Learning Outcome

By the end of this lecture, you will be able to:

- Form and interpret a confidence interval estimate for a single population proportion
- Determine the required sample size to estimate a single population proportion within a specified margin of error

Introduction

How to determine the required sample size needed to estimate a population proportion, when the margin error and the confidence level are given?

In this lecture, we will learn how to calculate the interval estimate and how to determine the required sample size for estimating the population proportion.

Interval Estimate > Calculation

An interval estimate for the population proportion $p = \frac{X}{N}$ can be calculated by adding a margin of error to the sample proportion $\bar{p} = \frac{x}{n}$.

Recall that:

The distribution of the sample proportion is approximately normal, if the sample size is large "enough" so that $n\bar{p} \geq 5$
and $n(1 - \bar{p}) \geq 5$

, with a

1. mean $\mu_{\bar{p}} = p$ and
2. standard deviation $\sigma_p = \sqrt{\frac{p(1-p)}{n}}$

But $p = \frac{X}{N}$ is unknown, so the standard deviation will be estimated by the sample data

$$s_p = \sqrt{\frac{\bar{p}(1 - \bar{p})}{n}} \quad (1)$$

Then, A large sample 100(1-α) % Confidence Interval for the population proportion *p* is given by:

$$\bar{p} \pm z_{\alpha/2} = \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} \qquad (2)$$

Determining the Required Sample Size

The required sample size for estimating the population proportion is determined by using the margin of error and the sample size. Click each term to see its details.

The margin of error	$e = \pm z_{\alpha/2}\sqrt{\frac{p(1-p)}{n}}$ **Note:** *p* can be estimated by a pilot sample, if necessary, or use *p* = 0.50.
The sample size	$n \geq \frac{z^2_{\alpha/2}\,p(1-p)}{e^2}$ Note: p can be estimated by a pilot sample, if necessary, or use p = 0.50.

Example

A random sample of 100 people shows that 25 are left-handed.

1. To construct a 94% confidence interval for the true proportion of left-handers:

$$\bar{p} = \frac{x}{n} = \frac{25}{100} = 0.25$$

$$n\bar{p} = 100(0.25) = 25 \geq 5 \ and$$

$$n(1-\bar{p}) = 100(0.75)75 \geq 5$$

$$\rightarrow A(1-\alpha)100\%\ C.I.\ for\ p\ is: \bar{p} \pm z_{\alpha/2}\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$1-\alpha = 0.94 \ \rightarrow \ z_{0.06/2} = z_{0.03} = 1.88$$

$$\rightarrow A\ 94\%\ \ C.I.\ for\ p\ is:$$

$$\bar{p} \pm z_{\alpha/2}\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.25 \pm 1.88\sqrt{\frac{0.25(1-0.25)}{100}}$$

$$= 0.25 \pm 0.0814 = [0.1686\ \ ,0.3314]$$

2. To construct a 94% confidence interval for the number of the left-handers:

$$[(0.1686\ )10000, (0.3314)10000] = [1686{,}3314]$$

Example

Determine the required sample size needed to estimate a population proportion, when the margin error ± 0.03, and the confidence level is 96%.

Solution :

$$n \geq \frac{z^2_{\alpha/2}\, p(1-p)}{e^2} = \frac{z^2_{0.04/2}\ 0.5(1-0.5)}{(0.03)^2}$$

$$= \frac{(2.05)^2(0.5)(1-0.5)}{(0.03)^2}$$

$$= 1167.36.11 \approx 1168$$

Recap

In this lecture, you have learned that:

- How to form and interpret a confidence interval estimate for a single population proportion
- How to determine the required sample size to estimate a single population proportion