Learning Outcome

By the end of this lecture, you will be able to:

- Describe the Binomial experiment and its characteristics
- Calculate the probabilities of a Binomial random variable

Introduction

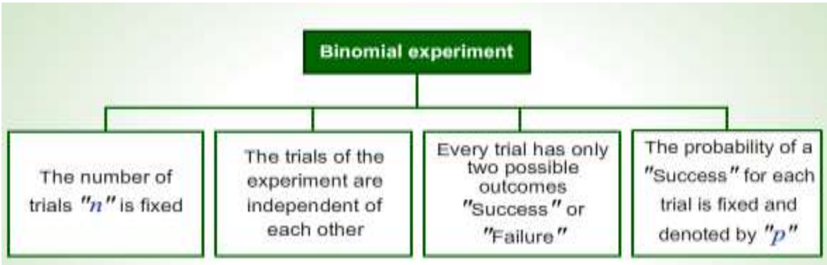
Ahmad, a quality engineer in an automotive spare parts manufacturing company, checks quality of spare parts produced in his company.

He rejects any products that do not meet quality standards of the company. He considers every rejected product as 'failure' event. All the remaining products, which satisfy quality standards, are considered as 'success' events.

This type of experiment, where an event can have only two possible outcomes, forms a different type of distribution, called binomial distribution. In this lecture you will learn about the characteristics of binomial distribution. You will also learn how to calculate the probabilities of a binomial random variable.

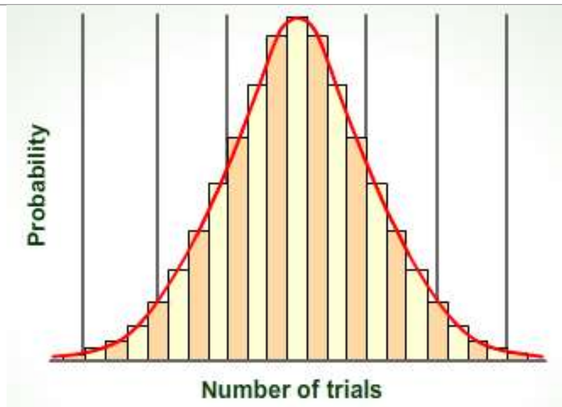
Binomial Experiment

Any experiment that satisfies all the four given characteristics is called binomial experiment.



Binomial probability distribution

If the random variable X counts the number (x) of successes in n trials in a binomial experiment then X is said to have the *Binomial Probability Distribution*.



Binomial probability distribution

Binomial Distribution Formula

Binomial distribution formula helps us to calculate the probabilities of getting *x* successes in *n* trials:

$$P(x) = C_x^n p^x \, (1-p)^{n-x} \quad , \qquad x = 0, 1, 2, \ldots n$$

Where:

- $C_x^n = \dfrac{n!}{x!\,(n-x)!}$ Number of combinations of *x* objects from *n* objects.

$$n! = n(n-1)(n-2) \ldots (2)(1)$$
$$x! = x(x-1)(x-2) \ldots (2)(1)$$
$$0! = 1$$

- $P(x)$: Probability of getting x successes in n trials.
- x: Number of successes in the experiment.
- n: Number of trials (sample size).
- p: Probability of success for *one* trial.

Mean and Variance for the Binomial Distribution

Mean and variance for the binomial distribution are computed by given formulae:

$$Mean(Expected\ value\ or\ average):\ \mu = np \qquad (2)$$

$$Variance:\ \sigma^2 = np(1-p) \qquad (3)$$

$$Standard\ deviation\ \sigma = \sqrt{np(1-p)} \qquad (4)$$

Example

Consider a tossing a *fair* coin 3 times and let X is number of Heads that comes up.

Coins

1. Write the sample space.

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

2. Find the probability to get exactly 3 heads.

$$P(3H) = 1/8$$

3. Find the probability of appearing at most 3 Heads.

$$P(at\ most\ 3\ H) = P(no\ H) + P(exactly\ 1\ H) + P(exactly\ 2\ H) + P(exactly\ 3\ H)$$
$$= 1/8 + 3/8 + 3/8 + 1/8 = 8/8 = 1$$

4. Write the probability distribution, and identify it, for the r.v. X.

X has the binomial probability distribution with $n = 3$ and $p = ½$ ➔

$$P(x) = C_x^3 \frac{1^x}{2} \frac{1^{3-x}}{2}$$
$$= C_x^3 \frac{1^3}{2} \quad , \quad x = 0, 1, 2, 3$$

5. Find the mean and the standard deviation for X.

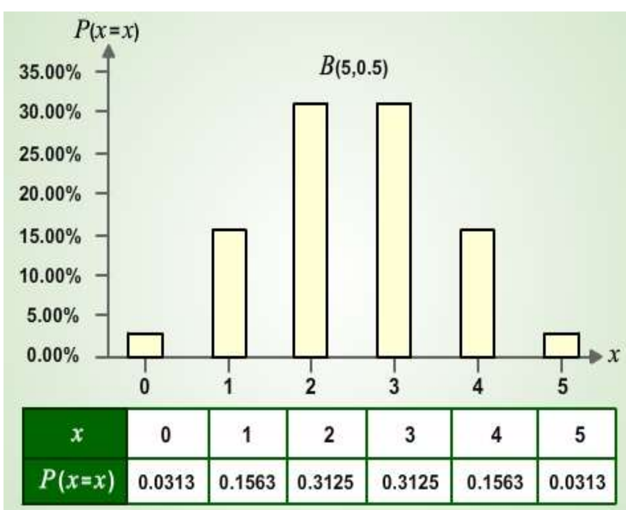
$$\mu = np$$
$$= (3)\left(\frac{1}{2}\right)$$
$$= \frac{3}{2} = 1.5$$
$$\sigma = \sqrt{np(1-p)}$$
$$= \sqrt{3\frac{1}{2}\frac{1}{2}} = \frac{\sqrt{3}}{2}$$

Binomial Distribution

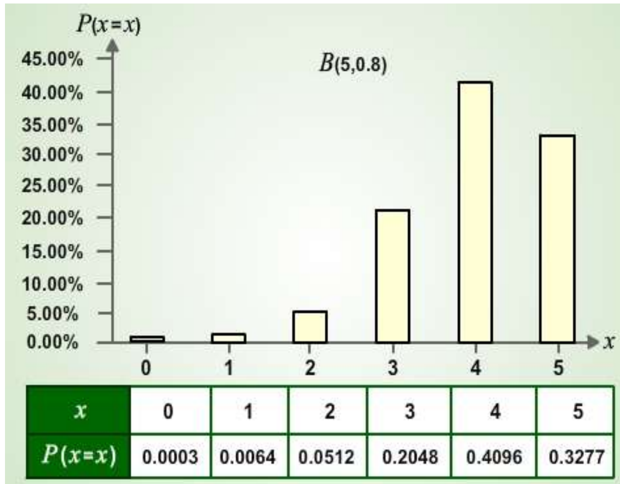
The shape of the binomial distribution depends on the value of *p*.

$p < 0.5$	$p > 0.5$	$p = 0.5$

If $\boldsymbol{p = 0.5}$ then the shape is *symmetric* regardless of the sample size. Here $n = 5$ and $p = 0.5$

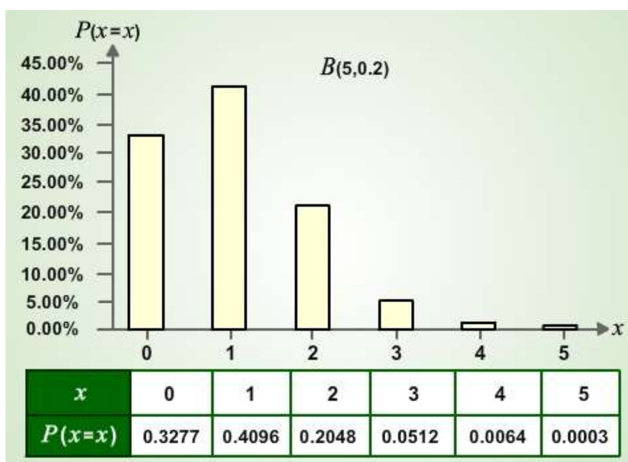


If $\boldsymbol{p}$ **< 0.5** then the shape is *right-skewed* regardless of the sample size. Here $n = 5$ and $p = 0.8$



Right-skewed distribution

If $\boldsymbol{p}$ **> 0.5** then the shape is *left-skewed* regardless of the sample size. Here $n = 5$ and $p = 0.2$



Left-skewed distribution

Activity 1

An Urn contains 10 calculators, 4 of the calculators are defective. If a sample, of size 4, is randomly selected *with replacement*, find:

1. The probability that two of them are defective.
2. The probability that at least two are defective.
3. The mean and the standard deviation for the number of defective calculators.
4. The coefficient of variation for the number of defective calculators.

Hint :

Use binomial distribution with $n=4$, $p=0.4$

Activity 2

A Sales Manager of a certain company conducted a survey that resulted in a percentage of 55% of the sales people preferred to use the email (and not the telephone) to contact with customers. If the manager selected 15 sales people randomly, then:

1. Find the probability that there will be at least two productive wells.
2. Find the probability that between two and four, inclusive, productive wells will be hit.
3. Complete the probabilities for the rest of the number of productive wells.
4. Compute the probability that the first two productive wells will be hit.
5. The company will stop drilling if they find oil, find the probability that the company will find oil in the 7th location.

Recap

In this lecture, you have learned that:

- binomial experiments have four characteristics:

    1. The number of trials **“n” is fixed**
    2. The **trials** of the experiment **are independent** of each other
    3. Every trial has only **two possible outcomes**, “Success” or “Failure”
    4. The probability of a “**Success” for each trial is fixed** and denoted by *p*