

4.1. Реализовать методы Эйлера, Рунге-Кутты и Адамса 4-го порядка в виде программ, задавая в качестве входных данных шаг сетки h . С использованием разработанного программного обеспечения решить задачу Коши для ОДУ 2-го порядка на указанном отрезке. Оценить погрешность численного решения с использованием метода Рунге – Ромберга и путем сравнения с точным решением.

№№	Задача Коши	Точное решение
1	$y'' + y - \sin 3x = 0$ Equation.3 , $y(0) = 1,$ $y'(0) = 1,$ Equation.3 $x \in [0,1], h = 0.1$	$y = \cos x + \frac{11}{8} \sin x - \frac{\sin 3x}{8}$
2	$y'' + y - 2 \cos x = 0$ Equation.3 , $y(0) = 1,$ $y'(0) = 0,$ Equation.3 $x \in [0,1], h = 0.1$	$y = x \sin x + \cos x$
3	$y'' - 2y - 4x^2 e^{x^2} = 0,$ $y(0) = 3,$ $y'(0) = 0,$ Equation.3 $x \in [0,1], h = 0.1$	$y = e^{x^2} + e^{x\sqrt{2}} + e^{-x\sqrt{2}}$
4	$x^2 y'' - x(x^2 - 1)y' - (x^2 + 1)y = 0,$ $y(1) = 1 + e^{1/2},$ Equation.3 $y'(1) = 2e^{1/2} - 1,$ $x \in [1,2], h = 0.1$	$y = \frac{1}{x}(1 + e^{x^2/2})$
5	$y'' - (1 + 2 \tan^2 x)y = 0,$ $y(0) = 1,$ $y'(0) = 2,$ Equation.3 $x \in [0,1], h = 0.1$	$y = \frac{1}{\cos x} + \sin x + \frac{x}{\cos x}$
6	$y'' + 4xy' + (4x^2 + 2)y = 0,$ $y(0) = 1,$ $y'(0) = 1,$ Equation.3 $x \in [0,1], h = 0.1$	$y = (1 + x)e^{-x^2}$
7	$y'' - 4xy' + (4x^2 - 2)y = 0,$ $y(0) = 1,$ $y'(0) = 1,$ Equation.3 $x \in [0,1], h = 0.1$	$y = (1 + x)e^{x^2}$
8	$y'' - 4xy' + (4x^2 - 3)y - e^{x^2} = 0,$ $y(0) = 1,$ Equation.3 $y'(0) = 0,$ $x \in [0,1], h = 0.1$	$y = (e^x + e^{-x} - 1)e^{-x^2}$

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9	$y'' - \left(\frac{1}{x^{1/2}}\right)y' + \left(\frac{1}{4x^2}\right)(x + x^{1/2} - 8)y = 0$ $y(1) = 2e$, Equation.3 $y'(1) = 2e$, $x \in [1, 2], h = 0.1$	$y = \frac{2}{e}x^2 + \frac{1}{x}e^{x^{1/2}}$
10	$y'' + y' \operatorname{tg}(x) + y \cos^2 x = 0$, $y(0) = 0$, Equation.3 $y'(0) = 1$, $x \in [0, 1], h = 0.1$	$y = \cos(\sin x) + \sin(\cos x)$
11	$y'' + y' \operatorname{tg} x - y \cos^2 x = 0$, $y(0) = 2$, Equation.3 $y'(0) = 0$, $x \in [0, 1], h = 0.1$	$y = e^{\sin x} + e^{-\sin x}$
12	$(x^2 + 1)y'' - 2xy' + 2y = 0$, $y(0) = 1$, Equation.3 $y'(0) = 1$, $x \in [0, 1], h = 0.1$	$y = x - x^2 + 1$
13	$y'' - 2(\operatorname{tg} x)y' - 3y = 0$, $y(0) = 1$, Equation.3 $y'(0) = 3$, $x \in [0, 1], h = 0.1$	$y = \cos^3 x + \sin x(1 + 2 \cos^2 x)$
14	$y'' + 2y' \operatorname{ctg} x + 3y = 0$, $y(1) = 1$, Equation.3 $y'(1) = 1$, $x \in [1, 2], h = 0.1$	$y = \frac{-0.9783 \cos 2x + 0.4776 \sin 2x}{\sin x}$
15	$xy'' + y' = 0$, $y(1) = 1$, Equation.3 $y'(1) = 1$, $x \in [1, 2], h = 0.1$	$y = 1 + \ln x $
16	$(x^2 - 1)y'' - 2xy' + 2y = 0$, $y(2) = 7$, Equation.3 $y'(2) = 5$, $x \in [2, 3], h = 0.1$	$y = x^2 + x + 1$
17	$xy'' - (x + 1)y' + y = 0$, $y(1) = 2 + e$, Equation.3 $y'(1) = 1 + e$, $x \in [1, 2], h = 0.1$	$y = x + 1 + e^x$

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18	$y'' - \frac{x+1}{x}y' - 2\frac{x-1}{x}y = 0,$ $y(1) = 1,$ $y'(1) = 1,$ $x \in [1, 2], h = 0.1$ Equation.3	$y = \frac{e^{2x}}{3e^2} + \frac{(3x+1)e^{-x}}{3e}$
19	$y'' + \frac{1}{x}y' + \frac{2}{x}y = 0,$ $y(1) = 1,$ $y'(1) = 1,$ $x \in [1, 2], h = 0.1$ Equation.3	$y = (\cos 2 - \sin 2)\cos(2x^{1/2}) +$ $(\cos 2 + \sin 2)\sin(2x^{1/2})$
20	$x(x-1)y'' + \frac{1}{2}y' - \frac{3}{4}y = 0,$ $y(2) = \sqrt{2},$ $y'(2) = \frac{3}{2}\sqrt{2},$ $x \in [2, 3], h = 0.1$ Equation.3	$y = x ^{3/2}$
21	$x^2y'' - 12y = 0,$ $y(1) = 2,$ $y'(1) = 1,$ $x \in [1, 2], h = 0.1$ Equation.3	$y = x^4 + x^{-3}$
22	$x^2y'' + (x^2 - 2)y = 0,$ $y(1) = 1,$ $y'(1) = 0,$ $x \in [1, 2], h = 0.1$ Equation.3	$y = \sin(x-1) + \frac{1}{x}\cos(x-1)$
23	$x^2y'' + xy' - y - 3x^2 = 0,$ $y(1) = 3,$ $y'(1) = 2,$ $x \in [1, 2], h = 0.1$ Equation.3	$y = x^2 + x + \frac{1}{x}$
24	$x^2y'' + (x+1)y' - y = 0,$ $y(1) = 2 + e,$ $y'(1) = 1,$ $x \in [1, 2], h = 0.1$ Equation.3	$y = x + 1 + xe^{1/x}$
25	$(x-2)^2y'' - (x-2)y' - 3y = 0,$ $y(3) = 2,$ $y'(3) = 2,$ $x \in [3, 4], h = 0.1$ Equation.3	$y = (x-2)^3 + \frac{1}{x-2}$

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26	$x^4 y'' + 2x^3 y' + y = 0,$ $y(1) = 1,$ $y'(1) = 1,$ $x \in [1, 2], h = 0.1$ Equation.3	$y = (\sin 1 + \cos 1) \cos \frac{1}{x} + (\sin 1 - \cos 1) \sin \frac{1}{x}$
27	$x^2 y'' - 2xy' + (x^2 + 2)y = 0,$ $y(p/2) = p/2,$ $y'(p/2) = 1 - p/2,$ $x \in [p/2, p/2 + 1], h = 0.1$ Equation.3	$y = x \cos x + x \sin x$
28	$x^2 y'' + 3xy' + 4y - 5x = 0,$ $y(1) = 6,$ $y'(1) = 8,$ $x \in [1, 2], h = 0.1$ Equation.3	$y = 5x + x^2 + x^2 \ln x $
29	$x^2 y'' - 3xy' - 5y - x^2 \ln x = 0,$ $y(1) = 1,$ $y'(1) = 1,$ $x \in [1, 2], h = 0.1$ Equation.3	$y = \frac{19}{54} x^5 + \frac{35}{54} \frac{1}{x} - \frac{x^2}{9} \ln x $
30	$x^2(x+1)y'' - x(2x+1)y' + (2x+1)y = 0$ $y(1) = 2,$ $y'(1) = 4,$ $x \in [1, 2], h = 0.1$ Equation.3	$y = x^2 + x + x \ln x$

