Hw 2/23

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Basic Function

Genderate X

We are getting a X every time

```
getX <- function(mean, sigma)
{
  return(rmultinom(1, mean, sigma))
}
```

Calculate distribution of theta

Since theta is following Dirichlet Distribution, the marginal distribution of theta i is following a beta distribution.

```
#calcConstant <- function(alpha){
#  return(exp(sum(lgamma(alpha)) - lgamma(sum(alpha))))
#}

calMarginalAlpha <- function(X, i){
  X.dim = length(X[,1])
  alpha <- rowSums(X)
  #balpha = calcConstant(alpha)
  return(c(alpha[i], sum(alpha) - alpha[i]))
}

calcDensity <- function( alpha )
{
  x <- seq(0,1,length = 200)
  return(dbeta(x, alpha[1], alpha[2]))
}
```

Plot function

```
plotCurrent <- function( data, n )
{
  par(mfrow = c(1,3))
  thetaValue <- seq(0,1, length = 200)
  length(data[1])
  plot(x = thetaValue, y = data[,1], type='l',  main= paste("Theta 1 in run ", n ))
  plot(x = thetaValue, y = data[,2], type='l',  main= paste("Theta 2 in run ", n ))
  plot(x = thetaValue, y = data[,3], type='l',  main= paste("Theta 3 in run ", n ))
}
```

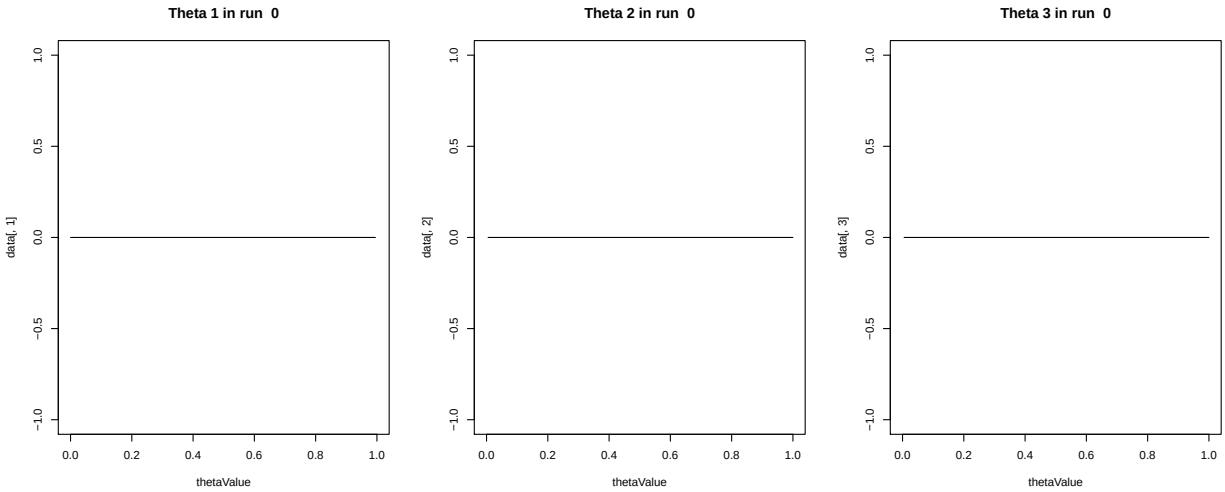
other supporting function

```
calcMean <- function( DensityData )
{
  thetaValue <- seq(0,1, length = 200)
  return(sum(DensityData * thetaValue / sum(DensityData)))
}

calcVar <- function( DensityData )
{
  mean <- calcMean(DensityData)
  thetaValue <- seq(0,1, length = 200)
  return(sum(DensityData*(thetaValue - mean)^2))
}

calcDensityAndPlot <- function(Data, i)
{
    alpha_1 <- calMarginalAlpha(Data, 1)
    alpha_2 <- calMarginalAlpha(Data, 2)
    alpha_3 <- calMarginalAlpha(Data, 3)

    DensityData_1 <- calcDensity(alpha_1)
    DensityData_2 <- calcDensity(alpha_2)
    DensityData_3 <- calcDensity(alpha_3)

    plotCurrent(cbind(DensityData_1,DensityData_2,DensityData_3), i)
}

calcDensityAndPrintMeanSn <- function(Data)
{
    alpha_1 <- calMarginalAlpha(Data, 1)
    alpha_2 <- calMarginalAlpha(Data, 2)
    alpha_3 <- calMarginalAlpha(Data, 3)

    DensityData_1 <- calcDensity(alpha_1)
    DensityData_2 <- calcDensity(alpha_2)
    DensityData_3 <- calcDensity(alpha_3)

    cat('Mean of Theta 1:')
    print(calcMean(DensityData_1))
    cat('Variance of Theta 1:')
    print(calcVar(DensityData_1)/10000)
    cat('Mean of Theta 2:')
    print(calcMean(DensityData_2))
    cat('Variance of Theta 2:')
    print(calcVar(DensityData_2)/10000)
    cat('Mean of Theta 3:')
    print(calcMean(DensityData_3))
    cat('Variance of Theta 3:')
    print(calcVar(DensityData_3)/10000)
}
```

Start Simulation

Start with theta ~ Dir(1)

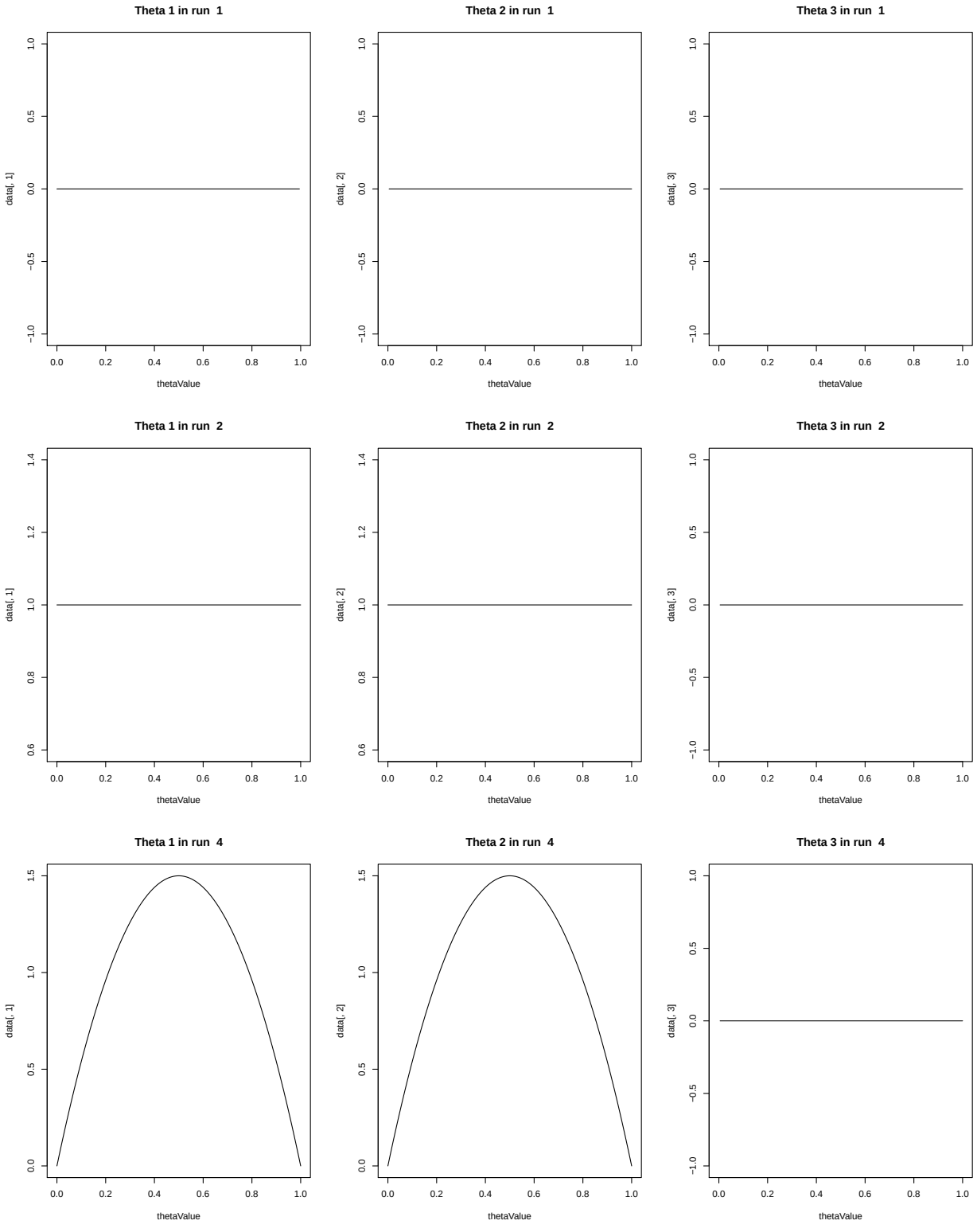
```
D <- numeric(length=3)
D <- matrix(D, nrow = 3, ncol = 1)
D[1,1] = 1

calcDensityAndPlot(D, 0)
```

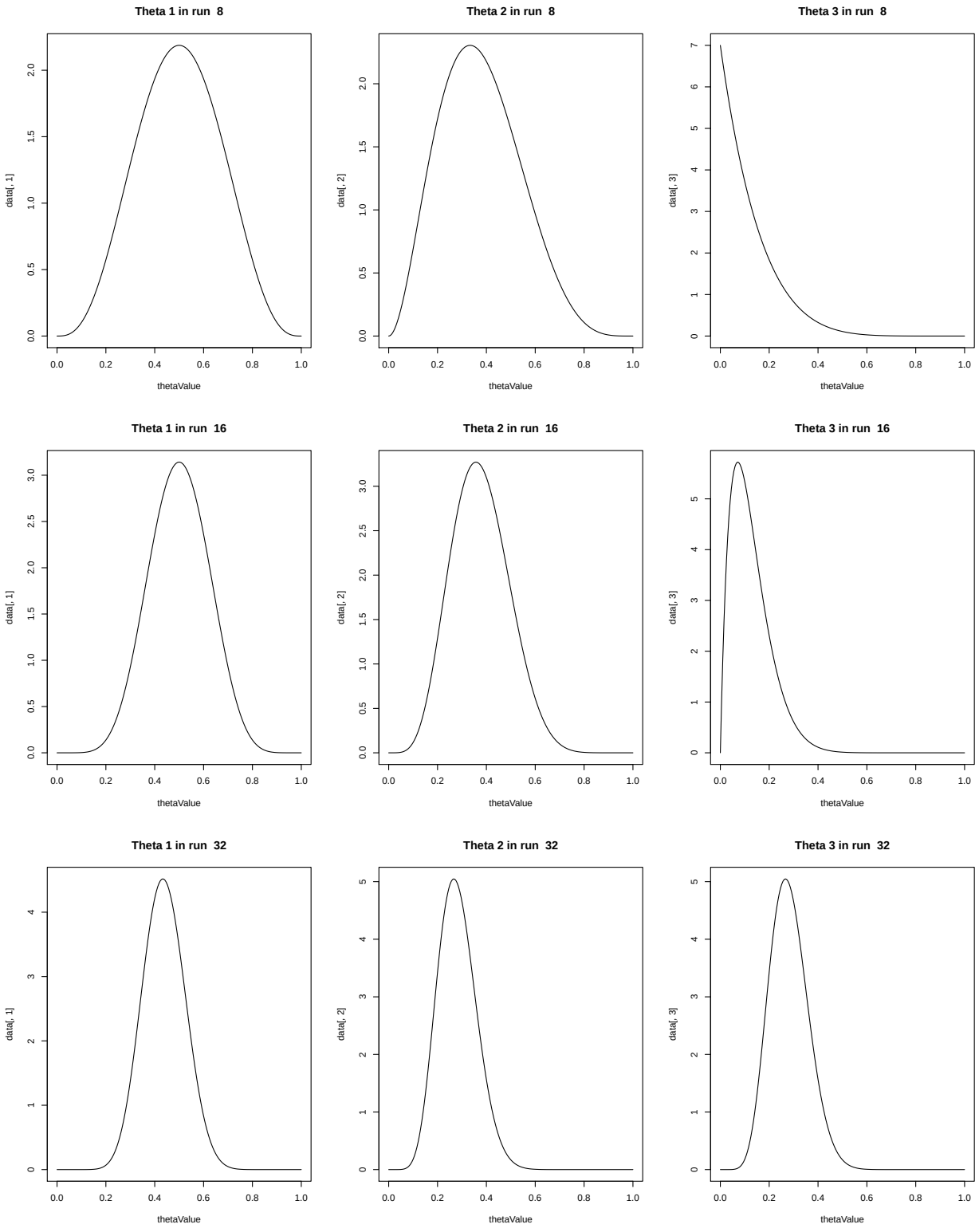


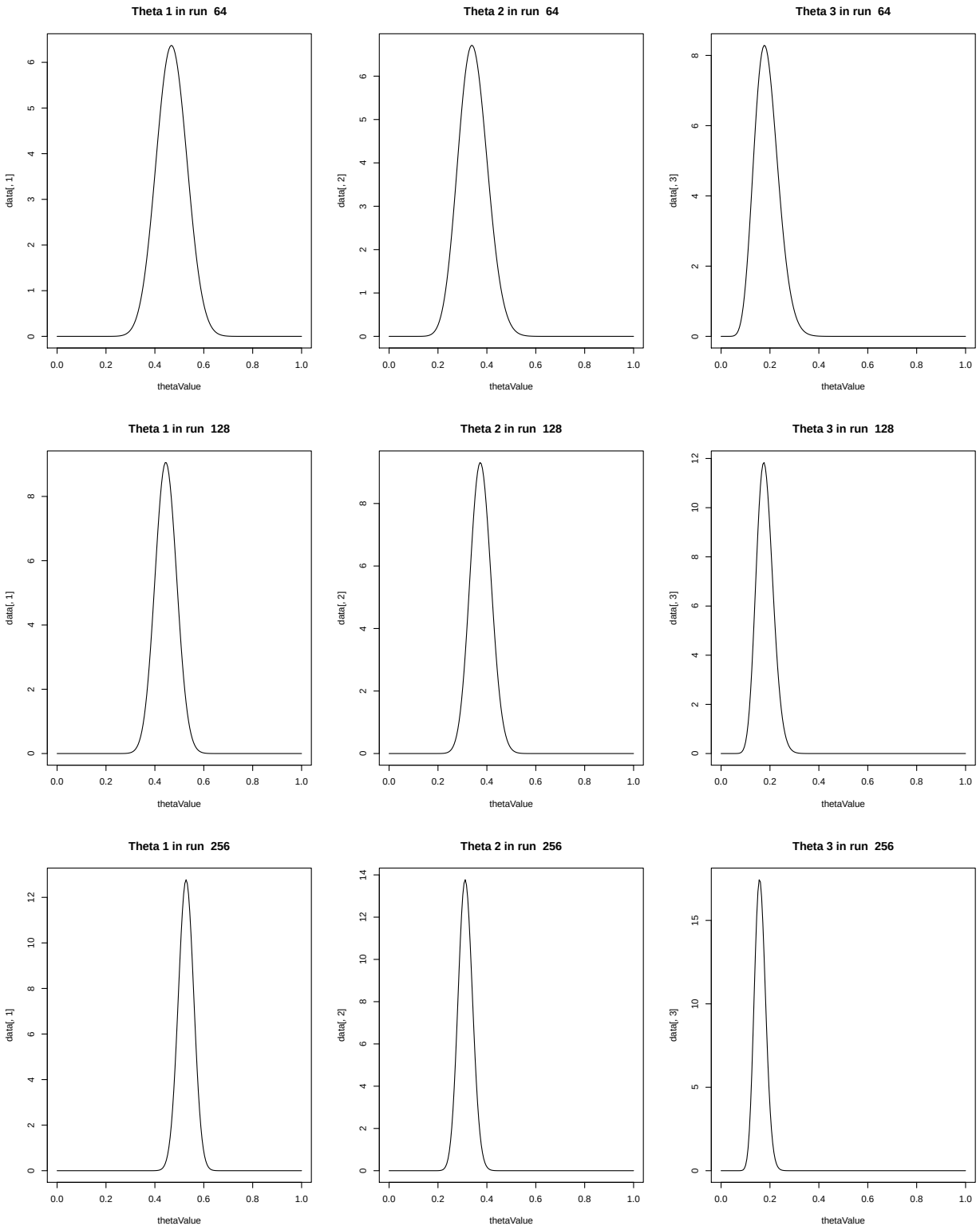
Sample X1 and plot the marginal distribution

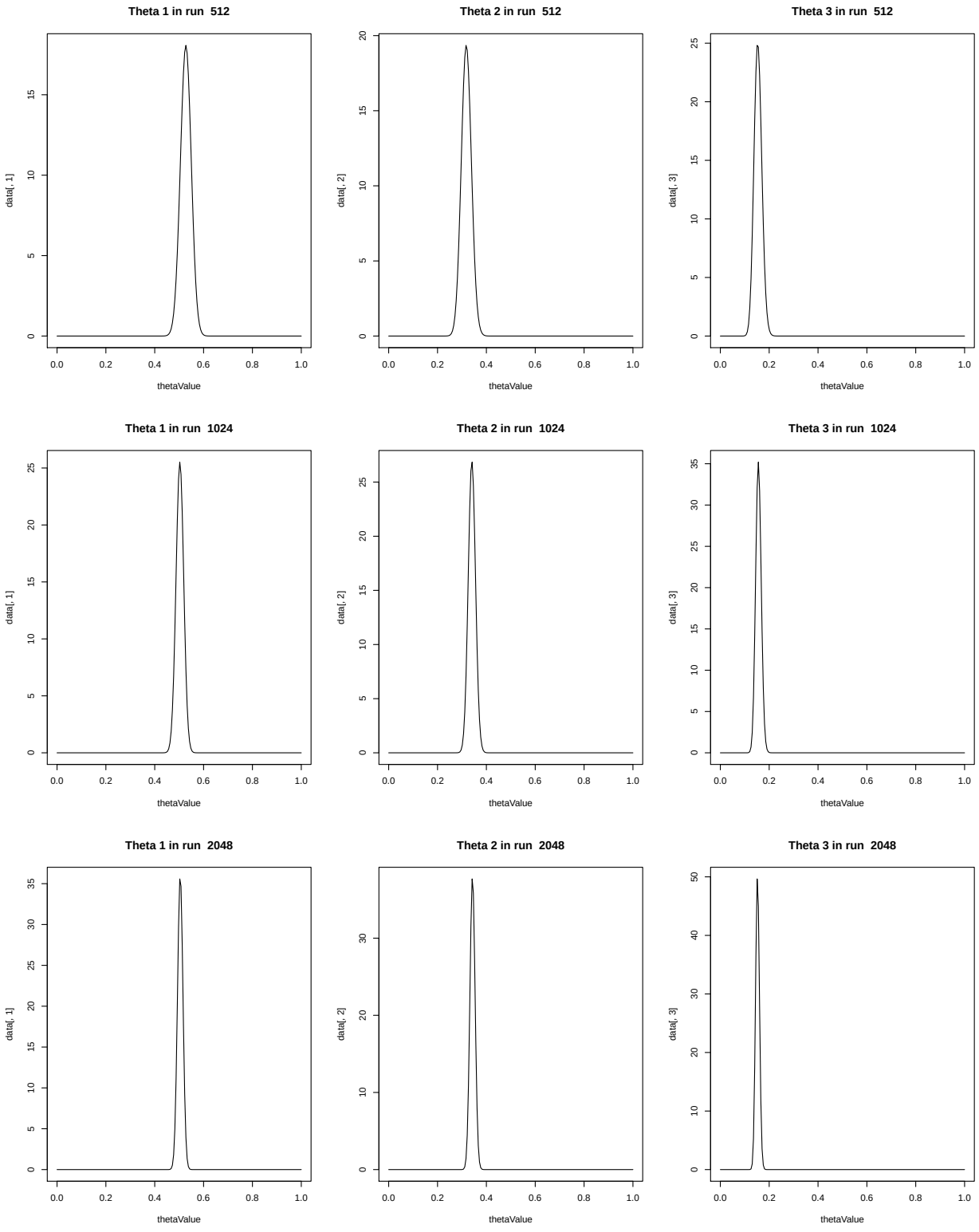
```
mean = 1
sigma = c(0.5, 0.35, 0.15)
Data <- 0
t = 1

for(i in seq(1, 50000))
{
  X <- getX(mean, sigma)
  if(i == 1)
  {
    Data = X
  }
  else
  {
    Data <- cbind(Data,X)
  }

  if(i %% t == 0)
  {
    t = t*2
    calcDensityAndPlot(Data, i)
  }
```

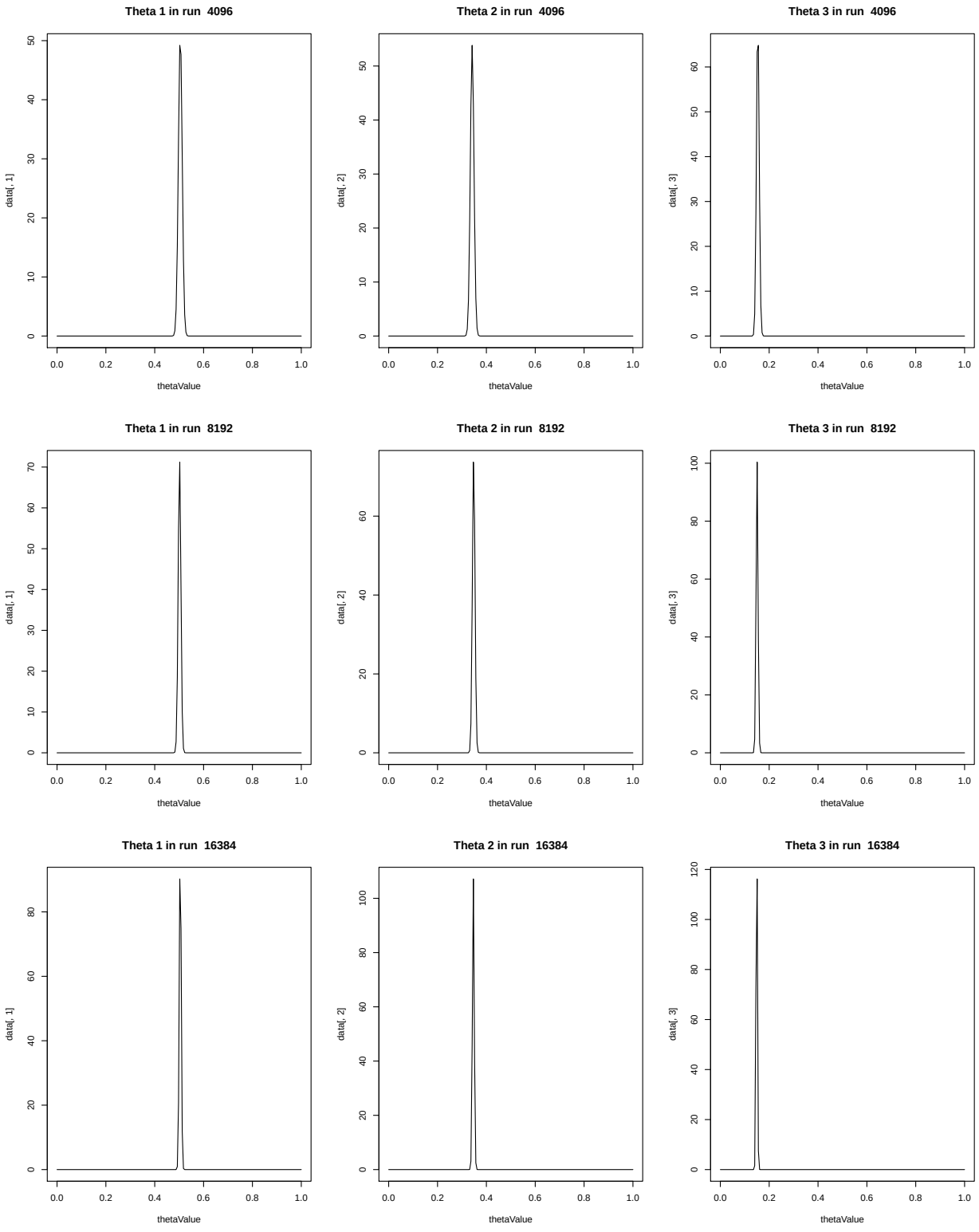
```
}
```

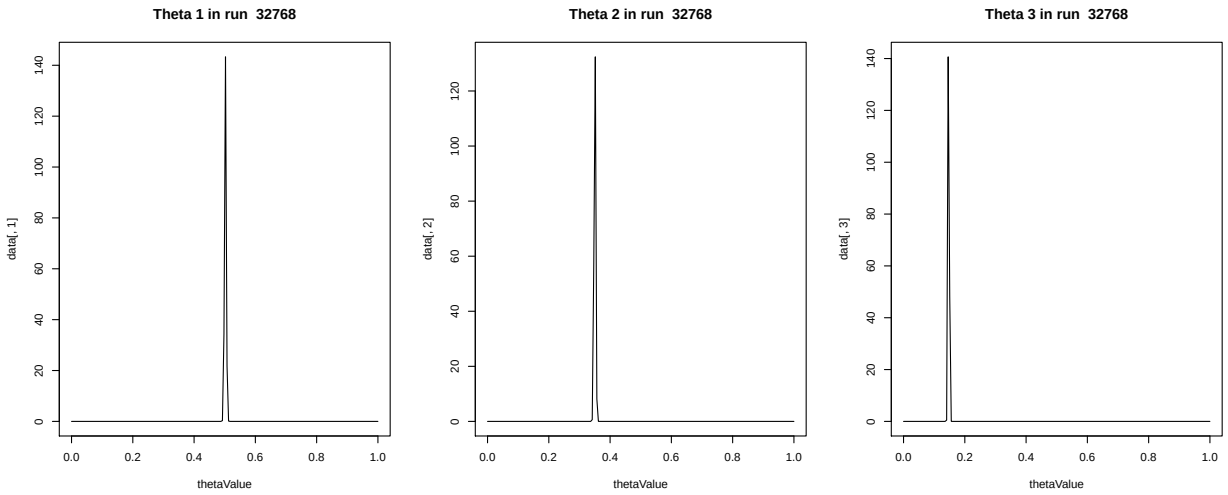

Theta 1 in run 1
Theta 2 in run 1
Theta 3 in run 1
Theta 1 in run 2
Theta 2 in run 2
Theta 3 in run 2
Theta 1 in run 4
Theta 2 in run 4
Theta 3 in run 4
data[, 1]
data[, 2]
data[, 3]
thetaValue


Theta 1 in run 8
Theta 2 in run 8
Theta 3 in run 8
Theta 1 in run 16
Theta 2 in run 16
Theta 3 in run 16
Theta 1 in run 32
Theta 2 in run 32
Theta 3 in run 32
data[, 1]
data[, 2]
data[, 3]
thetaValue

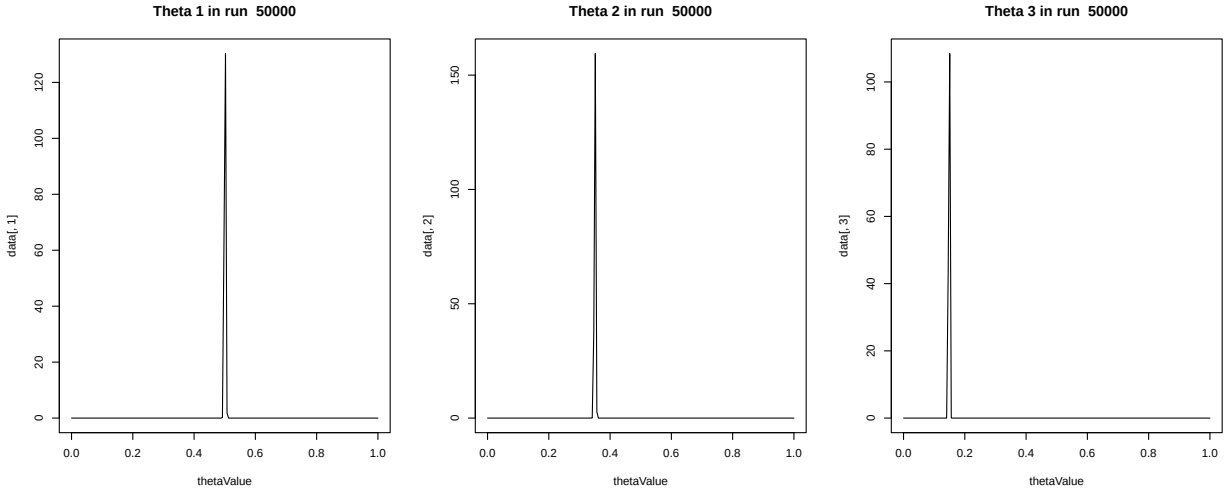

Theta 1 in run 64
Theta 2 in run 64
Theta 3 in run 64
Theta 1 in run 128
Theta 2 in run 128
Theta 3 in run 128
Theta 1 in run 256
Theta 2 in run 256
Theta 3 in run 256
data[, 1]
data[, 2]
data[, 3]
thetaValue


Theta 1 in run 512
Theta 2 in run 512
Theta 3 in run 512
Theta 1 in run 1024
Theta 2 in run 1024
Theta 3 in run 1024
Theta 1 in run 2048
Theta 2 in run 2048
Theta 3 in run 2048
data[, 1]
data[, 2]
data[, 3]
thetaValue


Theta 1 in run 4096
Theta 2 in run 4096
Theta 3 in run 4096
Theta 1 in run 8192
Theta 2 in run 8192
Theta 3 in run 8192
Theta 1 in run 16384
Theta 2 in run 16384
Theta 3 in run 16384
data[, 1]
data[, 2]
data[, 3]
thetaValue




```
calcDensityAndPlot(Data, 50000)
```



Calculate Mean and Sn

```
calcDensityAndPrintMeanSn(Data)
```

```
## Mean of Theta 1:[1] 0.5009454
## Variance of Theta 1:[1] 1.150597e-07
## Mean of Theta 2:[1] 0.3508824
## Variance of Theta 2:[1] 8.635919e-08
## Mean of Theta 3:[1] 0.1493105
## Variance of Theta 3:[1] 7.875904e-08
```

Conclusion

By simulation, we find the marginal posterior distribution of thetas.

It is a normal distribution, which is following Central Limit Theorem.