COMSATS University Islamabad, Wah Campus
Sessional-1 Examinations Spring 2021
Department of ___Mathematics__

Program/Class:__CVE(A, B) ___ Date: _____ 2021_____
Subject: _Differential Equations_____ Instructor:_____Dr. Adnan Jahangir_____
Total Time Allowed: ____60mins_____ Maximum Marks: _____(10)___
Student Name: _Muhammad Hamaad Kaleem_____ Registration (Section)#: FA17-BSE-106 SE-8B

IMPORTANT:

- Time to solve paper is 1hr, and extra 30 minutes for uploading the answer sheet.
- Write your, department, section and registration number on first page of question paper and registration number on each page of your answer sheet.
- Submit in PDF format on CUonline and as well submit through email within the time domain to adnan.jahangir@ciitwah.edu.pk

Q. No. 1) (4)

The population of student in a University grows at some specific rate at time *t.* After $(\ln\alpha)$ hours it is observed that the students are increased to the square of the initial amount. Find the initial population of students if After 2α hours the students are double the initial number.

where, $\alpha =$ Numeric digit of your registration number

Answer: ***Paste pictures of solution here***

Good Luck

Q1 $\frac{dP}{dt} = kP_0$

Integrating both sides

$\ln|P| = kt + c$

$P = ce^{kt}$ at $t=0$

$P(0) = P_0$

So, $P = P_0 e^{kt} \rightarrow (i)$

At $t = \ln|106|$

$t = 4.66$

$P = P_0^2$ at $t = 4.66$

eq (i) $P = P_0 e^{kt}$

$= P_0 e^{k(4.66)}$ —(ii)

At $t = 2\alpha$

$= 2(106)$

$= 212$

Now $2P_0 = P_0 e^{212k}$

Taking ln b/s

$\ln|2| = \ln|e^{212k}|$

$\ln 2 = 212 * k$

$k = \frac{\ln 2}{212} = \frac{0.69}{212} = 3.269 \times 10^{-3}$

$P_0 = e^{4.66(3.269 \times 10^{-3})}$

Good Luck

Q. No. 2) (3)

Find the general solution of the differential equation

$$x\frac{dy}{dx}+\alpha y=0: \qquad x(\alpha)=1,$$

where, $\alpha = numeric\,digits\,of\ \ your\,registration\,number$.

Answer:

Good Luck

Q02 $x\frac{dy}{dx} + ay = 0, \quad x(a) = 1$

$a = 106$

$x\frac{dy}{dx} + 106y = 0$

$\frac{dy}{dx} = -\frac{69y}{x}$ — (i)

let $y = ux$

$\frac{dy}{dx} = u + x\frac{du}{dx}$ — (a)

putting a in eq (i)

$u + x\frac{du}{dx} = -\frac{106(ux)}{x}$

$x\frac{du}{dx} = -106u - u$

$x\frac{du}{dx} = -107u$

$-\frac{du}{107u} = \frac{dx}{x}$

Applying integration on both sides

$-\frac{1}{107}\int \frac{1}{u}\,du = \int \frac{1}{x}\,dx$

$-\frac{1}{107}\ln|u| = \ln|x| + c$

$\ln(u)^{-1/107} = \ln|x| + c$ putting $u = y/x$

$\ln(y/x)^{-1/107} = \ln|x| + c$

Good Luck

Q. No. 3) (3)

Solve the following differential equation,

$$\frac{dy}{dx} + \frac{\alpha}{x} y = -x^{-9} y^{5}$$

Good Luck

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SE-8B
Sessional 1
Differential Equations

Q05 $\frac{dy}{dx} + \frac{4}{x} y = -x^{-9} y^5$

Dividing by y^5 on both sides

$\frac{1}{y^5} \frac{dy}{dx} + \frac{4}{x} y \times \frac{1}{y^5} = (-x^{-9} y^5) \times \frac{1}{y^5}$

$y^{-5} \frac{dy}{dx} + \frac{4}{x} y^{-4} = -x^{-9}$

Let $v = y^{-4}$

$\frac{dv}{dx} = -4y^{-5} \frac{dy}{dx}$

$y^{-5} \frac{dy}{dx} = -\frac{1}{4} \frac{dv}{dx}$

$-\frac{1}{4} \frac{dv}{dx} + \frac{v4}{x} = -x^{-9}$

Multiplying "-4" on both sides

$\frac{dv}{dx} - \frac{4v4}{x} = 4x^{-9}$

Taking I.F $= e^{\int -4 d/x} = e^{-4 \int 1/x} = e^{-4 \ln(x)}$

$= e^{\ln(x)^{-4}} = x^{-4} = x^{-4(106)} = x^{-424}$

Good Luck

Multiplying I.F with all equations :-

$$x^{4\alpha}\frac{dv}{dx} - (x^{4\alpha})\left(\frac{4\alpha v}{x}\right) = 4x^{-9}x^{4\alpha}$$

$$x^{4\alpha}\frac{dv}{dx} - 4\alpha x^{4\alpha-1}v = 4x^{-9}x^{4\alpha}$$

$$\left(x^{4\alpha}\frac{dv}{dx} - 4\alpha x\right)$$

$$\frac{d}{dx}\,vx^{4\alpha} = 4x^{4\alpha}x^{-9}$$

$$\frac{d}{dx}\,vx^{4\alpha} = 4x^{4\alpha-9}$$

Applying Integration

$$\int \frac{d}{dx}\,vx^{4\alpha} = \int 4x^{4\alpha-9}$$

$$vx^{4\alpha} = \frac{4x^{4\alpha-9+1}}{4\alpha-9+1} + C$$

$$= \frac{4x^{4\alpha-8}}{4\alpha-8} + C$$

$$= \frac{4(x^{4\alpha-8})}{4(\alpha-2)} + C$$

$$= \frac{x^{4\alpha-8}}{\alpha-2} + C$$

$$v = \left(\frac{x^{4\alpha-8}}{\alpha-2}\right)/x^{4\alpha} + \frac{C}{x^{4\alpha}}$$

$\alpha = 106$

$$= \left(\frac{x^{424-8}}{424-2}\right)/x^{424} + \frac{C}{x^{424}} \Rightarrow \left(\frac{x^{416}}{416}\right)/x^{424} + \frac{C}{x^{424}}$$

Good Luck