Learning Outcome

By the end of this lecture, you will able to:

- Learn how to construct a confidence interval estimate for the difference between two population proportions

Introduction

In this lecture, you will learn how to construct a confidence interval estimate for the difference between two population proportions.

Estimation for the Two Population Proportions

Confidence interval estimate for the difference between two means is constructed based on the nature of samples.

The aim of this lecture is to form a $(1 - \alpha)100\%$C.I for the difference between two population proportions, P_1 and P_2.

The assumptions are:

- $n_1P_1 \geq 5$, $n_1(1 - P_1) \geq 5$
- $n_2P_2 \geq 5$, $n_2(1 - P_2) \geq 5$

The point estimate for the difference between two population proportions $P_1 - P_2 = \bar{p}_1 - \bar{p}_2$

Where,

$$\bar{p}_1 = \frac{x_1}{n_1} \quad , \quad \bar{p}_2 = \frac{x_2}{n_2}$$

Therefore, the $(1 - \alpha)100\%$ C.I for the difference between two population proportions, $P_1 - P_2$ is:

$$(\bar{p}_1 - \bar{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\bar{p}_1(1-\bar{p}_1)}{n_1} + \frac{\bar{p}_2(1-\bar{p}_2)}{n_2}} \qquad (1)$$

The standard error for $\bar{p}_1$ - $\bar{p}_2$ is:

$$\sigma_{\bar{p}_1-\bar{p}_2} = \sqrt{\frac{\bar{p}_1(1-\bar{p}_1)}{n_1} + \frac{\bar{p}_2(1-\bar{p}_2)}{n_2}} \qquad (2)$$

The margin of error is:

$$e = \pm z_{\alpha/2}\sqrt{\frac{\bar{p}_1(1-\bar{p}_1)}{n_1} + \frac{\bar{p}_2(1-\bar{p}_2)}{n_2}} \qquad (3)$$

Example

We want to compare the proportion of defective bulbs turned out by two shifts of workers. From the large number of bulbs produced in a given week, $n_1 = 50$ bulbs were selected from the output of Shift I, and $n_2 = 40$ bulbs were selected from the output of Shift II. The sample from Shift I revealed five defective, and the sample from Shift II showed six faulty bulbs. Estimate, by a 95% confidence interval, the true difference between the proportions of defective bulbs produced.

Solution:

$n_1 = 50\,, n_2 = 40\,,\ x_1 = 5\,, x_2 = 6$

$\bar{p}_1 = \frac{5}{50} = 0.1\ \ and\ \ \bar{p}_2 = \frac{6}{40} = 0.15$

The assumptions are:

$$n_1\bar{p}_1 = 5$$

$$n_1(1-\ \bar{p}_1) = 45$$

$$n_2\bar{p}_2 = 6$$

$$n_2(1-\ \bar{p}_2) = 34$$

A $(1-\alpha)100\%$ C.I. for $P_1 - P_2$ is:

$$(\bar{p}_1 - \bar{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\bar{p}_1(1-\bar{p}_1)}{n_1} + \frac{\bar{p}_2(1-\bar{p}_2)}{n_2}}$$

$$1-\alpha = 0.95 \;\rightarrow\; z_{0.05/2} = z_{0.025} = 1.96$$

$$s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$$

$$= \sqrt{\frac{14(25)+9(36)}{23}}$$

$$= 5.41334904$$

A 95% C.I. for $P_1 - P_2$ is:

$$(\bar{p}_1 - \bar{p}_2) \pm z_{\alpha/2}\sqrt{\frac{\bar{p}_1(1-\bar{p}_1)}{n_1} + \frac{\bar{p}_2(1-\bar{p}_2)}{n_2}}$$

$$= (0.1 - 0.15) \pm 1.96\sqrt{\frac{0.1(0.9)}{50} + \frac{0.15(0.85)}{40}}$$

$$= -0.05 \pm 0.070622$$

$$= [-0.12062\,, 0.020622]$$

Recap

In this lecture, you have learned that:

- The point estimate for the difference between two population proportions, $P1 - P2$ is:

$$\bar{p}_1 - \bar{p}_2 \text{ , } where \text{ , } \bar{p}_1 = \frac{x_1}{n_1} \text{ , } \bar{p}_2 = \frac{x_2}{n_2}$$

- The $(1 - \alpha)100\%$ CI for difference between two population proportions, $P1 - P2$ is:

$$(\bar{p}_1 - \bar{p}_2) \pm z_{\alpha/2} \sqrt{\frac{\bar{p}_1(1 - \bar{p}_1)}{n_1} + \frac{\bar{p}_2(1 - \bar{p}_2)}{n_2}}$$