1.

$$J=\begin{bmatrix} -l_1\sin\theta_1 - l_2\sin(\theta_1+\theta_2) & -l_2\sin(\theta_1+\theta_2) \\ l_1\cos\theta_1 + l_2\cos(\theta_1+\theta_2) & l_2\cos(\theta_1+\theta_2) \end{bmatrix}$$

$$J^{-1} = \frac{1}{l_1 l_2 \sin\theta_2}\begin{bmatrix} l_2\cos(\theta_1+\theta_2) & l_2\sin(\theta_1+\theta_2) \\ -l_1\cos\theta_1 - l_2\cos(\theta_1+\theta_2) & -l_1\sin\theta_1 - l_2\sin(\theta_1+\theta_2) \end{bmatrix}$$

$$\dot{\Theta} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = J^{-1}V \qquad V=\begin{bmatrix} V_x \\ V_y \end{bmatrix}$$

$$\dot{\theta}_1 = \frac{V_x\cos(\theta_1+\theta_2) + V_y\sin(\theta_1+\theta_2)}{l_1\sin\theta_2}$$

① $$\dot{\theta}_1 = \frac{-\cos(30^\circ-60^\circ)}{0.5\sin(-60^\circ)} = 2\text{ rad/s}$$

$$\dot{\theta}_2 = \frac{\cos 30^\circ}{0.5\sin(-60^\circ)} + \frac{\cos(30^\circ-60^\circ)}{0.5\sin(-60^\circ)} = -4\text{ rad/s}$$

② $$\dot{\theta}_1 = \frac{\sin(30^\circ+120^\circ)}{0.5\sin(120^\circ)} = \frac{2\sqrt{3}}{3}\text{ rad/s}$$

$$\dot{\theta}_2 = \frac{-\sin 30^\circ}{0.5\sin 120^\circ} - \frac{\sin(30^\circ+120^\circ)}{0.5\sin 120^\circ} = -\frac{4\sqrt{3}}{3}\text{ rad/s}$$

③ $$\dot{\theta}_1 = \frac{\cos(30^\circ-30^\circ) + \sin(30^\circ-30^\circ)}{0.5\sin(-30^\circ)} = -4\text{ rad/s}$$

$$\dot{\theta}_2 = \frac{-\cos 30^\circ - \cos(30^\circ - 30^\circ) - \sin 30^\circ - \sin(30^\circ - 30^\circ)}{0.5\sin(-30^\circ)}$$

$$= 2\sqrt{3} + 6$$

3、$x_3 = l_1\cos\theta_1 + l_2\cos(\theta_1+\theta_2) + l_3\cos(\theta_1+\theta_2+\theta_3)$

$y_3 = l_1\sin\theta_1 + l_2\sin(\theta_1+\theta_2) + l_3\sin(\theta_1+\theta_2+\theta_3)$

$$\therefore J = \begin{bmatrix} -l_1 s\theta_1 - l_2 s(\theta_1+\theta_2) - l_3 s(\theta_1+\theta_2+\theta_3) & -l_2 s(\theta_1+\theta_2) - l_3 s(\theta_1+\theta_2+\theta_3) & -l_3 s(\theta_1+\theta_2+\theta_3) \\ l_1 c\theta_1 + l_2 c(\theta_1+\theta_2) + l_3 c(\theta_1+\theta_2+\theta_3) & l_2 c(\theta_1+\theta_2) + l_3 c(\theta_1+\theta_2+\theta_3) & l_3 c(\theta_1+\theta_2+\theta_3) \end{bmatrix}$$

$$\therefore J^T = \begin{bmatrix} -l_1 s\theta_1 - l_2 s(\theta_1+\theta_2) - l_3 s(\theta_1+\theta_2+\theta_3) & l_1 c\theta_1 + l_2 c(\theta_1+\theta_2) + l_3 c(\theta_1+\theta_2+\theta_3) \\ -l_2 s(\theta_1+\theta_2) - l_3 s(\theta_1+\theta_2+\theta_3) & l_2 c(\theta_1+\theta_2) + l_3 c(\theta_1+\theta_2+\theta_3) \\ -l_3 s(\theta_1+\theta_2+\theta_3) & l_3 c(\theta_1+\theta_2+\theta_3) \end{bmatrix}$$

$F = mg = 100\ N$ $\qquad F = \begin{bmatrix} F_x \\ F_y \end{bmatrix} = \begin{bmatrix} 0 \\ -100 \end{bmatrix}$

$$\tau_1 = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = J^T F = -100 \begin{bmatrix} l_1 c\theta_1 + l_2 c(\theta_1+\theta_2) + l_3 c(\theta_1+\theta_2+\theta_3) \\ l_2 c(\theta_1+\theta_2) + l_3 c(\theta_1+\theta_2+\theta_3) \\ l_3 c(\theta_1+\theta_2+\theta_3) \end{bmatrix}$$

$\therefore \tau_1 = -100\left[0.8\cos 60^\circ + 0.8\cos(60^\circ - 60^\circ) + 0.4\cos(60^\circ - 60^\circ - 90^\circ)\right]$

$= -100(0.4 + 0.8 + 0) = -120$

$\tau_2 = -100(0.8 + 0) = -80$

$\tau_3 = 0$