Learning Outcome

By the end of this lecture, you will be able to:

- Define Continuous Random Variable
- Calculate the probabilities of a Continuous Random Variable

Introduction

Consider the chart given here shows average temperatures recorded in the Dhahran in a year. The graph depicts continuous curve. It implies that temperature can acquire any value between a certain maximum and minimum values. This is an example of continuous random variable.

Dhahran, Saudi Arabia climate graph

Continuous random variables can potentially take on any value, depending only on the ability to measure accurately. In this lecture, we will learn about continuous random variables in detail. We will also learn how to calculate the probabilities of a continuous random variable.

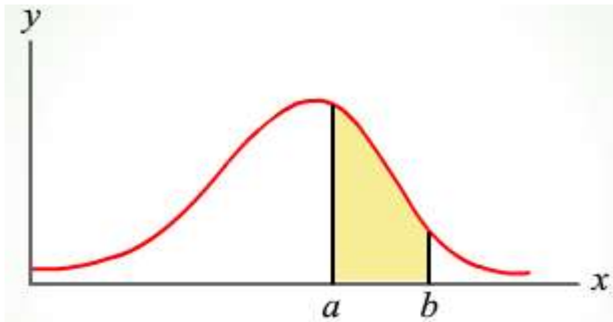
Probability Density Function (PDF)

Every continuous random variable (X) has a curve associated with it. This curve is formally known as a Probability Density Function (*PDF*), and it can be used to obtain the probabilities associated with the random variable.

Assume X is continuous random variable then the probability that X lies between **a** and **b** is equal to the area under the curve.

$$P(a \le X \le b) = \int_a^b f(x)\,dx = \text{area under the curve between } a \text{ and } b$$

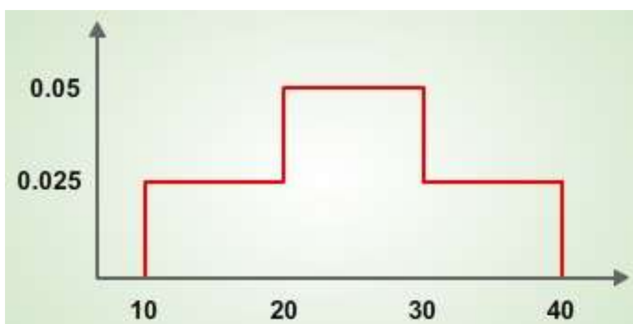
$$\text{The total area under the curve} = \int_{\text{all } x} f(x)\,dx = 1$$



Probability Density Function (PDF)

Example

Given here is an example of a probability density function. Suppose that the number of minutes of playing time of a certain college basketball player in a randomly chosen game has the given density curve



Example of a probability density function

a. Does the curve represent a probability density function?

If the area under the curve is 1, if is a PDF.
So,

$$area = (20 - 10)(0.025) + (30 - 20)(0.05) + (40 - 30)(0.025) = 1$$

b. Find the probability that the player plays over 20 minutes

$$P(X > 20) = 1 - P(X < 20) = 1 - (20 - 10)(0.025) = 0.75$$

c. Find the probability that the player plays less than 25 minutes

$$P(X < 25) = P(20 < X < 25) = (25 - 20)(0.025) = 0.125$$

d. Find the probability that the player plays between 15 and 35 minutes

$$\begin{aligned} P(15 < X < 35) &= P(15 < X < 20) + P(20 < X < 30) + P(30 < X < 35) \\ &= (20 - 15)(0.025) + (30 - 20)(0.05) + (35 - 30)(0.025) \\ &= 0.75 \end{aligned}$$

Example ...Contd.

If $f(x) = a(3x - 2)$,$2 < x < 12$ is a probability density function (PDF), then find ‘a’.

Solution:

To find a P(2<X<12) = 1

$$\int_2^{12} a(3x - 2)\, dx = 1$$

$$a\left(\frac{3x^2}{2} - 2x\right)_2^{12} = 1$$

$$a(192 - 2) = 1$$

$$190a = 1$$

$$a = \frac{1}{190}$$

Recap

In this lecture, you have learned that:

- A continuous Random Variable is continuous if its range is an interval. It can potentially take on any value, depending only on the ability to measure accurately

- Every continuous random variable X has a curve associated with it. This curve is formally known as a Probability Density Function (*PDF*), and it can be used to obtain the probabilities associated with the random variable