4) a) $.4 \text{ ms} = C \cdot 50$

$C = .4/50 = .008$

$1 \text{ hour} = 3{,}600{,}000 \text{ ms}$

$n = \frac{3{,}600{,}000}{.008} = \boxed{450{,}000{,}000 \text{ input size}}$ a)

b) $.4 \text{ ms} = C \cdot n\log n$

$50 \cdot \log_2 50 = 282.192$

$C = .001417$

$\frac{3{,}600{,}000}{.001417} = \boxed{2{,}540{,}578{,}687.36 = n\log n}$ b)

I don't know how to simplify this

c) $\frac{.4}{n^3} = C$

$C = .0000032$

$\frac{3{,}600{,}000}{.0000032} = 1.125 \times 10^{12}$

$\sqrt[3]{1.125 \times 10^{12}} = n$

$\boxed{n = 10{,}400.419}$ c)

D) $\frac{.4}{2^{50}} = C$

$C = 3.5527 \times 10^{-16}$

$\frac{3{,}600{,}000}{3.5527 \times 10^{-16}} = 1.01331 \times 10^{22}$

$2^n = 1.01331 \times 10^{22}$

$\boxed{n = 73.101}$ D)

3) is $2^{n+1} = O(2^n)$

$2^{n+1} = 2 \cdot 2^n = C \cdot g(n)$

$\boxed{\text{Yes, } 2^{n+1} = O(2^n)}$ a)

b) is $2^{2n} = O(2^n)$

$2^{2n} \le 2^n \cdot C$ Divide by 2^n

$2^n \le C$ 2^n can't be a constant

$\boxed{\therefore 2^{2n} \mathrel{!}= O(2^n)}$ b)

c) is $2^{2^{n+1}} = O\, 2^{2^n}$

$2^{2 \cdot 2^n} = O\, 2^{2^n}$

for $n \ge n_0$, $n_0 \ge n$

$2 \cdot 2^n > 2^n$

$2^{2^{n+1}} \le C \cdot 2^{2^n}$ $\boxed{\therefore 2^{2^{n+1}} \mathrel{!}= O\, 2^{2^n}}$ c)

Polynomials, raised to a higher power even by a little, ougrow those of a lower power.

There does not exist a constant such that

2) Assume $n_0, f(n), g(n) \ge 0$

$\therefore n \ge n_0$

$$f(n) + g(n) \ge \max(f(n), g(n))$$

If both numbers are postive or zero, then the lowest the sums of those numbers could be would be equal to the largest number.

$f(n) \le \max(f(n), g(n))$

$g(n) \le \max(f(n), g(n))$

Add both inequalities

$f(n) + g(n) \le 2\max(f(n), g(n))$

Divide by 2

$\frac{1}{2}(f(n) + g(n)) \le \max(f(n), g(n))$

$\therefore \frac{1}{2}(f(n) + g(n)) \le \max(f(n), g(n)) \le f(n) + g(n)$

by Definition!

$\max(f(n), g(n)) = \Theta(f(n) + g(n))$ because $C_1 = .5$, $C_2 = 1$

1) $n^2 - n + 5\sqrt{n} = O(n^2)$

Let $f(n) = n^2 - n + 5\sqrt{n}$

Let $g(n) = n^2$

Let $C = 1$, $N_0 = 25$

For all $n \ge n_0$

$-n + 5\sqrt{n} \le 0$ because n grows faster than $\sqrt{n}$.

$n^2 - n + 5\sqrt{n} \le n^2$ Add n^2

$\therefore$ by definition $0 \le f(n) \le C \cdot g(n)$ for all $n_0 < n$.

Let $f(n) = n^2 - n + 5\sqrt{n}$

let $g(n) = n^2$

Let $C = 5$, $N_0 = 5$

Show $g(n) = O\, f(n)$

$n^2 \le n^2 - n + 5\sqrt{n}$

$n^2 \le 5(n^2 - n + 5\sqrt{n})$

$25 \le n^2$

I have no idea where to go from here, I tried this problem for like 2 hours and couldn't figure it out.