ASSIGNMENT 10

DUE: Wednesday November 29, 7 PM. DO NOT COPY. ACKNOWLEDGE YOUR SOURCES.

Please read `http://www.student.cs.uwaterloo.ca/~cs341` for general instructions and policies. In particular, note that "giving" an algorithm includes justifying correctness and run time.

For this assignment, you may assume that the following problems are NP-complete:
3-SAT, Independent Set, Clique, Vertex Cover, Hamiltonian cycle (directed and undirected), Hamiltonian path, Subset Sum.

1. [10 marks] Prove that the following problems are NP-complete:

   (a) [5 marks] Given two graphs, $H = (V_H, E_H)$, and $G = (V_G, E_G)$, is H a subgraph of G, i.e. is there a mapping π of the vertices of H to the vertices of G such that π is one-to-one (it never maps two vertices of H to the same vertex of G) and such that for every pair of vertices $u, v \in V_H$, we have $(u, v) \in E_H$ iff $(\pi(u), \pi(v)) \in E_G$.

   (b) [5 marks] Prove that the following problem is NP-complete. Given an edge-weighted undirected graph $G = (V, E)$ and a number $k \in \mathbb{N}$, does G have a simple path of length $\geq k$? Recall that a "simple" path is one that does not repeat vertices. Hint: even the version where all edge weights are 1 is NP-complete.

2. [10 marks] Prove that the following problem is NP-complete. Given a list of n positive integers, $a_1, a_2, \ldots, a_n$, indexed by $S = \{1, \ldots, n\}$, is there a partition $S = A \cup B$ with $A \cap B = \phi$ such that $\sum_{i \in A} a_i = \sum_{i \in B} a_i$.

3. [10 marks] Prove that the following problem is NP-complete. Given a directed graph G and a number k, can we remove $\leq k$ edges to get a directed acyclic graph?

   Hints. Reduce from Vertex Cover. Suppose we have an input $G = (V, E)$, $k \in \mathbb{N}$, for vertex cover. Suppose graph G has m edges are n vertices $v_1, v_2, \ldots, v_n$. Construct a directed graph G' with $2n$ vertices $v_1, \ldots, v_n, v'_1, \ldots, v'_n$ and the following $n + 2m$ edges:

   - (v_i, v'_i) for all $i = 1, \ldots, n$
   - (v'_i, v_j) and (v'_j, v_i) for all $(v_i, v_j) \in E$.