Learning Outcome

By the end of this lecture, you will be able to:

- Distinguish between a point estimate and a confidence interval estimate
- construct and interpret a confidence interval estimate for a single population mean using both the z and t distributions

Introduction

Point and interval estimates allow approximate unknown population parameters.

In this lecture, we will study about the point estimate and the interval estimate in detail.

Point Estimate

A point estimate is a single number that approximates the value of an unknown population parameter (or characteristic). We can estimate a Population Parameter with a Sample Statistic.

Population Mean μ	The point estimate is the sample mean $\bar{x} = \frac{\sum x}{n}$
Population Variance σ^2:	The point estimate is the sample variance $s^2 = \frac{\sum x^2 - n\bar{x}^2}{n-1} \quad (2)$
Population Proportion P or p:	The point estimate is the sample proportion $\bar{p} = \frac{x}{n} \quad (3)$

Confidence Interval Estimate

An interval estimate provides more information about a population parameter than does a point estimate. A confidence interval provides additional information about variability. Such interval estimates are called confidence intervals.

Features

An interval estimate gives a range of values:

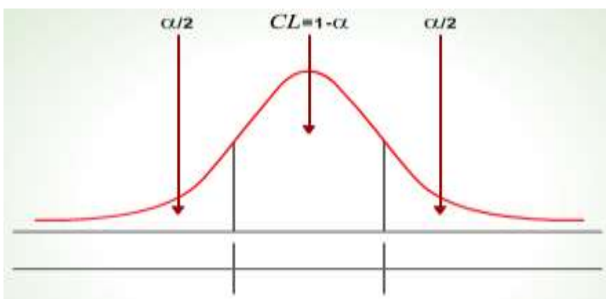
- Takes into consideration ***variation*** in sample statistics from sample to sample
- Based on observation from ***One*** sample
- Gives information about ***closeness*** to unknown population parameters
- Stated in terms of ***level of confidence***
- Never 100% sure

The general format of any interval estimate is given by:

$$\text{Point Estimate} \pm (\text{Critical Value})(\text{Standard Error}) \quad (4)$$

Confidence Level (1-α)

Confidence level is the confidence in which the interval will contain the unknown population parameter. This percentage is *always* less than 100%.



Confidence Level (1-α)

Suppose the confidence level = 95%, also written as $(1 - \alpha) = 0.95$, then $\alpha = 0.05$, and α is known as the *significance level*.

- A relative frequency interpretation:
  In the long run, 95% of all the confidence intervals that can be constructed will contain the unknown true parameter.
- A specific interval either will contain or will not contain the true parameter.
  No probability involved in a specific interval.

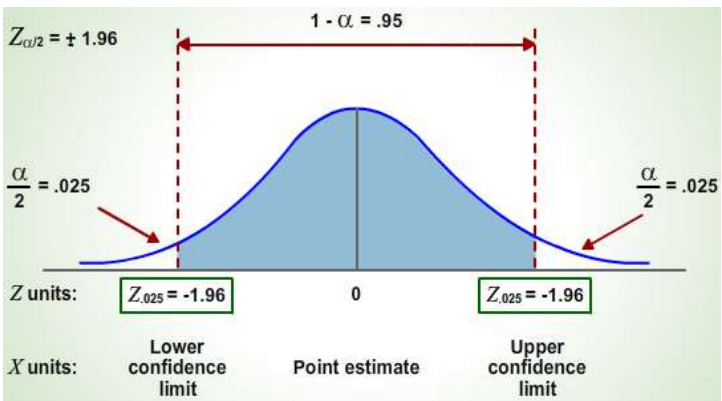
Finding the Critical Value

Shows critical values for different confidence levels:

95% Confidence Level

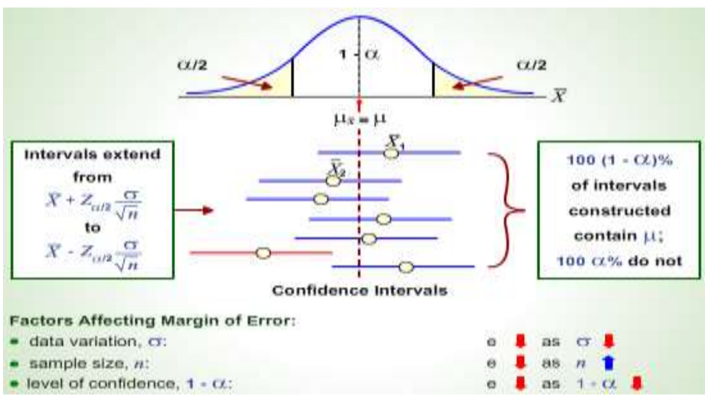
Confidence Level	$1-\alpha$	$z_{\alpha/2}$
80%	0.80	1.28
90%	0.90	1.645
95%	0.95	1.96
98%	0.98	2.33
99%	0.99	2.575
99.8%	0.998	3.08
99.9%	0.999	3.27

Consider a 95% confidence interval:



95% confidence interval

Interval and Level of Confidence



Interval and Level of confidence

Cases For Confidence Interval For The True Population Mean (μ)

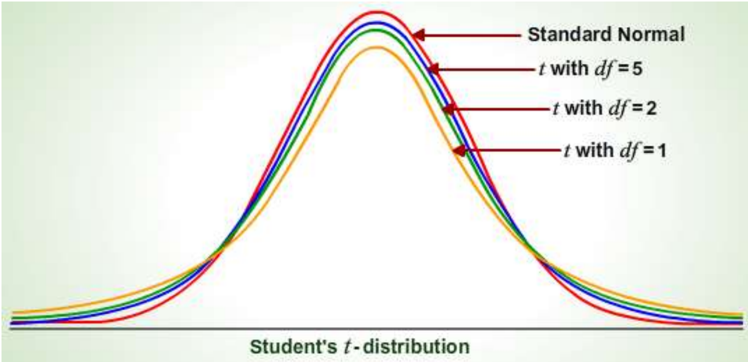
Given here are three cases for the confidence interval for the true population mean or mu

Case I	Assumptions 1. Sample size large ($n \geq 30$). Population standard deviation σ is unknown $\bar{x} \pm t_{\alpha/2,n-1} \frac{s}{\sqrt{n}}$ (5)
Case II	Assumption: 1. Sample size is small ($n < 30$). 2. Population is normally distributed. Population standard deviation σ is known $\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$ (6)
Case III	Assumption: 1. Sample size small ($n < 30$). 2. Population is normally distributed. 3. Population standard deviation σ is unknown. $\bar{x} \pm t_{\alpha/2,n-1} \frac{s}{\sqrt{n}}$ (7) **Note:** The term $z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$ or $t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}$ is called the *margin of error* and is denoted by *e*.

Student's t – Distribution

Student's t-distribution:

- It is bell shaped like the normal distribution
- It has the same properties as the normal distribution ; mean = median = mode
- It depends on degrees of freedom(d.f = $n - 1$)
- As n increases, t – distribution approaches the z – normal distribution



Student's t-distribution

Example 1

Assume a sample of size n has been obtained from a normal distribution. Determine the critical value from a t – distribution when you wish to estimate the population mean in each of the given cases:

1. Confidence level = 0.95 and n = 26

$$1-\ \alpha\ =\ 0.95\ \rightarrow \alpha/2 =\ 0.025 \rightarrow t_{0.025,25}\ = 2.0595$$

2. Confidence level = 0.90 and n = 31

$$1-\ \alpha\ =\ 0.90\ \rightarrow \alpha/2 =\ 0.05 \rightarrow t_{0.05,30}\ = 1.6973$$

3. Confidence level = 0.98 and n = 19

$$1-\ \alpha\ =\ 0.98\ \rightarrow \alpha/2 =\ 0.01 \rightarrow t_{0.01,18}\ = 2.5524$$

Example 2

Determine the margin of error:

1. Confidence level = 0.98, n = 13, and σ = 15.68

$$1 - \alpha = 0.98 \rightarrow t_{0.01,12} = 2.681 \rightarrow e = t_{\frac{\alpha}{2},n-1}\frac{\sigma}{\sqrt{n}} = 2.681\frac{15.68}{\sqrt{13}} = 11.6593$$

2. Confidence level = 0.99, n = 25, and σ = 3.47

$$1 - \alpha = 0.99 \rightarrow t_{0.005,24} = 2.7969 \rightarrow e = t_{\frac{\alpha}{2},n-1}\frac{\sigma}{\sqrt{n}} = 2.7969\frac{3.47}{\sqrt{25}} = 1.941$$

3. Confidence level = 0.95, n = 8, and standard error = 2.356

$$1 - \alpha = 0.95 \rightarrow t_{0.025,7} = 2.3646 \rightarrow e = t_{\frac{\alpha}{2},n-1}\frac{\sigma}{\sqrt{n}} = 2.3646\,(2.356) = 5.571$$

Example 3

A random sample of size 49 has mean 50 and standard deviation 8. Construct a 95% confidence interval for the population mean μ.

Determine the margin of error:

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A random sample of size 49 has mean 50 and standard deviation 8. Construct a 95% confidence interval for the population mean μ.

Solution:
Since $n \geq 30$ and σ is unknown ➔ A $(1 - \alpha)100\%$ C.I. for μ is:

$$\bar{x} \pm z_{\alpha/2}\frac{s}{\sqrt{n}}$$

$1 - \alpha = 0.95 \rightarrow z_{0.05/2} = z_{0.025} = 1.96 \rightarrow$ A 95% C.I. for μ is:

$1 - \alpha = 0.95 \rightarrow Z_{0.05/2} = Z_{0.25} = 1.96 \rightarrow$ A 95% C.I. for μ is:

$$\bar{x} \pm z_{\alpha/2}\frac{s}{\sqrt{n}} = 50 \pm 1.96\,\frac{8}{\sqrt{49}} = 50 \pm 2.24 = [47.65\,,52.24]$$

Recap

In this lecture, you have learned that:

- A point estimate is a single number that approximates the value of an unknown population parameter or characteristic
- An interval estimate provides more information about a population parameter than does a point estimate. A confidence interval provides additional information about variability
- Confidence level is the confidence in which the interval will contain the unknown population parameter. This percentage is always less than 100%