① Sum = 0

```
for i to n^2:
    for j to n^3:
        sum++
```

each n^2, do n^3

Runtime: $n^8 = n^2 \cdot n^3 + n^2$

b) Sum = 0

```
for i to n^2:
    j = 1
    while j ≥ 1:
        sum++
        j = j/5
```

ex: $J = 125$
$125/5 = 25$
$25/5 = 5$
$5/5 = 1$

$\log_5 n$

for each n^2

$n^2(\log_5 n)$

The base doesn't matter

Runtime: $n^2 \log n$

c)

```
sum = 0
i = 1
while i ≤ n:
    J = i
    while J ≥ 1
        sum++
        J = J/2
    i = i*4
```

i will occur in powers of 4

$i = 1, 4, 16 \ldots$

J will occur in powers of 2

$J = 1, 2, 4 \ldots$

$\log_4(n) \cdot \log_2(n) = (\log n)^2$

Runtime: $(\log n)^2$

② α/output: $\Sigma_1^n A[i]$ $\qquad Y \Rightarrow i > n$

Initialization:

Assignment statements, sum = 0 or sum = A[i-1]

$\Sigma_1^1 A[i-1] = A[0]$

Sum = A[0], the sum of an empty set of ints is zero, which equals our sum before the loop begins.

Maintenance:

Let sum' denote the updated value of sum after executing loop body

From code: $Sum' = Sum + A[i]$

$Sum' = \Sigma_1^{i-1} A[i] + A[i]$

$Sum' = \Sigma_1^{i} A[i]$

This proves that after each tstep of the loop, sum' equals the summation of all the previous terms in the sum i-1, plus the most recent term A[i].

Termination:

From α, $1 \leq i \leq n$ and $i \not> n$ from $\neg Y$

$\therefore$ At termination, $i = n$.

$Sum' = \Sigma_1^{n-1} A[i] + A[n]$

$= \Sigma_1^{n} A[i]$ which was the Goal

③ $P = 0$

for $i = n$ to 0

$P = A[i] + x \cdot P$

return P

a) $O(n)$

b) NaivePolyeval(A, n, x)

$P = 0$

for $i = 0$ to n

term $= A[i]$

for $j = 1$ to i

term = term * x

$P = P +$ term

return P

Runtime:

$O(n^2)$

Two loops

c) Initialization:

$P = 0$, $i = n$

$$P = \sum_{k=0}^{n} A[k] \cdot x^k$$

Once again, the array is empty, A[0]. The sum of an empty set it zero, which matches our P value.

Maintanence: Show loop holds at $i-1$

$P = A[i] + x \cdot P$

before update

$$P = \sum_{k=0}^{n-i-1} A[k+i+1] \cdot x^k$$

After

$$P' = A[i] + x \cdot \sum_{k=0}^{n-i-1} A[k+i+1] \cdot x^k$$

Simplify

$$P' = A[i] + \sum_{k=1}^{n-i} A[k+i] \cdot x^k$$

This matches our loop invariant when i is plugged in instead of i - 1, showing that it holds after the next term.

Termination:

Since $n \geq i \geq 0$ and $i \not> 0$ from ??

$i = -1$ at termination

$$\therefore \; P = \sum_{k=0}^{n} A[k] \cdot x^k$$

Which matches our starting polynomial