Network Simulation

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Gist:

https://gist.github.com/Halkenhaeusser/56b67cf4b04f6d0ce8753f83f6bb2392

Part 1 – Proposed Changes

1. Charm

   Each person has a charm attribute normally distributed around a mean of 0.5 with standard deviation of 0.1. The opinion is of a node is affected by the ratio of the other node's charm and their own charm. The weight is affected only by the node's charm to simulate how likely charismatic persons are to maintain connections.

2. Varying opinions

   Each person has a second opinion (creatively named opinion2) that is distributed uniformly between 0 and 1, i.e., the same way the first opinion was distributed. The topic of interaction between two nodes is chosen at random. The weight is adjusted using their mean opinions.

Part 2 – Local Analysis

Charm

Two nodes

When there are two nodes with one edge the change of their opinion changes will depend largely on the ratio of their charm. If the nodes have large charm differences e.g. 0.7 and 0.3 (two standard deviations from the mean) then the opinion change of the less charming person will be changed by a factor of 2.33 versus 0.429 for the more charming person compared to the original model. This means that the less charming node's opinion moves closer to the "common ground" at a faster rate. Consequently, the opinion of the more charming node will dominate the opinion change.

The weight of their connection will now change at a rate dependent also on their specified charm. Each iteration the weight change will be offset by the charisma of the node. A more charismatic nodes weight will usually increase the weight as charm of 0.5 positively offsets the weight and vice versa for the less charismatic nodes.

Formulae

The rules of the model are summarized in the two equations that change the opinion and weight.

$$\Delta o_i = \alpha * w_{ij} * (o_i - o_j)$$

$$\Delta w_{ij} = \beta w_{ij}\left(1 - w_{ij}\right)\left(1 - \gamma\left|o_i - o_j\right|\right)$$

The charm c of each node I and that of its neighbor j changes the opinion and weight equations through their ratio in the opinion equation and through a change in gamma in the weight equation.

$$\Delta o_i = \frac{c_j}{c_i}\alpha * w_{ij} * (o_i - o_j)$$

$$\Delta w_{ij} = \beta w_{ij}\left(1 - w_{ij}\right)\left(1 - (\gamma - (\gamma * c_i)\left|o_i - o_j\right|\right)$$

Hence, the opinion is changed depending on the charm of the own node but also that of the neighbor meaning it merely scales alpha (the rate at which nodes change their opinion) but does so in a way that is centered around mean of one (as the charm values are normally distributed around 0.5) thus not changing the overall pattern of opinion changes observed in the original model. The opinions will converge whenever alpha the rate of opinion change is smaller 1 (i.e. $\frac{c_j}{c_i}\alpha \leq 1$). The highest likely value for the ratio would be an uncharismatic node meeting a node with high charisma (at 3 standard deviations (i.e., probability 0.3%) high = 0.8 and low = 0.2) the ratio will be 4. Hence the value of alpha should be at most 0.25 for there to be an opinion change that is leads to convergence of opinion. A larger value for alpha could in an extreme case of w and opinion difference both being one would lead to an overshoot of Δo that leads to o_i changing so much that it goes beyond the value of o_j in the direction of change.

The weight change is being affected only by the current node's charm c_i (Note that in the implementation, the opinion o_i and w_{ij} are updated synchronously and hence the opinion change does not affect the weight change). Weights will always increase when the gamma term $(\gamma - (\gamma * c_i) \leq 1$ because it will make the term positive under all circumstances. All other terms in the weight change equation are nonnegative. The $(\gamma - (\gamma * c_i)$ could be below one if the charm value is at the upper end of the charm spectrum i.e., 0.8. Hence a reasonable value for gamma will be:

$$(\gamma - (\gamma * 0.8) > 1$$

$$0.2\,\gamma > 1$$

$$\gamma > 5$$

This would mean that the weight change is still moderated by the difference in opinion in almost all cases. If a node has charm higher than 0.8, it is a reasonable assumption that they will always strengthen their connections because they are so charismatic. By decreasing the value of gamma, the thresholds for node's charisma to lead to unconditional increase in weights can be adjusted. With charm being distributed around 0.5, the absolute lowest bound would be 2. In that case all nodes with above average charm will always have increasing weight.

To summarize, the values for alpha, beta and gamma will be alpha = 0.03 (as determined in the original simulation as a value that shows the real-life macro features), beta = 0.3 (also unchanged as it is just a scaling factor), and gamma = 5.

Vector fields

For various charm factors, the vector fields will be shown. For the opinion change equation, the neighbor's charm is taken to be the mean charm value. Figures 1, 2, and 3 show a low, mean, and high charm (See code output for smaller incremented charm changes).

The figures show the desire, change in weights depending on charm. Nodes with higher charm are less likely to have their edges be removed than those with low charm as seen easily by the increasing green proportion of paths that signals a smaller ratio of opinion differences leading to edges ultimately being removed. Hence, they show that charismatic people are less likely to lose connections and therefore influence the system more with their general opinion (especially since their opinion is less likely to change also).

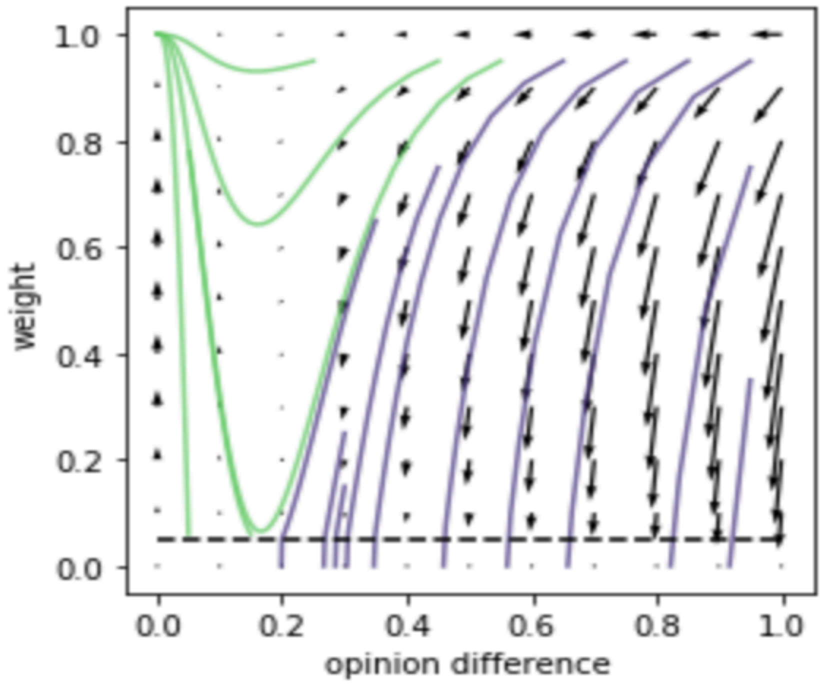

Figure 1. *Vector field at charm = 0.3 (low charm).*

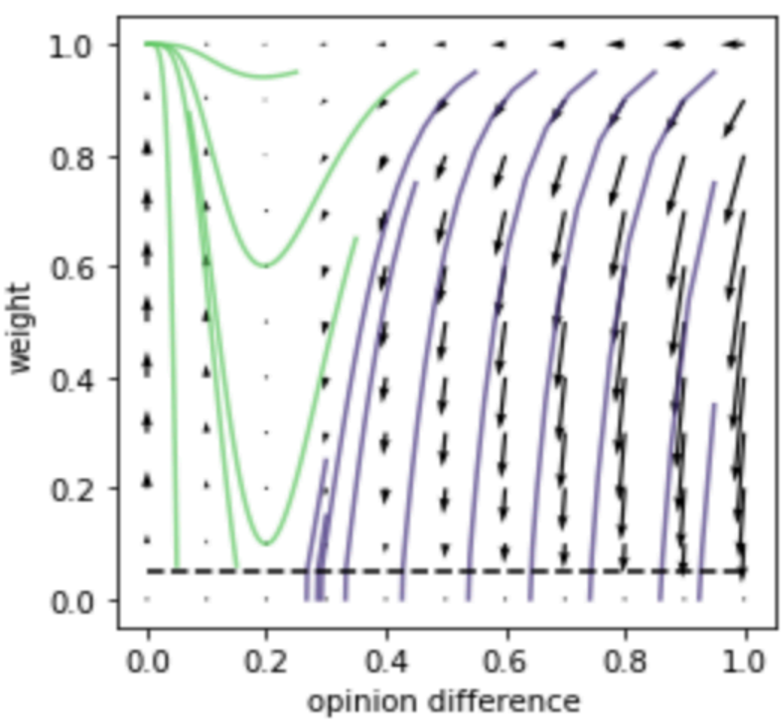

Figure 2. *Vector field at charm = 0.5 (mean charm).*

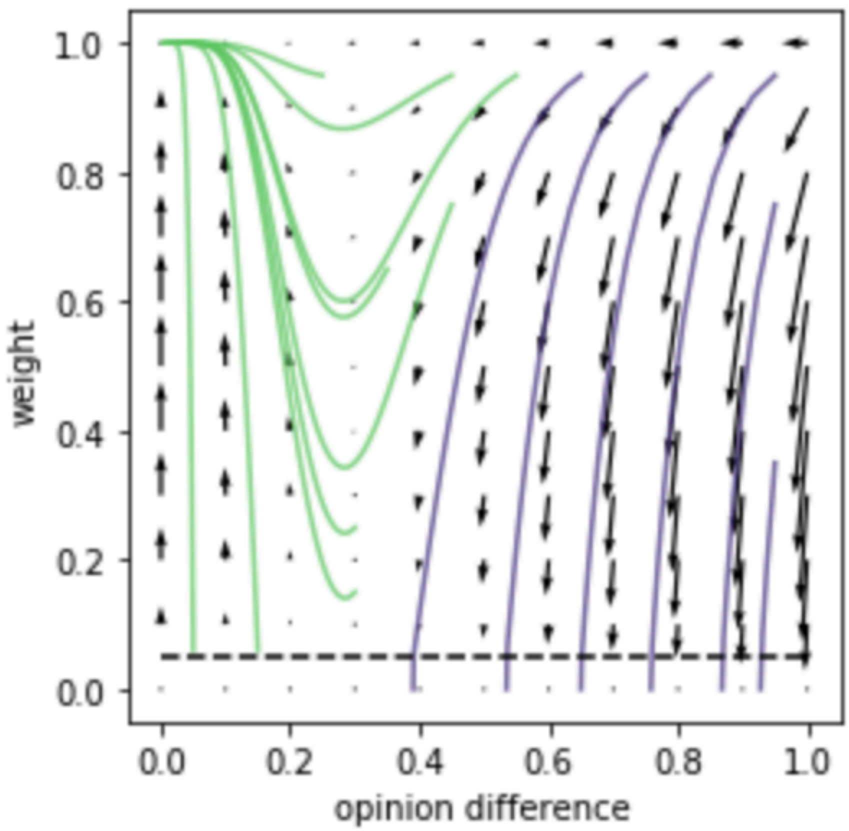

Figure 3. Vector field at charm = 0.8 (high charm).

2 opinions

Two nodes

With increasing number of opinions, the likelihood of two nodes having close mean opinion increases. I.e., from the start the entire system is more homogeneous. When two nodes interact the topic of interaction is chosen at random and will converge together in all cases (as the original equation will be used where opinions will converge). The more opinions there are, the smaller the impact of a single interaction on the mean opinion of that node will as there are more other opinions. Since the mean opinion will be used to adjust the edge weight, this change will in most cases be relatively small as the mean opinions have been initialized to be similar. Hence, the opinion change is relatively small, and it is likely to take more iterations for the edge wait to be decreased to the 0.05 threshold than the opinion difference to be small enough for the weights to increase again. Hence, a convergence of the mean opinions of two nodes becomes more likely the more opinions are added.

Formulae

The rules of opinion change are not modified except that the opinion o_{ir} to be discussed is chosen at random from a pool of opinions.

$$\Delta o_i = \alpha * w_{ij} * (o_{ir} - o_{jr})$$

Opinions will converge as long as alpha will not lead to the opinion change to overshoot which only occurs when alpha is larger 1 since that is the only case in which the opinion change is scaled up as weight is between 0.05 and 1. Hence, alpha will be keep the same as for the original simulation.

The weight change opinion is slightly modified to be the mean of all of a nodes' opinions where a node has opinions 1, …, n.

$$\Delta w_{ij} = \beta w_{ij}(1 - w_{ij})\ (1 - \gamma\ \left|\frac{1}{n}\sum_{x=1}^{n} o_{ix} - \frac{1}{n}\sum_{x=1}^{n} o_{jx}\right|)$$

As explained above, the weight change is going to be less extreme as the mean opinions are likely to be close together. The weight will increase for all values of $\gamma \left|\frac{1}{n}\sum_{x=1}^{n} o_{ix} - \frac{1}{n}\sum_{x=1}^{n} o_{jx}\right| < 1$ and decrease for $\gamma \left|\frac{1}{n}\sum_{x=1}^{n} o_{ix} - \frac{1}{n}\sum_{x=1}^{n} o_{jx}\right| > 1$. Hence, gamma will be increased to 3 + n to buffer for each extra opinion leading to more converging means. The parameters for alpha and beta will remain unchanged.

Part 3

The experimentation will be carried out using the Watts-Strogatz graph (WS) rather than a Barabasi-Albert graph (BA) for initialization because preferential attachment in the BA graph would detract from the effect of the charm factor in the first modification. WS, while still exhibiting small world properties, does not use preferential attachment. For sake of comparing both modifications, WS will be used for initialization in both modifications.

The real outcome of interest are the macro-features of the graph. Are there clusters of opinions and how connected are these clusters? These macro features are emergent properties that arise out of the interaction of the nodes and cannot be determined given the initial parameters as there are non-linear aspects (e.g. which random edge was chosen to compare). However, they can be identified analyzing large scale features. Further of interest on a local level is how similar the opinions of a given node are to that of their neighbors. This metric assesses how likely people are to self-select themselves into echo chambers of just their own opinion.

Clustering can be observed by comparing the distributions of opinions. If there is multimodality in the distribution, the opinions are clustered.

The local homogeneity is measured by looking each node's neighbors and taking the average of the absolute difference in opinion between the node and its neighbors. The absolute difference is needed to avoid opinion differences cancelling each other out. This average disagreement is then averaged for all nodes and recorded after every iteration.

The global connection of hubs of nodes (let them by hypernodes) is measured in the two-cluster example by dividing the nodes into two groups of above and below the mean opinion in the network. Then the number of edges between each hypernode is counted. The more edges the more connected the hypernodes. This approach has the limitation of not being able to identify more than a bimodal distribution (a possibility to extend this metric tool).

Part 4

Charm

The network with charms goes through separation and then reconnection within 30,000 iterations as shown visually in Fig. 4. The local disagreement drops drastically in the first iterations and then decreases at a slower rate to approach full local homogeneity (Fig. 5). The global separation into two hypernodes is shown by the decrease in edges for the first 5000 iterations. As seen in the other two visualizations, the hypernodes eventually merge back together into one cluster and the number of connections increases sharply (Fig. 6). The convergence can also be seen as the two clusters approach each other in the histograms.

This outcome is expected as the charismatic nodes' connections are robust against the separation of the clusters and converge the cluster opinions enough that a threshold of weights reducing is passed that leads to all opinions suddenly converging. This is realistic to the extent that charismatic leaders are able to unify separated groups. This analysis hence shows that adding leaders to the model will result in a more homogeneous system in the long run.

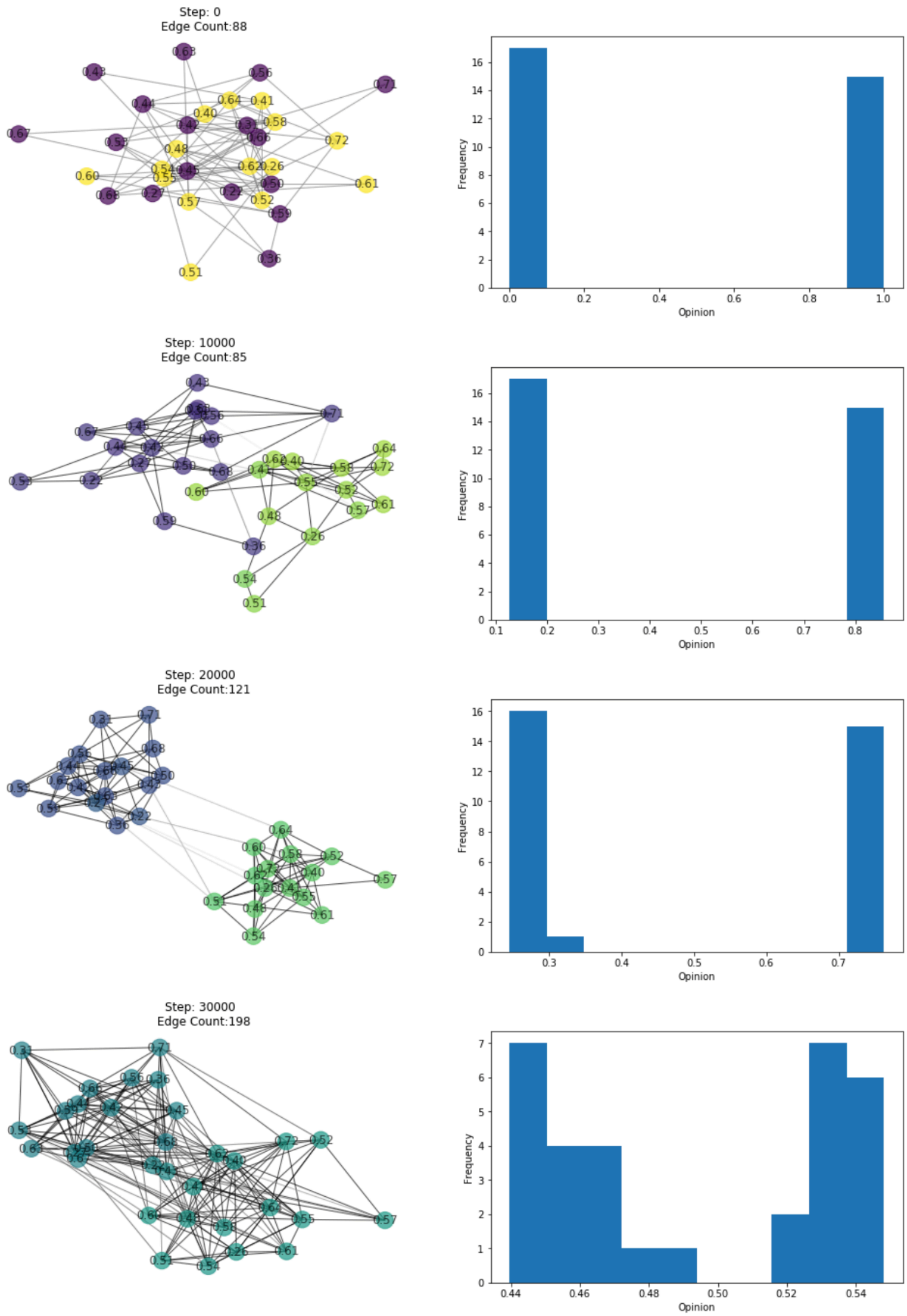

Figure 4. The graph and its corresponding histogram for 30000 iterations at the specified values.

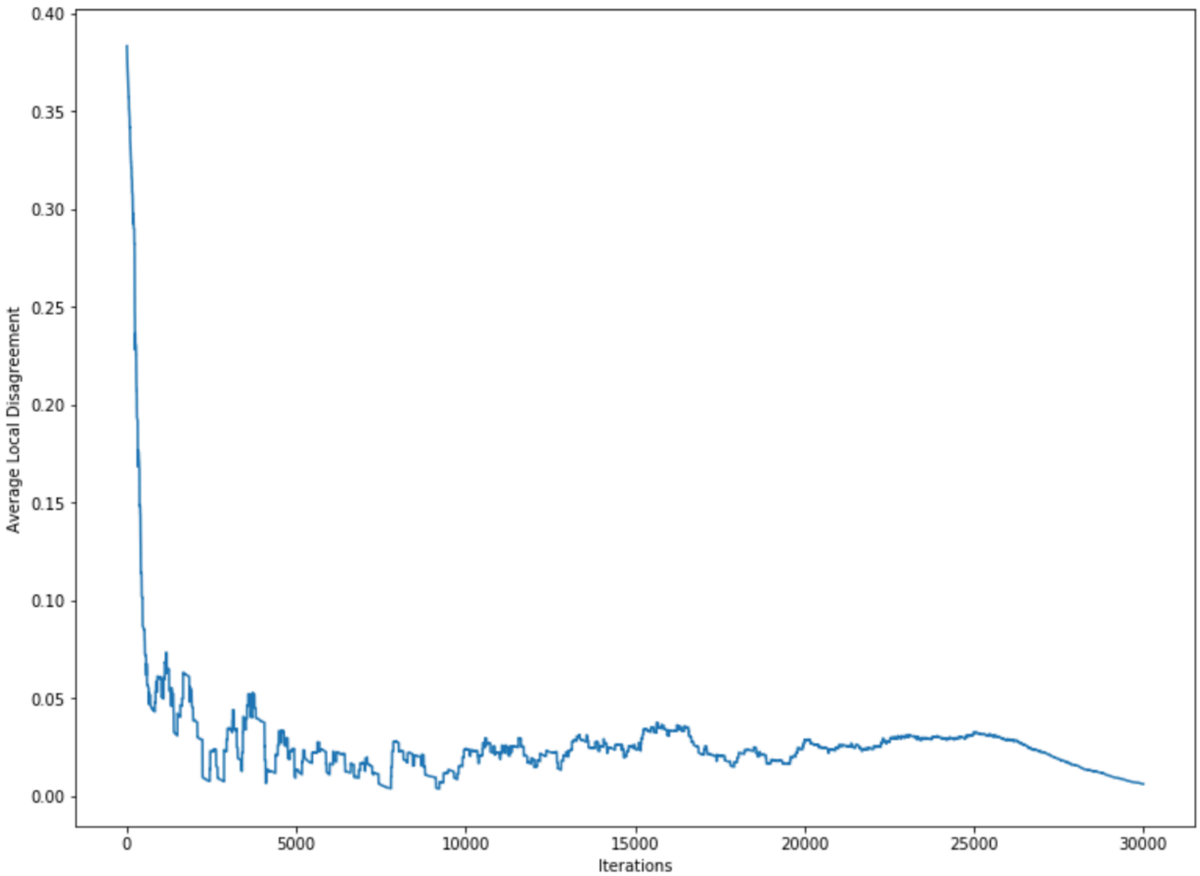

Figure 5. *Local disagreement decreases rapidly. The initial disagreement dissipates quickly as nodes self-select their neighborhoods. Once the subgroups merge back together the mean decreases further and smoothly (at around 25000 iterations) as nodes homogenize into one cluster.*

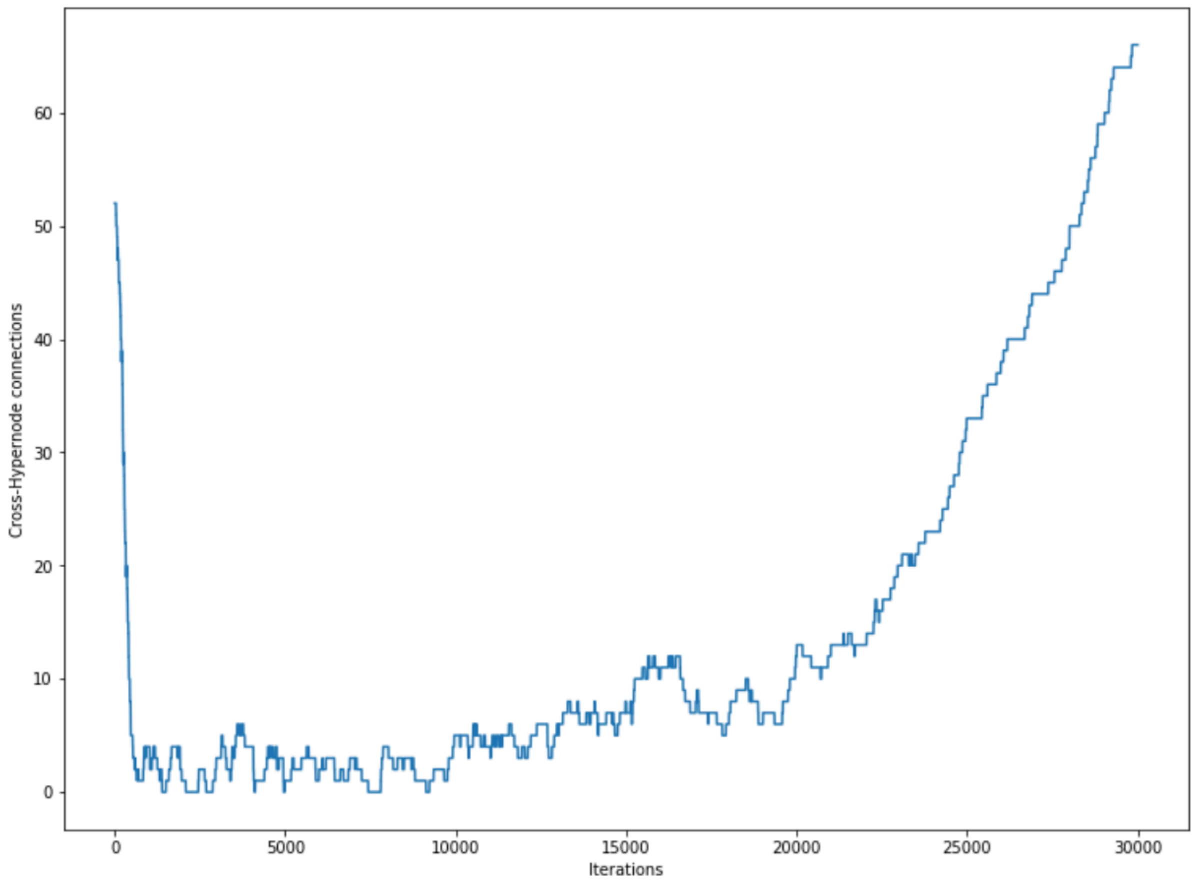

Figure 6. Global hub connections between the two hypernodes over 30000 iterations. The network separates and reconnects.

Multiple opinions

For multiple opinions we can observe that each opinion increases the number of clusters that form. Since gamma is increased with each individual opinion, the effect of the closer means is counteracted. This can be seen on just the two examples of having one opinion (Fig. 7), five opinions (Fig. 8) and ten opinions (Fig. 9). The histograms in the figure reveal the high-level formation of clusters of opinions. The more opinions there are the more clusters form at some point and the more non-deterministic is that cluster formation because there are more opinions that can be randomly drawn and affect the mean. Ultimately, however, these clusters converge back towards the mean as there will always be random nodes added throughout the simulation that mediate the difference in opinions. As we see multiple opinion clusters with non-deterministic boundaries the hypernode algorithm used for the charm-analysis is not applicable anymore.

This is also realistic as people will not cut the relationship with someone completely because of one disagreement. However, an extreme disagreement on a topic will significantly impact the mean and change it so that the connection is weakened. The increase in gamma makes a slight difference in the mean more pronounced.

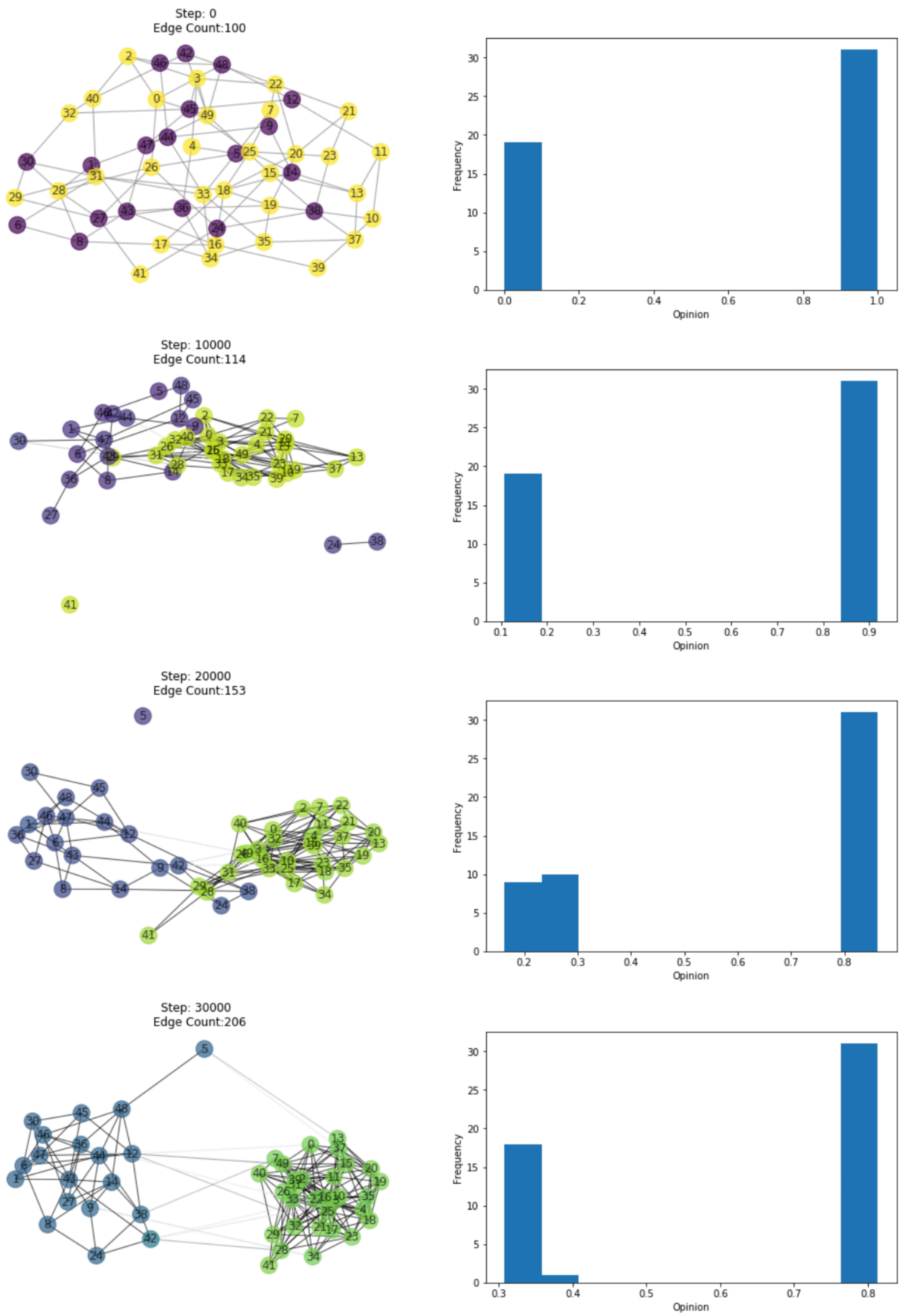

Figure 7. *Base case of one opinion. There is separation between the groups that decreases as means move closer together.*

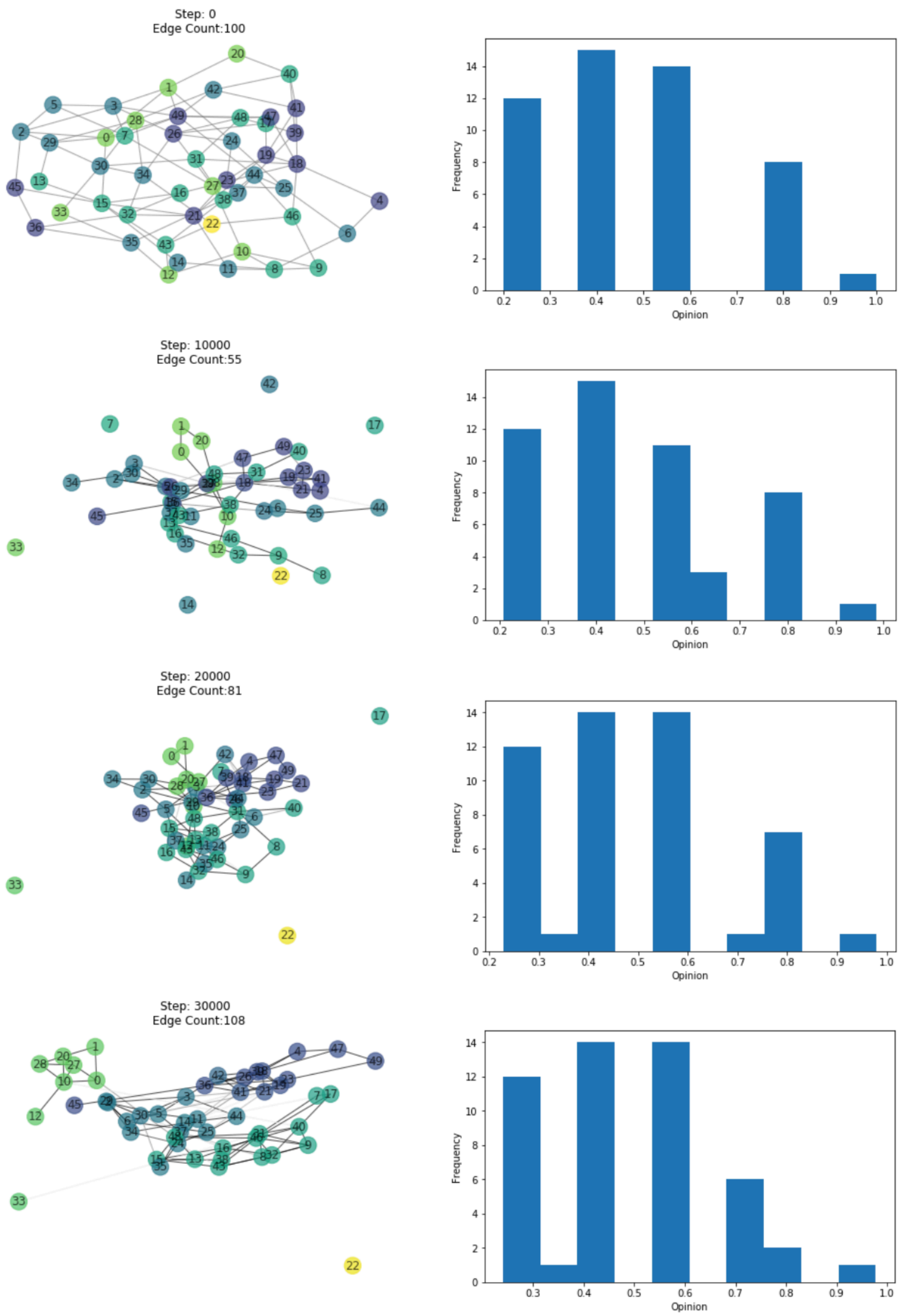

Figure 8. *Five opinions separating into five different groups over time. The histograms to the right of each graph show the intensifying modularity of the distributions*

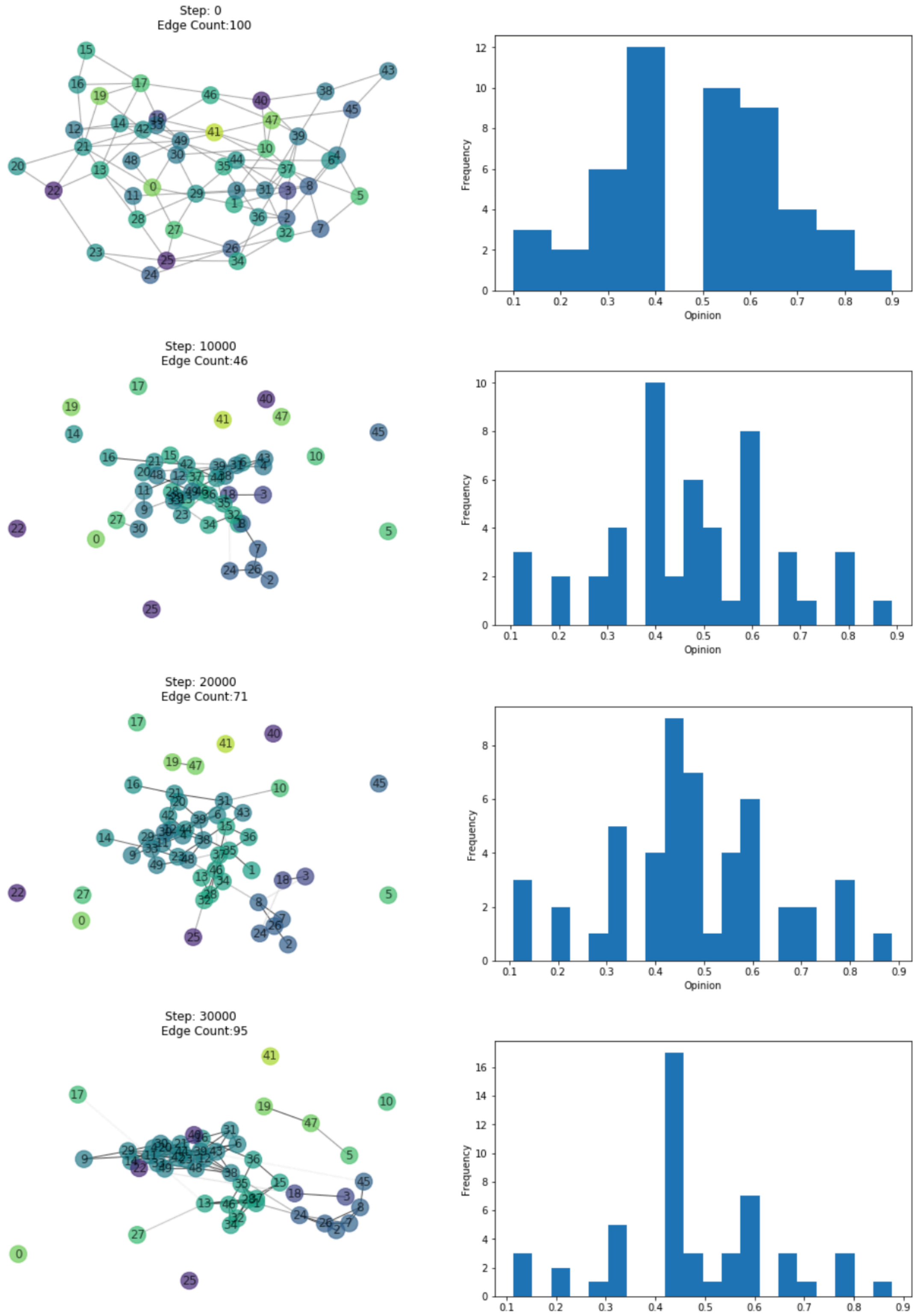

Figure 9. 10 opinions lead to the system split into six different subcamps.

HCs and LOs

#networkmodeling:

I accurately adapt the model to implement the given situation (i.e., charm or multiple opinions) and justify in depth of why and how I change my equations the way I did in Part 2 by referring to the real-life aspects the model is trying to capture. For example, I adjust alpha using the ratio of how convincing someone is and add a drift to the weight change or using a normal distribution to show charisma and adjusting the standard deviation fit the model I am using.

#networkanalysis:

Throughout Part 2 I give a detailed discussion of what I think the two proposed changes will change on the individual node level. Because the macro analysis is of real interest here, for Part 3/4 I implement a novel analysis algorithm to estimate the degree between two hypernodes that can be extended to further cluster sampling.

#systemdynamics:

Part 2 is a thorough analysis of how the different parameters will affect the dynamics of the system and how e.g. a change in opinion is dependent on the various terms that are part of the model. Part 4 visualizes these dynamics and shows how these parameters translate to real life and different aspects of the system (e.g. hypernodes in the charm-modification).

#emergentproperties:

In my analysis in Part 3 and 4 I discuss and identify how and why emergent properties arise in the network and discuss e.g. how the emergence of hypernodes leads to a convergence of opinions within those hypernodes and how the random edge or opinion drawing provides a non-deterministic aspect to the system.