Chapter 21-22 Electric Potential and Capacitance

§ 1 Electric Potential Energy (p515-)

- The similarity of electrostatic and gravitational force

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}\hat{r} \qquad \text{electrostatic}$$

$$\vec{F} = -G\frac{Mm}{r^2}\hat{r} \qquad \text{gravitational}$$

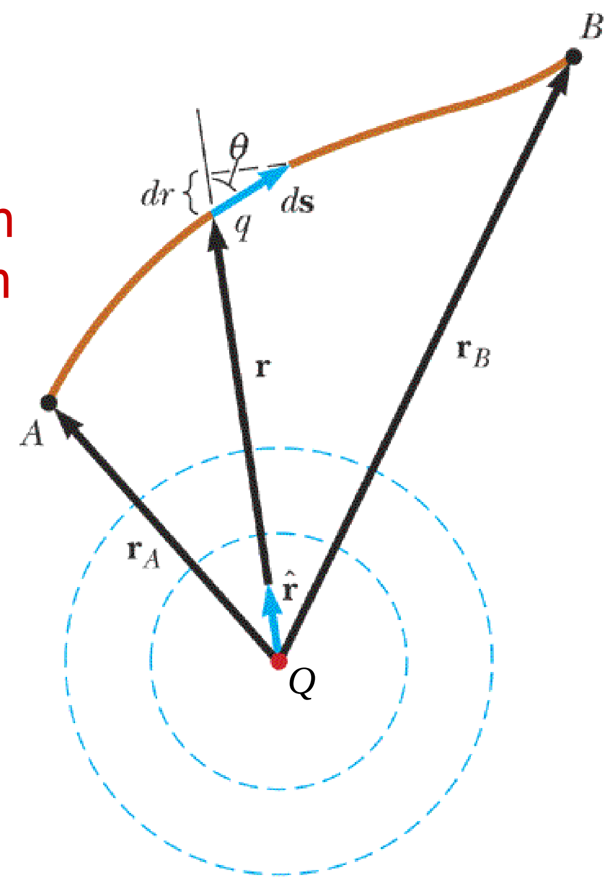
  - Both forces depend on the inverse square of the separation distance between the two objects.
  - The work done by the gravitational force on the object m depends only on the starting and finishing points and does not depend on the path taken between the points —— gravitational force is a conservative force.

$$\Delta U = U_f - U_i = -W_{if} = -\int_i^f \vec{F}\cdot d\vec{r}$$

the gravitational potential energy difference

$$\Delta U = \left(-G\frac{Mm}{r_f}\right) - \left(-G\frac{Mm}{r_i}\right)$$

- The electric potential energy
  - Because of the similarity of the electrostatic and gravitational force laws, the electrostatic force is also conservative, and therefore there is a potential energy associated with the configuration (the relative locations of the charges) of a system in which electrostatic forces act.



$$\vec{F}\cdot d\vec{r} = k_e\frac{Qq}{r^2}\hat{r}\cdot d\vec{r} = k_e\frac{Qq}{r^2}\left|d\vec{r}\right|\cos\theta = k_e\frac{Qq}{r^2}dr$$

electric potential energy difference

$$\Delta U = -\int_A^B \vec{F}\cdot d\vec{r} = -\int_{r_A}^{r_B} k_e\frac{Qq}{r^2}dr$$

$$= \left(\frac{Qq}{4\pi\varepsilon_0 r_B}\right) - \left(\frac{Qq}{4\pi\varepsilon_0 r_A}\right)$$

Choose the reference point A to correspond to an infinite separation between Q and q, and take

$$U_A(\infty) = 0$$

$$U(r) = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r}$$

Electric potential (p502-)

- Electric potential
  - A test charge q_0 in the field of charge q. The potential energy U associates with the test charge q_0. $\Delta U/q_0$ is independent of the value of q_0, and is characteristic only of the field of charge q —— we define the electric potential difference ΔV to be the electric potential energy difference per unit test charge.

$$\Delta U = \left(k_e \frac{q_0 q}{r_B}\right) - \left(k_e \frac{q_0 q}{r_A}\right) = -\int_A^B \vec{F} \cdot d\vec{r} = -q_0 \int_A^B \vec{E} \cdot d\vec{r}$$

$$\boxed{\Delta V = V_B - V_A = \frac{\Delta U}{q_0} = -\frac{1}{q_0}\int_A^B \vec{F} \cdot d\vec{r} = -\int_A^B \vec{E} \cdot d\vec{r}}$$

- SI unit: 1V=1 J/C

- Take infinity far away to be the reference point.

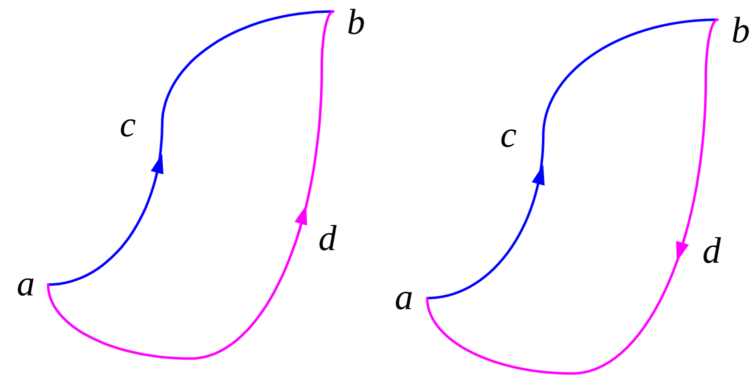
$$V_A = \int_A^\infty \vec{E} \cdot d\vec{r}$$

V_A equals to in magnitude work done by the $\vec{F}_e$ on unit positive charge during the move from *a* to infinity along any path.

ΔV is independent on the choice of reference potential

The value of electric potential is relative, but the one of electric potential difference is absolute.

- The Loop law of electric potential



$$\int_{acb} \vec{E}\cdot d\vec{r} = \int_{adb} \vec{E}\cdot d\vec{r}$$

$$\int_{acb} \vec{E}\cdot d\vec{r} - \int_{adb} \vec{E}\cdot d\vec{r} = \int_{acb} \vec{E}\cdot d\vec{r} + \int_{bda} \vec{E}\cdot d\vec{r} = 0$$

$$\boxed{\oint_L \vec{E}\cdot d\vec{r} = 0}$$

The line integral of the electrostatic field around a closed loop is zero.

It indicates that the electrostatic field is potential field.

§ 2 Calculating the Electric Potential (505-)

(a) Potential Due to a Point Charge (P507):

From definition of V, integral along radial direction:

$$V_P = \int_P^\infty \vec{E}\cdot d\vec{r} = \int_{r_p}^\infty E dr = \int_{r_p}^\infty \frac{q}{4\pi\varepsilon_0 r^2}\,\mathrm{d}r = \frac{q}{4\pi\varepsilon_0 r_p}$$

$$\boxed{\boldsymbol{V = \frac{q}{4\pi\varepsilon_0 r}}}$$

(b) Potential Due to a Group of Point Charges:

Based on the principle of superposition of $\vec{E}$

$$\boxed{\boldsymbol{V = \sum_{i=1}^{n} \frac{q}{4\pi\varepsilon_0 r_i}}}$$

- If the electric field is known

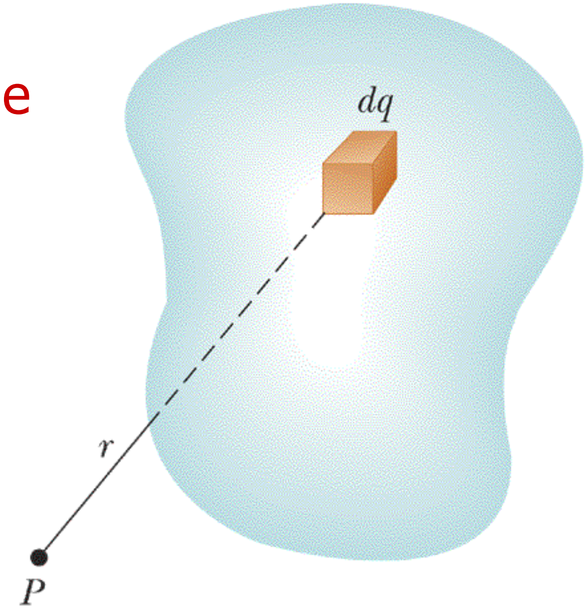
$$V_P = \int_P^\infty \vec{E} \cdot d\vec{r}$$

- If the charge distribution is known
  - The electric potential due to individual charge particles

$$V = \frac{1}{4\pi\varepsilon_0} \sum_i \frac{q_i}{r_i}$$

  - The electric potential due to continuous charge distributions

$$V = \frac{1}{4\pi\varepsilon_0} \int \frac{dq}{r}$$



Example —— The Electric Dipole

The electric potential of a dipole

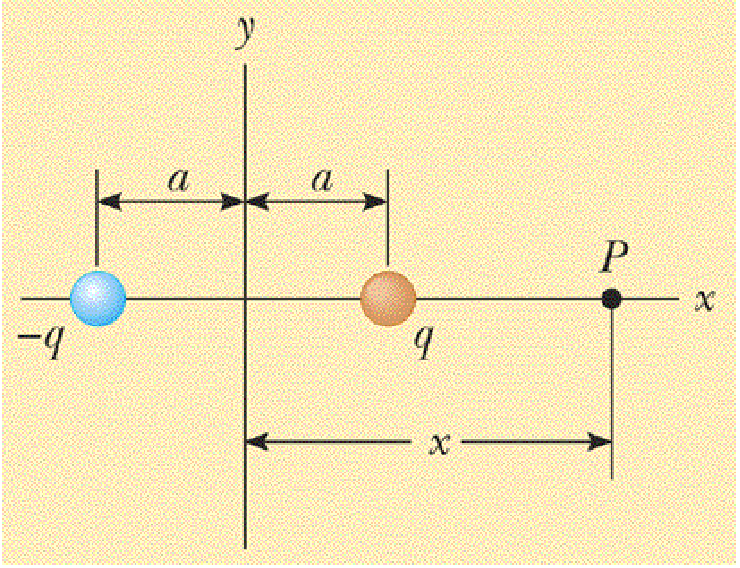
Example: The dipole is along the x axis and is centered at the origin. Calculating the electric potential at any point P along the x axis.

Solution:

$$V = k_e\left(\frac{q}{x-a} + \frac{-q}{x+a}\right) = \frac{2k_e qa}{x^2 - a^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{p}{x^2 - a^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{p}{x^2} \qquad x >> a$$



Example —— The Potential Energy of a Dipole in an External Field

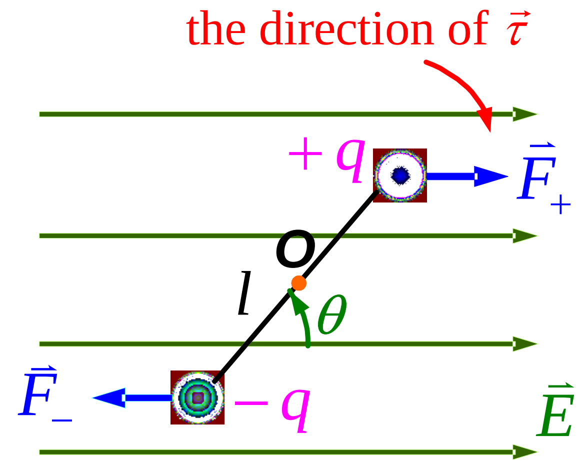
Example: Find the potential energy of an electric dipole in an external field.

Solution I:

The work done on the dipole by the electric field to change the angle θ from θ_1 to θ_2:

$$W = \int_{\theta_1}^{\theta_2} \tau d\theta = -pE\int_{\theta_1}^{\theta_2} \sin\theta d\theta$$

$$= pE\cos\theta_2 - pE\cos\theta_1$$



The work done by a conservative force decreases the potential energy.

$$U_f - U_i = -W = -pE\cos\theta_f + pE\cos\theta_i$$

Choose U=0 when $\vec{p} \perp \vec{E}$ $\quad \theta_i = 90^\circ, \quad \cos\theta_i = 0$

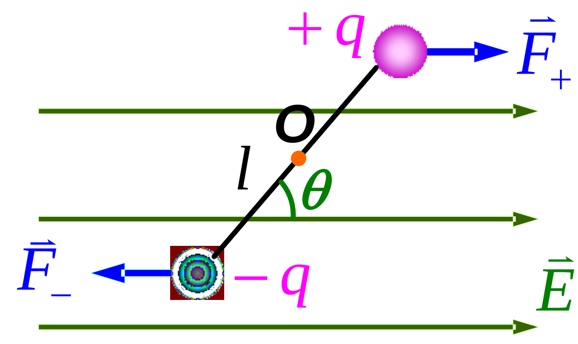
$$U = -pE\cos\theta$$

The vector description: $\boxed{U = -\vec{p}\cdot\vec{E}}$

Example —— The Potential Energy of a Dipole in an External Field Cont'd

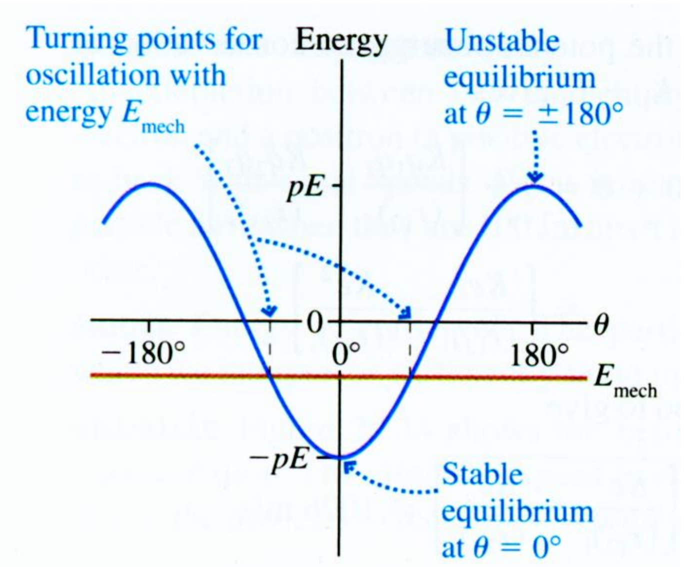
Solution II:

The potential energy of a dipole is the sum of the potential energies of positive and negative charges in the field.



$$U = U_+ + U_- = qV(P_+) - qV(P_-)$$
$$= q[V(P_+) - V(P_-)]$$
$$= -q\int_{P_-}^{P_+} \vec{E}\cdot d\vec{r} = -qlE\cos\theta = -\vec{p}\cdot\vec{E}$$

The potential energy is minimum at $\theta=0°$. This is the a point of stable equilibrium. The potential energy is maximum at $\theta=\pm180°$, which is at the point of unstable equilibrium. A dipole with mechanical energy E_{mech} will oscillates back and forth between turning points on either side of $\theta=0°$.

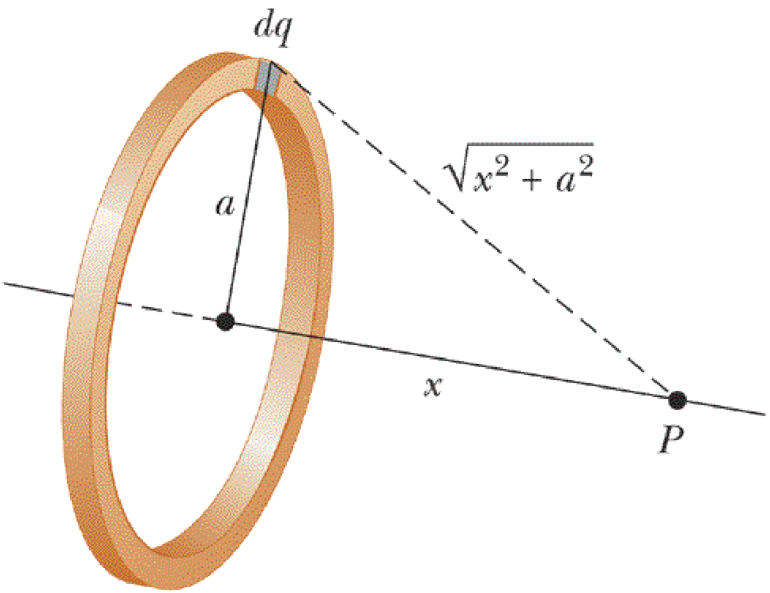


The electric potential due to a uniformly charged ring

Example: Find the electric potential at a point P located on the axis of a uniformly charged ring of radius a and total charge Q.

Solution:

$$V = \frac{k_e}{\sqrt{x^2+a^2}}\int dq = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{x^2+a^2}}$$



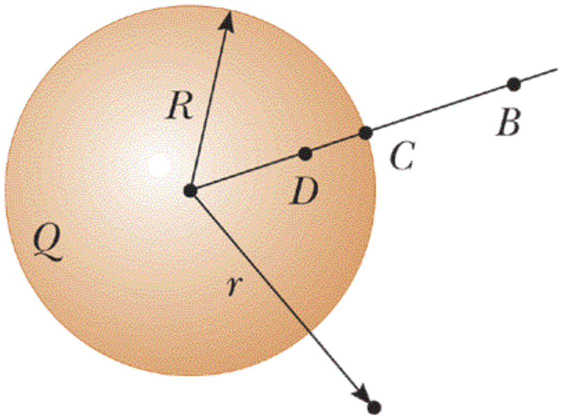
Example

The electric potential of a uniformly charged sphere

Example: An insulating solid sphere of radius ***R*** has a total charge ***Q***, which is distributed uniformly throughout the volume of the sphere.

(1) Find the electric potential at a point for r>R.

(2) Find the electric potential at a point for r<R.



Solution:

$$E=\begin{cases} k_e\dfrac{Q}{r^2} & \text{for } r>R \\ k_e\dfrac{Q}{R^3}r & \text{for } r<R \end{cases}$$

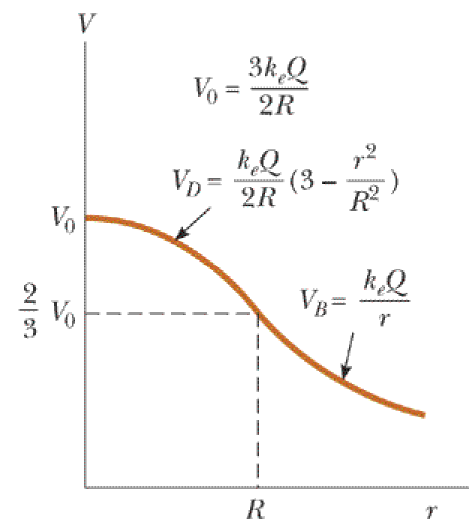
For r>R

$$V_B=\int_r^{\infty}\vec{E}\cdot d\vec{r}=k_eQ\int_r^{\infty}\frac{dr}{r^2}=\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

For r<R

$$V_D=\int_r^{R}\vec{E}\cdot d\vec{r}+\int_R^{\infty}\vec{E}\cdot d\vec{r}=\frac{k_eQ}{R^3}\int_r^{R}r\,dr+k_eQ\int_R^{\infty}\frac{dr}{r^2}$$

$$=\frac{k_eQ}{2R^3}\left(R^2-r^2\right)+\frac{k_eQ}{R}=\frac{k_eQ}{2R}\left(3-\frac{r^2}{R^2}\right)=\frac{Q}{8\pi\varepsilon_0 R}\left(3-\frac{r^2}{R^2}\right)$$



§ 3 Potential Gradient (p513)

$$-dV = -\left(\frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz\right) = \vec{E}\cdot d\vec{r}$$

$$= E_x dx + E_y dy + E_z dz$$

$$E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z}$$

$$\vec{E} = -\left(\hat{i}\frac{\partial V}{\partial x} + \hat{j}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right)$$

$$= -\nabla V$$

$\vec{E}$ is the negative of the gradient of V.

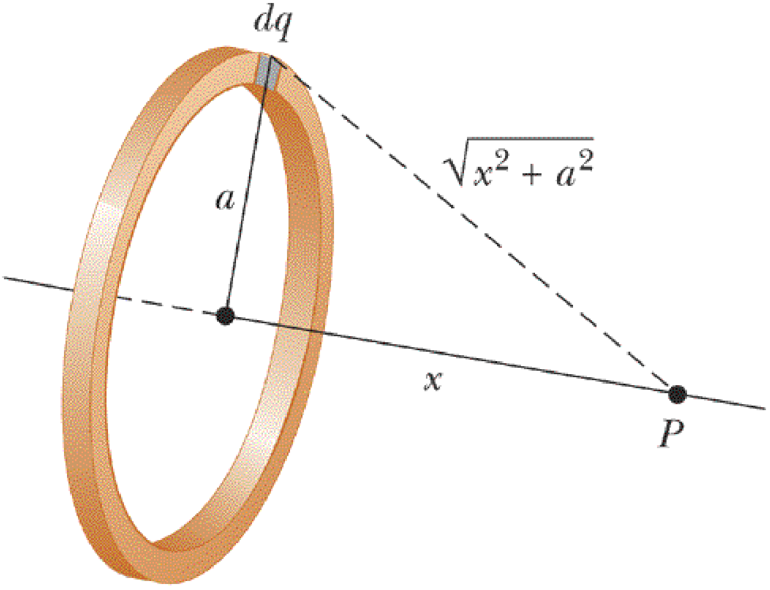
We therefore have two methods of calculating the electric field; one based on integrating Coulomb's law and another based on differentiating the potential.

Example

A uniformly charged ring

Example: Find the electric field at a point P located on the axis of a uniformly charged ring of radius a and total charge Q.

Solution: based on the electric potential:



$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{x^2+a^2}}$$

$$E = -\frac{\partial V}{\partial x} = -\frac{Q}{4\pi\varepsilon_0}\frac{d}{dx}\left(x^2+a^2\right)^{-1/2}$$

$$= -\frac{Q}{4\pi\varepsilon_0}\left(-\frac{1}{2}\right)\left(x^2+a^2\right)^{-3/2}(2x)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{xQ}{\left(x^2+a^2\right)^{3/2}}$$

§ 4 Equipotential Surface (p511)

- The equipotential surface
  - An equipotential surface is a three-dimensional surface on which the electric potential V is the same at every point.
- The properties of the equipotential surface
  - If a test charge moves over an equipotential surface, the electric field can do no work on such a charge.

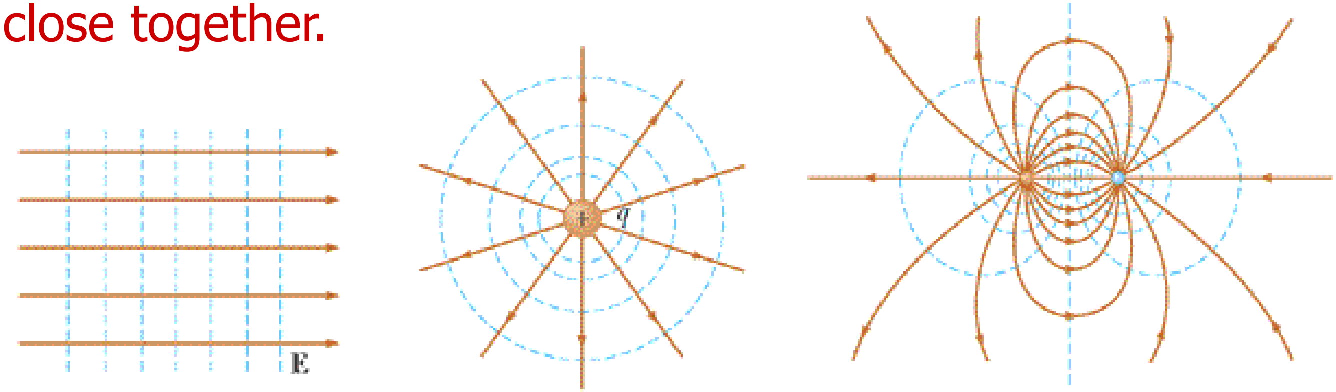
$$W_{ab} = -q_0 \Delta U = q_0 (U_a - U_b) = 0$$

  - Field lines and equipotential surface are always mutually perpendicular.

A test charge q_0 moves a distance $d\vec{l}$ on an equipotential surface

$$dW = q_0 \vec{E} \cdot d\vec{l} = 0 \quad \Rightarrow \quad \vec{E} \perp d\vec{l}$$

  - In regions where the magnitude of $\vec{E}$ is large, the equipotential surface are close together.



(a) (b) (c)

Review:

The properties that an isolated conductor in electrostatic equilibrium.

① The electric field is zero everywhere inside the conductor.

② If the isolated conductor carries a net charge, the net charge resides entirely on its surface.

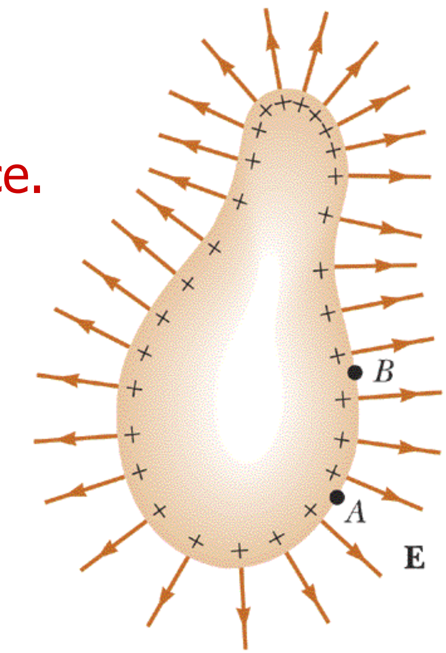
③ The electric field just outside the charged conductor is perpendicular to the conductor surface and has a magnitude σ / ε_0, where σ is the surface charge density at that point.

④ On an irregularly shaped conductor, the surface charge density is highest at locations where the radius of curvature of the surface is smallest.

§ 5 Electric Potential of A Charged Conductor

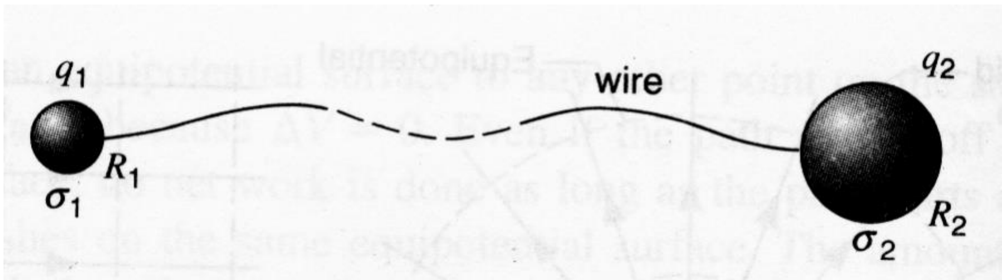
- The properties that an isolated conductor in electrostatic equilibrium
  - ⑤ The entire conductor is at the same potential. So the surface of a conductor is always an equipotential surface.
- The validity of property ④
  - ④ On an irregularly shaped conductor, the surface charge density is highest at locations where the radius of curvature of the surface is smallest.

    Consider two conducting spheres of different radii connected by a fine wire, let the entire assembly be raised to same arbitrary potential V.



$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{R_1} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{R_2}, \quad \text{which yields} \quad \frac{q_2}{q_1} = \frac{R_2}{R_1}$$

$$\frac{\sigma_2}{\sigma_1} = \frac{q_2/4\pi R_2^2}{q_1/4\pi R_1^2} = \frac{q_2}{q_1}\frac{R_1^2}{R_2^2} = \frac{R_1}{R_2}$$

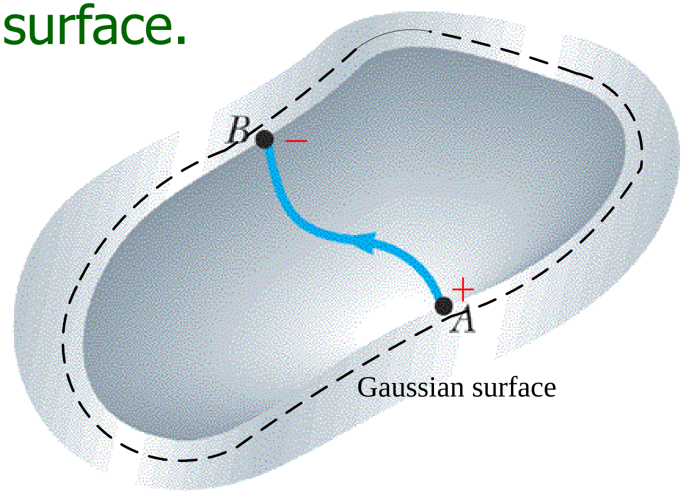


The property of an internal cavity in the conductor

- The validity of the statement "there is no charge in the internal cavity of the conductor ".

Draw a Gaussian surface just inside the inner surface.

$$\oint_S \vec{E} \cdot d\vec{A} = \frac{\sum q_{in}}{\varepsilon_0} = 0 \quad \Rightarrow \quad \sum q_{in} = 0$$



Is zero charge every where? If not, then

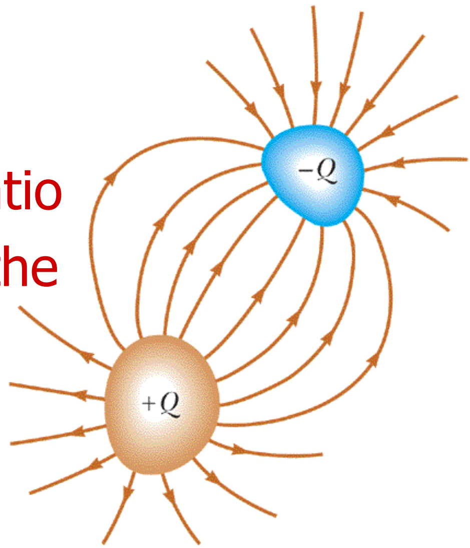
$$V_A - V_B = \int_A^B \vec{E} \cdot d\vec{r} > 0$$

It is contradictory to the fact that

the surface of a conductor is an equipotential surface.

§ 6 Capacitance (p525-)

- Definition
  - Any two conductors separated by an insulator (or a vacuum) form a capacitor, which can store amount of charge.
- Capacitance of a capacitor
  - The capacitance C of a capacitor is defined as the ratio of the charge on the capacitor to the magnitude of the potential difference across the capacitor.



$$C \equiv \frac{Q}{\Delta V}$$

  - The capacitance of a capacitor depends on the geometric arrangement of the conductors, and is independent of the charge Q or the potential difference ΔV. Because the potential difference is proportional to the charge, the ratio Q/ΔV is constant for a given capacitor.

Problem-Solving Strategy to Calculating The Capacitance of a Capacitor

- A convenient charge of magnitude Q is assumed.
- The potential difference is calculated.
- Use $C=Q/\Delta V$ to evaluate the capacitance.

Example

The parallel-plate capacitor

A parallel-plate capacitor consists of two parallel plates of equal area A separated by a distance d. Find the capacitance

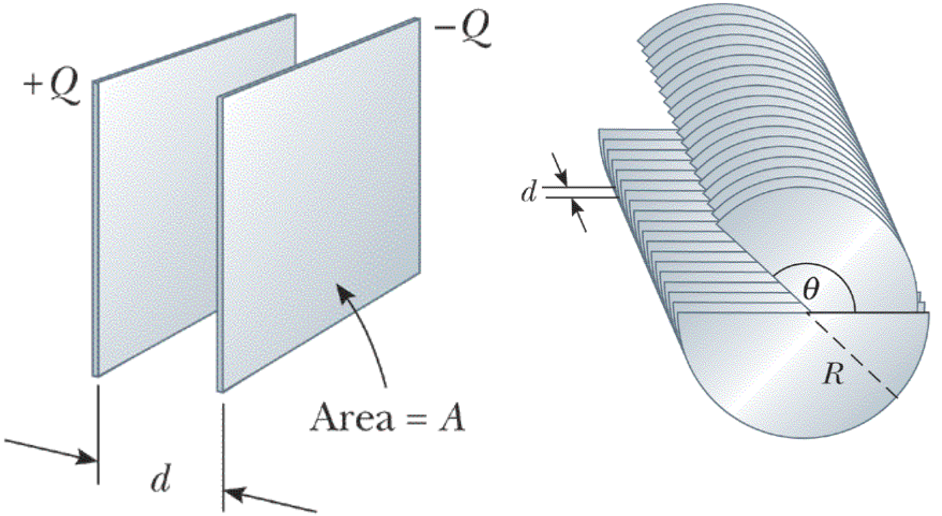
Solution: Assume the two plates have opposite charges +Q and –Q. An uniform electric field is:

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$

The potential difference:

$$\Delta V = \int_{+}^{-} \vec{E} \cdot d\vec{l} = Ed = \frac{Qd}{\varepsilon_0 A}$$

$$\boxed{C = \frac{Q}{\Delta V} = \frac{Q}{Qd/\varepsilon_0 A} = \frac{\varepsilon_0 A}{d}}$$



- The capacitance of a parallel-plate capacitor is proportional to the area of its plates and inversely proportional to the plate separation, which are the geometrical factors.
- The capacitance does not depend on the potential difference or the charge carried by the plates.
- The capacitance has form of ε_0 times a quantity with the dimension of length (A/d), which is essential form for all the capacitors. $\varepsilon_0 = 8.85\times10^{-12}\,\mathrm{F/m} = 8.85\,\mathrm{pF/m}$

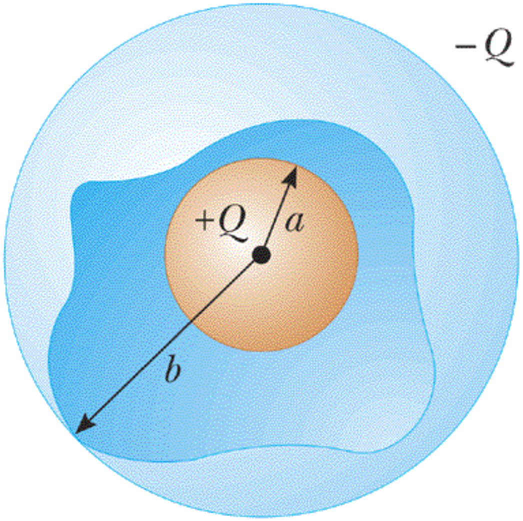
Example

The Spherical Capacitor

A spherical capacitor in which the inner conductor is a solid sphere of radius a, and outer conductor is a hollow spherical shell of inner radius b. Find the capacitance.

Solution: Assume the inner and outer sphere have opposite charges +Q and –Q. In the region a<r<b, we can use Gauss' law to determine:

$$E = \frac{k_e Q}{r^2}$$



The potential difference:

$$\Delta V = \int_{+}^{-} \vec{E} \cdot d\vec{r} = \int_{r_a}^{r_b} k_e Q \frac{dr}{r^2} = k_e Q \left(\frac{1}{a} - \frac{1}{b} \right)$$

$$= \frac{Q}{4\pi\varepsilon_0} \frac{b-a}{ab}$$

$$C = \frac{Q}{\Delta V} = 4\pi\varepsilon_0 \frac{ab}{b-a}$$

has the form of ε_0 times a quantity with dimension of length.

When $b \to \infty$, $C = 4\pi\varepsilon_0 a$

When $b-a \ll a$, $ab \approx a^2$, $d=b-a$, $A=4\pi a^2$, $C= \varepsilon_0 A/d$

Example

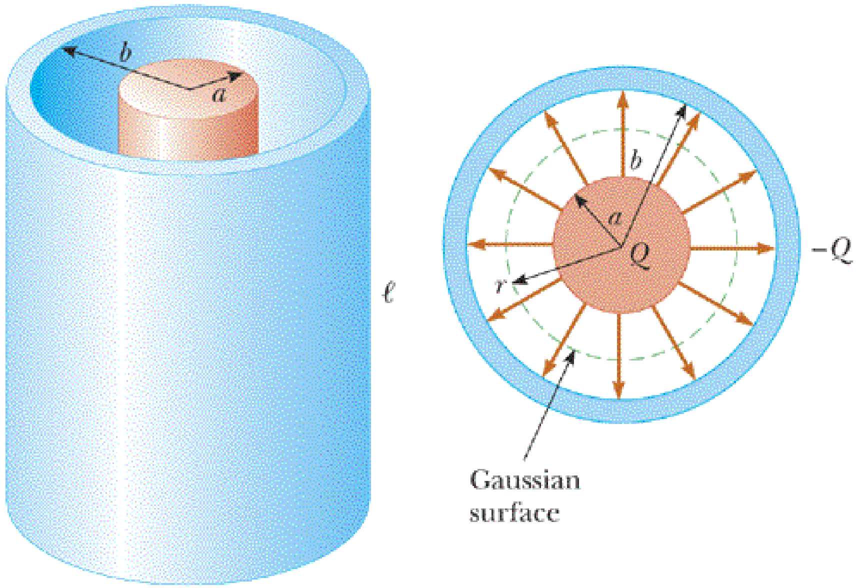
The Cylindrical Capacitor

A cylindrical capacitor consists of a cylindrical conductor of radius ***a*** coaxial with a larger cylindrical shell of radius ***b***. Find the capacitance of this device if its length is ***l***.

Solution: Assume the inner and outer conductors have opposite charges **+*Q*** and **–*Q***. In the region ***a<r<b***, we can use Gauss' law to determine:

$$\oint_S \vec{E}\cdot d\vec{A} = E2\pi rl = \frac{\lambda l}{\varepsilon_0}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$



The potential difference:

$$\Delta V = \int_+^- \vec{E}\cdot d\vec{r} = \int_a^b \frac{\lambda}{2\pi\varepsilon_0}\frac{dr}{r} = \frac{\lambda}{2\pi\varepsilon_0}\ln\left(\frac{b}{a}\right)$$

$$\boxed{C = \frac{Q}{\Delta V} = \frac{2\pi\varepsilon_0 l}{\ln(b/a)}}$$

has the form of ε_0 times a quantity with dimension of length.

When b-a=d<<a $\ln\left(\frac{b}{a}\right) = \ln\left(\frac{a+d}{a}\right) = \ln\left(1+\frac{d}{a}\right) \approx \frac{d}{a}$ $A = 2\pi al$ C= ε_0A/d

§ 7 Combinations of Capacitor (p529-532)

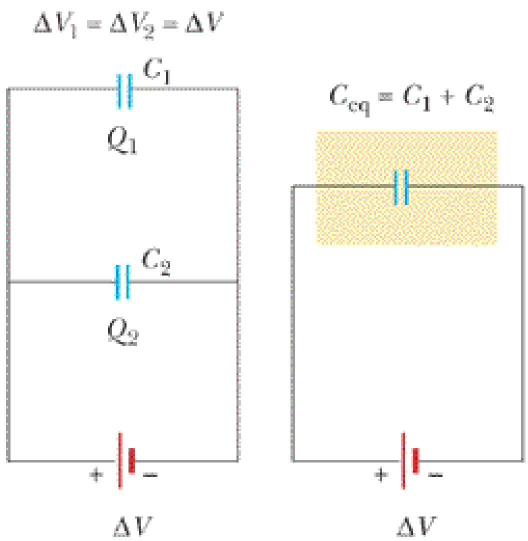
- Parallel Combination

$$Q = Q_1 + Q_2 \qquad \Delta V_1 = \Delta V_2 = \Delta V$$

$$Q = C_{eq}\Delta V = Q_1 + Q_2 = C_1\Delta V + C_2\Delta V$$

$$= (C_1 + C_2)\Delta V \qquad \boxed{C_{eq} = C_1 + C_2}$$

  - The equivalent capacitance of a parallel combination of capacitors is the algebraic sum of the individual capacitances.

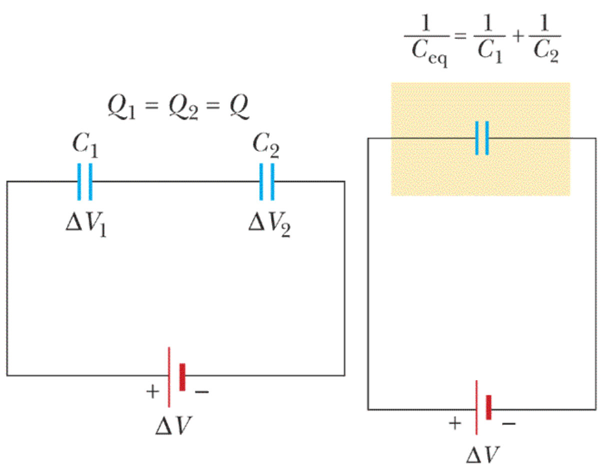


- Series Combination

$$Q_1 = Q_2 = Q \qquad \Delta V = \frac{Q}{C_{eq}} = \Delta V_1 + \Delta V_2 = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}} \qquad = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

  - The inverse of the equivalent capacitance is the algebraic sum of the inverse of the individual capacitances.



§ 8 Energy Stored in A Charged Capacitor

(p532)

A capacitor can store charge, and can also store energy!

Question: Which one is the storehouse of the energy, the charge or the electric field itself?

Energy of any charge configuration

Any charge configuration has a certain electric potential energy U, equal to the work W that is done by an external agent that assembles the charge configuration from its individual components, originally assumed to be infinitely far apart and at rest.

Energy Stored in A Charged Capacitor

- The potential energy of an isolated sphere conductor with charge Q
  - We evaluate the work of charging that an external agent continuously pulls charge dq from infinite until the conductor has the charge of Q.
  - Suppose that at a time t a charge q has already been transferred from infinite, the sphere conductor has the electric potential $V=q/4\pi\varepsilon_0 R=q/C$. If an increment of charge dq is now pulls from infinite, the resulting small change dU in the electric potential energy is:

$$dU = Vdq = \frac{q}{C}dq$$

  - If this process is continued until a total charge Q has been transferred, the total potential energy is:

$$U = \int dU = \int_0^Q \frac{q}{C}dq = \frac{Q^2}{2C}$$

$$U = \frac{1}{2}CV^2$$

Energy Stored in A Charged Capacitor

- The potential energy of a charged capacitor
  - We evaluate the work of charging that an external agent continuously pulls charge dq from negative plate to positive plate until the capacitor has the opposite charge of $\pm$Q.
  - Suppose that q is the charge on the capacitor at some instant during this charging process, the potential difference across the capacitor is $\Delta V=q/C$. Imaging that the external agent transfers an additional increment of charge dq from the plate of charge –q to the plate of charge q, the resulting small change dU in the electric potential energy is:

$$dU = \Delta V dq = \frac{q}{C} dq$$

  - If this process is continued until to charge the capacitor from q=0 to the final charge q=Q, the total potential energy is:

$$U = \int dU = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C} \qquad U = \frac{1}{2} Q \Delta V = \frac{1}{2} C (\Delta V)^2$$

Energy Stored in A Charged Capacitor

- Where does the potential energy reside?
  - From the equation $U=Q^2/2C$, we conclude that the energy relates to the charging.
  - Another point of view:

    $$C=\frac{\varepsilon_0 A}{d},\quad \Delta V=Ed \qquad U=\left(\frac{1}{2}\varepsilon_0 E^2\right)(Ad)$$

    U is proportional to the volume between the two plates.
  - Because the electric field is present in the space between the two plates, the energy is stored in the electric field that is present in this region.
  - The energy density: $$u=\frac{U}{Ad}=\frac{1}{2}\varepsilon_0 E^2$$
  - If an electric field $\vec{E}$ exists at any point in empty space, we can think of that point as the site of stored energy in amount of $\frac{1}{2}\varepsilon_0 E^2$.

$$U=\iiint du=\iiint_V\left(\frac{1}{2}\varepsilon_0 E^2\right)dV$$

Energy Stored in A Charged Capacitor

- In the case of electrostatic field, we can not answer which one is the storehouse of the energy.
  - Because in the case of electrostatic field, the electric field is always accompanied with the charge.
- In the case of time-varying electromagnetism field
  - The electromagnetic wave can exists in the vacuum, whether the charge exists or not.

Example

Example: How much energy is stored in the electric field of an isolated conducting sphere of radius R and charge Q.

Solution: The electric field distribution:

$$E = \begin{cases} 0 & \text{if} \quad r < R \\ \dfrac{Q}{4\pi\varepsilon_0 r^2} & \text{if} \quad r > R \end{cases}$$

$$U = \iiint \left[\frac{1}{2} \varepsilon_0 \left(\frac{Q}{4\pi\varepsilon_0 r^2} \right)^2 \right] dV = \int_R^\infty \left[\frac{1}{2} \varepsilon_0 \left(\frac{Q}{4\pi\varepsilon_0 r^2} \right)^2 \right] \left(4\pi r^2 dr \right)$$

$$= \frac{Q^2}{8\pi\varepsilon_0} \int_R^\infty \frac{dr}{r^2} = \frac{Q^2}{8\pi\varepsilon_0 R} = \frac{Q^2}{2C}$$

§ 9 Dielectric Materials (p533)

- Polar dielectric materials and nonpolar dielectric materials.
  - Polar dielectric material —— its molecule has a permanent electric dipole moment, such as water.

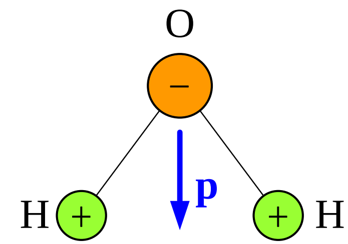
    The external electric field exerts a torque on the dipole that tries to align it with the field.

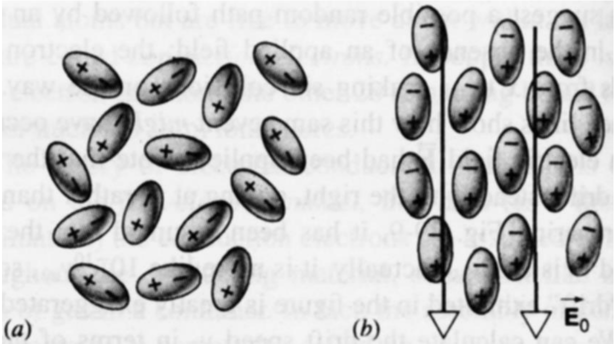
    $$\vec{\tau} = \vec{p} \times \vec{E}_0$$

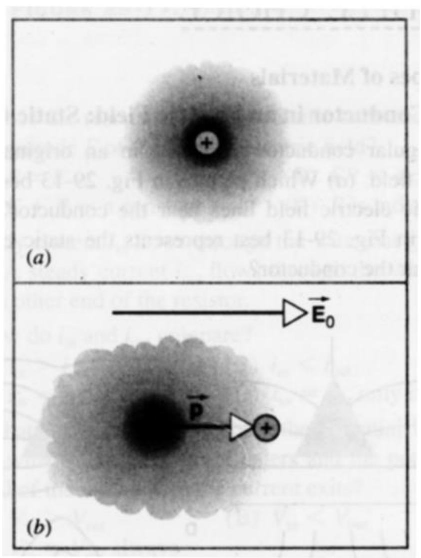
  - Nonpolar dielectric material —— its molecule has no permanent electric dipole.

    The atom acquires an induced dipole moment when the atom is placed in an external electric field.

    $$\vec{F}_e = -e\vec{E}_0$$





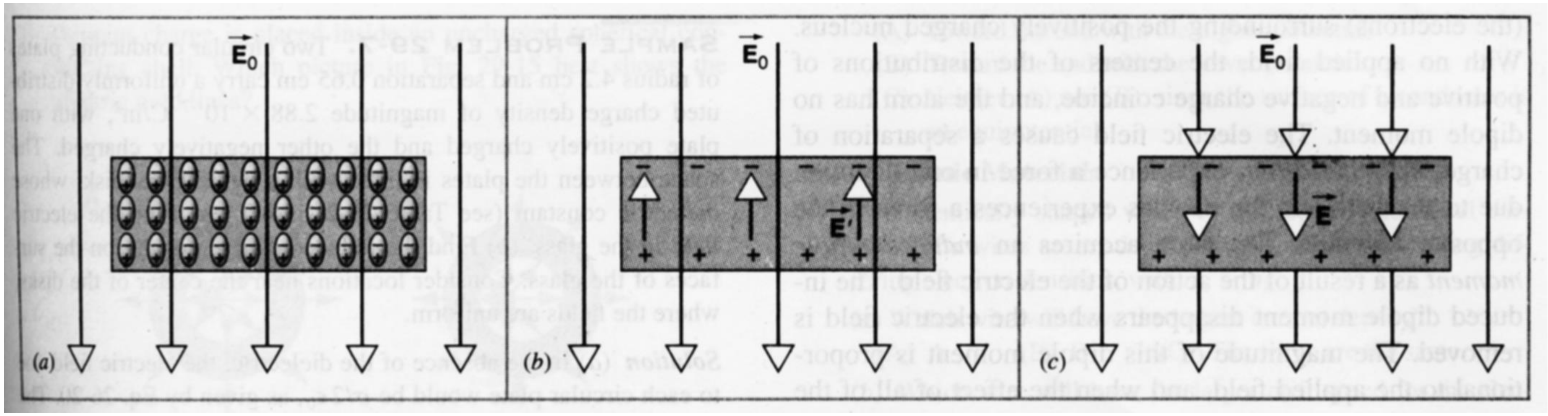


The induced surface charge and induced polarization field

- The induced surface charge and induced polarization field
  - When a dielectric material is placed in an external applied field E_0, induced surface charges q' appear that tend to weaken the original field E_0 by a polarization field E' within the material. For a linear material, the net field inside the material

$$\vec{E} = \vec{E}_0 + \vec{E'} \qquad E = \frac{1}{\kappa} E_0$$

  $\vec{E'}$ is called polarization field.
  - κ is called the dielectric constant, which is greater than 1.
  - The charge q_0, the origin of E_0, that resides in the conductors is called *free charge*, and induced charge q' that resides in the surface of dielectric materials, that not free to move and bound to a molecule, is called *bound charge*.



The polarization and the dielectric strength

- When either polar or nonpolar materials are put in an external field, the materials are said to be polarized.
- The dielectric strength: If we apply a large enough electric field to an insulator, we can ionize atoms or molecules of the insulator and thus create a condition for electric charge to flow, as in a conductor. The field necessary for the breakdown of the insulator is called the dielectric strength.

TABLE 20.1 Dielectric Constants and Dielectric Strengths of Various Materials at Room Temperature

Material	Dielectric Constant κ	Dielectric Strength[a] (V/m)
Vacuum	1.00000	—
Air (dry)	1.00059	3×10^6
Bakelite	4.9	24×10^6
Fused quartz	3.78	8×10^6
Pyrex glass	5.6	14×10^6
Polystyrene	2.56	24×10^6
Teflon	2.1	60×10^6
Neoprene rubber	6.7	12×10^6
Nylon	3.4	14×10^6
Paper	3.7	16×10^6
Strontium titanate	233	8×10^6
Water	80	—
Silicone oil	2.5	15×10^6

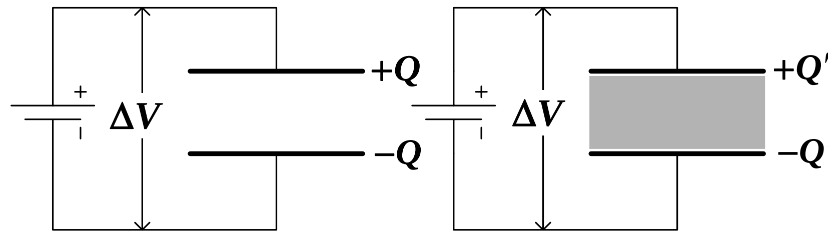
[a] The dielectric strength equals the maximum electric field that can exist in a dielectric without electrical breakdown.

§ 10 Capacitors With Dielectrics

Two identical capacitors, filling one with a dielectric material and leaving the other with air between its plates

- When both capacitors are connected to batteries with the same potential difference.

$$\Delta V = \Delta V' = \Delta V_0 \;\Rightarrow\; E = E' = E_0$$



$$E_0 = \frac{Q}{\varepsilon_0 A} = E' = \frac{Q'}{\kappa\varepsilon_0 A} \;\Rightarrow\; Q' = \kappa Q$$

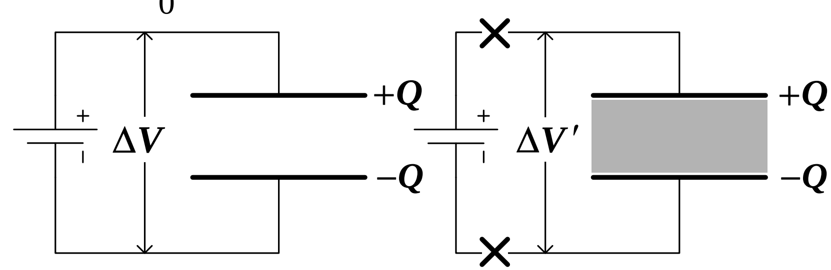
$$C' = \kappa C \;\Rightarrow\; C' = \frac{\kappa\varepsilon_0 A}{d} = \frac{\varepsilon A}{d}$$

$\varepsilon = \kappa\varepsilon_0$ permittivity

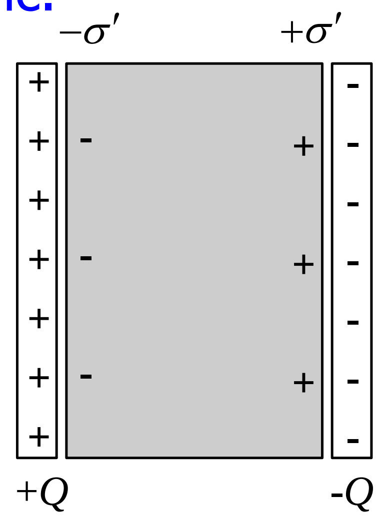
- When both are disconnected the batteries with the same charge

$$E_0 = \frac{Q}{\varepsilon_0 A}, \quad E = \frac{E_0}{\kappa} = \frac{Q}{\kappa\varepsilon_0 A} \qquad \Delta V' = \frac{\Delta V}{\kappa} = \frac{Qd}{\kappa\varepsilon_0 A}$$

$$C' = \frac{Q}{\Delta V'} = \frac{\kappa\varepsilon_0 A}{d} = \kappa C = \frac{\varepsilon A}{d}$$



- The electric field energy stored in a capacitor with dielectric.



$$E = \frac{\sigma}{\kappa\varepsilon_0} = \frac{Q}{\kappa\varepsilon_0 A}$$

$$U = \frac{Q^2}{2C} = \frac{Q^2 d}{2\kappa\varepsilon_0 A} = \frac{1}{2}\kappa\varepsilon_0\left(\frac{Q}{\kappa\varepsilon_0 A}\right)^2 (Ad) = \frac{1}{2}\kappa\varepsilon_0 E^2 (Ad)$$

- The electric field energy density in dielectric materials

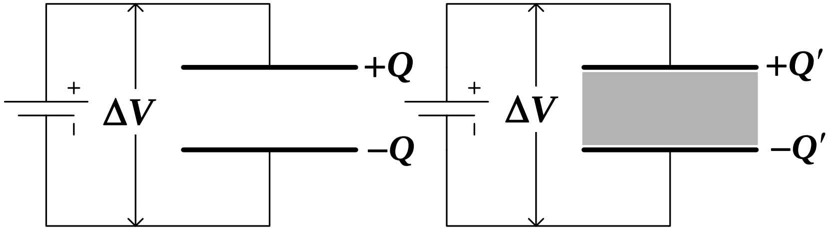
$$\boxed{u = \frac{1}{2}\kappa\varepsilon_0 E^2 = \frac{1}{2}\varepsilon E^2}$$

Example

Example: In following two cases, find the electric field energy stored in a parallel-plate capacitor before and after the dielectric is inserted. The capacitor without dielectric is C_0, and dielectric material has dielectric constant κ.

(1) From beginning to end, the capacitor is always connected to the battery of voltage ΔV;

(2) At beginning, the capacitor, with empty, is connected to the battery of voltage ΔV. The battery is then removed, and the capacitor is fill with the dielectric material.



Solution:

(1) Before inserting the dielectric:

$$U_b = \frac{1}{2} C_0 (\Delta V)^2$$

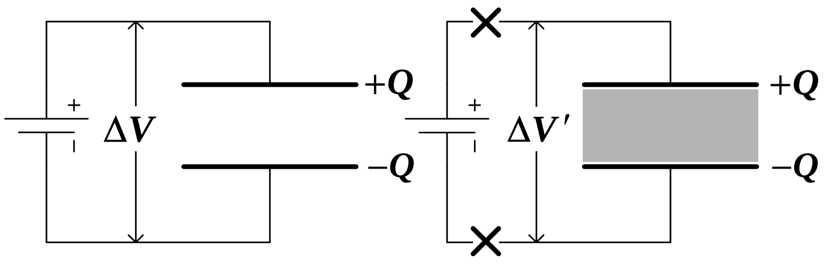
After inserting the dielectric: $U_a = \frac{1}{2} C (\Delta V)^2 = \frac{1}{2} \kappa C_0 (\Delta V)^2 = \kappa U_b$

$$\Delta U = U_{after} - U_{before} = (\kappa - 1) U_{before} > 0$$

The increase in energy is used for polarization of the dielectric material, and provided by the work done by the battery.

Example cont'd

(2) Before and after removing the battery, The charges in the capacitor are the same, and the voltage across the plates decreases after removing the battery $\Delta V' = \Delta V/\kappa$.



Before inserting the dielectric: $$U_b = \frac{1}{2} C_0 \left(\Delta V\right)^2$$

After inserting the dielectric: $$U_a = \frac{1}{2} C \left(\Delta V'\right)^2 = \frac{1}{2} \kappa C_0 \left(\frac{\Delta V}{\kappa}\right)^2 = \frac{U_b}{\kappa}$$

$$\Delta U = U_{after} - U_{before} = (1-\kappa) U_{before} < 0$$

The decrease in energy is used for polarization of the dielectric material. There is no work done because the battery was removed.