}000i_1 - 2000i_2 + 12{,}500i_3 + 5000i_\beta$$

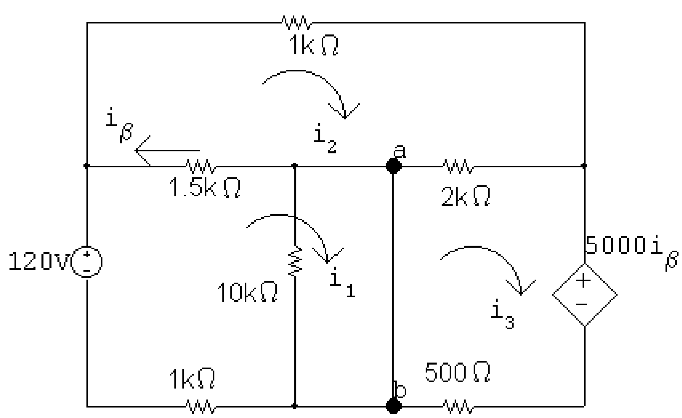
$$i_\beta = i_2 - i_1$$

Solving,

$i_1 = 99.6$ mA; $\quad i_2 = 78$ mA; $\quad i_3 = 100.8$ mA; $\quad i_\beta = -21.6$ mA

$$v_{\rm Th} = v_{\rm ab} = 10 \times 10^3(i_1 - i_3) = -12 \text{ V}$$

Short-circuit current:



$$120 = 2500i_1 - 1500i_2 + 0i_3$$

$$0 = -1500i_1 + 4500i_2 - 2000i_3$$

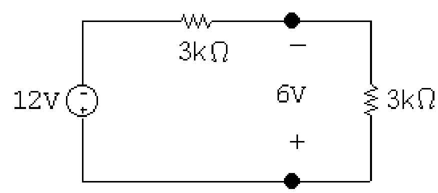
$$0 = 0i_1 - 2000i_2 + 2500i_3 + 5000i_\beta$$

$$i_\beta = i_2 - i_1$$

Solving,

$i_1 = 92$ mA; $\quad i_2 = 73.33$ mA; $\quad i_3 = 96$ mA; $\quad i_\beta = -18.67$ mA

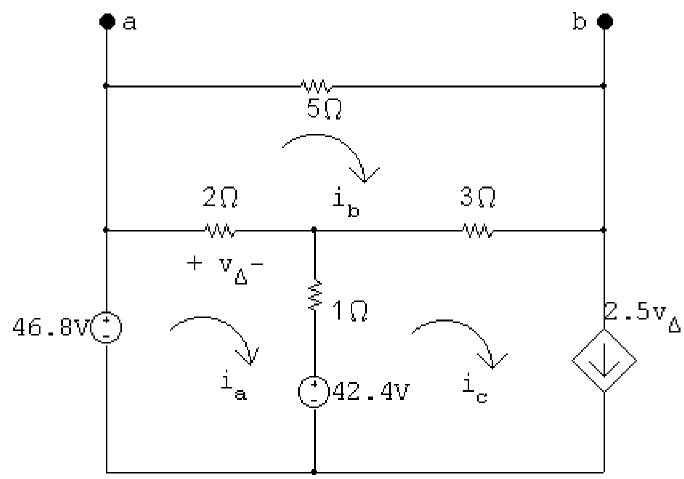
$i_{\rm sc} = i_1 - i_3 = -4$ mA; $\qquad R_{\rm Th} = \dfrac{v_{\rm Th}}{i_{\rm sc}} = \dfrac{-12}{-4 \times 10^{-3}} = 3$ kΩ



$R_{\rm L} = R_{\rm Th} = 3$ kΩ

[b] $p_{\rm max} = \dfrac{6^2}{3 \times 10^3} = 12$ mW

P 3.69 Find the Thévenin equivalent with respect to the terminals of R_o. Open circuit voltage:



$(46.8 - 42.4) = 3i_a - 2i_b - i_c$

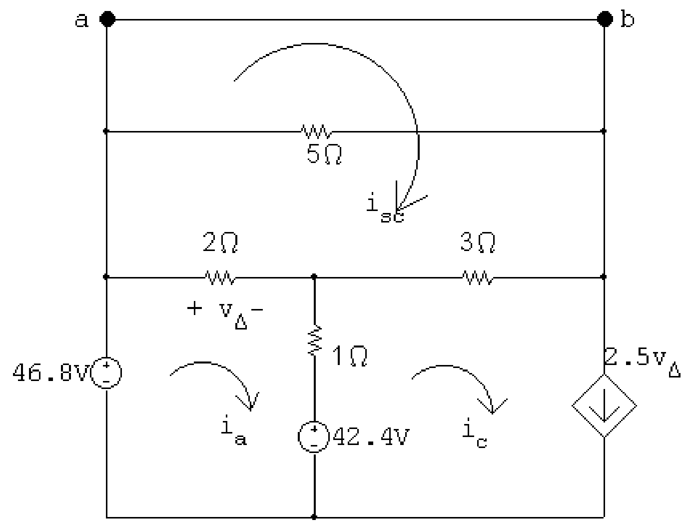
$0 = -2i_a + 10i_b - 3i_c$

$i_c = 2.5v_\Delta; \qquad v_\Delta = 2(i_a - i_b)$

Solving, $i_b = 74.8$ A

$\therefore \quad v_{\rm Th} = 5i_b = 374$ V

Short circuit current:



$46.8 - 42.4 = 3i_a - 2i_{\rm sc} - i_c$

$0 = -2i_a + 5i_{\rm sc} - 3i_c$

$i_c = 2.5v_\Delta;\qquad v_\Delta = 2(i_a - i_{\rm sc})$

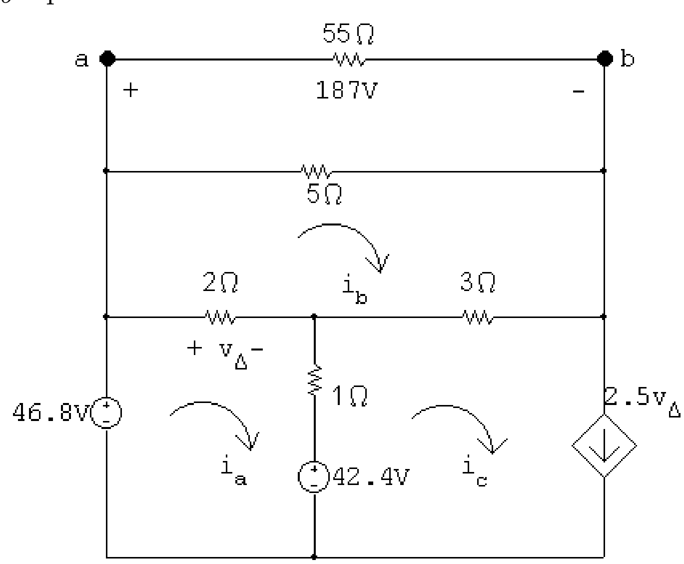
Solving,

$i_{\rm sc} = 6.8$ A; $\qquad i_a = 8$ A; $\qquad i_c = 6$ A; $\qquad v_\Delta = 2.4$ V

$R_{\rm Th} = v_{\rm Th}/i_{\rm sc} = 374/6.8 = 55\,\Omega$

$R_o = 55\,\Omega$

with R_o equal to $55\,\Omega$ the circuit becomes



$46.8 - 42.4 = 3i_a - 2i_b - 2.5(2)(i_a - i_b)$

$i_c = 2.5v_\Delta$

$v_\Delta = 2(i_a - i_b)$

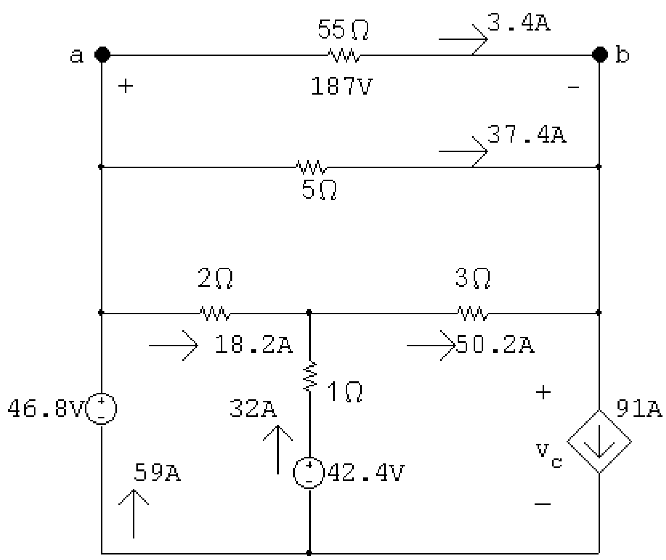
$187 + 3i_b - 3(2.5)(2)(i_a - i_b) + 2i_b - 2i_a = 0$

Solving, $i_a = 59$ A; $\qquad i_b = 40.8$ A

$v_\Delta = 2(59 - 40.80) = 36.4$ V

$i_c = 91$ A

Thus we have



$v_c = 42.4 - 32 - 150.6 = -140.20$ V

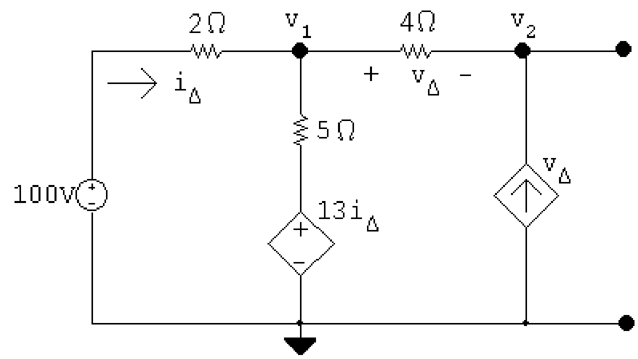
$\sum P_{\text{dev}} = 46.8(59) + 42.4(32) + 140.20(91) = 16{,}876.20$ W

CHECK:

$$\begin{aligned}\sum P_{\text{dis}} &= (18.2)^2(2) + (50.2)^2(3) + (32)^2(1) + 187(3.4) + 187(37.4)\\ &= 16{,}876.20 \text{ W}\end{aligned}$$

$$\%\text{ delivered } = \frac{(55)(3.4)^2(100)}{16{,}876.2} = 3.77\%$$

P 3.70 **[a]** Open circuit voltage



Node voltage equation:

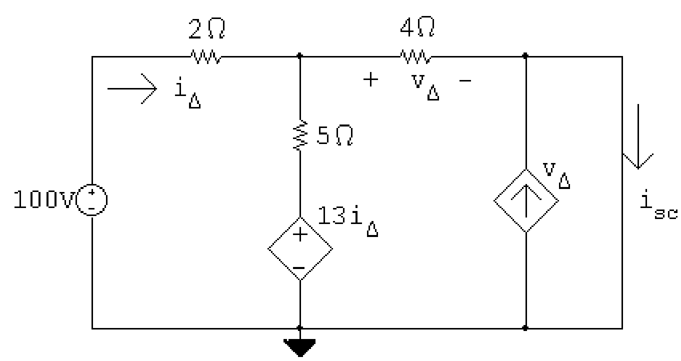
$$\frac{v_1 - 100}{2} + \frac{v_1 - 13i_\Delta}{5} + \frac{v_1 - v_2}{4} = 0$$

Constraint equations:

$$i_\Delta = \frac{100 - v_1}{2}; \qquad \frac{v_2 - v_1}{4} - v_\Delta = 0; \qquad v_\Delta = v_1 - v_2$$

Solving, $v_2 = 90$ V $= v_{\text{Th}}$; $\quad v_1 = 0$ V; $\quad v_\Delta = 0$ V; $\quad i_\Delta = 5$ A

Short circuit current:



$$\frac{v_1 - 100}{2} + \frac{v_1 - 13i_\Delta}{5} + \frac{v_1}{4} = 0$$

$$i_\Delta = \frac{100 - v_1}{2}$$

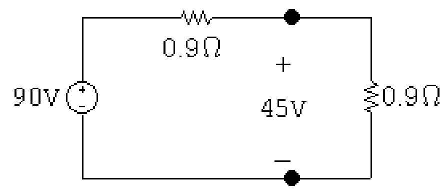
Solving, $v_1 = 80$ V $= v_\Delta$; $\quad i_\Delta = 10$ A

$$i_{\text{sc}} = \frac{v_1}{4} + v_\Delta = 20 + 80 = 100 \text{ A}$$

$$R_{\text{Th}} = \frac{v_{\text{Th}}}{i_{\text{sc}}} = \frac{90}{100} = 0.9\,\Omega$$

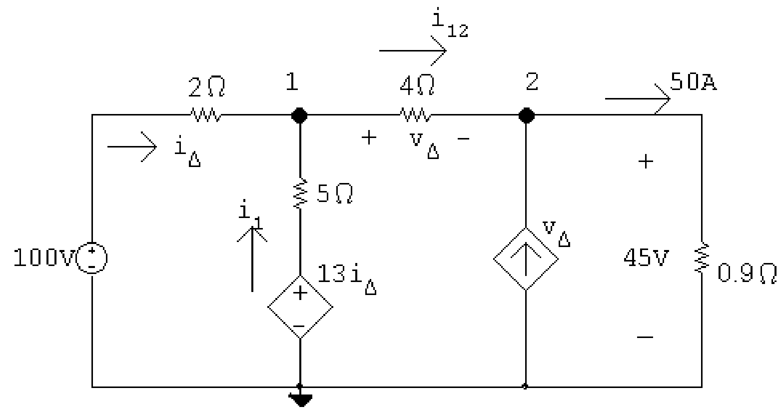
$$\therefore \quad R_o = R_{\text{Th}} = 0.9\,\Omega$$

[**b**]



$$p_{\max} = \frac{(45)^2}{0.9} = 2250 \text{ W}$$

[**c**]



$$\frac{v_1 - 100}{2} + \frac{v_1 - 13i_\Delta}{5} + \frac{v_1 - 45}{4} = 0$$

$$i_\Delta = \frac{100 - v_1}{2}$$

Solving, $v_1 = 85$ V; $\quad i_\Delta = 7.5$ A; $\quad v_\Delta = v_1 - v_2 = 85 - 45 = 40$ V

$i_{100\text{V}} = i_\Delta = 7.5$ A

$p_{100\text{V}}$ (dev) $= 100(7.5) = 750$ W

$i_{12} = v_\Delta/4 = 40/4 = 10$ A

$i_1 = i_{12} - i_\Delta - 10 - 7.5 = 2.5$ A

p_{13i_Δ} (dev) $= (97.5)(2.5) = 243.75$ W

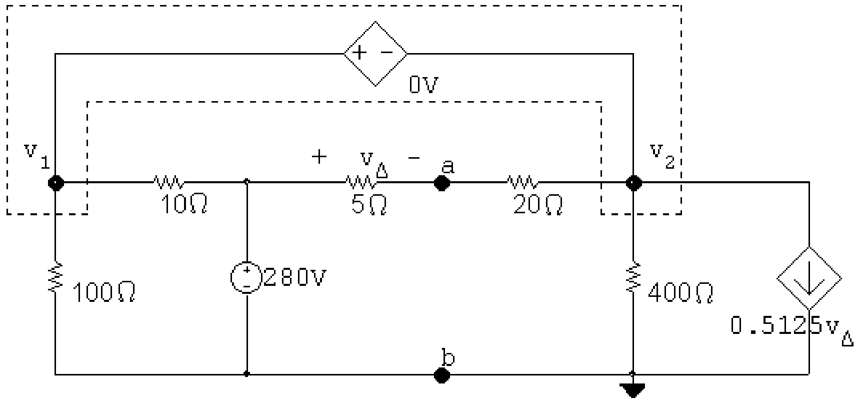
p_{v_Δ} (dev) $= (45)(40) = 1800$ W

$\sum p_{\text{dev}} = 750 + 243.75 + 1800 = 2793.75$ W

$$\% \text{ delivered } = \frac{2250}{2793.75} \times 100 = 80.54\%$$

P 3.71 [a] First find the Thévenin equivalent with respect to R_o.

Open circuit voltage: $i_\phi = 0$; $50i_\phi = 0$



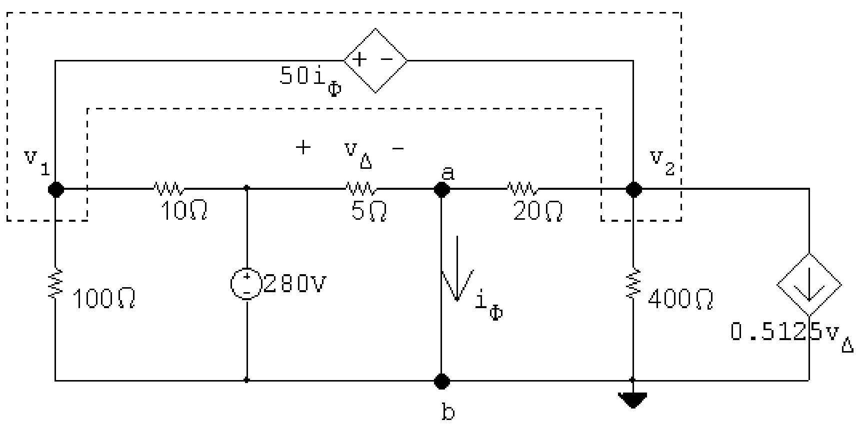
$$\frac{v_1}{100} + \frac{v_1 - 280}{10} + \frac{v_1 - 280}{25} + \frac{v_1}{400} + 0.5125v_\Delta = 0$$

$$v_\Delta = \frac{(280 - v_1)}{25}5 = 56 - 0.2v_1$$

$$v_1 = 210 \text{ V}; \qquad v_\Delta = 14 \text{ V}$$

$$v_{\text{Th}} = 280 - v_\Delta = 280 - 56 + 0.2(210) = 266 \text{ V}$$

Short circuit current



$$\frac{v_1}{100} + \frac{v_1 - 280}{10} + \frac{v_2}{20} + \frac{v_2}{400} + 0.5125(280) = 0$$

$$v_\Delta = 280 \text{ V}$$

$$v_2 + 50i_\phi = v_1$$

$$i_\phi = \frac{280}{5} + \frac{v_2}{20} = 56 + 0.05v_2$$

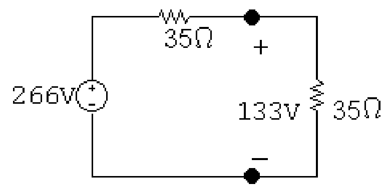
$$v_2 = -968 \text{ V}; \qquad v_1 = -588 \text{ V}$$

$i_\phi = i_{sc} = 56 + 0.05(-968) = 7.6$ A

$R_{Th} = v_{Th}/i_{sc} = 266/7.6 = 35\,\Omega$

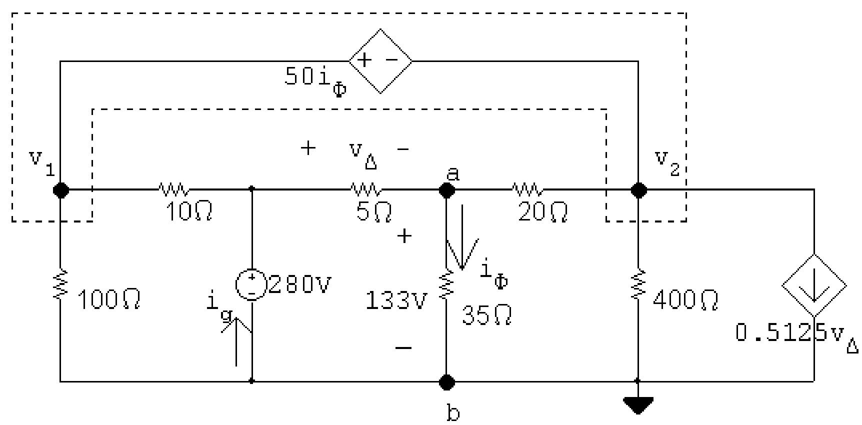
$\therefore \quad R_o = 35\,\Omega$

[b]



$p_{max} = (133)^2/35 = 505.4$ W

[c]



$$\frac{v_1}{100} + \frac{v_1 - 280}{10} + \frac{v_2 - 133}{20} + \frac{v_2}{400} + 0.5125(280 - 133) = 0$$

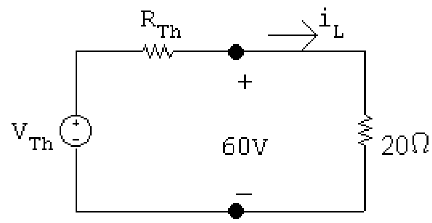
$v_2 + 50i_\phi = v_1; \qquad i_\phi = 133/35 = 3.8$ A

Therefore, $v_1 = -189$ V and $v_2 = -379$ V; thus,

$$i_g = \frac{280 - 133}{5} + \frac{280 + 189}{10} = 76.30 \text{ A}$$

p_{280V} (dev) $= (280)(76.3) = 21{,}364$ W

P 3.72 [a]



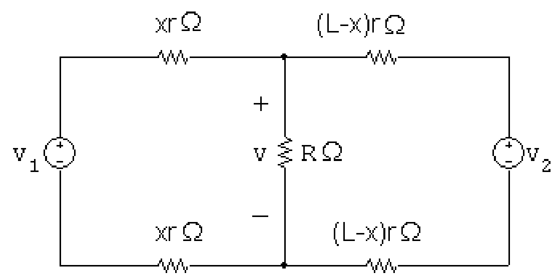
$$v_{oc} = V_{Th} = 75 \text{ V}; \qquad i_L = \frac{60}{20} = 3 \text{ A}; \qquad i_L = \frac{75 - 60}{R_{Th}} = \frac{15}{R_{Th}}$$

Therefore $\quad R_{\rm Th} = \dfrac{15}{3} = 5\,\Omega$

[b] $i_L = \dfrac{v_o}{R_L} = \dfrac{V_{\rm Th} - v_o}{R_{\rm Th}}$

Therefore $\quad R_{\rm Th} = \dfrac{V_{\rm Th} - v_o}{v_o/R_L} = \left(\dfrac{V_{\rm Th}}{v_o} - 1\right) R_L$

P 3.73 **[a]**



$$\frac{v - v_1}{2xr} + \frac{v}{R} + \frac{v - v_2}{2r(\ell - x)} = 0$$

$$v\left[\frac{1}{2xr} + \frac{1}{R} + \frac{1}{2r(L - x)}\right] = \frac{v_1}{2xr} + \frac{v_2}{2r(\ell - x)}$$

$$v = \frac{v_1 RL + xR(v_2 - v_1)}{RL + 2rLx - 2rx^2}$$

[b] Let $D = RL + 2rLx - 2rx^2$

$$\frac{dv}{dx} = \frac{(RL + 2rLx - 2rx^2)R(v_2 - v_1) - [v_1 RL + xR(v_2 - v_1)]2rL - 2x)}{D^2}$$

$$\frac{dv}{dx} = 0 \quad \text{when numerator is zero.}$$

The numerator simplifies to

$$x^2 + \frac{2L - v_1}{(v_2 - v_1)}x + \frac{RL(v_2 - v_1) - 2rv_1L^2}{2r(v_2 - v_1)} = 0$$

Solving for the roots of the quadratic yields

$$x = \frac{L}{v_2 - v_1}\left\{-v_1 \pm \sqrt{v_1v_2 - \frac{R}{2rL}(v_2 - v_1)^2}\right\}$$

[c] $x = \dfrac{L}{v_2 - v_1}\left\{-v_1 \pm \sqrt{v_1v_2 - \dfrac{R}{2rL}(v_1 - v_2)^2}\right\}$

$v_2 = 1200$ V, $\qquad v_1 = 1000$ V, $\qquad L = 16$ km

$r = 5 \times 10^{-5}\,\Omega/m; \qquad R = 3.9\,\Omega$

$$\frac{L}{v_2 - v_1} = \frac{16{,}000}{1200 - 1000} = 80; \qquad v_1 v_2 = 1.2 \times 10^6$$

$$\frac{R}{2rL}(v_1 - v_2)^2 = \frac{3.9(-200)^2}{(10 \times 10^{-5})(16 \times 10^3)} = 0.975 \times 10^5$$

$$x = 80\{-1000 \pm \sqrt{1.2 \times 10^6 - 0.0975 \times 10^6}\}$$

$$= 80\{-1000 \pm 1050\} = 80(50) = 4000 \text{ m}$$

[d]

$$v_{\min} = \frac{v_1 RL + R(v_2 - v_1)x}{RL + 2rLx - 2rx^2}$$

$$= \frac{(1000)(3.9)(16 \times 10^3) + 3.9(200)(4000)}{(3.9)(16{,}000) + 10 \times 10^{-5}(16{,}000)(4000) - 10 \times 10^{-5}(16 \times 10^6)}$$

$$= 975 \text{ V}$$

P 3.74 $$\frac{dv_1}{dI_{g1}} = \frac{-R_1[R_2(R_3 + R_4) + R_3R_4]}{(R_1 + R_2)(R_3 + R_4) + R_3R_4}$$

$$\frac{dv_1}{dI_{g2}} = \frac{R_1R_3R_4}{(R_1 + R_2)(R_3 + R_4) + R_3R_4}$$

$$\frac{dv_2}{dI_{g1}} + \frac{-R_1R_3R_4}{(R_1 + R_2)(R_3 + R_4) + R_3R_4}$$

$$\frac{dv_2}{dI_{g2}} = \frac{R_3R_4(R_1 + R_2)}{(R_1 + R_2)(R_3 + R_4) + R_3R_4}$$

P 3.75 From the solution to Problem 3.74 we have

$$\frac{dv_1}{dI_{g1}} = \frac{-25[5(125) + 3750]}{30(125) + 3750} = -\frac{175}{12} \text{ V/A}$$

and

$$\frac{dv_2}{dI_{g1}} = \frac{-(25)(50)(75)}{30(125) + 3750} = -12.5 \text{ V/A}$$

By hypothesis, $\Delta I_{g1} = 11 - 12 = -1$ A

$$\therefore \quad \Delta v_1 = (-\frac{175}{12})(-1) = \frac{175}{12} = 14.5833 \text{ V}$$

Thus, $v_1 = 25 + 14.5833 = 39.5833$ V

Also,

$$\Delta v_2 = (-12.5)(-1) = 12.5 \text{ V}$$

Thus, $v_2 = 90 + 12.5 = 102.5$ V
The PSpice solution is

$$v_1 = 39.5830 \text{ V}$$

and

$$v_2 = 102.5000 \text{ V}$$

These values are in agreement with our predicted values.

P 3.76 From the solution to Problem 3.74 we have

$$\frac{dv_1}{dI_{g2}} = \frac{(25)(50)(75)}{30(125) + 3750} = 12.5 \text{ V/A}$$

and

$$\frac{dv_2}{dI_{g2}} = \frac{(50)(75)(30)}{30(125) + 3750} = 15 \text{ V/A}$$

By hypothesis, $\Delta I_{g2} = 17 - 16 = 1$ A

$$\therefore \quad \Delta v_1 = (12.5)(1) = 12.5 \text{ V}$$

Thus, $v_1 = 25 + 12.5 = 37.5$ V
Also,

$$\Delta v_2 = (15)(1) = 15 \text{ V}$$

Thus, $v_2 = 90 + 15 = 105$ V
The PSpice solution is

$$v_1 = 37.5 \text{ V}$$

and

$$v_2 = 105 \text{ V}$$

These values are in agreement with our predicted values.

P 3.77 From the solutions to Problems 3.74 — 3.76 we have

$$\frac{dv_1}{dI_{g1}} = -\frac{175}{12} \text{ V/A}; \qquad \frac{dv_1}{dI_{g2}} = 12.5 \text{ V/A}$$

$$\frac{dv_2}{dI_{g1}} = -12.5 \text{ V/A}; \qquad \frac{dv_2}{dI_{g2}} = 15 \text{ V/A}$$

By hypothesis,

$$\Delta I_{g1} = 11 - 12 = -1 \text{ A}$$

$$\Delta I_{g2} = 17 - 16 = 1 \text{ A}$$

Therefore,

$$\Delta v_1 = \frac{175}{12} + 12.5 = 27.0833 \text{ V}$$

$$\Delta v_2 = 12.5 + 15 = 27.5 \text{ V}$$

Hence

$$v_1 = 25 + 27.0833 = 52.0833 \text{ V}$$

$$v_2 = 90 + 27.5 = 117.5 \text{ V}$$

The PSpice solution is

$$v_1 = 52.0830 \text{ V}$$

and

$$v_2 = 117.5 \text{ V}$$

These values are in agreement with our predicted values.

P 3.78 By hypothesis,

$$\Delta R_1 = 27.5 - 25 = 2.5\,\Omega$$

$$\Delta R_2 = 4.5 - 5 = -0.5\,\Omega$$

$$\Delta R_3 = 55 - 50 = 5\,\Omega$$

$$\Delta R_4 = 67.5 - 75 = -7.5\,\Omega$$

So

$\Delta v_1 = 0.5833(2.5) - 5.417(-0.5) + 0.45(5) + 0.2(-7.5) = 4.9168$ V

$\therefore \quad v_1 = 25 + 4.9168 = 29.9168$ V

$\Delta v_2 = 0.5(2.5) + 6.5(-0.5) + 0.54(5) + 0.24(-7.5) = -1.1$ V

$\therefore \quad v_2 = 90 - 1.1 = 88.9$ V

The PSpice solution is

$v_1 = 29.6710$ V

and

$v_2 = 88.5260$ V

Note our predicted values are within a fraction of a volt of the actual values.