CHITTAGONG UNIVERSITY OF ENGINEERING AND TECHNOLOGY
B.SC. ENGINEERING LEVEL-II TERM-II EXAMINATION '2016

DEPARTMENT	: ELECTRONICS AND TELECOMMUNICATION ENGINEERING
FULL TITLE OF PAPER	: Engineering Statistics and Complex variable
COURSE NO.	: MATH ~~281~~ 281
FULL MARKS	: 210
TIME	: 3 HOURS

The figures in the right margin indicate full marks. Answer any ***THREE*** *questions from each section. Use separate script for each section.*

Section-A

Q.1(a) Define moment, kurtosis, and Skewness of a data with example. 10

(b) Compute first four central moments from the following data. Also find the Beta co-efficient and make comment. 15

Value :	5	10	15	20	25	30	35
Frequency :	8	15	20	32	23	17	5

(c) Karl Pearson's co-efficients of skewness of a distribution is 0.32, its standard deviation is 6.5 and mean is 29.6. Find the mode of the distribution. 10

Q.2(a) The following table gives the data on rainfall and discharge in a certain river. Obtain the regression line of y on x and the line of x on y. 20

Rainfall x (inches)	1.53	1.78	2.60	2.95	3.42
Discharge y(1000 cc)	33.5	36.3	40.0	45.8	53.5

(b) Define mutually exclusive and exhaustive events. A and B are two mutually exhaustive events and P(B)=3/5, P(B/A)=2/7, find P(A). 07

(c) The random variable X has the following probability function 08

$$P(x=n)=\begin{cases} c/x; & x=1,2,.....6 \\ 0; & otherwise \end{cases}$$

Find E(4x+1) and Var(3x+2)

Q.3(a) Define Poisson distribution. Obtain the mean and variance of Poisson distribution. 18

(b) The longevity of a dry cells follows normal distribution. A dry cell has average longevity 12 hours and a standard deviation of life time is 3 hours. What percentage of dry cells is expected to have longevity? 17

i) Between 10 to 15 hours?

ii) More than 15 hours?

Q.4(a) Define null and alternative hypothesis. Write down the procedures consist of a statistical hypothesis. 10

(b) A certain type of cold vaccine is known to be only 20% effective after 2 years. To determine a new vaccine's efficiency 20 people were chosen from this group of people, if more than 8 people surpass 2 years without constructing the virus, then new vaccine will be considered superior than previous one. Write down null and alternative hypothesis. Find the test statistics. Make comment on your investigation. 15

(c) A coin was tossed 400 times and returned heads 2/6 times. Test the hypothesis that the coin is unbiased at 5% level of significance. 10

Section-B

Q.5(a) Define analytic function. Show that an analytic function of constant modulus is constant. 12

(b) Write down the Cauchy-Riemann equations in polar form. Find the complex potential $\omega(z) = \Phi(r,\theta) + i\Psi(r,\theta)$ of an electric field whose stream function is 14

$\Psi = (r - \frac{1}{r})\sin\theta$

09

(c) Evaluate $\int_0^{1+i} (x - y + ix^2)dz$ along z=0 to z=1 and z=1 to z=1+i

Q.6 (a) Define Mobius transformation. Find the Mobius transformation which maps Z=1, i, -1 to W=2, i, -2 12

(b) Find the image of infinite strip 1/4<y<1/2 under the mapping W=1/z. Sketch the region to obtain graphically and interpret. 13

(c) Find the nature and location of singularity of $f(z) = \frac{z - \sin z}{z^2}$. 10

Q.7(a) State Cauchy's integral formula. Using Cauchy's formula, evaluate $\oint_C \frac{\ln z}{(z-1)^3} dz$,where C is $|z-1| = \frac{1}{2}$. 15

(b) Give the statement of Laurent's theorem. expand the Laurent series if $f(z) = \frac{1}{z^2 + 4z + 3}$ in the following regions i) 1<|z|<3 ii) |z|<3 20

Q.8(a) Determine the poles and corresponding residues at each pole of the function 11

$$f(z) = \frac{Z^2 - 2Z}{(Z+1)^2(Z^2+1)}$$

(b) Evaluate the following by contour integration (any two) 24

(i) $\int_0^{2\pi} \frac{d\theta}{2 + \cos\theta}$

(ii) $\int_0^{\infty} \frac{\sin mx}{x^2 + 1} dx$

(iii) $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+2)(x^2+4)}$

THE END

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CHITTAGONG UNIVERSITY OF ENGINEERING AND TECHNOLOGY
B.Sc ENGINEERING LEVEL-II TERM-II EXAMINATION '2014

DEPARTMENT	: ELECTRONICS AND TELECOMMUNICATION ENGINEERING
FULL TITLE OF PAPER	: Engineering Statistics and Complex variable
COURSE NO.	: MATH~~283~~ 281
FULL MARKS	: 210
TIME	: 3 HOURS

The figures in the right margin indicate full marks. Answer any ***THREE*** *questions from each section. Use separate script for each section.*

Section-A

Q.1(a) Define each of the following : 17

i) Conditional probability
ii) Random variable with classification
iii) Estimator with properties of good estimator
iv) Hypothesis testing
v) Population with classification

(b) A bag contains 10 black and 8 white balls. Two successive drawing of 4 balls are made such that i) balls are replace before the second trail ii) the balls are not replace before the second trail. Find the probability that the first drawing will give 4 black and the second 4 white balls. 18

Q.2(a) Show that, $s^2 = \frac{1}{n-1} \left(\sum x^2 - n\bar{x}^2\right)$ is the unbiased estimator of the population variance ~~A=~~ σ^2. 10

(b) Given that four raw moments about 4 are -1.5, 17,-30,108. Show whether the distribution is leptokurtic or platykurtic. 15

(c) Define measures of Kurtosis. Prove that $\beta_2 \geq 1$. 10

Q.3(a) What do you mean by a regration model? If a and b are the least squares estimates of α and β of the regration model, $Y_i = \alpha + \beta X_i + \varepsilon_i$, then prove that, 20

i) a and b are linear functions of the observations
ii) E(a) = α
iii) E(b) = β
iv) $V(a) = \frac{\sum X_i^2}{n\sum (X_i - \bar{X})^2}\sigma^2$
v) $V(b) = \frac{\sigma^2}{\sum (X_i - \bar{X})^2}$
vi) $C_{ov}(a,b) = \frac{-\bar{X}}{\sum (X_i - \bar{X})^2}\sigma^2$

(b) If x_1, x_2 and x_3 are three correlated variables with equal variance s^2, show that the correlation between $x_1 + x_2$ and $x_2 + x_3$ is ½. 15

Q.4(a) Define Binomial experiment. Show that, p, $\sqrt{pq}$ and $3 + \frac{1-6pq}{pq}$ be the mean, standard deviation and kurtosis of the Bernoulli's distribution. 15

(b) In a certain manufacturing process it is known that, on the average 1 (one) in every 100 items is defective. What is the probability that the fifth item inspected is the 1st defective item found? 10

(c) Define standerized normal variate with its need and properties. 10

Section-B

Q.5(a) Define limit and continuity of a complex function $f(z)$. Prove that the function $f(z)$ defined by 25

$$f(z)=\frac{x^3(1+i)-y^3(1-i)}{x^2+y^2}\quad (z\neq 0),\ f(0)=0$$

Is continuous and the Cauchy-Riemann equations are satisfied at the origin, yet $f'(0)$ does not exit.

(b) If $w=\varphi+i\psi$ represents the complex potential for an electric field and $\psi=x^2-y^2+\frac{x}{x^2+y^2}$, determine the function φ. 10

Q.6 (a) If f(z) is analytic within and on a closed contour C and a is any point within C, then show that, 15

$$f(a)=\frac{1}{2\pi i}\int_C \frac{f(z)dz}{z-a}$$

(b) Evaluate the following: 20

i) $\oint_C \frac{\sin^2 z}{(z-\pi/6)^3}dz$, where C is the circle $|z|=1$

ii) $\oint_C \frac{e^z}{(z^2+\pi^2)^2}dz$, where C is the circle $|z|=4$

Q.7 (a) State Taylor's theorem in complex domain. Expand $f(z)=\frac{1}{(z+1)(z+3)}$ in the region $1<|z|<3$ in Larents series. 13

(b) Determine the poles of the function $f(z)=\frac{z^2}{(z-1)^2(z+2)}$ and the residue at each pole. Hence evaluate $\oint_C f(z)dz$, where C is the circle $|z|=2.5$. 12

(c) Define winding number with properties. 10

Q.8(a) Find the bilinear transformation which maps the points $z=1,\ i,\ -1$ onto the points $w=i,\ 0,\ -i$. Hence find the image of $|z|<1$. Also find the invariant points of this transformation. 17

(b) By the method of Contour integration find $\int_0^\infty \frac{\cos 5x}{4+x^2}dx$ 18

THE END

CHITTAGONG UNIVERSITY OF ENGINEERING AND TECHNOLOGY
B.SC. ENGINEERING LEVEL-II TERM-II EXAMINATION '2015

DEPARTMENT	: ELECTRONICS AND TELECOMMUNICATION ENGINEERING
FULL TITLE OF PAPER	: Engineering Statistics and Complex variable
COURSE NO.	: MATH~~283~~ 281
FULL MARKS	: 210
TIME	: 3 HOURS

The figures in the right margin indicate full marks. Answer any ***THREE*** *questions from each section. Use separate script for each section.*

Section-A

Q.1(a) What do mean by measures of dispersion? Find mean and variance of first n natural numbers. 18

(b) The students A and B obtained the following scores in a computer program course: 17

A	:	48	59	60	75	82	90	96	90
B	:	64	72	75	80	80	76	80	73

Who had a better average and who was more consistent?

Q.2(a) State conditional probability of two events. If A and B are independent events, then show that $\overline{A}$ and $\overline{B}$ are also independent. 15

(b) Write down the probability mass function of binomial distribution. Show that the mean and variance of Poisson distribution are same. 20

Q.3(a) Obtain the skewness and kurtosis of the normal distribution. 20

(b) How many fair coins must be tossed together to give at least 80% chance to get at least one head? 15

Q.4(a) Define hypothesis testing with the application. 08

(b) A research scientist reports that mice will live an average of 40 months when their diets are sharply restricted and then enriched with vitamins and proteins. Assuming that the lifetimes of such mice are normally distributed with an s.d of 6.3 months, find the probability that a given mouse will live (i) more than 32 months (ii) less than 28 months (iii) between 37 and 49 months. 14

(c) The s.d of the weight of 100 gms bread made by a certain bakery is 1 gm. On a certain day the owner claims that the production is out of control. To check whether its production is under control, employees select a random sample of 25 breads and find that there mean weight is 99.5 gms. Test the claim of the owner at 5% level of significance. 13

Section-B

Q.5(a) Define a complex variable. If $f : D \rightarrow \mathbb{C}$ is differentiable at a point $Z_0 \in D$, then show that f is continuous at Z_0. 10

(b) Define holomorphic function. Express Cauchy –Riemann equation in polar form. 12

(c) Define harmonic function. Show that u= ½ ln (x^2+y^2) is harmonic. Determine the conjugate of u. 13

Q.6 (a) If f(z) is analytic within and on a closed curve and if a is any point within C, prove that $f^n(a) = \frac{n!}{2\pi i}\oint_C \frac{f(z)}{(z-a)^{n+1}}dz$ 25

(b) Evaluate $\oint_C \frac{e^z}{(z^2+\pi^2)^2}dz$, where C is $|z| = 4$. 10

Q.7(a) Define pole and state Cauchy Residue theorem. Find pole and residue of 20

$f(z)=\dfrac{5z-2}{z(z-1)}$. Hence evaluate $\oint_C f(z)dz$,where C is $|z|=2$.

(b) State Laurents series. Expand $f(z)=(z^2-1)/(z+2)(z+3)$ for the region $2<|z|<3$. 15

Q.8(a) Define bilinear transformation and fixed point. Find the fixed point of the transformation $w=(2z-5)/(z+4)$. 15

(b) Define contour. Evaluate any one of the following using the method of contour integration. 20

(i) $\displaystyle\int_0^{2\pi}\frac{dx}{5-4\cos x}$

(ii) $\displaystyle\int_{-\infty}^{\infty}\frac{\cos mx}{(x^2+a^2)}dx$

THE END

CHITTAGONG UNIVERSITY OF ENGINEERING AND TECHNOLOGY
B.SC. ENGINEERING LEVEL-II TERM-II (15 Batch) EXAMINATION '2018

DEPARTMENT	: ELECTRONICS AND TELECOMMUNICATION ENGINEERING
FULL TITLE OF PAPER	: Engineering Statistics and Complex variable
COURSE NO.	: MATH ~~283~~ 281
FULL MARKS	: 210
TIME	: 3 HOURS

The figures in the right margin indicate full marks. Answer any <u>THREE</u> questions from each section. Use separate script for each section.

Section-A

Q.1(a) Define standard deviation and coefficient of variation. The following are the scores of two batsmen A and B in a series of innings: 17

A	:	12	115	6	73	7	19	119	30	84	35
B	:	47	112	76	42	4	51	37	48	13	0

Who is better and who was more consistent player?

(b) The following data gives the hardness (X) and tensile strength (Y) of 7 samples of a metal in certain units: 18

X	:	146	152	158	164	170	176	182
Y	:	75	78	77	79	80	85	86

(i) Obtain the correlation coefficient between hardness and tensile strength
(ii) Find the regression line of X on Y and also of Y on X

Q.2(a) Define random variable and probability mass function. From a box containing 4 black and 2 green balls, 3 balls are drawn random. Find the probability distribution of the number of green balls. 13

(b) In a continuous distribution, the probability density function is defined as $f(x) = kx(1-x); 0 \le x \le 1$
Find k, mean and variance of the distribution. 12

(c) The probability that a student pilot passes the written test is 0.7. Find the probability that the student will pass the test before the fourth try. 10

Q.3(a) What is binomial distribution? If the probability that a fluorescent light has a useful life of at least 800 hours is 0.9, find the probabilities that among 20 such lights 13

i) Exactly 18 will have a useful life of at least 800 hours.
ii) At least 15 will have a useful life of at least 800 hours.
iii) At least 2 will not have a useful life of least 800 hours.

(b) Define Poisson distribution. Show that Poisson distribution is the limiting form of the binomial distribution. 12

(c) A secretary makes 2 errors per page, on average. What is the probability that on the next page he or she will make 10

i) 4 or more errors?
ii) No errors?

Q.4(a) Define normal distribution. If a set of grades on a statistics examination are normally distributed with mean of 74 and standard deviation 7.9, find 16

i) The lowest passing grade if the lowest 15% of the students are given F grade.
ii) The lowest B if the top 10% of the students are given A and next 20% are given B grade.

(b) A sample of 1000 days is taken from a meteorological records of a certain city and 120 of them are found to be foggy. What are the probability limits to the percentage of foggy days in the city? 10

(c) A sample of 900 members is found to have a mean of 3.4 cm. Can it be regarded as a truly random sample from a large population with near 3.25 cm. and S.D 1.61 cm? 09

Section-B

Q.5(a) Define analytic functions. If the potential function is $\log\sqrt{x^2+y^2}$, find the flux function and the complex potential function. 20

(b) Evaluate $\int_C \bar{z}\,dz$ from z=0 to z= 4+2i among the curve C given by; 15

i) $z = t^2 + it$

ii) The line from z=0 to z=$2i$ and then the line from z=$2i$ to z=$4+2i$

Q.6 (a) State and prove the Cauchy's integral formula. 15

(b) Evaluate $\oint_C \frac{e^{zt}\,dz}{\left(z^2+1^2\right)^2}$, if $t > 0$ and C is the circle $x^2 + y^2 = 9$. 10

(c) Find the nature and location of singularities of the function $f(z) = \frac{\tan z}{z}$ 10

Q.7(a) Define conformal mapping. Find all points where the mapping $f(z) = \sin z$ is conformal. 10

(b) Construct a linear fractional transformation that maps the points i, $-i$, 1 of the z-plane into 0, 1, α of the w-plane respectively. 12

(c) Find a complex linear function that maps the equilateral triangle with vertices $1+i$, $2+i$, and $\frac{3}{2} + (1 + \frac{1}{2}\sqrt{3})i$ onto the equilateral triangle with vertices i, $\sqrt{3} + 2i$, $3i$. 13

Q.8(a) Determine the poles and the residue at each pole of 11

$$f(z) = \frac{z^2 - 2z}{(z+1)^2(z^2+1)}$$

(b) Evaluate the following by contour integration. 24

(i) $\int_0^{\pi} \frac{d\theta}{17 - 8\cos\theta}$

(ii) $\int_0^{\infty} \frac{dx}{\left(1+x^2\right)^2}$

THE END

Exam Date: 13/09/2022

CHITTAGONG UNIVERSITY OF ENGINEERING AND TECHNOLOGY
B.Sc. ENGINEERING LEVEL-2 TERM-I(19 Batch) EXAMINATION '2021

DEPARTMENT	: ELECTRONICS AND TELECOMMUNICATION ENGINEERING
FULL TITLE OF PAPER	: Engineering Statistics and Complex Variables
COURSE NO.	: MATH 281
FULL MARKS	: 210
TIME	: 3 HOURS

The figures in the right margin indicate full marks. Answer any THREE questions from each section. Use separate script for each section.

Section-A

Q.1(a) Define an analytic function. Show that the function $f(z) = u + iv$, where **25**

$$f(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2+y^2}, \quad z \neq 0$$
$$= 0, \quad z = 0$$

Satisfies the Cauchy-Riemann conditions at $z = 0$. Is the function analytic at $z = 0$? Justify your answer.

Q.1(b) If $W = \Phi + i\Psi$ represents the complex potential for an electric field and $\Psi = 3x^2y - y^3$, find the potential function Φ. **10**

Q.2(a) State the Cauchy's theorem. Evaluate $\oint_C \frac{e^{-z}dz}{(z+2)^5}$ where $|z| = 3$. **13**

Q.2(b) Determine the poles and residue at each pole of the function $f(z) = \frac{(z-3)}{(z-2)^2(z+1)}$ **12**

Q.2(c) Expand $\frac{1}{(z+1)(Z+3)}$ in the Laurent's series if $1 < z < 3$. **10**

Q.3(a) Define bilinear transformation. Find the bilinear transformation which maps the points $z = \infty, i, 0$ into $W = 0, i, \infty$ respectively. What are the invariant points of this transformation? **15**

Q.3(b) Find the invariant points of the transformation $\omega = \frac{z-1}{z+3}$ **10**

Q.3(c) Determine the analytic function whose real part is $u = xSinxCoshy - yCosxSinhy$. **10**

Q.4(a) Evaluate any two of the following by contour integration: **35**

(i) $\int_0^{2\pi} \frac{Cos3\theta d\theta}{5-4Cos\theta}$

(ii) $\int_0^{\infty} \frac{dx}{(x^2+1)(x^2+4)^2}$

(iii) $\int_0^{\infty} \frac{xSinx}{x^2+4}dx$

Section-B

Q.5(a) Define conditional probability. Box A contains 3 red and 6 green balls and box B contains 5 red and 4 green balls. One ball is taken at random from A and placed unseen in B. What is the probability that a ball now drawn from B is red? **12**

Q.5(b) There are three sets of people P_1, P_2 and P_3. P_1 contains 3 men and 2 boys, P_2 contains 2 men and 4 boys and P_3 contains 4 men and 3 boys. A set is selected and a people is chosen at random from the set. If the chosen people is boy, what is the probability that the boy has come from P_3? **13**

Q.5(c) A coin is biased so that a head is thrice as likely to occur as a tail. If the coin is tossed 4 times, what is the probability that 1 head and 3 tails will occur? **10**

Q.6(a) Define probability density function. Consider the following function **10**

$f(x) = K(x - x^2)$, for $0 < x < 1$

$= 0$, Otherwise

Find the mean and variance of the distribution.

Q.6(b) Define binomial distribution. Find the mean and variance of binomial distribution. **15**

Q.6(c) Out of 1000 families with 5 children each, how many family would you expect to have **10**

(i) 3 boys and 2 girls?

(ii) At least 3 boys?

(iii) At most 2 girls?

Assume equal probabilities for boys and girls.

Q.7(a) Define normal distribution. The I.Q.(Intelligence quotient) of a group of 1000 children has mean value of 96 and standard deviation 12. Assuming that the distribution of I.Q. is normal, find approximately the number of children having I.Q. **17**

(i) Less than 72

(ii) Between 80 and 120

(iii) More than 100

Q.7(b) The following data shows the marks in Mathematics and statistics of 10 students **18**

Math:X	92	89	87	86	83	77	71	63	77	95
Math:Y	86	88	91	77	68	85	52	82	75	76

(i) Find the correlation coefficient between the marks of Mathematics and statistics

(ii) Determine the regression of the line of Y on X and hence estimate the probable value of Y when X=75.

Q.8(a) Define Poisson distribution. Let x be a binomial random variable with parameters n and p. If n approaches infinity and p approaches 0 in such a way that np remains constant at same value $K > 0$, show that $P[X = x] = n_{C_x} P^x (1 - p)^{n-x} = \frac{e^{-k}k^x}{\lfloor x}$ **20**

Q.8(b) The probability that a student pilot passes the written test for a private pilot license is 80%. Find the probability that a student will pass the test **15**

i) On the fourth try

ii) Before the third try

THE END

$f(z)=\frac{5z-2}{z(z-1)}$. Hence evaluate $\oint_C f(z)dz$,where C is $|z|=2$.

(b) State Laurents series. Expand $f(z)=(z^2-1)/(z+2)(z+3)$ for the region $2<|z|<3$. 15

Q.8(a) Define bilinear transformation and fixed point. Find the fixed point of the transformation $w=(2z-5)/(z+4)$. 15

(b) Define contour. Evaluate any one of the following using the method of contour integration. 20

(i) $\int_0^{2\pi} \frac{dx}{5-4\cos x}$

(ii) $\int_{-\infty}^{\infty} \frac{\cos mx}{\left(x^2+a^2\right)}dx$

THE END