Øving 8 – TFY4125 – Hallvard

1)

Ser på kulene som punktladningen

$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2} \Rightarrow q^2 = 4\pi\varepsilon_0 F r^2$$

$$\Downarrow$$

$$q = \sqrt{4\pi\varepsilon_0 F r^2}$$

$$N = \frac{q}{e} = \frac{\sqrt{4\pi\varepsilon_0 F r^2}}{e}$$

$$\frac{1}{4\pi\varepsilon_0} = 8{,}988 \cdot 10^{9}\ N\cdot m^2/C^2 \Rightarrow 4\pi\varepsilon_0 = \frac{1}{8{,}988\cdot 10^{9}\ N\cdot m^2/C^2}$$

$$N = \frac{q}{e} = \frac{\sqrt{\frac{4{,}57\cdot 10^{-7} N\cdot\left(\frac{20}{100}\right)^2 m^2}{8{,}988\cdot 10^{9}\ N m^2/C^2}}}{1{,}602\cdot 10^{-19} C} = \frac{\sqrt{2{,}0338\ C^{-2}}}{1{,}602\cdot 10^{-19} C}$$

$$\underline{\underline{N = 8{,}9\cdot 10^{9}}}$$

2)

$k = 0{,}30m/2 = 0{,}15m \quad \lambda = 5.20\cdot 10^{-6}\ C/m \quad l = 0.05m$

$$\Phi = 2\pi r l E = \frac{\lambda l}{\varepsilon_0} \qquad E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$$F = qE = \lambda l E = \frac{\lambda^2 l}{2\pi\varepsilon_0 r} = \frac{(5.20\cdot 10^{-6})^2 C^2/m^2 \cdot 0{,}05m}{\frac{0{,}15m}{4{,}494\cdot 10^{9} N m^2/C^2}}$$

$$\underline{\underline{F = 4{,}1\cdot 10^{-2} N}}$$

3)

$q = 5.00 \cdot 10^{-6}\,C$; $m = 2{,}00 \cdot 10^{-4}\,kg$

$v_A = 5.0\,m/s$ $\qquad$ $v_B = ?$

O $\qquad$ O

$200V$ $\qquad$ $800V$

$$K_A + U_A = K_B + U_B$$

$$\frac{1}{2} m v_A^2 + qV_A = \frac{1}{2} m v_B^2 + qV_B$$

$$\Rightarrow v_B = \sqrt{v_A^2 + \frac{2q}{m}(V_A - V_B)}$$

$$\Rightarrow v_B = \sqrt{25\,m^2/s^2 + \frac{2 \cdot 5 \cdot 10^{-6}\,C}{2 \cdot 10^{-4}\,kg}(-600\,V)} = \sqrt{25\,m^2/s^2 - 30\,\frac{AS}{Kg} \cdot \frac{kg\,m^2}{s^3 A}}$$

$$v_B = \sqrt{-5\,m^2/s^2} = \sqrt{5}\,i\,m/s$$

Dette er en imaginær fart fordi det ikke er mulig at partikkelen beveger seg fra A til B gitt betingelsene.

4a) $\vec{F} = \sum \vec{F_i}$ for hver partikkel har vi at: $\vec{F_i} = \frac{1}{4\pi\varepsilon_0} \frac{Q_i q}{R^2} \hat{E_i}$

$\hat{E_i}$ er enhetsvektoren og er det eneste som skiller partiklene.

$$\Rightarrow \vec{F_i} = \frac{1}{4\pi\varepsilon_0} \frac{Q_i q}{R^2} \sum \hat{E_i}$$

Ladningene
$\hat{E}1: [0,-1]$
$\hat{E}2: [1,-1]/\sqrt{2}$
$\hat{E}3: [1,0]$
$\hat{E}4: [1,1]/\sqrt{2}$
$\hat{E}5: [0,1]$

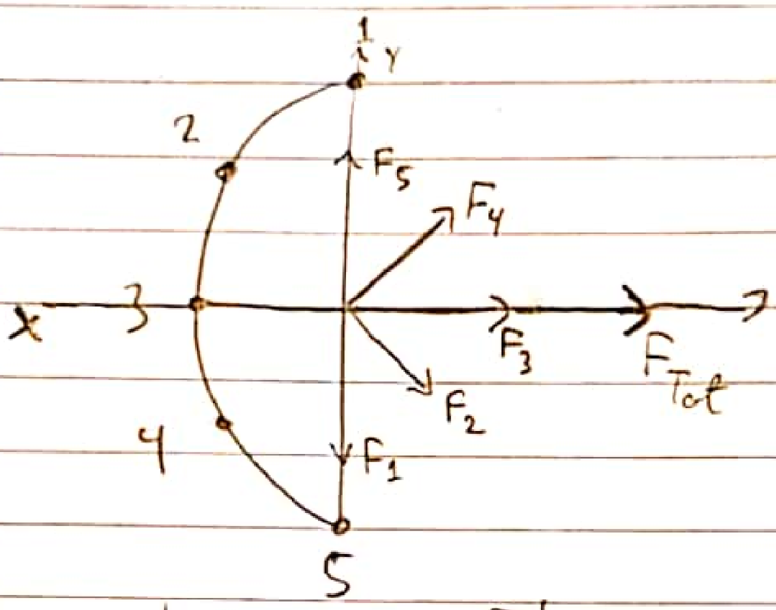
$$\Rightarrow \vec{F_i} = \frac{1}{4\pi\varepsilon_0} \frac{Q_i q}{R^2} \left(0+\frac{1}{\sqrt{2}}+1+\frac{1}{\sqrt{2}}+0,\ -1-\frac{1}{\sqrt{2}}+0+\frac{1}{\sqrt{2}}+1\right)$$

$$\Rightarrow \vec{F_i} = \frac{1}{4\pi\varepsilon_0} \frac{Q_i q}{R^2} \left(1+\sqrt{2},\ 0\right)$$

Ser at det kun gjelder i $\hat{x}$-retning

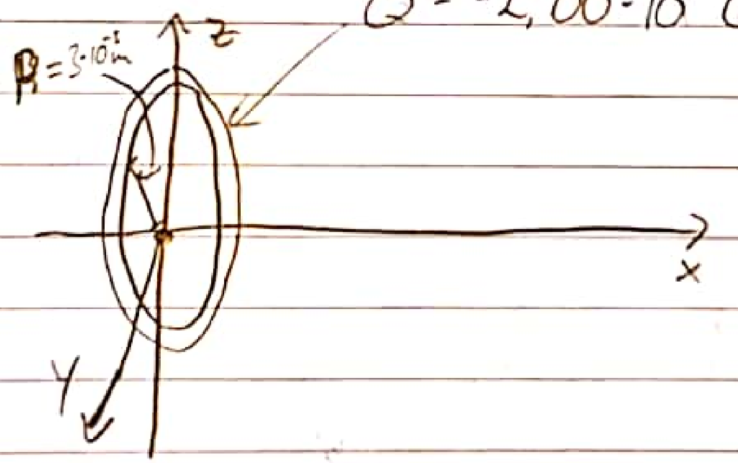
$$\Rightarrow \underline{\underline{\vec{F_i} = \frac{1}{4\pi\varepsilon_0} \frac{Q_i q}{R^2} \left(1+\sqrt{2}\right) \hat{x}}}$$

4b)



$$F_{tot} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{R^2}\left(1+\sqrt{2}\right)\hat{x}$$

5a) $Q = -2,00\cdot 10^{-9}\,C$ $\quad q_e = 1,60\cdot 10^{-19}\,C$



$$V(r) = kQ/r$$

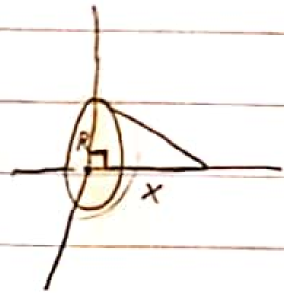
Fra boken Equation 23.16 har vi at

$V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}$ her endrer r seg ikke og $\frac{1}{4\pi\varepsilon_0}$ er en konstant

$$\Rightarrow V = k\cdot\frac{1}{r}\cdot\int dq = k\cdot Q/r$$

5b)

$$V(x) = k\frac{Q}{r} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{x^2+R^2}},$$



$$\frac{1}{4\pi\varepsilon_0} = 8{,}988\cdot10^{9}\ N\,m^2/C^2 -$$

$$V(0) = 8{,}988\cdot10^{9}\cdot\frac{-2.00\cdot10^{-9}C}{3\cdot10^{-3}} \qquad \left(\frac{kg\ m^2}{C\ s^2} = \frac{kg\ m^2}{A\,s^3} = V\right)$$

$$V(0) = -5992\,V = \underline{\underline{-5{,}99\,kV}}$$

5c)

$$U = qV$$

$$U(0) = q\cdot V(0) = -1{,}60\cdot10^{-19}C\cdot -5{,}99\cdot10^{3} = 9{,}6\cdot10^{-16}J$$

$$U(0{,}1) = qV(0{,}1) = -1{,}60\cdot10^{-19}\cdot 8{,}988\cdot10^{9}\cdot\frac{-2.00\cdot10^{-9}}{\sqrt{0{,}1^2+(3\cdot10^{-3})^2}}\ J$$

$$U(0{,}1) = 2{,}9\cdot10^{-17}J$$

$$W = -\Delta U = -\left(2{,}9\cdot10^{-17} - 9{,}6\cdot10^{-16}\right)J = \underline{\underline{9{,}3\cdot10^{-16}J}}$$

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5d)

$$9{,}3 \cdot 10^{-16} = \frac{1}{2} \cdot 9.109 \cdot 10^{-31} \cdot v^2$$

$$\Rightarrow v = \sqrt{\frac{2 \cdot 9{,}3 \cdot 10^{-16}}{9.109 \cdot 10^{-31}}} = \underline{\underline{4{,}5 \cdot 10^7 \, m/s}}$$

5e)

$E(r) = E(x)\hat{x}$ - fordi ladningen er lik i alle retninger vendt x-aksen. Potensialet er da kun avhengig av plasseringen på x-aksen.

$$E = -\nabla V = -\frac{\partial V}{\partial x} = \frac{1}{4\pi\varepsilon_0} \cdot Q \cdot -\frac{x}{(x^2+R^2)^{3/2}}$$

$$E(0) = -\frac{Q\,0}{4\pi\varepsilon_0\,(0^2+R^2)^{3/2}} = 0$$

Dette gir mening da elektronet ligger i origo og ladningene nuller hverandre ut.

5f)

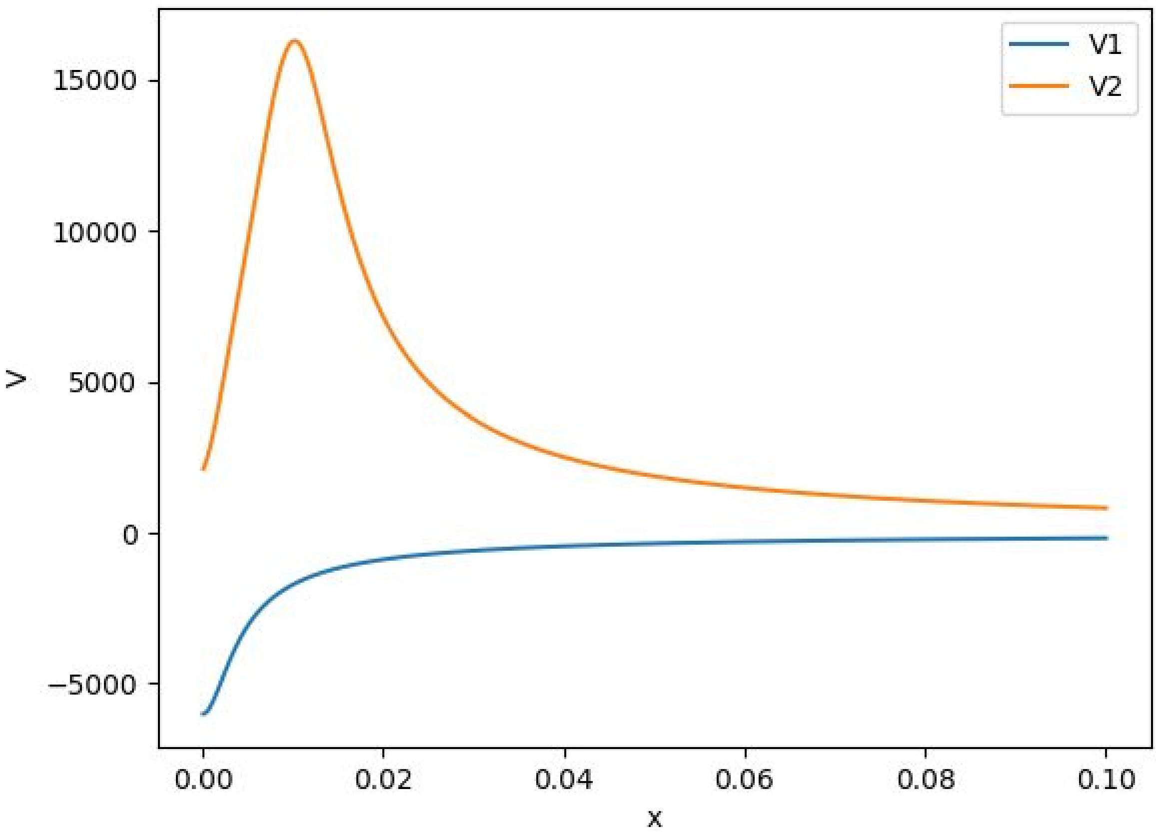
$$V(x) = \frac{1}{4\pi\varepsilon_0}\frac{Q_1}{\sqrt{x^2+R_1^2}}$$

$$V_{II}(x) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q_1}{\sqrt{x^2+R_1^2}} + \frac{Q_2}{\sqrt{(x-a)^2+R_2^2}}\right)$$

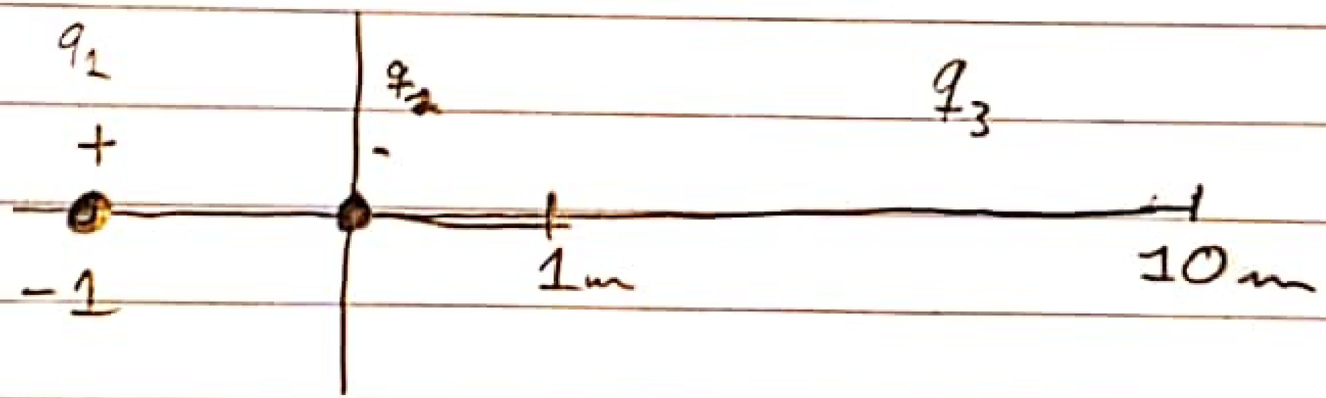
Fra plottet ser vi at det kan være fordelaktig med en ring ekstra hvis man ønsker en rask akselerasjon i starten. Ellers har det liten effekt da det flater ut.

5f)

```python
import numpy as np
import matplotlib.pyplot as plt

e0 = 8.85 * 10**-12  # m^3*kg^-1*s^4*A^2
R1 = 3*10**-3  # m
Q1 = -2*10**-9  # C
R2 = 5*10**-3  # m
Q2 = 10*10**-9  # C
a = 1*10**-2  # m


def V1(x):
    return 1/(4*np.pi*e0)*Q1/np.sqrt(x**2+R1**2)


def V2(x):
    return V1(x) + 1/(4*np.pi*e0)*Q2/np.sqrt((x-a)**2+R2**2)


x = np.linspace(0.0001, 0.1, 1000)
p1 = plt.plot(x, V1(x), label="V1")
p2 = plt.plot(x, V2(x), label="V2")
plt.legend(loc="upper right")
plt.xlabel("x")
plt.ylabel("V")
plt.show()
```



6a)

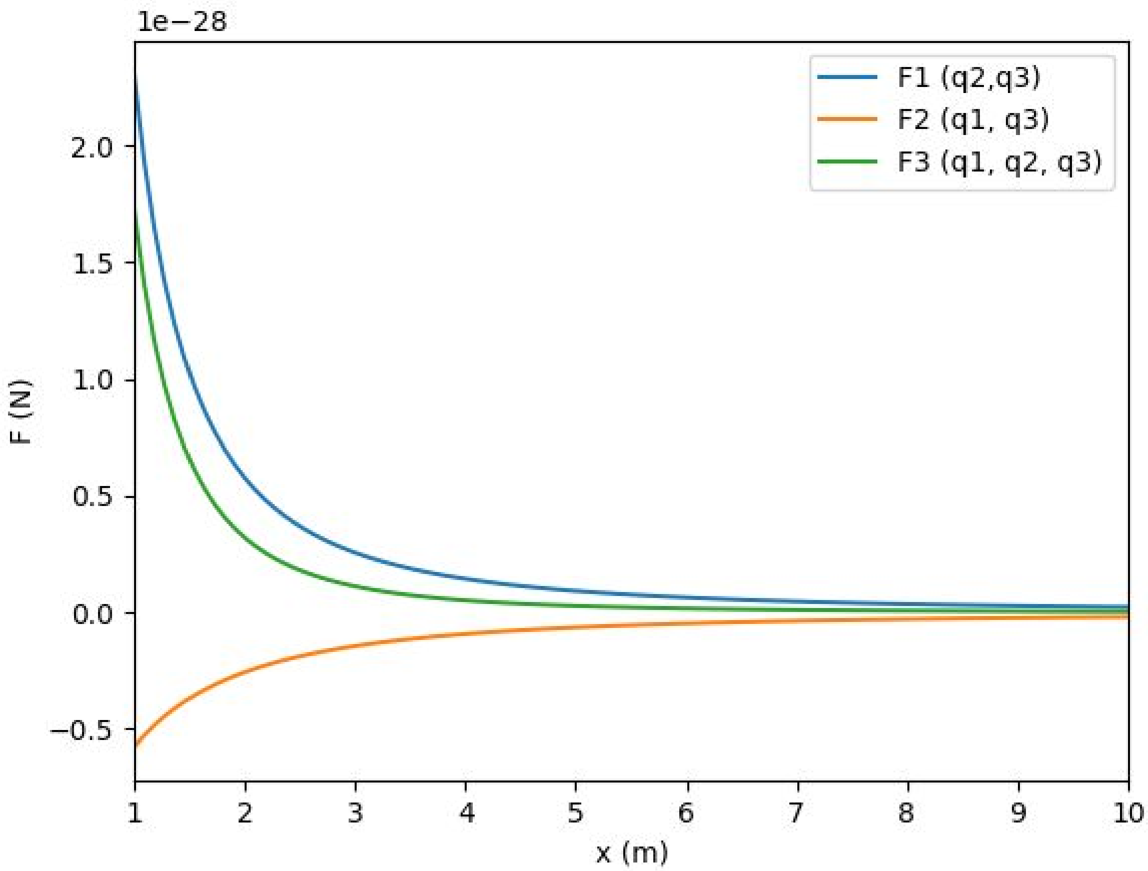


Elektron: $-1,602 \cdot 10^{-19}$ C
Proton: $+1,602 \cdot 10^{-19}$ C

En negativ ladning kan q_3 kan ikke være i likevekt siden q_2 er nærmere hvis q_3 ligger mellom 1m og 10m.

6a)

```python
import numpy as np
import matplotlib.pyplot as plt

e = 1.602 * 10**-19  # c
k = 8.988 * 10**9  # Nm^2C^-2


def F1(x):
    return k*-e*-e/(x**2)


def F2(x):
    return k*e*-e/(1+x)**2


def F3(x):
    return F1(x)+F2(x)


x = np.linspace(1, 10, 100)
p1 = plt.plot(x, F1(x), label="F1 (q2,q3)")
p2 = plt.plot(x, F2(x), label="F2 (q1, q3)")
p3 = plt.plot(x, F3(x), label="F3 (q1, q2, q3)")

plt.legend(loc="upper right")
plt.xlabel("x (m)")
plt.ylabel("F (N)")
plt.xlim([1, 10])
plt.show()
```



6b)

Like velit når $F1 = F2$

$$\Rightarrow \quad \frac{k\, q_1 q_3}{(x+1)^2} = \frac{k\, q_2 q_3}{x^2} \quad \Rightarrow \quad \underline{\underline{q_1 = \frac{(x+1)^2}{x^2} \cdot q_2}}$$

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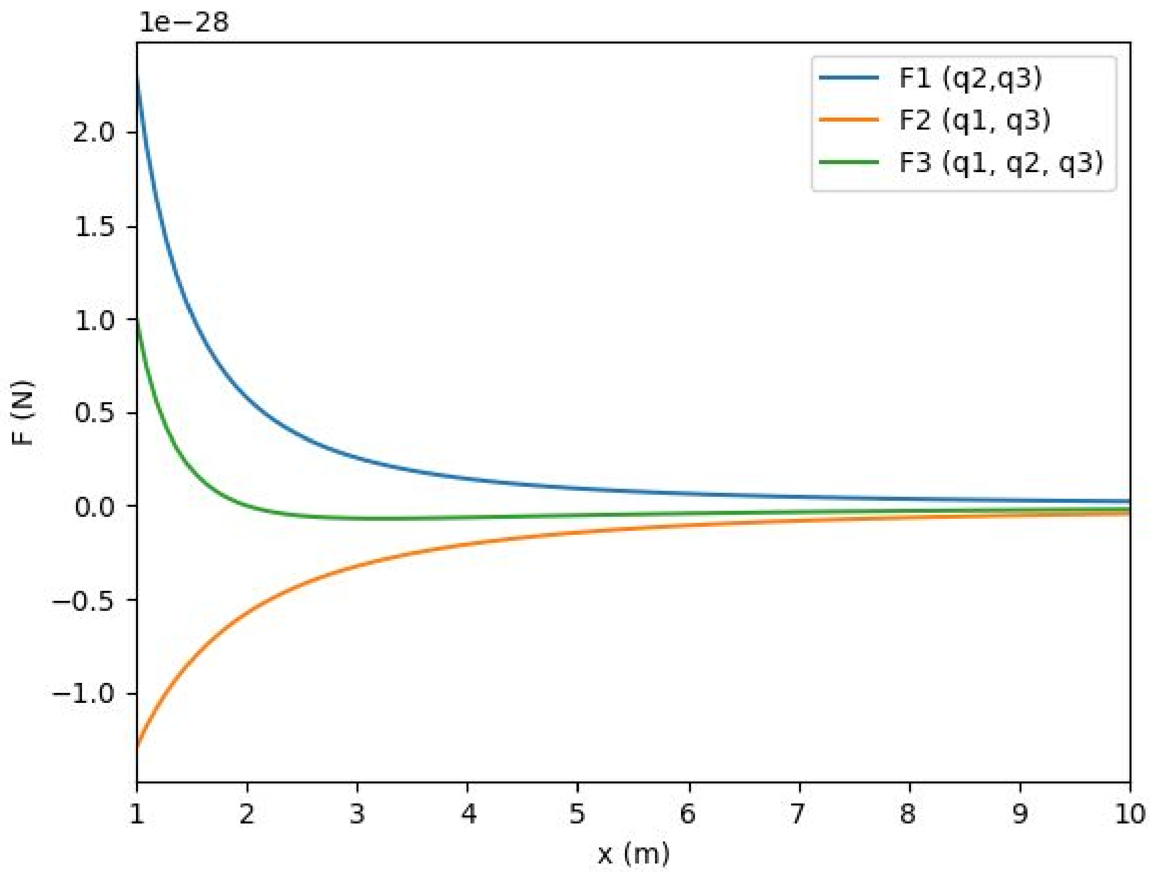
6b) Tilfelle q1 = 2.25*q2

```
import numpy as np
import matplotlib.pyplot as plt

e = 1.602 * 10**-19  # c
k = 8.988 * 10**9  # Nm^2C^-2

case = (2+1)**2/2**2


def F1(x):
    return k*-e*-e/(x**2)


def F2(x):
    return k*case*e*-e/(1+x)**2


def F3(x):
    return F1(x)+F2(x)


x = np.linspace(1, 10, 100)
p1 = plt.plot(x, F1(x), label="F1 (q2,q3)")
p2 = plt.plot(x, F2(x), label="F2 (q1, q3)")
p3 = plt.plot(x, F3(x), label="F3 (q1, q2, q3)")

plt.legend(loc="upper right")
plt.xlabel("x (m)")
plt.ylabel("F (N)")
plt.xlim([1, 10])
plt.show()
```



6c)

Det vil være stabil likevekt i $q_1 = \frac{(x+1)^2}{x^2} q_2$

6d)

Den elektriske permittiviteten har ingen ting å si

Ved dobling:

$$\frac{k\,2q_1\,2q_3}{(x+1)^2} = \frac{k\,2q_2\,2q_3}{x^2} \Rightarrow q_1 = \frac{(x+1)^2}{x^2} q_2$$

Som gir samme likevektspunkt, men kraften blir firedoblet.