VI. MITIGATION OF POWER SYSTEM HARMONICS

Mitigation of power system harmonics can be categorized as corrective solutions and precautionary solutions.

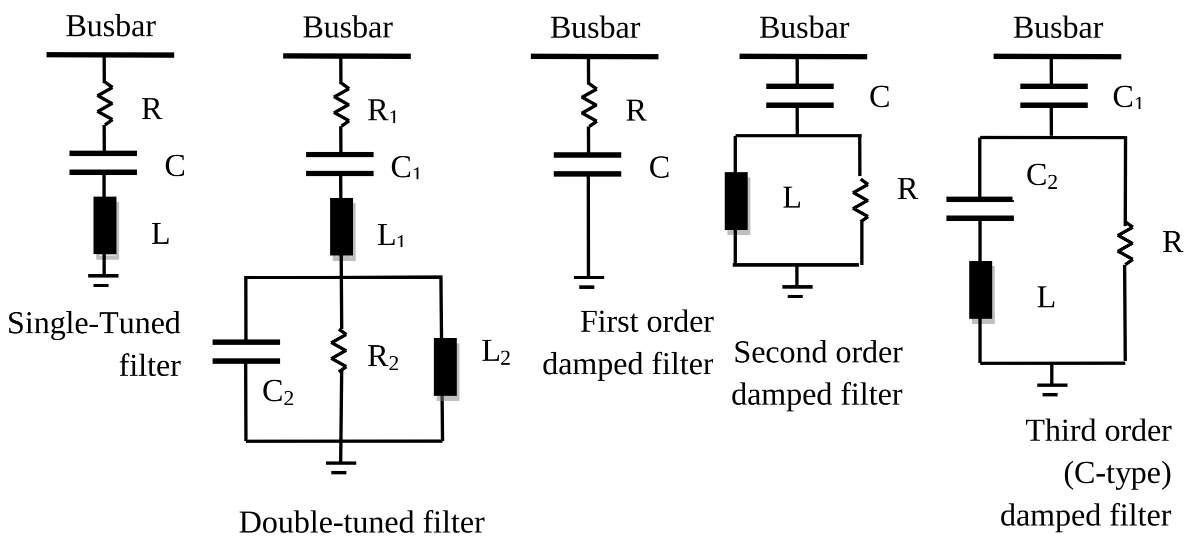
- **Corrective (remedial) solutions** are the techniques to overcome the existing problems.
  - The use of active and passive filters
  - Reconfiguration of the feeders or reallocation of capacitor banks to overcome the resonance.
- **Precautionary (Preventive) solutions** aim to avoid harmonics and their consequences.
  - Phase cancellation or harmonic control in power convertors.
  - Developing procedures and methods to control, reduce or eliminate harmonics in power system equipment; mainly capacitors, transformers and generators.

6.1 Passive Harmonic Filters

Precautionary solutions are not generally sufficient to eliminate the harmonics in power system, so we should use harmonic filters to eliminate or to reduce the effects of one or more orders of harmonic components. In a general context, we can refer to harmonic filters as passive and active filters.

Passive filters are inductance, capacitance, and resistance elements configured and tuned to control harmonics. Passive filtering techniques that make use of

- Single or double-tuned filters providing low impedance path to harmonic currents at certain frequencies or
- high or band-pass filters (damped filters) that can filter harmonics over a certain frequency bandwidth.



Passive filters are relatively inexpensive compared with other methods for eliminating harmonic distortion. However, they have the disadvantage of potentially interacting adversely with the power system, and it is important to check all possible system interactions when they are designed.

Passive filters work efficiently when they are located closer to harmonic generators (nonlinear loads). The resonant frequency must be safely away from any significant harmonic or other frequency component that may be produced by the load. Filters are commonly tuned ***slightly lower*** than the harmonic frequency for safety.

Passive filter design must take into account expected growth in harmonic current sources or load reconfiguration because it can otherwise be exposed to overloading, which can rapidly develop into extreme overheating and thermal breakdown. The design of a passive filter requires a precise knowledge of the harmonic-producing load and of the power system. A great deal of simulation work is often required to test its performance under varying load conditions or changes in the topology of the network.

Because passive filters always provide reactive compensation to a degree dictated by the voltampere size and voltage of the capacitor bank used, they can in fact be designed for the double purpose of providing the filtering action and compensating power factor to the desired level. If more than one filter is used — for example, sets of 5th and 7th or 11th and 13th branches — it will be important to remember that all of them will provide a certain amount of reactive compensation.

Passive filter is a series combination of an inductance and a capacitance. In reality, in the absence of a physically designed resistor, there will always be a series resistance, which is the intrinsic resistance of the series reactor sometimes used as a means to avoid filter overheating. All harmonic currents whose frequency coincides with that of the tuned filter will find a low impedance path through the filter.

6.1.1 Tuned filter design

Tuning a capacitor to a certain harmonic requires an additional reactor at the tuned harmonic.

$$X_{Ln} = \boldsymbol{h_n} X_L = X_{Cn} = \frac{X_C}{\boldsymbol{h_n}} = X_n \;\Rightarrow\; \boldsymbol{X_n} = \sqrt{X_L X_C} = \sqrt{L/\boldsymbol{C}}$$

$$h_n = \frac{\boldsymbol{f_n}}{\boldsymbol{f}_0} = \sqrt{\frac{X_C}{X_L}} = \frac{1}{\boldsymbol{w}_0\sqrt{\boldsymbol{LC}}} \;\Rightarrow\; X_L = \frac{X_C}{h_n^2}$$

The capacitor should withstand the total voltage across its terminals.

$$V_{C\text{-}LL}\Big|_{\textit{phases neglected}} = \sum_{h=1} V_{Ch\text{-}LL} = \sum_{h=1} \sqrt{3} V_{Ch\text{-}phase} = \sum_{h=1} \sqrt{3} X_{Ch} I_{Ch} = \sum_{h=1} \sqrt{3} \frac{X_C}{h} I_{Ch}$$

$$V_{C\text{-}RMS} = \sqrt{\sum_{h=1} V_{Ch\text{-}LL}^2} = \sqrt{3\sum_{h=1}\left[X_C I_{Ch}/h\right]^2}$$

The reactive power absorbed by the reactor is

$$Q_L = \sum_{h=1} V_{Lh} I_{Lh} = \sum_{h=1} X_{Lh} I_{Lh}^2 = \sum_{h=1} h X_L I_{Lh}^2 = \sum_{h=1} \frac{V_{Lh}^2}{h X_L}$$

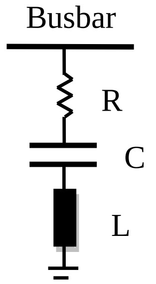
$$\frac{Q_L}{Q_{L1}} = \frac{X_L}{V_{L1}^2} \sum_{h=1} \frac{V_{Lh}^2}{h X_L} = \sum_{h=1} \frac{1}{h}\left[V_{Lh}/V_{L1}\right]^2 = \sum_{h=1} h\left[I_{Lh}/I_{L1}\right]^2$$

The reactive power delivered by the capacitor is

$$Q_C = \sum_{h=1} V_{Ch} I_{Ch} = \sum_{h=1} X_{Ch} I_{Ch}^2 = \sum_{h=1} \frac{X_C}{h} I_{Ch}^2 = \sum_{h=1} \frac{h}{X_C} V_{Ch}^2$$

$$\frac{Q_C}{Q_{C1}} = \frac{1}{X_C I_{C1}^2} \sum_{h=1} \frac{X_C}{\boldsymbol{h}} I_{Ch}^2 = \sum_{h=1} \frac{1}{h}\left[I_{Ch}/I_{C1}\right]^2 = \sum_{h=1} h\left[V_{Ch}/I_{C1}\right]^2$$

A series tuned filter is designed to trap a certain harmonic by adding a reactor to an existing capacitor. Design steps of the following single-tuned series filter is as follows.



Single Tuned filter

- Determine the value of the capacitance, Q_C to improve the power factor and to eliminate any penalty by the electric power company. Power factor compensation is generally applied to raise power factor to around 0.98 or higher.
- Evaluate the capacitor reactance at fundamental frequency $X_C = \dfrac{kV^2}{\boldsymbol{Q_C}}$
- Calculate the reactor size providing the resonance, $X_L = \dfrac{X_C}{\boldsymbol{h_n^2}}$

- Calculate the reactor resistance for a specified quality factor, Q, $R = \dfrac{X_n}{\boldsymbol{Q}}$; $30 < Q < 50$,

The characteristic reactance is $X_n = X_{Ln} = X_{Cn} = \sqrt{X_L X_C} = \sqrt{L/\boldsymbol{C}}$

Filter size $Q_{Filter} = \dfrac{\boldsymbol{kV}^2}{X_C - X_L} = \dfrac{\boldsymbol{kV}^2}{X_C - X_C/\boldsymbol{h}_n^2} = \dfrac{\boldsymbol{h}_n^2}{\boldsymbol{h}_n^2 - 1}\dfrac{\boldsymbol{kV}^2}{X_C} = \dfrac{\boldsymbol{h}_n^2}{\boldsymbol{h}_n^2 - 1}Q_C$

Filter impedance $Z_F(\boldsymbol{h}) = \boldsymbol{R} + \boldsymbol{j}(\boldsymbol{h}X_L - X_C/\boldsymbol{h})$; $|Z_F(\boldsymbol{h})| = \sqrt{\boldsymbol{R}^2 + [\boldsymbol{h}X_L - X_C/\boldsymbol{h}]^2}$

The voltage across the terminals of the capacitor will be,

at fundamental frequency $V_{C1} = \dfrac{V_{Bus1}}{\boldsymbol{j}[X_L - X_C]}[-\boldsymbol{j}X_C] \Rightarrow \dfrac{V_{C1}}{V_{Bus1}} = \dfrac{-\boldsymbol{j}X_C}{\boldsymbol{j}[X_L - X_C]} = \dfrac{X_C/X_L}{X_C/X_L - 1} = \dfrac{\boldsymbol{h}_n^2}{\boldsymbol{h}_n^2 - 1}$

at tuned frequency $V_{Cn} = \dfrac{V_{Busn}}{\boldsymbol{R} + \boldsymbol{j}\underbrace{[X_{Ln} - X_{Cn}]}_{0}}[-\boldsymbol{j}X_{Cn}] \Rightarrow \dfrac{V_{Cn}}{V_{Busn}} = \dfrac{-\boldsymbol{j}X_n}{\boldsymbol{R}} = -\boldsymbol{jQ}$

$$V_{Bus1} = \frac{\boldsymbol{h}_n^2 - 1}{\boldsymbol{h}_n^2}V_{C1} = V_{C1} - \frac{V_{C1}}{\boldsymbol{h}_n^2} = V_{C1} - V_{L1}$$

The following points summarize the relevant quality factor aspects in single-tuned filters:

- Typically, the resistance of a single-tuned harmonic filter is the intrinsic resistance of the reactor.
- However, R can be favorably used to vary the quality factor of the filter and provide a way to control the amount of desired harmonic current through it.
- A large Q value implies a prominent valley at the resonant (tuning) frequency of a filter and therefore the trapping of the largest amount of harmonic frequency.
- The best reduction of harmonic distortion will be achieved with large Q value filters. However, care should be exercised in assessing harmonic currents of frequencies other than the one for which the filter is tuned because they will also find a reduced impedance path. These currents will provide increased heat dissipation. It will often be necessary to conduct computer-aided harmonic simulation studies to predict the performance of the filters, especially when multiple harmonic sources exist.
- Lower quality factor filters could be used in situations in which harmonic distortion barely exceeds the limits and a small filtering action is all that is needed to bring it into compliance.

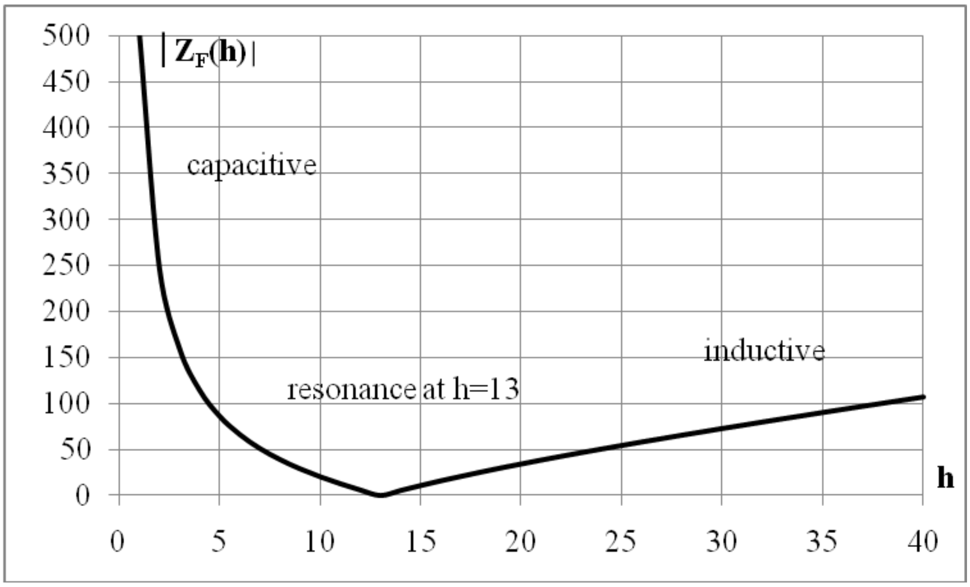
Example: A series filter is tuned to the 13th harmonic. Given X_C = 507 Ohm. Calculate the filter elements and plot the filter impedance.

$$X_L = \frac{X_C}{\boldsymbol{h}_n^2} = \frac{507}{13^2} = 3\ \Omega$$

$$X_n = X_{Ln} = X_{Cn} = \sqrt{X_L X_C} = \sqrt{507*3} = 39\ \Omega$$

For Q=100 $\quad R = \dfrac{X_n}{\boldsymbol{Q}} = \dfrac{39}{100} = 0.39\ \Omega$

$$Z_F(\boldsymbol{h}) = \boldsymbol{R} + \boldsymbol{j}(\boldsymbol{h}X_L - X_C\boldsymbol{h}) = 0.39 + \boldsymbol{j}(3\boldsymbol{h} - 507/\boldsymbol{h})\ ;\ \left|Z_F(\boldsymbol{h})\right| = \sqrt{0.39^2 + \left[3\boldsymbol{h} - 507/\boldsymbol{h}\right]^2}$$



Example: What is the tuning order and the quality factor for a 36 kV series-tuned filter with X_C = 544.5 Ohm, X_L = 4.5 Ohm and R=0.825 Ohm?

$$\boldsymbol{h_n} = \sqrt{X_C/X_L} = \sqrt{544.5/4.5} = 11$$

$$\boldsymbol{Q} = \frac{X_n}{R} = \frac{\sqrt{X_C X_L}}{\boldsymbol{R}} = \frac{\sqrt{544.5*4.5}}{0.825} = 60$$

$$\boldsymbol{Q_C} = \frac{kV^2}{X_C} = \frac{36^2}{544.5}10^6 = 2.38\ \boldsymbol{MVAr}$$

$$\boldsymbol{Q_{Filter}} = \frac{kV^2}{X_C - X_L} = \frac{36^2}{544.5 - 4.5}10^6 = 2.40\ \boldsymbol{MVAr}$$

$$Z_F(\boldsymbol{h}) = \boldsymbol{R} + \boldsymbol{j}(\boldsymbol{h}X_L - X_C\boldsymbol{h}) = 0.825 + \boldsymbol{j}(4.5\boldsymbol{h} - 544.5/\boldsymbol{h})\ ;\ |Z_F(\boldsymbol{h})| = \sqrt{0.825^2 + [4.5\boldsymbol{h} - 544.5/\boldsymbol{h}]^2}$$

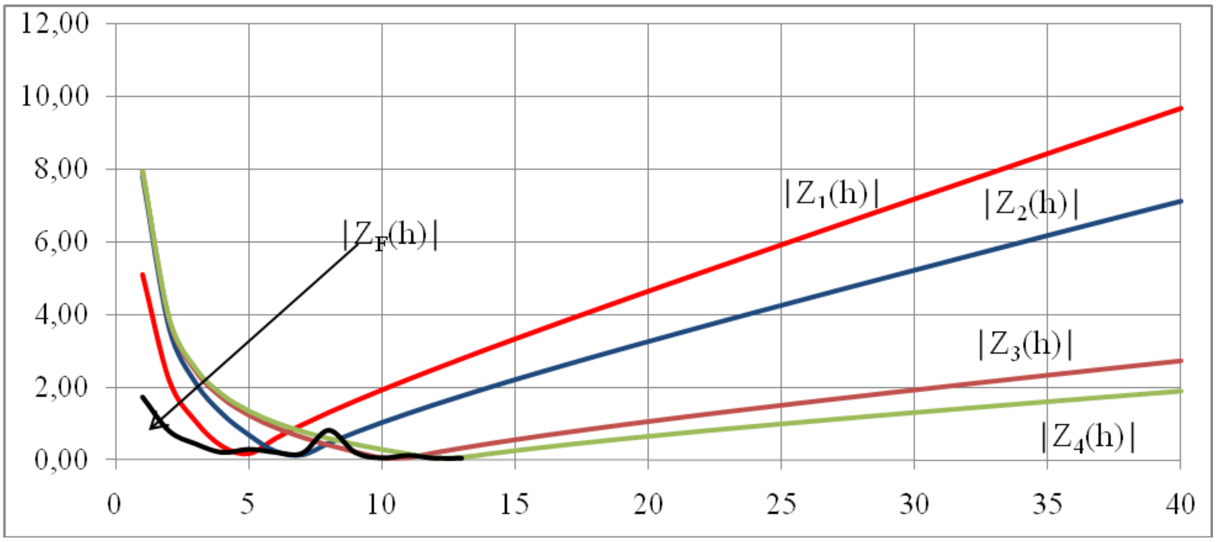
Example: Which harmonics will be trapped by the filter comprising four series-tuned branches with 3Φ, 50 Hz, 400 V, Y-connected, 1x30+3*20 kVAr capacitor banks and 0.779, 0.583, 0.233 and 0.166 mH reactors?

The following table can be constructed for the given C and L values.

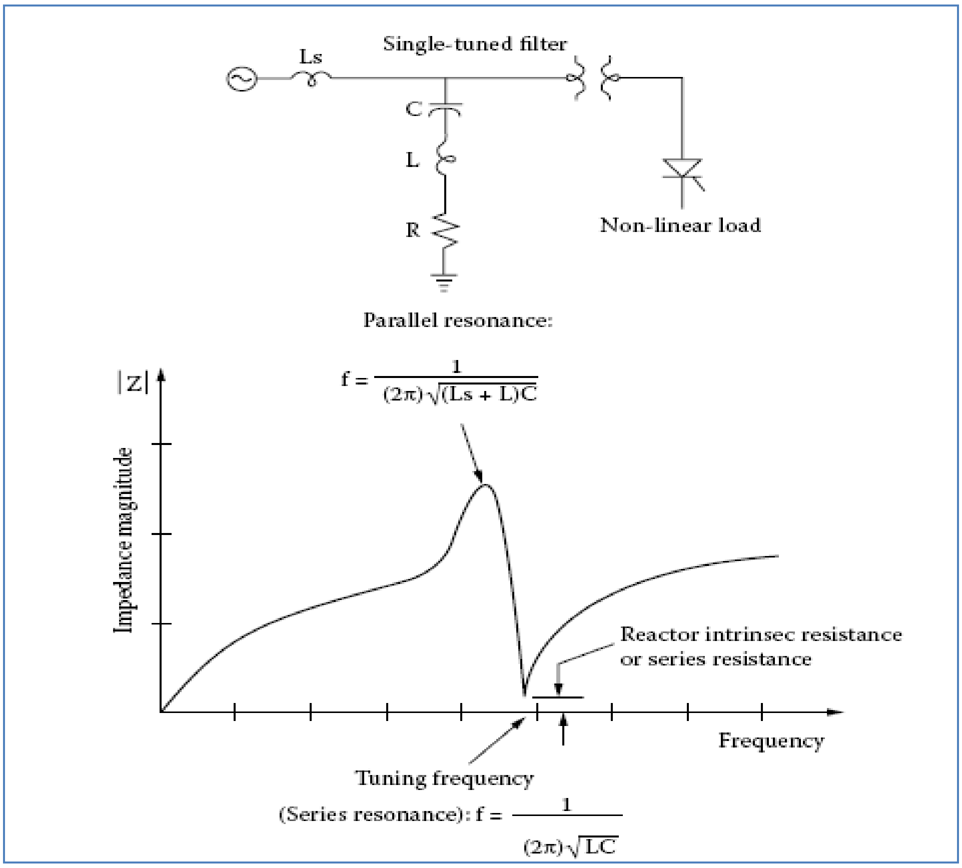
$$X_C = \frac{kV^2}{\boldsymbol{Q_C}}\ ,\ X_L = \boldsymbol{w}_0\boldsymbol{L}\ ,\ \boldsymbol{h_n} = \sqrt{X_C/X_L}\ ,\ X_n = \sqrt{X_C X_L}, \boldsymbol{Q}\ \textit{\textbf{given}},\ \boldsymbol{R} = X_n/\boldsymbol{Q}$$

	Filter-1	Filter-2	Filter-3	Filter-4
Q_C [kVAr]	30	20	20	20
X_C [Ohm]	5.33	8.00	8.00	8.00
L [mH]	0.779	0.583	0.233	0.166
X_L [Ohm]	0.2447	0.1832	0.0732	0.0522
h_n	4.67	6.61	10.45	12.39
X_n [Ohm]	1.142	1.210	0.765	0.646
Q	100	100	100	100
R [mOhm]	11.42	12.10	7.65	6.46

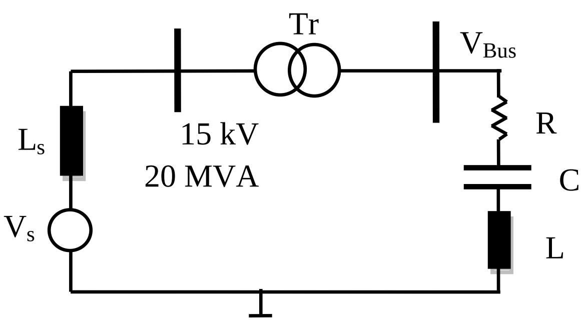
$$\left.\begin{aligned} Z_1(\boldsymbol{h}) &= 0.01142 + \boldsymbol{j}(0.2447\boldsymbol{h} - 5.33/\boldsymbol{h}) \\ Z_2(\boldsymbol{h}) &= 0.01210 + \boldsymbol{j}(0.1832\boldsymbol{h} - 8.00/\boldsymbol{h}) \\ Z_3(\boldsymbol{h}) &= 0.00765 + \boldsymbol{j}(0.0732\boldsymbol{h} - 8.00/\boldsymbol{h}) \\ Z_4(\boldsymbol{h}) &= 0.00646 + \boldsymbol{j}(0.0522\boldsymbol{h} - 8.00/\boldsymbol{h}) \end{aligned}\right\} Z_F(\boldsymbol{h}) = \frac{1}{\dfrac{1}{Z_1(\boldsymbol{h})} + \dfrac{1}{Z_2(\boldsymbol{h})} + \dfrac{1}{Z_3(\boldsymbol{h})} + \dfrac{1}{Z_4(\boldsymbol{h})}}$$



The interaction of the filter with the source reactance (L_s) always creates a parallel resonance condition addition to the series resonance frequency of the filter. Note that the frequency of the parallel resonant point would experience a shift whenever changes in filter elements (Lor C) or in source inductance occur. Source impedance is not constant and changes due to the changes in system configuration such as following the disconnection or addition of a transformer at the substation.



Example: Assume that a series filter tuned to fifth harmonic is going to be installed to the LV busbar in the following system. Determine the filter characteristics. Determine the parallel resonance conditions and plot the overall impedance diagram of the system.



Tr: 15/0.4 kV , 2 MVA , 6.5 %

C : 0.4 kV , 200 kVAr

$$X_C = \frac{kV^2}{Q_C} = \frac{0.4^2}{0.2} = 0.8\ \Omega \ , \ X_L = \frac{X_C}{h_n^2} = \frac{0.8}{25} = 0.032\ \Omega$$

$$X_n = X_{Ln} = X_{Cn} = \sqrt{X_L X_C} = 0.16\ \Omega$$

For Q=100 $\quad R = \dfrac{X_n}{\boldsymbol{Q}} = \dfrac{0.16}{100} = 1.6\ \boldsymbol{m}\Omega$

$$Z_F(\boldsymbol{h}) = \boldsymbol{R} + \boldsymbol{j}(\boldsymbol{h}X_L - X_C\boldsymbol{h}) = 0.0016 + \boldsymbol{j}(0.032\boldsymbol{h} - 0.8/\boldsymbol{h})$$

$$\sqrt{3}\boldsymbol{U} * \boldsymbol{I} = \sqrt{3}\boldsymbol{U}\frac{\boldsymbol{U}/\sqrt{3}}{\boldsymbol{X}_s} = \frac{\boldsymbol{U}^2}{\boldsymbol{X}_s} = 20\ \boldsymbol{MVA} \Rightarrow \boldsymbol{X}_s = \frac{\boldsymbol{U}^2}{20\ \boldsymbol{MVA}} = 11.25\ \ \Omega$$

$$\boldsymbol{X}_s(\boldsymbol{LV}) = 11.25 * (0.4/15)^2 = 0.008\ \Omega$$

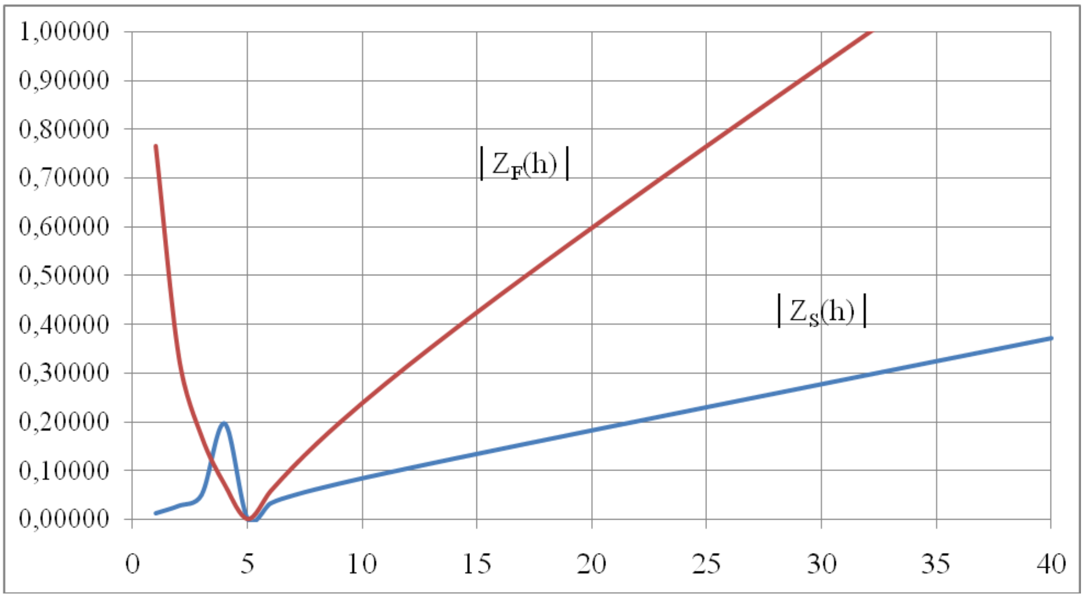
$$\boldsymbol{X}_{Tr}(\boldsymbol{LV}) = 0.065\frac{\boldsymbol{U}^2}{\boldsymbol{MVA}} = 0.065\frac{0.4^2}{2} = 0.0052\ \ \Omega$$

$$\boldsymbol{X}_{LV} = \boldsymbol{X}_s(\boldsymbol{LV}) + \boldsymbol{X}_{Tr}(\boldsymbol{LV}) = 0.0132\ \ \Omega$$

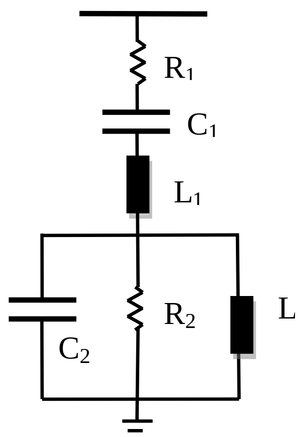
Parallel resonance frequency order $\boldsymbol{n}_p = \sqrt{\dfrac{\boldsymbol{X}_C}{\boldsymbol{X}_L + \boldsymbol{X}_{LV}}} = 4.21$

$$\boldsymbol{Z}_{LV}(\boldsymbol{h}) = \boldsymbol{Z}_{LV} * \boldsymbol{h} = \boldsymbol{j} * \boldsymbol{h} * 0.0132\ \ \Omega$$

$$Z_{System}(\boldsymbol{h}) = \frac{Z_F(\boldsymbol{h}) * Z_{LV}(\boldsymbol{h})}{Z_F(\boldsymbol{h}) + Z_{LV}(\boldsymbol{h})}$$



Sometimes double-tuned filters are preferred instead of two single-tuned series filter because of their low losses. An example double-tuned filter is shown below.

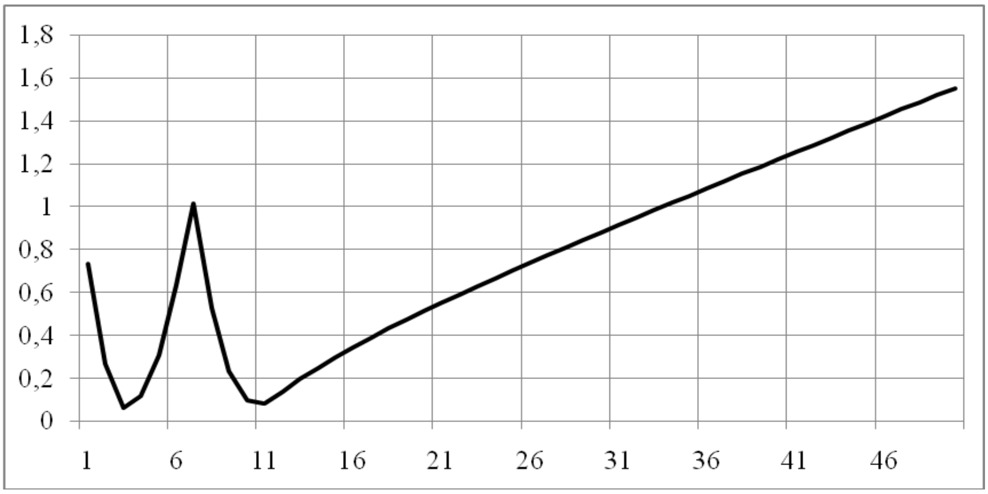


Double-tuned filter

Impedance scan of the double tuned filter is given below for the following parameters. Note that individual tuned frequencies of series and the parallel resonance circuits change when they are combined.

R_1 = 0.01 Ohm, C_1 = 3.98 mF L_1 = 0.102 mH

R_2 = 1.0 Ohm, C_2 = 2.07 mF L_2 = 0.10 mH



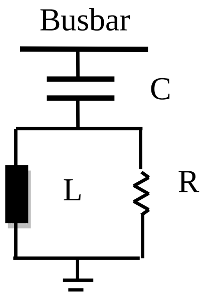
6.1.2 Damped filter design

Damped filters, high-pass in particular, show small impedance value above their corner frequency. This filter draws a considerable percentage of frequency harmonic currents above the corner frequency. Therefore, this frequency must be placed below all harmonic currents that have an important presence in the installation. In planning to adopt a high-pass filter as a harmonic mitigating measure, the following aspects should be considered:

- The impedance-frequency characteristic of a high-pass filter will entail a very different filtering action as compared with that provided by a single-tuned filter.

- Harmonic current elimination using a high-pass filter may require a quite different sizing of filter elements, particularly of the capacitor bank, compared with a single-tuned filter. For example, a 3-MVAr bank used in a fifth harmonic filter in a 50-Hz application may fall short in size when used as part of a high-pass filter with a corner frequency of 300 Hz. Obviously, this will very much depend on the additional harmonic currents that the high-pass filter will be draining off. First-order high-pass filters are characterized by large power losses at fundamental frequency, for which they are less common.

- The second-order high-pass filter is the simplest to apply; it provides a fairly good filtering action and reduces energy losses at fundamental frequency.
- The third-order high-pass filter presents greater operating losses than the second-order high-pass filter and is less effective in its filtering action. We will therefore concentrate ourselves on the second order damped filters.



Second order damped filter

Design steps of a second order damped filter is as follows.

- Determine the value of the capacitance, Q_C to improve the power factor and to eliminate any penalty by the electric power company. Power factor compensation is generally applied to raise power factor to around 0.98 or higher.
- Evaluate the capacitor reactance at fundamental frequency $\mathrm{X_C} = \dfrac{\mathrm{kV}^2}{\boldsymbol{Q_C}}$
- Calculate the reactor size trapping the h_n^{th} harmonic, $\mathrm{X_L} = \dfrac{\mathrm{X_C}}{\boldsymbol{h}_n^2}$
- Calculate the reactor resistance for a specified quality factor, Q, $\mathrm{R} = \mathrm{X_n}\boldsymbol{Q}$; $0.5 < \mathrm{Q} < 5$,

The characteristic reactance is $\mathrm{X_n} = \mathrm{X_{Ln}} = \mathrm{X_{Cn}} = \sqrt{\mathrm{X_L X_C}} = \sqrt{\mathrm{L}/\boldsymbol{C}}$

Filter size $\mathrm{Q_{Filter}} = \dfrac{\boldsymbol{kV}^2}{\mathrm{X_C} - \mathrm{X_L}} = \dfrac{\boldsymbol{kV}^2}{\mathrm{X_C} - \mathrm{X_C}/\boldsymbol{h}_n^2} = \dfrac{\boldsymbol{h}_n^2}{\boldsymbol{h}_n^2 - 1}\dfrac{\boldsymbol{kV}^2}{\mathrm{X_C}} = \dfrac{\boldsymbol{h}_n^2}{\boldsymbol{h}_n^2 - 1}\mathrm{Q_C}$

Filter impedance $\mathrm{Z_F}(\boldsymbol{h}) = \dfrac{\boldsymbol{jRh}\mathrm{X_L}}{\boldsymbol{R} + \boldsymbol{jRh}\mathrm{X_L}} - \boldsymbol{j}\dfrac{\mathrm{X_C}}{\boldsymbol{h}} = \dfrac{\boldsymbol{R}(\boldsymbol{h}\mathrm{X_L})^2}{\boldsymbol{R}^2 + (\boldsymbol{h}\mathrm{X_L})^2} + \boldsymbol{j}\left[\dfrac{\boldsymbol{R}^2\boldsymbol{h}\mathrm{X_L}}{\boldsymbol{R}^2 + (\boldsymbol{h}\mathrm{X_L})^2} - \dfrac{\mathrm{X_C}}{\boldsymbol{h}}\right]$

$$\mathrm{I_{Lh}} = \frac{\boldsymbol{R}}{\boldsymbol{R}^2 + (\boldsymbol{h}\mathrm{X_L})^2}\mathrm{I_{Fh}} = \frac{\boldsymbol{Q}}{\sqrt{\boldsymbol{Q}^2 + (\boldsymbol{h}/\boldsymbol{h}_n)^2}}\mathrm{I_{Fh}}$$

$$\mathrm{I_{Rh}} = \frac{\boldsymbol{h}\mathrm{X_L}}{\boldsymbol{R}^2 + (\boldsymbol{h}\mathrm{X_L})^2}\mathrm{I_{Fh}} = \frac{\boldsymbol{h}/\boldsymbol{h}_n}{\sqrt{\boldsymbol{Q}^2 + (\boldsymbol{h}/\boldsymbol{h}_n)^2}}\mathrm{I_{Fh}} = \frac{\boldsymbol{h}\mathrm{X_L}}{\boldsymbol{R}}\mathrm{I_{Lh}}$$

Power loss in the resistor:

$$P_R = \sum_{h=1} R I_{Rh}^2 = \frac{X_L^2}{R} \sum_{h=1} (h I_{Lh})^2$$

For typical high-pass filters, Q values between 0.5 and 2.0 are found. Filters with higher Q would provide a superior filtering action at the corner frequency, although at higher frequencies, the impedance would show a steady increase. Filters with smaller Q values would show an inferior performance at the corner frequency, although at frequencies higher than that, a less pronounced increase of impedance with frequency is obtained. Other factors that must be considered in the selection of Q are the following;

- the tuning frequency of the filter
- Concerns for telephone interference (if it exists)
- Power losses

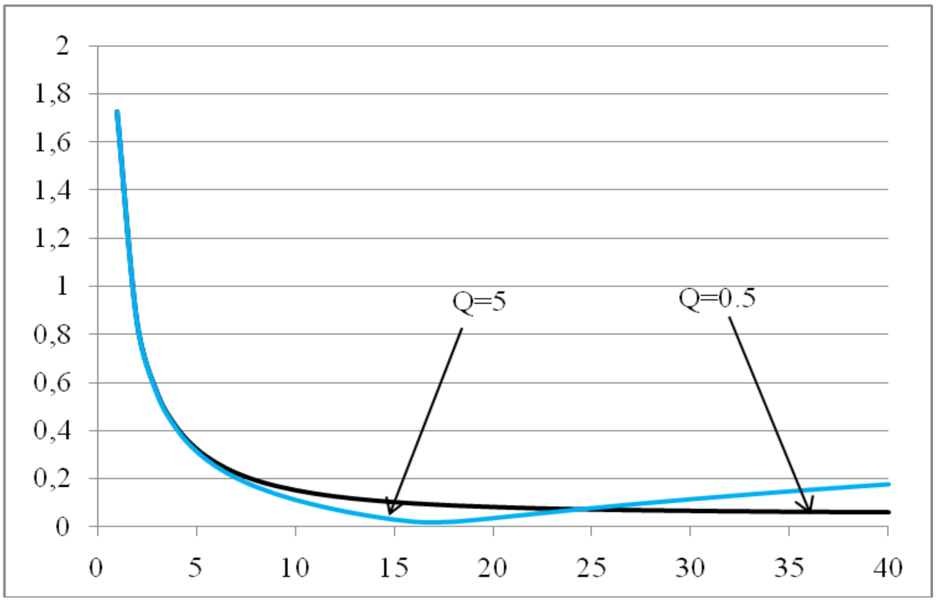
Example: A second order damped filter is tuned to $h_n \geq 17$. Knowing XC = 1.734 Ohm calculate the filter elements.

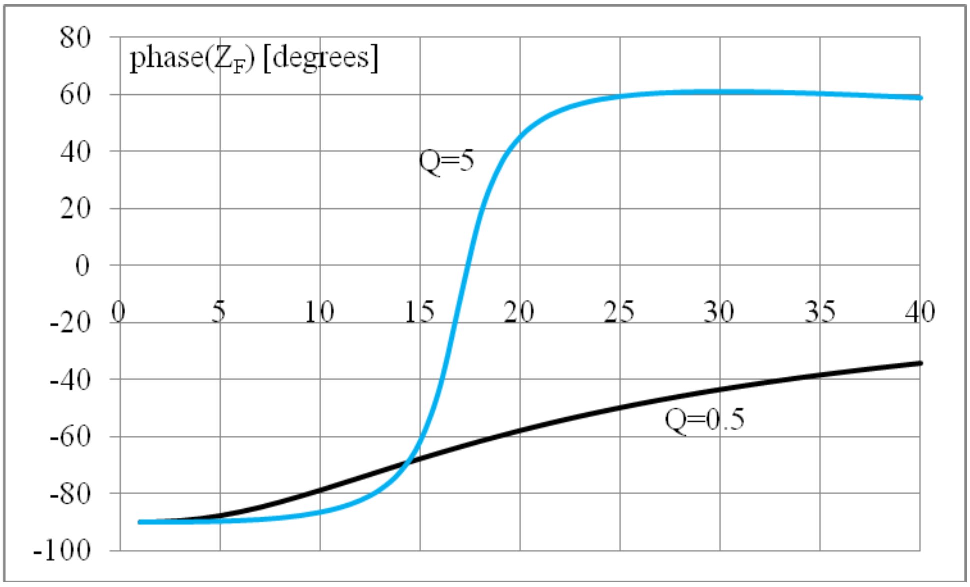
$$X_L = \frac{X_C}{h_n^2} = \frac{1.734}{17^2} = 0.006\ \Omega$$

$$X_n = \sqrt{X_C X_L} = \sqrt{1.734 * 0.006} = 0.102\ \Omega$$

$$R = X_n Q = \begin{cases} 0.051\ \Omega & \textit{for}\ \ Q = 0.5 \\ 0.51\ \Omega & \textit{for}\ \ Q = 5 \end{cases}$$

$$Z_F(h) = \frac{R(hX_L)^2}{R^2 + (hX_L)^2} + j\left[\frac{R^2 h X_L}{R^2 + (hX_L)^2} - \frac{X_C}{h}\right]$$



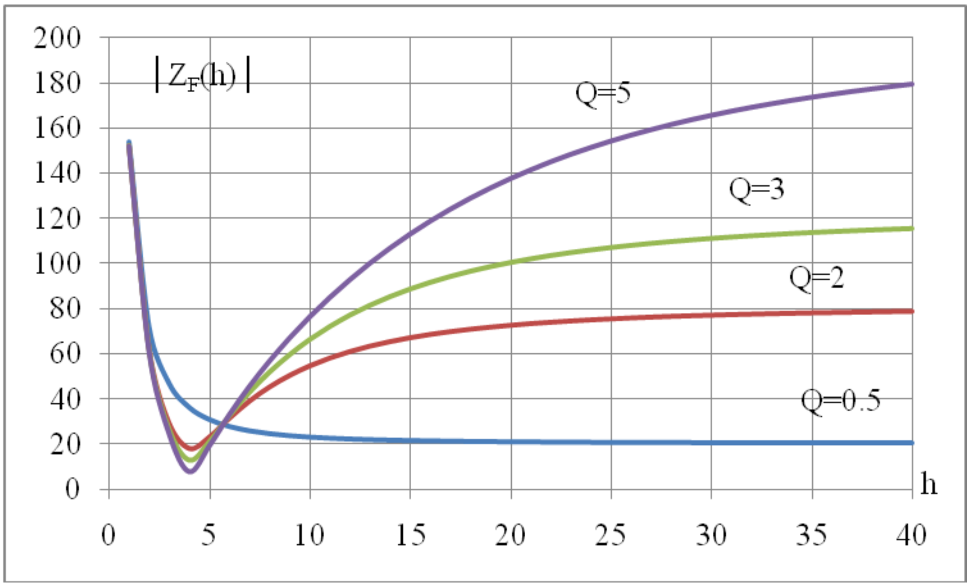


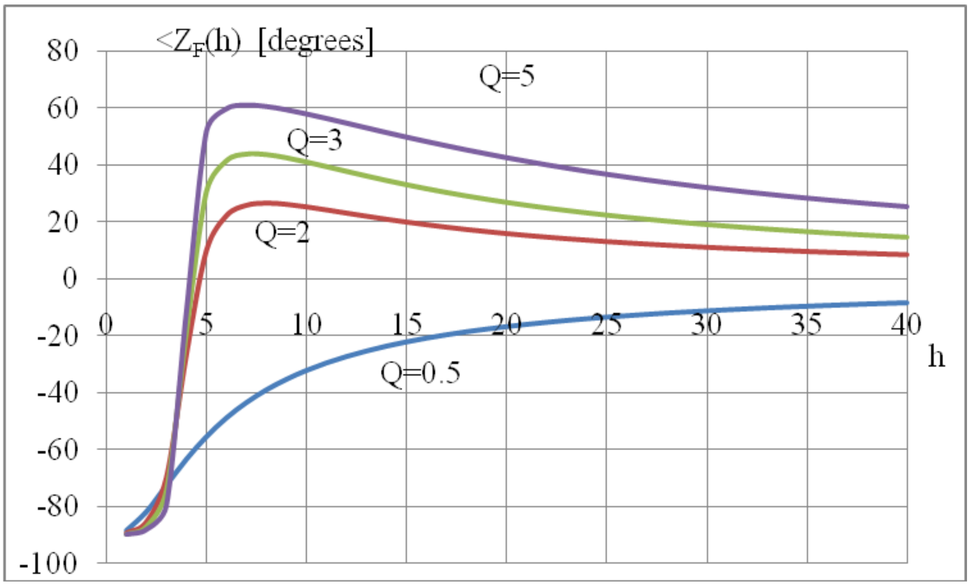
Example: A 36 kV, 8.0 MVar capacitor bank is to be used as a second order damped filter tuned to $\boldsymbol{h_n} \geq 4$. Find the elements of the filter.

$$\left.\begin{aligned} X_C &= \frac{kV^2}{Q_C} = \frac{36^2}{7} = 162\ \Omega \\ X_L &= \frac{X_C}{\boldsymbol{h}_n^2} = \frac{162}{4^2} = 10.125\ \Omega \end{aligned}\right\} X_n = \sqrt{X_C X_L} = 40.5\ \Omega$$

$$\boldsymbol{R} = X_n \boldsymbol{Q} = 20.25/81.0/121.5/202.5\ \Omega \quad \boldsymbol{for}\ \ \boldsymbol{Q} = 0.5/2/3/5$$

$$Z_F(\boldsymbol{h}) = \frac{\boldsymbol{R}(\boldsymbol{h}X_L)^2}{\boldsymbol{R}^2 + (\boldsymbol{h}X_L)^2} + \boldsymbol{j}\left[\frac{\boldsymbol{R}^2\boldsymbol{h}X_L}{\boldsymbol{R}^2 + (\boldsymbol{h}X_L)^2} - \frac{X_C}{\boldsymbol{h}}\right]$$





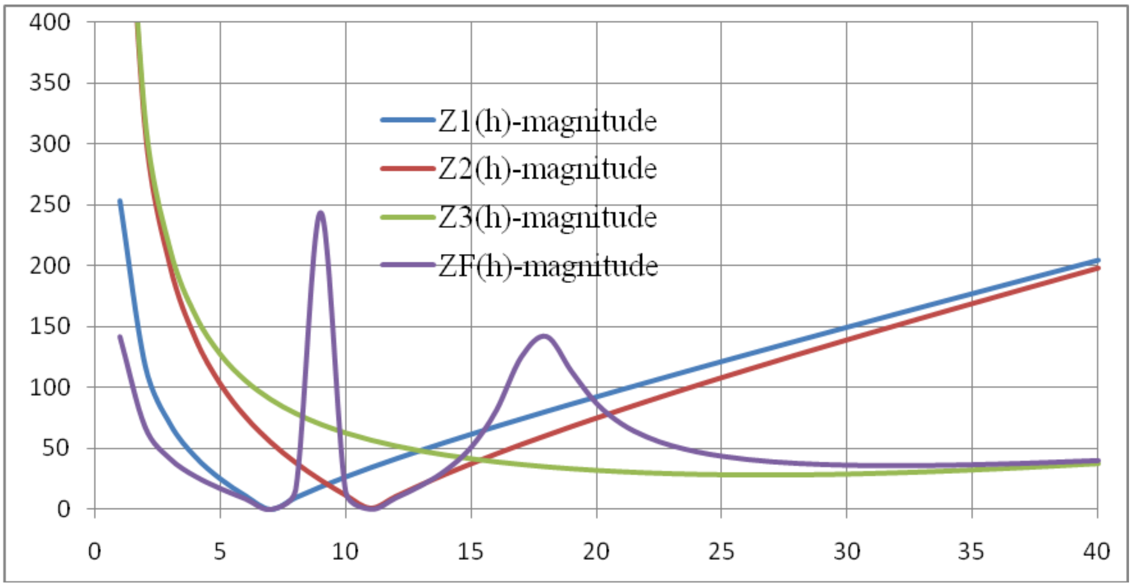
Example: 2 series filters tuned to 7 and 11th harmonics and a damped filter with $h_n \geq 17$ is to be installed to a 36 kV busbar. Design the filters for a capacitor bank of 2*2+5 MVAr.

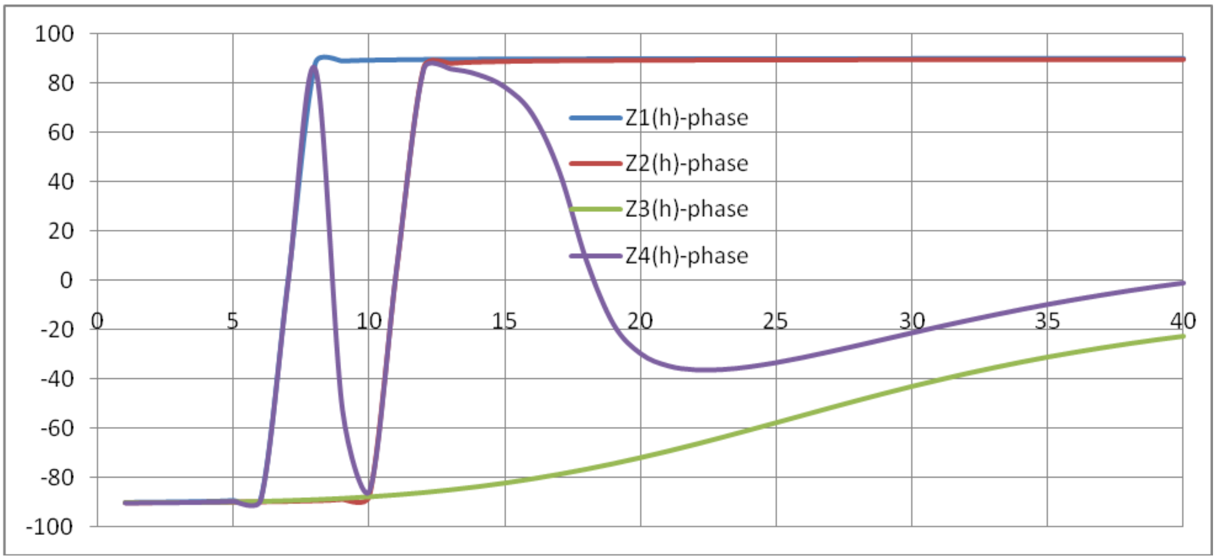
The values are given in the following table.

Filter Type	h	Q	Q_C [MVAr]	XC [Ω]	XL [Ω]	Xn [Ω]	R [Ω]	ZF(h) [Ω]
Tuned	7	100	5	259.2	5.29	37.03	0.37	0.37+j(5.29h-259.2/h)
Tuned	11	100	2	648.0	5.36	58.91	0.59	0.59+j(5.36h-648.0/h)
Damped	17	5	2	648.0	2.24	38.12	190.59	Given below

$$Z_3(\boldsymbol{h}) = \frac{958.2 * \boldsymbol{h}^2}{36323 + 5.03 * \boldsymbol{h}^2} + \boldsymbol{j}\left[\frac{81445.9\boldsymbol{h}}{36323 + 5.03 * \boldsymbol{h}^2} - \frac{648}{\boldsymbol{h}}\right]$$

Impedance scan are given below.





$$Q_{\text{Filter}} = \sum \frac{\boldsymbol{kV}^2}{\boldsymbol{X}_C - \boldsymbol{X}_L} = \sum \frac{\boldsymbol{h}_n^2}{\boldsymbol{h}_n^2 - 1}\boldsymbol{Q}_C = 9.13\boldsymbol{MVAr}$$

A number of aspects must be considered in the design stage of passive filters for controlling problems associated with harmonics. These are summarized as follows:

- The capacitive kVAR requirements for power factor correction. Some installations may benefit from the installation of harmonic filters because power factor will be improved. In other situations, power factor correction needs may dictate the size of the capacitor bank to use.
- In single-tuned filters, watch the resonant parallel peaks resulting from the interaction between the filter and the source.
- Consider tolerances of filter components. They may produce undesirable shifts of resonance frequencies.
- Also look for load and network impedance changes that may modify established worst-case harmonic scenarios.
- Oversized capacitor banks may be required in high-pass filters with low corner frequencies and significant higher order characteristic harmonics.
- Be aware of quality factor filters as a measure to control the amount of harmonic currents to be drawn from the system. Avoid overloading capacitor banks using a series resistor in single-tuned filters. A trade-off between decreased THD values and power factor correction assuring capacitor bank integrity will often decide the Qf value to adopt in a filter.
- Extensive electric networks may have nonlinear loads with different spectral content. Whenever possible, grouping loads by type of harmonic spectrum (for instance, 6-pulse converters, 12-pulse converters, arcing type devices, fluorescent lighting, etc.) can optimize the installation, location, and sizing of harmonic filters. Although this is a difficult task to achieve, especially when comparable types of loads are not on the same location, the idea should be considered as a way to reduce the number of harmonic filters to install. Load grouping could also help reduce telephone interference by trying to keep telephone lines as distant as possible from sites carrying higher-order harmonic currents.
- Minimum filters may be adopted under no reactive compensation needs. The parameters of a minimum filter must be chosen to reach the maximum recommended THD limit.
- Always watch for filter power losses.

6.2 Power Convertors

Power electronic devices are the main source of the harmonics in the power utilities. Harmonics can be minimized where they are created, if correct configurations are selected.

If passive elements are used like diodes in the converter sections, nothing can be done, except using additional active or passive filters. But, if active units are used, such as IGBTs instead of diodes, harmonics can be minimized.

On the other hand, harmonics of pulse convertors can be eliminated/minimized through the proper selection of phase shifts. This is called ***phase cancellation*** or ***phase multiplication***. For example harmonics of six pulse convertors can be eliminated in two six-pulse converters operating in series or in parallel with 0 and 30^0 phase shifts.

There are various types of frequency and/or voltage converters. The type of the converter depends on the application and the cost. Understanding the power converters is essential in order to minimize harmonics and compensate the reactive power.

6.3 Transformers

Transformers are manufactured for their rated frequencies and rtherefore the harmonic may create inadequate operating conditions. Oversizing the neutral connector and derated operation are temporary solutions.

Harmonics can be reduced by means of transformer connections. Delta connected transformers act as a two way filters protecting both the load and the source since they prevent the flow of zero sequence triplen harmonics. In addition, there are some special designed K-Rated transformers which tolerate the effects of harmonics. K-factor transformers are designed to reduce the heating effects of harmonic currents created by loads like those in the table below.

Load	K-Factor
Electric discharge lighting	K-4
UPS with optional input filtering	K-4
Welders	K-4
Induction heating equipment	K-4
PLCs and solid state controls (other than variable speed drives)	K-4
Telecommunications equipment (e.g. PBX)	K-13
UPS without input filtering	K-13
Multiwire receptacle circuits in general care areas of health care facilities and classrooms of schools, etc.	K-13
Multiwire receptacle circuits supplying inspection or testing equipment on an assembly or production line	K-13
Mainframe computer loads	K-20
Solid state motor drives (variable speed drives)	K-20
Multiwire receptacle circuits in critical care areas and operating/recovery rooms of hospitals	K-20

- They have lower than normal flux densities and can tolerate overvoltages coupled with circulation harmonic currents.
- They employ an electromagnetic shield between the primary and secondary windings of each coil, thus attenuating higher frequency harmonics.
- They provide a neutral with twice the size of a phase conductor, to account for increased neutral currents due to the flow of triplen harmonics.
- Windings are designed with several smaller sizes parallel conductors, therefore reducing skin effect at higher frequency harmonics.
- They use insulated and transposed conductors resulting in reduced losses.

K-factor is defined as follows:

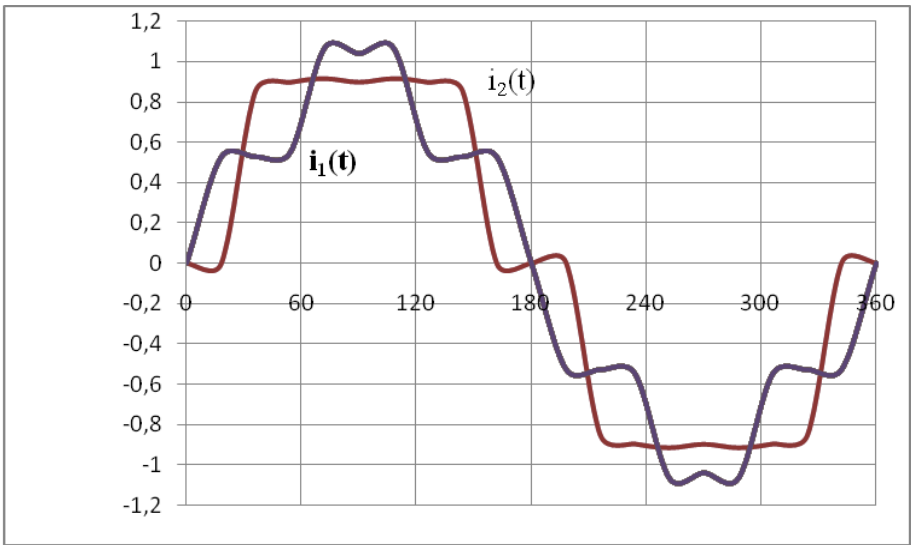
$$\mathrm{K} = \sum_{h=1} \left[h \frac{I_h}{I_1} \right]^2 \quad \text{or alternativelly} \quad \mathrm{K} = \frac{\sum_{h=1} [hI_h]^2}{\sum_{h=1} I_h^2} = \frac{\sum_{h=1} [hI_h]^2}{I_{rms}^2} = \frac{\sum_{h=1} [hI_h / I_1]^2}{[I_{rms} / I_1]^2} = \frac{\sum_{h=1} [hI_h / I_1]^2}{1 + THD_I^2}$$

Special transformers those will be used in harmonic environment are marked as “Suitable for non-sinusoidal current load with K-factor not to exceed 4 / 9 / 13 / 20 / 30 / 40 / 50. If the K-factor exceed 4 ,it becomes necessary to use a K-rated transformer or derate a standard transformer. The derating factor for standard non-harmonic rated transformer is

$$D = \frac{1.15}{1 + 0.15\boldsymbol{K}}$$

Example : A current through a 3 MVA 10/0.4 kV, Δ/Y_g, Z=1+j6.2 % transformer is given in the following table. Can the transformer operate in this harmonic environment? Plot the transformer current waveform neglecting the phases.

h	1	5	7	11	13	17	19	23	25
I_{1h} [%]	100	19	13	8	5	3	2	1	0.9
I_{2h} [%]	100	-19	-13	8	5	-3	-2	1	0.9



$$\mathrm{THD_I} \cong \frac{1}{\boldsymbol{I}_1}\sum_{2}^{25} \boldsymbol{I}_h^2 = 25.18\ \%$$

$$\mathrm{I_{rms}} = \boldsymbol{I}_{1-rms}\sqrt{1 + \boldsymbol{THD}_I^2} = 1.031$$

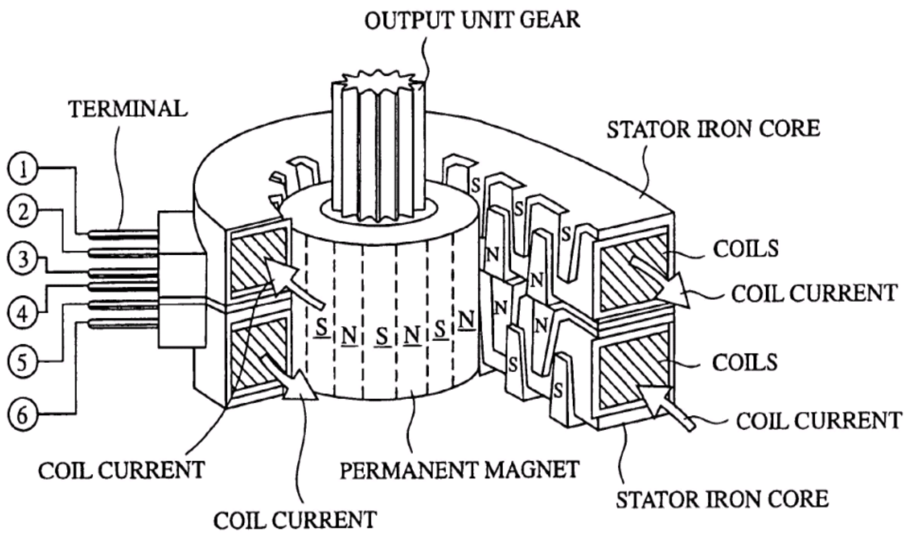
$$K = \sum_{h=1}\left[h\frac{I_h}{I_1}\right]^2 = 4.436 \text{ or normalized factor } K = \frac{\sum_{h=1}[hI_h/I_1]^2}{1+THD_I^2} = 4.171$$

K-factor is greater than 4 and therefore either use,

- a K-rated transformer with K=4 or
- a standard transformer with a derating factor of $D = \dfrac{1.15}{1+0.15K}$ =0.7074

6.4 Rotating Machines

Rotating machines are considered sources of harmonics because the windings are embedded in slots which can never be exactly sinusoidally distributed so that the mmf is distorted.



k_{ph} : pitch factor at the nth harmonic

k_{p1} : pitch factor at the fundamental frequency

k_{dh} : distribution factor at the nth harmonic

k_{p1} : distribution factor at the fundamental frequency

$$k_{ph} < k_{p1} < 1$$
$$k_{dh} < k_{p1} < 1$$

$$k_{ph} = Sin\left(\frac{\beta h}{2}\right) = 0 \Rightarrow \beta = \frac{360^0}{h} \text{ or } \beta = \frac{720^0}{h}$$

That is to say, a coil span of 4/5 pole pitch (144^0 electrical) results in eliminating the fifth harmonic; The third harmonic is suppressed through using a coil of 2/3 pole pitch (120^0 electrical) span. Furthermore, a coil span of 5/6 pole pitch (150^0 electrical) greatly reduces fifth and seventh harmonics.

6.5 Capacitor Banks

Relocating capacitors changes the source-to-capacitor inductive reactance thus avoiding parallel resonance with the supply. Varying the reactive power output of a capacitor bank will alter the resonant frequency.

Capacitors can be designed to trap a certain harmonic by employing a tuning reactor whose inductive reactance is equal to the capacitive reactance of the capacitor at the tuned frequency. Parallel resonance involving a capacitor and a source inductance is achieved when

$$\mathrm{X_{Cr}} = \frac{\mathrm{X_C}}{\boldsymbol{h_r}} = \mathrm{X_{sr}} = \boldsymbol{h_r}\mathrm{X_s} = \mathrm{X_r}$$

At a resonant frequency

$$\boldsymbol{w}_{\mathrm{r}} = \boldsymbol{h_r}\boldsymbol{w}_0 = \frac{1}{\sqrt{\mathrm{L_s}\boldsymbol{C}}} \quad \boldsymbol{rad}/\sec \quad , \quad \boldsymbol{f}_{\mathrm{r}} = \boldsymbol{h_r}\boldsymbol{f}_0 = \frac{1}{2\pi\sqrt{\mathrm{L_s}\boldsymbol{C}}} \quad \boldsymbol{Hz}$$

Resonance frequency order can be written as

$$\boldsymbol{h_r} = \frac{\boldsymbol{f}_{\mathrm{r}}}{\boldsymbol{f}_0} = \frac{1}{2\pi\boldsymbol{f}_0\sqrt{\mathrm{L_s}\boldsymbol{C}}} = \frac{1}{\boldsymbol{w}_0\sqrt{\mathrm{L_s}\boldsymbol{C}}} = \sqrt{\frac{\boldsymbol{X_C}}{\boldsymbol{X_s}}} = \sqrt{\frac{\boldsymbol{SCC_{pu}}}{\boldsymbol{Q_{C-pu}}}}$$

A capacitor with a reactance of $\mathrm{X_C} = \boldsymbol{h_r}^2\mathrm{X_s}$ excites a resonance at the $\boldsymbol{h_r^{th}}$ harmonic frequency. Tuning the capacitor to a certain harmonic requires the addition of a reactor.

$$\mathrm{X_{Ln}} = \boldsymbol{h_n}\mathrm{X_L} = \mathrm{X_{Cn}} = \frac{\mathrm{X_C}}{\boldsymbol{h_n}} = \mathrm{X_n} \Rightarrow \mathrm{X_n^2} = \mathrm{X_L X_C} = \frac{\boldsymbol{L}}{\boldsymbol{C}} \Rightarrow \mathrm{X_n} = \sqrt{\mathrm{X_L X_C}} = \sqrt{\boldsymbol{L}/\boldsymbol{C}}$$

The tuned frequency is then,

$$\boldsymbol{w_n} = \boldsymbol{h_n}\boldsymbol{w}_0 = \frac{1}{\sqrt{\mathrm{L}\boldsymbol{C}}} \quad ; \quad \boldsymbol{f_n} = \boldsymbol{h_n}\boldsymbol{f}_0 = \frac{1}{2\pi\sqrt{\mathrm{L}\boldsymbol{C}}}$$

The reactor’s inductance reactance is then,

$$\mathrm{X_L} = \frac{\mathrm{X_C}}{\boldsymbol{h_n^2}} = \frac{\boldsymbol{h_r^2}}{\boldsymbol{h_n^2}}\mathrm{X_s} \Rightarrow \mathrm{X_n^2} = \mathrm{X_L X_C} = \frac{\boldsymbol{L}}{\boldsymbol{C}} \Rightarrow \mathrm{X_n} = \sqrt{\mathrm{X_L X_C}} = \sqrt{\boldsymbol{L}/\boldsymbol{C}}$$

If the capacitor is tuned to the harmonic activating resonance,

$$\boldsymbol{h}_{\mathrm{n}} = \boldsymbol{h}_{\mathrm{r}} \;\Rightarrow\; \mathrm{X_L} = \mathrm{X_s} \;\Rightarrow\; \boldsymbol{h}_{\mathrm{n}} = \frac{\boldsymbol{f}_{\mathrm{n}}}{\boldsymbol{f}_0} = \sqrt{\frac{\mathrm{X_C}}{\mathrm{X_L}}} = \frac{\boldsymbol{h}_r^2}{\boldsymbol{h}_n^2}\mathrm{X_s} \qquad \mathrm{X_n^2} = \mathrm{X_L X_C} = \frac{\boldsymbol{L}}{\boldsymbol{C}} \Rightarrow \mathrm{X_n} = \sqrt{\mathrm{X_L X_C}} = \sqrt{\boldsymbol{L}/\boldsymbol{C}}$$

Example : A 33 kV, 6.2 MVAr capacitor bank is to be installed at a bus where the SCC is 750 MVA. Investigate the resonance and avoid the problem.

$$\boldsymbol{h}_{\mathrm{r}} = \frac{\boldsymbol{f}_{\mathrm{r}}}{\boldsymbol{f}_0} = \sqrt{\frac{\mathrm{X_C}}{\mathrm{X_S}}} = \sqrt{\frac{\mathrm{SCC}}{\mathrm{Q_C}}} = \sqrt{\frac{750}{6.2}} = 11 \;,\; \mathrm{X_C} = \frac{\boldsymbol{kV}^2}{\mathrm{Q_C}} = \frac{33^2}{6.2} = 175.645\ \Omega$$

Let’s design a filter to trap the eleventh harmonic by adding a reactor in series with the capacitor,

$$\mathrm{X_L} = \frac{\mathrm{X_C}}{\boldsymbol{h}_n^2} = \frac{175.645}{11^2} = 1.42\ \Omega$$

For a quality factor of Q=60

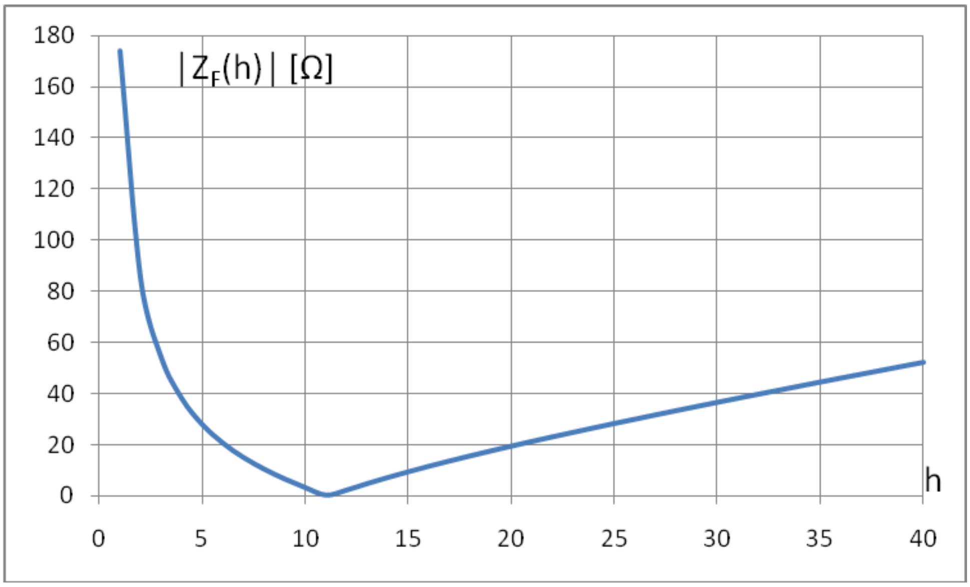
$$\mathrm{X_n} = \sqrt{\mathrm{X_L X_C}} = 17.968\ \Omega \quad , \; \boldsymbol{R} = \frac{\mathrm{X_n}}{\boldsymbol{Q}} = 0.266\ \Omega$$

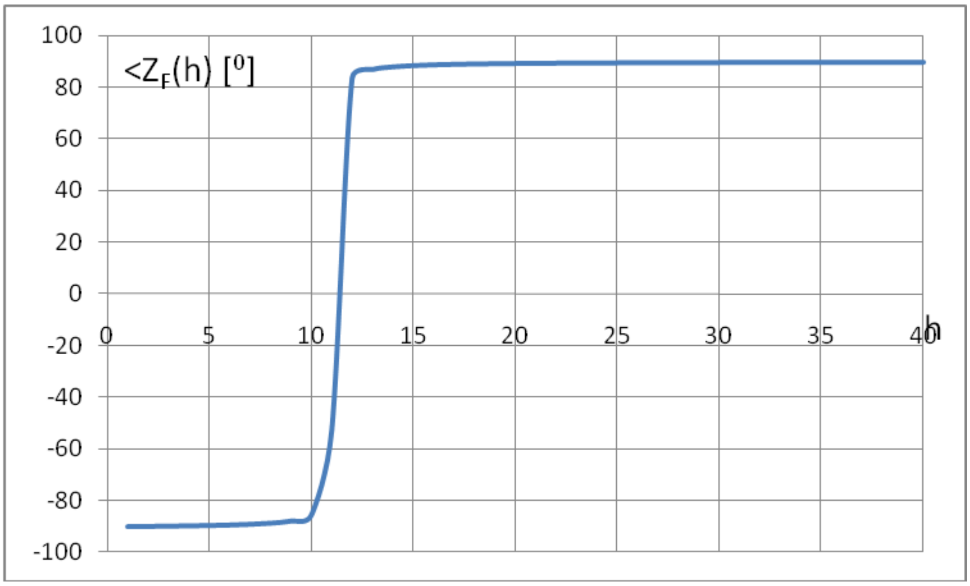
The filter impedance:

$$\mathrm{Z_F}(\boldsymbol{h}) = \boldsymbol{R} + \boldsymbol{j}(\boldsymbol{h}\mathrm{X_L} - \frac{\mathrm{X_C}}{\boldsymbol{h}}) = 0.266 + \boldsymbol{j}(1.42 * \boldsymbol{h} - \frac{175.645}{\boldsymbol{h}})\ \Omega$$

Magnitude and phase diagrams of the filter is given below.

$$\mathrm{Q_{Filter}} = \frac{\mathrm{kV^2}}{\mathrm{X_C - X_L}} = \frac{\mathrm{h_n^2}}{\mathrm{h_n^2 - 1}}\mathrm{Q_C} = 6.252\ \boldsymbol{MVAr}$$





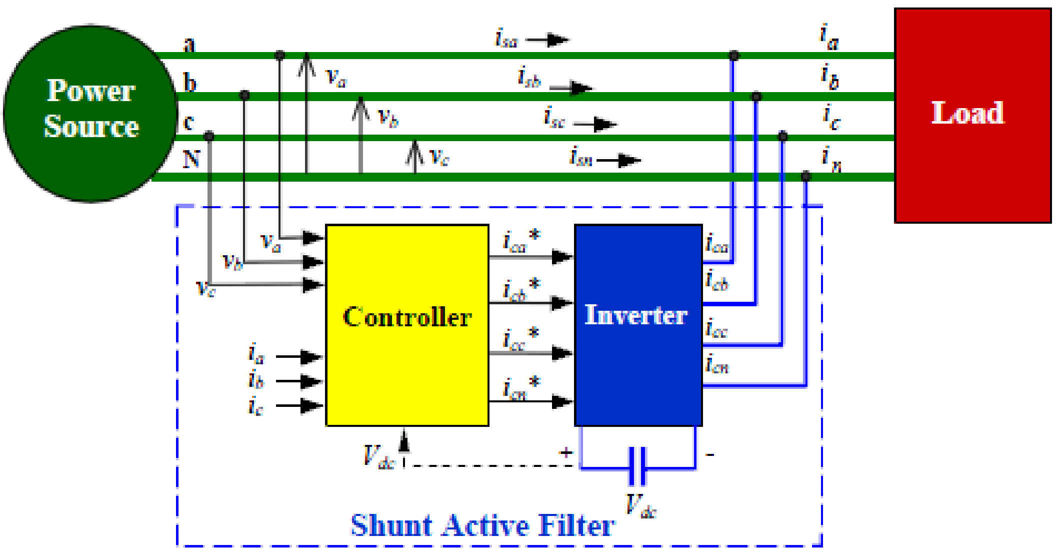
6.6 Active Filters

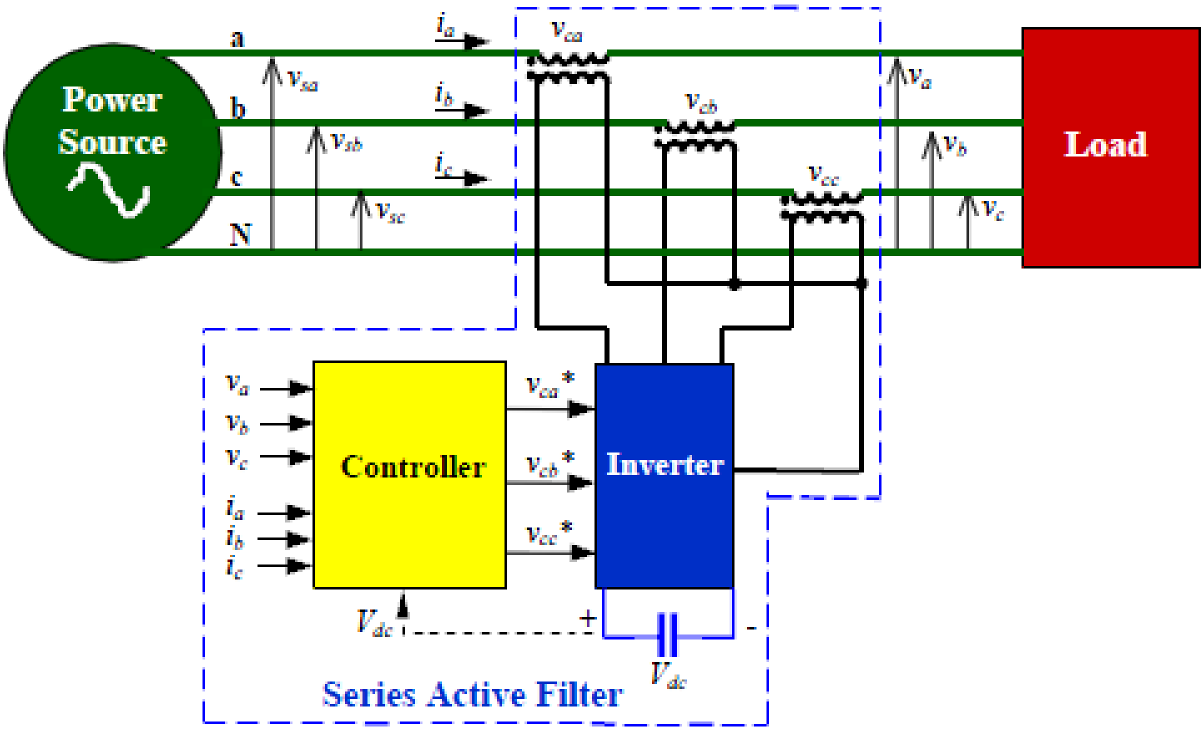
Recently, intensity of the usage of power electronic equipment has caused an increase of the harmonic disturbances in power systems. Conventionally, passive LC filters and capacitors have been used to eliminate line current harmonics and to increase the power factor. However, in some practical applications, in which the amplitude and the harmonic content of the distortion power can vary randomly, this conventional solution becomes ineffective. They only filter the frequencies that were calculated in the design section; its operation cannot be limited to a certain load or group of loads; Resonance can occur due to the interaction between the passive filters and other loads, with unexpected results. The source impedance is not really known and varies depending on the loading conditions and different configurations influence the characteristics of the shunt passive filter. A shunt passive filter may fall in series resonance with the source impedance. At a specific frequency, a parallel resonance may occur between the source impedance and the passive filter, called as “harmonic amplification.”

The more sophisticated active filtering concepts operate in a wide frequency range, adjusting their operation to the resultant harmonic spectrum. In this way, they are designed to inject harmonic currents to counterbalance existing harmonic components as they show up in the distribution system.

In 1982, 800kVA shunt active filter which consisted of current-source PWM inverters using GTO thyristors was put into practical use for harmonic compensation for the first time in the world. Nowadays, the objective of shunt active filters is to compensate reactive power, negative sequence, harmonics, and/or flicker. In addition, active power filters can also be combined with passive filters in hybrid structure in order to decrease its rated power.

There are mainly two types of active power filters: the shunt active filter and the series active filter which is shown in figures.





The shunt active filter is used to filter the line currents and the series active filter is used to filter the line voltages. It is also possible to combine both types to filter both current and voltage harmonics.

There are two types of power circuits of active power circuits. These are the current source and voltage source PWM inverters. They are physically almost the same circuit, but different in their behavior. The current or voltage source PWM inverter needs a dc reactor or a dc capacitor which is very important for energy storage, but it does not need any dc power supply on the dc side. The shunt active filter can be controlled so as to supply the losses in the PWM inverter from the ac source. The voltage source PWM inverter is preferred to the current source PWM inverter due to high efficiency of voltage source PWM inverter. Almost all shunt active power filters are the voltage source PWM inverters.