FA17-BSE-106

QUIZ#3

DE

SE-3B

Pg.01

$$x^2y''+xy'+\left(x^2-\frac{1}{4}\right)y = x^{3/2}$$

$$y_1 = x^{-1/2}\cos x, \; y_2 = x^{-1/2}\sin x$$

Sol:-

$$y_c = c_1x^{-1/2}\cos x + c_2x^{-1/2}\sin x$$

$$w\left(x^{-1/2}\cos x, x^{-1/2}\sin x\right) = \begin{vmatrix} x^{-1/2}\cos x & x^{-1/2}\sin x \\ \frac{-\sin x}{x^{-1/2}} - \frac{\cos x}{2x^{3/2}} & \frac{\cos x}{x^{-1/2}} - \frac{\sin x}{2x^{3/2}} \end{vmatrix}$$

$$= \left(x^{-1/2}\cos x\right)\left(\frac{\cos x}{x^{-1/2}} - \frac{\sin x}{2x^{3/2}}\right) + \left(x^{-1/2}\sin x\right)\left(\frac{\sin x}{x^{1/2}} \frac{-\cos x}{2x^{1/2}}\right)$$

$$= \left(\frac{\cos x}{x^{1/2}}\right)\left(\frac{2x\cos x - \sin x}{2x^{3/2}}\right) + \frac{\sin x}{x^{1/2}}\left(\frac{2x\sin x + \cos x}{2x^{3/2}}\right)$$

$$= \frac{2x(\cos^2 x) - \sin x\cos x + 2x\sin^2 x + \cos x\sin x}{2x^2}$$

$$= \frac{2x(\cos^2 x + \sin^2 x)}{2x}$$

$$\boxed{\rightarrow \sin^2 x + \cos^2 x = 1}$$

$$= \frac{2x(1)}{2x^2} \Rightarrow 1/x \neq 0$$

Scanned with CamScanner

$$x^2y'' + x^2y' + (x^2 - 1/4)y = x^{3/2}$$

dividing by x^2 on both sides

$$y'' + y'/x + \left(y - \frac{1}{4x^2}y\right) = x^{-1/2}$$

$$y'' + y'/x + y - \frac{1}{4x^2}y = \frac{1}{\sqrt{x}}$$

$$W_1 = \begin{vmatrix} 0 & \sin x/\sqrt{x} \\ 1/\sqrt{x} & 2x \cos x - \sin x/2x\sqrt{x} \end{vmatrix}$$

$$= 0 - x^{-1/2}(x^{-1/2}\sin x)$$

$$W_1 = -x^{-1}\sin x$$

$$W_1 = -\frac{\sin x}{x} \rightarrow A$$

$$W_2 = \begin{vmatrix} \cos x/\sqrt{x} & 0 \\ -2x\sin x - \cos x/2x\sqrt{x} & 1/\sqrt{x} \end{vmatrix}$$

$$= x^{-1/2}(x^{-1/2}\cos x)$$

$$= x^{-1}\cos x$$

$$W_2 = \frac{\cos x}{x} \text{ — } B$$

$u_1 = \int -\sin x$

$u_1 = -\int \sin x = \cos x$

$u_2 = \int \cos x$

$= \sin x$

The particular solution is $y_p = u_1 y_1 + u_2 y_2$

$y_p = \cos x \left(x^{-1/2} \cos x\right) + \sin x \left(x^{-1/2} \sin x\right)$

$y = y_c + y_p$

$y = c_1 x^{-1/2} \cos x + c_2 x^{-1/2} \sin x + x^{-1/2} \cos^2 x + x^{-1/2} \sin^2 x$