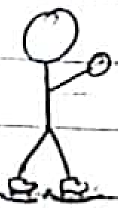
Øving 4 - Hallvard - TFY4125

Grunnleggende

1)



Om bevegelsesmengden er bevart kommer an på hvilket system man ser på. Hvis man kun ser på personen og isklumpen er det ikke bevart fordi f.eks tyngdekraften påvirker systemet. Anten at bevegelsesmengden er bevart.

$$P_{start,x} = 0 = m_k \cdot v_0 \cdot \cos(\theta) + m_d \cdot v_d$$

$$\Rightarrow \underline{\underline{\frac{-m_k \cdot v_0 \cdot \cos(\theta)}{m_d} = v_d}}$$

4.2) a)

$$F = At^2 \Leftrightarrow A = \frac{F}{t^2} = \frac{781,25\,N}{1,25^2\,s^2} = \underline{\underline{500\,N/s^2}}$$

4,2 b)

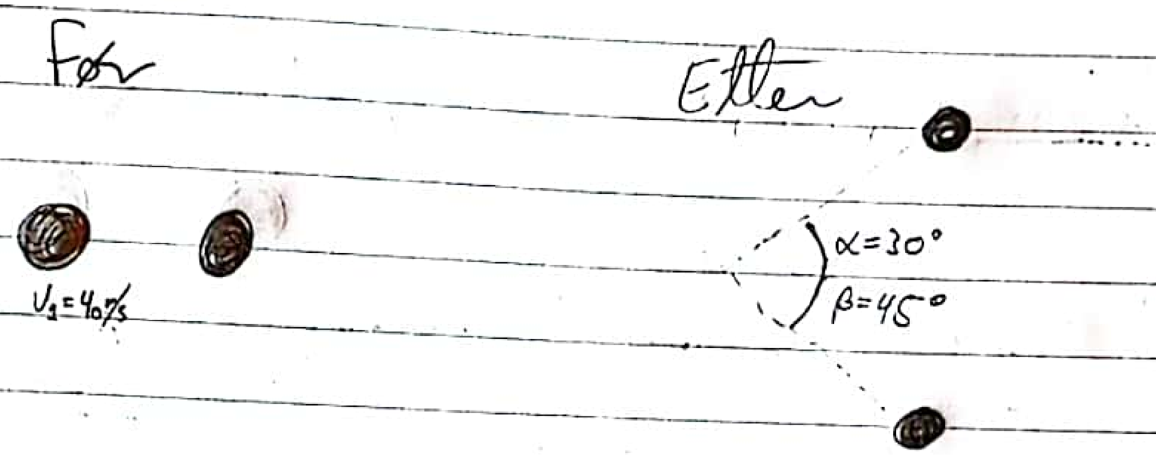
$$\vec{J} = \int_{t_1}^{t_2} \sum F = \int_{t_1}^{t_2} At^2\,dt = A\left[\tfrac{1}{3}t^3\right]_{t_1}^{t_2} = \frac{A}{3}\left[2^3 - 1,5^3\right]$$

$$J = \frac{500}{3} \cdot (2^3 - 1,5^3) = 770,83 = \underline{\underline{771\,Ns}}$$

4.2c)

$770Ns / 2150kg = \underline{\underline{0{,}358 m/s}}$

4.3a)



b)

$$m \cdot v_1 + m \cdot 0 = m \cdot v_1 \cdot \cos(30°) + m \cdot v_2 \cos(45°)$$

$$40 m/s = v_1 \cdot \cos(30°) + v_2 \cdot \cos(45°)$$

$$0 = m\, v_1 \sin(30°) - m\, v_2 \sin(45°)$$

$$\Rightarrow \quad v_2 = v_1 \frac{\sin(30°)}{\sin(45°)}$$

$$\Rightarrow \quad 40 m/s = v_1 \cos(30°) + v_1 \frac{\sin(30°)}{\sin(45°)} \cos(45°)$$

$$40 m/s = \frac{1+\sqrt{3}}{2} v_1 \quad \Rightarrow \quad v_1 = \underline{\underline{29{,}3 m/s}}$$

$$v_2 = v_1 \cdot \frac{\sin(30°)}{\sin(45°)} = 29{,}3 \cdot \frac{\sin(30°)}{\sin(45)} = \underline{\underline{20{,}7 m/s}} = v_2$$

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4,3c)

Før $\frac{1}{2} m v_0^2 = \frac{1}{2} \cdot m \cdot 40 m/s^2 = 800 m$

Etter $\frac{1}{2} m\ 29,3^2 + \frac{1}{2} m\ 20,7^2 = 643,5 m$

$$\left(1 - \frac{643,5 m}{800 m}\right) \cdot 100\% = \underline{\underline{19,6\%}}$$

19,6% går tapt.

4.4a) Elastisk støt $E_{k\,før} = E_{k\,etter}$

$mgh = m_1 g l$ $\quad m_1 g l = \frac{1}{2} m_1 v_{1f}^2$

$\Rightarrow v_{1f} = \pm\sqrt{2gl}$

$$\underline{\underline{v_{1f} = \sqrt{2gl}}}$$

$$T - G = m\frac{v^2}{r} \Rightarrow T = m\frac{v^2}{r} + G$$

$$T = m\left(\frac{2gl}{l} + g\right) = \underline{\underline{3mg}}$$

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4b)

V_{1e}, V_{2e} $\qquad \frac{1}{2} m_1 v_{1f}^2 = \frac{1}{2} m_1 v_{1e}^2 + \frac{1}{2} m_2 V_{2e}^2$

$m_1 V_{1f} = m_1 V_{1e} + m_2 V_{2e}$

$m_1 \cdot \sqrt{2gl} = m_1 V_{1e} + m_2 V_{2e}$ $\qquad \overbrace{m_1 \cdot 2gl = m_1 V_{1e}^2 + m_2 V_{2e}^2}^{**}$

$$\overbrace{V_{1e} = \frac{m_1 \sqrt{2gl} - m_2 V_{2e}}{m_1}}^{*}$$

$$m_1 2gl = m_1 \left(\frac{m_1 \sqrt{2gl} - m_2 V_{2e}}{m_1}\right)^2 + m_2 V_{2e}^2$$

$$m_1 2gl = \frac{m_1}{m_1^2}\left(m_1^2 2gl - 2m_1 \sqrt{2gl} \cdot m_2 V_{2e} + m_2^2 V_{2e}^2\right) + m_2 V_{2e}^2$$

$$\cancel{m_1^2 2gl} = \cancel{m_1^2 2gl} - 2m_1 \sqrt{2gl}\, m_2 V_{2e} + m_2^2 V_{2e}^2 + \cancel{m_2} m_1 V_{2e}^2$$

$$2m_1 \sqrt{2gl}\, \cancel{V_{2e}} = m_2 V_{2e}^{\cancel{2}} + m_1 V_{2e}$$

$$V_{2e} = \frac{2^{3/2} m_1 \sqrt{gl}}{m_1 + m_2}$$

$$V_{1e} = \frac{m_1 \sqrt{2gl} - m_2 V_{2e}}{m_1} = \sqrt{2gl} - \frac{m_2 \cdot 2^{3/2} \cdot \sqrt{gl}}{m_1 + m_2}$$

Taks 4.4b)

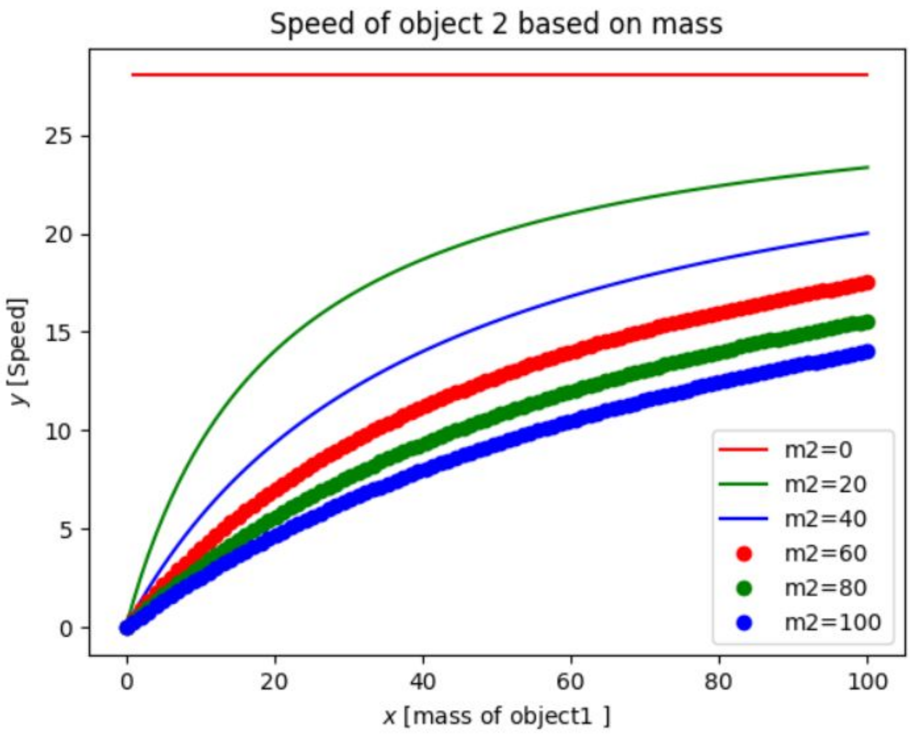
```
import numpy as np
import matplotlib.pyplot as plt

m1 = np.linspace(0, 100, 100)
l = 10
g = 9.81


def v2e(m1, m2):
    return (2**(3/2)*m1*(g*l)**0.5)/(m1+m2)


plt.plot(m1, v2e(m1, 0), 'r-', label="m2=0")
plt.plot(m1, v2e(m1, 20), 'g-', label="m2=20")
plt.plot(m1, v2e(m1, 40), 'b-', label="m2=40")
plt.plot(m1, v2e(m1, 60), 'ro', label="m2=60")
plt.plot(m1, v2e(m1, 80), 'go', label="m2=80")
plt.plot(m1, v2e(m1, 100), 'bo', label="m2=100")

plt.title("Speed of object 2 based on mass")
plt.legend()
plt.xlabel(r'$x$ [mass of object1 ]')
plt.ylabel(r'$y$ [Speed]')
plt.show()
```



Ser at hastigheten synker når massen til objekt 2 blir større og at hastigheten synker når massen til objekt en blir mindre. Grensene gir det jeg forventer.

4c)

$$T_{1e} = m_1 g + \frac{m_1}{l}\left(\sqrt{2gl} - \frac{m_2 \cdot 2^{3/2}\sqrt{gl}}{m_1+m_2}\right)^2$$

$$T_{2e} = m_1 g + \frac{m_2}{l}\left(\frac{2^{3/2} m_1 \sqrt{gl}}{m_1+m_2}\right)^2$$

4d)

$m_1 = 10g$ $\quad m_2 = 20g \quad l = 1m \quad g = 9{,}8 m/s^2$

$= 0{,}01kg \qquad = 0{,}02kg$

$$V_{1e} = \sqrt{2 \cdot 9{,}8 m/s^2 \cdot 1m} - \frac{0{,}02kg \cdot 2^{3/2} \cdot \sqrt{9{,}8 m/s^2 \cdot 1m}}{0{,}01kg + 0{,}02kg} = \underline{\underline{-1{,}48 m/s}}$$

$$V_{2e} = \frac{2^{3/2} \cdot 0{,}01kg \sqrt{9{,}8 m/s^2 \cdot 1m}}{0{,}01kg + 0{,}02kg} = \underline{\underline{2{,}95 m/s}}$$

$$T_{1e} = 0{,}01kg \cdot 9{,}8 m/s^2 + \frac{0{,}01kg}{1m}\left(\sqrt{2 \cdot 9{,}8 m/s^2 \cdot 1m} - \frac{0{,}02kg \cdot 2^{3/2}\sqrt{9{,}8 m/s^2 \cdot 1m}}{0{,}01kg + 0{,}02kg}\right)^2$$

$$T_{1e} = \underline{\underline{0{,}12N}}$$

$$T_{2e} = \underline{\underline{0{,}20N}}$$

Dimensjonene passer!

5a) $[x, y]$

$$P_{f1} = mv\,[1, 0]$$
$$P_{f2} = mv\,[\cos\theta, \sin\theta]$$

$$mV_{1x} + mV_{2x} = 2mV_{1x}$$

$$V_1 + V_2\cos\theta = 2V_{1x}$$

$$mV_2\sin\theta = 2mV_{1y}$$

$$V + V\cos\theta = 2V_{1x} \qquad V_{1x} = \frac{V + V\cos\theta}{2}$$

$$V\sin\theta = 2V_{1y} \qquad V_{1y} = \frac{V\sin\theta}{2}$$

$$V_x = \frac{V(1+\cos\theta)}{2} \qquad V_y = \frac{V\sin\theta}{2}$$

$$P = 2mV = 2m\frac{V}{2}[1+\cos\theta, \sin\theta]$$

$$P = mV\,[1+\cos\theta, \sin\theta]$$

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5 b)

$$K_i = \frac{1}{2} m v^2 + \frac{1}{2} m v^2 = m v^2$$

$$K_f = \frac{1}{2}(2m) V_f^2 = m V_f^2$$

$$V_f = \sqrt{\left(\frac{V(1+\cos\theta)}{2}\right)^2 + \left(\frac{V\sin\theta}{2}\right)^2}$$

$$V_f = \sqrt{\frac{V^2(1+2\cos\theta+\cos^2\theta)}{4} + \frac{V^2\sin^2\theta}{4}}$$

$$V_f = \frac{V}{2}\sqrt{1+2\cos\theta + \underbrace{\cos^2\theta+\sin^2\theta}_{=1}}$$

$$V_f = \frac{V}{2}\sqrt{2+2\cos\theta}$$

$$K_f = m\left(\frac{V}{2}\sqrt{2+2\cos\theta}\right)^2 = m\frac{V^2}{4}(2+2\cos\theta)$$

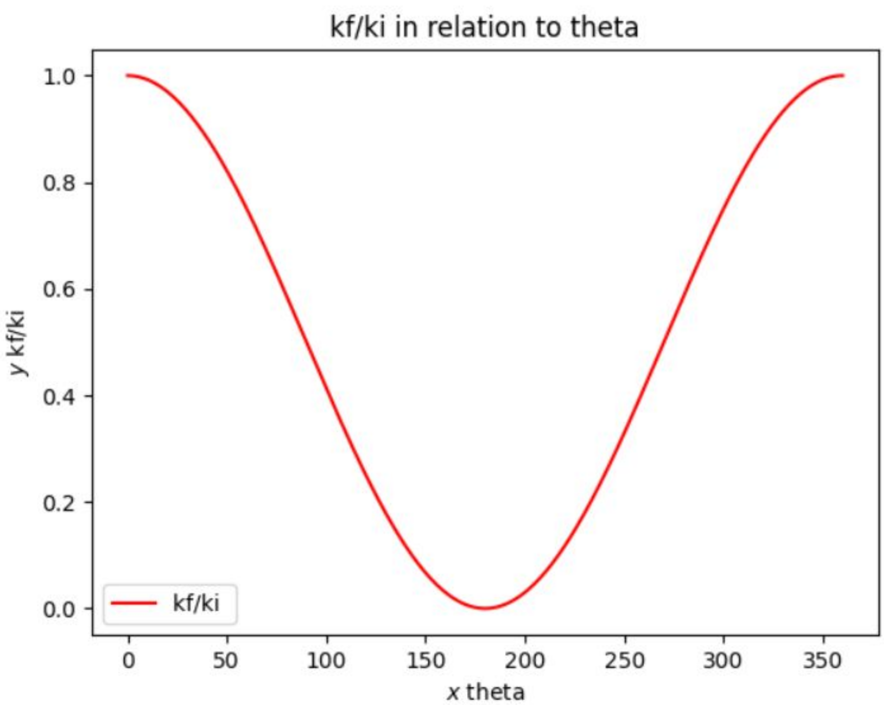
$$K_f = \frac{mV^2}{2}(1+\cos\theta)$$

$$\frac{K_f}{K_i} = \frac{\frac{mv^2}{2}(1+\cos\theta)}{mv^2} = \frac{1}{2}(1+\cos\theta)$$

Task 4.5c)

task45.py > ...

```python
import numpy as np
import matplotlib.pyplot as plt

theta = np.linspace(0, 360, 360)


def function(theta):
    return 0.5*(1+np.cos(np.deg2rad(theta)))


plt.plot(theta, function(theta), 'r-', label="kf/ki ")

plt.title("kf/ki in relation to theta")
plt.legend()
plt.xlabel(r'$x$ theta')
plt.ylabel(r'$y$ kf/ki')
plt.show()

```



5.6)

$V_0 = 50\,km/t = 13{,}9\,m/s$

$V_e = 0\,m/s$

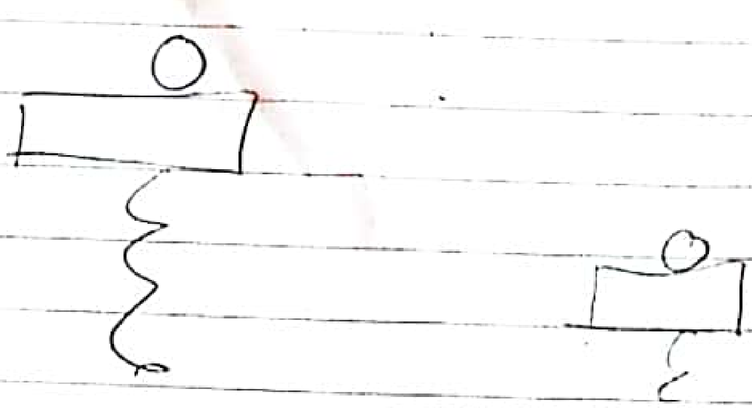
$$\underset{=0}{V^2} = V_0^2 + 2a(x - \underset{=0}{x_0})$$

$$x = \frac{V^2 - V_0^2}{2a}$$

$$x = \frac{-V_0^2}{2a} = \frac{-13{,}9^2\,m^2/s^2}{2 \cdot 250\,m/s^2} = \underline{\underline{-0{,}386\,m}}$$

$x \geq 0{,}386\,m$ for at personen ikke skal ~~dø~~ dø.

S,7a)



$$F_{fjær} = Kd = ma \iff a = \frac{kd}{m}$$

$$\frac{Kd}{m} = g \iff d = \frac{mg}{K} = \frac{(1,5+0,225)\cdot 9,8}{125} = 0,094\,m$$

$$0,094\,m + 0,015\,m = 0,244\,m = \underline{\underline{24,4\,cm}}$$

S,7b)

$$x(t) = A\cos(\omega t + \phi), \quad \phi = 0$$

$$0,094 = -0,15\cos\left(\sqrt{\frac{k}{m}}\,t\right) \qquad \omega = \sqrt{\frac{k}{m}}$$

$$t = \frac{1}{\omega} \arccos\left(\frac{x}{A}\right) \leftarrow \text{Radianer!}$$

$$t = \frac{1}{\sqrt{\frac{k}{m}}} \arccos\left(\frac{x}{A}\right) = \frac{1}{\sqrt{\frac{125N}{1,5kg + 0,275kg}}} \arccos\left(\frac{0,094\,m}{-0,15\,m}\right)$$

$$t = \underline{\underline{0,221\,s}}$$

5,7 c)

$$E_{p_{fjær}} = \frac{1}{2} k d^2$$

Hele massen

$$\frac{1}{2} k A^2 = \frac{1}{2} k \underset{0,094}{\underset{\shortparallel}{x^2}} + \frac{1}{2} m v^2$$

$$\Rightarrow \quad v = \sqrt{\frac{k}{m} (A^2 - x^2)}$$

$$\Rightarrow \quad v = \sqrt{\frac{185\,N/m}{1{,}5\,kg + 0{,}275\,kg} \left((0{,}15)^2 - 0.094^2\right)}$$

$$\underline{\underline{v = 1{,}19\,m/s}}$$