Øving 3 - Hallvard - TFY4125

3.1)

$E_{kin} = W = Fs = ma \cdot \frac{1}{2} at^2 = \frac{1}{2} m v^2$

$KE_i = KE$

$V_i = V$

$KE_f = 2KE$

$\frac{KE_i}{KE_f} = \frac{V_i^2}{V_f^2} \Rightarrow \frac{1}{2} = \frac{V^2}{V_f^2}$

$\Rightarrow V_f^2 = 2V^2$

$\Rightarrow V_f = \sqrt{2V^2}$

$\underline{\underline{V_f = \sqrt{2}\, V^2}}$

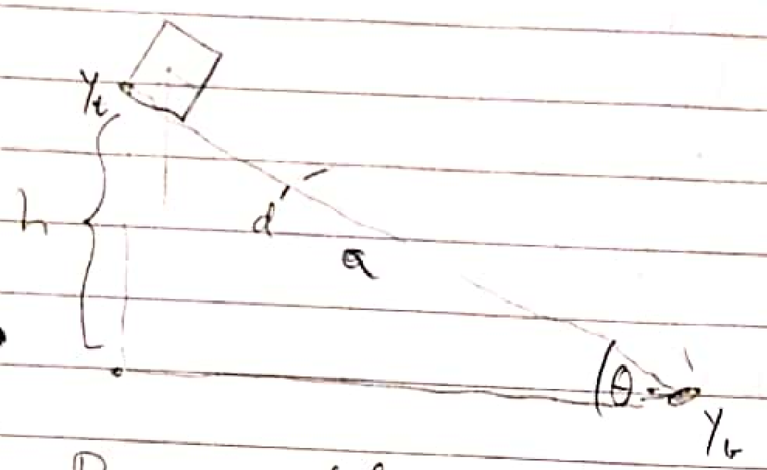
Hvis vi dobbler arbeid øker farten med en faktor $\sqrt{2}$.

3.2)

Det resulterende arbeidet blir null hvis man ser på det samlet, men arbeidet virker på hvert sitt objekt og det fører til at klossen blir gjort et arbeid på og beveger seg framover.

3.3

Antar at det ikke er luftmotstand og den totale energien er bevart.



t : topp
B : bunn

$$\sin(\theta) = \frac{h}{a} \Rightarrow h = a\sin(\theta)$$

Bunnpunkt:

$$mgh = mg(a\sin\theta)$$
$$mg(a\sin\theta) = \frac{1}{2}mv_b^2$$
$$\Rightarrow \quad v_b = \sqrt{2g(a\sin\theta)}$$

Topppunkt:

$$y_t = 0$$
$$y_b = (y_t - a\sin\theta) = -a\sin\theta$$
$$\frac{1}{2}mv_b^2 = mg(-a\sin\theta)$$
$$\Rightarrow \quad v_b = -\sqrt{2g(a\sin\theta)}$$

Farten i d:

$$mg(a\sin\theta) = mg(a\sin\theta - (a-d)\sin\theta) + \frac{1}{2}mv_d^2$$
$$\Rightarrow ga\sin\theta - ga\sin\theta + g(a-d)\sin\theta = \frac{1}{2}v_d^2$$

$\Rightarrow$

$$g a \sin\theta - g d \sin\theta = \frac{1}{2} v_d^2$$

$$\Rightarrow \underline{\underline{v_d = \sqrt{2ga\sin\theta - 2gd\sin\theta}}}$$

3.4 a)

Hookes lov: $F = -kx$

$$W = \int_c \vec{F}\, d\vec{s} \qquad d\vec{r} = d\vec{s}$$

$$\frac{d\vec{r}}{dt}\, dt = d\vec{s}$$

$$W = \int_{x_0=0}^{x=\Delta x} F\, dx$$

$$W = -k \int_{x_0=0}^{x=\Delta x} x\, dx = -k\left[\frac{1}{2}x^2\right]_0^{\Delta x} = \underline{\underline{-k\frac{1}{2}\Delta x^2}}$$

b)

$$\underline{\underline{E_{kin} = W = -k\frac{1}{2}\Delta x^2}}$$

c)

$$F = -k_1 x - k_2 x^3$$

$$W = \int_c F \, ds$$

$$\Rightarrow \quad W = \int_{x_0=0}^{x=\Delta x} F \, dx$$

$$\Rightarrow \quad W = -k_1 \left[\frac{1}{2} x^2\right]_{x_0=0}^{x=\Delta x} - k_2 \left(\frac{1}{4} x^4\right]_{x_0=0}^{x=\Delta x}$$

$$W = -k_1 \frac{1}{2} \Delta x^2 - k_2 \frac{1}{4} \Delta x^4$$

d)

$$U(x) = \int_0^{\Delta x} F \, dx = -k_1 \frac{1}{2} \Delta x^2 - k_2 \frac{1}{4} \Delta x^4$$

$$U(x) = -k_1 \frac{1}{2} \Delta x^2 - k_2 \frac{1}{4} \Delta x^4 - \text{Motstand}$$

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4) 5.a)

V_G : Fart uten friksjon

V_A : Fart med friksjon (luftmotstand)

$F_G = mg \Rightarrow V_G = V_0 + gt,\ v_0 = 0 \Rightarrow V_G = gt$

$F_A = mg - kv \Rightarrow ma = mg - kv$

$\Rightarrow ma = mg - kv$

$\Rightarrow m\frac{dv}{dt} = mg - kv \quad | :k$

$\Rightarrow \frac{m}{k}\frac{dv}{dt} = \frac{mg}{k} - v \qquad mg/k = v_t$

$v_0 = 0$

$\Rightarrow \int_0^v \frac{1}{v - v_t}\,dv = -\frac{k}{m}\int_0^t dt$

$\Rightarrow \ln\frac{v_t - v}{v_t} = -\frac{k}{m}t \Rightarrow 1 - \frac{v}{v_t} = e^{-\frac{kt}{m}}$

$\Rightarrow v_A = v_t - v_t\, e^{-\frac{kt}{m}}$

$\Rightarrow V_A = \frac{mg}{k}\left(1 - e^{-\frac{kt}{m}}\right)$

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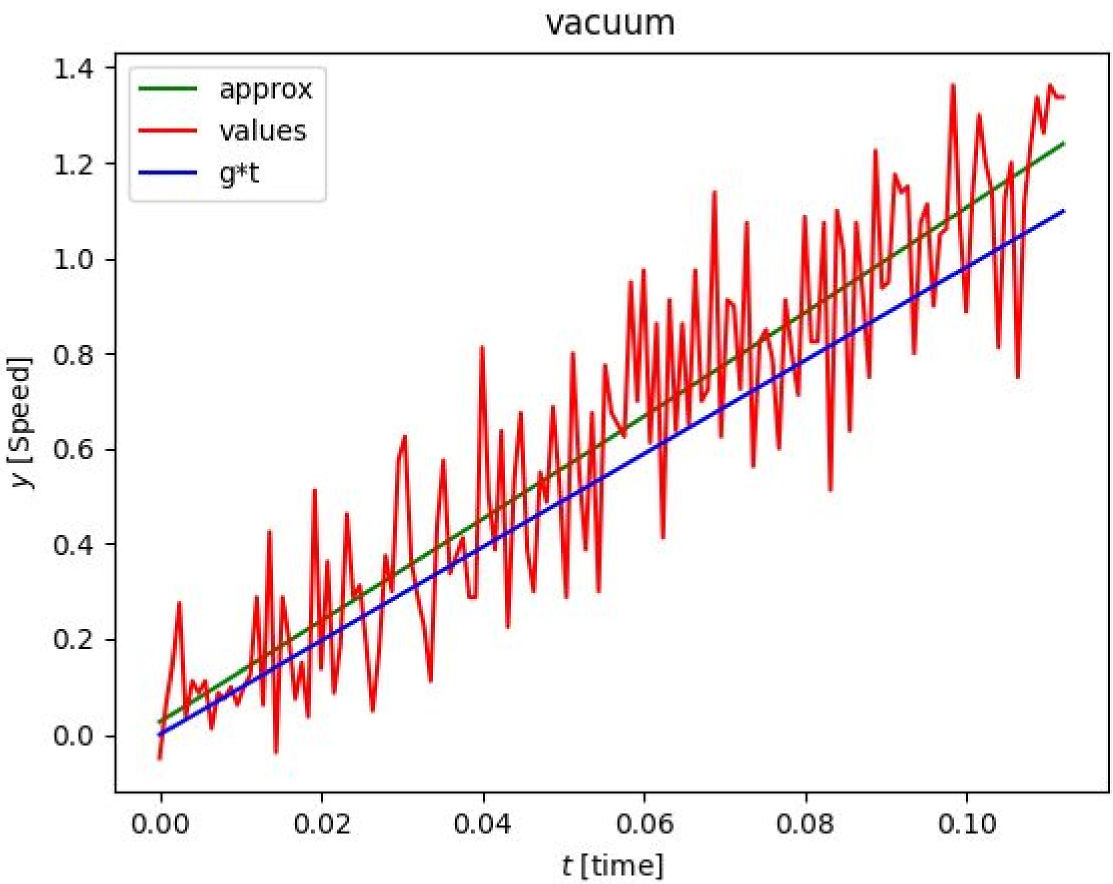
Oppgave 4.5b)i)

```python
import numpy as np
import matplotlib.pyplot as plt

# Trimming av data før fall
t_novac = np.genfromtxt("tnonvacuum.txt")[600:]
x_novac = np.genfromtxt("xnonvacuum.txt")[600:]
# Trimming av data før fall og lik lengde
t_vac = np.genfromtxt("tvacuum.txt")[600:741]
x_vac = np.genfromtxt("xvacuum.txt")[600:741]

# find smallest number
min_t_novac = min(t_novac)
min_t_vac = min(t_vac)

# Starte på t=0
t_vac = [time - min_t_vac for time in t_vac]
t_novac = [time - min_t_novac for time in t_novac]


def andregrad(time, pvalues):
    returnarray = []
    for i in range(len(time)):
        returnarray.append(pvalues[0]*time[i]**2+pvalues[1]*time[i]+pvalues[2])
    return returnarray


v_novac = np.diff(x_novac)/np.diff(t_novac)
v_novac = np.append(v_novac, v_novac[v_novac.size-1])
p_novac = np.polyfit(t_novac, v_novac, 2)
print(p_novac)

v_vac = np.diff(x_vac)/np.diff(t_vac)
v_vac = np.append(v_vac, v_vac[v_vac.size-1])
p_vac = np.polyfit(t_vac, v_vac, 2)
print(p_vac)  # Verdier for 5bii)


plt.plot(t_vac, andregrad(t_vac, p_vac), 'g-', label="approx")
plt.plot(t_vac, v_vac, 'r-', label="values")
plt.plot(t_vac, np.multiply(t_vac, 9.81), 'b-', label="g*t")
plt.title("vacuum")
plt.legend()
plt.xlabel(r'$t$ [time]')
plt.ylabel(r'$y$ [Speed]')
plt.show()

```



Sammenlikn polynomet du får fra målingen xvacuum.txt med fritt fall løsningen.
Hvilken verdi får du for tyngdens akselerasjon?

- Linjene passer ganske bra, men approksimasjonen har en tyngdekraftskonstant som er nesten lik 11 som virker for høyt.

Kan du si noe om massen til objektet fra denne målingen?

- Nei siden det ikke er en del av funksjonen b)

$V_G = gt \qquad V_A = \frac{mg}{k}\left(1 - e^{-\frac{kt}{m}}\right)$

$e^t = \frac{t^0}{0!} + \frac{t^1}{1!} + \frac{t^2}{2!}$

$\Rightarrow V_A = \frac{mg}{k}\left(1 - \left(1 + \frac{kt}{m} + \frac{\left(\frac{-kt}{m}\right)^2}{2}\right)\right)$

$\Rightarrow V_A = \frac{mg}{k}\left(\frac{\left(\frac{-kt}{m}\right)^2}{2} - \frac{kt}{m}\right)$

$V_A = \frac{mg}{k}\left(\frac{k^2t^2}{m^2 \cdot 2} - \frac{kt}{m}\right) = g\left(\frac{kt^2}{2m} - t\right)$

$V_A = g\frac{k}{2m}t^2 - gt + 0$

P verdier jeg fikk : [2.910, 10.50, 0.0267]

Polynom $2.91t^2 + 10.5t + 0,0267$

- Kan ikke si noe om terminal hastighet da målingen stopper før det.
- Kan ikke si noe om k eller m alene
- forholdet mellom k og m

$\Rightarrow g\frac{k}{2m} \approx 2.91$

$\Rightarrow k \approx 0.593m$

6) a) "N"

$$\frac{1}{2}(m_1+m_2)v_A^2 = m_2 g h - \mu \cdot (m_1 \cdot g) \cdot h$$

$$v_A = \sqrt{\frac{2}{m_1+m_2}\left(m_2 g h - \mu \cdot m_1 \cdot g \cdot h\right)}$$

6) b)

$$\frac{1}{2} m_1 v_b^2 = \frac{1}{2} m_1 v_A^2 - \mu (m_1 \cdot g) \cdot (l-h)$$

$$\Rightarrow v_b = \sqrt{v_A^2 - 2\mu g (l-h)}$$

6 c)

$$V_a = \sqrt{\frac{2}{3{,}00\,kg}\left(2{,}00\,kg \cdot 9{,}81\,m/s^2 \cdot 1{,}00\,m - 0{,}300 \cdot 1{,}00\,kg \cdot 9{,}81 \cdot 1{,}00\,m\right)}$$

$$V_a = \sqrt{\frac{2}{3{,}00\,kg} \cdot 16{,}677\,kg\,m^2/s^2}$$

$$V_a = \sqrt{11{,}118\,m^2/s^2} = 3{,}33\,m/s$$ Enhet stemmer!

$$V_b = \sqrt{(3{,}33\,m/s)^2 - 2 \cdot 0{,}300 \cdot 9{,}81\,m/s^2 (2{,}00\,m - 1{,}00\,m)}$$

$$V_b = \sqrt{11{,}0889\,m^2/s^2 - 5{,}886\,m^2/s^2}$$

$$V_b = 2{,}28\,m/s$$ Enhet stemmer!

7a) $mgL = mg(2x) - \frac{1}{2}mv^2$

Energien i start går over til en lavere potensiell energi og en høyere kinetisk energi.

$mgL = mg(2x) - \frac{1}{2}mv^2 \quad |:m$

$\Rightarrow \frac{1}{2}v^2 = g(2x) - gL$

$\Rightarrow v^2 = 2g(2x-L)$

$\underline{\underline{v = \sqrt{2g(2x-L)}}}$

7b) Stram snor $\Leftrightarrow$ sentripetalakselerasjon ≥ 0

$a = \frac{v^2}{r}$

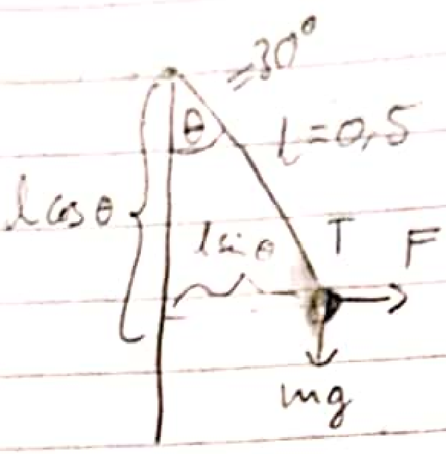
$a = \frac{\sqrt{2g(2x-L)}^2}{x} = \frac{2g(2x-L)}{x}$

$\Rightarrow a = \frac{4gx - 2gL}{x} = 4g - \frac{2gL}{x} > 0$

$\Rightarrow x = \frac{2gL}{4g} = \underline{\underline{\frac{1}{2}L = x}}$

X må være minst $\frac{1}{2}L$ for stram snor.

5.8a)



$m = 0{,}1\,kg$

$$T\cos\theta = mg \Rightarrow T = \frac{mg}{\cos\theta}$$

$$T\sin\theta = F$$

$$\Rightarrow F = mg \cdot \frac{\sin\theta}{\cos\theta} = mg\tan\theta = \underline{\underline{0{,}57\,N}}$$

5.9b)

$$0{,}2\,N = 0{,}1\,kg \cdot 9{,}81\,m/s^2 \cdot \tan\theta$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{0{,}2}{0{,}1 \cdot 9{,}81}\right) = \underline{\underline{11{,}52°}}$$

5.9c)

$$T = 1\,s \qquad v = \frac{2\pi r}{T} \qquad r = l\sin\theta$$

$$a_r = \frac{-v^2}{r} \qquad F = -m a_r = m \frac{\left(\frac{2\pi l\sin\theta}{T}\right)^2}{l\sin\theta}$$

$$\tan\theta = \frac{F}{mg} = \frac{\frac{m\left(\frac{2\pi l\sin\theta}{T}\right)^2}{l\sin\theta}}{mg}$$

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$$\tan\theta = \frac{(2\pi l \sin\theta)^2}{l\sin\theta\, g\, T^2} = \frac{4\pi^2 l^2 \sin^2\theta}{l\sin\theta\, g\, T^2}$$

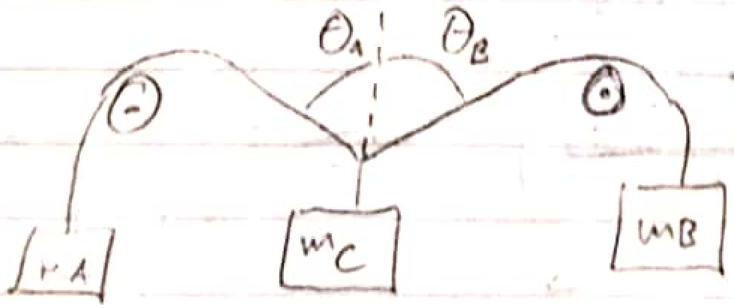
$$\tan\theta = \frac{4\pi^2 l \sin\theta}{g T^2}$$

$$\Rightarrow \frac{\tan\theta}{\sin\theta} = \frac{1}{\cos\theta} = \frac{4\pi^2 l}{g T^2}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{g T^2}{4\pi^2 l}\right)$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{9.81 \cdot 1}{4 \cdot \pi^2 \cdot 0.5}\right) = \underline{\underline{60.2^\circ}}$$

9) a)



$$m_c g = \sin(45^\circ) m_b g + \sin(60^\circ) m_a g$$

$$10\,kg = \frac{\sqrt{2}}{2} m_b + \frac{\sqrt{3}}{2} m_a \Rightarrow m_b = \frac{20 - \sqrt{3} m_a}{\sqrt{2}}$$

$$m_a g \cos(90 - \theta_A) = m_b g \cos(90 - \theta_B)$$

$$\Rightarrow m_b = m_a \frac{\cos(90 - \theta_A)}{\cos(90 - \theta_B)}$$

$$\Rightarrow m_a \frac{\cos(90-\theta_A)}{\cos(90-\theta_B)} = \frac{20-\sqrt{3}\, m_a}{\sqrt{2}}$$

$$\Rightarrow m_a \left(\frac{\cos(90-\theta_A)}{\cos(90-\theta_B)} + \frac{\sqrt{3}}{\sqrt{2}} \right) = 20$$

$$\Rightarrow m_a = \frac{20}{\frac{\cos(90-30)}{\cos(90-45)} + \frac{\sqrt{3}}{\sqrt{2}}} = \underline{\underline{10{,}35 \text{ kg} = m_a}}$$

$$m_b = 10{,}35 \text{ kg} \cdot \frac{\cos(90-30)}{\cos(90-45)} = \underline{\underline{7{,}32 \text{ kg} = m_b}}$$

9b)

$$10 = 8 \sin(\theta_B) + 6 \sin(\theta_A)$$

$$8 \cos(90-\theta_B) = 6 \cos(90-\theta_A)$$
$$8 \sin(\theta_B) = 6 \sin(\theta_A)$$

$$\sin(\theta_A) = \frac{4}{3} \sin(\theta_B)$$

$$\Rightarrow 10 = 8 \sin(\theta_B) + 6 \cdot \frac{4}{3} \sin(\theta_B)$$
$$\Rightarrow \frac{10}{16} = \sin(\theta_B) \quad \Rightarrow \quad \theta_B = 38.7^\circ$$
$$\Rightarrow \theta_A = \sin^{-1}\left(\frac{4}{3} \sin(38{,}7^\circ) \right) = 56{,}4^\circ$$

$$\underline{\underline{\theta_A = 56{,}4^\circ \qquad \theta_B = 38{,}7^\circ}}$$

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10)

$F = -bv \qquad m = (1,0 \pm 0,2)\,kg$

$b = (0,51 \pm 0,05)\,Ns/m \qquad g = (9,8 \pm 0,2)\,m/s^2$

$\Rightarrow \quad mg = -bv$

$$\Rightarrow \quad v = -\frac{mg}{b} = -\frac{(1,0 \pm 0,2)(9,8 \pm 0,2)}{(0,51 \pm 0,05)}\ m/s$$

$\frac{0,2}{1,0} \cdot 100\% = 20\%$, $\quad \frac{0,2}{9,8} \cdot 100\% = 2,1\%$

$\frac{0,05}{0,51} \cdot 100\% = 9,8\%$

$\Rightarrow$

$$v = \frac{9,8 \pm 22,1\%}{0,51 \pm 9,8\%}$$

$$v = 19 \pm 31,9\% = \underline{\underline{19 \pm 6,1\ m/s}} = v$$

Vekten gir størst bidrag til usikkerheten.

11) a)

$$ma = mg \sin\theta - \mu_k mg \cos\theta$$

$$a = 9,81 \cdot \sin(30°) - 0,42 \cdot 9,81 \cdot \cos(30°) = \underline{\underline{1,34\ m/s^2}}$$

11b)

$$ma = mg\sin\theta - \mu mg\cos\theta - F$$

$$a = g\sin\theta - \mu g\cos\theta - \frac{F}{m}$$

$$\Rightarrow a = 9{,}81 \cdot \sin(30) - 0{,}42 \cdot 9{,}81 \cdot \cos(30) - \frac{1}{1}$$

$$\Rightarrow a = \underline{\underline{0{,}34\, m/s^2}}$$

11c)

Da blir den negativ og klossen vil bevege seg opp planet.

11d)

$$ma = mg\sin\theta - \mu_k mg\cos\theta$$

$$\Rightarrow a = g(\sin\theta - \mu_k\cos\theta)$$

$$\Rightarrow a = 9{,}81\left(\sin(30^\circ) - (0{,}42 + 0{,}13v) \cdot \cos(30^\circ)\right)$$

$$\Rightarrow a = 9{,}81 \cdot \left(\frac{1}{2} - (0{,}36 + 0{,}11v)\right)$$

$$a = 4{,}91 - 3{,}5 + 1{,}1v \Rightarrow a = 1{,}41 + 1{,}1v$$

$$\Rightarrow \frac{dv}{dt} = 1{,}41 + 11v$$

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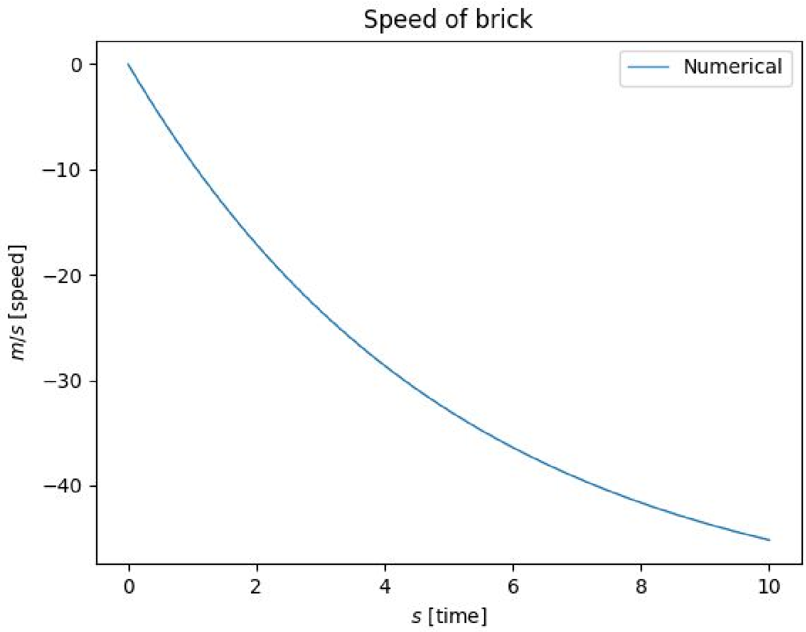
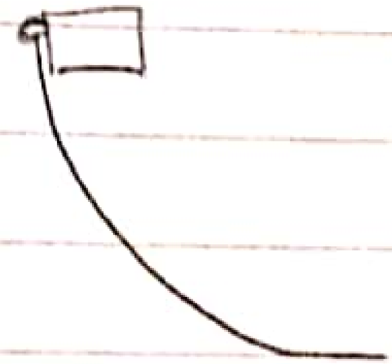
5.11d)

```python
import numpy as np
import matplotlib.pyplot as plt  # Used for plotting results


def step_Euler(y, h, f):
    next_y = y + h * f(y)
    return next_y


def full_Euler(h, f, y_0=0, start_t=0, end_t=10):
    N = int((end_t - start_t) / h)
    t_list = np.linspace(start_t, end_t, N + 1)
    y_list = np.zeros(N + 1)
    y_list[0] = y_0
    for i in range(0, N):
        y_list[i + 1] = step_Euler(y_list[i], h, f)
    return y_list, t_list


def g(v):
    speed = 9.81*(np.sin(30)-(0.42+0.13*v)*np.cos(30))
    return speed


y_0 = 0
h = 0.01
t_0 = 0
t_N = 10


y_list, t_list = full_Euler(h, g, y_0, t_0, t_N)
plt.plot(t_list, y_list, label="Numerical", linewidth=1)


# Making the plot look nice
plt.legend()
plt.title("Speed of brick")
plt.xlabel(r'$s$ [time]')
plt.ylabel(r'$m/s$ [speed]')
plt.show()

```



6 Utledning



$$W = Fs$$

$$F = \mu N = \mu m \frac{v^2}{r}$$

$$W = \mu m \frac{v^2}{r} \cdot \frac{2\pi r}{4}$$

$$\Rightarrow \underline{\underline{W = \frac{1}{2} \mu m v^2 \pi}}$$

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