distribution.

185 to 215

$\mu + d$

$\mu = 200$

$\sigma = 15\,ml$

200ml

$X \sim N(\mu, \sigma^2)$ $X \sim N(200, 15^2)$

W2a

(a) What is the probability that the individual waits more than 10 minutes?

(b) What is the probability that the individual waits between 6 and 12 minutes?

6.5 Given a standard normal distribution, find the area under the curve that lies

(a) to the left of $z = -1.10$;

(b) to the right of $z = 1.645$;

(c) between $z = -2.43$ and -0.45;

(d) to the left of $z = 0.45$;

(e) to the right of $z = -0.45$;

(f) between $z = -0.45$ and $z = 0.45$.

4/15

6.6 Find the value of z if the area under a standard normal curve

(a) to the right of z is 0.3745;

(b) to the left of z is 0.3050;

(c) between $-z$ and 0, with $-z < 0$, is 0.4838;

(d) between $-z$ and z with $z > 0$, is 0.90.

6.7 Given the standard normal distribution, find the value of k such that

(a) $P(Z > k) = 0.9625$;

6.11 A soft-drink machine is regulated so that it discharges an average of 200 milliliters per cup. If the amount of drink is normally distributed with a standard deviation equal to 15 milliliters,

(a) what fraction of the cups will contain more than 224 milliliters?

(b) what is the probability that a cup contains between 191 and 209 milliliters?

(c) how many cups will probably overflow if 230-milliliter cups are used for the next 1000 drinks?

(d) below what value do we get the smallest 25% of the drinks?

$P(X > 224)$

$P\left(\frac{X-\mu}{\sigma} > \frac{224-200}{15}\right)$

$= P\left(Z > \frac{4}{15}\right)$

$= 0.5 - P\left(Z < \frac{4}{15}\right)$

$= 0.5 - \text{Table}$

6.12 The average length of steel nails is 5 centimeters, with a standard deviation of 0.05 centimeters. Assuming that the lengths are normally distributed, what percentage of the nails are

(a) longer than 5.05 centimeters?

(b) between 4.95 and 5.05 centimeters in length?

(c) shorter than 4.90 centimeters?

6.13 A group of individuals with standard health conditions are put on an experimental diet for one month. The gain in weights of these individuals after a month is normally distributed. They average 1450 grams, with a standard deviation of 250 grams. Find the probability

distribution.

185 to 215

$\mu \pm \sigma$

$\mu = 200$

$\sigma = 15\,ml$

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(d) below what value do we get the smallest 25% of the drinks?

$X \sim N(200, 15^2)$

$P(X > 224)$

$P\left(\frac{X-\mu}{\sigma} > \frac{224-200}{15}\right)$

$= P\left(Z > \frac{4}{15}\right)$

$= 0.5 - P\left(Z < \frac{4}{15}\right)$

$= 0.5 - \text{Table}$

$P(191 < X < 209)$

$= P\left(\frac{191-200}{15} < Z < \ldots\right)$

$= P\left(-\frac{9}{15} < Z < \frac{9}{15}\right)$

$= 2P\left(Z < \frac{9}{15}\right)$

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185 to 215

$\mu + \sigma$

$\mu = 200$

$\sigma = 15$ml

200ml

walpole

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$P(191 < X < 209)$

$= P\left(\frac{191-200}{15} < Z < \frac{209-200}{15}\right)$

$= P\left(-\frac{9}{15} < Z < \frac{9}{15}\right)$

$= 2P\left(Z < \frac{9}{15}\right)$

$P(Z < a) = 0.025$

$P(X > 230)$

$P\left(\frac{X-\mu}{\sigma} > \frac{230-200}{15}\right)$

$= P(Z > 2)$

$= 0.5 - P(Z < 2)$

1000

Example 6.15: The probability that a patient recovers from a rare blood disease is 0.4. If 100 people are known ~~to have contracted~~ this disease, what is the probability that fewer than 30 survive?

***Solution*:** Let the binomial variable X represent the number of patients who survive. Since $n = 100$, we should obtain fairly accurate results using the normal-curve approximation with

$$\mu = np = (100)(0.4) = 40 \text{ and } \sigma = \sqrt{npq} = \sqrt{(100)(0.4)(0.6)} = 4.899.$$

To obtain the desired probability, we have to find the area to the left of $x = 29.5$.

The z value corresponding to 29.5 is

$$z = \frac{29.5 - 40}{4.899} = -2.14,$$

and the probability of fewer than 30 of the 100 patients surviving is given by the shaded region in Figure 6.26. Hence,

$$P(X < 30) \approx P(Z < -2.14) = 0.0162.$$

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$$P(X < 30) \approx P(Z < -2.14) = 0.0162.$$

6.6 Gamma and Exponential Distributions

Although the normal distribution can be used to solve many problems in engineering and science, there are still numerous situations that require different types of density functions. Two such density functions, the **gamma** and **exponential distributions**, are discussed in this section.

It turns out that the exponential distribution is a special case of the gamma distribution. Both find a large number of applications. The exponential and gamma distributions play an important role in both queuing theory and reliability problems. Time between arrivals at service facilities and time to failure of component parts and electrical systems often are nicely modeled by the exponential distribution. The relationship between the gamma and the exponential allows the gamma to be used in similar types of problems. More details and illustrations will be supplied later in the section.

The gamma distribution derives its name from the well-known **gamma function**, studied in many areas of mathematics. Before we proceed to the gamma distribution, let us review this function and some of its important properties.

Definition 6.2: The **gamma function** is defined by

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1}e^{-x}\,dx, \qquad \text{for } \alpha > 0.$$

The following are a few simple properties of the gamma function.

(a) $\Gamma(n) = (n-1)(n-2)\cdots(1)\Gamma(1)$, for a positive integer n.

To see the proof, integrating by parts with $u = x^{\alpha-1}$ and $dv = e^{-x}\,dx$, we obtain

$$\Gamma(\alpha) = -e^{-x}\,x^{\alpha-1}\Big|_0^\infty + \int_0^\infty e^{-x}(\alpha-1)x^{\alpha-2}\,dx = (\alpha-1)\int_0^\infty x^{\alpha-2}e^{-x}\,dx,$$

for $\alpha > 1$, which yields the recursion formula

$$\Gamma(\alpha) = (\alpha-1)\Gamma(\alpha-1).$$

for $\alpha > 1$, which yields the recursion formula

$$\Gamma(\alpha) = (\alpha - 1)\Gamma(\alpha - 1).$$

The result follows after repeated application of the recursion formula. Using this result, we can easily show the following two properties.

(b) $\Gamma(n) = (n - 1)!$ for a positive integer n.

(c) $\Gamma(1) = 1$.

Furthermore, we have the following property of $\Gamma(\alpha)$, which is left for the reader to verify (see Exercise 6.39 on page 226).

(d) $\Gamma(1/2) = \sqrt{\pi}$.

The following is the definition of the **gamma distribution**.

Gamma Distribution The continuous random variable X has a **gamma distribution**, with parameters α and β, if its density function is given by

$$f(x; \alpha, \beta) = \begin{cases} \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta}, & x > 0, \\ 0, & \text{elsewhere}, \end{cases}$$

where $\alpha > 0$ and $\beta > 0$.

Graphs of several gamma distributions are shown in Figure 6.28 for certain specified values of the parameters α and β. The special gamma distribution for which $\alpha = 1$ is called the **exponential distribution**.

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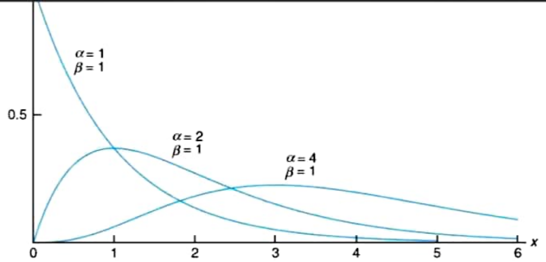


Figure 6.28: Gamma distributions.

Exponential Distribution

The continuous random variable X has an **exponential distribution**, with parameter β, if its density function is given by

$$f(x;\beta) = \begin{cases} \frac{1}{\beta} e^{-x/\beta}, & x > 0, \\ 0, & \text{elsewhere,} \end{cases}$$

where $\beta > 0$.

The following theorem and corollary give the mean and variance of the gamma and exponential distributions.

Theorem 6.4: The mean and variance of the gamma distribution are

$$\mu = \alpha\beta \text{ and } \sigma^2 = \alpha\beta^2.$$

The proof of this theorem is found in Appendix A.26.

In Case of exponential distribu

Corollary 6.1: The mean and variance of the exponential distribution are

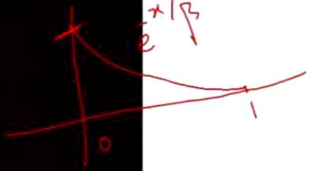
$$\mu = \beta \text{ and } \sigma^2 = \beta^2.$$

Relationship to the Poisson Process

We shall pursue applications of the exponential distribution and then return to the gamma distribution. The most important applications of the exponential distribution are situations where the Poisson process applies (see Section 5.5). The reader should recall that the Poisson process allows for the use of the discrete distribution called the Poisson distribution. Recall that the Poisson distribution is used to compute the probability of specific numbers of "events" during a particular *period of time or span of space*. In many applications, the time period or span of space is the random variable. For example, an industrial engineer may be interested in the Poisson process, and the exponential distribution does apply. Other applications include survival times in biomedical experiments and computer response time.

In the following example, we show a simple application of the exponential distribution to a problem in reliability. The binomial distribution also plays a role in the solution.

G 1 years



Example 6.17: Suppose that a system contains a certain type of component whose time, in years, to failure is given by T. The random variable T is modeled nicely by the exponential distribution with mean time to failure $\beta = 5$. If 5 of these components are installed in different systems, what is the probability that at least 2 are still functioning at the end of 8 years?

Solution: The probability that a given component is still functioning after 8 years is given by

$$P(T > 8) = \frac{1}{5}\int_8^\infty e^{-t/5}\,dt = e^{-8/5} \approx 0.2.$$

Let X represent the number of components functioning after 8 years. Then using the binomial distribution, we have

$$P(X \geq 2) = \sum_{x=2}^{5} b(x; 5, 0.2) = 1 - \sum_{x=0}^{1} b(x; 5, 0.2) = 1 - 0.7373 = 0.2627.$$

There are exercises and examples in Chapter 3 where the reader has already encountered the exponential distribution. Others involving waiting time and reliability include Example 6.24 and some of the exercises and review exercises at the end of this chapter.

The Memoryless Property and Its Effect on the Exponential Distribution

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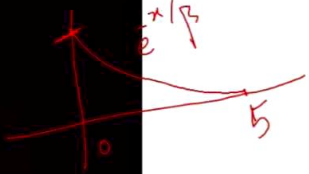

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The Memoryless Property and Its Effect on the Exponential Distribution

The types of applications of the exponential distribution in reliability and component or machine lifetime problems are influenced by the **memoryless** (or lack-of-memory) **property** of the exponential distribution. For example, in the case of, the Poisson process, and thus the exponential distribution does apply. Other applications include survival times in biomedical experiments and computer response time.

In the following example, we show a simple application of the exponential distribution to a problem in reliability. The binomial distribution also plays a role in the solution.

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There are exercises and examples in Chapter 3 where the reader has already encountered the exponential distribution. Others involving waiting time and reliability include Example 6.24 and some of the exercises and review exercises at the end of this chapter.

$P(X \geq 2)$
$= P(2) + P(3) + P(4) + P(5)$

$= \frac{1}{\beta}\int_8^\infty e^{-t/\beta}\,dt$

$X \sim B(n, p)$

The Memoryless Property and Its Effect on the Exponential Distribution

The types of applications of the exponential distribution in reliability and component or machine lifetime problems are influenced by the **memoryless** (or lack-of-memory) **property** of the exponential distribution. For example, in the case of,

=

reader. The following is a numerical example of the use of the gamma distribution in a waiting-time application.

Example 6.18: Suppose that telephone calls arriving at a particular switchboard follow a Poisson process with an average of 5 calls coming per minute. What is the probability that up to a minute will elapse by the time 2 calls have come in to the switchboard?

Solution: The Poisson process applies, with time until 2 Poisson events following a gamma distribution with $\beta = 1/5$ and $\alpha = 2$. Denote by X the time in minutes that transpires before 2 calls come. The required probability is given by

$$P(X \le 1) = \int_0^1 \frac{1}{\beta^2} x e^{-x/\beta}\, dx = 25 \int_0^1 x e^{-5x}\, dx = 1 - e^{-5}(1+5) = 0.96.$$

While the origin of the gamma distribution deals in time (or space) until the occurrence of α Poisson events, there are many instances where a gamma distribution works very well even though there is no clear Poisson structure. This is particularly true for **survival time** problems in both engineering and biomedical applications.

Example 6.19: In a biomedical study with rats, a dose-response investigation is used to determine the effect of the dose of a toxicant on their survival time. The toxicant is one that is frequently discharged into the atmosphere from jet fuel. For a certain dose of the toxicant, the study determines that the survival time, in weeks, has a gamma distribution with $\alpha = 5$ and $\beta = 10$. What is the probability that a rat survives no longer than 60 weeks?

Hence, $x = 51.265$. This means that only 5% of the controls will have lifetimes less than 51,265 miles.

6.10 Weibull Distribution (Optional)

Modern technology has enabled engineers to design many complicated systems whose operation and safety depend on the reliability of the various components making up the systems. For example, a fuse may burn out, a steel column may buckle, or a heat-sensing device may fail. Identical components subjected to identical environmental conditions will fail at different and unpredictable times. We have seen the role that the gamma and exponential distributions play in these types of problems. Another distribution that has been used extensively in recent years to deal with such problems is the **Weibull distribution**, introduced by the Swedish physicist Waloddi Weibull in 1939.

Weibull Distribution

The continuous random variable X has a **Weibull distribution**, with parameters α and β, if its density function is given by

$$f(x; \alpha, \beta) = \begin{cases} \alpha\beta x^{\beta-1} e^{-\alpha x^{\beta}}, & x > 0, \\ 0, & \text{elsewhere,} \end{cases}$$

where $\alpha > 0$ and $\beta > 0$.

The graphs of the Weibull distribution for $\alpha = 1$ and various values of the parameter β are illustrated in Figure 6.30. We see that the curves change considerably in shape for different values of the parameter β. If we let $\beta = 1$, the Weibull distribution reduces to the exponential distribution. For values of $\beta > 1$, the curves become somewhat bell shaped and resemble the normal curve but display some skewness.

The mean and variance of the Weibull distribution are stated in the following theorem. The reader is asked to provide the proof in Exercise 6.52 on page 226.

Like the gamma and exponential distributions, the Weibull distribution is also applied to reliability and life-testing problems such as the **time to failure** or

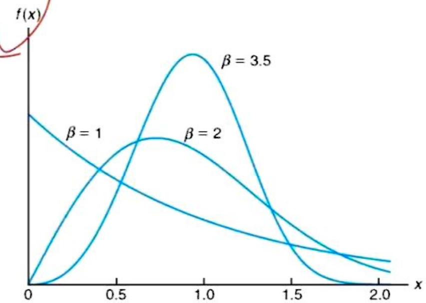


Figure 6.30: Weibull distributions ($\alpha = 1$).

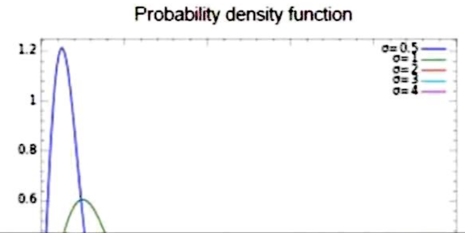
life length of a component, measured from some specified time until it fails. Let us represent this time to failure by the continuous random variable T, with probability density function $f(t)$, where $f(t)$ is the Weibull distribution. The Weibull distribution has inherent flexibility in that it does not require the lack

The Rayleigh Distribution:

The Rayleigh distribution is a continuous probability distribution named after the English Lord Rayleigh. It is a special case of the Weibull distribution with a scale parameter of 2.

The notation *X* Rayleigh(σ) means that the random variable *X* has a Rayleigh distribution with shape parameter σ. The probability density function (X > 0) is:

$$\frac{x}{\sigma^2} e^{-x^2/2\sigma^2}.$$



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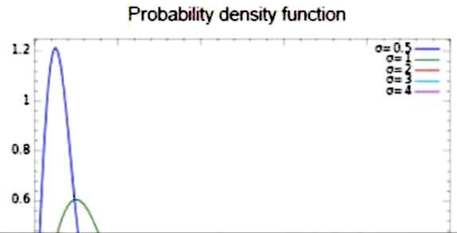

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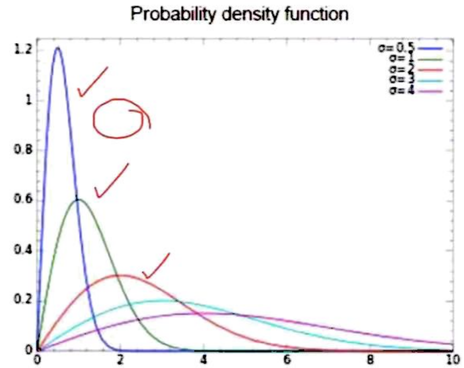

The Rayleigh distribution is widely used:

The Rayleigh distribution is widely used:

- In communications theory, to model multiple paths of dense scattered signals reaching a receiver.
- In the physical sciences to model wind speed, wave heights and sound/light radiation.
- In engineering, to measure the lifetime of an object, where the lifetime depends on the object's age. For example: resistors, transformers, and capacitors in aircraft radar sets.
- In medical imaging science, to model noise variance in magnetic resonance imaging.

How this equation is derived involves solving an integral, using calculus: The expected value of a probability distribution is: $E(x) = \int x\, f(x)dx$.

Substituting in the Rayleigh probability density function, this becomes the improper integral:

$$\mathbb{E}[x] = \int_0^{\infty} x \frac{x}{\sigma^2} \exp\left(-\frac{x^2}{2\sigma^2}\right) dx$$

Where:

- *exp* is the exponential function,
- dx is the differential operator.

Solving the integral for you gives the Rayleigh expected value of $\sigma\sqrt{(\pi/2)}$

The variance of a Rayleigh distribution is derived in a similar way, giving the variance formula of:
$Var(x) = \sigma^2[(4 - \pi)/2]$.

1. Find the Value of expected value and variance of Rayleigh distribution when the parameter $\sigma = 4$.

6.41 If a random variable X has a gamma distribution, with $\alpha = 2$, $\beta = 1$, find $P(1.6 > X > 2.8)$.

6.42 Suppose that the time, in hours, required to service a motorbike is a random variable X having a gamma distribution, with $\alpha = 2$ *and* $\beta = \frac{1}{2}$. What is the probability that on the next service call,

(a) at most 2 hours of service will be required?

(b) at least 1 hour of service will be required?

6.43 (a) Find the mean and variance of the daily water consumption in Exercise 6.40.

(b) According to Chebyshev's theorem, there is a probability of at least 3/4 that the water consumption on any given day will fall within what interval?

6.44 The water supply board of a metropolitan city reveals that the each family consumes an average of 20 liters of drinking water per day, with a standard deviation of $\sqrt{200}$ liters. Let X denote the drinking water consumption per family and follow the gamma distribution.

(a) Find α and β.

(b) Find the probability that a randomly selected family consumes more than 20 liters on a particular day.

6.45 At a train reservation counter, one man completes his reservation with a mean time of 3 minutes. Service completion time is assumed to follow exponential distribution. Out of the 5 customers in queue, what is the probability that at least 4 will complete their reservation within 3 minutes?

6.46 The life of a street bulb follows an exponential distribution, with an average life $\beta = 3$ years. The bulbs are replaced whenever they fail. Out of the 1000 street bulbs installed in a city, what is the probability that at most 250 of them will need to be replaced

(b) Determine the variance of X.

(c) Find the probability that $X > 1/3$.

6.50 If the proportion of a brand of television requiring service during the first year of operation is a random variable having a beta distribution with $\alpha = 3$ and $\beta = 2$, what is the probability that at least 80% of the new models of this brand sold this year will require service during their first year of operation?

6.51 The lives of a certain automobile seal have the Weibull distribution with failure rate $Z(t) = 1/\sqrt{t}$. Find the probability that such a seal is still intact after 4 years.

6.52 Derive the mean and variance of the Weibull distribution.

6.53 In a biomedical research study, it was determined that the survival time, in weeks, of an animal subjected to a certain exposure of gamma radiation has a gamma distribution with $\alpha = 5$ and $\beta = 10$.

(a) What is the mean survival time of a randomly selected animal of the type used in the experiment?

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6.41 If a random variable X has a gamma distribution, with $\alpha = 2$, $\beta = 1$, find $P(1.6 > X > 2.8)$.

6.42 Suppose that the time, in hours, required to service a motorbike is a random variable X having a gamma distribution, with $\alpha = 2$ and $\beta = \frac{1}{2}$. What is the probability that on the next service call,

(a) at most 2 hours of service will be required?

(b) at least 1 hour of service will be required?

6.43 (a) Find the mean and variance of the daily water consumption in Exercise 6.40.

(b) According to Chebyshev's theorem, there is a probability of at least 3/4 that the water consumption on any given day will fall within what interval?

$\mu = 20$

$\sigma = \sqrt{200}$

6.44 The water supply board of a metropolitan city reports that the each family consumes an average of 20 liters of drinking water per day, with a standard deviation of $\sqrt{200}$ liters. Let X denote the drinking water consumption per family and follow the gamma distribution.

(a) Find α and β.

(b) Find the probability that a randomly selected family consumes more than 20 liters on a particular day.

6.45 At a train reservation counter, one man completes his reservation with a mean time of 3 minutes. Service completion time is assumed to follow exponential distribution. Out of the 5 customers in queue, what is the probability that at least 4 will complete their reservation within 3 minutes?

6.46 The life of a street bulb follows an exponential distribution, with an average life $\beta = 3$ years. The bulbs are replaced whenever they fail. Out of the 1000 street bulbs installed in a city, what is the probability that at most 250 of them will need to be replaced

(b) Determine the variance of X.

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