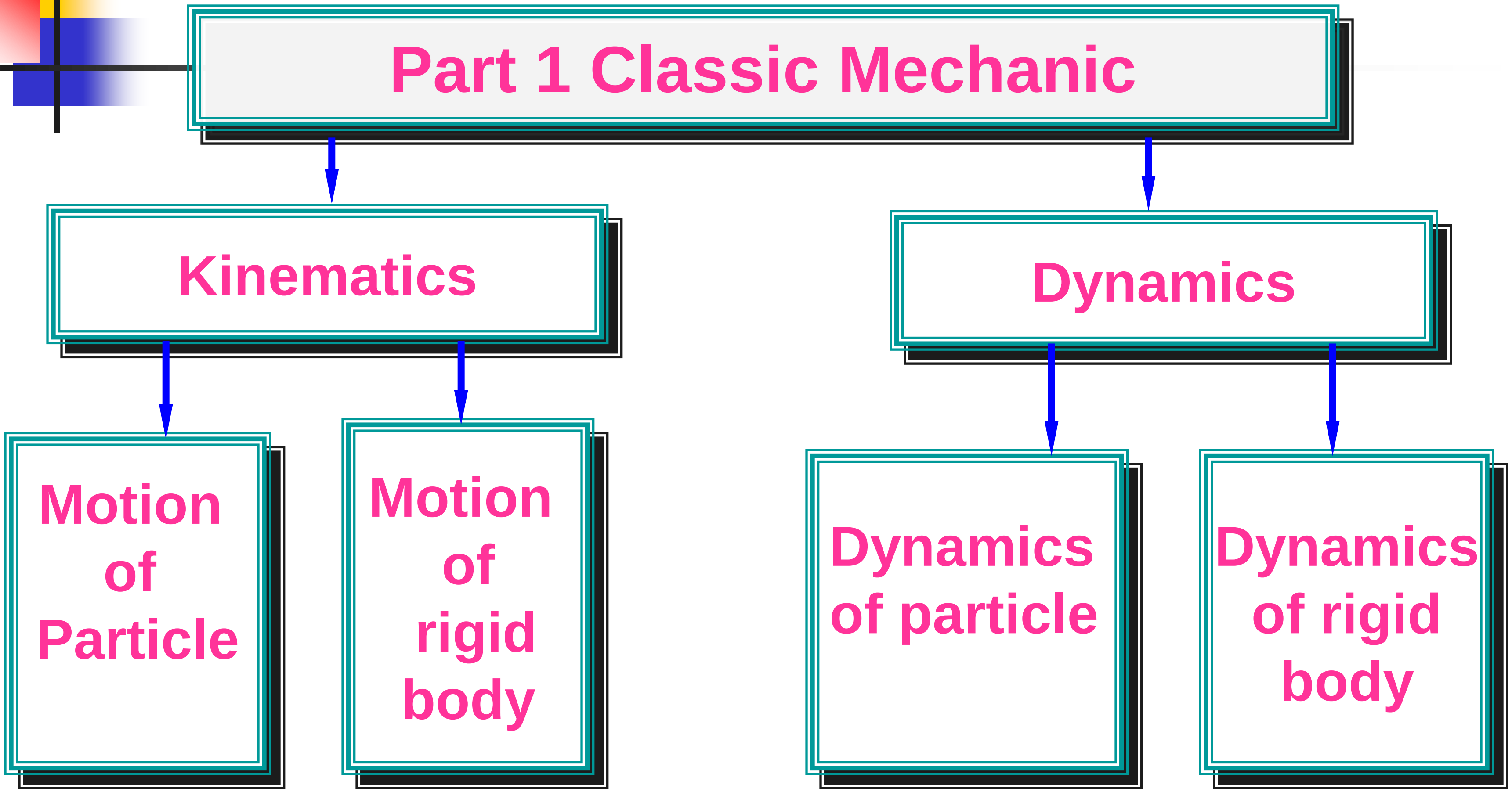

- Kinematics: the part of mechanics that deals with the description of motion.
- Dynamics: the relation of motion to its causes.

Position, velocity, and Acceleration Vectors

- Position vector and motional equation

$$\vec{r} = \vec{r}(t)$$

- Velocity and speed

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

- Acceleration

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

- In cartesian coordinate

$$\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$$

$$\vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}$$

$$= \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j} + \frac{d^2z}{dt^2}\hat{k}$$

- In natural coordinate

$$s = s(t)$$

$$\vec{v} = \frac{ds}{dt}\hat{\tau}$$

$$\vec{a} = \vec{a}_n + \vec{a}_t = \frac{v^2}{r}\hat{n} + \frac{dv}{dt}\hat{\tau}$$

- Two categories of problems in Kinematics
  - The position of particle is known quantity, Find its velocity and acceleration——By way of derivatives
  - The acceleration of particle is known quantity, Find its velocity and position——By way of integrals.

Known quantities **Unknown quantities**

$$\vec{r}(t) \xrightarrow{\text{derivative}} \vec{v}(t),\quad \vec{a}(t)$$

$$\vec{a}(t) + \text{initial conditions } (\vec{v}(0), \vec{r}(0)) \xrightarrow{\text{integral}} \vec{v}(t),\quad \vec{r}(t)$$

Newton's Second Law

- The acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass.

$$\sum \vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = m\frac{d^2\vec{r}}{dt^2}$$

The component expressions:

$$\sum F_x = ma_x \quad \sum F_y = ma_y \quad \sum F_z = ma_z$$

$$\sum F_t = ma_t = m\frac{dv}{dt} \quad \sum F_n = ma_n = m\frac{v^2}{\rho}$$

It is only suitable for the inertial frame, and particles or particle-like bodies.

Work Done by a Varying Force along a Curve Path

$$dW = \vec{F} \cdot d\vec{r} \qquad \boxed{W = \int_A^B \overrightarrow{F} \cdot d\vec{r}}$$

- The work done by a pair of internal forces

$$dW = \vec{f}_{12} \cdot d\vec{r}_1 + \vec{f}_{21} \cdot d\vec{r}_2 = \vec{f}_{21} \cdot d\vec{r}_{21}$$

- The work done by a conservative force and potential energy.

$$\boxed{\Delta U = U(\vec{r}_b) - U(\vec{r}_a) = -W = -\int_a^b \overrightarrow{F} \cdot d\vec{r}}$$

- Work – kinetic energy theorem:

$$\boxed{\sum W_{i-\text{external}} + \sum W_{i-\text{internal}} = \sum K_f - \sum K_i}$$

- Work – energy theorem:

$$\sum W_{i-\text{external}} + \sum W_{i-\text{internal-nonconserv}} = \Delta E_{\text{mech}} = E_{\text{mech}\,f} - E_{\text{mech}\,i}$$

- Conservation of Mechanical Energy

For a system, if $\sum W_{i-\text{external}} + \sum W_{i-\text{internal-nonconserv}} = 0$ then

$$\Delta E_{\text{mech}} = 0 \quad \text{or} \quad K_f + U_f = K_i + U_i = \text{constant}$$

Impulse-momentum theorem

$$\boxed{\sum_i \vec{F}_{i-\text{ext}} = \frac{d\vec{p}_{\text{tot}}}{dt}}$$

The derivative form

$$\boxed{\int_{t_1}^{t_2} \sum_i \vec{F}_{i-\text{ext}}\, dt = \vec{p}_{\text{tot}\,2} - \vec{p}_{\text{tot}1}}$$

The integral form

Conservation of Momentum

When $\sum_i \vec{F}_{i-\text{ext}} = 0$ then $\vec{p}_{\text{tot}} = \sum_i \vec{p}_i = \text{constant}$

Conservation of momentum in component form

When $\sum_i F_{i-\text{ext-}x} = 0$ then $p_{\text{tot-}x} = \sum_i p_{i-x} = \text{constant}$

Angular Momentum $\boxed{\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v}}$

Torque $\boxed{\vec{\tau} = \vec{r} \times \vec{F}}$

Torque-Angular Momentum Theorem

$$\boxed{\sum \vec{\tau}_{\text{ext}} = \sum_i \frac{d\vec{L}_i}{dt} = \frac{d}{dt} \sum_i \vec{L}_i = \frac{d\vec{L}_{\text{tot}}}{dt}}$$

Conservation of Angular Momentum

When $\sum \vec{\tau}_{\text{ext}} = 0$ Then $\vec{L}_{\text{tot}} = \text{constant}$

Kinematics of Rigid Bodies

Angular position: θ

Angular velocity $$\omega = \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t} = \frac{d\theta}{dt}$$

Angular acceleration $$\alpha = \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

moment of inertia

$$I = \sum_i \Delta m_i R_i^2$$

$$I = \int R^2 dm$$

Angular Momentum for a Rigid Body about a fixed axis

$$L_\omega = \sum_i l_{iz} = \left(\sum_i m_i R_i^2 \right) \omega = I\omega$$

Rotational Kinetic Energy

$$K = \sum_i \left(\frac{1}{2} m_i v_i^2 \right) = \sum_i \left(\frac{1}{2} m_i R_i^2 \omega^2 \right) = \frac{1}{2} \sum_i \left(m_i R_i^2 \right) \omega^2 = \frac{1}{2} I\omega^2$$

The Rotational Form of Newton's Second Law

$$\boxed{\sum \tau_{\text{net-axis}} = I\alpha}$$

Angular Momentum theorem

$$\boxed{\int_{\theta_1}^{\theta_2} \tau_{\text{net}} dt = \int_{\omega_1}^{\omega_2} I d\omega = I\omega_2 - I\omega_1}$$

Work-kinetic energy theorem

$$\boxed{W_{\text{net}} = \int_{\theta_1}^{\theta_2} \tau_{\text{net}} d\theta = \int_{\omega_1}^{\omega_2} I\omega d\omega = \frac{1}{2} I\omega_2^2 - \frac{1}{2} I\omega_1^2}$$

The Electric Field

The definition of electric field $\vec{E} \equiv \dfrac{\vec{F}_e}{q_0}$

The definition of Electric potential

$$\boxed{\Delta V = V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{r}}$$

Take infinity far away to be the reference point

$$V_A = \int_A^\infty \vec{E} \cdot d\vec{r}$$

Gauss's Law: $$\boxed{\Phi_E = \oint_S \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}}$$

The Loop law $$\boxed{\oint_L \vec{E} \cdot d\vec{r} = 0}$$

The Electric Field

Calculating Electric field

Model each element as a point charge. $d\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\hat{r}$

Apply the superposition principle to get the total field at P.

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r^2}\hat{r}$$

Using Gauss's Law:

$$\boxed{\Phi_E = \oint_S \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}}$$

Calculating the Electric Potential

- If the electric field is known

$$V_P = \int_P^\infty \vec{E} \cdot d\vec{r}$$

- If the charge distribution is known
  - The electric potential due to individual charge particles

$$V = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{r_i}$$

  - The electric potential due to continuous charge distributions

$$V = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r}$$

Conductors in Electrostatic Equilibrium

The properties that an conductor in electrostatic equilibrium.

- The electric field is zero everywhere inside the conductor.
- If the isolated conductor carries a net charge, the net charge resides entirely on its surface.
- The electric field just outside the charged conductor is perpendicular to the conductor surface and has a magnitude σ / ε_0, where σ is the surface charge density at that point.
- The entire conductor is at the same potential. So the surface of a conductor is always an equipotential surface

Capacitance

Capacitance of a capacitor $$C \equiv \frac{Q}{\Delta V}$$

Problem-Solving Strategy to Calculating The Capacitance of a Capacitor

- A convenient charge of magnitude Q is assumed.
- The potential difference is calculated.
- Use $C=Q/\Delta V$ to evaluate the capacitance.

Energy Stored in A Charged Capacitor

$$U = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2 = \frac{Q^2}{2C}$$

The energy density: $$u = \frac{1}{2}\varepsilon_0 E^2$$

Dielectric Materials

When a dielectric material is placed in an external applied field E_0, induced surface charges q' appear that tend to weaken the original field E_0 by a polarization field E' within the material. For a linear material, the net field inside the material

$$\vec{E} = \vec{E}_0 + \vec{E'} = \frac{1}{\kappa}\vec{E}_0$$

κ is called the dielectric constant, which is greater than 1.

Magnetic fields

Definition of B

Direction: the north pole of a compass needle would point when placed at that point.

Magnitude: $$B = \frac{F_{\max}}{qv}$$

Gauss's law for magnetism $$\boxed{\oint_S \vec{B} \cdot d\vec{A} = 0}$$

Ampère's Law: $$\boxed{\oint_L \vec{B} \cdot d\vec{s} = \mu_0 I_{\mathrm{encl}}}$$

Calculating the magnetic field

Biot-Savart law:

$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{s} \times \hat{r}}{r^2}}$$

The total magnetic field due to entire wire:

$$\vec{B} = \int d\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I d\vec{s} \times \hat{r}}{r^2}$$

Using Ampère’s Law:

$$\boxed{\oint_L \vec{B} \cdot d\vec{s} = \mu_0 I_{\mathrm{encl}}}$$

Magnetic Forces

- The magnetic force on a moving charged particle

$$\boxed{\vec{F}_B = q\vec{v} \times \vec{B}}$$

- The magnetic force on a non-straight current-carrying wire

$$\boxed{d\vec{F}_B = I d\vec{s} \times \vec{B}} \qquad \boxed{\vec{F}_B = I \int_a^b d\vec{s} \times \vec{B}}$$

- Torque on the current loop in a uniform magnetic field

$$\boxed{\vec{\tau} = I\vec{A} \times \vec{B} = \vec{\mu} \times \vec{B}}$$

a vector magnetic moment $\boxed{\vec{\mu} \equiv I\vec{A}}$

Faraday's Law

Complete Faraday's law:

$$\boxed{\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\iint\limits_{\substack{\text{surrounding}\\ \text{surface}}} \vec{B}\cdot d\vec{A}}$$

motional emf:

$$\boxed{\varepsilon = \int_L (\vec{v}\times\vec{B})\cdot d\vec{s}}$$

non-electric force

$$\vec{F} = q\vec{v}\times\vec{B}$$

Induced emf

$$\varepsilon = \oint_L \vec{E}_i\cdot d\vec{s}$$

non-electric force

$$\vec{F} = q\vec{E}_i$$

The Features of Induced Electric Field

The Comparison between the electrostatic field and induced electric field

	Electrostatic field $\vec{E}_s$	Induced electric field $\vec{E}_i$
The source of the field	The charges	The changing magnetic field
Line integral around a closed path	$\oint_L \vec{E}_s \cdot d\vec{s} = 0$ Conservative	$\oint_L \vec{E}_i \cdot d\vec{s} = -\iint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A}$ Non-conservative
Gauss's law	$\oiint_S \vec{E}_s \cdot d\vec{A} = \frac{q_{\text{encl}}}{\varepsilon_0}$ Field lines begin and end on charge	$\oiint_S \vec{E}_i \cdot d\vec{A} = 0$ Field lines form closed loops

Self-Inductance

Self-induced emf: $$\boxed{\varepsilon_L = -L\frac{dI}{dt}}$$

The self-inductance $$\boxed{L = \frac{N\Phi_B}{I}}$$

Energy stored in the inductor $$U_B = \frac{1}{2}LI^2$$

Energy density $$\boxed{u_B = \frac{U_B}{V} = \frac{B^2}{2\mu_0}}$$

For a non-uniformed magnetic field

$$U_B = \iiint du_B = \iiint_V \left(\frac{B^2}{2\mu_0}\right) dV$$

displacement current Id

$$I_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \frac{d}{dt} \iint_S \vec{E} \cdot d\vec{A} = \varepsilon_0 \iint_S \frac{\partial \vec{E}}{\partial t} \cdot d\vec{A}$$

Displacement current density:

$$\vec{J}_d = \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \qquad I_d = \iint_S \vec{J}_d \cdot d\vec{A}$$

Extended Ampére's law

$$\boxed{\oint_L \vec{B} \cdot d\vec{A} = \mu_0 (I + I_d)_{\text{encl}} = \mu_0 I_{\text{encl}} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}}$$

Maxwell's Equations

$$\oiint_S \vec{E} \cdot d\vec{A} = \frac{q_{\text{encl}}}{\varepsilon_0}$$

Gauss's law for electricity

$$\oiint_S \vec{B} \cdot d\vec{A} = 0$$

Gauss's law for magnetism

$$\oint_L \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} = -\iint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A}$$

Faraday's law of induction

$$\oint_L \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{encl}} + \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt} = \mu_0 I_{\text{encl}} + \varepsilon_0 \mu_0 \iint_S \frac{\partial \vec{E}}{\partial t} \cdot d\vec{A}$$

Ampére-Maxwell law

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

Lorentz force

Maxwell's equations and Lorentz force give the fundamental relations of electromagnetism! They are fundamental in the sense that Newton's three laws are for mechanics.

The Physical Meaning Embodied in Maxwell's Equations

$$\oint\!\!\!\oint_S \vec{E}\cdot d\vec{A} = \frac{q_{\text{encl}}}{\varepsilon_0}$$ —— Charged particles create an electric field (electrostatic).

$$\oint_L \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt} = -\iint_S \frac{\partial \vec{B}}{\partial t}\cdot d\vec{A}$$ —— An electric field (non-electrostatic) can also be created by a changing magnetic field.

$$\oint\!\!\!\oint_S \vec{B}\cdot d\vec{A} = 0$$ —— There are no magnetic monopoles.

$$\oint_L \vec{B}\cdot d\vec{s} = \mu_0 I_{\text{encl}} + \varepsilon_0\mu_0 \frac{d\Phi_E}{dt} = \mu_0 I_{\text{encl}} + \varepsilon_0\mu_0 \iint_S \frac{\partial \vec{E}}{\partial t}\cdot d\vec{A}$$

—— A magnetic field can either be created by currents or a changing electric field.