

Homework 8: Report

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1 Introduction

In this assignment we will describe the details of a dry-run of a homomorphic evaluation. The cleartext computation is as follows:

Given three bits $a, b, c \in \{0, 1\}$, output $AND(XOR(a, b), c)$.

2 Choose Input Bits and Security Parameter

- Input bits: $a = 1, b = 0, c = 1$.
- Security parameter: $\lambda = 128$.

3 Step 1: Generate Keys

Generate keys $sk \leftarrow \text{Gen}(1^\lambda)$.

The input: $\lambda = 3$

The details of the computation: Generate sk , an odd number with a length of λ^2 bits.

The resulting outcome of the step: $sk = 471$.

4 Step 2: Encrypt the Inputs

Encrypt the input $c_a \leftarrow \text{Enc}_{sk}(a)$, $c_b \leftarrow \text{Enc}_{sk}(b)$, $c_c \leftarrow \text{Enc}_{sk}(c)$.

The input: $a = 1, b = 0, c = 1$ and $sk = 471$

The details of the computation: For each input bit b , generate q_b , a "large" number with a length of λ^5 bits and r_b , a "small" even number with a length of λ bits. compute $c_b = p \cdot q_b + 2 \cdot r_b + b$ where $p = sk$.

The parameters for each input bit:

1. $q_a = 13227508180736365427652432425341829447350829616451546311965174612433249107$
2. $r_a = 4$

3. $q_b = 12002215933213370183707946188374698613314017595349859285056644318303495188$
4. $r_b = 4$
5. $q_c = 13702469671669046184834548173749448833879217564627451534214593734031651248$
6. $r_c = 6$

The resulting outcome of the step:

$$c_a = 6230156353126828116424295672336001669702240749348678312935597242456060329406$$

$$c_b = 5653043704543497356526442654724483046870902287409783723261679473920946233556$$

$$c_c = 6453863215356120753057072189835990400757111472939529672615073648728907737821$$

5 Step 3: Homomorphically evaluate the Aforementioned Cleartext Computation

Homomorphically evaluate the aforementioned cleartext computation to obtain a result ciphertext c_{res} .

The input: Encrypted values

$$c_a = 6230156353126828116424295672336001669702240749348678312935597242456060329406$$

$$c_b = 5653043704543497356526442654724483046870902287409783723261679473920946233556$$

$$c_c = 6453863215356120753057072189835990400757111472939529672615073648728907737821$$

The details of the computation: Homomorphically evaluate $\text{AND}(\text{XOR}(c_a, c_b), c_c)$ to obtain a result ciphertext c_{res} .

$$c_{res} = (c_a + c_b) \cdot c_c$$

The resulting outcome of the step: $c_{res} = 153222266277439889145067278920780277329449860799982992619964353065717569136826807959957008066704941488410702989174619469650071949675185407104459733704906$

6 Step 4: Decrypt

Decrypt to obtain a cleartext result $res = \text{Dec}_{sk}(c_{res})$.

The input: $c_{res} = 153222266277439889145067278920780277329449860799982992619964353065717569136826807959957008066704941488410702989174619469650071949675185407104459733704906$ and $sk = 471$

The details of the computation: Compute $res = \text{LSB}(c_{res} \bmod sk)$.

The resulting outcome of the step: $res = 1$.

7 Step 5: Verify the Result

Verify that the result is correct, i.e., check that $res = \text{AND}(\text{XOR}(a, b), c)$ if yes output res else output *Error*.

The input: $a = 1, b = 0, c = 1$ and $res = 1$

The details of the computation: $\text{AND}(\text{XOR}(a, b), c) = 1$ and therefore, $res = \text{AND}(\text{XOR}(a, b), c)$.

The resulting outcome of the step: $res = 1$.