Signals and Systems Project

Phase 2

Instructor: Prof. Hamid Aghajan

Sharif University of Technology

Analysis of Phase-Amplitude Coupling during Olfactory Stimulation as a Biomarker for Alzheimer's Disease in EEG Signals

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Grading

The grading will be based on the following criteria, with a total of 0.6 points for Phase 1 and 1.4 points **(0.4 mandatory + 1.0 bonus)** for Phase 2.

Task	**Points**
Phase 1 — Total: 0.6 Points	
Data and Preprocessing	0.30
Power Analysis of EEG Signals	0.15
Online Q&A Session (Evaluation)	0.12
Clarity and Quality of Material (Code + Report)	0.03
Bonus: Writing your report in LaTeX	0.03
Phase 2 — Total: 1.4 Points	
Mandatory (0.4 Points):	
PAC Analysis Using MVL (Code + Explanation)	0.20
Interpretation of Results (Clarity and Insight)	0.10
Online Q&A Session (Evaluation)	0.10
Bonus (1.0 Points):	
PAC Analysis Using MI (Code + Explanation)	0.40
Interpretation and Comparison (MI vs. MVL)	0.20
Spatial Analysis (Electrode-wise PAC)	0.10
Online Q&A Session (Evaluation)	0.10
Additional Insights/Visualizations (e.g., polar plots)	0.10
Clarity and Quality of Bonus Material (Code + Report)	0.10

1 Phase-Amplitude Coupling (PAC) Analysis

1.1 Building on Phase 1: Toward PAC

In the first phase of the project, the EEG data recorded during an olfactory stimulation task were preprocessed to remove artifacts, extract events, and prepare the signals for analysis. This included filtering, re-referencing, ICA-based artifact rejection, epoching, and trial-level cleaning. The preprocessed data were then analyzed to investigate power dynamics over time using time-frequency decomposition (Short-Time Fourier Transform), focusing particularly on the theta and gamma bands. These bands were selected based on their relevance to sensory processing and memory-related neural activity.

That initial power analysis provided insight into the activation levels of specific brain rhythms under different conditions and subject groups (Healthy Control, MCI, Mild Alzheimer's Disease). However, power alone does not reveal how these rhythms interact. The second and final phase now focuses on **Phase-Amplitude Coupling (PAC)** to explore how the amplitude of high-frequency oscillations (e.g., gamma) may be modulated by the phase of slower rhythms (e.g., theta). PAC reflects cross-frequency communication in the brain and is a candidate biomarker for cognitive decline. This phase introduces two methods to quantify PAC: **Mean Vector Length (MVL)** as the required method, and **Modulation Index (MI)** as a bonus extension.

1.2 What is Phase-Amplitude Coupling?

Phase-Amplitude Coupling (PAC) is a form of cross-frequency coupling in which the amplitude envelope of a high-frequency brain oscillation is systematically modulated by the phase of a slower oscillation. In functional terms, it reflects how integrative processes (associated with low frequencies like theta) structure the timing of local, fast computations (associated with high-frequency gamma activity). For example, in healthy cognition, gamma bursts related to perception or memory encoding may consistently occur near the *trough* of a theta wave. In neurodegenerative conditions, this structure may degrade, leading to disorganized communication between brain regions.

PAC can be interpreted as a biological form of amplitude modulation (AM): the low-frequency phase acts as the carrier, and the high-frequency amplitude envelope is the modulated signal.

By studying how gamma amplitude fluctuates in alignment with theta phase, we aim to uncover differences in brain coordination patterns across olfactory stimuli (chocolate, rose) and subject groups (HC, MCI, AD). Two complementary methods—MVL and MI—are introduced for this analysis.

1.3 Phase and Amplitude Extraction: Wavelet vs. Hilbert Transform

To compute PAC, it is first necessary to extract the instantaneous *phase* of a low-frequency rhythm (such as theta) and the *amplitude envelope* of a higher-frequency rhythm (such as gamma). Two standard approaches for this are the Hilbert transform and wavelet decomposition, each with distinct strengths.

The **Hilbert transform** requires first applying a narrow bandpass filter to isolate the desired frequency component. The analytic signal is then derived, from which the instantaneous phase and amplitude can be directly calculated. This method provides high temporal precision and is computationally efficient, making it well-suited for clean, stationary frequency bands.

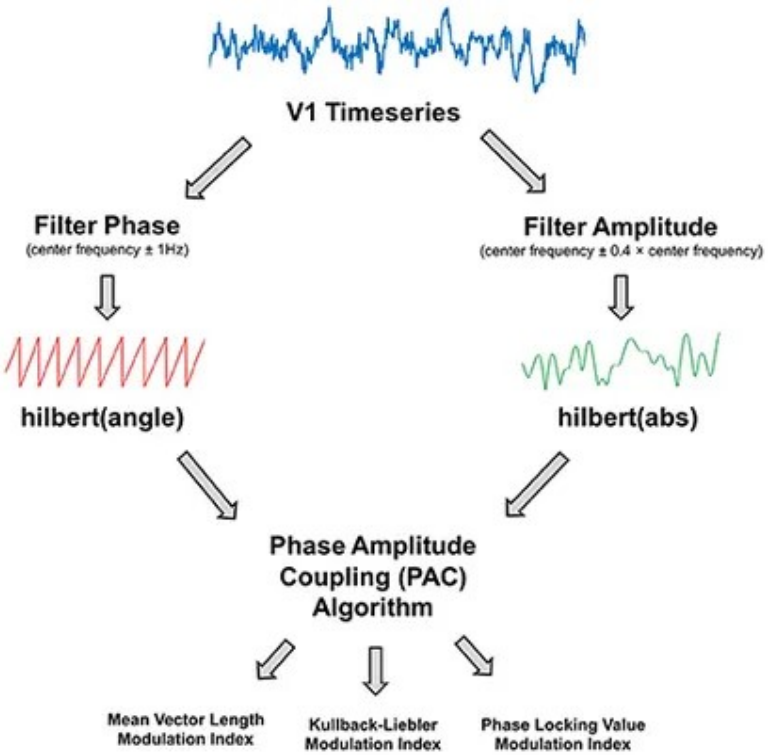

Figure 1: Illustration of the PAC analysis procedure using Hilbert Transform

In contrast, the **wavelet transform**, particularly using complex Morlet wavelets, offers a more flexible and robust time-frequency decomposition. It allows for smooth, continuous tracking of oscillatory activity with better frequency resolution, especially important for non-stationary signals like EEG. Wavelets can handle transient events and overlapping frequency content more gracefully than fixed-window filters.

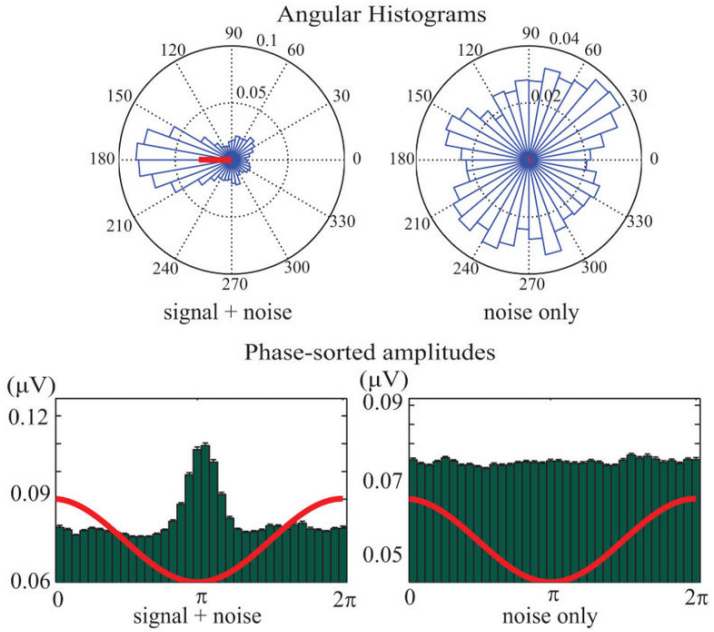

Figure 2: Wavelet Transform Histogram

Given the non-stationary nature of brain activity during cognitive tasks, and the goal of tracking PAC over time, **wavelet-based decomposition is recommended for this project**. However, if you are more comfortable using the Hilbert method, or wish to compare the outcomes of both, you may do so—provided you clearly justify your choice in your analysis.

1.4 PAC via Mean Vector Length (MVL)

Why Use MVL?

The Mean Vector Length (MVL) is a widely used and conceptually straightforward method to quantify PAC. It captures how consistently the amplitude of a high-frequency signal aligns with a specific phase of a slower rhythm. This method is ideal for time-resolved analysis of PAC and allows for direct comparison across trials, stimuli, and groups.

Procedure and Output

To begin, filter the EEG signal to isolate the theta band (4–8 Hz) for phase and the gamma band (30–50 Hz) for amplitude. Then compute the analytic signal using wavelet or Hilbert transform:

$$\phi_\theta(t) = \arg\left[\mathcal{H}(x_\theta(t))\right], \quad A_\gamma(t) = \left|\mathcal{H}(x_\gamma(t))\right|$$

Construct the complex vector:

$$z(t) = A_\gamma(t) \cdot e^{j\phi_\theta(t)}$$

Compute the MVL over a window of N samples:

$$\text{MVL} = \left| \frac{1}{N} \sum_{t=1}^{N} z(t) \right|$$

To track PAC changes over time, apply a sliding window (e.g., 1-second width, 95% overlap). Perform this for each trial, then average over:

- Theta and gamma frequency ranges
- Trials within each condition

The output is a time-resolved MVL curve per subject per condition. By averaging these across Trials, you generate group-level PAC curves for each stimulus (chocolate, rose) and cognitive group (HC, MCI, AD).

Interpretation

These curves are expected to show increased PAC strength (higher MVL) following odor onset. Healthy participants may display stronger and more sharply time-locked PAC than MCI or AD subjects. Comparisons between chocolate and rose conditions may also reveal stimulus-specific differences.

Questions to consider include:

- Is there a clear increase in PAC during stimulus delivery?
- Are coupling dynamics consistent across trials and subjects?
- Does the PAC pattern distinguish between cognitive states?

1.5 Bonus Part: PAC via Modulation Index (MI)

Why Use MI?

While MVL summarizes overall coupling strength, it does not reveal how gamma amplitude is distributed across the full phase cycle of a slower oscillation. The Modulation Index (MI), introduced by Tort et al., captures this structural aspect by measuring the deviation of the phase-amplitude distribution from uniformity. This enables a more detailed characterization of the coupling and supports visualization through polar (circular) histograms. MI is particularly useful when interpreting whether certain frequencies preferentially interact at specific phases and how this differs across brain regions or cognitive states.

Procedure and Output

To compute MI, filter the EEG data into the theta band (4–8 Hz) and gamma band (30–50 Hz), then extract instantaneous phase and amplitude using the Hilbert or wavelet transform.

Next, divide the phase range $(-\pi, \pi]$ into n equal-width bins (e.g., $n = 18$). For each bin θ_i, calculate the average gamma amplitude A_i corresponding to the theta phases that fall within that bin. Normalize the resulting amplitudes to obtain a probability distribution:

$$P(\theta_i) = \frac{A_i}{\sum_{j=1}^{n} A_j}$$

The Modulation Index is then computed as the Kullback-Leibler divergence between this empirical distribution and a uniform one:

$$MI = \frac{D_{KL}(P(\theta) \parallel U(\theta))}{\log(n)}$$

where D_{KL} represents the Kullback-Leibler divergence and $U(\theta)$ is the uniform distribution.

The expected output includes a scalar MI value and a polar histogram of amplitude vs. phase. In healthy participants, these polar plots may show a clear peak near the trough of theta, indicating structured coupling. In cognitively impaired groups, the distribution may appear flatter or more dispersed, reflecting degraded coupling.

Electrode Selection and Anatomical Considerations

For additional insight, consider examining PAC across electrode sites aligned with known processing pathways. Using frontal (e.g., **Fz**) and parietal (e.g., **Pz**) electrodes allows investigation of PAC along the rostro-caudal axis. Coupling where theta phase is from Fz and gamma amplitude from Pz may reflect a *feedforward* interaction, while the reverse could suggest *feedback* communication.

You can also explore local PAC at single electrodes such as **Fp1**, **Fp2**, **Fz**, or **Pz**, which may reflect localized sensory or integrative processes. Choosing electrode pairs based on task relevance—such as olfactory or memory-related regions—can help reveal whether PAC is spatially organized or disrupted across cognitive states.

Interpretation

MI-based analyses complement MVL by clarifying the structure and specificity of PAC. In addition to evaluating MI values across odors and subject groups, consider how phase preferences are organized across the scalp. Are gamma bursts aligned with particular theta phases? Do polar plots reveal consistent coupling in healthy subjects but disrupted or less focal patterns in MCI or AD?

You may also explore how MI-based observations correspond to MVL time courses—do strong MVL peaks coincide with sharply defined phase distributions? This multi-angle approach can offer deeper insights into how rhythmic coordination is altered in neurodegeneration.

1.6 Conclusion

This phase of the project explores how oscillatory components of brain activity interact during olfactory processing through the lens of Phase-Amplitude Coupling. By first establishing a dynamic baseline using MVL, and optionally expanding the analysis with MI, a multifaceted perspective on the organization of brain rhythms emerges.

MVL serves as a robust, time-resolved measure of PAC strength, offering immediate insight into whether coupling increases with stimulus onset. MI complements this view by providing a detailed structural profile of how amplitude is distributed across phase, and supports intuitive interpretation through visual plots.

Together, these methods allow for the exploration of coordinated brain dynamics with greater nuance than power analysis alone. They help reveal whether disruptions in PAC might serve as early electrophysiological indicators of cognitive impairment, and demonstrate the value of signal processing tools in neuroscience research.

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