ASSIGNMENT 3

DUE: Wednesday October 4, 7 PM. DO NOT COPY. ACKNOWLEDGE YOUR SOURCES.

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1. [5 marks] **Making Change.** In the Canadian coin system (with pennies), all coins have a value of the form $2^i 5^j$. For example, a penny is $2^0 5^0$, a quarter is 5^2, a loonie is $2^2 5^2$, etc. As mentioned in class, for the Canadian coin system the greedy algorithm always gives change consisting of the fewest number of coins.

   (a) [5 marks] Consider a different coin system with the property that all coins have a value of the form $2^i 5^j$. Suppose the system includes the penny, so we can make change for any amount. In such a coin system, does the greedy algorithm always gives change consisting of the fewest number of coins? If you answer NO, you should give an example of a coin system and an amount where the greedy algorithm does not give the fewest number of coins. If you answer YES, you must prove that for any such coin system and any amount, the greedy algorithm gives the fewest number of coins.

   (b) [BONUS 5 marks, only given to clean solutions] Answer the same question if all coins have a value of the form 2^i. (One example of such a coin system would be a 1 cent, 4 cent, and 32 cent coin.)

2. [15 marks] **A Scheduling Problem.** Suppose you have n activities $a_1, a_2, \ldots, a_n$ where each activity a_i is given by a start time, end time pair, $a_i = (s_i, t_i)$ with $t_i > s_i$. You want to schedule these activities into the fewest number of rooms so that the activities scheduled into any one room do not overlap in time. Let the rooms be $R_1, R_2, \ldots$.

   In this question you will explore greedy approaches where you sort the activities (by some criterion), and then go through the ordered list, putting each activity into the first room from $R_1, R_2, \ldots$ in which it fits.

   For example, if you have the ordered list $a_1 = (6, 8), a_2 = (9, 12), a_3 = (7, 9), a_4 = (8, 10)$ then the greedy strategy would assign $a_1 = (6, 8)$ and $a_2 = (9, 12)$ to room R_1. Then $a_3 = (7, 9)$ would have to go in R_2 and $a_4 = (8, 10)$ in R_3. However, an alternative solution uses two rooms: R_1 has $a_1 = (6, 8), a_4 = (8, 10)$ and R_2 has $a_3 = (7, 9), a_2 = (9, 12)$. These two solutions are illustrated below.
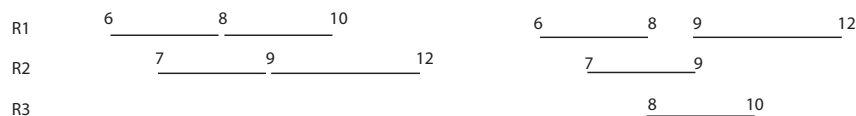
   (a) [5 marks] Show that sorting the activities by end time, t_i, does not always give the smallest number of rooms.

(b) [10 marks] Show that sorting the activities by start time, s_i, gives the smallest number of rooms. HINT: You will not need a full-fledged exchange proof. Use the following observation: if there are k activities that overlap in time then any schedule must use at least k rooms.