Adı ve Soyadı : Hamdi Utku Parali

Numara : 19253510

1) $a=0$ ve $b=1$ alınırsa $f(a) = e^0 - \cos 0 - 1 = -1$

$f(b) = e^1 - \cos 1 - 1 = 1,1780$

$f(a) \cdot f(b) < 0$

$$x_0 = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)} = \frac{0 \cdot 1,1780 - 1 \cdot (-1)}{1,1780 - (-1)} = \frac{1}{2,1780} = 0,4591$$

$f(x_0) = -0,3138$ $f(x_0) \cdot f(b) < 0$ olduğundan dolayı

$$x_1 = \frac{0,4591 \cdot 1,1780 - 1 \cdot (-0,3138)}{1,1780 - (-0,3138)} = 0,5729$$

$f(x_1) = -0,0669$ $f(x_1) \cdot f(b) < 0$ olduğundan dolayı

$$x_2 = \frac{0,5729 \cdot 1,1780 - 1 \cdot (-0,0669)}{1,1780 - (-0,0669)} = 0,5959$$

$f(x_2) = -0,0130$ $f(x_2) \cdot f(b) < 0$

$$x_3 = \frac{0,5959 \cdot 1,1780 - 1 \cdot (-0,0130)}{1,1780 - (-0,0130)} = 0,6003$$

$f(x_3) = -0,0025$ $f(x_3) \cdot f(b) < 0$

$$x_4 = \frac{0,6003 \cdot 1,1780 - 1 \cdot (-0,0025)}{1,1780 - (-0,0025)} = 0,6011$$

$f(x_4) = -0,0006$

$f(x_4) \cdot f(b) < 0$

$$x_5 = \frac{0,6011 \cdot 1,1780 - 1 \cdot (-0,0006)}{1,1780 - (-0,0006)} = 0,6013$$

$f(x_5) = -0,0001$

$f(x_5) \cdot f(b) < 0$

$$x_6 = \frac{0,6013 \cdot 1,1780 - 1 \cdot (-0,0001)}{1,1780 - (-0,0001)} = 0,6013$$

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2)

$$7x + 2y - 3z = -32$$
$$2x + 5y - 3z = -31$$
$$x - y - 6z = -23$$

$$\begin{bmatrix} 7 & 2 & -3 \\ 2 & 5 & -3 \\ 1 & -1 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -32 \\ -31 \\ -23 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 2 & -3 \\ 2 & 5 & -3 \\ 1 & -1 & -6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$A = L \cdot U$$

$1 \cdot u_{11} + 0 \cdot 0 + 0 \cdot 0 = 7$ $\quad 1 \cdot u_{12} + 0 \cdot u_{22} + 0 \cdot 0 = 2$ $\quad 1 \cdot u_{13} + 0 \cdot u_{23} + 0 \cdot u_{33} = -3$

$l_{21} \cdot u_{11} + 1 \cdot 0 + 0 \cdot 0 = 2$ $\quad l_{21} \cdot u_{12} + 1 \cdot u_{22} + 0 = 5$ $\quad l_{21} \cdot u_{13} + 1 \cdot u_{23} + 0 \cdot u_{33} = -3$

$l_{31} \cdot u_{11} + l_{32} \cdot 0 + 1 \cdot 0 = 1$ $\quad l_{31} \cdot u_{12} + l_{32} \cdot u_{22} + 1 \cdot 0 = 1$ $\quad l_{31} \cdot u_{13} + l_{32} \cdot u_{23} + 1 \cdot u_{33} = -6$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2/7 & 1 & 0 \\ 1/7 & -9/31 & 1 \end{bmatrix} \quad \text{ve} \quad U = \begin{bmatrix} 7 & 2 & -3 \\ 0 & 31/7 & -15/7 \\ 0 & 0 & -192/31 \end{bmatrix}$$

$x = x_1 \quad y = x_2 \quad z = x_3$ olarak adlandırırsak

$$\begin{bmatrix} 1 & 0 & 0 \\ 2/7 & 1 & 0 \\ 1/7 & -9/31 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} -32 \\ -31 \\ -23 \end{bmatrix}$$

$1 \cdot y_1 + 0 \cdot y_2 + 0 \cdot y_3 = -32$ $\qquad y_1 = -32$

$2/7 \cdot y_1 + 1 \cdot y_2 + 0 \cdot y_3 = -31$ $\qquad y_2 = -\frac{153}{7}$

$1/7 \cdot y_1 + (-9/31) \cdot y_2 + 1 \cdot y_3 = -23$ $\qquad y_3 = -\frac{5376}{217}$

$$\begin{bmatrix} 7 & 2 & -3 \\ 0 & 31/7 & -15/7 \\ 0 & 0 & -\frac{192}{31} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -32 \\ -\frac{153}{7} \\ -\frac{5376}{217} \end{bmatrix}$$

$7 \cdot x_1 + 2 \cdot x_2 - 3 \cdot x_3 = -32$ $\qquad x_1 = x = -2$

$0 \cdot x_1 + 31/7 \cdot x_2 - 15/7 \cdot x_3 = \frac{-153}{7}$ $\qquad x_2 = y = -3$

$0 \cdot x_1 + 0 \cdot x_2 - \frac{192}{31} \cdot x_3 = -\frac{5376}{217}$ $\qquad x_3 = z = 4$

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3) $f(x) = x - \sqrt{x^2 - 1}$ fonksiyonunun

mutlak hatası

$$\Delta f(x_0) \cong |f'(x_0)| \cdot \Delta x_0$$

$$\Delta f(x_0) \cong \left| 1 - \frac{2x_0}{2\sqrt{x_0^2 - 1}} \right| \cdot \Delta x_0 \quad //$$

Bağıl Hatası

$$\varepsilon_r(f(x_0)) = \frac{\Delta f(x_0)}{|f(x_0)|}$$

$$\varepsilon_r(f(x_0)) = \frac{\left| 1 - \frac{2x_0}{2\sqrt{x_0^2 - 1}} \right|}{|x_0 - \sqrt{x_0^2 - 1}|} \cdot \Delta x_0 \quad //$$

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4) $-(x_i+1, y_i+1) = f(x_i,y_i) + (x_{i+1}-x_i)\frac{df}{dx}\Big|_{(x_i,y_i)} + (y_{i+1}-y_i)\frac{df}{dx}\Big|_{(x_i,y_i)}$

$-f(x_i,y_i) = (x_{i+1}-x_i)\frac{df}{dx}\Big|_{(x_i,y_i)} + (y_{i+1}-y_i)\frac{df}{dx}\Big|_{(x_i,y_i)}$ (1)

$g(x,y)$ için uygulanırsa

$-g(x_i,y_i) = (x_{i+1}-x_i)\frac{dg}{dx}\Big|_{(x_i,y_i)} + (y_{i+1}-y_i)\frac{dg}{dx}\Big|_{(x_i,y_i)}$ (2)

$x_0 = 0,4$, $y_0 = -0,75$ için işleme başlayalım

$f(x_0,y_0) = -0,853$ $\quad \frac{df}{dx}\Big|_{(x_0,y_0)} = 2\cdot\cos(2x-y) - 1,2 = 0,8$

$g(x_0,y_0) = -0,028$ $\quad \frac{df}{dy}\Big|_{(x_0,y_0)} = -1\cdot\cos(2x-y) = -1$

$\frac{dg}{dx}\Big|_{(x_0,y_0)} = 1,6 - x = 0,64$

$\frac{dg}{dy}\Big|_{(x_0,y_0)} = 3y = -2,25$

1. ve 2. denklemde yerine koyarsak

$+0,853 = (x_1-x_0)\cdot 0,8 + (y_1-y_0)\cdot(-1)$

$+0,028 = (x_1-x_0)\cdot 0,64 + (y_1-y_0)\cdot(-2,25)$

$-0,8$ | $1,923 = 0,8x_1 - 1y_1$

$1,972 = 0,64x_1 - 2,25y_1$

$-1,5384 = -0,64x_1 + 0,8y_1$

$+\quad 1,972 = 0,64x_1 - 2,25y_1$

$0,4336 = -1,45y_1$ $\qquad x_1 = 2,118$ $\qquad y_1 = -0,299$