Date: 14.01.2

Engineering Statistics and Complex variable

Statistics → data নিয়ে কাজ করে

- → collection
- → summarize / Organize
- → Analyze
- → Decision
- → Valid conclusion

Statistics is a branch of ~~engineering~~ science which deals with collection of data, summarizing analyzing and after analysing it makes decision and draw a valid conclusion.

Data
↳ variable

- quantitive (numerical)
  - → height, weight
  - → shoe size
  - Discrete (whole number)
    - → only take specific whole number
    - · number of student
  - continuous
    - height, weight, time.
- qualitive (non-numerical)
  - → hair colour
  - eye "

Mean, $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$

Mean use করবো যখন extreme value থাকবেনা

median " " " " " থাকবে।

Mod " " " qualitive value নিয়ে কাজ করবো।

1.1 → walpole

↳ overview

Step devialion method.

Abu Kawsere Sir

Math 281
16.01.24

$L = 50$

$D = 500$
$M = 1000$

$C = 100$

$2024 = \text{max} \times \text{IV}$

$N = \{1, 2, 3, \ldots\}$

$W = \{0, 1, 2, \ldots\}$

Integer / Integral number →

$I = \{\ldots -3, -2, -1, 0, 1, \ldots\}$

I_+ or $I^+ = \{1, 2, 3, \ldots\} = N$

Negative integers

I_- or $I^- = \{-1, -2, -3, \ldots\}$

neutral → 0

$x \geq 0$ → non negative $x > 0$ → n, positive

$x \leq 0$ → " posi

Rational number.

$Q = \{\frac{a}{b} : a, b \in I, b \neq 0\}$

Ujjal sir 21.01.2024

mean
- direct method
- Step deviation
- Short cut

median

where l is the lower limit of the median group.

$$\text{median}, \; m = l + \frac{\frac{N}{2} - c}{f} \cdot i$$

$$\text{mod} = l + \frac{f + f_{-1}}{2f + f_{-1} + f_1}$$

Ex-12

$$\text{mean} - \text{mode} = 3[\text{mean} - \text{median}]$$

সবচেয়ে বেশি value এর মান modal plus

10.10: Average deviation or mean deviation

Defn term
- mean deviation $= \dfrac{\sum f_i |x_i - \bar{x}|}{\sum f_i}$
- Standard "

Standard deviation: Standard deviation is defined as the square root of the mean of the square of the deviation from the arithmatic mean.

$$S.D. = \sigma = \sqrt{\frac{\sum f (x - \bar{x})^2}{\sum f}}$$

Note:

① The square of the SD. σ^2 is called variance

② σ^2 is called the second moment about the mean and is denoted by μ_2.

$$S.D. = \sigma = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$$

Note → coefficient of variation $= \frac{\sigma}{\bar{x}} \times 100\%$

Ex.12

x	f	$d = x_i - a$	$f.d$	$f.d^2$
6	3	~~-6~~ -3	~~-18~~ -9	27
7	6	~~-3~~ -2	-12	24
8	9	-1	-9	9
9	13	0	~~52~~ 0	0
10	8	+1	+8	8
11	5	+2	+10	20
12	4	+~~5~~ 3	+12	36
$\Sigma x =$ 63	$\Sigma = 48$		$\Sigma fd = 0$	$\Sigma fd^2 = 124$

$a = \frac{6+12}{2} = 9$

$$S.D = \sqrt{\frac{\Sigma fd^2}{n} - \left(\frac{\Sigma fd}{n}\right)^2}$$

$$= \sqrt{\frac{124}{48}} = 1.6$$

$\bar{x} = \frac{63}{48} = \frac{21}{16}$

Coefficient of variation $= \frac{\sigma}{\bar{x}} \times 100\%$

→ class time

CT → group data

↳ mean

median, mode, coefficient of variation

<u>Moment</u>: The rth moment of a variable x about the mean $\bar{x}$ is usually denoted by μ_r is given by

$$\mu_r = \frac{1}{N} \sum f_i (x_i - \bar{x})^r, \quad \sum f_i = N$$

The rth moment of a variable x about any point a is defined by

$$\mu_r' = \frac{1}{N} f_i (x_i - a)^r$$

$$\mu_0 = \frac{1}{N} \sum f_i (x - \bar{x})^0 = \frac{1}{N} \sum f_i = \frac{N}{N} = 1$$

↳ data প্রকৃতি আলোচনার জন্য কয়েকটি ভ বুঝাও।

* Rln b/w moments about mean and moment about any point:

$\mu_r = \frac{1}{N}$

$$\mu_2 = \mu_2' - \mu_1'^2$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1^3$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1^2 - 3\mu_1^4$$

$$\mu_r = \frac{1}{N}\Sigma f_i (x-\bar{x})^r = \frac{1}{N}\Sigma f_i\left[(x-a)-(\bar{x}-a)\right]^r$$

$$= \frac{1}{N}\Sigma f_i (X_i - d)^r$$ where

$X_i = x - a$ and

$d = \bar{x} - a$

$$= \frac{1}{N}\left[\Sigma f_i x_i^r - {}^rC_1 (\Sigma \ldots\right.$$

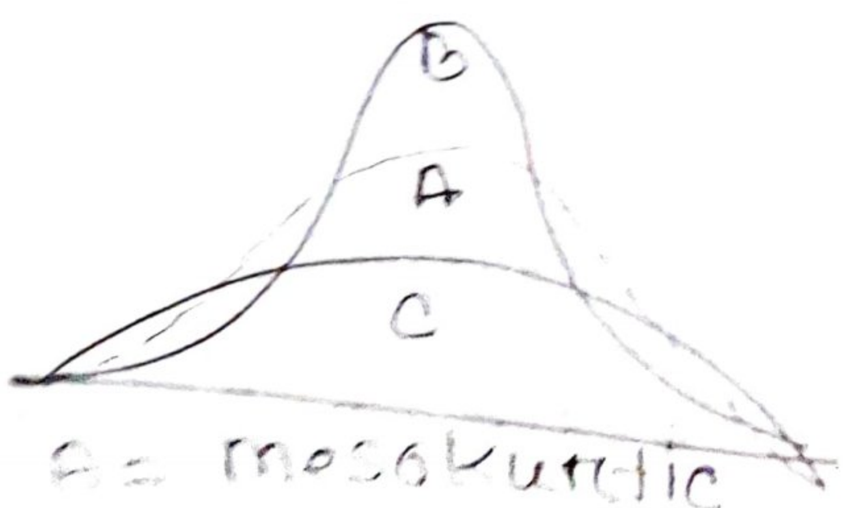
Skewness: (opposite of symmetry)

[illegible] symmetry then mean, [illegible] values are same

[illegible] moment, skewness, [illegible]

Kurtosis: It measures the degree of peakness

-ve skewness) (+ve skewness)



A = mesokurtic
B = leptokurtic
C = platykurtic

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$\mu_2 = \frac{\Sigma(x-\bar{x})^2}{N}$$

$$\mu_4 = \frac{\Sigma(x-\bar{x})^4}{N}$$

if

If

If

Exce

(1)

if $\beta_2 = 3$, the curve is normal or

If $\beta_2 > 3$; " " " peaked or Le

If $\beta_2 < 3$ " " " flat topped or platy kurtic

Excercise 10.1

① x_i	$x_i - \bar{x}$	d^2 $(x_i - \bar{x})^2$	d^3 $(x_i - \bar{x})^3$	d^4 $(x_i - \bar{x})^4$
52	2	4	8	16
57	7	49	343	2401
40	-10	100	-1000	10000
70	20	400	8000	160000
43	-7	49	-343	2401
40	-10	100	-1000	10000
65	15	225	3375	50625
35	-15	225	-3375	50625
48	-2	4	-8	16
$\Sigma = 0$		$\Sigma d^2 = 1156$	$\Sigma d^3 = 6000$	$\Sigma d^4 = 286084$

$\bar{x} \to 50$

$N = 9$

$$\mu_2 = \frac{1156}{9}$$

$$\mu_4 = \frac{286084}{9}$$

$$\beta_2 = 1.92$$

$<$

platykurtic

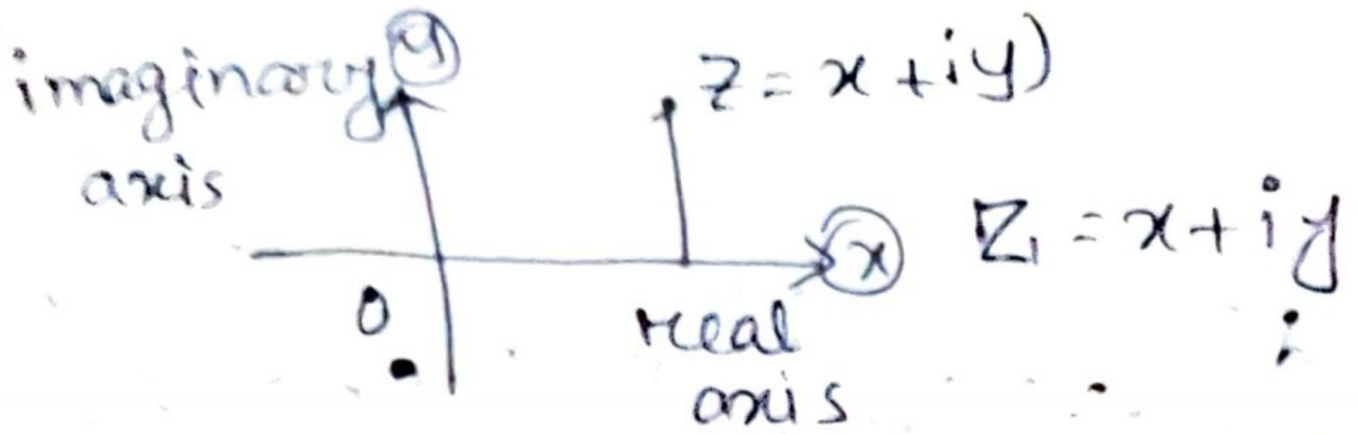
23.01.24

Kawsar sir.

$i^2 = -1$ $\quad x^2 + 4 = 0$

$i = \sqrt{-1}$ $\quad x = \pm 2\sqrt{-1}$

$= \pm 2i$



$z = x + iy$

$= r\cos\theta + ir\sin\theta$

$= r(\cos\theta + i\sin\theta)$

$= re^{i\theta}$

$e^{i\theta} = \cos\theta + i\sin\theta$

$e^{-i\theta} = \cos\theta - i\sin\theta$

$|e^{i\theta}| = |\cos\theta + i\sin\theta| = \sqrt{\cos^2\theta + \sin^2\theta} = 1$

$|z_1| = |re^{i\theta}|$

$= |r||e^{i\theta}|$

$= r$

$|z| = |x + iy| = e^{i\theta} r = r$ (circle represent করে.

$|x + iy| = r$

$\sqrt{x^2 + y^2} = r$

$x^2 + y^2 = r^2$

Descaste's rule ab sign- দিয়ে eqn এর imaginary roots সংখ্যা বের করবো।

$f(x) = x^4 + x^2 + 1$

$f(-x) = (-x)^4 + (-x)^2 + 1$

চিহ্নের পরিবর্তন নেই - কোনো real or imag. root নাই

De Moivre's thm:

- Statement: For all rational values of n:

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$$

Proof

Case-1: when n is a positive ~~number~~ integer.

We have $(\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2) =$

$\cos\theta_1\cos\theta_2 + i\cos\theta_1\sin\theta_2 + i\sin\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2$

$= (\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2) + i(\cos\theta_1\sin\theta_2 + \sin\theta_1\cos\theta_2)$

$= \cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)$

Also $(\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2)(\cos\theta_3 + i\sin\theta_3)$

$= \{\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)\}(\cos\theta_3 + i\sin\theta_3)$

~~Case 1: When~~

proceeding in this way, the product of the n factors we get,

$(\cos\theta_1 + i\sin\theta_1)(\cos\theta_2 + i\sin\theta_2) \ldots (\cos\theta_n + i\sin\theta_n)$

$= \cos(\theta_1 + \theta_2 + \ldots \theta_n) + i\sin(\theta_1 + \theta_2 + \ldots + \theta_n)$

$\Rightarrow (\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

Case-2: When n is a negative number

let, $n = -m$, where m is a positive integer

Now $(\cos\theta + i\sin\theta)^n = (\cos\theta + i\sin\theta)^{-m}$

$= \dfrac{1}{(\cos\theta + i\sin\theta)^m}$ $\Big|$ $a^{-n} = \dfrac{1}{a^n}$

$= \dfrac{1}{\cos m\theta + i\sin m\theta}$ [using case-I]

$= \dfrac{\cos m\theta - i\sin m\theta}{\cos^2 m\theta + \sin^2 m\theta}$

$(\cos\theta + i\sin\theta)^n$

$= \cos(m\theta) - i\sin(m\theta)$ $[\because m = -n]$

$= \cos(-n\theta) - i\sin(-n\theta)$

$= \cos n\theta + i\sin n\theta$

Case 3: When n is a fraction, +ve or -ve.

Let $n = \frac{p}{q}$, when q is a positive integer and p is any integer. +ve or -ve.

Now,

$$\left(\cos\frac{p}{q}\theta + i\sin\frac{p}{q}\theta\right)^q = \cos\left(q\cdot\frac{p}{q}\theta\right) + i\sin\left(q\cdot\frac{p}{q}\theta\right)$$

$$= \cos p\theta + i\sin p\theta$$

$$= (\cos\theta + i\sin\theta)^p$$

Taking qth root, we get

$\cos\frac{p}{q}\theta + i\sin\frac{p}{q}\theta =$ one of the values of $(\cos\theta + i\sin\theta)^{p/q}$

or, $\cos n\theta + i\sin n\theta =$ one of the values of $(\cos\theta + i\sin\theta)^n$

Thus, De moivre's Theorem is completely established for all rational values of n.

1. If $x_r = \cos\frac{\pi}{2^r} + i\sin\frac{\pi}{2^r}$ prove that

$$x_1 x_2 x_3 \ldots \infty = -1$$

2. $x_r = \cos\frac{\pi}{3^r} + i\sin\frac{\pi}{3^r}$ prove,

$$x_1 x_2 x_3 \ldots \infty = i.$$

given

$$x_r = \cos\frac{\pi}{3^r} + i\sin\frac{\pi}{3^r}$$

Now LHS $= x_1 x_2 x_3 \ldots \infty$

$$= \left(\cos\frac{\pi}{3^1} + i\sin\frac{\pi}{3^1}\right)\left(\cos\frac{\pi}{3^2} + i\sin\frac{\pi}{3^2}\right)$$

$$\left(\cos\frac{\pi}{3^3} + i\sin\frac{\pi}{3^3}\right)\ldots\infty$$

$$= \cos\left(\frac{\pi}{3^1} + \frac{\pi}{3^2} + \frac{\pi}{3^3} + \cdots \infty\right) + i\sin\left(\frac{\pi}{3^1} + \frac{\pi}{3^2} + \frac{\pi}{3^3} + \cdots \infty\right)$$

$$= \cos \frac{\frac{\pi}{3}}{\left(1 - \frac{1}{3}\right)} + i \sin\left(\frac{\frac{\pi}{3}}{1 - \frac{1}{3}}\right)$$

$$\cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = 0 + i1 = i = RHS$$

For exam →

If n be a positive integer, prove that (i) $(1+i)^n + (1-i)^n = 2^{n+1}\cos\frac{n\pi}{4}$

(ii) $(\sqrt{3}+i)^n + (\sqrt{3}-i)^n = 2^{n+1}\cos\frac{n\pi}{6}$

$$(\sqrt{3}+i)^n = r\cos\theta + ir\sin\theta$$
$$= r(\cos\theta + i\sin\theta)$$
$$= r\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$$

$\sqrt{3} = r\cos\theta$

$1 = r\sin\theta$

$\tan\theta = \frac{1}{\sqrt{3}}$

$\theta = \frac{\pi}{6}$

$u = r^2$

$r = 2$

$$(\sqrt{3}+i)^n = 2^n\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)^n$$
$$= 2^n\cos n\frac{\pi}{6} + i\sin\frac{n\pi}{6}$$

similarly

$$(\sqrt{3}-i)^n = 2^n\cos\frac{n\pi}{6} - i\sin n\frac{\pi}{6}$$

$$(\sqrt{3}+i)^n + (\sqrt{3}-i)^n = 2\cdot 2^n \cos\frac{n\pi}{6}$$

$$= 2^{n+1}\cos\frac{n\pi}{6}$$

30.01.2024

Kawser sir

$$1 = e^{i2n\pi} = \cos 2n\pi + i\sin n2\pi$$

$$\boxed{e^{i2n\pi} = \cos 2n\pi = 1}$$

$$\cos n\pi = (-1)^n$$
$$\sin n\pi = 0$$

$$-1 = e^{i(2n+1)\pi} = \cos(2n+1)\pi + i\sin(2n+1)\pi$$

$$i = e^{i\frac{\pi}{2}} = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$$

$$-i = e^{-i\frac{\pi}{2}} = \cos\frac{\pi}{2} - i\sin\frac{\pi}{2}$$

(*) Using De moivre's thm, solve the following

(i) $x^9 = 1$ (ii) $(x+1)^5 + (x-1)^5 = 0$

(iii) $x^4 + x^2 + 1 = 0$ ⇒ (i) (ii) এর form এ নেয়া যাবে, (x^2-1) নিয়ে সুন

(Pi)

$$(x+1)^5 = -(x-1)^5$$

$$\Rightarrow \left(\frac{x+1}{x-1}\right)^5 = -1$$

$$\Rightarrow \left(\frac{x+1}{x-1}\right)^5 = e^{i(2n+1)\pi}$$

$$\Rightarrow \left(\frac{x+1}{x-1}\right)^5 = \cos(2n+1)\pi + i\sin(2n+1)\pi$$

$$\Rightarrow \frac{x+1}{x-1} = \left\{\cos(2n+1)\pi + i\sin(2n+1)\pi\right\}^{1/5}$$

or, $\frac{x+1}{x-1} = \cos(2n+1)\frac{\pi}{5} + i\sin(2n+1)\frac{\pi}{5}$

or $\frac{x+1}{x-1} = \cos\theta + i\sin\theta$ | $\theta = (2n+1)\frac{\pi}{5}$

or, $\frac{x+1+x-1}{x+1-x+1} = \frac{\cos\theta + i\sin\theta + 1}{\cos\theta + i\sin\theta - 1}$

or, $x = \frac{2\cos^2\frac{\theta}{2} + i2\sin\theta/2\cos\theta/2}{-2\sin^2\theta/2 + i2\sin\theta/2\cos\theta/2}$

$= \frac{2\cos^2\theta/2 + 2i\sin\theta/2\cos\theta/2}{i^2 2\sin^2\theta/2 + 2i\sin\theta/2\cos\theta/2}$

$= \frac{\cancel{2}\cos\theta/2\,\cancel{(\cos\theta/2 + i\sin\theta/2)}}{\cancel{2}i\sin\theta/2\,\cancel{(i\sin\theta/2 + \cos\theta/2)}}$

$= \frac{\cos\theta/2}{i\sin\theta/2} = -i\cot\theta/2$

$\therefore x = -i\cos(2n+1)\frac{\pi}{10}$ where $n = 0, 1, 2, \ldots$

$-i\cos\frac{\pi}{10}, \ldots$

Find values of (i) $(1+i)^{1/5}$ (ii) $(-i)^{1/6}$

~~Let~~

$e^{i\frac{\pi}{2}} = i = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$

$e^{-i\frac{\pi}{2}} = -i = \cos\frac{\pi}{2} - i\sin\frac{\pi}{2}$

$-i \cdot 1 = e^{\frac{\pi}{2}i} e^{i2n\pi}$
$= e^{i(2n-1)\pi}$

$x^6 = 1 = -i$

$x = \sqrt[3]{1}$

$x^3 = 1$

(ii)

$-i = -i \cdot 1 = e^{\pi/2\, i} \cdot e^{i2n\pi}$
$= e^{i(2n-\frac{1}{2})\pi}$
$= e^{i(4n-1)\frac{\pi}{2}}$

$= \cos(4n-1)\frac{\pi}{2} + i\sin(4n-1)\frac{\pi}{2}$

$(-1)^6 = \left\{\cos(4n-1)\frac{\pi}{2} + i\sin(4n-1)\frac{\pi}{2}\right\}^{1/6}$

$= \cos(4n-1)\frac{\pi}{2} + i\sin(4n-1)\frac{\pi}{2}$

(i)

$(1+i)^{1/5}$ $\quad (a+bi)^{1/5}$ $\quad n = 0, 1, 2, \ldots$

Ans: $1 = r\cos\theta$
$1 = r\sin\theta$

$r^2 = 2$
$\therefore r = \sqrt{2} = 2^{1/2}$

Also $\tan\theta = 1$
$\theta = \pi/4$

$1 + i = r\cos\theta + i r\sin\theta$
$= r(\cos\theta + i\sin\theta$
$= re^{i\theta} \cdot 1$
$= re^{i\theta} \cdot e^{i2n\pi}$ $\quad [e^{i2n\pi} = 1]$

$$= re^{i(2n\pi+\theta)}$$

$$\cancel{re^{i(2n\pi+\frac{\pi}{4})}}$$

$$(1+i) = 2^{1/2}\, e^{i(8n+1)\frac{\pi}{4}}$$

$$= 2^{1/2}\left\{\cos(8n+1)\frac{\pi}{4} + i\sin(8n+1)\frac{\pi}{4}\right\}$$

$$(1+i)^{1/5} = 2^{1/10}\left\{\cos(8n+1)\frac{\pi}{20} + i\sin(8n+1)\frac{\pi}{20}\right\}$$

Analytic function:

$z = x+iy$ $\qquad$ $f(z) = z^3+z$

$$f(z) = u+iv$$
$$= u(x,y) + iv(x,y)$$

$$f(z) = z^3+z$$

$$= (x+iy)^3 + (x+iy)$$

$$= x^3 + 3x^2 iy + 3x(iy)^2 + (iy)^3 + x + iy$$

$$= x^3 - i3x^2y - 3xy^2 - iy^3 + x + iy$$

$$= (x^3 - 3xy^2 + x) + i(-3x^2y - y^3 + y)$$

$$= u(x,y) + iv(x,y)$$

• $y = f(x)$ $\quad$ $x = a$

$$\frac{dy}{dx} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

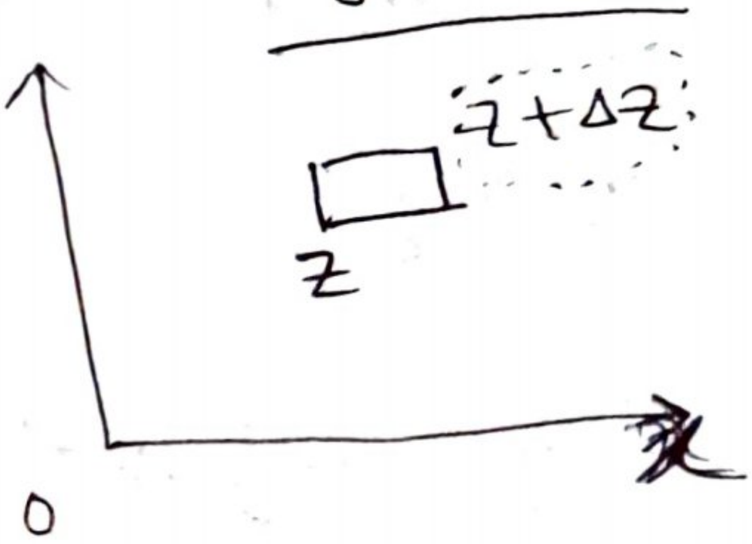
$x = a$ ⟹ derivative

$$Rf'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$Lf'(a) = \lim_{h \to 0} \frac{f(a-h) - f(a)}{-h}$$

$$w = f(z) = \frac{f(z+\Delta z) - f(z)}{\Delta z}$$

$$\frac{dw}{dz} = f'(z) = \lim_{\Delta z \to 0}$$



$\boxed{f(z) = u + iv}$

$$\left.\begin{aligned} u_x = v_y \quad & \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \\ u_y = -v_x \quad & \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \end{aligned}\right\} \text{Cauchy-Riemann (C-R) equation}$$

$z = x + iy$

$\Delta z = \Delta x + i\Delta y$

$\Delta z \to 0$

$\Delta x \to 0$

$\Delta y \to 0$

• CR eqn হচ্ছে একটা condn

• Is the func. $f(z) = z^3 + z$ analytic?

where $u = x^3 - 3xy^2 + x$

$v = 3x^2y - y^3 + y$

$$\frac{\partial u}{\partial x} = 3x^2 - 3y^2 + 1 \quad ; \quad \frac{\partial v}{\partial y} = 6xy$$

$\frac{\partial v}{\partial x} = 6xy$; $\frac{\partial v}{\partial y} = \cancel{3x^2}\ 3x^2 - 3y^2 + 1$

Since,

$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{x}$ so

the given func. is analytical.

Examine whether the following functions are analytic or not:

(i) $e^{-x}(\cos y - i \sin y)$

(ii) $\frac{1}{z}\ (z \neq 0)$ .... $f(z) = \frac{1}{z}$

$= \frac{1}{x+iy}$

(iii) $\bar{z}$

(iv) $2xy + i(x^2 - y^2)$

$= \frac{x - iy}{x^2 + y^2}$

$= \frac{x}{x^2+y^2} - i\frac{y}{x^2+y^2}$

$f(z) = \bar{z}$

$= x - iy$

$z = x + iy$

$\bar{z} = x - iy$

04.02.2024

Ujjal Sir

Co-relation ~~and~~ and regression

whenever 2 variables x and y are so related that an increase in the one . . . .

Karl pearson's coefficient of correlation:

$$r = \frac{\Sigma xy}{\sqrt{(\Sigma x^2)(\Sigma y^2)}} = \frac{p}{\sigma_x \sigma_y} = \frac{\text{Covariance}(x, y)}{\sqrt{\text{variance } x}\ \sqrt{\text{variance } y}}$$

$X = x - \bar{x}$; $Y = y - \bar{y}$

i.e. X, Y are the deviations measured from their respective means.

$$p = \left(\frac{\Sigma XY}{n}\right) = \text{co-variance.}$$

2. mean of x, $\bar{x} = \frac{548}{8} = 68.5$

mean of y, $\bar{y} = \frac{546}{8} = 68.25$

$\Sigma X^2 = 40$

$\Sigma Y^2 = 50.875$

$\Sigma XY = 23.75$

x	y	$X = x - 68.5$	$Y = y - 68.25$	X^2	Y^2	XY
67	65	-1.5	-3.25	2.25	10.5625	4.875
68	66	-0.5	-2.25	0.25	5.0625	1.125
64	67	-4.5	-1.25	20.25	2.25	6.75
68	67	-0.5	-1.25	0.25	2.25	2.25
72	68	3.5	-0.25	12.25	.0625	-0.875
70	69	1.5	0.75	2.25	.5625	1.125
69	71	0.5	2.75	0.25	7.5625	1.375
70	73	1.5	4.75	2.25	22.5625	7.125

$$r = \frac{23.75}{\sqrt{40 \times 50.875}}$$

$$= 0.52$$

Spearman's rank correlation coefficient

$$r = 1 - \frac{6\Sigma d^2}{n(n^2-1)}$$

Regression:

Line of regression:

$$\begin{cases} \frac{\Sigma y}{n} = a + b\frac{\Sigma x}{n} \\ \bar{y} = a + b\bar{x} \end{cases}$$

~~$\Sigma(x-$~~ This shows that $(\bar{x}, \bar{y})$ lie on line of regression ① shifting origion to $(\bar{x}, \bar{y})$

$$\Sigma(x-\bar{x})(y-\bar{y}) = a\Sigma(x-\bar{x}) + b\Sigma(x-\bar{x})^2$$

But, $\Sigma(x-\bar{x}) = 0$ i.e.

$$\Sigma(x-\bar{x})(y-\bar{y}) = b\Sigma(x-\bar{x})^2$$

$$b = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2} = \frac{\Sigma XY}{\Sigma X^2}$$

$$r = \frac{\Sigma XY}{\sqrt{\Sigma X^2 \, \Sigma Y^2}} = \frac{\Sigma XY}{n\sqrt{\frac{\Sigma X^2}{n} \, \frac{\Sigma Y^2}{n}}}$$

$$= \frac{\Sigma XY}{n \, \sigma_x \, \sigma_y}$$

x	y	$X = x - \bar{x}$	$Y = y - \bar{y}$	XY	X^2

[+] another way

$Y = a + bx$ be the eqn ab. line ab regression ab y on x, where a and b are given by following eqn.

$$\Sigma Y = na + b\Sigma x$$
$$\Sigma xy = a\Sigma x + b\Sigma x^2$$

Example 30 :

$\cancel{\Sigma XY = \Sigma X \times \Sigma Y}$

$\Sigma XY = 1022250$

$\Sigma X = 15000 \qquad \Sigma Y = 6800$

$\Sigma X^2 = 2272500$

$\Sigma Y^2 = 463025$

$\cancel{r = \dfrac{\Sigma xy}{\sqrt{\Sigma x^2 \times \Sigma y^2}}}$

$\cancel{= 99.65}$

another method →

$\Sigma y = na + b\Sigma x$

$\Rightarrow 6800 = 100a + 15000b$ — ①

$1022250 = 15000a + 2272500b$ — ②

$a = 53$

$b = 0.1$

2nd ne —

$\cancel{68 = a + 150b}$ — ①

$\cancel{6800 = a + 15000b}$

$b = \dfrac{\Sigma XY}{\Sigma X^2}$

What do you understand by error of prediction?

→ The deviation of the predicted value from the observed value is known as standard error of prediction. It is given by

$$E_{yx} = \sqrt{\frac{\Sigma(y - y_r)^2}{n}} \qquad y_r = a + bx$$

Table of error of prediction

① x	② y	③ $X = x - \bar{x}$	④ $Y = y - \bar{y}$	⑤ X^2	⑥ Y^2	⑦ XY	⑧ y_r	⑨ $y - y_r$	⑩ $(y - y_r)^2$

Number 6 error of prediction

06.02.2021

Kawser sir

Analytical func: A single valued function $f(x)$ which is differentiable at every point at region R is called as an analytical function at ~~z~~ in R

Define an alytic function find the condn so that $f(z) = u + iv$ is to be analyted.

$f(z) = u + iu = u(x, y) + i\,u(x, y)$ | $z = x + iy$, $\Delta dz = \Delta x + i\Delta y$

$f(z + dz)$

$u(x + \Delta xy + \Delta y) + (x + \Delta xy + \Delta y)$

$$f'(z) = \frac{d f(z)}{dz} = \lim_{\Delta z \to 0} \frac{f(z + \Delta z) - f(z)}{\Delta z}$$

2

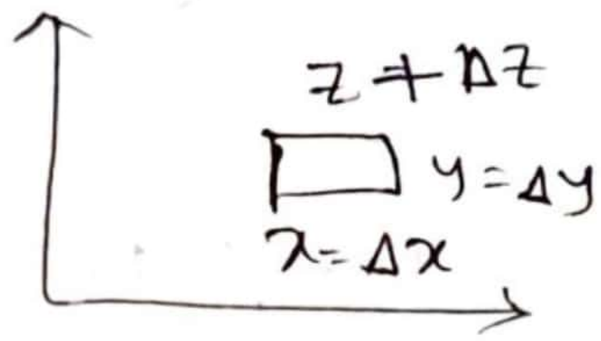
2

$\Delta z \to$

$$f'(z) = \frac{df(z)}{dz} = \lim_{\substack{\Delta x \to 0 \\ \Delta y \Rightarrow 0}} \frac{[u(x+\Delta x, y+\Delta y) - u(x,y)] + i[v(x+\Delta x, y+\Delta y) - v(x,y)]}{\Delta x + i\Delta y} \quad \cdots (1)$$

$y = x$

$\Delta x = 0$



Along x axis, $\Delta y = 0$ then

$$f'(z) = \lim_{\Delta x \to 0} \frac{[u(x+\Delta x, y) - u(x,y)] + i[v(x+\Delta x, y) - v(x,y)]}{\Delta x}$$

$$\frac{\partial u}{\partial x} = \lim_{h \to 0} \frac{u(x+h, y) - u(x,y)}{h}$$

$$\frac{\partial u}{\partial y} = \lim_{k \to 0} \frac{u(x, y+k) - u(x,y)}{k}$$

Along y axis, $\Delta x = 0$ then

$$f'(z) = \lim_{\Delta y \Rightarrow 0} \frac{[u(x, y+\Delta y) - u(x,y)] + i[v(x, y+\Delta y), v(x,y)]}{i\Delta y}$$

$$= \frac{i}{i}\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y}$$

$$= \frac{\partial u}{\partial x}$$

Since the given func. is analytic, so (2) and (3) must be equal.

$$\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} = \frac{1}{i}\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y}$$

$$= -i\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y}$$

Equating real & imaginary part we get

$$\left.\begin{aligned} \frac{\partial u}{\partial x} &= \frac{\partial v}{\partial y} \\ \frac{\partial v}{\partial x} &= -\frac{\partial u}{\partial y} \end{aligned}\right\} \quad \left.\begin{aligned} \frac{\partial u}{\partial x} &= \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} &= -\frac{\partial v}{\partial x} \end{aligned}\right\} \quad \begin{aligned} u_x &= v_y \\ u_y &= -v_x \end{aligned}$$

which are known as Cauchy-Riemnann (C-R) equation

$$f(z) = u + iv$$

$$u = u(x, y)$$

$$v = v(x, y)$$

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

$$\frac{\partial v}{\partial y} = -\frac{\partial u}{\partial x}$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

↳ Laplacian ~~equation~~ operator

so,

$$\nabla^2 u = 0$$

$$\nabla^2 v = 0$$

analytic func.?
" for check] CT
harmonic u? তার conjugate harmonic

- Find the constants a, b, c if $f(z) = x + ay + i(bx + cy)$ is analytic

Here

$u = x + ay$

$v = bx + cy$

$\frac{\partial u}{\partial y} = a$

$\frac{\partial v}{\partial y} = c$

Since the given function is analytic so,

$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$, $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

or $1 = c$ $\quad a = -b$

$c = 1$ $\quad a = -b$; b may be any value.

- Determine b such that $e^{bx}\cos 5y$ is harmonic

$$\frac{\partial u}{\partial x} = be^{bx}\cos 5y$$

$$\frac{\partial^2 u}{\partial x^2} = b^2 e^{bx}\cos 5y$$

$$\frac{\partial u}{\partial y} = \cancel{5\cos 5y} - e^{bx}\, 5\sin 5y$$

$$\frac{d^2 u}{dy^2} = -e^{bx}\, 25\cos 5y$$

Since u is a harmonic function, u satisfies Laplace's eqn

i.e. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

or $b^2 e^{bx} \cos 5y - 25 e^{bx} \cos 5y = 0$

or $(b^2 - 25) e^{bx} \cos 5y = 0$

or $b^2 - 25 = 0$ $[\because e^{bx} \cos 5y \neq 0]$

$b = \pm 5$.

. Show that $u = 3x^2y + 2x^2 - y^2 - 2y^2$ is harmonic function. Also find its conjugate harmonic function.

$$\frac{\partial u}{\partial x} = 6xy + 4x$$

$$\frac{\partial^2 u}{\partial x^2} = 6y + 4 \quad \text{--- (1)}$$

$$\frac{\partial u}{\partial y} = 3x^2 - 3y^2 - 4y$$

$$\frac{\partial^2 u}{\partial y^2} = 0 - 6y - 4 \quad \text{--- (2)}$$

adding (1) and (2), we get,

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

satisfies Laplace equation

Let v be the conjugate of u

We have $du =$

$$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz$$

$$= -\frac{\partial u}{\partial y} dx + \frac{\partial u}{\partial x} dy$$

$$= -(3x^2 - 3y^2 - 4y)\, dx + (6xy + 4x)\, dy$$

$$dv = -3x^2 dx + 3y^2 dx + 4y\, dx + 6xy\, dy + 4x\, dy$$

$$= -3x^2 dx + 3\, d(xy^2) + 4\, d(xy)$$

Integrating it, we set

$$v = -3 \cdot \frac{x^3}{3} + 3xy^2 + 4xy + c$$

<u>Milnes method</u>

$$\frac{\partial u}{\partial x} = 6xy + 4x = u_1(x, y) \text{ say}$$

$$\frac{\partial u}{\partial y} = 3x^2 - 3y^2 - 4y = u_2(x, y) \text{ say}$$

$$f(z) = \frac{\partial u}{\partial x} + i \frac{\partial u}{\partial y}$$

$$= u_1(x, y) - i\, u_2(x, y)$$

$f = u + iv$

$$f'(z) = u_1(z, 0) - u_2(z, 0)$$

$$f'(z) = v_1(z, 0) + i\, (v_2(z, 0))$$

By Milne's method

we have,

$$f'(z) = u_1(z,0) - u_2(z,0)$$

$$= 0 + 4z - i\,3z^2$$

Integrating it we get.

$$f(z) = \frac{2z^2}{2} - iz \cdot \frac{z^3}{3} + c$$

$$c = c_1 + ic_2$$

$$f(z) = 2(x+iy)^2 - i(x+iy)^3 + c_1 + ic_2$$

$$= 2(x^2 - y^2 + i2xy) - i(x^3 + i3x^2y - 3xy^2 - iy^3) + c_1 + c_2$$

$$= (2x^2 - 4^2 + 3x^2y - y^3 + 4) + i(4x + x^3 + 3xy^2 + c_2)$$

$$\therefore v = 4xy - x^3 + 3xy^2 + c_2$$

11.02.2024

H.W Ujjal sir

(6)

S.No	x	y	xy	x^2
1	40	2.5	100	1600
2	70	6	420	4900
3	50	4.5	225	2500
4	60	5	300	3600
5	80	4.5	360	6400
6	50	2	100	2500
7	90	5.5	495	8100
8	40	3	120	1600
9	60	4.5	270	3600
10	60	3	180	3600
Total =	600	40.5	2570	38400

Let $y = a + bx$ be the line of regression of y on x, where a and b are given by the following eqn:

$$\Sigma y = na + b\Sigma x \quad \text{or, } 40.5 = 10a + 600b \quad \text{--- (i)}$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2 \quad \text{or, } 2570 = 600a + 38400b \quad \text{--- (ii)}$$

$$a = \frac{11}{20}, \quad b = \frac{7}{120}$$

line eqn, $y = \frac{11}{20} + \frac{7}{120}x$

$y = 0.55 + 0.0583x$ (regression line)

~~The~~ The eqn. of line of regression of y on x is

$$y = \frac{7x}{120} + \frac{11}{20} = y_n$$

Serial no	x	y	y_n	$y - y_n$	$(y-y_n)^2$
1	40	2.5	$\frac{173}{60}$	$-23/60$	$529/3600$
2	70	6	$139/60$	$221/60$	13.57
3	50	4.5	$52/15$	$31/30$	$961/900$
4	60	5	$81/20$	$19/20$	$361/400$
5	80	4.5	$313/60$	$-43/60$	$1849/3600$
6	50	2	$52/15$	$-22/15$	$484/225$
7	90	5.5	$29/5$	$-3/10$	$9/100$
8	40	3	$173/60$	$7/60$	$49/3600$
9	60	4.5	$81/20$	$9/20$	$81/400$
10	60	3	$81/20$	$-21/20$	$441/400$
					19.76

$$E_{yx} = \sqrt{\frac{(y-y_n)^2}{n}} = \sqrt{\frac{19.76}{10}}$$

$$= 1.4$$

11.02.2021
Ujjal sir

$2x + 3y + 5 = 0$

$x + 4y + 8 = 0$

$r = \sqrt{b_{yx} \times b_{xy}}$

$\hookrightarrow -1 < r < 1$

b_{yx} ~~b_{xy}~~ $= r \frac{\sigma_y}{\sigma_x}$

$b_{xy} = r \frac{\sigma_x}{\sigma_y}$

$y - \bar{y} = r \cdot \frac{\sigma_y}{\sigma_x}(x - \bar{x})$

$x - \bar{x} = r \frac{\sigma_x}{\sigma_y}(y - \bar{y})$

Find the corelation coefficient between x and y when the lines of regression are $2x - 9y + 6 = 0$ and $x - 2y + 1 = 0$

Ans: $2x - 9y + 6 = 0$

y on x: $y = \frac{2}{9}x + \frac{2}{3}$

$\therefore b_{yx} = \frac{2}{9}$

$x - 2y + 1 = 0$

x on y: $x = 2y + 1$

$\therefore b_{xy} = 2$

$r = \sqrt{b_{yx} \times b_{xy}} = \sqrt{\frac{2}{9} \times 2} = \sqrt{\frac{4}{9}} = \frac{2}{3} = 0.67$

(13) In a partially destroyed labrotory record at an analysis at corelation data the following results only are legible

(illegible) $\sigma_x^2 = 9$

Var at $x = 9$

regression eq^{n}s: $8x - 10y + 66 = 0$

$40x - 18y - 214 = 0$

(i) $\bar{x}, \bar{y}$ (ii) σ_y (iii) r

$8x - 10y + 66 = 0$

$y = \frac{8}{10}x + \frac{66}{10}$

y on x:

$b_{yx} = \frac{8}{10}$

$40x - 18y - 214 = 0$

$x = \frac{18}{40}y + \frac{214}{40}$

x on y:

$b_{xy} = \frac{18}{40}$

$r = \sqrt{\frac{8}{10} \times \frac{18}{40}}$

$= 0.6$

$b_{yx} = r\frac{\sigma_y}{\sigma_x}$

$\sigma_x = \cancel{\frac{r\sigma_y}{b_{yx}}}$

$\sigma_y = \frac{b_{yx} \cdot \sigma_x}{r}$

$= 4$

$y - \bar{y} = 0.6 \times \frac{4}{3}(x - \bar{x})$

$= \frac{4}{5}(x - \bar{x})$ $\Rightarrow \frac{5}{4}(y - \bar{y}) = x - \bar{x}$

$x - \bar{x} = 0.6 \times \frac{3}{4}(y - \bar{y})$

$= \frac{9}{20}(y - \bar{y})$

$0 = \cancel{(y - \bar{y})}\left(\cancel{\frac{5}{4}} - \cancel{\frac{9}{20}}\right)$

- Experiment
- Event
- Sample space
- Exaustive event
- mutually exclusive event
- Conditional probability
- Baye's Theorem.

$$\text{Probability} = \frac{\text{Possible outcomes}}{\text{Total outcomes}}$$

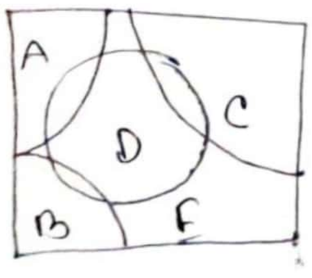
যত element এখানে ফিক্সড থাকা হবে তত exaustive.

Probability of the event A given that the event B is already happened

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$ conditional

* Independent event $P(A \cap B) = P(A) \times P(B)$

* Exclusive event $P(A \cap B) = 0$



Baye's Thm: (~~Total~~ term ও আসবে)

$$P(B_i/A) = \frac{P(B_i)\,P(A/B_i)}{\sum_{i=1}^{n} P(B_i)\,P(A/B_i)}$$

$$P(B_2/A) = \frac{P(B_2)\,P(A/B_2)}{P(B_1)\,P(A/B_1) + P(B_2)\,P(A/B_2) + P(B_3)\,P(A/B_3)}$$

Ques: (fig1)

	Defect
$P(A) = 0.2$	$A \to 20\% \to 2\%$
$P(B) = 0.25$	$B \to 25\% \to 3\%$
$P(C) = 0.35$	$C \to 35\% \to 4\%$
$P(E) = 0.2$	$E \to 20\% \to 5\%$

$P(A/D) = 0.02$ $P(B/D) = 0.03$

$P(C/D) = 0.04$ $P(E/D) = 0.04$

$$P(D/B) = \frac{P(D)\,P(B/D)}{\cancel{P(D)\,[P(A/D+}}$$

480 page

~~Ex-e~~ Defect)

A → 25% → 5%

B → 35% → 4%

C → 40% → 2%

$P(A) = 0.25$ $\quad P(D|A) = 0.05$

$P(B) = 0.35$ $\quad P(D|B) = .04$

$P(C) = .4$ $\quad P(D|C) = .02$

$$P(B/D) = \frac{P(B)\,P(D/B)}{\frac{1}{80} + \frac{7}{500} + \frac{1}{125}}$$

$$= \frac{.35 \times .04}{\frac{69}{2000}}$$

$$= .405$$

13.07.2024

$f(z) = u + iv$

$f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$ $\quad \frac{\partial v}{\partial y}$

$= \frac{\partial u}{\partial x} - i\frac{\partial u}{\partial y}$

$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$

$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

$\frac{\partial u}{\partial x} = u_1(x,y)$

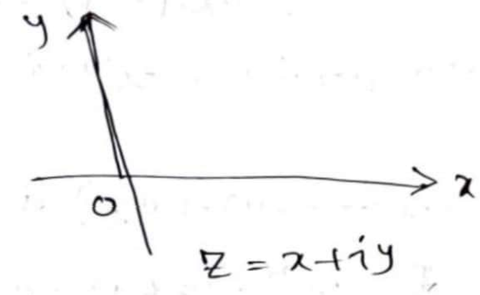
$\frac{\partial u}{\partial y} = u_2(x,y)$

$f'(z) = v_1(x,y) + iv_2(x,y)$

$x = z,\ y = 0$

$= u_1(\ ) - iu_2(x,y)$

$= u_1(z,0) + iu_2(z,0)$



$z = x + iy$

$z = x$

$w = \phi + i\psi$

$= \phi(x,y) + i\psi(x,y)$

complex potential function

Stream function

Velocity potential

* If $w = \phi + i\psi$ represents, the complex potential for an electric field and $\psi = 3x^2y - y^3$ the potential function ϕ

given, $\psi = 3x^2y - y^3$

$\therefore \frac{\partial \psi}{\partial y} = 3x^2 - 3y^2 = \psi_1(x,y)$ (say)

$$\frac{\partial \psi}{\partial x} = 6xy = \psi_2(x,y), \text{ say}$$

By milne's method, we have,

$$w'(z) = \psi_1(z,0)^{x=z,\ y=0} + i\,\psi_2(z,0)$$

$$= 3z^2 + i \times 0$$

$$= 3z^2$$

Integrating it we get,

$$w(z) = 3 \cdot \frac{z^3}{3} + c = z^3 + c$$

$$= (x+iy)^3 + c_1 + ic_2$$

or, $\phi + i\psi = x^3 + i\,3x^2y - 3xy^2 - iy^3 + c_1 + c_2 i$

$$= (x^3 - 3xy^2 + c_1) + (3x^2y - y^3 + c_2)$$

$\phi = x^3 - 3xy^2 + c_1$; is the required potential function.

In a two dimentional flow at a fluid, the velocity potential $\phi = x^2 - y^2$ find the stream function ψ.

prove that $u = e^{-x}(x\sin y - y\cos y)$ is harmonic. Also find v such that $f(z) = u + iv$ is analytic.

given,

$$u = e^{-x}(x\sin y - y\cos y)$$

$$\frac{\partial u}{\partial x} = e^{-x}(1\sin y - 0) + (-e^{-x})(x\sin y - y\cos y)$$

$$= e^{-x}(\sin y - x\sin y + y\cos y)$$

$$= u_1(x,y), \text{ say}$$

$$\frac{\partial^2 u}{\partial x^2} = e^{-x}(0 - 1.\sin y + 0) + (e^{-x})(\sin y - x\sin y + y\cos y)$$

$$= e^{-x}(-2\sin y + x\sin y - y\cos y) \quad \text{--- (1)}$$

Also, $\frac{\partial u}{\partial y} = e^{-x}(x\cos y - 1.\cos y + y\sin y)$

$$= u_2(x,y), \text{ say}$$

$$\frac{\partial^2 u}{\partial y^2} = e^{-x}(-x\sin y + \sin y + 1.\sin y + y\cos y)$$

$$= e^{-x}(-x\sin y + 2\sin y + y\cos y) \quad \text{--- (2)}$$

adding (1) and (2) we get,

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

or $\nabla^2 u = 0$

since u satisfies laplace's eqn, so u is a harmonic function.

By milne's theorem we have,

$$f'(z) = u_1(z,0) + i u_2(z,0)$$

$$= 0 - i e^{-z}(z \cdot 1 - 1 + 0)$$

$$= -i e^{-z}(z-1)$$

Integrating it, we get,

$$f(z) = -i \int z e^{-z} dz + i \int e^{-z} dz$$

$$= -i \cdot z \cdot \frac{e^{-z}}{(-1)} + i \int \cancel{1 \frac{e^{-z}}{(-1)}} + i \int \cancel{e^{-z} dz} + c$$

$$= iz e^{-z} + c$$

$$= i(x+iy) e^{-(x+iy)} + c_1 + ic_2$$

$$= e^{-x}(ix - y)(\cos y - i \sin y) + c_1 + ic_2$$

$$= e^{-x}(ix\cos y + x\sin y - y\cos y - iy\sin y) + c_1 + ic_2$$

$$= e^{-x}(x\sin y - y\cos y) + i e^{-x}(x\cos y + y\sin y) + c_1 + ic_2$$

~~∂^2/∂~~ Equating imaginary part, we get,

$$e^{i\theta} = \cos\theta + i\sin\theta$$
$$e^{-i\theta} = \cos\theta - i\sin\theta$$

$$v = e^{-x}(x\cos y + y\sin y) + c_2$$

is ~~the~~ the required value.

III Show that an analytic function w/ constant absolute value/ modulus is constant

$$f(z) = u + iv$$

$$|f(z)| = \sqrt{u^2+v^2} = k$$

$$u^2 + v^2 = k^2 \quad \text{—— (1)}$$

$$2uu_x + 2vv_x = 0$$

$$uu_x - vu_y = 0 \quad \text{—— (2)}$$

$u_x = v_y$

$u_y = -v_x$

$$uu_y + vv_y = 0$$

$$uu_y + vu_x = 0 \quad \text{—— (3)}$$

(2) $\times u$ + (3) $\times v \Rightarrow$

$$(u^2+v^2)u_x = 0$$

(2) $\times(-v)$ + (3) $\times u \Rightarrow (u^2+v^2)u_y = 0$

If, $k^2 = u^2 + v^2 = 0$

$\Rightarrow u^2 = 0$, $v^2 = 0$

$u = 0$, $v = 0$

$u_x = 0$, $u_y = 0$

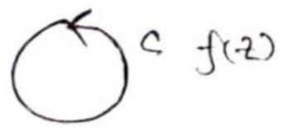
Integrating it u = constant.

Chapter 4:

Complex integration

State and prove cauchy's Theorem / Cauchy's integral theorem.

Statement: If $f(z)$ is analytic inside and on a simple closed curve c then $\oint_c f(z)\,dz = 0$



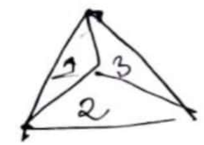
Cauchy's Integral formula: If $f(z)$ is analytic inside and on a simple closed curve c. and 'a' is any point within c.

Then $\oint_c \frac{f(z)\,dz}{z-a} = 2\pi i f(a)$

$F(z) = \frac{f(z)}{z-a}$

Ujjal sir
20.02.2024

Random variable: Experiment: Three fair coins & crossed together



1st coin	H_1	T_1
2nd coin		
H_2	H_1H_2 (2)	T_1H_2 (1)
T_2	H_1T_2 (1)	T_1T_2 (0)

Random var → 0, 1, 2

1st & 2nd coin

	H_1H_2	T_1H_2	H_1T_2	T_1T_2
H_3	$H_1H_2H_3$ (3)	$T_1H_2H_3$ (2)	$H_1T_2H_3$ (2)	$T_1T_2H_3$ (1)
T_3	$H_1H_2T_3$ (2)	$H_2T_1T_3$ (1)	$H_1T_2T_3$ (1)	$T_1T_2T_3$ 0

3rd coin

Sample space

Random var → 0, 1, 2, 3

X → number of head

Probability distribution

$\forall$ → for all

x	0	1	2	3
$P(X=x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

$\frac{6}{4} = \frac{3}{2}$

number similar mean

$\sum_{\forall x} P(X=x) = 1.$

Expected value: $\mu = E(x) = \sum_{\forall x} x_i P(x=x_i)$

$= 0\times\frac{1}{8} + 1\times\frac{3}{8} + 2\times\frac{3}{8} + 3\times\frac{1}{8}$

$= \frac{1}{8}(0+3+6+3)$

$= \frac{12}{8} = 3/2$

$$Var(x) = E(x^2) - [E(x)]^2$$

$$= 0^2 \times \frac{1}{8} + 1^2 \times \frac{3}{8} + 2^2 \times \frac{3}{8} + 3^2 \frac{1}{8} - \left(\frac{3}{2}\right)^2$$

$$\sigma^2 = \frac{3}{4}$$

Standard deviation : $\sigma = \sqrt{\frac{3}{4}}$

x	0	1	2	3
$P(X=x)$	1/8	3/8	3/8	1/8

$$p(X=x) = \begin{cases} \frac{1}{8} & ; x = 0, 3 \\ \frac{3}{8} & ; x = 1, 2 \end{cases}$$

Random variable: A random variable is a function that associates a real number with each element in the sample space. We shall use a capital letter, say X, to denote the random variable and its corresponding small letter x, in the cases for one its values.

Ques: $\boxed{10}$ h.w

book: Walpole 164

A shipment 8 similar microcomputers to a ~~detail~~ retail outlet contains 3 $\boxed{4}$ that are defective. If a school makes a random purchase of 2 of these computers. Find the probability $\boxed{3}$ distribution

Ans: $X : 0, 1, 2$

~~$P(X=0)$~~ =

x	0	1	2
$P(X=x)$	$\frac{10}{28}$	$\frac{15}{28}$	$\frac{3}{28}$

$$P(X=0) = \frac{{}^3C_0 \times {}^5C_2}{{}^8C_2} = 5/14$$

$$P(X=1) = \frac{{}^3C_1 \times {}^5C_1}{{}^8C_2} = 15/28$$

$$P(X=2) = \frac{{}^3C_2 \times {}^5C_0}{{}^8C_2} = 3/28$$

$$\mu = E(x) = \sum_{\forall x} x_i \, P(X = x_i)$$

$$= 0 \times \frac{5}{14} + 1 \times \frac{15}{28} + 2 \times \frac{3}{28}$$

$$= \frac{3}{4}$$

$$Var(X) = E(x^2) - [E(x)]^2$$

$$= 0^2 \times \frac{5}{14} + 1^2 \times \frac{15}{28} + 2^2 \times \frac{3}{28} - \left(\frac{3}{4}\right)^2$$

$$\sigma^2 = \frac{45}{112}$$

$\sigma = 0.63$

Probability Function: The set of ordered pairs $(x, f(x))$ is a probability function of the ~~disaster~~ discrete random variable x, if for each possible outcome x,

1. $f(x) \geq 0$
2. $\sum_{\forall x} f(x) = 1$
3. $p(X = x) = f(x)$

<u>Cumulative distribution function:</u>

x	0	1	2	3
$P(X=x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$
$F(x)$	$\frac{1}{8}$	$\frac{4}{8}$	$\frac{7}{8}$	1

$F(x) = \sum P(X \leq x)$

$F(2) = \sum P(X \leq 2)$

$= 7/8$

$F(3) = 1$

Two ~~technical~~ tetrahedral dice have the number 1, 2, 3, 4 on their faces. The dice are thrown together. Let S = the sum of their two scores and D = the difference ~~left~~ betn two scores)

a) Show that $(S = 6) = 3/16$

b) Find the probability distn for the variable S and D.

c) find $P(S \leq 7)$

d) show that, $P(D=1) = 3/8$

e) Find $P(D \geq 2)$

Sample space for S:

2nd dice \ 1st dice	1	2	3	4
1	2	3	4	5
2	3	6	5	6
3	4	5	6	7
4	5	6	7	8

Probability distn for

X	2	3	4	5	6	7	8
$P(X=x)$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

Sample space for D:

2nd dice \ 1st dice	1	2	3	4
1	0	1	2	3
2	1	0	1	2
3	2	1	0	1
4	3	2	1	0

Probability distn for

X	0	1	2	3
$P(X=x)$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{3}{16}$	$\frac{2}{16}$

The discrete random variable X has c.d.f. $F(x)$ defined by

$$F(x) = \frac{2+x}{7} \; ; \; x = 1, 2, 3, 4, 5$$

a) Find $P(x \leq 3)$

b) Show that $P(X = 4) = \frac{1}{7}$

c) Find probability distribution.

Ans

$$P(X=1) = F(1) = \frac{2+1}{7} = \frac{3}{7}$$

$$P(X=2) = F(2) - F(1) = \frac{4}{7} - \frac{3}{7} = \frac{1}{7}$$

$$P(X \leq 3) = F(3) = \frac{2+3}{7} = 5/7$$

X	1	2	3	4	5
$P(X=x)$	$\frac{3}{7}$	$\frac{1}{7}$	$1/7$	$1/7$	$1/7$
$F(x)$	$\frac{3}{7}$	$\frac{4}{7}$	$\frac{5}{7}$	$\frac{6}{7}$	$7/7$

$2P \to \frac{14}{40}$

$5P \to 11/40$

Probability distn for 2P

X	-2	4
$P(X=x)$	$\frac{21}{40}$	$\frac{14}{40}$

$$E(x) = \sum_{\forall x} x_i P(X = x_i)$$

$$= -5 \times \frac{29}{40} + 10 \times \frac{11}{40}$$

$$= \cancel{-1/24} = -7/8$$

5P

X	-5	4
$P(X=x)$	$\frac{29}{40}$	$\frac{11}{40}$

$$E(x) = \sum_{\forall x} x_i P(X = x_i)$$

$$= -2 \times \frac{21}{40} + 4 \times \frac{19}{40}$$

$$= 17/20$$

$\boxed{\square}$ $E(ax) = a\,E(x)$

$E(ax+b) = a\,E(x) + b$

$Var(ax) = a^2\, var(x)$

$Var(ax+b) = a^2\, var(x)$

$var(3x+1) = 3^2\, Var(x)$

$= 3^2 [E(x^2) - (E(x))^2]$

Udal sir. 25.02.24

not all random variable can be handled in a same way.

Probability function →

~~P(r~~ P(r)

Ex27 Ten percent screw

imp for exam } at best (সর্বোচ্চ) 0, 1, 2
at least (সর্বনিম্ন) 1 - (0, 1, 2)

Ex 29 10% bolt

Ex 34 ** security

defective $p = 0.2$ $n = 4$

non " $q = 0.8$

① $r = 1$

~~P(r~~ $P(1) = {}^nC_r p^r . q^{n-r}$

$= {}^4C_1 (0.2)^1 \times (0.8)^3$

$= 0.4096$

② $r = 0$ $P(0) = {}^4C_0 (0.2)^0 \times (0.8)^4$

$= 0.4096$

③ $r = 2$; $P(2) = {}^4C_2 \times (0.2)^2 \times (0.8)^2 = 0.1536$

at most $2 = P(0) + P(1) + P(2)$
$= 0.9728$

। n এর মান 35 এর উপরে গেলে poison এ— যাবো।

27.02.2024
Kawsere sir

MATH 281

Cauchy integral formula

$$\oint_C \frac{f(z)}{z-a}\,dz = \oint_C \frac{f a}{}$$

P-97

~~proof~~ statement
proof } → final term

Evaluate $\oint_C \frac{e^{iz}}{z^3}\,dz$ where c is the circle $|z| = z$. Here $f(z) = e^{iz}$ -1

Kawsar sir MATH 281 10.03.2024

Page no 27
P-158

91; P-166: Expand $f(z) = \frac{1}{z-3}$ in a Laurent series valid for (a) $|z|<3$ (b) $|z|>3$

$(1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 + \dots$

$(1-x)^{-1} = 1 + x + x^2 + x^3 + x^4 + \dots$

$|x|<1$

$(x+a)^n = x^n + {}^nC_1 x^{n-1} a^1 + \dots + a^n$

We have, $f(z) = \frac{1}{z-3}$

(a) since $|z|<3$, so from (1) we get

$$f(z) = \frac{-1}{3}\left(1 - z/3\right)^{-1}$$

(b) since $|z|>3$ so from (1) we get

$$f(z) = \frac{1}{z}\left(1 - \frac{3}{z}\right)^{-1}$$

$$= \frac{1}{z}\left(1 + \frac{3}{z} + \frac{9}{z^2} + \dots\right.$$

$$= \frac{1}{z} + \frac{3}{z^2} + \frac{9}{z^3} + \dots$$

Q2; P-166 Expand $f(z) = \frac{z}{(z-1)(2-z)}$ in a Laurentz series valid for (a) $|z|<1$ (b) $1<|z|<2$ (c) $|z|>2$ (d) $|z-1|>1$ (e) $0<|z-2|<1$

we have, $f(z) = \frac{z}{(z-1)(2-z)}$

$$= \frac{1}{z-1} + \frac{2}{2-z} \quad \text{--- (1)}$$

(a) Since $|z|<1$ so from (1) we get

$f(z) =$

Ujjal sir

19.03.2024

Binomial distribution, $P(r) = {}^nC_r p^r q^{n-r}$

failure

success

$X \sim B(n, p)$

page 787 → $\boxed{\text{prob 21:}}$

x	0	1	2	3	4
$P(X=x)$	$\frac{15}{200}$	$\frac{35}{100}$			

$\mu = E(x) = \sum x P(X=x)$

$\mu = np$

$p = x$

$q = 1 - p$

Binom. distri.

mean $= np$

standard devi.

$\sigma = \sqrt{npq}$

$\boxed{\text{Ex - 36}}$ $\boxed{\text{Ex 37}}$

$\boxed{\text{Exercise 11.4(1)}}$

poison distribution is a particular limiting form of the binomial distri. when p(or q) is very small and n is large enough

like $n = 35$

$$P(r) = \frac{m^r e^{-m}}{r!}$$

in case of poison distri.

mean and variance are same.

$\sigma^2 = m$

$S.D = \sqrt{m}$

Page 800, Ex 43: ★

→ poison এ mean direct না দিলে n এবং P আছে কিনা check করে mean বের করবো।

if $n > P$

$m = n \times P$

Ex-45: 46, 50

Ex-51: → Data to formula

Exercise 11.5 (4), (6)

Normal distribution
↳ continuous

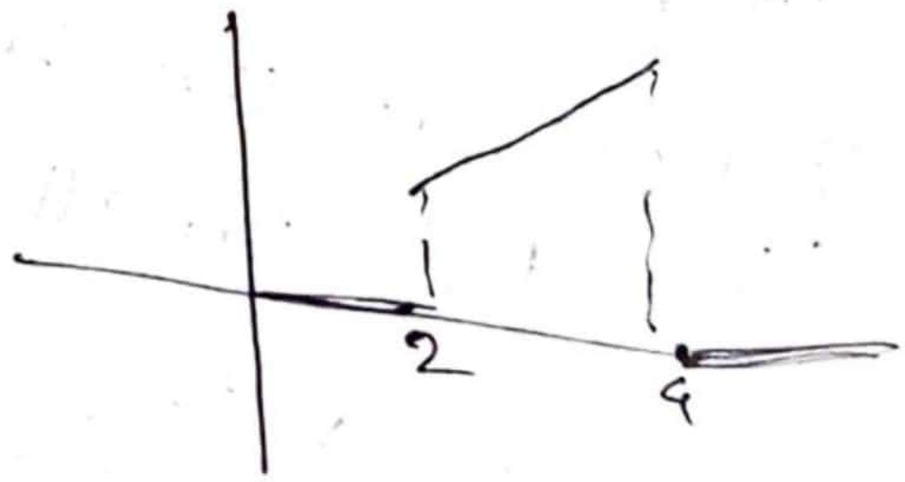
$$E(x) = \int_{-\infty}^{\infty} x\, P(X=x)$$

mean

$$\text{Variance} = \int_{-\infty}^{+\infty} (x-\bar{x})^2 f(x)\, dx$$

$$\int_{-\infty}^{+\infty} x f(x)\, dx$$

Probability density func.



1) $f(x) \geq 0$

2) $\int_{-\infty}^{\infty} f(x)\, dx = 1$

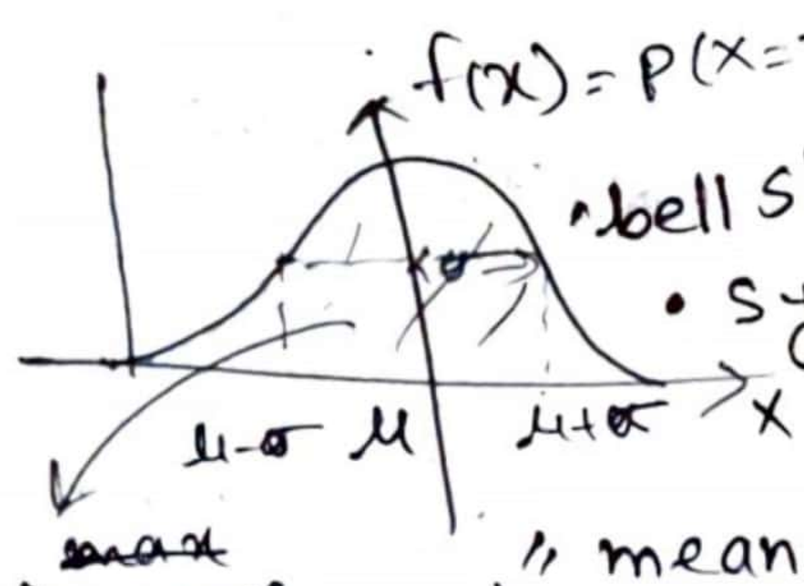
3) $\int_{a}^{b} f(x)\, dx = P(a < x < b$

Q: Define p.d.f. Show that f(x) is pdf or not. Exm. 53

20.03.2024 Walpole

Normal distribution → Continuous probability

↳ Gaussian "



- bell shaped
- symmetrical about the mean.

$X \sim N(\mu, \sigma^2)$

~~max~~ (concave downward)

" mean = median = mode

" Area under the curve is unity.

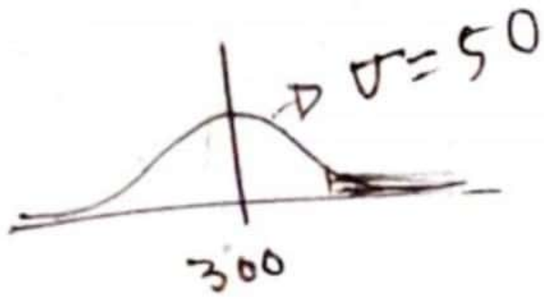
↳ $Z \sim N(0, 1^2)$.

$Z = \frac{X-\mu}{\sigma}$] table / chart.

" ~~Pro~~ write down the properties of Normal distribution! [Exam]

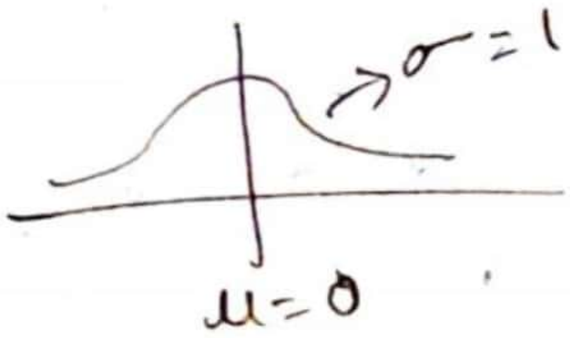
Area under normal curve:

Exm 6.5:



$Z = \frac{362-300}{50}$

$= 1.24$



Exm 6.8: $P(778 < X < 834) = ?$

$P\left(\frac{778-800}{40} < Z < \frac{834-800}{40}\right)$

$P(-0.55 < Z < 0.85)$

$= P(Z < .85) - P(Z < 0.55)$

: chart থেকে value

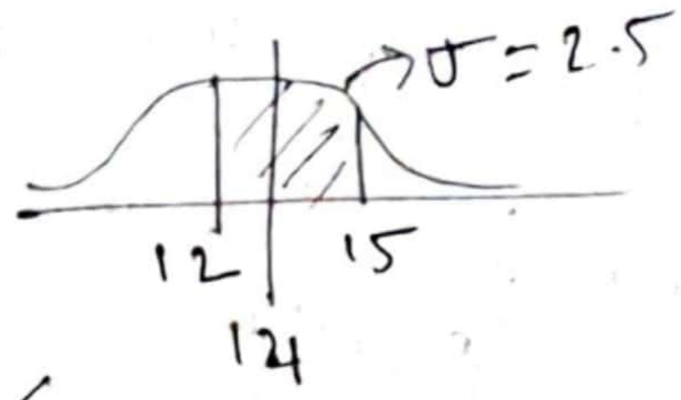
Exm: 6.9 (walpole)

H.K dass →

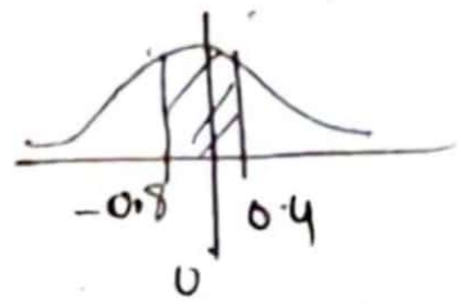
Exm 57 → $X \sim N(72, 15^2)$

Exm 61 →

$X \sim N(14, 2.5^2)$



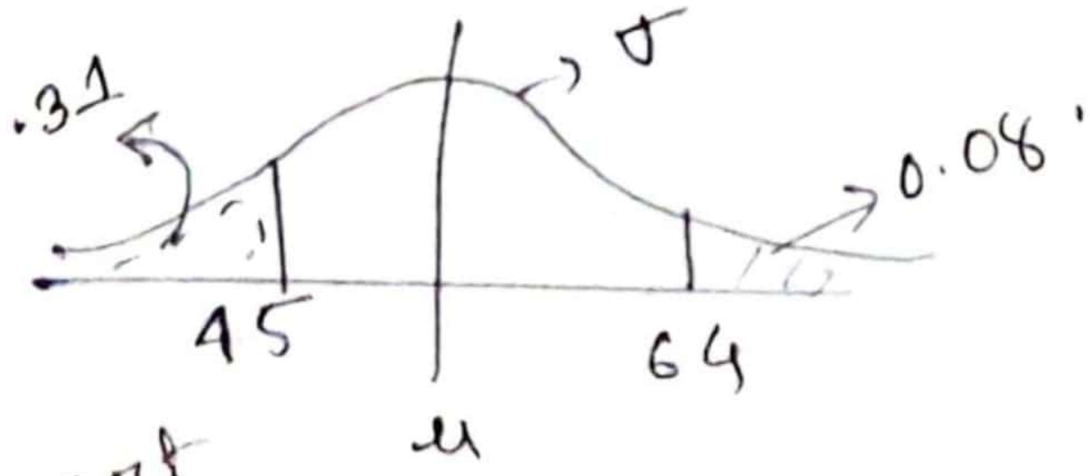
Z ও convert করে



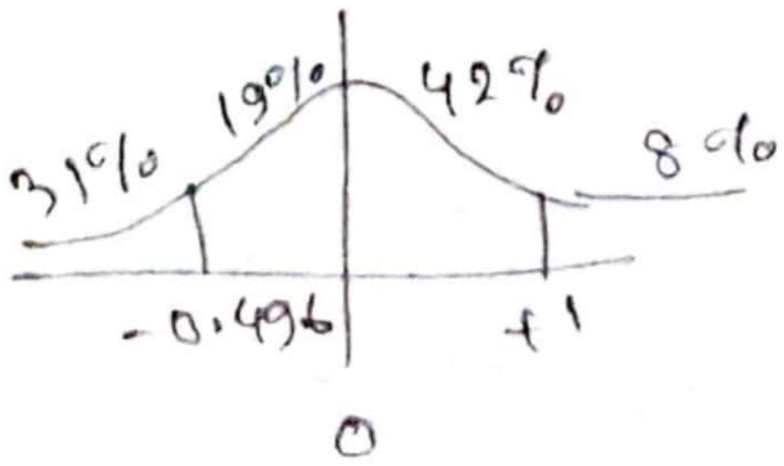
Exm 64 ; 68

Exm 69 ; 70

Exm 70



Z ও convert করে

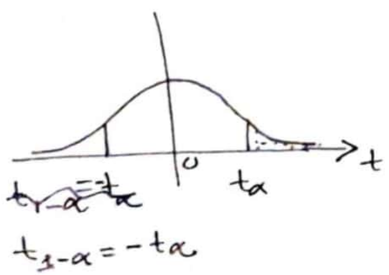


Student / t - distribution:

let $x_1, x_2, \ldots x_n$ be independent random variable that are all normal mean μ and standard deviation σ lets

$$\bar{x} = \sum_{i=1}^{n} \frac{x_i}{n} \quad \text{and} \quad s^2 = \sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{n-1}$$

Then the random variable $T = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$ has a t - distribution with (n-1) degree of freedom.



* Name of sample

71 → Ex
page → 827 } exam

11.38

ex - 76,

value নিচে গাওয়া

not table.

Q pattern (HK)

Q.1
a) Stand. deviation, variance
b) ~~varie~~ moment, skewness, kurtosis

Q2. a) Correlation
&
b) regression.

$Y = a + bx$

" Standard error.

" Rank Correlation.

Q3. a) probability / random var / Bayes thm.

b) Binomial / poison

Q.4
a) Normal / sample / population
b) Hypothesis / exponential / Reylish
c) t - distribution / exponential.

Hypothesis
- type 1, 2 error.

Kawser sir Suggestions

- Evaluate by Contour integration.
- unit circle, semi circle (sheet fn(a))