Combined Transmission and Distribution Grid Infeasibility Analysis

Combined Transmission and Distribution Dual variable formulation

In the formulation, consider four bus positive sequence transmission network is connected with four bus three phase distribution network as highlighted in the Fig. (1).

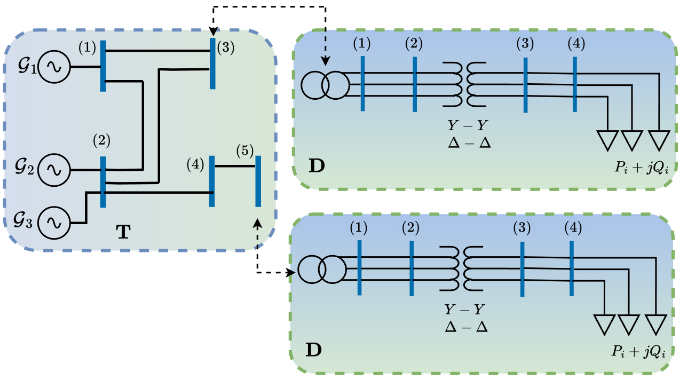

Fig. 1: Combined T&D network

The circuit that couples the transmission and distribution network is highlighted in the Fig.(2)

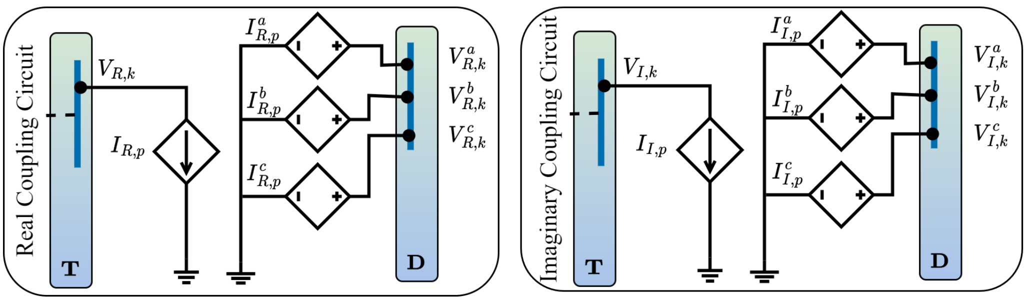

Fig. 2: Equivalent circuit for transmission and distribution point of interconnection.

The equations governing the expression is shown as follows;

$$\begin{bmatrix} \mathbf{I}_R^P \\ \mathbf{I}_I^P \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 0 & \alpha & 0 & \alpha^2 & 0 \\ 0 & 1 & 0 & \alpha & 0 & \alpha^2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_R^A \\ \mathbf{I}_I^A \\ \mathbf{I}_R^B \\ \mathbf{I}_I^B \\ \mathbf{I}_R^C \\ \mathbf{I}_I^C \end{bmatrix} \tag{1}$$

Similarly, the (2) represents the distribution end voltage as a function of transmission end voltage at the point of interconnection.

$$\begin{bmatrix} \mathbf{V}_R^A \\ \mathbf{V}_I^A \\ \mathbf{V}_R^B \\ \mathbf{V}_I^B \\ \mathbf{V}_R^C \\ \mathbf{V}_I^C \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \alpha^2 & 0 \\ 0 & \alpha^2 \\ \alpha & 0 \\ 0 & \alpha \end{bmatrix} \begin{bmatrix} \mathbf{V}_R^P \\ \mathbf{V}_I^P \end{bmatrix} \tag{2}$$

Notations

Table 1: Symbols and definitions

Symbol	Interpretation
i_p^R, i_p^I	Real and Imaginary coupling current at the transmission side
$i_D^{R\Omega}$,$i_D^{I\Omega}$	Distribution side coupling current, $\Omega \in a, b, c$
$V_{T,c}^R$, $V_{T,c}^I$	Coupling port voltage at the transmission
$V_{D,c}^{R\Omega}$,$V_{D,c}^{I\Omega}$	Distribution side coupling voltage $\Omega = \{a, b, c\}$
X, Y	Transmission and distribution state variable (V^R, V^I), $(V^{R\Omega}, V^{I\Omega})$, $\Omega \in a, b, c$
$I_{f,T}^R$, $I_{f,T}^I$ $I_{f,T}^{R\Omega}$, $I_{f,T\Omega}^I$	Transmission side infeasibility current Distribution side infeasibility current
λ^R, λ^I $\beta^{R\Omega}$,$\beta^{I\Omega}$ $\gamma^{R\Omega}$, $\gamma^{I\Omega}$	Transmission side dual variables Distribution side dual variables Coupling constraint dual variables

Converting the equation into the real and imaginary component;

$$i_p^R = i_D^{R,a} - 0.5 i_D^{R,b} - 0.866 i_D^{I,b} - 0.5 i_D^{R,c} + 0.866 i_D^{I,c} \tag{3a}$$

$$i_p^I = i_D^{I,a} - 0.5 i_D^{I,b} + 0.866 i_D^{R,b} - 0.5 i_D^{I,c} - 0.866 i_D^{R,c} \tag{3b}$$

Also, $i_p^R = \dfrac{i_p^R}{k}$, $i_p^I = \dfrac{i_p^I}{k}$, where $k = 3 \times I_{base}$. The coupling constraint after converting the eq(2) into the real and imaginary components are as follows;

$$V_{D,c}^{R,a} - V_{T,c}^R = 0 \tag{4a}$$

$$V_{D,c}^{R,b} + 0.5 V_{T,c}^R - 0.866 V_{T,c}^I = 0 \tag{4b}$$

$$V_{D,c}^{R,c} + 0.5 V_{T,c}^R + 0.866 V_{T,c}^I = 0 \tag{4c}$$

$$V_{D,c}^{I,a} - V_{T,c}^I = 0 \tag{4d}$$

$$V_{D,c}^{I,b} + 0.5 V_{T,c}^I + 0.866 V_{T,c}^R = 0 \tag{4e}$$

$$V_{D,c}^{I,c} + 0.5 V_{T,c}^I - 0.866 V_{T,c}^R = 0 \tag{4f}$$

Optimization Problem

The optimization problem is defined as follows;

$$\min ||I^R_{f,T}||^2_2 + ||I^I_{f,T}||^2_2 + ||I^{I\Omega}_{f,T}||^2_2 + ||I^{I\Omega}_{f,T}||^2_2 \tag{5a}$$

s.t

$$\mathcal{H}(X) - (I^R_{f,T} + I^I_{f,T}) = 0 \tag{5b}$$

$$\mathcal{G}(Y) - (I^{R\Omega}_{f,D} + I^{I\Omega}_{f,D}) = 0 \tag{5c}$$

$$\mathcal{C}(X, Y, I^R_{f,T}, I^I_{f,T}, I^{R\Omega}_{f,D}, I^{I\Omega}_{f,D}) = 0 \tag{5d}$$

Where, $\mathcal{H}(X)$, $\mathcal{G}(Y)$, and $\mathcal{C}(X, Y, I^R_{f,T}, I^I_{f,T}, I^{R\Omega}_{f,D}, I^{I\Omega}_{f,D})$ are the transmission, distribution side, and coupling constraints respectively.
Consider, transmission network is a four bus system, contains , four real KCL equations, four imaginary KCL equations. In this case, the coupling is at bus-4, therefore $V^R_{T,c}, V^I_{T,c} \in X$, and $V^{R\Omega}_{D,c}, V^{I\Omega}_{D,c} \in Y$. In the following Lagrangian equation, $\odot$ denotes the dot product.

$$\begin{aligned}
\mathcal{L}(X, Y, I^R_{f,T}, I^I_{f,T}, I^{R\Omega}_{f,D}, I^{I\Omega}_{f,D}, i^{R\Omega}_D, i^{I\Omega}_D) &= ||I^R_{f,T}||^2_2 + ||I^I_{f,T}||^2_2 + ||I^{I\Omega}_{f,T}||^2_2 + ||I^{I\Omega}_{f,T}||^2_2 \\
+ \begin{bmatrix} \lambda^R_1 \\ \lambda^R_2 \\ \lambda^R_3 \\ \lambda^R_4 \end{bmatrix} &\odot \begin{bmatrix} \mathcal{H}_1(X) - I^R_{f1,T} \\ \mathcal{H}_2(X) - I^R_{f2,T} \\ \mathcal{H}_3(X) - I^R_{f3,T} \\ \mathcal{H}_4(X) - I^R_{f1,T} + i^{R,a}_D - 0.5 i^{R,b}_D - 0.866 i^{I,b}_D - 0.5 i^{R,c}_D + 0.866 i^{I,c}_D \end{bmatrix} \\
+ \begin{bmatrix} \lambda^I_1 \\ \lambda^I_2 \\ \lambda^I_3 \\ \lambda^I_4 \end{bmatrix} &\odot \begin{bmatrix} \mathcal{H}_1(X) - I^I_{f1,T} \\ \mathcal{H}_2(X) - I^I_{f2,T} \\ \mathcal{H}_3(X) - I^I_{f3,T} \\ \mathcal{H}_4(X) - I^I_{f1,T} + i^{I,a}_D - 0.5 i^{I,b}_D + 0.866 i^{R,b}_D - 0.5 i^{I,c}_D - 0.866 i^{R,c}_D \end{bmatrix} \\
+ \begin{bmatrix} \beta^{R,a}_1 \\ \beta^{R,b}_2 \\ \beta^{R,c}_3 \\ . \\ \beta^{R,c}_{12} \end{bmatrix} &\odot \begin{bmatrix} \mathcal{G}_1(Y) + i^{R,a}_D \\ \mathcal{G}_2(Y) + i^{R,b}_D \\ \mathcal{G}_2(Y) + i^{R,c}_D \\ .. \\ \mathcal{G}_{12}(Y) \end{bmatrix} + \begin{bmatrix} \beta^{I,a}_1 \\ \beta^{I,b}_2 \\ \beta^{I,c}_3 \\ . \\ \beta^{I,c}_{12} \end{bmatrix} \odot \begin{bmatrix} \mathcal{G}_1(Y) + i^{I,a}_D \\ \mathcal{G}_2(Y) + i^{I,b}_D \\ \mathcal{G}_2(Y) + i^{I,c}_D \\ .. \\ \mathcal{G}_{12}(Y) \end{bmatrix} \\
+ \begin{bmatrix} \gamma^{R,a}_1 \\ \gamma^{R,b}_2 \\ \gamma^{R,c}_3 \\ \gamma^{I,a}_4 \\ \gamma^{I,b}_5 \\ \gamma^{I,c}_6 \end{bmatrix} &\odot \begin{bmatrix} V^{R,a}_{D,c} - V^R_{T,c} \\ V^{R,b}_{D,c} + 0.5 V^R_{T,c} - 0.866 V^I_{T,c} \\ V^{R,c}_{D,c} + 0.5 V^R_{T,c} + 0.866 V^I_{T,c} \\ V^{I,a}_{D,c} - V^I_{T,c} \\ V^{I,b}_{D,c} + 0.5 V^I_{T,c} + 0.866 V^R_{T,c} \\ V^{I,c}_{D,c} + 0.5 V^I_{T,c} - 0.866 V^R_{T,c} \end{bmatrix}
\end{aligned} \tag{6}$$

Let $\hat{X} = (X, Y, I^R_{f,T}, I^I_{f,T}, I^{R\Omega}_{f,D}, I^{I\Omega}_{f,D}, i^{R\Omega}_D, i^{I\Omega}_D)$, started taking the derivatives;

$$\frac{\partial \mathcal{L}(\hat{X})}{\partial X^R} = \begin{bmatrix} \lambda^R_1 \\ \lambda^R_2 \\ \lambda^R_3 \\ \lambda^R_4 \end{bmatrix} \odot \begin{bmatrix} \nabla \mathcal{H}_1(X) \\ \nabla \mathcal{H}_2(X) \\ \nabla \mathcal{H}_3(X) \\ \nabla_{V^R_{T,c}} \mathcal{H}_4(X) \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\gamma^{R,a}_1 + 0.5\gamma^{R,b}_2 + 0.5\gamma^{R,c}_3 + 0.866\gamma^{I,b}_5 - 0.866\gamma^{I,c}_6 \end{bmatrix} \tag{7}$$

$$\frac{\partial \mathcal{L}(\hat{X})}{\partial X^I} = \begin{bmatrix} \lambda_1^I \\ \lambda_2^I \\ \lambda_3^I \\ \lambda_4^I \end{bmatrix} \odot \begin{bmatrix} \nabla \mathcal{H}_1(X) \\ \nabla \mathcal{H}_2(X) \\ \nabla \mathcal{H}_3(X) \\ \nabla_{V_{T,c}^I} \mathcal{H}_4(X) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\gamma_4^{I,a} + 0.5\gamma_5^{I,b} + 0.5\gamma_6^{I,c} + 0.866\gamma_3^{R,c} - 0.866\gamma_2^{R,b} \end{bmatrix} \tag{8}$$

$$\frac{\partial \mathcal{L}(\hat{X})}{\partial Y^R} = \begin{bmatrix} \beta_1^{R,a} \\ \beta_2^{R,b} \\ \beta_3^{R,c} \\ . \\ \beta_{12}^{R,c} \end{bmatrix} \odot \begin{bmatrix} \nabla_{V_{D,c}^{R,a}} \mathcal{G}(Y) \\ \nabla_{V_{D,c}^{R,b}} \mathcal{G}(Y) \\ \nabla_{V_{D,c}^{R,c}} \mathcal{G}(Y) \\ . \\ \nabla \mathcal{G}(Y) \end{bmatrix} + \begin{bmatrix} \gamma_1^{R,a} \\ \gamma_2^{R,b} \\ \gamma_2^{R,c} \\ 0 \\ 0 \end{bmatrix} \tag{9}$$

$$\frac{\partial \mathcal{L}(\hat{X})}{\partial Y^I} = \begin{bmatrix} \beta_1^{I,a} \\ \beta_2^{I,b} \\ \beta_3^{I,c} \\ . \\ \beta_{12}^{R,c} \end{bmatrix} \odot \begin{bmatrix} \nabla_{V_{D,c}^{I,a}} \mathcal{G}(Y) \\ \nabla_{V_{D,c}^{I,b}} \mathcal{G}(Y) \\ \nabla_{V_{D,c}^{I,c}} \mathcal{G}(Y) \\ . \\ \nabla \mathcal{G}(Y) \end{bmatrix} + \begin{bmatrix} \gamma_4^{I,a} \\ \gamma_5^{I,b} \\ \gamma_6^{R,c} \\ 0 \\ 0 \end{bmatrix} \tag{10}$$

$$\frac{\partial \mathcal{L}(\hat{X})}{\partial I_{f,T}^R} = 2(I_{f,T}^R)_i - \lambda_i^R, \quad i \in \text{All buses except slack} \tag{11}$$

$$\frac{\partial \mathcal{L}(\hat{X})}{\partial I_{f,T}^I} = 2(I_{f,T}^I)_i - \lambda_i^I, \quad i \in \text{All buses except slack} \tag{12}$$

$$\frac{\partial \mathcal{L}(\hat{X})}{\partial I_{f,D}^{R,\Omega}} = 2(I_{f,D}^{R\Omega})_i - \beta_i^{R\Omega}, \quad i \in \text{All buses except slack} \tag{13}$$

$$\frac{\partial \mathcal{L}(\hat{X})}{\partial I_{f,D}^{I,\Omega}} = 2(I_{f,D}^{I\Omega})_i - \beta_i^{I\Omega}, \quad i \in \text{All buses except slack} \tag{14}$$

$$\frac{\partial \mathcal{L}(\hat{X})}{\partial I_D^{R,a}} = \lambda_4^R + \beta_1^R \tag{15}$$

$$\frac{\partial \mathcal{L}(\hat{X})}{\partial I_D^{R,b}} = -0.5\lambda_4^R + 0.866\lambda_4^I + \beta_2^R \tag{16}$$

$$\frac{\partial \mathcal{L}(\hat{X})}{\partial I_D^{R,c}} = -0.5\lambda_4^R - 0.866\lambda_4^I + \beta_3^R \tag{17}$$

$$\frac{\partial \mathcal{L}(\hat{X})}{\partial I_D^{I,a}} = \lambda_4^I + \beta_1^I \tag{18}$$

$$\frac{\partial \mathcal{L}(\hat{X})}{\partial I_D^{I,b}} = -0.866\lambda_4^R - 0.5\lambda_4^I + \beta_2^I \tag{19}$$

$$\frac{\partial \mathcal{L}(\hat{X})}{\partial I_D^{I,c}} = 0.866\lambda_4^R - 0.5\lambda_4^I + \beta_3^I \tag{20}$$

From the eq.(15) to eq.(20), $\alpha = 2\pi/3$, the relation can be described as;

$$\begin{bmatrix} \beta_1^R \\ \beta_1^I \\ \beta_2^R \\ \beta_2^I \\ \beta_3^R \\ \beta_3^I \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \alpha^2 & 0 \\ 0 & \alpha^2 \\ \alpha & 0 \\ 0 & \alpha \end{bmatrix} \begin{bmatrix} \lambda_4^R \\ \lambda_4^I \end{bmatrix} \tag{21}$$

Consider, the eq.(7) and (8),

$$\lambda_4^R(\nabla_{V_{T,C}^R}\mathcal{H}_4(X)) - \gamma_1^{R,a} + 0.5\gamma_2^{R,b} + 0.5\gamma_3^{R,c} + 0.866\gamma_5^{I,b} - 0.866\gamma_6^{I,c} = 0 \tag{22}$$

$$\lambda_4^I(\nabla_{V_{T,C}^I}\mathcal{H}_4(X)) - \gamma_4^{I,a} + 0.5\gamma_5^{I,b} + 0.5\gamma_6^{I,c} + 0.866\gamma_3^{R,c} - 0.866\gamma_2^{R,b} = 0 \tag{23}$$

The equations ((22)) and ((23)) can be further simplified by substituting the value of γ from the equations (9)-(10). The overall expression will become, where λ_4^R and λ_4^I are the dual variables at the coupling port.

$$\begin{bmatrix} \lambda_4^R \\ \lambda_4^I \end{bmatrix} = \begin{bmatrix} \frac{-1}{\nabla_{V_{T,c}^R}\mathcal{H}(X)} & 0 \\ 0 & \frac{-1}{\nabla_{V_{T,c}^I}\mathcal{H}(X)} \end{bmatrix} \begin{bmatrix} 1 & 0 & \alpha^2 & 0 & \alpha & 0 \\ 0 & 1 & 0 & \alpha^2 & 0 & \alpha \end{bmatrix} \begin{bmatrix} \beta_1^{R,a}\nabla_{V_{D,c}^{R,a}}\mathcal{G}^R(Y) \\ \beta_1^{I,a}\nabla_{V_{D,c}^{I,a}}\mathcal{G}^I(Y) \\ \beta_2^{R,b}\nabla_{V_{D,c}^{R,b}}\mathcal{G}^R(Y) \\ \beta_2^{I,b}\nabla_{V_{D,c}^{I,b}}\mathcal{G}^I(Y) \\ \beta_3^{R,c}\nabla_{V_{D,c}^{R,c}}\mathcal{G}^R(Y) \\ \beta_3^{I,c}\nabla_{V_{D,c}^{I,c}}\mathcal{G}^I(Y) \end{bmatrix} \tag{24}$$

Test Results - Combined GC-12.47-1 (D) with IEEE-118 (T)

I conducted a test using the IEEE-118 test case single-phase transmission network in conjunction with the three-phase GC-14.57-1. The test involved injecting infeasible reactive power at various locations within the transmission network, specifically targeting ten locations associated with PQ buses. The applied reactive power in-feasibility is highlighted with the different color in the attached Fig.3. Also, in the transmission network, at these ten locations, voltage bounds are applied.

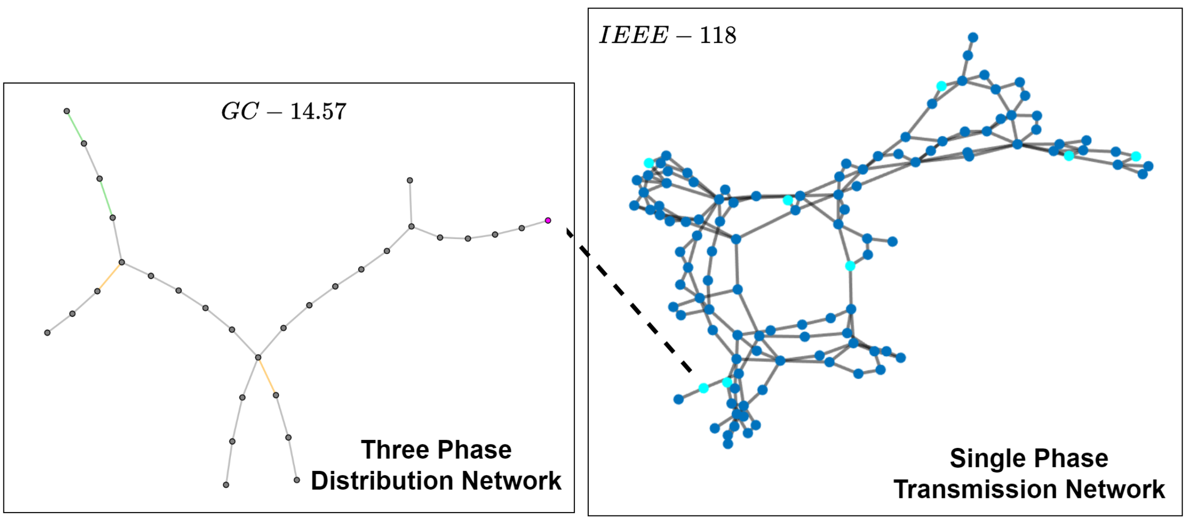

Fig. 3: Combined T&D network (IEEE-118, GC14.57.1)

Location	Q^{inf}
13	-0.80
9	2.3×10^{-5}
103	1.32×10^{-5}
116	3.36×10^{-8}
109	4.25×10^{-8}
24	7.12×10^{-12}
53	8.3×10^{-5}
85	6.57×10^{-6}
104	1.12×10^{-8}

Table 2: Q^{inf} results.

The decentralized algorithm converges in 3 epochs, and in-feasibility results are shown in the table;