

3.2)

```
# 3.2

import matplotlib.pyplot as plt
import numpy as np

array100 = np.arange(1, 101)

sum100 = np.sum(array100)

print("Sum av tallene til 100: ", sum100)
```

```
Sum av tallene til 100: 5050
```

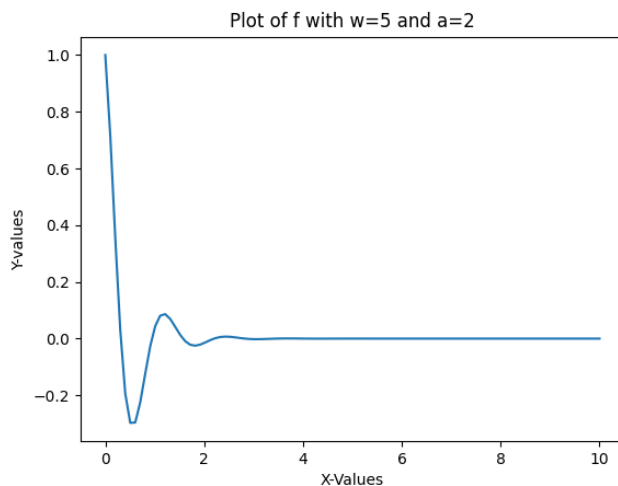
3.3)

```
# 3.3

w = 5
a = 2
time = np.linspace(0, 10, 100)
values = np.cos(w*time)*np.exp(-a*time)

plt.plot(time, values)
plt.title("Plot of f with w=5 and a=2")
plt.ylabel("Y-values")
plt.xlabel("X-Values")
plt.show()
```

Figure 1



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4) OPPGAVE 4

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$$|A| = \sqrt{4^2 + 3^2} = 5m$$

$$|B| = \sqrt{2^2 + (-3)^2} = \sqrt{13}m$$

ii)

$$A+B = 4m\hat{i} + 3m\hat{j} + 2m\hat{i} - 3m\hat{j} = 6m\hat{i}$$

$$A-2B = 4m\hat{i} + 3m\hat{j} - 2(2m\hat{i} - 3m\hat{j}) = 9m\hat{j}$$

iii)

$$A \cdot B = 4m\hat{i} \cdot 2m\hat{i} + 3m\hat{j} \cdot -3m\hat{j} = 8m^2\hat{i}^2 - 9m^2\hat{j}^2$$

$$A \cdot B = 8m^2 - 9m^2 = -1m^2$$

iv)

$$F \times B = (8N\hat{i} - 15N\hat{j}) \times (2m\hat{i} - 3m\hat{j})$$

$$F \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8N & -15N & 0 \\ 2m & -3m & 0 \end{vmatrix} = \hat{i}(0 \cdot -15N - 0 \cdot -3m) - \hat{j}(0 \cdot 8N - 0 \cdot 2m) + \hat{k}(8N \cdot -3m - -15N \cdot 2m)$$

$$F \times B = \hat{k}(-24Nm + 30Nm) = \hat{k} \cancel{6Nm} = \hat{k} 6Nm$$

v)

$$A-F = (4m-8N)\hat{i} + (3m+15N)\hat{j}$$

↖ m-N gir ikke mening.

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4b)

$$A = A \hat{a}$$

$$4m\hat{i} + 3m\hat{j} = \sqrt{4m^2 + 3m^2} \hat{a}$$

$$4m\hat{i} + 3m\hat{j} = 5m\hat{a}$$

$$\Rightarrow \hat{a} = \frac{4}{5}m\hat{i} + \frac{3}{5}m\hat{j}$$

4c)

$$K = \frac{1}{2}mv^2, m = 9 \text{ kg}$$

$$K = \frac{1}{2} \cdot 9 \text{ kg} \left(2 \frac{m}{s} \hat{i} \cdot 2 \frac{m}{s} \hat{i} + -3 \frac{m}{s} \hat{j} \cdot -3 \frac{m}{s} \hat{j} \right)$$

$$K = 58,5 \text{ kg} \frac{m^2}{s^2} = 58,5 \text{ J} = 58,5 \text{ Nm}$$

Hvis $v \rightarrow -v$ så hadde svaret vært det samme siden $v^2 = (-v)^2$

5i)

$$v(t): \quad v' = a$$

$$\int_0^t v'(\tau) d\tau = \int_0^t a(\tau) d\tau$$

$$v(t) - v(0) = \int_0^t a(\tau) d\tau$$

$$x(t): \quad \int_0^t x'(\tau) d\tau = \int_0^t v(\tau) d\tau$$

$$x(t) - x(0) = \int_0^t v(\tau) d\tau$$

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Si) enhet

Hastighet: m/s

Akselerasjon: $m/s^2 = m \cdot s^{-2}$

Sii) Hvis a er konstant

$$v(t) - v(0) = \int_0^t a(\tau) d\tau$$

$$v(t=0) = v(0) = v_0$$

$$\Rightarrow v(t) = v_0 + \int_0^t a(\tau) d\tau$$

Siden a er konstant

$$\Rightarrow v(t) = v_0 + a \int_0^t \mathbf{1} d\tau$$

$$v(t) = v_0 + at \quad \square$$

$$x(t) - x(0) = \int_0^t v(\tau) d\tau$$

a er konstant, $x_0 = x(0) = x(t=0)$

$$\Rightarrow x(t) = x_0 + \int_0^t (v_0 + a\tau) d\tau$$

$$\Rightarrow x(t) = x_0 + v_0 t + a \int_0^t \tau d\tau$$

$$\Rightarrow x(t) = x_0 + v_0 t + a \frac{t^2}{2}$$

$$\Rightarrow x(t) = x_0 + v_0 t + \frac{1}{2} a t^2 \quad \square$$

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6i)

$$a = c_1 t - c_2 t^3 \quad c_1 = 3,0 \text{ m/s}^3 \quad c_2 = 0,8 \text{ m/s}^5$$

$$v_0 = 0$$

~~$$v(t) = v_0 + \int_0^t (c_1 \tau - c_2 \tau^3) d\tau$$~~

$$v(t) = 0 + \frac{1}{2} c_1 t^2 - \frac{1}{4} c_2 t^4$$

$$x(t) = x(0) + \int_0^t \left(\frac{1}{2} c_1 \tau^2 - \frac{1}{4} c_2 \tau^4 \right) d\tau$$

$$x(t) = 0 + \frac{1}{6} c_1 t^3 - \frac{1}{20} c_2 t^5$$

$$x(2\text{ s}) = \frac{1}{6} \cdot 3 \frac{\text{m}}{\text{s}^3} \cdot 2^3 \text{ s}^3 - \frac{1}{20} \cdot 0,8 \frac{\text{m}}{\text{s}^5} \cdot 2^5 \text{ s}^5$$

$$x(2\text{ s}) = 2,72 \text{ m} = \underline{\underline{2,7 \text{ m}}}$$

6ii)

Gjennomsnittshastighet:

$$\frac{2,7 \text{ m} - 0 \text{ m}}{2 \text{ s} - 0 \text{ s}} = 1,35 \text{ m/s}$$

6iii)

Analytisk Areal ^{av} $\int v = x \Rightarrow \underline{\underline{2,72 \text{ m}}}$

Numerisk analyse i ~~Python~~ Python

med trapesintegrasjon, gir $2,7199 \text{ m} \approx \underline{\underline{2,72 \text{ m}}}$
og $N=100$

4.6iii)

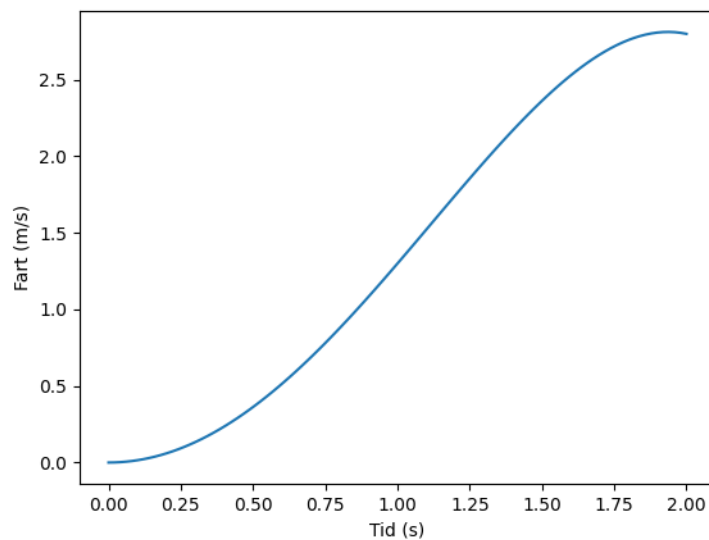
```
# Task 6

# 6.3
t0 = 0
tf = 2
N = 100 # Antall punkter
c1 = 3
c2 = 0.8
t = np.linspace(t0, tf, N)
v = 0.5*c1*t**2-0.25*c2*t**4
plt.plot(t, v)
plt.ylabel("Fart (m/s)")
plt.xlabel("Tid (s)")
plt.show()

def trapezoidal(f, a, b, n):
    h = float(b - a) / n
    s = 0.0
    s += f(a)/2.0
    for i in range(1, n):
        s += f(a + i*h)
    s += f(b)/2.0
    return s * h

print("Areal: ", trapezoidal(lambda t: 0.5*c1*t**2-0.25*c2*t**4, 0, 2, N))
```

Figure 1



Areal: 2.7199866688000003

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7)

$$\frac{36.2 \text{ m}}{1.7 \text{ s}} = \frac{362}{17} \text{ m/s} \approx \underline{\underline{21 \text{ m/s}}}$$

Oppgave 5

8)

$$a_t = 4.0 \text{ m/s}^2 \quad v_t(0) = 0 \quad x_t(0) = 0$$

$$v_t(t) = 0 \text{ m/s} + 4.0 \text{ m/s}^2 \cdot t = 4.0 \frac{\text{m}}{\text{s}^2} \cdot t$$

$$x_t(t) = 0 \text{ m} + 0 \frac{\text{m}}{\text{s}} t + \frac{1}{2} \cdot 4.0 \text{ m/s}^2 \cdot t^2 = 2.0 \text{ m/s}^2 \cdot t^2$$

$$a_p = 0.0 \text{ m/s}^2$$

$$x_p(t) = 0 + (x_t - x) + 0$$

$$x_t(t) = x_p(t)$$

$$2t^2 = xt - x \Rightarrow 2t^2 = x(t-1)$$

$$\Rightarrow \frac{2t^2}{t-1} = x$$

Må finne nullpunktet til den deriverte av $\frac{2t^2}{t-1}$ for å finne minste konstante hastighet

$$\frac{2t^2}{t-1} \frac{d}{dt} = \frac{4t}{t-1} - \frac{2t^2}{(t-1)^2}$$

$$\Rightarrow \frac{4t}{t-1} - \frac{2t^2}{(t-1)^2} = 0 \Rightarrow t = 2$$

$$\text{Putter } t=2 \text{ inn i } \frac{2t^2}{t-1} \Rightarrow \frac{2 \cdot 2^2}{2-1} = x = \underline{\underline{8 \text{ m/s}}}$$

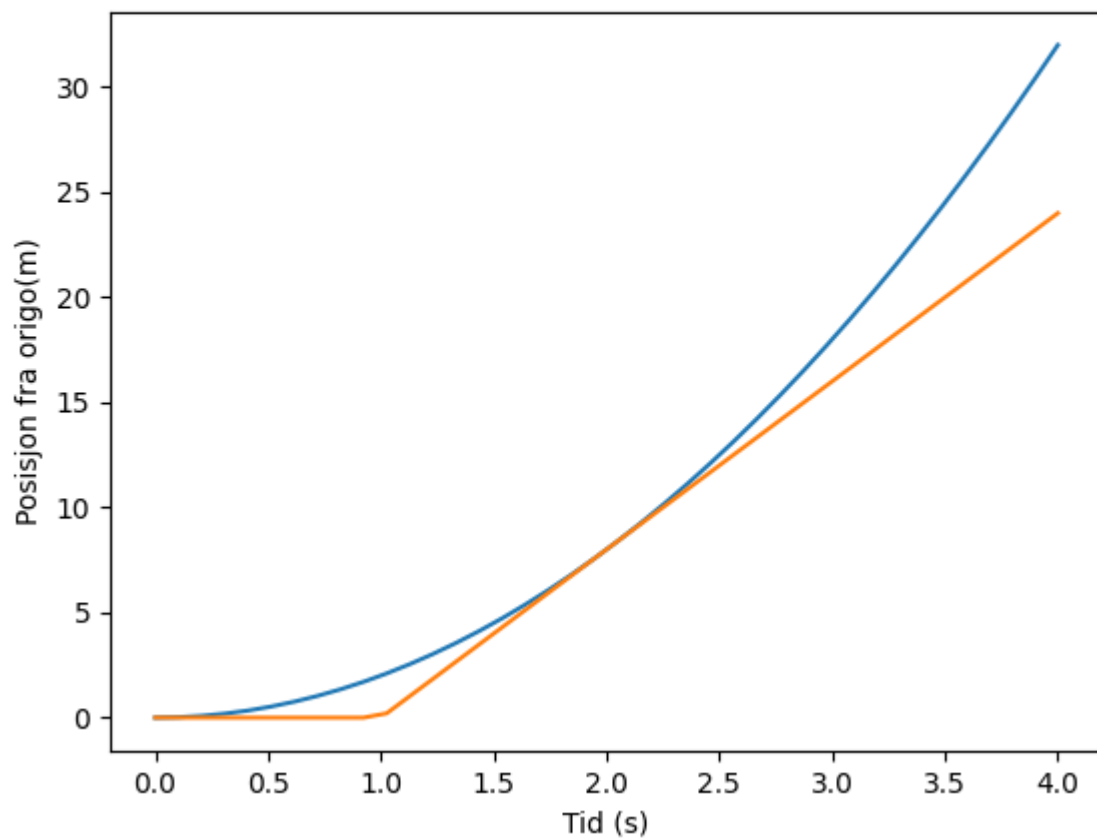
5.8)

```
# Task 8
t = np.linspace(start=0, stop=4, num=40)
pt = 2*t**2
pp = 8*t-8
pp = np.hstack((np.zeros(10), pp[10:]))

plt.plot(t, pt, t, pp)

plt.ylabel("Posisjon fra origo(m)")
plt.xlabel("Tid (s)")
plt.show()
```

Figure 1



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9) i)

$$t = 4 \text{ min} = 240 \text{ s}$$

$$v(240) = 30.0 \text{ m/s} \cdot \frac{(120)^2 \text{ s}^2}{(240)^2 \text{ s}^2} = \underline{\underline{7.50 \text{ m/s}}}$$

När $t \rightarrow \infty$ vil $v(t) = 0$ fordi $\frac{1}{t^2} = 0$
när $t \rightarrow \infty$.

9) ii)

$$a(t) = v'(t) = \frac{v_0 t_1^2}{t^2} \frac{d}{dt} = \frac{-2 \cdot v_0 \cdot t_1^2}{t^3}$$

Graf i python

for $t \geq 120 \text{ s}$

~~men~~

$$a(t) = \begin{cases} 0, & t < t_1 \\ \frac{-2 \cdot v_0 \cdot t_1^2}{t^3}, & t \geq t_1 \end{cases}$$

5.9i)

```
# Task 9

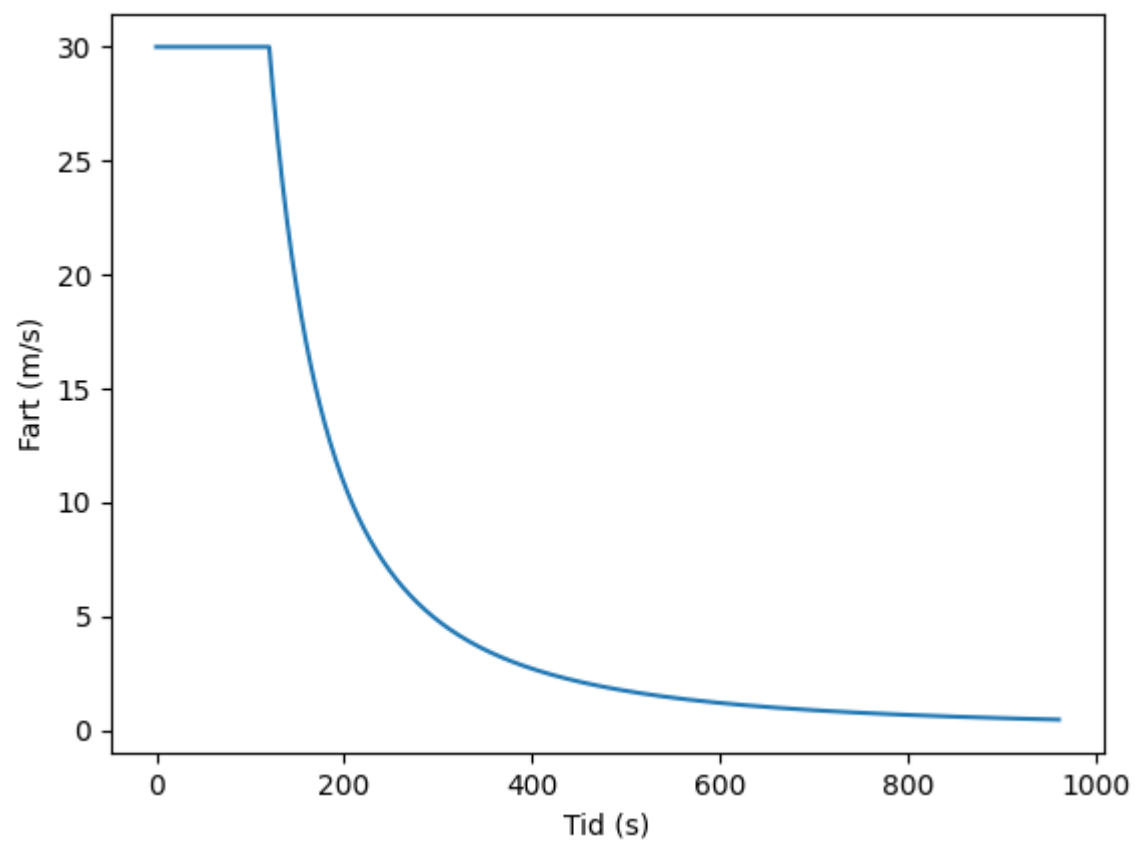
# 9.1
t0 = 0
t120 = 120
v0 = 30

t = np.linspace(t120, 960, 840)
b = v0*t120**2/t**2

a = np.ones(120)*30

t = np.linspace(0, 960, 960)
c = np.hstack((a, b))

plt.plot(t, c)
plt.ylabel("Fart (m/s)")
plt.xlabel("Tid (s)")
plt.show()
```



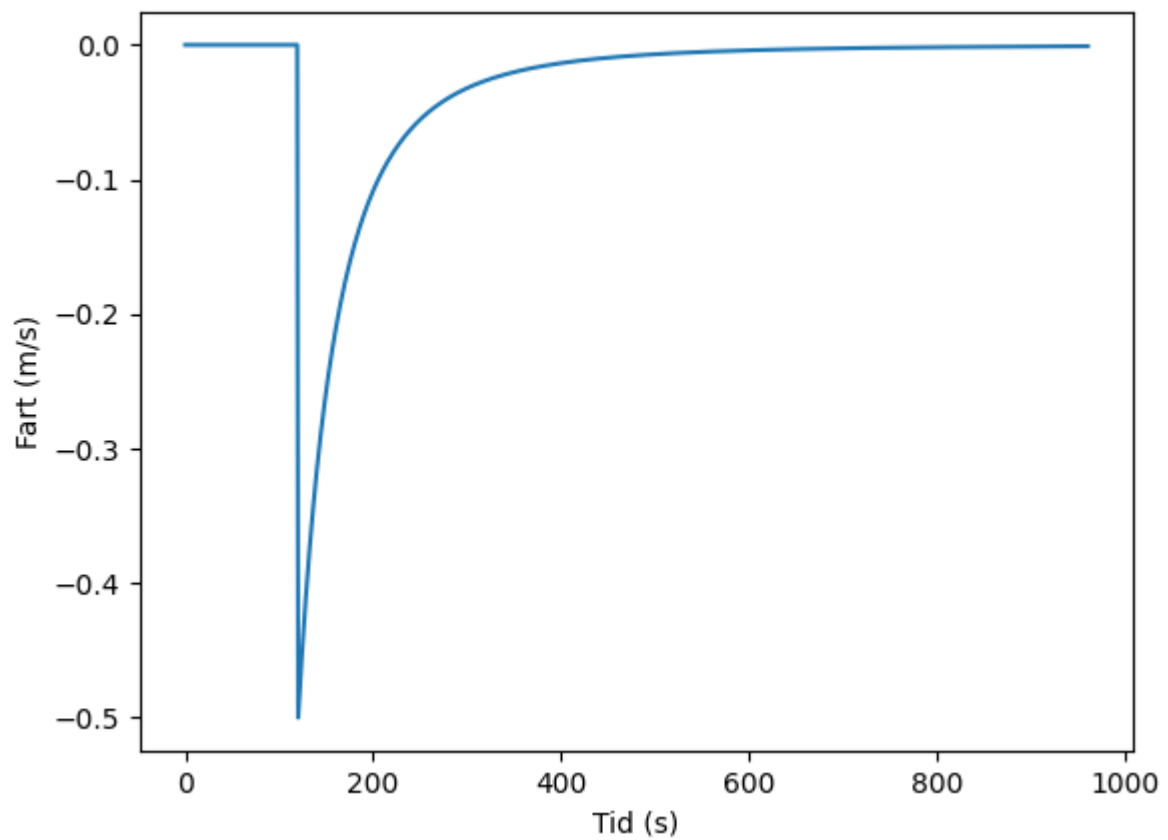
5.9ii)

```
# 9.2

a = np.ones(t120)*0
t = np.linspace(t120, 960, 840)
b = -2*v0*t120**2/t**3

t = np.linspace(0, 960, 960)
c = np.hstack((a, b))

plt.plot(t, c)
plt.ylabel("Fart (m/s)")
plt.xlabel("Tid (s)")
plt.show()
```



9iii)

$$S(t) = S(0) + \int_0^t v \, dt = \int_0^{120} V_0 + \int_{120}^t \frac{V_0 t_1^2}{t^2} \, dt$$

$$S(t) = [V_0 t]_0^{120} + \left[\frac{-V_0 t_1^2}{t} \right]_{120}^t$$

$$S(t) = \begin{cases} V_0 t, & t \leq 120 \\ V_0 \cdot 120 + \left(\frac{-V_0 t_1^2}{t} + \frac{V_0 t_1^2}{120} \right), & t > 120 \end{cases}$$

$$S(240) = 30 \cdot 120 + \left(\frac{-30 \cdot 120^2}{240} + \frac{30 \cdot 120^2}{120} \right) = \underline{\underline{5400 \, \text{m}}} \\ = \underline{\underline{5,4 \, \text{km}}}$$

$$S(t \rightarrow \infty) = 30 \cdot 120 + \left(0 + \frac{30 \cdot 120^2}{120} \right) = \underline{\underline{7,2 \, \text{km}}}$$

5.9iii)

```
# 9.3

t0 = 0
t120 = 120
v0 = 30

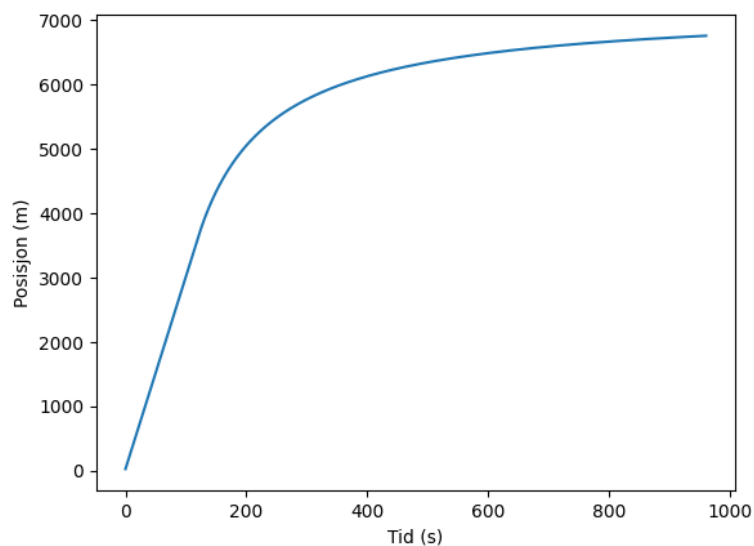
t = np.linspace(t120, 960, 840)
b = v0*t120**2/t**2

a = np.ones(120)*30

t = np.linspace(0, 960, 960)
c = np.hstack((a, b))

def integration(arrayC):
    temp = 0
    integration = []
    for i in range(len(arrayC)):
        integration.append(temp+arrayC[i])
        temp += arrayC[i]
    return integration

plt.plot(t, integration(c))
plt.ylabel("Posisjon (m)")
plt.xlabel("Tid (s)")
plt.show()
```



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10)

$$V_{oy} = 150 \text{ m/s} \cdot \sin(30^\circ)$$

$$x_y(t) = 0 \text{ m} + 150 \cdot \sin(30^\circ) t + \frac{1}{2} \cdot -9,8 \cdot t^2$$

$$0 = 150 \cdot \sin(30^\circ) t + \frac{1}{2} \cdot -9,8 t^2$$

$$\Rightarrow t_1 = 0 \quad t_2 = \underline{\underline{15,3 \text{ s}}}$$

Det tar ~~18,8~~ 15 s før kula treffer bakken.

Lengde i Horizontal retning

$$x_x(t) = 150 \text{ m/s} \cdot \cos(30^\circ) \cdot t$$

$$x_x(15 \text{ s}) = 150 \text{ m/s} \cdot \cos(30^\circ) \cdot 15 \text{ s} = \underline{\underline{2.0 \text{ km}}}$$

11)

Her behøves kun å regne akselerasjonsleddet
for $x_x(t) = x_0 + v_0 t + \frac{1}{2} a t^2$

$$\frac{1}{2} \cdot 0.03 \text{ m/s}^2 \cdot 15^2 \text{ s}^2 = \underline{\underline{3,4 \text{ m}}}$$

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12)

$$X(t) = 10 \text{ m/s} \cdot t + \frac{1}{2} \cdot -9,8 \frac{\text{m}}{\text{s}^2} t^2$$

$$X(t) = 5$$

$$\Rightarrow \frac{1}{2} \cdot -9,8 \frac{\text{m}}{\text{s}^2} t^2 + 10 \text{ m/s} \cdot t - 5 = 0$$

$$\Rightarrow t_1 = 0.876100 \text{ s}$$

$$t_2 = 1.164715 \text{ s}$$

$$t_2 - t_1 = \underline{\underline{0.29 \text{ s}}}$$