データ科学特論　課題 1

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まず，$\boldsymbol{A\theta} = \boldsymbol{B}$ の形に変形する．問題より

$$L(\theta) = \frac{1}{2}\sum_{i=1}^{2}(y_i - f(x_i;\theta))^2 \tag{1}$$

$f(x_i;\theta) = \theta_1 x_i + \theta_2$ を式 (1) に代入すると

$$L(\theta) = \frac{1}{2}\sum_{i=1}^{2}(y_i - (\theta_1 x_i + \theta_2))^2 = \frac{1}{2}\sum_{i=1}^{2}(y_i - \theta_1 x_i - \theta_2)^2 \tag{2}$$

ここで式 (2) を θ_1, θ_2 で偏微分すると

$$\left(\begin{array}{c} \frac{\partial L(\boldsymbol{\theta})}{\partial \theta_1} = 2\frac{1}{2}\sum_{i=1}^{2}(y_i - \theta_1 x_i - \theta_2)(-x_i) = 0 \\ \frac{\partial L(\boldsymbol{\theta})}{\partial \theta_1} = 2\frac{1}{2}\sum_{i=1}^{2}(y_i - \theta_1 x_i - \theta_2)(-1) = 0 \end{array} \right) \tag{3}$$

計算すると

$$\left(\begin{array}{c} \sum_{i=1}^{2}(y_i - \theta_1 x_i - \theta_2)x_i = 0 \\ \sum_{i=1}^{2}(y_i - \theta_1 x_i - \theta_2) = 0 \end{array} \right) \tag{4}$$

$$= \left(\begin{array}{c} (\sum_{i=1}^{2} x_i^2)\theta_1 + (\sum_{i=1}^{2} x_i)\theta_2 = (\sum_{i=1}^{2} y_i x_i) \\ (\sum_{i=1}^{2} x_i)\theta_1 + (\sum_{i=1}^{2} 1)\theta_2 = (\sum_{i=1}^{2} y_i) \end{array} \right) \tag{5}$$

ここで

$$\boldsymbol{A} = \left[\begin{array}{cc} \sum_{i=1}^{2} x_i^2 & \sum_{i=1}^{2} x_i \\ \sum_{i=1}^{2} x_i & \sum_{i=1}^{2} 1 \end{array} \right], \boldsymbol{\theta} = \left[\begin{array}{c} \theta_1 \\ \theta_2 \end{array} \right], \boldsymbol{B} = \left[\begin{array}{c} \sum_{i=1}^{2} y_i x_i \\ \sum_{i=1}^{2} y_i \end{array} \right] \tag{6}$$

とすると $\boldsymbol{A\theta} = \boldsymbol{B}$ に変形できる．問題より $\mathcal{D} = (x_i, y_i)_{i=1}^{2} = (1,1),(a,b)$ を与え $\boldsymbol{A}, \boldsymbol{B}$ をそれぞれ計算すると

$$\boldsymbol{A} = \left[\begin{array}{cc} x_1^2 + x_2^2 & x_1 + x_2 \\ x_1 + x_2 & 2 \end{array} \right] = \left[\begin{array}{cc} 1 + a^2 & 1 + a \\ 1 + a & 2 \end{array} \right] \tag{7}$$

$$\boldsymbol{B} = \left[\begin{array}{c} x_1 y_1 + x_2 y_2 \\ y_1 + y_2 \end{array} \right] = \left[\begin{array}{c} 1 + ab \\ 1 + b \end{array} \right] \tag{8}$$

問 1 終了

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