

Brussels Sprouts Game

In this video the presenter is introducing the audience to a game called Brussels Sprouts. The point of the game is to start out with two or more crosses on paper, then each player takes turns connecting any two ends of the crosses without intersecting the lines that are already there. After connecting the lines the player that made the connection is supposed to make a line in the middle of the connection that was made. The presenter proceeds to play the game with the cameramen and after a certain amount of moves the presenter wins the game because the camera men did not have any more legal moves that could be made. As it turns out, this game will always have a definite winner and a definite loser, there is no strategy that can be used to win this game. The presenter states that if the game is started out with two crosses, then the person who makes the first move always wins the game. It is possible to predict the number of moves that could be made before the game is even started. Since starting off with two crosses leads to eight moves, then the player that moves first will win.

If each cross is made into a vertex and each connection is an edge, then what we end up with is a planar graph – a graph where none of the edges intersect. For all planar graphs there is a theorem called Euler Characteristic, which states that if V is number of vertices, E is a number and edges, and F is a number if faces, then $V - E + F = 2$. We can use this theorem to calculate the number of moves any given planar graph will have. Let M be the number of moves we make during a game, then $V = 2 + M$, because we start out with 2 vertices and we create one more every time we make a move. $E = 2M$, because every time we make a move we create 2 edges since we cut the line we created in half. Faces are calculated slightly differently. If we look at any game of Brussels Sprouts, we will see that each graph face (closed off area of the graph) has a piece of the cross inside it. For example a cross has 4 pieces in it, two crosses have 8 and so on. This number depends on the amount of crosses we start off with, and since we are starting of with 2 crosses then we have 8 pieces. Now since we know all the variables, we can calculate the amount of all possible moves.

$$\begin{aligned}V - E + F &= 2 \\(2+M) - (2M) + 8 &= 2 \\M &= 8\end{aligned}$$

Now, if you are not the first person to start the game then in the scenario the game could be lost, however if the number of crosses we start off with is changed to 3, then by using the same formula the number of moves is increased to 13, so the player that will go second will win the game. In the end, if the number of crosses that we start with is even then then player that makes the first move wins, if the number of crosses is odd then player who makes the second move wins.

I found this topic interesting because graphs theory has always been interesting for me. A couple of years ago in school, I was studying some of the algorithms of graph theory, and on top of that I had to implement them in C++. I remember spending days trying to figure out how to do it, and I let all my classmates turn in my projects as their own because nobody could be bothered to code all of the algorithms. In the end, all of my classmates got an A for the course while I received a B, even though I was the author of all the labs that were turned in. Since then I have been very interested in figuring out graph theory algorithms and learning how to apply them in the real world, hence why I chose to watch this video. Besides, it is always nice to find a game that you can always win.