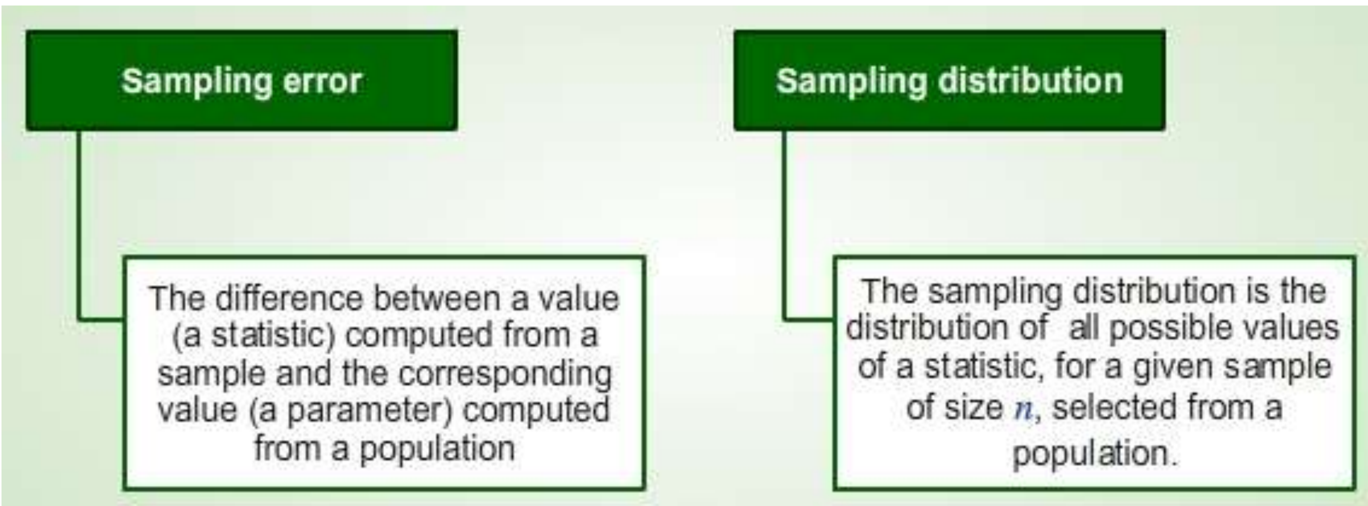
Learning Outcome

By the end of this lecture, you will be able to:

- Determine the sampling distribution of the sampling population, p
- Determine the mean and standard deviation for the sampling distribution of the sample proportion, p

Introduction

Recall sampling error and sampling distribution.



By using sampling error and sampling distribution, a selected sample's parameters can be estimated based on its population's parameters. In this lecture, we will learn about sampling distribution for the proportion.

Population Proportion And Sample Proportion

Let's look at population and sample proportions.

Population proportion	*Population proportion* (π *):* The ratio of number of items having a certain attribute in the population to the size of the population $\left(\pi = \frac{X}{N}\right)$.
Sample proportion	*Sample proportion* (p *or p hat is better than* p *bar* $\bar{P}$): The ratio of number of items having a certain attribute in the sample to the size of the sample $\left(p = \frac{x}{n}\right)$ and the sampling error: $p - \pi$.

Sampling Distribution for $\bar{P}$

Sampling distribution for $\bar{P}$ for large size sample ($n \geq 30$):

If $n\,\pi \geq 5$ and $nq = n(1-\pi) \geq 5$ then p has approximately a normal distribution with mean $\mu_{\bar{p}} = \pi$ and standard deviation $\sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}}$

Standardize p to a z value with the formula:

$$z = \frac{p-\pi}{\sigma_p} = \frac{p-\pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} \qquad (1)$$

<u>*Note*</u>: If sampling is **without replacement** and n is greater than 5% of the population size, then use the **finite population correction factor** to calculate σ_p.

$$\sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}}\sqrt{\frac{N-n}{N-1}} \qquad (2)$$

Example

If the true proportion of voters who support Proposition A is 40%, then

1. What is the probability that a sample of size 200 yields a sample proportion between 0.40 and 0.45? (i.e.: if $\pi = 0.4$ and $n = 200$, what is $P\ (0.40 \le p \le 0.45)$)?

Solution:

Since $n\ \pi \geq 5$ (200*0.4=80>5) and $nq = n(1-\pi) \geq 5$ (200*0.6=120>5), p (200*0.6=120>5), p has approximately a normal distribution with mean $\mu_{\bar{p}} = \pi = 0.4$ and standard deviation $\sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}} = 0.034641$

$$P(0.4 \leq p \leq 0.45) = P(0 < Z < 1.44) = 0.4251$$

2. What is the probability that more than 85 voters will vote for Proposition A?

Solution:

Proportion of the 85 voter is 85/200=0.425

$$P(p > 0.425) = P(Z > 0.72) = 0.2358$$

Recap

In this lecture, you have learned that:

- Population proportion (π): is the ratio of number of items having a certain attribute in the population to the size of the population
- Sample proportion, p or $\bar{P}$: is the ratio of number of items having a certain attribute in the sample to the size of the sample and the sampling error
- If sampling is without replacement and n is greater than 5% of the population size, then the finite population correction factor is used to calculate σ_p