PROGRAMMING ASSIGNMENT 1

DUE: Wednesday October 4, 7 PM. DO NOT COPY. ACKNOWLEDGE YOUR SOURCES.

Please read `http://www.student.cs.uwaterloo.ca/~cs341` for general instructions and policies.

1. [20 marks] **Divide-and-conquer.** The input for this problem consists of n radar stations where station i is given by its integer coordinates x_i and y_i in the plane. We say that station i can transmit to station j if station j is south-west of station i, i.e., $x_i \geq x_j$ and $y_i \geq y_j$. The "load factor" of station i is defined to be the number of stations it can transmit to (not counting itself). The goal is to compute the load factor of each station. You will develop and implement two algorithms for this problem

   Input and output. The input consists of $n + 1$ lines. The first line has n, the number of stations. The following n lines have three numbers each: the ID number of station i, and its x_i and y_i coordinates. All station IDs are different. We will not be testing whether your program detects input errors.

   The output must have n lines, each giving a station ID number and the load factor for that station. Your output must be sorted by station ID (in increasing order).

   Sample input

   4
   2 3 3
   10 4 2
   13 5 6
   7 2 1

   Sample output

   2 1
   7 0
   10 1
   13 3

   (a) Implement a brute-force algorithm for this problem. For each radar station i, go through the whole list of radar stations to find the ones that i can transmit to. Your algorithm will have worst-case run time $\Theta(n^2)$.

   (b) Implement an efficient divide-and-conquer algorithm for this problem. Your algorithm should have worst-case run time $\Theta(n \log n)$. (Our tests will check that your algorithm is fast enough.) Hints for the algorithm are as follows:

Sort the stations by x_i and by y_i, creating arrays X and Y.

Write a recursive routine that takes as input a set of radar stations given as two arrays, array X sorted by x_i and array Y sorted by y_i:

- Divide the stations into Left half and Right half based on x_i
- Construct arrays X_1, X_2 of stations on the Left/Right sorted by x_i.
- Construct arrays Y_1, Y_2 of stations on the Left/Right sorted by y_i. Be careful to take linear time.
- Recurse on X_1, Y_1 and on X_2, Y_2.
- Combine the solutions to the two halves. Be careful to take linear time.
  - Observe that if station i is in the Left half then you have the correct load factor.
  - If a station i is in the Right half, then i can also transmit to all stations j in the Left half with $y_j \leq y_i$. You can find the number of such stations by using array Y_1.