Big Data Computing

Master's Degree in Computer Science

2019-2020

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SAPIENZA
UNIVERSITÀ DI ROMA

Recommendation Strategies

3 approaches to recommender systems

Content-based filtering

Collaborative filtering

- **Neighborhood-based** (a.k.a. memory-based)
- **Latent-factor-based** (a.k.a. model-based)
- **Hybrid** (memory-model-based)

Hybrid

COLLABORATIVE FILTERING

Collaborative Filtering (CF)

Idea

Recommend items to user u based on preferences of other users similar to u

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Core concept:

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No need for explicit creation of user/item profiles

Collaborative Filtering: Approaches

3 main approaches to collaborative filtering

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Neighborhood-based (Memory-based) CF

Compute the relationship between **users** or **items**

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Evaluates a user's preference for an item based on ratings of "neighboring" users for that item

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Compute the relationship between **users** or **items**

User-based

Evaluates a user's preference for an item based on ratings of "neighboring" users for that item

Item-based

Evaluates a user's preference for an item based on ratings of "neighboring" items by the same user

USER-BASED COLLABORATIVE FILTERING

User-based Neighborhood

Given a user u and an item i not rated by u, we want to estimate $r(u, i)$

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Given a user *u* and an item *i* not rated by *u*, we want to estimate *r(u, i)*

From the set of users $\{u': u' \neq u\}$ who have already rated *i*
extract a subset of *k* **neighbours** of *u*

k-neighborhood of *u* is found on the basis of the similarity between user ratings without the need of explicit user profiles

Estimate *r(u, i)* based on the ratings of users in the *k*-neighborhood of *u*

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In theory, rating prediction *r(u,i)* could be defined on *any* item *i* not rated by *u*

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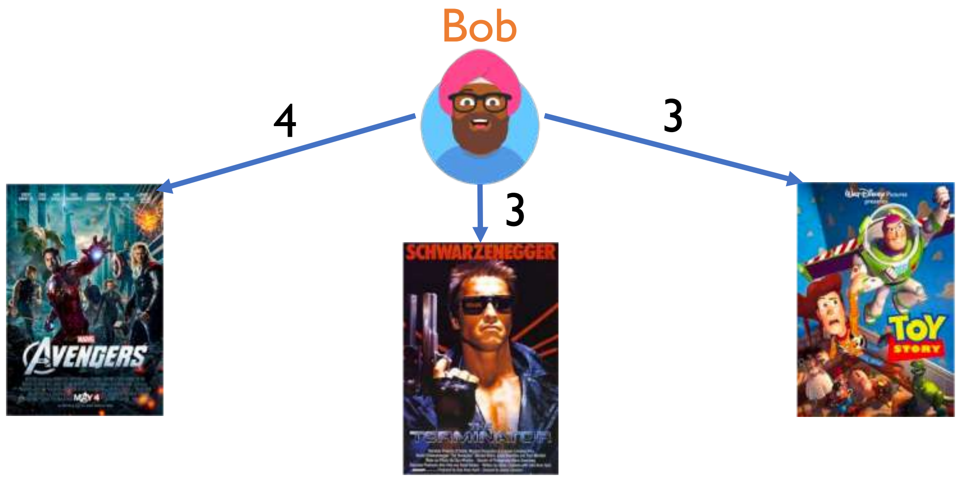
Intuitively, if a user v is not in the u's k-neighborhood then very likely u will not be interested in any item that **only** v has rated

In other words, the u's k-neighborhood must be computed first to narrow down the set of items which we must predict the rating of

User-based Neighborhood: Example

		MOVIES							
									
USERS	Alice	2		5	4	5	4		4
	Bob	4					3		3
	Carl	5	5	3	4	5	4		5
	…	…	…	…	…	…	…	…	…
	Zoe		1	3				5	4

User-based Neighborhood: Example



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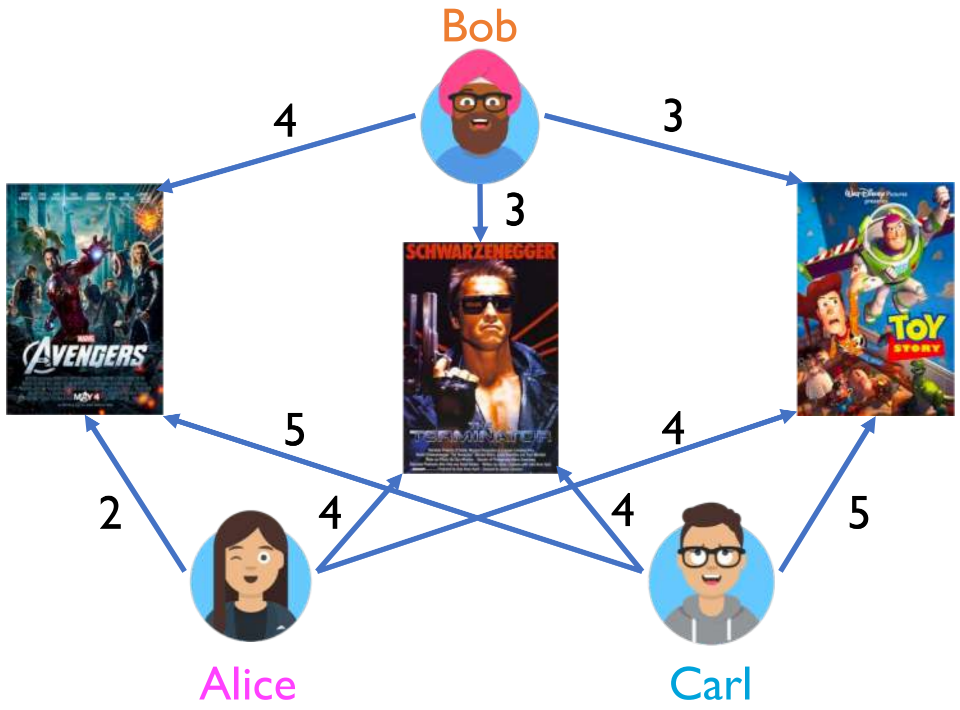
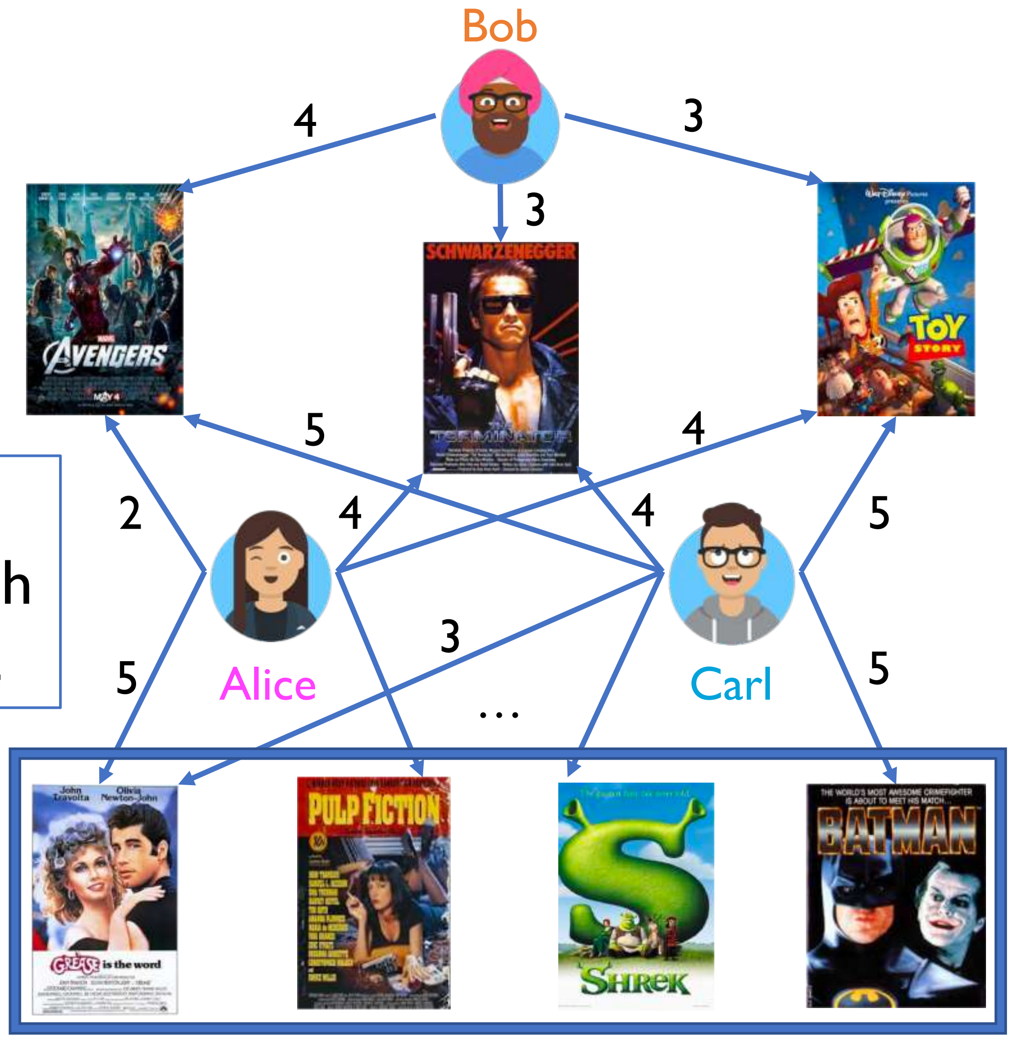


Alice and Carl are the 2-**nearest neighbours** of Bob
if we look at their rating behaviours

User-based Neighborhood: Example

Consider all the movies rated by Alice or Carl which Bob **haven't** watched yet



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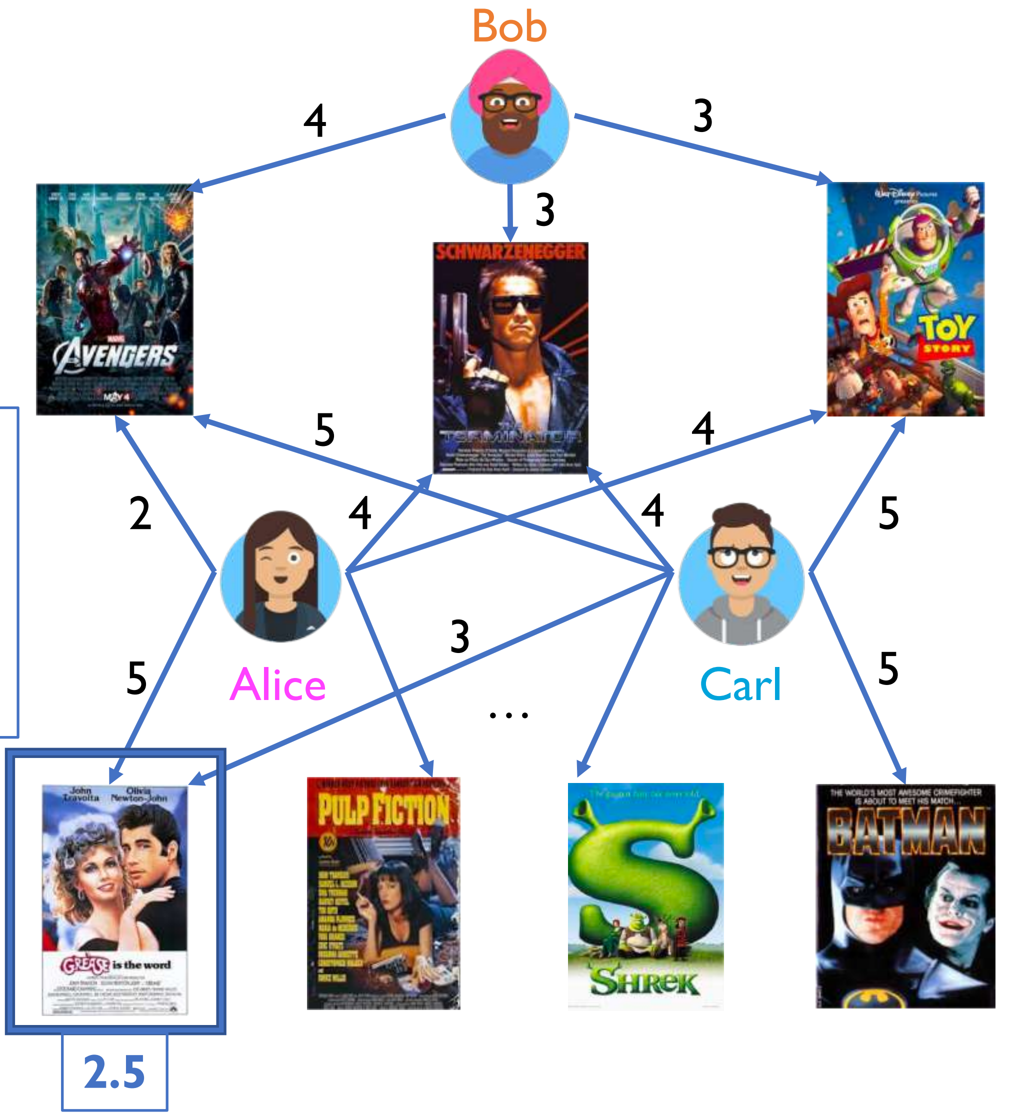

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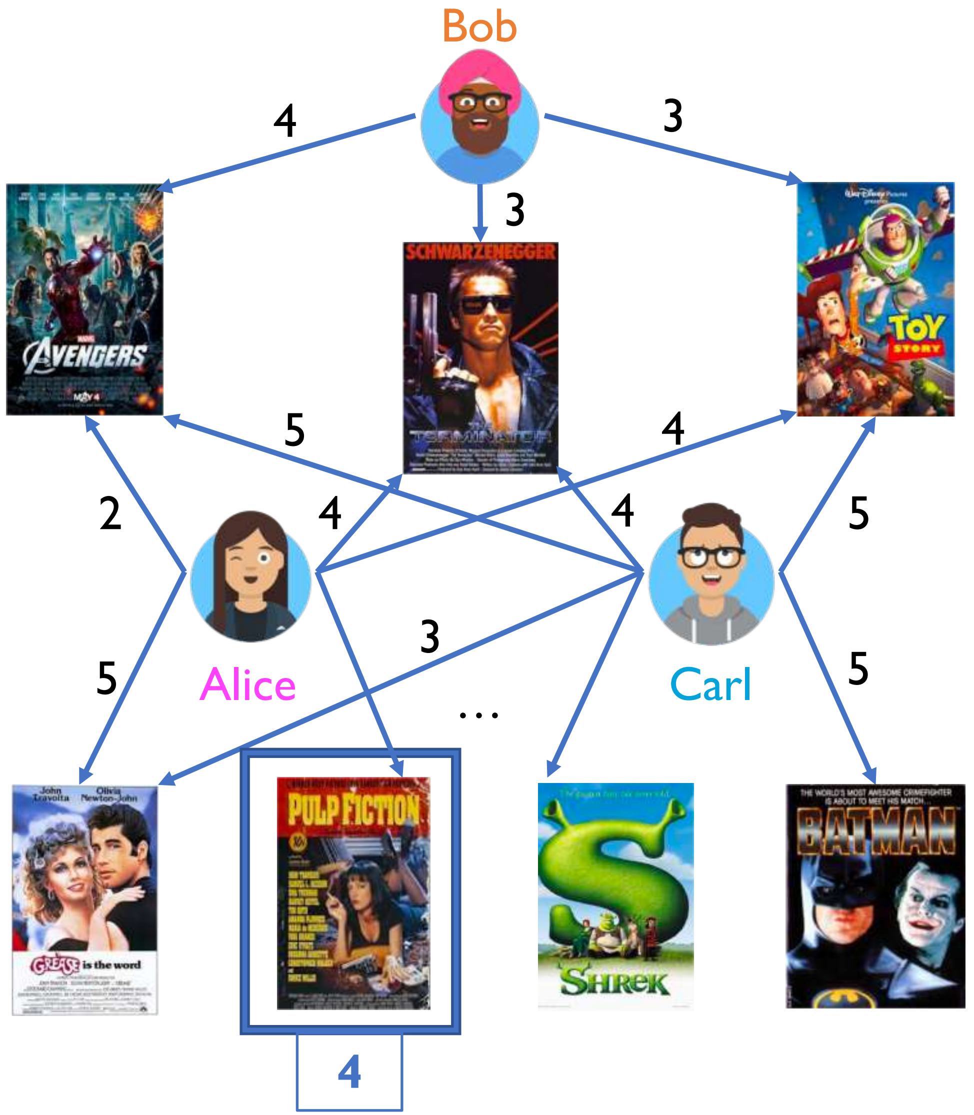

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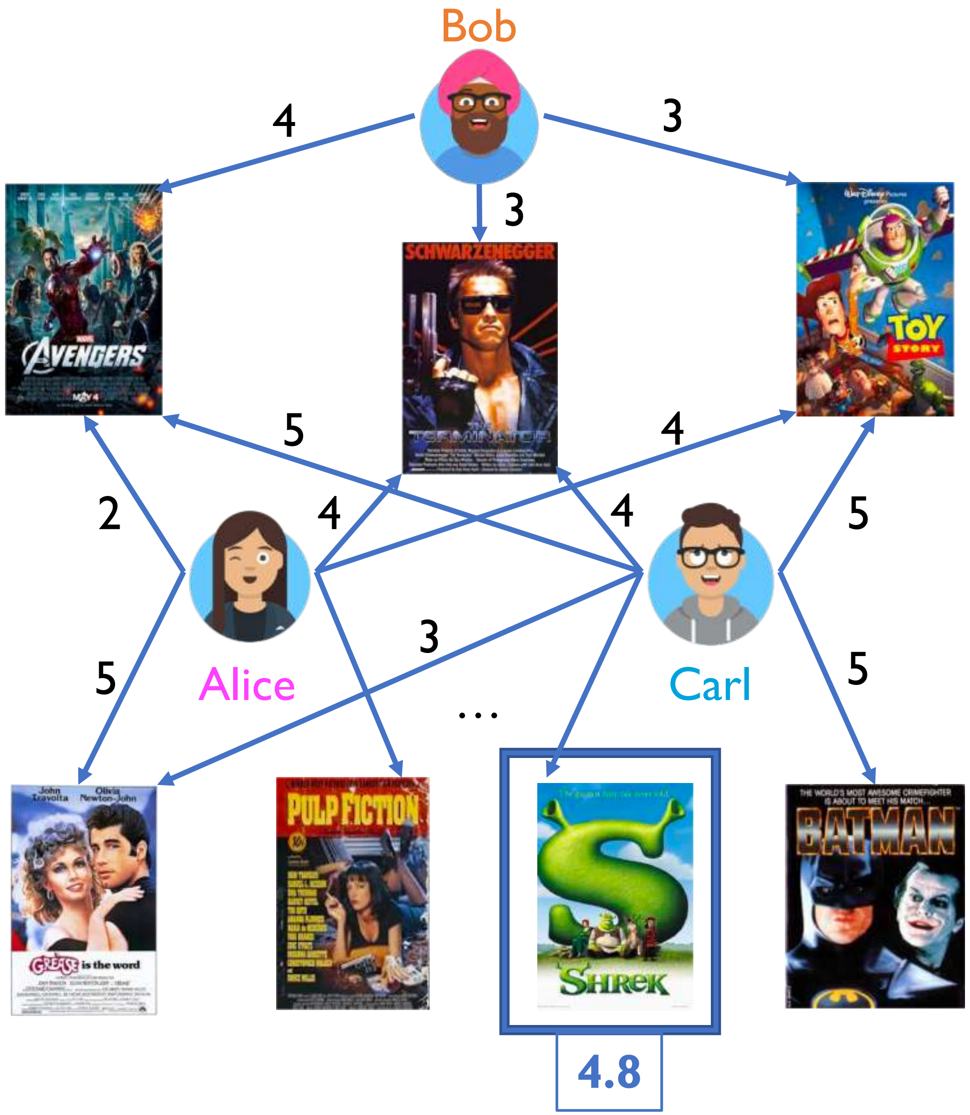

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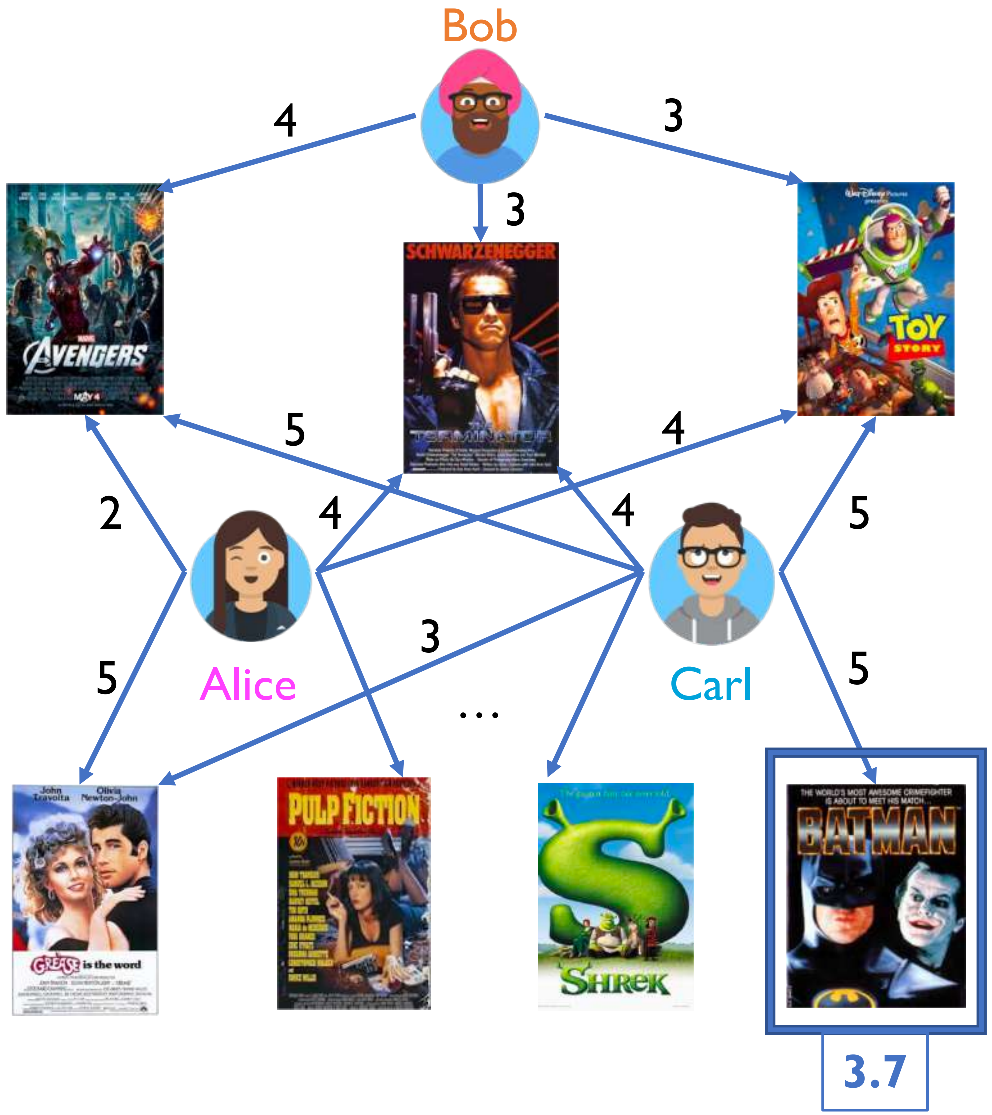
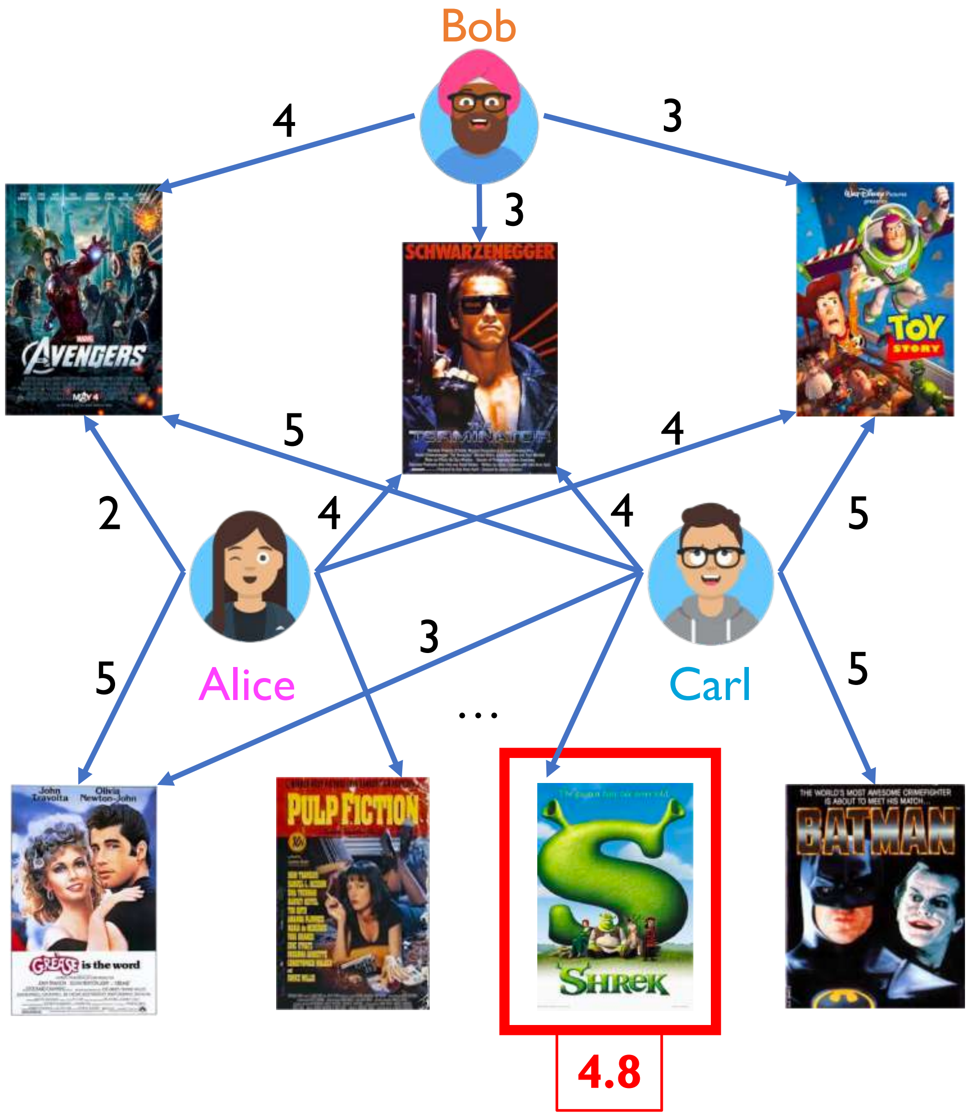


User-based Neighborhood: Example

Recommend the highest rated movie(s) to Bob!



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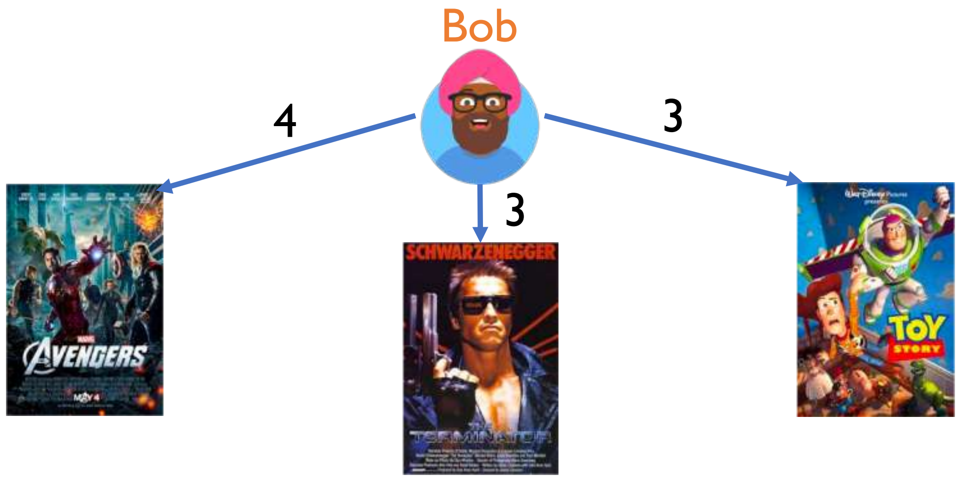
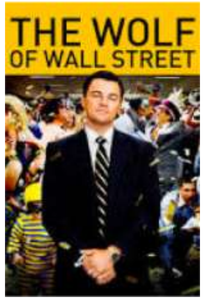


There is no point in predicting the rating of a movie which has only been rated by a user (Zoe) who is **not** in the Bob's neighborhood



User-to-User Similarity

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- Each user represented by her/his rating vector and similarity between them is measured in the item (rating) space

User-to-User Similarity

$\text{sim}(u, v)$ Similarity metric between any pair of users

		MOVIES							
		Avengers	Batman	Grease	Pulp Fiction	Shrek	Schwarzenegger	The Wolf of Wall Street	Toy Story
USERS	Alice	2		5	4	5	4		4
	Bob	4					3		3
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Must capture the intuition that $\mathrm{sim}(\mathrm{Alice}, \mathrm{Carl}) > \mathrm{sim}(\mathrm{Alice}, \mathrm{Bob})$

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$\mathbf{r}_u$ *n*-dimensional vector of ratings provided by user *u* (*n* = #movies)

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$\mathbf{r}_{\text{Bob}}$

User-to-User Similarity: Jaccard Similarity

$$\mathrm{sim}(u, v) = J(\mathbf{r}_u, \mathbf{r}_v) = \frac{|\mathbf{r}_u \cap \mathbf{r}_v|}{|\mathbf{r}_u \cup \mathbf{r}_v|}$$

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$$\mathrm{sim}(\text{Alice}, \text{Bob}) = \frac{|\mathbf{r}_{\text{Alice}} \cap \mathbf{r}_{\text{Bob}}|}{|\mathbf{r}_{\text{Alice}} \cup \mathbf{r}_{\text{Bob}}|}$$

$$= \frac{3}{6} = 0.5$$

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$$= \frac{6}{7} \approx 0.86$$

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Problem!

Jaccard ignores rating values

User-to-User Similarity: Cosine Similarity

$$\text{sim}(u, v) = \text{cosine}(\mathbf{r}_u, \mathbf{r}_v) = \frac{\mathbf{r}_u \cdot \mathbf{r}_v}{||\mathbf{r}_u|| \, ||\mathbf{r}_v||}$$

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$$\text{sim}(\text{Alice}, \text{Bob}) = \frac{\mathbf{r}_{\text{Alice}} \cdot \mathbf{r}_{\text{Bob}}}{||\mathbf{r}_{\text{Alice}}|| \, ||\mathbf{r}_{\text{Bob}}||}$$

$$= \frac{32}{\sqrt{102}\sqrt{44}} \approx 0.48$$

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$$= \frac{102}{\sqrt{102}\sqrt{141}} \approx 0.85$$

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Problem!
Missing rating values are treated as 0s and have a negative effect

User-to-User Similarity: Pearson Correlation

$$\mathrm{sim}(u,v) = \mathrm{Pearson}(\mathbf{r}_u, \mathbf{r}_v) = \frac{(\mathbf{r}_u - \bar{\mathbf{r}}_u) \cdot (\mathbf{r}_v - \bar{\mathbf{r}}_v)}{\sqrt{(\mathbf{r}_u - \bar{\mathbf{r}}_u)^T \cdot (\mathbf{r}_u - \bar{\mathbf{r}}_u)} \times \sqrt{(\mathbf{r}_v - \bar{\mathbf{r}}_v)^T \cdot (\mathbf{r}_v - \bar{\mathbf{r}}_v)}}$$

		MOVIES							
USERS	Alice	-2		1	0	1	0		0
	Bob	2/3					-1/3		-1/3
	Carl	4/7	4/7	-10/7	-3/7	4/7	-3/7		4/7
	…	…	…	…	…	…	…	…	…
	Zoe		-9/4	-1/4				7/4	-1/4

Solution:
Normalize ratings by subtracting the mean rating

Now 0 means neutral, and if we treat missing ratings as 0, it doesn't mean it's negative

User-to-User Similarity: Pearson Correlation

$\mathbf{r}'_u = \mathbf{r}_u - \bar{\mathbf{r}}_u$ mean-scaled rating vector of *u*

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$$= \frac{(\mathbf{r}_u - \bar{\mathbf{r}}_u) \cdot (\mathbf{r}_v - \bar{\mathbf{r}}_v)}{\sqrt{(\mathbf{r}_u - \bar{\mathbf{r}}_u)^T \cdot (\mathbf{r}_u - \bar{\mathbf{r}}_u)} \times \sqrt{(\mathbf{r}_v - \bar{\mathbf{r}}_v)^T \cdot (\mathbf{r}_v - \bar{\mathbf{r}}_v)}} = \text{Pearson}(\mathbf{r}_u, \mathbf{r}_v)$$

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Top-*k* most "similar" users to *u*

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Predicted rating given by user *u* to item *i*

$$\mathbf{r}_u[i] = r(u, i) = r_{u,i}$$

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plain average

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weighted average

User-based CF: Drawbacks

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Aging

user profiles changed quickly and the entire system model had to be recomputed

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- The model doesn't suffer from aging and therefore it does not need to be recomputed frequently

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- Remember that we are not building any content-based user/item profile
- Intuitively, 2 items i_1 and i_2 are similar to each other if users who rated them are similar
- Each item represented by the user ratings vector and similarity between them is measured in the user (rating) space

Item-to-Item Similarity

$\mathbf{r}_i$ *m*-dimensional vector of ratings provided for item *i* (*m* = #users)

		MOVIES							
									
USERS	Alice	2		5	4	5	4		4
	Bob	4					3		3
	Carl	5	5	3	4	5	4		5
	...	...	...	...	...	...	...	...	...
	Zoe		1	3				5	4

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$\mathbf{r}_{\text{Shrek}}$

Item-based Neighborhood: Example

Let's consider again Bob!

USERS	MOVIES							
Alice	2		5	4	5	4		4
Bob	4					3		3
Carl	5	5	3	4	5	4		5
…	…	…	…	…	…	…	…	…
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Item-based Neighborhood: Example

Suppose we want to predict the rating Bob would give to Shrek

		MOVIES							
									
USERS	Alice	2		5	4	5	4		4
	Bob	4				$\boxed{?}$	3		3
	Carl	5	5	3	4	5	4		5
	…	…	…	…	…	…	…	…	…
	Zoe		1	3				5	4

Item-based Neighborhood: Example

We first extract the subset of *k* most similar items to Shrek which have been rated by Bob

		MOVIES							
									
USERS	Alice	2		5	4	5	4		4
	Bob	4					3		3
	Carl	5	5	3	4	5	4		5
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$\mathbf{r}_{\text{Shrek}}$

Item-based Neighborhood: Example

Suppose those are: The Avengers and The Terminator

		MOVIES							
USERS	Alice	2		5	4	5	4		4
	Bob	4					3		3
	Carl	5	5	3	4	5	4		5
	…	…	…	…	…	…	…	…	…
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For example, item similarity is measured using Pearson's correlation

Item-based Neighborhood: Example

The predicted rating is computed as an **aggregating function** of the ratings that Bob gave to the *k* most similar movies to Shrek

		MOVIES							
USERS	Alice	2		5	4	5	4		4
	Bob	4				?	3		3
	Carl	5	5	3	4	5	4		5
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Item-based Neighborhood: Predictions

$\mathbf{r}_i$ Vector of ratings given to item *i*

$$\mathcal{I}_u = \{i \in \mathcal{I} : r_{u,i} = \downarrow\}$$

Set of items rated by *u*

$$\mathcal{I}_u^k = \mathrm{argmax}_{\mathcal{I}'_u \subseteq \mathcal{I}_u, |\mathcal{I}'_u| = k} \sum_{i' \in \mathcal{I}'_u} \mathrm{sim}(i, i')$$

Top-*k* most "similar" items to *i* among those rated by *u*

i's *k*-neighborhood

Predicted rating given by user *u* to item *i*

$$\mathbf{r}_u[i] = r(u, i) = r_{u,i}$$

Item-based Neighborhood: Predictions

2 possible ways of aggregating neighbors ratings

$$\forall i' \in \mathcal{I}_u^k$$

$$r_{u,i} = \frac{1}{k} \sum_{i' \in \mathcal{I}_u^k} r_{u,i'}$$

plain average

$$r_{u,i} = \frac{\sum_{i' \in \mathcal{I}_u^k} \text{sim}(i, i') \cdot r_{u,i'}}{\sum_{i' \in \mathcal{I}_u^k} \text{sim}(i, i')}$$

weighted average

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In general, **item-based** works better than **user-based** CF

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  - Locality-Sensitive Hashing (LSH) approximation

Latent Factor (Model-based) CF

Tries to predict ratings by representing both items and users with a number of **hidden factors** inferred from observed ratings

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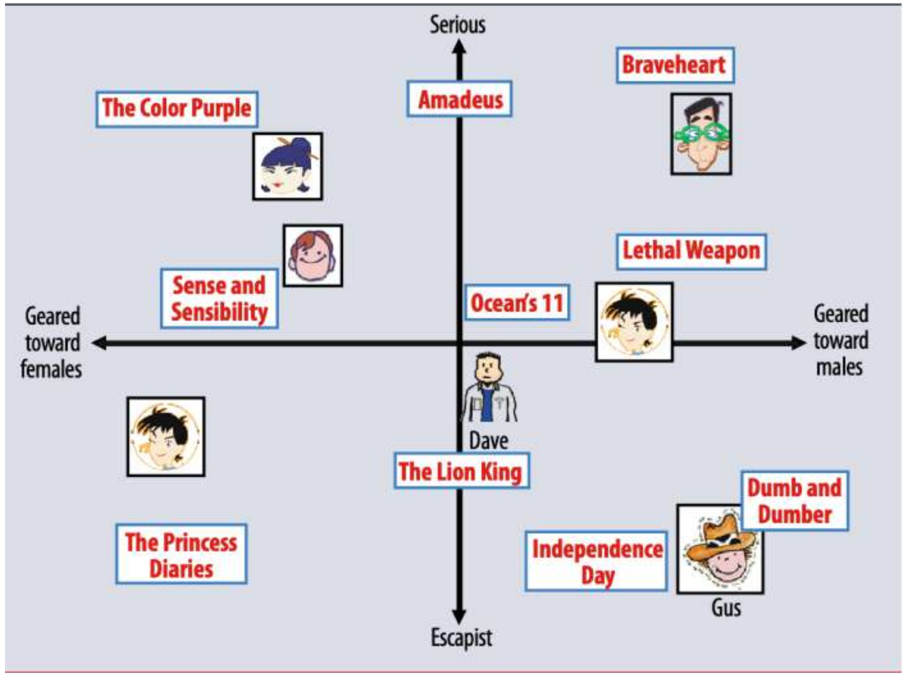

Example: 2 hidden factors

- Dim. 1: Male vs. Female
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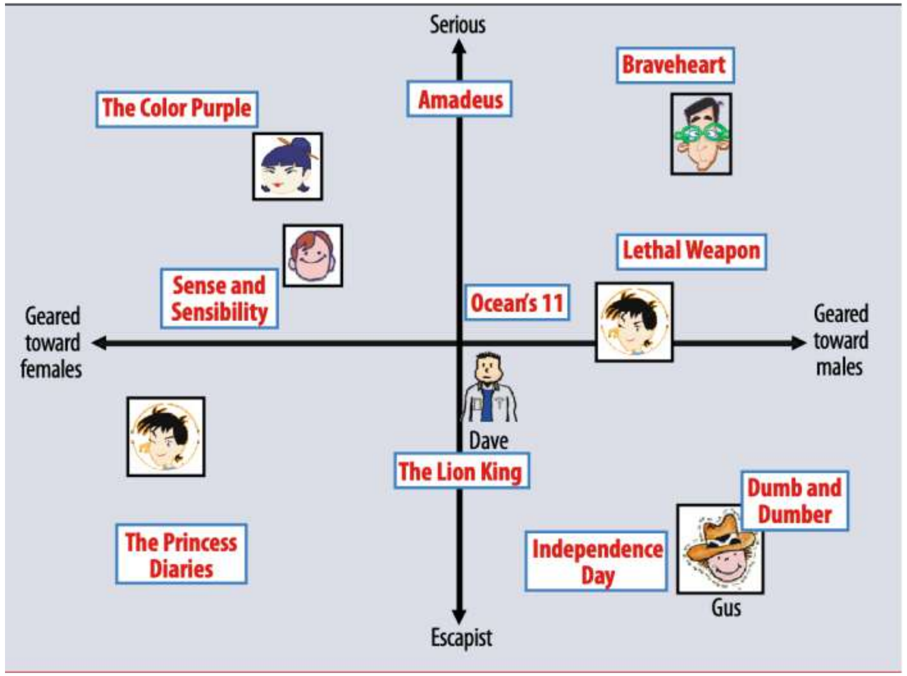

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A user's predicted rating for an item (movie) would equal the **dot product** of the movie and user vectors on the plot

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- Such vectors are inferred (i.e., learned) from observed item ratings
- High correspondence between item and user factors leads to a recommendation

Matrix Factorization Framework

- Map both items and users to a **joint latent factor** d-dimensional space

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- That is why these features are often refer to as **latent features**

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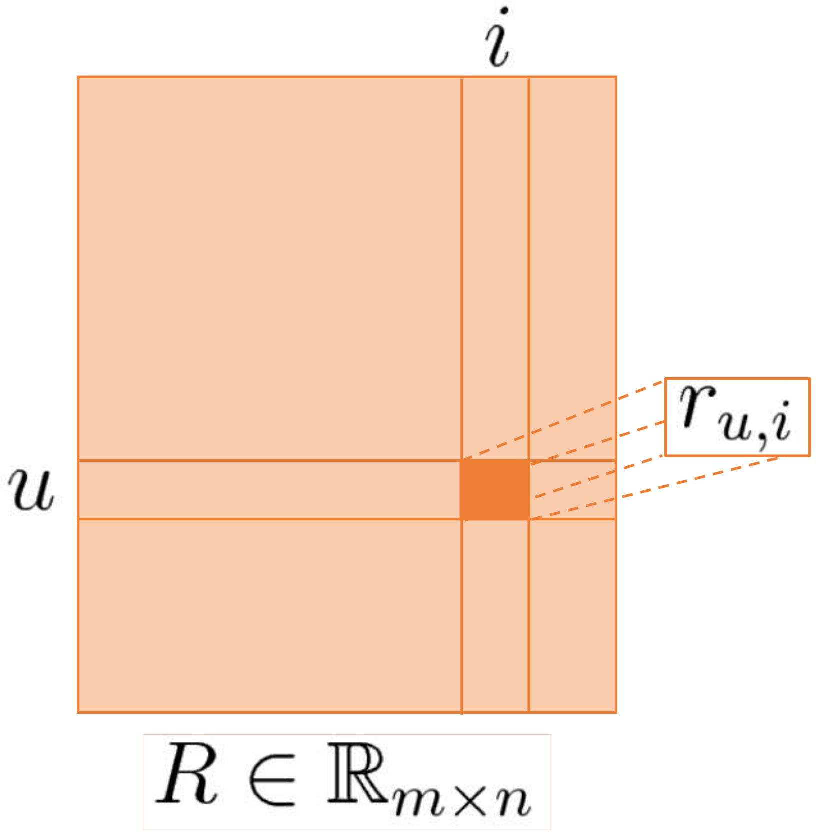
Recommendations for a user are generated by computing the estimated ratings for unseen items, and by taking the **top-*k* highest rated** ones

Matrix Factorization Framework



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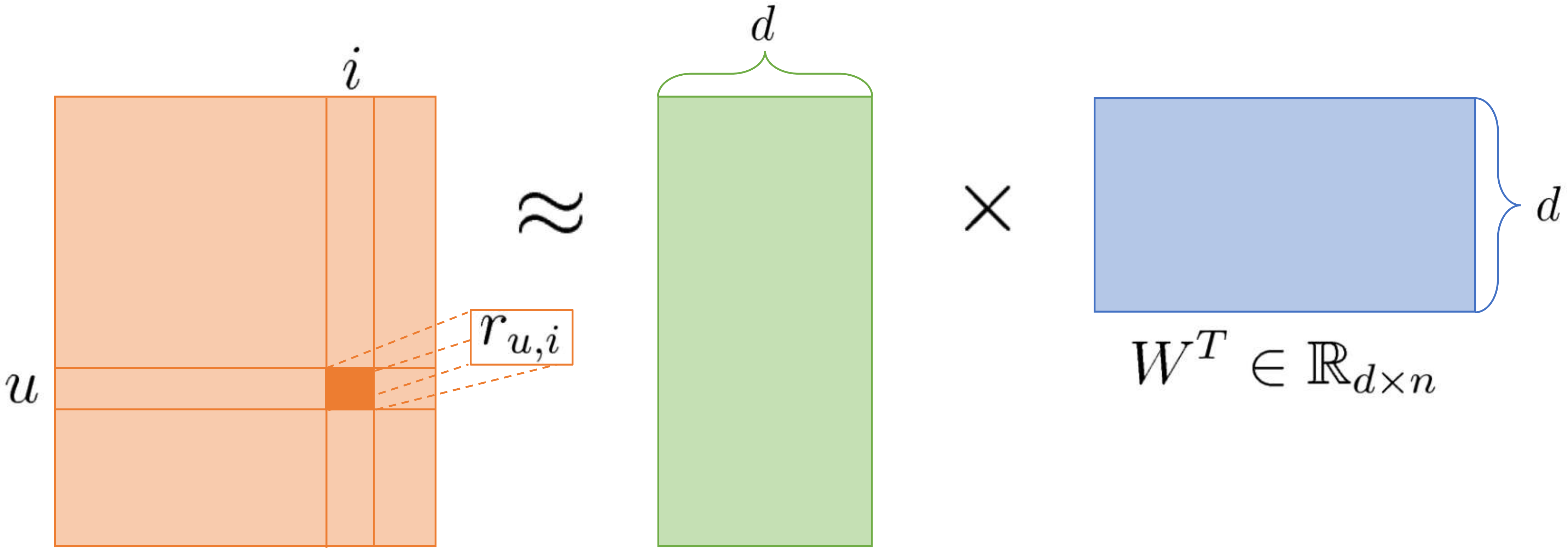
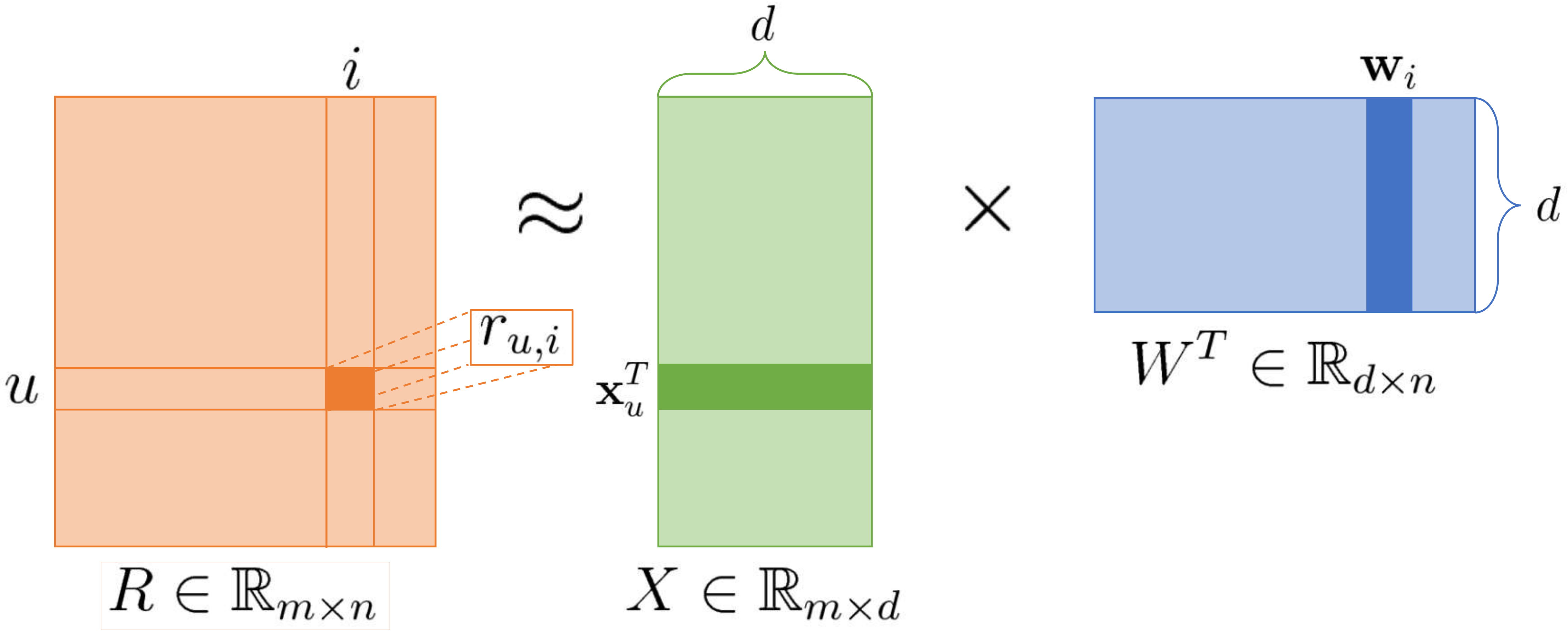


$R \in \mathbb{R}_{m\times n}$ $X \in \mathbb{R}_{m\times d}$

Matrix Factorization Framework



Approximate the user-item rating matrix R with the product of X x W^T

How Do We Learn X and W?

Assuming we have access to a **dataset** of **observed ratings**

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To actually learn the latent factor representations $\mathbf{x}_u$ and $\mathbf{w}_i$ we **minimize** the following **loss function**

$$L(X, W) = \sum_{(u,i)\in\mathcal{D}} \left(r_{u,i} - \mathbf{x}_u^T \cdot \mathbf{w}_i\right)^2 + \lambda\left(\sum_{u\in\mathcal{D}} ||\mathbf{x}_u||^2 + \sum_{i\in\mathcal{D}} ||\mathbf{w}_i||^2\right)$$

Training set of observed ratings

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$$X^*, W^* = \text{argmin}_{X,W} L(X, W)$$

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Mathematically convenient

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Still, how do we solve this?

Learning Algorithms

2 main optimization methods

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We know that the updating strategy for SGD is as follows:

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At each iteration, both user and item latent vectors are updated by a magnitude proportional to η in the **opposite direction** of the gradient

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We define the **prediction error** associated with each training instance (*u*, *i*)

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- Indeed, there are $d(m+n)$ parameters to optimize
- In real life problems, this number can get very large quite often, requiring both a parallelization mechanism or an alternative optimizer

Alternating Least Squares (ALS)

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- Each alternating iteration reduces to traditional least squares and can be solved using OLS or its regularized variant (e.g., pseudo-inverse)

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$$L(X,W) = \sum_{(u,i)\in\mathcal{D}} \left(r_{u,i} - \mathbf{x}_u^T \cdot \mathbf{w}_i\right)^2 + \lambda\Big(\sum_{u\in\mathcal{D}} ||\mathbf{x}_u||^2 + \sum_{i\in\mathcal{D}} ||\mathbf{w}_i||^2\Big)$$

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We want to set this to 0 $\boxed{- \sum_{i \in \mathcal{D}} (r_{u,i} - \mathbf{x}_u^T \cdot \mathbf{w}_i) \mathbf{w}_i + \lambda \mathbf{x}_u = 0}$

ALS: Item Vector Fixed

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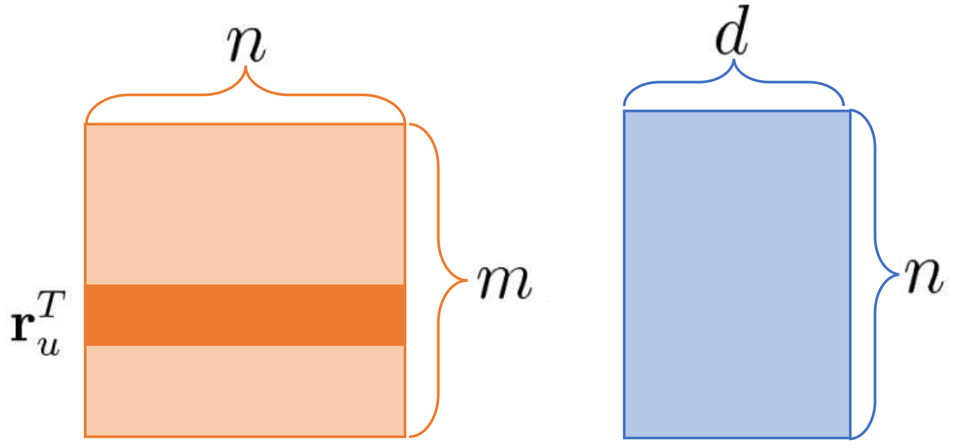

user-item rating matrix R

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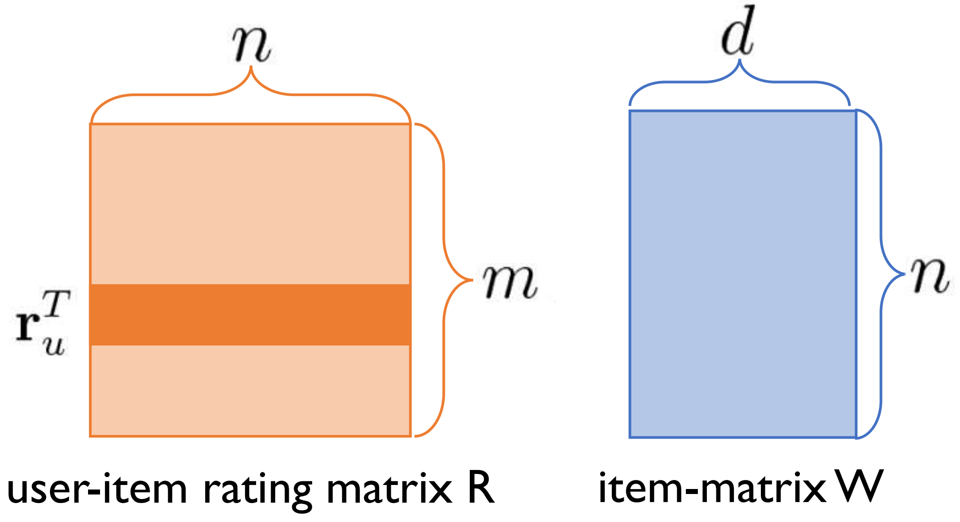

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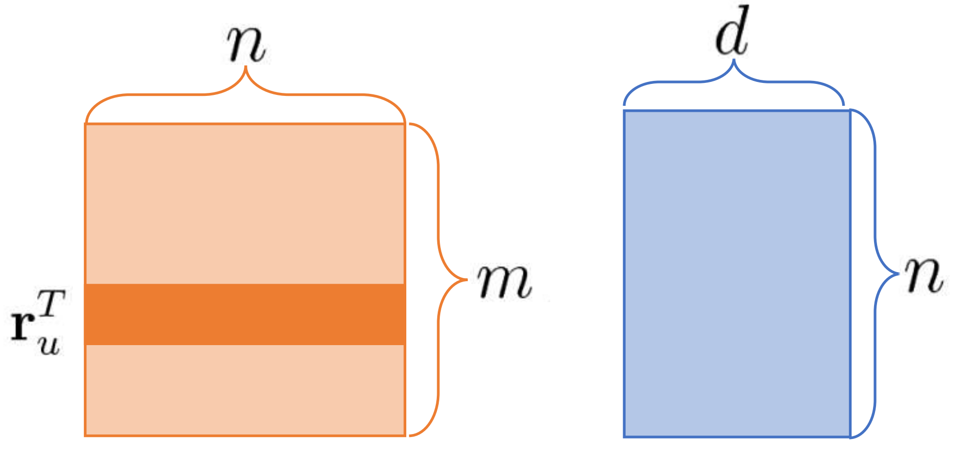

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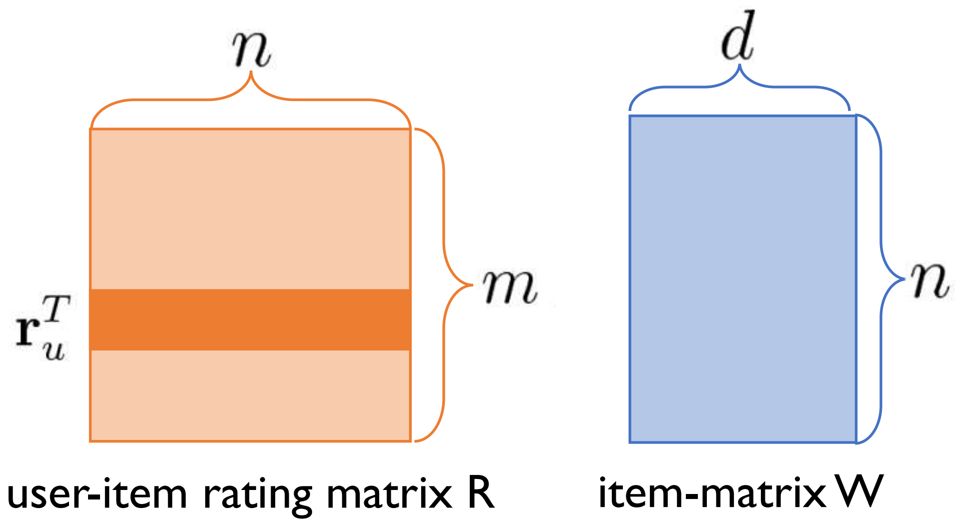

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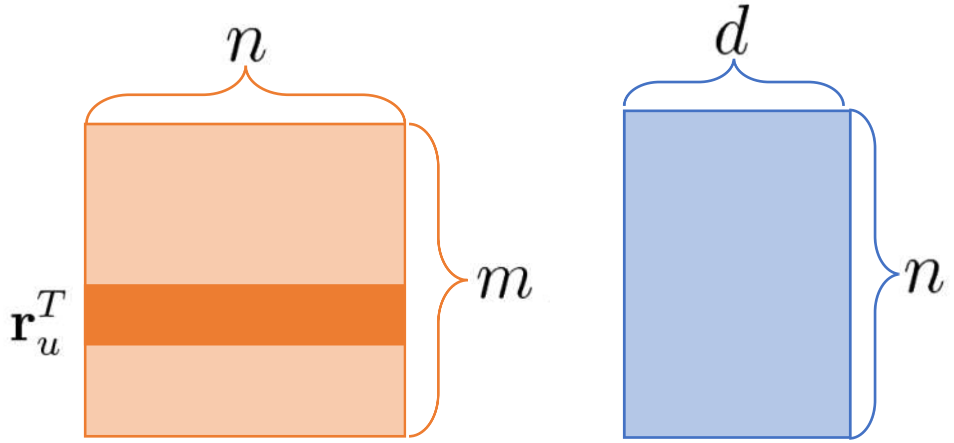

user-item rating matrix R item-matrix W

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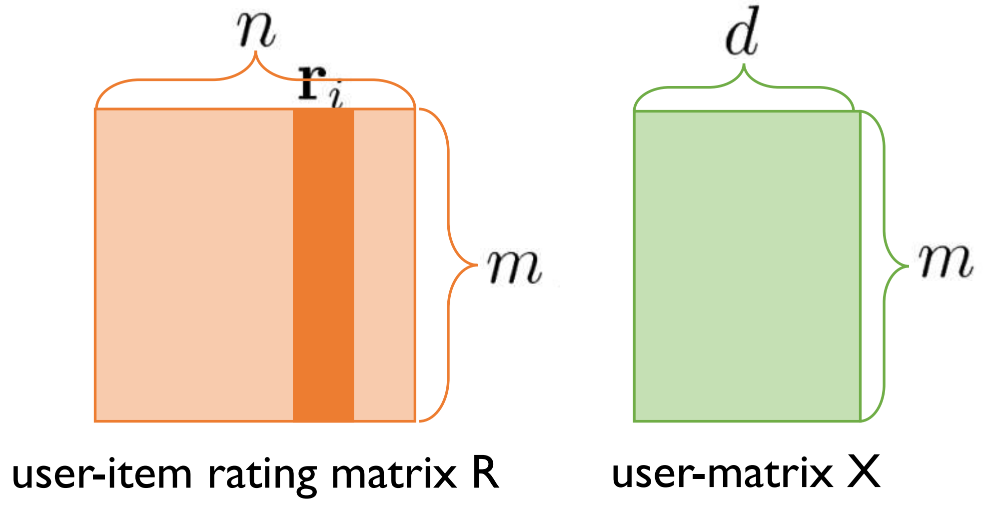

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Convergence is guaranteed because in each step the loss function can either decrease or stay unchanged, but never increase

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  - **Parallelization:** each $\mathbf{x}_u$ and $\mathbf{w}_i$ is computed independently of user/item factors
  - **Implicit Data:** the training set is dense and looping over each single instance - as SGD does - would be unfeasible

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A well-known technique to decompose a matrix into the product of **3 matrices**

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SVD solution is unique

Applying SVD to Collaborative Filtering

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Each factor *k* is the result of the **similarity** between user *i* and item *j* and its overall effect on ratings across all users and items

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- For example, some users systematically tend to give higher ratings than others, and some items receive higher ratings than others

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$$\boxed{b_{\text{Joe,Titanic}} = 3.7 - 0.3 + 0.5 = 3.9}$$

Bias term

Including Bias into the Optimization

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The estimated rating of an item *i* for the user *u* is now made of **2 components**

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Overall, the original optimization problem becomes as follows

$$X^*, W^* = \text{argmin}_{X,W} \left\{ \frac{1}{2} \sum_{(u,i)\in\mathcal{D}} \left[r_{u,i} - (\mathbf{x}_u^T \cdot \mathbf{w}_i + \mu + b_u + b_i) \right]^2 + \lambda \Big(\sum_{u\in\mathcal{D}} ||\mathbf{x}_u||^2 + \sum_{i\in\mathcal{D}} ||\mathbf{w}_i||^2 + \sum_{u\in\mathcal{D}} b_u^2 + \sum_{i\in\mathcal{D}} b_i^2 \Big) \right\}$$

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Can still be solved using ALS

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- Netflix is a good example of hybrid recommender systems

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[Netflix: What Happens When You Press Play?]

For more details about how Netflix actually works

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- Participating teams submit predicted ratings for a test set of approximately 3M ratings

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- According to the contest website, more than 48,000 teams from 182 different countries have downloaded the data

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A combination of 100 different predictor sets, mostly factorization models

Evaluation Metrics

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Online

A/B testing measuring CTR, ROI, and other "live" metrics

Evaluation Metrics: RMSE

$$\mathrm{RMSE} = \frac{1}{|\mathcal{D}_{\text{test}}|} \sqrt{\sum_{(u,i) \in \mathcal{D}_{\text{test}}} (r_{u,i} - \hat{r}_{u,i})^2}$$

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The RMSE might penalize a method that does well for high ratings and badly for others

Evaluation Metrics: Precision & Recall

For a binary classifier predicting a condition (y = 1) or not, we define

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Mapping of binary classification terminology to recommender systems

binary classifier	recommender system
# with condition (y = 1)	# of all possible relevant items for a user
# predicted positive (TP + FP)	# of recommended items
# correct positives (TP)	# of recommended items that are relavant

Evaluation Metrics: Precision & Recall

For a recommender system, we can therefore define

$$P = \frac{\#\ \text{relevant item recommendations}}{\#\ \text{items recommended}} \qquad R = \frac{\#\ \text{relevant item recommendations}}{\#\ \text{items actually relevant}}$$

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A recommender system generates k=5 items to recommend

There are only 3 relevant items

The success/failure of our recommendations: [0, 1, 1, 0, 0] 0=not relevant/1=relevant

$$P = \frac{2}{5} \qquad R = \frac{2}{3}$$

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- Precision and Recall don't seem to care about ordering
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- Imagine taking our list of N recommendations and considering only the first element, then only the first two, then only the first three, and so on
- P@k and R@k are simply the precision and recall calculated only from the subset of the first k recommendations

P@k: Example

$$k = 3$$

$$P@3 = \frac{1}{3}$$

Rank	Product Recommended	Result
1	Credit card	Correct positive
2	Christmas Fund	False positive
3	Debit Card	False positive
~~4~~	~~Auto loan~~	~~False positive~~
~~5~~	~~HELOC~~	~~Correct Positive~~
~~6~~	~~College Fund~~	~~Correct positive~~
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$$k = 6$$

$$P@6 = \frac{3}{6}$$

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indicator function $$\mathbf{1}_{\mathrm{Rel}}(k) = \begin{cases} 1 & \text{if item } k \in \mathrm{Rel} \\ 0 & \text{otherwise} \end{cases}$$

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$$MAP@N = \frac{1}{|\mathcal{U}|}\sum_{u=1}^{|\mathcal{U}|} AP@N(u) = \frac{1}{|\mathcal{U}|}\sum_{u=1}^{|\mathcal{U}|}\frac{1}{|\mathrm{Rel}|}\sum_{k=1}^{N} P@k(u) \times \mathbf{1}_{\mathrm{Rel}}(k,u)$$

Personalization

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Intuitively, a high personalization score indicates the recommender system is able to provide a **highly personalized** experience to the users

Personalization

Suppose 3 users are recommended the following lists of items

u_1 = [A, B, C, D] u_2 = [A, B, C, E] u_3 = [A, B, F, G]

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	A	B	C	D	E	F	G
u_1	1	1	1	1	0	0	0
u_2	1	1	1	0	1	0	0
u_3	1	1	0	0	0	0	1

Personalization

Compute the 3-by-3 triangular matrix containing the cosine similarity between each pair of user's recommendation binary vector

$$M_{i,j} = \text{cosine}(\mathbf{u}_i, \mathbf{u}_j)$$

	u_1	u_2	u_3
u_1	1	0.75	0.58
u_2	0.75	1	0.58
u_3	0.58	0.58	1

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~0.64

Take the average of the upper triangle of the matrix M above

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Personalization = 1 − 0.64 = 0.36

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- Recommender systems as tools for dealing with information overload

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- Hybrid approaches combining both usually work better in practice
- New Neural-Network-based approaches have been proposed recently
- Evaluation metrics must capture the accuracy, personalization and serendipity of recommendations

Recommended Readings and Information :)

- A huge body of work on recommender systems is available out there!
- Surveys:
  - Adomavicius & Tuzhilin [2005]
  - Koren & Volinsky [2009]
  - Bobadilla *et al.* [2013]
  - Zhang *et al.* [2019]
- Well-renowned series of Conferences: RecSys, KDD, SIGIR, TheWebConf