Journal of the Textile Institute Transactions

Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/jtit20

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Version of record first published: 22 Jan 2013.

To cite this article: (1926): 31—IX.—Tensile Tests for Cotton Yarns iv.—The Dynamics of Some Testing Instruments, Journal of the Textile Institute Transactions, 17:7, T342-T354

To link to this article: http://dx.doi.org/10.1080/19447027.1926.10599952

31—IX.—TENSILE TESTS FOR COTTON YARNS
iv.—THE DYNAMICS OF SOME TESTING INSTRUMENTS

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INTRODUCTION AND SUMMARY

The operation of several testing instruments is analysed in order to detect possible errors due to momentum or friction, to find the degree of constancy attained in rate of loading and in sensitivity, to evaluate the time of break, and to show the possibilities of improvement in any of these respects. The treatment is mathematical throughout, practical applications being detailed in the earlier papers of this series.

Momentum errors do not attain serious proportions in the pendulum type of machine, especially when the lower jaw begins to move gradually as the specimen comes under tension—a condition realised in the single-thread tester operated by an oil plunger—and when the period of free swing of the pendulum is short. The effect of an eccentric roller on sensitivity, steadiness of movement and constancy of rate of loading is examined, and it is shown that an eccentricity one half the radius[1] greatly improves the efficiency of the instrument.

The variable velocity of the loading jaw of the Moscrop machine introduces slight momentum errors, which are not important at speeds which obviate more serious dangers, and a negligible variation of the rate of loading. An error is introduced by the fling of the pointer, which increases rapidly as the operation is accelerated but which can be measured. The jerk caused by starting the slide of the recording jaw into sudden movement may seriously affect the result and cannot be allowed for in a correction. The danger is avoided by slowing the machine down to 6 cycles per minute for strong yarns, or still more for very weak ones, using springs such that the specimens break in the second half of the record. Formulæ are given for the correction for fling and for a criterion of safe speed.

The load-extension instrument is free from error if the carriage be moved steadily and suitable dimensions of the spring chosen according to the analysis given by Shorter and Hall.[3]

Calculations are given for designing a ballistic tester to avoid impulsive reactions on the support and in the pendulum, for calibration in absolute units and for evaluating the rate and time of break.

PENDULUM TESTERS

The pendulum of a single-thread or lea tester of the standard type has an appreciable inertia and if its motion be not steady, the force on the yarn differs by the effective force of the acceleration from that given by static calibration.

In particular, if the lower grip be moving when the specimen comes under tension, the pendulum starts with a jerk, producing an oscillation which is imposed on the steady increase of load. Shorter[2] has evaluated this error for the case where the lower grip is moving at full speed when the tension begins, and the specimen is perfectly elastic. This case is reproduced in practice if a fine copper wire be placed slackly between the jaws of a single-thread tester, and the oscillations are then very evident to eye and ear. With cotton yarn, however, such oscillations damp out very quickly, only three periods being detectable whether from the initial jerk or if restarted by momentarily checking the pendulum. The single-thread machine used in the present work is operated by a plunger in oil, which starts into movement as the tension begins, so that the initial jerk is quite negligible.

Shorter[1] also suggests an eccentric roller to obtain an even scale. As the very uneven scale of the concentric roller so restricts the range, the makers (Messrs. Goodbrand), at request, incorporated this modification in the single-thread tester used for this research. Incidentally a pendulum was thus obtained with a very short free period, 1·0 second, which further reduces the possibility of error from oscillations even with more elastic specimens. A vibration of a fraction of a second can be detected with sized yarns but dies out very early.

It is desirable that three quantities should be constant over the range of a tester, closeness of scale or sensitivity, momentum of the pendulum, and rate of loading. Constancy in all cannot be attained with the dead-weight principle, so it is necessary to inquire how close an approximation can be made, quite appreciable variations being allowable before the soundness of the test is impaired.

Let the radius of the roller be r, the axis be at a distance x from the centre, and the line joining them be horizontal when the tension is zero. If θ be the inclination of the pendulum from the zero position, the vertical movement of the upper grip (positive downwards) is given by—

$$y_2 = r(\theta + \lambda \sin \theta) \text{ where } \lambda = x/r. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If the lower grip moves at a constant rate, its movement is given by—

$$y_1 = vt. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

As the lateral movement of the upper grip does not appreciably increase the distance between the two grips for a long specimen, the extension of the yarn is—

$$y_1 - y_2 = f/E \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where f is the tension on the yarn and E a constant when the load-extension curve is a straight line, which is usually nearly enough the case for yarn under steady loading.

The tension according to static calibration is—

$$f - I . \frac{\delta^2 \theta}{\delta t^2} \cdot \frac{1}{r(1 + \lambda \cos \theta)} = Mgh . \frac{\sin \theta}{r(1 + \lambda \cos \theta)} = R \frac{\sin \theta}{(1 + \lambda \cos \theta)} \quad (4)$$

where $R = Mgh/r$ is the maximum scale reading ($\theta = 90°$), M is the mass, I the moment of inertia, and h the distance from the axis to the centre of gravity of the pendulum. As oscillations damp out so quickly and the only cases of interest are where the movement is very nearly steady, no appreciable error is caused by neglecting the term in $\delta^2\theta/\delta t^2$ to obtain, from Equations 1, 2, 3, and 4—

$$vt = r(\theta + \lambda \sin \theta) + \frac{R}{E} \cdot \frac{\sin \theta}{1 + \lambda \cos \theta} \quad . \quad . \quad . \quad (5)$$

Then the three quantities which ideally should be constant are—

Closeness of scale, $\frac{\delta f}{\delta \theta} = R \cdot \frac{\lambda + \cos \theta}{(1 + \lambda \cos \theta)^2}$ . . . . (6)

Movement of pendulum, $\frac{\delta \theta}{\delta t} = \frac{v}{r(1 + \lambda \cos \theta)} \cdot \left[1 + \frac{R}{Er} \cdot \frac{\lambda + \cos \theta}{(1 + \lambda \cos \theta)^3}\right]^{-1}$ (7)

Rate of loading, $\frac{\delta f}{\delta t} = vR \cdot \frac{\lambda + \cos \theta}{r(1 + \lambda \cos \theta)^3} \cdot \left[1 + \frac{R}{Er} \cdot \frac{\lambda + \cos \theta}{(1 + \lambda \cos \theta)^3}\right]^{-1}$ (8)

The possible error due to effective force can be judged more directly from

$$I \cdot \frac{\delta^2 \theta}{\delta t^2}$$

where I can be found from the period of free swing of the pendulum T, as

$$I = Mgh \cdot \frac{T^2}{4\pi^2} = Rr \cdot \frac{T^2}{4\pi^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (9a)$$

and

$$\frac{\delta^2 \theta}{\delta t^2} = \frac{v^2}{r^2} \cdot \frac{\lambda \sin \theta}{(1 + \lambda \cos \theta)^3} \cdot \left[1 + \frac{R}{Er} \cdot \frac{1/\lambda \sin \theta + \sin \theta \,.\, \cos \theta - 2 \cos \theta - 2\lambda}{(1 + \lambda \cos \theta)^3}\right]$$

$$\times \left[1 + \frac{R}{Er} \cdot \frac{\lambda + \cos \theta}{(1 + \lambda \cos \theta)^3}\right]^{-3} \quad . \quad . \quad . \quad . \quad . \quad (9)$$

It may be seen from these equations that none of the three quantities is strictly constant for any value of λ. With a concentric roller, $\lambda = 0$, the scale opens out rapidly at higher angles, the rate of loading decreases, and the rate of movement increases slightly according to the extensibility of the specimen. If the scale is made even, the rate of loading and velocity of pendulum both increase with θ. The practical compromise is to allow the scale to open out slowly, keeping the rate of loading practically constant and to ensure that the accelerations do not introduce appreciable errors.

The three conditions are fully satisfied in practice by the round value $\lambda = 0{\cdot}50$, rather less than the optimum when evenness of scale alone is considered. In order to obtain numerical values from the above formulæ, the value of R/Er must be known. As the variations are not very sensitive to this value, it may be taken as 0·724, which is that for a yarn breaking at 325 gms. with an extension of 1 in. on a machine of range 470 gms., the pulley radius being 2 in.

On this point, it may be noticed that Equation 8 may be written—

$$\frac{\delta f}{\delta t} = \frac{vk}{1 + k/E}$$

where

$$k = \frac{R}{r} \cdot \frac{\lambda + \cos \theta}{(1 + \lambda \cos \theta)^3}$$

is a constant of the machine. Comparison between machines is usually made by a quantity called "the machine rate of loading," which is the change of tension on an inextensible specimen for unit traverse of the loading jaw. This quantity is the constant k for, when $1/E$ is zero, $\delta f/v\delta t = k$.

On the other hand, for very extensible threads, the rate of loading tends to become independent of the machine and approach the value $\delta f/\delta t = vE$.

In practice, breaks usually occur when the lever is about the middle of the scale and the extensibility of single cotton yarn rarely varies outside 4—6%, so that the representative value taken for R/Er gives a much better approximation to actual conditions than the limiting values of zero and infinity, when the subject is the variation over the scale of any one machine.

The momentum error in the machine with eccentric roller was evaluated to be under 0·1 milligm. at 60° inclination, the free period being 1·0 second. The condition of steady motion may therefore be ignored. The variations of sensitivity and rate of loading are shown in Table I. and, when $\lambda = 0{\cdot}50$, are too small to be any disadvantage. With a concentric pulley, specimens should be tested on a machine of such range that the inclination does not exceed 60°.

Table I.

Ratio of Value of Quantities to that at Zero

Inclination, θ	60°	60°	90°	90°	0°
Eccentricity, λ	0	0·5	0	0·5	–
Scale divisions, $\delta\theta/\delta f$	2·0	1·04	∞	1·33	1
Rate of loading, $\delta f/\delta t$	0·63	1·11	0	1·09	1

In studying the effect of rate of loading on breaking load, the results are conveniently expressed against the time which would be occupied in breaking the specimen at the rate of loading which obtains at rupture, which is given by—

$$T = f / \frac{\delta f}{\delta t} = \frac{r}{v} \cdot \left[\frac{f}{R} \cdot \frac{(1 + \lambda \cos \dot{\theta})^3}{\lambda + \cos \theta} + \frac{e}{r}\right] \quad . \quad . \quad . \quad (10)$$

where the values of f, $\dfrac{\delta f}{\delta t}$, θ, and e are those at rupture.

For the particular machine used for the work of Paper III.—

$$T = \frac{120}{v} \cdot \left[\frac{f}{470} \cdot \frac{(1 + \frac{1}{2} \cos \theta)^3}{\frac{1}{2} + \cos \theta} + \frac{e}{5{\cdot}08}\right] \text{ seconds} \quad . \quad . \quad (10a)$$

The value of $\dfrac{(1 + \frac{1}{2} \cos \theta)^3}{\frac{1}{2} + \cos \theta}$

changes slowly and in the neighbourhood of 340 gms. is 1·94.

$$\therefore T = (0{\cdot}495\, f + 23{\cdot}6\, e)/v \text{ seconds} \quad . \quad . \quad . \quad . \quad (10b)$$

Before leaving the question of deadweight machines, the possible error due to overswinging should be considered. Suppose the pointer to continue to move after the specimen is ruptured, through an angle $\Delta\theta$, then neglecting friction—

$$\tfrac{1}{2}\, I\, \dot{\theta}^2 = Mgh - \Delta \cos \theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad (Xa)$$

Substituting values given by Equations 6, 7, and 9a, and simplifying, the false increment to the breaking load is given by—

$$\Delta F = R \frac{v^2 T^2}{8 \pi^2 r^2} \cdot \frac{\lambda + \cos \theta}{\sin \theta} \cdot \left[(1 + \lambda \cos \theta)^2 + \frac{R}{Er} \cdot \frac{\lambda + \cos \theta}{1 + \lambda \cos \theta}\right]^{-2} \quad (X)$$

For the particular instrument described above, the factors independent of θ reduce to 470/7896 gms. and the error, assuming

$$\frac{R}{Er} \text{ is } 0{\cdot}724,$$

is 0·015 gm. at 60°, 0·021 gm. at 30°. This is far below the limit of reading but with a smaller roller, longer pendulum, higher speed, and less extensible material the error may become appreciable, and it is well to check this point by Equation (X) for any given instrument used in research testing.

MOSCROP MACHINE

The tension is measured by the extension of a spring working well within the limits of perfect elasticity, so the scale is accurately even. It is also sensitive enough for practical purposes.

As the inertia of the spring itself is very small and the extension too small to introduce elastic after-effects, the ratio of tension to extension is also independent of the speed. This may be experimentally verified over the large interval of speed between static calibration and free vibration. The period of free vibration is given by

$$T = 2\pi\sqrt{M/\mu}$$

where M is the load, μ the tension per unit extension during oscillation. Testing a spring of set No. 3 (maximum 16 oz.), steady calibration gave 0·316 cm. per oz. The time of vibration under an 8 oz. load was 0·32 sec., under a 13 oz. load 0·405 sec., giving 0·314 and 0·316 cm. per oz. respectively.

It follows also that the rate of loading would be constant if the loading jaw moved at constant speed and the tension on the yarn was equal to that on the spring. Any variations and errors must therefore be introduced by accelerations of the loading jaw and by the momentum and friction of the recording jaw.

The loading jaw is moved by a lever passing through a swivelled nut on the carriage. This lever is oscillated by a pin, on a uniformly rotating wheel, which passes freely up and down a slot in the lever. The geometrical conditions are shown in the diagram, Fig. 1; AQ, a fixed distance r, rotates at a constant angular velocity ω about A; OC moves so that it always passes through Q; P is the intersection of OC and the fixed horizontal line BP, and is a fixed point on the carriage. The length of BP, x, therefore defines the displacement of the loading jaw (positive to the right) from its position when the lever is vertical. Let x_0 be its value when the jaw closes at the beginning of loading, and let p be the number of complete cycles per minute. Then—

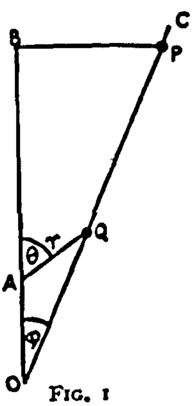

FIG. 1

$$x = \text{OB} \tan \varphi, \text{ where } \tan \varphi = \frac{\text{AQ} \sin \theta}{\text{AO} + \text{AQ} \cos \theta}$$

$$= b \cdot \frac{r \sin \omega t}{a + r \cos \omega t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

$$\dot{x} = \omega b r \cdot \frac{a \cos \omega t + r}{(a + r \cos \omega t)^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

$$\ddot{x} = -\omega^2 b r \sin \omega t \cdot \frac{a^2 - r^2}{(a + r \cos \omega t)^3}$$

$$= -\omega^2 x \cdot \frac{a^2 - r^2}{(a + r \cos \theta)^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

If y be the displacement of the recording slide from its zero position and the extension of the thread be $e = f/E$, where f is the tension in the thread—

$$x - x_0 = e + y \text{ and } f = E(x - x_0 - y). \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

As the spring is slightly extended at the zero position, let y_0 be the extension corresponding to the initial tension. The tension of the spring is

$$f' = \mu\,(y + y_0) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

The slide of the recording jaw is mounted on rollers with a very slight slope forwards, nicely adjusted so that the small forces due to friction and gravity neutralise each other in forward movement. Expressing the tensions in gravitational units, $f' = f - M\ddot{y}/g$, where M is the mass of the slide.

$$\therefore \mu\,(y + y_0) = E(x - x_0 - y) - M\ddot{y}/g$$

or $\quad M/g\,.\,\ddot{y} + (E + \mu)\,y = Ex - (\mu y_0 + Ex_0)$

$$= Ebr \cdot \frac{\sin\,\omega t}{a + r\,\cos\,\omega t} - (\mu y_0 + Ex_0) \quad . \quad . \quad . \quad (16)$$

Equation 16 is the differential equation of motion, with this reserve, that E is not a constant under oscillating tension. The initial jerk is rapidly damped out by elastic imperfection and the motion of the slide is thereafter steady except for the slow variation in speed of the loading jaw. Under conditions defined below, when the effective force of the slide is small in comparison with the tension, the term $M/g.\ddot{y}$ may beneg lected, giving—

$$y = \frac{E}{E+\mu} \cdot x - \frac{\mu y_0 + Ex_0}{E+\mu} = \frac{E}{E+\mu}(x - x_0) - \frac{\mu}{E+\mu} \cdot y_0 \qquad (17)$$

$$\dot{y} = \frac{E}{E+\mu} \cdot \dot{x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

$$\ddot{y} = \frac{E}{E+\mu} \cdot \ddot{x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

The errors introduced by the momentum at the moment of rupture are—

(1) The difference between the tension of the yarn and of the spring due to acceleration of the slide which, denoting final tensions by the capital F, is—

$$F' - F = -M\ddot{y}/g \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

and either (2) or (3), whichever be greater.

(2) The extension of the spring after rupture due to the momentum of the slide; the amount of this error, $\Delta' F$, is given by the energy equation—

$$\Delta' F/\mu \times F' \,.\, g = \tfrac{1}{2} M \,.\, \dot{y}^2$$

or

$$\Delta' F = \frac{\mu M}{2F'} \cdot \frac{\dot{y}^2}{g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

(3) The fling of the recording pin, $\Delta F/\mu$, beyond the point where it is pushed by the slide. If m be its mass, ν the frictional coefficient—

$$\nu mg \,.\, \Delta F/\mu = \tfrac{1}{2} m\dot{y}^2$$

or

$$\Delta F = \frac{\mu}{2\nu g} \cdot \dot{y}^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

The ratio $\Delta F/\Delta' F = F'/\nu M$ is of the order 10, hence errors (1) and (3) only need be considered. The difference between the indicated strength, F_i, and the maximum tension in the yarn, F_m, is—

$$F_i - F_m = \frac{1}{g}\left(\frac{\mu}{2\nu} \cdot \dot{y}^2 - M\ddot{y}\right) \quad . \quad . \quad . \quad (23)$$

all the quantities referring to the moment of rupture. From Equations (18) and (19)—

$$F_i - F_m = \frac{1}{g} \cdot \frac{E}{E + \mu}\left(\frac{\mu}{2\nu} \cdot \dot{x}^2 - M\ddot{x}\right) \quad . \quad . \quad . \quad (23a)$$

All the quantities on the right of the equation are known except ν and E, which depends on the specimen.

Taking an actual case of a yarn breaking at 16 oz., extending 2·0 cms., tested at 8 cycles per minute against springs for a maximum strength of 32 oz.—M=4·17 oz.; ω=0·84 radian per second; E=8·0 oz. per cm.; μ=6·33 oz. per cm.; $E/(E+\mu)$ = 0·558; μy_0= 2 oz.; y = 2·21 cm.; $x - x_0$= 4·21 cm.; x = 14·41 cm. at break;

$$\text{Cos } \theta = (\sqrt{1 - 9{\cdot}7x^2/b^2} - 3{\cdot}27x^2/b^2) \,/\, (1 + x^2/b^2) = 0{\cdot}5369.$$

$$\dot{x} = 0{\cdot}10472\, p \times \frac{3{\cdot}27 \cos \theta + 1}{(3{\cdot}27 + \cos \theta)^2} \times 65{\cdot}5 = 10{\cdot}43 \text{ cm. per second.}$$

$$\ddot{x} = -\,\omega^2 x \cdot \frac{9{\cdot}7}{(3{\cdot}27 + \cos \theta)^2} = -\,6{\cdot}77 \text{ cm. per second.}$$

$$\text{The effective force error} = \frac{1}{g} \cdot \frac{E}{E + \mu} \cdot -\,M\ddot{x} = 0{\cdot}020 \text{ oz. on 16 ozs.}$$

This is equal to 1/100th of an interval on the record and is negligible.

The error due to the sliding of the pin is—

$$\frac{1}{g} \cdot \frac{E}{E + \mu} \cdot \frac{\mu}{2\nu} \cdot \dot{x}^2 = 0{\cdot}196/\nu \text{ oz.}$$

A slight upward inclination of the bed of the recording pin increases the effective value of ν, but, as the coefficient of friction between polished metal surfaces is of the order 0·2, this error may attain serious proportions in normal working.

As the research on rate of loading demanded enhanced speeds, the error had to be measured. This was done by placing a metal bar across the machine to stop the slides at a definite position near the breaking point of the yarn used in that research. The position of rupture being kept constant, the fling of the pin should be proportional to p^2. On Fig. 2 are shown the measured flings plotted against p^2. Below 5 cycles per minute, these are too small to be measured, but they increase rapidly in linear relation with p^2, the deviations being irregular and of the order of 0·2 mm., which cannot be regarded as significant. The fling was shorter and more regular when the pin was vaselined, being then 0·0064 p^2 mm. At the usual rate of 8 cycles per minute, this gives a fling of 0·4 mm.—rather more than 1/10th of an interval. It would be safer to make the standard rate 6 cycles per minute, 1 in 10 seconds, when the error is inappreciable.

The recording slide starts into motion with a jerk which may damage the yarn, for the moving jaw closes when travelling at 11·86 cm. per second (at 8 cycles per minute). This sets up an oscillation, but as it is very highly damped, the effective force in the first half period will show the likelihood of injury. Its magnitude may be found from Equation (16), simplified by taking the velocity of the loading jaw as constant at the value when the jaw closes.

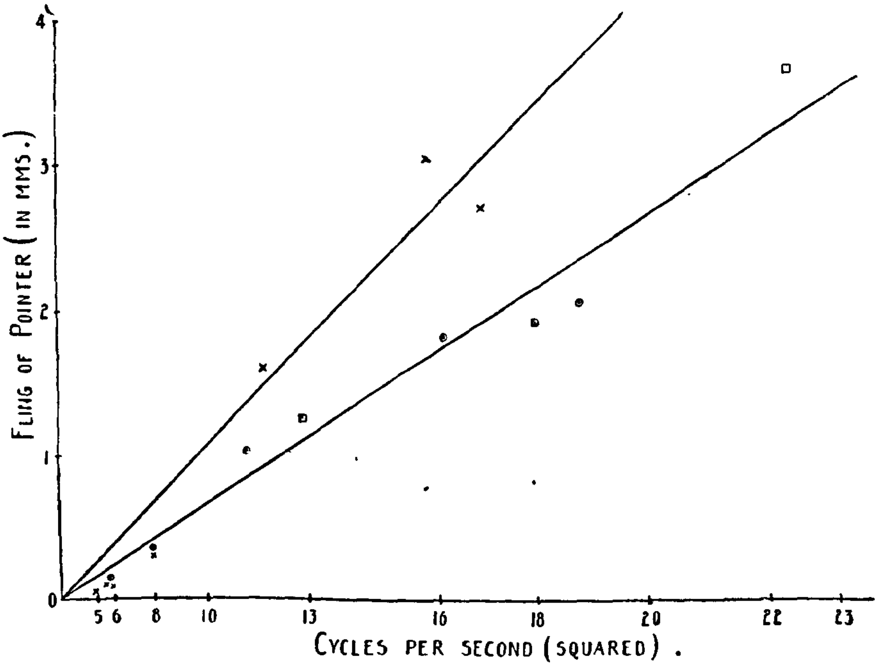

FIG. 2

Then $M/g \cdot \ddot{y} + (E + \mu)y = E(x - x_0) - \mu y_0$

$$= E\dot{x}_0(t - t_0) - \mu y_0 \quad \ldots\ldots \quad (16a)$$

which may be written in the form—

$$(D^2 + a)y = bt' \quad \ldots\ldots\ldots \quad (16b)$$

where $a = \frac{g}{M}(E + \mu)$, $b = \frac{gE}{M} \cdot \dot{x}_0$, and $t' = t_0 + \frac{\mu y_0}{E x_0}$, $dt' = dt$

The solution obtained by the general method is—

$$y = A \cos \sqrt{a} \,.\, t' + B \sin \sqrt{a} \,.\, t' + bt'/a$$

$$\text{and } \dot{y} = - A\sqrt{a} \sin \sqrt{a} \,.\, t' + B\sqrt{a} \,.\, \cos \sqrt{a} \,.\, t' + b/a$$

When the tension of the thread just equals the initial tension of the spring, $t' = 0$, $y = 0$, and $\dot{y} = 0$, whence $A = 0$ and

$$B = -\frac{b}{a\sqrt{a}} \cdot$$

Substituting, $y = \frac{b}{a}\left(t' - \frac{1}{\sqrt{a}} \cdot \sin \sqrt{a}\, t'\right)$

$$\dot{y} = \frac{b}{a}(1 - \cos \sqrt{a}\, t')$$

$$\ddot{y} = \frac{b}{\sqrt{a}} \cdot \sin \sqrt{a}\, t'$$

$$M/g\, \ddot{y} = E\dot{x}_0 \cdot \frac{1}{\sqrt{a}} \sin . \, 2\pi \cdot \frac{t'}{2\pi/\sqrt{a}} \cdot \quad \ldots\ldots \quad (24)$$

The oscillation of stress set up by the initial jerk has then an amplitude $E\dot{x}/\sqrt{a}$ and a period $2\pi/\sqrt{a}$. A similar oscillation is set up if the slide be held at rest under a slight tension of yarn and spring, and then disturbed. By trial the period is then a small fraction of a second as given by Equation (17), but the vibration is so highly damped that two periods can scarcely be detected. The amplitude under testing conditions is greatest when $t' = \pi/2\sqrt{a}$, succeeding oscillations being much less than given by the formula.

An extension of 5% and a break at the middle of the record give a value of E, $0{\cdot}3R$, which should be close to the value in most cases, while μ is always $0{\cdot}2R$ approximately, where R is the maximum reading of the record.

The amplitude of the initial jerk stress—

$$E\dot{x}_0/\sqrt{a} = 0{\cdot}0967 \frac{E}{\sqrt{E+\mu}} \cdot p \qquad (25)$$

$$= 0{\cdot}1368\ E\ p/\sqrt{R}$$

$$= 0{\cdot}041\ p\sqrt{R} \text{ oz. approx.}$$

The period of the oscillation—

$$2\pi/\sqrt{a} = 0{\cdot}41/\sqrt{E+\mu} \qquad (26)$$

$$= 0{\cdot}58/\sqrt{R} \text{ sec. approx.}$$

The time from loading to rupture $(\theta - \theta_0)/\omega$

$$= (\theta - 0{\cdot}6816)/0{\cdot}1047\ p \text{ sec.}$$

$$= 3{\cdot}08/p \text{ sec. for a break at } \tfrac{1}{2}R.$$

For higher speeds and weaker springs therefore the initial jerk is more serious and the oscillations have less time to die down. As examples—

When $R = 4$ oz., $p = 8$ per minute,

then the amplitude $= \tfrac{3}{4}$ oz. on a 2 oz. breaking load.

the period $= 0{\cdot}29$ sec.,

and the time of break $= 0{\cdot}39$ sec.

Results under these conditions would be quite untrustworthy and the machine must be slowed down very considerably when using these weaker springs. As an arbitrary criterion it might be laid down that the period should not exceed one fourth of the time of break, nor the amplitude one fourth of the breaking load, the safe speed then being found from Equations 25 and 26—

When $R = 32$ oz., $p = 6$ per minute,

the amplitude $= 1{\cdot}85$ oz. on a 16 oz. breaking load.

the period $= 0{\cdot}10$ sec.,

and the time of break $= 0{\cdot}51$ sec.

Under these conditions results will be unaffected. The frequency curve of breaking loads in this case is shown in Fig. 3, Curve I., and the effect of increased speed is clearly shown in the Curves II. and III.

The rate of loading, ignoring momentum errors, is given by—

$$\delta f/\delta t = \mu\dot{y} = \frac{\mu E}{E+\mu} \cdot \dot{x} \qquad (27)$$

The velocity of the loading jaw, $\dot{x}$ (Equation 12), decreases during the loading from 11·9 cm. per second to 10·4 cm. per second at $R/2$, and 8·0 cm. per second at the top of the record, but this is not enough to affect the strength seriously. Using the same approximations as before, the rate of loading at rupture is 0·156 $p.R.$ oz. per second.

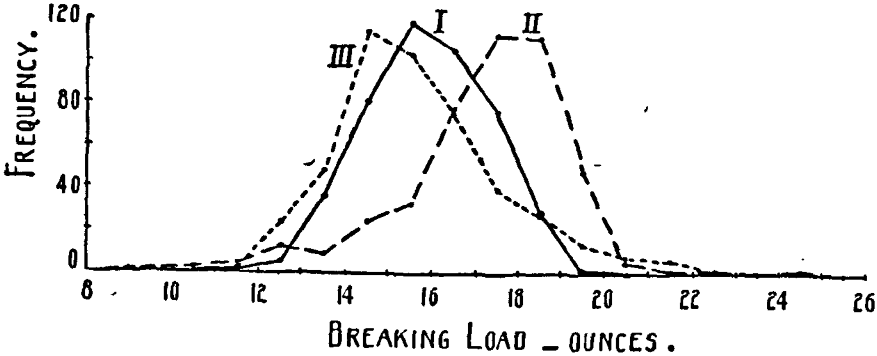

FIG. 3

Final extension 6·5%; 32 oz. spring; period of oscillation 0·11 sec.

I.—At 6 cycles per minute. Initial jerk 2·7 oz. Time of break 0·51 sec. Correction −0·15 oz. Normal frequency curve.

II.—At 18 cycles per minute. Initial jerk 3·7 oz. Time of break 0·17 sec. Correction −1·33 oz. Breaking load of most frequent group increased by greater speed but some 50 of the weaker threads affected by initial jerk.

III.—At 22·5 cycles per minute. Initial jerk 4·6 oz. Time of break 0·14 sec. Correction −2·06 oz. Most frequent group affected by initial jerk but many of the strong threads showing the normal increase of breaking load.

LOAD-EXTENSION INSTRUMENT

The equations of motion for the type of machine designed by Shorter and Hall[3] are very similar to those for the Moscrop tester, but there is more opportunity to vary the parts in order to eliminate errors. Oscillating stresses are overcome by making the period of the spring with recording jaw small in comparison with the time of break, the necessary calculations being given in the paper quoted. The driving mechanism can be arranged at will; the velocity of the moving grip can be made constant and the initial jerk avoided by using an oil-plunger drive.

The rate of loading is given by Equation 27, as for the Moscrop machine. When the carriage is moved steadily, the openness of scale, rate of loading, and momentum of recording system are all very nearly constant.

There is no friction to disturb the measurement, that of the recording needle on the glass plate being almost infinitesimal.

BALLISTIC TESTER

The essential point in the construction of a ballistic tester is to ensure that no appreciable energy is lost in the swing apart from that absorbed by the specimen. To this end, the anchorage, pendulum, and frame must be made rigid, that is, the distortion due to the breaking stress must be quite negligible compared with the extension of the specimen. The line of application of the stress must pass through the centre of percussion of the pendulum in order to avoid impulsive bending moments in the pendulum and reactions on the support.

The rigidity of the parts, friction of the bearings and pointer, dead stop of the latter, and uniformity of release are arranged for in the present instrument by mechanical details which were considered in Paper II. and which ensure freedom from the corresponding errors. Dynamical analysis is necessary to calculate the position of the centre of percussion, the energy and the rate of break.

If, as in the present case, the pendulum has two planes of symmetry intersecting in the longitudinal axis, its dynamics can be fully described by its mass, radius of gyration, and the position on the axis of the centres of suspension, of gravity, and of percussion. The grip for the specimen is made to coincide with the last point, preliminary calculations for the design of the pendulum having been made to ensure that its position would be convenient.

The mass M was found by weighing to be 3524·1 gms., the distance from the centre of suspension to the centre of gravity l was found by balancing on a knife edge to be 26·16 cm. The distance to the centre of percussion l_0 is obtained from the period of the pendulum—

$$t = 2\pi\sqrt{l_0/g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (28)$$

The time of vibration was 1·24 secs., giving l_0 38·2 cm. Measurement from the centre of the axis to the centre of the rod on which the grip is placed gave 37·8 cm. in the finished machine. A rider was provided to adjust any such difference, but was not used as the disparity is sufficiently small. The radius of gyration, k, is given by the formula—

$$k^2 = l\,l_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (29)$$

as 31·6 cm. The moment of inertia $I = Mk^2 = 1{\cdot}114 \times 10^5$ gm. cm.^{2}

If θ_0 be the angular displacement of the pendulum at release, the height of the centre of gravity above its position at the bottom of the swing—

$$h_0 = l\,(1 - \cos\,\theta_0) \quad . \quad . \quad . \quad . \quad . \qquad (30)$$

In falling this height, the kinetic energy acquired is $Mgh_0 = \frac{1}{2}Mk^2\omega_0^2$, and the angular velocity at the bottom of the swing—

$$\omega_0 = \sqrt{2gh}/k. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (31)$$

The energy gained in falling from the horizontal position—

$$\begin{aligned} Mgl &= 9{\cdot}05 \text{ joules.} \\ &= 92{\cdot}17 \times 10^3 \text{ gm. cm.} \\ &= 6{\cdot}67 \text{ feet-lbs.} \\ &= 80{\cdot}1 \text{ inch-lbs.} \end{aligned}$$

The scale is graduated in fractions of the total height of fall (equal intervals of $1 - \cos\,\theta$), and absolute values of the work are found by multiplying readings by the above values for the total capacity.

If a specimen be broken at the bottom of the swing, the centre of gravity rises to a height h_1 and the work of rupture—

$$\begin{aligned} W_1 &= Mg(h_0 - h_1) \\ &= Mgl\,(\cos\,\theta_1 - \cos\,\theta_0) \\ &= \int f.\,de \end{aligned}$$

where f is the force on the specimen at any moment, e its extension.

When the specimen is a lea of yarn in positive grips, the manner of break is given by the case analysed in Paper V. of gripped threads originally at the same tension. The complications involved in measuring the breaking load do not concern the measurement of work of rupture, but affect the precise rate at which the tension increases. Up till the rupture of the first thread, the tension increases proportionally with the extension, the ratio of load to extension ($E = f/e$) being the sum of the ratios for the individual threads. Thereafter the rate of increase diminishes to zero at the maximum load, the tension decreases, and suddenly ceases. The change of tension with extension during rupture is given by Equation (8c), *loc. cit.*, but not in a form suitable for calculation.

The rate of break and of loading for a specimen obeying Hooke's Law to rupture is more generally applicable and close enough to actual cases for practical purposes. The fixed grip of the tester is placed so that the specimen comes under tension just before the bottom of the swing and rupture is complete just after. During the extension, the pendulum grip is moving practically in a straight line with velocity $v = \omega l_0$, and

$$Mk^2 \frac{d\omega}{dt} = -f \,.\, l_0$$

$$\therefore f = Ee = -M \,.\, \frac{k^2}{l_0^2} \cdot \frac{dv}{dt}$$

$$\text{and } \frac{d^2e}{dt^2} = -\frac{l_0}{l} \cdot \frac{E}{M} \cdot e = -Ae \quad . \quad . \quad (32)$$

Whence $$v = \frac{de}{dt} = \sqrt{C - Ae^2} = \left[\frac{2l_0}{Ml}(Mgh - \tfrac{1}{2}Ee)\right]^{\frac{1}{2}} \quad . \quad . \quad . \quad (33)$$

where C is a constant of integration whose value is given by—

$$C = v^2 = 2gh_0 \cdot \frac{l_0^2}{k^2} = 2gh_0 \cdot \frac{l_0}{l} \cdot$$

Integrating $t\sqrt{A} = \sin^{-1}\dfrac{e}{\sqrt{C/A}} + C_1$

When $t = 0$, $e = 0$, and $C_1 = 0$

$$\therefore t = \frac{1}{\sqrt{A}} \cdot \sin^{-1}\frac{e}{\sqrt{C/A}}$$

$$= \sqrt{\frac{l_0}{l} \cdot \frac{M}{E}} \cdot \sin^{-1} e\sqrt{\frac{E}{2Mgh_0}} \quad . \quad . \quad (34)$$

The duration of the process of rupture is given by substituting the limiting extension for e. In the case of a lea, the mean breaking extension of single threads will give a sufficiently accurate measure of this slightly indeterminate time. It can also be expressed in the forms—

$$t = e\sqrt{\frac{Ml}{2Wl_0}} \cdot \sin^{-1}\sqrt{\frac{W}{W_0}} = \frac{0{\cdot}116e}{\sqrt{R - R_0}} \cdot \sin^{-1}\sqrt{\frac{R_0 - R}{R_0}} \text{ sec.} \quad (34a)$$

the last applying to the particular machine, where R is the reading in 1/1000ths of the total capacity corresponding to the kinetic energy at the time t.

H

The rate of extension—

$$\frac{de}{dt} = \sqrt{\frac{2l_0}{Ml}} \cdot \sqrt{W_0 - W} = 8{\cdot}614\sqrt{R} \text{ . cm./sec.} \quad . \quad (35)$$

and the tension at time t—

$$f = \frac{2}{e_1}\sqrt{W_1 W_0} \cdot \sin\sqrt{\frac{2l_0}{Ml}} \cdot \frac{\sqrt{W_1}}{e_1} \cdot t$$

$$= 184{\cdot}34\sqrt{R_0(R_0 - R_1)} \,/\, e_1 \,.\, \sin 8{\cdot}614\sqrt{R_0 - R_1}/e_1 \,.\, t \quad (36)$$

where e_1 is the final extension.

The time which would be taken in stretching to rupture at the rate of extension obtaining just before rupture is—

$$T = e_1 \bigg/ \frac{\delta e_1}{\delta t} = \frac{0{\cdot}116 e_1}{\sqrt{R_1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

REFERENCES

[1] Shorter. J. Text. Inst., 1923, **14**, T508.
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Received for publication, 9th October 1925.