Øving 5 - Hallvard - TFY4125

1a)

$F = ma \quad F_g = mg \quad$ Hookes law $= -kx$

$F_g =$ Hookes law $\Rightarrow \quad mg = -kx$

$\Rightarrow \quad mg + kx = 0$

$mx'' + kx = 0 \quad , \quad \omega = \sqrt{\frac{k}{m}}$

$\Rightarrow \quad x'' + \omega^2 x = 0$

$\Rightarrow \quad x(t) = A \sin(\omega t) + B \cos(\omega t)$

1b)

$C \cdot \cos(\omega t + \theta) \frac{d}{dt} = -C\omega \sin(\omega t + \theta)$

$-C\omega \sin(\omega t + \theta) \frac{d}{dt} = -C\omega^2 \cos(\omega t + \theta)$

$m(-C\omega^2 \cos(\omega t + \theta)) + kC\cos(\omega t + \theta) = 0$

$C \cdot \cos(\omega t + \theta)(-m\omega^2 + k) = 0$

$-m\omega^2 + k = 0 \quad \Rightarrow \omega^2 = \frac{k}{m} \quad \Rightarrow \underline{\underline{\omega = \sqrt{\frac{k}{m}}}}$

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$$A\sin(\omega_0 t) + B\cos(\omega_0 t) = C\cos(\omega_0 t + \theta)$$

$$\cos(\omega_0 t + \theta) = \cos(\omega_0 t)\cos(\theta) - \sin(\omega_0 t)\sin(\theta)$$

$$A\sin(\omega_0 t) + B\cos(\omega_0 t) = C\cos(\omega_0 t)\cos\theta - C\sin(\omega_0 t)\sin(\theta)$$

$$\Rightarrow A = -C\sin(\theta) \quad , B = C\cos(\theta)$$

$$A^2 + B^2 = C^2\sin^2\theta + C^2\cos^2(\theta)$$

$$A^2 + B^2 = C^2\underbrace{(\sin^2\theta + \cos^2\theta)}_{=1} = C^2$$

$$A^2 + B^2 = C^2 \Rightarrow \underline{\underline{C = \sqrt{A^2+B^2}}}$$

$$\frac{A}{B} = -\frac{C\sin(\theta)}{C\cos(\theta)} = -\tan(\theta) \Rightarrow \underline{\underline{\theta = \tan^{-1}\left(-\frac{A}{B}\right)}}$$

c)

$$v(0) = v_0 \quad , x(0) = x_0$$

$$x_0 = C\cos(\omega\cdot 0 + \theta) \Rightarrow x_0 = C\cos(\theta)$$

$$v_0 = -C\omega\sin(\omega 0 + \theta) \Rightarrow v_0 = -C\omega\sin(\theta)$$

$$\Rightarrow C = \frac{x_0}{\cos(\theta)} \qquad v_0 = -\frac{x_0\omega}{\cos\theta}\sin(\theta)$$

$$v_0 = -x_0\omega\tan\theta \Rightarrow \underline{\underline{\theta = \tan^{-1}\left(-\frac{v_0}{x_0\omega}\right)}}$$

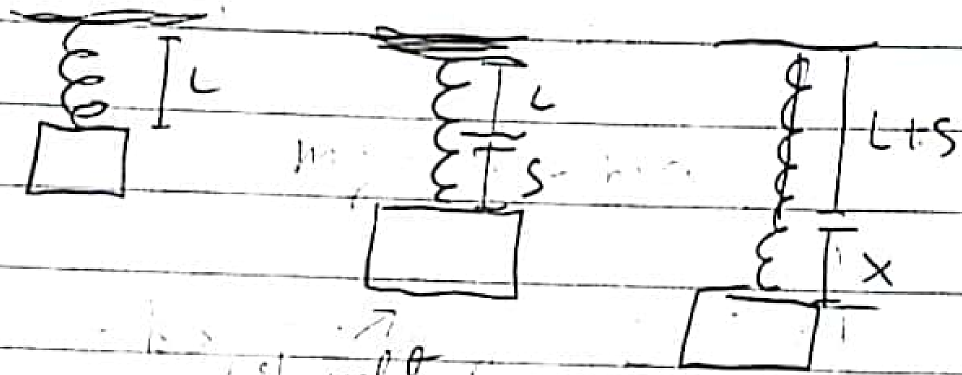
$$\underline{\underline{C = \frac{x_0}{\cos\left(\tan^{-1}\left(-\frac{v_0}{x_0\omega}\right)\right)}}}$$

1d)

$$\omega = \frac{2\pi}{T} = 2\pi f = \frac{|v|}{|r|} \qquad T = \frac{1}{f}$$

$$\underline{\underline{f = \frac{\omega}{2\pi}}}$$

1e)



Likevekt i

$$m x'' = -k(s+x) + mg$$

$$m x'' = -ks - kx + mg$$

$$m x'' + kx = 0$$

$$x'' + \omega^2 x = 0$$

Detle er samme ligning som i 1a gir samme frekvens.

2) $mx'' = -kx - bx' \Rightarrow$

For et udempet system har vi:

$$\omega_D = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$

Og for et system uten demping

$$\omega_H = \sqrt{\frac{k}{m}}$$

Vi ser at $\omega_D < \omega_H$ siden $\left(\frac{b}{2m}\right)^2$ alltid er positiv.

Vi har da at $f_D = \frac{\omega_D}{2\pi} < \frac{\omega_H}{2\pi} = f_H$

Dette er intuitivt fordi vi kan øke motstanden til dempingen ved f. eks ha fjæren og klossen i et glass med olje og det vil gi en lavere frekvens.

3) Inhomogen 2. ordens differansial ligning.

$$mx'' + bx' + kx = F(t)$$ se video.

$$x'' + \frac{b}{m}x' + \frac{k}{m} = \frac{F_0 \cos(\omega_d t)}{m}, \quad \omega_0 = \sqrt{\frac{k}{m}}$$

$$\Rightarrow x(t) = X_0 \cos(\omega t - \phi)$$

hvor $X_0 = \dfrac{F_0}{\sqrt{(k - m\omega^2)^2 + (b\omega)^2}}$

Ser at systemet ender opp å svinge med samme frekvens som drivfrekvensen. Da det er en fase forskjell siden $X(t) \neq F(t)$ grunnet ϕ-leddet. $X(t)$ vil ligge validette gir også mening da den "responderer" på drivfrekvensen.

4a)

Resonans vil si at et system som kan svinge kommer i store svingninger når det blir påvirket av en periodisk kraft med samme frekvens som egenfrekvensen til systemet.

Intuitiv forklaring: Tenk på en sanger som knuser glass med stemmen. Det skjer når sangeren synger noten som er egenfrekvensen til glasset.

b)

$$A(\omega_d) = \frac{F_0}{\sqrt{m^2(\omega_0^2-\omega_d^2)^2+b^2\omega_d^2}} \quad , \quad \omega_0 = \sqrt{\frac{k}{m}}$$

$$A_{max} = \frac{F_0}{b\omega}$$

c)

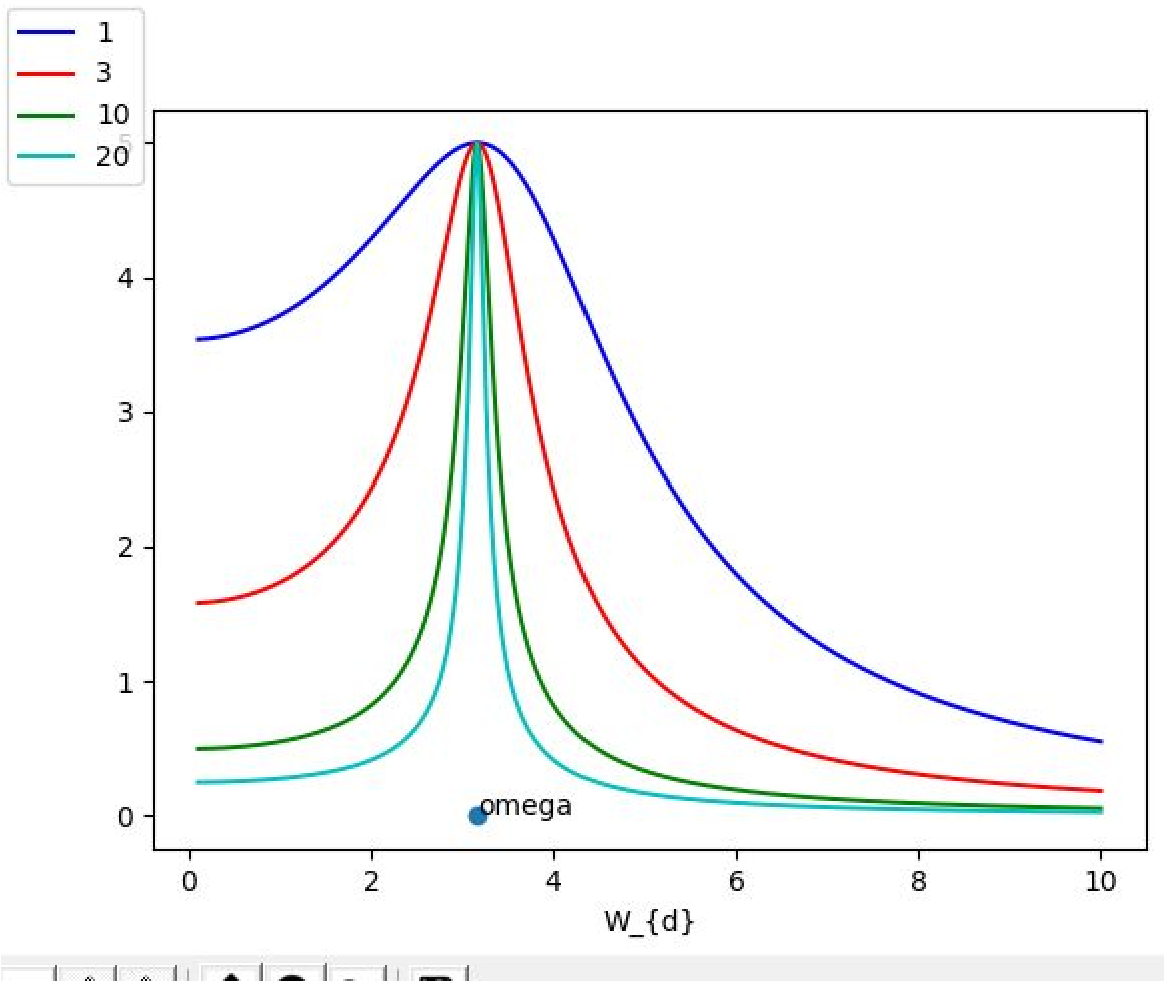
Q-faktor er en dimensjonsløs parameter som sier hvor underdempet et svinge system er.

$$Q = \frac{k}{b\omega} \Rightarrow b = \frac{k}{Q\omega}$$

$$F_0 = b\omega Q$$

Oppgave 4d)

```python
import numpy as np
import matplotlib.pyplot as plt  # Used for plotting results

m = 0.01  # kg
k = 0.1  # N/m
F0 = 5  # usikker på om man skulle finne f0 eller bare velge
OMEGA = np.sqrt(k/m)
WD = np.linspace(0.1, 10, 1000)


def b(k, Q, w):
    return k/(Q*w)


def A(wd, q):
    b_value = b(k, q, wd)
    return F0*wd*b_value/np.sqrt(m**2*(OMEGA**2-wd**2)**2+b_value**2*wd**2)


fig = plt.figure()
l1, l2, l3, l4 = plt.plot(WD, A(WD, 1), 'b-', WD, A(WD, 3), 'r-',
                          WD, A(WD, 10), 'g-', WD, A(WD, 20), 'c-')
fig.legend((l1, l2, l3, l4), ('1', '3', '10', '20'), 'upper left')
plt.scatter([OMEGA], [0])
plt.annotate('omega', (OMEGA, 0))
plt.xlabel('W_{d}')
plt.show()
```



a)

$$F = -k_1 x - k_2 x = -kx$$

$$mx'' + \frac{k}{m} x = 0$$

$$\Rightarrow \omega = \sqrt{\frac{k_1 + k_2}{m}}$$

b)

$$F = -k_1 x_1 = -k_2 x_2 \Rightarrow x_1 = \frac{k_2}{k_1} x_2$$

$$F = -k(x_1 + x_2)$$

$$\Rightarrow k_2 x_2 = k\left(\frac{k_2}{k_1} x_2 + x_2\right) \Rightarrow k_2 = k\left(\frac{k_2}{k_1} + 1\right)$$

$$\Rightarrow k = \left(\frac{1}{k_1} + \frac{1}{k_2}\right)^{-1} \Rightarrow \omega = \sqrt{\frac{k_1 k_2}{(k_1 + k_2) m}}$$

c)

$$mx'' = -k_1 x - k_2 x = -(k_1 + k_2) x$$

$$x'' = \frac{-k_1 + k_2}{m} x \Rightarrow k = k_1 + k_2$$

$$\Rightarrow \omega = \sqrt{\frac{k_1 + k_2}{m}}$$

6)

$k = 4.0 \pm 0.3\ N/m$

$m = 200 \pm 4\ g = 0{,}200 \pm 0.004\ kg$

$\omega = \sqrt{\frac{k}{m}}$

$k = 4.0 \pm 7{,}5\%\ N/m$

$m = 0{,}200 \pm 2\%\ kg$

$\omega = \sqrt{\frac{4.0 \pm 7{,}5\%\ N/m}{0{,}200 \pm 2\%\ kg}} = \sqrt{20.0 \pm (7{,}5+2)\%^2\ s^{-2}}$

Siden $\sqrt{x} = x^{0{,}5}$

$\omega = 4{,}5 \pm (0{,}5 \cdot 9{,}5)\%\ s^{-1}$

$\omega = 4{,}5 \pm 4{,}75\%\ s^{-1}$

$\underline{\underline{\omega = 4{,}5 \pm 0{,}2\ s^{-1}}}$

→ c)

$$x(0) = 0{,}01\,m = A\cos(\omega \cdot 0 + \theta) = A\cos(\theta)$$

$$x'(0) = 0{,}05\,m/s = -\omega A \sin(\omega \cdot 0 + \theta) = -\omega A \sin(\theta)$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{0{,}1\,N/m}{0{,}01\,kg}} = \sqrt{10\ s^{-2}} = \sqrt{10}\,s^{-1}$$

$$\Rightarrow 0{,}01\,m = A\cos(\theta)$$

$$0{,}05\,m/s = \sqrt{10}\,s^{-1} \cdot -A\sin(\theta) \Rightarrow -\frac{0{,}05}{\sqrt{10}}\,m = A\sin(\theta)$$

$$A = \frac{0{,}01\,m}{\cos(\theta)} \Rightarrow -\frac{0{,}05}{\sqrt{10}}\,m = 0{,}01\,m \cdot \tan(\theta)$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{-0{,}05}{\sqrt{10} \cdot 0{,}01}\right) = -1{,}007\,rad \,\hat{=}\, \underline{\underline{-57{,}7^\circ = \theta}}$$

$$\Rightarrow A = \frac{0{,}01\,m}{\cos(-1{,}51)} = \underline{\underline{0{,}019 = A}}$$

7b)

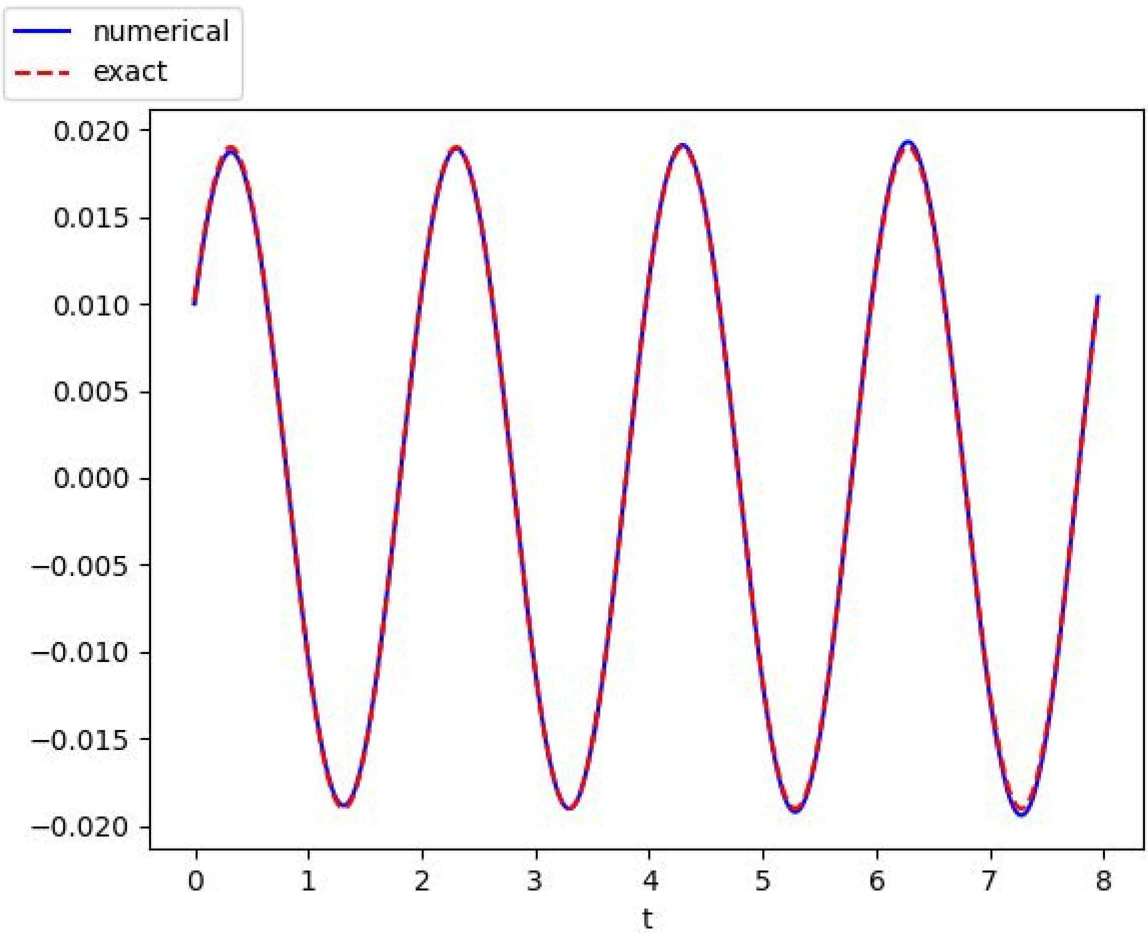
$$T = \frac{2\pi}{\sqrt{\frac{k}{m}}} = \frac{2\pi}{\sqrt{\frac{0,1}{0,01}}} = \underline{\underline{1,99}}$$

7c)

Tidsintervallet må være 0,001 eventuelt 1000 punkter per sekund.

Oppgave 7a og c)

```python
import numpy as np
import matplotlib.pyplot as plt  # Used for plotting results

m = 0.01  # kg
k = 0.1  # N/
A = 0.019  # Amplitude
theta = -1.007  # rads
omega = np.sqrt(k/m)
P = 2*np.pi/omega
dt = P/2000
print(1/dt)
T = 4*P
N_t = int(round(T/dt))
print(N_t)
t = np.linspace(0, N_t*dt, N_t+1)

u = np.zeros(N_t+1)
v = np.zeros(N_t+1)

# Initial condition
X_0 = 0.01
u[0] = X_0
v[0] = 0.05

# Step equations forward in time
for n in range(N_t):
    u[n+1] = u[n] + dt*v[n]
    v[n+1] = v[n] - dt*omega**2*u[n]

fig = plt.figure()
l1, l2 = plt.plot(t, u, 'b-', t, A*np.cos(omega*t+theta), 'r--')
fig.legend((l1, l2), ('numerical', 'exact'), 'upper left')
plt.xlabel('t')
plt.show()
```



8) a)

$f = -bv$

$mx'' = -kx - bv \iff mx'' + bx' + kx = 0$

Overdempet	$b > 2\sqrt{km}$
Kritisk dempet	$b = 2\sqrt{km}$
Underdempet	$b < 2\sqrt{km}$

$$b = 0{,}0010\ Ns/m \quad < \quad 2\sqrt{0.1\ N/m \cdot 0.01\ kg} = 0{,}063\ Ns/m$$

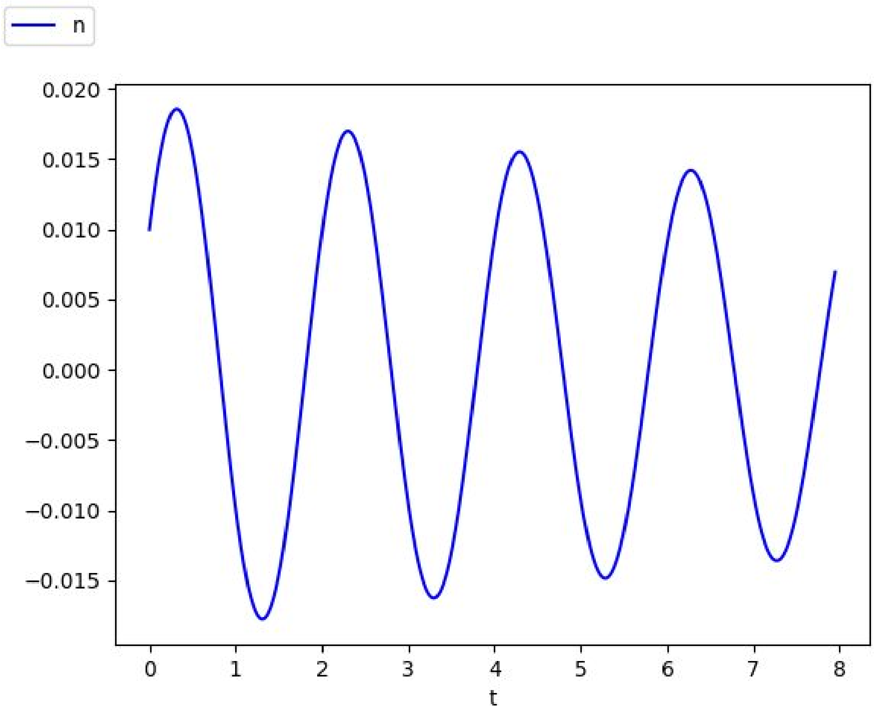
$\Rightarrow$ Systemet er underdempet.

b)

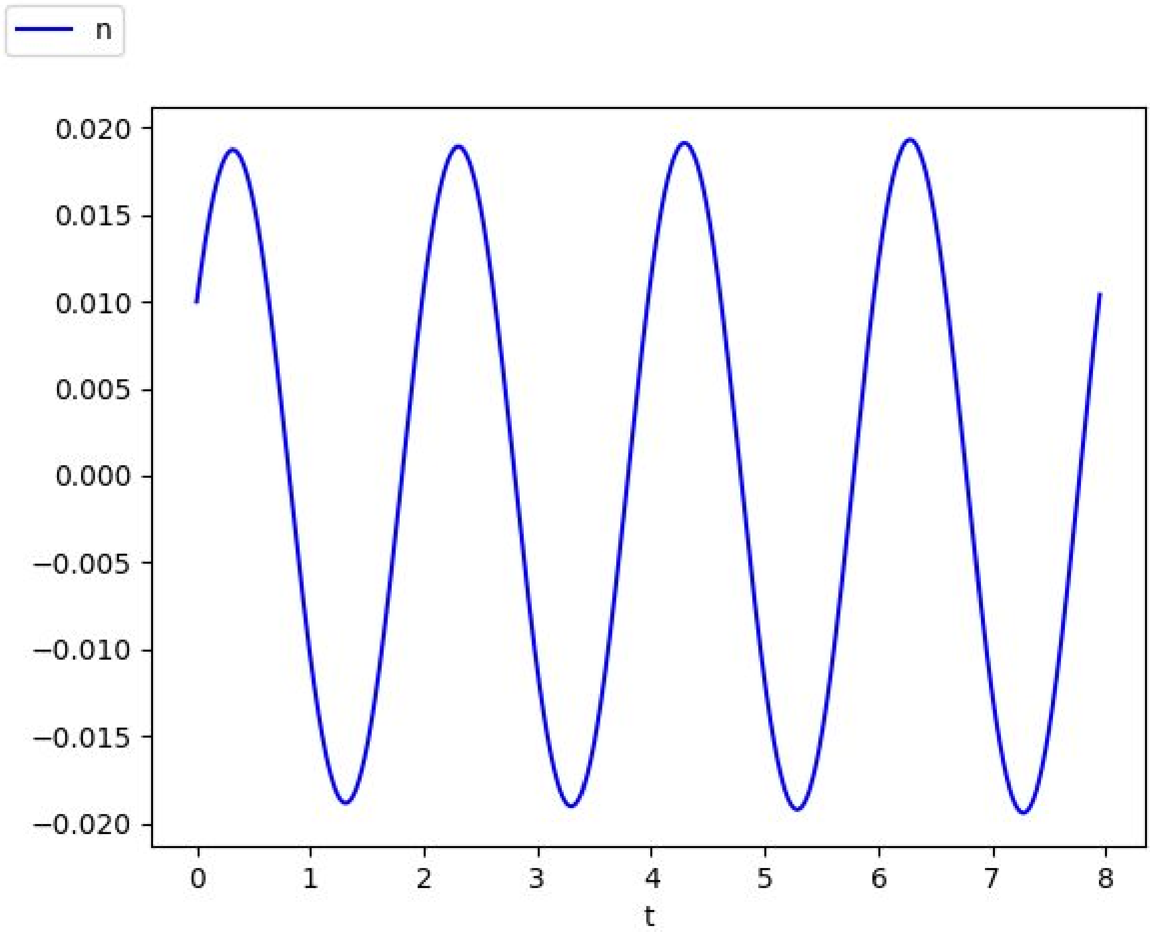
$b = 0 \Rightarrow$ Kritisk dempet.

Oppgave 8a)

```
import numpy as np
import matplotlib.pyplot as plt  # Used for plotting results

m = 0.01  # kg
k = 0.1  # N/m
b = 0.001  # Ns/m
A = 0.019  # Amplitude
theta = -1.007  # rads
omega = np.sqrt(k/m)
P = 2*np.pi/omega
dt = P/2000
print(1/dt)
T = 4*P
N_t = int(round(T/dt))
print(N_t)
t = np.linspace(0, N_t*dt, N_t+1)


u = np.zeros(N_t+1)
v = np.zeros(N_t+1)

# Initial condition
X_0 = 0.01
u[0] = X_0
v[0] = 0.05

# Step equations forward in time
for n in range(N_t):
    u[n+1] = u[n] + dt*v[n]
    v[n+1] = v[n]*(1-(dt*(b/m))) - dt*omega**2*u[n]

fig = plt.figure()
l1 = plt.plot(t, u, 'b-')
fig.legend((l1), ('numerical'), 'upper left')
plt.xlabel('t')
plt.show()

```



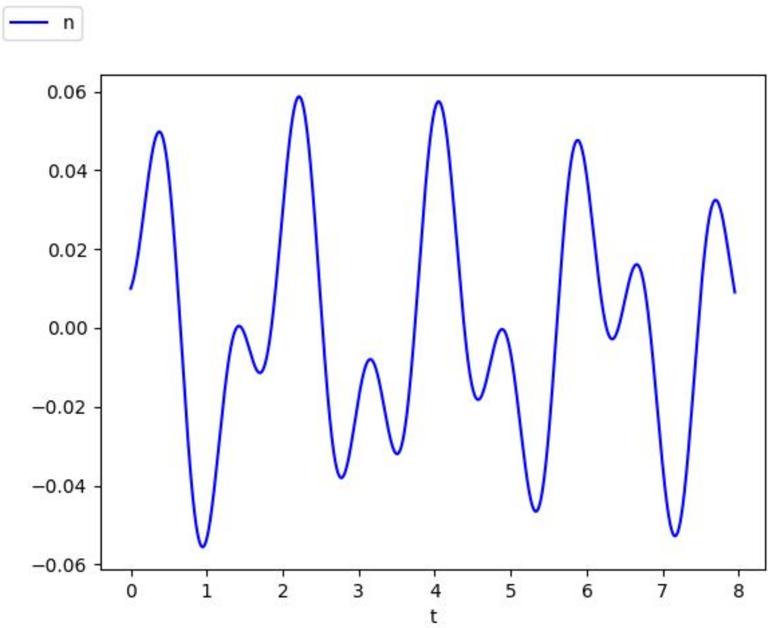
8b)
b=0



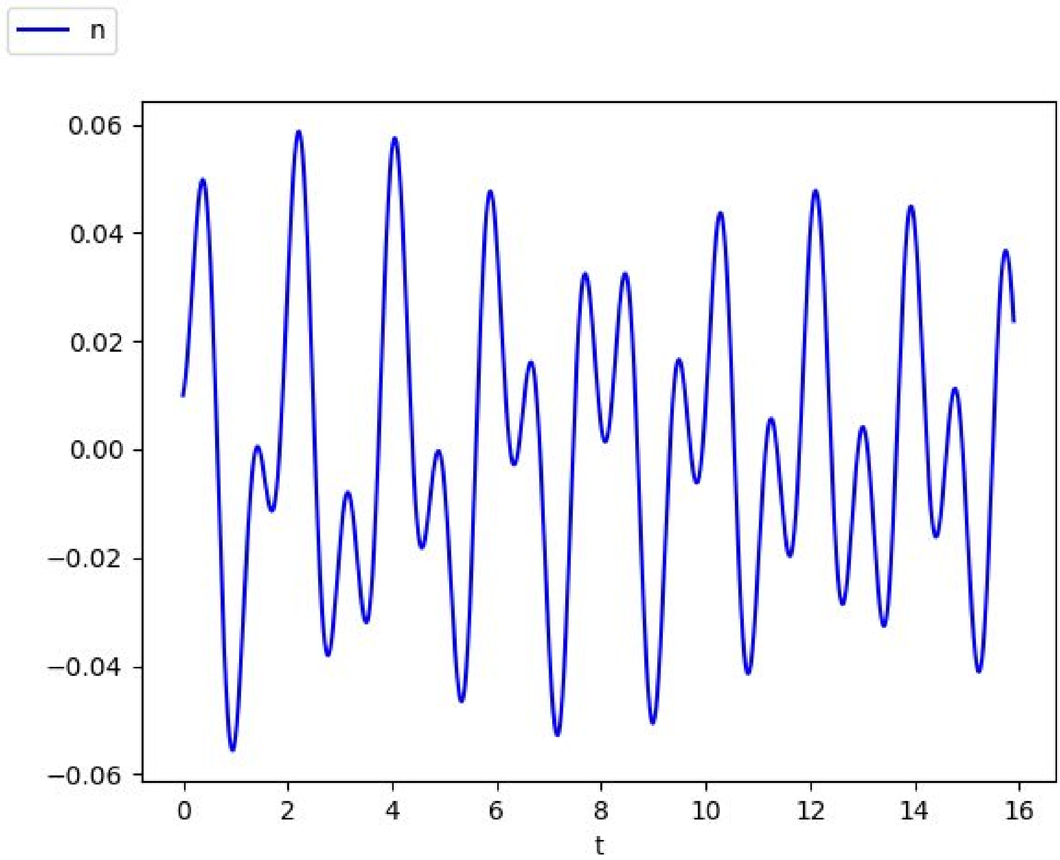
9a)

task9.py > ...

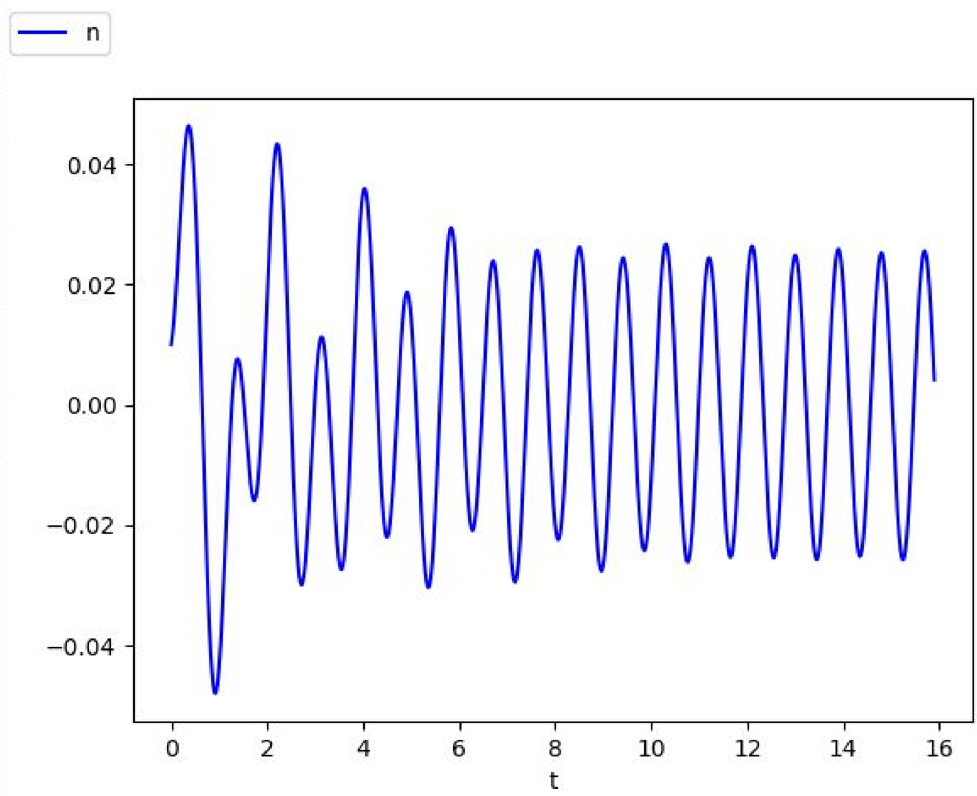
```python
import numpy as np
import matplotlib.pyplot as plt  # Used for plotting results

m = 0.01  # kg
k = 0.1  # N/m
b = 0.001  # Ns/m
A = 0.019  # Amplitude
theta = -1.007  # rads
wd = 7  # rad/s
F0 = 0.01  # N
omega = np.sqrt(k/m)
P = 2*np.pi/omega
dt = P/2000
print(1/dt)
T = 4*P
N_t = int(round(T/dt))
print(N_t)
t = np.linspace(0, N_t*dt, N_t+1)


u = np.zeros(N_t+1)
v = np.zeros(N_t+1)

# Initial condition
X_0 = 0.01
u[0] = X_0
v[0] = 0.05

# Step equations forward in time
for n in range(N_t):
    u[n+1] = u[n] + dt*v[n]
    v[n+1] = v[n]*(1-(dt*(b/m))) - dt*omega**2*u[n] + (F0*dt/m)*np.cos(wd*n*dt)

fig = plt.figure()
l1 = plt.plot(t, u, 'b-')
fig.legend((l1), ('numerical'), 'upper left')
plt.xlabel('t')
plt.show()

```



9b)
b=0 og 8 perioder



b=0.1*2*sqrt(km) cirka lik 0.00632



9d)

$$Q = \frac{k}{bw}$$

$$b = 2\sqrt{km}$$

$$\Rightarrow Q = \frac{k}{2\sqrt{km}\,w} = \frac{k}{2\sqrt{km}\sqrt{\frac{k}{m}}} = \frac{k}{2 \cdot k^{0{,}25} \cdot m}$$

$$\underline{\underline{Q = \frac{1}{2} k^{0{,}75}}}$$