Learning Outcome

By the end of this lecture, you will be able to:

- Define some basic concepts in probability
- Explain approaches of assigning probability
- Explain types and relationships between events

Defining Probability

Probability is a numeric value between 0 and 1 representing the chance or possibility that an uncertain event will occur.

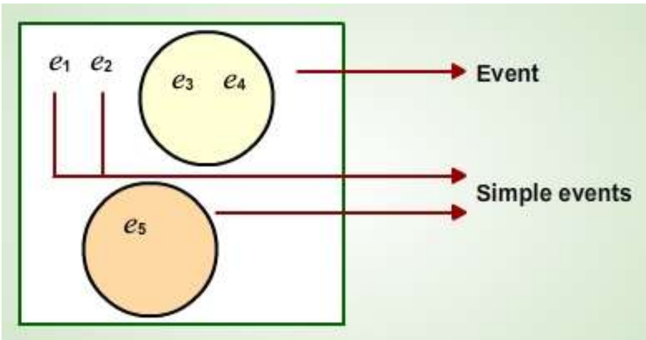
What is the chance of the Kingdom of Saudi Arabia winning the football world cup?

How likely is a child to complete work in time?

When is the stock market likely to rise?

Basic Terms

The basic terms associated with probability are:

Random experiment	The process of obtaining an outcome, which is uncertain or that cannot be exactly predictable. For example, tossing a coin is a random experiment. We know the possible outcomes are head or tail, but we do not know exactly which one will occur. In contrast, when we mix Hydrogen (H_2) and Oxygen (O_2), under the appropriate conditions, we get Water (H_2O). So, such an experiment is not random.
Elementary outcome	Each individual outcome of the random experiment.
Sample space	The collection of all possible elementary outcomes, denoted by S or $\mathbf{\Omega}$. For instance, $S = \{e_1, e_2, e_3, e_4, e_5\}$.
Event	A set of possible outcomes from an experiment.  Possible outcomes of the experiment
Elementary event	An event with only one individual outcome.

Example

Write a sample space for tossing a coin once and twice.

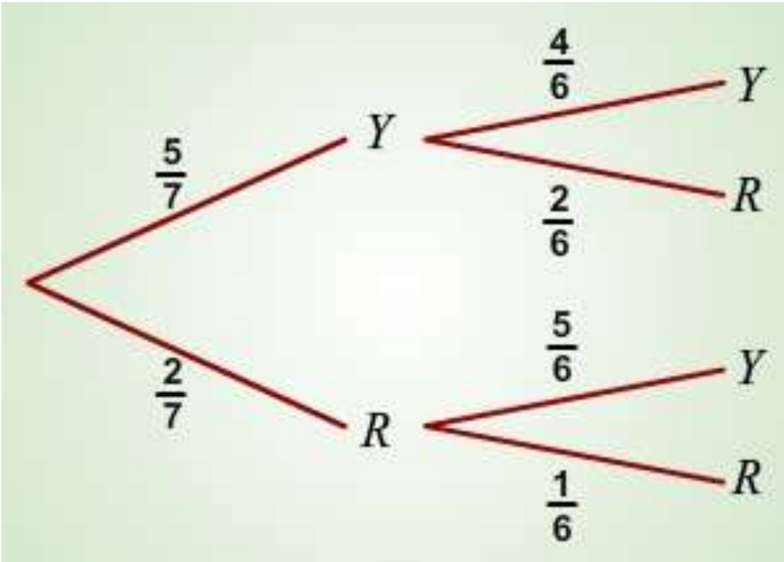
Solution:

Tossing a coin once:
$S = \{H, T\}$

Tossing a coin twice:
$S = \{HH, HT, TH, TT\}$

Sample Space Tree Diagram

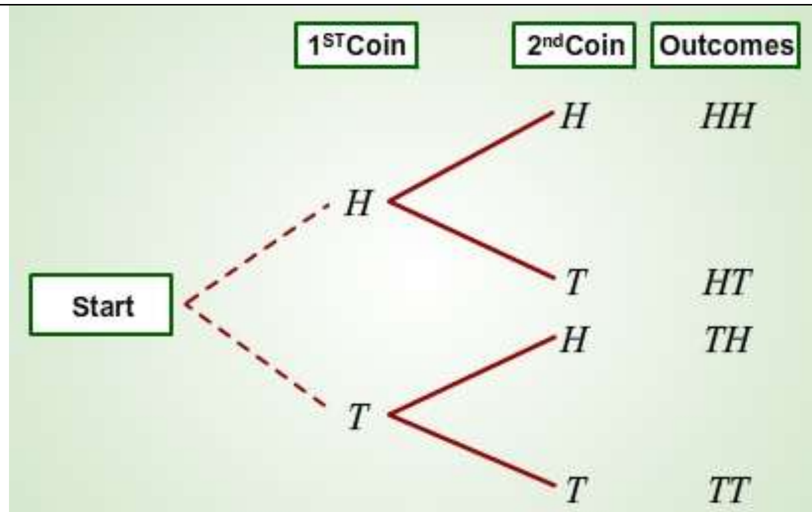
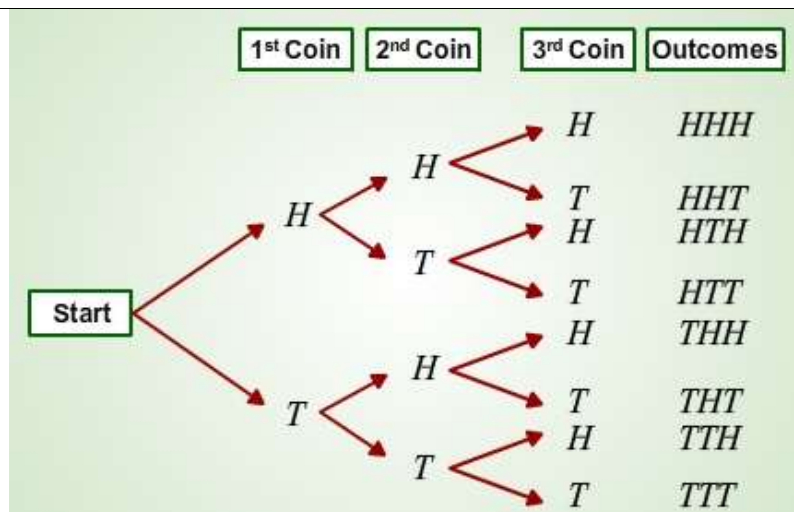
Let's look at how to write a sample space using a tree diagram. A tree diagram can be used to write a sample space for a given experiment. Tree diagrams permit us to recognize all the possible outcomes of a random experiment. Each branch in a tree diagram leads to a likely outcome. Here is a sample tree diagram.



A model tree diagram

Sample Space Tree Diagram ...Contd.

Here is an example that shows how to write a sample space using a tree diagram.

Tossing a coin twice	 Tossing the coin twice
Tossing coin thrice	 Tossing the coin three times

Example

Two students were asked to list their choice of ice cream flavor from among vanilla, chocolate, and strawberry. List the sample space showing the possible outcomes.

Ice creams

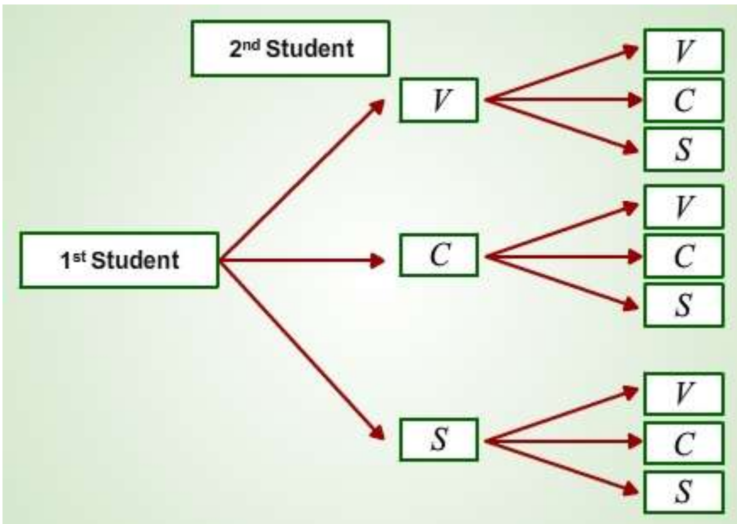
Solution:

The experiment comprises asking people to express their choice among three flavors of ice cream.

Define the possible outcomes for one trial of the experiment. Let,

V = Vanilla
C = Chocolate
S = Strawberry

Tree diagram:



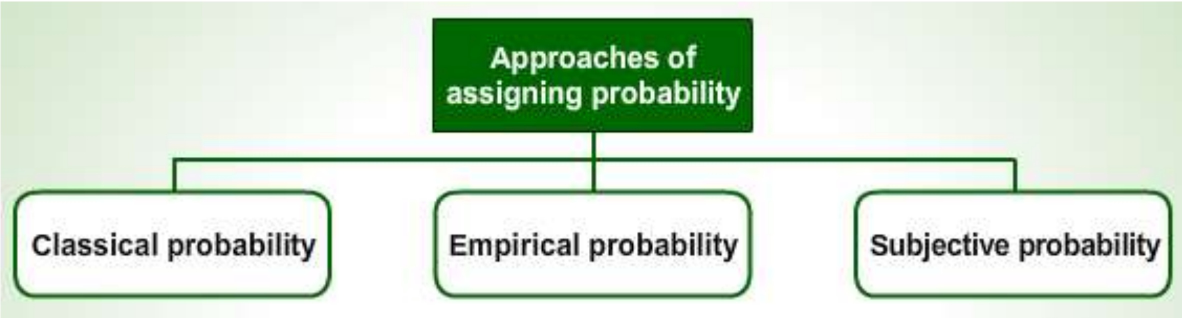
Tree diagram

Then the sample space is:

$$\Omega = \{V\,V, V\,C, V\,S, C\,V, C\,C, C\,S, S\,V, S\,C, S\,S\}$$

Approaches of Assigning Probability

Let's look at the approaches of assigning probability. They are classical probability, empirical probability, and subjective probability.

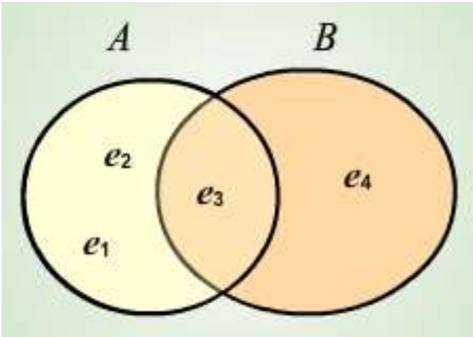


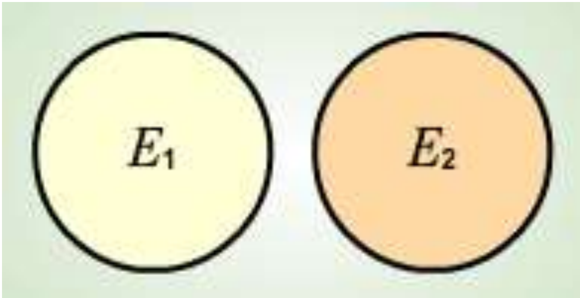
Approaches of assigning probability

Classical probability	Assuming all the outcomes are equally likely, the chance of occurrence of the event is defined by: $$\text{Probabiltiy of occurrence for event } E = \frac{\text{Number of elements in } E}{\text{Total number of elements in } \Omega}$$ Here, Ω is a finite sample space.
Empirical probability	It is also known as Relative Frequency of Occurrence. The outcomes are based on observed data. Assuming all the outcomes are equally likely, the chance of occurrence of the event is defined by: $$\text{Probabiltiy of occurrence} = \frac{\text{frequency}}{\text{Total frequency}}$$
Subjective probability	It is based on a combination of an individual's past experience, personal opinion, and analysis of a particular situation.

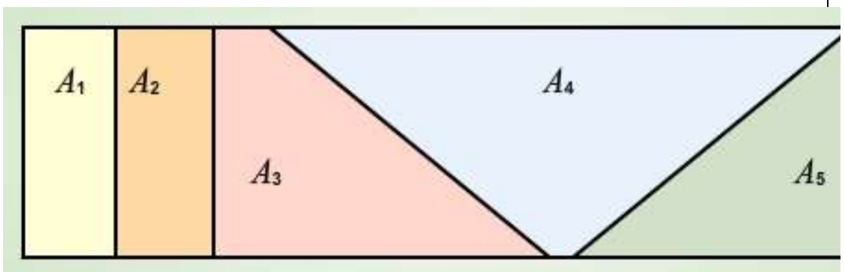
Types of Events

The different types of events are:

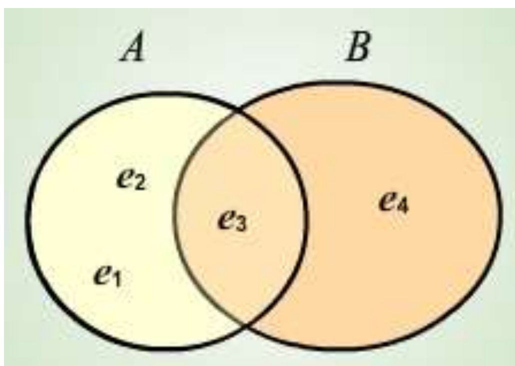
<table>
<tr><td>Intersection of events</td><td>If the elementary outcome e_i occurred and it belongs to the events A and B, then we say both A and B occurred. That is, the intersection of events A and B is the event that occurs when both A and B occur.


Intersection of the two events A and B is $\{e_3\}$ A and B or $A \cap B = \{e_3\}$.
Intersection of events</td></tr>
<tr><td>Mutually exclusive events</td><td>The events E_1 and E_2 are mutually exclusive if they cannot occur together or simultaneously.


Mutually exclusive events
• If the event E_1 occurs, then the event E_2 cannot occur
• The events E_1 and E_2 have no common elements</td></tr>
</table>

<table>
<tr><td>Collectively exhaustive events</td><td>One of the events must occur and the set of all events together equals the entire sample space.


Collectively exhaustive events

The events A_1 to A_5 are mutually exclusive and collectively exhaustive.</td></tr>
<tr><td>Independent and Dependent events</td><td>• Two events are independent if the occurrence of one does not affect the occurrence of the other. That is, it does not affect the probability of the other
• Two events are dependent if the occurrence of one affects the probability of the other.</td></tr>
<tr><td>Union of events</td><td>If the elementary outcome e_i occurred and it belongs to the events A or B or both, then we say both A or B occurred. That is, the union of events A and B is the event that occurs when at least A or B occur.


Union of the two $\{e_1, e_2, e_3, e_4\}$

$(A \text{ or } B) = A \cup B$

Union of events</td></tr>
</table>

Example

Consider the two events given here.

$$E_1 = \{e_1, e_3, e_5, e_7\}, \quad E_2 = \{e_2, e_4, e_6, e_8\}$$

Are the two events mutually exclusive?

- **Yes**
- No

Example

Assume that someone is drawing one card from a deck of cards. Events are:
A = aces; *B* = black cards; *C* = diamonds; *D* = hearts

1. Are all events collectively exhaustive?
2. Are all events mutually exclusive?
3. Are the events *B*, *C* and *D* collectively exhaustive?
4. Are the events *B*, *C* and *D* mutually exclusive?

Solution:

1. Events *A*, *B*, *C*, and *D* are collectively exhaustive since one of the events must occur or all together give the sample space.
2. Events *A*, *B*, *C*, and *D* are not mutually exclusive since an ace may also be a heart or a diamond.
3. Events *B*, *C*, and *D* are collectively exhaustive since one of the events must occur or all together give the sample space.
4. Events *B*, *C*, and *D* are mutually exclusive since there is no intersection between the events.

Example

The table given here shows number of sophomore and junior students in KFUPM from the Eastern province and not from the Eastern province.

	Eastern province	**Not from Eastern province**	**Total**
Sophomore	256	940	1196
Junior	340	784	1124
Total	596	1724	2320

If a student is randomly selected:

1. The probability that the student is from the eastern province = 596 / 2320 = 0.26
2. The probability that the student is sophomore = 1196 / 2320 = 0.52

Recap

In this lecture, you have learned that:

- Probability is a numeric value between 0 and 1 representing the chance or possibility that an uncertain event will occur
- The basic terms associated with probability are random experiment, elementary outcome, sample space, event, and elementary event
- The approaches of assigning probability are classical probability, empirical probability, and subjective probability
- The different types of events are intersection of events, mutually exclusive events, collectively exhaustive events, and independent and dependent events