ALGORITHMS FOR THE DETERMINATION OF POLYNOMIALS WITH SMALL MAHLER MEASURE

by

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DISSERTATION

Presented to the Faculty of the Graduate School of

The University of Texas at Austin

in Partial Fulfillment

of the Requirements

for the Degree of

DOCTOR OF PHILOSOPHY

THE UNIVERSITY OF TEXAS AT AUSTIN

August 1995

Acknowledgements

I would like to thank my supervisor, Jeffrey Vaaler, for his expert advice, keen insights, and thoughtful direction. I would also like to thank him for his generosity with his time, and for his patience during a time when I completed work on another project.

I would like to thank all the members of the committee. Their careful reading of the manuscript improved its clarity and accuracy.

I would like to thank David Boyd for many helpful suggestions, especially for pointing out an improvement to proposition 3.9. I would also like to thank him for directing me to several valuable references, and for making his list of polynomials having small Mahler measure available to me.

I would also like to thank Christopher Pinner for sharing his insights on Lehmer's problem.

Finally, I would like to thank my wife, Kristine, for her constant support and encouragement.

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Publication No. ____________________

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The University of Texas at Austin, 1995

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Lehmer's conjecture states that there is an absolute constant $c > 1$ so that if $f(x)$ is any polynomial with integer coefficients, then its Mahler measure $M(f)$ satisfies $M(f) = 1$ or $M(f) \geq c$. We search for polynomials with small Mahler measure by considering slightly perturbed products of cyclotomic polynomials. By analyzing a problem in the theory of partitions, we show that our algorithm has computational complexity that is subexponential in the degree. We report on the implementation of this algorithm and describe its results through degree 64. Finally, we discover a new limit point of Mahler measures of polynomials. Its value is approximately 1.3091.

An appendix lists 1560 irreducible, noncyclotomic polynomials with Mahler measure less than 1.3 and degree at most 64. None of these has Mahler measure less than Lehmer's degree 10 example reported in 1933.

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CHAPTER 1

Introduction

Suppose that $f(x)$ is a monic polynomial with integer coefficients. The Mahler measure of $f(x)$, denoted $M(f)$, is the product of the absolute values of the roots of $f(x)$ that lie outside the unit circle. Clearly, $M(f) \geq 1$, and if $f(x)$ is a product of cyclotomic polynomials, then $M(f) = 1$. A result of Kronecker (theorem 2.9) shows that this is essentially the only case that the minimum value of $M(f)$ is attained.

In 1933, D. H. Lehmer [22] asked if it was possible to find a noncyclotomic polynomial with integer coefficients having Mahler measure arbitrarily close to 1. Lehmer's conjecture asserts that this is in fact not possible.

Lehmer's Conjecture. There exists a constant $c > 1$ such that any irreducible, noncyclotomic polynomial $f(x)$ with integer coefficients and $f(0) \neq 0$ satisfies $M(f) \geq c$.

Lehmer discovered that the polynomial

$$f(x) = x^{10} + x^9 - x^7 - x^6 - x^5 - x^4 - x^3 + x + 1 \tag{1}$$

has Mahler measure

$$M(f) = 1.1762808\ldots \tag{2}$$

Lehmer's conjecture remains open, and (2) remains the smallest known Mahler measure of an irreducible, noncyclotomic polynomial with integer coefficients. Thus a strong version of Lehmer's conjecture takes c to be this value.

In this thesis, we are interested in finding polynomials with small Mahler measure. Chapter 2 gives some elementary properties of the Mahler measure and describes several applications of Lehmer's conjecture. Chapter 3 reports some theoretical lower bounds for the Mahler measure and describes some previous computational searches for polynomials with small Mahler measure, including extensive searches performed by Boyd.

Pinner observed that many of the polynomials that Boyd found may be written as a product of cyclotomic polynomials, plus or minus a power of x. We employ a new search strategy based on this observation. We search for polynomials with small Mahler measure by forming all possible products of cyclotomic polynomials, subject to some restrictions on the maximum multiplicity of any cyclotomic factor, then adjusting each product by altering one or more of the coefficients.

In order to determine the computational complexity of our algorithm, we consider some partition problems in chapter 4. We analyze the number of ways to write a given integer n as a sum of the form

$$n = \sum_{m \geq 1} a_m \varphi(m)$$

where φ is the Euler totient function and a_m is a nonnegative integer no larger than a given parameter k. This counts the number of polynomials of degree n that are products of cyclotomic polynomials, where no factor has multiplicity greater than k. Note that we have replaced the integer summands of the classical partition problem with the image of the integers under φ.

In chapter 4 we review some results of Boyd and Montgomery giving asymptotic estimates for the cases $k = 1$ and $k = \infty$, and we derive an asymptotic estimate for the case $k = 2$. This result allows us to conclude that our algorithm for searching for polynomials with small Mahler measure has complexity that is subexponential in the degree D. We determine in chapter 5 that our algorithm is $O(C^{\sqrt{D}})$, where C is a constant.

We have implemented this algorithm and have used it to search for polynomials with Mahler measure at most 1.3 through degree 64. Chapter 5 gives a detailed description of our algorithm and a summary of our results. Appendix A gives a complete list of the polynomials that we find.

We also investigate some small limit points of Mahler measures of polynomials. We document many polynomials with Mahler measure below 1.3 by considering the polynomials occurring in the sequences corresponding to the three known limit points below 1.324. Finally, we report a new limit point of Mahler measures near 1.3091. By considering the polynomials associated with this limit point, we find many additional polynomials with Mahler measure at most 1.3 and degree at most 64. These results are given in chapter 6.

In spite of these searches, no polynomial we discovered has Mahler measure less than that of Lehmer's degree 10 example given in (1).

CHAPTER 2

The Mahler Measure of a Polynomial

1. Some Basic Properties

Let $f(x)$ be a polynomial with complex coefficients, and denote the degree of $f(x)$ by D. We shall use a_k to denote the coefficients of $f(x)$, and α_k to denote its roots. Thus,

$$\begin{aligned} f(x) &= \sum_{0\le k\le D} a_k x^k \\ &= a_D \prod_{1\le k\le D} (x-\alpha_k). \end{aligned}$$

We define the *Mahler measure* of $f(x)$, denoted $M(f)$, by

$$M(f) = |a_D| \prod_{1\le k\le D} \max\{1, |\alpha_k|\}. \tag{3}$$

Many elementary properties of the Mahler measure may be found in [26, chapter 1] and [28, chapter 4]. We note some of these below.

LEMMA 2.1. *The Mahler measure satisfies the following:*

(i) $M(f(x)g(x)) = M(f(x))M(g(x))$.

(ii) $M(f(x^n)) = M(f(x))$.

(iii) $M(x^D f(1/x)) = M(f(x))$.

PROOF. Statements (i) and (ii) are immediate. For (iii), suppose first that $f(0) \neq 0$. Then

$$\begin{aligned} M(x^D f(1/x)) &= |a_0| \prod_{1 \le k \le D} \max\left\{1, |\alpha_k|^{-1}\right\} \\ &= \left(|a_D| \prod_{1 \le k \le D} |\alpha_k|\right)\left(\prod_{1 \le k \le D} \max\left\{1, |\alpha_k|^{-1}\right\}\right) \\ &= |a_D| \prod_{1 \le k \le D} \max\{1, |\alpha_k|\} \\ &= M(f(x)). \end{aligned}$$

If $f(0) = 0$, write $f(x) = x^r g(x)$ where $g(0) \neq 0$. Then

$$M(x^D f(1/x)) = M(x^{D-r} g(1/x)) = M(g(x)) = M(f(x)).$$

$\square$

The polynomial $x^D f(1/x)$ appearing in (iii) is called the *reciprocal polynomial of* $f(x)$. A polynomial satisfying $x^D f(1/x) = f(x)$ is called a *reciprocal polynomial.*

We define the *length* of a polynomial by

$$L(f) = \sum_{0 \le k \le D} |a_k|,$$

the *Euclidean norm* by

$$\|f\| = \left(\sum_{0 \le k \le D} |a_k|^2\right)^{1/2},$$

and the *height* by

$$H(f) = \max_{0 \le k \le D} |a_k|.$$

The following lemmas exhibit relationships among the length, Euclidean norm, and Mahler measure of a polynomial.

Lemma 2.2. *Suppose $f(x)$ is a polynomial with degree D. The k-th coefficient a_k of $f(x)$ satisfies*

$$|a_k| \le \binom{D}{k} M(f). \tag{4}$$

Proof. The k-th elementary symmetric function evaluated at the roots of $f(x)$ is a_{D-k}/a_D. Thus

$$\begin{aligned} |a_{D-k}| &= |a_D \sum_{1 \le i_1 < i_2 < \cdots < i_k \le D} \alpha_{i_1}\alpha_{i_2}\cdots\alpha_{i_k}| \\ &\le |a_D| \sum_{1 \le i_1 < i_2 < \cdots < i_k \le D} |\alpha_{i_1}\alpha_{i_2}\cdots\alpha_{i_k}| \\ &\le \binom{D}{k} M(f). \end{aligned}$$

The lemma follows. $\square$

We immediately obtain the following corollaries.

Corollary 2.3. *Suppose $f(x)$ is a polynomial with degree D. Then*

$$L(f) \le 2^D M(f). \tag{5}$$

Corollary 2.4. *For any positive integer D and positive real number M, there are only finitely many polynomials $f(x) \in \mathbb{Z}[x]$ with $\deg(f) \le D$ and $M(f) \le M$.*

Next, we prove an inequality of Gonçalves. The proof given here is due to Mignotte [27]. We require the following lemma.

Lemma 2.5. *If $g(x) \in \mathbb{C}[x]$ and $z \in \mathbb{C}$, then*

$$\|(x-z)g(x)\| = \|(\bar{z}x - 1)g(x)\|.$$

PROOF. Write $g(x) = \sum_{0 \le k \le D} c_k x^k$. Then

$$\begin{aligned}
\|(x-z)g(x)\|^2 &= \left\| \sum_{0\le k\le D} c_k x^{k+1} - \sum_{0\le k\le D} c_k z x^k \right\|^2 \\
&= \left\| \sum_{0\le k\le D+1} (c_{k-1} - c_k z) x^k \right\|^2 \\
&= \sum_{0\le k\le D+1} (c_{k-1} - c_k z)(\bar{c}_{k-1} - \bar{c}_k \bar{z}) \\
&= (1+|z|^2)\|g\|^2 - \sum_{0\le k\le D+1} (c_{k-1}\bar{c}_k \bar{z} + \bar{c}_{k-1} c_k z).
\end{aligned}$$

Similarly,

$$\begin{aligned}
\|(\bar{z}x-1)g(x)\|^2 &= \left\| \sum_{0\le k\le D} c_k \bar{z} x^{k+1} - \sum_{0\le k\le D} c_k x^k \right\|^2 \\
&= \left\| \sum_{0\le k\le D+1} (\bar{z} c_{k-1} - c_k) x^k \right\|^2 \\
&= \sum_{0\le k\le D+1} (\bar{z} c_{k-1} - c_k)(z\bar{c}_{k-1} - \bar{c}_k) \\
&= (1+|z|^2)\|g\|^2 - \sum_{0\le k\le D+1} (c_{k-1}\bar{c}_k \bar{z} + \bar{c}_{k-1} c_k z).
\end{aligned}$$

□

We now prove Gonçalves' inequality.

LEMMA 2.6. *Let $f(x) = \sum_{0\le k\le D} a_k x^k$ be a nonconstant polynomial with complex coefficients. Then*

$$M(f)^2 + |a_0 a_D| M(f)^{-2} \le \|f\|^2. \tag{6}$$

PROOF. Suppose $\alpha_1, \alpha_2, \ldots, \alpha_n$ are the roots of $f(x)$ that lie outside the unit circle. Consider the polynomial

$$\begin{aligned}
g(x) &= a_D \prod_{1\le k\le n} (\bar{\alpha}_k x - 1) \prod_{n+1\le k\le D} (x - \alpha_k) \\
&= \sum_{0\le k\le D} b_k x^k.
\end{aligned}$$

Applying lemma 2.5 n times, we conclude $\|g\| = \|f\|$. Also, we see that

$$\begin{aligned}\|g\|^2 &\geq |b_0|^2 + |b_D|^2 \\ &= |a_D|^2 \prod_{n+1\leq k\leq D} |\alpha_k|^2 + |a_D|^2 \prod_{1\leq k\leq n} |\alpha_k|^2 \\ &= |a_0 a_D|^2 M(f)^{-2} + M(f)^2.\end{aligned}$$

The desired inequality follows. $\square$

An inequality of Landau follows immediately.

COROLLARY 2.7. *Let $f(x)$ be a polynomial with complex coefficients which is not a monomial. Then*

$$M(f) < \|f\|. \tag{7}$$

We have the following theorem.

THEOREM 2.8. *Let $f(x)$ be a nonconstant polynomial with integer coefficients and degree D. Then*

$$M(f) \leq \|f\| \leq L(f) \leq 2^D M(f). \tag{8}$$

Let $f(x)$ be a polynomial with integer coefficients and nonzero constant term. Clearly, $M(f) \geq 1$, and if $f(x)$ is a product of cyclotomic polynomials, then $M(f) = 1$. The following theorem of Kronecker [21] shows that this is the only situation when this lower bound is attained.

THEOREM 2.9. *Suppose $f(x)$ is a polynomial of degree D with integer coefficients and $f(0) \neq 0$. Then $M(f) = 1$ if and only if $f(x)$ is a product of cyclotomic polynomials.*

PROOF. Suppose $M(f) = 1$. Because $|f(0)| \neq 0$, all the roots of $f(x)$ lie on the unit circle. Let α be a root of $f(x)$, and let $f_n(x)$ be the minimal polynomial of α^n, for $n \geq 1$. The conjugates of α^n all lie on the unit circle, since they are all n-th powers of roots of $f(x)$. So $M(f_n) = 1$ for $n \geq 1$. Therefore, by (8),

$$\|f_n\| \leq 2^D.$$

Now there are only finitely many polynomials $g(x)$ with integer coefficients having $\deg(g) \leq D$ and $\|g\| \leq 2^D$, so there must exist distinct integers n and m so that $\alpha^n = \alpha^m$. Therefore α is a root of unity. □

We note an analytic formulation of the Mahler measure. If $f(x) \in \mathbb{C}[x]$, then by Jensen's formula,

$$\log M(f) = \int_0^1 \log \left| f\big(e(t)\big) \right| dt \tag{9}$$

where $e(t) = \exp(2\pi i t)$.

Finally, if α is an algebraic number, we often use $M(\alpha)$ to denote the Mahler measure of the minimal polynomial of α over $\mathbb{Z}$.

2. Some Applications

In this section, we briefly describe several mathematical problems where polynomials with small Mahler measure play an important role.

2.1. Finding Large Prime Numbers. In his 1933 paper, D. H. Lehmer [22] determined that finding a polynomial with small Mahler measure would enable one to construct a sequence of integers whose prime divisors must lie in a very restricted set. Let $f(x)$ be a monic polynomial with integer coefficients and roots α_1, α_2, …, α_D. For each positive integer N, Lehmer defines the integer

$$\begin{aligned} \Delta_N(f) &= \operatorname{Res}\big(x^N - 1, f(x)\big) \\ &= \prod_{1 \leq k \leq D} (\alpha_k^N - 1). \end{aligned} \tag{10}$$

So $\Delta_n(f)$ divides $\Delta_N(f)$ if n divides N. Lehmer defines a *characteristic prime factor* of $\Delta_N(f)$ as a prime number p such that $p \mid \Delta_N(f)$ but $p \nmid \Delta_n(f)$ for any n dividing N. He then proves the following results:

(i) A characteristic prime factor p of $\Delta_N(f)$ does not divide N.

(ii) Suppose $f(x)$ is irreducible, p is a characteristic prime factor of $\Delta_N(f)$, and $p^e \parallel \Delta_N(f)$. If w is the smallest positive integer for which $p^w \equiv 1 \pmod N$ (that is, p has order w in $(\mathbb{Z}/N\mathbb{Z})^*$), then $w \le D$ and $w \mid e$.

(iii) If $f(x)$ is a reciprocal polynomial then $\Delta_N(f)$ is a perfect square.

(iv) If no root of $f(x)$ lies on the unit circle, then $\lim_{N\to\infty} \frac{\Delta_{N+1}(f)}{\Delta_N(f)} = M(f)$.

We note that the Mahler measure is a good indicator of the rate of growth of $\Delta_N(f)$, even when $f(x)$ has roots on the unit circle. From (10), it is easy to show

$$\limsup_{N\to\infty} |\Delta_N(f)|^{1/N} = M(f). \tag{11}$$

Suppose then that we are given a polynomial $f(x)$ and an integer N and that we have completely factored $\Delta_n(f)$ for all divisors n of N. In order to factor $\Delta_N(f)$, we need to test only a few numbers as possible divisors: for each w between 1 and D, we test only the primes $p \le \Delta_N(f)^{1/2w}$ that have order $w \pmod N$. If $f(x)$ has small Mahler measure, then $\Delta_N(f)$ grows slowly, and the conditions on possible prime factors are quite restrictive. These conditions are even more restrictive if $f(x)$ is a reciprocal polynomial.

Using this strategy, Lehmer remarks that the following numbers have been shown to be prime:

$$\Delta_{127}(x^3 - x - 1) = 3\,23351\,42510\,32733$$

$$\left(\Delta_{379}(x^{10} + x^9 - x^7 - x^6 - x^5 - x^4 - x^3 + x + 1)\right)^{1/2} = 179\,43271\,40357.$$

We remark that (11) remains true when the limit supremum is replaced by a limit. Riley [34] uses the Gel'fond-Baker theory of linear forms in logarithms of algebraic numbers to show this. The quantity $|\Delta_N(f)|$ is analyzed by Riley because it appears in knot theory as the order of the first homology group of an N-sheeted cyclic cover of S^3 branched over a knot whose Alexander polynomial is $f(x)$.

2.2. The Number of Noncyclotomic Factors of a Polynomial. Let $f(x)$ be a polynomial with integer coefficients and $f(0) \neq 0$, and write

$$f(x) = \Phi(x) \prod_{1 \leq j \leq J} g_j(x)^{m_j}$$

where $\Phi(x)$ is a product of cyclotomic polynomials and the $g_j(x)$ are distinct, irreducible, noncyclotomic polynomials. Following Schinzel [38], we note that if Lehmer's conjecture is true, then

$$M(f) \geq \prod_{1 \leq j \leq J} c^{m_j}$$

where c is the constant appearing in Lehmer's conjecture. From (7) it follows that

$$\sum_{1 \leq j \leq J} m_j \leq \frac{\log \|f\|}{\log c}. \tag{12}$$

Pinner [29] shows that the weaker inequality

$$\max_{1 \leq j \leq J} m_j \leq \frac{\log \|f\|}{\log c}$$

is sharp if Lehmer's conjecture holds. He also proves that this inequality is in fact equivalent to Lehmer's conjecture. More information on the relationship between the Mahler measure of a polynomial and the number of its irreducible noncyclotomic factors appears in [30].

2.3. PV Numbers and Salem Numbers. Some special types of algebraic numbers play an important role in the study of polynomials with small Mahler measure. A *Pisot-Vijayaraghavan number*, or *PV number*, is an algebraic integer $\alpha > 1$, all of whose conjugates lie strictly inside the unit circle. A *Salem number* is an algebraic integer $\alpha > 1$, all of whose conjugates lie inside or on the unit circle, with at least one conjugate on the unit circle. We record two simple consequences of this definition:

- All the conjugates of a Salem number α except one occur on the unit circle. The remaining conjugate is $1/\alpha$.
- The minimal polynomial of a Salem number is reciprocal.

Let S denote the set of PV numbers and T the set of Salem numbers. Salem [35] proved that S is closed, and Siegel [39] showed that the smallest element of S is the real root of $x^3 - x - 1$. Its value is approximately 1.32472. Salem [36] also proved that every point in S is a limit point from both sides of elements of T. It is unknown whether $S \cup T$ is closed, and whether T has a smallest element.

These numbers also arise in the study of the distribution of certain real sequences modulo 1. A theorem of Koksma states that the sequence $\{\beta^n\}_{n\geq 1}$ is uniformly distributed modulo 1 for almost all real numbers $\beta > 1$. But neither PV nor Salem numbers have this property, so $S \cup T$ represents a nontrivial exceptional set in Koksma's theorem. A proof of this fact, as well as many other interesting properties of these numbers, may be found in [1] or [37].

We mention in closing that the three smallest known Mahler measures of polynomials with integer coefficients are Salem numbers. Boyd [4, 5] has conducted extensive computational searches for small Salem numbers. There are 43 known Salem numbers smaller than 1.3.

2.4. Ergodic Theory. Polynomials with small Mahler measure also find application in ergodic theory. Let X be a compact topological space, and let m be a probability measure on the Borel sets of X. Let $\mathcal{A} = \{A_1, A_2, \dots, A_N\}$ be a finite partition of X where each A_n is measurable. Define the *entropy* of $\mathcal{A}$ by

$$H(\mathcal{A}) = -\sum_{1 \le n \le N} m(A_n) \log m(A_n).$$

If $\mathcal{A}$ and $\mathcal{B}$ are finite partitions of X, define

$$\mathcal{A} \vee \mathcal{B} = \{A \cap B : A \in \mathcal{A}, B \in \mathcal{B}\}.$$

Let T be a measure-preserving function on X. Define the entropy of T by

$$h(T) = \sup_{\mathcal{A}} \left\{ \lim_{L \to \infty} \frac{1}{L} H\Big(\mathcal{A} \vee T^{-1}(\mathcal{A}) \vee T^{-2}(\mathcal{A}) \vee \ldots \vee T^{-(L-1)}(\mathcal{A})\Big) \right\}$$

where

$$T^{-1}(\mathcal{A}) = \{T^{-1}(A_1), T^{-1}(A_2), \dots, T^{-1}(A_N)\}.$$

Let $f(x)$ be a monic polynomial of degree D with integer coefficients and $f(0) = \pm 1$. Let A be the companion matrix of $f(x)$, and define a continuous map T_A on the D-torus $\mathbb{R}^D/\mathbb{Z}^D$ by $T_A\vec{x} = A\vec{x}$. A theorem of Sinai ([43], [47, chapter 6]) shows

$$h(T_A) = \log_2 M(f). \tag{13}$$

Therefore, finding a polynomial with small Mahler measure allows us to construct a continuous map on a D-torus with small entropy.

A measurable function T on X is *ergodic* if every measurable set B in X satisfying $T^{-1}(B) = B$ has $m(B) = 0$ or $m(B) = 1$. Let $\mathbb{T}^\infty$ denote the *infinite torus*, a countable direct product of copies of the circle group $\mathbb{R}/\mathbb{Z}$. Lind [24] proves that ergodic automorphisms of $\mathbb{T}^\infty$ with finite entropy exist if and only if Lehmer's conjecture is false.

2.5. Exceptional Units. A unit u in a number field is an *exceptional unit* if $1-u$ is also a unit. Evertse [18] proved that a number field of degree D contains at most $3\cdot 7^{3D}$ exceptional units. Silverman [40, 42] investigates how often α^n is an exceptional unit when α is a Salem number. He verifies that many powers of small Salem numbers are exceptional units, and proves that if α is an algebraic integer of degree D, then at most $O(D^{1+\epsilon})$ powers of α are exceptional units.

It is therefore of interest to investigate the frequency of exceptional units among powers of roots of polynomials with small Mahler measure.

2.6. Polylogarithm Relations. Let z be a complex number satisfying $|z|<1$, and let m be a positive integer. The m-th order polylogarithm of z is defined by

$$\mathrm{Li}_m(z) = \sum_{n\geq 1}\frac{z^n}{n^m}.$$

This function extends analytically to the whole complex plane slit along the real axis from 1 to infinity.

Cohen, Lewin, and Zagier [12] investigate $\mathbb{Z}$-linear relations among polylogarithms of powers of certain algebraic numbers. They find that the real root larger than 1 of Lehmer's degree 10 polynomial, $\alpha\approx 1.17628$, satisfies a large number of cyclotomic relations of the form

$$\alpha^N = \prod_{n\geq 1}(\alpha^n-1)^{c_n}$$

where the c_n are integers. They use these relations to show that $\mathbb{Z}$-linear relations among polylogarithms of powers of α exist for orders up to 16, and conjecturally construct these relations. These sixteenth order polylogarithm relations are by far the highest order such relations ever found.

Additional details on the role of polynomials with small Mahler measure in polylogarithm relations may be found in [23, chapters 7 and 16].

CHAPTER 3

Lower Bounds on the Mahler Measure

In this chapter we describe three kinds of results on bounding the Mahler measure of a polynomial with integer coefficients. The first section lists lower bounds for the general case. The second section describes some better lower bounds that hold for particular classes of polynomials. The third section discusses some computational searches for polynomials with small Mahler measure.

1. General Lower Bounds

In 1971, Blanksby and Montgomery [2] proved the following lower bound on the Mahler measure.

THEOREM 3.1. *Let α be an algebraic integer of degree $D > 1$ over the rationals. If α is not a root of unity, then*

$$M(\alpha) > 1 + \frac{1}{52D\log 6D}.$$

Their proof uses an averaging technique in Fourier analysis.

In 1978, Stewart [45] gave a new proof of this result, although with a different constant.

THEOREM 3.2. *Let α be an algebraic integer of degree $D > 1$ over the rationals. If α is not a root of unity, then*

$$M(\alpha) > 1 + \frac{1}{10^4 D \log D}.$$

Stewart's proof uses a method common on transcendence theory involving the construction of an auxiliary polynomial with small coefficients and prescribed zeros.

Dobrowolski [14] in 1979 improved the general lower bound considerably.

THEOREM 3.3. *Let ϵ be an arbitrary positive constant. If α is an algebraic integer of degree $D > 1$, α is not a root of unity, and $D > D_0(\epsilon)$, then*

$$M(\alpha) > 1 + (1 - \epsilon)\left(\frac{\log\log D}{\log D}\right)^3.$$

Dobrowolski's proof is based on Stewart's method, but employs the p-th powers of the conjugates of α for many small primes p. The key step in Dobrowolski's argument is based on the following observation.

LEMMA 3.4. *Let $f(x)$ be an irreducible polynomial over the integers of degree D. Let $f_p(x)$ be the polynomial whose roots are the p-th powers of the roots of $f(x)$. Then $p^D \mid \mathrm{Res}(f(x), f_p(x))$.*

A short time later, Cantor and Strauss [11] and Rausch [32] independently simplified Dobrowolski's argument and improved his result slightly, replacing the constant term $1 - \epsilon$ by $2 - \epsilon$. In 1983, a further refinement by Louboutin [25] achieved a constant of $9/4 - \epsilon$.

2. Special Lower Bounds

Lehmer's conjecture is known to hold for certain classes of polynomials. The most important result in this vein is due to Smyth [44], who in 1971 proved the following theorem.

THEOREM 3.5. *Suppose $f(x)$ is an irreducible polynomial with integer coefficients, $f(0) \neq 0$, and $f(1) \neq 0$. If $f(x)$ is not reciprocal, then*

$$M(f) \geq \theta_0 \tag{14}$$

where θ_0 is the real root of $x^3 - x - 1$, $\theta_0 = 1.32472\ldots$

Smyth's theorem immediately yields Siegel's result [39] that θ_0 is the smallest PV number.

In 1994, Silverman [41] proved Lehmer's conjecture for a special class of polynomials. Recall that if k is a number field of degree D over $\mathbb{Q}$ and p is a rational prime, then p is said to *split completely* in k if there are D distinct prime ideals in the ring of integers O_k of k lying over the ideal (p) in $\mathbb{Z}$.

THEOREM 3.6. *If α is an algebraic integer which is not a root of unity, and if there is a rational prime $p \leq \sqrt{D \log D}$ which splits completely in $\mathbb{Q}(\alpha)$, then*

$$M(\alpha) \geq 1 + \frac{1}{200}.$$

3. Computational Searches

In this section we introduce some of the techniques used to search for polynomials with small Mahler measure.

3.1. Boyd's Searches. In 1980, Boyd [7] developed an algorithm for searching for polynomials with small Mahler measure. He used it to find all polynomials with integer coefficients, Mahler measure at most 1.3, and degree at most 16, as well as all polynomials with height 1, Mahler measure at most 1.3, and degree at most 26. In 1989 [9], he extended these computations to include degree at most 20 for the exhaustive search and degree at most 32 for the height 1 search.

Searching among polynomials with ± 1 coefficients is of interest in view of the following result.

THEOREM 3.7. *If $f(x)$ is a polynomial with integer coefficients having Mahler measure less than 2, then there exists a polynomial $g(x)$ with integer coefficients such that $H(f(x)g(x)) = 1$.*

This result is a consequence of Siegel's lemma. From [3], one determines that there exists such a $g(x)$ with degree no larger than cD^2, where D is the degree of f and c satisfies the following inequality:

$$2\log(cD+1) + 3 - 4\log 2 < 4c\log(2/M(f)).$$

For example, if $M(f) \leq 1.3$ and $D \leq 32$, we may select c to be 6.3. On the other hand, nothing is known about the Mahler measure of $g(x)$. In his height 1 search, Boyd finds many examples where $g(x)$ may be taken to be cyclotomic. A theorem showing that such a cyclotomic $g(x)$ always exists (or at least that a $g(x)$ exists with $M(g)$ fairly small) would greatly assist in the determination of all polynomials of a given degree with small Mahler measure.

Lemma 2.2 immediately gives us an algorithm for finding all polynomials $f(x)$ of degree D with integer coefficients whose Mahler measure is at most M. We simply construct all polynomials whose k-th coefficient satisfies $|a_k| \le \binom{D}{k} M$, then compute the Mahler measure of each one. This is a very impractical algorithm. Boyd's method relies on sharper bounds for the coefficients of polynomials having small Mahler measure, and a fast technique for computing a lower bound on the Mahler measure of a polynomial.

Boyd proves the following inequalities in [7].

PROPOSITION 3.8. *Let $f(x)$ be a polynomial with complex coefficients, $f(x) = \sum_{0 \le k \le D} a_k x^k$, and suppose $M(f) \le M$. Then*

$$|a_k| \le \binom{D-2}{k-1} \left(M + |a_0 a_D| M^{-1}\right) + \binom{D-2}{k} |a_D| + \binom{D-2}{k-2} |a_0|. \tag{15}$$

This bound is sharp, for all D.

Boyd's second inequality gives an improved estimate in special situations.

PROPOSITION 3.9. *Let $f(x)$ be a monic, reciprocal polynomial with real coefficients, $f(x) = \sum_{0 \le k \le D} a_k x^k$, and suppose $M(f) \le M$. Suppose that all the negative real roots of $f(x)$ have multiplicity greater than or equal to 2, and that $a_1 \ge D - 4$. Then*

$$\begin{aligned} |a_k| \le \left(M + M^{-1} + 4\right) \binom{D-4}{k-2} + \binom{D-4}{k-4} + \binom{D-4}{k} \\ + 2\left(M^{1/2} + M^{-1/2}\right) \left\{ \binom{D-4}{k-3} + \binom{D-4}{k-1} \right\}. \end{aligned} \tag{16}$$

This proposition applies to a broader class of polynomials than the result proved in [7, lemma 6], as Boyd's original statement requires that $f(x)$ have no negative real roots. The proof of the more general proposition requires only minor modifications to the original argument.

SKETCH OF PROOF. Suppose α is a root of $f(x)$ and $|\alpha| > M^{1/2}$. Then α is real and positive, since the complex roots of $f(x)$ occur in conjugate pairs and the negative real roots have multiplicity at least 2. Let $\alpha_1, \alpha_2, \ldots, \alpha_r$ denote the positive real roots of $f(x)$. Then

$$\sum_{1\le i\le r} \alpha_i \ge \alpha + \alpha^{-1} > M^{1/2} + M^{-1/2}.$$

Let $\beta_1, \beta_2, \ldots, \beta_s$ be the complex and negative real roots of $f(x)$. Then $|\beta_i| < M^{1/4}$ for $1 \le i \le s$, so by [7, lemma 4],

$$|\sum_{1\le i\le s} \beta_i| \le \sum_{1\le i\le s} |\beta_i| \le 2\left(M^{1/4} + M^{-1/4}\right) + D - 6.$$

Therefore,

$$\begin{aligned} -a_1 &= \sum_{1\le i\le r} \alpha_i + \sum_{1\le i\le s} \beta_i \\ &> M^{1/2} + M^{-1/2} - 2\left(M^{1/4} + M^{-1/4}\right) - (D-6) \\ &> -(D-4). \end{aligned}$$

Thus $a_1 \ge D-4$ implies $|\alpha| \le M^{1/2}$, for every root α of $f(x)$. Now the proposition follows by applying [7, lemma 5]. $\square$

Boyd's fast technique for computing a lower bound on the Mahler measure of a polynomial is based on the root-squaring method of Graeffe. We describe this algorithm as follows. Let $f(x)$ be a polynomial with complex coefficients, and write $f(x)$ as the sum of its even-exponent and odd-exponent parts:

$$f(x) = f_e(x^2) + x f_o(x^2).$$

Now define a polynomial $f_1(x)$ by

$$f_1(x) = f_e(x)^2 - x f_o(x)^2.$$

Then

$$f_1(x^2) = f(x)\Big(f_e(x^2) - x f_o(x^2)\Big).$$

Thus if α is a zero of $f(x)$ of multiplicity k, then α^2 is a zero of $f_1(x)$ of multiplicity k. Now $f(x)$ and $f_1(x)$ have the same degree, so the zeros of $f_1(x)$ are precisely the squares of the zeros of $f(x)$.

We remark that it is also possible to compute the polynomial whose roots are the k-th powers of the roots of $f(x)$, for any integer k [48, pp. 153–154].

Let $f_m(x)$ denote the polynomial whose roots are the 2^m-th powers of the roots of $f(x)$. These successive root-squared polynomials provide us with a bound on $M(f)$.

PROPOSITION 3.10. *As $m \to \infty$, $L(f_m(x))^{2^{-m}}$ approaches $M(f)$ from the right.*

PROOF. Let D denote the degree of $f(x)$. By theorem 2.8,

$$M(f_m) \leq L(f_m) \leq 2^D M(f_m).$$

Since $M(f_m) = M(f)^{2^m}$,

$$M(f) \leq L(f_m)^{2^{-m}} \leq 2^{D/2^m} M(f).$$

The proposition follows. $\square$

This algorithm also detects cyclotomic polynomials.

PROPOSITION 3.11. *If $f(x)$ is a product of cyclotomic polynomials and $m > \log_2 \deg(f)$, then $f_m(x) = f_{m+1}(x)$.*

PROOF. Suppose $f(x) = \Phi_n(x)$, the n-th irreducible cyclotomic polynomial, and write $n = 2^r q$ with q odd. Then

$$f_m(x) = \begin{cases} \left(\Phi_{2^{r-m}q}(x)\right)^{2^m} & m < r \\ \left(\Phi_q(x)\right)^{2^{r-1}} & m \geq r. \end{cases}$$

Thus $f_m(x) = f_{m+1}(x)$ if $m \geq r$, and the statement follows. $\square$

The root-squaring algorithm allows Boyd to discard most polynomials with Mahler measure exceeding a given bound M very quickly. Let $f(x)$ be a monic, reciprocal polynomial with integer coefficients, and let $a_{k,m}$ denote the k-th coefficient of $f_m(x)$. If

$$|a_{k,m}| > \binom{D-2}{k-1}\left(M^{2^m} + M^{2^{-m}}\right) + \binom{D-2}{k} + \binom{D-2}{k-2}$$

then $M(f) > M$. We get a similar test when $f(x)$ satisfies the hypotheses of proposition 3.9.

The coefficients of $f_m(x)$ grow rapidly, and Boyd performs this test only as long as the $a_{k,m}$ can be stored exactly in a double-precision real variable. He reports that this normally restricts m to be at most 7 for $D \leq 16$.

Using his algorithm, Boyd finds 437 noncyclotomic, irreducible polynomials with degree at most 32 and Mahler measure at most 1.3. These polynomials are included in table 1 of appendix A.

3.2. Ray's Search. In 1994, Ray [33] developed a different algorithm for searching for polynomials with small Mahler measure. Let $\Phi(x)$ be a product of distinct, irreducible, cyclotomic polynomials with $\Phi(\pm 1) \neq 0$, and let $f(x) = \Phi(x)(x-1)^2$. Let $D = 2d$ denote the degree of $f(x)$. Let $q(x) \in \mathbb{Z}[x]$ be the polynomial

$$q(x) = \sum_{1 \leq k \leq D-1} q_k x^k$$

with $q_k = q_{D-k}$. Let t be a real parameter, and consider the reciprocal polynomial $f_t(x) \in \mathbb{R}[x]$ defined by

$$f_t(x) = f(x) + tq(x).$$

Then

$$f_t(x) = (x - s(t))(x - 1/s(t)) \prod_{1 \le j \le d-1} (x - r_j(t))(x - 1/r_j(t))$$

where $s(0) = 1$ and $r_j(0) = \zeta_j$, a complex root of $f(x)$. For small t, $s(t)$ and $r_j(t)$ may be approximated by the first terms in their series expansions:

$$s(t) \approx 1 + a\sqrt{t}$$

$$r_j(t) \approx \zeta_j(1 + ib_jt).$$

A matrix equation relates $\{a^2, b_1, \ldots, b_{d-1}\}$ to $\{q_1, \ldots, q_d\}$. Ray then selects q_d to minimize a, then selects the remaining q_k to minimize the Euclidean norm of the b_j. His technique uses Choleski factorization or the LLL lattice reduction algorithm to find the minimum of an inhomogeneous quadratic form. Finally, he sets t to 1 and computes the measure of the resulting polynomial.

Ray gives examples where this method gives sequences of polynomials whose Mahler measures converge to the two smallest known limit points. (We discuss small limit points of Mahler measures in chapter 6.) Ray also reports finding many of the Salem numbers listed in [4].

CHAPTER 4

Restricted Cyclotomic Partitions

In this chapter, we derive asymptotic estimates for the number of polynomials of a given degree that are products of cyclotomic polynomials, with restrictions on the maximum multiplicity of any factor. These estimates are used in chapter 5 to determine the computational complexity of some algorithms for searching for polynomials with small Mahler measure.

1. Asymptotic Estimates

Let $c(n)$ denote the number of polynomials of degree n that are products of cyclotomic polynomials, and let $c_k(n)$ denote the number of polynomials of degree n that are products of cyclotomic polynomials, where the maximum multiplicity of any factor is k. Let $P(z)$ and $P_k(z)$ be generating functions for these numbers:

$$P(z) = \sum_{n\geq 0} c(n)e^{-nz} \tag{17}$$

$$P_k(z) = \sum_{n\geq 0} c_k(n)e^{-nz}. \tag{18}$$

It is convenient to take $c(0) = c_k(0) = 1$.

Boyd and Montgomery [10] determine asymptotic estimates for $c(n)$ and $c_1(n)$. To do this, they first estimate $P(z)$ and $P_1(z)$ for z near 0.

LEMMA 4.1. *Let* $\mathcal{R} = \left\{ z \in \mathbb{C} : \operatorname{Re} z > 0, |\arg z| \le \frac{\pi}{2} - \frac{C}{\log\log\frac{1}{|z|}} \right\}$, *where* C *is a fixed positive constant. As* z *tends to* 0 *in* $\mathcal{R}$, *we have*

$$P(z) = a\left(z/\log\frac{1}{z}\right)^{1/2}\exp(b/z)E(z) \tag{19}$$

where

$$E(z) = 1 - \frac{\log\log\frac{1}{z}}{2\log\frac{1}{z}} + \frac{\log a}{\log\frac{1}{z}} + O\left(\frac{\left(\log\log\frac{1}{|z|}\right)^3}{\left(\log\frac{1}{|z|}\right)^2}\right),$$

$$a = (2\pi e^{\gamma})^{-1/2},$$

$$b = \frac{\zeta(2)^2\zeta(3)}{\zeta(6)},$$

and γ *is Euler's constant. Also,*

$$P_1(z) = \frac{1}{\sqrt{2}}\exp(b/2z)E_1(z) \tag{20}$$

where

$$E_1(z) = 1 - \frac{\log 2}{2\log\frac{1}{z}} + O\left(\frac{\left(\log\log\frac{1}{|z|}\right)^3}{\left(\log\frac{1}{|z|}\right)^2}\right).$$

To derive (19), Boyd and Montgomery determine a generating function for $\log P(z)$, apply an inverse Mellin transform to express it as an integral of a certain function, then adjust the contour of integration and estimate this integral. Statement (20) follows easily from (19) since $P_1(z) = P(z)/P(2z)$.

By Cauchy's formula, they express $c(n)$ as

$$c(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} P(z)e^{nz}\,dy \tag{21}$$

where $z = x + iy$. They find that the main contribution to $c(n)$ occurs near $y = 0$, so their estimate is essentially an application of the circle method of Hardy and Littlewood using just one major arc. The analysis of $c_1(n)$ is similar although trickier: again, one major arc suffices. They find:

THEOREM 4.2. *With $c(n)$ and $c_1(n)$ as above, we have*

$$c(n) = A(\log n)^{-1/2}n^{-1}\exp(B\sqrt{n}) \cdot \left(1 - \frac{\log\log n}{\log n} - \frac{c_0}{\log n} + O\left(\frac{(\log\log n)^3}{(\log n)^2}\right)\right) \tag{22}$$

as $n \to \infty$, where

$$A = \frac{1}{4\pi^2}\left(\frac{105\zeta(3)}{e^\gamma}\right)^{1/2} \approx 0.213234,$$
$$B = \frac{1}{\pi}\Big(105\zeta(3)\Big)^{1/2} \approx 3.57608,$$

and

$$c_0 = 2\log\pi + \gamma - \frac{1}{2}\log\left(\frac{105}{4}\zeta(3)\right) \approx 1.14083.$$

Also,

$$c_1(n) = A_1 n^{-3/4}\exp(B_1\sqrt{n})\left(1 - \frac{\log 2}{2\log n} + O\left(\frac{(\log\log n)^3}{(\log n)^2}\right)\right) \tag{23}$$

where

$$A_1 = \frac{1}{4\pi}\left(\frac{105\zeta(3)}{2}\right)^{1/4} \approx 0.224291$$

and

$$B_1 = \frac{1}{\pi}\left(\frac{105\zeta(3)}{2}\right)^{1/2} \approx 2.52867.$$

We are interested in estimating $c_2(n)$. However, Boyd and Montgomery's method does not apply directly to the case $k = 2$, since we require more than one major arc when estimating the integral analogous to (21).

We can however determine a qualitative asymptotic estimate for $c_2(2n)$ by appealing to Ingham's Tauberian theorem for partitions [19, 31]. We state here a special case of this theorem.

THEOREM 4.3. *Suppose* $f(s) = \int_0^\infty e^{-us}\,dA(u)$, *and the following conditions hold:*

(i) $A(u)$ *is nondecreasing as* $u \to \infty$, *and*

(ii) $f(s) \sim C(M/s)^{m\beta-1/2}\exp\left(\frac{1}{\beta}(M/s)^\beta\right)$ *uniformly as* $s \to 0$ *in any angle of the form* $|t| < \Delta\sigma$, *where* $s = \sigma + it$, Δ *is a positive constant,* C, M, *and* β *are positive constants, and* m *is a real constant.*

Then as $u \to \infty$,

$$A(u) \sim C\sqrt{\frac{1-\alpha}{2\pi}}(uM)^{m\alpha-1/2}\exp\left(\frac{(uM)^\alpha}{\alpha}\right) \tag{24}$$

where $\alpha = \beta/(\beta+1)$.

In the sequel, we are particularly interested in the number of reciprocal polynomials of even degree. With this in mind, we define several quantities that we wish to estimate.

- $r(n)$ is the number of reciprocal polynomials of degree $2n$ that are products of cyclotomic polynomials. $R(z)$ is its generating function: $R(z) = \sum_{n\geq 0} r(n)e^{-nz}$.
- $r_2(n)$ is the number of reciprocal polynomials of degree $2n$ that are products of cyclotomic polynomials, where the maximum multiplicity of any factor is 2. $R_2(z)$ is the generating function for $r_2(n)$.
- $q_2(n)$ is the number of reciprocal polynomials of degree $2n$ that are products of cyclotomic polynomials, where the maximum multiplicity of any factor except $x-1$ and $x+1$ is 2. These two linear cyclotomics may have multiplicity as high as 4. $Q_2(z)$ is the generating function for $q_2(n)$.
- $t_2(n)$ denotes $c_2(2n)$ for convenience and $T_2(n)$ is the generating function for $t_2(n)$.

To apply Ingham's theorem, we must find asymptotic estimates for the generating functions as z approaches 0 in the prescribed domain. We then establish the monotonicity of the functions $r_2(n)$, $q_2(n)$, and $t_2(n)$ in order to deduce asymptotic estimates for these quantities from estimates of their summatory functions.

LEMMA 4.4. *Let* $\mathcal{R} = \left\{z \in \mathbb{C} : \operatorname{Re} z > 0, |\arg z| \le \frac{\pi}{2} - \frac{C}{\log\log\frac{1}{|z|}}\right\}$, *where* C *is a fixed positive constant. As* z *approaches* 0 *in* $\mathcal{R}$, *we have*

$$R(z) \sim \frac{a}{4\sqrt{2}}\left(z/\log\frac{1}{z}\right)^{1/2}\exp(2b/z) \tag{25}$$

$$R_2(z) \sim \frac{4}{9\sqrt{3}}\exp(4b/3z) \tag{26}$$

$$Q_2(z) \sim \frac{1}{\sqrt{3}}\exp(4b/3z) \tag{27}$$

$$T_2(z) \sim \frac{5}{9\sqrt{3}}\exp(4b/3z) \tag{28}$$

where $a = (2\pi e^{\gamma})^{-1/2}$ *and* $b = \zeta(2)^2\zeta(3)/\zeta(6)$.

PROOF. We first note a product representation for $R(z)$. Because we are only interested in reciprocal polynomials of even degree, the two linear cyclotomic polynomials $x-1$ and $x+1$ must appear with even exponents. All other cyclotomic polynomials may appear with any exponent. Hence

$$\begin{aligned} R(z) &= \Big(\sum_{l\ge 0}\exp(-lz)\Big)^2\prod_{m\ge 3}\Big(\sum_{l\ge 0}\exp(-l\varphi(m)z/2)\Big) \\ &= \Big(1-\exp(-z)\Big)^{-2}\prod_{m\ge 3}\Big(1-\exp(-\varphi(m)z/2)\Big)^{-1}. \end{aligned} \tag{29}$$

Likewise, we see from (17) that

$$P(z) = \prod_{m\ge 1}\Big(1-\exp\left(-\varphi(m)z\right)\Big)^{-1}$$

so

$$P(z/2) = \Big(1 - \exp(-z/2)\Big)^{-2} \prod_{m\geq 3} \Big(1 - \exp\big(-\varphi(m)z/2\big)\Big)^{-1}$$
$$= \Big(1 + \exp(-z/2)\Big)^{2} R(z).$$

Applying (19),

$$R(z) = a\left(z/2\log\frac{2}{z}\right)^{1/2} \exp(2b/z)\Big(1+\exp(-z/2)\Big)^{-2} E(z/2)$$
$$\sim \frac{a}{4\sqrt{2}}\left(z/\log\frac{1}{z}\right)^{1/2} \exp(2b/z).$$

This shows (25).

The polynomials counted by the coefficients of $Q_2(z)$ may have multiplicity as great as 4 at $x-1$ and $x+1$ but only 2 everywhere else. Therefore,

$$\begin{aligned} Q_2(z) &= (1+e^{-z}+e^{-2z})^2 \prod_{m\geq 3}\Big(1+\exp(-\varphi(m)z/2)+\exp(-\varphi(m)z)\Big) \\ &= \frac{R(z)}{R(3z)} \end{aligned} \tag{30}$$

by (29). Thus by (25) we have

$$Q_2(z) \sim \frac{1}{\sqrt{3}} \exp(4b/3z)$$

which is (27).

In $R_2(z)$, $x-1$ and $x+1$ must appear with multiplicity 0 or 2, so

$$R_2(z) = \left(\frac{1+e^{-z}}{1+e^{-z}+e^{-2z}}\right)^2 Q_2(z) \tag{31}$$

and so

$$R_2(z) \sim \frac{4}{9\sqrt{3}} \exp(4b/3z)$$

as $z \to 0$ in $\mathcal{R}$. This gives (26).

Finally, the factor (x^2-1) is allowed in the polynomials counted by $T_2(z)$, so

$$T_2(z) = \frac{1+3e^{-z}+e^{-2z}}{(1+e^{-z}+e^{-2z})^2} Q_2(z)$$

hence

$$T_2(z) \sim \frac{5}{9\sqrt{3}} \exp(4b/3z).$$

This is (28). $\square$

LEMMA 4.5. *$r_2(n)$, $q_2(n)$, and $t_2(n)$ are monotonically increasing functions of n.*

PROOF. Since

$$(1-e^{-z})R_2(z) = 1 + \sum_{n\geq 1} \Big(r_2(n) - r_2(n-1)\Big) e^{-nz} \tag{32}$$

we need to show that $(1-e^{-z})R_2(z)$ is a generating function for a sequence of nonnegative integers. We prove this by deriving another product representing $R_2(z)$. The key step uses the identity

$$\prod_{l\geq 0} \Big(1 + w^{3^l} + (w^2)^{3^l}\Big) = \frac{1}{1-w}. \tag{33}$$

We have, by (30) and (31),

$$
\begin{aligned}
R_2(z) &= (1+e^{-z})^2 \prod_{m\geq 3} \bigl(1+\exp\left(-\varphi(m)z/2\right)+\exp\left(-\varphi(m)z\right)\bigr) \\
&= (1+e^{-z})^2 \left(\prod_{l\geq 1} \bigl(1+\exp(-\varphi(3^l)z/2)+\exp(-\varphi(3^l)z)\bigr)\right) \\
&\quad \cdot \left(\prod_{l\geq 1} \bigl(1+\exp(-\varphi(2\cdot 3^l)z/2)+\exp(-\varphi(2\cdot 3^l)z)\bigr)\right) \\
&\quad \cdot \left(\prod_{\substack{m\geq 4\\ 3\nmid m}} \bigl(1+\exp(-\varphi(m)z/2)+\exp(-\varphi(m)z)\bigr)\right. \\
&\qquad \left. \cdot \prod_{l\geq 1} \bigl(1+\exp(-\varphi(3^l m)z/2)+\exp(-\varphi(3^l m)z)\bigr)\right) \\
&= (1+e^{-z})^2 \left(\prod_{l\geq 0} \bigl(1+\exp(-3^l z)+\exp(-2\cdot 3^l z)\bigr)\right)^2 \\
&\quad \cdot \left(\prod_{\substack{m\geq 4\\ 3\nmid m}} \bigl(1+\exp(-\varphi(m)z/2)+\exp(-\varphi(m)z)\bigr)\right. \\
&\qquad \left. \cdot \prod_{l\geq 0} \bigl(1+\exp(-3^l\varphi(m)z)+\exp(-2\cdot 3^l\varphi(m)z)\bigr)\right) \\
&= \left(\frac{1+e^{-z}}{1-e^{-z}}\right)^2 \prod_{\substack{m\geq 4\\ 3\nmid m}} \frac{1+\exp(-\varphi(m)z/2)+\exp(-\varphi(m)z)}{1-\exp(-\varphi(m)z)}.
\end{aligned}
$$

The last step follows from (33). To compute $(1-e^{-z})R_2(z)$, we cancel one factor of $(1-e^{-z})$. Each remaining term in the product is a generating function for a series with nonnegative coefficients. In view of (32) it follows that $r_2(n)$ is monotonically increasing.

The proofs for $q_2(n)$ and $t_2(n)$ are similar. $\square$

THEOREM 4.6. *Suppose D is even, and r_2, q_2, and c_2 are defined as above.*

Then

$$r_2(D/2) \sim \frac{4}{9\sqrt{3}\pi}\left(\frac{35}{2}\zeta(3)\right)^{1/4} D^{-3/4}\exp\left(\frac{1}{\pi}\sqrt{70\zeta(3)D}\right) \tag{34}$$
$$\approx 0.174923 D^{-3/4}\exp(2.91986\sqrt{D})$$

$$q_2(D/2) \sim \frac{1}{\sqrt{3}\pi}\left(\frac{35}{2}\zeta(3)\right)^{1/4} D^{-3/4}\exp\left(\frac{1}{\pi}\sqrt{70\zeta(3)D}\right) \tag{35}$$
$$\approx 0.393578 D^{-3/4}\exp(2.91986\sqrt{D})$$

$$c_2(D) \sim \frac{5}{9\sqrt{3}\pi}\left(\frac{35}{2}\zeta(3)\right)^{1/4} D^{-3/4}\exp\left(\frac{1}{\pi}\sqrt{70\zeta(3)D}\right) \tag{36}$$
$$\approx 0.218654 D^{-3/4}\exp(2.91986\sqrt{D}).$$

PROOF. By lemma 4.4, we may apply Ingham's theorem with $f(z) = R_2(z)$, $\beta = 1$, $M = 4b/3$, $C = 4/9\sqrt{3}$, and $m = 1/2$. This gives $\alpha = 1/2$, and we conclude

$$A(u) \sim \frac{C}{\sqrt{4\pi}}(uM)^{-1/4}\exp\left(2(uM)^{1/2}\right)$$

where

$$A(u) = \sum_{n \le u} r_2(n).$$

By lemma 4.5, $r_2(n)$ is monotone increasing, so we can determine an asymptotic estimate for $r_2(n)$ by estimating $A(n) - A(n-1)$. Thus

$$\begin{aligned} r_2(n) &\sim \frac{C}{2\sqrt{\pi}}M^{-1/4}\left(n^{-1/4}\exp(2\sqrt{Mn}) - (n-1)^{-1/4}\exp\left(2\sqrt{M(n-1)}\right)\right) \\ &= \frac{C}{2\sqrt{\pi}}(Mn)^{-1/4}\exp(2\sqrt{Mn}) \\ &\quad \cdot \left(1 - \left(\frac{n}{n-1}\right)^{1/4}\exp\left(2\sqrt{M}(\sqrt{n-1} - \sqrt{n})\right)\right). \end{aligned}$$

Now

$$\left(\frac{n}{n-1}\right)^{1/4} = \left(1 + \frac{1}{n-1}\right)^{1/4} = 1 + O\left(\frac{1}{n}\right)$$

and

$$\begin{aligned}\exp\left(2\sqrt{M}(\sqrt{n-1}-\sqrt{n})\right) &= \exp\left(2\sqrt{Mn}\left(\left(1-\frac{1}{n}\right)^{1/2}-1\right)\right)\\ &= \exp\left(2\sqrt{Mn}\left(-\frac{1}{2n}+O\left(\frac{1}{n^2}\right)\right)\right)\\ &= \exp\left(-\sqrt{\frac{M}{n}}+O\left(n^{-3/2}\right)\right)\\ &= 1-\sqrt{\frac{M}{n}}+O\left(\frac{1}{n}\right).\end{aligned}$$

Thus

$$r_2(n) \sim \frac{C}{2\sqrt{\pi}}M^{1/4}n^{-3/4}\exp(2\sqrt{Mn}).$$

Since $\zeta(2)=\pi^2/6$ and $\zeta(6)=\pi^6/945$, we compute

$$2\sqrt{M}=\frac{2}{\pi}\sqrt{35\zeta(3)}$$

and

$$\frac{C}{2\sqrt{\pi}}M^{1/4}=\frac{2}{9\sqrt{3}\pi}(35\zeta(3))^{1/4}.$$

Hence

$$r_2(n) \sim \frac{2}{9\sqrt{3}\pi}(35\zeta(3))^{1/4}n^{-3/4}\exp\left(\frac{2}{\pi}\sqrt{35\zeta(3)n}\right).$$

Substituting $D=2n$ yields (34). The proofs of (35) and (36) are similar. □

Remark. Following Boyd and Montgomery, we find that the contribution from the major arc near zero of the integral

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}P_2(z)e^{Dz}\,dy$$

yields a main term of only

$$\frac{1}{2\sqrt{3}\pi}\left(\frac{35}{2}\zeta(3)\right)^{1/4}D^{-3/4}\exp\left(\frac{1}{\pi}\sqrt{70\zeta(3)D}\right).$$

This is smaller than the term obtained in (36), and indicates that more than one major arc must be considered in order to obtain a more precise estimate for $c_2(D)$ similar to (23).

2. Exact Values

In view of the slowly decaying error terms in theorem 4.2, we might expect rather slow convergence of the functions r_2, q_2, and c_2 to their asymptotic estimates listed in theorem 4.6. In this section we give a formula for calculating $q_2(n)$ explicitly, then compare its value at a large value of n with that given by the asymptotic expression of (35). Similar formulas may be given for $r_2(n)$ and $c_2(2n)$.

Let $\Phi_n(x)$ denote the n-th irreducible cyclotomic polynomial. For $d \geq 2$, define

$$S_d = \{\Phi_n(x) : \varphi(n) = 2d\}$$

and let

$$S_1 = \left\{(x-1)^2, (x+1)^2, x^2+x+1, x^2+1, x^2-x+1\right\}.$$

Also, for $m \geq 0$ and $d \geq 1$ define

$$T_{m,d} = \left\{ \prod_{\Phi \in S_d} \Phi(x)^{e_\Phi} : \sum_{\Phi \in S_d} e_\Phi = m, 0 \leq e_\Phi \leq 2 \right\}.$$

Let $\vec{m} = (m_1, m_2, \ldots, m_n)$ denote a partition of n:

$$\sum_{1 \leq d \leq n} m_d d = n.$$

Then we have

$$q_2(n) = \sum_{\vec{m}} \prod_{1 \leq d \leq n} |T_{m_d,d}|$$

where the sum is over all partitions $\vec{m}$ of n. Thus we need to determine the size of the set $T_{m,d}$. To accomplish this, we consider a combinatorial problem.

Let $\begin{bmatrix} n \\ k \end{bmatrix}$ denote the number of ways to select k objects from a collection of n objects, allowing each object to be selected at most twice. Then we have the recurrence formula

$$\begin{bmatrix} n \\ k \end{bmatrix} = \begin{bmatrix} n-1 \\ k \end{bmatrix} + \begin{bmatrix} n-1 \\ k-1 \end{bmatrix} + \begin{bmatrix} n-1 \\ k-2 \end{bmatrix}$$

grounded by the base conditions

$$\begin{bmatrix} n \\ 0 \end{bmatrix} = 1, \quad n \geq 0,$$
$$\begin{bmatrix} 0 \\ k \end{bmatrix} = 0, \quad k \geq 1.$$

Comtet [13, pp. 77–78] calls these numbers the *trinomial coefficients*. They were first considered by Euler [17]. We list some of their properties.

LEMMA 4.7. *The numbers* $\begin{bmatrix} n \\ k \end{bmatrix}$ *satisfy the following:*

(i) $\begin{bmatrix} n \\ k \end{bmatrix}$ *is the coefficient of* x^k *in* $(1 + x + x^2)^n$.

(ii) $\begin{bmatrix} n \\ k \end{bmatrix} = \begin{bmatrix} n \\ 2n-k \end{bmatrix}$.

(iii) *If* $0 \leq k \leq n$ *then*

$$\begin{bmatrix} n \\ k \end{bmatrix} = \sum_{0 \leq i \leq k/2} \binom{n}{k-i}\binom{k-i}{i}.$$

PROOF. Statement (i) is immediate from the recurrence relation. Statement (ii) is a consequence of the symmetry of the definition, since there is a one-to-one correspondence between the multisets selected and their complements in the multiset consisting of two copies of each object. Statement (iii) follows easily from the binomial theorem. $\square$

Therefore,

$$q_2(n) = \sum_{\vec{m}} \prod_{1 \le d \le n} \begin{bmatrix} |S_d| \\ m_d \end{bmatrix} \tag{37}$$

and $\begin{bmatrix} |S_d| \\ m_d \end{bmatrix}$ may be evaluated according to (ii) and (iii) of lemma 4.7. Using this formula, we compute $q_2(1500)$:

2385753783987905281502413981937925622349509368675889581151794867226.

The asymptotic expression in (35) with $D = 3000$ yields approximately $2.7713 \cdot 10^{66}$. Our estimate is about 16.2% larger than the actual value. At $D = 4000$, our estimate is still about 15.1% larger than the actual value.

CHAPTER 5

A Subexponential Algorithm

1. Introduction

C. Pinner observed that many of the polynomials in Boyd's list are slightly perturbed products of cyclotomic polynomials. For instance, Lehmer's tenth degree example is

$$(x-1)^2(x+1)^2(x^2+x+1)^2(x^2-x+1)-x^5.$$

We define a *Pinner polynomial* as a product of cyclotomic polynomials with the middle coefficient shifted by ± 1. We find that all 13 of the irreducible polynomials with measure at most 1.3 and degree at most 12 are Pinner polynomials, as are all but three of the polynomials with measure at most 1.3 and degree 14. In all, 225 of the 440 previously indicated polynomials with Mahler measure at most 1.3 and degree at most 32 are Pinner polynomials.

Pinner's observation inspires an algorithm for searching for polynomials with small Mahler measure. In this chapter, we describe this algorithm in detail and show that its running time is subexponential in the degree. We also summarize the results of running this algorithm through high degree.

2. Description of the Algorithm

We describe an algorithm for searching for polynomials with small Mahler measure. We construct all reciprocal polynomials with even degree D that are products of cyclotomic polynomials, where the maximum multiplicity of any factor is 2. We then adjust some of the coefficients of these polynomials and test the Mahler measure of these adjusted products. We are interested in finding polynomials with Mahler measure below 1.3.

In forming our products of cyclotomic polynomials, we restrict the multiplicity to 2 for two reasons:

- 216 of the 225 Pinner polynomials on Boyd's list have maximum multiplicity 2 in their associated cyclotomic factors. (Seven of the remaining ones require $(x+1)^4$; the other two need a quadratic cyclotomic with multiplicity 3.)
- If $f(x)$ is a reciprocal polynomial of degree $D = 2d$ with all its roots on the unit circle, and some root of $f(x)$ has multiplicity at least 3, then $M(f(x) + cx^d) > 1$ for any $c \neq 0$. If we limit the maximum multiplicity of any cyclotomic factor to 2, the roots may remain on the unit circle after a small change to the middle coefficient.

We proceed to describe the algorithm. We are given an even integer $D = 2d$.

STEP 1. Enumeration

Let $\Phi_r(x)$ denote the r-th irreducible cyclotomic polynomial. For $n \leq d$ define

$$S_n = \{\Phi_r(x) : \varphi(r) = 2n\}.$$

For each $n \leq d$, we construct a sequence of sets $T_{m,n}$. These sets are used repeatedly by the algorithm and are constructed during initialization. For

$n \geq 2$, $T_{m,n}$ is the collection of products of m irreducible cyclotomic polynomials of degree $2n$, where no factor appears more than twice:

$$T_{m,n} = \Big\{ \prod_{\Phi \in S_n} \Phi(x)^{e_\Phi} : \sum_{\Phi \in S_n} e_\Phi = m, 0 \leq e_\Phi \leq 2 \Big\}, \quad n \geq 2.$$

We construct this set for each m between 1 and $2|S_n|$. When $n = 1$, we must account for the linear cyclotomics:

$$T_{m,1} = \Big\{ (x-1)^{2e_1}(x+1)^{2e_2} \prod_{\Phi \in S_1} \Phi(x)^{e_\Phi} : e_1 + e_2 + \sum_{\Phi \in S_1} e_\Phi = m, \\ 0 \leq e_1, e_2 \leq 1, 0 \leq e_\Phi \leq 2 \Big\}.$$

We construct the $T_{m,1}$ for $1 \leq m \leq 2|S_1| + 2 = 8$. We remark that allowing multiplicity as great as 4 on the linear cyclotomic polynomials simplifies the construction of the $T_{m,1}$.

Let $\vec{m} = (m_1, m_2, \ldots, m_d)$ be a partition of d:

$$\sum_{1 \leq n \leq d} m_n n = d$$

where $0 \leq m_1 \leq 8$ and $0 \leq m_n \leq 2|S_n|$ for $2 \leq n \leq d$. For each such partition, we construct all polynomials of the form

$$f(x) = \prod_{1 \leq n \leq d} f_n(x)$$

where

$$f_n(x) \in T_{m_n,n}.$$

Step 2. Adjustments

For each polynomial $f(x)$ constructed in step 1, we test the Mahler measure of the Pinner polynomials:

$$f(x) \pm x^d. \tag{38}$$

For all but the largest degrees, we also consider other adjustments to the middle coefficients:

$$f(x) \pm x^{d-1}(x^2 + x + 1) \tag{39}$$

$$f(x) \pm x^{d-1}(x^2 + 1) \tag{40}$$

$$f(x) \pm x^{d-1}(x^2 - x + 1). \tag{41}$$

At this point, we use two symmetries of the Mahler measure to reduce the number of polynomials we must test. First, we need only test one of $g(x)$ and $g(-x)$, since their Mahler measures are obviously identical. Second, if we create a polynomial of the form $g(x^k)$ with $k \geq 2$, we may reduce to the primitive polynomial $g(x)$ in view of lemma 2.1. We call a polynomial $f(x)$ *primitive* if it is not of the form $g(x^k)$ for any $k \geq 2$.

STEP 3. Root-Squaring

Let $f(x)$ be an adjusted product of cyclotomic polynomials created in step 2, and let $f_m(x)$ denote the polynomial whose roots are the 2^m-th powers of the roots of $f(x)$. Let $a_{k,m}$ for $1 \leq k \leq D$ denote the coefficients of $f_m(x)$. We use the Graeffe algorithm to compute

$$a_{k,m} = (-1)^k a_{k,m-1}^2 + 2 \sum_{1 \leq i \leq k} (-1)^{i+k} a_{k-i,m-1} a_{k+i,m-1} \tag{42}$$

for $0 \leq k \leq d$. Note that $f_m(x)$ is also a reciprocal polynomial, so $a_{D-k,m} = a_{k,m}$. Once we compute $f_m(x)$, we test its coefficients. If $a_{1,m} < D - 4$, we check that the $a_{k,m}$ satisfy the inequality of proposition 3.8, with M replaced with $(1.3)^{2^m}$. Otherwise, we check that the $a_{k,m}$ satisfy the inequality of proposition 3.9, again using $(1.3)^{2^m}$ for M.

Recall that proposition 3.9 requires that all negative real roots of $f_m(x)$ have multiplicity greater than or equal to 2. This is indeed the case for $m \geq 1$, for suppose $f_m(\alpha) = 0$ and $\alpha < 0$. Let $g(x)$ be the minimal polynomial for α. Then $g(x^2)$ is the minimal polynomial for $\sqrt{\alpha}$, so $g(x^2) \mid f_{m-1}(x)$. Thus $g(x)^2 \mid f_m(x)$, so α is a zero of multiplicity at least 2 of $f_m(x)$.

We perform the root-squaring operation ten times before accepting a polynomial. In our experience, most polynomials with measure larger than 1.3 are rejected after far fewer iterations. However, the number of polynomials that survive even nine iterations of the Graeffe algorithm becomes too large in searches of higher degrees.

Performing the Graeffe algorithm ten times presents a problem of precision. Boyd [7, p. 1367] demonstrates the importance of recording the coefficients of $f_m(x)$ exactly. Initially, we store the $a_{k,m}$ as double-precision, floating point numbers. This gives 53 bits of precision, which normally suffices for m up to about 6. For larger m, we use a big integer format implemented in software. Arithmetic with double-precision, floating point variables is much faster than that with big integers, so it behooves us to remain in the fast representation as long as possible. We determine when to switch representations using a simple criterion.

LEMMA 5.1. *Let $g(x)$ be a polynomial with integer coefficients. Let $g_e(x)$ and $g_o(x)$ be the even and odd parts of $g(x)$, so*

$$g(x) = g_e(x^2) + xg_o(x^2).$$

Suppose $H(g) \leq N$. Let $g_1(x)$ be the polynomial whose roots are the squares of the roots of $g(x)$. If

$$L(g_e)^2 + L(g_o)^2 \leq N \tag{43}$$

then $g_1(x)$ may be computed exactly using integers no larger than N in absolute value.

The proof is immediate from the definition of $g_1(x)$:

$$g_1(x) = g_e(x)^2 - x g_o(x)^2$$

and the observation that $H(g_1) \leq L(g_1)$.

We use the criterion (43) with $N = 2^{53} - 1$ to determine when to switch to the big integer representation.

STEP 4. Remove Cyclotomic Factors

Any polynomial $f(x)$ surviving the root-squaring tests of step 3 is then checked for cyclotomic factors. Three observations speed this check.

First, we know that $f(x)$ is not a product of cyclotomic polynomials, as this would have been detected in step 3. Since the smallest degree where a polynomial exists with measure greater than 1 but smaller than 1.3 is 8, we need to check for cyclotomic factors of degree at most $D - 8$.

Second, a cyclotomic factor $\Phi_n(x)$ of $f(x)$, where $n = 2^r q$ with q odd, stabilizes as a factor of $\Phi_q(x)$ with multiplicity 2^{r-1} of $f_m(x)$ when $m \geq r$. Therefore we can speed the check for cyclotomic factors by testing one of the $f_m(x)$ for cyclotomic factors with odd index first. We select m so that $f_m(x)$ is the last root-squared polynomial computed in the fast representation. For each odd q with $\varphi(q) \leq D - 8$, we test whether $\Phi_q(x)$ divides $f_m(x)$. If it does not, we know that $\Phi_{2^r q}(x)$ with $r \leq m$ does not divide $f(x)$.

Third, we avoid a trial division whenever $\Phi_q(1)$ does not divide $f_m(1)$. We must therefore keep track of the value of $f_m(1)$. This is simplified by the following two propositions. The first is a fact from elementary number theory.

PROPOSITION 5.2. *Let $\Phi_n(x)$ denote the n-th irreducible cyclotomic polynomial. Then*

$$\Phi_n(1) = \begin{cases} 0 & n = 1 \\ p & n = p^r, \quad p \text{ a prime} \\ 1 & \text{otherwise.} \end{cases}$$

In view of this proposition, we need only keep track of the prime divisors of $f_m(1)$.

PROPOSITION 5.3. *Let $f(x)$ be a polynomial, and let $f_1(x)$ be the polynomial whose roots are the squares of the roots of $f(x)$. Then*

$$f_1(1) = f(1)f(-1). \tag{44}$$

Also, if $f(x)$ is a reciprocal polynomial of even degree, then $f_1(-1)$ is a perfect square.

PROOF. The first statement follows easily from the Graeffe algorithm. Write

$$f(x) = f_e(x^2) + xf_o(x^2).$$

Then

$$f_1(x) = f_e(x)^2 - xf_o(x)^2$$

so

$$\begin{aligned} f_1(1) &= f_e(1)^2 - f_o(1)^2 \\ &= \big(f_e(1) + f_o(1)\big)\big(f_e(1) - f_o(1)\big) \\ &= f(1)f(-1). \end{aligned}$$

For the second statement, write $D = 2d$ for the degree of $f(x)$, and suppose d is odd. Then

$$\begin{aligned} f_e(-1) &= \sum_{0\le k\le d} a_{2k}(-1)^k \\ &= \sum_{0\le k<d/2} a_{2k}(-1)^k + \sum_{d/2<k\le d} a_{2k}(-1)^k \\ &= \sum_{0\le k<d/2} a_{2k}(-1)^k + \sum_{0\le k<d/2} a_{D-2k}(-1)^{d-k} \\ &= 0 \end{aligned}$$

since $a_{D-2k} = a_{2k}$. Therefore

$$f_1(-1) = f_o(-1)^2.$$

Similarly, if d is even, we find

$$f_1(-1) = f_e(-1)^2.$$

□

So for each m we compute two integers s_m and t_m associated with $f_m(x)$. Let

$$s_0 = f(1)$$

and

$$t_0 = f(-1).$$

Then for $m \geq 1$ let

$$s_m = s_{m-1}t_{m-1}$$

and

$$t_m = \sqrt{f_m(-1)}.$$

If $f_m(-1) = 0$, we remove all factors of $x+1$ from $f_m(x)$ before computing t_m. Then s_m contains all the prime divisors of $f_m(1)$. We check whether $\Phi_q(1)$ divides s_m before testing if $\Phi_q(x)$ divides $f_m(x)$.

STEP 5. Compare

For each positive even integer $n \le D$, we maintain a binary tree of the polynomials of degree n that have previously been identified as having small Mahler measure. We use the lexicographic order on coefficients to compare polynomials of the same degree. The polynomials in these trees come from several sources:

1. All of the polynomials in Boyd's lists [4, 5, 7, 9] with Mahler measure below 1.3.
2. Polynomials from the known sequences whose Mahler measures converge to limit points below 1.3247. All of the polynomials given in (52), (53), (54), and (55) in chapter 6 were computed through degree 100 and factored. Each noncyclotomic factor having Mahler measure below 1.3 is placed in the proper tree.
3. Polynomials with Mahler measure below 1.3 that are factors of a polynomial of the form

   $$g(x) = x^{2d+1} - x^{2d-k} - x^{d+s+1} + x^{d-s} + x^{k+1} - 1$$

   with $d \le 50$. By Gonçalves' inequality (lemma 2.6),

   $$M(g)^2 + M(g)^{-2} \le 6.$$

   This yields

   $$M(g) \le 1 + \sqrt{2}.$$

Boyd [7] remarks that all known small Salem numbers are measures of irreducible factors of polynomials from this family.

4. All new polynomials previously produced by this algorithm.

When step 4 produces a polynomial $f(x)$, we search the appropriate tree for that polynomial. If it is found, we increment some counters associated with the polynomial. Otherwise, we insert it.

STEP 6. Compute Mahler Measures

After we have finished enumerating all of the adjusted cyclotomic products of a given degree, processed each, and placed the new polynomials in our binary trees, we compute the Mahler measure of our new polynomials. We search our trees for polynomials whose Mahler measures have not been computed. Each new polynomial is handled according to the following procedure. First, we apply Bairstow's method to compute a good approximation of the Mahler measure. Bairstow's method [46, section 5.7] is a technique for computing the roots of a real polynomial without using complex arithmetic. It is a two-dimensional Newton-Raphson method that finds a quadratic factor of a polynomial using an iterative technique. Given a monic polynomial $f(x)$ with real coefficients, this algorithm begins by selecting real numbers t and u and computing A and B so that

$$f(x) = (x^2 - tx + u)g(x) + Ax + B.$$

Then it adjusts t and u so that the remainder terms become smaller, and continues this process until the changes to t and u become insignificant.

We implement Bairstow's method exploiting the fact that our polynomials are reciprocal. We use the computer's standard double-precision arithmetic in our implementation. This provides a very fast method for computing Mahler measures to a modest accuracy.

Next, polynomials found to have sufficiently small measure by Bairstow's method are passed to a slower but more accurate algorithm. We use PARI to compute the Mahler measures of these polynomials to 12 decimal places. PARI's root-finding algorithm is a modified Newton-Raphson method. The PARI routine is invoked only if Bairstow's method indicates that the Mahler measure is below 1.3 plus a small positive number.

We remark that the root-finding algorithm in PARI (version 1.38) is not robust: it is designed to work quickly on most polynomials, but fails on a few exceptional polynomials. As a result, occasionally PARI cannot compute the Mahler measure of a polynomial we give it. We use Maple to compute the Mahler measures of these exceptional polynomials.

3. Complexity

We review some notation from the analysis of algorithms. Let P be an algorithm, and let $g_P(n)$ denote the number of operations performed by P given input n. We say that the computational complexity of P is $O(f(n))$, or that P is $O(f(n))$, if $g_P(n) = O(f(n))$, that is, if there exists a positive constant C and an n_o so that

$$g_P(n) \leq C|f(n)|$$

whenever $n \geq n_0$. Similarly, we say P is $\Omega(f(n))$ is there exists a positive constant c and an n_1 such that

$$g_P(n) \geq c|f(n)|$$

whenever $n \geq n_1$. Finally, P is $\Theta(f(n))$ if P is both $O(f(n))$ and $\Omega(f(n))$.

We have the following result on the complexity of our algorithm.

THEOREM 5.4. *The computational complexity of the algorithm described in section* 2 *given input* D *is*

$$\Omega\left(D^{5/4}\exp\left(\frac{1}{\pi}\sqrt{70\zeta(3)D}\right)\right) \tag{45}$$

and

$$O\left(\exp\left(\frac{1+\epsilon}{\pi}\sqrt{70\zeta(3)D}\right)\right). \tag{46}$$

PROOF. Theorem 4.6 gives us an asymptotic estimate on the number of polynomials of degree D that we construct in step 1. We consider between 2 and 8 adjustments to each of these polynomials in step 2, and pass half of these to the Graeffe root-squaring algorithm in step 3. The root-squaring process is clearly $\Theta(D^2)$, and this yields (45).

Step 4, removing cyclotomic factors, is $O(D^3)$, since there are $\Theta(D)$ cyclotomic polynomials with degree at most D [15, 16], and a single trial division is $O(D^2)$. Step 5, inserting a polynomial into a binary tree, is $O(D^{3/2})$ for the following reasons. First, comparing two polynomials of degree D is $\Theta(D)$. Second, the average depth of our binary tree is no worse than $O(D^{1/2})$ because we consider $O(C^{\sqrt{D}})$ polynomials of degree D. From [20], we see that the roots of a polynomial $f(x)$ may be calculated in time that is polynomial in $\deg(f)$, $\log H(f)$, and the number of bits required. By (4), the polynomials we consider have $\log H(f) = O(D)$, so step 6 requires time that is polynomial in the degree. Statement (46) follows. $\square$

We remark that step 6 is fast in practice. Our implementations of the Newton-Raphson techniques for computing roots of polynomials are $O(D^2)$.

However, these implementations occasionally fail, so we cannot assert this complexity in general for step 6.

Steps 4 through 6 are performed only in the relatively rare case that a polynomial survives all 10 iterations of the Graeffe algorithm. A quantitative result on the frequency of this occurrence would allow us to determine the complexity of our algorithm more precisely. However, it may well be the case that the number of iterations of the Graeffe process must increase slowly as D grows larger in order to keep the latter steps of our algorithm from executing too frequently. We therefore suspect that, after possible slight adjustments to step 3, we have an algorithm with complexity

$$O\left(D^{5/4+\epsilon}\exp\left(\frac{1}{\pi}\sqrt{70\zeta(3)D}\right)\right). \tag{47}$$

In closing, we remark on the complexity of some other algorithms for searching for polynomials with small Mahler measure. These algorithms perform tasks that are different from the algorithm we describe, so their complexities are not directly comparable to those stated in theorem 5.4.

One algorithm is inspired by lemma 2.2 and theorem 3.5: in order to find all polynomials of degree D and Mahler measure at most M, where M is less than the smallest PV number, we test all reciprocal polynomials whose k-th coefficient a_k satisfies

$$|a_k| \leq \binom{D}{k} M.$$

Using Stirling's approximation and Euler's summation formula, we determine that the number of polynomials considered by this algorithm is

$$\Theta\left(\exp\left(D^2/4\right)\right).$$

Boyd's exhaustive search reduces the number of polynomials that must be tested. From [7, p. 1369], we find that the number of polynomials considered in this search is

$$\Omega\left(\exp\left(\frac{\log M}{8}D^2\right)\right)$$

and

$$O\left(\exp\left(\frac{\log M}{4}D^2\right)\right).$$

Boyd's second search considers reciprocal polynomials with height 1. Enumerating these polynomials clearly has complexity $\Theta(3^{D/2})$.

4. Summary of Results

Using our algorithm, we have checked all polynomials of degree at most 64 formed as in (38) by adjusting the middle coefficient of a product of cyclotomic polynomials with maximum multiplicity 2. We also checked the secondary adjustments (40) and (41) through degree 56, and (39) through degree 54. We summarize our results below. More details may be found in appendix A.

1. There are 440 polynomials with degree at most 32 and Mahler measure at most 1.3 that arise from the families of polynomials discussed in 1–3 in step 5 of section 2. We find 376 of these, or about 85%.
2. We assemble 1481 primitive, irreducible, noncyclotomic polynomials with Mahler measure below 1.3 and degree at most 64. All of these are listed in table 1 of appendix A. Our algorithm finds 1368 of these. The 113 missing polynomials fall into three categories:
   (a) The 64 missing polynomials with degree at most 32.
   (b) The two polynomials with degree greater than 32 yielding Salem numbers less than 1.3.

(c) 47 polynomials that are factors of members of one of the sequences whose Mahler measures tend to limit points below 1.3247.

The polynomials in the last category all have rather large degree and we expect the algorithm to find many of these if we continue the search through higher degrees.

3. We find 176 new polynomials, which appear neither in previously published lists nor among sequences of polynomials previously identified as having small Mahler measure, as discussed in step 5 of the algorithm. These new polynomials are marked with a "$*$" in table 1 of appendix A.
4. We find only eleven new polynomials with degree at most 32: four with degree 30 and seven with degree 32. This seems to support Boyd's heuristic of restricting to polynomials with height 1 for higher degrees.
5. The smallest Mahler measure among the new polynomials is approximately 1.23608. It is associated with the polynomial

$$x^{32} + x^{31} + x^{30} + x^{29} - x^{27} - x^{26} - 2x^{25} - x^{24} + x^{21} + x^{20} \\ - x^{16} + x^{12} + x^{11} - x^{8} - 2x^{7} - x^{6} - x^{5} + x^{3} + x^{2} + x + 1.$$

   This is the smallest Mahler measure known among primitive, irreducible, noncyclotomic polynomials of degree 32. Table 3 in appendix A lists the polynomials with the smallest known Mahler measure of each degree. It is also the 47th smallest Mahler measure known among all noncyclotomic polynomials with integer coefficients. Eleven of the new polynomials found have Mahler measure below 1.27; 27 have Mahler measure below 1.28. These are listed in table 2 of appendix A.
6. The Pinner polynomials (38) are more successful at generating polynomials with small Mahler measure than any of the other proposed adjust-

ments to coefficients. Table 5.1 summarizes the efficacy of the secondary adjustments. Column A lists the number of polynomials with Mahler measure below 1.3 found using each secondary strategy through degree 54. Column B displays the number of these not detected using Pinner's adjustment over the same range. Column C shows the number of these not found by any of the other adjustments over this range.

	A	B	C
$f(x) \pm x^{d-1}(x^2+x+1)$	445	15	10
$f(x) \pm x^{d-1}(x^2+1)$	914	19	16
$f(x) \pm x^{d-1}(x^2-x+1)$	936	32	28

TABLE 5.1. Efficiency of secondary coefficient adjustments

In contrast, Pinner's adjustment finds 973 polynomials with Mahler measure below 1.3 through degree 54, including 54 not found by any of the secondary adjustments through this stage.

7. We find no new Salem numbers, and we do comparatively poorly finding known Salem numbers. We find only 18 of the 43 known Salem numbers below 1.3. We also find that the average number of roots outside the unit circle among the known polynomials with small Mahler measure increases with the degree. Table 5 of appendix A summarizes the distribution of roots outside the unit circle among polynomials with small Mahler measure, and table 4 lists the polynomials with the smallest known Mahler measure by the number of roots outside the unit circle.

In closing, we remark that the algorithm is implemented in C++. All computations were performed on a Sun SPARC-20 or an Intel Pentium 90.

5. A Variation

We found in chapter 4 that it is more natural to count the number of reciprocal polynomials of a given degree that are products of cyclotomic polynomials, allowing multiplicity at most 2 at each cyclotomic factor except the linear ones, where we allow multiplicity 4. We also remarked at the beginning of section 2 that seven polynomials on Boyd's lists are Pinner polynomials whose decomposition includes the factor $(x+1)^4$. Because of these facts, we experimented with a variation on our algorithm, allowing multiplicity 4 on $x-1$ and $x+1$ in the construction of our products of cyclotomics. We checked these polynomials through degree 50 and found 11 of the 66 polynomials found by Boyd that were missed before, including 5 that yield missing Salem numbers. We also found one additional new polynomial:

$$\begin{aligned} x^{42} &+ x^{41} + x^{40} + x^{39} - x^{36} - x^{35} - x^{34} - x^{33} - x^{32} - x^{31} - x^{30} \\ &- x^{29} + x^{26} + x^{25} + x^{24} + x^{23} + x^{21} + x^{19} + x^{18} + x^{17} + x^{16} - x^{13} \\ &- x^{12} - x^{11} - x^{10} - x^{9} - x^{8} - x^{7} - x^{6} + x^{3} + x^{2} + x + 1. \end{aligned}$$

Its Mahler measure is approximately 1.26638.

The polynomials found using this variation that were not detected using our original algorithm are marked with a "q" in table 1 of appendix A.

CHAPTER 6

Limit Points

In this chapter we discuss small limit points of Mahler measures of polynomials. The first section describes the method used to evaluate limit points and lists the small limit points that were previously known. The second section gives the derivation of a new limit point.

1. Known Limit Points

We first define a generalization of the Mahler measure to polynomials in two variables. Recall from (9) that the Mahler measure of a polynomial $f(x)$ in one variable is the geometric mean of $f(x)$ on the unit circle. If $f(x, y)$ is a polynomial in two variables, we define its Mahler measure by

$$\log M\Big(f(x,y)\Big) = \int_0^1 \int_0^1 \log \Big| f\Big(e(s), e(t)\Big)\Big| \, ds\, dt. \tag{48}$$

We may extend the definition to polynomials in m variables in a similar way.

Boyd proves that the Mahler measure of a polynomial in two variables is the limit of the Mahler measures of a sequence of polynomials in one variable [8]:

THEOREM 6.1. *If $f(x, y)$ is a polynomial, then*

$$\lim_{n\to\infty} M\Big(f(x, x^n)\Big) = M\Big(f(x,y)\Big).$$

A similar result holds for polynomials in m variables.

We are interested in small limit points of Mahler measures of polynomials. Since each PV number is a limit of Salem numbers, we are particularly interested in limit points less than the smallest PV number, approximately 1.32472. Boyd and Smyth [6, 8] have found three such limit points.

Boyd [8] reports three polynomials in two variables with Mahler measure less than the smallest PV number:

$$M(x^2y^2 + x^2y + xy^2 + xy + x + y + 1) = 1.25542\ldots \tag{49}$$

$$M(x^2y + xy^2 + xy + x + y) = 1.28573\ldots \tag{50}$$

$$\begin{aligned} &M(x^4y + x^3y^2 + x^3y + x^3 + x^2y^2 + x^2y + x^2 + xy^2 + xy + x + y) \\ &\qquad = 1.31566\ldots \end{aligned} \tag{51}$$

By theorem 6.1, each of these yields a family of polynomials in one variable whose Mahler measures are fairly small. We replace x with ax^k and y with bx^n, where a and b may be ± 1, then remove extraneous powers of x to obtain the three families. The polynomials corresponding to (49) and (50) are respectively

$$x^{2n} + ax^{2n-k} + bx^{n+k} + abx^n + bx^{n-k} + ax^k + 1, \quad 0 < k < n/2, \tag{52}$$

and

$$x^{2n} + ax^{n+k} + bx^n + ax^{n-k} + 1, \quad 0 < k < n. \tag{53}$$

The following polynomials arise from (51):

$$\begin{aligned} &x^{2n} + ax^{2n-k} + x^{2n-2k} + bx^{n+2k} + abx^{n+k} + bx^n \\ &\qquad + abx^{n-k} + bx^{n-2k} + x^{2k} + ax^k + 1, \quad 0 < k < n/2, \end{aligned} \tag{54}$$

$$\begin{aligned} &x^{4n} + ax^{3n+k} + bx^{3n} + ax^{3n-k} + abx^{2n+k} + x^{2n} \\ &\qquad + abx^{2n-k} + ax^{n+k} + bx^n + ax^{n-k} + 1, \quad 0 < k < n. \end{aligned} \tag{55}$$

2. A New Limit Point

Studying the polynomials with small Mahler measure produced by our algorithm, we find many having the shape

$$(x^{3n-1}+1)(x^{n+1}+1)+x^{2n-1}(x^2-x+1).$$

Replacing x^n with y and multiplying by x produces the polynomial

$$f(x,y)=x^2y^2+x^2y+xy^4-xy^2+x+y^3+y^2. \tag{56}$$

We compute the Mahler measure of this polynomial. By Jensen's formula,

$$\begin{aligned}\log M(f) &= \int_0^1\int_0^1 \log\left|f\Big(e(s),e(t)\Big)\right|\,ds\,dt\\ &= \int_0^1\left(\log|e(3t)+e(2t)|+\sum_{\substack{f(\alpha,e(t))=0\\ |\alpha|<1}}\log\frac{1}{|\alpha|}\right)dt.\end{aligned} \tag{57}$$

The first term of (57) is simple:

$$\int_0^1 \log|e(t)+1|\,dt=\log M(y+1)=0.$$

For the second term, a straightforward calculation shows that one of the roots of $f(x,e(t))$, considered as a polynomial in x, is given by

$$\alpha(t)=\frac{1-2\cos(4\pi t)+\sqrt{(1-2\cos(4\pi t))^2-16\cos^2(\pi t)}}{4\cos(\pi t)}e(t/2). \tag{58}$$

Now $f(x,y)$ is quadratic in x, and the product of its two roots for any fixed y is y. Thus the absolute value of the second root of $f(x,e(t))$ is $1/|\alpha(t)|$. These facts, combined with the observation that $|\alpha(t)|=|\alpha(1-t)|$, allow us to simplify the second term in (57). We have

$$\log M(f)=2\int_0^{1/2}\Big|\log|\alpha(t)|\Big|\,dt.$$

We find that the expression under the radical in (58) is zero when $z = \cos(2\pi t)$ satisfies

$$(2z+1)(8z^3 - 4z^2 - 10z + 1) = 0.$$

This gives three solutions for t between 0 and 1/2:

$$\begin{aligned} t_1 &= 0.23454\ldots \\ t_2 &= \frac{1}{3} \\ t_3 &= 0.45028\ldots \end{aligned}$$

We determine that the expression under consideration is positive for $t_1 < t < t_2$ and $t_3 < t \leq 1/2$. When it is negative, we have $|\alpha(t)| = 1$. Therefore,

$$\log M(f) = 2\int_{t_1}^{t_2} \Big|\log|\alpha(t)|\Big|\,dt + 2\int_{t_3}^{1/2} \Big|\log|\alpha(t)|\Big|\,dt.$$

This yields

$$M(f) = 1.30909838\ldots \tag{59}$$

As in section 1, the polynomial $f(x,y)$ gives rise to a family of polynomials in one variable whose Mahler measures approach the value in (59) as the degree becomes large. These polynomials are given by

$$x^{4n} + ax^{3n+k} + bx^{2n+k} - x^{2n} + bx^{2n-k} + ax^{n-k} + 1 \tag{60}$$

where $-n/2 < k < n$, and

$$x^{2n} + ax^{2n-k} + bx^{n+2k} - bx^{n} + bx^{n-2k} + ax^{k} + 1 \tag{61}$$

where $0 < k < n/2$. Here, a and b may be ± 1.

We compute all of the polynomials having the form (60) or (61) through degree 100. We factor each and compute the Mahler measure of each factor with degree at most 64. Using this strategy, we find 78 additional new polynomials.

None of these polynomials was found using the algorithm described in chapter 5, and none arises from any of the other families of polynomials discussed. These 78 polynomials are marked with "$**$" in table 1 of appendix A.

We also rediscover 44 of the new polynomials found by our algorithm, including the new polynomial having the smallest Mahler measure. Let $f(x)$ denote this polynomial, which is listed on page 52. Let $\Phi_n(x)$ denote the n-th irreducible cyclotomic polynomial. We find

$$\Phi_{10}(x)\Phi_{20}(x)f(x) = x^{44} - x^{37} + x^{36} - x^{22} + x^8 - x^7 + 1. \tag{62}$$

The right side of (62) is obtained from (61) by setting $a = -1$, $b = 1$, $k = 7$, and $n = 22$.

APPENDIX A

Polynomials with Small Mahler Measure

1. Complete List of Polynomials

This table lists all the primitive, irreducible, noncyclotomic polynomials with Mahler measure less than 1.3 that were found during this investigation. The first column gives the degree D of the polynomial. The second column contains a code that indicates how the polynomial was found. The meanings of the different codes are given below.

(blank)	Previously known polynomial that was also found using our algorithm.
$*$	New polynomial discovered by our algorithm.
$**$	New polynomial found using the new limit point.
b	Previously known polynomial that was not found using our algorithm.
$*$q	New polynomial discovered by the variation discussed in section 5 of chapter 5.
bq	Previously known polynomial that was not found using our original algorithm, but was found using the variation.

The third column shows the approximate Mahler measure M of the polynomial. The fourth column, ν, is the number of roots of the polynomial that lie outside the unit circle. The remaining columns give the first half of the coefficients of each polynomial.

D	M	ν	Coefficients
8	1.280638156268	1	1 0 0 1 −1
10	1.176280818260	1	1 1 0 −1 −1 −1
10	1.216391661138	1	1 0 0 0 −1 1
10	1.230391434407	1	1 0 0 1 0 1
10	1.261230961137	1	1 0 −1 0 0 1
10	1.267233859440	2	1 0 1 1 0 1
10	1.283582360621	2	1 1 0 0 0 −1
10	1.293485953125	1	1 0 −1 1 0 −1
12	1.227785558695	2	1 1 1 0 −1 −1 −1
12	1.240726423653	1	1 1 1 1 0 0 −1
12	1.251046617204	2	1 1 0 0 0 −1 −1
12	1.264393854743	2	1 0 1 0 0 1 −1
12	1.272818365083	2	1 0 1 1 1 2 1
14	1.200026523987	1	1 0 0 1 −1 0 0 −1
14	1.202616743689	1	1 0 −1 0 0 0 0 1
14	1.255093516764	1	1 0 −1 1 0 −1 0 1
14	1.265122072380	2	1 0 0 1 1 1 1 1
14	1.267296442523	1	1 1 0 0 0 0 −1 −1
14	1.278043016131	2	1 1 1 0 0 −1 −1 −1
14	1.280123027338	2	1 1 0 −1 0 0 0 −1
14	1.290178036769	2	1 0 0 0 0 1 0 −1
14	1.291562760024	3	1 1 1 0 −1 −1 −1 −1
14	1.293072888328	3	1 0 0 0 −1 0 0 1
14	1.297398316823	2	1 1 1 1 1 0 −1 −1
16	1.224278907222	2	1 1 0 −1 −1 0 1 1 1
16	1.235256705642	2	1 1 0 0 0 −1 0 0 −1
16	1.236317931803	1	1 1 0 0 0 0 0 0 −1
16	1.243477618690	2	1 1 0 −1 −1 −1 0 1 1
16	1.248013608327	2	1 1 1 1 0 −1 −1 −1 −1
16	1.251732411297	2	1 1 1 0 0 0 0 0 −1
16	1.252828663032	2	1 0 1 1 0 1 0 0 1
16	1.257316625602	2	1 1 0 −1 0 1 1 0 −1
16	1.266677526509	2	1 1 1 1 1 0 −1 −2 −2
16	1.281341129377	2	1 1 0 −1 0 0 −1 0 1
16	1.289590102270	2	1 1 0 −1 −1 0 1 0 0
16	1.293493112485	2	1 1 1 0 0 0 1 0 0
16	1.297064774519	2	1 0 0 1 0 0 0 0 1
16	1.298032254275	4	1 0 0 1 −1 0 0 −1 1
18	1.188368147508	1	1 1 1 1 0 0 −1 −1 −1 −1
18	1.201396186235	2	1 1 1 0 0 −1 0 −1 0 −1
18	1.219446875941	3	1 1 1 0 −1 −1 −1 0 0 1
18	1.219720859040	1	1 1 0 0 0 0 0 0 −1 −1

18		1.225503424104	2	1	1	0	0	1	0	−1	0	0	−1	
18		1.231342769993	3	1	0	0	0	−1	1	0	0	0	−1	
18		1.240770634960	2	1	0	1	0	1	0	0	1	−1	1	
18		1.244617058976	3	1	0	−1	0	1	0	−1	0	0	1	
18	b	1.252775937410	1	1	0	0	0	0	0	−1	1	−1	1	
18		1.256221154392	1	1	1	0	0	−1	−1	0	0	0	1	
18		1.266334328968	3	1	1	0	0	0	−1	−1	0	0	−1	
18		1.278742424399	2	1	0	0	0	1	1	0	0	0	1	
18		1.280082037220	3	1	1	0	−1	−1	0	0	0	0	1	
18	b	1.284616550926	1	1	0	0	0	−1	0	−1	1	0	1	
18		1.285787140645	3	1	0	−1	0	0	0	0	0	0	1	
18		1.286395966836	1	1	2	2	2	2	2	2	3	3	3	
18		1.286478694652	2	1	0	1	0	1	1	0	1	0	1	
18		1.287893487041	2	1	1	1	0	0	0	0	−1	−1	−1	
18		1.289140044883	3	1	1	1	1	1	0	−1	−1	−2	−3	
18	bq	1.295675371944	1	1	1	0	0	−1	−1	−1	0	1	1	
18		1.295738636398	2	1	1	1	0	0	−1	0	0	1	1	
18		1.296078447471	2	1	0	0	0	1	0	0	1	0	1	
18		1.299243183982	3	1	2	2	1	0	−1	−1	−1	−2	−3	
20		1.212824180989	2	1	1	0	0	1	1	0	−1	−1	−1	−1
20		1.214995700776	2	1	1	1	1	1	0	−1	−1	−1	−1	−1
20		1.218396362520	2	1	1	0	−1	0	0	0	−1	0	0	1
20		1.226993758166	2	1	1	−1	−2	0	2	1	−1	−1	0	1
20		1.232613548593	1	1	1	0	0	0	1	1	0	0	1	1
20		1.246108142719	2	1	1	1	0	0	−1	−1	−1	0	0	1
20		1.246878271453	2	1	1	1	1	0	0	0	−1	−1	−1	−1
20	bq	1.253330650201	1	1	0	−1	0	0	1	0	0	0	0	0
20		1.253635565709	2	1	1	1	1	0	0	−1	−1	−1	−1	−1
20		1.254700427115	2	1	0	1	0	0	1	0	1	1	0	1
20		1.256293862611	2	1	0	1	0	0	0	−1	0	0	1	1
20		1.259849432481	2	1	0	0	1	0	0	0	0	0	0	1
20		1.261360997649	2	1	0	0	0	0	1	−1	0	0	−1	1
20		1.262509762460	2	1	0	−1	1	1	−1	0	0	0	0	−1
20		1.262564740671	4	1	2	2	1	−1	−3	−3	−2	0	2	3
20		1.263945856410	3	1	0	0	1	0	0	0	1	0	0	1
20		1.268737648104	2	1	0	0	1	0	0	0	−1	0	0	−1
20		1.269331428837	2	1	2	2	1	0	−1	−1	−1	−1	−1	−1
20		1.271841655431	2	1	1	1	0	−1	−1	0	1	1	0	−1
20		1.274612798280	2	1	1	1	0	0	0	0	0	−1	−1	−1
20		1.280386221566	2	1	0	1	0	1	0	0	0	0	1	0
20		1.282495560640	1	1	2	2	2	2	2	1	0	−1	−1	−1
20		1.284487312196	3	1	0	0	0	0	0	0	0	0	1	−1
20		1.284541307234	2	1	1	1	1	1	1	2	2	2	2	1
20		1.287881336401	2	1	0	−1	0	0	1	0	−1	0	0	1
20		1.287884568067	2	1	1	0	0	1	1	0	−1	−1	0	1
20		1.288359664537	4	1	1	0	−1	−1	−1	−1	−1	0	2	3
20		1.289289736514	2	1	0	0	1	−1	0	0	−1	0	0	−1
20		1.289462523106	2	1	1	0	−1	0	1	1	0	−1	−1	−1
20	b	1.292039106018	1	1	0	−1	0	0	1	0	0	−1	0	1
20		1.292503334264	4	1	0	0	1	1	0	1	1	0	1	1
20		1.293993740440	4	1	1	0	−1	−1	−1	0	1	1	0	−1
20		1.296754236842	4	1	0	0	0	0	0	0	1	0	0	1
20		1.297584816017	3	1	0	0	0	0	0	0	1	−1	0	−1

20		1.298272824288	4	1 1 0 0 0 −1 −1 0 0 0 1
22		1.205019854225	2	1 0 1 0 0 1 −1 1 0 0 1 −1
22		1.229566456617	3	1 1 0 −1 −1 −1 −1 −1 0 1 1 1
22	bq	1.235664580390	1	1 0 −1 1 0 0 0 −1 1 0 −1 1
22		1.244802445450	2	1 0 0 1 0 0 0 0 0 0 0 −1
22		1.245181138298	3	1 1 0 −1 −1 0 1 1 0 −1 −1 −1
22		1.245602317112	2	1 1 1 1 0 −1 −1 −1 −1 0 1 1
22		1.248611165686	3	1 1 0 0 0 −1 −1 −1 −1 0 1 1
22		1.250461871099	3	1 0 −1 0 0 0 0 1 0 0 0 −1
22		1.252721020130	2	1 0 0 1 0 0 0 0 1 0 0 1
22		1.254142519162	3	1 0 −1 0 0 0 0 0 0 1 0 −1
22		1.257259068491	3	1 1 1 0 −1 −2 −2 −1 0 1 1 1
22		1.259433606774	2	1 0 −1 0 1 0 0 0 0 0 0 1
22		1.260283958937	3	1 0 0 1 −1 0 0 −1 0 0 0 1
22	bq	1.260284236896	1	1 1 0 1 1 0 0 0 −1 −1 −1 −1
22		1.261526266069	3	1 1 0 −1 −1 0 1 0 −1 −1 0 1
22		1.262460975915	3	1 1 1 0 0 −1 −1 −1 −1 0 0 1
22		1.264051196102	3	1 0 0 0 0 0 0 0 0 0 −1 1
22		1.270231008664	3	1 0 0 0 0 0 0 0 −1 0 0 1
22		1.270616018981	3	1 1 1 0 0 −1 0 −1 0 −1 0 −1
22		1.272433437809	3	1 0 0 1 0 1 0 0 0 0 0 −1
22		1.272557587236	2	1 0 1 0 0 0 0 0 0 1 0 1
22		1.272935781547	3	1 1 1 1 1 0 −1 −1 −1 −1 −1 −1
22		1.273310275572	2	1 1 1 1 1 1 0 0 0 −1 −1 −1
22		1.275872162255	3	1 0 0 0 0 0 0 1 0 0 0 1
22		1.276284296398	3	1 1 0 0 0 0 0 −1 0 0 −1 −1
22	bq	1.276779674019	1	1 1 −1 −1 0 0 0 0 0 1 0 −1
22		1.277380143641	2	1 1 1 1 1 0 0 0 0 0 0 1
22		1.277572123045	2	1 1 0 −1 −1 −1 −1 −1 0 1 2 2
22		1.280267454300	3	1 1 0 −1 −1 −1 0 1 1 0 −1 −1
22		1.283326411512	3	1 0 0 0 −1 0 −1 0 0 0 1 1
22		1.283410490448	3	1 0 0 0 0 0 −1 0 0 0 0 1
22		1.283926281329	4	1 0 0 1 0 0 0 0 0 −1 0 0
22		1.283960069703	3	1 0 0 1 0 0 0 0 −1 0 0 −1
22		1.284066283372	3	1 1 0 0 1 1 0 −1 −1 −1 −1 −1
22		1.284829254718	3	1 0 0 1 0 1 0 0 1 0 0 1
22		1.284897491464	4	1 1 0 0 0 −1 −1 0 0 0 1 1
22		1.285465569282	4	1 1 0 −1 0 0 0 0 0 −1 0 1
22		1.286735477551	3	1 1 0 0 0 0 0 0 0 0 0 1
22		1.289043270830	3	1 0 0 0 −1 0 0 1 0 0 0 −1
22		1.289153079653	5	1 1 0 0 0 −1 −1 0 0 −1 0 1
22		1.289397453818	3	1 0 0 0 0 1 −1 0 0 0 0 −1
22		1.292723128748	2	1 1 0 0 0 0 0 0 0 0 1 1
22		1.292918185636	3	1 0 0 0 0 1 0 0 0 0 0 1
22		1.293955335404	3	1 2 2 2 2 1 0 −1 −2 −3 −3 −3
22	b	1.296346045959	2	1 0 0 1 1 0 0 0 0 −1 −1 0
22	b	1.296421365195	1	1 1 0 0 0 1 0 0 0 0 0 −1
22		1.296992814015	4	1 1 1 1 1 0 0 0 −1 −1 −1 −1
22		1.299313007733	2	1 0 1 0 1 0 0 1 0 0 0 0
24		1.218855150304	2	1 0 0 0 0 1 0 −1 0 0 0 0 −1
24		1.219057507826	2	1 1 0 −1 −1 −1 −1 −1 0 1 1 1 1
24		1.234443834873	2	1 0 0 0 −1 0 −1 1 0 0 1 −1 1

24		1.253743761231	2	1	1	1	1	1	0	0	0	−1	−1	−1	−1	−1	
24		1.254104305576	2	1	1	0	−1	−1	0	0	0	1	1	0	−1	−1	
24		1.254491781942	2	1	0	0	1	−1	0	0	−1	0	0	0	0	1	
24		1.256201731791	2	1	1	1	0	0	0	0	0	0	0	−1	−1	−1	
24		1.257606104952	4	1	1	1	0	−1	−1	−1	0	0	0	0	0	1	
24	b	1.260103540355	1	1	1	0	0	−1	−1	0	1	1	1	0	−1	−1	
24		1.261552552254	4	1	0	1	1	0	2	0	1	1	0	1	0	1	
24		1.261622390372	2	1	1	1	1	0	0	−1	−1	−1	−1	0	0	1	
24		1.264833080300	2	1	0	−1	0	0	1	0	−1	0	0	1	0	−1	
24		1.266538700017	2	1	1	0	0	1	1	0	−1	−1	0	1	0	−1	
24		1.270608190764	2	1	1	1	0	−1	−1	0	1	1	0	−1	−1	−1	
24		1.273094845325	4	1	1	1	0	0	0	0	−1	−1	0	0	0	−1	
24		1.273252334303	4	1	1	1	1	1	1	0	−1	−1	−1	−1	−1	−1	
24		1.273797810522	2	1	1	0	0	1	1	0	−1	0	0	−1	−1	−1	
24		1.274016985626	4	1	1	0	0	0	−1	−1	0	0	0	1	0	−1	
24		1.275963094358	2	1	0	−1	1	0	−1	1	0	−1	1	0	−1	1	
24		1.277168708382	2	1	0	0	0	−1	1	0	−1	1	−1	0	1	−1	
24		1.277515976502	3	1	1	1	0	0	0	0	0	0	1	0	0	−1	
24	b	1.277880456611	2	1	0	1	0	0	1	0	1	0	1	1	0	1	
24		1.278063398983	2	1	1	0	−1	−1	−1	0	0	1	1	1	−1	−1	
24		1.279415986109	4	1	1	1	0	0	−1	−1	−2	−1	0	1	1	1	
24		1.281656726661	2	1	1	0	0	1	0	−1	−1	0	0	−1	0	1	
24	b	1.281824295349	2	1	1	0	0	0	−1	−1	0	1	1	0	−1	−1	
24		1.282228408314	2	1	1	0	0	0	0	0	0	0	0	0	−1	−1	
24		1.283668286046	4	1	0	−1	1	1	−1	0	1	0	−1	0	0	−1	
24		1.284029047869	4	1	1	1	0	0	−1	0	0	1	0	0	−1	0	
24		1.284078466121	2	1	1	0	−1	0	1	0	−1	0	1	0	−1	−1	
24		1.284098295177	3	1	0	0	0	0	1	0	1	0	0	0	0	1	
24		1.286884070865	2	1	0	1	1	0	1	0	0	0	−1	−1	−1	−1	
24		1.288507402632	4	1	1	0	−1	−1	0	1	1	0	−1	0	1	1	
24		1.289179589678	2	1	0	0	0	1	0	0	1	0	0	1	0	0	
24		1.290066152643	2	1	0	0	0	0	0	0	1	0	0	0	−1	−1	
24		1.290821129117	3	1	1	2	2	2	2	1	1	0	0	−1	−1	−1	
24		1.291240454188	2	1	1	0	0	1	1	1	1	1	2	3	2	1	
24		1.291424801069	2	1	1	0	−1	0	2	1	−2	−2	1	2	−1	−3	
24	b	1.291741425714	1	1	1	0	0	0	0	−1	0	0	0	0	0	0	
24		1.292259056428	2	1	0	−1	1	0	−1	0	0	0	−1	1	1	−1	
24	b	1.292448862134	2	1	1	0	0	0	0	1	1	1	1	0	−1	−1	
24		1.292984094421	6	1	1	1	1	0	−1	−2	−2	−2	−1	1	2	3	
24		1.293602150874	4	1	0	1	0	1	0	1	0	0	1	−1	1	−1	
24		1.295587903566	4	1	1	0	0	0	0	0	0	1	0	−1	−1	−1	
24		1.296865864684	2	1	1	0	0	1	0	−1	0	0	−1	0	0	−1	
24	b	1.299560949999	2	1	1	2	1	1	0	0	−1	−1	−1	−1	−1	−1	
26		1.223777454948	3	1	1	1	0	0	−1	−1	−1	−1	0	0	1	0	1
26		1.226092894512	2	1	0	0	1	0	0	0	0	0	0	1	0	0	1
26		1.234500336789	3	1	0	−1	1	0	−1	1	0	−1	1	0	−1	0	1
26	b	1.237504821217	1	1	0	−1	0	0	1	0	0	−1	0	1	0	0	−1
26		1.240254178706	3	1	1	0	−1	−1	0	0	0	−1	0	1	1	0	−1
26	b	1.246393202356	2	1	2	3	3	2	1	0	−1	−1	−1	−1	−1	−1	−1
26		1.247608702636	2	1	0	1	0	1	0	0	0	0	1	0	1	0	1
26		1.248818753713	3	1	1	1	1	0	0	0	0	0	0	−1	−1	−1	−1
26		1.249499333193	3	1	1	0	0	0	0	0	0	0	0	0	0	−1	−1
26		1.257383216738	3	1	0	0	0	0	0	−1	0	0	0	0	0	0	1

26		1.260765028236	2	1 1 1 0 0 −1 0 0 1 1 1 0 −1 −1
26		1.261347529349	2	1 1 0 0 0 −1 0 0 0 0 0 0 0 −1
26	b	1.263038139930	1	1 1 0 0 0 0 −1 0 0 0 0 0 0 −1
26		1.264161172368	2	1 2 3 4 4 3 2 1 0 −1 −1 −1 −1 −1
26		1.265230670506	3	1 1 0 −1 −1 0 0 −1 −1 0 1 1 0 −1
26		1.268333890036	3	1 0 0 0 0 1 0 0 0 0 0 0 0 1
26		1.271748284801	2	1 1 0 0 0 0 0 0 0 0 0 0 1 1
26		1.272019269348	2	1 0 0 1 0 1 0 0 0 0 0 −1 0 −1
26		1.272561259437	3	1 0 0 0 0 0 −1 1 0 0 0 0 0 −1
26		1.275162594340	3	1 0 0 0 0 1 0 0 −1 0 0 0 0 −1
26		1.275208223427	5	1 0 0 0 0 0 0 0 −1 0 0 0 0 1
26		1.277343467933	3	1 1 2 2 2 2 1 1 0 0 −1 −1 −1 −1
26		1.278469667285	4	1 0 0 1 1 1 0 1 1 1 1 0 1 1
26		1.279832880056	5	1 1 0 0 0 0 0 0 0 0 0 1 0 −1
26		1.279957171286	3	1 0 0 0 −1 0 0 0 0 1 0 0 0 −1
26		1.281627245595	3	1 0 0 0 −1 0 0 0 0 0 0 0 0 1
26	b	1.281691371528	1	1 0 0 0 0 0 −1 1 −1 1 −1 1 −1 1
26		1.284224708139	4	1 0 0 1 0 0 0 0 0 0 0 0 0 −1
26	b	1.284746821545	1	1 2 1 −1 −2 −1 0 0 −1 −1 0 1 1 1
26		1.285123263199	3	1 0 0 1 0 0 0 0 0 0 −1 0 0 −1
26	b	1.285196726770	1	1 0 −1 1 0 0 0 −1 0 1 −1 0 1 −1
26		1.286253592873	4	1 0 0 0 0 0 0 0 0 0 0 1 0 −1
26		1.286308436738	5	1 1 1 1 1 1 0 0 −1 −1 −1 −1 −1 −1
26		1.286364473710	3	1 1 1 0 0 0 0 −1 −1 −1 0 0 0 −1
26	b	1.286730182048	1	1 1 0 0 −1 −1 −1 0 1 1 1 0 −1 −1
26		1.287157601101	4	1 1 0 0 0 −1 −1 0 1 1 1 1 0 −1
26		1.287755460611	3	1 0 0 0 0 0 −1 0 −1 0 0 0 1 1
26		1.288460008663	3	1 0 0 0 −1 0 0 0 1 1 0 0 −1 −1
26	bq	1.290051521254	3	1 0 0 1 0 0 0 1 −1 0 0 −1 0 −1
26		1.290083190494	2	1 0 1 1 1 1 1 1 1 1 1 1 0 1
26		1.290363818013	4	1 0 1 1 0 1 0 0 1 −1 1 0 0 1
26		1.290488770334	4	1 1 0 0 1 0 −1 0 1 −1 −1 1 0 −2
26		1.291631617175	4	1 0 0 0 0 0 0 0 0 1 0 0 0 −1
26		1.293551483051	3	1 1 0 −1 0 0 −1 −1 0 1 0 0 0 1
26		1.294793259550	5	1 0 0 0 0 0 0 0 0 0 −1 0 0 1
26		1.295540495364	4	1 0 0 0 0 0 0 0 1 0 0 0 0 1
26		1.295871269617	4	1 1 0 0 0 −1 −1 0 1 1 1 0 −1 −1
26		1.296403391978	5	1 0 −1 0 0 0 0 0 0 1 0 −1 0 1
26		1.296536720532	3	1 0 0 1 0 0 0 0 0 0 0 0 0 1
26		1.296998603555	5	1 1 1 1 1 1 0 −1 −1 −1 −1 −1 −1 −1
26	bq	1.297266841131	3	1 1 0 0 0 0 −1 −1 0 1 0 −1 0 1
26		1.297273682164	4	1 1 1 1 1 0 0 0 0 0 0 0 −1 −1
26	b	1.297315623802	2	1 0 0 0 0 0 −1 0 1 1 0 −1 0 0
26	b	1.297417306157	2	1 0 0 0 1 0 1 0 0 1 0 1 0 1
26		1.297426869772	5	1 0 0 0 0 0 0 0 0 0 0 1 0 1
26		1.297715235385	4	1 1 1 1 2 2 2 2 2 2 2 2 1 1
26		1.297851226537	5	1 0 1 0 0 0 −1 0 −1 0 0 1 1 1
26		1.298568632349	5	1 0 1 0 1 0 1 0 0 1 −1 1 −1 1
26		1.298611109885	5	1 0 0 0 0 0 0 0 0 0 0 0 −1 1
26		1.299521136729	5	1 0 1 0 0 0 −1 1 −1 1 0 0 1 −1
26	b	1.299744869472	1	1 1 −1 0 2 0 −2 1 2 −2 −2 2 0 −3
28		1.207950028412	2	1 1 1 1 0 0 0 −1 −1 −1 −1 0 0 0 1
28	b	1.232628775929	2	1 2 2 1 1 1 0 −1 −1 −1 −1 −1 −1 −1 −1

28		1.236808305865	2	1	1	0	−1	0	0	0	0	0	−1	0	0	0	0	1
28		1.240699637594	2	1	2	3	3	2	1	0	−1	−1	−1	−1	−1	−2	−3	−3
28		1.243878801656	2	1	1	1	1	1	1	0	0	0	−1	−1	−1	−1	−1	−1
28		1.244901680004	2	1	1	1	1	0	0	−1	−1	−1	−1	0	0	1	1	1
28		1.245613552973	3	1	0	1	0	0	1	−1	1	−1	0	0	0	1	0	1
28	bq	1.249352148260	3	1	1	1	0	−1	−2	−2	−2	−1	0	1	1	1	1	1
28		1.251950020826	2	1	0	0	0	0	0	0	0	1	0	0	0	0	1	−1
28		1.253516636634	4	1	0	0	1	−1	−1	0	−1	0	0	0	1	0	0	1
28		1.254225340086	4	1	1	0	−1	0	0	−1	−1	0	1	1	0	−1	0	1
28		1.254457547229	4	1	1	1	1	0	0	−1	−2	−1	−1	0	0	0	1	1
28		1.254774086064	4	1	0	0	0	0	1	0	0	0	−1	0	0	0	0	−1
28		1.256440300936	4	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	1	1
28		1.257879174708	4	1	0	−1	1	0	−2	1	1	−1	1	1	−1	0	0	−1
28		1.257981943424	4	1	1	0	0	0	0	0	0	0	0	0	0	−1	−1	−1
28		1.258041637335	3	1	1	0	−1	−1	0	0	0	0	1	1	0	−1	0	0
28		1.260774506622	4	1	1	0	0	0	−1	−1	0	0	−1	0	1	0	0	1
28		1.262375997895	4	1	1	0	−1	−1	0	1	1	0	−1	−1	−1	0	1	1
28		1.263460794381	4	1	1	1	0	0	0	1	1	0	−1	−1	0	0	0	−1
28		1.265190265700	3	1	0	0	0	0	1	0	0	0	1	0	0	0	0	1
28		1.267197450352	2	1	0	0	0	0	0	1	0	0	0	0	0	0	1	−1
28		1.268965256129	4	1	1	0	0	0	0	0	−1	0	0	−1	0	0	0	1
28		1.269117837458	3	1	1	0	0	0	0	0	0	0	0	0	0	0	1	1
28		1.269625267089	4	1	0	−1	0	1	0	0	1	−1	−2	1	2	−1	−1	1
28		1.270149462492	2	1	1	1	0	−1	−2	−1	0	1	1	1	0	0	−1	−1
28		1.270556906505	4	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1
28		1.273964670586	4	1	0	1	0	0	0	−1	1	−1	1	0	0	1	−1	1
28		1.275851372894	4	1	1	1	1	1	1	1	0	0	0	−1	−1	−2	−2	−1
28		1.276268906562	4	1	1	0	−1	−1	0	1	1	1	0	−1	−1	0	1	1
28		1.277818561663	3	1	0	1	0	0	0	0	0	0	0	−1	0	−1	1	−1
28		1.279546943404	3	1	0	0	1	0	0	0	0	0	0	0	1	0	0	1
28	b	1.279715121393	2	1	0	0	0	0	1	0	0	0	0	0	0	1	0	−1
28	bq	1.279741143454	3	1	0	0	0	−1	1	0	0	0	0	0	0	0	0	1
28		1.281077834940	4	1	0	0	0	0	0	0	0	0	1	0	0	0	0	1
28		1.281494670580	4	1	0	1	0	1	0	1	1	1	2	1	2	1	2	1
28		1.282817230735	4	1	1	0	−1	−1	0	1	0	−1	−1	0	1	1	0	−1
28		1.283042525998	4	1	1	1	1	1	1	0	0	0	0	0	−1	−1	−1	−1
28		1.284899766447	2	1	1	1	0	0	0	0	0	0	0	0	0	−1	−1	−1
28		1.285423017864	4	1	1	1	1	1	1	1	0	−1	−1	−1	−1	−1	−1	−1
28		1.286866091754	3	1	0	0	1	0	0	0	0	0	0	0	0	0	0	−1
28		1.288066055820	4	1	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	1
28		1.288142930775	4	1	0	0	1	0	−1	0	1	−1	−1	1	0	−1	0	1
28	b	1.289200261605	3	1	1	1	1	1	1	1	1	1	1	1	2	1	1	1
28		1.289257756564	4	1	1	0	0	1	1	0	−1	−1	0	0	−1	−1	0	1
28		1.290180372525	4	1	1	0	0	1	1	0	−1	0	1	0	−1	−1	0	1
28	b	1.292337015275	2	1	1	1	0	−1	0	1	1	0	−1	−1	0	0	−1	−1
28	b	1.292392346874	2	1	1	1	0	0	0	0	0	1	1	1	1	1	0	0
28		1.293360747224	4	1	1	2	1	1	0	0	−1	−1	−1	−1	0	−1	0	−1
28		1.294485550557	5	1	0	0	1	0	0	0	0	−1	0	0	0	0	0	1
28		1.295676843823	4	1	0	0	0	0	0	−1	1	0	0	0	0	0	−1	1
28		1.296452326801	3	1	0	1	0	1	1	1	1	1	2	1	2	1	2	1
28	b	1.296821373715	1	1	0	0	0	−1	1	−1	1	−1	0	0	0	1	−1	1
28		1.296827935200	3	1	0	0	1	0	0	1	0	−1	1	0	−1	0	0	−1
28		1.298363672419	3	1	0	0	0	0	0	0	0	0	0	−1	0	0	1	0
28		1.298384957045	3	1	1	1	0	0	0	0	0	0	0	0	−1	−1	−1	−1

28		1.298532294692	4	1	1	0	0	0	0	1	0	−1	0	0	−1	0	0	−1	
28	b	1.299604492248	2	1	1	0	0	0	0	1	0	0	1	0	0	0	0	1	
28		1.299729886570	2	1	2	3	4	4	3	2	1	0	−1	−1	−1	−1	−1	−1	
30		1.225619851977	2	1	0	1	0	0	0	0	0	0	0	0	0	0	1	0	1
30		1.225810532354	3	1	0	1	0	0	0	−1	1	−1	1	−1	0	0	−1	1	−1
30		1.228140772740	3	1	1	0	0	0	−1	−1	−1	−1	0	0	0	1	0	0	1
30		1.243128704866	3	1	1	1	1	1	0	−1	−1	−1	−1	−1	0	0	0	0	1
30		1.247574755048	3	1	0	0	0	0	0	0	1	−1	0	0	0	0	0	0	−1
30		1.252001448745	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1
30		1.252124204475	3	1	2	2	1	0	−1	−1	−1	−1	−1	−1	−1	0	0	0	0
30		1.255638129256	3	1	0	1	0	0	1	−1	0	−1	−1	0	−1	0	0	0	1
30		1.256815487640	5	1	0	−1	1	1	−1	0	1	−1	0	1	−1	−1	1	0	−1
30	b	1.259524516835	3	1	0	0	0	0	1	−1	0	−1	1	0	0	0	−1	1	−1
30		1.259619357073	3	1	1	0	−1	−1	−1	−1	0	1	1	1	1	0	−1	−1	−1
30		1.260218697373	4	1	0	0	0	1	0	0	0	0	0	0	1	0	0	0	1
30		1.260952057343	3	1	0	0	0	−1	0	0	0	0	0	0	0	0	0	0	1
30		1.262936982450	3	1	0	0	0	−1	0	0	0	0	0	0	1	0	0	0	−1
30		1.264580284423	4	1	1	1	0	−1	−1	−1	0	0	0	0	0	1	1	1	1
30		1.267387724694	2	1	1	1	1	0	0	0	0	0	0	1	0	0	0	−1	−1
30	b	1.267813458869	3	1	0	0	1	−1	0	0	−1	1	0	−1	1	−1	0	1	−1
30		1.268335144250	3	1	1	0	−1	−1	0	0	0	0	0	1	0	−1	−1	0	1
30		1.270358661378	3	1	0	1	1	0	1	0	1	0	0	0	−1	−1	−1	−1	−1
30		1.270476144115	3	1	1	0	−1	0	1	1	0	−1	0	1	1	−1	−1	0	1
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30		1.275324679391	4	1	0	0	0	0	0	0	1	1	0	0	0	0	0	0	1
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30		1.276496975884	3	1	1	0	0	0	0	0	0	0	0	0	0	0	0	−1	−1
30		1.276789738378	3	1	0	1	1	0	1	0	0	1	0	0	1	−1	0	0	−1
30	b	1.277324548849	3	1	0	0	1	0	1	0	0	0	0	0	0	−1	−1	0	−1
30		1.278420197823	5	1	1	0	0	0	−1	−1	0	0	0	1	1	0	−1	−1	−1
30		1.279042808401	3	1	0	0	0	−1	0	0	1	1	0	0	−1	−1	0	0	1
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30		1.280627662390	3	1	0	−1	0	0	0	0	0	0	0	0	0	0	1	0	−1
30		1.280910420246	4	1	0	0	0	−1	0	0	1	1	0	0	−1	0	0	0	1
30		1.280950587799	3	1	0	0	0	0	1	0	0	−1	0	1	0	−1	−1	0	1
30		1.283825588028	5	1	1	0	0	0	−1	−1	0	0	0	1	0	−1	0	0	−1
30		1.284263099264	3	1	0	−1	0	1	0	−1	0	0	0	0	0	0	0	0	1
30	b	1.285099363652	1	1	0	0	0	0	1	−1	1	−1	1	−1	0	0	0	0	−1
30	b	1.285121520153	1	1	2	2	2	1	0	−1	−2	−2	−1	0	1	1	1	1	1
30	b	1.285185670753	1	1	1	0	0	0	0	0	0	−1	0	0	0	−1	0	0	1
30		1.285213389477	4	1	0	0	1	1	0	1	1	0	0	1	0	−1	1	0	−1
30	b	1.285235436229	1	1	0	−1	0	0	1	−1	0	0	0	1	0	0	−1	0	1
30	*	1.285530553671	4	1	0	1	1	1	1	1	2	0	1	1	0	0	0	0	−1
30	b	1.286394910684	3	1	1	1	1	0	0	0	0	0	−1	−1	−1	−1	−1	−1	−1
30		1.287259535069	5	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	0	−1	−1
30		1.288162507069	4	1	1	1	0	−1	−2	−2	−1	1	2	2	1	0	−1	−1	−1
30		1.288747816048	4	1	1	1	0	0	0	1	0	0	−1	0	−1	0	−1	0	−1
30	bq	1.289705971330	3	1	1	0	0	0	0	0	1	1	0	−1	−1	−1	−1	0	1
30		1.290232081368	2	1	0	0	1	0	0	1	0	1	0	1	1	0	1	1	0
30		1.290373429988	4	1	0	1	0	1	0	1	0	1	0	0	1	−1	1	−1	1
30		1.290434368539	4	1	0	1	1	1	1	1	1	1	0	1	0	0	0	−1	0

30		1.290932373762	4	1	1	1	0	0	0	1	1	1	0	−1	−2	−2	−1	0	1	
30	b	1.292018120924	2	1	1	0	0	0	1	1	0	0	0	1	0	−1	−1	0	1	
30		1.292576727533	3	1	0	1	0	1	0	0	0	0	1	0	1	0	1	0	1	
30	*	1.292745216074	4	1	0	1	0	1	1	0	2	0	2	0	1	1	0	2	−1	
30		1.293551871324	5	1	0	−1	0	0	0	0	0	0	1	0	0	0	−1	0	1	
30		1.293827358375	3	1	0	−1	0	0	1	0	−1	0	0	1	0	−1	0	0	1	
30		1.293958817838	3	1	0	−1	0	0	0	0	0	0	0	0	0	0	0	0	1	
30	b	1.294490857906	2	1	1	1	0	0	0	0	−1	0	0	0	−1	0	0	0	−1	
30		1.294677129704	5	1	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	1	
30		1.295059494142	3	1	0	−1	0	1	0	−1	0	0	0	0	1	0	−1	0	1	
30		1.295089002057	4	1	1	1	1	1	1	1	0	0	−1	−1	−1	−1	−1	−1	−1	
30		1.295358609548	5	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	
30	*	1.295830812559	3	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	3	
30		1.296409599888	4	1	0	1	1	0	1	0	0	0	0	0	0	1	0	0	1	
30	*	1.296432383243	6	1	0	1	1	1	2	1	3	2	3	3	3	4	3	4	3	
30		1.297069706103	6	1	1	0	0	0	−1	−1	0	0	0	1	1	0	0	0	−1	
30		1.297111614628	5	1	1	0	−1	−1	0	0	0	0	1	1	0	−1	−1	0	1	
30	b	1.297213674819	4	1	0	−1	1	1	0	0	0	1	0	0	0	0	1	0	−1	
30	b	1.297599482921	3	1	1	0	−1	−2	−2	−1	0	1	2	2	1	0	−1	−1	−1	
30	b	1.297671779398	2	1	0	0	0	0	0	0	0	0	1	−1	0	0	0	1	−1	
30	b	1.298033076609	3	1	1	0	0	0	0	1	1	0	−1	−1	−1	−1	0	0	−1	
30	b	1.298782465682	4	1	1	1	0	0	0	0	0	0	1	1	1	0	−1	−1	−1	
30		1.299365237935	4	1	0	1	0	1	0	1	1	1	1	0	1	0	1	0	1	
30	bq	1.299672830907	3	1	0	−1	1	0	−1	1	0	−2	1	1	−1	0	0	0	1	
30		1.299749608400	3	1	0	−1	1	0	−1	1	0	−1	1	0	−1	1	0	−1	1	
30		1.299764321006	2	1	1	0	0	0	0	0	1	1	0	0	0	0	0	1	1	
30		1.299933148743	4	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	1	
32	*	1.236083368052	4	1	1	1	1	0	−1	−1	−2	−1	0	0	1	1	0	0	0	−1
32		1.236198469859	3	1	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	1
32		1.236227922245	4	1	2	2	1	0	−1	−1	−1	−1	−1	−1	−1	−1	−1	0	2	3
32		1.242940115199	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	−1
32	*	1.249688298465	4	1	2	2	1	0	−1	−1	−1	0	1	2	2	2	1	0	−1	−1
32		1.253625444787	4	1	0	0	0	1	1	0	0	0	1	0	−1	−1	0	0	−1	−1
32		1.253720739388	4	1	0	0	1	0	0	0	0	0	0	0	0	0	−1	0	0	−1
32		1.257186114941	4	1	1	0	0	0	−1	−1	−1	−1	0	1	1	1	1	0	−1	−1
32		1.259073676582	2	1	0	0	1	1	0	0	1	0	0	0	0	−1	−1	0	−1	−1
32		1.262317250431	4	1	1	0	−1	0	0	0	−1	−1	−1	0	0	0	0	1	1	1
32		1.262957374128	5	1	1	0	0	1	1	0	0	0	0	0	0	−1	0	1	0	−1
32	b	1.264804665986	2	1	1	0	0	0	0	−1	−1	0	0	−1	0	1	0	−1	0	1
32		1.266283513991	4	1	1	1	1	1	1	1	1	0	−1	−1	−1	−1	−1	−1	−1	−1
32		1.268277679022	3	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	−1
32		1.268867282818	4	1	1	1	1	0	−1	−2	−2	−2	−2	−1	−1	0	1	2	3	3
32		1.270758381361	4	1	1	0	0	1	1	0	−1	−1	0	0	−1	−1	0	1	0	−1
32		1.270839337139	4	1	1	0	0	0	0	0	−1	−1	−1	0	1	0	0	0	0	1
32	*	1.270932746058	4	1	1	1	0	0	−1	−1	−1	0	1	2	2	2	1	0	−1	−1
32		1.272555064914	4	1	1	0	0	1	1	0	0	1	1	0	−1	−1	−1	−1	−1	−1
32		1.272841831223	4	1	1	1	1	0	−1	−1	−1	0	1	1	1	0	−1	−1	−1	−1
32		1.274587884424	4	1	0	0	0	0	0	0	1	0	−1	0	0	0	0	0	0	−1
32		1.275190870757	4	1	0	0	0	0	1	0	0	0	0	0	−1	0	0	0	0	−1
32		1.278087702910	4	1	1	0	−1	0	1	1	0	−1	0	1	1	0	−1	−1	0	1
32		1.278292317564	3	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
32	b	1.278912378418	2	1	1	0	−1	−1	−1	0	1	1	0	0	0	0	0	1	1	1
32		1.279025595513	2	1	1	1	0	0	0	0	0	1	1	1	0	0	0	1	1	1

32	b	1.281860716209	2	1	1	0	0	0	1	1	0	0	0	1	1	1	0	0	1	1	
32		1.283569816684	4	1	1	0	−1	−1	−1	0	1	1	0	−1	−1	0	1	1	0	−1	
32		1.284418973401	6	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	
32		1.285123329626	5	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	−1	
32		1.285602913826	4	1	0	0	0	0	1	−1	0	0	0	1	−1	0	0	0	0	−1	
32		1.286650909902	6	1	0	−1	1	0	−2	1	1	−2	1	2	−2	0	2	−2	−1	3	
32		1.286667320764	4	1	0	1	0	1	1	1	1	0	1	0	1	0	0	0	0	1	
32	b	1.287598964144	3	1	1	0	0	0	−1	−1	0	1	1	0	−1	−1	−1	−1	0	1	
32		1.287792177375	3	1	0	0	0	0	1	0	0	1	0	0	1	0	1	0	0	1	
32		1.288299915050	3	1	1	1	1	1	1	1	1	0	0	0	0	−1	−1	−1	−1	−1	
32		1.288313664818	4	1	0	1	1	0	1	0	0	1	0	1	1	0	1	0	0	1	
32		1.290107330245	5	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	−1	
32		1.290836487717	4	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	
32	b	1.290904110639	2	1	0	0	1	−1	0	0	0	0	0	0	0	−1	0	0	−1	1	
32	b	1.290968940881	4	1	0	−1	0	1	0	−1	0	0	0	1	1	−1	−1	1	0	−1	
32		1.291024122419	4	1	1	1	0	−1	−1	−1	−1	−1	0	1	2	1	0	−1	−1	−1	
32		1.291378694968	5	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	−1	
32		1.291927330235	4	1	1	1	0	−1	−1	−1	0	0	1	0	0	−1	−1	0	0	1	
32		1.292525363644	6	1	1	0	0	0	−1	−1	0	0	0	1	1	0	0	0	−1	−1	
32	b	1.292555422601	2	1	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	−1	
32		1.293282731109	4	1	2	2	1	0	−1	−1	0	1	1	0	−1	−1	−1	−1	−1	−1	
32		1.294117542560	4	1	1	1	1	0	0	0	−1	−1	−1	−1	0	0	0	1	1	1	
32	*	1.294553682172	4	1	1	0	−1	−1	−1	0	1	2	2	0	−2	−3	−2	0	2	3	
32		1.294555564144	4	1	1	1	1	0	−1	−1	−1	−1	0	0	0	0	0	0	0	1	
32	*	1.294774730521	4	1	1	−1	−2	0	2	1	−1	−1	0	0	0	1	1	0	−1	−1	
32		1.295126300814	6	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	
32		1.295252179924	5	1	1	0	−1	−1	0	1	0	−1	−1	0	1	1	−1	−1	0	1	
32		1.295905964885	6	1	1	0	−1	−1	0	1	1	0	−1	−1	−1	0	1	1	0	−1	
32	b	1.296527895517	2	1	0	0	1	0	0	0	0	0	0	0	−1	0	0	0	0	−1	
32		1.296644676541	3	1	0	1	1	0	1	0	1	0	1	1	1	1	1	1	0	1	
32		1.296819969436	5	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	−1	
32		1.297376616087	6	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	1	1	0	−1	
32		1.297510362433	4	1	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	−1	
32	*	1.298335890166	6	1	1	1	1	0	−1	−1	−1	−1	−1	−1	−1	−1	0	1	2	3	
32		1.298996124311	5	1	1	1	1	1	1	1	0	0	−1	−1	−1	−1	−1	−1	−1	−1	
32	*	1.299312144051	4	1	1	0	0	1	1	1	1	1	1	2	2	1	1	2	2	1	
32		1.299605093725	4	1	0	1	0	0	0	0	0	1	0	1	1	0	1	0	0	1	
32		1.299771300797	4	1	1	0	−1	−1	0	0	0	0	0	1	1	1	0	−1	−1	−1	
34		1.220287441693	3	1	0	1	0	0	1	−1	1	−1	0	0	−1	1	−1	0	0	−1	1
34		1.229999039700	3	1	0	0	0	0	0	−1	0	0	0	0	1	0	0	0	0	0	−1
34		1.236579223637	3	1	0	0	0	0	1	0	0	0	0	0	0	−1	0	0	0	0	−1
34		1.245831463605	3	1	0	0	0	−1	0	0	0	0	0	0	0	0	1	0	0	0	−1
34		1.249030943524	3	1	1	0	−1	−1	0	1	0	−1	0	1	0	−1	−1	0	1	0	−1
34		1.257706711683	3	1	0	0	1	0	0	0	0	0	0	0	0	0	0	−1	0	0	−1
34		1.257821010070	3	1	0	0	0	−1	0	−1	0	0	0	1	0	1	0	0	0	−1	1
34		1.258592254216	4	1	1	0	0	0	−1	0	1	0	0	1	0	−1	0	0	−1	0	1
34		1.263309248640	5	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	1	0	−1	−1	−1
34		1.265382309806	4	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	−1
34	*	1.265417146469	4	1	0	1	1	0	1	0	0	1	−1	1	0	0	1	0	0	1	−1
34		1.267551051017	4	1	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	1
34		1.267882006285	5	1	1	1	0	0	−1	−1	−1	−1	−1	−1	0	0	0	0	1	1	1
34		1.268159848408	5	1	1	0	0	0	−1	−1	0	0	0	1	0	−1	0	0	−1	0	1
34		1.269204757935	5	1	1	0	0	0	0	0	−1	−1	−1	−1	−1	−1	0	1	1	1	1
34		1.269559384492	5	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	0	−1	−1	0	1

34		1.270056645459	4	1	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	1
34		1.270192450594	3	1	0	−1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	−1
34		1.272005913843	4	1	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	1
34	*	1.273336860029	5	1	0	−1	0	0	0	0	0	0	0	0	1	0	0	0	−1	0	1
34		1.274116383833	4	1	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	1
34		1.274352628389	5	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	1
34		1.274606834747	3	1	0	0	1	0	0	1	0	−1	1	0	−1	0	0	−1	0	0	−1
34		1.275164361030	5	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1
34		1.275963554763	5	1	1	0	−1	−1	0	1	1	0	−1	−1	−1	0	1	1	0	−1	−1
34		1.276519776778	5	1	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	0	0	1
34		1.277067971252	3	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	−1
34		1.277998798066	3	1	0	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
34	**	1.278390605526	5	1	0	0	0	0	1	−1	0	0	0	0	0	0	0	0	0	0	−1
34		1.280861542152	5	1	0	0	0	0	0	0	0	0	0	0	0	−1	0	0	0	0	1
34	*	1.280971739447	6	1	0	0	0	0	1	0	0	0	0	0	−1	0	0	0	0	0	0
34		1.281011299022	3	1	0	−1	0	0	1	0	−1	0	0	1	0	−1	0	0	1	0	−1
34	*	1.281167623534	3	1	1	1	0	−1	−1	−1	0	0	0	−1	−1	−1	0	1	1	1	1
34	*	1.283030701841	4	1	0	0	1	0	0	0	0	0	0	0	−1	0	0	0	0	0	1
34	*	1.283121967286	3	1	0	0	0	−1	1	0	0	1	−1	0	0	−1	1	0	0	0	−1
34	*	1.283137116832	4	1	1	1	2	2	2	2	2	2	1	1	1	1	1	1	1	1	1
34		1.283188551242	5	1	0	0	0	−1	0	0	0	0	0	0	0	0	0	0	0	0	1
34		1.284434252867	4	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1
34	*	1.284997148629	5	1	0	−1	0	1	0	−1	0	1	0	−1	0	1	0	−1	0	0	1
34		1.285309794591	6	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	−1
34	b	1.285409064765	1	1	1	0	0	−1	−1	−1	0	1	1	1	0	−1	−1	−1	0	1	1
34	*	1.286672436337	4	1	0	0	1	0	0	0	0	0	−1	0	0	0	0	0	0	0	0
34	*	1.287107005970	5	1	0	0	0	−1	0	0	0	1	0	0	0	−1	1	0	0	0	−1
34	*	1.287249318440	6	1	0	1	1	0	1	0	1	0	1	1	0	2	0	1	1	0	1
34		1.287263174286	5	1	0	0	0	0	0	0	0	0	0	−1	0	0	0	0	0	0	1
34		1.287496388185	3	1	0	−1	1	0	−1	1	0	−1	1	0	−1	1	0	−1	1	0	−1
34		1.288962220151	5	1	1	0	0	0	−1	−1	−1	−1	0	1	1	1	1	0	−1	−1	−1
34		1.289442027376	5	1	1	1	0	0	0	0	0	−1	0	0	0	−1	−1	0	0	0	−1
34		1.291061253422	5	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1
34	*	1.291268281129	5	1	1	1	1	0	0	0	0	0	−1	−1	−1	−1	0	0	0	0	−1
34	*	1.291960508478	4	1	1	0	−1	−1	0	1	1	1	1	1	0	−1	−1	0	1	1	1
34		1.292129748486	5	1	0	1	0	0	1	−1	0	−1	−1	0	−1	0	0	0	1	0	1
34		1.292143406357	4	1	0	0	1	0	0	1	1	0	0	1	0	−1	1	1	−1	0	1
34	*	1.292289993727	5	1	1	1	0	−1	−2	−2	−1	0	1	1	1	0	−1	−1	0	1	1
34		1.292525888839	7	1	1	0	0	0	−1	−1	0	0	0	1	1	0	0	0	−1	−1	−1
34		1.292877673519	6	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1
34		1.292915833166	6	1	1	0	0	0	−1	−1	0	0	0	1	1	1	1	0	−1	−1	−1
34	*	1.293085206229	4	1	1	1	0	0	0	0	0	0	0	0	0	0	0	−1	−1	0	0
34		1.293271981798	4	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1
34		1.293515495631	5	1	1	1	1	1	0	0	0	0	0	0	0	−1	−1	−1	−1	−1	−1
34		1.294204500560	5	1	1	0	−1	−1	0	1	0	−1	−1	0	1	1	0	−1	−1	0	1
34	*	1.294659324357	4	1	1	0	0	1	0	0	1	1	0	1	1	0	0	1	0	0	0
34		1.294753219289	7	1	1	1	1	0	−1	−2	−2	−2	−1	1	2	3	2	1	−1	−3	−3
34		1.294976012991	5	1	0	0	0	0	0	0	0	−1	0	0	0	0	0	0	0	0	1
34	*	1.296322615212	3	1	0	−1	0	1	0	0	0	0	0	0	1	0	0	0	0	0	1
34		1.296973405444	5	1	1	1	1	1	1	1	1	0	−1	−1	−1	−1	−1	−1	−1	−1	−1
34		1.297088692167	6	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1
34		1.297213664669	6	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	−1
34		1.297419746944	6	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	−1
34	*	1.298107355647	4	1	1	1	0	0	−1	−1	−1	0	0	1	1	1	0	0	−1	−1	−1

34		1.298306536961	5	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	
34		1.298490064152	5	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	
34		1.298898433769	4	1	1	1	1	0	−1	−1	−1	−1	0	1	1	1	1	0	−1	−1	−1	
34		1.299134951609	3	1	0	0	1	0	0	0	−1	0	0	−1	0	0	0	0	0	0	1	
34	*	1.299714243227	5	1	1	1	0	0	0	1	0	0	−1	0	−1	0	−1	0	−1	0	−1	
34		1.299969894819	5	1	1	0	−1	−1	−1	0	1	1	0	−1	−1	0	1	1	0	−1	−1	
36		1.226493301473	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	−1
36		1.229482810200	4	1	1	1	0	0	−1	−1	−1	0	1	1	1	0	−1	−2	−1	0	1	1
36		1.247049516003	5	1	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	1
36	*	1.252046815621	4	1	0	0	0	0	1	−1	0	0	0	0	−1	1	−1	0	0	0	1	−1
36	*	1.258484141432	4	1	1	0	−1	−1	−1	0	1	1	1	0	−1	−1	0	1	1	0	−1	−1
36		1.263910438145	4	1	0	0	0	0	0	0	1	0	0	0	−1	0	0	0	0	0	0	−1
36		1.265527334805	3	1	1	1	1	0	0	−1	−1	−1	−1	0	0	1	1	1	1	0	0	−1
36		1.266206109348	4	1	1	1	0	0	−1	−1	−1	−1	−1	−1	−1	−1	0	0	1	1	2	1
36	**	1.266948881320	2	1	1	1	1	0	−1	−1	−1	−1	−1	0	1	1	1	1	0	−1	−1	−1
36		1.267356853096	4	1	0	−1	1	1	−2	0	2	−2	−1	3	−1	−2	3	0	−3	2	1	−3
36		1.268398494855	6	1	1	1	0	−1	−1	−1	0	0	0	0	0	1	1	1	0	−1	−1	−1
36		1.268720479717	5	1	0	1	0	0	0	−1	1	−1	1	0	0	1	0	1	0	0	0	−1
36		1.270620037890	4	1	0	0	0	0	1	0	0	0	0	0	0	0	−1	0	0	0	0	−1
36	*	1.272095458988	6	1	1	1	0	−1	−1	−1	0	0	1	0	0	−1	−1	0	0	1	0	1
36		1.274102213282	3	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
36		1.276242776442	4	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1
36		1.276506747067	4	1	1	1	1	1	1	1	1	1	0	0	0	−1	−1	−1	−1	−1	−1	−1
36		1.277128539936	4	1	1	1	1	1	1	0	−1	−1	−1	−1	−1	−1	0	0	0	0	0	1
36		1.278255875713	6	1	0	1	0	0	0	−1	0	−1	0	0	0	1	0	1	0	0	1	−1
36	*	1.278286859233	6	1	1	0	−1	−1	−1	−1	0	1	1	1	1	0	−1	0	0	−1	0	1
36	*	1.279501364240	2	1	1	1	0	0	1	1	1	0	0	0	0	0	−1	−1	−1	−1	−1	−1
36		1.280364121994	4	1	1	0	−1	0	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	1
36	*	1.281223267290	4	1	1	0	0	1	1	0	−1	−1	−1	−1	−1	−1	−1	0	1	1	1	1
36	**	1.281226915272	3	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	−1	0	−1
36	*	1.281650403601	4	1	0	1	1	0	1	0	0	0	−1	0	−1	−1	0	−1	0	0	0	1
36	*	1.281813401501	6	1	1	2	1	1	0	0	−1	−1	−2	−1	−2	0	−1	1	0	1	0	1
36		1.282367506749	4	1	1	0	−1	−1	−1	0	1	1	1	1	0	−1	−1	−1	−1	0	1	1
36	*	1.282948673059	4	1	1	0	−1	0	0	0	0	1	1	1	0	−1	−1	0	0	0	1	2
36	*	1.283954126983	4	1	1	1	1	1	0	0	0	0	−1	−1	−1	−1	−1	0	0	0	0	1
36	*	1.284355377035	4	1	1	1	1	1	0	0	0	0	0	1	2	2	2	2	2	1	1	1
36	*	1.284467885334	4	1	1	0	0	0	−1	0	0	−1	0	1	0	0	1	0	−1	0	0	−1
36	*	1.284730613601	3	1	0	0	0	0	0	0	0	0	0	−1	0	0	0	0	0	0	1	−1
36	*	1.285151994477	4	1	0	1	1	1	2	1	2	1	1	1	0	0	−1	−1	−2	−2	−2	−3
36	*	1.285265883003	4	1	1	1	0	−1	−1	0	1	1	1	0	0	0	0	0	0	0	0	1
36		1.285614961565	6	1	1	1	0	−1	−1	−1	0	0	0	0	−1	0	0	1	1	0	0	−1
36	*	1.285777884660	6	1	0	0	1	0	1	0	0	1	0	0	0	0	0	0	−1	0	0	−1
36	*	1.286781207957	4	1	1	1	1	1	1	1	0	−1	−1	−1	−1	−1	−1	0	1	1	1	1
36	*	1.286789765625	4	1	0	0	0	0	0	−1	1	0	0	0	−1	1	−1	0	0	0	1	−1
36		1.289114294025	6	1	1	1	1	1	1	1	1	1	0	−1	−1	−1	−1	−1	−1	−1	−1	−1
36		1.289987020338	6	1	1	0	−1	−1	0	1	0	−1	−1	0	1	1	0	−1	−1	0	1	1
36		1.290636353835	6	1	0	1	0	0	0	−1	0	−1	0	0	1	1	1	1	0	0	−1	−1
36	**	1.290932031408	3	1	1	0	0	0	0	0	0	0	0	−1	0	0	0	0	0	0	0	−1
36		1.291042045419	6	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	0	−1	−1	0	1	1
36	*	1.291586819889	4	1	1	0	−1	−1	0	0	−1	−1	0	1	1	0	−1	0	1	1	0	−1
36		1.291782143712	4	1	0	0	0	0	0	0	1	0	−1	0	0	0	0	0	0	−1	0	1
36		1.291786719277	6	1	1	1	0	0	0	0	0	0	1	0	0	−1	−1	−1	−1	0	0	1
36	*	1.292310164279	4	1	0	1	0	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1
36	*	1.292859183747	8	1	1	0	−1	−1	−1	0	1	2	1	0	−1	−1	−1	0	0	0	0	1

36	*	1.293466190304	4	1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 −1
36	*	1.293705260637	6	1 0 1 1 0 2 0 1 1 0 1 0 1 0 1 0 0 1 −1
36	*	1.294046792658	3	1 1 1 0 0 0 0 0 0 0 −1 −1 −1 0 0 1 0 0 −1
36		1.295099319407	4	1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
36		1.295203229454	6	1 0 −1 1 0 −2 1 1 −1 1 1 −1 0 0 −1 0 0 0 1
36	*	1.295268095332	4	1 1 0 0 0 −1 0 0 −1 0 0 −1 0 0 0 1 0 0 1
36	*	1.295281245283	4	1 0 0 1 1 0 1 1 1 0 1 1 0 0 1 0 0 0 1
36	*	1.295443530108	6	1 1 1 0 −1 −1 −1 −1 −1 −1 0 0 0 0 0 1 1 1 1
36		1.295512283270	6	1 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1
36	*	1.295811497729	3	1 0 0 0 0 0 0 0 −1 0 0 0 0 0 0 0 0 1 −1
36	*	1.296134198791	6	1 1 0 0 1 0 0 0 0 0 1 0 0 0 1 0 0 0 1
36		1.296357985088	6	1 1 0 −1 −1 0 1 1 0 −1 0 1 1 0 −1 −1 0 1 1
36	**	1.296634017049	2	1 1 0 1 1 0 0 0 0 −1 0 0 −1 0 0 0 0 0 1
36		1.297184459189	6	1 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 1
36	*	1.297650990308	4	1 0 0 0 0 1 0 0 0 0 0 0 0 0 1 0 0 0 −1
36		1.297997370758	4	1 0 0 0 0 1 0 0 −1 0 1 0 0 −1 0 0 0 0 −1
36	**	1.298710408743	4	1 1 1 0 0 −1 −1 −1 0 0 0 0 0 −1 −1 0 1 1 1
36		1.298713083842	4	1 0 −1 1 0 −1 1 0 −1 1 0 −1 1 0 −1 1 0 −1 1
36	*	1.298727031611	5	1 1 1 0 0 −1 −1 −1 0 0 1 0 0 −1 0 −1 0 0 1
36	*	1.299141530165	6	1 0 0 0 0 1 0 0 1 0 0 0 0 0 0 0 0 0 −1
36	*	1.299801978491	4	1 0 0 0 0 1 1 1 0 0 1 1 1 1 1 1 1 1 1
38		1.223447381400	3	1 1 0 −1 −1 0 0 −1 −1 0 1 1 0 0 1 1 0 −1 −1 −1
38		1.230263271363	3	1 0 0 0 −1 0 0 0 0 0 0 0 0 0 0 1 0 0 0 −1
38		1.233672001767	3	1 1 0 −1 −1 −1 −1 0 1 1 0 0 0 0 0 1 1 0 −1 −1
38		1.253114419836	4	1 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1
38		1.257125950692	4	1 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 0 0 1
38		1.257292512932	5	1 0 0 0 0 0 −1 0 0 0 0 0 0 1 0 0 0 0 0 −1
38		1.257799926024	4	1 0 1 0 0 0 0 1 1 1 0 0 −1 0 0 1 1 0 0 −1
38		1.257817801855	4	1 0 0 1 0 1 0 0 1 −1 1 0 0 1 −1 1 0 0 1 −1
38		1.261034951978	4	1 0 0 0 0 0 1 0 0 0 0 0 0 1 0 0 0 0 0 1
38		1.261911160974	3	1 0 0 1 −1 0 0 −1 0 0 0 0 1 0 0 1 −1 0 0 −1
38		1.262714497932	3	1 0 −1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
38	*	1.264139103163	5	1 1 1 1 0 −1 −1 −2 −2 −1 −1 −1 0 0 0 1 1 1 1 1
38		1.264247719549	5	1 0 0 0 0 0 0 1 0 0 0 0 −1 0 0 0 0 0 0 −1
38		1.264283879463	5	1 0 0 0 0 0 0 0 0 0 0 0 −1 0 0 0 0 0 0 1
38		1.265850257613	4	1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1
38		1.265884892635	4	1 0 0 0 0 1 0 0 0 0 0 0 0 0 1 0 0 0 0 1
38		1.266833775946	5	1 0 0 0 0 0 0 0 −1 0 0 1 0 0 0 0 0 0 0 −1
38		1.267722943437	5	1 0 0 0 0 0 0 0 0 1 −1 0 0 0 0 0 0 0 0 −1
38		1.268141866493	3	1 1 1 1 0 0 −1 −1 −1 −1 0 0 1 1 1 1 0 0 −1 −1
38		1.268308536615	5	1 1 0 0 0 −1 −1 −1 −1 0 0 0 1 1 1 1 0 −1 −1 −1
38		1.269031561548	4	1 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 −1
38		1.269123041240	3	1 1 1 1 1 1 1 1 0 0 0 0 −1 −1 −1 −1 −1 −1 −1 −1
38		1.270112038412	3	1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 −1 −1
38		1.271425839753	5	1 1 1 0 −1 −1 −1 −1 −1 −1 0 1 2 2 1 0 −1 −1 −1 −1
38		1.272625119342	5	1 0 0 0 0 0 0 0 0 0 −1 0 0 0 0 0 0 0 0 1
38		1.273069252937	4	1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 1
38	**	1.273972436701	3	1 1 1 0 −1 −2 −2 −1 0 1 1 0 −1 −1 −1 0 1 1 1 1
38		1.277285119570	4	1 1 0 −1 0 0 0 0 1 0 −1 −1 0 0 0 1 1 0 −1 −1
38		1.277660275574	5	1 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1
38		1.280587394047	7	1 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 1
38	*	1.280627569854	6	1 2 2 1 0 −1 −1 0 1 1 1 0 −1 −2 −2 −2 −1 0 1 1
38	*	1.282412632108	4	1 1 1 0 0 −1 0 0 1 1 1 0 0 0 0 1 1 1 0 0
38		1.282424559611	7	1 1 0 −1 −1 0 1 1 0 −1 −1 −1 0 1 1 0 −1 −1 0 1

38		1.283180815626	5	1 0 0 0 0 0 0 0 -1 0 0 0 0 0 0 0 0 0 0 1
38	*	1.283664773348	5	1 1 1 0 -1 -2 -2 -2 0 1 2 2 1 0 -1 -1 -1 0 0 1
38		1.285263104550	6	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1
38		1.285786959951	6	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 -1
38		1.286318444041	7	1 1 1 0 -1 -1 -1 0 0 0 0 0 1 1 1 0 -1 -1 -1 -1
38		1.286943773303	7	1 0 0 0 0 0 0 0 0 0 0 0 0 0 -1 0 0 0 0 1
38		1.287475467945	5	1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
38		1.287838353412	6	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 -1
38		1.289319427711	6	1 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1
38	**	1.289691998306	3	1 0 0 1 0 0 0 0 -1 0 0 0 0 0 0 0 0 0 0 -1
38		1.289986074876	7	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 1
38	*	1.290480446964	5	1 0 1 0 0 0 -1 0 -1 0 -1 1 -1 1 0 0 1 -1 1 -1
38		1.291262164722	4	1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -1
38		1.291401153147	7	1 1 0 0 0 -1 -1 0 0 0 1 1 0 0 0 -1 -1 0 0 -1
38	*	1.291455925499	5	1 0 1 0 1 0 1 0 1 0 1 0 0 1 -1 1 -1 1 -1 1
38		1.291516284726	5	1 0 1 0 0 1 -1 1 -1 0 0 -1 1 -1 1 0 0 1 -1 1
38		1.291629010984	7	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -1 0 0 1
38	*	1.292011339807	7	1 1 0 0 0 -1 -1 0 0 0 0 1 1 0 0 1 0 -1 -1 -1
38		1.292455106353	7	1 0 1 0 0 0 -1 0 -1 0 0 0 1 0 1 0 0 1 -1 1
38		1.292973461685	7	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -1 1
38	*	1.293156089367	4	1 1 1 0 -1 -1 0 0 1 0 0 0 0 0 0 -1 0 0 1 1
38		1.293346162002	7	1 1 1 0 -1 -1 -1 0 0 0 0 0 1 0 0 -1 -1 0 0 1
38		1.293832501813	4	1 0 0 0 0 1 0 0 0 1 1 0 0 0 1 0 0 0 0 1
38		1.294481726642	5	1 0 0 0 0 0 -1 0 0 0 0 0 0 0 0 0 0 0 0 1
38	*	1.294647631718	5	1 0 1 1 1 1 1 2 1 1 2 1 1 2 1 2 1 2 2 1
38		1.294782502187	6	1 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 -1
38		1.294791033007	7	1 1 0 -1 -1 0 1 1 0 -1 -1 0 1 1 0 -1 -1 -1 0 1
38		1.295336758537	5	1 1 0 0 0 -1 -1 0 0 0 0 -1 0 1 0 0 1 0 -1 -1
38		1.295861582470	5	1 0 0 1 0 0 0 -1 0 -1 -1 1 -1 0 1 -1 1 0 -1 1
38	*	1.295866963944	5	1 0 -1 1 0 -1 1 0 -1 1 0 -1 1 0 -1 1 0 -1 0 1
38		1.295930973312	6	1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 1
38		1.297108084756	6	1 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 -1
38	*	1.297643033972	4	1 0 -1 0 1 0 -1 0 1 0 0 0 0 0 0 0 0 0 0 1
38	*	1.298161777342	5	1 0 0 1 0 0 0 0 -1 -1 0 -1 -1 0 0 0 0 1 1 1
38	*	1.298173024273	4	1 0 0 1 0 0 0 0 0 -1 0 -1 0 0 -1 1 0 0 1 0
38		1.298577749726	5	1 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 1
38		1.298658936938	5	1 0 0 0 -1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
38	*	1.299167911184	5	1 1 1 0 -1 -1 -1 0 1 1 1 0 -1 -1 -1 0 0 0 0 -1
40		1.236249557300	3	1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 1
40		1.244414501983	4	1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -1 -1
40		1.244914151807	4	1 0 0 0 0 0 0 0 0 1 0 -1 0 0 0 0 0 0 0 0 -1
40		1.249209965697	4	1 0 1 0 0 1 -1 1 -1 0 0 -1 1 -1 1 0 0 0 -1 0 -1
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40		1.255554899986	4	1 0 0 1 -1 0 0 -1 1 -1 0 1 -1 1 0 -1 1 -1 0 1 -1
40		1.259217258460	5	1 1 1 0 0 -1 -1 -2 -1 -1 0 0 1 1 1 1 1 0 -1 -1 -1
40		1.261000567100	4	1 1 0 -1 -1 0 0 -1 -1 0 1 1 0 0 1 1 0 -1 -1 0 1
40		1.261837651283	6	1 1 0 0 0 0 0 -1 -1 0 1 1 0 -1 0 1 0 -1 -1 0 1
40	*	1.265651031527	6	1 0 1 1 0 1 0 0 0 -1 0 -1 -1 0 -1 0 0 0 1 1 1
40		1.267859394095	6	1 1 0 -1 -1 0 1 1 0 -1 -1 0 1 1 0 -1 -1 0 1 1 1
40	*	1.269725034034	8	1 0 0 0 -1 1 0 0 0 -1 0 0 0 0 0 0 0 0 0 0 1
40		1.270564369213	6	1 1 0 -1 -1 0 1 1 0 -1 -1 0 1 1 0 -1 -1 -1 0 1 1
40		1.270799676624	5	1 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 -1
40		1.271543601093	4	1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 -1 0 0 0 -1
40		1.272196640216	6	1 1 0 -1 -1 0 1 1 0 -1 -1 0 1 1 0 -1 0 1 1 0 -1

40	**	1.272780237961	2	1 1 1 1 0 −1 −1 −1 −1 0 1 1 1 0 −1 −1 −1 −1 0 1 1
40		1.272988605889	6	1 1 0 0 0 −1 −1 0 0 0 1 1 0 −1 −1 −1 −1 0 1 1 1
40		1.274080098621	4	1 1 0 −1 −1 −1 −1 0 1 1 1 1 0 −1 −1 −1 −1 0 1 1 1
40		1.274527905997	3	1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 −1
40	*	1.276128762738	6	1 1 1 1 1 0 0 0 −1 −1 −1 −1 −1 0 0 0 0 0 0 0 1
40		1.276792803139	6	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1
40	*	1.276955576128	2	1 0 1 0 1 1 1 0 1 0 1 0 0 −1 0 −1 0 −2 −1 −1 −1
40	*	1.277333466563	4	1 1 1 1 0 −1 −1 −1 −1 0 0 −1 −1 −1 −1 0 1 1 1 1 1
40		1.279278846309	6	1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 0 −1 −1 0 1 1 0 −1
40		1.281194037226	4	1 0 0 1 −1 −1 1 −1 −1 2 0 −1 2 0 −2 1 0 −2 1 1 −1
40		1.282525594863	6	1 1 0 −1 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1
40		1.283251551377	5	1 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 −1
40	**	1.285066056972	2	1 1 0 1 1 0 0 0 0 −1 −1 −1 −1 −1 −1 0 0 0 1 1 1
40		1.285411068709	6	1 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 1
40		1.285662153518	4	1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
40	**	1.286066935872	3	1 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 −1
40		1.287184439618	4	1 1 2 2 2 2 1 1 0 0 −1 −1 −1 −2 −1 −2 −1 −1 −1 0 −1
40		1.287240651486	4	1 0 1 1 0 1 0 0 0 0 −1 0 0 −1 1 −1 0 0 −1 0 −1
40		1.287361243737	8	1 0 −1 1 0 −2 1 1 −2 1 2 −2 0 2 −1 −1 1 0 −1 0 1
40	*	1.288244616730	5	1 1 1 0 −1 −2 −2 −1 0 1 1 1 1 0 0 −1 −1 −1 0 1 1
40	*	1.288352444032	6	1 0 1 1 1 1 1 2 1 1 2 1 1 2 1 2 1 2 2 1 3
40	*	1.288918014880	4	1 1 1 0 0 −1 −1 −1 0 0 1 1 1 0 0 −1 −1 −1 0 0 1
40	*	1.288978745843	3	1 0 0 1 0 1 0 0 1 0 1 0 0 1 0 1 0 1 1 0 1
40		1.289050603803	6	1 1 0 −1 −1 0 1 1 0 −1 0 1 1 0 −1 −1 0 1 1 0 −1
40	*	1.289445397428	4	1 0 1 0 0 0 0 0 1 0 0 0 −1 0 0 1 1 1 0 0 −1
40		1.290905920345	4	1 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1
40		1.291534271918	8	1 1 0 0 0 −1 −1 0 0 0 1 1 0 0 0 −1 −1 0 0 0 1
40	*	1.291702653642	6	1 0 0 0 −1 1 0 0 1 −1 0 0 −1 1 0 −1 1 −1 0 1 −1
40		1.292664281558	4	1 1 1 1 1 1 1 1 0 0 0 0 −1 −1 −1 −1 −1 −1 −1 −1 −1
40	*	1.292765482065	6	1 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 −1 0
40		1.292789783629	7	1 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 −1
40		1.292826580456	4	1 0 0 1 0 0 1 0 0 0 0 0 0 0 −1 0 0 −1 0 0 −1
40	*	1.293503916102	5	1 0 0 1 0 0 0 −1 0 −1 −1 0 −1 0 0 −1 1 0 0 1 0
40	*	1.294242444420	4	1 1 1 0 0 0 1 1 1 0 −1 −1 0 1 1 1 0 0 0 1 1
40	*	1.294503800033	4	1 0 0 0 −1 0 0 0 0 1 0 0 0 −1 0 0 0 0 0 0 1
40		1.294594236601	7	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 −1
40		1.294754520814	4	1 0 −1 0 0 1 0 −1 0 0 1 0 −1 0 0 1 0 −1 0 0 1
40		1.294981483889	4	1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 −1 −1 −1
40		1.295175291940	5	1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 −1
40		1.295239073785	6	1 1 0 0 0 0 1 1 0 0 0 −1 0 0 −1 0 0 −1 0 0 −1
40		1.295589151844	6	1 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 1
40		1.295674216376	6	1 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 1
40		1.296174324257	5	1 0 0 0 0 0 0 1 0 0 1 0 0 1 0 0 0 1 0 0 1
40		1.296343642174	5	1 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 1 1
40	*	1.296645461842	4	1 1 1 1 0 −1 −1 −1 0 1 1 1 0 −1 −1 −1 −1 0 0 0 1
40	*	1.297027438813	6	1 0 1 1 0 1 0 0 1 −1 0 1 −2 1 0 −1 2 −1 1 2 −1
40		1.297327948474	6	1 1 1 1 1 1 1 1 1 0 0 0 0 0 −1 −1 −1 −1 −1 −1 −1
40		1.297458779984	4	1 0 −1 1 0 −1 1 0 −1 1 0 −1 1 0 −1 1 0 −1 1 0 −1
40	**	1.297740386711	4	1 1 1 1 1 0 0 0 −1 −1 −1 −2 −2 −1 −1 0 1 1 1 1 1
40	*	1.298591244814	5	1 1 1 0 0 −1 −1 −1 0 0 0 0 0 0 −1 0 0 1 0 1 0
40		1.298901196503	6	1 1 1 1 1 1 0 −1 −1 −1 −1 −1 −1 0 0 0 0 0 0 0 −1
40	*	1.298974498763	5	1 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 1 −1
40		1.299021805152	4	1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
40		1.299555555459	6	1 1 1 1 1 1 1 1 1 1 0 −1 −1 −1 −1 −1 −1 −1 −1 −1 −1

40	*	1.299597928983	4	1 0 1 0 1 0 1 0 1 0 0 0 0 0 0 1 0 1 0 1 0
40		1.299950661447	6	1 0 −1 0 0 0 0 0 0 1 0 −1 0 0 0 0 0 0 1 0 −1
42		1.230295468643	4	1 0 1 0 0 0 −1 1 −1 1 0 0 1 −1 1 −1 1 0 0 1 −1 1
42		1.235761099712	4	1 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 1
42		1.239505770490	4	1 0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 0 0 0 1
42		1.245372900786	3	1 0 −1 0 0 0 0 1 0 −1 0 0 0 0 1 0 −1 0 0 0 0 1
42		1.245423918110	3	1 0 −1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 −1
42		1.250446676244	3	1 1 1 1 0 0 −1 −1 −1 −1 0 0 1 1 1 1 0 0 −1 −1 −1 −1
42		1.252119156007	4	1 0 1 0 1 0 0 0 −1 0 −1 0 0 0 1 0 1 0 0 1 −1 1
42		1.253409159129	5	1 0 0 0 −1 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 −1
42		1.254905118937	4	1 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1
42		1.256720845374	5	1 0 1 0 0 0 −1 0 −1 0 −1 0 0 1 1 1 1 0 0 −1 −1 −1
42		1.259725739226	4	1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1
42		1.260616712978	3	1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 −1 −1
42		1.262137128019	4	1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 1
42		1.265574117468	5	1 1 0 0 0 −1 −1 −1 −1 0 1 1 1 1 0 −1 −1 −1 −1 0 1 1
42		1.265957174444	5	1 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 −1 0 0 0 0 −1
42	*q	1.266378567293	5	1 1 1 1 0 0 −1 −1 −1 −1 −1 −1 −1 −1 0 0 1 1 1 1 0 1
42		1.266505761622	5	1 0 0 0 0 0 0 0 0 0 −1 1 0 0 0 0 0 0 0 0 0 −1
42		1.270422473895	5	1 0 0 0 0 0 0 0 −1 0 0 0 0 0 0 0 0 0 0 0 0 1
42		1.270455996464	6	1 1 0 0 0 −1 −1 0 0 0 1 1 1 1 0 −1 −1 −1 −1 0 1 1
42		1.270978180827	4	1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1
42		1.274542386386	5	1 1 1 0 0 0 0 0 0 1 0 0 −1 0 0 0 −1 −1 0 0 0 −1
42	**	1.279404456563	6	1 1 0 0 0 −1 −1 −1 0 1 1 1 1 0 −1 −1 −1 −1 0 1 1 1
42		1.280419176662	7	1 1 0 0 0 −1 −1 0 0 0 1 1 0 0 0 −1 −1 0 0 −1 0 1
42		1.282967134586	7	1 1 0 0 0 −1 −1 0 0 0 1 1 0 0 0 −1 −1 −1 −1 0 1 1
42	*	1.283275505290	6	1 1 0 −1 −1 −1 0 1 1 0 0 0 −1 −1 1 1 0 0 0 −1 0 1
42	*	1.284595194965	5	1 1 0 −1 −1 −1 0 1 1 0 −1 −1 −1 0 1 1 0 −1 −1 0 1 1
42	**	1.284878221572	6	1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 −1
42	*	1.285103180421	4	1 1 1 1 1 0 0 −1 −1 −1 0 0 1 1 1 0 0 −1 −1 −1 0 0
42		1.285454619530	4	1 0 0 0 0 1 0 0 −1 0 0 0 0 −1 0 0 1 0 0 0 0 1
42	*	1.286019694797	5	1 0 0 1 0 0 0 0 0 0 1 0 0 1 −1 −1 0 −1 −1 0 0 −1
42		1.286616944129	5	1 0 −1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
42		1.286725738688	6	1 1 0 0 1 0 −1 0 0 −1 0 1 0 0 1 0 −1 0 0 −1 0 1
42		1.287438291579	7	1 0 0 0 0 0 0 0 0 0 −1 0 0 0 0 0 0 0 0 0 0 1
42	*	1.288308657648	7	1 0 0 0 −1 0 0 0 1 1 0 0 −1 −1 0 0 0 1 0 0 0 −1
42	**	1.288324508616	3	1 0 0 1 −1 0 0 −1 0 −1 −1 0 −1 −1 1 −1 0 1 0 1 1 1
42	*	1.288429733375	4	1 1 1 0 0 0 0 −1 −1 −1 0 0 0 0 1 1 1 0 0 −1 −1 −1
42	**	1.290357388877	3	1 0 −1 0 1 1 −1 −1 0 1 1 −1 −2 0 2 1 −2 −1 1 1 0 −1
42		1.291117710385	7	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 −1 1
42	**	1.292230412813	6	1 0 1 0 0 0 0 1 0 0 −1 −1 −1 0 0 0 0 −1 0 0 1 1
42		1.292534389006	7	1 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1
42		1.292905763880	8	1 1 0 0 0 −1 −1 0 0 0 1 1 0 0 0 −1 −1 0 0 0 1 1
42		1.293148891533	7	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 −1 0 0 0 0 1
42		1.293598887911	7	1 1 0 0 0 −1 −1 0 0 −1 0 1 0 0 1 0 −1 0 0 −1 0 1
42		1.296741473115	5	1 0 0 0 −1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
42		1.296847253421	4	1 1 1 1 1 1 1 1 1 1 0 0 0 0 0 −1 −1 −1 −1 −1 −1 −1
42	*	1.297012934408	5	1 0 −1 0 1 0 −1 0 0 0 0 0 0 0 0 1 0 −1 0 1 0 −1
42	*	1.297165927757	3	1 1 0 0 0 0 0 0 0 1 1 0 0 0 0 0 −1 −1 0 0 −1 −1
42	*	1.297314338274	8	1 1 1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1
42	*	1.297777212109	5	1 0 1 0 1 0 0 0 0 0 0 0 0 0 0 1 0 1 0 1 0 1
42		1.299220086382	5	1 1 2 2 2 2 1 1 0 0 0 0 1 1 1 1 0 0 −1 −1 −1 −1
44		1.236674812187	4	1 0 0 0 0 0 0 1 0 0 0 0 0 0 0 −1 0 0 0 0 0 0 −1

44		1.255552437923	3	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
44		1.257123779345	6	1	1	0	0	0	0	1	0	-1	0	0	0	0	-1	0	1	0	0	0	-1	0	0	-1
44		1.260646516263	4	1	1	1	1	1	1	0	0	0	0	0	-1	-1	-1	-1	-1	-1	0	0	0	0	0	1
44		1.262571853868	4	1	1	1	0	-1	-2	-2	-1	0	1	1	1	0	-1	-1	0	1	2	2	1	-1	-2	-3
44		1.263071975206	5	1	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	1
44		1.263095875491	4	1	0	-1	0	1	0	-1	0	0	0	0	1	-1	-1	1	2	-1	-2	1	1	0	-1	1
44		1.264648132493	6	1	1	0	0	0	0	0	-1	-1	0	0	0	1	0	0	1	0	0	0	-1	0	0	-1
44		1.265086776269	4	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	-1
44		1.265808404363	4	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1
44		1.266592565791	5	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1
44	*	1.268281868499	6	1	1	0	-1	0	0	-1	-1	0	1	1	1	0	0	0	0	-1	-1	0	0	0	0	1
44		1.268305756791	4	1	0	0	0	0	0	-1	0	0	1	0	-1	0	0	0	-1	0	1	1	0	-1	0	1
44		1.268940307634	6	1	1	0	0	0	-1	-1	0	0	0	1	0	-1	0	0	-1	0	1	0	0	1	0	-1
44		1.274564102329	6	1	1	0	-1	-1	0	1	1	0	-1	-1	-1	0	1	1	0	-1	-1	0	1	1	0	-1
44		1.275631248724	4	1	0	0	0	0	0	-1	0	0	0	0	1	0	0	0	0	-1	-1	0	0	0	0	1
44		1.275926381318	8	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1
44		1.277182381092	4	1	0	-1	1	1	-1	0	1	0	-1	0	1	-1	-1	1	0	-1	0	1	0	-1	0	1
44	*	1.277465838963	6	1	0	-1	1	0	-1	1	0	-1	1	0	-1	1	0	-1	1	0	-1	1	0	-1	0	1
44	**	1.282910509372	4	1	1	1	0	0	-1	-1	-1	0	0	1	1	1	0	0	0	0	0	0	0	-1	-1	-1
44		1.283915814250	6	1	1	1	0	0	0	0	0	-1	0	0	0	-1	-1	0	0	0	-1	0	1	1	0	-1
44		1.284355030821	4	1	0	-1	0	0	0	0	0	0	0	0	1	0	0	0	-1	0	0	0	0	0	0	1
44	b	1.285199179206	1	1	1	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	-1	0	0	1
44		1.285356658303	7	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1
44		1.285779586136	4	1	0	0	1	0	0	0	0	0	0	0	-1	0	0	-1	0	0	0	0	0	0	0	1
44	*	1.286829673139	4	1	1	0	0	0	0	1	1	1	1	0	-1	-1	0	1	1	0	-1	-2	-2	-1	0	1
44		1.286964650897	5	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1
44	*	1.287051201923	4	1	1	1	0	-1	-1	-1	-1	-1	0	1	2	2	1	0	-1	-1	-1	-1	-1	0	1	2
44		1.287462932793	4	1	0	-1	0	0	1	0	-1	0	0	1	0	-1	0	0	1	0	-1	0	0	1	0	-1
44		1.287469219772	6	1	0	0	0	0	0	0	0	0	0	-1	1	-1	0	0	0	0	0	0	0	0	-1	1
44		1.287530378379	6	1	1	1	1	1	1	1	1	1	1	1	0	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
44		1.287728384082	8	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1
44		1.288167876582	7	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	-1
44		1.288281222483	6	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1
44		1.288328604719	6	1	1	0	-1	-1	0	1	1	1	0	-1	-1	0	1	1	0	-1	-1	0	1	1	0	-1
44		1.288956473681	7	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	-1
44		1.289105367212	6	1	1	0	0	0	-1	-1	-1	-1	0	1	1	1	1	0	-1	-1	-1	-1	0	1	1	1
44		1.290624832983	8	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1
44		1.290692872764	7	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	-1
44		1.290712226726	6	1	1	1	0	-1	-1	-1	-1	-1	0	1	2	2	1	0	-1	-2	-2	-1	0	1	1	1
44		1.290944502554	4	1	0	0	0	-1	0	0	0	0	0	0	1	0	0	0	-1	0	0	-1	0	0	0	1
44	*	1.291273677148	4	1	0	-1	0	0	0	0	0	0	1	0	-1	0	1	0	-1	0	0	1	0	-1	0	1
44		1.292194283087	6	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
44		1.292618501256	8	1	1	0	0	0	-1	-1	0	0	0	1	1	0	0	0	-1	-1	-1	-1	0	1	1	1
44		1.292742855945	8	1	1	0	-1	-1	0	1	1	0	-1	-1	0	1	1	0	-1	-1	-1	0	1	1	0	-1
44		1.293040554717	8	1	1	0	-1	-1	0	1	1	0	-1	-1	0	1	1	0	-1	-1	0	1	1	1	0	-1
44	*	1.294482337197	4	1	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
44		1.294490470026	4	1	0	0	0	-1	0	0	0	0	1	0	0	0	-1	0	0	0	0	1	0	0	0	-1
44	*	1.294843896222	5	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1	0	0	0	0	0	0	0	-1
44		1.295223623432	6	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
44	**	1.295242940629	4	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	-1
44		1.295255431611	6	1	1	0	-1	-1	-1	0	1	1	0	-1	-1	0	1	1	0	-1	-1	0	1	1	0	-1
44	**	1.296191873495	4	1	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	-1
44	*	1.296336454420	6	1	1	0	0	0	0	0	0	0	-1	0	1	0	-1	-1	0	0	0	1	0	-1	0	1
44		1.296587119740	6	1	0	-1	1	0	-1	1	0	-1	0	1	-1	-1	2	-1	-1	2	-1	0	1	-1	0	1

44	**	1.296624048224	4	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	−1
44	*	1.297217345767	6	1	1	1	0	0	0	0	−1	−1	0	1	1	0	0	1	2	1	0	0	1	1	0	−1
44		1.297251049552	5	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1
44	*	1.298178762707	6	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0	−1	−1	−1	−1	−1	−1	−1	−1
44	*	1.298633042214	6	1	1	0	−1	−1	0	1	0	−1	0	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1
44	*	1.298674483402	6	1	0	0	0	1	1	0	0	0	1	1	0	−1	0	1	1	0	−1	0	1	1	0	−1
44	*	1.299604009970	6	1	1	1	1	1	0	0	−1	−1	−1	0	0	0	0	0	−1	−1	−1	−1	0	1	1	1

46		1.230743009076	3	1	0	0	1	0	1	0	0	1	0	0	0	0	0	0	−1	0	0	−1	0	−1	0	0	−1
46		1.235496042193	3	1	0	−1	0	0	0	0	1	0	−1	0	0	0	0	1	0	−1	0	0	0	0	1	0	−1
46		1.237634830280	5	1	1	0	0	0	0	0	−1	−1	0	0	−1	−1	0	1	0	−1	0	1	1	0	−1	0	1
46		1.243564793293	4	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1
46		1.243682745689	5	1	0	1	0	0	0	−1	0	−1	0	0	1	1	1	0	0	−1	−1	−1	−1	0	0	1	1
46	**	1.251881911571	5	1	1	1	0	0	−1	−1	−1	−1	−1	−1	−1	−1	0	1	2	2	2	1	0	−1	−1	−1	−1
46		1.252075672711	5	1	0	0	0	−1	0	−1	0	0	0	1	0	1	0	0	0	−1	0	−1	0	0	0	1	1
46		1.253844152357	6	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1
46		1.255615612455	7	1	1	1	0	0	−1	−1	−1	0	0	0	−1	−1	−1	0	1	1	1	0	0	−1	0	0	1
46		1.255827515020	5	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	0	0	−1
46		1.255932551000	5	1	0	0	0	0	0	0	0	0	0	0	1	−1	0	0	0	0	0	0	0	0	0	0	−1
46		1.258269489378	5	1	0	0	0	0	0	0	0	0	1	0	0	0	0	−1	0	0	0	0	0	0	0	0	−1
46		1.259502724664	6	1	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	1
46		1.260501777115	5	1	0	0	0	0	0	0	0	−1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	−1
46		1.262726526149	6	1	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	1
46		1.263237413879	5	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	−1	0	0	0	0	0	0	−1
46		1.265282569944	6	1	0	1	0	0	0	−1	0	0	0	1	0	1	0	0	0	−1	1	−1	1	0	0	1	−1
46		1.266066319232	5	1	0	0	0	0	0	−1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	−1
46		1.266965083066	5	1	0	0	0	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	−1
46		1.267102727539	5	1	1	1	0	−1	−2	−2	−1	0	1	1	1	1	1	1	0	−1	−2	−2	−1	0	1	1	1
46		1.267870559386	4	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1
46		1.268053428519	5	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	−1	0	0	0	0	−1
46	*	1.268085560681	3	1	0	0	1	0	1	0	0	1	−1	0	0	−1	0	−1	0	0	−1	0	0	0	0	0	1
46		1.268128795510	4	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
46		1.268325995901	4	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1
46		1.271055503381	7	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	1	0	−1	−1	−1
46	*	1.273654870918	5	1	1	0	−1	−1	0	0	−1	−1	0	1	1	1	1	0	−1	−1	−1	−1	−1	0	1	1	1
46		1.273718045414	5	1	0	0	0	0	0	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
46	**	1.273870574713	2	1	0	−1	0	0	1	0	−1	0	0	1	0	0	−1	0	1	0	0	−1	0	1	0	0	−1
46		1.274212741546	7	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	0	−1	−1	0	1
46		1.277127353326	5	1	1	0	−1	−1	−1	0	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	1	0	−1	−1
46		1.277359096327	7	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	1	0	−1	−1	−1	0	1	1	0	−1	−1
46		1.278470344249	7	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	1
46		1.278862037902	7	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1
46		1.279003871397	6	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1
46	**	1.279258997870	3	1	0	0	1	0	0	0	0	0	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	−1
46		1.279446011920	8	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	−1
46		1.279512867618	7	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	0	0	1

46		1.280133105070	7	1	1	1	1	0	−1	−2	−2	−2	−1	1	1	2	2	1	0	−1	−1	−2	−1	0	0	1	1
46		1.281187929201	4	1	0	0	1	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	1	0	0	1
46		1.281574377491	7	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	0	0	0	0	1
46		1.282115962834	5	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
46	*	1.282675643571	7	1	0	0	1	0	−1	0	0	−1	−1	0	0	−1	0	1	0	−1	1	1	−1	0	1	0	−1
46		1.282912172322	5	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
46		1.283666443171	6	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	−1
46		1.284226466391	7	1	1	1	0	−1	−1	−1	0	0	0	0	−1	0	0	1	1	0	0	−1	0	0	0	0	−1
46	*	1.284468338213	5	1	1	1	0	−1	−1	−1	0	0	0	0	0	0	0	0	0	1	1	1	0	−1	−1	−1	−1
46		1.284578419272	7	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	0	0	0	0	0	0	1
46	*	1.284623836253	5	1	1	0	−1	−1	0	0	0	0	0	0	−1	0	0	0	0	0	1	0	0	0	0	0	−1
46	*	1.285156948585	4	1	0	1	0	1	0	1	0	1	0	0	0	0	0	0	1	0	1	0	1	0	1	0	1
46	*	1.286041875200	3	1	0	0	1	0	1	0	0	1	0	1	0	0	1	−1	1	0	−1	1	−1	0	0	−1	1
46		1.286356097538	7	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	0	−1	−1	0	1	1	0	−1	−1	0	1
46		1.286376735848	7	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1
46		1.286759916526	5	1	0	0	1	0	−1	0	0	−1	−1	0	0	−1	1	1	0	1	1	0	0	0	−1	−1	−1
46		1.286864985298	7	1	1	0	−1	−1	0	1	0	−1	−1	0	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1
46		1.287186158204	4	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	−1	−1
46		1.287638583256	5	1	1	0	−1	−1	0	1	1	0	−1	−1	−1	−1	−1	0	1	1	0	−1	−1	0	1	1	1
46		1.288310251669	7	1	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	0	0	0	0	0	0	0	0	1
46		1.288727028845	9	1	1	1	0	−1	−1	−1	0	0	0	0	0	1	1	1	0	−1	−2	−2	−1	0	1	1	1
46		1.288737172525	7	1	0	0	0	0	0	0	0	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
46	**	1.289148923322	4	1	0	0	0	0	1	0	0	0	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	−1
46		1.289908537588	5	1	0	0	0	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
46		1.290129786277	9	1	1	1	1	0	−1	−2	−2	−2	−1	1	2	3	3	2	0	−2	−3	−4	−3	−1	1	3	3
46		1.290293430320	7	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1
46		1.290516910813	8	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	−1
46		1.291084823468	5	1	0	−1	1	0	−1	1	0	−1	1	0	−1	1	0	−1	1	0	−1	1	0	−1	1	0	−1
46		1.291317205618	7	1	1	0	−1	−1	0	1	1	0	−1	−1	−1	0	1	1	0	−1	−1	0	1	1	0	−1	−1
46		1.291333844073	4	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1
46		1.291790363329	9	1	1	1	0	−1	−1	−1	0	0	0	0	0	1	1	1	0	−1	−1	−1	0	0	0	0	−1
46		1.292032098880	8	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1
46		1.292183287996	7	1	0	0	0	0	0	0	0	0	0	0	0	−1	0	0	0	0	0	0	0	0	0	0	1
46		1.292549784478	6	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1
46		1.292966111535	8	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	−1
46		1.293025845482	8	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1
46		1.293084963049	8	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	−1
46		1.293734257577	7	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1

46		1.293933853998	5	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	−1	−1	
46		1.294502449422	7	1	0	0	0	0	0	0	0	0	0	−1	0	0	0	0	0	0	0	0	0	0	0	0	1	
46	*	1.294739634441	6	1	1	1	1	1	1	1	0	0	0	0	−1	−1	−1	−1	−1	−1	−1	0	0	0	0	1	1	
46		1.295459930754	5	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	
46		1.295462247202	6	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	
46		1.295933728636	5	1	0	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	
46	*	1.296465798292	5	1	1	0	−1	0	1	1	0	−1	−1	0	1	0	−1	−1	1	1	0	−1	0	1	1	0	−1	
46	*	1.296949332627	5	1	0	1	0	0	1	0	1	1	0	1	0	0	1	−1	1	0	0	1	−1	0	0	−1	1	
46	*	1.296975645272	5	1	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	1	
46	*	1.298706963798	4	1	1	1	1	0	0	0	0	0	1	0	0	0	−1	0	0	0	0	0	−1	0	0	0	1	
46	*	1.298913908702	8	1	1	1	0	0	−1	−1	−2	−1	−1	0	0	1	1	1	1	1	1	0	0	−1	0	−1	0	
46	**	1.299693633603	7	1	0	0	0	0	0	−1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	−1	
48		1.232202952743	4	1	1	0	0	1	1	0	−1	−1	0	0	−1	−1	0	1	1	0	0	1	1	0	−1	−1	−1	−1
48		1.238040100176	4	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	0	0	0	0	−1
48		1.249834231934	5	1	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	1
48		1.254161546188	4	1	1	1	1	0	0	0	−1	−1	−1	−1	0	0	0	1	1	1	1	0	0	0	−1	−1	−1	−1
48		1.257401302933	6	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	−1	0	0	0	0	0	0	−1
48		1.258901369893	6	1	1	0	0	0	0	0	−1	−1	0	0	−1	−1	0	1	1	1	1	1	1	0	−1	−1	−1	−1
48		1.265220494631	6	1	0	1	0	0	0	−1	1	−1	1	0	0	1	−1	1	−1	0	0	−1	0	−1	0	0	0	1
48		1.265389131919	4	1	1	1	1	0	0	−1	−1	−1	−1	0	0	1	1	1	1	0	0	−1	−1	−1	−1	0	0	1
48		1.265623874102	6	1	0	0	0	0	0	0	0	0	0	0	1	0	−1	0	0	0	0	0	0	0	0	0	0	−1
48		1.266312134392	5	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1
48		1.268359961801	4	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	−1
48		1.268826390898	6	1	1	0	0	0	−1	−1	0	0	−1	0	1	0	0	1	0	−1	0	0	−1	0	1	0	0	1
48		1.270152773799	6	1	0	−1	1	0	−2	1	1	−1	1	1	−1	0	0	−1	0	0	0	1	0	0	1	−1	−1	1
48		1.273401338513	8	1	0	−1	1	0	−2	1	1	−2	1	2	−2	0	2	−2	−1	2	−1	−1	2	0	−1	1	0	−1
48		1.273699978331	8	1	1	1	0	−1	−1	−1	0	0	0	0	0	1	1	1	0	−1	−1	−1	0	0	0	0	0	1
48		1.274633632152	4	1	0	0	1	−1	0	0	−1	0	0	0	0	1	0	0	1	−1	0	0	−1	0	0	0	0	1
48		1.274788185023	8	1	1	1	1	0	−1	−2	−2	−2	−1	1	2	3	3	2	0	−2	−3	−4	−4	−2	0	2	4	5
48	**	1.277269415260	2	1	0	−1	0	0	1	0	−1	0	0	1	0	−1	0	1	0	0	−1	0	1	0	0	−1	0	1
48	*	1.277760723777	4	1	0	0	0	1	0	0	0	0	1	0	0	−1	1	0	0	−1	0	0	0	0	−1	0	−1	1
48	*	1.278270773806	6	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	−1
48	*	1.279464310958	4	1	1	1	1	0	0	0	0	0	0	−1	−1	−1	−1	−1	−1	−1	−1	0	0	1	1	1	1	1
48		1.279603840840	4	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	−1	−1	−1	−1	−1	−1	−1
48	*	1.279702474008	4	1	1	1	1	1	1	0	0	−1	−1	−2	−2	−2	−2	−1	−1	0	0	1	1	1	1	1	1	1
48		1.280455598962	8	1	1	0	0	0	−1	−1	0	0	0	1	1	0	0	0	−1	−1	0	0	0	1	1	0	−1	−1
48	**	1.280492259520	3	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	−1

48	*	1.282890007720	4	1 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 −1 −1
48		1.283005649176	8	1 1 1 0 −1 −1 −1 0 0 0 0 0 1 1 1 0 −1 −2 −2 −1 0 1 1 1 1
48		1.283188413802	4	1 1 1 1 1 1 1 1 1 1 0 0 0 0 0 −1 −1 −1 −1 −1 −1 −1 −1 −1 −1
48		1.283316524592	4	1 0 −1 1 0 −1 1 0 −1 1 0 −1 1 0 −1 1 0 −1 1 0 −1 1 0 −1 1
48		1.284204191533	8	1 1 0 0 0 −1 −1 0 0 0 1 1 0 0 0 −1 −1 0 0 −1 0 1 0 0 1
48		1.284897642578	8	1 0 −1 1 0 −2 1 1 −2 1 2 −2 0 2 −2 −1 3 −1 −2 3 0 −3 2 1 −3
48		1.285198763783	6	1 1 0 −1 −1 0 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1
48		1.286121890563	8	1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 0 1 1
48		1.286606287635	6	1 1 0 −1 −1 0 1 1 0 0 0 −1 −2 −2 0 2 2 0 −1 −1 −1 −1 0 2 3
48		1.287657646625	4	1 0 1 0 1 1 0 1 0 1 0 0 0 0 0 −1 0 −1 0 −1 −1 0 −1 0 −1
48	*	1.288322643005	6	1 1 1 0 −1 −1 −1 0 0 1 1 1 1 0 0 −1 0 0 1 1 1 1 0 0 −1
48		1.288345240480	8	1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 0 −1 −1 0 1 1
48	*	1.289054443162	4	1 0 0 1 0 0 0 0 −1 0 0 −1 0 0 0 0 1 0 0 1 0 −1 0 0 −1
48		1.289119636663	6	1 1 1 0 −1 −2 −2 −1 0 1 1 1 1 1 1 0 −1 −2 −2 −1 0 1 1 1 1
48	*	1.289554301520	6	1 1 0 0 0 0 0 0 0 −1 0 1 0 −1 −1 0 0 0 0 −1 0 1 1 0 −1
48	**	1.289638381651	2	1 1 0 0 0 0 0 0 0 0 0 0 0 −1 0 0 0 0 0 0 0 0 0 0 −1
48		1.290137551856	8	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1
48		1.290487178386	8	1 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 1
48		1.290585716745	6	1 1 0 0 0 −1 −1 0 0 0 1 0 −1 0 0 0 1 1 0 −1 −1 −1 0 1 1
48		1.290601882359	8	1 1 1 0 −1 −1 −1 0 0 0 0 0 1 0 0 −1 −1 0 0 1 0 0 0 0 1
48	*	1.290715266971	4	1 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 −1 −1
48		1.291370605980	8	1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 0 1 1 0 −1 −1 0 1 1
48		1.291593267327	8	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1
48		1.292171937915	6	1 1 0 −1 −1 −1 −1 0 1 1 1 1 0 −1 −1 −1 −1 0 1 1 1 1 0 −1 −1
48		1.292847785849	8	1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1
48	*	1.293495834782	4	1 0 1 1 0 1 0 0 1 0 1 1 0 1 0 0 1 0 1 1 0 1 0 0 1
48	**	1.293851240863	6	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 −1 0 0 0 0 −1
48	*	1.293948375358	6	1 1 1 1 0 0 −1 −1 0 −1 0 −1 −1 0 −1 1 0 0 1 −1 1 0 0 1 −1
48		1.294758017761	6	1 1 0 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1
48	*	1.295304756161	4	1 1 1 0 0 0 0 −1 −1 0 0 0 0 1 1 0 −1 0 0 −1 −1 0 1 0 −1
48		1.295513327396	4	1 0 −1 1 0 −1 1 0 −1 1 0 −1 0 0 0 0 0 0 0 0 0 0 0 0 1
48		1.295520000815	6	1 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
48		1.295788689188	6	1 1 1 1 1 1 1 1 0 −1 −1 −1 −1 −1 −1 −1 −1 0 0 0 0 0 0 0 1
48	*	1.296107678325	8	1 1 1 0 −1 −1 0 1 2 1 0 −1 −1 −1 0 0 1 0 0 −1 −1 −1 0 0 1
48		1.296537832691	8	1 1 1 1 1 1 1 1 1 1 1 1 0 −1 −1 −1 −1 −1 −1 −1 −1 −1 −1 −1 −1
48		1.297136707875	6	1 1 1 1 1 1 1 1 0 0 0 0 −1 −1 −1 −1 −1 −1 −1 −1 0 0 0 0 1
48	*	1.297339094555	6	1 0 0 1 0 0 0 −1 0 0 −1 0 0 0 1 0 0 1 0 0 0 −1 0 0 −1
48		1.297375827355	4	1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 −1 −1 −1 −1

48	*	1.297862950017	6	1 1 1 0 0 0 0 0 -1 0 0 1 0 -1 -1 -1 0 -1 0 0 1 1 0 0 -1
48	*	1.298098615173	6	1 0 0 1 -1 0 0 -1 1 -1 0 1 -1 1 0 -1 1 -1 0 1 -1 1 0 -1 1
48	*	1.298694109853	4	1 1 0 0 0 0 1 1 0 0 1 1 0 -1 -1 0 1 1 0 -1 0 1 0 -1 -1
48	*	1.299455885267	6	1 0 0 0 0 0 0 0 0 0 0 0 -1 1 0 0 0 0 0 0 0 0 0 -1 1
48	**	1.299999380792	9	1 0 1 1 0 1 0 0 0 0 -1 0 -1 -1 0 -1 0 0 0 0 1 0 0 1 -1
50		1.240379074717	3	1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -1 -1
50		1.241974375265	4	1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 1
50		1.242878658278	5	1 0 0 0 0 0 0 0 0 0 0 0 -1 1 0 0 0 0 0 0 0 0 0 0 0 -1
50		1.247712370498	5	1 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 -1 0 0 0 0 0 0 0 0 -1
50		1.250665272660	6	1 1 1 0 0 -1 -1 -1 0 0 0 -1 -1 -1 0 1 2 2 2 1 0 -1 -1 -1 -1 -1
50		1.250918653907	5	1 0 0 0 0 0 0 0 -1 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 -1
50		1.252174127597	4	1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 1
50		1.254838200790	5	1 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 -1 0 0 0 0 0 0 -1
50		1.259261368486	5	1 0 0 0 0 0 -1 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 -1
50		1.261636972527	4	1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1
50		1.262217783207	5	1 1 1 1 1 0 -1 -1 -1 -1 -1 0 0 0 0 1 1 1 1 1 0 -1 -1 -1 -1 -1
50		1.263435761239	6	1 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 1
50		1.264880682900	6	1 0 0 0 0 0 0 0 0 0 0 1 0 0 1 0 0 0 0 0 0 0 0 0 0 1
50		1.265544449230	6	1 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 1
50		1.266077998775	5	1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -1 0 0 -1
50		1.266859819691	5	1 0 0 0 -1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 -1
50		1.267296723888	4	1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1
50		1.271070558922	4	1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
50		1.271615657033	7	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 1
50		1.272509828343	7	1 1 0 0 0 -1 -1 0 0 0 1 1 0 -1 -1 -1 -1 0 1 1 1 1 0 -1 -1 -1
50		1.273464959636	4	1 1 1 1 0 0 0 0 0 0 -1 -1 -1 -1 -1 -1 -1 -1 0 0 1 1 1 1 1 1
50		1.273815684063	6	1 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -1
50		1.273838848022	7	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -1 0 0 0 0 0 0 0 0 1
50	*	1.278680927394	7	1 1 0 -1 -1 0 1 0 -1 -1 0 1 0 -1 -1 0 1 1 0 -1 -1 0 1 1 0 -1
50		1.279028762933	7	1 0 0 0 0 0 0 0 0 0 0 0 0 0 -1 0 0 0 0 0 0 0 0 0 0 1
50		1.279495728421	6	1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
50		1.281312149340	5	1 0 0 0 -1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
50		1.281745153290	9	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -1 0 0 0 0 0 0 1
50		1.281938224541	7	1 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 1
50		1.282042757003	7	1 0 0 0 0 0 -1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
50	*	1.282648606457	7	1 1 0 -1 -1 -1 -1 0 1 1 0 0 0 0 0 1 1 0 -1 -1 -1 -1 0 1 1 1
50		1.283216026797	6	1 1 1 0 0 -1 -1 -1 -1 -1 -1 0 0 0 0 1 2 2 2 1 0 -1 -1 -1 -1 -1

50	*	1.284720212342	7	1	1	0	0	0	−1	−1	−1	−1	0	1	1	1	0	−1	−1	−1	−1	0	1	1	1	1	0	−1	−1	
50		1.284977285843	7	1	0	0	0	0	0	0	0	0	0	0	0	−1	0	0	0	0	0	0	0	0	0	0	0	0	1	
50		1.285422891839	8	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	
50		1.285686864089	8	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	−1	
50		1.285838394319	9	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1	
50		1.286719822870	8	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	−1	
50		1.287014872899	6	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	
50		1.287845622896	8	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	
50		1.288027126855	7	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	
50		1.289140686038	9	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	
50		1.289322607087	8	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	
50		1.289405676655	5	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	
50		1.289867053334	9	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	0	0	1	
50		1.290292987656	9	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	
50		1.290354216216	8	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	−1	
50		1.290426368557	6	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	
50		1.290516266093	9	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	1	
50		1.291638418996	9	1	0	1	0	0	0	−1	0	−1	0	0	0	1	0	1	0	0	0	−1	1	−1	1	0	0	1	−1	
50		1.291654102407	8	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	
50		1.292341505690	7	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	
50		1.292856591623	8	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	
50		1.293159734770	7	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	
50		1.293445491675	7	1	0	1	0	0	0	−1	1	−1	1	0	0	1	−1	1	−1	0	0	−1	1	−1	1	0	0	1	−1	
50	*	1.293603107085	4	1	1	1	0	0	0	0	−1	−1	0	1	1	1	1	1	0	−1	−1	0	0	0	0	1	1	0	−1	
50		1.293915394580	5	1	0	−1	0	0	1	0	−1	0	0	1	0	−1	0	0	1	0	−1	0	0	1	0	−1	0	0	1	
50		1.294057120593	5	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	
50		1.294161913154	7	1	0	0	0	0	0	0	0	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	
50		1.294602151071	5	1	0	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	
50	**	1.295663140557	7	1	0	0	0	0	0	−1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	−1	
50		1.298578703043	5	1	0	0	0	0	1	0	0	0	0	1	0	−1	0	0	1	0	−1	−1	0	1	0	−1	−1	0	1	
50	*	1.299080911034	6	1	1	0	0	1	0	−1	0	0	−2	−1	1	0	−1	1	1	−1	0	2	0	−1	1	1	−2	−1	1	
50		1.299202550621	9	1	1	1	1	1	1	1	1	1	1	1	1	0	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	
50	*	1.299667589902	4	1	0	0	0	1	1	0	0	0	1	1	0	0	0	1	1	0	0	0	1	1	1	0	0	1	1	
52		1.234348374876	4	1	1	0	−1	−1	0	0	−1	−1	0	1	1	0	0	1	1	0	−1	−1	0	0	−1	−1	0	1	1	1
52		1.242362139933	5	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1
52		1.247654794902	4	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	0	0	−1
52		1.249043167061	6	1	1	0	0	0	0	0	−1	−1	0	0	0	1	1	1	1	0	−1	−1	−1	−1	−1	−1	0	1	1	1

52		1.250286509814	5	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1
52		1.251490388246	6	1	1	1	1	0	0	−1	−2	−1	−1	0	1	1	2	2	1	0	−1	−2	−2	−2	−2	0	0	1	2	1
52		1.252792295944	6	1	1	0	0	0	0	0	−1	−1	0	1	1	0	0	1	1	0	−1	−1	−1	−1	−1	−1	0	1	1	1
52		1.253179234057	5	1	0	1	0	0	1	−1	1	−1	0	0	−1	1	−1	1	0	0	1	−1	1	−1	0	0	0	1	0	1
52		1.253234895431	6	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	0	0	0	0	−1
52		1.260468569031	5	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1
52		1.261806159182	7	1	0	1	0	0	0	−1	0	−1	0	0	1	1	1	1	0	0	0	−1	0	−1	0	0	0	1	0	1
52		1.262366415512	6	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	−1	0	0	0	0	0	0	0	0	0	0	−1
52		1.264314130613	6	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	−1	0	0	0	0	0	0	0	0	−1
52		1.264619159112	6	1	1	0	0	0	−1	−1	−1	−1	0	0	0	1	1	1	1	0	0	0	−1	−1	−1	−1	0	1	1	1
52		1.266710186839	5	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1
52		1.270235862562	4	1	1	1	1	0	0	−1	−1	−1	−1	0	0	1	1	1	1	0	0	−1	−1	−1	−1	0	0	1	1	1
52		1.273481203906	9	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	−1
52		1.274892843328	8	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	1	0	−1	−1	−1	0	1	1
52		1.275784965834	8	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	1	0	−1	0	1	1	0	−1
52		1.276704016603	6	1	1	0	−1	−1	0	1	0	−1	−1	0	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	1	0	−1
52		1.278146743378	8	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	1	1	0	−1	−1	0	1	1
52		1.278701577669	8	1	1	0	0	0	−1	−1	0	0	0	1	1	0	0	0	−1	−1	−1	−1	0	1	1	1	1	0	−1	−1
52		1.279012875519	4	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
52		1.279604062572	8	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	0	−1	−1	0	1	1	0	−1
52		1.279681804162	8	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
52	*	1.280727170449	6	1	1	0	−1	0	0	0	0	0	−1	0	1	0	−1	0	0	0	1	1	−1	−1	0	0	0	1	0	−1
52		1.281023032783	6	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
52	b	1.281697241774	6	1	1	0	−1	0	1	1	0	−1	0	1	0	−2	−1	1	1	−1	−2	−1	1	1	−1	−1	1	2	0	−1
52		1.281982330228	8	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1
52		1.282714951752	6	1	0	0	1	0	0	0	1	0	−1	1	0	−1	0	1	−1	−1	1	0	−1	0	1	−1	−1	1	0	−1
52	**	1.283710835955	3	1	0	0	0	−1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	−1
52		1.285372874376	6	1	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	1
52		1.285703696090	5	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1
52		1.286982985987	8	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1
52		1.287935162333	8	1	0	1	0	0	0	−1	0	−1	0	0	0	1	1	1	1	0	0	−1	−1	−1	−1	0	0	1	1	1
52		1.288800309908	8	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	1
52		1.289282270742	10	1	0	1	0	0	0	−1	0	−1	0	0	0	1	0	1	0	0	0	−1	0	−1	0	0	0	1	1	1
52	**	1.289340321841	4	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	−1
52	**	1.289716496566	5	1	1	0	0	0	−1	−1	0	0	0	1	1	0	0	0	−1	−1	0	0	−1	0	1	0	−1	0	0	−1
52		1.289764940945	10	1	1	0	0	0	−1	−1	0	0	0	1	1	0	0	0	−1	−1	0	0	0	1	1	0	0	0	−1	−1
52		1.289993668651	8	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1
52		1.290503740984	4	1	0	−1	0	0	0	0	0	0	0	0	0	0	1	0	−1	0	0	0	0	0	0	0	0	−1	0	1

52		1.290860627837	10	1 0 -1 1 0 -2 1 1 -2 1 2 -2 0 2 -2 -1 2 -1 -1 2 0 -1 1 0 -1 0 1
52		1.290888275697	7	1 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -1
52		1.291240760565	9	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 -1
52		1.291632902429	8	1 1 0 0 0 -1 -1 0 0 -1 0 1 0 0 1 0 -1 0 0 -1 0 1 0 0 1 0 -1
52		1.291693397214	9	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 -1
52		1.291804049321	9	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 -1
52		1.291979261516	6	1 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
52		1.291995153902	8	1 1 0 -1 -1 0 1 1 0 -1 0 1 1 0 -1 -1 0 1 1 0 -1 -1 0 1 1 0 -1
52		1.292027618526	8	1 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
52		1.292757892429	8	1 1 0 -1 -1 0 1 1 0 -1 -1 -1 0 1 1 0 -1 -1 0 1 1 0 -1 -1 0 1 1
52		1.292801136040	6	1 0 0 1 -1 -1 1 -1 -1 2 0 -1 2 0 -2 1 0 -2 1 1 -1 1 1 -1 0 0 -1
52		1.293308368018	6	1 1 0 -1 0 1 1 0 -1 -1 0 1 1 0 -1 -1 0 1 1 0 -1 -1 0 1 1 0 -1
52		1.293849341249	7	1 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -1
52		1.294671309884	5	1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -1
52	**	1.295423732288	7	1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -1 0 0 0 0 0 1
52	*	1.296659463082	4	1 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 1
52	*	1.298381333842	6	1 1 1 1 1 1 1 0 0 0 0 0 0 -1 -1 -1 -1 -1 -1 -1 0 0 0 0 0 0 1
52	*	1.298399397452	6	1 1 1 1 1 1 1 1 1 1 0 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 0 1 1 1 1 1
52		1.299211061414	6	1 0 0 1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 -1 0 0 -1 0 0 -1

54		1.236566917569	5	1 0 1 0 0 0 -1 1 -1 1 0 0 1 -1 1 -1 0 0 -1 1 -1 1 -1 0 0 -1 1 -1
54		1.239747974875	6	1 1 1 0 0 -1 -1 -1 0 0 0 0 1 1 1 1 1 0 -1 -2 -2 -2 -1 0 1 1 1 1
54		1.240983403460	5	1 0 0 0 0 0 0 0 -1 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 -1
54		1.245678970730	5	1 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 -1 0 0 0 0 0 0 -1
54		1.247953027873	6	1 0 1 0 0 0 -1 0 -1 0 0 0 1 1 1 1 1 0 0 -1 -1 -1 -1 0 0 1 1 1
54		1.249319539914	5	1 1 1 0 -1 -2 -2 -1 0 1 1 1 0 0 0 1 1 1 0 -1 -2 -2 -1 0 1 1 1 1
54		1.249896621001	7	1 0 0 0 0 0 0 0 0 0 -1 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 -1
54		1.256128929153	7	1 1 1 1 0 0 -1 -2 -2 -2 -1 0 1 1 2 2 1 1 -1 -2 -2 -3 -2 -1 0 2 3 3
54		1.256924817375	5	1 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -1 0 0 0 0 -1
54		1.256963641913	4	1 1 1 1 0 0 -1 -1 -1 -1 0 0 1 1 1 1 0 0 -1 -1 -1 -1 0 0 1 1 1 1
54		1.257451908723	6	1 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 1
54	*	1.258079413341	7	1 1 0 0 0 -1 -1 0 0 -1 0 0 -1 0 1 0 0 1 0 -1 0 0 -1 0 1 0 0 1
54		1.258882142249	7	1 0 0 0 0 0 0 0 0 0 0 0 0 1 -1 0 0 0 0 0 0 0 0 0 0 0 0 -1
54		1.260267357727	6	1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1
54		1.260942158309	6	1 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1
54		1.261336986244	6	1 0 1 0 1 0 0 0 -1 0 -1 0 0 0 1 0 1 0 0 0 -1 0 -1 0 0 1 1 1
54		1.261717349530	7	1 1 0 0 0 0 0 -1 -1 0 0 0 0 -1 0 1 0 -1 -1 0 1 0 -1 0 1 1 0 -1
54		1.262429924944	5	1 0 0 0 -1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 -1

54		1.262878024605	7	1 0 1 0 0 0 −1 0 −1 0 −1 0 0 0 1 0 1 0 0 1 −1 1 −1 0 0 −1 1 −1
54		1.262917418612	4	1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1
54		1.263631528146	6	1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 1
54		1.264132389613	5	1 0 −1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 −1
54		1.264932486096	6	1 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 1
54		1.271572921706	7	1 1 0 0 0 −1 −1 0 0 0 1 0 −1 0 0 −1 0 1 0 0 1 0 −1 0 0 −1 0 1
54		1.272191177616	5	1 0 0 0 −1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
54		1.274357763329	5	1 1 0 −1 −1 0 0 0 0 0 0 −1 −1 0 1 1 1 1 0 −1 −1 0 1 1 0 −1 −1 −1
54		1.275236058262	6	1 1 0 0 0 −1 0 1 0 0 1 0 −1 0 0 −1 0 1 0 0 1 0 −1 0 0 −1 0 1
54		1.276683500746	8	1 1 0 0 0 −1 −1 0 0 0 1 1 0 0 0 −1 0 1 0 0 1 0 −1 0 0 −1 0 1
54		1.280813300357	9	1 1 0 0 0 −1 −1 0 0 0 1 1 0 0 0 −1 −1 0 0 0 1 1 0 0 0 −1 −1 −1
54	**	1.280842546458	7	1 0 0 0 −1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 −1
54		1.284105838728	7	1 0 0 0 0 0 0 0 0 0 −1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
54		1.284152259946	9	1 0 0 0 0 0 0 0 0 0 0 0 0 0 −1 0 0 0 0 0 0 0 0 0 0 0 0 1
54		1.285075974903	9	1 1 0 0 0 −1 −1 0 0 0 1 1 0 0 0 −1 −1 0 0 0 1 0 −1 0 0 −1 0 1
54		1.285998712043	9	1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 0 −1 −1
54		1.286083095042	7	1 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
54		1.286792706210	9	1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1
54		1.287459375692	8	1 1 0 0 0 −1 −1 0 0 0 1 1 1 1 0 −1 −1 −1 −1 0 1 1 1 1 0 −1 −1 −1
54		1.287755652734	6	1 1 1 0 −1 −2 −2 −1 1 2 2 1 0 −1 −1 0 1 1 0 −1 −2 −2 −1 0 1 1 1 1
54	**	1.288530074471	9	1 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 −1
54		1.289422759050	9	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 −1 1
54		1.289571169659	9	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1
54		1.289589050959	5	1 0 −1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
54		1.289686123306	7	1 1 0 0 0 −1 −1 −1 −1 0 1 1 1 1 0 −1 −1 −1 −1 0 1 1 1 1 0 −1 −1 −1
54		1.289720990343	9	1 1 0 0 0 −1 −1 0 0 0 1 1 0 0 0 −1 −1 −1 −1 0 1 1 1 1 0 −1 −1 −1
54		1.290032926031	9	1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 0 −1 −1 0 1 1 0 −1 −1
54		1.290062889368	5	1 0 −1 0 0 1 0 −1 0 0 1 0 −1 0 0 1 0 −1 0 0 1 0 −1 0 0 1 0 −1
54		1.290508408147	9	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 −1 0 0 0 0 1
54		1.290545515646	10	1 1 0 0 0 −1 −1 0 0 0 1 1 0 0 0 −1 −1 0 0 0 1 1 1 1 0 −1 −1 −1
54		1.290824289040	9	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 −1 0 0 0 0 0 0 0 0 0 0 1
54		1.291282750424	7	1 0 0 0 0 0 0 0 −1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
54		1.291315858110	9	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 −1 0 0 0 0 0 0 1
54	**	1.291363818801	5	1 1 0 −1 −1 −1 −1 0 1 2 2 0 −2 −2 −1 −1 0 2 3 2 0 −2 −3 −2 −1 0 2 3
54		1.291608020359	9	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1
54		1.292282851197	5	1 0 −1 1 0 −1 1 0 −1 1 0 −1 1 0 −1 1 0 −1 1 0 −1 1 0 −1 1 0 −1 1
54	**	1.294652435792	9	1 1 0 −1 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 −1 0 1 1 0 −1 −1 0 1 1
54		1.295311024901	7	1 0 1 1 0 2 0 1 1 0 1 0 0 0 0 −1 0 −1 −1 0 −2 0 −1 −1 0 −1 0 −1

54		1.295556890710	8	1	0	0	1	0	0	0	1	0	-1	1	0	-1	0	1	0	0	1	1	0	0	1	0	0	0	0	0	-1	
54	*	1.296678789733	4	1	1	0	0	1	1	0	0	0	0	1	1	0	0	1	1	0	-1	-1	0	1	0	-1	0	1	0	-1	-1	
54		1.297452318639	6	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	
54	*	1.297657045330	4	1	1	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	1	1	
54		1.297731226233	6	1	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	1	
54		1.299458117128	6	1	0	0	0	0	0	0	1	0	0	0	0	0	1	1	0	0	0	0	0	1	0	0	0	0	0	0	1	
56		1.238431627359	4	1	1	1	1	0	0	0	-1	-1	-1	-1	0	0	0	1	1	1	1	0	0	0	-1	-1	-1	-1	0	0	0	1
56		1.238708978554	4	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	-1
56		1.243935933206	6	1	1	1	0	0	-1	-1	-1	0	1	1	1	0	0	-1	0	0	1	0	0	-1	-1	-1	-1	0	0	1	0	1
56		1.247816215528	6	1	1	0	0	0	0	0	-1	-1	-1	-1	0	0	0	1	1	1	1	0	0	0	-1	0	0	-1	0	0	0	1
56		1.250334196141	4	1	0	-1	0	0	0	0	1	0	-1	0	0	0	0	1	0	-1	0	0	0	0	1	0	-1	0	0	0	0	1
56		1.252816515888	6	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	-1	0	0	0	0	0	0	0	0	0	0	0	0	-1
56		1.254901282850	5	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
56		1.254917837018	6	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0	-1
56		1.255384658420	8	1	1	0	0	0	0	0	-1	-1	0	0	0	0	0	1	0	0	1	0	0	0	-1	0	0	-1	0	0	0	1
56		1.255678303917	7	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	1
56		1.258845767054	6	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	-1
56		1.260066595750	4	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1
56		1.262960021026	7	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1
56		1.263875629370	6	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	-1
56		1.264393924384	5	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1
56		1.274238478136	8	1	1	0	-1	-1	0	1	1	0	-1	-1	0	1	1	0	-1	-1	-1	0	1	1	0	-1	-1	0	1	1	0	-1
56		1.275885130333	7	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1
56		1.277702182440	5	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1
56		1.279283237262	6	1	0	0	1	0	1	0	0	1	-1	1	0	-1	1	-1	0	0	-1	1	-1	0	0	-1	1	-1	0	0	-1	1
56	**	1.280086980269	8	1	1	1	1	0	-1	-1	-1	-1	-1	-1	-1	-1	0	1	1	2	2	1	1	1	0	0	0	-1	-1	-1	-1	-1
56	**	1.280406441267	7	1	0	0	0	0	0	0	0	0	1	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
56		1.281305744894	8	1	1	1	1	0	-1	-2	-2	-2	-1	1	1	2	2	1	0	-1	-1	-2	-1	0	0	1	1	1	0	0	0	-1
56	**	1.281650682888	4	1	1	0	0	1	0	-1	0	0	-1	0	1	0	-1	0	0	-1	0	1	0	0	1	0	-1	0	0	-1	0	1
56		1.281999544164	8	1	1	0	-1	-1	0	1	1	0	-1	-1	0	1	1	1	0	-1	-1	0	1	1	0	-1	-1	0	1	1	0	-1
56		1.282355072566	6	1	1	0	-1	-1	-1	0	1	1	0	-1	-1	0	1	1	0	-1	-1	0	1	1	0	-1	-1	0	1	1	0	-1
56		1.282385080265	6	1	0	-1	1	1	-2	0	2	-2	-1	3	-1	-2	3	0	-3	2	1	-3	1	2	-2	0	2	-1	-1	1	0	-1
56		1.283214322688	8	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1
56		1.284581358929	7	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1
56		1.285471808311	9	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	-1
56		1.285702445977	10	1	1	1	1	0	-1	-2	-2	-2	-1	1	2	3	3	2	0	-2	-3	-4	-3	-1	1	3	3	3	1	-1	-2	-3
56		1.285864072493	6	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

56		1.285917992729	10	1 0 −1 1 0 −2 1 1 −2 1 2 −2 0 2 −2 −1 2 −1 −1 2 0 −1 1 0 −1 0 0 0 1
56		1.287014751256	9	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 −1
56		1.287908591120	10	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1
56		1.289051027565	10	1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 1 0 −1 −1 0 1 1 0 −1
56		1.289906267218	8	1 1 0 −1 −1 0 1 1 0 −1 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1
56		1.290792755305	10	1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 −1 0 1 1 0 −1
56		1.290805257309	8	1 1 0 −1 −1 0 1 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1
56		1.291198515789	7	1 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 −1
56		1.291405324275	9	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 −1
56	∗	1.291442439853	6	1 1 1 1 0 −1 −1 −1 −1 0 0 0 0 0 0 0 0 0 0 −1 0 0 0 1 1 0 0 0 −1
56		1.291508742027	9	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 −1
56		1.291818431779	8	1 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
56	∗∗	1.292228836706	5	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 1 0 0 −1
56	∗	1.292277074243	6	1 1 0 −1 −1 0 1 1 0 0 1 1 0 −1 −1 0 1 1 0 0 1 1 0 −1 −1 0 1 1 1
56		1.292785064098	5	1 0 0 1 0 0 0 0 0 0 0 0 0 0 1 0 0 1 0 0 0 0 0 0 0 1 0 0 1
56		1.293830249986	6	1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
56	∗∗	1.293926504587	8	1 0 0 1 0 0 0 −1 0 −1 −1 0 −1 0 0 0 1 1 1 1 1 1 0 0 0 −1 −1 −1 −1
56	∗∗	1.295444946800	10	1 1 0 −1 −1 −1 −1 0 2 3 2 0 −2 −3 −3 −2 0 2 4 4 2 −1 −3 −4 −4 −2 1 4 5
56		1.297065215211	7	1 1 1 0 0 −1 −1 −1 0 0 0 0 0 −1 −1 0 0 0 0 0 −1 −1 0 1 1 1 1 0 −1
56	∗∗	1.297937402938	6	1 1 1 0 0 −1 −1 −1 0 0 0 0 0 0 0 1 1 1 0 0 −1 −1 −1 −1 −1 0 1 1 1
56		1.298666384012	8	1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 −1 −1 −1 −1 −1 −1 −1 −1 −1 −1 −1 −1 −1 −1
56		1.299131367919	6	1 0 0 0 0 1 0 0 0 0 1 0 0 0 0 0 0 0 −1 0 0 0 0 −1 0 0 0 0 −1
56	∗∗	1.299813146526	4	1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 −1 0 0 0 0 0 0 0 0 0 0 0 0 −1
58		1.237684127894	5	1 1 0 0 0 −1 −1 −1 −1 0 0 0 1 1 1 1 0 0 0 −1 −1 −1 −1 0 0 0 1 0 0 1
58		1.241541076676	5	1 0 1 0 0 1 −1 1 −1 0 0 −1 1 −1 1 0 0 1 −1 1 −1 0 0 −1 1 −1 0 0 −1 1
58		1.241902161200	6	1 0 1 0 0 0 −1 0 −1 0 0 1 1 1 1 0 0 −1 −1 −1 0 0 1 1 1 1 0 0 −1 −1
58		1.242217045566	5	1 1 1 0 −1 −2 −2 −1 0 1 1 1 0 0 0 1 1 1 0 −1 −2 −2 −1 0 1 1 1 0 0 −1
58		1.242610289442	5	1 0 0 0 0 0 −1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 −1
58		1.246289657523	4	1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1
58		1.247406043552	6	1 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 1
58		1.249326739190	5	1 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 −1 0 0 0 0 −1
58	b	1.251576165895	7	1 1 1 0 0 −1 −1 −1 0 0 0 0 0 0 −1 0 0 1 0 0 −1 −1 −1 0 1 1 1 0 0 −1
58		1.255364249872	6	1 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 1
58		1.256227883912	5	1 0 0 0 −1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 −1
58		1.257101710157	7	1 0 0 0 0 0 0 0 −1 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 −1
58		1.258254888403	6	1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 1
58		1.258595404159	7	1 0 1 0 0 0 −1 0 −1 0 0 0 1 0 0 0 −1 1 −1 1 0 0 1 −1 1 −1 0 0 −1 1

58		1.260722599588	5	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	−1
58		1.261205969114	6	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1
58		1.261349465395	7	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	−1	0	0	0	0	0	0	0	0	−1
58		1.262551693469	5	1	0	0	0	−1	0	−1	0	0	0	1	0	1	0	0	0	−1	0	−1	0	0	0	1	0	1	0	0	0	−1	1
58		1.263144654250	7	1	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	−1
58		1.263233624977	7	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	−1	0	0	0	0	0	0	0	0	0	0	0	0	−1
58		1.263330550711	7	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	−1	0	0	0	0	0	0	0	0	0	0	−1
58		1.263346699147	7	1	0	0	0	0	0	0	0	0	0	0	0	−1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	−1
58		1.263692746525	6	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1
58		1.265222703407	5	1	0	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	−1
58		1.273334904173	7	1	1	0	0	0	−1	−1	0	0	−1	0	1	0	0	1	0	−1	0	0	−1	0	1	0	0	1	0	−1	0	0	−1
58		1.273571073280	5	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
58		1.274256714395	8	1	1	0	0	0	−1	−1	0	0	0	1	1	0	0	1	0	−1	0	0	−1	0	1	0	0	1	0	−1	0	0	−1
58		1.275182952121	9	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	1	0	−1	−1	−1
58		1.276216416568	7	1	0	0	0	0	0	0	0	0	0	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
58	b	1.277275812849	9	1	1	1	1	0	−1	−2	−2	−2	−1	1	2	3	3	2	0	−2	−4	−5	−4	−2	1	4	6	6	4	1	−3	−6	−7
58		1.278864060776	9	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	1	0	−1	−1	−1	0	1	1	0	−1	−1
58	*	1.278934747587	6	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	1	1	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	1	1
58		1.279284153763	8	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	−1
58	**	1.279320724447	7	1	0	0	1	0	0	0	0	0	−1	−1	−1	−1	−1	−1	0	0	0	1	1	1	1	1	1	0	0	0	−1	−1	−1
58		1.280463197895	9	1	1	0	0	0	−1	−1	0	0	0	1	1	0	0	0	−1	−1	0	0	−1	0	1	0	0	1	0	−1	0	0	−1
58		1.280590313739	9	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	1
58		1.280959576176	7	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
58		1.281183304230	9	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	0	0	1
58		1.281629825308	8	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1
58		1.281697636700	9	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1
58		1.281732098365	6	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1
58		1.282009994078	11	1	1	0	0	0	−1	−1	0	0	0	1	1	0	0	0	−1	−1	0	0	0	1	1	0	−1	−1	−1	−1	0	1	1
58		1.282352048235	9	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	0	0	0	0	1
58		1.282996610699	5	1	0	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
58		1.283140168294	9	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1
58		1.283343723504	10	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1
58		1.284052661797	9	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	0	0	0	0	0	0	1
58		1.284093097761	9	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	0	−1	−1	0	1	1	0	−1	−1	0	1
58		1.284761861591	8	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1
58		1.285581320954	7	1	0	0	0	0	0	0	0	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
58	*	1.285718821138	9	1	0	0	0	−1	1	0	0	0	−1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	−1	1	0	0	0	−1
58		1.286190461112	9	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	0	0	0	0	0	0	0	0	1

58		1.286238790823	7	1	1	0	−1	−1	−1	−1	0	1	1	1	1	0	−1	−1	−1	−1	0	1	1	1	1	0	−1	−1	−1	−1	0	1	1
58		1.286897490009	7	1	0	0	0	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
58		1.286985146432	7	1	1	0	−1	−1	0	1	0	−1	−1	0	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1
58		1.287193438732	9	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	1	0	−1	−1	−1	0	1	1	0	−1	−1	0	1	1	0	−1	−1
58		1.287208982020	10	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	−1
58	**	1.287230477142	3	1	0	0	0	−1	1	0	0	0	−1	0	0	0	0	0	0	0	1	−1	1	0	0	1	−1	0	0	0	0	0	−1
58		1.287367534849	9	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1
58		1.287440915549	5	1	0	−1	1	0	−1	1	0	−1	1	0	−1	1	0	−1	1	0	−1	1	0	−1	1	0	−1	1	0	−1	1	0	−1
58		1.287764925703	9	1	0	0	0	0	0	0	0	0	0	0	0	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
58		1.287811625661	10	1	1	0	0	0	−1	−1	0	0	0	1	1	0	0	0	−1	−1	0	0	0	1	1	0	0	1	0	−1	0	0	−1
58	**	1.287823339597	7	1	1	0	0	0	−1	−1	0	0	−1	0	0	−1	0	1	0	0	1	0	−1	0	0	−1	0	1	0	0	1	0	−1
58		1.287841375380	9	1	1	0	−1	−1	0	1	1	0	−1	−1	−1	0	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	1	0	−1	−1
58		1.288565229569	9	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	0	0	0	0	0	0	0	0	0	0	1
58		1.288565599895	6	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
58		1.288888767752	11	1	1	1	1	0	−1	−2	−2	−2	−1	1	2	3	3	2	0	−2	−3	−4	−3	−1	1	3	4	4	2	0	−2	−4	−5
58		1.289051629437	8	1	0	1	0	0	0	−1	0	−1	1	0	1	1	1	1	0	0	−1	0	−1	0	0	1	1	1	1	0	0	−1	−1
58		1.289061953395	11	1	1	0	0	0	−1	−1	0	0	0	1	1	0	0	0	−1	−1	0	0	0	1	1	0	0	0	−1	−1	0	0	−1
58	**	1.289075427851	7	1	1	1	1	0	−1	−1	−1	−1	−1	−1	−1	−1	−1	0	0	0	1	1	1	2	2	1	1	0	−1	−1	−1	−1	−1
58		1.290030350301	10	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	−1
58	**	1.290475356814	9	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	−1
58		1.290739310915	9	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	0	0	0	0	0	0	0	0	0	0	0	0	1
58		1.290788176755	10	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
58		1.290809675525	9	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
58		1.290819654658	10	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	−1
58		1.290822148766	10	1	1	0	0	0	−1	−1	0	0	0	1	1	0	0	0	−1	−1	0	1	1	1	1	0	−1	−1	−1	−1	0	1	1
58		1.290884906851	10	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1
58		1.290892316970	10	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	−1
58		1.290893470079	9	1	1	0	0	0	−1	−1	0	0	0	1	1	0	−1	−1	−1	−1	0	1	1	1	1	0	−1	−1	−1	−1	0	1	1
58		1.291468740808	9	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1
58		1.291609476255	9	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	0	−1	−1	0	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1
58		1.291653539470	9	1	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
58		1.291784092709	8	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
58		1.292015367286	5	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
58		1.292047661070	8	1	1	0	0	0	−1	−1	0	1	1	1	1	0	−1	−1	−1	−1	0	1	1	1	1	0	−1	−1	−1	−1	0	1	1
58		1.292508057656	8	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1
58		1.292569173973	7	1	0	0	0	0	0	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
58		1.292702718356	6	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1
58		1.292884356004	7	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

58	*	1.294438401999	11	1 1 0 −1 −1 −1 −1 0 1 1 0 0 0 0 0 1 1 0 −1 −1 −1 −1 0 1 1 0 0 0 0 −1
58	**	1.294716649888	5	1 1 1 0 −1 −1 0 1 1 1 −1 −1 −1 0 1 1 0 −1 −1 −1 1 1 1 0 −1 −1 −1 0 0 1
58	*	1.294798231363	6	1 1 0 −1 −1 0 1 1 1 0 −1 −1 0 1 1 1 0 −1 −1 0 1 1 1 1 0 −1 −1 0 1 1
58	*	1.297080156222	8	1 0 0 0 −1 0 0 0 1 1 0 0 −1 −1 0 0 1 1 0 0 0 −1 0 0 0 1 0 0 0 −1
60		1.240061859037	4	1 0 0 1 0 1 0 0 1 0 0 0 0 0 0 −1 0 0 −1 0 −1 0 0 −1 0 0 0 0 0 0 1
60		1.243027765980	6	1 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 −1 0 0 0 0 0 0 0 0 0 0 0 0 −1
60		1.243486256656	4	1 0 −1 0 0 0 0 1 0 −1 0 0 0 0 1 0 −1 0 0 0 0 1 0 −1 0 0 0 0 1 0 −1
60	b	1.244271476892	6	1 1 0 0 0 0 0 −1 0 1 0 0 0 0 1 0 −1 0 0 0 0 −1 0 1 0 −1 −1 0 1 0 −1
60		1.246295155833	6	1 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 −1 0 0 0 0 0 0 0 0 0 0 −1
60		1.253698073656	4	1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 −1 −1
60		1.256400013906	7	1 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 1
60	b	1.256460630524	8	1 2 2 1 0 −1 −2 −3 −3 −2 −1 0 1 1 1 2 3 3 2 1 0 −2 −4 −4 −2 0 1 1 1 1 1
60		1.258491665790	6	1 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 −1 0 0 0 0 0 0 −1
60	b	1.262277810304	6	1 1 0 0 1 1 0 −1 −1 0 0 −1 −1 0 1 1 0 0 1 1 0 −1 −1 0 0 −1 −1 0 1 0 −1
60		1.262680696526	7	1 0 1 0 0 0 −1 1 −1 1 0 0 1 −1 1 −1 0 0 −1 1 −1 1 0 0 1 0 1 0 0 0 −1
60		1.262797635404	7	1 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 1
60		1.263070916381	6	1 1 1 1 1 0 −1 −1 −1 −1 −1 0 0 0 0 1 1 1 1 1 0 −1 −1 −1 −1 −1 0 0 0 0 1
60		1.263246708286	5	1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1
60		1.263821626752	5	1 1 1 1 0 0 −1 −1 −1 −1 0 0 1 1 1 1 0 0 −1 −1 −1 −1 0 0 1 1 1 1 0 0 −1
60		1.271148136598	6	1 0 1 0 0 1 −1 1 −1 0 0 −1 1 −1 1 0 0 1 −1 1 −1 0 0 −1 1 −1 1 0 0 1 −1
60		1.273845911088	8	1 1 0 0 0 −1 −1 0 0 0 1 1 0 −1 −1 −1 −1 0 1 1 1 1 0 −1 −1 −1 −1 0 1 1 1
60		1.275588071323	6	1 0 −1 1 1 −2 0 2 −2 −1 3 −1 −2 3 0 −3 2 1 −3 1 2 −2 0 2 −1 −1 1 0 −1 0 1
60		1.276573566904	10	1 0 −1 1 0 −2 1 1 −2 1 2 −2 0 2 −2 −1 2 −1 −1 2 0 −1 1 0 −1 0 0 0 0 0 1
60		1.276758695801	10	1 1 1 0 −1 −1 −1 0 0 0 0 0 1 1 1 0 −1 −1 −1 0 0 0 0 0 1 1 1 0 −1 −1 −1
60		1.277611624883	8	1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1
60	**	1.280769309560	3	1 0 0 0 −1 0 0 0 0 0 −1 0 0 0 1 0 0 0 0 1 1 1 0 0 −1 0 0 0 0 −1 −1
60		1.281278497039	8	1 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
60		1.281292275419	10	1 1 0 0 0 −1 −1 0 0 0 1 1 0 0 0 −1 −1 0 0 0 1 1 0 0 0 −1 −1 0 0 0 1
60		1.281758555031	10	1 0 1 0 0 0 −1 0 −1 0 0 0 1 0 1 0 0 0 −1 0 −1 0 0 0 1 0 1 0 0 1 −1
60	b	1.282480557364	10	1 1 1 0 −1 −1 −1 0 0 0 0 0 1 1 1 0 −1 −1 −1 0 0 0 0 −1 0 0 1 1 0 0 −1
60	*	1.283048118529	6	1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 −1 −1 −1 −1 −1 −1 −1 −1 0 0 0 0 0 0 0 1
60	**	1.283179853119	10	1 1 1 0 0 −1 −1 −2 −1 −1 0 0 1 1 1 1 1 1 0 0 0 0 −1 −1 −1 −1 −1 0 1 1 1
60	**	1.284016780410	4	1 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 −1
60		1.285922383579	6	1 1 0 −1 −1 −1 −1 0 1 1 1 1 0 −1 −1 −1 −1 0 1 1 1 1 0 −1 −1 −1 −1 0 1 1 1
60		1.287226719102	10	1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 0 −1 −1 0 1 1
60		1.287385037167	8	1 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
60	b	1.287418784999	10	1 1 1 0 −1 −1 −1 0 0 0 0 0 1 1 1 0 −1 −1 −1 −1 −1 −1 0 1 2 2 1 0 −1 −1 −1

60		1.287529106708	10	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 1
60		1.287608676997	10	1 1 0 0 0 −1 −1 0 0 0 1 1 0 0 0 −1 −1 0 0 0 1 0 −1 0 0 −1 0 1 0 0 1
60		1.287688009081	8	1 1 0 −1 −1 0 1 1 0 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1
60	**	1.288166009066	5	1 0 0 0 0 0 0 0 0 0 0 0 0 0 −1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 −1
60		1.288858711067	6	1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
60		1.288873027322	6	1 0 0 1 −1 −1 1 −1 −1 2 0 −1 2 0 −2 1 0 −2 1 1 −1 1 1 −1 0 0 −1 0 0 0 1
60		1.288882550359	10	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1
60		1.289095555871	10	1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 0 1 1 0 −1 −1 0 1 1
60		1.289119195174	8	1 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
60		1.289140359037	10	1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1
60		1.289711465078	6	1 0 0 0 0 0 0 0 −1 0 0 1 0 0 0 0 0 0 −1 0 0 1 0 0 0 0 0 0 0 0 −1
60	**	1.289735260718	6	1 1 1 0 0 −1 −1 −1 0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 −1 −1 −1 0 0 1
60	b	1.290094862514	10	1 0 −1 1 0 −2 1 1 −2 1 2 −2 0 2 −1 −1 1 0 −1 0 1 0 −1 1 1 −2 0 2 −2 −1 3
60		1.290363797021	10	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 1
60	b	1.290428231211	8	1 1 1 0 0 0 0 0 −1 0 0 1 0 0 0 0 0 −1 0 −1 0 −1 0 0 1 1 0 0 −1 0 −1
60		1.290502028694	10	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 1
60		1.290698674736	10	1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1
60		1.290706142805	10	1 0 1 0 0 0 −1 0 −1 0 0 0 1 0 1 0 0 1 −1 1 −1 0 0 −1 1 −1 1 0 0 1 −1
60	**	1.291301378798	8	1 1 0 −1 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 1 0 −1 −1 0 1 1 0 −1 −1
60	b	1.291404119218	8	1 1 1 0 −1 −1 −1 −1 −1 −1 0 1 2 2 1 0 −1 −1 −1 −1 −1 −1 0 1 2 2 1 0 −1 −1 −1
60		1.291478472931	6	1 0 −1 1 0 −1 1 0 −1 1 0 −1 1 0 −1 1 0 −1 1 0 −1 1 0 −1 1 0 −1 1 0 −1 1
60		1.291557365229	8	1 1 0 −1 −1 0 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1
60	**	1.291889333827	5	1 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 −1 0 −1
60		1.292222571686	6	1 0 −1 0 0 1 0 −1 0 0 1 0 −1 0 0 1 0 −1 0 0 1 0 −1 0 0 1 0 −1 0 0 1
60	b	1.294957928223	8	1 1 1 0 0 −1 −1 −1 −1 −1 −1 0 0 0 0 1 1 1 1 1 0 0 0 0 −1 −1 0 0 0 0 1
60	**	1.297791895394	8	1 1 0 −1 −1 0 1 1 1 0 −1 −1 0 1 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1
60	b	1.298428340334	6	1 0 1 0 1 1 0 1 0 1 0 0 0 0 0 −1 0 −1 0 −1 −1 0 −1 0 −1 0 0 0 0 0 1
60	**	1.298514308222	3	1 0 −1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 −1
60		1.298575862081	6	1 0 −1 0 0 0 0 0 0 0 0 0 0 0 0 1 0 −1 0 0 0 0 0 0 0 0 0 0 −1 0 1
60	**	1.299264244307	4	1 1 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1
62		1.244598861428	6	1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1
62		1.251520881744	8	1 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 1
62	b	1.252883509372	7	1 1 0 0 0 0 0 −1 −1 0 0 −1 −1 0 1 1 0 0 1 1 0 −1 −1 −1 −1 −1 −1 0 1 1 1 1
62		1.253129657745	7	1 0 0 0 0 0 −1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 −1
62		1.254322870881	8	1 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 1
62		1.255471485039	6	1 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 1
62		1.256812463699	5	1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 −1 0 0 −1

62	b	1.257678956991	7	1	1	0	0	0	0	0	−1	−1	−1	−1	0	0	0	1	1	1	1	0	0	0	−1	−1	−1	−1	−1	−1	0	1	1	1	1
62		1.258277491621	7	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1
62		1.259211666083	7	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	−1	0	0	0	0	0	0	0	0	0	0	0	0	−1
62		1.259439009697	6	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1
62		1.260081371015	7	1	0	0	0	0	0	0	0	0	0	0	0	−1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	−1
62		1.260143325512	6	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1
62		1.261039389839	7	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	0	0	0	0	0	0	−1
62		1.261120453503	7	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	−1	0	0	0	0	0	0	0	0	0	0	−1
62		1.262177761213	7	1	0	0	0	0	0	0	0	0	0	−1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	−1
62		1.262739624863	6	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1
62		1.262977765160	7	1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	−1	0	0	0	0	0	0	0	0	−1
62		1.262989350570	7	1	0	0	0	0	0	0	0	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	−1
62		1.264045948708	6	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1
62		1.264062691441	5	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	−1
62	b	1.274013743951	7	1	1	1	0	−1	−1	−1	−1	−1	−1	0	1	2	2	1	0	−1	−1	−1	−1	−1	−1	0	1	2	2	1	0	−1	−1	−1	−1
62		1.274938359464	6	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1
62		1.274960793159	5	1	0	0	1	−1	0	0	−1	0	0	0	0	1	0	0	1	−1	0	0	−1	0	0	0	0	1	0	0	1	−1	0	0	−1
62		1.275703877077	9	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	0	0	0	0	0	0	0	0	1
62		1.275746936760	5	1	0	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
62		1.277159925856	9	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	1
62	b	1.277425795420	7	1	1	0	0	0	−1	−1	−1	−1	0	1	1	1	1	0	−1	−1	−1	−1	0	1	1	1	1	0	−1	−1	−1	−1	0	1	1
62	b	1.278247029007	5	1	1	1	1	0	0	−1	−1	−1	−1	0	0	1	1	1	1	0	0	−1	−1	−1	−1	0	0	1	1	1	1	0	0	−1	−1
62		1.278734474285	7	1	0	0	0	0	0	0	0	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
62		1.278750930677	9	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	0	0	0	0	0	0	0	0	0	0	1
62	**	1.278969119714	8	1	0	1	0	0	0	0	0	0	0	−1	1	−1	0	0	−1	0	0	0	0	1	−1	1	0	0	1	0	0	0	0	−1	1
62		1.280808709011	9	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1	1	0	−1	−1	−1	0	1	1	0	−1	−1	0	1	1	0	−1	−1	0	1
62		1.281052526268	8	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
62		1.282275740033	9	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	0	0	0	0	0	0	0	0	0	0	0	0	1
62		1.283975618424	7	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
62		1.284156631967	9	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1
62		1.284471658861	8	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1
62		1.284542670355	5	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
62	b	1.284584612639	9	1	1	1	0	−1	−1	−1	0	0	0	0	0	1	0	0	−1	−1	0	0	1	0	0	0	0	1	0	0	−1	−1	0	0	1
62	b	1.284716249936	5	1	0	−1	0	0	0	0	0	0	0	0	1	0	−1	0	0	0	0	1	0	−1	0	1	0	−1	0	0	0	0	1	0	−1
62		1.285183489377	11	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	0	0	0	0	0	0	1
62		1.285509147456	10	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
62		1.285560388545	8	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
62		1.285663487884	10	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	−1

62		1.285789163161	10	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 −1
62		1.285863207287	11	1 1 1 0 −1 −1 −1 0 0 0 0 0 1 1 1 0 −1 −1 −1 0 0 0 0 0 1 1 1 0 −1 −1 −1 −1
62		1.286057305365	9	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 −1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
62		1.286268303454	10	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 −1
62	b	1.286310561328	11	1 1 0 0 0 −1 −1 0 0 0 1 1 0 0 0 −1 −1 0 0 −1 0 1 0 0 1 0 −1 0 0 −1 0 1
62		1.286707754113	10	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1
62		1.286820564153	11	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 1
62	b	1.287123533290	9	1 0 1 0 0 0 −1 0 −1 0 0 0 1 1 1 1 0 0 −1 −1 −1 −1 0 0 1 1 1 1 0 0 −1 −1
62		1.287148701204	9	1 0 0 0 0 0 0 0 0 0 −1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
62		1.287806083106	11	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 −1 0 0 0 0 1
62		1.287903260905	9	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
62	b	1.288055521652	7	1 0 1 0 0 1 −1 1 −1 0 0 −1 1 −1 1 0 0 1 −1 1 −1 0 0 −1 1 −1 1 0 0 1 −1 1
62	∗	1.288236206497	5	1 0 0 1 0 0 0 0 0 −1 −1 0 −1 −1 0 0 0 0 1 1 0 1 1 0 0 0 0 0 −1 0 0 −1
62	b	1.288418208184	11	1 1 1 0 −1 −1 −1 0 0 0 0 0 1 1 1 0 −1 −1 −1 0 0 0 0 0 1 0 0 −1 −1 0 0 1
62		1.288427544595	11	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 1
62		1.288443469725	10	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 −1
62		1.288561849720	6	1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 −1
62		1.288607761362	7	1 0 0 0 0 0 −1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
62		1.288821310139	11	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 −1 0 0 1
62		1.289010367264	11	1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 −1 0 1 1 0 −1 −1 0 1
62		1.289191756301	11	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 −1 1
62	b	1.289561238506	11	1 1 1 0 −1 −1 −1 0 0 0 0 0 1 1 1 0 −1 −1 −1 −1 −1 −1 0 1 2 2 1 0 −1 −1 −1 −1
62		1.289578079808	9	1 0 0 0 0 0 0 0 0 0 0 0 0 0 −1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
62		1.289729955686	10	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 −1
62	b	1.289892481549	11	1 0 1 0 0 0 −1 0 −1 0 0 0 1 0 1 0 0 0 −1 0 −1 0 0 0 1 1 1 1 0 0 −1 −1
62		1.290146946870	11	1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 −1 0 1
62		1.290188795697	7	1 1 0 −1 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1
62		1.290291705898	10	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 1
62		1.290394501894	8	1 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 −1
62		1.290444149501	10	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 −1
62		1.290838744499	9	1 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
62	∗	1.291382684887	9	1 0 −1 1 0 −1 1 0 −1 1 0 −1 1 0 −1 1 0 −1 1 0 −1 1 0 −1 1 0 −1 1 0 −1 0 1
62		1.291441197926	9	1 1 0 −1 −1 0 1 1 0 −1 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1
62		1.291525030588	9	1 0 0 0 0 0 0 0 0 0 0 0 −1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
62		1.291868633146	7	1 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
62	∗	1.291940705476	9	1 1 0 −1 −1 0 0 0 0 1 1 0 −1 −1 0 0 0 0 1 1 0 −1 −1 0 0 0 0 1 1 0 −1 −1
62		1.292277360733	6	1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
62		1.292380876770	7	1 0 0 0 −1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1

62	b	1.293689599847	5	1 1 0−1−1 0 0 0 0 1 1 0−1−1 0 0 0−1 0 1 1 0−1 0 0 0−1 0 1 1 0−1
62	**	1.294450052440	9	1 0−1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0−1
62		1.297037676402	6	1 1 0 0 0 0 1 1 0−1−1 0 1 1 0−1−1 0 1 1 0−1−1 0 1 1 1 0−1 0 1 1
62		1.297963122576	6	1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0−1−1−1
62	**	1.298012697894	3	1 1 0 1 1 0 0 0 0−1−1 0−1−1 0 0 0 0 0 1 0 0 1 0 0 0 0 0−1 0 0−1
62		1.299507549580	9	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0−1−1−1−1−1−1−1−1−1−1−1−1−1−1−1−1
64		1.246918808403	4	1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0−1−
64	b	1.250924464000	9	1 1 1 0 0−1−1−1 0 0 0 0 0−1−1 0 1 1 1 1 0−1−1−1−1−1 0 0 0 0 1 1
64		1.252259930126	6	1 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0−1 0 0 0 0 0 0−
64	b	1.252785187146	8	1 1 1 1 0 0−1−2−2−2−1 0 1 2 3 3 2 0−2−3−3−3−2 0 2 4 4 3 2 0−2−4−
64		1.253629337816	8	1 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0−1 0 0 0 0 0 0 0 0 0 0−
64		1.253801022820	5	1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0
64		1.254705574543	7	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
64	b	1.256053579322	6	1 0 1 0 0 1−1 1−1 0 0−1 1−1 1 0 0 1−1 1−1 0 0−1 1−1 1 0 0 0−1 0−
64		1.258751618297	7	1 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0
64		1.259486793306	8	1 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0−1 0 0 0 0 0 0 0 0 0 0 0 0−
64		1.260792371381	8	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0−1 0 0 0 0 0 0 0 0 0 0 0 0 0 0−
64		1.260845566367	6	1 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0−1 0 0 0 0−
64	b	1.261519394031	8	1 0 1 0 0 0−1 0−1 0 0 1 1 1 1 0 0−1−1−1−1 0 0 0 1 0 1 0 0 0−1 0−
64		1.261806449479	7	1 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
64		1.262551237880	6	1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0−1 0 0−
64	b	1.274564591470	8	1 1 0 0 0−1−1 0 0 0 1 0−1 0 0−1 0 1 0 0 1 0−1 0 0−1 0 1 0 0 1 0−
64		1.276633074201	9	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0−
64	b	1.277496544947	10	1 1 0−1−1 0 1 1 0−1−1 0 1 1 0−1−1 0 1 1 0−1−1 0 1 1 0−1−1−1 0 1
64	b	1.278046827072	10	1 1 0−1−1 0 1 1 0−1−1 0 1 1 0−1−1 0 1 1 0−1−1 0 1 1 0−1 0 1 1 0−
64	b	1.279498634670	10	1 1 0−1−1 0 1 1 0−1−1 0 1 1 0−1−1 0 1 1 0−1−1 0 1 1 1 0−1−1 0 1
64	b	1.280392211600	10	1 1 0−1−1 0 1 1 0−1−1 0 1 1 0−1−1 0 1 1 0−1−1 0 1 0−1−1 0 1 1 0−
64		1.280764259179	7	1 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0−
64		1.281456703223	5	1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0−
64	b	1.281795308650	10	1 1 0 0 0−1−1 0 0 0 1 1 0 0 0−1−1 0 0 0 1 0−1 0 0−1 0 1 0 0 1 0−
64	b	1.282005358260	8	1 1 0−1−1 0 1 1 0−1 0 1 1 0−1−1 0 1 1 0−1−1 0 1 1 0−1−1 0 1 1 0−
64		1.282677163920	10	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0
64		1.284014398800	10	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0
64	**	1.284368847918	4	1 1 1 1 1 0 0 0−1−1−1−1−1 0 0 0 0 0 0 0 1 1 1 0 0−1−1 0 0 0 0 0−
64		1.285226835617	9	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0−
64		1.285394832562	9	1 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0−
64	b	1.285791738509	12	1 1 0 0 0−1−1 0 0 0 1 1 0 0 0−1−1 0 0 0 1 1 0 0 0−1−1−1−1 0 1 1

64		1.285811530402	11	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 −
64	b	1.286154577276	10	1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1
64	b	1.286636704956	8	1 1 0 −1 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1
64	**	1.286646742999	4	1 0 −1 0 0 1 0 −1 0 0 1 0 −1 0 0 1 1 −1 −1 −1 1 2 0 −1 −2 0 2 1 0 −2 −1 1
64	b	1.287540780029	10	1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −
64		1.288309527609	8	1 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
64	b	1.288451012888	12	1 1 0 0 0 −1 −1 0 0 0 1 1 0 0 0 −1 −1 0 0 0 1 1 0 0 0 −1 −1 0 0 0 1 0 −
64	b	1.289151170343	8	1 1 0 −1 −1 0 1 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1
64		1.289290544186	9	1 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 −
64		1.289467483862	10	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0
64		1.289678644705	10	1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1
64	**	1.289801641641	10	1 0 −1 1 1 −2 0 2 −2 −1 3 −1 −2 3 0 −3 2 1 −3 1 2 −2 0 2 −1 −1 1 −1 −1 1 0 0
64		1.289831566943	10	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
64		1.289960564610	11	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 −
64	b	1.290066372212	8	1 1 0 0 0 −1 −1 −1 −1 0 1 1 1 1 0 −1 −1 −1 −1 0 1 1 1 1 0 −1 −1 −1 −1 0 1 1
64		1.290091156250	11	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 −
64		1.290174315600	11	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 −
64		1.290280734667	7	1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 −
64		1.290456051673	10	1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1 0 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −
64		1.290831126027	6	1 0 −1 0 0 1 0 −1 0 0 1 0 −1 0 0 1 0 −1 0 0 1 0 −1 0 0 1 0 −1 0 0 1 0 −
64	**	1.291549687407	8	1 0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 −
64	**	1.291843157128	11	1 1 1 1 0 0 0 0 0 0 −1 −1 −1 −1 0 0 0 0 0 0 1 1 1 1 0 0 0 0 0 0 −1 −1 −
64		1.291880932079	6	1 0 −1 1 0 −1 1 0 −1 1 0 −1 1 0 −1 1 0 −1 1 0 −1 1 0 −1 1 0 −1 1 0 −1 1 0 −
64		1.292034196809	8	1 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
64	**	1.292407280968	10	1 1 1 1 0 −1 −1 −1 −1 0 0 0 1 0 0 0 −1 −1 0 0 1 2 1 1 0 −1 −1 −1 −1 0 0 0
64	**	1.292750021121	6	1 1 1 1 0 −1 −1 −1 −1 0 0 0 0 −1 −1 −1 −1 0 1 1 2 2 1 1 0 −1 −1 −1 −1 0 0 0
64	**	1.294044120166	10	1 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 −
64		1.295287177117	7	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 −1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 −1
64	b	1.295845457388	8	1 2 2 1 0 −1 −2 −3 −3 −2 −1 −1 −1 0 1 2 3 3 2 1 0 −1 −1 0 1 1 0 −1 −1 −1 −1 0
64	**	1.296043065491	4	1 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 −1 0 0 0 0 0 0 0 0 0 0 −
64	*	1.298175455504	12	1 1 1 0 −1 −2 −2 −1 0 1 1 1 0 0 0 0 0 0 0 0 1 1 1 0 −1 −2 −2 −1 0 1 1 1
64	**	1.299097527219	6	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 −1 0 0 0 0 0 −
64	b	1.299689675856	8	1 1 0 0 0 0 1 1 0 0 0 −1 0 1 0 0 0 −1 −1 0 0 0 1 0 −1 −1 −1 0 1 1 0 −1 −
64	**	1.299695644404	4	1 1 0 0 1 0 −1 0 0 −1 0 1 0 0 1 0 −1 0 0 −1 0 0 −1 0 1 0 0 1 0 −1 0 0 −

2. New Polynomials with Smallest Measure

The following table lists all the new polynomials having Mahler measure less than 1.28. The ke gives the meanings of the codes appearing in the second column.

D		M	ν	Coefficients
32	*	1.236083368052	4	1 1 1 1 0 −1 −1 −2 −1 0 0 1 1 0 0 0 −1
32	*	1.249688298465	4	1 2 2 1 0 −1 −1 −1 0 1 2 2 2 1 0 −1 −1
46	**	1.251881911571	5	1 1 1 0 0 −1 −1 −1 −1 −1 −1 −1 −1 0 1 2 2 2 1 0 −1 −1 −1 −1
36	*	1.252046815621	4	1 0 0 0 0 1 −1 0 0 0 0 −1 1 −1 0 0 0 1 −1
54	*	1.258079413341	7	1 1 0 0 0 −1 −1 0 0 −1 0 0 −1 0 1 0 0 1 0 −1 0 0 −1 0 1 0 0 1
36	*	1.258484141432	4	1 1 0 −1 −1 −1 0 1 1 1 0 −1 −1 0 1 1 0 −1 −1
38	*	1.264139103163	5	1 1 1 1 0 −1 −1 −2 −2 −1 −1 −1 0 0 0 1 1 1 1 1
34	*	1.265417146469	4	1 0 1 1 0 1 0 0 1 −1 1 0 0 1 0 0 1 −1
40	*	1.265651031527	6	1 0 1 1 0 1 0 0 0 −1 0 −1 −1 0 −1 0 0 0 1 1 1
42	*q	1.266378567293	5	1 1 1 1 0 0 −1 −1 −1 −1 −1 −1 −1 −1 0 0 1 1 1 1 0 1
36	**	1.266948881320	2	1 1 1 1 0 −1 −1 −1 −1 −1 0 1 1 1 1 0 −1 −1 −1
46	*	1.268085560681	3	1 0 0 1 0 1 0 0 1 −1 0 0 −1 0 −1 0 0 −1 0 0 0 0 0 1
44	*	1.268281868499	6	1 1 0 −1 0 0 −1 −1 0 1 1 1 0 0 0 0 −1 −1 0 0 0 0 1
40	*	1.269725034034	8	1 0 0 0 −1 1 0 0 0 −1 0 0 0 0 0 0 0 0 0 0 1
32	*	1.270932746058	4	1 1 1 0 0 −1 −1 −1 0 1 2 2 2 1 0 −1 −1
36	*	1.272095458988	6	1 1 1 0 −1 −1 −1 0 0 1 0 0 −1 −1 0 0 1 0 1
40	**	1.272780237961	2	1 1 1 1 0 −1 −1 −1 −1 0 1 1 1 0 −1 −1 −1 −1 0 1 1
34	*	1.273336860029	5	1 0 −1 0 0 0 0 0 0 0 0 1 0 0 0 −1 0 1
46	*	1.273654870918	5	1 1 0 −1 −1 0 0 −1 −1 0 1 1 1 1 0 −1 −1 −1 −1 −1 0 1 1 1
46	**	1.273870574713	2	1 0 −1 0 0 1 0 −1 0 0 1 0 0 −1 0 1 0 0 −1 0 1 0 0 −1
38	**	1.273972436701	3	1 1 1 0 −1 −2 −2 −1 0 1 1 0 −1 −1 −1 0 1 1 1 1
40	*	1.276128762738	6	1 1 1 1 1 0 0 0 −1 −1 −1 −1 −1 0 0 0 0 0 0 0 1
40	*	1.276955576128	2	1 0 1 0 1 1 1 0 1 0 1 0 0 −1 0 −1 0 −2 −1 −1 −1
48	**	1.277269415260	2	1 0 −1 0 0 1 0 −1 0 0 1 0 −1 0 1 0 0 −1 0 1 0 0 −1 0 1
40	*	1.277333466563	4	1 1 1 1 0 −1 −1 −1 −1 0 0 −1 −1 −1 −1 0 1 1 1 1 1
44	*	1.277465838963	6	1 0 −1 1 0 −1 1 0 −1 1 0 −1 1 0 −1 1 0 −1 1 0 −1 0 1
48	*	1.277760723777	4	1 0 0 0 1 0 0 0 0 1 0 0 −1 1 0 0 −1 0 0 0 0 −1 0 −1 1
48	*	1.278270773806	6	1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 −1

36	*	1.278286859233	6	1 1 0 −1 −1 −1 −1 0 1 1 1 1 0 −1 0 0 −1 0 1
34	**	1.278390605526	5	1 0 0 0 0 1 −1 0 0 0 0 0 0 0 0 0 0 −1
50	*	1.278680927394	7	1 1 0 −1 −1 0 1 0 −1 −1 0 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 0 −1
58	*	1.278934747587	6	1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 1 1 1 0 −1 −1 0 1 1 0 −1 −1 0 1 1 1
62	**	1.278969119714	8	1 0 1 0 0 0 0 0 0 0 −1 1 −1 0 0 −1 0 0 0 0 1 −1 1 0 0 1 0 0 0 0
46	**	1.279258997870	3	1 0 0 1 0 0 0 0 0 0 −1 0 0 0 0 0 0 0 0 0 0 0 0 −1
58	**	1.279320724447	7	1 0 0 1 0 0 0 0 0 −1 −1 −1 −1 −1 −1 0 0 0 1 1 1 1 1 1 0 0 0 −1 −1 −1
42	**	1.279404456563	6	1 1 0 0 0 −1 −1 −1 0 1 1 1 1 0 −1 −1 −1 −1 0 1 1 1
48	*	1.279464310958	4	1 1 1 1 0 0 0 0 0 0 −1 −1 −1 −1 −1 −1 −1 −1 0 0 1 1 1 1 1
36	*	1.279501364240	2	1 1 1 0 0 1 1 1 0 0 0 0 0 −1 −1 −1 −1 −1 −1
48	*	1.279702474008	4	1 1 1 1 1 1 0 0 −1 −1 −2 −2 −2 −2 −1 −1 0 0 1 1 1 1 1 1 1

3. Smallest Measures by Degree

For each degree $D \leq 64$, the following table shows the polynomial of degree D having the sm
Mahler measure among all primitive, irreducible, noncyclotomic polynomials of this degree. The "*
new polynomial found using our algorithm.

D		M	ν	Coefficients
8		1.280638156268	1	1 0 0 1 −1
10		1.176280818260	1	1 1 0 −1 −1 −1
12		1.227785558695	2	1 1 1 0 −1 −1 −1
14		1.200026523987	1	1 0 0 1 −1 0 0 −1
16		1.224278907222	2	1 1 0 −1 −1 0 1 1 1
18		1.188368147508	1	1 1 1 1 0 0 −1 −1 −1 −1
20		1.212824180989	2	1 1 0 0 1 1 0 −1 −1 −1 −1
22		1.205019854225	2	1 0 1 0 0 1 −1 1 0 0 1 −1
24		1.218855150304	2	1 0 0 0 0 1 0 −1 0 0 0 0 −1
26		1.223777454948	3	1 1 1 0 0 −1 −1 −1 −1 0 0 1 0 1
28		1.207950028412	2	1 1 1 1 0 0 0 −1 −1 −1 −1 0 0 0 1
30		1.225619851977	2	1 0 1 0 0 0 0 0 0 0 0 0 0 1 0 1
32	*	1.236083368052	4	1 1 1 1 0 −1 −1 −2 −1 0 0 1 1 0 0 0 −1
34		1.220287441693	3	1 0 1 0 0 1 −1 1 −1 0 0 −1 1 −1 0 0 −1 1
36		1.226493301473	2	1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 −1 −1
38		1.223447381400	3	1 1 0 −1 −1 0 0 −1 −1 0 1 1 0 0 1 1 0 −1 −1 −1
40		1.236249557300	3	1 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 1
42		1.230295468643	4	1 0 1 0 0 0 −1 1 −1 1 0 0 1 −1 1 −1 1 0 0 1 −1 1
44		1.236674812187	4	1 0 0 0 0 0 0 1 0 0 0 0 0 0 0 −1 0 0 0 0 0 0 −1
46		1.230743009076	3	1 0 0 1 0 1 0 0 1 0 0 0 0 0 0 −1 0 0 −1 0 −1 0 0 −1
48		1.232202952743	4	1 1 0 0 1 1 0 −1 −1 0 0 −1 −1 0 1 1 0 0 1 1 0 −1 −1 −1 −1
50		1.240379074717	3	1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 −1 −1
52		1.234348374876	4	1 1 0 −1 −1 0 0 −1 −1 0 1 1 0 0 1 1 0 −1 −1 0 0 −1 −1 0 1 1 1
54		1.236566917569	5	1 0 1 0 0 0 −1 1 −1 1 0 0 1 −1 1 −1 0 0 −1 1 −1 1 −1 0 0 −1 1 −1
56		1.238431627359	4	1 1 1 1 0 0 0 −1 −1 −1 −1 0 0 0 1 1 1 1 0 0 0 −1 −1 −1 −1 0 0 0 1
58		1.237684127894	5	1 1 0 0 0 −1 −1 −1 −1 0 0 0 1 1 1 1 0 0 0 −1 −1 −1 −1 0 0 0 1 0 0 1

60	1.240061859037	4	1 0 0 1 0 1 0 0 1 0 0 0 0 0 0 −1 0 0 −1 0 −1 0 0 −1 0 0 0 0 0 0
62	1.244598861428	6	1 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
64	1.246918808403	4	1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

4. Smallest Measures by Roots Outside the Unit Circle

In our investigation, we find polynomials with Mahler measure less than 1.3 having as many as lie outside the unit circle. For each $\nu \leq 12$, the following table gives the polynomial with ν roots the unit circle having the smallest known Mahler measure among all primitive, irreducible polynom same number of roots outside the unit circle.

D	M	ν	Coefficients
10	1.176280818260	1	1 1 0 −1 −1 −1
18	1.201396186235	2	1 1 1 0 0 −1 0 −1 0 −1
18	1.219446875941	3	1 1 1 0 −1 −1 −1 0 0 1
36	1.229482810200	4	1 1 1 0 0 −1 −1 −1 0 1 1 1 0 −1 −2 −1 0 1 1
54	1.236566917569	5	1 0 1 0 0 0 −1 1 −1 1 0 0 1 −1 1 −1 0 0 −1 1 −1 1 −1 0 0 −1 1 −1
54	1.239747974875	6	1 1 1 0 0 −1 −1 −1 0 0 0 0 1 1 1 1 1 0 −1 −2 −2 −2 −1 0 1 1 1 1
54	1.249896621001	7	1 0 0 0 0 0 0 0 0 0 −1 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 −1
62	1.251520881744	8	1 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0
64	1.250924464000	9	1 1 1 0 0 −1 −1 −1 0 0 0 0 0 −1 −1 0 1 1 1 1 0 −1 −1 −1 −1 −1 0 0 0 0 1
60	1.276573566904	10	1 0 −1 1 0 −2 1 1 −2 1 2 −2 0 2 −2 −1 2 −1 −1 2 0 −1 1 0 −1 0 0 0 0 0 1
58	1.282009994078	11	1 1 0 0 0 −1 −1 0 0 0 1 1 0 0 0 −1 −1 0 0 0 1 1 0 −1 −1 −1 −1 0 1 1
64	1.285791738509	12	1 1 0 0 0 −1 −1 0 0 0 1 1 0 0 0 −1 −1 0 0 0 1 1 0 0 0 −1 −1 −1 −1 0 1

5. Summary of Roots Outside the Unit Circle

For each degree $D \leq 64$, the following table shows the number of known primitive, irreducible polynomials with degree D and Mahler measure less than 1.3 having exactly ν roots outside the unit circle. The columns of the table correspond to different values of ν.

D	1	2	3	4	5	6	7	8	9	10	11	12	Total
8	1												1
10	5	2											7
12	1	4											5
14	4	5	2										11
16	1	12		1									14
18	7	8	8										23
20	4	22	3	6									35
22	4	12	27	4	1								48
24	2	29	3	11		1							46
26	7	11	21	11	11								61
28	1	15	14	28	1								59
30	4	9	34	20	8	2							77
32		9	7	34	7	7							64
34	1		17	18	30	8	2						76
36		4	7	34	3	20		1					69
38			10	16	25	8	12						71
40		3	4	27	10	23	2	3					72
42			7	12	15	6	8	2					50
44	1		1	22	5	23	4	6					62
46		1	5	10	30	9	23	7	3				88
48		2	1	21	2	24		17	1				68
50			1	8	14	10	16	8	8				65
52			1	7	8	19	4	15	4	3			61
54				4	13	13	13	3	15	1			62
56				6	5	15	7	11	4	6			54
58			1	1	14	11	20	9	25	9	4		94
60			2	5	4	18	3	13		18			63
62			1		9	11	20	7	19	9	13		89
64				5	2	8	6	15	5	16	5	3	65
Total	43	148	177	311	217	236	140	117	84	62	22	3	1560

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This dissertation was typeset with $\mathcal{AMS}$-LaTeX[†] by the author.

[†] $\mathcal{AMS}$-LaTeX is a document preparation system developed for the American Mathematical Society by Romesh Kumar, Frank Mittelbach, and Rainer Schöpf, with assistance from Michael Downes. It is based on $\mathcal{AMS}$-TeX, developed by Michael Spivak, and LaTeX, written by Leslie Lamport, which are extensions of TeX, the typesetting system written by Donald Knuth.