CS 341, Fall 2017 A. Lubiw, B. Ma, E. Zima

ASSIGNMENT 7

DUE: Wednesday November 8, 7 PM. DO NOT COPY. ACKNOWLEDGE YOUR SOURCES.

Please read `http://www.student.cs.uwaterloo.ca/~cs341` for general instructions and policies. In particular, note that "giving" an algorithm includes justifying correctness and run time.

1. [10 marks] **Depth First Search and Topological Sort.** Let G be a directed acyclic graph. We want a linear time algorithm to decide if there is a directed path in G that goes through every vertex exactly once. For each of the following approaches, either prove or disprove that it works correctly.

   (a) [5 marks] Run DFS from some vertex. If the DFS tree is a path, then output the path, otherwise output FAIL.

   (b) [5 marks] Run DFS to find a topological sort of the graph and test if that ordering is a directed path. If it is, then output the path, otherwise output FAIL. (Recall that we get a topological sort by ordering the vertices in decreasing order of finish time.)

2. [10 marks] **Path length.** Given an undirected graph G with non-negative weights on the edges and a start node s, there may be more than one shortest path from s to v. In this case we prefer the shortest path with the fewest edges. Define the *hop number*, $h(v)$ to be the minimum number of edges in a shortest path from s to v. Define $h(s)$ to be 0. Give an $O(m \log n)$ time algorithm to compute the hop numbers of all the vertices of G. Here n is the number of vertices and m is the number of edges.

3. [10 marks] **MST.** Suppose you have an edge weighted undirected graph G and a minimum spanning tree T. Let the weight function be $w : E \to \mathbb{R}^{\geq 0}$. One edge $e = (u, v)$ changes its weight and we want to update the minimum spanning tree. There are 4 cases. In each case give a linear time algorithm to update T. You may assume that T is stored as adjacency lists.

   (a) $e \notin T$ and its weight goes up

   (b) $e \notin T$ and its weight goes down

   (c) $e \in T$ and its weight goes down

   (d) $e \in T$ and its weight goes up