

Engineering Statistics

and

Complex variable

Statistics की?

Statistics Data নিয়ে কথা বলো.

সা collect করি (numbers এর information)

Qualitative → numbers দিয়ে যথে নাই

Quantitative → পরিমাণ এ numbers দিয়ে express করা সাধ-

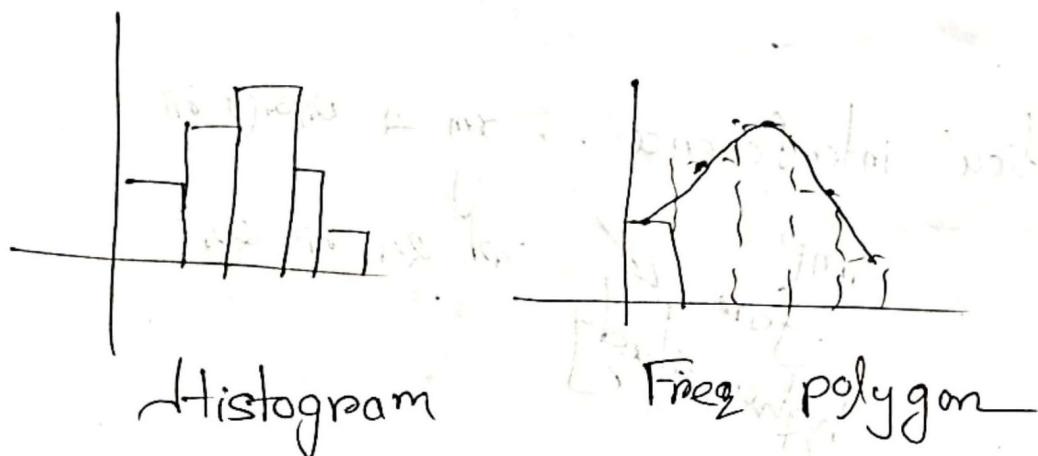
statistics for Engineers
and scientists
(Walpole)Advanced Engineering
M. H. K. DassStatisticsData

- Collection
- Summarize / Organize
- Analyze (proper way to present কর)
- ↓
- Data analyze
- Decision (পদ্ধতি নির্মাণ)
- Valid Conclusion (সম্ভাবিত বলা সত্য)

S is a branch of science dealing with the collection of data, organising, summarising, presenting and analysing data and drawing valid conclusions and thereafter

Graphical Representation

i) Histogram → દર્શાવતી માફિ ગપ નાથ
continuous data value



3) Cumulative freq. curve or the ogive

class interval	f_i	cumulative freq.
0-10	12	12
10-20	32	44
20-30	67	111
30-40	82	193

Avg or Measures of Central tendency:

- i) Arithmetic avg of mean
- ii) Median
- iii) Mode (125.00)
- iv) Geometric Mean
- v) Harmonic Mean

Mean can't use major or → extreme value Diffr.

Median suitable for continuous data

Mode

Walpole

→ qualitative data

1) Statistical Interference → $\bar{x}_m \rightarrow$ average of imp. for higher study

2) Step deviation method → example 3 → HkDass

example: 9

- Mean: Date: 21.02
• Direct method
• Short-cut memory : Data handle page
Date: 21.02
Cls: 02
use this.

$$\text{AM} = a + \frac{\sum f d}{\sum f}$$

$$d = x - a$$

- Updated method: Step deviation method

$$\text{AM} = a + \frac{\sum f d}{\sum f}$$

i = interval or difference

MEDIAN : \rightarrow extreme value \therefore median
the mean of the values of $\frac{1}{2}n$ th and $(\frac{1}{2}n+1)$ th items is the median.

Grouped data, median = $l + \frac{\frac{1}{2}N - F}{f} \cdot i$

l = median class \Rightarrow lower value

F = original \Rightarrow cumulative freq

N = total frequency

i = class interval

f = freq for the median \Rightarrow

MODE :

$$\downarrow \quad \text{MODE} = l + \frac{f - f_{-1}}{2f - f_{-1} - f_1} \cdot i$$

Empirical formula: \therefore $3[\text{Mean} - \text{Median}]$

$$\text{Mean} - \text{Mode} = 3[\text{Mean} - \text{Median}]$$

Modal \Rightarrow largest freq value

f = modal \Rightarrow freq

f_{-1} = $n - n - 1$ original freq

f_1 = $n - n - 1$ original freq

- Geometric mean: ~~mean of products of different numbers~~
- Harmonic mean: ~~mean of reciprocals of numbers~~
- Avg Deviation or mean deviation

It is the mean of the absolute values of the deviations of a given set of numbers from their arithmetic mean.

Absolute value among +ve & -ve?

$$|a| = \begin{cases} a; & a > 0 \\ 0; & a = 0 \\ -a; & a < 0 \end{cases}$$

$$|-3| = -(-3) = 3$$

Mean deviation: $\frac{\sum f_i |x_i - \bar{x}|}{\sum f_i}$

CT & Find ex 3 pg imp

□ Standard Deviation: Standard deviation is defined as the square root of the mean of the square of the deviation from the arithmetic mean.

$$S.D. = \sigma = \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}}$$

important

Notes: i) The square of the standard deviation σ^2 is called variance

ii) σ^2 is called the 2nd moment about the mean and is denoted by M_2

$(x - \bar{x})^2 \rightarrow$ deviation \Rightarrow square

mean $\rightarrow \frac{(x - \bar{x})^2}{n}$

$$S.D = \sigma = \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}}$$

$$SD = \sigma = \sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2}$$

Note: Coefficient of Variation = $\frac{\sigma}{\bar{x}} \times 100$

Problem - 12 freq. distribution group data.

$$\text{Mean } \bar{x} = \frac{\sum fd}{N}$$

Calc. σ \rightarrow minimum to maximum

$$35 \times 100 = 3500$$

$$a = \frac{0+9}{2} = 9$$

x	f	d = x - a	f.d.	f.d ²
6	3	-3	-9	27
7	6	-2	-12	24
8	9	-1	-9	9
9	13	0	0	0
10	8	1	8	8
11	5	2	10	20
12	9	3	12	36
$\sum f = 98$			$\sum fd = 0$	$\sum fd^2 = 124$

$$S.D = \sigma = \sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2}$$

$$\sqrt{\frac{124}{98}} - \sqrt{\left(\frac{0}{98}\right)^2} = 1.6$$

$$\text{Mean} = a + \frac{\sum fd}{\sum f} = 9 + 0 = 9$$

$$S.D = \sqrt{\frac{\sum (x-\bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fd^2}{\sum f}}$$

$$\therefore \text{Coefficient of variation} = \frac{\sigma}{\bar{x}} \times 100$$

$$= \frac{1.6}{9} \times 100 = 17.78$$

Q) grouped data no. 21: x : mid value
 $60-62 \rightarrow x = 61$

Sunday CT \rightarrow Grouped Data G.I. 20
 Mean, Median, Mode, Coefficient
 of variation \rightarrow Q.S. line

A) Moment: The n th moment of a variable x about the mean \bar{x} is usually denoted by m_n , is given by

$$m_n = \frac{1}{N} \sum f_i (x_i - \bar{x})^n ; \sum f_i = N$$

The n th moment of a variable x about any point a is defined by

$$m_n' = \frac{1}{N} \sum f_i (x_i - a)^n ; \sum f_i = N$$

In particular: $m_0 = \frac{1}{N} \sum f_i (x - \bar{x})^0 = \frac{1}{N} \sum f_i$

$$= \frac{N}{N} = 1$$

Moment \rightarrow Data and ungrouped data \rightarrow Mean

$$M_1 = \frac{1}{N} \sum f_i (x - \bar{x}) = 0, M'_1 = \frac{1}{n} \sum f_i (x - a) = \bar{x} - a$$

$$M_2 = \frac{1}{N}$$

Reltn b/w moments about mean and moment about any point

mean \Rightarrow respect to M_2
 point \Rightarrow M'_2 respect to

$$M_2 = M'_2 + M'_1$$

$$M_3 = M'_3 - 3M'_2 M'_1 + 2M'_1^3$$

$$M_4 = M'_4 - 4M'_3 M'_1 + 6M'_2 M'_1^2 - 3M'_1^4$$

Note: $(\bar{x} - a)^2 = \frac{1}{N} \sum (x - a)^2 = M_2$

① The sum of the coefficients of the varians

$$\frac{1}{N} \sum \frac{1}{n}$$

is called sum of squares of deviations

Q Skewness:

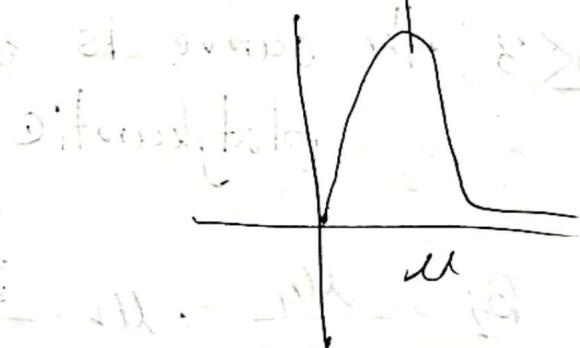
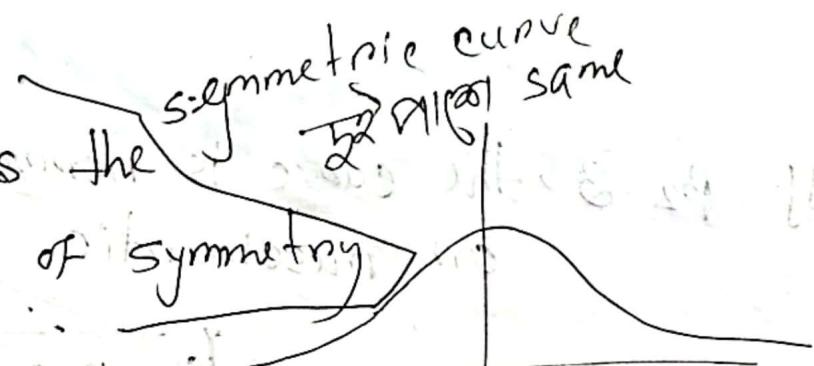
i) Skewness → denotes the opposite of symmetry

• Mean, median, mode

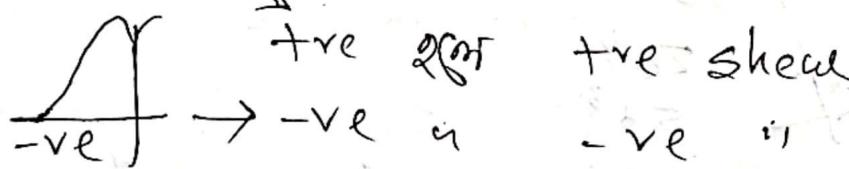
same → in case of symmetrical mean, median, mode

→ points about the value

→ value



Coefficient of skewness: $\frac{\text{Mean} - \text{Mode}}{\text{Standard deviation}}$

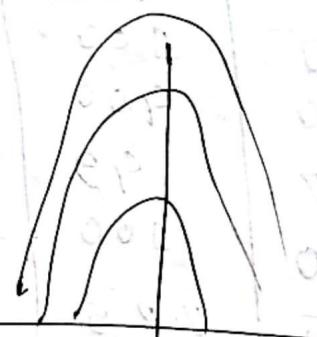


Q. Define moment, skewness, kurtosis.

IMP

i) KURTOSIS:

→ High & Low → kurtosis



If $B_2 = 3$, the curve is normal
or mesokurtic

If $B_2 > 3$, the curve is [A = mesokurtic, B = leptokurtic]
peaked or leptokurtic [C = platykurtic]

If $B_2 < 3$, the curve is flat-topped or
platykurtic

$$B_2 = \frac{M_2}{M_1^2}, M_2 = \frac{\sum (x - \bar{x})^2}{N}, M_3 = \frac{\sum (x - \bar{x})^3}{\sqrt{N}}$$

Page - 745

<u>1</u>	x_i	$d = x_i - \bar{x}_0$	$(d)^2$	$(d)^3$	$(d)^4$
52	2	9	8		16
57	7	49	343		2401
40	-10	100	-1000		10000
70	20	400	8000		160000
43	-7	49	-343		2401
40	-10	100	-1000		10000
65	15	225	3375		50625
35	-15	225	-3375		50625
48	-2	4	-8		16
		$\sum d = 0$	$\sum d^2 = 1156$	$\sum d^3 = 6000$	$\sum d^4 = 986084$

$$N = 9$$

$$\bar{m}_2 = \frac{1156}{9} = 128.44$$

$$\bar{m}_4 = \frac{286089}{9} = 31787.33$$

$$\beta_2 = \frac{\bar{m}_4}{\bar{m}_2^2} = \frac{31787.33}{(128.44)^2} = 1.927 < 3$$

platykurtic

Date: 28.01.24
cls: 03

Complete the first four moments about mean

Class interval: 0-10 10-20 20-30 30-40

Frequency : 1 3 4 2

\Rightarrow Raw moment $\times A = 25$

Mean $A = \frac{10+20}{2} = 15$
 $\bar{m}_1 = 15 + \frac{10 \times 1}{4} = 17.5$
 $\bar{m}_2 = 15 + \frac{10 \times 10}{4} = 22.5$
 $\bar{m}_3 = 15 + \frac{10 \times 30}{4} = 27.5$
 $\bar{m}_4 = 15 + \frac{10 \times 100}{4} = 32.5$

Determination of β_1 and β_2 and comment on that

$$P.P. = 0.8 \times 8 + (0.2) \times 0.8 \times 8 = 0.8 \times 8 = 6.4$$

Class	f	Mid_x	$\mu = \frac{x-25}{10}$	fu	fu^2	fu^3	fu^4
0-10	1	5	-2	-2	9	-8	16
10-20	3	15	-1	-3	3	-3	3
20-30	4	25	0	0	0	0	0
30-40	2	35	1	1	15	5	2
					-3	9	-9
							21

$$M_m' = \frac{1}{n} \sum f_i (x_i - a)^n$$

$$M_1' = \frac{-3}{10} \times 10 \rightarrow \text{Mean } 10 \text{ पर } 16 \text{ वाले } 3 \text{ तक } 25 \text{ का }$$

$$fu^2 \rightarrow 5 \text{ वाले } 16^2$$

$$M_2' = -\frac{3}{10} \times 10 \rightarrow fu^3 \rightarrow 10^3$$

$$= 0 - 3$$

$$M_2' = \frac{-9 \times 10^2}{10} = 90$$

$$M_3' = \frac{-9 \times 10^3}{10} = -900$$

$$M_4' = \frac{2 \times 10^4}{10} = 200$$

Moments about A = 25

Moments about the mean

$$M_1 = 0 \quad M_2 = 90 - 9$$

$$M_3 = -900 - 3 \times 90 \times (-3) + 2 \times (-3)^3 = -199$$

$$\beta_1, \beta_2 \text{ ? } \rightarrow \text{Shewness} \quad \beta_1 = \frac{\mu_3}{\mu_2^3} = \frac{(-144)^2}{(81)^3} = 0.03902$$

$$\beta_2 \rightarrow \text{kurtosis} \quad \beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{19817}{(81)^2} = 2.2583$$

A The 1st four central moments of a distribution's are

mean $\bar{x} = 21(7)(25) - 0, 2.5, 0.7$ and 18.75
Find Beta coefficients.

$$\Rightarrow \mu_1 = 0, \mu_2 = 2.5, \mu_3 = 0.7, \mu_4 = 18.75 \rightarrow \text{+ve shewness}$$

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = \frac{(0.7)^2}{(2.5)^3} = 0.03136$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{18.75}{(2.5)^2} = 3$$

mean $\bar{x} = 21(7)(25) - 0, 2.5, 0.7$
(fast)

mesokurtic

Coefficient of co-relation

0-1 → +ve

0-1 → -ve

Date: 04.02.24
Qs: 04

0 means there's no relation b/w two variables.

variable: variable শুল্ক এবং আরক্ষণ্য প্রভাবিত-

$$\text{losses} = \frac{(P_{H=1})}{(28)} \cdot \frac{\text{ক্ষেত্রফল}}{\text{মুল}} = 6 \quad 22 \text{ মিলি টা টা}$$

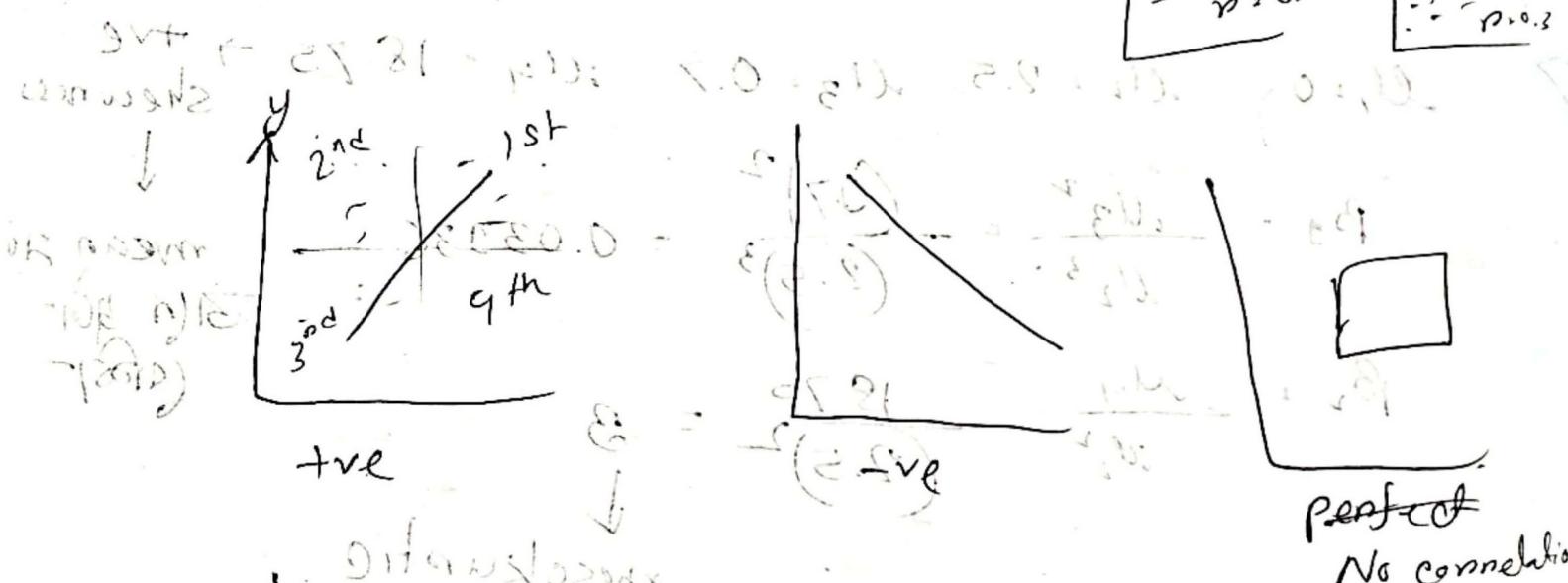
$$\text{Walpole - এর ঘর}: \text{বর্ণনা শুল্ক পরিধান মেটেরিয়াল টা টা$$

$$\text{losses} = \frac{(P_{H=1})}{(28)} \cdot \frac{\text{পুরো ক্ষেত্রফল}}{10} = 6$$

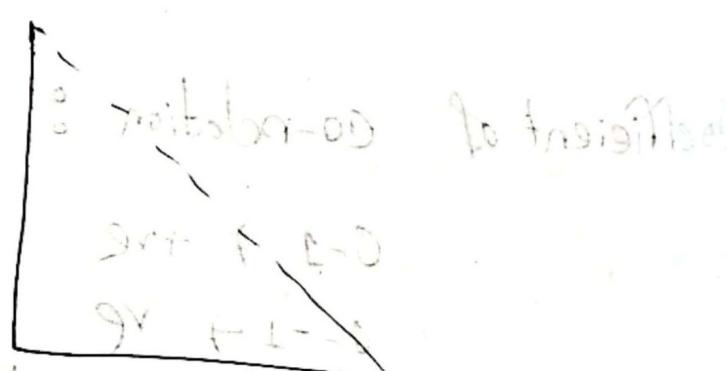
Correlation: এটির মুদ্দিতে আরেকটি বাড়ি অবস্থা কর্মসূচি
এটি বাড়ির আরেকটি ও বাড়ির $\rightarrow +ve$
ক্ষেত্রফল $\rightarrow -ve$

Perfect correlation: এটি variable এর ratio always
same রাশি

$$r = 0.8$$



perfect
No correlation



Coefficient of correlation = +1

Coefficient of correlation = -1
ক্ষেত্রফল এবং স্থান সহজেই সমান

Methods of Determining Simple Correlations:

→ KARL Pearson's coefficient of correlation.

r between two variables x and y is defined by the relation:

$$r = \frac{\sum xy}{\sqrt{(\sum x^2)(\sum y^2)}} = \frac{P}{\sigma_x \sigma_y} = \frac{\text{Covariance}(x,y)}{\text{Variance}_x \text{ Variance}_y}$$

Where, $X = x - \bar{x}$, $Y = y - \bar{y}$.

i.e X, Y are the deviations measured from their respective means.

$$P = \left(\frac{\sum xy}{n} \right) = \text{Co-variance}$$

and σ_x, σ_y being the standard deviations of these series.

$\bar{x} = 70$

$\bar{y} = 68.5$

$y = 67$

$\bar{y} = 68.25$

x	y	$x - \bar{x}$	$y - \bar{y}$	x^2	y^2	xy
67	65	-3	-2	9	9	6
68	66	-2	-1	4	1	2
69	67	-1	0	36	0	0
68	67	-2	0	4	0	2
72	68	2	1	9	1	0
70	69	0	2	0	9	-9
69	71	-1	2	1	16	0
70	73	0	6	0	36	0
				$\sum x^2 = 58$	$\sum y^2 = 62$	$\sum xy = 6$

$$\therefore r_{\rho_2} = \frac{\sum xy}{\sqrt{(\sum x^2)(\sum y^2)}} = \frac{6}{\sqrt{58 \times 62}} = 0.1000556$$

x	Xy	$x - \bar{x}$	$y - \bar{y}$	x^2	y^2	xy
67	65	-1.5				
68	66	-0.5				
69	67	-0.5				
68	67	-0.5				
72	68	3.5				
70	69	1.5				
69	71	0.5				
70	73	1.5				

Spearman's Rank Correlation Coefficient

$$P = 1 - \frac{6 \sum d^2}{n(n^2 - 1)}$$

where $r = \text{rank coefficient of correlation}$ and
 $d = \text{the difference of ranks b/w paired items}$
 in 2 series.

• Example 18:

- Theory of Superconductivity

Line of Regression

interpolation → data-? f(x)

extrapolation → data- ର ସାରିଟିକେ ଦିଆଯାଇଥାଏ ମୁଣ୍ଡର ପରିମାଣ ହେବାରେ

When the curve is a straight line, it is called a line of Regression.

~~Q1~~ What do u understand by regression and what is the purpose of BIS in engg field

$$y = a + bx$$

$$b = \frac{\sum xy}{\sum x^2}$$

$$\bar{y} = a + b\bar{x}$$

$$a = \bar{y} - b\bar{x}$$

$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{y} = \frac{\sum y}{n}$$

x	y	\bar{x}	\bar{y}	xy
1	2			
2	3			
3	4			
4	5			
5	6			
6	7			
7	8			
8	9			
9	10			
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96	97			
97	98			
98	99			
99	100			

x	y	\bar{x}	\bar{y}	xy
1	2			
2	3			
3	4			
4	5			
5	6			
6	7			
7	8			
8	9			
9	10			
10	11			
11	12			
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97	98			
98	99			
99	100			

Example 29: det, $y = a + bx$.

$$\text{summation } \sum y = na + b\sum x$$

$$\sum x$$

$$\text{multiply}$$

Example 30: $n = 100$

$$\sum y = na + b\sum x$$

$$\Rightarrow 6800 = 100a + 15000b \quad \text{(1)}$$

$$\sum xy = a\sum x + b\sum x^2$$

$$\Rightarrow 1022250 = a \cdot 15000 + 2272500b \quad \text{(2)}$$

Solving (1) & (2) \Rightarrow

$$a = 53, b = 0.1$$

∴ Equation of the line regression is $y = 53 + 0.1x$

$$b = \frac{\sum xy}{\sum x^2} = \frac{15000}{100} = 150$$

$$b = \frac{\sum xy}{\sum x^2} = 68.15$$

$$\bar{x} = \frac{\sum x}{n} = \frac{15000}{100} = 150$$

$$\bar{y} = \frac{\sum y}{n} = \frac{6800}{100} = 68$$

$$a = \bar{y} - b\bar{x} = 68 - 150 \times 150 = -22390$$

Q: What do you understand by error of prediction?

Table for error of Prediction: $y_p = a + bx$

x	y	$x - \bar{x}$	$y - \bar{y}$	x^2	y^2	xy	y_p	$y - y_p$
10	10	-140	-58	19600	100	-680	100	-10
20	20	-120	-38	4800	400	-480	20	-20
30	30	-100	-18	900	900	-300	30	-30
40	40	-80	2	1600	1600	-160	40	-42
50	50	-60	42	2500	2500	-210	50	-40
60	60	-40	62	3600	3600	-240	60	-50
70	70	-20	82	4900	4900	-140	70	-60
80	80	0	102	6400	6400	-160	80	-80
90	90	20	122	8100	8100	-180	90	-100
100	100	40	142	10000	10000	-200	100	-100
110	110	60	162	12100	12100	-220	110	-100
120	120	80	182	14400	14400	-240	120	-100
130	130	100	202	16900	16900	-260	130	-100
140	140	120	222	19600	19600	-280	140	-100
150	150	140	242	22500	22500	-300	150	-100
160	160	160	262	25600	25600	-320	160	-100
170	170	180	282	28900	28900	-340	170	-100
180	180	200	302	32400	32400	-360	180	-100
190	190	220	322	36100	36100	-380	190	-100
200	200	240	342	40000	40000	-400	200	-100
210	210	260	362	44100	44100	-420	210	-100
220	220	280	382	48400	48400	-440	220	-100
230	230	300	402	52900	52900	-460	230	-100
240	240	320	422	57600	57600	-480	240	-100
250	250	340	442	62500	62500	-500	250	-100
260	260	360	462	67600	67600	-520	260	-100
270	270	380	482	72900	72900	-540	270	-100
280	280	400	502	78400	78400	-560	280	-100
290	290	420	522	84100	84100	-580	290	-100
300	300	440	542	90000	90000	-600	300	-100
310	310	460	562	96100	96100	-620	310	-100
320	320	480	582	102400	102400	-640	320	-100
330	330	500	602	108900	108900	-660	330	-100
340	340	520	622	115600	115600	-680	340	-100
350	350	540	642	122500	122500	-700	350	-100
360	360	560	662	130000	130000	-720	360	-100
370	370	580	682	1376900	1376900	-740	370	-100
380	380	600	702	145600	145600	-760	380	-100
390	390	620	722	153600	153600	-780	390	-100
400	400	640	742	161600	161600	-800	400	-100
410	410	660	762	170000	170000	-820	410	-100
420	420	680	782	178400	178400	-840	420	-100
430	430	700	802	187200	187200	-860	430	-100
440	440	720	822	196400	196400	-880	440	-100
450	450	740	842	206000	206000	-900	450	-100
460	460	760	862	216000	216000	-920	460	-100
470	470	780	882	226400	226400	-940	470	-100
480	480	800	902	237600	237600	-960	480	-100
490	490	820	922	249600	249600	-980	490	-100
500	500	840	942	262400	262400	-1000	500	-100
510	510	860	962	275600	275600	-1020	510	-100
520	520	880	982	289600	289600	-1040	520	-100
530	530	900	1002	304400	304400	-1060	530	-100
540	540	920	1022	320000	320000	-1080	540	-100
550	550	940	1042	336400	336400	-1100	550	-100
560	560	960	1062	353600	353600	-1120	560	-100
570	570	980	1082	371600	371600	-1140	570	-100
580	580	1000	1102	390400	390400	-1160	580	-100
590	590	1020	1122	410000	410000	-1180	590	-100
600	600	1040	1142	430400	430400	-1200	600	-100
610	610	1060	1162	451600	451600	-1220	610	-100
620	620	1080	1182	473600	473600	-1240	620	-100
630	630	1100	1202	496400	496400	-1260	630	-100
640	640	1120	1222	520000	520000	-1280	640	-100
650	650	1140	1242	544400	544400	-1300	650	-100
660	660	1160	1262	569600	569600	-1320	660	-100
670	670	1180	1282	595600	595600	-1340	670	-100
680	680	1200	1302	622400	622400	-1360	680	-100
690	690	1220	1322	650000	650000	-1380	690	-100
700	700	1240	1342	678400	678400	-1400	700	-100
710	710	1260	1362	707600	707600	-1420	710	-100
720	720	1280	1382	737600	737600	-1440	720	-100
730	730	1300	1402	768400	768400	-1460	730	-100
740	740	1320	1422	800000	800000	-1480	740	-100
750	750	1340	1442	832400	832400	-1500	750	-100
760	760	1360	1462	865600	865600	-1520	760	-100
770	770	1380	1482	900000	900000	-1540	770	-100
780	780	1400	1502	935600	935600	-1560	780	-100
790	790	1420	1522	972400	972400	-1580	790	-100
800	800	1440	1542	1010000	1010000	-1600	800	-100
810	810	1460	1562	1048400	1048400	-1620	810	-100
820	820	1480	1582	1087600	1087600	-1640	820	-100
830	830	1500	1602	1127600	1127600	-1660	830	-100
840	840	1520	1622	1168400	1168400	-1680	840	-100
850	850	1540	1642	1210000	1210000	-1700	850	-100
860	860	1560	1662	1252400	1252400	-1720	860	-100
870	870	1580	1682	1295600	1295600	-1740	870	-100
880	880	1600	1702	1340000	1340000	-1760	880	-100
890	890	1620	1722	1385600	1385600	-1780	890	-100
900	900	1640	1742	1432400	1432400	-1800	900	-100
910	910	1660	1762	1480000	1480000	-1820	910	-100
920	920	1680	1782	1528400	1528400	-1840	920	-100
930	930	1700	1802	1577600	1577600	-1860	930	-100
940	940	1720	1822	1627600	1627600	-1880	940	-100
950	950	1740	1842	1678400	1678400	-1900	950	-100
960	960	1760	1862	1730000	1730000	-1920	960	-100
970	970	1780	1882	1782400	1782400	-1940	970	-100
980	980	1800	1902	1835600	1835600	-1960	980	-100
990	990	1820	1922	1890000	1890000	-1980	990	-100
1000	1000	1840	1942	1946400	1946400	-2000	1000	-100

Number: 6 → ~~7.007~~ → 2701 check
 Ans: $y = 0.55 + 0.0583$ → ~~7.0726~~

Practice

$$2x + 3y + 5 = 0 \quad \text{--- (1)}$$

$$x + 4y + 8 = 0 \quad \text{--- (2)}$$

$$\rightarrow y = a + bx$$

$$\therefore x = a + by$$

$$\therefore r = \sqrt{b_{yx} \times b_{xy}}$$

$$\therefore b_{yx} = r \frac{\sigma_y}{\sigma_x} \quad \text{standard deviation of } y$$

$$\therefore b_{xy} = r \frac{\sigma_x}{\sigma_y}$$

$$y - \bar{y} = r \frac{\sigma_y}{\sigma_x} (x - \bar{x}) \quad \bar{x}, \bar{y} \text{ are lines}$$

$$x - \bar{x} = r \frac{\sigma_x}{\sigma_y} (y - \bar{y})$$

Q Find the correlation co-efficient b/w x and y when the lines of regression are $2x - 3y + 6 = 0$ and $x - 2y + 1 = 0$

$\Rightarrow y$ of x :

$$2x - 3y + 6 = 0$$

$$3y = 2x + 6$$

$$y = \frac{2}{3}x + \frac{2}{3}$$

$$\therefore b_{yx} = \frac{2}{3}$$

$$x \text{ of } y: x - 2y + 1 = 0$$

$$x = 2y - 1$$

$$b_{xy} = 2$$

$$\therefore r = \sqrt{b_{yx} \times b_{xy}} = \sqrt{\frac{2}{3} \times 2} = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}} < 1$$

~~অবস্থার অভিযন্তা~~

~~xx-190~~
 In a partially destroyed laboratory record of an analysis of correlation data the following results only are legible:

* Variance of $x = 9 \rightarrow \sigma_x^2 = 9, \sigma_x = 3$

Regression eqns: $8x - 10y + 6.6 = 0$

$$90x - 18y - 214 = 0$$

$$i) \bar{x} - \bar{y} \quad ii) \sigma_y$$

$$iii) r_x$$

$$\Rightarrow y \text{ of } x: \quad \begin{cases} i) \bar{x} - \bar{y} \\ ii) \sigma_y \\ iii) r_x \end{cases} \quad \begin{cases} x \text{ of } y \\ iv) \end{cases}$$

$$\begin{aligned} 8x - 10y + 6.6 &= 0 \\ 10y &= 8x + 6.6 \\ y &= \frac{8}{10}x + 0.66 \end{aligned}$$

$$\begin{aligned} 90x - 18y - 214 &= 0 \\ 18y &= 90x - 214 \\ y &= \frac{90}{18}x - \frac{214}{18} \\ y &= 5x - \frac{107}{9} \end{aligned}$$

$$\therefore b_{yx} = \frac{8}{10} \quad \therefore b_{xy} = \frac{18}{90}$$

$$\therefore r = \sqrt{b_{yx} \times b_{xy}} = \sqrt{\frac{8}{10} \times \frac{18}{40}} = \frac{3}{5}$$

$$\therefore b_{yx} = r \frac{\sigma_y}{\sigma_x}$$

$$\Rightarrow \frac{8}{10} = \frac{3}{5} \times \frac{\sigma_y}{\sigma_x}$$

$$\Rightarrow \frac{8}{10} = \frac{3}{5} \times \frac{\sigma_y}{\sigma_x}$$

$\Rightarrow \sigma_y$ is called homothetic coefficient

Solving the 2 eqn \Rightarrow

$$\begin{cases} x = 13 \\ y = 17 \end{cases}$$

$$\therefore y - \bar{y} = r \frac{\sigma_y}{\sigma_x} (x - \bar{x})$$

$$\Rightarrow 17 - \bar{y} = b_{yx} (\bar{x} - x)$$

$$\Rightarrow 17 - \bar{y} = \frac{8}{10} \times (13 - \bar{x})$$

$$\Rightarrow 17 - \bar{y} = 10 - 8\bar{x}$$

$$\Rightarrow 8\bar{x} - 10\bar{y} + 6 = 0 \quad \text{--- (1)}$$

$$\therefore x - \bar{x} = b_{xy} (y - \bar{y}) \Rightarrow 13 - \bar{x} = 2(17 - \bar{y})$$

$$\Rightarrow 13 - \bar{x} = 34 - 2\bar{y} \Rightarrow \bar{x} - 2\bar{y} + 21$$

$$\begin{aligned} \therefore x - \bar{x} &= b_{xy} (y - \bar{y}) \\ \Rightarrow 13 - x &= -\frac{18}{90} (y - \bar{y}) \\ \Rightarrow 520 - 40x &= 306 - 18y \\ \Rightarrow 40x - 18y - 214 &= 0 \quad \text{(iv)} \end{aligned}$$

Solving (ii) & (iv)

$$x = 13$$

$$y = 17$$

Probability

Correlation (সম্পর্ক)

অন্তর & সমান



$\rightarrow 1, 2, 3, 4, 5, 6$

↓
Sample Space

Experiment

Event \rightarrow ঘটনা. $P(\text{even num}) = \frac{3}{6} = \frac{1}{2}$

Sample space \rightarrow exp (সম্পর্ক সত্ত্বেও) possible outcome (সম্ভব)

Exhaustive event \rightarrow সম্পূর্ণ ঘটনা

Mutually exclusive event \rightarrow একই ঘটনা ঘটে না।
অন্যান্য ঘটনা ঘটে না। এবং একটি ঘটনা ঘটে তখন অন্য ঘটনা ঘটে না।

Conditional probability:

defn মনে রেখা $\rightarrow P(A|B)$
Probability of the
event A, given that the event B is
already happened

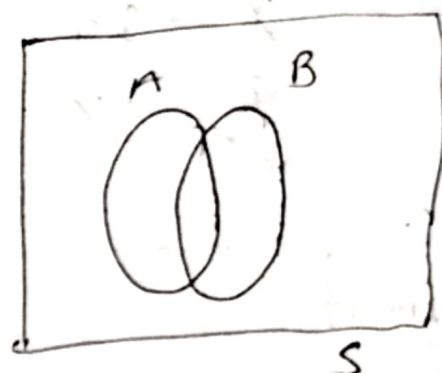
Baye's Thm

$$\boxed{\text{Probability} = \frac{\text{Possible outcomes}}{\text{Total outcomes}}}$$

$P(A|B) \rightarrow$ Probability of the event A

Given that the event B is already happened.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$



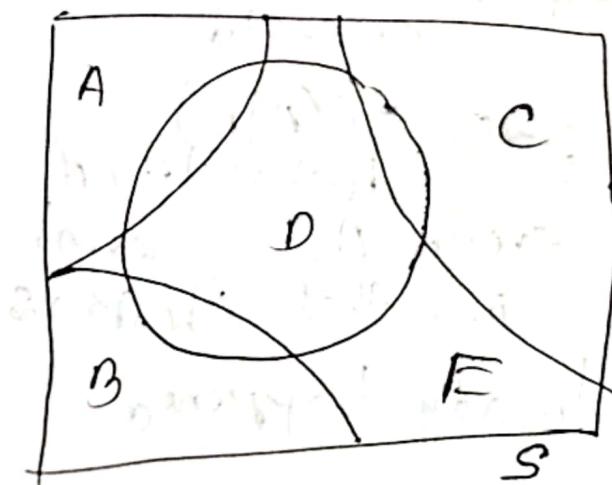
$$A \cap B$$

$$A \cup B$$

⊗ Independent events: 2 events independent of each other

$$P(A \cap B) = P(A) \times P(B)$$

Exclusive: $P(A \cap B) = 0$



$$20\% \rightarrow 27.$$

$$25\% \rightarrow 5\%$$

$$33\% \rightarrow 20\%, 47.$$

$$47 \rightarrow 47.$$

$$20\% \rightarrow 5\%$$

$$P(B_i|A) = \frac{P(B_i) P(A|B_i)}{\sum_{i=1}^3 P(B_i) P(A|B_i)}$$

$$P(B_2|A) = \frac{P(B_2) P(A|B_2)}{P(B_1) P(A|B_1) + P(B_2) P(A|B_2) + P(B_3) P(A|B_3)}$$

$B_1, B_2, B_3 \rightarrow 3$ रूपांकन करना

$$P(A) = 0.2 \quad P(A|D) = 0.002$$

$$P(B) = 0.25 \quad P(B|D) = 0.03$$

$$P(C) = 0.35 \quad P(C|D) = 0.04$$

$$P(E) = 0.2 \quad P(E|D) = 0.05$$

$$P(D|B) = ?$$

$$P(D|B) = \frac{P(D), P(B|D)}{P(D|A)P(A) + P(D|B)P(B|D) + P(D|C)P(C|D) + P(D|E)P(E|D)}$$

=

$$\frac{0.002 \times 0.25 + 0.03 \times 0.25}{0.002 \times 0.2 + 0.03 \times 0.25 + 0.04 \times 0.25 + 0.05 \times 0.2}$$

0.005 + 0.0075

④ বহুমুক্ত মানব: $\left\{ \begin{array}{l} \text{bullets} \\ \text{similar} \end{array} \right. \rightarrow \text{target} \right\}$
 780 page

Example-C

$A \rightarrow 25\%$, $B \rightarrow 35\%$, $C \rightarrow 40\%$
 if three output 5%, 47%, 27% are found to
 be defective

$$A \rightarrow 25\% \quad 5\% \quad P(A) = 0.25$$

$$B \rightarrow 35\% \quad 47\% \quad P(B) = 0.35$$

$$C \rightarrow 40\% \quad 2\% \quad P(C) = 0.4$$

$$P(D|A) = 0.05$$

$$P(D|B) = 0.04$$

$$P(D|C) = 0.02$$

$$\therefore P(B|D) = \frac{P(B)P(D|B)}{P(A)P(D|A) + P(B)P(D|B) + P(C)P(D|C)}$$

$$= \frac{0.35 \times 0.04}{0.25 \times 0.05 + 0.35 \times 0.04 + 0.4 \times 0.02}$$

$$= 0.4058$$

Date: 25.02.24

BINOMIAL DISTRIBUTION Cls: 7

P → probability of success

q → & failure $\rightarrow \frac{1}{2}$

$$\rightarrow P(n) = {}^n C_0 p^n q^{n-p}, \rightarrow \text{probability fn}$$

→ no of trials

no of trials should be fixed in Binom. Dist

su

$$n=10, n=3$$

$$P(3) = {}^{10} C_3 \cdot \left(\frac{1}{2}\right)^3 \cdot \left(\frac{1}{2}\right)^7 \quad F(n) \cdot \left\{ \begin{array}{l} 1/2 \\ 1/2 \end{array} \right.$$

→ n fixed, success failure एकी optn

↓
Condtn fill up → Binomial Distribution

- fair coin $\rightarrow p \& q = 1/2$

Advanced Eng

example 27

Sol: best \rightarrow अधिक 2 \rightarrow at most $\rightarrow 0, 1, 2$
at least \rightarrow अधिक 2 $\rightarrow 0, 1, 2$

Exam: 39 99% \rightarrow 100% का का full success का का

Exercise 11.3

$$P = 0.2 \quad Q = 0.8 \quad n = 4$$

a)

$$P(1) = {}^4C_1 \cdot P(0.2) \cdot Q^3 = {}^4C_1 \cdot (0.2)^1 \cdot (0.8)^3$$

$$= 4 \cdot 0.9096$$

b)

$$P(0) = {}^4C_0 \cdot (0.2)^0 \cdot (0.8)^4$$

$$= 0.9096$$

c) At most 2 (0 or 1 or 2)

$$P(\text{at most } 2) = P(0) + P(1) + P(2)$$

$$\therefore P(2) = {}^4C_2 \cdot (0.2)^2 \cdot (0.8)^2$$

$$= 0.1536$$

$$\therefore P(0) + P(1) + P(2) = 0.9096 + 0.4096 + 0.1536$$

$$= 0.9728$$

Six dice are thrown 729 times

Binomial (216)⁶ slightly differ \rightarrow Poisson Distribution

 Poisson Distribution: $P(\text{on } \mathbb{S})$, very small
 $n \rightarrow \text{large}$

- $n \gg m$ or $m \ll n$: \rightarrow Bin. D. \rightarrow Poisson

- Verteilungsfunktion: $\sum_{k=0}^n P(X=k)$

$$P(n) = \frac{m^n e^{-m}}{n!}$$

- Verteilungsfunktion: $P(X \leq k) = \sum_{i=0}^k P(i)$ mean: m std. dev.: \sqrt{m}

$$\text{Bin}(n, p) \text{ kann Pois}(np) \text{ sein}, P = \frac{m}{n}$$

Example: 39