Learning Outcome

By the end of this lecture, you will be able to:

- Define Exponential distribution.
- Calculate the probabilities of an Exponential Random Variable

Exponential Distribution

Exponential distribution can be used to measure the time that elapses between two occurrences of an event (the time between two successive arrivals), for example:

Time between trucks arriving at an unloading dock

Time between transactions at an ATM machine

Time between phone calls to the main operator

In this lecture, you will explore about the exponential distribution.

Probability Density Function

The probability density function given by:

$$f(x) = \lambda e^{-\lambda x}, \quad x \geq 0 \qquad (1)$$

Where,

- λ the mean time between events
- Mean $\mu = \frac{1}{\lambda}$
- Variance $\sigma^2 = \frac{1}{\lambda^2}$ and the standard deviation $\sigma = \frac{1}{\lambda}$

Probability Density Function > Determining Arrival Time

The different probabilities related to arrival time can be determined as given here:

- The probability that an arrival time is equal to or less than some specified time a is:

$$P(0 \leq x \leq a) = 1 - e^{-\lambda a} \qquad (2)$$

- The probability that an arrival time between two specified times b and a is:

$$P(b \leq x \leq a) = e^{-\lambda b} - e^{-\lambda a} (where\ b < a) \qquad (3)$$

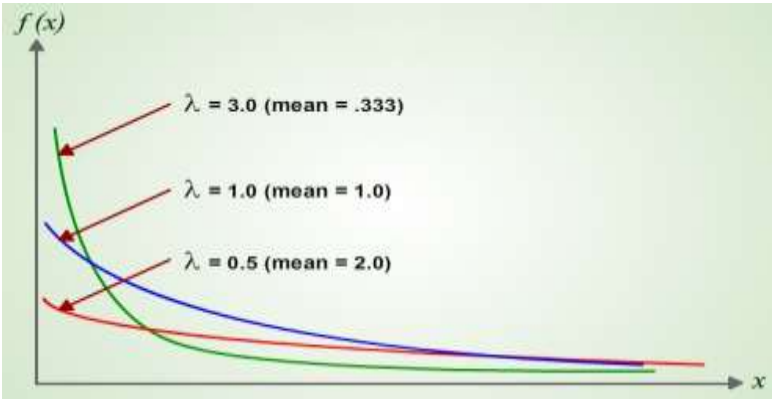
- The probability that an arrival time is greater than some specified time a is:

$$P(x > a) = e^{-\lambda a} \qquad (4)$$

Note: If the number of occurrences per time period is Poisson with mean λ, then the time between occurrences is exponential with mean time $1/\lambda$

Shape of the Exponential Distribution

For any exponential distribution, with density function $f(x)$, $f(0)=\lambda$ and as x increases, $f(x)$ approaches to zero.



Shape of the exponential distribution

Recap

In this lecture, you have learned that:

- Exponential distribution can be used to measure the time that elapses between two occurrences of an event