KU LEUVEN

FACULTEIT WETENSCHAPPEN

Evaluating measures of partisan gerrymandering in U.S. redistricting

Halyna ALEKSANDROVYCH

Promotor: Prof. Dr. Geert Molenberghs

Proefschrift ingediend tot het behalen van de graad van Master of Science in Statistics

Academiejaar 2017-2018

Preface

The aim of this work was to examine, evaluate, and compare the currently existing measures of partisan gerrymandering as applied to the United States. This was done by applying a number of them to a dataset of Congressional elections from 1896 to 1992 and evaluating them in the light of existing redistricting regulations. I wish to thank Professor Dr. Molenberghs of KU Leuven for his support, guidance, and enthusiasm for the subject in writing this thesis.

Summary

Gerrymandering is the act of using the electoral redistricting process to benefit a certain interested party. Partisan gerrymandering is to use this process to benefit a political party in future elections and is an issue gaining increasing recognition as a danger to the electoral process which ought to be regulated. The aim of this work was to examine, evaluate, and compare the currently existing measures of partisan gerrymandering as applied to the United States. First, key definitions and concepts of redistricting and gerrymandering were covered, including the main goals and tactics used. Next, the already established redistricting criteria to be satisfied were briefly described, such as the population equality principle, preventing ethnicity-based discrimination in redistricting, and ensuring geographical compactness.

Following this, measures of partisan gerrymandering were introduced, including the efficiency gap, seats-votes curves, and Gelman and King's model which can be used to measures a number of quantities of interest related to elections and redistricting. All of these measures were based on the concept of partisan symmetry which has been proposed to evaluate partisan gerrymandering - the idea that a fair electoral system is to treat all parties in the same manner. This is manifested in comparing the seat-share that parties obtain given they win the same proportion of the vote.

The measures were applied to 49 Congressional elections from 1896 to 1992 for the entire United States and their applications were compared. The efficiency gap and uniform partisan swing seats-votes curves could be used as cross-sectional methods to evaluate elections which have already occurred and only require election results to be computed. Measures of partisan bias and responsiveness obtained using Gelman and King's model could evaluate electoral systems longitudinally and predict future results or evaluate counterfactual scenarios.

Further research is needed in applying these measures to more recent data, as well as considering other measures. In addition, these measures need to be evaluated in the context of the other redistricting criteria which will need to be met.

Contents

1 Introduction

Redistricting is the process of redrawing electoral district lines, a practice usually performed to compensate for demographic shifts but which presents political dangers or opportunities as consequences (Grofman, 2015). 'Gerrymandering', an associated term and often used interchangeably, refers to the use of redistricting for political gain and is a portmanteau of the words 'Gerry' and 'salamander' as a nod to Elbridge Gerry who, as governor of Massachusetts in 1810-1812, oversaw the redrawing of districts in his state. One appeared to be such an odd shape that opponents compared it to that of a salamander and the word was born (Cox & Katz, 2002).

This paper will focus on gerrymandering specifically in the United States and its two-party, winner-take-all system characteristics. However, the described concepts could be applied to other similar two-party systems and to some extent also in multi-party or proportional systems. This paper will first describe the main goals and tactics of gerrymandering, along with the key redistricting norms that have evolved over time. Next, measures of implementing those norms will be examined, with a focus on measures of partisan gerrymandering. This is because partisan gerrymandering has gotten increasing recognition by the courts as an issue of legal concern (see *Vieth v. Jubelirer*, *League of Women Voters v. The Commonwealth of Pennsylvania*, *Gill v. Whitford* and more) but one that has yet to have a widely accepted measurement method, let alone a threshold for courts to use.

1.1 Definitions

First, to clarify some key terminology and concepts. The United States Congress is comprised of two chambers - the Senate with 100 members (two from each state), and the House of Representatives with 435 members, allocated proportionally among the states based on population. Apportionment is the process of allocating those 435 seats among the states, something which takes place every ten years after the latest census (Grofman, 2015). Based on that, districting is done - the drawing of lines to demarcate each representative's district within the state. This can either be done because national population trends have resulted in a different number of seats being given to a state, or due to within-state shifts and no longer sufficiently equal districts in terms of population (Grofman, 2015). Having a great inequality in voter-to-representative ratios, beyond the accepted margins, across districts is known as malapportionment (Grofman, 2015). This process is overseen by different parties depending on the state - most have the state legislature draw the plans (usually with the help of outside consultants), while a few have separate commissions and the governor either approves or vetos the plan (Grofman, 2015). This paper will work with congressional districts, although similar redistricting processes take place to allocate districts for state legislatures.

The map on the left demonstrates the number of seats allocated to each state after the 2010 Census, totaling to 435 House of Representatives seats. Based on the demographic shifts reflected in the census results, some states received extra seats at the expense of others. The map on the right is that of Texas, demarcating its 36 districts, each of which elect a single representative. The size of the districts varies greatly based on population density.

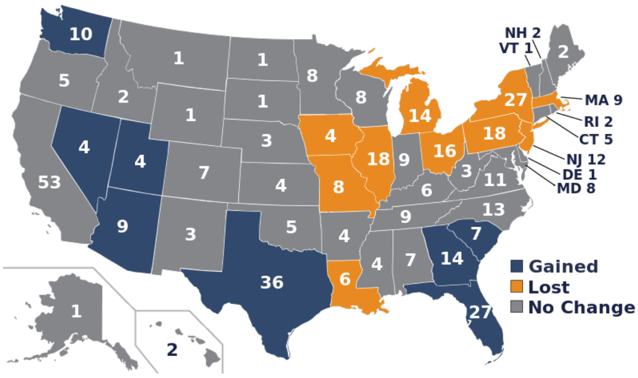

Figure 1: Map of 2010 Congressional district apportionment

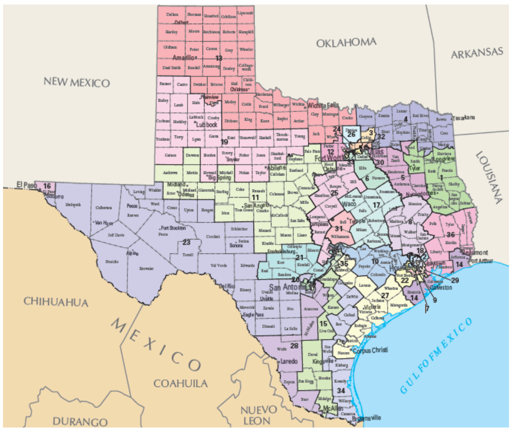

Figure 2: Map of Texas Congressional districts

1.2 Goals and tactics

The redistricting process and its current requirements and procedures creates opportunities for gerrymandering. The most commonplace goals of it can be summarized in five categories - partisan, bipartisan or incumbent, ethnicity-based, personal, or affirmative action gerrymandering (Grofman, 2015). A partisan goal aims to help one particular political party at the expense of the others, while a bipartisan or incumbent tactic aims to protect those already in office, to preserve the status quo (Grofman, 2015). Race- or ethnicity-based gerrymandering is one that attempts to disadvantage such a group by, for example, diluting that group's voting power (Grofman, 2015). The passage of the 1965 Voting Rights Act made such attempts illegal and imposed strict scrutiny on states and local governments that have demonstrated discriminatory voter regulations in the past. Personal gerrymandering refers to actions meant to benefit a single individual, not necessarily related to their party or other political associations, and lastly, redistricting has sometimes been used to purposefully benefit a previously disadvantaged group through what is called affirmative action or benign gerrymandering (Grofman, 2015).

The main tactics employed in reaching any of these goals can also be categorized. 'Cracking' refers to spreading one's opponent thin across enough districts to ensure they cannot win a seat, always finding themselves in the minority (Grofman, 2015). When such a tactic is not feasible, the opposite method can be employed - 'packing' - concentrating an opponent into as few districts as possible, having them win by large margins but restricting the total number of seats that group could obtain (Grofman, 2015). A more targeted tactic is 'kidmapping' - using the fact that a Congressperson must reside within their district, maps can be redrawn to either exclude a potential candidate from their district, or to lump multiple candidates or incumbents into a single district, forcing competition and ensuring that at most one would win (Grofman, 2015). A combination of these goals and tactics, aided by the precision and power of modern software and using publicly available voter information as well as additionally gathered data forms the basis of modern-day gerrymandering in the US.

The following graphics demonstrate these three basic tactics:

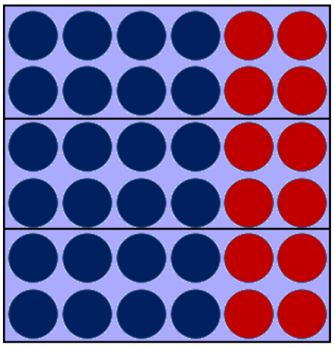

Figure 3: Cracking

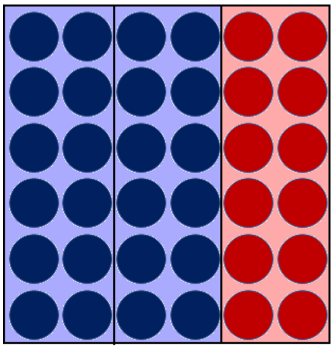

Figure 4: Packing

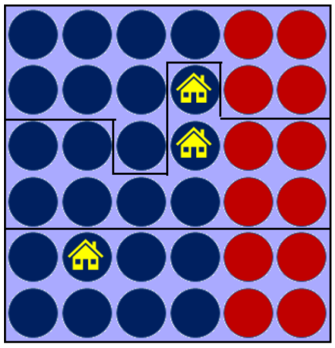

Figure 5: Kidmapping

The first shows 'cracking' of the red minority in such a way which prevents it from controlling any district. The second shows the opposite, 'packing' of the red constituents into a district they win but ensuring that it is the only one. Finally, the right-most graphic shows a redistricting scheme which would exclude the incumbent from his or her top district, and lump the blue incumbents of the top and middle districts into a competition with each other.

1.3 Fairness

Being that redistricting is a governmental process, statistical approaches to describe, dictate, or improve it must look towards the legal field for the criteria to use, the considerations that can be made as well as those that cannot, as well as the resources that may be used in formulating such approaches. While it can be easy to identify an unfair district, it has been more difficult to identify key priorities, their hierarchy, or formulate clear quantifiable criteria as to what constitutes a fair district. Looking at the evolution of this issue in the U.S., some principles and their order of priority can be identified through legislation, court rulings, and related texts.

The most important principle stems from Art. 1 Sec. 2 of the U.S. Constitution that "Representatives ... shall be apportioned among the several States ... according to their respective Numbers" as well as the 14th Amendment which repeats that statement. It has been identified as the "one person, one vote" principle - the idea that each person's vote should weigh as much as anyone elses or, in a quantifiable manner that there should be equality of voter-to-representative ratios as much as possible. This was further enforced through the decisions of *Baker v. Carr* (1962), which first recognized redistricting as a justiciable issue falling under the courts and not a purely political matter, as well as *Reynolds v. Sims* (1964) and *Wesberry v. Sanders* (1964). This principle is achieved by apportioning the 435 House representatives among the states based on population, though this does still result in some discrepancy, and then creating districts within each state that are as equal in population size as possible (Crocker, 2012). Despite these efforts though, the largest district is in Montana with 994,416 people and the smallest in Rhode Island with just 527,624 (Grofman, 2015). Thus, given that both have one representative, the voting power of someone in Rhode Island is nearly twice as large as of someone in Montana.

The second most important principle has been identified as the protection of racial and language minorities from vote dilution (Crocker, 2012). This is less specific but can include balancing packing and cracking to ensure that minorities have a strong enough hold on some districts to elect their own representatives but not to the extent where they are grouped into a single district to limit the number of seats they are able to win. Another measure would be to see if certain minorities live disproportionately in lower voting power districts as the Montana district mentioned above. If, for example all Native Americans lived only in Montana their group as a whole would have less strength on the national level. Lastly, some states have put in place rules that a minority cannot be worse off after a redistricting process (Crocker, 2012).

Another principle has been of geographical compactness and contiguity, and overall district "normalcy", arguing that bizarrely-shaped districts are more likely to be so as a result of some manipulation (Crocker, 2012). No single recognized meausure of geographical normalcy exists but measures have been proposed based on perimeter-to-area ratios, standard shapes like circles or squares, dispersion and compactness measures, along with convexity measures (Maceachren, 1985; Hodge et al., 2010). Adjustments have also been devised to adjust for natural boundaries such as coastlines, islands, or mountain ranges as well as state or national boundaries which cannot be changed for redistricting (Ansolabehere & Palmer, 2015; Hodge et al., 2010).

Principles to strive for, but which can be sacrificed in order to achieve the above-mentioned, are the protection or preservation of political subdivisions and communities of interest (Crocker, 2012). Examples of political subdivisions include cities, counties, school districts, etc. The more districts overlap with already existing political subdivisions, the greater the likelihood that constituents will be familiar with others in their district and the groups that it encompasses (Crocker, 2012). Preserving communities of interest serves a similar goal - communities of interest are groups of people which are believed to share a certain set of values, ideas, or be affected similarly by policy and thus could have their interests better represented as a group (Crocker, 2012). Examples of such communities could include urban versus rural populations, a community which heavily relies on a particular industry, such as coal mining, or an area with a high immigrant population.

Lastly, the principle of protection of incumbents exists with some states supporting it and others opposing. Some argue that protecting the status quo leads to more stability in the long run and allows for politicians to legislate rather than campaign, while others argue against and for the need to encourage political competition and engagement (Crocker, 2012). However, in a two-party system, heavily contested districts that frequently switch from one party control to the other, result in less stable governing and the repeal of work done by the previous administration (Crocker, 2012).

1.4 Partisan gerrymandering

Following these developments, beginning with the 1986 case of *Davis v. Bandemer*, making the issue justiciable for courts, partisan gerrymandering entered the field. Partisan gerrymandering is the use of the redistricting process to benefit one's political party and until 1986 was argued to be a political issue and not one to be regulated with laws or looked at by the courts (*Davis v. Bandemer*, 1986). However, in this decision the judges granted that if the effective intent of voter dilution of a particular group could be proven, it would violate the Equal Protections Clause of the Constitution's 14th Amendment, though they were at a loss as to how to measure such a phenomenon (*Davis v. Bandemer*, 1986). Since then, the issue has been repeatedly argued, generally on the grounds of freedom of speech - not punishing someone based on how they have voted in the past, or freedom of association - not punishing someone based on which (political) group they choose to associate with, the legal system recognizing the potential unconstitutionality of partisan gerrymandering, but finding itself helpless in finding a clear standard (see *Vieth v. Jubelirer*, 2004 and *Jackson v. Perry*, 2006).

The one successful case happened at the state level in Pennsylvania, but was argued using its state Constitution in early 2018 (*League of Women Voters v. Commonwealth of Pennsylvania*) and does not extend nationally. However, given these developments and acknowledgement that technology has made it increasingly possible to achieve partisan gains in the redistricting process, it is likely that the Supreme Court will be forced into ruling on this issue in greater detail. Currently, it is deciding on *Gill v. Whitford*, a case on appeal from Wisconsin in which the plaintiffs have utilized the 'efficiency gap' measure to argue proof of partisan gerrymandering. This case is expected to provide more guidelines on the issue as well as potentially accept the efficiency gap as a convincing measure of partisan gerrymandering (Schwinn, 2017). A number of state level cases with similar arguments have been paused, awaiting the Supreme Court's decision (see *Benisek V. Lamone* and *Common Cause v. Rucho*).

Ultimately, these legal actions show the issue gaining traction and an increasing need for statistical measures which could take into account both the many limitations and rules imposed on redistricting, as well as serve as "clear, manageable, and politically neutral" (*Vieth v. Jubelirer*, 2004, p.307-308) tools.

Apart from the general multitude of approaches which could be used, one key obstacle to formulating a measure of partisan gerrymandering is that it cannot be based on proportional representation. Though proportional votes-to-seats results could logically be seen as fair election outcomes, the Supreme Court ruled in *Vieth v. Jubelirer* (2004) that "the Constitution provides no right to proportional representation" (p. 268), and hence a deviation from proportional representation cannot be used as proof of partisan gerrymandering - e.g. showing that a given party consistently receives 40% of the vote but only 5% of the seats is irrelevant.

A solution which has been widely accepted by both parties as well as non-partisan entities is the standard of partisan symmetry (King et al., 2006). Partisan symmetry is satisfied when different political parties, if given the same conditions, have the same results (King et al., 2006). In other words, the parties are treated symmetrically. Practically, it is when, given a certain percentage of votes won, they would translate into the same number of seats regardless of which party received them. For example, borrowing from the previous case, receiving 40% of the vote and 5% of the seats would be considered fair if this was true for any party receiving that number of votes. This standard is thus politically neutral as it does not evaluate any specific party, and in fact those can be anonymous. Furthermore, it does not rely on proportional representation, so long as the votes-to-seats conversion is the same regardless of party.

1.4.1 Efficiency Gap

Multiple measures of partisan symmetry have been proposed, offering different approaches to measure deviation from symmetry, also known as partisan bias (King et al., 2004). One such measure is the efficiency gap, devised by Dr. Stephanopoulos and Dr. McGhee, and could be recognized in the upcoming *Gill v. Whitford* decision as an acceptable measure.

It is based on so called “wasted votes” - any votes over the 50% threshold for the winner and all of the votes received by the losing party (Stephanopoulos & McGhee, 2014). In other words, they are the votes that did not further help a party gain more seats and were thus “wasted”. The larger the gap between the two parties in wasted votes, the greater the partisan advantage in favor of the party that wasted the least (Stephanopoulos & McGhee, 2014).

The efficiency gap formula for a given state and election is as follows:

$$\frac{(W_d - W_a)}{Total} = \%\, advantage\, for\, P_a \tag{1}$$

Where W_d are the number of wasted votes across all districts in the state for the party at a disadvantage, W_a are the number of wasted votes across all districts in the state for the party at an advantage ($W_d > W_a$) and the difference is divided by the total number of votes cast to obtain the partisan advantage.

The advantages of this method are, firstly, that it is very simple in terms of information needed, computation, as well as interpretation. Election results are public domain and available for every election. Secondly, this measure is based on actual, concrete results rather than simulations, or extrapolated from historical trends which can instill more confidence in the public. Lastly, the concept and computations are simple enough to explain to the general public as well as allowing to make the process of evaluating redistricting plans more democratic.

The disadvantages are that this is clearly a method for an exclusively two-party system and though the U.S. is dominated by the Republican and Democratic parties, smaller ones do exist. The stronger the presence of third parties in a given election, the more this measure would be affected and the more difficult it would be to compare those results to more traditional two-party cases. A second disadvantage of this method is that low values of the efficiency gap follows an S-shaped curve (like most seats-votes curves) and would assign greater levels of bias to proportional systems than to those with a steeper seats-votes curve (Duchin, 2017). This would disadvantage proportional states, which many find fair (Duchin, 2017). One last disadvantage of this measure is that it cannot be used to account for historical trends. A single instance of partisan advantage for a given party is not strong evidence of entrenched unfairness as a result of gerrymandering.

For this reason, though the efficiency gap measure gives clear and simple snapshots of individual elections, more complex methods are needed to prove trends.

1.4.2 Seats-votes curve symmetry

Seats-votes curves are plots which show the percentage or number of votes won versus the number of seats that that translated to for a given party (Duchin, 2017). In a two-party system, one seats-votes curve describes the entire election and the symmetry of the curve can be used to evaluate partisan bias. Because most U.S. election results fall close to the 50% mark, the middle section of seats-votes curves can be estimated best and are of most interest (pragmatically, partisan symmetry matters little at the 90% of votes point if such a result is highly unlikely to ever take place). Some measures of symmetry near the center are the mean-median test, or the horizontal distance from the curve to the center point, or partisan bias, the vertical distance from the curve to the center point (Duchin, 2017).

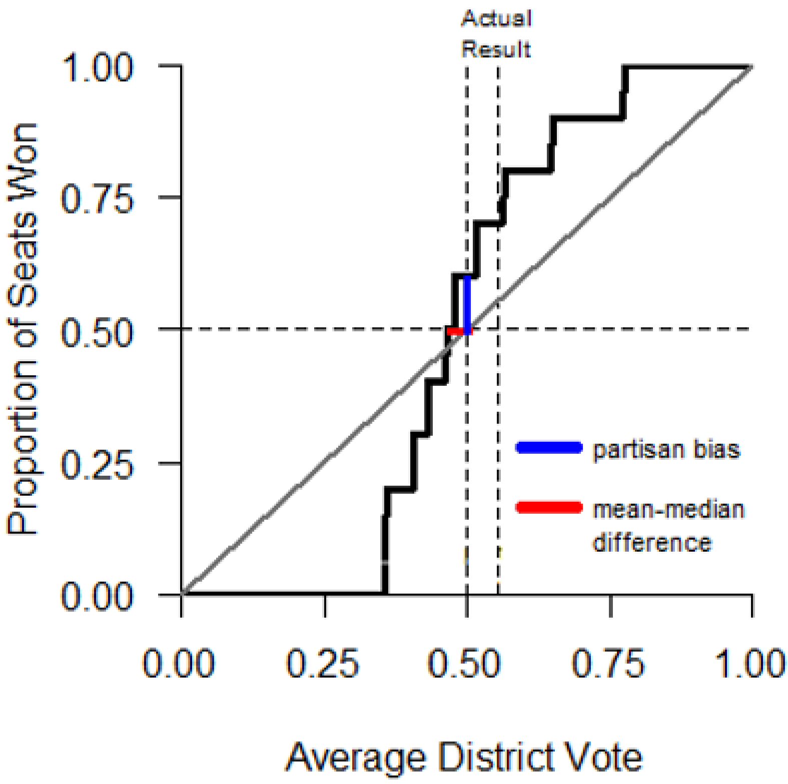

Figure 6: Seats-votes curve of Indiana 1992

The most common method of generating seats-votes curves is to use the uniform partisan swing hypothesis, which states that if a given party's vote share changes, it will change by the same amount in each district (Duchin, 2017). For example, if Republicans are expected to nationally do better by 5% in the next election, the uniform partisan swing hypothesis would assume that Republicans would increase their voter share in every district by 5% as compared to the last election. Uniform partisan swing is often assumed in presidential elections - after two presidential terms of one party, a partisan swing is given to the other party across all districts (Gelman & King, 1994). Generating a seats-votes curve from an election for a given state is done by recording the vote shares in each of the state's districts and then taking turns applying uniform partisan swing under hypothetical conditions - e.g. how would the seats be distributed had Republicans done better by 5% in each district? by 10%? and so forth to complete the curve.

Seats-votes curves make it easy to assess symmetry through a number of measures and even more simply, visually. However, measures such as the mean-median or partisan bias only assess deviation from symmetry at a single point rather than assessing the entirety of a curve (though that too is of lesser importance along the stretches of the curve that are unlikely to occur in reality). Furthermore, generating seats-votes curves requires making assumption about voter-behavior which may not always have large evidential support.

1.4.3 Sampling and outliers

The sampling and outliers method involves generating hypothetical samples of districts and comparing the district in question to the simulated sample. The reasoning behind it being that if the district in question is an outlier, it is likely that it was not due to random chance, but rather the direct result of gerrymandering (Duchin, 2017). There are many ways of generating samples of simulated districts - for example by using Markov Chain Monte Carlo (MCMC) random walk sampling, starting with existing districts, imposing a contiguity restriction, and flipping neighbouring geographical units according to a given probability (Duchin, 2017).

The difficulty with sampling is, that while it is fairly easy to generate possible districts, it is less so to generate plausible ones (Cho & Liu, 2016). Plausible districts which fulfill the necessary requirements - equal population is still fairly simple, but the principle of compact geography, for example, is not. Districts are usually based with at least some consideration to political subdivisions, making it more likely that they are kept whole than split (Crocker, 2012). Furthermore, what might look like a non-compact, irregular district might seem much more normal when the geography of the state is considered, or the history of the area. For these reasons, comparing a district in question to samples of possible districts may often be misleading (Cho & Liu, 2016).

1.4.4 Gelman and King's model

Dr. Gelman and Dr. King have done particularly extensive research in this field having developed an unbiased estimator of incumbency advantage (1990a), measures of partisan bias and responsiveness (1990b) along with building upon others' work through their unified model (1994). They have built a linear regression model to predict any quantity of interest based on a set of existing variables to better understand past elections, predict future elections, or to run hypothetical scenarios by altering variable values. The linear model takes the following form:

$$v = X\beta + \gamma + \varepsilon \tag{2}$$

$$\gamma \sim N(0, \sigma_\gamma^2) \tag{3}$$

$$\varepsilon \sim N(0, \sigma_\varepsilon^2) \tag{4}$$

Where v is the district vote outcome (set as either of the two parties' vote share), X is the set of existing variables, β are the parameter estimates, γ is the systematic error and ε is the random error, independent and both normally distributed around a mean zero and with variances of σ^2_γ and σ^2_ε, respectively, which add up to the total variance of the model, σ^2. Additionally, the proportion of variance due to γ is also computed, denoted by λ.

$$\sigma^2 = \sigma^2_\gamma + \sigma^2_\varepsilon \tag{5}$$

$$\lambda = \frac{\sigma^2_\gamma}{\sigma^2_\gamma + \sigma^2_\varepsilon} \tag{6}$$

The γ term accounts for any variance between districts by variables that were not included in the model. Given two districts, i and j with identical values of X, the difference in their outcomes v_i and v_j would be due to systematic as well as random error. Systematic error can be a number of variables not included in the model that would stay constant if the election was repeated - for example campaign spending, number of ads run by either side, etc., while ε encompasses the remaining stochastic elements which could affect results, such as weather on election day, for example.

Gelman and King then use a Bayesian approach to simulate distributions of possible outcomes under the assumption that an observed election result does not reflect a single true population value, but rather just one of many possible outcomes that could have occurred given the same covariate values.

Thus, a distribution is simulated and its mean used as the predicted quantity of interest:

Table 1: Model structure

District number	Actual election results	Hypothetical replications of each district election 1	2	...	m
1	v_1	$v_1^{(hyp)1}$	$v_1^{(hyp)2}$	...	$v_1^{(hyp)m}$
2	v_2	$v_2^{(hyp)1}$	$v_2^{(hyp)2}$	...	$v_2^{(hyp)m}$
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
n	v_n	$v_n^{(hyp)1}$	$v_n^{(hyp)2}$	...	$v_n^{(hyp)m}$
Quantity of interest	Q	$Q^{(hyp)1}$	$Q^{(hyp)2}$	...	$Q^{(hyp)m}$

For each district, given its actual election results, a hypothetical set is generated of possible results had the election been re-run multiple times under the exact same conditions. Based on these, posterior distributions can be calculated and used to estimate any quantity of interest, from vote share to the number of incumbency seats retained, or partisan bias. Once a quantity of interest is selected and the hypothetical outcomes generated, a point estimate and standard error for Q can be computed from the average and variance of $Q^{(hyp)}$ values:

$$\overline{Q} = \frac{1}{m}\Sigma^m_{j=1} Q^{(hyp)j} \tag{7}$$

$$Var(Q) = \frac{1}{m-1}\Sigma^m_{j=1}(Q^{(hyp)j} - \overline{Q})^2 \tag{8}$$

Where $\overline{Q}$ is the point estimate and the square root of $Var(Q)$ is the standard error. The hypothetical election outcomes can then also be derived:

$$v^{(hyp)} = X^{(hyp)}\beta + \delta^{(hyp)} + \gamma + \varepsilon^{(hyp)} \tag{9}$$

Where $X^{(hyp)}$ is used instead of X in case hypothetical covariate values are of interest instead of actual conditions, $\delta^{(hyp)}$ denotes state-wide partisan swing which can be specified, and $\varepsilon^{(hyp)}$ denotes the variability in $v^{(hyp)}$ values. The other error term, γ stays fixed because the hypothetical election results are simulated for constant circumstances, which includes keeping the systematic error constant.

If $\sigma_\gamma^2 = 0$, meaning also $\lambda = 0$, then the only difference in values of $v^{(hyp)}$ would be due to the stochastic element ε. On the other hand, if variance in the model is completely attributable to γ with $\sigma_\varepsilon^2 = 0$ and $\lambda = 1$, then all hypothetical election outcomes will be identical and the same as the actual election results, if available.

The covariates used in such a model serve mainly the purpose of predicting results, rather than inferring causal relationships. While those relationships can be important to study, the ultimate goal of this model is to best predict election outcomes and variable selection is aimed at that. Covariates can be anything from previous election results, current party in control, average age of the district or voting population, campaign contribution amounts, etc. but variables such as incumbency, uncontestedness, and party control have been found to be strong predictors (Gelman & King, 1994). Additionally, for optimal results, the variables chosen need to have information available for all elections being studied. The quantities of interest Gelman and King used the model to estimate were partisan bias and electoral responsiveness for the House of Representatives since 1900 and to evaluate proposed Ohio redistricting plans in 1992. This model was found to be advantageous in that it could be used for a multitude of variables of interest as well as more accurate uncertainty estimates than prior models (Gelman & King, 1994).

Before calculating the posterior distributions of hypothetical outcomes $v^{(hyp)}$ or estimating a quantity of interest Q, preliminary estimation needs to be carried out for the parameters β, σ^2, and λ. This can be done by with existing election results and using equation (2) to estimate them. If the model is being used for prediction and the election results have yet to occur, then σ^2 is estimated through the regression of the most recent election for which data is available. Lastly, λ is estimated by data from a pair of consecutive election results, first estimating the covariate coefficients, and then regressing the election outcome over the prior results, estimating how much a prior result contributes to election outcomes, holding other covariates X, constant. The systematic part in v which helps predict a subsequent election result on top of the already included covariates, is the share of variance due to γ.

In general, the σ^2 and λ estimates can be better estimated by pooling them over time, if the data is available and the same covariate information is used for each election. This cannot be done for the covariate coefficients β as they have been found to be more volatile across election cycles and pooling them does not generally improve the fit or predictive ability of the model (Gelman & King, 1994).

As previously stated, the given model can be used for three types of purposes - to predict future election results, to evaluate elections which have occurred, or to model hypothetical cases by setting specific covariate values. These require different methods of estimation of the posterior distribution of $v^{(hyp)}$. In the predictive case, the actual results are not available and thus the hypothetical outcomes are only conditional on the covariate estimates of β and the state-wide partisan swing δ:

$$P(v^{(hyp)}) = N(v^{(hyp)}|X^{(hyp)}\hat{\beta} + \delta, X^{(hyp)}\Sigma_\beta X^{(hyp)\prime} + \sigma^2 I) \tag{10}$$

In the prediction case, the parameters γ or λ do not appear and the variance in the model is due to the variability in election results conditional on the covariates, as well as the variability in estimating β. The mean, meanwhile, is based on the covariates and their coefficient estimates as well as the state-wide partisan swing (if included).

When election results are available and the goal is to either evaluate that election as it happened, or evaluate a counterfactual scenario, a different model, conditional on the observed outcome is used. Given that evaluating the election with its true covariate values falls under the set of models with hypothetical covariate values, the same model is used for both cases:

$$P(v^{(hyp)}|v) = N(v^{(hyp)}|\lambda v+(X^{(hyp)}-\lambda X)\hat{\beta}+\delta^{(hyp)}, (1-\lambda^2)\sigma^2 I+(X^{(hyp)}-\lambda X)\Sigma_\beta(X^{(hyp)}-\lambda X)\prime) \tag{11}$$

Which, when evaluating an actual election, $X^{(hyp)}$ is replaced by X and $\delta^{(hyp)} = 0$ and simplifies to:

$$E(v_i^{(hyp)}|v) = \lambda v + (1-\lambda)X\beta \tag{12}$$

The resulting equation is a weighted average of the outcome predicted by the covariates, $X\beta$, as well as by the remaining systematic components in v that are not covered by the variables included. The weights are based on λ, the proportion of variability attributed to systematic error γ. Thus, the posterior distribution of outcomes of an actual election, which describes the range of possible outcomes which could have occured had the same election been re-run, is based both on the predictive ability of the covariates, as well as the observed outcome.

Often, it is the estimated coefficients which are of interest in a model analysis. However, in the purposes that this model is intended for, the variables estimates serve mainly an intermediary role for insight into the outcome v or a range of potential quantities of interest associated with elections. After obtaining the distributions of $v^{(hyp)}$ values, the quantities of interest, Q, can be derived. For example, vote share outcomes or related quantities of interest, such as the number of Democrats or incumbents that win, can be obtained from the average of hypothetical outcomes $E(v_i^{(hyp)})$. Another possibility is to, rather than estimate the expected vote shares, compute the probability of a district going to either party (falling above or below 50%) using the expected outcome and the associated standard error. Given that v reflects the vote share obtained by Democrats, the probability of a Democrat winning a given district i would be as follows:

$$P(Democrat \quad wins) = P(v_i^{(hyp)} > 0.5) = \Phi\left[\frac{v_i^{(hyp)} - 0.5}{\sqrt{var(v_i^{(hyp)})}}\right] \tag{13}$$

Where the normal distribution is used to evaluate the probability of that a Democrat will win in district i given the computed posterior, its mean and variance.

This model can also be used to generate seats-votes curves, by setting the quantity of interest to be the number of seats obtained by one of the parties and computing it based on the posterior distribution of hypothetical election outcomes. In order to create a curve, this would need to be done for a range of $\overline{v}^{(hyp)}$, based on the data available. The hypothetical outcomes are then simulated using partisan swing (or by fixing different values of $\overline{v}^{(hyp)}$) and recording the subsequent seats distribution. Most election results in practice fall around the 50% mark, making for a rule-of-thumb seats-votes curve range of [0.4;0.6] (Gelman & King, 1994). This is broad enough to be informative for the most likely to occur outcomes, but narrow enough to be sufficiently accurate because it is based on enough data. The benefit of using this model is that it is not only able to generate seats-votes curves as described in an above section, but to also do so taking into account covariates that might be of interest or the values of which are known for a future election. An additional benefit of using the Bayesian approach rather than simply applying the uniform partisan swing hypothesis, is that by working with posterior distributions, standard errors are also computed and can be used to create, and then test, a 95% confidence region.

Lastly, two quantities of interest related to partisan gerrymandering which can be estimated with this model is responsiveness and bias. In simple terms, responsiveness can be explained as the steepness of the seats-votes curve, the steeper it is over the range of most plausible outcomes, the more effect a difference in outcome has on the resulting seats allocation. Given a point v_0 on the seats-votes curve, its local responsiveness can be calculated using the following:

$$\frac{[E(\overline{s}^{(hyp)}|\overline{v}^{(hyp)} = v_0 + 0.01) - [E(\overline{s}^{(hyp)}|\overline{v}^{(hyp)} = v_0 - 0.01)]}{0.02} \tag{14}$$

Where $E(\overline{s}^{(hyp)}|\overline{v}^{(hyp)})$ is the expected seats allocation given the average of the posterior of hypothetical outcomes, and the slope is evaluated over a range of 0.02 around the point of interest, v_0. The responsiveness is thus the percentage change in seats allocated as the result of a 1% change in the vote. The shape of the curve and location the point dictate how confident one needs to be in the location of this point of interest, but popular choices are the actual or predicted outcome (Gelman & King, 1994).

Partisan bias is a measure of partisan symmetry and is derived from the symmetry of the seats-votes curves. For any given point along the curve, the difference in seat-share can be compared for both parties given the same vote-share. Assuming, that the curve represents the vote- and seat- shares obtained by Democrats, one could evaluate partisan bias at 45% of the vote by comparing the associated seat-share of the Democrats to those of the Republicans if they were to win 45% (when Democrats win 55%). This can be done for any point, though given the closeness of many elections, it is classically evaluated at the midpoint:

$$E(\overline{s}^{(hyp)}|\overline{v}^{(hyp)} = 0.5) - 0.5 \tag{15}$$

Which is interpreted as the percentage of seats obtained, above or below what is fair, given an average of hypothetical votes of 50%. However, just as with other methods, it is only reliable within the ranges close to those of historical outcomes and cannot be used to make reliable predictions about outcomes far from observed values.

Having given an overview of the existing measures of partisan gerrymandering and the methods currently under consideration, the aim of this work can be better formulated. The goal of this thesis is to apply, evaluate, and compare some of the above-mentioned measures of partisan gerrymandering. This will be done by applying them to historical data and studying the behavior of these measures and their advantages and shortcomings in practice.

The efficiency gap measure will be focused on because it is currently the most likely measure to be recognized by the courts as an acceptable tool for measuring partisan gerrymandering (*Gill v. Whitford*). Additionally, Gelman and King's model will be used to produce seats-votes curves using the uniform partisan swing hypothesis and the mean-median as well as partisan bias measures applied. Lastly, the model will be used to evaluate the past elections through Bayesian estimation and gain insight into the quantities of interest of responsiveness and partisan bias.

2 Methods

The data used in this thesis was a dataset of House of Representative election results for each district between the years 1896 and 1992. Elections took place every two years in all districts, for a total of 49 election cycles. The representatives serve two-year terms with no term restrictions, meaning that incumbents can always run for re-election. The dataset was titled ICPSR 6311 and was included in Gelman and King's R package JudgeIt and was originally compiled by the Inter-university Consortium for Political and Social Research (ICPSR). All statistical analysis was performed using R software and particularly Gelman and King's package JudgeIt when applying their model.

When looking at the results, it is important to note that at the beginning of 1896, when the dataset begins, the U.S. was comprised of 44 states, a number which grew to 50 by 1959 when Hawaii joined the Union (Encylopaedia Britannica, 2018) (see Appendix A). Furthermore, the total number of districts during that time fluctuated between 357 and 437, settling on 435 from 1963 onwards (U.S. House of Representatives, 2018) (see Appendix B).

Lastly, unless otherwise specified, all results are reported for the Democratic side, e.g. a partisan bias of 5% represents a 5% bias in favor of the Democratic party and putting the Republicans at a 5% disadvantage. Similarly, a vote-share of 0.4 represents an outcome of 40% for the Democrats and 60% for the Republicans.

3 Results

3.1 Efficiency gap

The efficiency gap was calculated using equation (1) for each state at each election time point, reflecting the efficiency (dis)advantage for Democrats in the given race. Plotting the average efficiency gap for each election cycle with the respective standard deviations resulted in the following plot:

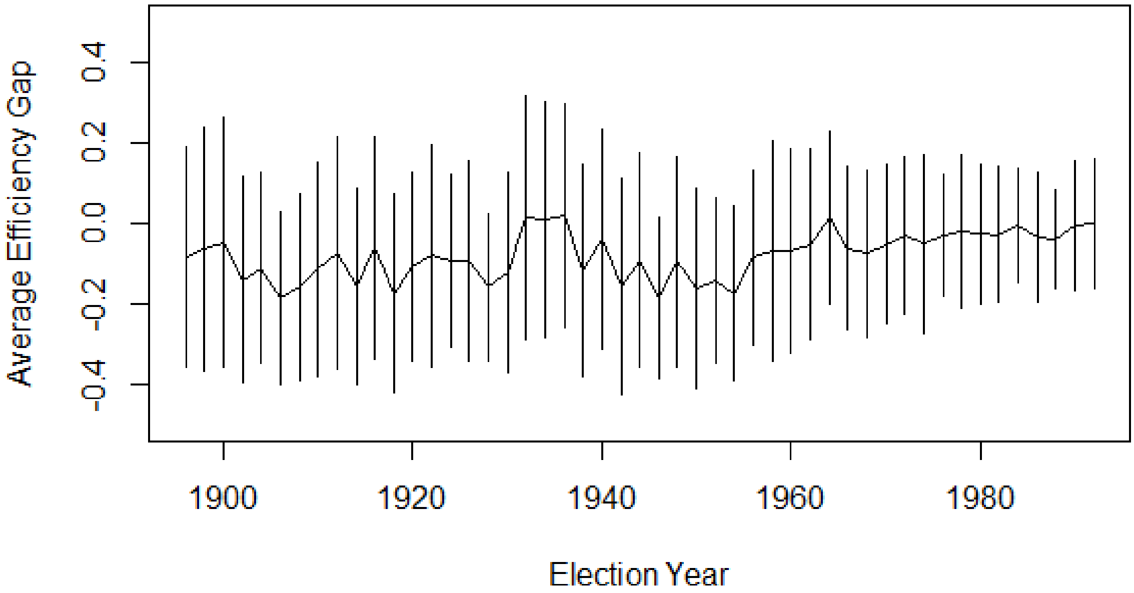

Figure 7: Average efficiency gap values over time with standard deviations

The average efficiency gap fluctuated between a 20% disadvantage for the Democrats and a little over the break-even point. The earlier years showed greater year-to-year variation in values, as well as larger standard deviations at each time point, evening out from the 1970s onwards to values closer to zero and with smaller deviation.

In order to observe trends among individual states or groups of states, the efficiency gap plots for each were also examined (see Appendix C). The ICPSR grouped states into categories based on both geography and historical development. Given that the United States started with the original thirteen colonies on the east coast and expanded farther westward with time these categories were deemed logical, and preserved.

What can be seen is that most states do indeed show trends of negative efficiency gaps. However, some states show much steadier values, such as Massachusetts or Pennsylvania, while others show great volatility, an example being Delaware or South Dakota. Similarly, an interesting trend could be observed among the Southern States, most of which showed very negatives efficiency gaps, particularly before the 1960s. This is likely due to race conflicts in the South during the mid-20th century - the tendency of black voters to vote Democratic and to be heavily discriminated against, including using redistricting (Crocker, 2012). It was not until the passage of the Voting Rights Act of 1965, which prohibited redistricting purely based on race or to disadvantage a racial minority, that this trend can be seen to alter.

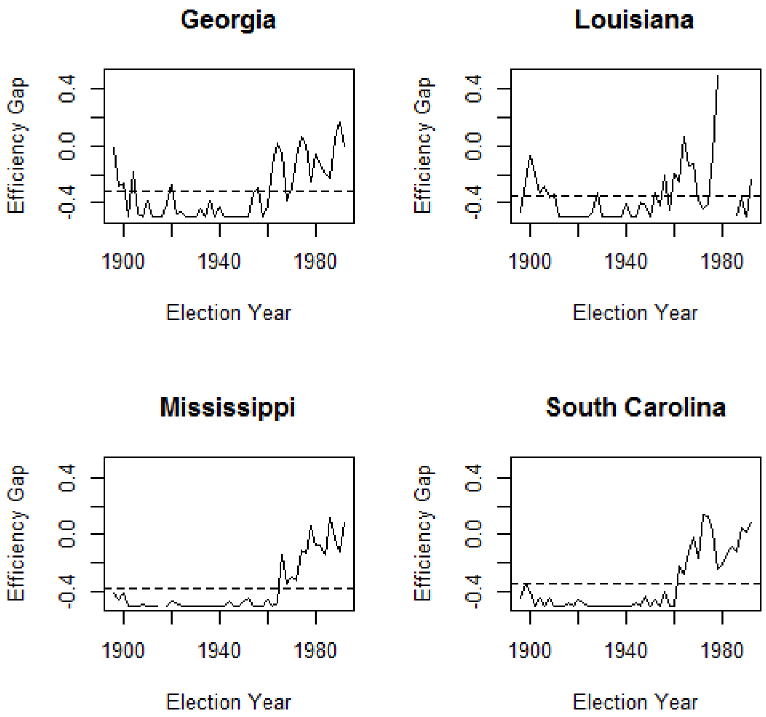

Figure 8: Efficiency gap plots of Southern States (means in dashed line)

This method has a number of clear advantages - values can be calculated for virtually all cases with the most easily available and non-controversial data - the actual results. The reasoning behind the measure is simple to follow and easy to visualize, and does not require extensive statistical knowledge or software to compute, making this one of the most accessible measures. Furthermore, because it is based on actual results and is not actually a model applied to data, it does not include any measures of uncertainty or deviation, which can be a positive when communicating results to the general public (however, some cases suggest the Supreme Court wants measures of uncertainty, see *Castaneda v. Partida*, 1977). It also serves as a good tool to catch potential packing and cracking behavior - showing that certain groups are either being packed into few districts and winning them overwhelmingly, thus wasting all of the votes they did not need to succeed, or being spread so thinly that all of the group's votes end up wasted. However it does not, on its own, show that such manipulation (and with partisan intent) is the reason for such a voter distribution. Packing can occur naturally, for example, rural areas tend to lean more Republican while urban areas vote more Democratically.

In addition, such a measure treats election results as the the distribution of Democratic and Republican voters in a given state, rather than as results in a particular election. A Democratic win might be due to a large proportion of staunch, consistently-Democratic voting party-members, but it is often also due to the swing of independent voters (Schuck, 2016). It therefore overlooks the effect of independent voters in helping a party win, as well as their greater willingness to switch parties between elections (Schuck, 2016). Of course, this measure is also intended for two-party systems and thus states with a stronger third party field, or types of elections that are less partisan (e.g. local positions where the individual can matter more than his/her party) would not have that required, strong, two-party structure (Schleicher, 2007). One last concern which can be observed in the in the plots is that the stability of the efficiency gap measure over time seems to be associated with the number of seats allocated to that state, with less-populous states seeing greater volatility. This makes sense for close, competitive states with few districts, such as for example Delaware, where parties alternate control and win by small margins, resulting in ever-switching 40% and greater efficiency gaps. In such a case, the efficiency gap is more a measure of competitiveness or the effect of national partisan swing rather than partisan gerrymandering.

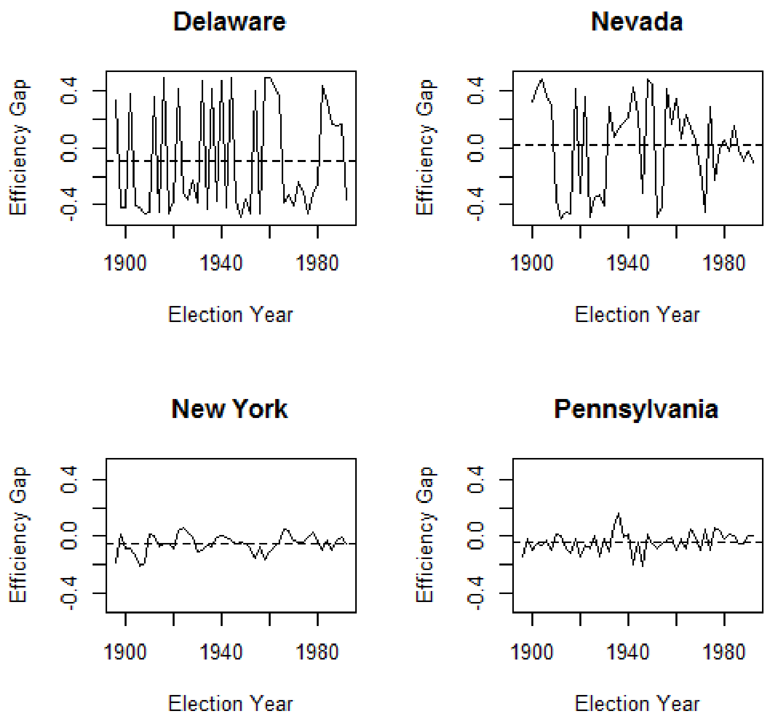

Figure 9: Efficiency gap plots of high and low variance

3.2 Seats-votes curves

Using another approach to measure partisan symmetry, seats-votes curves were generated for the states of New Jersey and Texas using 1992 election results and the uniform partisan swing hypothesis. For a given state with k districts, with corresponding vote-share election results of $v_1 ... v_k$, the same percentage was added or subtracted from each district to simulate outcomes under different values of uniform partisan swing, forming the curve. The curve plots include dashed reference lines at 50% of the vote and 50% of the seats, as well as 45° diagonal which corresponds to proportional representation. It is not an objective, but serves as a visual aid in this case to evaluate the symmetry of curves. Two measures of partisan symmetry can be derived from the center points - partisan bias (in blue), and the mean-median difference (in red). Partisan bias, as described previously was the deviation of the seat-share above or below 50% obtained with 50% of the vote, while the mean-median difference was the deviation of the vote-share above or below 50% obtained with 50% of the seats. The mean-median difference gets its name because it is equivalent to the difference of the average district vote $\overline{v}$ in a state and the district with the median outcome. While a perfectly symmetric curve would results in values of zero for both of these measures, curves that show a large partisan bias or mean-median difference at the mid-point can still be fairly symmetric in other ranges. These measures are taken at the center-point because so many outcomes in the U.S. two-party system fall close to that point. However, if for a given state, the results are more likely to fall in a different range, it would be more logical to evaluate the symmetry of the curve on that range.

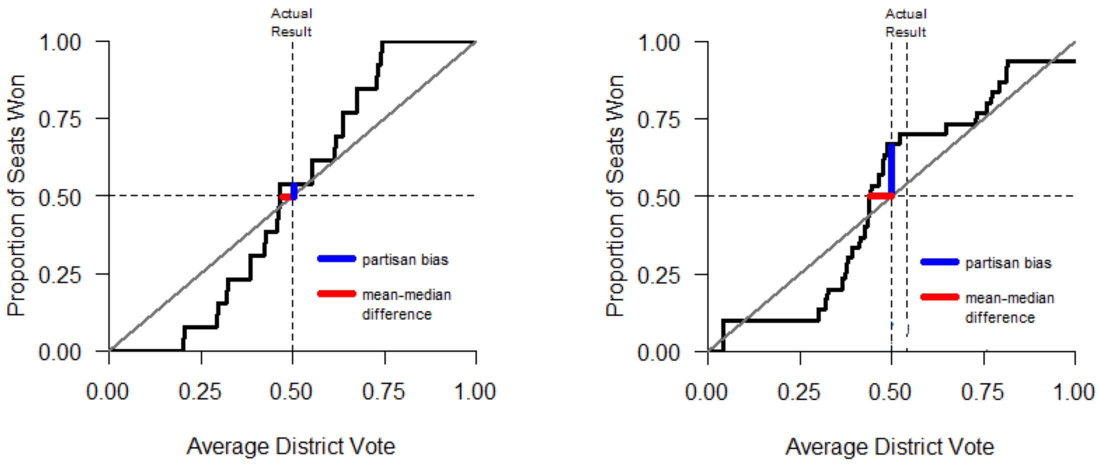

Figure 10: Seats-votes curves using uniform partisan swing for New Jersey (left) and Texas (right) 1992 election

The New Jersey curve on the left visually looks much more symmetric than that of Texas, with a fairly even reflection over the 45° diagonal even if it does not go precisely through the center point. The Texas curve shows a range of values, where, before having obtained a majority of the vote, the Democrats can already control a majority of the seats (mean-median difference), something the Republicans would not be able to do. To see how partisan symmetry faired in these specific two elections, however, it makes more sense to evaluate the curves at the actual results (or the range of most common outcomes). The symmetry at the actual outcome point was calculated by comparing the seat-share results had the two parties switched results:

Table 2: Vote- and seat-shares for actual and mirror outcomes

		Vote-share	Seat-share
New Jersey	Democrats (actual)	0.4977	0.5385
	Republicans (hypothetical)	0.4977	0.4615
Texas	Democrats (actual)	0.5402	0.7000
	Republicans (hypothetical)	0.5402	0.4667

Had the Republicans won the same number of votes as Democrats in the same elections, it can be seen that the seat share they would have won was different from what the

Democrats received. In the case of New Jersey and its very close results, the Democrats won 49.77% of the vote and received 53.85% of the seats. Had the Republicans received 49.77% of the vote under these same conditions, they would have received 46.15% of the seats, showing a discrepancy between the two parties. In Texas, the discrepancy is larger with the Democrats gaining 70% of the seats with just 54.02% of the vote, when Republicans would only have won 46.67% of the seats with the same election outcome. This type of scenario is what is described as a lack of partisan symmetry - given the same conditions, the parties would fare differently.

It is one method of evaluating elections which have already happened but is limited in some key ways. Firstly, seats-votes curves are based on actual results, meaning they have more information in the vote ranges which have occurred most often (usually around the center) and are less accurate at the tails. This makes them a poor choice to make predictions or evaluations of hypothetical scenarios where there is a large discrepancy between the party outcomes. Another aspect of seats-votes curves are that their smoothness depends on the number of districts in a state - the more districts, the smoother the curve. This creates the possibility for misleading results when evaluating a state with few districts and not fully explaining the step-like behavior of the curve. Therefore, determining an acceptable threshold of bias would need to take into account a number of factors. Firstly, ensuring that the higher-priority requirements of redistricting are met, equal population, non-discrimination on racial or ethnic grounds, and geographical compactness. Secondly, it would need to be related to the state's number of districts. Just as with the efficiency gap, it can be seen that states with a high number of districts cannot be directly compared to those with fewer districts on some of these partisan gerrymandering measures.

A further aspect of using seats-votes curves is that they are not predictive models and cannot use a lot of supplemental information which plays a role in election outcomes, such as by including covariates. Thus, though hypothetical outcomes can be considered by moving along the curve, it does not provide information about future elections. Lastly, these curves are generated using the uniform partisan swing assumption, which has had support for some instances (such as alternating presidential wins), but cannot be assumed to apply broadly to all election cycles (Gelman & King, 1994).

3.3 Partisan bias

The Gelman and King model was used to model partisan bias in a different way - as a quantity of interest derived from a linear regression where the uncertainty of election outcomes was modeled through a Bayesian posterior. Each district's outcome was regressed on the available variables of whether that district was contested or not (whether the Democrats won >95% of the vote, Republicans, or neither), whether an incumbent is running, and the previous election cycle's results:

$$v = \beta_0 + \beta_1 * Uncontested + \beta_2 * Incumbent + \beta_3 * LastResults + \gamma + \varepsilon \tag{16}$$

The uncontestedness of a district takes on a value of 1 for a > 95% Democratic win, -1 for a Republican landslide, and 0 for neither. Incumbency is measured with a binary variable where 1 indicates that the incumbent is running for re-election, versus 0 is not. The results of the previous election are included as the Democratic vote-share from the previous election cycle. Both error terms are normally distributed $\gamma \sim N(0, \sigma_\gamma^2)$, and $\varepsilon \sim N(0, \sigma_\varepsilon^2)$ and pooled over the years as it has been shown to produce more reliable estimates, while the more volatile predictor coefficients were kept for each election cycle (Gelman & King, 1994).

This model was applied to the set of all states over the 49 election cycles from 1896 to 1992 as well as to subsets of 'Southern' and 'non-Southern' states (Southern states are those classified into "Solid South", see Appendix C) to compare with efficiency gap results. Having estimated the regression coefficients for each election cycle (see Appendix D) and the error parameters - with σ indicating to within which percentage the covariates were able to estimate v and λ which indicated how much of the variation in v could be explained by systematic factors not included in the chosen covariates - the posteriors could be computed.

Table 3: Values of variance parameters

	σ	λ
Whole U.S.	0.0777	0.5566
Southern states	0.0737	0.4735
Non-Southern states	0.0723	0.5925

The model assumes that election results are not fixed parameters and any recorded result is but one possible outcome, if the election were to be re-run under the same conditions once again. Thus, the Bayesian approach is used which instead models parameters with distributions of possible values and this was done in this case to estimate posteriors of $v^{(hyp)}$ for each election cycle, using equation (11). Having obtained those distributions, partisan bias values were computed for both at the center point 0.5 as well as for the range between 0.45 and 0.55 where many results fall.

The following are plots of the two measures of partisan bias, along with 95% credible intervals plotted over time for the three cases evaluated:

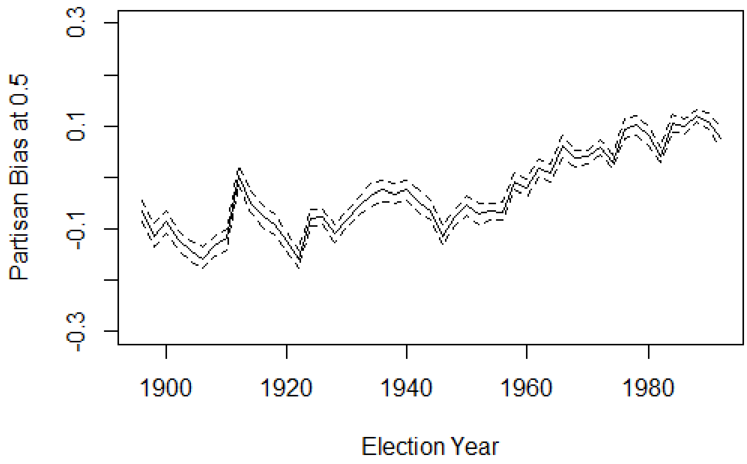

Figure 11: Bias at 0.5 plot for entire U.S.

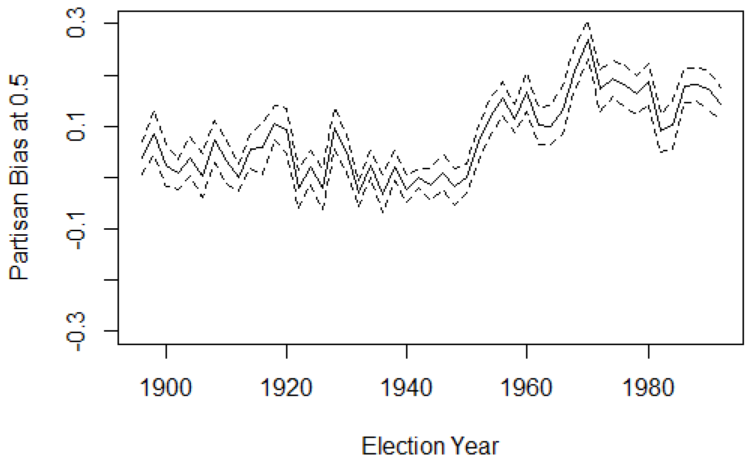

Figure 12: Bias at 0.5 plot for Southern states

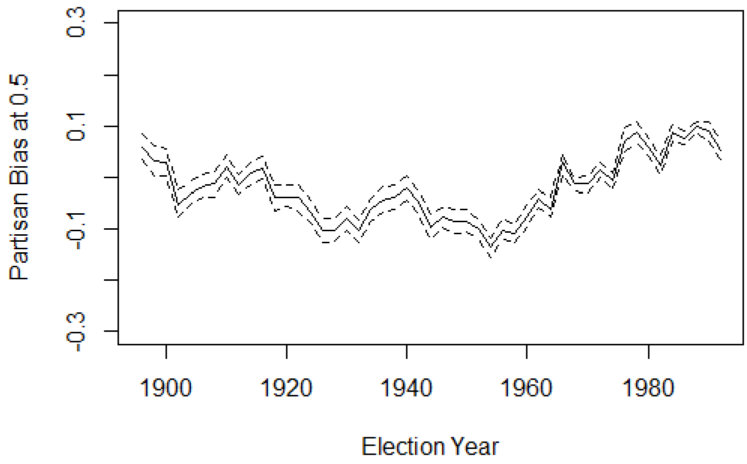

Figure 13: Bias at 0.5 plot for non-Southern states

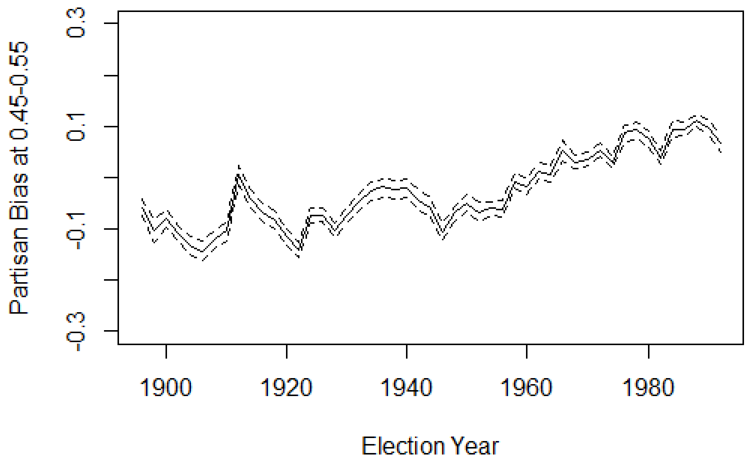

Figure 14: Bias at 0.45-0.55 plot for entire U.S.

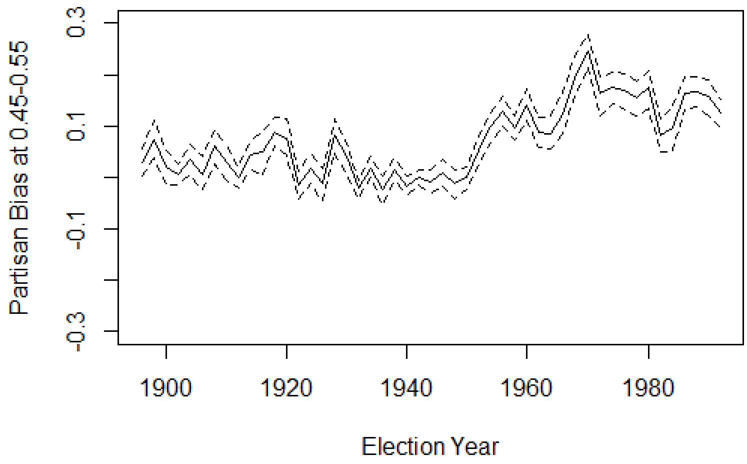

Figure 15: Bias at 0.45-0.55 plot for Southern states

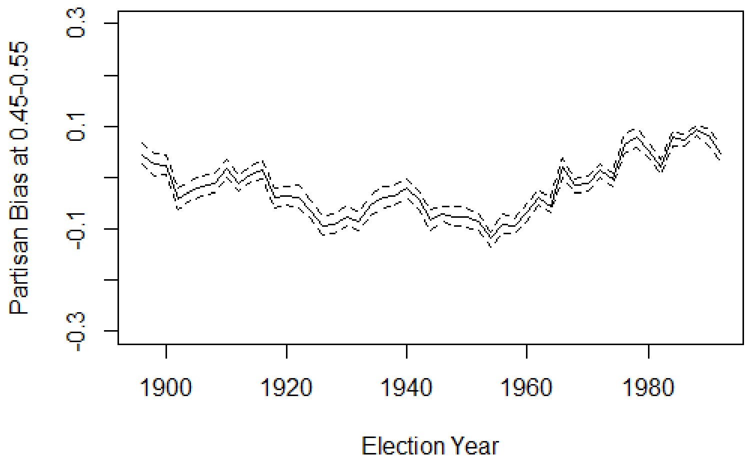

Figure 16: Bias at 0.45-0.55 plot for non-Southern states

A general trend could be observed of partisan bias in favor of the Republicans to a Democratic bias from the 1960s onwards. In addition, Southern states consistently showed higher values of partisan bias than non-Southern states, staying close to zero until the 1950s and then showing Democratic advantage.

3.4 Responsiveness

The posteriors obtained in the previous section were also used to estimate responsiveness, which measures partisan bias through the effect that a 1% change in the vote-share outcome has on the seat-share (slope of the seats-votes curve). It was calculated for both the traditional interval of 0.45-0.55 as well as at the points of actual results along the curve:

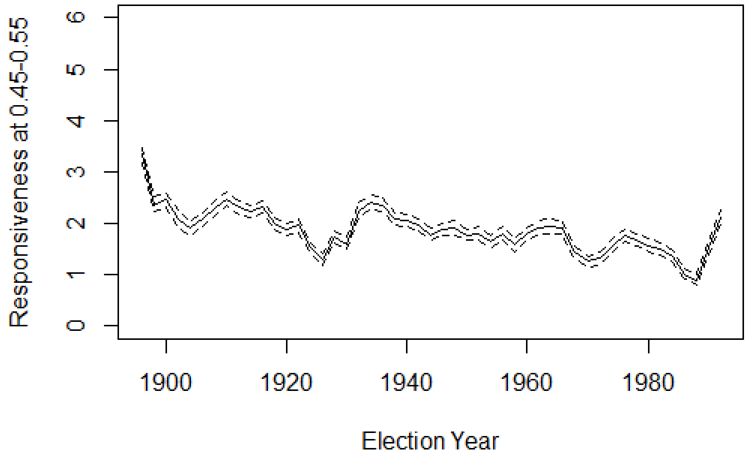

Figure 17: Responsiveness at 0.45-0.55 plot for entire U.S.

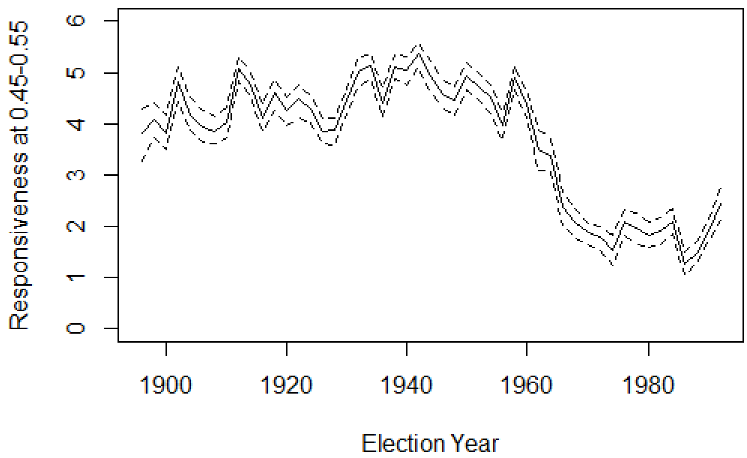

Figure 18: Responsiveness at 0.45-0.55 plot for Southern states

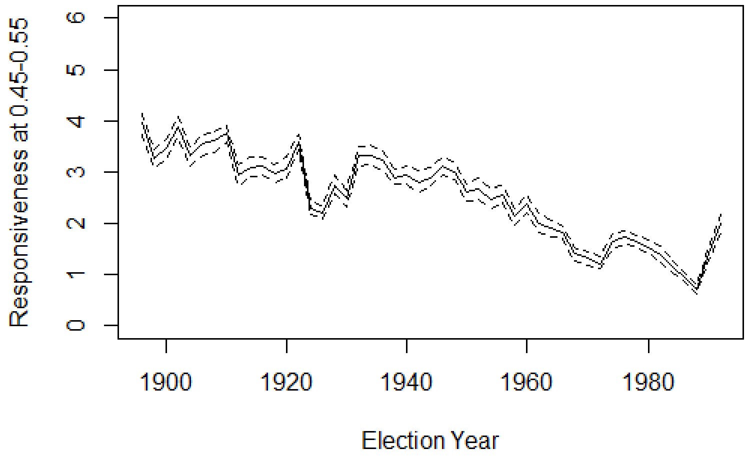

Figure 19: Responsiveness at 0.45-0.55 plot for non-Southern states

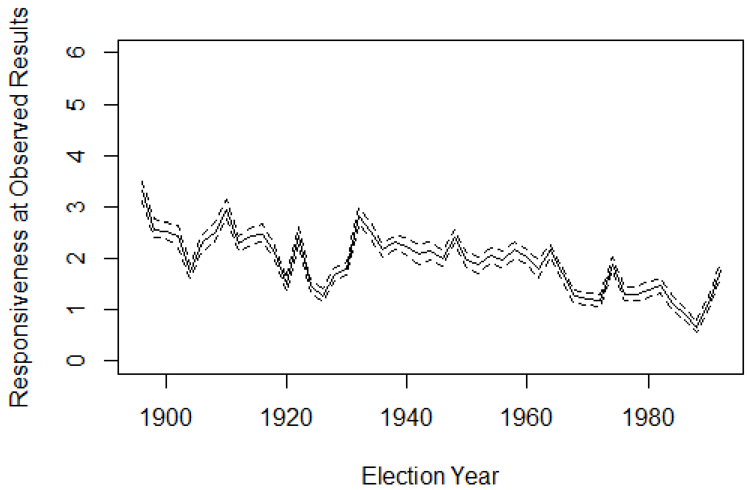

Figure 20: Responsiveness at observed results plot for entire U.S.

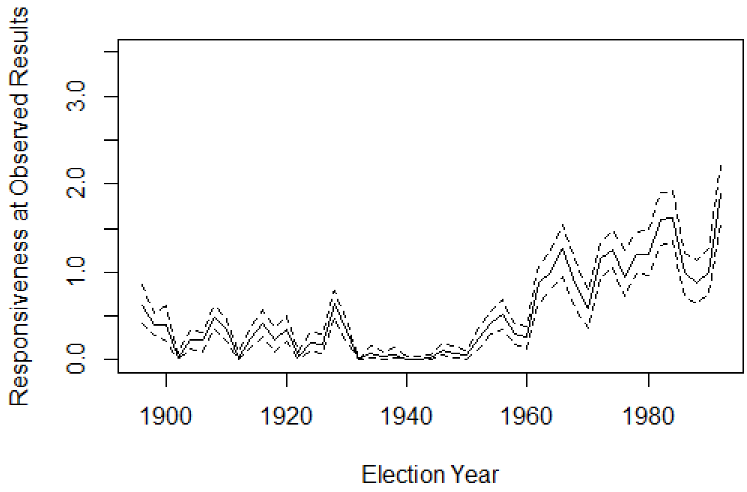

Figure 21: Responsiveness at observed results plot for Southern states

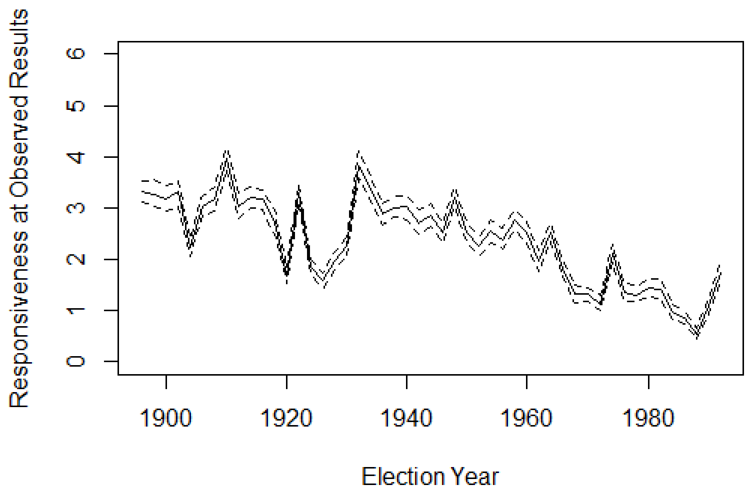

Figure 22: Responsiveness at observed results plot for non-Southern states

Higher values signify a steeper slope at that point or interval, with a larger percentage of slopes being gained for additional votes won (a slope of 1 would show a proportional relationship between the two), an expected behavior in the 0.45-0.55 interval where curves are usually steepest.

The results show interesting trends. For both responsiveness at 0.45-0.55 as well as at the points of actual results, the values have slowly decreased over time, with an increase in the past few cycles. This is the case for both the overall U.S. as well as non-Southern states. However, the behavior changes when looking at the two plots of Southern states.

Responsiveness values at 0.45-0.55 are high up until the 1960s to between 1 and 2, indicating that middle part of the seats-votes curve for southern states became less steep since then. Looking at the plot of values at actual result points, however, the opposite is seen - values of responsiveness have been increasing since the 1960s. This suggests that a significant number of election outcomes in the Southern states do not fall within the 0.45-0.55 range and thus while the slope there has decreased in recent decades, competitiveness around the actual outcomes has increased. Higher values of responsiveness can prevent incumbents from from creating comfortable districts - when the slope around the expected outcome is steep, every small change in the results impact seat-share to a greater extent. As for which point or interval to use to measure responsiveness, it is best to measure it at the actual result points in order to evaluate elections which have happened, while intervals (that are believed to be plausible) provide more insight for predicting future elections.

4 Discussion

A number of measures striving to evaluate partisan gerrymandering have been considered in this paper and applied to the same data. Based on these analyses, no single measure can be said to be overwhelmingly better than the rest at identifying partisan gerrymandering due to the nature of redistricting and the many other considerations and requirements that are included in the redistricting process. However, a number of recommendations and guidelines can be formulated based on the circumstances, the available resources and the specific purpose.

The first distinction which can be made is between cross-sectional and longitudinal methods. The efficiency gap and uniform partisan swing seats-votes curves are both cross-sectional approaches, while the Gelman and King model is able to capture longitudinal trends. Of course, the efficiency gap and seats-votes curves can be calculated for multiple subsequent elections as was done here, but they cannot be used for prediction nor do they model historical trends, but serve as exploratory or descriptive measures. Whether to choose a cross-sectional or longitudinal method can depend on the data available, whether the interest is to study a single point in time or longitudinal trends, as well as based on possible imposed guidelines. If it is established, for example, that partisan gerrymandering may only be evaluated for elections which have already occurred and for which results are available, a predictive model will not pass the criteria. Another aspect to consider would be that though districts stay the same for 10-year periods in between redistricting processes, every election cycle is a reflection of a new voter population, and thus measuring the same district over multiple cycles could, in certain cases, not be valid.

A second aspect to consider is which variables are to be used in determining evidence of partisan gerrymandering - only election outcomes, or additional variables which are believed to impact the results. There can be debate over which are relevant and whether redistricting can be influenced by anything other than the actual outcomes - e.g. proposing different redistricting strategies based on whether an incumbent is running or how much money is usually spent in the state on political ads. While these are the types of variables which can be included in Gelman and King's model and potentially further improve its predictive ability, they might not be accepted in justifying a particular redistricting plan.

Thirdly, the measures can be used to both evaluate scenarios which have already occurred as well as attempt to predict outcomes or associated quantities of interest. Prediction is important in evaluating a proposed redistricting plan and its potential consequences on electoral fairness. On the other hand, those voting results have yet to happen, and though it might be possible to predict them to a certain degree, such reasoning could be rejected on the grounds that people's speech expressions are being assumed and voters are being treated differently based on those assumptions.

One clear shortcoming of these methods is that they are intended for two-party systems and evaluate results purely in those terms - if the Democrats win, it must be at the expense of the Republicans. While those two are undoubtedly the two main parties, districts and states exist where third-party candidates capture a significant share of the vote (Berman & McGill, 2016). That would make it difficult to make direct comparisons of some states with others. Furthermore, these measures ignore independent voters which are less strongly set on a certain party than long-time supporters and are more likely to switch between parties. Given that the measures examined categorize the electorate into the two parties, they fail to capture or model the more flexible population.

Lastly, this analysis came across additional points for nuance. For example, the effect of the number of districts in a state on the efficiency gap measure and its ability to measure packing and cracking versus competitiveness. Another is the use of the uniform partisan swing hypothesis which is a more appropriate assumption under certain circumstances (Gelman & King, 1994) than others and could thus not always be applied to a whole set of elections, but rather be more appropriate for just some individual ones.

5 Conclusion

The goal of this paper was to evaluate a set of partisan gerrymandering measures - the efficiency gap, seats-votes curves using uniform partisan swing and the associated measures of partisan bias and mean-median difference, and use the Gelman and King model to compute measures of partisan bias and responsiveness using predictors and at multiple voter-share points. Various aspects of these measures of examined and compared, including the variables they require, the cases which they can evaluate, whether certain state characteristics impact the measure results, and which measures could be used under certain regulations or court guidelines.

More research is needed in applying these as well as other potential measures in order to formulate workable thresholds which could indicate partisan gerrymandering in the United States. Additional research is needed into whether any of these measures interact with the current three main redistricting criteria of population equality, ethnic discrimination, and geographical compactness, as these will need to be met first. For that reason, it is important to understand how these measures will behave once those conditions have been satisfied. Secondly, this paper used data only up until 1992 and it is possible that it has not captured recent developments, which could be technology-driven, or due to population or political shifts.

At this day, partisan gerrymandering has yet to be recognized as an official criteria in redistricting, but legal developments suggest that the courts will with time have to rule in greater detail on this issue. It has been recognized as being potentially unconstitutional and thus it is likely a matter of time and further research into this issue until the courts feel confident enough to accept certain statistical measures (*Vieth v. Jubelirer*, 2004). In that case, and for the defendants arguing any such cases, what is ultimately most important is to understand how these measures work, what affects them, and the precise boundaries of what they suggest.

6 References

Ansolabehere, S. & Palmer, M. (2015). A two hundred-year statistical history of the Gerrymander. Congress & History Conference, Vanderbilt University. Retrieved from https://www.brennancenter.org/sites/default/files/legal-work/Ansolabehere_Palmer_200_Year_History_Gerrymandering_2015.pdf

Baker v. Carr, 369 U.S. 186 (1962).

Benisek v. Lamone, U.S., No. 17-333, argued Mar. 28th, 2018, pending.

Berman, R. & McGill, A. (2016). The states where third-party candidates perform best, The Atlantic. Retrieved from https://www.theatlantic.com/politics/archive/2016/08/third-party-candidates-2016-clinton-trump-johnson/493931/

Brief for Professors Gary King et. al. as Amici Curiae in support of neither party, Jackson v. Perry, 548 U.S. 399 (2006).

Castaneda v. Partida, 430 U.S. 482 (1977).

Cho, W. K. T., & Liu, Y. Y. (2016). Towards a talismanic redistricting tool: A Computational method for identifying extreme redistricting plan, Election Law Journal, 15(4), 351-366.

Common Cause v. Rucho.

Crocker, R. (2012). Congressional redistricting: An overview. CRS Report for Congress, Congressional Research Service.
Retrieved from https://fas.org/sgp/crs/misc/R42831.pdf

Davis v. Bandemer, 478 U.S. 109 (1986).

Duchin, M. (2017). Redistricting and Representation Conference, American Academy of Arts & Sciences, 2062nd Stated Meeting, Nov. 8th, 2017.
Retrieved from https://www.amacad.org/binaries/video/streamPlayer.aspx?i=20901

Gelman, A. & King, G. (1990a). Estimating incumbency advantage without bias. American Journal of Political Science, 34(4), 1142-1164. Retrieved from http://www.stat.columbia.edu/~gelman/research/published/ajps1990.pdf

Gelman, A. & King, G. (1990b). Estimating the electoral consequences of legislative redistricting. Journal of the American Statistical Association, 85(410), 274-282. Retrieved from https://gking.harvard.edu/files/svstat.pdf

Gelman, A. & King, G. (1994). A unified method of evaluating electoral systems and redistricting plans. American Journal of Political Science, 38(2), 514-554. Retrieved from https://gking.harvard.edu/files/writeit.pdf

Gill v. Whitford , U.S., No. 16-1161 , argued Oct. 3rd, 2017, pending.

Grofman, B. (2015). Elections: Apportionment. International Encyclopedia of the Social & Behavioral Sciences (2nd ed.), 7, 342-344.

Hodge, J. K., Marshall, E., & Patterson, G. (2010). Gerrymandering and convexity. The College Mathematics Journal, 41(4), 312-324.
Retrieved from http://www.jstor.org/stable/10.4169/074683410x510317

Jackson v. Perry, 548 U.S. 399 (2006).

King, G., Grofman, B., Gelman, A., Katz, J. N. (2006). Brief for Professors Gary King et. al. as Amici Curiae in support of neither party, Jackson v. Perry, 548 U.S. 399 (2006)

Lewis, J. B., DeVine, B., Pritcher, L., & Martis, K. C. (2013). Digital Boundary Definitions of United States Congressional Districts, 1789-2012. [Data file and code book].Retrieved from http://cdmaps.polisci.ucla.edu/

Maceachren, A. M. (1985). Compactness of geographic shape: Comparison and evaluation of methods, Geografiska Annaler Series B, Human Geography 67, 53-67.

Miller v. Johnson, 515 U.S. 900 (1995).

Party divisions of the House of Representatives. United States House of Representatives, Office of the Historian. Retrieved from http://history.house.gov/Institution/Party-Divisions/Party-Divisions/

Perry v. Perez, 565 U.S. 388 (2012).

Reynolds v. Sims, 377 U.S. 533 (1964).

Schleicher, D. (2007). Why is there no partisan competition in city council elections?: The Role of election law, Yale Law School Faculty Scholarship Series, paper 4950. Retrieved from
https://pdfs.semanticscholar.org/1dd7/06ddab992003bba54725cdc4efbf0df1495a.pdf

Schuck, P. (2016). Still in the thickest thicket. Election Law Blog, De. 6th, 2016. Retrieved from https://electionlawblog.org/?p=89747

Schwinn, S. D. (2017). Election Law: Gill v. Whitford, Preview of United States Supreme Court Cases, 45(1), 26-29.

Shaw v. Reno, 509 U.S. 630 (1993).

Shelby County v. Holder, 570 U.S. 2 (2013).

Stephanopoulos, N. & McGhee, E. (2014). Partisan gerrymandering and the efficiency gap, Public Law and Legal Theory Working Papers, 493. Retrieved from https://chicagounbound.uchicago.edu/cgi/viewcontent.cgi?article=1946&context=public_law_and_legal_theory

United States: State maps, flags, and seals. Encyclopaedia Britannica. Retrieved from https://www.britannica.com/place/United-States/Presidents-of-the-United-States#ref276430

U.S. Const. art. I, 2.

U.S. Const. amend. XIV.

Vieth v. Jubelirer, 541 U.S. 267 (2004).

Voting Rights Act of 1965, Pub. L. 89-110, 79 Stat. 437.

Wesberry v. Sanders, 376 U.S. 1 (1964).

Figure 1 (public domain)
https://www.census.gov/library/visualizations/2010/dec/2010-map.html

Figure 2 (public domain)
https://nationalmap.gov/small_scale/printable/images/pdf/congdist/pagecgd113_tx.pdf

Figures 3-38 were created by the author

Appendix A: Date of accession to the Union

Source: Encyclopaedia Britannica

Table 4: Date of accession of each U.S. state (states which joined after 1896 marked in bold)

State	Date	State	Date
Delaware	De. 7th, 1787	Michigan	Jan. 26th, 1837
Pennsylvania	De. 12th, 1787	Florida	Mar. 3rd, 1845
New Jersey	De. 18th, 1787	Texas	Dec. 29th, 1845
Georgia	Jan. 2nd, 1788	Iowa	Dec. 28th, 1846
Connecticut	Jan. 9th, 1788	Wisconsin	May 29th, 1848
Massachusetts	Fe. 6th, 1788	California	Sep. 9th, 1850
Maryland	Apr. 28th, 1788	Minnesota	May 11th, 1858
South Carolina	May 23rd, 1788	Oregon	Feb. 14th 1859
New Hampshire	Jun. 21st, 1788	Kansas	Jan. 29th, 1861
Virginia	Jun. 25th, 1788	West Virginia	Jun. 20th, 1863
New York	Jul. 26th, 1788	Nevada	Oct. 31st, 1864
North Carolina	Nov. 21, 1788	Nebraska	Mar. 1st, 1867
Rhode Island	May 29th, 1790	Colorado	Aug. 1st, 1876
Vermont	Mar. 4th, 1791	North Dakota	Nov. 2nd, 1889
Kentucky	Jun. 1st, 1792	South Dakota	Nov. 2nd, 1889
Tennessee	Jun. 1st, 1796	MOntana	Nov. 8th, 1889
Ohio	Mar. 1st, 1803	Washington	Nov. 11th, 1889
Louisiana	Apr. 30th, 1812	Idaho	Jul. 3rd, 1890
Indiana	Dec. 11th, 1816	Wyoming	Jul. 10th, 1890
Mississippi	De. 10th, 1817	**Utah**	Jan. 4th, 1896
Illinois	Dec. 3rd, 1818	**Oklahoma**	Nov. 16th, 1907
Alabama	Dec. 14th, 1819	**New Mexico**	Jan. 6th, 1912
Maine	Mar. 15th, 1820	**Arizona**	Feb. 14th, 1912
Missouri	Aug. 10th, 1821	**Alaska**	Jan. 3rd, 1959
Arkansas	Jun. 15th, 1836	**Hawaii**	Aug. 21st, 1959

Appendix B: Number of Congressional districts since 1896

The districts are single-member, therefore the terms 'districts', 'members', and 'seats' are used interchangeably when referring to their numbers. Source: U.S. House of Representatives

Table 5: Changes in the number of Congressional districts since 1896

Year of change	Number of seats
1893	357
1901	386
1911	391
1913	435
1959	436
1961	437
1963	435

Appendix C: Efficiency gap state plots

The plots show the efficiency gap values across election cycles for each state, with an overall historical average for each state plotted in a dotted line.

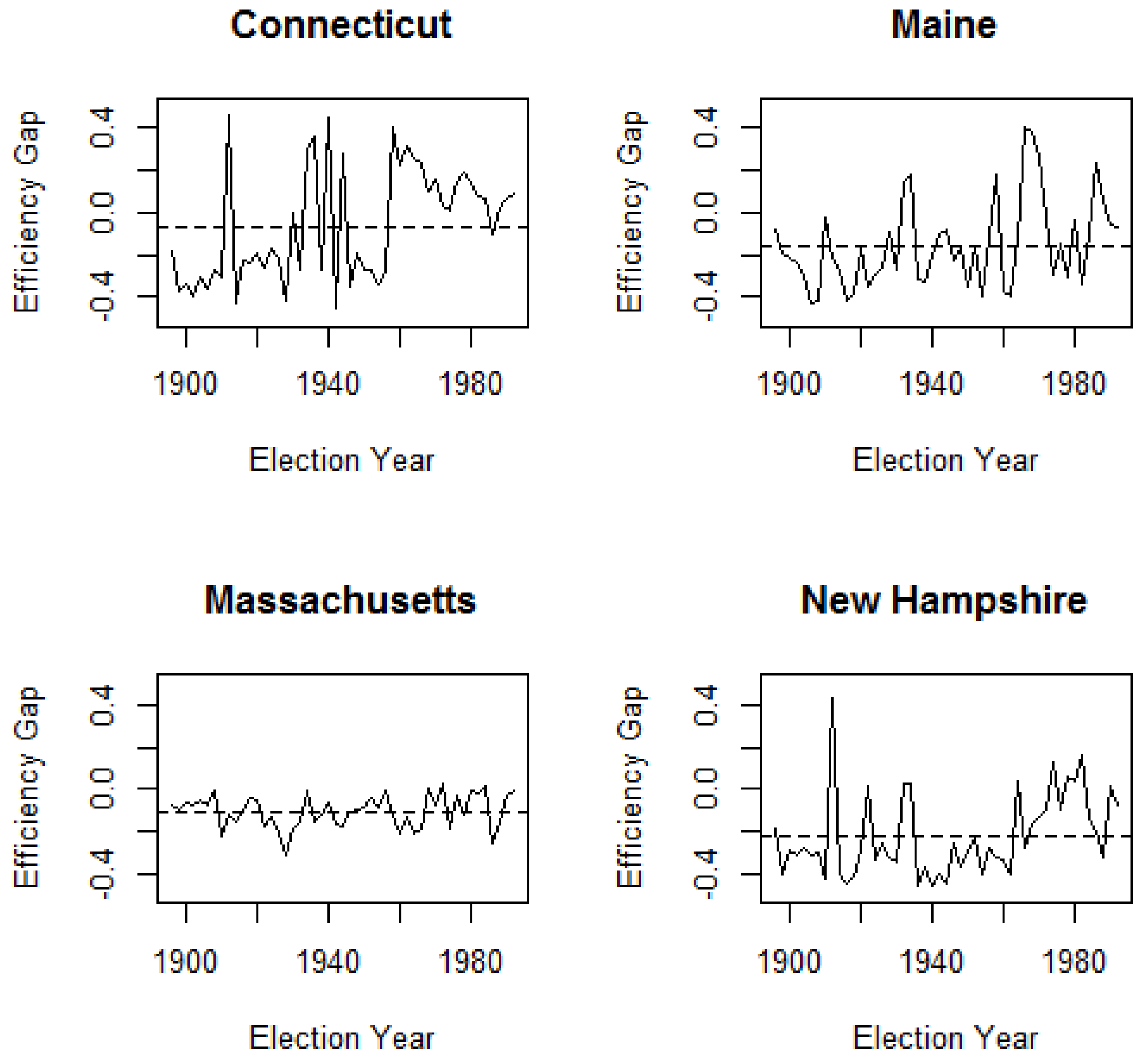

Figure 23: New England

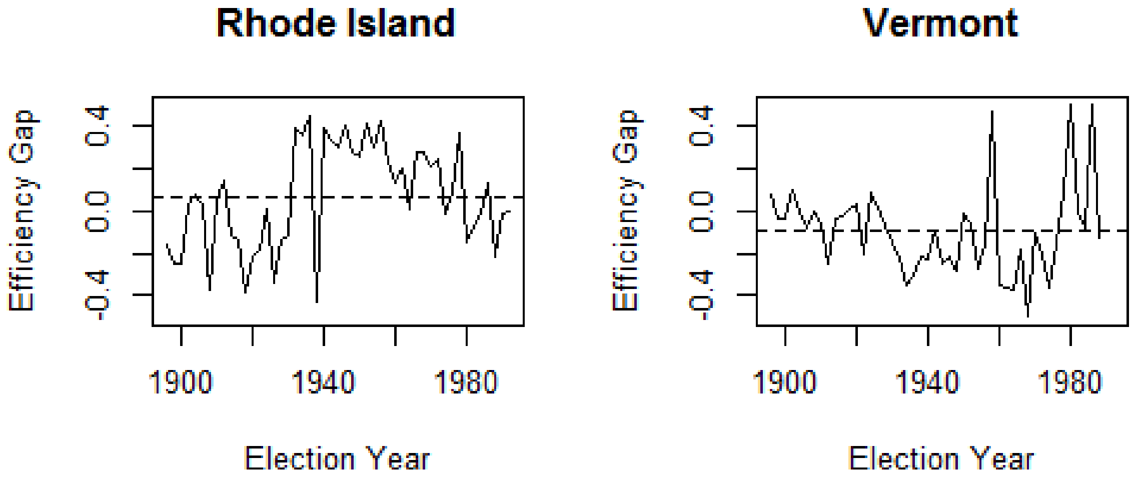

Figure 24: New England

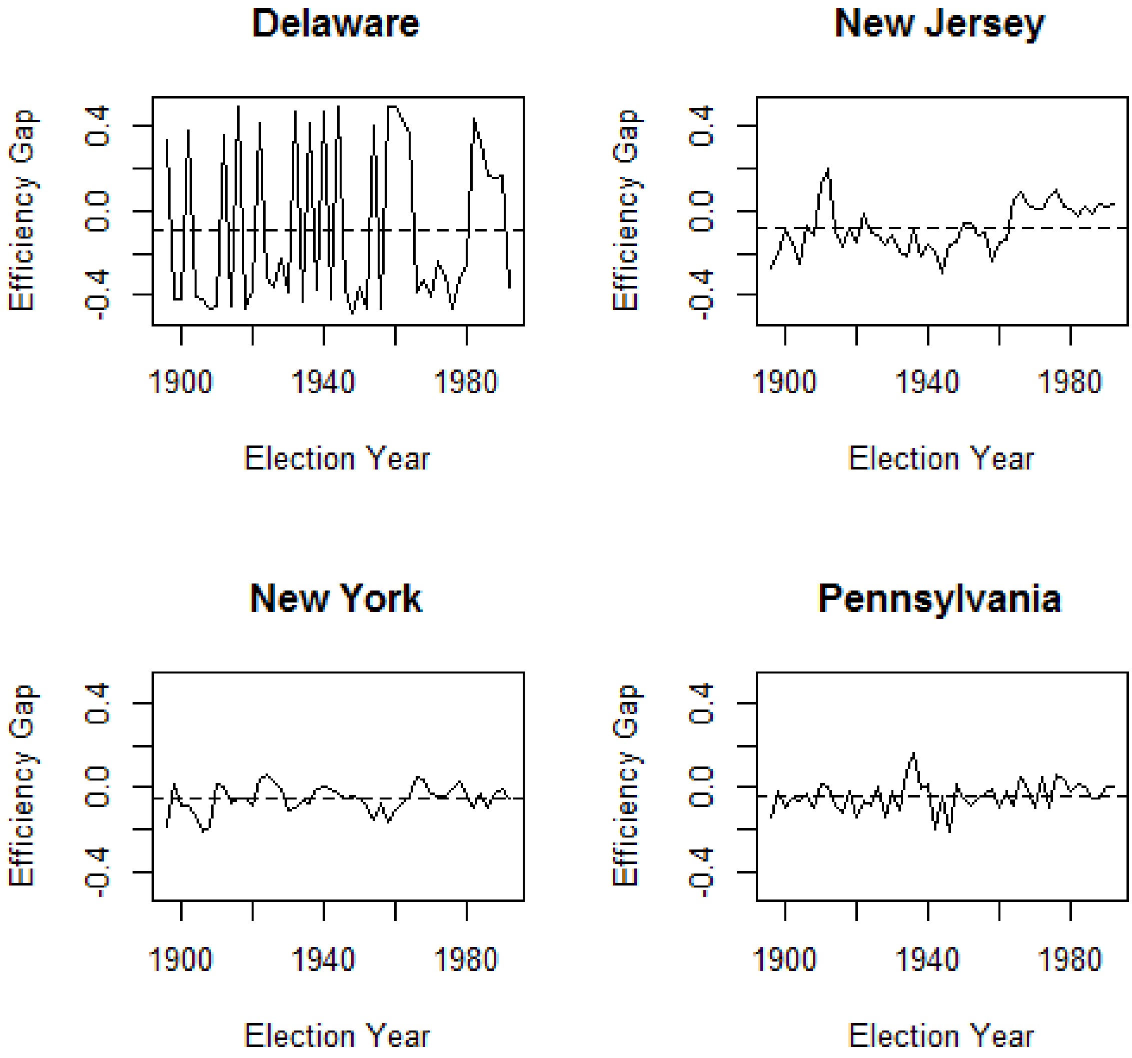

Figure 25: Middle Atlantic

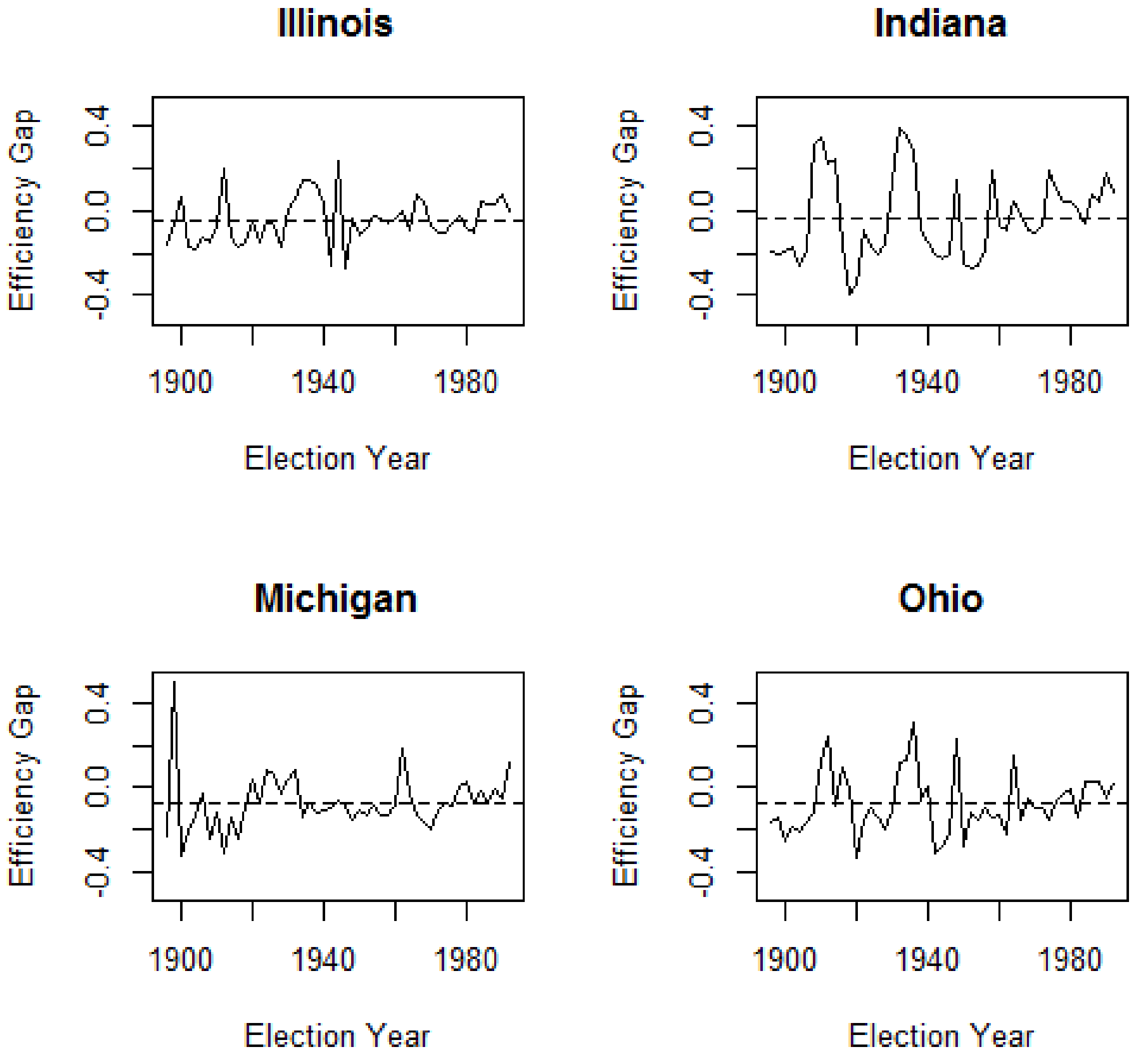

Figure 26: East North Central

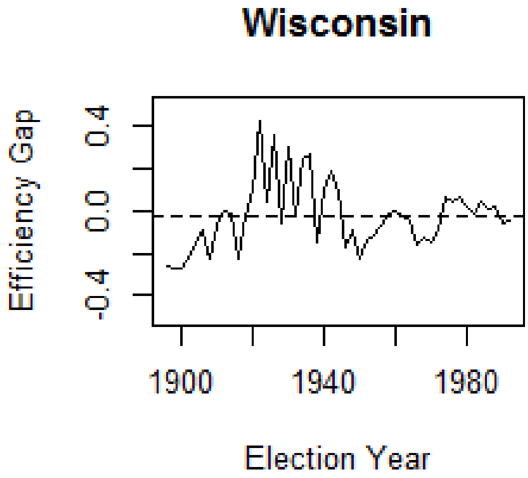

Figure 27: East North Central

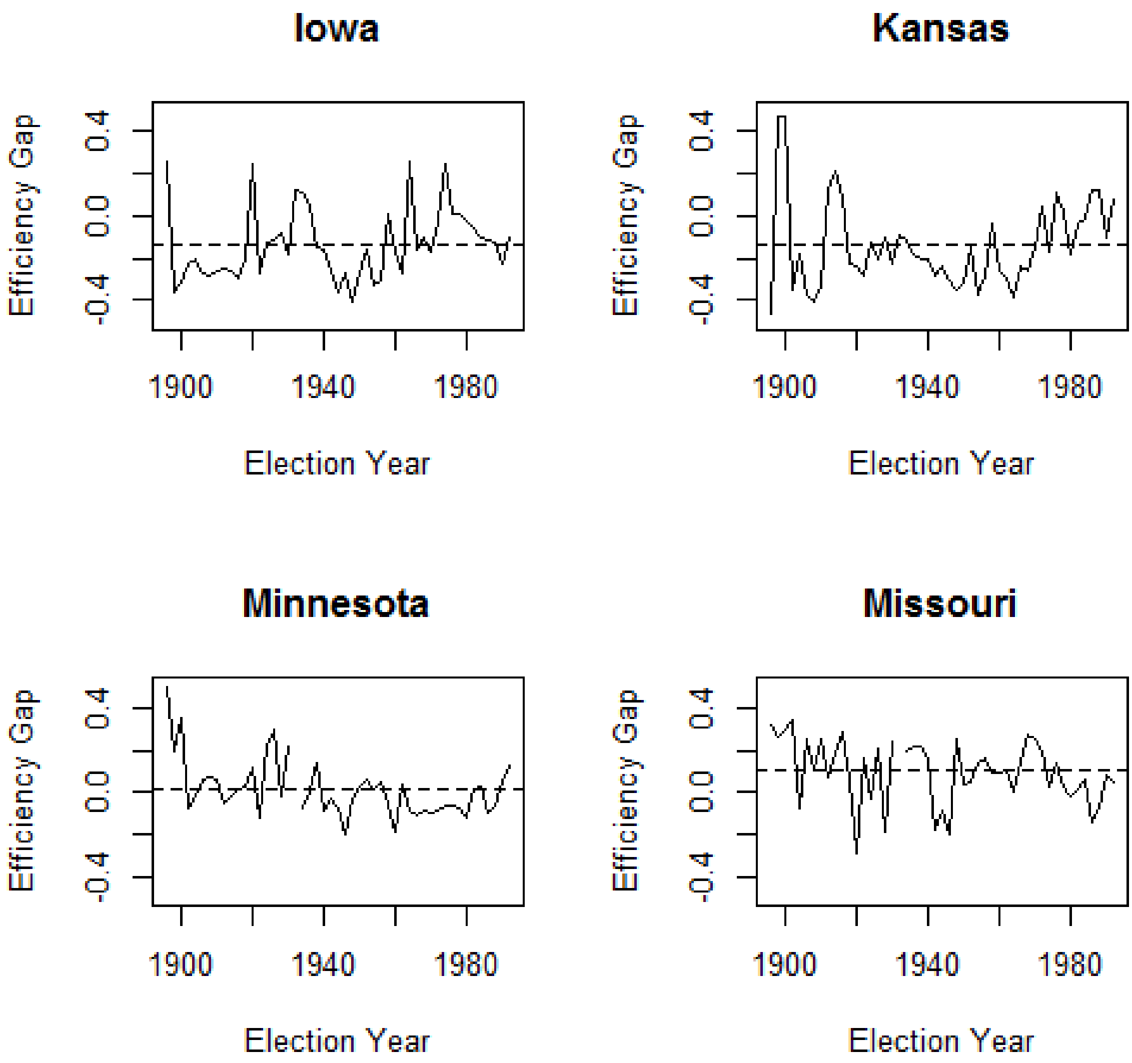

Figure 28: West North Central

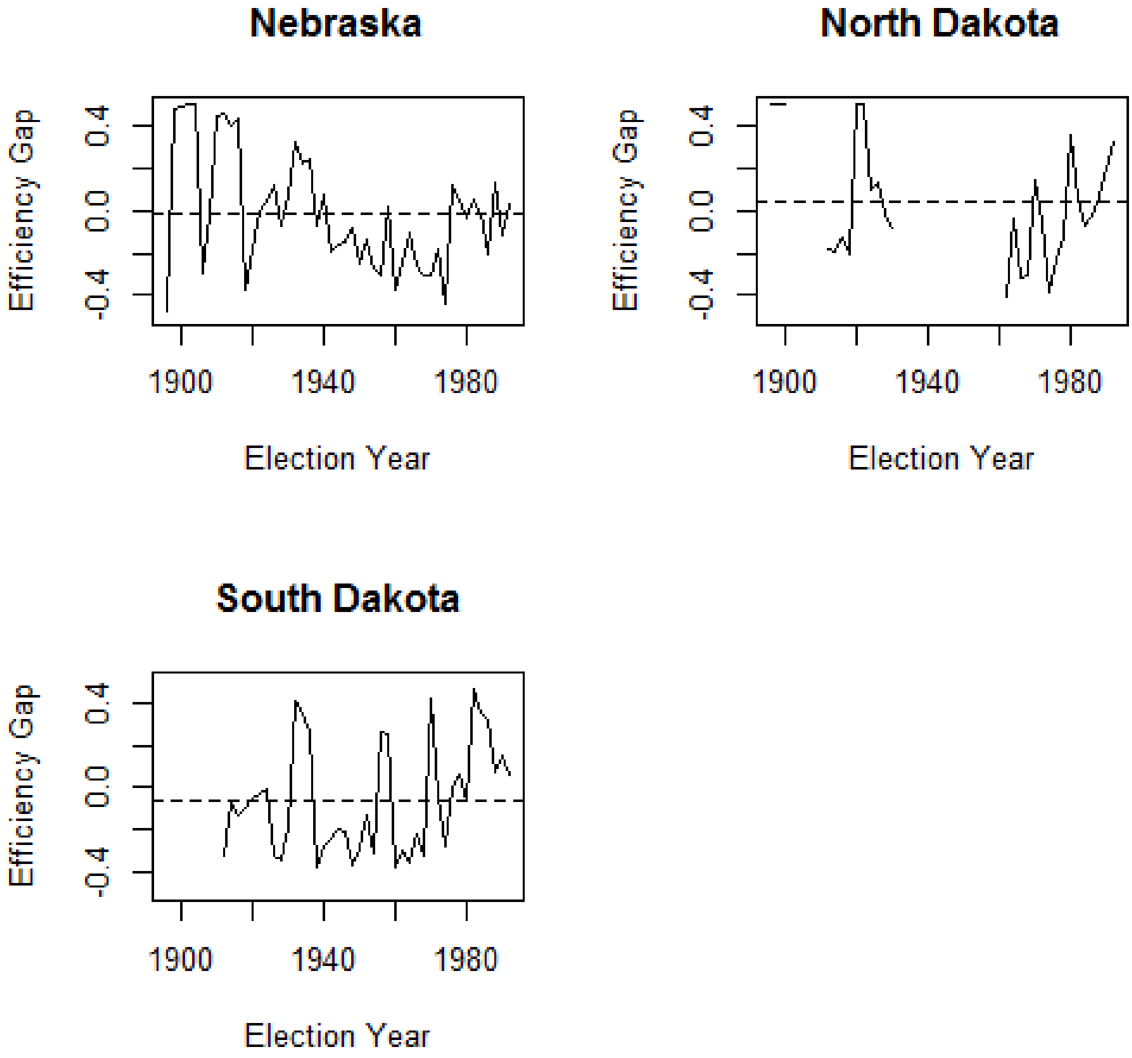

Figure 29: West North Central

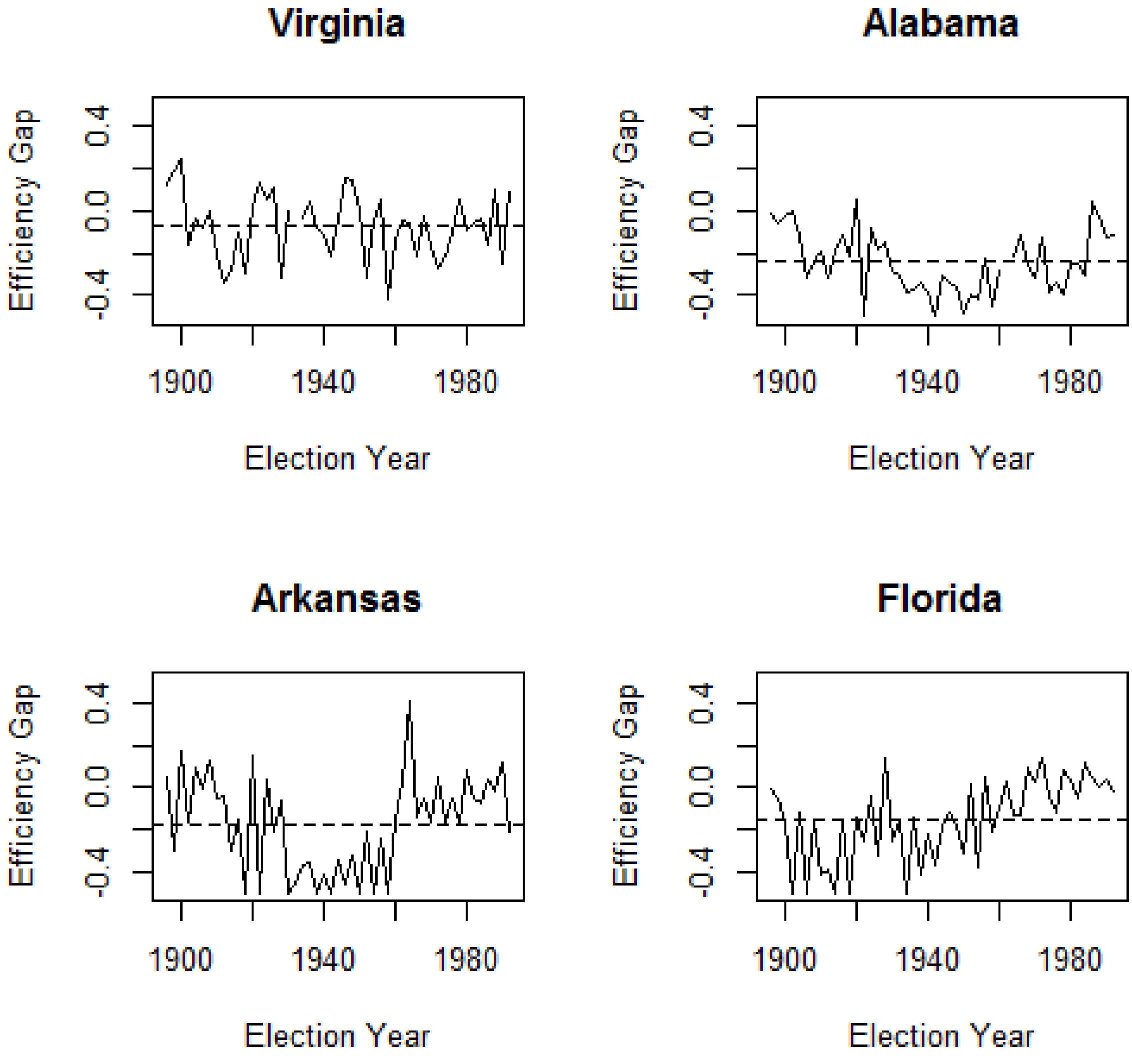

Figure 30: Solid South

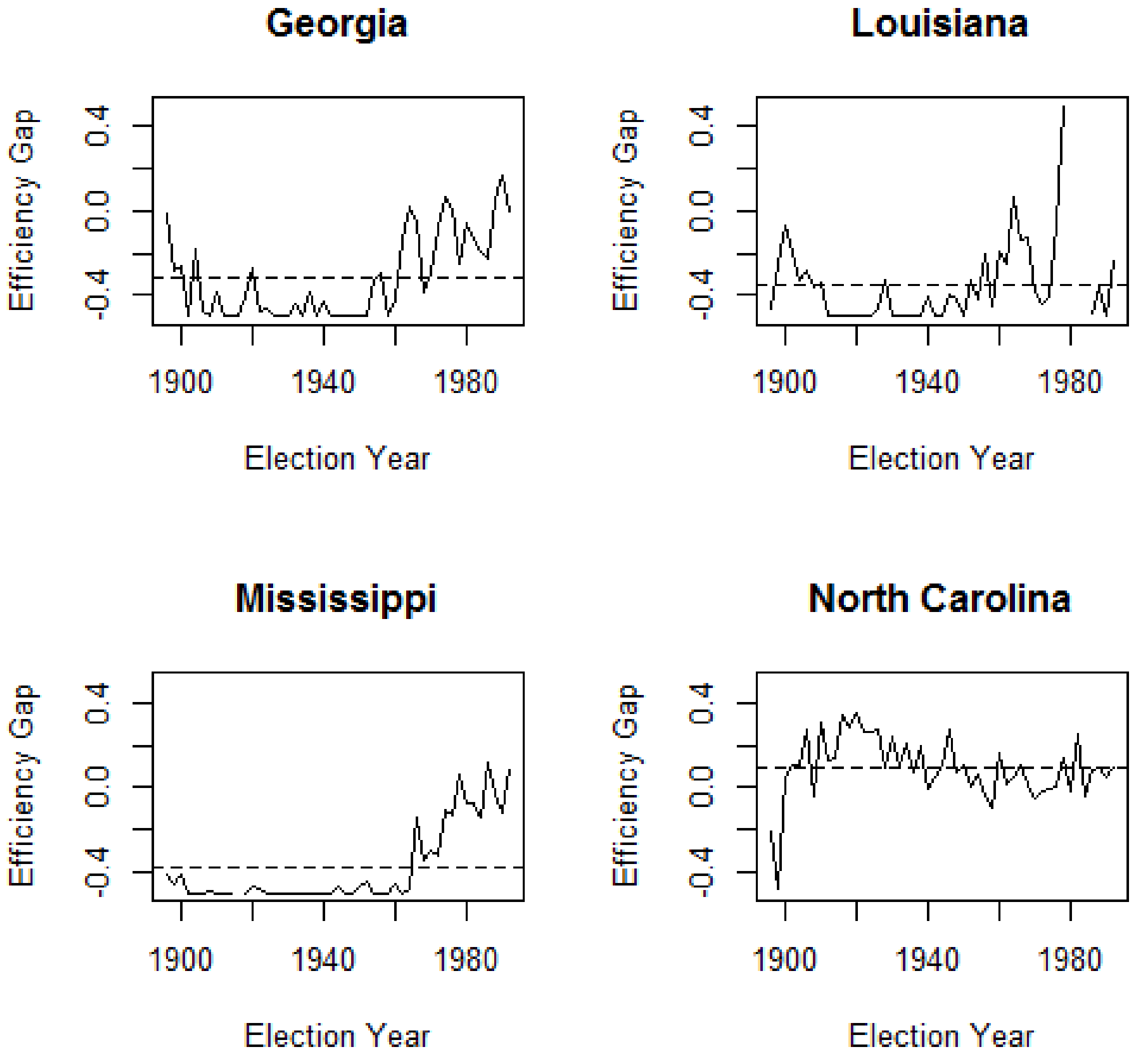

Figure 31: Solid South

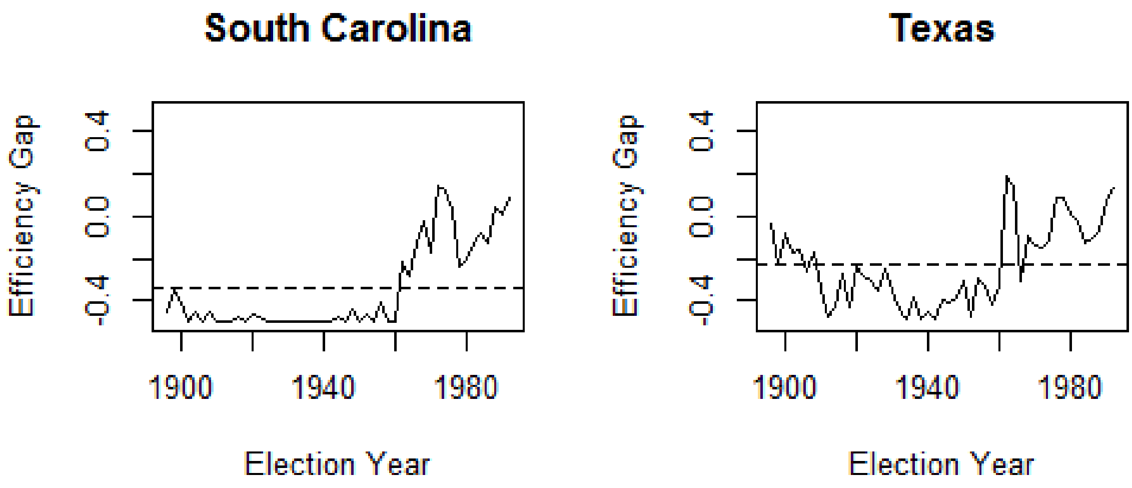

Figure 32: Solid South

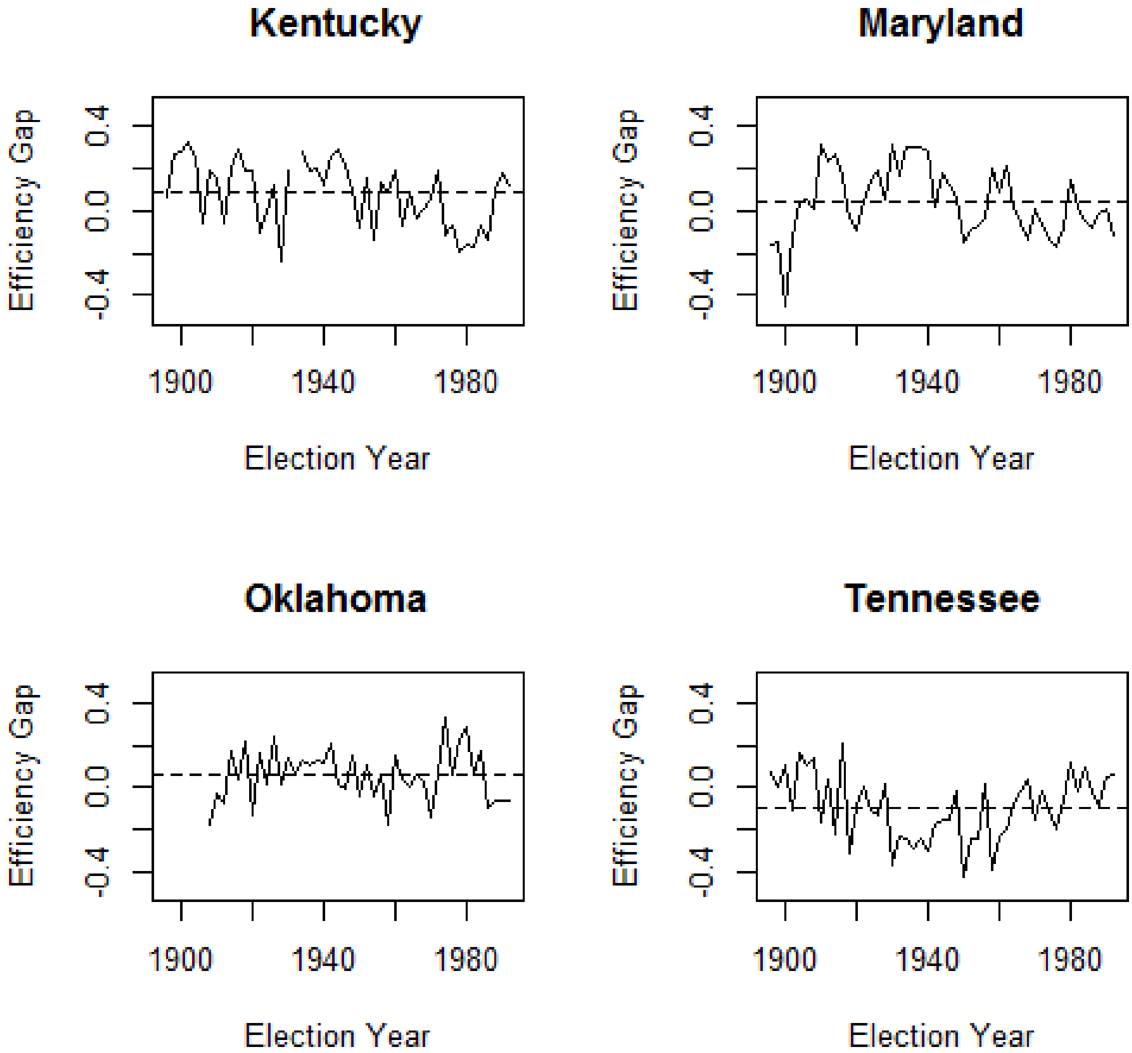

Figure 33: Border States

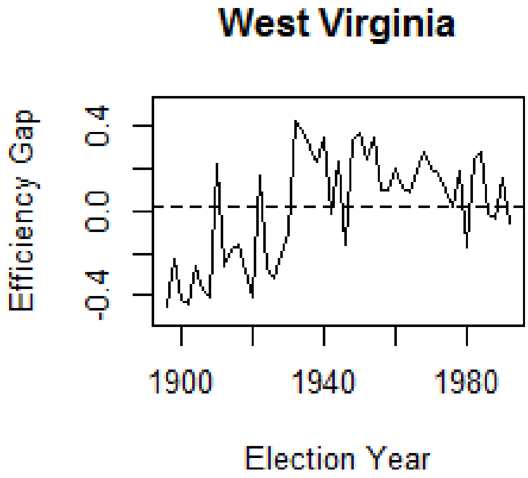

Figure 34: Border States

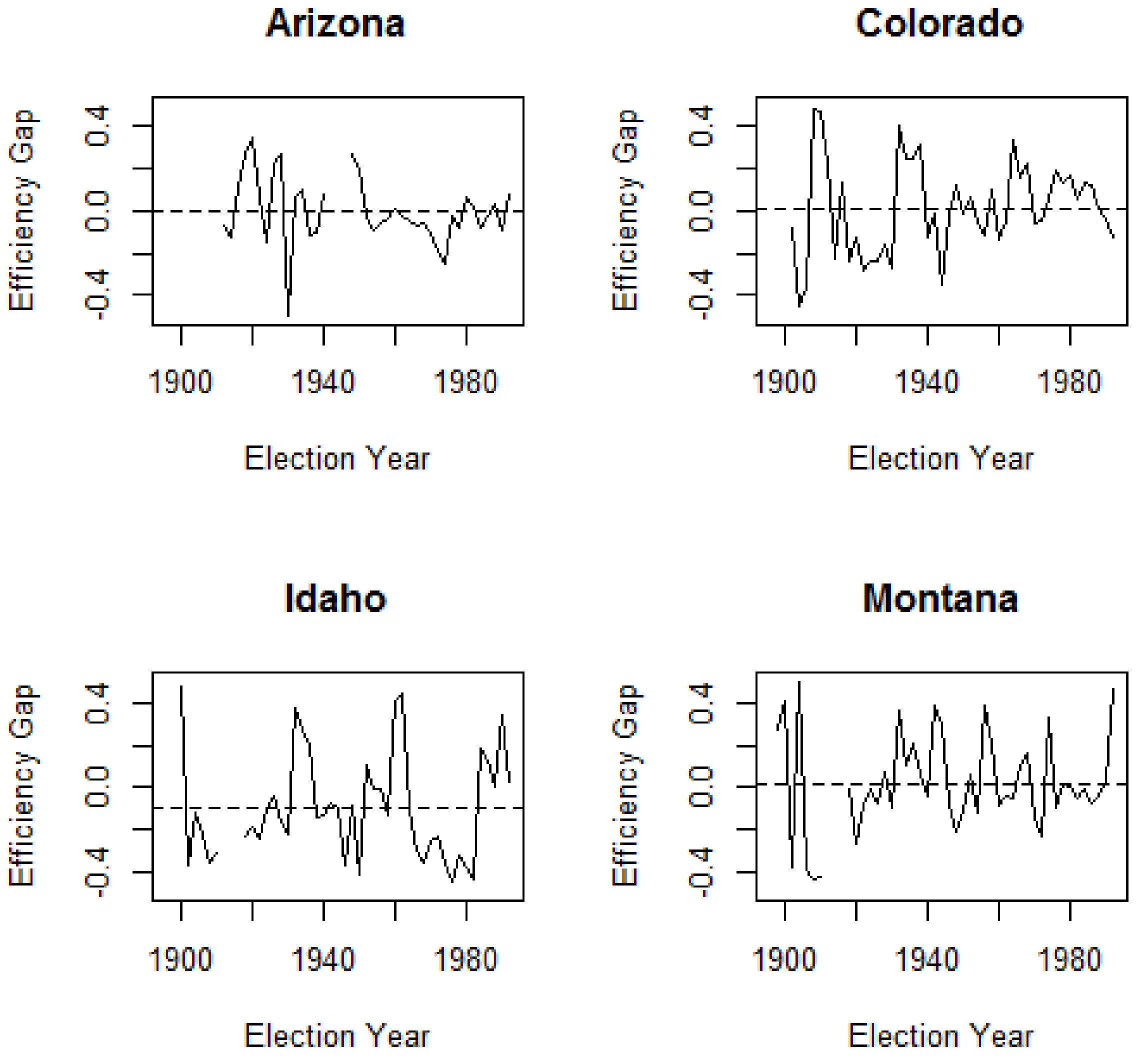

Figure 35: Mountain States

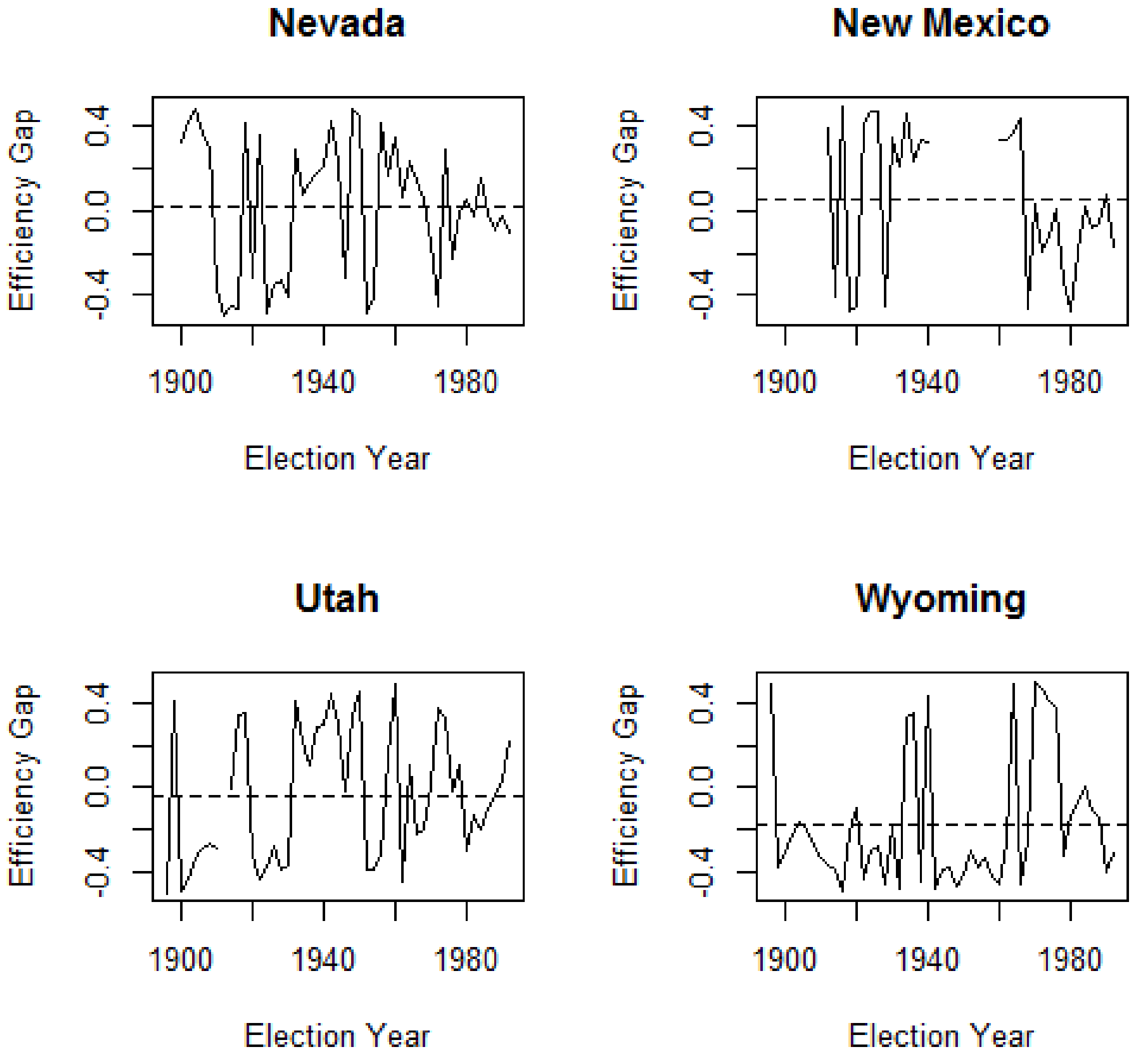

Figure 36: Mountain States

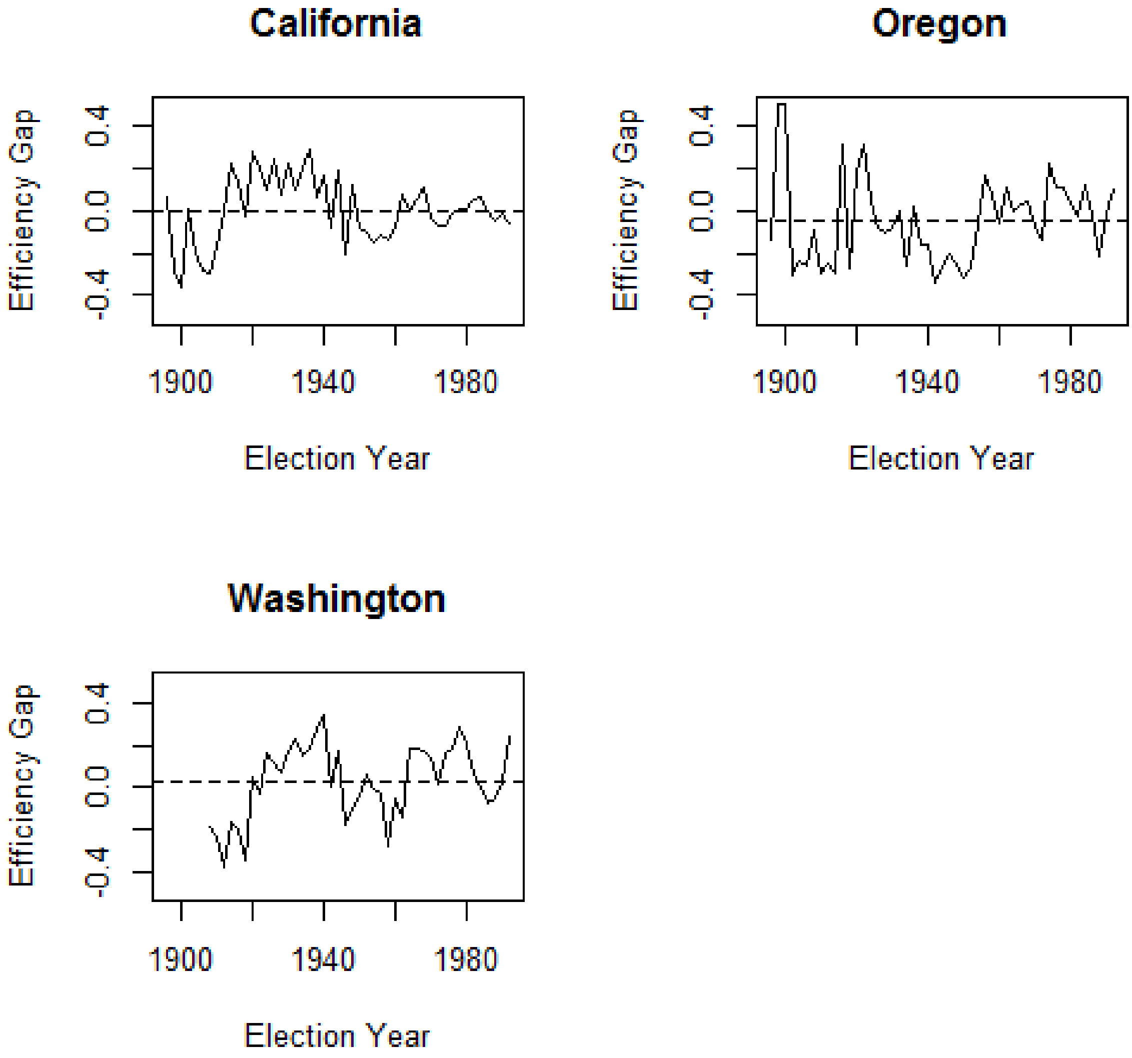

Figure 37: Pacific States

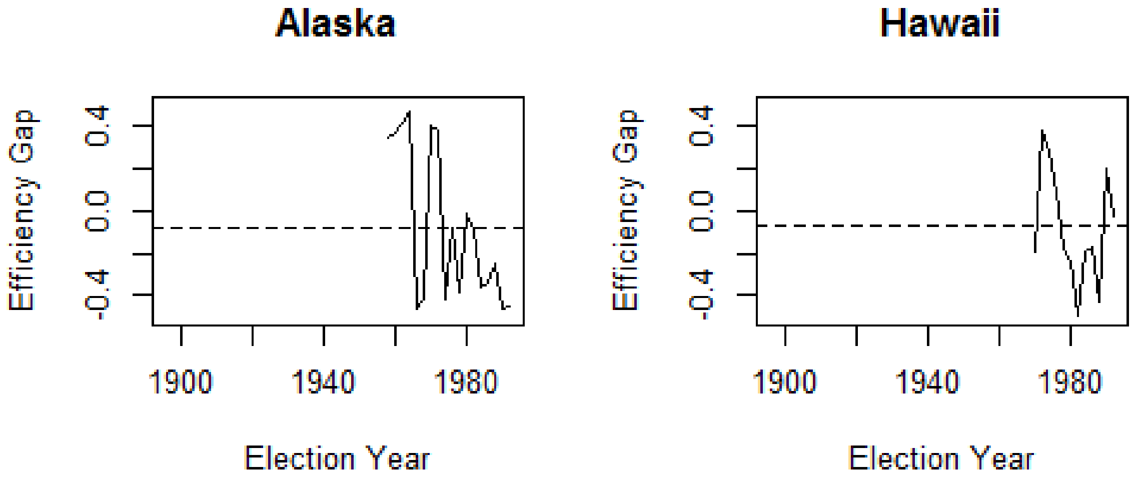

Figure 38: External States

Appendix D: Regression coefficients for the Gelman & King model

unc(VOTE) refers to the effect of the district being uncontested (neither party wins > 95%, INC refers to the effect of an incumbent running, and lastvote stands for the previous cycle's outcome in the given district.

Table 6: Regression coefficients of the Gelman & King model on the whole United States

1896		1898		1900		1902	
(Intercept)	0.4782847	(Intercept)	0.4782847	(Intercept)	0.13678744	(Intercept)	0.5269928
unc(VOTE)	0.2522464	unc(VOTE)	0.1276482	unc(VOTE)	0.07624165	unc(VOTE)	0.1678146
		INC	0.0216392	INC	0.01509419	INC	0.1130562
		lastvote	0.6415978	lastvote	0.70357583		
1904		1906		1908		1910	
(Intercept)	0.03715875	(Intercept)	0.148152660	(Intercept)	0.110317890	(Intercept)	0.11870012
unc(VOTE)	0.03112685	unc(VOTE)	0.005407013	unc(VOTE)	0.043442720	unc(VOTE)	0.02055946
INC	0.02811954	INC	0.024715849	INC	0.008462296	INC	0.01071377
lastvote	0.84818251	lastvote	0.777309870	lastvote	0.775171466	lastvote	0.82428607
1912		1914		1916		1918	
(Intercept)	0.54310203	(Intercept)	0.11933807	(Intercept)	0.059119087	(Intercept)	0.19698808
unc(VOTE)	0.13560852	unc(VOTE)	0.09696454	unc(VOTE)	0.057373871	unc(VOTE)	0.08391486
INC	0.09732019	INC	0.02858110	INC	0.004815989	INC	0.04687006
		lastvote	0.67439089	lastvote	0.853970593	lastvote	0.57400751
1920		1922		1924		1926	
(Intercept)	0.02224955	(Intercept)	0.5609308	(Intercept)	0.02141152	(Intercept)	0.079453217
unc(VOTE)	0.08091319	unc(VOTE)	0.1095178	unc(VOTE)	0.01772376	unc(VOTE)	-0.009139105
INC	0.01218511	INC	0.1359776	INC	0.03539703	INC	0.027565257
lastvote	0.82853439			lastvote	0.87613198	lastvote	0.872914858
1928		1930		1932		1934	
(Intercept)	0.15725884	(Intercept)	0.17972529	(Intercept)	0.58009780	(Intercept)	0.18008530
unc(VOTE)	0.05934671	unc(VOTE)	0.02369762	unc(VOTE)	0.09012243	unc(VOTE)	0.03241573
INC	0.02295441	INC	0.04702918	INC	0.10232622	INC	0.01714314
lastvote	0.65773279	lastvote	0.71290559			lastvote	0.67536542
1936		1938		1940		1942	
(Intercept)	0.025208032	(Intercept)	6.002441e-05	(Intercept)	0.10649186	(Intercept)	0.4803749
unc(VOTE)	-0.022566425	unc(VOTE)	4.337787e-02	unc(VOTE)	0.02857763	unc(VOTE)	0.1848587
INC	0.001256031	INC	2.118873e-02	INC	0.01122389	INC	0.1005462
lastvote	0.995219983	lastvote	8.823676e-01	lastvote	0.80974279		
1944		1946		1948		1950	
(Intercept)	0.25293283	(Intercept)	-3.595622e-03	(Intercept)	0.17703147	(Intercept)	0.09873106
unc(VOTE)	0.03645038	unc(VOTE)	4.720147e-02	unc(VOTE)	-0.02464576	unc(VOTE)	0.04398922
INC	0.05251771	INC	2.355072e-05	INC	0.02129925	INC	0.02764102
lastvote	0.55381068	lastvote	9.185649e-01	lastvote	0.78353140	lastvote	0.76453830
1952		1954		1956		1958	
] (Intercept)	0.4927863	(Intercept)	0.19837634	(Intercept)	0.11487556	(Intercept)	0.25798284
unc(VOTE)	0.1532994	unc(VOTE)	0.01022413	unc(VOTE)	0.06596127	unc(VOTE)	-0.01379285
INC	0.1155031	INC	0.03858090	INC	0.02319793	INC	0.05691973
		lastvote	0.68135244	lastvote	0.73662661	lastvote	0.61608748
1960		1962		1964		1966	
] (Intercept)	0.11273498	(Intercept)	0.5123568	(Intercept)	0.25626875	(Intercept)	0.1925772
unc(VOTE)	0.05640123	unc(VOTE)	0.1193099	unc(VOTE)	0.01599731	unc(VOTE)	0.1145985
INC	0.02670754	INC	0.1212526	INC	0.04605929	INC	0.0808164
lastvote	0.74231687			lastvote	0.59184508	lastvote	0.5080180
1968		1970		1972		1974	
] (Intercept)	0.21970676	(Intercept)	0.26320681	(Intercept)	0.50497444	(Intercept)	0.41805766
unc(VOTE)	0.06638095	unc(VOTE)	0.02090574	unc(VOTE)	0.09287336	unc(VOTE)	0.02141884
INC	0.07012257	INC	0.07948832	INC	0.15697382	INC	0.09984032
lastvote	0.55228415	lastvote	0.53564252			lastvote	0.30316511
1976		1978		1980		1982	
] (Intercept)	0.18967237	(Intercept)	0.23218855	(Intercept)	0.15925440	(Intercept)	0.53884323
unc(VOTE)	0.05340083	unc(VOTE)	0.06959552	unc(VOTE)	0.05841372	unc(VOTE)	0.08719011
INC	0.06970857	INC	0.08460569	INC	0.06601438	INC	0.14744627
lastvote	0.61031490	lastvote	0.51292728	lastvote	0.62639197		
1984		1986		1988		1990	
] (Intercept)	0.19754623	(Intercept)	0.21387579	(Intercept)	0.20162688	(Intercept)	0.29314038
unc(VOTE)	0.06569785	unc(VOTE)	0.02924835	unc(VOTE)	0.04650462	unc(VOTE)	0.08442702
INC	0.08705319	INC	0.07588650	INC	0.07741414	INC	0.06384484
lastvote	0.54078609	lastvote	0.61355449	lastvote	0.58582798	lastvote	0.43671055
1992							
] (Intercept)	0.5138432						
unc(VOTE)	0.1478337						
INC	0.1306844						

R Code

```
library(JudgeIt)
data(house6311)
head(house6311)
attach(house6311)
names(house6311)
names(house6311$`1898`)
house6311[[1]]
head(house6311[[4]])

egfun<- function(x){
  DEMCOUNT<-round(VOTE*TURNOUT)
  REPCOUNT<-TURNOUT-DEMCOUNT
  DEMWIN<- ifelse(VOTE>0.5,1,0)
  REPWIN<-ifelse(VOTE<0.5,1,0)
  DEMWASTE<-ifelse(DEMWIN==0, DEMCOUNT,
  DEMCOUNT-TURNOUT*0.5)
  REPWASTE<-ifelse(REPWIN==0, REPCOUNT,
  REPCOUNT-TURNOUT*0.5)
  TDEMWASTE<-xtabs(formula=DEMWASTE~STATE,
   data=x)
  TREPWASTE<-xtabs(formula=REPWASTE~STATE,
   data=x)
  TTURNOUT<-xtabs(formula=TURNOUT~STATE, data=x)
  EG<-cbind(TDEMWASTE, TREPWASTE)
  EG<-cbind(EG, TTURNOUT)
  DEMEG<-(EG[,2]-EG[,1])/EG[,3]

}
attach(house6311$`1896`)
house1<-egfun(house6311[[1]])
attach(house6311$`1898`)
house2<-egfun(house6311[[2]])
attach(house6311$`1900`)
house3<-egfun(house6311[[3]])
house1<-data.frame(house1)
house2<-data.frame(house2)
house3<-data.frame(house3)
house1$statecode<-rownames(house1)
house2$statecode<-rownames(house2)
house3$statecode<-rownames(house3)
eg1896<-merge(house1,house2,all=TRUE)
eg1896<-merge(eg1896,house3,all=TRUE)
setwd("C:\\Users\\Halyna\\Google Drive\\
KU Leuven\\Thesis\\Thesis R")
save(eg1896, file="eg1896.Rda")
attach(house6311$`1902`)
house4<-egfun(house6311[[4]])
attach(house6311$`1904`)
house5<-egfun(house6311[[5]])
attach(house6311$`1906`)
house6<-egfun(house6311[[6]])
attach(house6311$`1908`)
house7<-egfun(house6311[[7]])
attach(house6311$`1910`)
house8<-egfun(house6311[[8]])
house4<-data.frame(house4)
house5<-data.frame(house5)
house6<-data.frame(house6)
house7<-data.frame(house7)
house8<-data.frame(house8)
house4$statecode<-rownames(house4)
house5$statecode<-rownames(house5)
house6$statecode<-rownames(house6)
house7$statecode<-rownames(house7)
house8$statecode<-rownames(house8)
eg1902<-merge(house4,house5,all=TRUE)
eg1902<-merge(eg1902,house6,all=TRUE)
eg1902<-merge(eg1902,house7,all=TRUE)
eg1902<-merge(eg1902,house8,all=TRUE)
save(eg1902, file="eg1902.Rda")
attach(house6311$`1912`)
house9<-egfun(house6311[[9]])
attach(house6311$`1914`)
house10<-egfun(house6311[[10]])
attach(house6311$`1916`)
house11<-egfun(house6311[[11]])
attach(house6311$`1918`)
house12<-egfun(house6311[[12]])
attach(house6311$`1920`)
house13<-egfun(house6311[[13]])
house9<-data.frame(house9)
house10<-data.frame(house10)
house11<-data.frame(house11)
house12<-data.frame(house12)
house13<-data.frame(house13)
house9$statecode<-rownames(house9)
house10$statecode<-rownames(house10)
house11$statecode<-rownames(house11)
house12$statecode<-rownames(house12)
house13$statecode<-rownames(house13)
eg1912<-merge(house9,house10,all=TRUE)
eg1912<-merge(eg1912,house11,all=TRUE)
eg1912<-merge(eg1912,house12,all=TRUE)
eg1912<-merge(eg1912,house13,all=TRUE)
save(eg1912, file="eg1912.Rda")
attach(house6311$`1922`)
house14<-egfun(house6311[[14]])
attach(house6311$`1924`)
house15<-egfun(house6311[[15]])
attach(house6311$`1926`)
house16<-egfun(house6311[[16]])
attach(house6311$`1928`)
house17<-egfun(house6311[[17]])
attach(house6311$`1930`)
house18<-egfun(house6311[[18]])
house14<-data.frame(house14)
house15<-data.frame(house15)
house16<-data.frame(house16)
house17<-data.frame(house17)
house18<-data.frame(house18)
house14$statecode<-rownames(house14)
house15$statecode<-rownames(house15)
house16$statecode<-rownames(house16)
house17$statecode<-rownames(house17)
house18$statecode<-rownames(house18)
eg1922<-merge(house14,house15,all=TRUE)
eg1922<-merge(eg1922,house16,all=TRUE)
eg1922<-merge(eg1922,house17,all=TRUE)
eg1922<-merge(eg1922,house18,all=TRUE)
save(eg1922, file="eg1922.Rda")
attach(house6311$`1932`)
house19<-egfun(house6311[[19]])
attach(house6311$`1934`)
house20<-egfun(house6311[[20]])
attach(house6311$`1936`)
house21<-egfun(house6311[[21]])
attach(house6311$`1938`)
house22<-egfun(house6311[[22]])
attach(house6311$`1940`)
house23<-egfun(house6311[[23]])
house19<-data.frame(house19)
house20<-data.frame(house20)
house21<-data.frame(house21)
house22<-data.frame(house22)
house23<-data.frame(house23)
house19$statecode<-rownames(house19)
house20$statecode<-rownames(house20)
house21$statecode<-rownames(house21)
house22$statecode<-rownames(house22)
house23$statecode<-rownames(house23)
eg1932<-merge(house19,house20,all=TRUE)
eg1932<-merge(eg1932,house21,all=TRUE)
eg1932<-merge(eg1932,house22,all=TRUE)
eg1932<-merge(eg1932,house23,all=TRUE)
save(eg1932, file="eg1932.Rda")
attach(house6311$`1942`)
house24<-egfun(house6311[[24]])
attach(house6311$`1944`)
house25<-egfun(house6311[[25]])
attach(house6311$`1946`)
house26<-egfun(house6311[[26]])
attach(house6311$`1948`)
house27<-egfun(house6311[[27]])
attach(house6311$`1950`)
house28<-egfun(house6311[[28]])
house24<-data.frame(house24)
house25<-data.frame(house25)
house26<-data.frame(house26)
house27<-data.frame(house27)
house28<-data.frame(house28)
house24$statecode<-rownames(house24)
house25$statecode<-rownames(house25)
house26$statecode<-rownames(house26)
house27$statecode<-rownames(house27)
house28$statecode<-rownames(house28)
eg1942<-merge(house24,house25,all=TRUE)
eg1942<-merge(eg1942,house26,all=TRUE)
eg1942<-merge(eg1942,house27,all=TRUE)
eg1942<-merge(eg1942,house28,all=TRUE)
save(eg1942, file="eg1942.Rda")
attach(house6311$`1952`)
house29<-egfun(house6311[[29]])
attach(house6311$`1954`)
house30<-egfun(house6311[[30]])
attach(house6311$`1956`)
house31<-egfun(house6311[[31]])
attach(house6311$`1958`)
house32<-egfun(house6311[[32]])
attach(house6311$`1960`)
house33<-egfun(house6311[[33]])
house29<-data.frame(house29)
house30<-data.frame(house30)
house31<-data.frame(house31)
house32<-data.frame(house32)
house33<-data.frame(house33)
house29$statecode<-rownames(house29)
house30$statecode<-rownames(house30)
house31$statecode<-rownames(house31)
```

```
house32$statecode<-rownames(house32)
house33$statecode<-rownames(house33)
eg1952<-merge(house29,house30,all=TRUE)
eg1952<-merge(eg1952,house31,all=TRUE)
eg1952<-merge(eg1952,house32,all=TRUE)
eg1952<-merge(eg1952,house33,all=TRUE)
save(eg1952, file="eg1952.Rda")
attach(house6311$`1962`)
house34<-egfun(house6311[[34]])
attach(house6311$`1964`)
house35<-egfun(house6311[[35]])
attach(house6311$`1966`)
house36<-egfun(house6311[[36]])
attach(house6311$`1968`)
house37<-egfun(house6311[[37]])
attach(house6311$`1970`)
house38<-egfun(house6311[[38]])
house34<-data.frame(house34)
house35<-data.frame(house35)
house36<-data.frame(house36)
house37<-data.frame(house37)
house38<-data.frame(house38)
house34$statecode<-rownames(house34)
house35$statecode<-rownames(house35)
house36$statecode<-rownames(house36)
house37$statecode<-rownames(house37)
house38$statecode<-rownames(house38)
eg1962<-merge(house34,house35,all=TRUE)
eg1962<-merge(eg1962,house36,all=TRUE)
eg1962<-merge(eg1962,house37,all=TRUE)
eg1962<-merge(eg1962,house38,all=TRUE)
save(eg1962, file="eg1962.Rda")
attach(house6311$`1972`)
house39<-egfun(house6311[[39]])
attach(house6311$`1974`)
house40<-egfun(house6311[[40]])
attach(house6311$`1976`)
house41<-egfun(house6311[[41]])
attach(house6311$`1978`)
house42<-egfun(house6311[[42]])
attach(house6311$`1980`)
house43<-egfun(house6311[[43]])
house39<-data.frame(house39)
house40<-data.frame(house40)
house41<-data.frame(house41)
house42<-data.frame(house42)
house43<-data.frame(house43)
house39$statecode<-rownames(house39)
house40$statecode<-rownames(house40)
house41$statecode<-rownames(house41)
house42$statecode<-rownames(house42)
house43$statecode<-rownames(house43)
eg1972<-merge(house39,house40,all=TRUE)
eg1972<-merge(eg1972,house41,all=TRUE)
eg1972<-merge(eg1972,house42,all=TRUE)
eg1972<-merge(eg1972,house43,all=TRUE)
save(eg1972, file="eg1972.Rda")
attach(house6311$`1982`)
house44<-egfun(house6311[[44]])
attach(house6311$`1984`)
house45<-egfun(house6311[[45]])
attach(house6311$`1986`)
house46<-egfun(house6311[[46]])
attach(house6311$`1988`)
house47<-egfun(house6311[[47]])
attach(house6311$`1990`)
house48<-egfun(house6311[[48]])
attach(house6311$`1992`)
house49<-egfun(house6311[[49]])
house44<-data.frame(house44)
house45<-data.frame(house45)
house46<-data.frame(house46)
house47<-data.frame(house47)
house48<-data.frame(house48)
house49<-data.frame(house49)
house44$statecode<-rownames(house44)
house45$statecode<-rownames(house45)
house46$statecode<-rownames(house46)
house47$statecode<-rownames(house47)
house48$statecode<-rownames(house48)
house49$statecode<-rownames(house49)
eg1982<-merge(house44,house45,all=TRUE)
eg1982<-merge(eg1982,house46,all=TRUE)
eg1982<-merge(eg1982,house47,all=TRUE)
eg1982<-merge(eg1982,house48,all=TRUE)
eg1982<-merge(eg1982,house49,all=TRUE)
save(eg1982, file="eg1982.Rda")

#load saved eg data
egfull<- merge(eg1896, eg1902, all=TRUE)
egfull<- merge(egfull, eg1912, all=TRUE)
egfull<- merge(egfull, eg1922, all=TRUE)
egfull<- merge(egfull, eg1932, all=TRUE)
egfull<- merge(egfull, eg1942, all=TRUE)
egfull<- merge(egfull, eg1952, all=TRUE)
egfull<- merge(egfull, eg1962, all=TRUE)
egfull<- merge(egfull, eg1972, all=TRUE)
egfull<- merge(egfull, eg1982, all=TRUE)
egfull<-data.matrix(egfull)
#add row and column means
statemeandata <- egfull[,2:50]
statemean <- rowMeans(statemeandata, na.rm=TRUE)
statecode <- egfull[,1]
statemean
yearmean <- colMeans(egfull, na.rm=TRUE)
yearmean <- yearmean[2:50]
yearvar <- colVars(egfull, na.rm=TRUE)
yearvar <- yearvar[2:50]
yearstd <- colStdevs(egfull, na.rm=TRUE)
yearstd <- yearstd[2:50]
year<-seq(1896,1992,2)
plot(year, yearmean, type="l", ylim=c(-0.5,0.5),
     xlab = "Election Year", ylab = "Average Efficiency Gap")
segments(year, yearmean-yearstd,year, yearmean+yearstd)

#state plots for appendix
egfull[,1]
#new england
par(mfrow=c(2,2))
plot(year, egfull[1,2:50], type="l", ylim=c(-0.5,0.5),
     xlab = "Election Year", ylab = "Efficiency Gap",
     main="Connecticut")
abline(h=-0.064584155, lty=2)
plot(year, egfull[6,2:50], type="l", ylim=c(-0.5,0.5),
     xlab = "Election Year", ylab = "Efficiency Gap",
     main="Maine")
abline(h=-0.154781171, lty=2)
plot(year, egfull[12,2:50], type="l", ylim=c(-0.5,0.5),
     xlab = "Election Year", ylab = "Efficiency Gap",
     main="Massachusetts")
abline(h=-0.104870963, lty=2)
plot(year, egfull[20,2:50], type="l", ylim=c(-0.5,0.5),
     xlab = "Election Year", ylab = "Efficiency Gap",
     main="New Hampshire")
abline(h=-0.223222712, lty=2)
plot(year, egfull[31,2:50], type="l", ylim=c(-0.5,0.5),
     xlab = "Election Year", ylab = "Efficiency Gap",
     main="Rhode Island")
abline(h=0.066870198, lty=2)
plot(year, egfull[37,2:50], type="l", ylim=c(-0.5,0.5),
     xlab = "Election Year", ylab = "Efficiency Gap",
     main="Vermont")
abline(h=-0.097004349, lty=2)

#Middle Atlantic
plot(year, egfull[2,2:50], type="l", ylim=c(-0.5,0.5),
     xlab = "Election Year", ylab = "Efficiency Gap",
     main="Delaware")
abline(h=-0.088456786, lty=2)
plot(year, egfull[3,2:50], type="l", ylim=c(-0.5,0.5),
     xlab = "Election Year", ylab = "Efficiency Gap",
     main="New Jersey")
abline(h=-0.080425644, lty=2)
plot(year, egfull[4,2:50], type="l", ylim=c(-0.5,0.5),
     xlab = "Election Year", ylab = "Efficiency Gap",
     main="New York")
abline(h=-0.053420260, lty=2)
plot(year, egfull[5,2:50], type="l", ylim=c(-0.5,0.5),
     xlab = "Election Year", ylab = "Efficiency Gap",
     main="Pennsylvania")
abline(h=-0.040847569, lty=2)
#East North Central
plot(year, egfull[7,2:50], type="l", ylim=c(-0.5,0.5),
     xlab = "Election Year", ylab = "Efficiency Gap",
     main="Illinois")
abline(h=-0.043348158, lty=2)
plot(year, egfull[8,2:50], type="l", ylim=c(-0.5,0.5),
     xlab = "Election Year", ylab = "Efficiency Gap",
     main="Indiana")
abline(h=-0.033884140, lty=2)
plot(year, egfull[9,2:50], type="l", ylim=c(-0.5,0.5),
     xlab = "Election Year", ylab = "Efficiency Gap",
     main="Michigan")
abline(h=-0.074438168, lty=2)
plot(year, egfull[10,2:50], type="l", ylim=c(-0.5,0.5),
     xlab = "Election Year", ylab = "Efficiency Gap",
     main="Ohio")
abline(h=-0.077010670, lty=2)
plot(year, egfull[11,2:50], type="l", ylim=c(-0.5,0.5),
     xlab = "Election Year", ylab = "Efficiency Gap",
     main="Wisconsin")
abline(h=-0.025417738, lty=2)

#West North Central
plot(year, egfull[13,2:50], type="l", ylim=c(-0.5,0.5),
     xlab = "Election Year", ylab = "Efficiency Gap",
     main="Iowa")
abline(h=-0.137221226, lty=2)
plot(year, egfull[14,2:50], type="l", ylim=c(-0.5,0.5),
     xlab = "Election Year", ylab = "Efficiency Gap",
     main="Kansas")
abline(h=-0.140501863, lty=2)
```

```
plot(year, egfull[15,2:50], type="l",
ylim=c(-0.5,0.5),xlab = "Election Year",
      ylab = "Efficiency Gap",main="Minnesota")
      abline(h=0.013831048, lty=2)
plot(year, egfull[16,2:50], type="l",
ylim=c(-0.5,0.5),xlab = "Election Year",
      ylab = "Efficiency Gap",main="Missouri")
      abline(h=0.098990417, lty=2)
plot(year, egfull[17,2:50], type="l",
ylim=c(-0.5,0.5),xlab = "Election Year",
      ylab = "Efficiency Gap",main="Nebraska")
      abline(h=-0.00858882, lty=2)
plot(year, egfull[18,2:50], type="l",
ylim=c(-0.5,0.5),xlab = "Election Year",
ylab = "Efficiency Gap",main="North Dakota")
      abline(h=0.038354617, lty=2)
plot(year, egfull[19,2:50], type="l",
ylim=c(-0.5,0.5),xlab = "Election Year",
ylab = "Efficiency Gap",main="South Dakota")
      abline(h=-0.061828799, lty=2)

#Solid South
plot(year, egfull[21,2:50], type="l",
ylim=c(-0.5,0.5),xlab = "Election Year",
      ylab = "Efficiency Gap",main="Virginia")
      abline(h=-0.071229899, lty=2)
plot(year, egfull[22,2:50], type="l",
ylim=c(-0.5,0.5),xlab = "Election Year",
      ylab = "Efficiency Gap",main="Alabama")
      abline(h=-0.243942853, lty=2)
plot(year, egfull[23,2:50], type="l",
ylim=c(-0.5,0.5),xlab = "Election Year",
      ylab = "Efficiency Gap",main="Arkansas")
      abline(h=-0.171997697, lty=2)
plot(year, egfull[24,2:50], type="l",
ylim=c(-0.5,0.5),xlab = "Election Year",
      ylab = "Efficiency Gap",main="Florida")
      abline(h=-0.152335086, lty=2)
plot(year, egfull[25,2:50], type="l",
ylim=c(-0.5,0.5),xlab = "Election Year",
      ylab = "Efficiency Gap",main="Georgia")
      abline(h=-0.316281848, lty=2)
plot(year, egfull[26,2:50], type="l",
ylim=c(-0.5,0.5),xlab = "Election Year",
      ylab = "Efficiency Gap",main="Louisiana")
      abline(h=-0.353544697, lty=2)
plot(year, egfull[27,2:50], type="l",
ylim=c(-0.5,0.5),xlab = "Election Year",
      ylab = "Efficiency Gap",main="Mississippi")
      abline(h=-0.377728444, lty=2)
plot(year, egfull[28,2:50], type="l",
ylim=c(-0.5,0.5),xlab = "Election Year",
ylab = "Efficiency Gap",main="North Carolina")
      abline(h=0.091541655, lty=2)
plot(year, egfull[29,2:50], type="l",
ylim=c(-0.5,0.5),xlab = "Election Year",
ylab = "Efficiency Gap",main="South Carolina")
      abline(h=-0.345974473, lty=2)
plot(year, egfull[30,2:50], type="l",
ylim=c(-0.5,0.5),xlab = "Election Year",
      ylab = "Efficiency Gap",main="Texas")
      abline(h=-0.231481856, lty=2)

#Border States
plot(year, egfull[32,2:50], type="l",
ylim=c(-0.5,0.5),xlab = "Election Year",
      ylab = "Efficiency Gap",main="Kentucky")
      abline(h=0.082317362, lty=2)
plot(year, egfull[33,2:50], type="l",
ylim=c(-0.5,0.5),xlab = "Election Year",
      ylab = "Efficiency Gap",main="Maryland")
      abline(h=0.041326146, lty=2)
plot(year, egfull[34,2:50], type="l",
ylim=c(-0.5,0.5),xlab = "Election Year",
      ylab = "Efficiency Gap",main="Oklahoma")
      abline(h=0.056206719, lty=2)
plot(year, egfull[35,2:50], type="l",
ylim=c(-0.5,0.5),xlab = "Election Year",
      ylab = "Efficiency Gap",main="Tennessee")
      abline(h=-0.093890643, lty=2)
plot(year, egfull[36,2:50], type="l",
ylim=c(-0.5,0.5),xlab = "Election Year",
ylab = "Efficiency Gap",main="West Virginia")
      abline(h=0.016033386, lty=2)

#Mountain states
plot(year, egfull[38,2:50], type="l",
ylim=c(-0.5,0.5),xlab = "Election Year",
      ylab = "Efficiency Gap",main="Arizona")
      abline(h=-0.004518149, lty=2)
plot(year, egfull[39,2:50], type="l",
ylim=c(-0.5,0.5),xlab = "Election Year",
      ylab = "Efficiency Gap",main="Colorado")
      abline(h=0.012070312, lty=2)
plot(year, egfull[40,2:50], type="l",
ylim=c(-0.5,0.5),xlab = "Election Year",
      ylab = "Efficiency Gap",main="Idaho")
      abline(h=-0.097571621, lty=2)
plot(year, egfull[41,2:50], type="l",
ylim=c(-0.5,0.5),xlab = "Election Year",
      ylab = "Efficiency Gap",main="Montana")
      abline(h=0.018463902, lty=2)
plot(year, egfull[42,2:50], type="l",
ylim=c(-0.5,0.5),xlab = "Election Year",
      ylab = "Efficiency Gap",main="Nevada")
      abline(h=0.023429845, lty=2)
plot(year, egfull[43,2:50], type="l",
ylim=c(-0.5,0.5),xlab = "Election Year",
      ylab = "Efficiency Gap",main="New Mexico")
      abline(h=0.058674234, lty=2)
plot(year, egfull[44,2:50], type="l",
ylim=c(-0.5,0.5),xlab = "Election Year",
      ylab = "Efficiency Gap",main="Utah")
      abline(h=-0.038365731, lty=2)
plot(year, egfull[45,2:50], type="l",
ylim=c(-0.5,0.5),xlab = "Election Year",
      ylab = "Efficiency Gap",main="Wyoming")
      abline(h=-0.177975456, lty=2)

#pacific states
plot(year, egfull[46,2:50], type="l",
ylim=c(-0.5,0.5),xlab = "Election Year",
      ylab = "Efficiency Gap",main="California")
      abline(h=0.002938470, lty=2)
plot(year, egfull[47,2:50], type="l",
ylim=c(-0.5,0.5),xlab = "Election Year",
      ylab = "Efficiency Gap",main="Oregon")
      abline(h=-0.046046009, lty=2)
plot(year, egfull[48,2:50], type="l",
ylim=c(-0.5,0.5),xlab = "Election Year",
      ylab = "Efficiency Gap",main="Washington")
      abline(h=0.030782904, lty=2)

#external states
plot(year, egfull[49,2:50], type="l",
ylim=c(-0.5,0.5),xlab = "Election Year",
      ylab = "Efficiency Gap",main="Alaska")
      abline(h=-0.076260500, lty=2)
plot(year, egfull[50,2:50], type="l",
ylim=c(-0.5,0.5),xlab = "Election Year",
      ylab = "Efficiency Gap",main="Hawaii")
      abline(h=-0.073505088, lty=2)

#uniform swing curves
library(pscl)
house6311$`1992`
#indiana
state1year1992<-data.frame(house6311$`1992`[110:119,4])
curve<-seatsVotes(state1year1992, method="uniformSwing")
plot(curve, type = c("seatsVotes", "density"))
plot(curve)
#NJ
state1year1992<-data.frame(house6311$`1992`[25:37,4])
curve<-seatsVotes(state1year1992, method="uniformSwing")
plot(curve, type = c("seatsVotes", "density"))
plot(curve)
curve #bias5 0.0385
#D got v=0.4977 s=0.5385
#if R got v=0.4977, s=0.4615
curve$s[252]
curve$s[249]
#Texas
house6311$'1992'$STATE==49
state1year1992<-data.frame(house6311$`1992`[281:310,4])
curve<-seatsVotes(state1year1992, method="uniformSwing")
plot(curve, type = c("seatsVotes", "density"))
plot(curve)
curve #bias5 0.16667; average vote 0.5402
curve$s[271]
curve$s[230] #actual results, D get v=0.5402 s=0.7,
curve$v[271] # if R got v=0.5402, s=0.4667
curve$v[230]

#king & gelman model
unc <- function(inp) -1*(inp<0.05)+1*(inp>0.95)
elecyears <- as.numeric(names(house6311))
house6311$'1896'
#all US
j.ob<-judgeit(model.form=VOTE~unc(VOTE)+INC,
vote.form=TURNOUT~1,data=house6311,
same.districts=(elecyears%%10!=2), use.last.votes=T)
#south
j.ob<-judgeit(model.form=VOTE~unc(VOTE)+INC,
vote.form=TURNOUT~1,data=house6311,
same.districts=(elecyears%%10!=2), use.last.votes=T,
subset=DELSOUTH==1)
#nonsouth
j.ob<-judgeit(model.form=VOTE~unc(VOTE)+INC,
vote.form=TURNOUT~1,data=house6311,
same.districts=(elecyears%%10!=2), use.last.votes=T,
subset=DELSOUTH==0)
j.ob$beta
```

```
summary(j.ob)
a<-bias.resp(j.ob, year=1896)
summary1896<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1898)
summary1898<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1900)
summary1900<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1902)
summary1902<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1904)
summary1904<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1906)
summary1906<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1908)
summary1908<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1910)
summary1910<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1912)
summary1912<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1914)
summary1914<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1916)
summary1916<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1918)
summary1918<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1920)
summary1920<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1922)
summary1922<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1924)
summary1924<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1926)
summary1926<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1928)
summary1928<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1930)
summary1930<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1932)
summary1932<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1934)
summary1934<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1936)
summary1936<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1938)
summary1938<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1940)
summary1940<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1942)
summary1942<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1944)
summary1944<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1946)
summary1946<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1948)
summary1948<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1950)
summary1950<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1952)
summary1952<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1954)
summary1954<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1956)
summary1956<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1958)
summary1958<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1960)
summary1960<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1962)
summary1962<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1964)
summary1964<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1966)
summary1966<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1968)
summary1968<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1970)
summary1970<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1972)
summary1972<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1974)
summary1974<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1976)
summary1976<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1978)
summary1978<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1980)
summary1980<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1982)
summary1982<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1984)
summary1984<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1986)
summary1986<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1988)
summary1988<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1990)
summary1990<-data.frame(a$svsums)
a<-bias.resp(j.ob, year=1992)
summary1992<-data.frame(a$svsums)
summary<-rbind(summary1896, summary1898, summary1900,
summary1902, summary1904, summary1906, summary1908,
summary1910, summary1912, summary1914, summary1916,
summary1918, summary1920, summary1922, summary1924,
summary1926, summary1928, summary1930, summary1932,
summary1934, summary1936, summary1938, summary1940,
summary1942, summary1944, summary1946, summary1948,
summary1950, summary1952, summary1954, summary1956,
summary1958, summary1960, summary1962, summary1964,
summary1966, summary1968, summary1970, summary1972,
summary1974, summary1976, summary1978,summary1980,
summary1982, summary1984, summary1986, summary1988,
summary1990, summary1992)
f1<-seq(1,196,4)
f2<-seq(2,196,4)
f3<-seq(3,196,4)
f4<-seq(4,196,4)
#for all of US, bias at 0.5, 0.45-0.55
#resp at 0.45-55, resp at obs
bias5<-data.frame(summary[f1,])
bias4555<-data.frame(summary[f2,])
resp4555<-data.frame(summary[f3,])
respobs<-data.frame(summary[f4,])

#plots
year<-seq(1896, 1992, 2)
plot(year,bias5[,1], 'l', ylab = "Partisan Bias at 0.5",
xlab="Election Year", ylim=c(-0.2,0.2))
lines(year, bias5[,3], 'l', lty=2)
lines(year, bias5[,5], 'l', lty=2)
plot(year,bias4555[,1], 'l',
ylab = "Partisan Bias at 0.45-0.55", xlab="Election Year",
ylim=c(-0.2,0.2))
lines(year, bias4555[,3], 'l', lty=2)
lines(year, bias4555[,5], 'l', lty=2)
plot(year,resp4555[,1], 'l',
ylab = "Responsiveness at 0.45-0.55", xlab="Election Year",
ylim=c(0,3.5))
lines(year, resp4555[,3], 'l', lty=2)
lines(year, resp4555[,5], 'l', lty=2)
plot(year,respobs[,1], 'l',
ylab = "Responsiveness at Observed Results",
xlab="Election Year", ylim=c(0,3.5))
lines(year, respobs[,3], 'l', lty=2)
lines(year, respobs[,5], 'l', lty=2)

#which states are southern
south<-lapply(house6311, subset, DELSOUTH==1)
south$`1992`$STATE
```

AFDELING
Straat nr bus 0000
3000 LEUVEN, BELGIË
tel. + 32 16 00 00 00
fax + 32 16 00 00 00
www.kuleuven.be