Localized modes for acoustic waves in a bubbly crystal with a defect

Author: Erik Orvehed Hiltunen, Master student in Engineering physics, Uppsala University
Supervisor: Habib Ammari, Professor of Applied Mathematics, Department of Mathematics, ETH Zürich

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1 Purpose and goals

We consider acoustic waves propagation through a liquid with bubbles. These bubbles resonates at a frequency which correspond to wavelengths much larger than the bubble, the so called Minnaert resonance frequency. Specifically, we consider a periodic array of bubbles, where the size of one of the bubbles is perturbed. The goal is to analytically and numerically show that this crystal has a localized eigenmode close to the defect bubble.

It has previously been shown that an unperturbed, periodic bubble crystal has a bandgap close to the Minnaert resonance [2]. Physically, this means that waves with frequencies inside the bandgap will be exponentially decaying inside the crystal. Furthermore, it has been experimentally shown that a perturbed crystal will have a localized mode, i.e. for waves of certain frequencies inside the bandgap, the wave will be localized around the perturbation and will be exponentially decaying away from the perturbation [3].

2 Planning

zThe following plan shows the work which needs to be done and the time plan for the project.

Week	Planned work
2017 - 37	Reading old papers, especially [1] and [2]
38	Reading old papers, especially [1] and [2]
39	Working on the essential spectrum for the original and perturbed crystals
40	Continue work on essential spectrum, formulating the perturbed problem
41	Formulating the perturbed problem and the boundary integral equations
42	Solving the model problem with two different methods
43	Working on equations for the numerical implementation, Taylor expansions of the quasi-periodic Green's function
44	Starting the numerical implementation
45	Continuing with the numerical implementation, starting writing the paper
46	Deriving the correct jump conditions for crystal single layer potential, debugging the code
47	Finish the code, start the analysis. Read on Taylor expansions for single layer potentials for perturbed boundaries.
48	Use Taylor expansions to reduce the 4x4 system to a 2x2 system
49	Perform asymptotic analysis for the radius to write the system as the original + perturbation
50	Show that the system has a characteristic value inside the bandgap
51	Finish the analysis
2018 - 03	Write the paper
04	Write the paper
05	Write the paper
06	Finish the paper

References

[1] H. Ammari, B. Fitzpatrick, D. Gontier, H. Lee, and H. Zhang. Minnaert resonances for acoustic waves in bubbly media. *ArXiv e-prints*, March 2016.

[2] H. Ammari, B. Fitzpatrick, H. Lee, S. Yu, and H. Zhang. Subwavelength phononic bandgap opening in bubbly media. *ArXiv e-prints*, February 2017.

[3] Fabrice Lemoult, Nadège Kaina, Mathias Fink, and Geoffroy Lerosey. Soda cans metamaterial: A subwavelength-scaled phononic crystal. *Crystals*, 6(7), 2016.