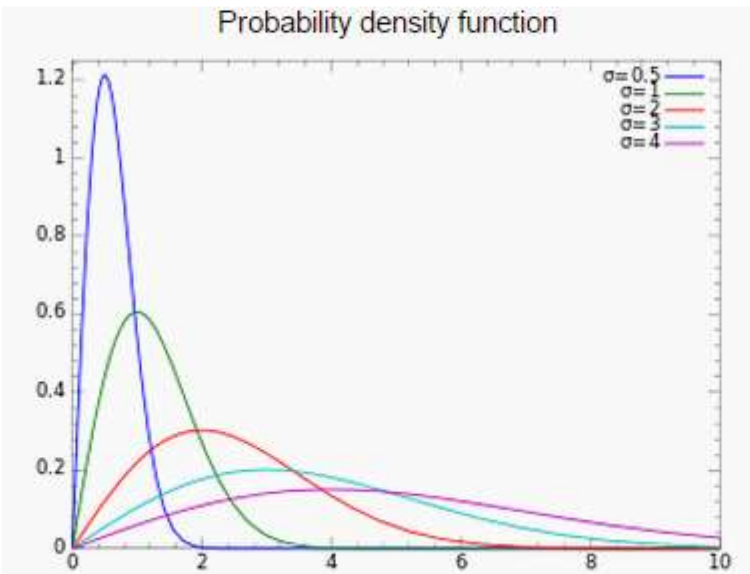
The Rayleigh Distribution:

The Rayleigh distribution is a continuous probability distribution named after the English Lord Rayleigh. It is a special case of the Weibull distribution with a scale parameter of 2.

The notation X Rayleigh(σ) means that the random variable X has a Rayleigh distribution with shape parameter σ. The probability density function (X > 0) is: $\frac{x}{\sigma^2}e^{-x^2/2\sigma^2}$.



The Rayleigh distribution is widely used:

- In communications theory, to model multiple paths of dense scattered signals reaching a receiver.
- In the physical sciences to model wind speed, wave heights and sound/light radiation.
- In engineering, to measure the lifetime of an object, where the lifetime depends on the object's age. For example: resistors, transformers, and capacitors in aircraft radar sets.
- In medical imaging science, to model noise variance in magnetic resonance imaging.

Variance and Mean (Expected Value) of a Rayleigh Distribution

The expected value (the mean) of a Rayleigh is:

$$\mathrm{E}[x] = \sigma\sqrt{\frac{\pi}{2}}$$

How this equation is derived involves solving an integral, using calculus:
The expected value of a probability distribution is:
$E(x) = \int x\, f(x)dx$.

Substituting in the Rayleigh probability density function, this becomes the improper integral:

$$\mathrm{E}[x] = \int_0^\infty x \frac{x}{\sigma^2} \exp\left(-\frac{x^2}{2\sigma^2}\right) dx$$

Where:

- *exp* is the exponential function,
- dx is the differential operator.

Solving the integral for you gives the Rayleigh expected value of $\sigma \sqrt{(\pi/2)}$

The variance of a Rayleigh distribution is derived in a similar way, giving the variance formula of:
$Var(x) = \sigma^2[(4 - \pi)/2]$.

1. Find the Value of expected value and variance of Rayleigh distribution when the parameter $\sigma = 4$.