butions like binomial, Poisson, continuous distribution like Gaussian distribution, Rayleigh distribution, Nakagami distribution, characteristics of distributions, Test of hypothesis like T-test, Chi-square test, Z-test.

Complex Variable: General functions of a complex variable with physical significance, Limits and continuity of a function of complex variable and related theorems; Complex differentiation and the Cauchy-Riemann equations, Mapping by elementary functions, Line Integral of a complex function, Cauchy's Integral theorem, Cauchy's Integral formula, Liouville's theorem, Taylor's theorem and Laurent's theorem. Singular points, Residue, Cauchy's Residue theorem. Evaluation of residues, Contour integration, Conformal mapping.

Binomial distribution:-

$$P(r) = {}^nC_r \, p^r q^{n-r}$$

$$x \sim B(n, p)$$

21. $n = 1200$

- Random variable

$E(x)$

Page: 804/1332

35 For Bino. Distribution

mean $= np$

Standard deviation, $\mu_2 = \sigma^2 = npq$

36

$$= np\,[\{(q+p)^{n-1}\} + (n-1)\,p\,(q+p)^{n-2}] = np\,[1 + (n-1)\,p]$$

$$= np\,[np + (1-p)] = np[np + q] = n^2p^2 + npq$$

Putting these values in (1), we have

$$\text{Variance } = \sigma^2 = \frac{n^2p^2 + n\,pq}{1} - \left(\frac{np}{1}\right)^2 = npq\,,$$

$$S.D. = \sigma = \sqrt{n\,p\,q}$$

Hence for the binomial distribution, Mean $= np$, $\mu_2 = \sigma^2 = n\,p\,q$

$E(X) =$ Mean np

Standard Deviation

$\sigma = \sqrt{npq}$

Example 35. *Find the first four moments of the binomial distribution.* (AMIETE, Summer 2000)

Solution. First moment about the origin

$P(X=r) = {}^nC_r\, p^r q^{n-r}$

$$\mu_1' = \sum_{r=0}^{n} {}^nC_r\, p^r q^{n-r}.r = \sum_{r=0}^{n} r.\frac{n(n-1)(n-2).......(n-r+1)}{r!}\, p^r q^{n-r}$$

$$= n\sum_{r=1}^{n} \frac{(n-1)(n-2).......(n-r+1)}{(n-r)!}\, p^r q^{n-r} = np\sum_{r=1}^{n} {}^{n-1}C_{r-1}\, p^{r-1} q^{n-r}$$

$$= np\,(q+p)^{n-1} = np$$

x	0	1	2	3	4
$P(X=x)$	$\frac{15}{200}$	$\frac{35}{200}$			

Thus, the mean of the Binomial distribution is np.

$E(X)$

Second moment about the origin

$E(X) = \sum x\, P(X=x)$

$\mu = np$

$\Rightarrow$ $p = ?$

$$\mu_2' = \sum_{r=0}^{n} {}^nC_r\, p^r q^{n-r}\, r^2 \qquad [r^2 = r\,(r-1) + r]$$

$$= \sum_{r=0}^{n}\{r(r-1)+r\}\, {}^nC_r\, p^r q^{n-r} = \sum_{r=0}^{n} r(r-1)\, {}^nC_r\, p^r q^{n-r} + \sum_{r=0}^{n} r\cdot {}^nC_r\, p^r q^{n-r}$$

$$= \sum_{r=0}^{n} \frac{r(r-1)\,n(n-1)(n-2).......(n-r+1)}{r!}\, p^r q^{n-r}$$

$$= \sum_{r=0}^{n} \frac{r\,n(n-1)(n-2).......(n-r+1)}{r!}\, p^r q^{n-r}$$

$$n(n-1)p^2\sum \frac{(n-2)(n-3).......(n-r+1)}{} p^{r-2} q^{n-r}$$

$$\text{Mean } = np = 3 \times \frac{1}{3} = 1$$

$$\text{Variance } = npq = 3 \times \frac{1}{3} \times \frac{2}{3} = \frac{2}{3} \qquad \textbf{Ans..}$$

11.13 RECURRENCE RELATION FOR THE BINOMIAL DISTRIBUTION

By Binomial distribution, $P(r) = {}^nC_r \, p^r \, q^{n-r}$ ...(1) (*A.M.I.E., Summer 2002*)

$P(r+1) = {}^nC_{r+1} \, p^{r+1} \, q^{n-r-1}$ ...(2)

On dividing (2) by (1), we get

$$\frac{P(r+1)}{P(r)} = \frac{{}^nC_{r+1} \, p^{r+1} q^{n-r-1}}{{}^nC_r \quad p^r q^{n-r}}$$

$$= \frac{n(n-1)(n-2).....(n-r)}{(r+1)!} \cdot \frac{r!}{n(n-1)(n-2).....(n-r+1)} \cdot \frac{p}{q}$$

$$\frac{P(r+1)}{P(r)} = \frac{n-r}{r+1}\frac{p}{q} \Rightarrow P(r+1) = \frac{n-r}{r+1}\frac{p}{q} P(r) \qquad \textbf{Ans.}$$

$\mu = E(x) = \sum x p(x = n)$

$\mu = np$

$= 100p$

$\boxed{q = 1 - p}$

p, Σ

Exercise 11.4

1. Fit a binomial distribution to the following frequency data:

x	0	1	3	4
f	28	62	10	4

$= n = 104$

$P(x=r)$ $\frac{28}{104}$ $\frac{62}{104}$ $\frac{10}{104}$ $\frac{4}{104}$

(*U. P III Sem. Dec. 2004*)

Ans. $P(r) = {}^{104}C_r (0.00999)^r (0.99111)^{104-r}$

2. Fill in the blanks :

(*a*) A coin is biased so that a head is twice as likely to occur as a tail. If the coin is n এর মান > 35 হলে

avoid binomial

then use poisson

Exp - 43

p - 816

⇒ poisson distr..

m/ mean দেওয়া না থাকলে

n ও p check করতে হবে

if n > p

$m = n \times p$

H.K. Das

45/ 46/ 50/ 51

→ data থেকে formula

06,

☞ continuous distribution:-

11.21 CONTINUOUS DISTRIBUTION

So far we have dealt with discrete distributions where the variate takes only the integral values. But the variates like temperature, heights and weights can take all values in a given interval. Such variables are called continuous variables.

Distribution function.

If $F(x) = P(X \le x) = \int_{-\infty}^{x} f(x)dx$, then $f(x)$ is defined as the Distribution Function.

	6!	

Example 51. *Fit a Poisson distribution to the following data which gives the number of yeast cells per square for 400 squares.*

No. of cells per square (x)	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*Total*
No. of squares	103	143	98	42	8	4	2	0	0	0	0	400

It is given that $e^{-1.32} = 0.2674$ *(A.M.I.E., Summer 2000)*

Solution.

x	0	1	2	3	4	5	6	7	8	9	10	Total
f	103	143	98	42	8	4	2	0	0	0	0	400
$f.x$	0	143	196	126	32	20	12	0	0	0	0	529

$$m = \text{Mean} = \frac{\Sigma f.x}{\Sigma f} = \frac{529}{400} = 1.32$$

But Poisson distribution is $P(x) = \dfrac{e^{-m} \cdot m^x}{r!} = \dfrac{e^{-1.32}(1.32)^x}{x!} \Rightarrow P(r) = \dfrac{0.2674(1.32)^x}{x!}$

For continuos fun

probability density function - 3 টি শর্ত

(I)

$E(x) = \int_{-\infty}^{\infty} x \, p(X=x)$

Let $f(x)$ be a continuous function, then Mean $= \int_{-\infty}^{+\infty} x f(x)dx$

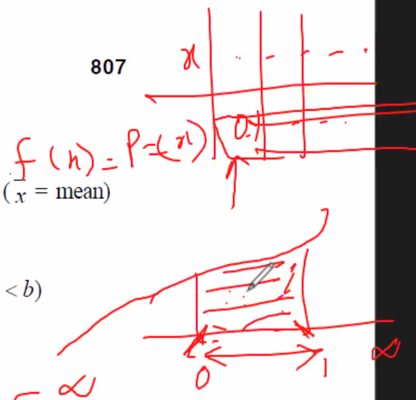
Variance $= \int_{-\infty}^{+\infty} (x-\bar{x})^2 . f(x)dx.$ $(\bar{x}$ = mean)

(pdf)

Note. $f(x)$ is called probability density function if

(1) $f(x) \geq 0$ for every value of x. (2) $\int_{-\infty}^{\infty} f(x)dx = 1$ (3) $\int_a^b f(x)dx = P, (a < x < b)$

$f(x) = P(x)$ [0.1] $\in 1$

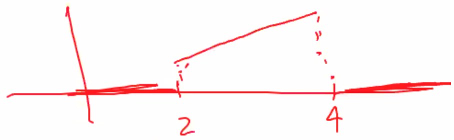


Example 52. *A function $f(x)$ is defined as follows*

$$f(x) = \begin{cases} 0, & x < 2 \\ \frac{1}{\ }(2x+3), & 2 \leq x \leq 4 \end{cases}$$

(II)

$$\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{2} f(x)dx + \int_{2}^{4} f(x)dx + \int_{4}^{\infty} f(x)dx$$



$$= \int_2^4 \frac{1}{18}(2x+3)dx = \frac{1}{18}\left[x^2+3x\right]_2^4$$

$$= \frac{1}{18}\left[(16+12) - (4+6)\right]$$

$$= \frac{1}{18} \times 18 = 1$$

Probability 807

Let $f(x)$ be a continuous function, then Mean $= \int_{-\infty}^{+\infty} x f(x)dx$

Variance $= \int_{-\infty}^{+\infty} (x-\bar{x})^2 . f(x)dx.$ $(\bar{x}$ = mean)

Note. $f(x)$ is called probability density function if

(1) $f(x) \geq 0$ for every value of x. (2) $\int_{-\infty}^{\infty} f(x)dx = 1$ (3) $\int_a^b f(x)dx = P, (a < x < b)$

Example 52. *A function $f(x)$ is defined as follows*

$0, \quad x < 2$

(III)

* Q: Define PDF

: Show that $f(x)$ is pdf or not

Exm 53 |