EXTENSION OF FIBER BUNDLE MODELS FOR CREEP RUPTURE AND INTERFACE FAILURE

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Abstract. We present two extensions of the classical fiber bundle model to study the creep rupture of heterogeneous materials and the shear failure of glued interfaces of solid blocks. To model creep rupture, we assume that the fibers of a parallel bundle present time dependent behaviour under an external load and fail when the deformation exceeds their local breaking threshold. Assuming global load sharing among fibers, analytical and numerical calculations showed that there exists a critical load below which only partial failure occurs while above which the system fails globally after a finite time. Approaching the critical point from both sides the system exhibits scaling behaviour which implies that creep rupture is analogous to continuous phase transitions. To describe interfacial failure, we model the interface as an array of elastic beams which experience stretching and bending under shear load and break if the two deformation modes exceed randomly distributed breaking thresholds. The two breaking modes can be independent or combined in the form of a von Mises type breaking criterion. In the framework of global load sharing, we obtain analytically the macroscopic constitutive behaviour of the system and describe the microscopic process of the progressive failure of the interface.

Keywords: fiber bundle, creep rupture, beam, interface failure.

1.Introduction

The failure of heterogeneous materials under various types of external load has attracted continuous interest during the last two decades [1,2]. One of the most important approaches to the fracture and damage problem is the well-known Fiber Bundle Model (FBM), which was introduced a long time ago by Daniels [3] and has been the subject of extensive research over the past years [4-14]. These models consist of a set of parallel fibers having statistically distributed strengths. The sample is loaded parallel to the fibers' direction, and the fibers fail if the load on them exceeds their threshold value. In stress controlled experiments, after each fiber failure the load carried by the broken fiber is redistributed among the intact ones. The behaviour of a fiber bundle under external loading strongly depends on the range of interaction, *i.e.* on the range of load sharing among fibers. Exact analytic results on FBM have been achieved in the framework of the mean field approach, or global load sharing, which means that after each fiber breaking the stress is equally distributed on the intact fibers implying an infinite range of interaction and a neglect of stress enhancement in the vicinity of failed regions [3-11]. In spite of their simplicity, FBMs capture the most important aspects of material damage and they provide a deep insight into the fracture process. Based on their success, FBMs have served as a starting point for more complex models like the micromechanical models of the failure of fiber reinforced composites [14,15,16,17]. Over the past years several extensions of FBM have been carried out by considering stress localization (local load transfer) [15,16,17], the effect of matrix material between fibers [14,15,16,17], possible non-linear behaviour of fibers [18,21], coupling to an elastic block [19], and thermally activated breakdown [20].

Under high steady stresses, materials may undergo time dependent deformation resulting in failure called creep rupture, which determines the long time performance of construction elements. Creep failure tests are usually performed under uniaxial tensile loading when the specimen is subjected to a constant load σ_o and the time evolution of the damage process is followed by recording the strain ε of the specimen and the acoustic signals emitted by microscopic failure events. Theoretical studies of creep rupture encounter various challenges: on the one hand, applications of materials in construction components require the development of analytical and numerical models which are able to predict the damage histories of loaded specimens in terms of the characteristic parameters of the constituents. On the other hand, it is important to reveal universal aspects of creep rupture of disordered materials, which are independent of specific material properties relevant on the microlevel. Based on these universal features, methods of forecasting catastrophic failure events can be worked out.

In applications solid blocks are often joined together by welding or glueing of the interfaces which are expected to sustain various types of external loads. Interfacial failure also occurs in fiber reinforced composites, where debonding of the fiber-matrix interface is an important damage mechanism. Since disordered properties of the glue play a crucial role in the failure of interfaces, most of the theoretical studies in this field rely on discrete models [1,2,18,19,20,21,27,28] which are able to capture heterogeneities and can account for the complicated interaction of nucleated cracks.

In this paper we present two extensions of the classical fiber bundle model to study the creep rupture of heterogeneous materials and the shear failure of glued interfaces of solid blocks. To model creep rupture, we assume that the microscopic mechanism of creep is the viscoelastic behaviour of fibers giving rise to time dependent deformation. The fibers of a parallel bundle fail during the time evolution of the system when the deformation exceeds their local breaking threshold. Assuming global load sharing among fibers, we analyze the macroscopic response of the bundle and the microscopic process of failure. Analytical and numerical calculations showed that there exists a critical load which determines the final state of the material. Approaching the critical point from below and above the system exhibits scaling behaviour which implies that creep rupture is analogous to continuous phase transitions.

To describe interfacial failure, we model the interface as an array of elastic beams which experience stretching and bending under shear load and break if the two deformation modes exceed randomly distributed breaking thresholds. The two breaking modes can be independent or combined in the form of a von Mises type breaking criterion. Assuming global load sharing following the beam breaking, we obtain analytically the macroscopic constitutive behaviour of the system and describe the microscopic process of the progressive failure of the interface.

2. Fiber bundle model of creep rupture Our model consists of N parallel fibers having viscoelastic constitutive behaviour. For simplicity, the pure viscoelastic behaviour of fibers is modelled by a Kelvin-Voigt element which consists of a spring and a dashpot in parallel and results in the constitutive equation $\sigma_0 = \beta\dot{\varepsilon} + E\varepsilon$, where σ_0 is the external load, β denotes the damping coefficient, and E is the Young modulus of fibers, respectively. In order to capture failure in the model a strain controlled breaking criterion is imposed, *i.e.* a fiber fails during the time evolution of the system when its strain exceeds a breaking threshold ε_i, $i=1,...N$ drawn from a probability distribution $P(\varepsilon)=\int_0^{\varepsilon} p(x)dx$. For the stress transfer between fibers following fiber failure we assume that the excess load is equally shared by all the remaining intact fibers (global load sharing), which provides a satisfactory description of load redistribution in continuous fiber reinforced composites. For the breaking thresholds of fibers a uniform distribution between 0 and 1, and a Weibull distribution of the form $P(\varepsilon) = 1 - \exp\left[-(\varepsilon/\lambda)^{\rho}\right]$ were considered. The construction of the model is illustrated in Fig. 1a). In the framework of global load sharing most of the quantities describing the behaviour of the fiber bundle can be obtained analytically. In this case the time evolution of the system under a steady external load σ_o is described by the differential equation

$$\frac{\sigma_o}{1-P(\varepsilon)} = \beta\dot{\varepsilon} + E\varepsilon \quad , \tag{1}$$

where the viscoelastic behaviour is coupled to the failure of fibers [22,23]. The viscoelastic fiber bundle model with the equation of motion Eq. (1) can provide an adequate description of natural fiber composites like wood subjected to a constant load [28]. For the behaviour of the solutions ε(t) of Eq. (1) two distinct regimes can be distinguished depending on the value of the external load σ_o: when σ_o falls below a critical value σ_c, Eq. (1) has a stationary solution ε_s, which can be obtained by setting $\dot{\varepsilon} = 0$, *i.e.* $\sigma_o = E\varepsilon_s[1-P(\varepsilon_s)]$. It means that as long as this equation can be solved for ε_s at a given external load σ_o, the solution $\varepsilon(t)$ of Eq. (1) converges to ε_s when $t \to \infty$, and the system suffers only a partial failure. However, when σ_o exceeds the critical value σ_c no stationary solution exists, furthermore, $\dot{\varepsilon}$ remains always positive, which implies that for $\sigma_o > \sigma_c$ the strain of the system $\varepsilon(t)$ monotonically increases until the system fails globally at a finite time t_f [22,23].

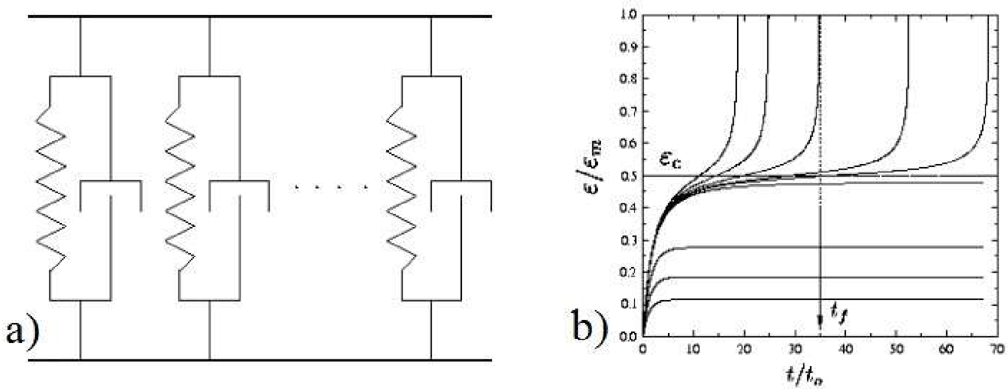

Figure 1: *a)* The viscoelastic fiber bundle model. Intact fibers are modelled by Kelvin-Voigt elements. *b)* $\varepsilon(t)$ for several different values of the external load σ_0 below and above σ_c. t_0 denotes the characteristic timescale of the system $t_0 = \beta / E$.

The behaviour of $\varepsilon(t)$ is illustrated in Fig. 1b) for several values of σ_0 below and above σ_c with uniformly distributed breaking thresholds between 0 and ε_m. It follows from the above argument that the critical value of the load σ_c is the static fracture strength of the bundle. The creep rupture of the viscoelastic bundle can be interpreted so that for $\sigma_o \leq \sigma_c$ the bundle is partially damaged implying an infinite lifetime $t_f = \infty$ and the emergence of a stationary macroscopic state, while above the critical load $\sigma_0 > \sigma_c$ global failure occurs at a finite time t_f, but in the vicinity of σ_c the global failure is preceded by a long lived stationary state. The nature of the transition occurring at σ_c can be characterized by analyzing how the creeping system behaves when approaching the critical load both from below and above. For $\sigma_0 \leq \sigma_c$ the fiber bundle relaxes to the stationary deformation ε_s through a gradually decreasing breaking activity. It can be shown analytically that $\varepsilon(t)$ has an exponential relaxation to ε_s with a characteristic time scale τ that depends on the external load σ_0 as $\tau \propto (\sigma_c - \sigma_o)^{-1/2}$ for $\sigma_0 < \sigma_c$, *i.e.* when approaching the critical point from below the characteristic time of the relaxation to the stationary state diverges according to a universal power law with an exponent -1/2 independent on the form of the disorder distribution *P*.

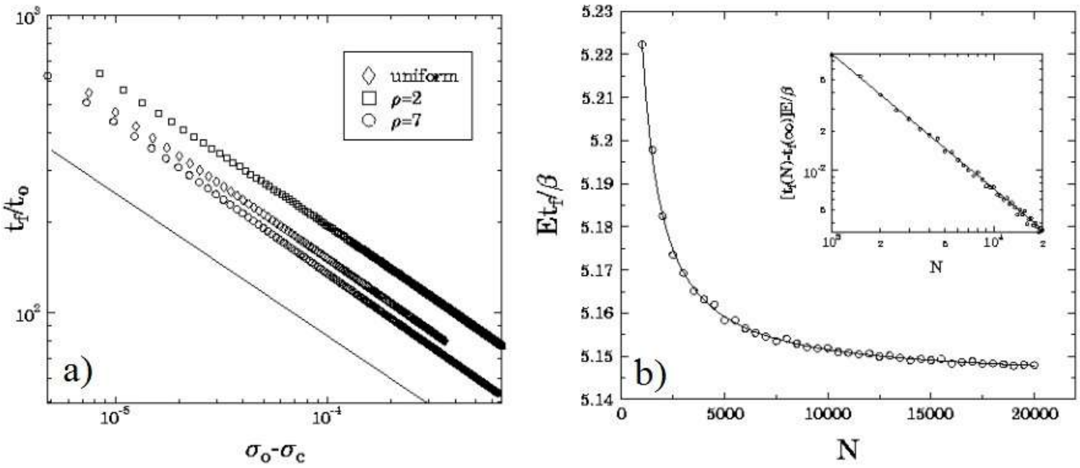

Figure 2: *a)* Lifetime t_f of the bundle as a function of the distance from the critical point σ_0-σ_c for three different disorder distributions. *b)* t_f as a function of the number of fibers *N* at a fixed value of the external load σ_0. Results of computer simulations (symbols) are in good agreement with the analytic predictions (solid lines).

Above the critical point the lifetime t_f defines the characteristic time scale of the system which can be cast in the form $t_f \propto (\sigma_o - \sigma_c)^{-1/2}$ for $\sigma_0 > \sigma_c$, so that t_f also has a power law divergence at σ_c with a universal exponent -1/2 like τ below the critical point, see Fig. 2a). Hence, for global load sharing the system exhibits scaling behaviour on both sides of the critical point indicating a continuous transition at the critical load σ_c. It can also be shown analytically that fixing the external load above the critical point, the lifetime t_f of the system exhibits a universal scaling $t_f(N) - t_f(\infty) \propto 1/N$ with respect to the number *N* of fibers of the bundle (Fig. 2b) [22,23].

The process of fiber breaking on the micro level can easily be monitored experimentally by means of the acoustic emission techniques. Except for the primary creep regime, where a large amount of fibers break in a relatively short time, the time of individual fiber failures can be recorded with high precision. In order to characterize the process of fiber breaking in our viscoelastic fiber bundle model, we calculated numerically the distribution *f* of waiting times Δt between consecutive breaks [24]. A detailed analysis revealed that *f(*Δ*t)* shows a power law behavior $f(\Delta t) \sim \Delta t^{-b}$ on both sides of the critical point [24]. The value of the exponent b is different on the two sides of the critical point, *i.e.* $b = 1.5 \pm 0.05$ and $b = 1.95 \pm 0.05$ were measured below and above the critical load; however, b proved to be independent of the disorder distribution of fibers, see Fig. 5.

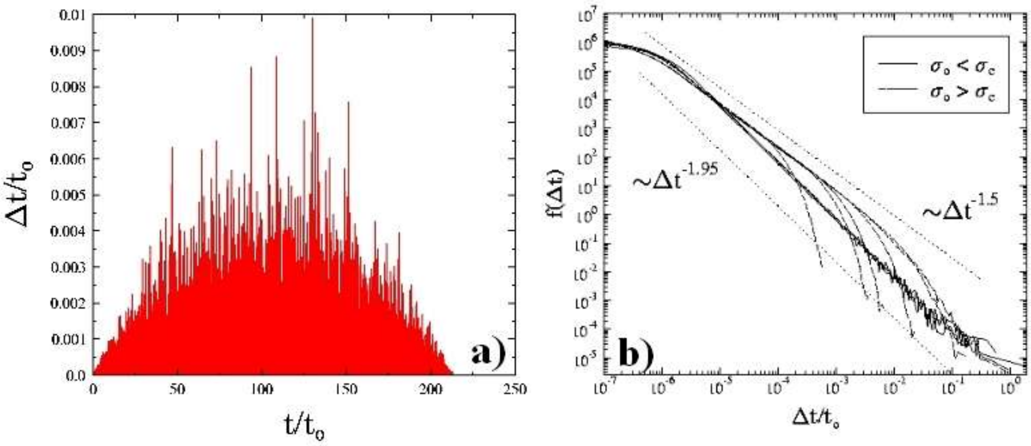

Figure 3: *a)* Waiting times Δt between consecutive fiber breakings for a load $\sigma_0 > \sigma_c$ plotted at the time t of breakings. *b):* Distribution of Δt below and above the critical load. Simulation results are presented for 10 million fibers with uniformly distributed threshold values.

3. Shear failure of glued interfaces

We propose a novel approach to the shear failure of glued interfaces by extending the classical fiber bundle model to model interfacial failure [19,25,26]. Our model represents the interface as an ensemble of parallel beams connecting the surface of two rigid blocks [27], see Fig. 4. The beams are assumed to have identical geometrical extensions (length l and width d) and linearly elastic behaviour characterized by the Young modulus E. In order to capture the failure of the interface, the beams are assumed to break when their deformation exceeds a certain threshold value. Under shear loading of the interface, beams suffer stretching and bending deformation resulting in two modes of breaking. The stretching and bending deformation of beams can be expressed in terms of a single variable, *i.e.* longitudinal strain $\varepsilon = \Delta l / l$, which enables us to map the interface model to the simpler fiber bundle models. The two breaking modes can be considered to be independent or combined in the form of a von Mises type breaking criterion. The strength of beams is characterized by the two threshold values of stretching ε_1 and bending ε_2 a beam can withstand. The breaking thresholds are assumed to be randomly distributed variables of the joint probability distribution $p(\varepsilon_1,\varepsilon_2)$. The randomness of the breaking thresholds is supposed to represent the disorder of the interface material. After breaking of a beam the excess load has to be redistributed over the remaining intact elements. Coupling to the rigid blocks ensures that all the beams have the same deformation giving rise to global load sharing, *i.e.* the load is equally shared by all the elements, stress concentration in the vicinity of failed beams cannot occur.

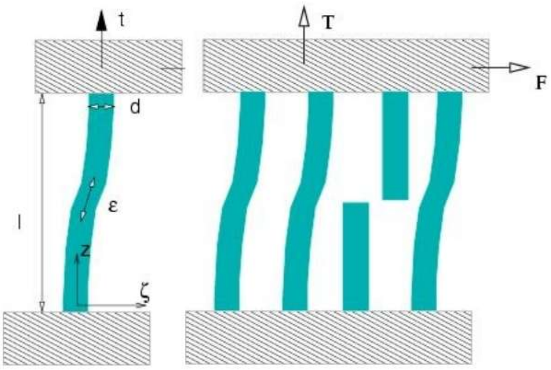

Figure 4: Illustration of the model construction. Beams of the sheared interface suffer stretching and bending (*left*) and fail due to the two deformation modes (*right*).

Assuming the breaking modes to be independent, a single beam breaks if either its stretching or bending deformation exceeds the respective breaking threshold ε_1 or ε_2, *i.e.* failure occurs if $f(\varepsilon)/\varepsilon_1 \geq 1$ or $g(\varepsilon)/\varepsilon_2 \geq 1$, where $f(\varepsilon)$ and $g(\varepsilon)$ describe the stretching and bending breaking modes, respectively. Later on this case will be called the *OR* criterion. The failure functions $f(\varepsilon)$ and $g(\varepsilon)$ can be determined from the elasticity equations of beams, but in general the only restriction for them is that they have to be monotonous. For our specific case of sheared beams they take the form $f(\varepsilon) = \varepsilon$ and $g(\varepsilon) = \sqrt{\varepsilon}$, where *E=1* is assumed [27]. In the plane of breaking thresholds each beam is represented by a point with coordinates ($\varepsilon_1,\varepsilon_2$). The constitutive behaviour of the interface can be obtained by integrating the load of single beams over the intact ones in the plane of breaking thresholds, see Fig. 5a). For the *OR* criterion one gets $\sigma = \varepsilon \int_{g(\varepsilon)}^{\varepsilon_2^{max}} d\varepsilon_2 \int_{f(\varepsilon)}^{\varepsilon_1^{max}} d\varepsilon_1 p(\varepsilon_1,\varepsilon_2)$. Assuming the thresholds of the two breaking modes to be independently distributed, the disorder distribution factorizes $p(\varepsilon_1,\varepsilon_2) = p_1(\varepsilon_1)p_2(\varepsilon_2)$ and $\sigma(\varepsilon)$ takes the simple form $\sigma(\varepsilon) = \varepsilon[1 - P_1(f(\varepsilon))][1 - P_2(g(\varepsilon))]$. The terms $1 - P_1(f(\varepsilon))$ and $1 - P_2(g(\varepsilon))$ provide the fraction of beams failed under the stretching and bending breaking modes, respectively. When the two breaking modes are coupled by a von Mises type breaking criterion, a single beam breaks if its strain ε fulfils the condition $(f(\varepsilon)/\varepsilon_1)^2 + g(\varepsilon)/\varepsilon_2 \geq 1$, which is illustrated by Fig. 5a).

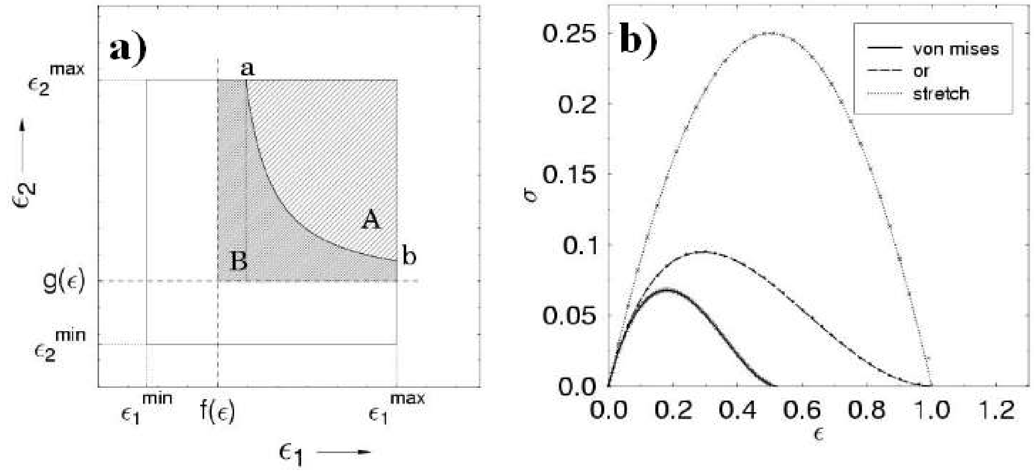

Figure 5: *a)* The plane of breaking thresholds. Fibers which are intact at a deformation ε fall in the rectangle bounded by $f(\varepsilon), g(\varepsilon)$ and the maximum values of the two thresholds $\varepsilon_1^{max}, \varepsilon_2^{max}$ for the *OR* criterion (area *A+B*), and in the area bounded by the curve connecting *a* and *b* and the maximum thresholds for the von Mises type criterion (area *A*), respectively. The fibers which break due to the coupling of the two breaking modes in the von Mises criterion fall in area *B*. *b)* Comparison of the constitutive curves of a simple fiber bundle and of the beam model with different breaking criteria using uniformly distributed breaking thresholds.

In this case the constitutive integral $\sigma = \varepsilon \int_a^{\varepsilon_1^{max}} d\varepsilon_1 \int_{\varepsilon_2^*(\varepsilon_1,\varepsilon)}^{\varepsilon_2^{max}} d\varepsilon_2 p(\varepsilon_1, \varepsilon_2)$ cannot be performed explicitly with the integration limit $\varepsilon_2^*(\varepsilon_1, \varepsilon) = \varepsilon_1^2 g(\varepsilon)/(\varepsilon_1^2 - f^2(\varepsilon))$ in general.

Simulation techniques Based on the above equations analytic results can only be obtained for the simplest forms of disorder like the uniform distribution. In order to determine the behaviour of the system for complicated disorder distributions and explore the microscopic failure process of the sheared interface, it is necessary to work out a computer simulation technique. For the simulations we consider an ensemble of *N* beams arranged on a square lattice. Two breaking thresholds $\varepsilon_1^i, \varepsilon_2^i$ are assigned to each beam *i* ($i = 1, \ldots, N$) of the bundle from the joint probability distribution $p(\varepsilon_1, \varepsilon_2)$. For the *OR* breaking rule, the failure of a beam is caused either by stretching or bending depending on which one of the conditions is fulfilled at a lower value of the external load. In this way an effective breaking threshold ε_c^i can be defined for the beams as $\varepsilon_c^i = \min(f^{-1}(\varepsilon_1^i), g^{-1}(\varepsilon_2^i)), i = 1,\ldots,N$, where f^{-1} and g^{-1} denote the inverse of *f, g*, respectively. A beam *i* breaks during the loading process of the interface when the load on it exceeds its effective breaking threshold ε_c^i. For the case of the von Mises type breaking criterion the effective breaking threshold ε_c^i of beam *i* can be obtained as the solution of the algebraic equation $\left(f(\varepsilon_c^i)/\varepsilon_1^i\right)^2 + g(\varepsilon_c^i)/\varepsilon_2^i = 1$. In the case of global load sharing, the load and deformation of beams is everywhere the same along the interface, which implies that beams break in increasing order of their effective breaking thresholds. In the simulation, after determining ε_c^i for each beam, they are sorted in increasing order. Quasi-static loading of the beam bundle is performed by increasing the external load to break only a single element. Due to the subsequent load redistribution on the intact beams, the failure of a beam may trigger an avalanche of breaking beams. This process has to be iterated until the avalanche stops, or leads to catastrophic failure at the critical stress and strain. In Fig. 5b) the analytic results on the constitutive behaviour obtained with uniform distribution of the breaking thresholds are compared to the corresponding results of computer simulations. As a reference, we also plotted the constitutive behaviour of a bundle of fibers where the fibers fail solely due to simple stretching [3-11]. It can be seen in the figure that the simulation results are in perfect agreement with the analytical predictions. It is important to note that the presence of two breaking modes substantially reduces the critical stress σ_c and strain ε_c (σ_c and ε_c are the value of the maximum of the constitutive curves) with respect to the case when failure of elements occurs solely under stretching [3-11]. Since one of the failure functions $g(\varepsilon)$ is non-linear, the shape of the constitutive curve $\sigma(\varepsilon)$ also changes, especially in the post-peak regime. The coupling of the two breaking modes in the form of the von Mises criterion gives rise to further reduction of the strength of the interface, see Fig. 5b).

Microscopic damage process During the quasi-static loading process of an interface, avalanches of simultaneously failing beams occur. Inside an avalanche, however, the beams can break solely under one of the breaking modes when the *OR* criterion is considered, or the breaking can be dominated by one of the breaking modes in the coupled case of the von Mises type criterion. In order to study the effect of the disorder distribution $p(\varepsilon_1,\varepsilon_2)$ of beams on the relative importance of the two breaking modes and on the progressive failure of the interface, we considered independently distributed breaking thresholds $\varepsilon_1,\varepsilon_2$ both with a Weibull distribution of exponents m_1 and m_2, and scale parameters λ_1 and λ_2 for the stretching and bending modes, respectively. It can be seen in Fig. 6a) that increasing λ_2 of the bending mode, the beams become more resistant against bending so that the stretching mode starts to dominate the breaking of beams, which is indicated by the increasing fraction of stretching failure. In the limiting case of $\lambda_2 >> \lambda_1$ the beams solely break under stretching.

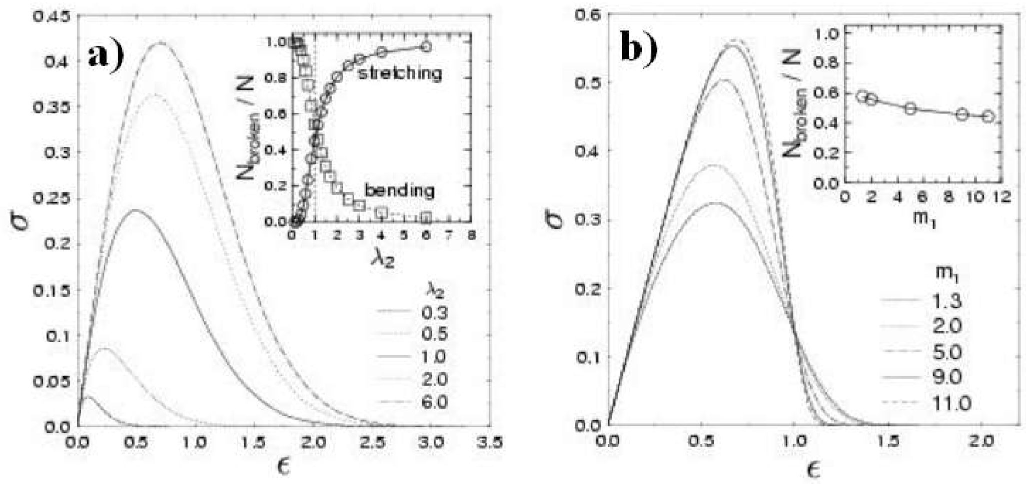

Figure 6: a) Constitutive behaviour of a bundle of *N=90000* beams using the *OR* criterion for different values of the scale parameter λ_2 of the bending mode. The other parameters were fixed $m_1 = m_2 = 2.0$ and $\lambda_1 = 1.0$. Inset: fraction of beams broken under the two breaking modes. *b)* Constitutive behaviour of the same bundle changing the Weibull exponent m_1 of the stretching mode with $\lambda_1 = \lambda_2 = 1.0$ and $m_2 = 2.0$. Inset: the fraction of beams broken under the bending mode.

It is interesting to note that varying the relative importance of the two failure modes gives also rise to a change of the macroscopic constitutive behaviour of the system. Shifting the strength distributions of beams, the functional form of the constitutive behaviour remains the same, however, the value of the critical stress and strain vary in a relatively broad range. Varying the amount of disorder in the breaking thresholds, *i.e.* the Weibull exponents, has a similar strong effect on the macroscopic response of the system, see Fig. 6b). Applying the von Mises breaking criterion, the microscopic and macroscopic response of the interface show similar behaviour. Our careful numerical analysis of the microscopic failure process of the interface revealed that the size of avalanches of simultaneously failing beams under a stress controlled loading of the interface has a power law distribution. The exponent of the power law proved to be 5/2, it is equal to the mean field exponent characterizing the distribution of bursts in simple fiber bundle models [7,8].

4. Conclusions Fiber bundle models have been applied to describe various aspects of the failure of heterogeneous materials. However, fibers of the classical fiber bundle model can sustain solely stretching deformation and present a time independent behaviour. In order to study time dependent failure phenomena like creep rupture, and the damage process of materials occurring under complex deformation like in sheared interfaces, we proposed two extensions of fiber bundle models in the present paper. To model creep rupture, we represent the fibers of a parallel bundle by Kelvin-Voigt elements which exhibit time dependent deformation under an external load [22,23,24]. We showed analytically and by computer simulations that creep rupture occurs analogously to second order phase transitions, *i.e.* approaching the critical load the system is characterized by scaling laws with exponents independent of specific material properties [22,23,24]. The microscopic mechanism of creep of our model is relevant for natural fiber composites like wood [28], but can also be simply extended to more complex material behaviours [29].

To describe the shear failure of interfaces, we model the interface as an array of elastic beams which experience stretching and bending under shear load and break if the two deformation modes exceed randomly distributed breaking thresholds. The two breaking modes can be independent or combined in the form of a von Mises type breaking criterion [27]. We showed that the model provides a broad spectrum of interface responses varying the relative importance of the two breaking modes in qualitative agreement with experiments [27]. Both models presented here can be further extended by taking into account localized load redistribution after fiber failure [24,27].

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