

I'm Chris Grossack, and I study the geometric structure of certain groups of functions. By this, I mean I have a bunch of functions that I carry around with me (who doesn't?) and I want to position my functions around me in a way that makes sense (So randomly putting functions all over the place is not helpful, I want to be meaningful with where I put things). I then want to use the geometric structure of where the functions are positioned in order to better understand the functions themselves.

If you'll indulge me, as a mathematician I feel obligated to say big words, so I'll need to give some definitions. However, as a mathematician, I almost never get to have as much fun with how I present definitions as I like, because it's not "serious" enough. I will try here to simultaneously have fun, while still explaining what is happening.

**Definition:** A *binary string* is a (possibly infinite) sequence of 0s and 1s. 01101110011001010111001001100100, for instance, is a binary string.

**Definition:** If I have a bunch of functions, this collection of functions is called a *Group* of functions if it satisfies the following bonus properties:

1. There is a fancy function called  $I$  (for identity) so that for every  $x$ ,  $I(x) = x$ .  
 $I$  is the "do nothing" function, and it is in our collection.
2. For any two functions in our bag,  $f$  and  $g$  there is another function called  $f + g$  which is also in our bag. Here,  $(f + g)(x) = f(g(x))$ . We also ask that addition not behave stupidly.  
Namely  $(f + g) + h = f + (g + h)$ , and  $f + g = g + f$ .
3. For every function  $f$  which is in our bag, there is another function in our bag (which I will suggestively call  $-f$ ) such that for every  $x$ ,  $f + (-f) = (-f) + f = I$  (that is,  $-f$  undoes  $f$ ).

Now, to preemptively stop certain kinds of mathematicians from trying to revoke my Math Card™, I am aware this makes us an *Abelian* Group, and is slightly less general than a regular group. To those who care, I kindly ask you to get over it.

There are a lot of ways to define functions which are slightly more involved than the standard definitions. For instance, here are the objects which create the functions I study:

**Definition:** A *Finite State Automaton* is a machine like the ones shown below:

