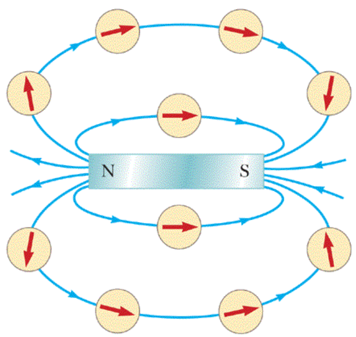
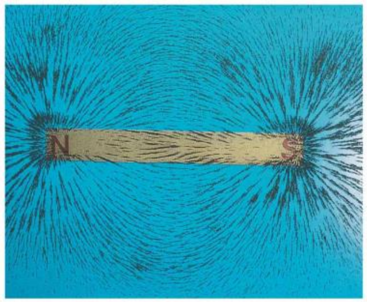
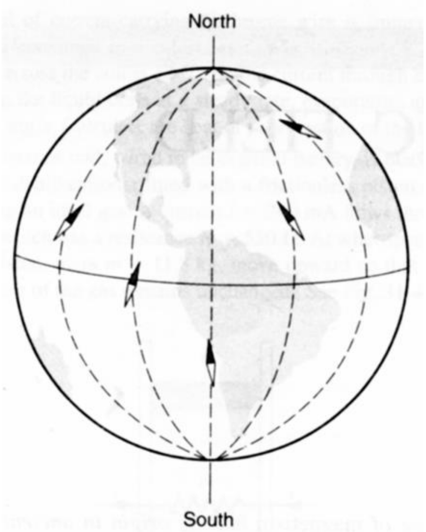
Chapter 25-26 Magnetic Forces and Magnetic fields

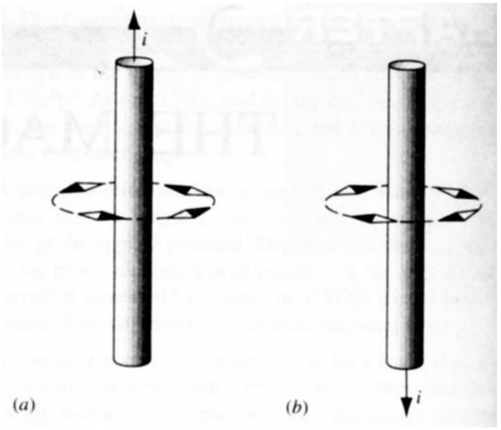
§ 1 Magnetic Fields and Magnetic Forces (580-586)

Magnetic phenomena









Any magnet has two poles: north pole and south pole.

When two magnets are brought near one another, each exerts a force on the other.

No magnetic monopole has ever been observed.

The Comparison between Electronic and Magnetic Interaction Models

Electric interaction model

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{r} \qquad \vec{F}_e = q\vec{E}$$

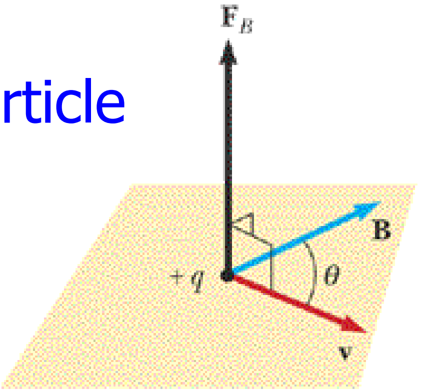
charge $\rightleftarrows$ electric field $\rightleftarrows$ charge

Magnetic interaction model

moving charge current $\overset{?}{\rightleftarrows}$ magnetic field $\overset{?}{\rightleftarrows}$ moving charge current

- How does a moving charge or a current create the magnetic field throughout the space?
- How does the magnetic field exert a force on any other moving charge or current that presents in the field?

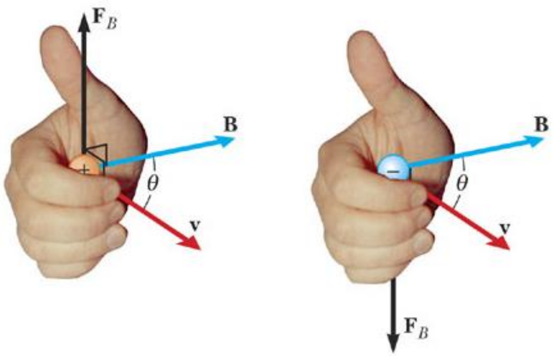
- The magnetic force on a moving charged particle

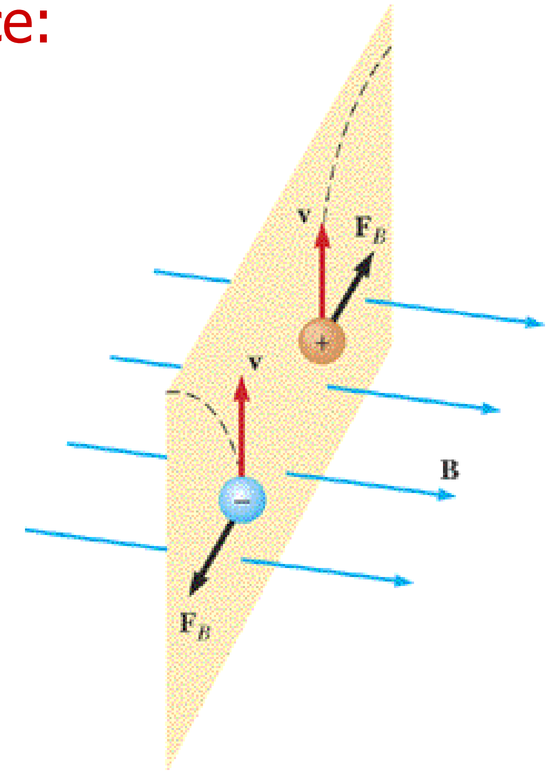


$$\boxed{\vec{F}_B = q\vec{v} \times \vec{B}}$$

$$F_B = |q| vB \sin\theta$$

- Right hand rule for the magnetic force:





- **Definition of B**

  Direction: the north pole of a compass needle would point when placed at that point.

  Magnitude: $$B = \frac{F_{\max}}{qv}$$

- The unit of magnetic field
  - SI unit: tesla or T. $1\ \mathrm{T} = 1\ \mathrm{N \cdot s/C \cdot m}$
  - cgs unit: gauss or G. $1\ \mathrm{G} = 10^{-4}\ \mathrm{T}$

  The magnetic field of the earth is of the order of 1G or 10^{-4}T

- The important differences between electric force and magnetic forces
  - The electric force is always parallel or anti-parallel to the direction of the electric field ($\vec{F}_e = q\vec{E}$), whereas the magnetic force is perpendicular to the magnetic field ($\vec{F}_B = q\vec{v} \times \vec{B}$).
  - The electric force acts on a charged particle is independent of the particle's velocity, whereas the magnetic force acts on a charged particle only when the particle is in motion.
  - The electric force does work in displacing a charge particle, whereas the magnetic force does no work when a charged particle is displaced (because the magnetic force is always perpendicular to its velocity $\vec{F}_B \perp \vec{v}$).

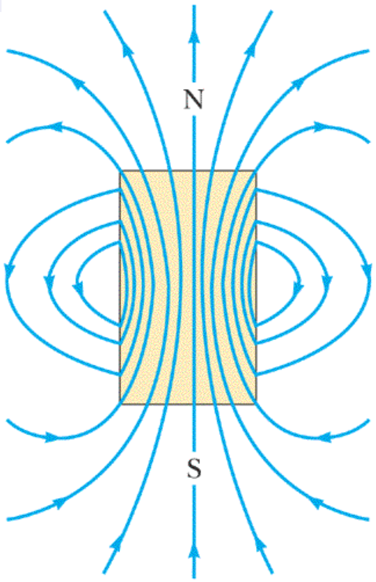
§ 2 Magnetic Field Lines and Magnetic Flux

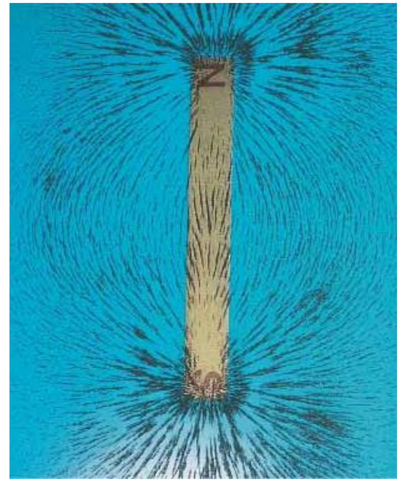
- Magnetic field lines, a graphical way, are related to the magnetic field in the following manner:

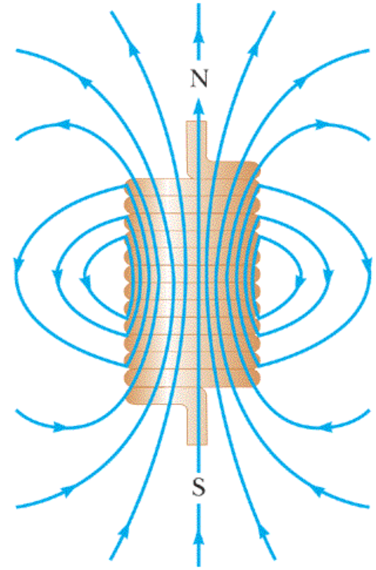
  Magnetic field in space:

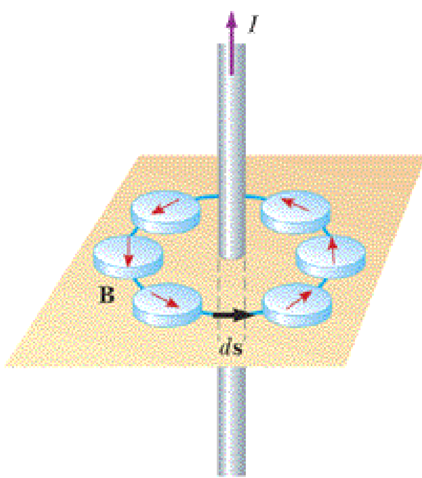
  - Direction —— is tangent to the magnetic field line at that point.
  - Magnitude —— is proportional to the number of magnetic field lines per unit area through the cross-sectional surface in that region. The magnitude of B is larger where the adjacent field lines are close together and small where they are far apart.

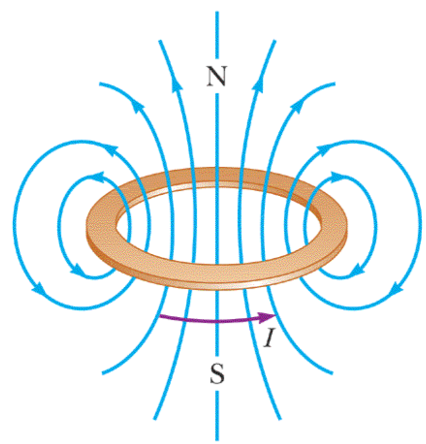
Magnetic field lines produced by some typical sources







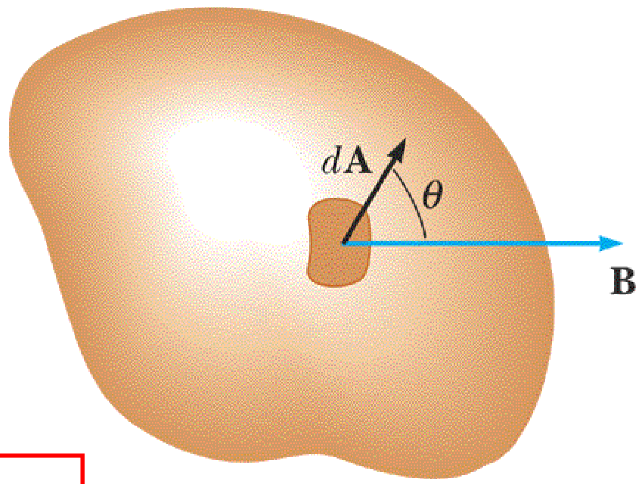




- The fundamental properties for magnetic field lines
  - Unlike electric field lines that begin and end on electric charges, magnetic field lines never have end point, and always form closed loops; (If a magnetic field line had end point, such a point would indicate the existence of a magnetic monopole (磁单极)).
  - Because the direction of magnetic field at each point is unique, field lines never intersect.

- Magnetic flux:
  - Magnetic flux through a surface:

$$\boxed{\Phi_B = \int_{surface} \vec{B}\cdot d\vec{A}}$$



  - SI unit: weber(Wb)

Gauss's law for magnetism

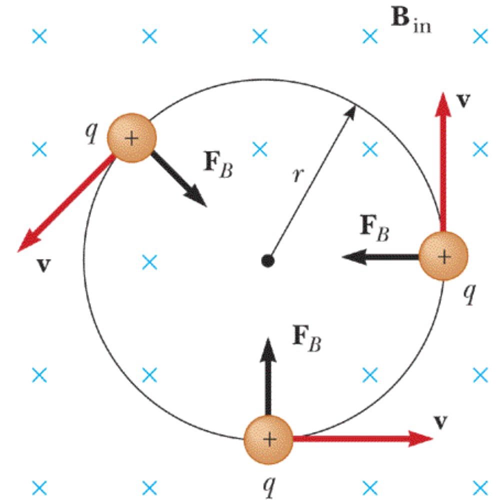
$$\boxed{\oint_S \vec{B}\cdot d\vec{A} = 0}$$

The total magnetic flux through a closed surface is always zero.

In other word, the magnetic flux penetrating a closed surface is always equal to the flux leaving the closed surface.

§ 3 Motion of A Charged Particle in A Uniform Magnetic Field (p586-588)

Magnetic force: $\vec{F}_B = q\vec{v} \times \vec{B}$

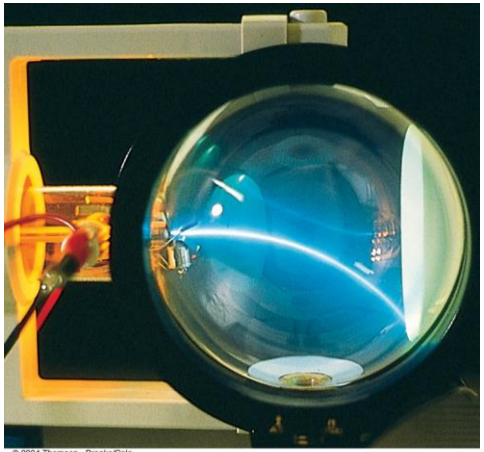


- For the case that the initial velocity of the particle is perpendicular to the magnetic field:
  - $\vec{v}_0 \perp \vec{F}_B$, the magnetic force provides the centripetal force. The particle is in uniform circular motion:

$$F_B = qvB = ma = m\frac{v^2}{r}$$

$$r = \frac{mv}{qB}$$

$$\omega = \frac{v}{r} = \frac{qB}{m}, \quad T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$$

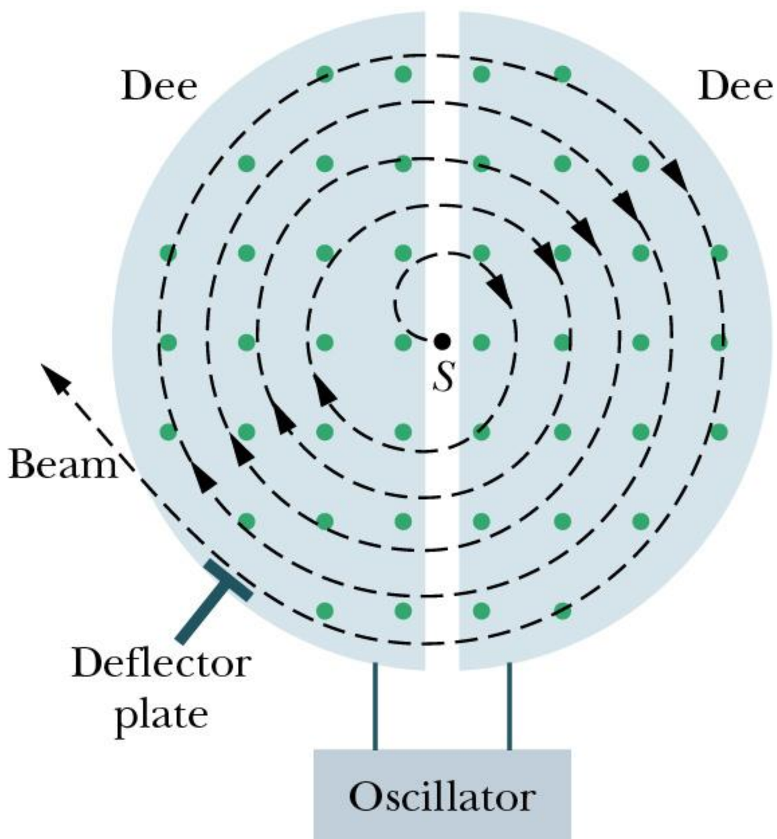


The bending of an electron beam in a magnetic field

Magnetic force Cont'd

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$$

- The angular speed or the period of the circular motion do not depend on speed *v* or radius of the orbit *r*, which is the basis for the cyclotron ([ˈsaikləˌtrɔn] 回旋加速器). ω is often called cyclotron frequency.

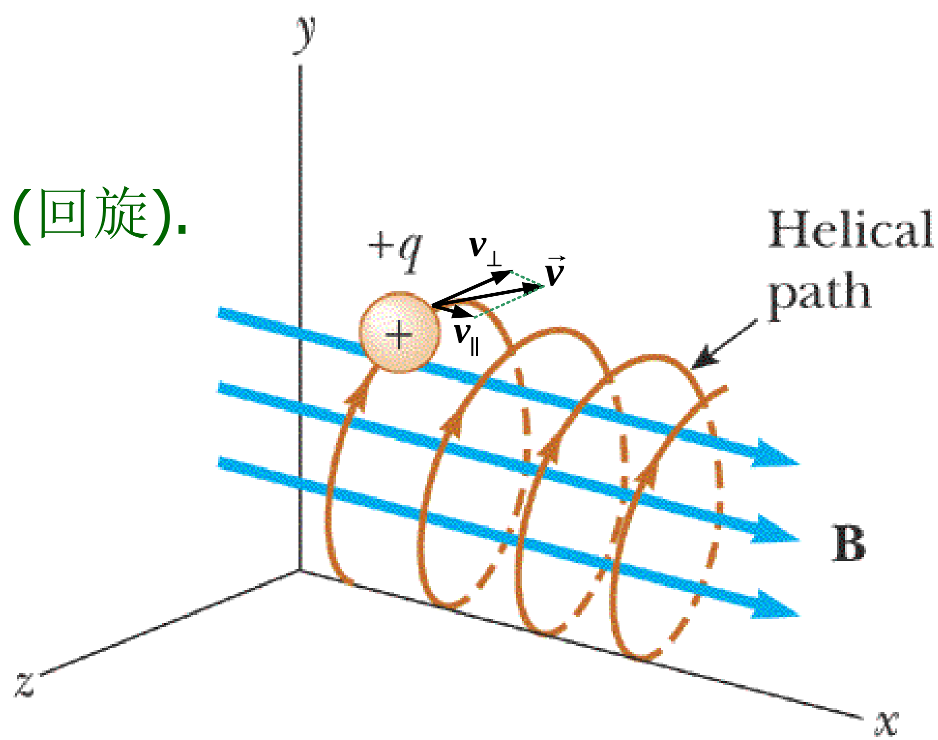


Magnetic force Cont'd

- For the case the initial velocity of the particle is not perpendicular to the magnetic field:
  - The parallel component of acceleration $a_{\parallel} = 0$.
  - The perpendicular component of acceleration

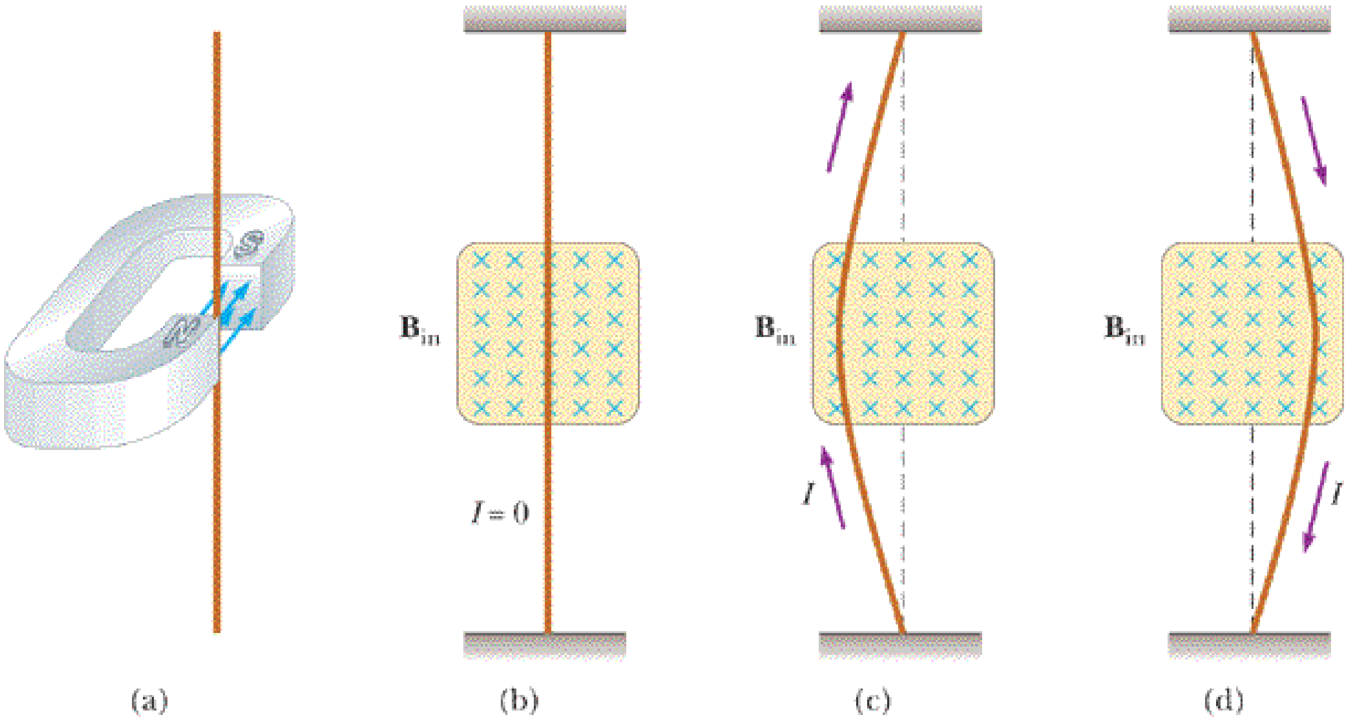
$$a_{\perp} = \frac{v_{\perp}^2}{r} \qquad r = \frac{mv_{\perp}}{qB}$$

  - The particle moves in a helix (回旋).



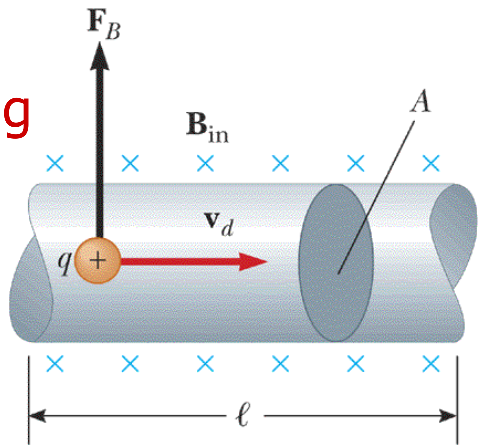
§ 4 Magnetic Force on A Current-Carrying Conductor (p583-585)

The phenomena of the magnetic force on the current-carrying conductor acted by an external magnetic field.



The magnetic force on a straight current-carrying wire

- The magnetic force on a straight current-carrying wire with segment of length l in uniform magnetic field:



  - The magnetic force on a charge q in the wire moving with drift velocity v_d is: $$q\vec{v}_d \times \vec{B}$$
  - The total magnetic force on the wire segment: the number of charges in the segment is nAl, where n is the number of charges per unit volume, A and l are the cross-sectional area and length of the wire.

$$\vec{F}_B = (q\vec{v}_d \times \vec{B})nAl$$

  - The current in the wire is $I = nqv_dA$. So the $\vec{F}_B$ can be expressed as

$$\boxed{\vec{F}_B = I\vec{l} \times \vec{B}}$$

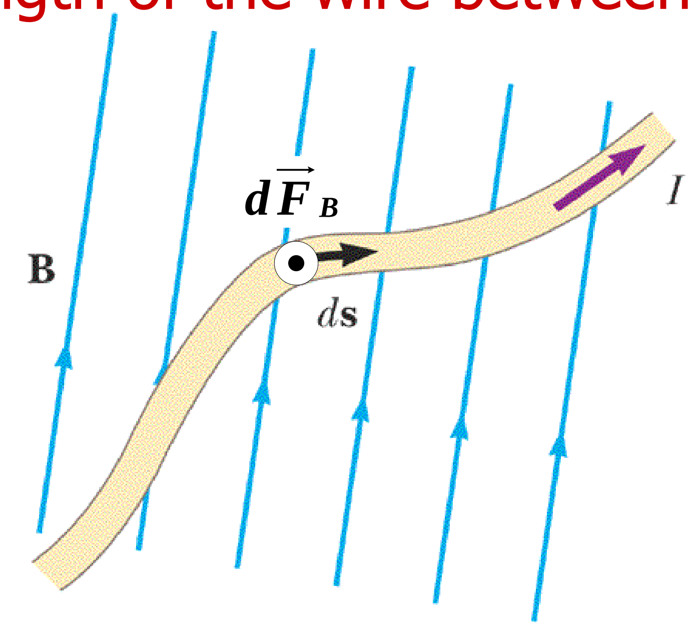
$\vec{l}$ is the length vector in the direction of the current I.

- If the wire is not straight or the magnetic field is not uniform
  - Imaging the wire to be broken into small segments of length $d\vec{s}$. For each small segment:

$$\boxed{d\vec{F}_B = I d\vec{s} \times \vec{B}}$$

  - The total magnetic force on a length of the wire between arbitrary point ***a*** and ***b***:

$$\boxed{\vec{F}_B = I\int_a^b d\vec{s} \times \vec{B}}$$



Magnetic force on a semicircular conductor

Example: A wire bent the shape of a semicircle of radius ***R*** forms a closed circuit and carries a current ***I***. The circuit lies in the *xy* plane, and a uniform magnetic field is present along the positive *y* axis as in the figure. Find the magnetic force on the straight portion of wire and on the curved portion.

Solution: (1) The force on the straight portion.

$$F_1 = IlB = 2IRB$$

The direction of $\vec{F}_1$ is outward.

(2) The magnetic force $d\vec{F}_2$ on the element $d\vec{s}$

$$dF_2 = I\,|d\vec{s}\times\vec{B}| = IB\sin\theta\, ds$$

ds = R dθ

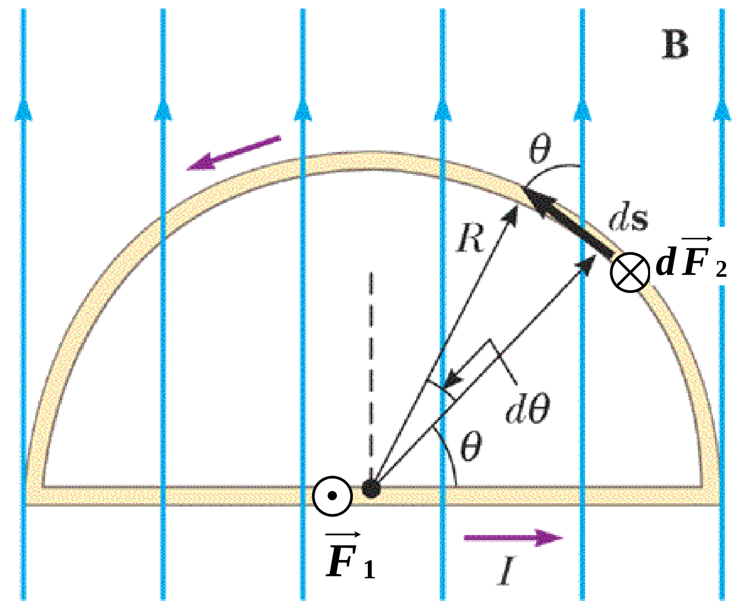
$$dF_2 = IRB\sin\theta\, d\theta$$

$$F_2 = IRB\int_0^{\pi}\sin\theta d\theta$$

$$= -IRB(\cos\pi - \cos 0) = 2IRB$$

The magnetic force $\vec{F}_2$ is inward.

We see that the net magnetic force on the closed loop is zero when the magnetic field is uniform.



§ 5 Torque on A Current Loop (589-590)

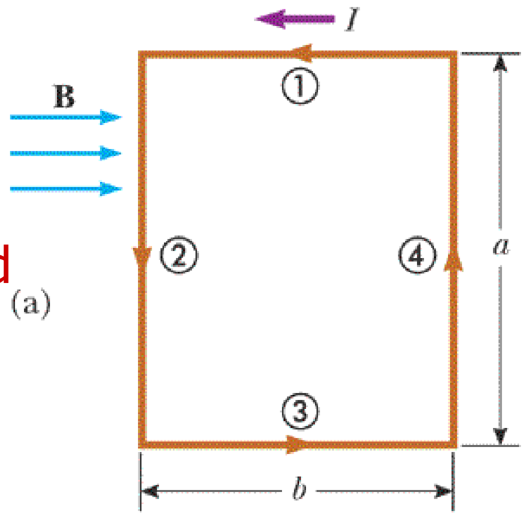
- The net force on a current loop in a uniform magnetic field is zero.
- However, the net torque is not generally zero.
  - Example: a rectangular current loop of wire, with side length ***a*** and ***b***:
  - When the loop is oriented so that the magnetic field is in the plane of the loop, according to

$$d\vec{F}_B = Id\vec{s}\times\vec{B}$$

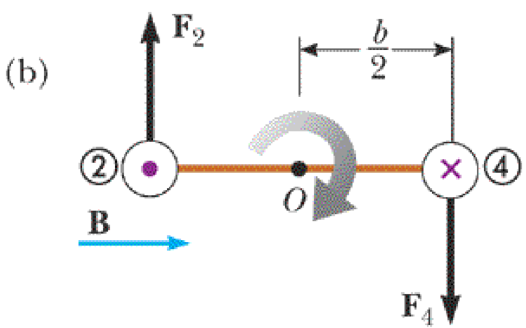
the magnetic forces on the short ends are zero.

On the long ends, the force are equal but in opposite directions. $|\vec{F}_2| = |\vec{F}_4| = IaB$

The net force on the loop is zero.



  - The net torque: tend to rotate the loop clockwise.



$$|\vec{\tau}| = \left(\frac{b}{2}\right)F_2 + \left(\frac{b}{2}\right)F_4 = 2\left(\frac{b}{2}\right)IaB = I(ab)B = (IA)B$$

If we define the A as a vector $\vec{A}$ perpendicular to the plane of the loop

$$\vec{\tau} = (I\vec{A})\times\vec{B}$$

Torque on A Current Loop

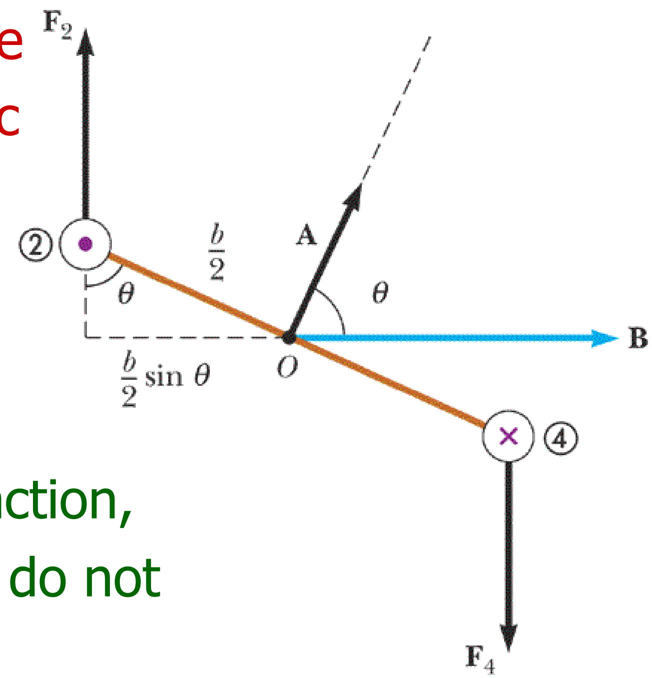
- When the loop is oriented so that the loop plane makes an angle θ with the direction of magnetic field, F_1 is inward and has a magnitude of

$$F_1 = IbB\sin(90^\circ + \theta) = IbB\cos\theta$$

  F_2 is outward and has a magnitude of

$$F_3 = IbB\sin(90^\circ - \theta) = IbB\cos\theta$$

  The forces of F_1 and F_3 have the same line of action, not only they create a total zero force, but also do not contribute to the net torque.



- The forces F_2 and F_4:

  They also create a total zero force, but they create a torque:

$$|\vec{\tau}| = F_2\left(\frac{b}{2}\right)\sin\theta + F_4\left(\frac{b}{2}\right)\sin\theta$$

$$= 2\left(\frac{b}{2}\right)IaB\sin\theta = (IA)B\sin\theta$$

$$\vec{\tau} = (I\vec{A})\times\vec{B}$$

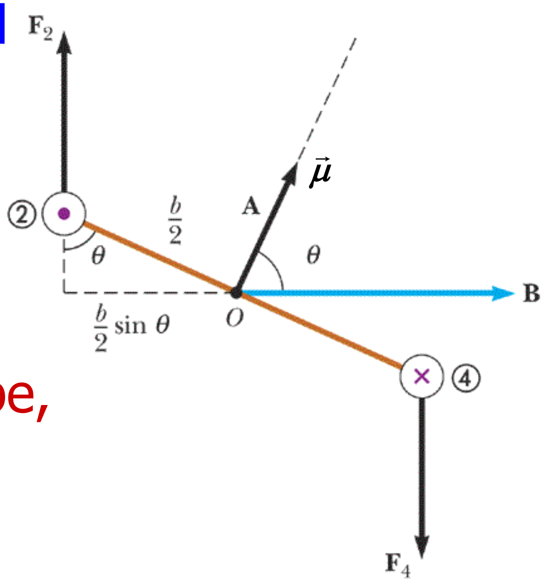
Magnetic Dipole

- Introducing the magnetic dipole and magnetic dipole moment.
  - For any current loop with any shape, we can defined a vector magnetic moment $\vec{\mu}$ with magnitude ***IA***. The direction of $\vec{\mu}$ is determined by right-hand rule.

$$\boxed{\vec{\mu} \equiv I\vec{A}}$$

  - If a coil consists of *N* turns of wire, the total magnetic moment of the coil is: $\vec{\mu} = NI\vec{A}$
- Torque on the current loop in a uniform magnetic field

$$\boxed{\vec{\tau} = I\vec{A}\times\vec{B} = \vec{\mu}\times\vec{B}}$$

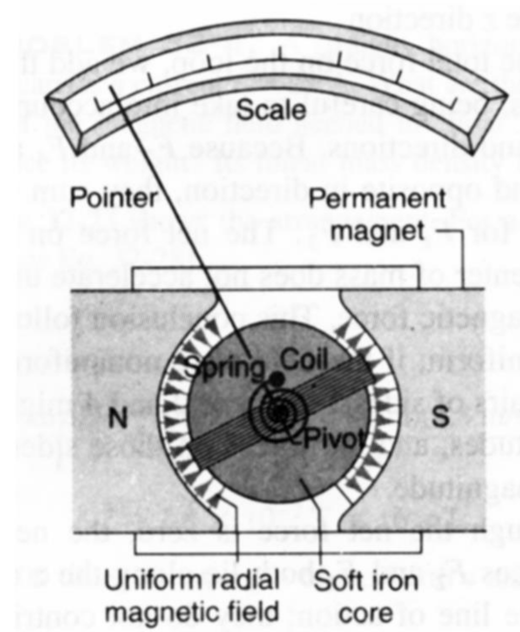


  - The torque tries to rotate the loop so that $\vec{\mu}$ is brought into alignment with $\vec{B}$.
  - The torque expression is valid for loop of any shape, although it was derived for a rectangular loop.

Application of magnetic torque

Application of magnetic torque

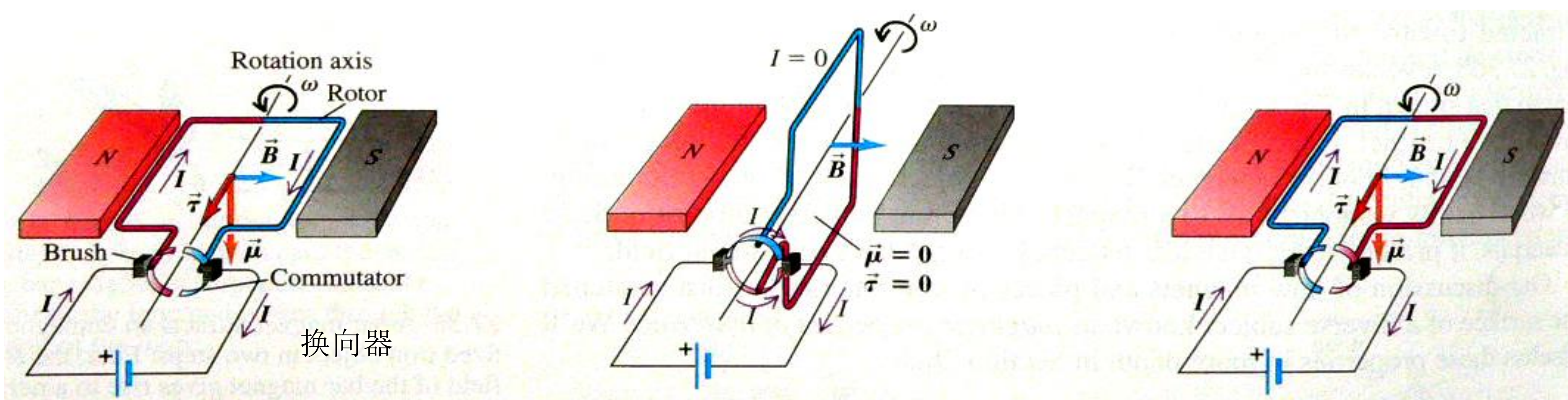
—— Galvanometer ([ˌɡælvəˈnɔmitə] 检流计)



$$\tau = NIAB\sin\theta = \kappa\phi$$

$$\phi = \frac{NIAB\sin\theta}{\kappa} \propto I$$

—— The direct-current motor

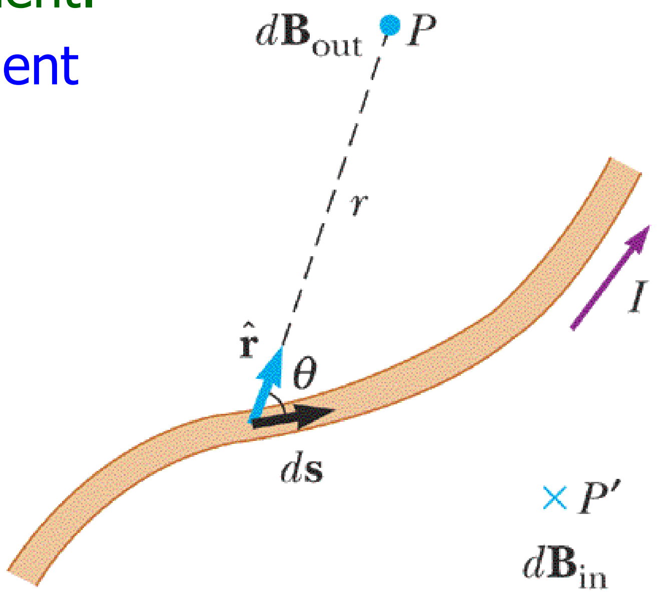


§ 6 The Biot-Savart Law (p613-616)

- If we will find the magnetic field due to a current in a wire, our strategy is first to find the field due to the current in a short element of the wire.
  - The total magnetic field caused by entire wire is the vector sum of the fields caused by individual current element.
- The magnetic field produced by a current element

  —— Biot-Savart Law
  - Definition of vector of current element $I d\vec{s}$
  - Biot-Savart law:

$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{s}\times\hat{r}}{r^2}}$$



  - The total magnetic field due to entire wire:

$$\vec{B} = \int d\vec{B} = \frac{\mu_0}{4\pi}\int\frac{I d\vec{s}\times\hat{r}}{r^2}$$

μ_0 is called the permeability of free space.

$$\frac{\mu_0}{4\pi} = 10^{-7}\ \mathrm{T\cdot m/A}$$

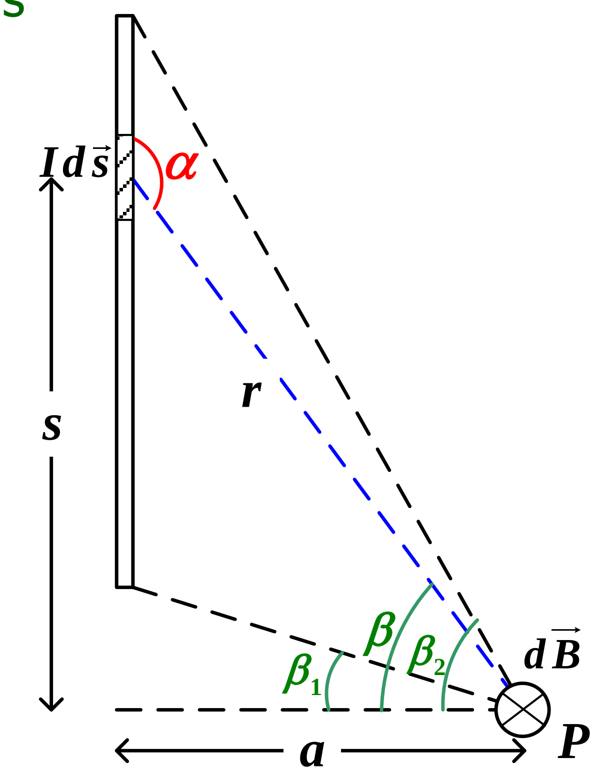
Example

Magnetic field of a straight current wire segment

Example: Find the magnetic field at the point *P*, located a distance a from the wire. The straight wire carries a constant current I. Assume the lines connecting two ends of wire and point *P* make the angles β_1 and β_2 with the horizontal line.

Solution: $d\vec{B}$ produced by the current element $Id\vec{s}$ is always inward.

$$dB = \frac{\mu_0}{4\pi}\frac{Ids\sin\alpha}{r^2}$$



$$B = \frac{\mu_0}{4\pi}\int\frac{Ids\sin\alpha}{r^2}$$

Find r, l, α in terms of β

$$\alpha = 90° + \beta, \quad \sin\alpha = \cos\beta$$

$$r = \frac{a}{\cos\beta} = a\sec\beta, \quad s = a\tan\beta, \quad ds = a\sec^2\beta d\beta$$

$$B = \frac{\mu_0 I}{4\pi}\int_{\beta_1}^{\beta_2}\frac{(\cos\beta)(a\sec^2\beta d\beta)}{a^2\sec^2\beta}$$

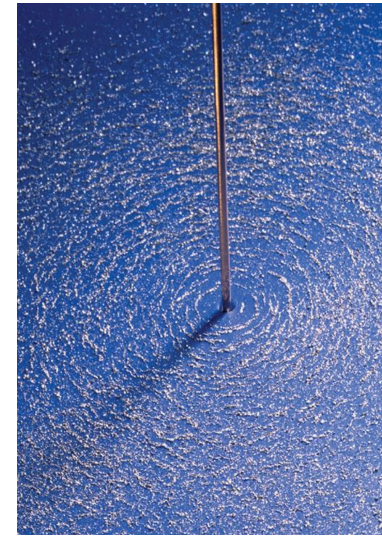
$$= \frac{\mu_0 I}{4\pi a}\int_{\beta_1}^{\beta_2}\cos\beta d\beta = \frac{\mu_0 I}{4\pi a}(\sin\beta_2 - \sin\beta_1)$$

Example Cont'd

For a very long wire, s>>a

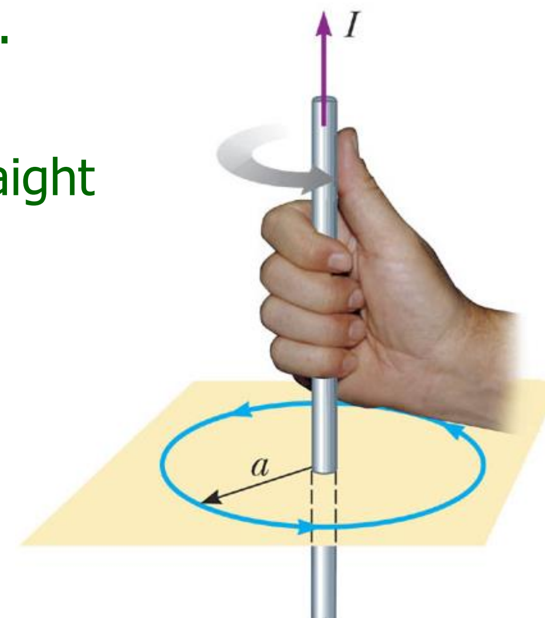
$$\beta_1 \to -\frac{\pi}{2}, \quad \beta_2 \to \frac{\pi}{2}$$

$$B \to \frac{\mu_0 I}{2\pi a} \propto \frac{1}{a}$$



For a long, straight, current-carrying wire, a set of magnetic lines form concentric circles around the wire.

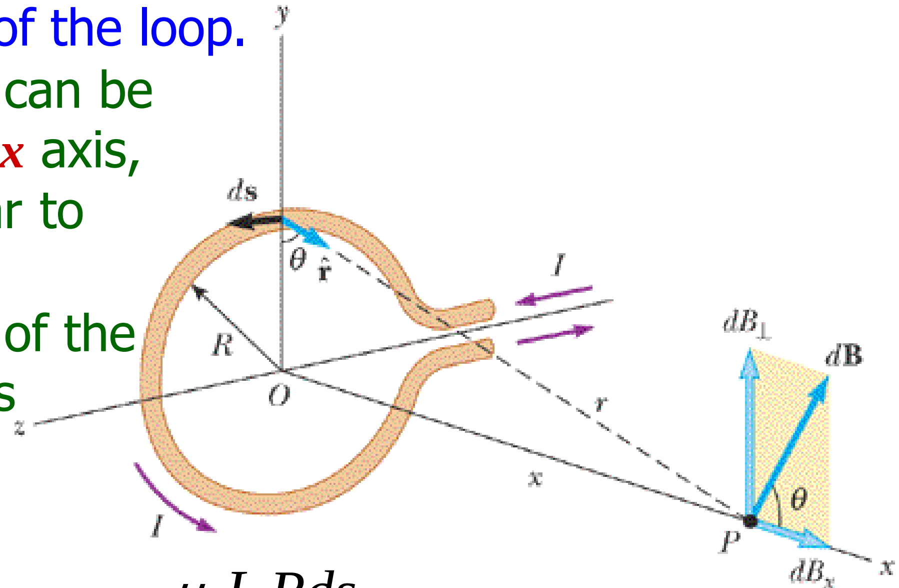
We can use the right-hand rule to determine the direction of the magnetic field surrounding a long, straight wire carrying a current.



Example

Magnetic field on the axis of a circular current loop

Example: Consider a circular loop of wire of radius ***R*** located in the yz plane and carrying a steady current ***I***. Calculate the magnetic field at an axial point ***P*** a distance ***x*** from the center of the loop.



Solution: The $d\vec{B}$ due to the element $d\vec{s}$ can be resolved into a component dB_x, along the ***x*** axis, and a component $dB_\perp$, which perpendicular to the x axis.

By symmetry, any element on one side of the loop sets up a component $dB_\perp$ that cancels the component set up by an element diametrically opposite it.

$$dB = \frac{\mu_0 I}{4\pi}\frac{|d\vec{s}\times\hat{r}|}{r^2} = \frac{\mu_0 I}{4\pi}\frac{ds}{r^2}, \quad dB_x = dB\cos\theta = \frac{\mu_0 I}{4\pi}\frac{Rds}{r^3}$$

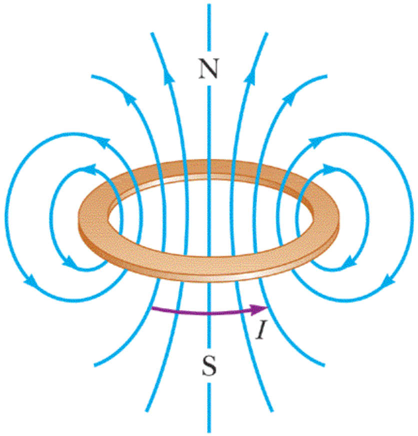
$$B = \oint dB_x = \frac{\mu_0 I}{4\pi}\frac{R}{r^3}\oint ds = \frac{\mu_0 I}{4\pi}\frac{R}{\left(x^2+R^2\right)^{3/2}}(2\pi R) = \frac{\mu_0 I R^2}{2\left(x^2+R^2\right)^{3/2}}$$

At the center of the loop:

$$B = \frac{\mu_0 I}{2R} \quad (\text{at } x = 0)$$

The direction is determined by the right-hand rule.

Example Cont'd



It is interesting to determine the behavior of the magnetic field far from the loop, x>>R

$$B = \frac{\mu_0 I R^2}{2\left(x^2 + R^2\right)^{3/2}} \xrightarrow{x>>R} \frac{\mu_0 I R^2}{2x^3}$$

Consider the magnetic dipole moment of the loop $\mu = I(\pi R^2)$

$$B = \frac{\mu_0}{2\pi}\frac{\mu}{x^3} \propto \frac{\mu}{x^3}$$

Compare the electric field due to a electric dipole:

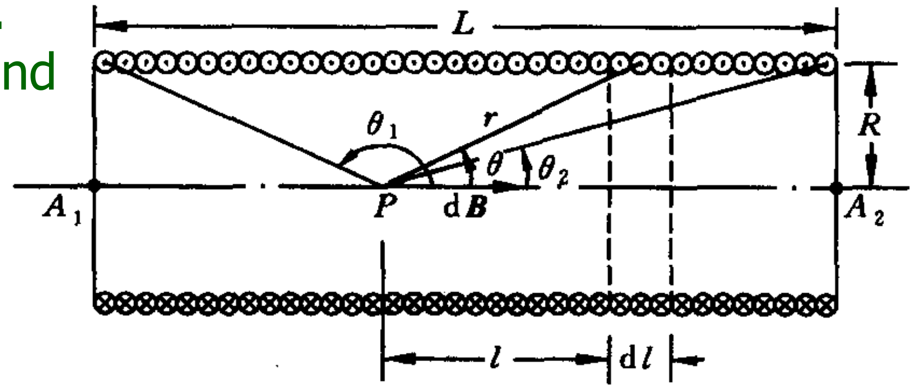
$$E = \frac{1}{2\pi\varepsilon_0}\frac{p}{x^3} \propto \frac{p}{x^3}$$

Example

Magnetic field on the axis of a solenoid

Example: A solenoid is a helical winding of wire on a cylindrical core of radius ***R***. The wire carries a current ***I***. The number of the turns per unit length is ***n*** **=*N*/*L***. Consider a point P on the central axis of the solenoid make the angles of θ_1 and θ_2 from axis up to the edges of two ends.

Solution: Consider a thin ring of width ***dl***. The number of turns in that ring is ***ndl***, and so the total current carried by the ring is ***nIdl***. The field at P due to this ring is:



$$dB = \frac{\mu_0 R^2 dI}{2\left(l^2+R^2\right)^{3/2}} = \frac{\mu_0 R^2 (nIdl)}{2\left(l^2+R^2\right)^{3/2}}$$

Express the x in terms of θ: $l = R\cot\theta,\quad dl = -R\csc^2\theta d\theta,\quad l^2+R^2 = R^2\csc^2\theta$

$$B = \frac{\mu_0 nI}{2}\int_{\theta_1}^{\theta_2}\frac{R^2(-R\csc^2)d\theta}{R^3\csc^3\theta} = -\frac{\mu_0 nI}{2}\int_{\theta_1}^{\theta_2}\sin\theta d\theta$$

$$= \frac{\mu_0 nI}{2}(\cos\theta_2 - \cos\theta_1)$$

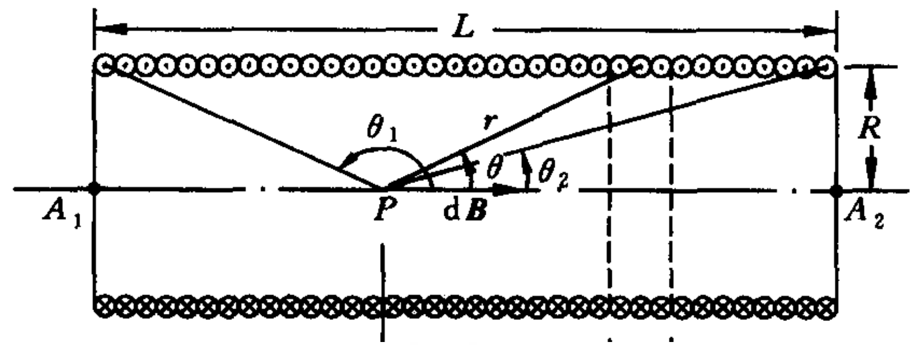
The direction of the field is determined using right-hand rule.

Example Cont'd

$$B = \frac{\mu_0 nI}{2}(\cos\theta_2 - \cos\theta_1)$$

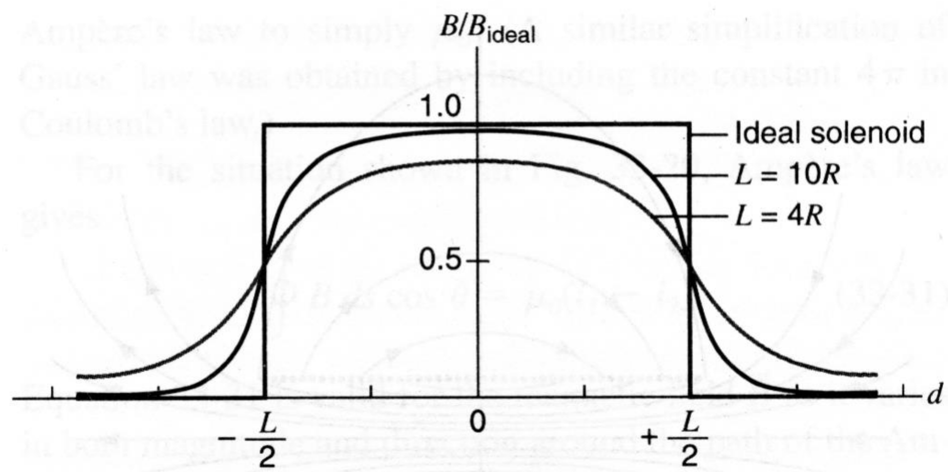
For an ideal solenoid, whose length is very long, L>>R

$$\theta_2 \to 0,\quad \theta_1 \to \pi,\quad B \xrightarrow{L>>R} \mu_0 nI$$



At the end at point A_1 of the solenoid:

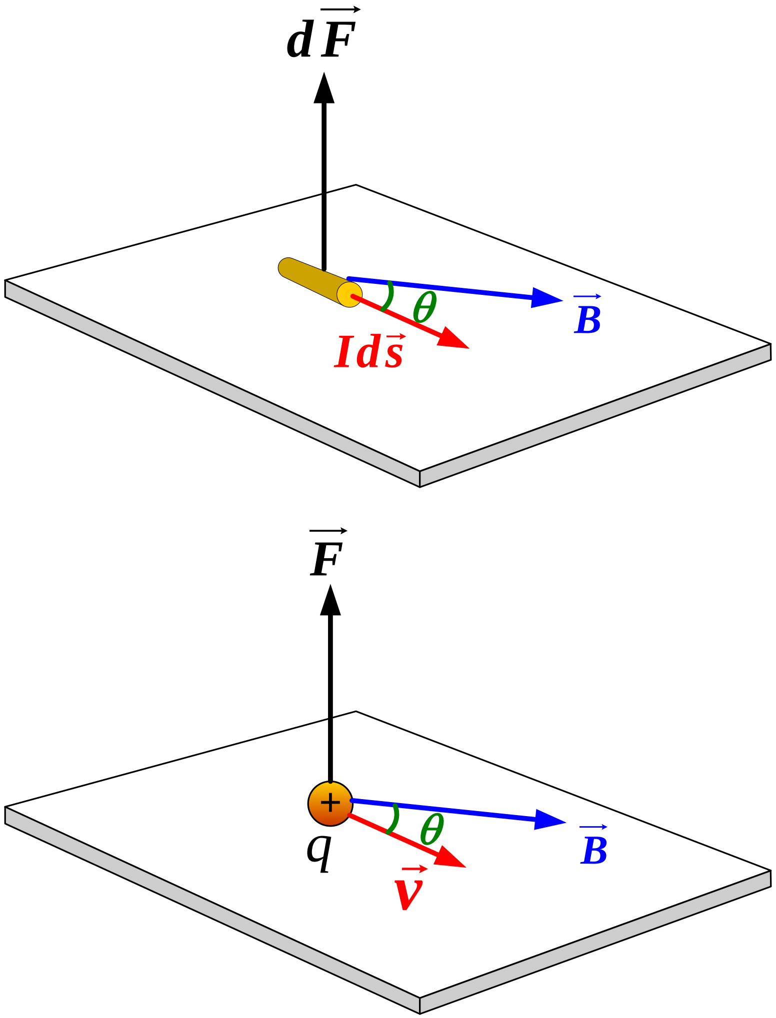
$$\theta_2 \to 0,\quad \theta_1 \to \frac{\pi}{2},\quad B = \frac{1}{2}\mu_0 nI$$



At the end at point A_2 of the solenoid:

$$\theta_2 \to \frac{\pi}{2},\quad \theta_1 \to \pi,\quad B = \frac{1}{2}\mu_0 nI$$

§ 7 Magnetic field of A Moving Charge



Magnetic force on a current element

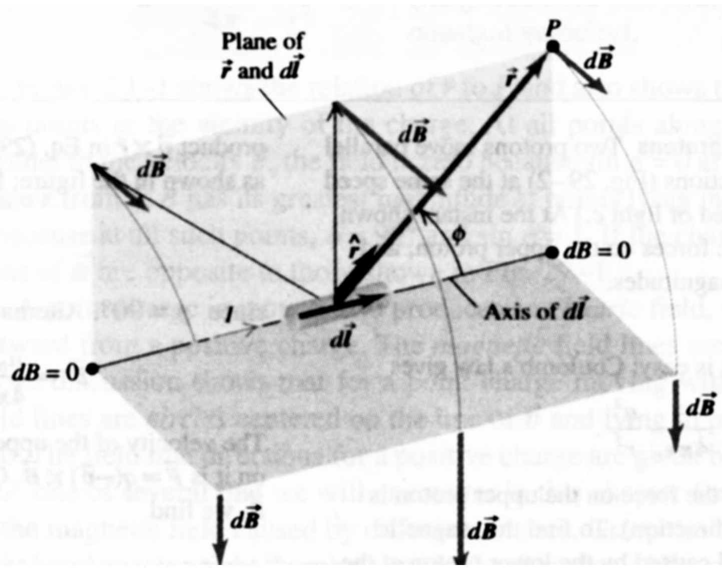
$$\boxed{d\vec{F} = Id\vec{s} \times \vec{B}}$$

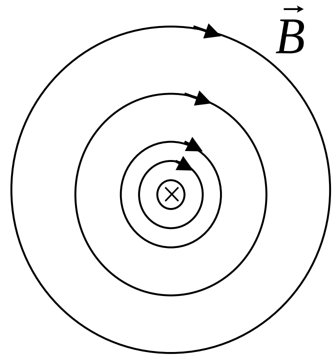
$$Id\vec{s} \Leftrightarrow q\vec{v}$$

Magnetic force on a moving charge

$$\boxed{\vec{F} = q\vec{v} \times \vec{B}}$$

Magnetic field of a moving charge

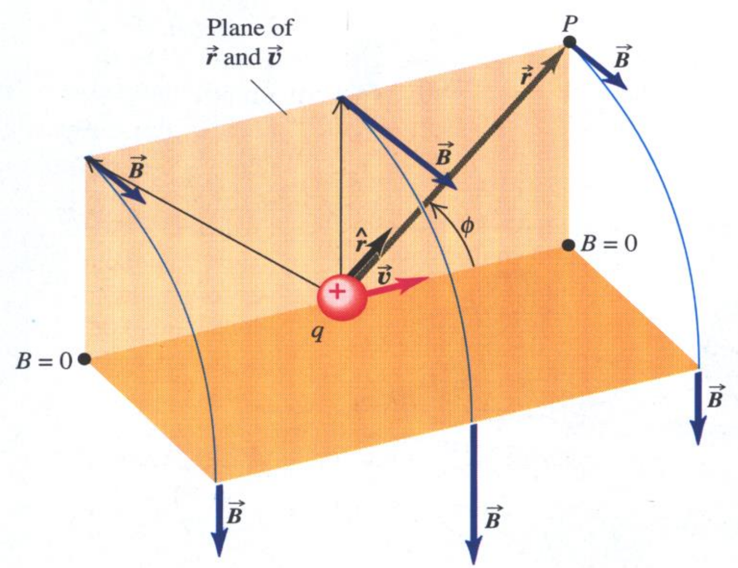


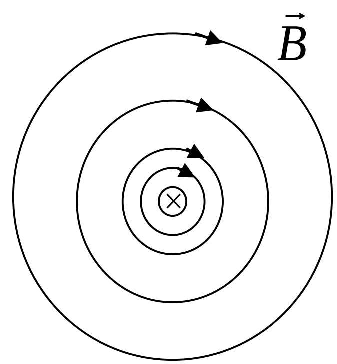


Magnetic field of a current element

$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{s}\times\hat{r}}{r^2}}$$

$$Id\vec{s} \Leftrightarrow q\vec{v}$$





Magnetic field of a moving charge

$$\boxed{\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}}$$

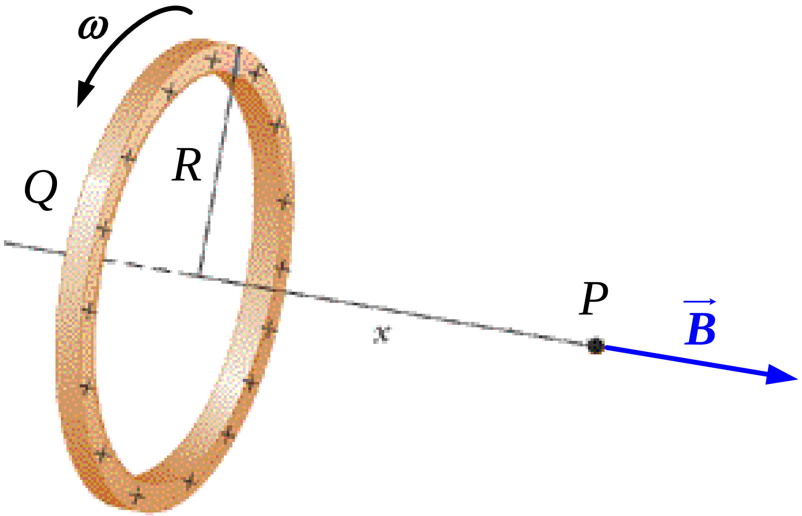
Example

Example: A ring of radius ***a*** has a uniform positive charge distribution, with a total charge ***Q***. Now the ring rotates anti-clockwise about its central axis. Calculate the magnetic field at the point *P* located on the axis a distance ***x*** from the center of the ring.

Solution I: The a rotating charge ring is equivalent to a circular current loop.

For a circular current loop:

$$B = \frac{\mu_0 I R^2}{2\left(x^2 + R^2\right)^{3/2}}$$



Here, the current:

$$I = \frac{Q}{T} = \frac{Q\omega}{2\pi}$$

So:

$$B = \frac{\mu_0 Q \omega R^2}{4\pi\left(x^2 + R^2\right)^{3/2}}$$

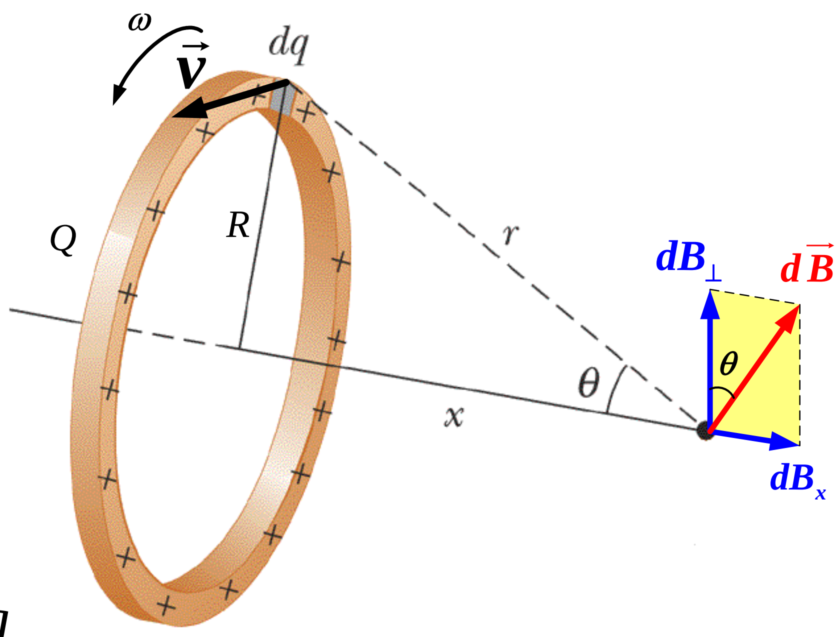
$$\mu = IA = \frac{Q\omega}{2\pi}\pi R^2 = \frac{Q\omega R^2}{2}, \quad B = \frac{\mu_0}{2\pi}\frac{\mu}{\left(x^2 + R^2\right)^{3/2}}$$

Example

Solution II: Dividing the ring into small segment of charge ***dq***.

$d\vec{B}$ is the field due to the charge ***dq*** , which can be resolved into a component dB_x, along the x axis, and a component $dB_\perp$, which is perpendicular to the x axis.

By symmetry, the vector sum of all $dB_\perp$ vanishes. The total field is only contributed by the sum of dB_x.



$$dB = \frac{\mu_0 dq}{4\pi}\frac{|\vec{v}\times\hat{r}|}{r^2} = \frac{\mu_0 v}{4\pi}\frac{dq}{r^2} \qquad v = \omega R$$

$$dB_x = dB\cos\theta = \frac{\mu_0 \omega R}{4\pi}\frac{R}{r}\frac{dq}{r^2} = \frac{\mu_0\omega}{4\pi}\frac{R^2 dq}{r^3}$$

$$B = \oint dB_x = \frac{\mu_0\omega}{4\pi}\frac{R^2}{r^3}\oint dq = \frac{\mu_0\omega Q}{4\pi}\frac{R^2}{\left(x^2+R^2\right)^{3/2}} = \frac{\mu_0}{2\pi}\frac{\mu}{\left(x^2+R^2\right)^{3/2}}$$

§ 8 Ampère's Law (p607-610)

- The line integral around a loop near a long, straight current-carrying wire.
  - The circle loop is centered on the wire, the direction of loop is right-hand related to the direction of the current.

$$\oint_{L_1} \vec{B}\cdot d\vec{s} = B\oint_{L_1} ds = \frac{\mu_0 I}{2\pi r_1}(2\pi r_1) = \mu_0 I$$

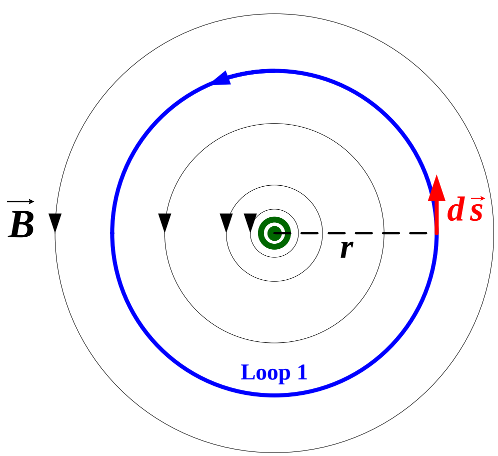
  - The same circle loop but in opposite direction.

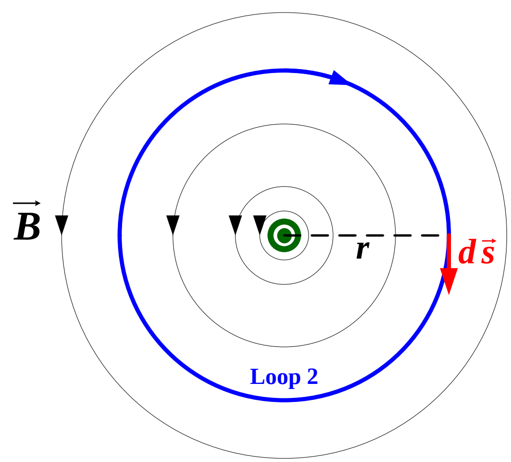
$$\oint_{L_2} \vec{B}\cdot d\vec{s} = -B\oint_{L_2} ds = -\frac{\mu_0 I}{2\pi r_1}(2\pi r_1) = -\mu_0 I$$

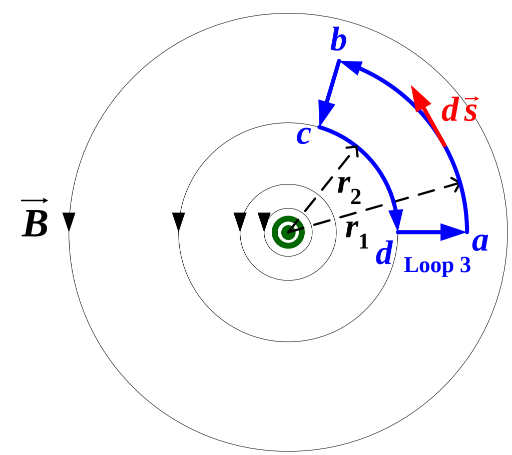
  - An integration loop does not enclose the wire.

$$\vec{B}_1\cdot d\vec{s}_1 = \frac{\mu_0 I}{2\pi r_1}(r_1 d\theta) = \frac{\mu_0 I}{2\pi r_2}(r_2 d\theta) = -\vec{B}_2\cdot d\vec{s}_2$$

$$\oint_{L_3} \vec{B}\cdot d\vec{s} = \int_a^b B_1 ds + \int_b^c (0)ds + \int_c^d (-B_2)ds + \int_d^a (0)ds = 0$$







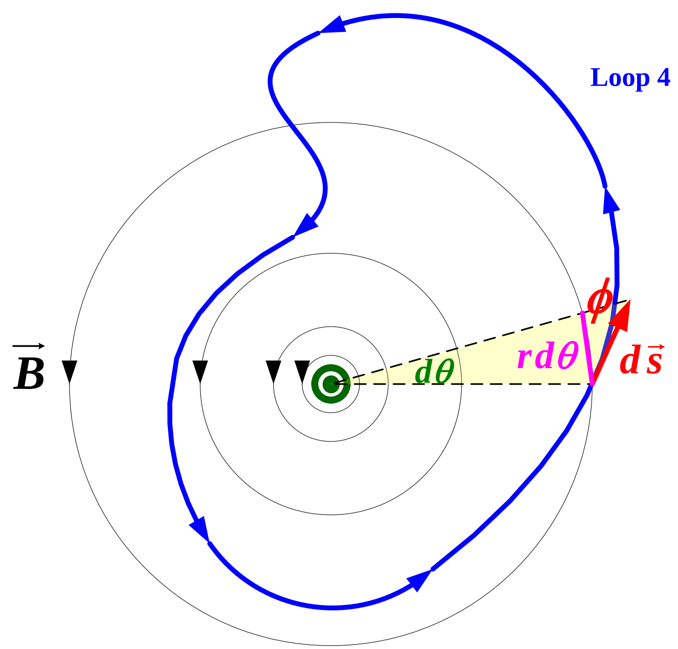
Ampère's Law

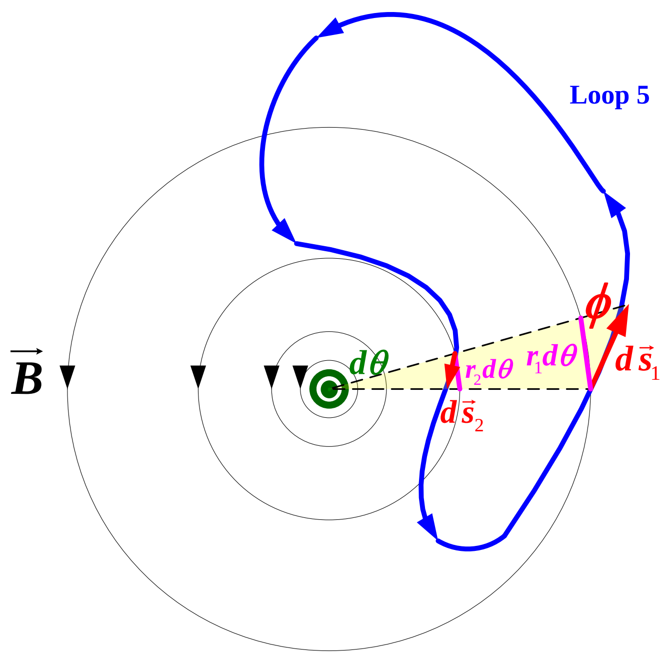
- A more general loop that encloses the wire. $ds\cos\phi = r\,d\theta$

$$\oint_{L_4} \vec{B}\cdot d\vec{s} = \oint_{L_4} B\,ds\cos\phi = \oint_{L_4} \frac{\mu_0 I}{2\pi r}(r\,d\theta) = \mu_0 I$$

- A more general loop that does not enclose the wire.

$$\vec{B}_1\cdot d\vec{s}_1 = B_1 ds_1\cos\phi_1 = \frac{\mu_0 I}{2\pi r_1}(r_1 d\theta) = \frac{\mu_0 I}{2\pi r_2}(r_2 d\theta) = -\vec{B}_2\cdot d\vec{s}_2 \qquad \oint_{L_5} \vec{B}\cdot d\vec{s} = 0$$





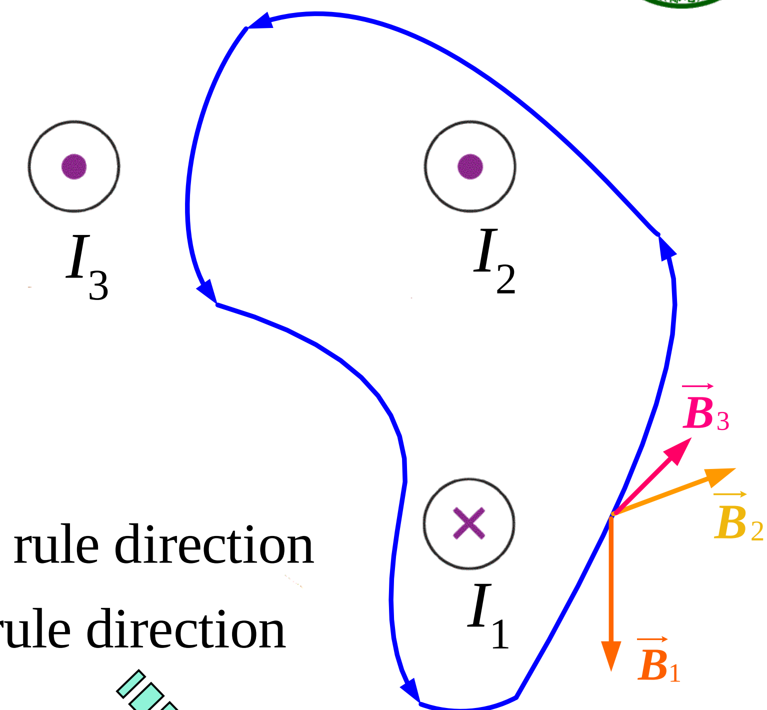
Ampère's Law

- Ampère's Law
  - For any loop with any shape.

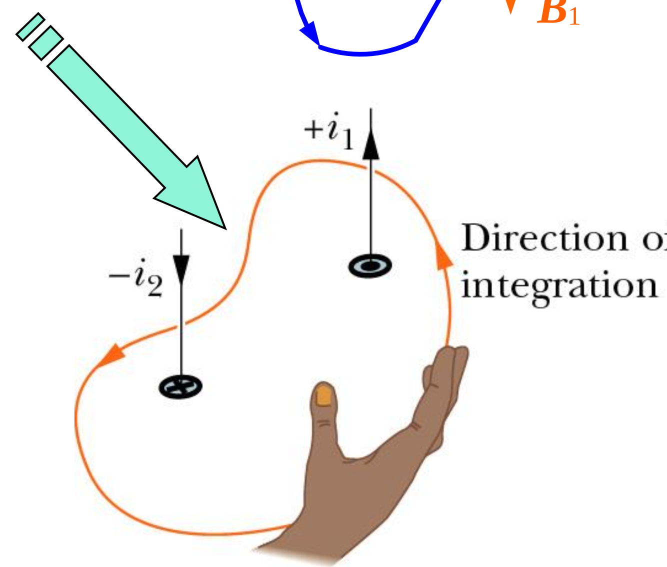
$$\vec{B}=\sum_i \vec{B}_i$$

$$\oint_L \vec{B}\cdot d\vec{s}=\oint_L \sum_i \vec{B}_i\cdot d\vec{s}=\sum_i \oint_L \vec{B}_i\cdot d\vec{s}$$

$$\oint_L \vec{B}_i\cdot d\vec{s}=\begin{cases}\mu_0 I & I \text{ within the loop, right-hand rule direction}\\ -\mu_0 I & I \text{ within the loop, left-hand rule direction}\\ 0 & I \text{ not within the loop}\end{cases}$$



  - Ampère's Law: $\boxed{\oint_L \vec{B}\cdot d\vec{s}=\mu_0 I_{\text{encl}}}$

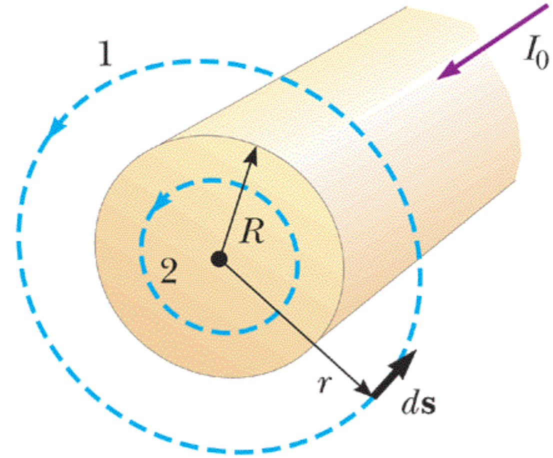


The line integral of magnetic field along a loop equals μ_0 times the algebra sum of the currents enclosed or linked by the loop.

Example

The magnetic field created by a long, straight cylindrical wire

Example: A long, straight cylindrical wire of radius R carries a steady current I_0 that is uniformly distributed through the cross-section of the wire. Calculate the magnetic field a distance r from the center of the wire in the regions $r \geq R$ and $r < R$.



Solution: For $r \geq R$, we choose loop 1, a circle of radius r centered at wire.

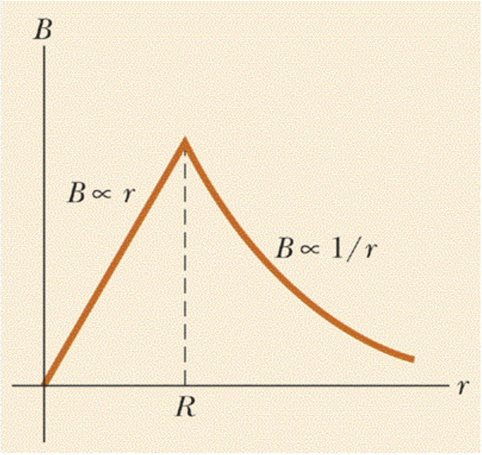
$$\oint_1 \vec{B} \cdot d\vec{s} = B \oint_1 ds = B(2\pi r) = \mu_0 I_0$$

$$B = \frac{\mu_0 I_0}{2\pi r} \qquad (\text{for } r \geq R)$$

For, we choose circular loop 2. $I_{encl} = \dfrac{r^2}{R^2} I_0$

$$\oint_2 \vec{B} \cdot d\vec{s} = B(2\pi r) = \mu_0 I_{encl} = \mu_0 \left(\frac{r^2}{R^2} I_0 \right)$$

$$B = \frac{\mu_0 I_0}{2\pi R^2} r \qquad (\text{for } r < R)$$



Example

The magnetic field created by a solenoid

Example: A ideal solenoid: its turns are closely spaced and its length is large compared with its radius. For an ideal solenoid, the field outside the solenoid is zero, and the field inside is uniform. Calculate the field inside an ideal solenoid carrying a current ***I***. The number of turns per unit length is ***n***.

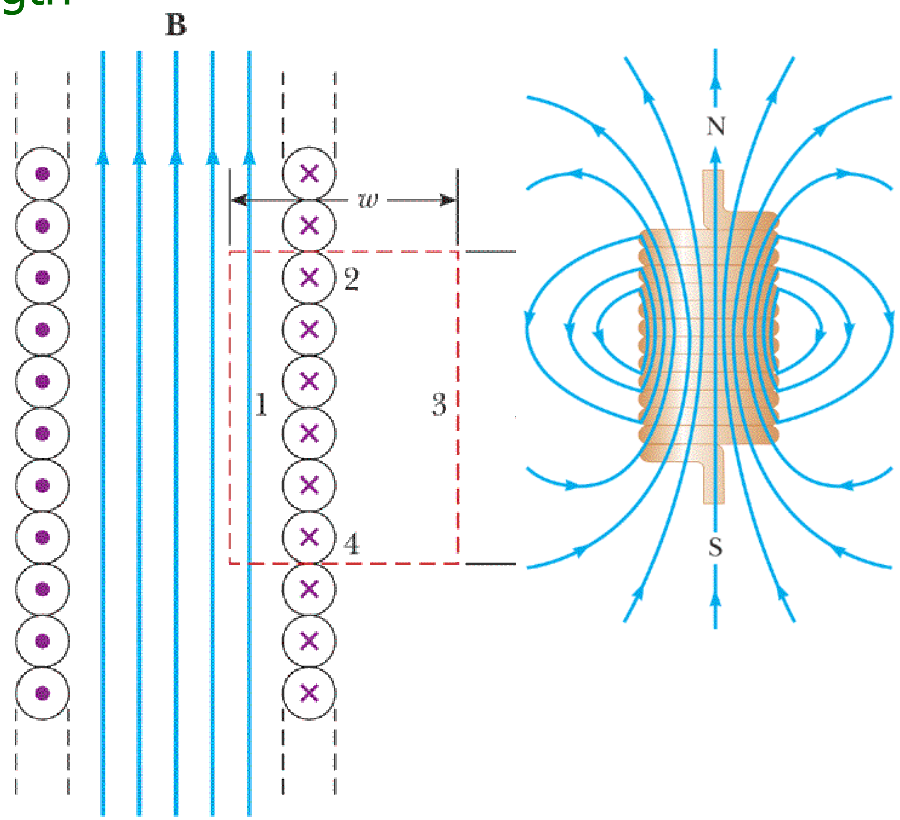
Solution: Choose a rectangular loop of length ***l*** and width ***w***.

$$\oint_L \vec{B}\cdot d\vec{s} = \int_1 + \int_2 + \int_3 + \int_4 \vec{B}\cdot d\vec{s}$$

$$= \int_1 \vec{B}\cdot d\vec{s} = B\int_1 ds = Bl$$

$$= \mu_0 NI$$

$$B = \mu_0 \frac{N}{l} I = \mu_0 nI$$



Example

The magnetic field created by a toroid solenoid (螺绕环)

Example: A toroid has N closely spaced turns of wire carrying a current I. Calculate the magnetic field in the region occupied by the torus (圆环体), a distance r from the center.

Solution: Choose a circular loop of radius of r.

$$\oint_L \vec{B}\cdot d\vec{s} = B\oint_L ds = B(2\pi r) = \mu_0 NI$$

$$B = \frac{\mu_0 NI}{2\pi r} \xrightarrow{a<<r} \frac{\mu_0 NI}{2\pi r_{mid}} = \mu_0 nI$$