

Big Data Computing

Master's Degree in Computer Science

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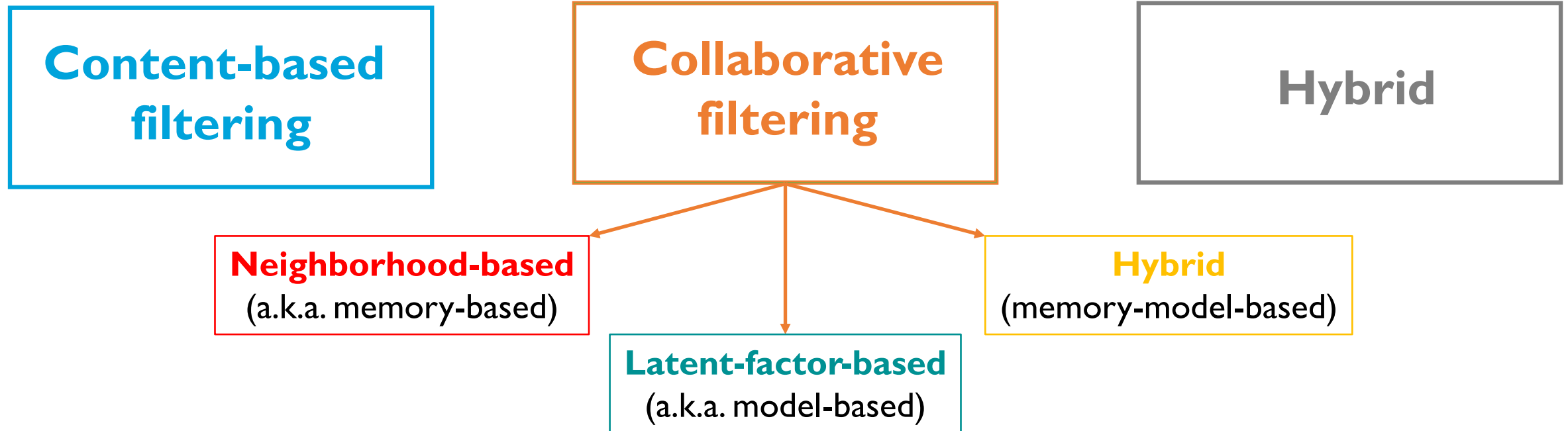
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SAPIENZA
UNIVERSITÀ DI ROMA

Recommendation Strategies

3 approaches to recommender systems



COLLABORATIVE FILTERING

Collaborative Filtering (CF)

Idea

Recommend items to user u based on preferences of other users similar to u

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Core concept:

User-to-User or **Item-to-Item similarity**

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Recommend items to user u based on preferences of other users similar to u

Core concept:

User-to-User or **Item-to-Item similarity**

No need for explicit creation of user/item profiles

Collaborative Filtering: Approaches

3 main approaches to collaborative filtering

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Neighborhood-based
(a.k.a. memory-based)

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Latent-factor-based
(a.k.a. model-based)

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3 main approaches to collaborative filtering

Neighborhood-based
(a.k.a. memory-based)

Hybrid
(memory-model-based)

Latent-factor-based
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Neighborhood-based (Memory-based) CF

Compute the relationship between **users** or **items**

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User-based

Evaluates a user's preference for an item based on ratings of "neighboring" users for that item

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Compute the relationship between **users** or **items**

User-based

Evaluates a user's preference for an item based on ratings of "neighboring" users for that item

Item-based

Evaluates a user's preference for an item based on ratings of "neighboring" items by the same user

USER-BASED COLLABORATIVE FILTERING

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Given a user u and an item i not rated by u , we want to estimate $r(u, i)$

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k -neighborhood of u is found on the basis of the similarity between user ratings
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Estimate $r(u, i)$ based on the ratings of users in the k -neighborhood of u

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In theory, rating prediction $r(u,i)$ could be defined on *any* item i not rated by u

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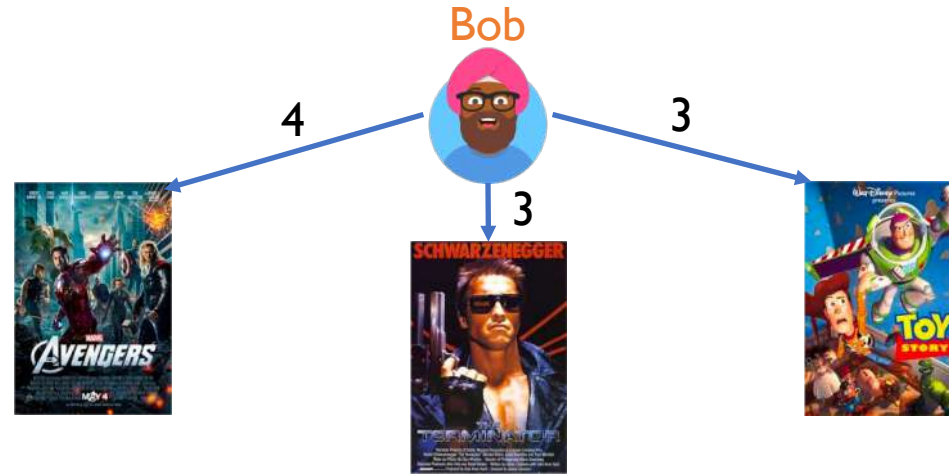
In other words, the u 's k -neighborhood must be computed first to narrow down the set of items which we must predict the rating of

User-based Neighborhood: Example

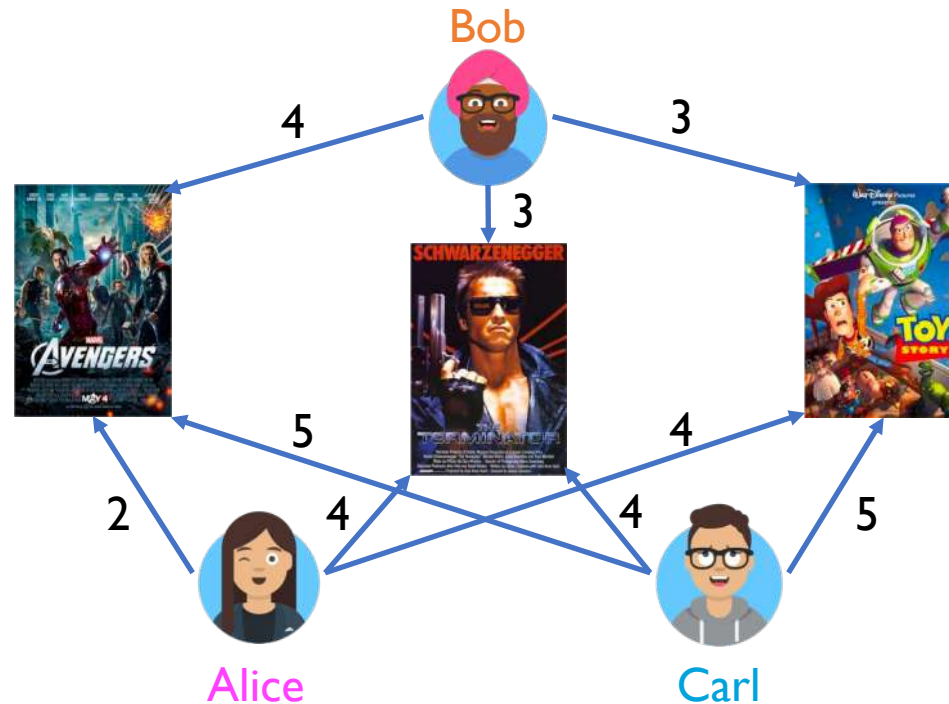
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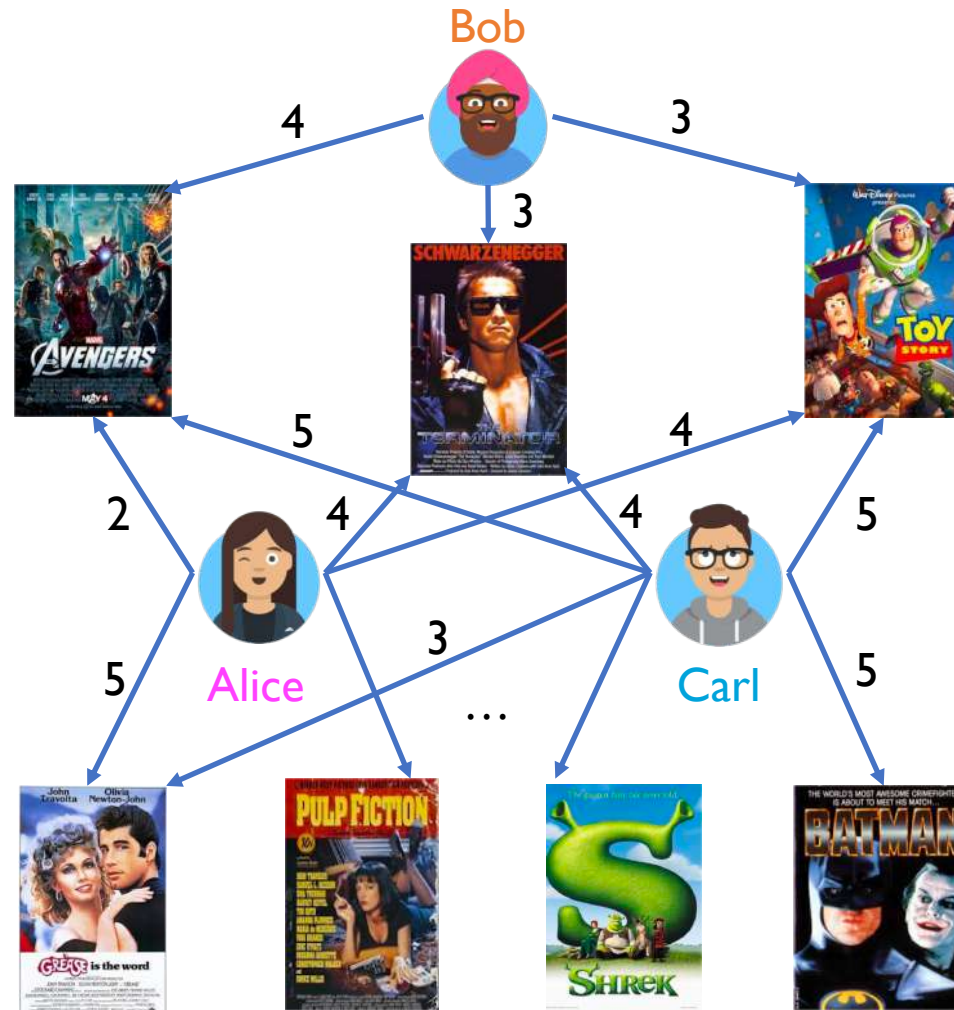
Alice and Carl are the 2-nearest neighbours of Bob if we look at their rating behaviours

User-based Neighborhood: Example



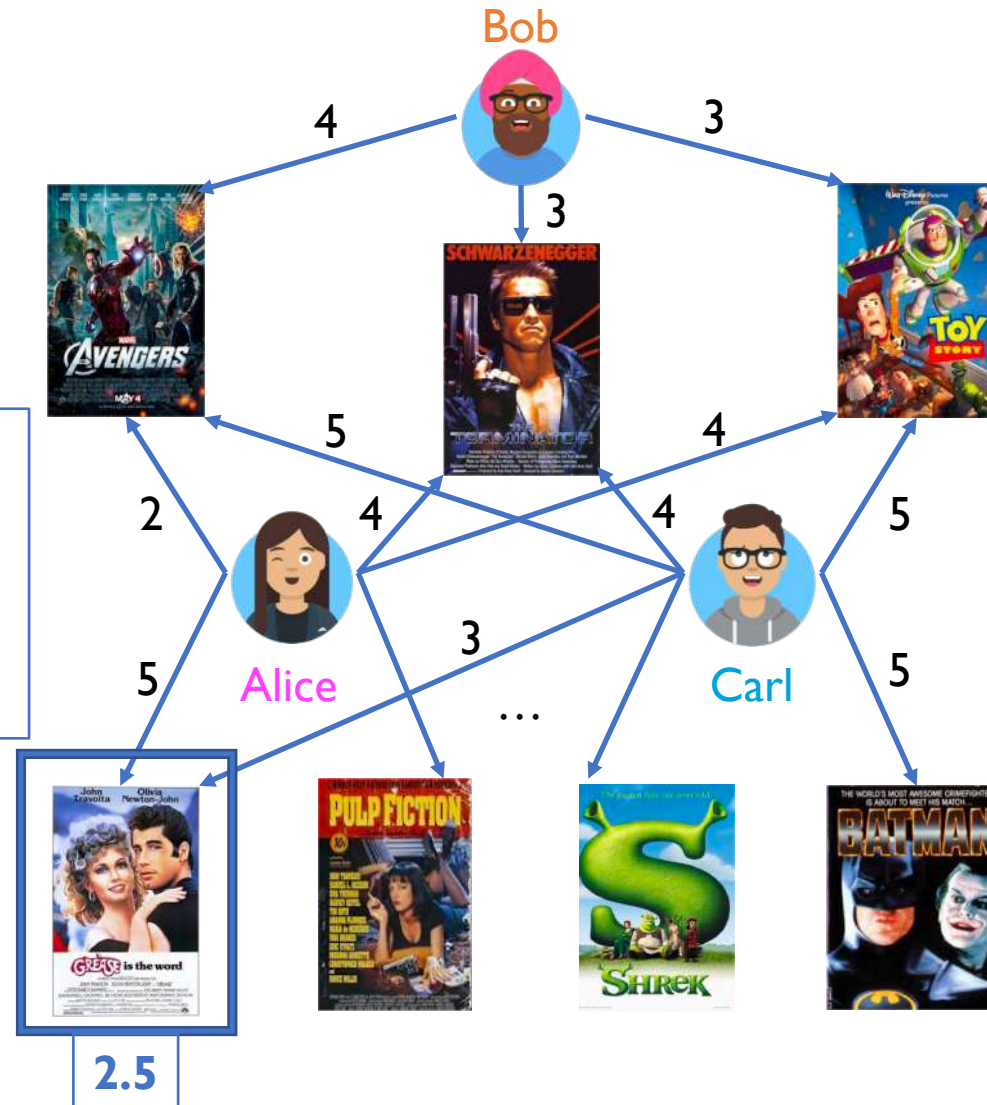
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Predict the rating that **Bob** would give to each of those movies on the basis of **Alice's** and **Carl's** ratings



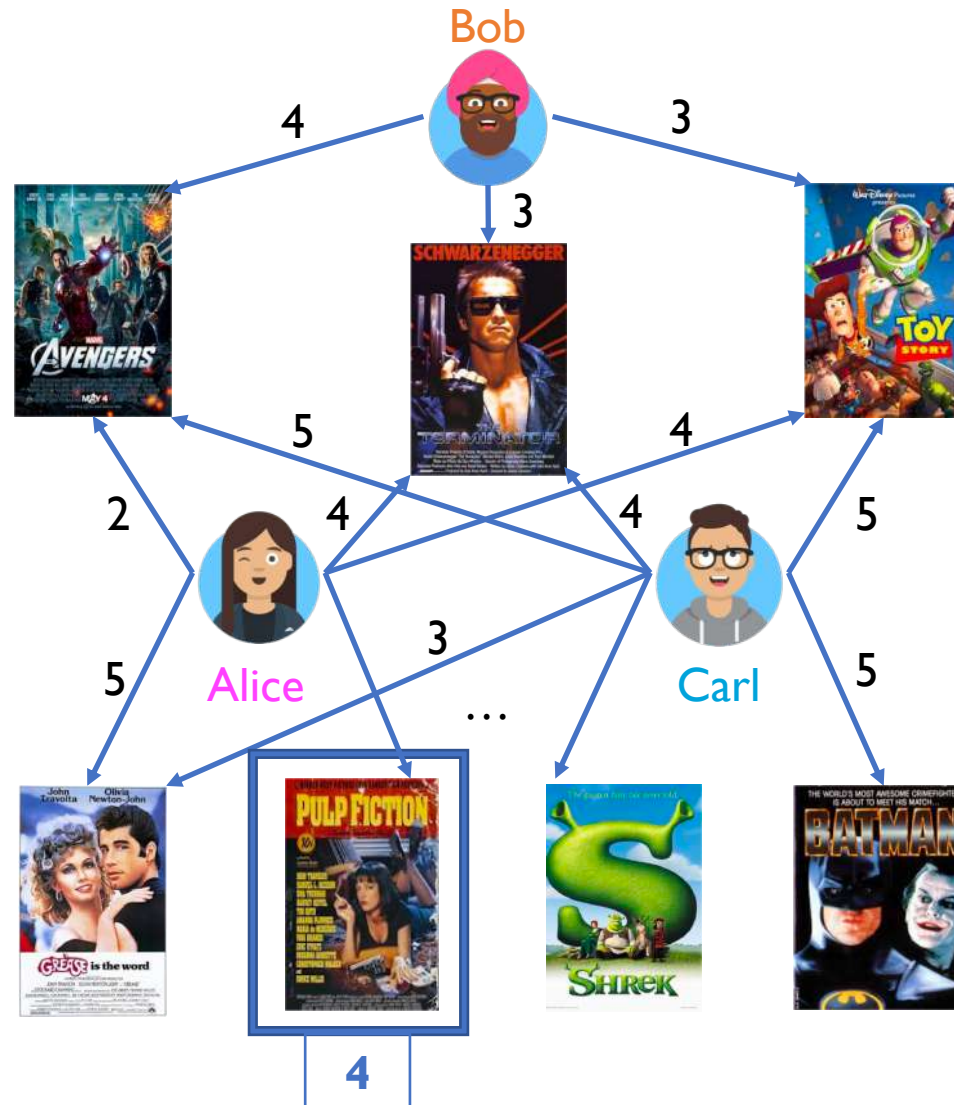
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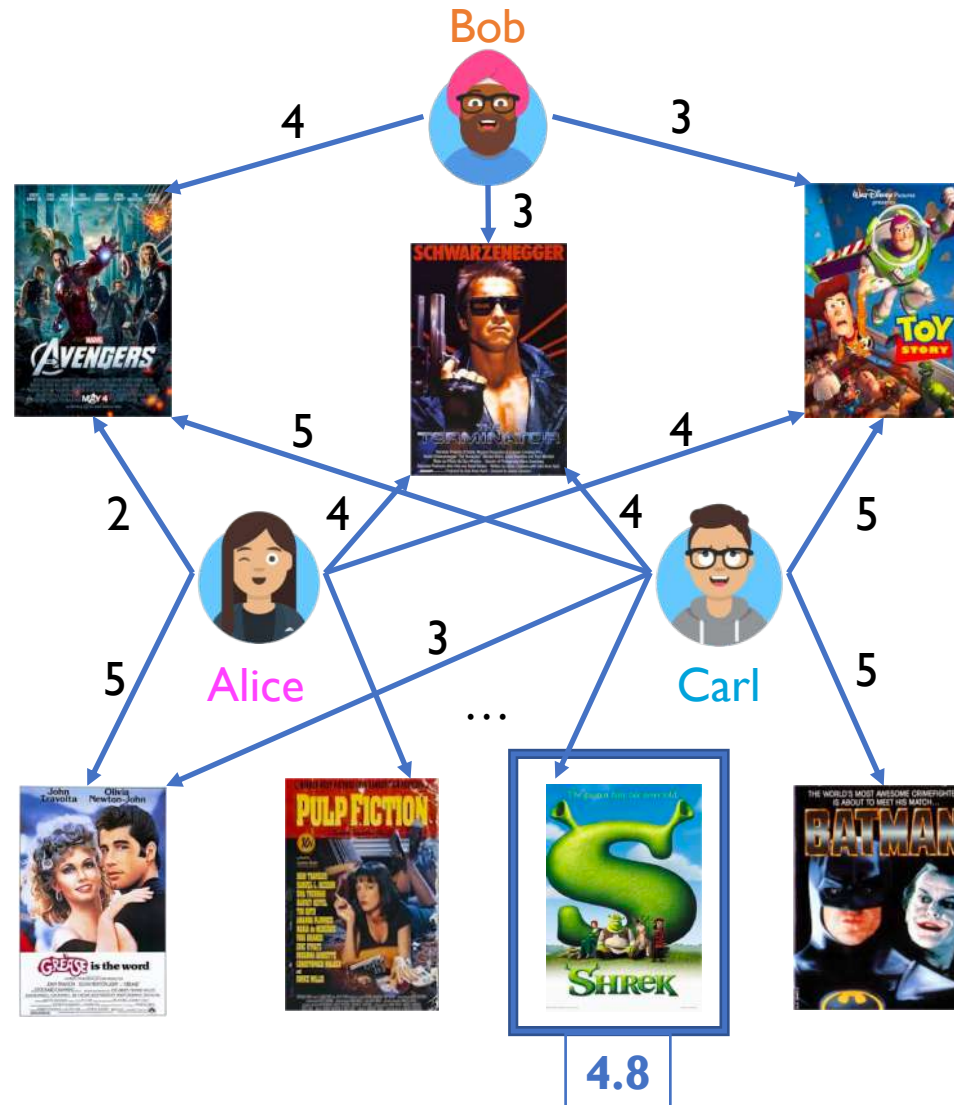
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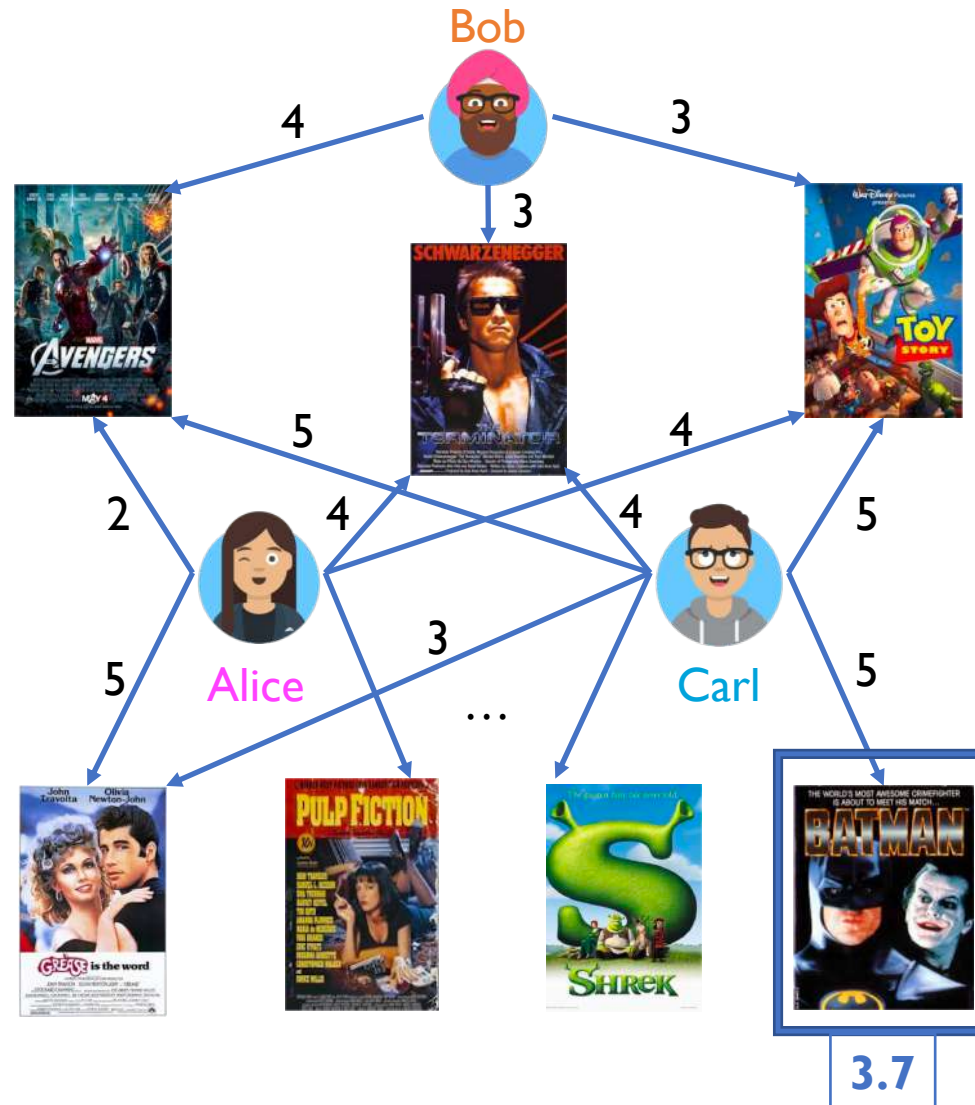
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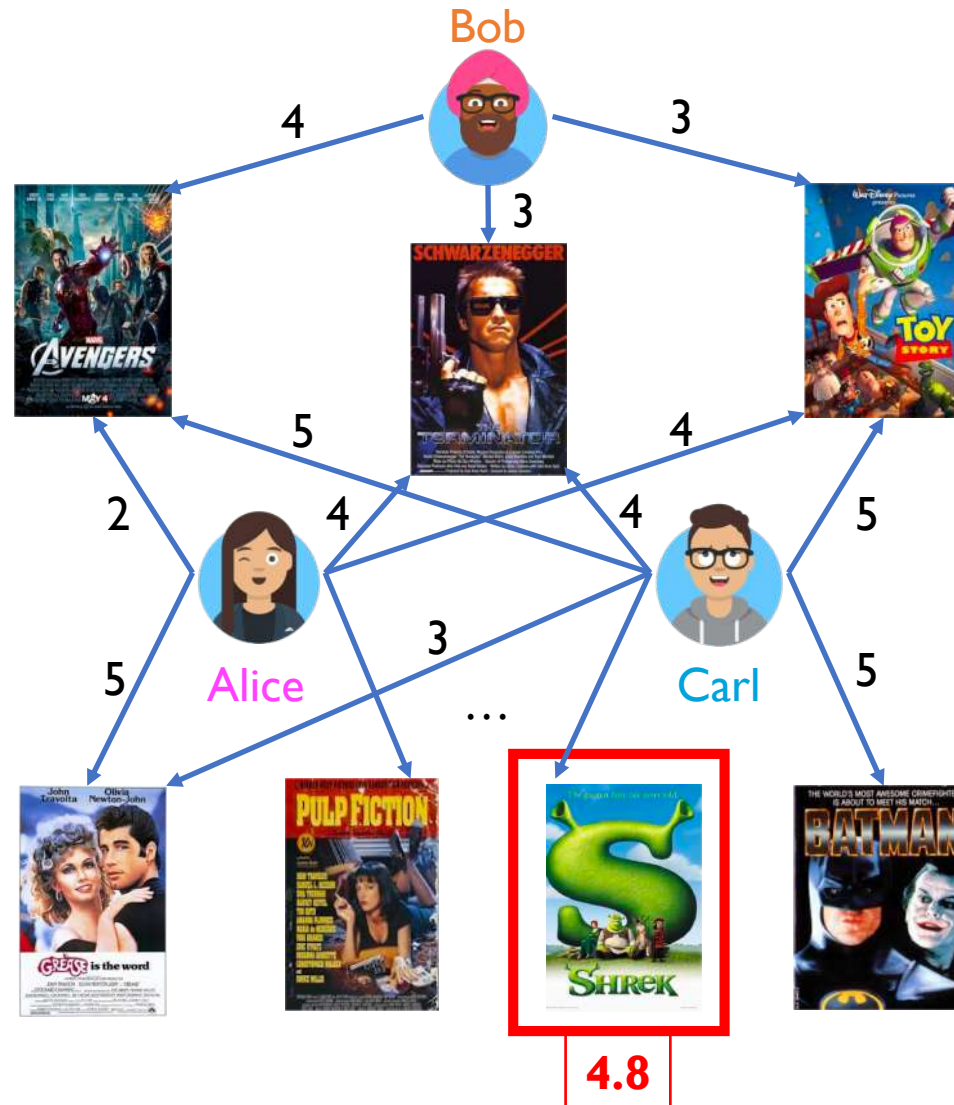
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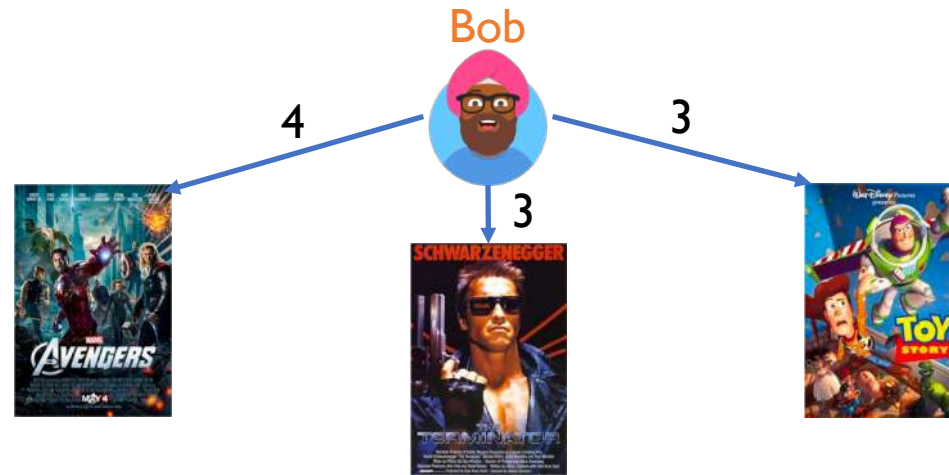


User-based Neighborhood: Example

Recommend the highest rated movie(s) to **Bob**!



User-based Neighborhood: Example



There is no point in predicting the rating of a movie which has only been rated by a user (**Zoe**) who is **not** in the **Bob's** neighborhood



User-to-User Similarity

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- Each user represented by her/his rating vector and similarity between them is measured in the item (rating) space

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$\text{sim}(u, v)$ Similarity metric between any pair of users

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Must capture the intuition that $\text{sim}(\text{Alice}, \text{Carl}) > \text{sim}(\text{Alice}, \text{Bob})$

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\mathbf{r}_{Bob}

User-to-User Similarity: Jaccard Similarity












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Problem!
Jaccard ignores rating values

User-to-User Similarity: Cosine Similarity

$$\text{sim}(u, v) = \text{cosine}(\mathbf{r}_u, \mathbf{r}_v) = \frac{\mathbf{r}_u \cdot \mathbf{r}_v}{\|\mathbf{r}_u\| \|\mathbf{r}_v\|}$$












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$$= \frac{102}{\sqrt{102} \sqrt{141}} \approx 0.85$$

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Problem!

Missing rating values are treated as 0s and have a negative effect

User-to-User Similarity: Pearson Correlation

$$\text{sim}(u, v) = \text{Pearson}(\mathbf{r}_u, \mathbf{r}_v) = \frac{(\mathbf{r}_u - \bar{\mathbf{r}}_u) \cdot (\mathbf{r}_v - \bar{\mathbf{r}}_v)}{\sqrt{(\mathbf{r}_u - \bar{\mathbf{r}}_u)^T \cdot (\mathbf{r}_u - \bar{\mathbf{r}}_u)} \times \sqrt{(\mathbf{r}_v - \bar{\mathbf{r}}_v)^T \cdot (\mathbf{r}_v - \bar{\mathbf{r}}_v)}}$$

		MOVIES							
									
USERS	 Alice	-2		1	0	1	0		0
	 Bob	2/3					-1/3		-1/3
	 Carl	4/7	4/7	-10/7	-3/7	4/7	-3/7		4/7

	 Zoe		-9/4	-1/4				7/4	-1/4

Solution:
Normalize ratings by
subtracting the mean rating

Now 0 means neutral, and if we treat missing ratings as 0, it doesn't mean it's negative

User-to-User Similarity: Pearson Correlation

$$\mathbf{r}'_u = \mathbf{r}_u - \bar{\mathbf{r}}_u \quad \text{mean-scaled rating vector of } u$$

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$$= \frac{(\mathbf{r}_u - \bar{\mathbf{r}}_u) \cdot (\mathbf{r}_v - \bar{\mathbf{r}}_v)}{\sqrt{(\mathbf{r}_u - \bar{\mathbf{r}}_u)^T \cdot (\mathbf{r}_u - \bar{\mathbf{r}}_u)} \times \sqrt{(\mathbf{r}_v - \bar{\mathbf{r}}_v)^T \cdot (\mathbf{r}_v - \bar{\mathbf{r}}_v)}} = \text{Pearson}(\mathbf{r}_u, \mathbf{r}_v)$$

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Top- k most "similar" users to u

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Set of items rated by u 's neighbors

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Predicted rating given by user u to item i

$$\mathbf{r}_u[i] = r(u, i) = r_{u,i}$$

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$$r_{u,i} = \frac{\sum_{v \in \mathcal{U}^k} \text{sim}(u, v) \cdot r_{v,i}}{\sum_{v \in \mathcal{U}^k} \text{sim}(u, v)}$$

weighted average

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- The model doesn't suffer from aging and therefore it does not need to be recomputed frequently

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$\mathbf{r}_{\text{Shrek}}$

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In general, **item-based** works better than **user-based** CF

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 - Locality-Sensitive Hashing (LSH) approximation

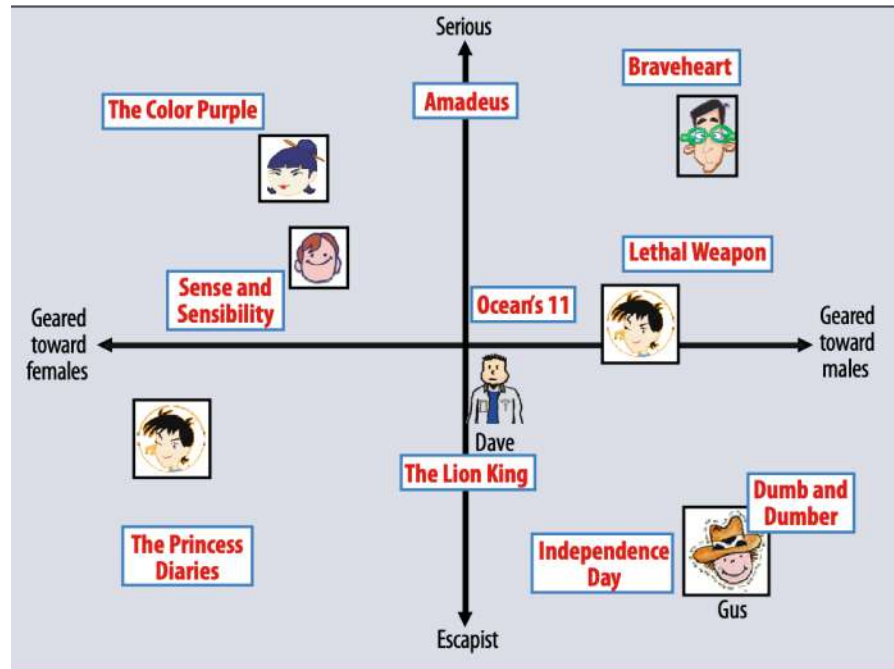


Latent Factor (Model-based) CF

Tries to predict ratings by representing both items and users with a number of **hidden factors** inferred from observed ratings

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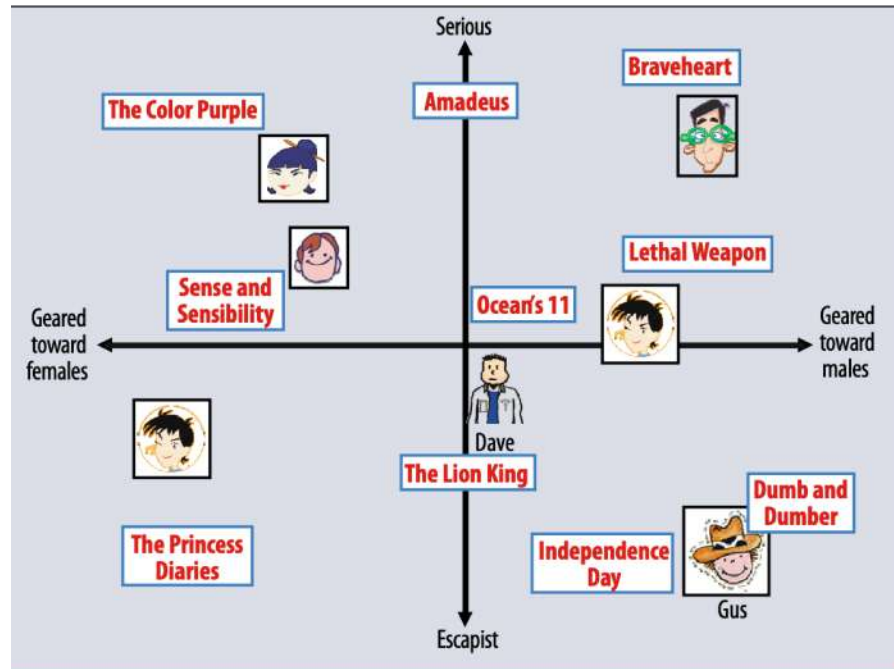


Example: 2 hidden factors

- Dim. 1: Male vs. Female
- Dim. 2: Serious vs. Escapist

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A user's predicted rating for an item (movie) would equal the **dot product** of the movie and user vectors on the plot

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- High correspondence between item and user factors leads to a recommendation

Matrix Factorization Framework

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- That is why these features are often refer to as **latent features**

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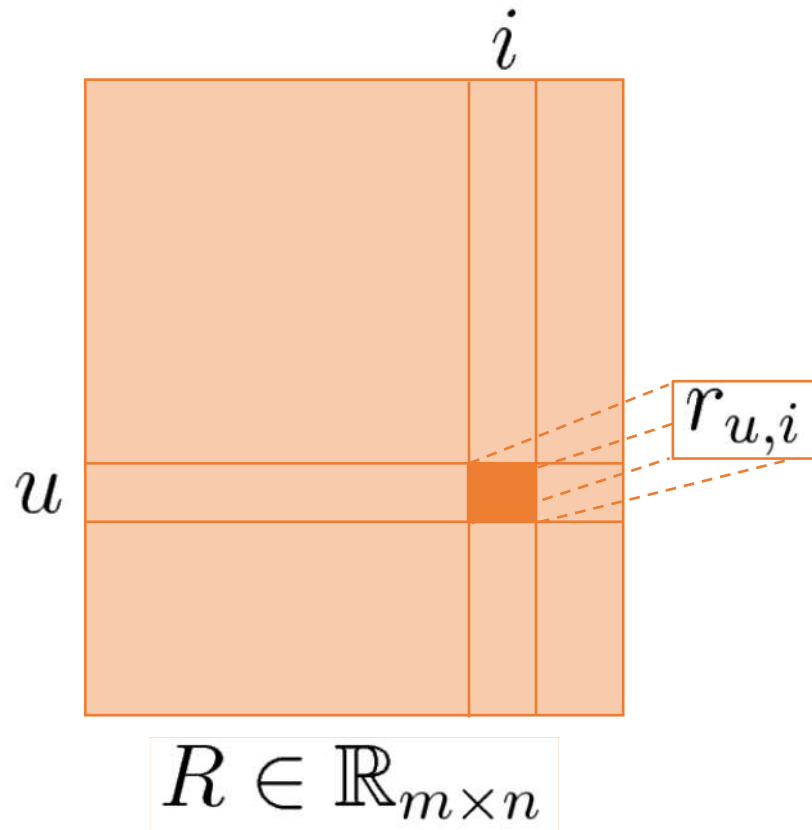
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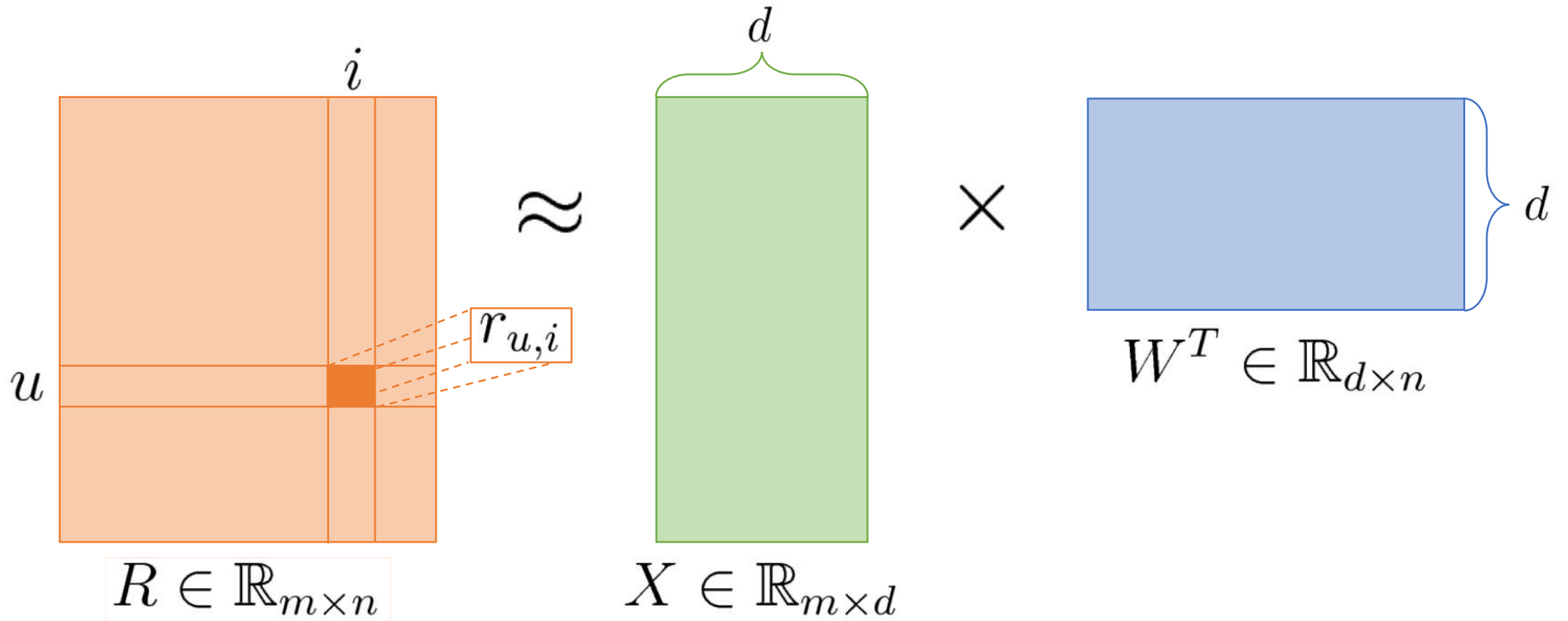
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Recommendations for a user are generated by computing the estimated ratings for unseen items, and by taking the **top-k highest rated** ones

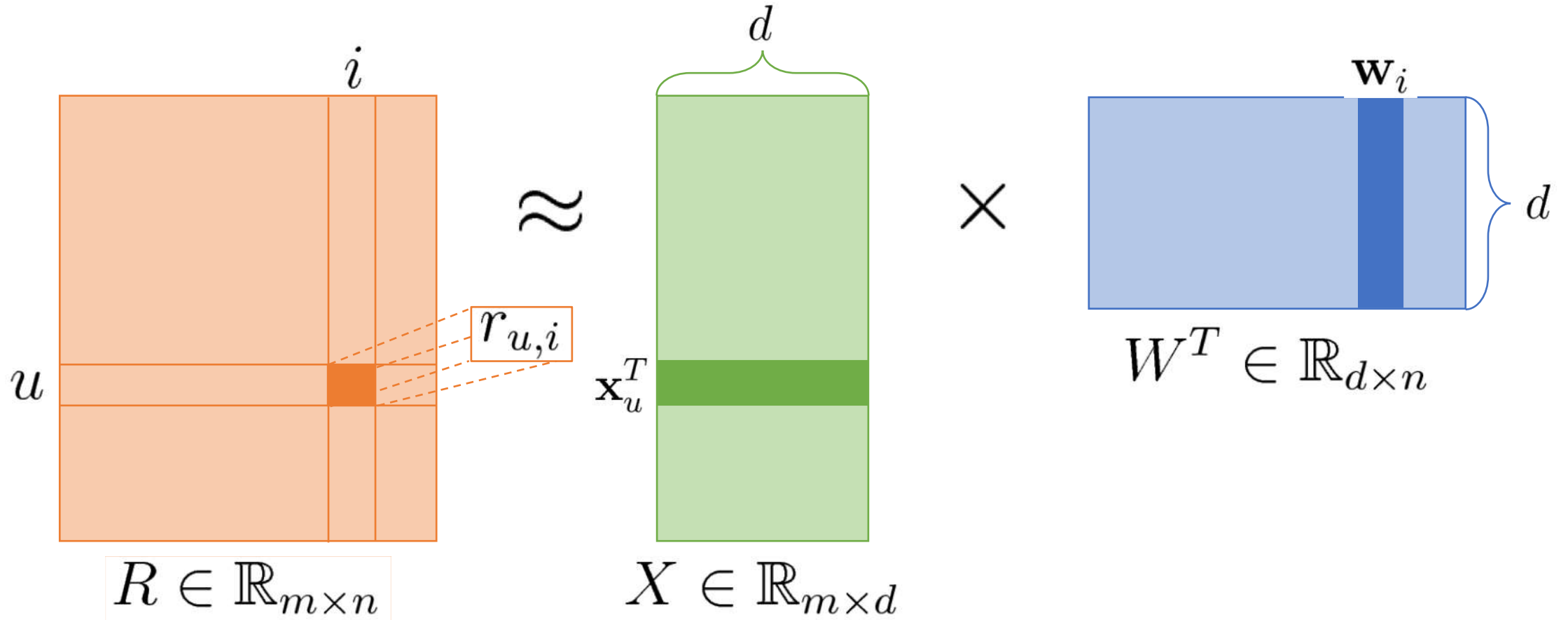
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Matrix Factorization Framework



Approximate the user-item rating matrix R with the product of $X \times W^T$

How Do We Learn X and W ?

Assuming we have access to a **dataset** of **observed ratings**

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Mathematically convenient

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Still, how do we solve this?

Learning Algorithms

2 main optimization methods

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Stochastic Gradient Descent (SGD)

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```
graph TD; A["2 main optimization methods"] -- blue arrow --> B["Stochastic Gradient Descent (SGD)"]; A -- green arrow --> C["Alternating Least Squares (ALS)"];
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We know that the updating strategy for SGD is as follows:

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At each iteration, both user and item latent vectors are updated by a magnitude proportional to η in the **opposite direction** of the gradient

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- In real life problems, this number can get very large quite often, requiring both a parallelization mechanism or an alternative optimizer

Alternating Least Squares (ALS)

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- Each alternating iteration reduces to traditional least squares and can be solved using OLS or its regularized variant (e.g., pseudo-inverse)

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We want to set this to 0

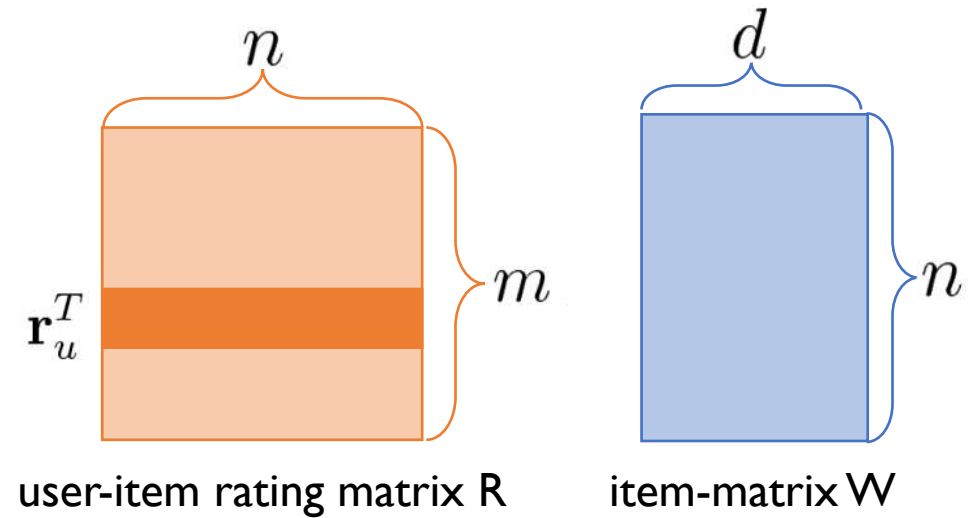
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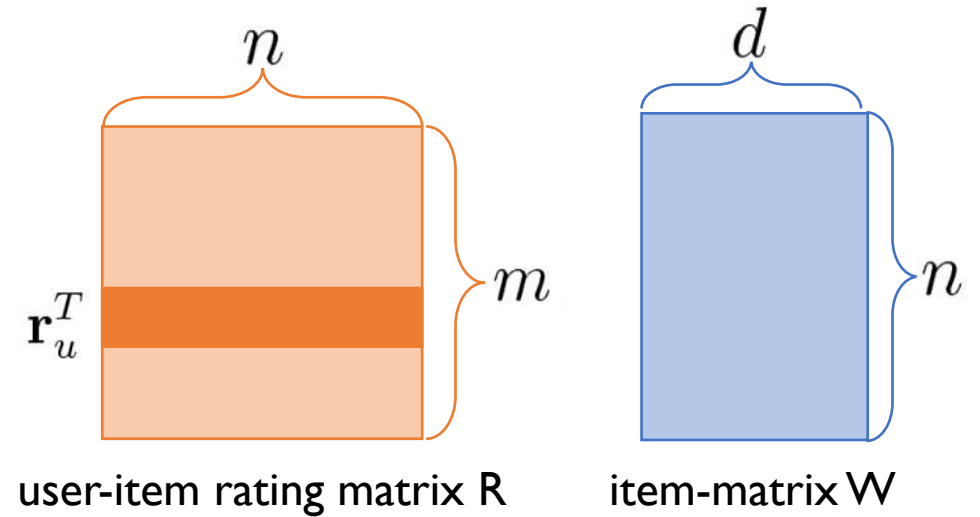
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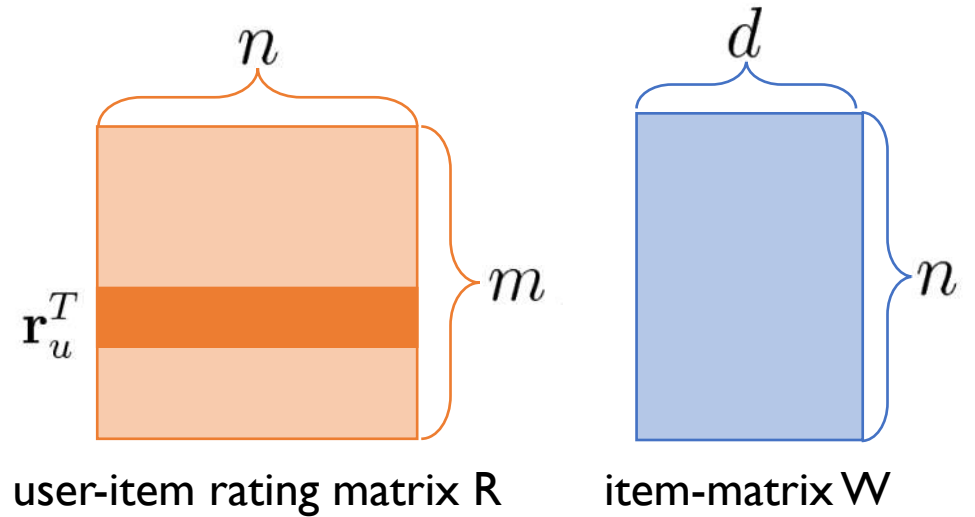
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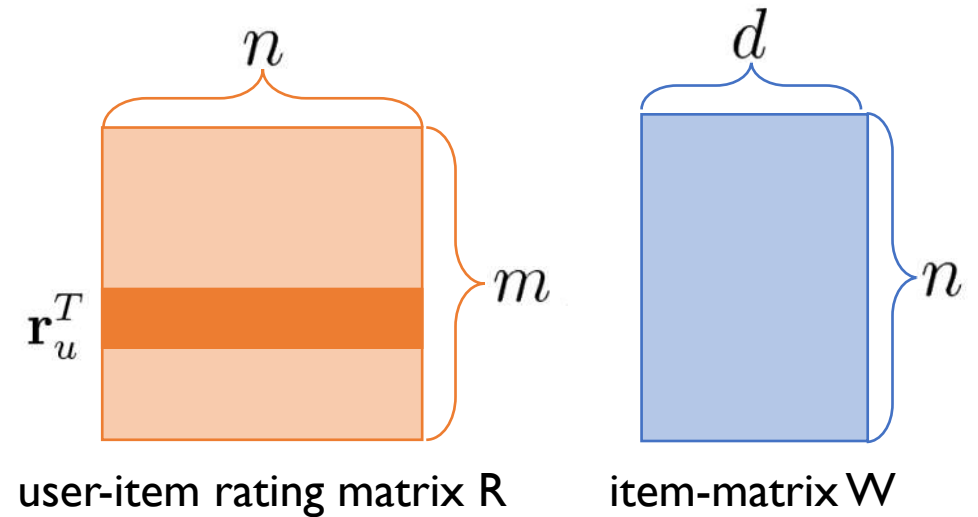
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$$-\sum_{i \in \mathcal{D}} (r_{u,i} - \mathbf{x}_u^T \cdot \mathbf{w}_i) \mathbf{w}_i + \lambda \mathbf{x}_u = 0$$

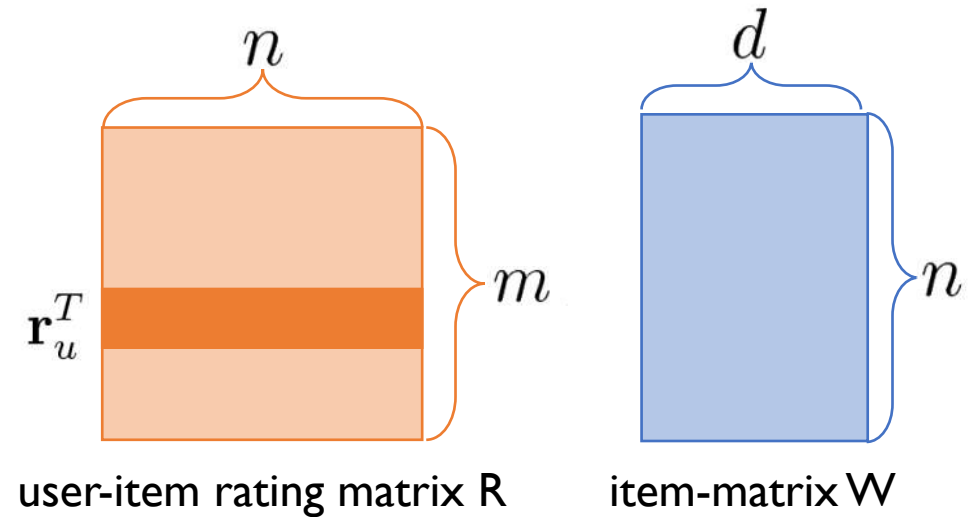
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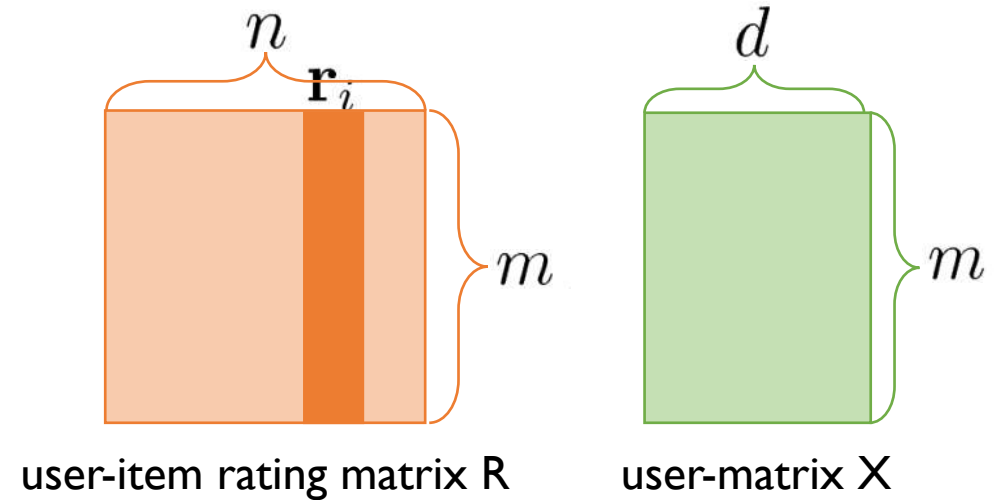
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Convergence is guaranteed because in each step the loss function can either decrease or stay unchanged, but never increase

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 - **Implicit Data:** the training set is dense and looping over each single instance - as SGD does - would be unfeasible

Singular Value Decomposition (SVD)

A well-known technique to decompose a matrix
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SVD solution is unique

Applying SVD to Collaborative Filtering

- If we let the matrix A be the user-item ratings R
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Each factor k is the result of the **similarity** between user i and item j and its overall effect on ratings across all users and items

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- Possible workaround to apply SVD: use **imputation** to fill missing values in the matrix R

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- For example, some users systematically tend to give higher ratings than others, and some items receive higher ratings than others

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$$b_{\text{Joe,Titanic}} = 3.7 - 0.3 + 0.5 = 3.9$$

Bias term

Including Bias into the Optimization

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Overall, the original optimization problem becomes as follows

$$X^*, W^* = \operatorname{argmin}_{X, W} \left\{ \frac{1}{2} \sum_{(u,i) \in \mathcal{D}} \left[r_{u,i} - (\mathbf{x}_u^T \cdot \mathbf{w}_i + \mu + b_u + b_i) \right]^2 + \lambda \left(\sum_{u \in \mathcal{D}} \|\mathbf{x}_u\|^2 + \sum_{i \in \mathcal{D}} \|\mathbf{w}_i\|^2 + \sum_{u \in \mathcal{D}} b_u^2 + \sum_{i \in \mathcal{D}} b_i^2 \right) \right\}$$

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Can still be solved using ALS

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- **Netflix** is a good example of hybrid recommender systems

Netflix's Hybrid Recommender System

Recommendations are generated

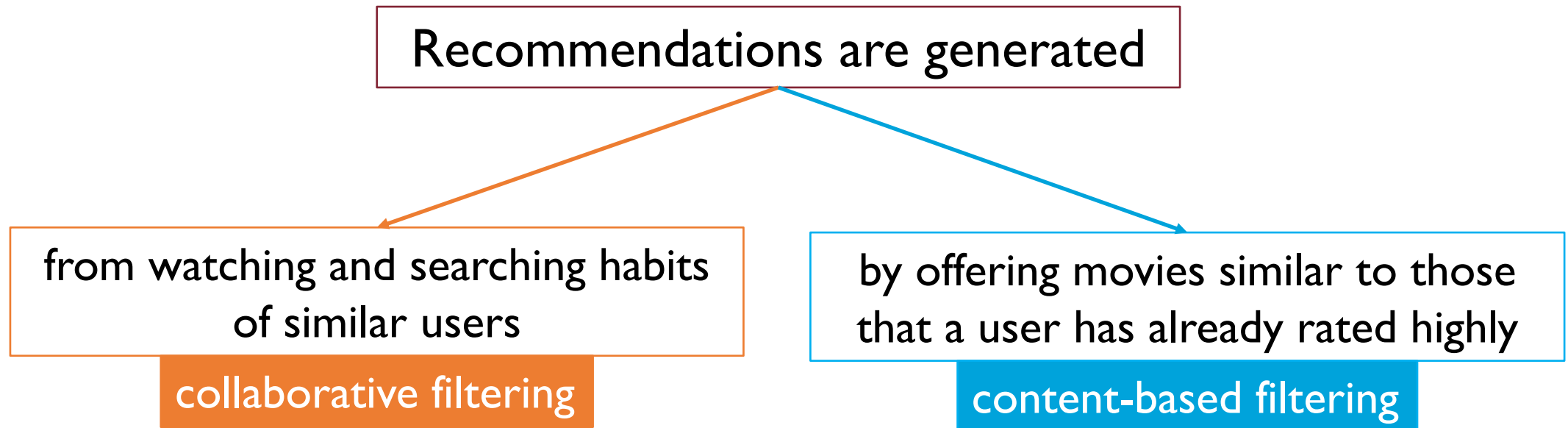
Netflix's Hybrid Recommender System

Recommendations are generated

from watching and searching habits
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collaborative filtering

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- Participating teams submit predicted ratings for a test set of approximately 3M ratings

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- According to the [contest website](#), more than 48,000 teams from 182 different countries have downloaded the data

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A combination of 100 different predictor sets, mostly factorization models

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A/B testing measuring CTR,
ROI, and other "live" metrics

Evaluation Metrics: RMSE

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The RMSE might penalize a method that does well for high ratings and
badly for others

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Mapping of binary classification terminology to recommender systems

binary classifier	recommender system
# with condition ($y = 1$)	# of all possible relevant items for a user
# predicted positive ($TP + FP$)	# of recommended items
# correct positives (TP)	# of recommended items that are relevant

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A recommender system generates k=5 items to recommend

There are only 3 relevant items

The success/failure of our recommendations: [0, 1, 1, 0, 0] 0=not relevant/1=relevant

$$P = \frac{2}{5} \quad R = \frac{2}{3}$$

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- $P@k$ and $R@k$ are simply the precision and recall calculated only from the subset of the first k recommendations

P@k: Example

$k = 3$

$P@3 = \frac{1}{3}$

Rank	Product Recommended	Result
1	Credit card	Correct positive
2	Christmas Fund	False positive
3	Debit Card	False positive
4	Auto loan	False positive
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with $|\text{Rel}|$ actually relevant items

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$$AP@N = \frac{1}{|\text{Rel}|} \sum_{k=1}^N P@k \times \mathbf{1}_{\text{Rel}}(k)$$

indicator function $\mathbf{1}_{\text{Rel}}(k) = \begin{cases} 1 & \text{if item } k \in \text{Rel} \\ 0 & \text{otherwise} \end{cases}$

Mean Average Precision (MAP)

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$$MAP@N = \frac{1}{|\mathcal{U}|} \sum_{u=1}^{|\mathcal{U}|} AP@N(u) = \frac{1}{|\mathcal{U}|} \sum_{u=1}^{|\mathcal{U}|} \frac{1}{|\text{Rel}|} \sum_{k=1}^N P@k(u) \times \mathbf{1}_{\text{Rel}}(k, u)$$

Personalization

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Intuitively, a high personalization score indicates the recommender system is able to provide a **highly personalized** experience to the users

Personalization

Suppose 3 users are recommended the following lists of items

$$u_1 = [A, B, C, D] \quad u_2 = [A, B, C, E] \quad u_3 = [A, B, F, G]$$

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	A	B	C	D	E	F	G
u_1	1	1	1	1	0	0	0
u_2	1	1	1	0	1	0	0
u_3	1	1	0	0	0	0	1

Personalization

Compute the 3-by-3 triangular matrix containing the cosine similarity between each pair of user's recommendation binary vector

$$M_{i,j} = \text{cosine}(\mathbf{u}_i, \mathbf{u}_j)$$

	\mathbf{u}_1	\mathbf{u}_2	\mathbf{u}_3
\mathbf{u}_1	1	0.75	0.58
\mathbf{u}_2	0.75	1	0.58
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Take the average of the upper triangle of the matrix M above

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$$\text{Personalization} = 1 - 0.64 = 0.36$$

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- Evaluation metrics must capture the accuracy, personalization and serendipity of recommendations

Recommended Readings and Information :)

- A huge body of work on recommender systems is available out there!
- Surveys:
 - [Adomavicius & Tuzhilin](#) [2005]
 - [Koren & Volinsky](#) [2009]
 - [Bobadilla et al.](#) [2013]
 - [Zhang et al.](#) [2019]
- Well-renowed series of Conferences: [RecSys](#), [KDD](#), [SIGIR](#), [TheWebConf](#)