Learning Outcome

By the end of this lecture, you will be able to:

- State the Central limit Theorem
- Recognize the significance of Central limit theorem

Introduction

Consider a researcher aiming to measure the average height of people living in Riyadh. It is a complex task for the researcher to select an appropriate sample for the study as the population of Riyadh is 7,000,000.

The task becomes even more complex if researcher aims to compare the average height of people living Riyadh with average height of people living in Jeddah.

City of Riyadh

City of Jeddah

This is exactly where the central limit theorem comes into picture. It helps us reliably estimate a parameter like the mean, by using an average derived from a much smaller sample. Also, it helps validate the use of polls, and marketing research. In this lecture, let's learn more about the central limit theorem.

The Central Limit Theorem (CLT)

Let $X_1, X_2, X_3, \ldots, X_n$ be a random sample from a population having mean μ and standard deviation σ. Then the sample means from the population will be *approximately* normal as long as the sample size is large enough $(n \geq 30)$ and the sampling distribution will have:

$$\mu_{\bar{x}} = \mu \ \ and \ \ \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} \qquad (1)$$

Notes:

- For most distributions, $n > 30$ will give a sampling distribution that is nearly normal
- For fairly symmetric distributions, $n > 15$

Example

A random sample of size 100 is taken from an infinite population having a mean, 76 and a variance, 256.

What is the sampling distribution of the sample mean?	The sample size n =100 is large (n >30), by CLT the sampling distribution of the sample mean is approximately Normal with mean 76 and variance 2.56.
What is the probability that the sample mean will be between 75 and 78?	$P(75 < \bar{x} < 78) = P\left(\frac{75-76}{1.6} < Z < \frac{78-76}{1.6}\right)$ $= P(-0.63 < Z < 1.25)$ $= P(Z < 1.25) - P(Z < -0.63)$ $= 0.8944 - 0.2643 = 0.6301$

Recap

In this lecture, you have learned that:

- In central limit theorem, the sample means from the population having mean μ and standard deviation σ, will be *approximately* normal as long as the sample size is large enough