4 The Operational Amplifier

Drill Exercises

DE 4.1 **[a]** $v_o = (-80/16)v_s, \qquad v_o = -5v_s$

v_s(V)	0.4	2.0	3.5	−0.6	−1.6	−2.4
v_o(V)	−2.0	−10.0	−15.0	3.0	8.0	10.0

[b] $-15 = -5v_s, \quad v_s = 3\ \text{V}; \qquad 10 = -5v_s, \qquad v_s = -2\ \text{V}$

Therefore $\quad -2 \le v_s \le 3\ \text{V}$

DE 4.2 $v_o = (-R_x/16)v_s = (0.64R_x/16) = 10\ \text{V}$

Therefore $\quad R_x = \dfrac{160}{0.64} = 250\,\text{k}\Omega, \qquad 0 \le R_x \le 250\,\text{k}\Omega$

DE 4.3 **[a]** $v_o = -\dfrac{250}{5}v_\text{a} - \dfrac{250}{25}v_\text{b} = -50(0.1) - 10(0.25) = -5 - 2.5 = -7.5\ \text{V}$

[b] $v_o = -50v_\text{a} - 2.5 = -10\ \text{V}; \qquad \text{therefore} \quad 50v_\text{a} = 7.5, \quad v_\text{a} = 0.15\ \text{V}$

[c] $v_o = -5 - 10v_\text{b} = -10\ \text{V}; \qquad 10v_\text{b} = 5, \quad v_\text{b} = 0.5\ \text{V}$

[d] $v_o = -50v_\text{a} + 10v_\text{b} = -5 + 2.5 = -2.5\ \text{V}$

$v_o = -50v_\text{a} + 2.5 = -10\ \text{V};$

$50v_\text{a} = 12.5, \quad v_\text{a} = 0.25\ \text{V}$

$v_o = -5 + 10v_\text{b} = 15\ \text{V}; \qquad 10v_\text{b} = 20; \qquad v_\text{b} = 2.0\ \text{V}$

DE 4.4 **[a]** $\dfrac{v_n}{4500} + \dfrac{v_n - v_o}{63{,}000} = 0, \qquad \text{therefore} \quad v_o = 15v_n, \quad v_n = v_p$

Thus $\quad v_o = 15v_p, \qquad v_p = \dfrac{0.4R_x}{15{,}000 + R_x}$

So when $\quad R_x = 60\,\text{k}\Omega, \qquad v_p = 0.32\ \text{V}, \qquad v_o = 4.8\ \text{V}$

[b] $\dfrac{15(0.4R_x)}{15{,}000 + R_x} = 5, \qquad R_x = 75\,\text{k}\Omega$

DE 4.5 **[a]** Assume v_a is acting along. Replacing v_b with a short circuit yields $v_p = 0$, therefore $v_n = 0$ and we have

$$\frac{0 - v_\text{a}}{R_\text{a}} + \frac{0 - v_o'}{R_\text{b}} + i_n = 0, \qquad i_n = 0$$

Therefore

$$\frac{v_o'}{R_\text{b}} = -\frac{v_\text{a}}{R_\text{a}}, \qquad v_o' = \frac{R_\text{b}}{R_\text{a}} v_\text{a}$$

Assume v_b is acting alone. Replace v_a with a short circuit. Now

$$v_p = v_n = \frac{v_\text{b} R_\text{d}}{R_\text{c} + R_\text{d}}$$

$$\frac{v_n}{R_\text{a}} + \frac{v_n - v_o''}{R_\text{b}} + i_n = 0, \qquad i_n = 0$$

$$\left(\frac{1}{R_\text{a}} + \frac{1}{R_\text{b}}\right)\left(\frac{R_\text{d}}{R_\text{c} + R_\text{d}}\right) v_\text{b} - \frac{v_o''}{R_\text{b}} = 0$$

$$v_o'' = \left(\frac{R_\text{b}}{R_\text{a}} + 1\right)\left(\frac{R_\text{d}}{R_\text{c} + R_\text{d}}\right) v_\text{b} = \frac{R_\text{d}}{R_\text{a}}\left(\frac{R_\text{a} + R_\text{b}}{R_\text{c} + R_\text{d}}\right) v_\text{b}$$

$$v_o = v_o' + v_o'' = \frac{R_\text{d}}{R_\text{a}}\left(\frac{R_\text{a} + R_\text{b}}{R_\text{c} + R_\text{d}}\right) v_\text{b} - \frac{R_\text{b}}{R_\text{a}} v_\text{a}$$

[b] $\dfrac{R_\text{d}}{R_\text{a}}\left(\dfrac{R_\text{a} + R_\text{b}}{R_\text{c} + R_\text{d}}\right) = \dfrac{R_\text{b}}{R_\text{a}}, \qquad$ therefore $\quad R_\text{d}(R_\text{a} + R_\text{b}) = R_\text{b}(R_\text{c} + R_\text{d})$

$R_\text{d} R_\text{a} = R_\text{b} R_\text{c}, \qquad$ therefore $\quad \dfrac{R_\text{a}}{R_\text{b}} = \dfrac{R_\text{c}}{R_\text{d}}$

When $\dfrac{R_\text{d}}{R_\text{a}}\left(\dfrac{R_\text{a} + R_\text{b}}{R_\text{c} + R_\text{d}}\right) = \dfrac{R_\text{b}}{R_\text{a}}$

Eq. (4.22) reduces to $\quad v_o = \dfrac{R_\text{b}}{R_\text{a}} v_\text{b} - \dfrac{R_\text{b}}{R_\text{a}} v_\text{a} = \dfrac{R_\text{b}}{R_\text{a}}(v_\text{b} - v_\text{a})$.

DE 4.6 **[a]** $v_o = \dfrac{20(60)}{10(24)} v_\text{b} - \dfrac{50}{10} v_\text{a} = 5(v_\text{b} - v_\text{a}) = 20 - 5v_\text{a}$

$20 - 5v_\text{a} = \pm 10\ \text{V}$

$5v_\text{a} = 20 \mp 10, \qquad v_\text{a} = 2\ \text{V}, \qquad v_\text{a} = 6\ \text{V}$

Therefore $\quad 2 \le v_\text{a} \le 6\ \text{V}$

[b] $v_o = \dfrac{8(60)}{10(12)}v_{\rm b} - 5v_{\rm a} = 4v_{\rm b} - 5v_{\rm a}$

$4v_{\rm b} - 5v_{\rm a} = 16 - 5v_{\rm a} = \pm 10\ \ {\rm V}$

$16 \mp 10 = 5v_{\rm a}, \qquad v_{\rm a} = 1.2\ \ {\rm V}, \qquad v_{\rm a} = 5.2\ \ {\rm V}$

Therefore $\quad 1.2 \leq v_{\rm a} \leq 5.2\ \ {\rm V}$

DE 4.7 **[a]** $A_{\rm dm} = \dfrac{(24)(26) + (25)(25)}{(2)(1)(25)} = 24.98$

[b] $A_{\rm cm} = \dfrac{(1)(24) - 25(1)}{1(25)} = -0.04$

[c] $\text{CMRR} = \left|\dfrac{24.98}{0.04}\right| = 624.50$

DE 4.8 $A_{\rm cm} = \dfrac{(20)(50) - (50)R_x}{20(50 + R_x)}$

$$A_{\rm dm} = \frac{50(20 + 50) + 50(50 + R_x)}{2(20)(50 + R_x)}$$

$$\frac{A_{\rm dm}}{A_{\rm cm}} = \frac{R_x + 120}{2(20 - R_x)}$$

$$\therefore \quad \frac{R_x + 120}{2(20 - R_x)} = \pm 1000$$

If we use $+1000 \quad R_x = 19.93\,\text{k}\Omega$

If we use $-1000 \quad R_x = 20.07\,\text{k}\Omega$

Problems

P 4.1 Since the current into the inverting input terminal of an ideal op-amp is zero, the voltage across the 3.3 MΩ resistor is (2.5)(3.3) or 8.25 V. Therefore the voltmeter reads 8.25 V.

P 4.2 $v_p = \frac{18}{24}(12) = 9 \text{ V} = v_n$

$$\frac{v_n - 24}{30} + \frac{v_n - v_o}{20} = 0$$

$$v_o = (45 - 48)/3 = -1.0 \text{ V}$$

$$i_{\text{L}} = \frac{v_o}{5} \times 10^{-3} = -\frac{1}{5} \times 10^{-3} = -200 \times 10^{-6}$$

$$i_{\text{L}} = -200\,\mu\text{A}$$

P 4.3 $\frac{v_{\text{b}} - v_{\text{a}}}{20} + \frac{v_{\text{b}} - v_o}{160} = 0,$ therefore $v_o = 9v_{\text{b}} - 8v_{\text{a}}$

[a] $v_{\text{a}} = 1.5$ V, $v_{\text{b}} = 0$ V, $v_o = -12$ V

[b] $v_{\text{a}} = 3.0$ V, $v_{\text{b}} = 0$ V, $v_o = -18$ V (sat)

[c] $v_{\text{a}} = 1.0$ V, $v_{\text{b}} = 2$ V, $v_o = 10$ V

[d] $v_{\text{a}} = 4.0$ V, $v_{\text{b}} = 2$ V, $v_o = -14$ V

[e] $v_{\text{a}} = 6.0$ V, $v_{\text{b}} = 8$ V, $v_o = 18$ V (sat)

[f] If $v_{\text{b}} = 4.5$ V, $v_o = 40.5 - 8v_{\text{a}} = \pm 18$

$$\therefore \quad 2.8125 \leq v_{\text{a}} \leq 7.3125 \text{ V}$$

P 4.4 **[a]** $i_{\text{a}} = \frac{120}{6} \times 10^{-6} = 20\,\mu\text{A}$

$$v_{\text{a}} = -20 \times 10^3 i_{\text{a}} = -400\,\text{mV}$$

[b] $\frac{v_{\text{a}}}{60{,}000} + \frac{v_{\text{a}}}{20{,}000} + \frac{v_{\text{a}} - v_o}{240{,}000} = 0$

$$\therefore \quad v_o = 17v_{\text{a}} = -6.8 \text{ V}$$

[c] $i_{\text{a}} = 20\,\mu\text{A}$

[d] $i_o = \frac{-v_o}{80{,}000} + \frac{v_a - v_o}{240{,}000} = 111.67\,\mu$ A

P 4.5 $v_o = (1)(9) = 9\ \text{V}; \qquad i_{15k\Omega} = \frac{9}{15{,}000} = 0.6\,\text{mA};$

$$i_{6k\Omega} = \frac{9}{6000} = 1.5\,\text{mA}; \qquad i_{9k\Omega} = \frac{9}{9000} = 1\,\text{mA}$$

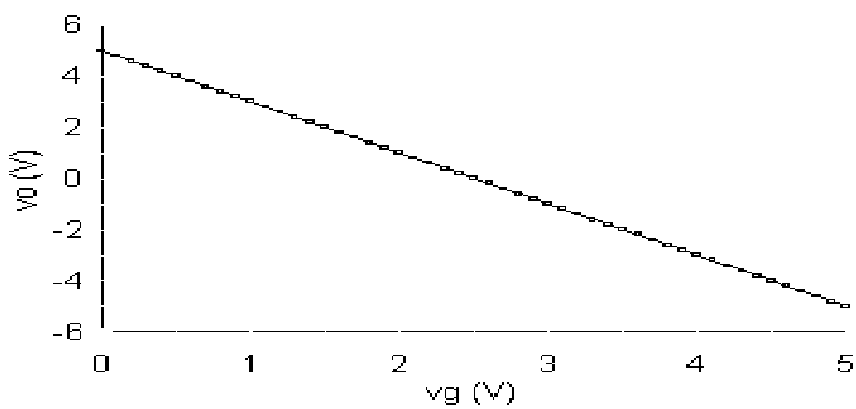
$$\therefore \quad i_o = -0.6 - 1.5 - 1 = -3.1\,\text{mA}$$

P 4.6 **[a]** First, note that $v_n = v_p = 2.5\ \text{V}$

Let v_{o1} equal the voltage output of the op-amp. Then

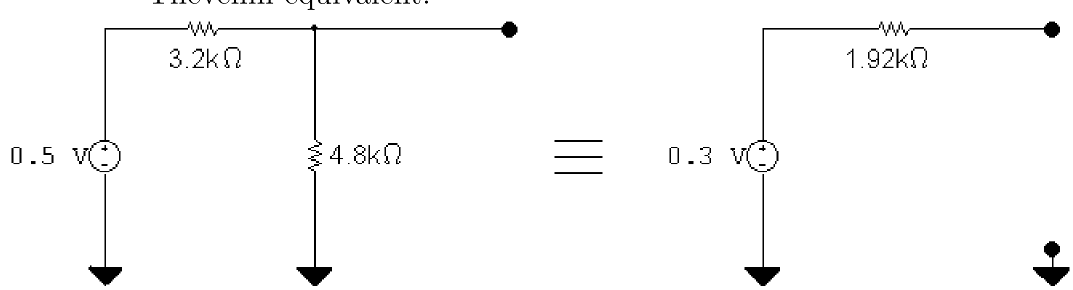
$$\frac{2.5 - v_g}{5000} + \frac{2.5 - v_{o1}}{10{,}000} = 0, \qquad \therefore \quad v_{o1} = 7.5 - 2v_g$$

Also note that $v_{o1} - 2.5 = v_o, \qquad \therefore \quad v_o = 5 - 2v_g$



[b] Yes, the circuit designer is correct!

P 4.7 **[a]** Replace the combination of v_g, 3.2 kΩ, and the 4.8 kΩ resistors with its Thévenin equivalent.



Then $v_o = \frac{-[30 + \sigma 170]}{1.92}(0.30)$

At saturation $v_o = -10\ \text{V}; \qquad$ therefore

$$-\left(\frac{30 + \sigma 170}{1.92}\right)(0.3) = -10, \quad \text{or} \quad \sigma = 0.2$$

Thus for $0 \le \sigma < 0.20$ the operational amplifier will not saturate.

[b] When $\sigma = 0.12, \qquad v_o = \frac{-(30+20.4)}{1.92}(0.30) = -7.875\ \text{V}$

Also $\frac{v_o}{180} + \frac{v_o}{50.4} + i_o = 0$

$$\therefore \quad i_o = -\frac{v_o}{180} - \frac{v_o}{50.4} = \frac{7.875}{180} + \frac{7.875}{50.4}\,\text{mA} = 200\,\mu\text{A}$$

P 4.8 **[a]** Let v_Δ be the voltage from the potentiometer contact to ground. Then

$$\frac{0-v_g}{5} + \frac{0-v_\Delta}{15} = 0$$

$$-3v_g - v_\Delta = 0, \qquad \therefore \quad v_\Delta = -150\,\text{mV}$$

$$\frac{v_\Delta}{\alpha R_\Delta} + \frac{v_\Delta - 0}{15{,}000} + \frac{v_\Delta - v_o}{(1-\alpha)R_\Delta} = 0$$

$$\frac{v_\Delta}{\alpha} + 10v_\Delta + \frac{v_\Delta - v_o}{1-\alpha} = 0$$

$$v_\Delta\left(\frac{1}{\alpha} + 10 + \frac{1}{1-\alpha}\right) = \frac{v_o}{1-\alpha}$$

$$\therefore \quad v_o = -0.15\left[1 + 10(1-\alpha) + \frac{(1-\alpha)}{\alpha}\right]$$

When $\alpha = 0.3, \qquad v_o = -0.15(1+7+7/3) = -1.55\ \text{V}$

When $\alpha = 0.75, \qquad v_o = -0.15(1+2.5+1/3) = -0.575\ \text{V}$

$$\therefore \quad -1.55\ \text{V} \le v_o \le -0.575\ \text{V}$$

[b] $-0.15\left[1 + 10(1-\alpha) + \frac{(1-\alpha)}{\alpha}\right] = -6$

$$\alpha + 10\alpha(1-\alpha) + (1-\alpha) = 40\alpha$$

$$\alpha + 10\alpha - 10\alpha^2 + 1 - \alpha = 40\alpha$$

$$\therefore \quad 10\alpha^2 + 30\alpha - 1 = 0 \qquad \text{so} \qquad \alpha \cong 0.033$$

P 4.9 **[a]** $\frac{v_\text{d} - v_\text{a}}{72} + \frac{v_\text{d} - v_\text{b}}{120} + \frac{v_\text{d} - v_\text{c}}{450} + \frac{v_\text{d}}{600} + \frac{v_\text{d} - v_o}{180} = 0$

$$v_o = 180\left(-\frac{10}{72{,}000} + \frac{2}{120{,}000} + \frac{23}{450{,}000} + \frac{8}{600{,}000} + \frac{8}{180{,}000}\right) = -2.4\ \text{V}$$

[b] $v_o = -8.4 - 0.4v_c$

$$-8.4 - 0.4v_c = -16; \qquad v_c = 19 \text{ V}$$

$$-8.4 - 0.4v_c = 16; \qquad v_c = -61 \text{ V}$$

$$-61 \text{ V} \le v_c \le 19 \text{ V}$$

P 4.10 **[a]** $\dfrac{v_d - v_a}{72{,}000} + \dfrac{v_d - v_b}{120{,}000} + \dfrac{v_d - v_c}{450{,}000} + \dfrac{v_d}{600{,}000} + \dfrac{v_d - v_o}{R_f} = 0$

$$(25/3)v_d - (25/3)v_a + 5v_d - 5v_b + (4/3)v_d - (4/3)v_c + v_d + \frac{600}{R_f}v_d = \frac{600}{R_f}v_o$$

$$(47/3)v_d + \frac{600}{R_f}v_d - (25/3)v_a - 5v_b - (4/3)v_c = \frac{600}{R_f}v_o$$

$$(376/3) + \frac{4800}{R_f} - 150 - 30 + 20 = \frac{600}{R_f}v_o$$

$$14400 - 104R_f = 1800v_o \quad \text{or} \quad 104R_f = 14400 - 1800v_o$$

$$v_o = \pm 16 \text{ V}, \qquad \text{but} \quad R_f > 0$$

$$\therefore \quad 104R_f = 14400 - 1800(-16) \quad \text{or} \quad R_f = 415.38\,\text{k}\Omega$$

[b] $i_f = \dfrac{8 - (-16)}{415.38 \times 10^3} = 57.78\,\mu\text{A}$

$$i_{27\,\text{k}\Omega} = \frac{v_o}{0.027 \times 10^6} = -592.59\,\mu\text{A}$$

$$i_o - i_f + i_{27\,\text{k}\Omega} = 0$$

$$i_o = 57.78 - (-592.59) = 650.37\,\mu\text{A}$$

P 4.11 **[a]** $v_o = -\dfrac{220}{33}v_a - \dfrac{220}{22}v_b - \dfrac{220}{80}v_c = -8 + 15 - 11 = -4 \text{ V}$

[b] $v_o = -19 - 10v_b = \pm 6$

$$\therefore \quad v_b = -1.3 \text{ V} \quad \text{when} \quad v_o = -6 \text{ V};$$

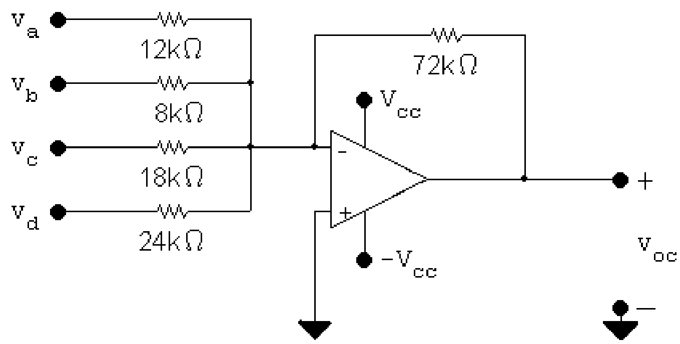
$$v_b = -2.5 \text{ V} \quad \text{when} \quad v_o = 6 \text{ V}$$

$$\therefore \quad -2.5 \text{ V} \le v_b \le -1.3 \text{ V}$$

P 4.12 $v_o = -\left[\dfrac{R_f}{3000}(0.15) + \dfrac{R_f}{5000}(0.1) + \dfrac{R_f}{25{,}000}(0.25)\right]$

$$-6 = -8 \times 10^{-5}R_f; \qquad R_f = 75\,\text{k}\Omega; \qquad \therefore \quad 0 \le R_f \le 75\,\text{k}\Omega$$

P 4.13



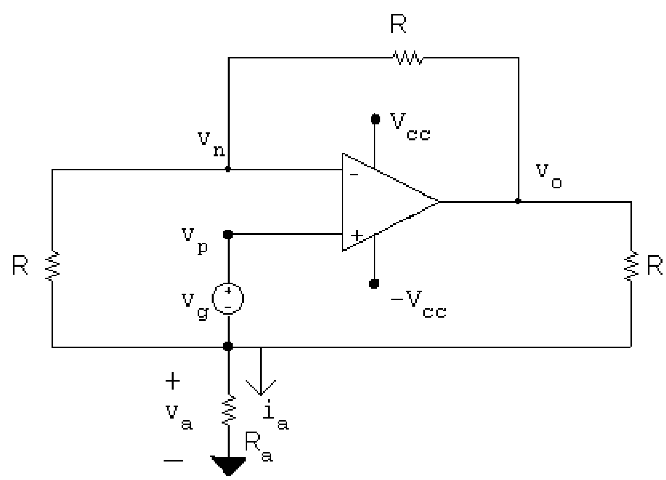
$$v_o = -(6v_a + 9v_b + 4v_c + 3v_d)$$

$$v_o = -\left[\frac{72}{R_\mathrm{a}}v_\mathrm{a} + \frac{72}{R_\mathrm{b}}v_\mathrm{b} + \frac{72}{R_\mathrm{c}}v_\mathrm{c} + \frac{72}{R_\mathrm{d}}v_\mathrm{d}\right]$$

$$\therefore \quad R_\mathrm{a} = 72{,}000/6 = 12\,\mathrm{k}\Omega \qquad R_\mathrm{c} = 72{,}000/4 = 18\,\mathrm{k}\Omega$$

$$R_\mathrm{b} = 72{,}000/9 = 8\,\mathrm{k}\Omega \qquad R_\mathrm{d} = 72{,}000/3 = 24\,\mathrm{k}\Omega$$

P 4.14 [a]



$$\frac{v_n - v_\mathrm{a}}{R} + \frac{v_n - v_o}{R} = 0$$

$$2v_n - v_\mathrm{a} = v_o$$

$$\frac{v_\mathrm{a}}{R_\mathrm{a}} + \frac{v_\mathrm{a} - v_n}{R} + \frac{v_\mathrm{a} - v_o}{R} = 0$$

$$v_\mathrm{a}\left[\frac{1}{R_\mathrm{a}} + \frac{2}{R}\right] - \frac{v_n}{R} = \frac{v_o}{R}$$

$$v_\mathrm{a}\left(2 + \frac{R}{R_\mathrm{a}}\right) - v_n = v_o$$

$$v_n = v_p = v_\mathrm{a} + v_g$$

$$\therefore \quad 2v_n - v_\text{a} = 2v_\text{a} + 2v_g - v_\text{a} = v_\text{a} + 2v_g$$

$$\therefore \quad v_\text{a} - v_o = -2v_g \qquad (1)$$

$$2v_\text{a} + v_\text{a}\left(\frac{R}{R_\text{a}}\right) - v_\text{a} - v_g = v_o$$

$$\therefore \quad v_\text{a}\left(1 + \frac{R}{R_\text{a}}\right) - v_o = v_g \qquad (2)$$

Now combining equations (1) and (2) yields

$$-v_\text{a}\frac{R}{R_\text{a}} = -3v_g$$

$$\text{or} \quad v_\text{a} = 3v_g\frac{R_\text{a}}{R}$$

$$\text{Hence} \quad i_\text{a} = \frac{v_\text{a}}{R_\text{a}} = \frac{3v_g}{R} \qquad \text{Q.E.D.}$$

[b] At saturation $\quad \mathrm{V}_o = \pm\, \mathrm{V}_\text{cc}$

$$\therefore \quad v_\text{a} = \pm\, \mathrm{V}_\text{cc} - 2v_g \qquad (3)$$

and

$$\therefore \quad v_\text{a}\left(1 + \frac{R}{R_\text{a}}\right) = \pm\, \mathrm{V}_\text{cc} + v_g \qquad (4)$$

Dividing Eq (4) by Eq (3) gives

$$1 + \frac{R}{R_\text{a}} = \frac{\pm\, \mathrm{V}_\text{cc} + v_g}{\pm\, \mathrm{V}_\text{cc} - 2v_g}$$

$$\therefore \quad \frac{R}{R_\text{a}} = \frac{\pm\, \mathrm{V}_\text{cc} + v_g}{\pm\, \mathrm{V}_\text{cc} - 2v_g} - 1 = \frac{3v_g}{\pm\, \mathrm{V}_\text{cc} - 2v_g}$$

$$\text{or} \quad R_\text{a} = \frac{(\pm\, \mathrm{V}_\text{cc} - 2v_g)}{3v_g}R \qquad \text{Q.E.D.}$$

P 4.15 **[a]** Assume the op-amp is operating within its linear range, then

$$i_L = \frac{3}{1.5} = 2\,\text{mA}$$

For $R_L = 2.5\,\text{k}\Omega \qquad v_o = (2.5 + 1.5)(2) = 8\ \text{V}$

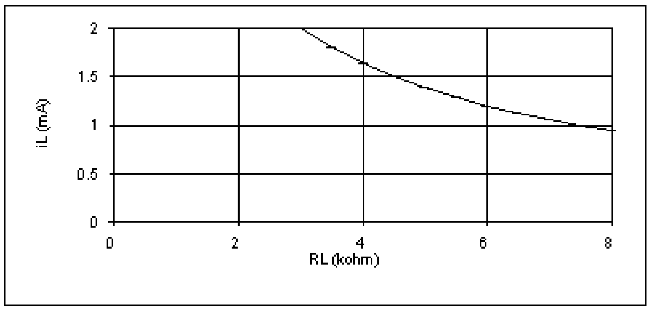
Now since $v_o < 9\ \text{V}$ our assumption of linear operation is correct, therefore

$$i_L = 2\,\text{mA}$$

[b] $9 = 2(1.5 + R_L); \qquad R_L = 3\,\text{k}\Omega$

[c] As long as the op-amp is operating in its linear region i_L is independent of R_L. From (b) we found the op-amp is operating in its linear region as long as $R_L \leq 3\,\text{k}\Omega$. Therefore when $R_L = 6.5\,\text{k}\Omega$ the op-amp is saturated. We can estimate the value of i_L by assuming $i_p = i_n \ll i_L$. Then $i_L = 9/(1.5+6.5) = 1.125\,\text{mA}$. To justify neglecting the current into the op-amp assume the drop across the 47 kΩ resistor is negligible, and the input resistance to the op-amp is at least $500\,\text{k}\Omega$. Then $i_p = i_n = (3-1.5)/500\times 10^{-3} = 3\,\mu\text{A}$. But $3\,\mu\text{A} \ll 1.125\,\text{mA}$, hence our assumption is reasonable.

[d]



P 4.16 [a] The output voltage of the first op-amp is $v_{o1} = -(80/20)v_g = -4v_g$
The output voltage of the second op-amp is $v_{o2} = -1.6v_{o1} = 6.4v_g$
When v_g has its largest value, i.e., 1.2 V,

$$v_{o1} = -4.8 \text{ V} \quad \text{and} \quad v_{o2} = 7.68 \text{ V}$$

Therefore neither op-amp saturates. The expression for i_g is

$$i_g = \frac{v_g}{20{,}000} + \frac{v_g - 6.4v_g}{R_o} = v_g\left[\frac{1}{20{,}000} - \frac{5.4}{R_o}\right]$$

$$i_g = 0 \quad \text{when} \quad \left(\frac{1}{20{,}000} - \frac{5.4}{R_o}\right) = 0, \quad \text{or} \quad R_o = 108\,\text{k}\Omega$$

[b] $$i_{R_o} = \frac{6.4v_g - v_g}{R_o} = \frac{5.4v_g}{R_o} = 50v_g\,\mu\text{A} = 50\,\mu\text{A}$$

$$p_{R_o} = (50\times 10^{-6})^2(108\times 10^3) = 270\,\mu\text{W}$$

P 4.17 Let v_{o1} be the output voltage of the first operational amplifier and v_{o2} the output voltage of the second operational amplifier. Then

$$\frac{0-1}{12{,}000} + \frac{0-v_{o1}}{48{,}000} + \frac{0-v_{o2}}{100{,}000} = 0$$

$$-50 - 12.5v_{o1} - 6v_{o2} = 0$$

$$\frac{v_{o1}}{30{,}000} + \frac{v_{o1} - v_{o2}}{6000} = 0$$

$$\therefore \quad 6v_{o1} = 5v_{o2}$$

$$\therefore \quad -50 - 12.5[(5/6)v_{o2}] - 6v_{o2} = 0 \quad \text{so} \quad v_{o2} = -3.05 \text{ V}$$

$$i_\text{a} = \frac{v_{o2}}{36{,}000} = -0.0846\,\text{mA}$$

$$i_\text{a} = -84.6\,\mu\text{A}$$

P 4.18 **[a]** Let v_{o1} = output voltage of the amplifier on the left. Let v_{o2} = output voltage of the amplifier on the right. Then

$$v_{o1} = \frac{-47}{10}(1) = -4.7 \text{ V}; \qquad v_{o2} = \frac{-220}{33}(-0.15) = 1.0 \text{ V}$$

$$i_\text{a} = \frac{v_{o2} - v_{o1}}{1000} = 5.7\,\text{mA}$$

[b] $i_\text{a} = 0$ when $v_{o1} = v_{02}$ so from (a) $v_{o2} = 1$ V

Thus

$$\frac{-47}{10}(v_\text{L}) = 1$$

$$v_\text{L} = -\frac{10}{47} = -212.77 \text{ mV}$$

P 4.19 **[a]** $p_{16\,\text{k}\Omega} = \dfrac{(320 \times 10^{-3})^2}{(16 \times 10^3)} = 6.4\,\mu\text{W}$

[b] $v_{16\,\text{k}\Omega} = \left(\dfrac{16}{64}\right)(320) = 80\,\text{mV}$

$$p_{16\,\text{k}\Omega} = \frac{(80 \times 10^{-3})^2}{(16 \times 10^3)} = 0.4\,\mu\text{W}$$

[c] $\dfrac{p_\text{a}}{p_\text{b}} = \dfrac{6.4}{0.4} = 16$

[d] Yes, the operational amplifier serves several useful purposes:

- First, it enables the source to control 16 times as much power delivered to the load resistor. When a small amount of power controls a larger amount of power, we refer to it as *power amplification*.
- Second, it allows the full source voltage to appear across the load resistor, no matter what the source resistance. This is the *voltage follower* function of the operational amplifier.

- Third, it allows the load resistor voltage (and thus its current) to be set without drawing any current from the input voltage source. This is the *current amplification* function of the circuit.

P 4.20 **[a]** $v_p = v_s, \qquad v_n = \dfrac{R_1 v_o}{R_1 + R_2}, \qquad v_n = v_p$

$$\text{Therefore} \quad v_o = \left(\frac{R_1 + R_2}{R_1}\right) v_s = \left(1 + \frac{R_2}{R_1}\right) v_s$$

[b] $v_o = v_s$

[c] Because $v_o = v_s$, thus the output voltage follows the signal voltage.

P 4.21 **[a]** $v_p = v_n = \dfrac{45}{75} v_g = 0.6 v_g$

$$\therefore \quad \frac{0.6v_g}{15} + \frac{0.6v_g - v_o}{48} = 0$$

$$\therefore \quad v_o = 2.52 v_g = 2.52(3), \quad v_o = 7.56 \text{ V}$$

[b] $v_o = 2.52 v_g = \pm 10$

$$v_g = \pm 3.97 \text{ V}, \qquad -3.97 \le v_g \le 3.97 \text{ V}$$

[c] $\dfrac{0.6v_g}{15} + \dfrac{0.6v_g - v_o}{R_\text{f}} = 0$

$$\left(\frac{0.6R_\text{f}}{15} + 0.6\right) v_g = v_o = \pm 10$$

$$\therefore \quad 3R_\text{f} + 45 = \pm 150; \qquad 3R_\text{f} = 150 - 45; \qquad R_\text{f} = 35\,\text{k}\Omega$$

P 4.22 **[a]** $\dfrac{v_p - v_\text{a}}{R_\text{a}} + \dfrac{v_p - v_\text{b}}{R_\text{b}} + \dfrac{v_p - v_\text{c}}{R_\text{c}} + \dfrac{v_p}{R_g} = 0$

$$\therefore \quad v_p = \frac{R_\text{b}R_\text{c}R_g}{D} v_\text{a} + \frac{R_\text{a}R_\text{c}R_g}{D} v_\text{b} + \frac{R_\text{a}R_\text{b}R_g}{D} v_\text{c}$$

$$\text{where} \quad D = R_\text{b}R_\text{c}R_g + R_\text{a}R_\text{c}R_g + R_\text{a}R_\text{b}R_g + R_\text{a}R_\text{b}R_\text{c}$$

$$\frac{v_n}{R_s} + \frac{v_n - v_o}{R_\text{f}} = 0$$

$$v_n \left(\frac{1}{R_s} + \frac{1}{R_\text{f}}\right) = \frac{v_o}{R_\text{f}}$$

$$\therefore \quad v_o = \left(1 + \frac{R_\text{f}}{R_s}\right) v_n = k v_n$$

$$\text{where} \quad k = \left(1 + \frac{R_\text{f}}{R_s}\right)$$

$$v_p = v_n$$

$$\therefore \quad v_o = kv_p$$

or

$$v_o = \frac{kR_gR_\mathrm{b}R_\mathrm{c}}{D}v_\mathrm{a} + \frac{kR_gR_\mathrm{a}R_\mathrm{c}}{D}v_\mathrm{b} + \frac{kR_gR_\mathrm{a}R_\mathrm{b}}{D}v_\mathrm{c}$$

$$\frac{kR_gR_\mathrm{b}R_\mathrm{c}}{D} = 3 \qquad \therefore \quad \frac{R_\mathrm{b}}{R_\mathrm{a}} = 1.5$$

$$\frac{kR_gR_\mathrm{a}R_\mathrm{c}}{D} = 2 \qquad \therefore \quad \frac{R_\mathrm{c}}{R_\mathrm{b}} = 2$$

$$\frac{kR_gR_\mathrm{a}R_\mathrm{b}}{D} = 1 \qquad \therefore \quad \frac{R_\mathrm{c}}{R_\mathrm{a}} = 3$$

Since $\quad R_\mathrm{a} = 2\,\mathrm{k}\Omega \qquad R_\mathrm{b} = 3\,\mathrm{k}\Omega \qquad R_\mathrm{c} = 6\,\mathrm{k}\Omega$

$$\therefore \quad D = [(3)(6)(4) + (2)(6)(4) + (2)(3)(4) + (2)(3)(6)] \times 10^9 = 180 \times 10^9$$

$$\frac{k(4)(3)(6) \times 10^9}{180 \times 10^9} = 3$$

$$k = \frac{540 \times 10^9}{72 \times 10^9} = 7.5$$

$$\therefore \quad 7.5 = 1 + \frac{R_\mathrm{f}}{R_s}$$

$$\frac{R_\mathrm{f}}{R_s} = 6.5$$

$$R_\mathrm{f} = (6.5)(12{,}000) = 78\,\mathrm{k}\Omega$$

[b] $v_o = 3(0.8) + 2(1.5) + 2.10 = 7.5\ \mathrm{V}$

$$v_n = v_p = \frac{7.5}{7.5} = 1.0\ \mathrm{V}$$

$$i_\mathrm{a} = \frac{0.8 - 1}{2000} = \frac{-0.2}{2000} = -0.1\,\mathrm{mA} = -100\,\mu\mathrm{A}$$

$$i_\mathrm{b} = \frac{1.5 - 1.0}{3000} = \frac{0.5}{3000} = 166.67\,\mu\mathrm{A}$$

$$i_\mathrm{c} = \frac{2.10 - 1.0}{6000} = \frac{1.1}{6000} = 183.33\,\mu\mathrm{A}$$

$$i_g = \frac{1}{4000} = 250\,\mu\mathrm{A}$$

$$i_s = \frac{v_n}{12{,}000} = \frac{1}{12{,}000} = 83.33\,\mu\mathrm{A}$$

P 4.23 **[a]** $\frac{v_p - v_\text{a}}{80 \times 10^3} + \frac{v_p - v_\text{b}}{64 \times 10^3} = 0$

$\therefore \quad 9v_p = 4v_\text{a} + 5v_\text{b}$

$\frac{v_n}{18{,}000} + \frac{v_n - v_o}{72{,}000} = 0$

$\therefore \quad v_o = 5v_n = 5v_p = (20/9)v_\text{a} + (25/9)v_\text{b} = 4.44 \text{ V}$

[b] $v_p = v_n = \frac{v_o}{5} = 0.889 \text{ V}$

$i_\text{a} = \frac{v_\text{a} - v_p}{80 \times 10^3} = -4.86\,\mu\text{A}$

$i_\text{b} = \frac{v_\text{b} - v_p}{64 \times 10^3} = 4.86\,\mu\text{A}$

[c] (20/9) for v_a

(25/9) for v_b

P 4.24 **[a]** $\frac{v_p - v_\text{a}}{R_\text{a}} + \frac{v_p - v_\text{b}}{R_\text{b}} + \frac{v_p - v_\text{c}}{R_\text{c}} = 0$

$\therefore \quad v_p = \frac{R_\text{b}R_\text{c}}{D}v_\text{a} + \frac{R_\text{a}R_\text{c}}{D}v_\text{b} + \frac{R_\text{a}R_\text{b}}{D}v_\text{c}$

where $\quad D = R_\text{b}R_\text{c} + R_\text{a}R_\text{c} + R_\text{a}R_\text{b}$

$\frac{v_n}{10{,}000} + \frac{v_n - v_o}{R_\text{f}} = 0$

$\left(\frac{R_\text{f}}{10{,}000} + 1\right) v_n = v_o$

Let $\quad \frac{R_\text{f}}{10{,}000} + 1 = k$

$v_o = kv_n = kv_p$

$\therefore \quad v_o = \frac{kR_\text{b}R_\text{c}}{D}v_\text{a} + \frac{kR_\text{a}R_\text{c}}{D}v_\text{b} + \frac{kR_\text{a}R_\text{b}}{D}v_\text{c}$

$\therefore \quad \frac{kR_\text{b}R_\text{c}}{D} = 5 \qquad \therefore \quad \frac{R_\text{c}}{R_\text{a}} = 5$

$\frac{kR_\text{a}R_\text{c}}{D} = 4$

$\frac{kR_\text{a}R_\text{b}}{D} = 1 \qquad \therefore \quad \frac{R_\text{c}}{R_\text{b}} = 4$

$$\therefore \quad R_{\mathrm{c}} = 5R_{\mathrm{a}} = 5\,\mathrm{k}\Omega$$

$$R_{\mathrm{b}} = R_{\mathrm{c}}/4 = 1.25\,\mathrm{k}\Omega$$

$$\therefore \quad D = (1.25)(5) + (1)(5) + (1.25)(1) = 12.5 \times 10^{6}$$

$$\therefore \quad k = \frac{5D}{R_{\mathrm{b}}R_{\mathrm{c}}} = \frac{(5)(12.5) \times 10^{6}}{(1.25)(5) \times 10^{6}} = 10$$

$$\therefore \quad \frac{R_{\mathrm{f}}}{10{,}000} + 1 = 10, \qquad R_{\mathrm{f}} = 90\,\mathrm{k}\Omega$$

[b] $v_o = 5(0.5) + 4(1) + 1.5 = 8\ \mathrm{V}$

$$v_n = v_o/10 = 0.8\ \mathrm{V} = v_p$$

$$i_{\mathrm{a}} = \frac{v_{\mathrm{a}} - v_p}{1000} = \frac{0.5 - 0.8}{1000} = -300\,\mu\mathrm{A}$$

$$i_{\mathrm{b}} = \frac{v_{\mathrm{b}} - v_p}{1250} = \frac{1 - 0.8}{1250} = 160\,\mu\mathrm{A}$$

$$i_{\mathrm{c}} = \frac{v_{\mathrm{c}} - v_p}{5000} = \frac{1.5 - 0.8}{5000} = 140\,\mu\mathrm{A}$$

P 4.25 **[a]** $v_o = \dfrac{R_{\mathrm{d}}(R_{\mathrm{a}} + R_{\mathrm{b}})}{R_{\mathrm{a}}(R_{\mathrm{c}} + R_{\mathrm{d}})}v_{\mathrm{b}} - \dfrac{R_{\mathrm{b}}}{R_{\mathrm{a}}}v_{\mathrm{a}} = \dfrac{33(100)}{20(80)}(0.90) - 4(0.45)$

$$v_o = 1.8563 - 1.8 = 56.25\ \mathrm{mV}$$

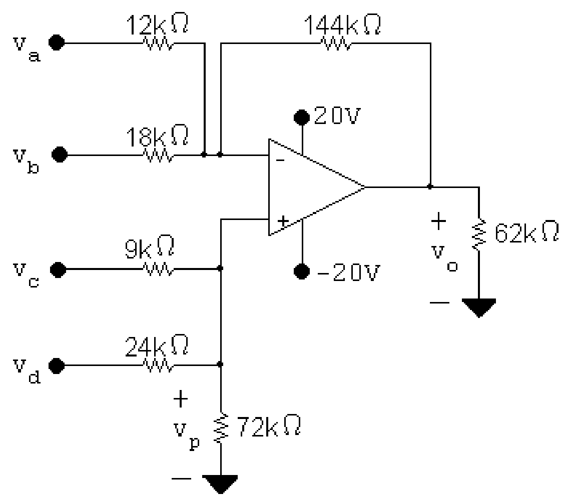
[b] $v_n = v_p = \dfrac{(0.90)(33)}{80} = 371.25\,\mathrm{mV}$

$$i_{\mathrm{a}} = \frac{(450 - 371.25)10^{-3}}{20 \times 10^{3}} = 3.9375\,\mu\mathrm{A}$$

$$R_{\mathrm{a}} = \frac{v_{\mathrm{a}}}{i_{\mathrm{a}}} = \frac{450 \times 10^{-3}}{3.9375 \times 10^{-6}} = 114.3\,\mathrm{k}\Omega$$

[c] $R_{\mathrm{in\,b}} = R_{\mathrm{c}} + R_{\mathrm{d}} = 80\,\mathrm{k}\Omega$

P 4.26 **[a]**



$$\frac{v_p}{72{,}000} + \frac{v_p - v_c}{9{,}000} + \frac{v_p - v_d}{24{,}000} = 0$$

$$\therefore \quad v_p = (2/3)v_c + 0.25v_d = v_n$$

$$\frac{v_n - v_a}{12{,}000} + \frac{v_n - v_b}{18{,}000} + \frac{v_n - v_o}{144{,}000} = 0$$

$$\begin{aligned} \therefore \quad v_o &= 21v_n - 12v_a - 8v_b \\ &= 21[(2/3)v_c + 0.25v_d] - 12v_a - 8v_b \\ &= 21(0.4 + 0.2) - 12(0.5) - 8(0.3) = 4.2 \text{ V} \end{aligned}$$

[b] $v_o = 14v_c + 4.2 - 6 - 2.4$

$$\pm 15 = 14v_c - 4.2$$

$$\therefore \quad 14v_c = \pm 15 + 4.2$$

$$\therefore \quad v_c = 1.371 \text{ V} \qquad \text{and} \qquad v_c = -0.771 \text{ V}$$

$$\therefore \quad -771 \le v_c \le 1371 \,\text{mV}$$

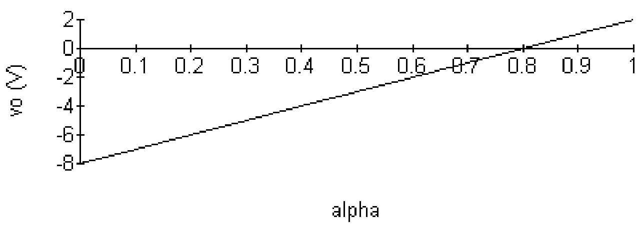
P 4.27 **[a]** $v_n = v_p = \alpha v_g$

$$\frac{v_n - v_g}{R_1} + \frac{v_n - v_o}{R_f} = 0$$

$$(v_n - v_g)\frac{R_f}{R_1} + v_n - v_o = 0$$

$$\begin{aligned} v_o &= (\alpha v_g - v_g)4 + \alpha v_g \\ &= [(\alpha - 1)4 + \alpha]v_g \\ &= (5\alpha - 4)v_g \\ &= (5\alpha - 4)(2) = 10\alpha - 8 \end{aligned}$$

α	v_o	α	v_o	α	v_o
0.0	-8 V	0.4	-4 V	0.8	0 V
0.1	-7 V	0.5	-3 V	0.9	1 V
0.2	-6 V	0.6	-2 V	1.0	2 V
0.3	-5 V	0.7	-1 V		



[b] Rearranging the equation for v_o from (a) gives

$$v_o = \left(\frac{R_f}{R_1} + 1\right) v_g \alpha - \left(\frac{R_f}{R_1}\right) v_g$$

Therefore,

$$\text{slope } = \left(\frac{R_f}{R_1} + 1\right) v_g; \qquad \text{intercept } = -\left(\frac{R_f}{R_1}\right) v_g$$

[c] Using the equations from (b),

$$-6 = \left(\frac{R_f}{R_1} + 1\right) v_g; \qquad 4 = -\left(\frac{R_f}{R_1}\right) v_g$$

Solving,

$$v_g = -2 \text{ V}; \qquad \frac{R_f}{R_1} = 2$$

P 4.28 $v_p = v_n = R_\text{b} i_\text{b}$

$$\frac{R_\text{b} i_\text{b} - 3000 i_\text{a}}{3000} + \frac{R_\text{b} i_\text{b} - v_o}{R_\text{f}} = 0$$

$$\left(\frac{R_\text{b}}{3000} + \frac{R_\text{b}}{R_\text{f}}\right) i_\text{b} - i_\text{a} = \frac{v_o}{R_\text{f}}$$

$$v_o = \left[\frac{R_\text{b} R_\text{f}}{3000} + R_\text{b}\right] i_\text{b} - R_\text{f} i_\text{a}$$

$$\therefore \quad R_\text{f} = 2000\,\Omega$$

$$(2/3)R_\text{b} + R_\text{b} = 2000$$

$$\therefore \quad R_\text{b} = 1200\,\Omega$$

P 4.29 $$v_p = \frac{v_\text{b} R_\text{b}}{R_\text{a} + R_\text{b}} = v_n$$

$$\frac{v_n - v_\text{a}}{4700} + \frac{v_n - v_o}{R_\text{f}} = 0$$

$$v_n\left(\frac{R_\text{f}}{4700} + 1\right) - \frac{v_\text{a} R_\text{f}}{4700} = v_o$$

$$\therefore \quad \left(\frac{R_\text{f}}{4700} + 1\right)\frac{R_\text{b}}{R_\text{a} + R_\text{b}} v_\text{b} - \frac{R_\text{f}}{4700} v_\text{a} = v_o$$

$$\therefore \quad \frac{R_\text{f}}{4700} = 10; \qquad R_\text{f} = 47\,\text{k}\Omega$$

$$\therefore \quad \frac{R_\text{f}}{4700} + 1 = 11$$

$$\therefore \quad 11\left(\frac{R_\text{b}}{R_\text{a} + R_\text{b}}\right) = 10$$

$$11R_\text{b} = 10R_\text{b} + 10R_\text{a} \qquad R_\text{b} = 10R_\text{a}$$

$$R_\text{a} + R_\text{b} = 220\,\text{k}\Omega$$

$$11R_\text{a} = 220\,\text{k}\Omega$$

$$R_\text{a} = 20\,\text{k}\Omega$$

$$R_\text{b} = 220 - 20 = 200\,\text{k}\Omega$$

P 4.30 $$v_o = \frac{R_\text{d}(R_\text{a} + R_\text{b})}{R_\text{a}(R_\text{c} + R_\text{d})} v_\text{b} - \frac{R_\text{b}}{R_\text{a}} v_\text{a}$$

By hypothesis: $R_\text{b}/R_\text{a} = 5$; $R_\text{c} + R_\text{d} = 600\,\text{k}\Omega$; $\dfrac{R_\text{d}(R_\text{a} + R_\text{b})}{R_\text{a}(R_\text{c} + R_\text{d})} = 2$

$$\therefore \quad \frac{R_\text{d}}{R_\text{a}}\frac{(R_\text{a} + 5R_\text{a})}{600{,}000} = 2 \quad \text{so} \quad R_\text{d} = 200\,\text{k}\Omega; \quad R_\text{c} = 400\,\text{k}\Omega$$

Also, when $v_o = 0$ we have

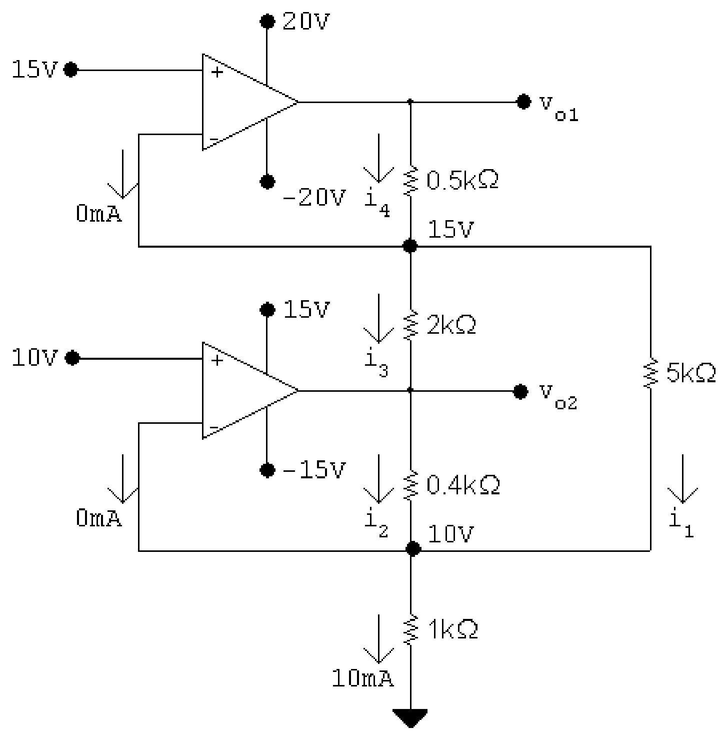
$$\frac{v_n - v_\mathrm{a}}{R_\mathrm{a}} + \frac{v_n}{R_\mathrm{b}} = 0$$

$$\therefore \quad v_n\left(1 + \frac{R_\mathrm{a}}{R_\mathrm{b}}\right) = v_\mathrm{a}; \qquad v_n = (5/6)v_\mathrm{a}$$

$$i_\mathrm{a} = \frac{v_\mathrm{a} - (5/6)v_\mathrm{a}}{R_\mathrm{a}} = \frac{1}{6}\frac{v_\mathrm{a}}{R_\mathrm{a}}; \qquad R_\mathrm{in} = \frac{v_\mathrm{a}}{i_\mathrm{a}} = 6R_\mathrm{a} = 18\,\mathrm{k\Omega}$$

$$\therefore \quad R_\mathrm{a} = 3\,\mathrm{k\Omega}; \qquad R_\mathrm{b} = 15\,\mathrm{k\Omega}$$

P 4.31



$$i_1 = \frac{15 - 10}{5000} = 1\,\mathrm{mA}$$

$$i_2 + i_1 + 0 = 10\,\mathrm{mA}; \qquad i_2 = 9\,\mathrm{mA}$$

$$v_{o2} = 10 + (400)(9) \times 10^{-3} = 13.6 \;\; \mathrm{V}$$

$$i_3 = \frac{15 - 13.6}{2000} = 0.7\,\mathrm{mA}$$

$$i_4 = i_3 + i_1 = 1.7\,\mathrm{mA}$$

$$v_{o1} = 15 + 1.7(0.5) = 15.85 \;\; \mathrm{V}$$

P 4.32 Let v_{o1} be the output voltage of the first op-amp. Then

$$\frac{0-1.1}{3000}+\frac{0-v_{o1}}{18{,}000}+\frac{0-v_o}{24{,}000}=0$$

$$-26.4-4v_{o1}-3v_o=0$$

But $\quad v_{o1}=\frac{v_o}{30}(27)=0.9v_o$

$$\therefore\quad -3.6v_o-3v_o=26.4 \quad\text{or}\quad v_o=-4\ \text{V}$$

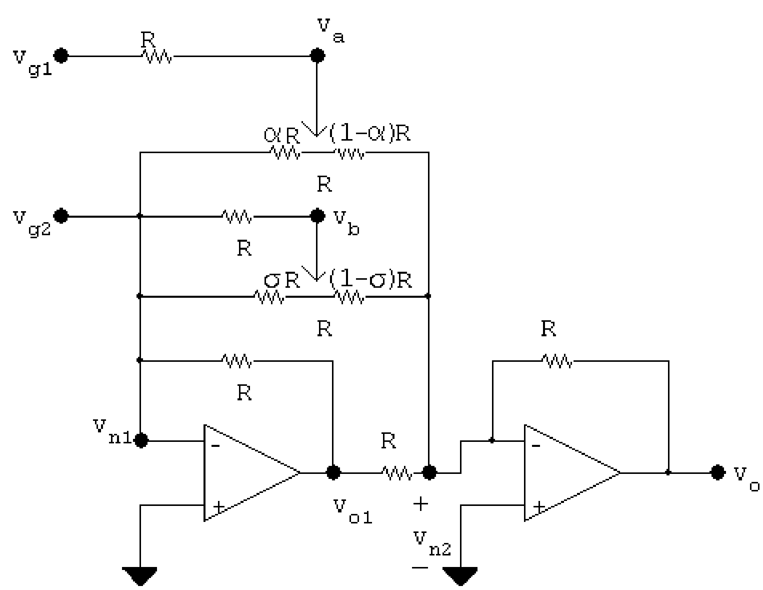
$$i_{24\,\text{k}\Omega}=\frac{0-(-4)}{24}=(1/6)\,\text{mA}$$

$$i_{3\,\text{k}\Omega}=\frac{-4}{30}=(2/15)\,\text{mA}$$

$$i_{4.5\,\text{k}\Omega}=\frac{-4}{4.5}=(8/9)\,\text{mA}$$

$$\frac{1}{6000}=i_o-\frac{2}{15{,}000}-\frac{8}{9000}; \qquad i_o=1.1889\,\text{mA}$$

P 4.33 **[a]** The circuit of Fig. P4.33 is redrawn with intermediate voltages defined to facilitate the analysis.



$$v_{n1}=v_{n2}=0$$

$$\frac{0-v_{o1}}{R}+\frac{0-v_{\text{b}}}{\sigma R}+\frac{0-v_{\text{a}}}{\alpha R}=0$$

therefore $\quad v_{o1} = -\frac{v_\mathrm{a}}{\alpha} - \frac{v_\mathrm{b}}{\sigma}$

$$\frac{0 - v_\mathrm{b}}{(1-\sigma)R} + \frac{0 - v_\mathrm{a}}{(1-\alpha)R} + \frac{0 - v_{o1}}{R} + \frac{0 - v_o}{R} = 0$$

therefore $\quad v_o = -v_{o1} - \frac{v_\mathrm{a}}{1-\alpha} - \frac{v_\mathrm{b}}{1-\sigma}$

$$v_o = \frac{v_\mathrm{a}}{\alpha} + \frac{v_\mathrm{b}}{\sigma} - \frac{v_\mathrm{a}}{1-\alpha} - \frac{v_\mathrm{b}}{1-\sigma} = \frac{v_\mathrm{a}(1-2\alpha)}{\alpha(1-\alpha)} + v_\mathrm{b}\frac{(1-2\sigma)}{\sigma(1-\sigma)}$$

$$\frac{v_\mathrm{a} - v_{g1}}{R} + \frac{v_\mathrm{a} - 0}{\alpha R} + \frac{v_\mathrm{a} - 0}{(1-\alpha)R} = 0$$

$$v_\mathrm{a} + \frac{v_\mathrm{a}}{\alpha} + \frac{v_\mathrm{a}}{1-\alpha} = v_{g1}$$

$$v_\mathrm{a}\left(\frac{\alpha(1-\alpha) + (1-\alpha) + \alpha}{\alpha(1-\alpha)}\right) = v_{g1}$$

$$v_\mathrm{a} = \frac{v_{g1}\alpha(1-\alpha)}{(\alpha - \alpha^2 + 1)}$$

By symmetry $\quad v_\mathrm{b} = \frac{v_{g2}\sigma(1-\sigma)}{\sigma - \sigma^2 + 1)}$

therefore $\quad v_o = \frac{(1-2\alpha)}{(\alpha - \alpha^2 + 1)} v_{g1} + \frac{(1-2\sigma)}{(\sigma - \sigma^2 + 1)} v_{g2}$

[b] $\alpha = \sigma = 1:$

$v_o = -v_{g1} - v_{g2} = -(v_{g1} + v_{g2});$ inverted summing amplifier

[c] $\alpha = \sigma = 0:$

$v_o = v_{g1} + v_{g2};$ noninverted summing amplifier

P 4.34 $\quad v_p = \frac{5.6}{8.0} v_g = 0.7 v_g = 7\sin(\pi/3)t$ V

$$\frac{v_n}{15{,}000} + \frac{v_n - v_o}{75{,}000} = 0$$

$6v_n = v_o; \qquad v_n = v_p$

$\therefore \quad v_o = 42\sin(\pi/3)t$ V $\qquad 0 \le t \le \infty$

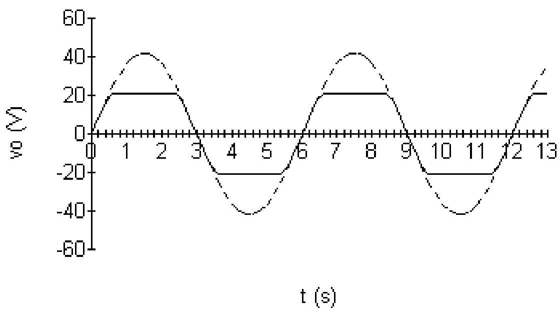
$v_o = 0 \qquad t \le 0$

At saturation

$$42\sin\left(\frac{\pi}{3}\right)t = \pm 21; \qquad \sin\frac{\pi}{3}t = \pm 0.5$$

$$\therefore \quad \frac{\pi}{3}t = \frac{\pi}{6}, \quad \frac{5\pi}{6}, \quad \frac{7\pi}{6}, \quad \frac{11\pi}{6}, \quad \text{etc.}$$

$t = 0.50\,\text{s}, \quad 2.50\,\text{s}, \quad 3.50\,\text{s}, \quad 5.50\,\text{s}, \quad \text{etc.}$



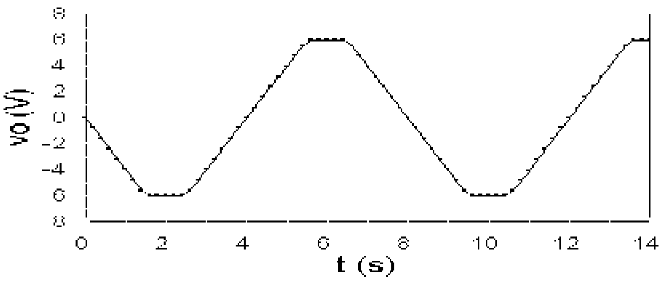
P 4.35 It follows directly from the circuit that $v_o = -16v_g$
From the plot of v_g we have $\quad v_g = 0, \quad t < 0$

$$\begin{aligned}
v_g &= (1/4)t & 0 \le t \le 2 \\
v_g &= -(1/4)t + 1 & 2 \le t \le 6 \\
v_g &= (1/4)t - 2 & 6 \le t \le 10 \\
v_g &= -(1/4)t + 3 & 10 \le t \le 14 \\
v_g &= (1/4)t - 4 & 14 \le t \le 18, \quad \text{etc.}
\end{aligned}$$

Therefore

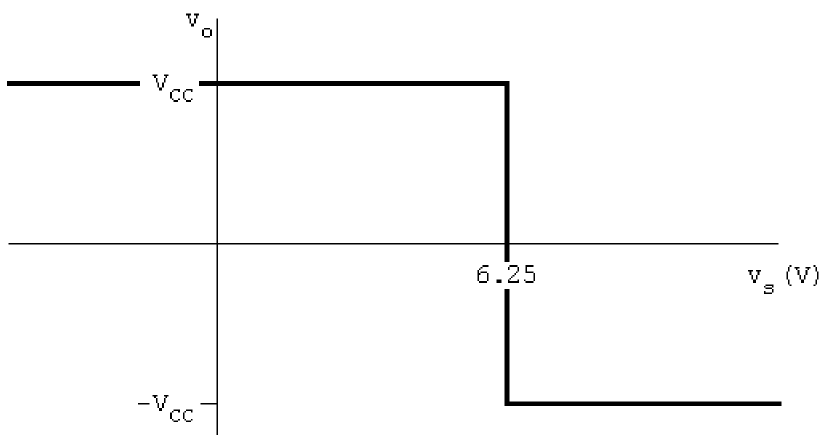
$$\begin{aligned}
v_o &= -4t & 0 \le t \le 2 \\
v_o &= 4t - 16 & 2 \le t \le 6 \\
v_o &= -4t + 32 & 6 \le t \le 10 \\
v_o &= 4t - 48 & 10 \le t \le 14 \\
v_o &= -4t + 64 & 14 \le t \le 18, \quad \text{etc.}
\end{aligned}$$

These expressions for v_o are valid as long as the op amp is not saturated. Since the peak values of v_o are ± 6, the output is clipped at ± 6. The plot is shown below.



P 4.36 **[a]** v_o will equal $V_{\rm CC}$ when $v_n < v_{\rm ref}$. Thus

$$v_s\left(\frac{40}{50}\right) < v_{\rm ref} \qquad \text{or} \qquad v_s < 6.25\ \text{V}$$

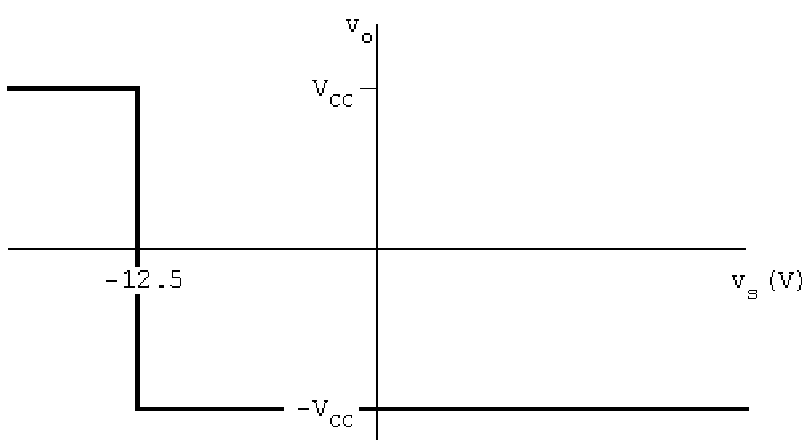


[b] v_o will equal $V_{\rm CC}$ when $v_n < v_{\rm ref}$. Hence

$v_s < -12.5$ V

v_o will equal $-V_{\rm CC}$ when $v_n > v_{\rm ref}$. Thus

$v_s < -12.5$ V



[c] Observe that in Problem 4.36 the inputs to the comparator are interchanged with thode in Example 4.2. Hence the v_o versus v_s plots in Problem 4.36 are interchanged with those in Example 4.2. For example, when $v_{\text{ref}} = 5$ V

$$
\begin{aligned}
v_o = V_{\text{CC}} \quad &\text{when} \quad v_s > 6.25 \text{ (Example 4.2)}\\
&\phantom{\text{when}} \quad v_s < 6.25 \text{ (Problem 4.36)}\\
v_o = -V_{\text{CC}} \quad &\text{when} \quad v_s < 6.25 \text{ (Example 4.2)}\\
&\phantom{\text{when}} \quad v_s > 6.25 \text{ (Problem 4.36)}
\end{aligned}
$$

when $v_{\text{ref}} = -10$ V

$$
\begin{aligned}
v_o = V_{\text{CC}} \quad &\text{when} \quad v_s > -12.5 \text{ (Example 4.2)}\\
&\phantom{\text{when}} \quad v_s < -12.5 \text{ (Problem 4.36)}\\
v_o = -V_{\text{CC}} \quad &\text{when} \quad v_s < -12.5 \text{ (Example 4.2)}\\
&\phantom{\text{when}} \quad v_s > -12.5 \text{ (Problem 4.36)}
\end{aligned}
$$

P 4.37 **[a]** The output of the comparator will be zero when $v_n = 0$. Summing the currents away from the inverting input terminal yields

$$\frac{0 - v_s}{R_1} + \frac{0 - v_{\text{ref}}}{R_2} = 0$$

Solving for v_s gives

$$v_s = -\frac{R_1}{R_2} v_{\text{ref}}$$

[b] The threshold value of v_s is

$$v_s = -\frac{10}{20}(-10) = 5 \text{ V}$$

Assume v_s is slightly less than 5 V, say

$$v_s = (5 - \epsilon) \text{ V}$$

Then

$$\frac{v_n - (5 - \epsilon)}{10} \frac{v_n + 10}{20} = 0$$

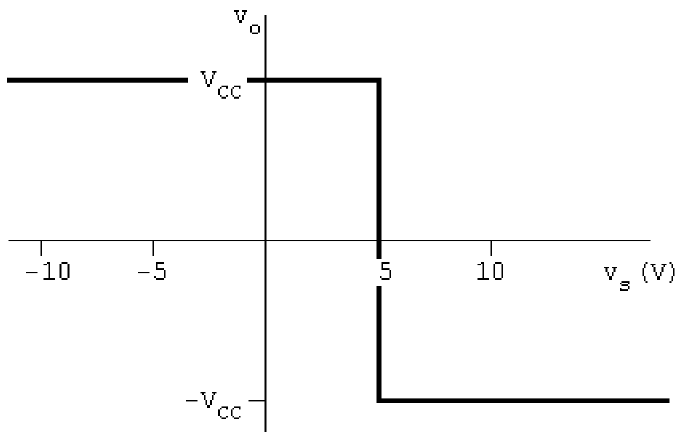
or

$$v_n = -\frac{2}{3}\epsilon$$

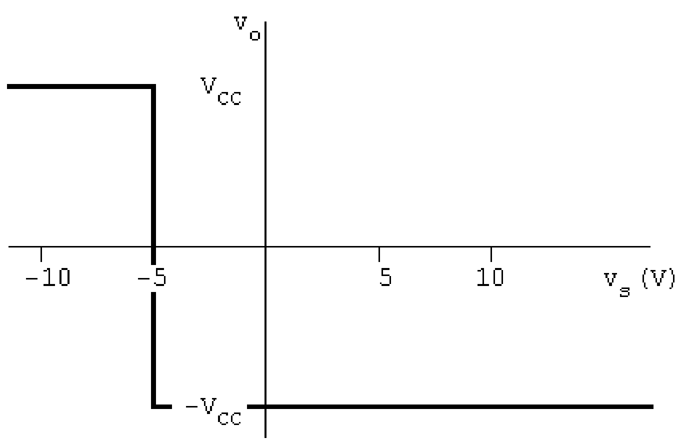
With v_n slightly negative $v_o = V_{\text{CC}}$
If $v_s = (5 + \epsilon)$ then

$$v_n = \frac{2}{3}\epsilon$$

Therefore, when v_s is slightly larger thanthe threshold value v_n goes positive and $v_o = -V_{\text{CC}}$. Thus, the v_o versus v_s sketch is



[**c**] When $v_{\text{ref}} = 10$ V, the thrshold value of v_s is -5 V and the sketch of v_o versus v_s is



P 4.38 The voltages at the inverting input terminal of the comparators, starting with the lower comparator, are: 0.875 V, 1.75 V, 2.625 V, 3.5 V, 4.375 V, 5.25 V, and 6.125 V. When $v_s = 1$ V, all comparator output voltages are low except the lowest one. Therefore, the thermometer code is 0 0 0 0 0 0 1.

When $v_s = 3$ V, the comparator output voltages of the three lowest comparators are high, hence the code is 0 0 0 0 1 1 1.

For $v_s = 5$ V the code is 0 0 1 1 1 1 1 and for $v_s = 7$ V the code is 1 1 1 1 1 1 1.

The results are summarized in the following table:

v_s (V)	Thermometer Code						
1	0	0	0	0	0	0	1
3	0	0	0	0	1	1	1
5	0	0	1	1	1	1	1
7	1	1	1	1	1	1	1

P 4.39 Since the current into the terminals of the ideal comparators is zero the current oriented down through the string of resistors is

$$i = \frac{v_{\text{ref}} - (-v_{\text{ref}})}{8R} = \frac{v_{\text{ref}}}{4R}$$

It follows that

$$v_1 = -v_{\text{ref}} + \frac{v_{\text{ref}}}{4R}(R) = -\frac{3}{4}v_{\text{ref}}$$

$$v_2 = -v_{\text{ref}} + \frac{v_{\text{ref}}}{4R}(2R) = -\frac{1}{2}v_{\text{ref}}$$

$$v_3 = -v_{\text{ref}} + \frac{v_{\text{ref}}}{4R}(3R) = -\frac{1}{4}v_{\text{ref}}$$

$$v_4 = -v_{\text{ref}} + \frac{v_{\text{ref}}}{4R}(4R) = 0$$

$$v_5 = -v_{\text{ref}} + \frac{v_{\text{ref}}}{4R}(5R) = \frac{1}{4}v_{\text{ref}}$$

$$v_6 = -v_{\text{ref}} + \frac{v_{\text{ref}}}{4R}(6R) = \frac{1}{2}v_{\text{ref}}$$

$$v_7 = -v_{\text{ref}} + \frac{v_{\text{ref}}}{4R}(7R) = \frac{3}{4}v_{\text{ref}}$$

P 4.40 From the solution to Problem 4.39 we have

$$v_1 = (-3/4)(7) = -5.25 \text{ V}$$

$$v_2 = (-1/2)(7) = -3.5 \text{ V}$$

$$v_3 = (-1/4)(7) = -1.75 \text{ V}$$

$$v_4 = 0$$

$v_5 = (1/4)(7) = 1.75$ V

$v_6 = (1/2)(7) = 3.5$ V

$v_7 = (3/4)(7) = 5.25$ V

When $v_s = -7$ V all the comparator output voltages will be low, thus the thermometer code is 0 0 0 0 0 0 0.

When $v_s = -5$ V, all except the first comparator (counting from the bottom up) output voltage will be low, thus the code is 0 0 0 0 0 0 1.

When $v_s = -3$ V, all except the first two comparator output voltages will be low, hence the code is 0 0 0 0 0 1 1.

When $v_s = -1$ V, the output voltages of the first three comparators will be high, thus the thermometer code is 0 0 0 0 1 1 1.

Then $v_s = 1$ V the output voltages of the first four comparators will be high (0 0 0 1 1 1 1); when $v_s = 3$ V the first five comparators will be high (0 0 1 1 1 1 1); when $v_s = 5$ V the first six comparators will be high (0 1 1 1 1 1 1); and when $v_s = 7$ V the output voltages of all seven comparators will be high (1 1 1 1 1 1 1). Our results are summarized in the following table:

v_s (V)	Thermometer Code						
-7	0	0	0	0	0	0	0
-5	0	0	0	0	0	0	1
-3	0	0	0	0	0	1	1
-1	0	0	0	0	1	1	1
1	0	0	0	1	1	1	1
3	0	0	1	1	1	1	1
5	0	1	1	1	1	1	1
7	1	1	1	1	1	1	1

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