



A Comprehensive Study of an Electricity Market Problem using MATLAB® and GAMS®

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Course: Electricity Markets

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1. Introduction

1.1. Why Electricity Markets, OPF and LMP?

Electricity markets are critical to the optimal use of resources in the power system. They provide as a forum for power producers and customers to interact and trade power at competitive costs. Electricity markets operate on the basis of supply and demand, with prices decided by the balance between the two.

The optimal dispatch of power generation and the resulting pricing for energy at various points in the power system are determined using price-based optimal power flow (OPF), a crucial instrument in electricity markets. The electricity system is operated with price-based OPF to assure efficiency, cost-effectiveness, dependability, and security. By determining the ideal amounts of generation and load within a range of constraints including generator output limits, transmission line capacity, and bus voltage limits, it improves the flow of power across the system.

A key idea in electricity markets is the locational marginal price (LMP), which denotes the marginal cost of supplying an extra unit of power at a particular location in the power system. The LMP takes into account transmission limitations and power system losses to reflect the cost of producing and distributing electricity at that location. The ideal generation and transmission levels in the system are estimated using price-based OPF, subject to a number of constraints. To monitor and analyze the performance of the electricity market and to make choices on transmission upgrades, market design, and other aspects of power system planning and operation, market participants, regulators, and policy makers could apply LMPs.

The determination of LMPs is critical for the effective operation of electricity markets, as it provides signals to generators and consumers about the cost of electricity at different locations in the system. LMPs can be used to incentivize generators to locate their facilities in areas where the LMP is high, and to encourage consumers to reduce their electricity consumption during periods of high LMPs. LMPs also provide important information for market participants to make decisions about bidding strategies, resource planning, and other aspects of market participation. In summary, price-based OPF and the calculation of LMPs are essential tools for the efficient and effective operation of electricity markets, and for the optimization of the power system as a whole.

1.2. This Report

In this work, the problems have been solved and the desired items are found using MATLAB® and GAMES®. The MATLAB® program code includes a visualizing code that generates a power flow diagram automatically that helps to easily find the congested, overloaded and uncongested lines.

The approach to solving the problem is systematic and simple enough to be applied to any other system with a different configuration and set of specifications.

NOTE: We focus on a one-hour interval and make the following assumptions: $S_{base}=100$ (MVA), $V_{mbase}=400$ (kV).

2. Problem

There is a 15-bus, 3-area power grid with the topology shown below:

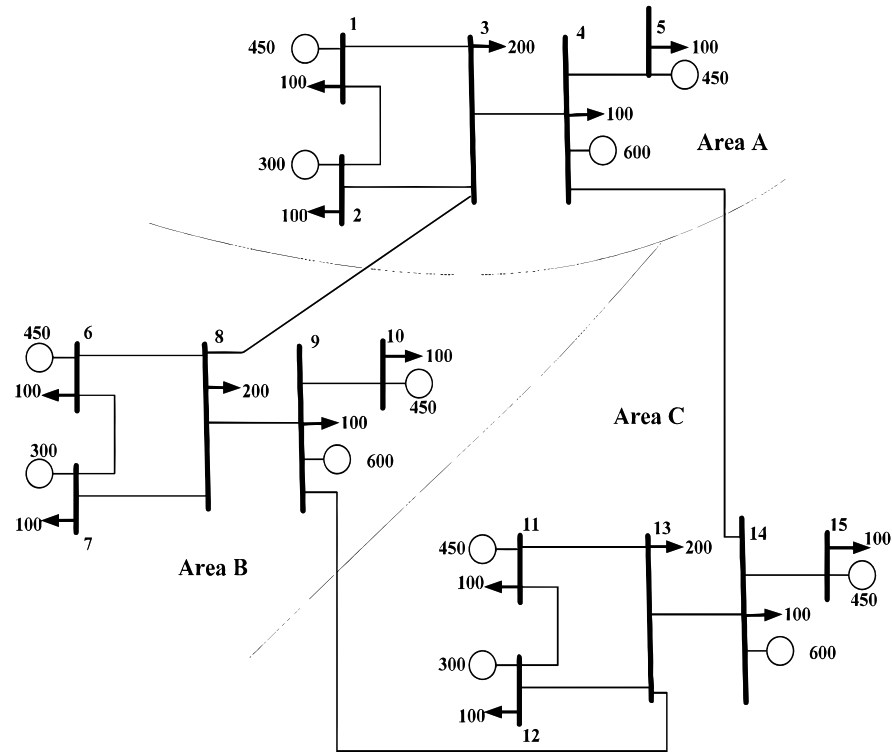


Fig. 2-1: Configuration of the system mentioned in the problem

Branches, Generators and Loads data are as following ($S_{base} = 100$ MVA):

From Bus	To Bus	X_{Line} (p.u.)	R_{Line} (p.u.)	B	Rating (p.u.)
1	2	0.020851	0.002085	0	1
1	3	0.024241	0.002424	0	1.5
2	3	0.024241	0.002424	0	1.5
3	4	0.069502	0.006950	0	4
4	5	0.069502	0.006950	0	4
6	7	0.020851	0.002085	0	1
6	8	0.024241	0.002424	0	1.5
7	8	0.024241	0.002424	0	1.5
8	9	0.069502	0.006950	0	4
9	10	0.069502	0.006950	0	4
11	12	0.020851	0.002085	0	1
11	13	0.024241	0.002424	0	1.5
12	13	0.024241	0.002424	0	1.5
13	14	0.069502	0.006950	0	4
14	15	0.069502	0.006950	0	4
3	8	0.069502	0.006950	0	2
4	14	0.069502	0.006950	0	2
9	13	0.069502	0.006950	0	2

Bus	P_d (p.u.)	Q_d (p.u.)
1	1	0.2
2	1	0.2
3	2	0.41
4	1	0.2
5	1	0.2
6	1	0.25
7	1	0.25
8	2	0.5
9	1	0.25
10	1	0.25
11	1	0.14
12	1	0.14
13	2	0.28
14	1	0.14
15	1	0.14

Gen	P_{max} (p.u.)	P_{min} (p.u.)	bid (\$/MWh)
1	4.5	0	5
2	3	0	6
4	6	0	12
5	4.5	0	8
6	4.5	0	13
7	3	0	11
9	6	0	21
10	4.5	0	18
11	4.5	0	22
12	3	0	23
14	6	0	25
15	4.5	0	26

3. Results

3.1. Simple manual solution by Supply and Demand equilibrium

By sorting the generators from the cheapest to the most expensive we have the table below:

Table 3-1: Generators capacity sorted by their marginal costs

Marginal Cost (\$/MW-hr)	Generator Bus	Pmax (MW)
5	1	450
6	2	300
8	5	450
11	7	300
12	4	600
13	6	450
18	10	450
21	9	600
22	11	450
23	12	300
25	14	600
26	15	450

We can create supply and demand curves based on the table while taking into account the system's total fixed demand of 1800 MW; the intersection of the two curves shows the equilibrium point, which has the x value of the system demand and the y value of the System Marginal Price (SMP), as shown in Fig. 3-1:

Supply and demand curves show that the SMP in the simplest condition would be 12 (\$/MW-hr). Therefore, we could predict that the obtained SMP from the DC OPF solution will be 12 (\$/MW-hr).

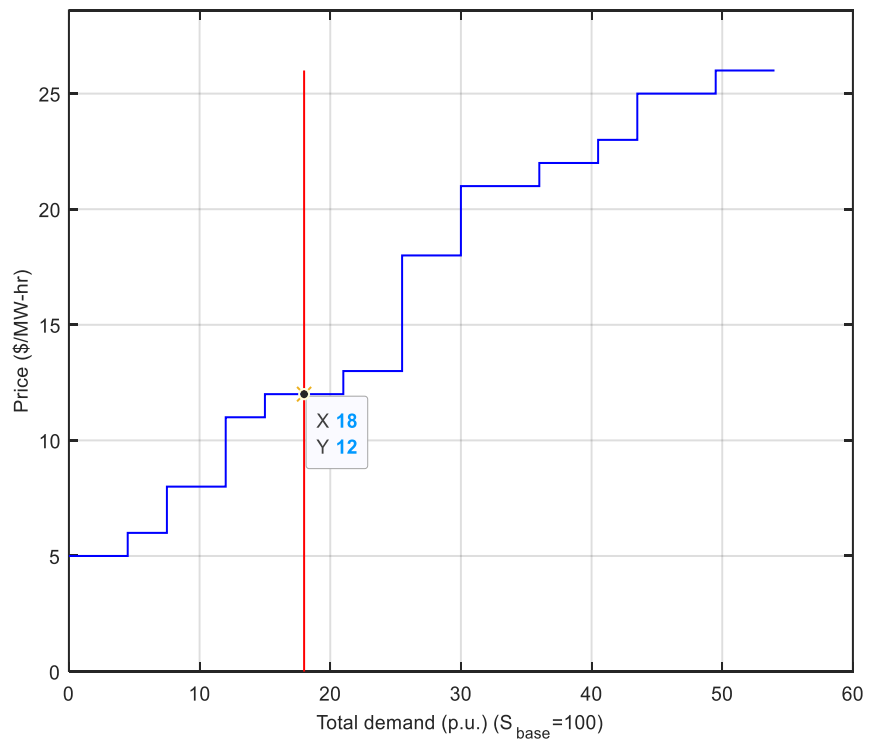


Fig. 3-1: Supply and demand curves in the simplest conditions

3.2. Unconstrained DC OPF using MATPOWER

The diagram below shows the flow of real power (p.u.) in lines (green and red colors indicate uncongested and overloaded lines, respectively) and not to note that all buses have a voltage magnitude of 1 p.u. (because of DC PF):

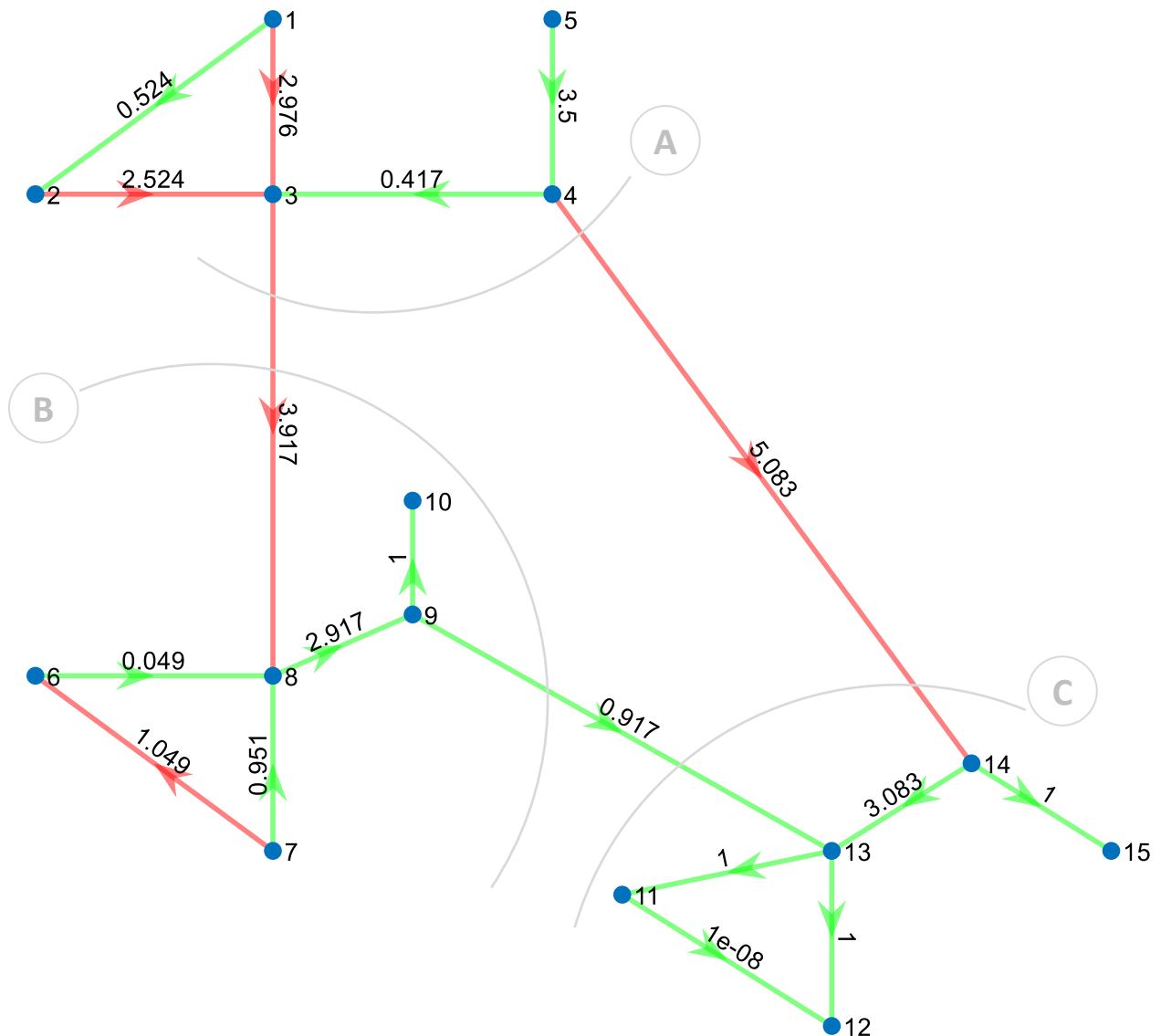


Fig. 3-2: Flow of P in lines resulted from Unconstrained DC OPF using MATPOWER

Information derived from Fig. 3-2:

1. 5 lines are overloaded and the rest are uncongested.

2. Power exchange between areas is as follows (in MW):

A->B: 391.7 A->C: 508.3 ➔ A->*: 900 B->C: 91.7
 ➔ B->*: -300, C->*: -600

Generators dispatch information table is also automatically generated by “Visualizing_results.m” code:

Table 3-2: Generators dispatch resulted from Unconstrained DC OPF using MATPOWER

Gen	Pg (MW)	MC (\$/MW-hr)	λ_p (\$/MW-hr)	Generation Income (Pg \times λ_p)	Generation Cost (Pg \times MC)	Generators Profit (Income – Cost)
1	450	5	12	5,400	2,250	3,150
2	300	6	12	3,600	1,800	1,800
4	300	12	12	3,600	3,600	0
5	450	8	12	5,400	3,600	1,800
6	0	13	12	0	0	0
7	300	11	12	3,600	3,300	300
9	0	21	12	0	0	0
10	0	18	12	0	0	0
11	0	22	12	0	0	0
12	0	23	12	0	0	0
14	0	25	12	0	0	0
15	0	26	12	0	0	0
Total:	1,800	-	-	21,600 (\$)	14,550 (\$)	7,050 (\$)

The SMP is 12 (\$/MW-hr), as predicted in section 3.1.

3.3. Constrained DC OPF using MATPOWER

Flow of real power (p.u.) in lines is shown below, green and black colors indicate uncongested and congested lines, respectively. *Why is there no red line? :)

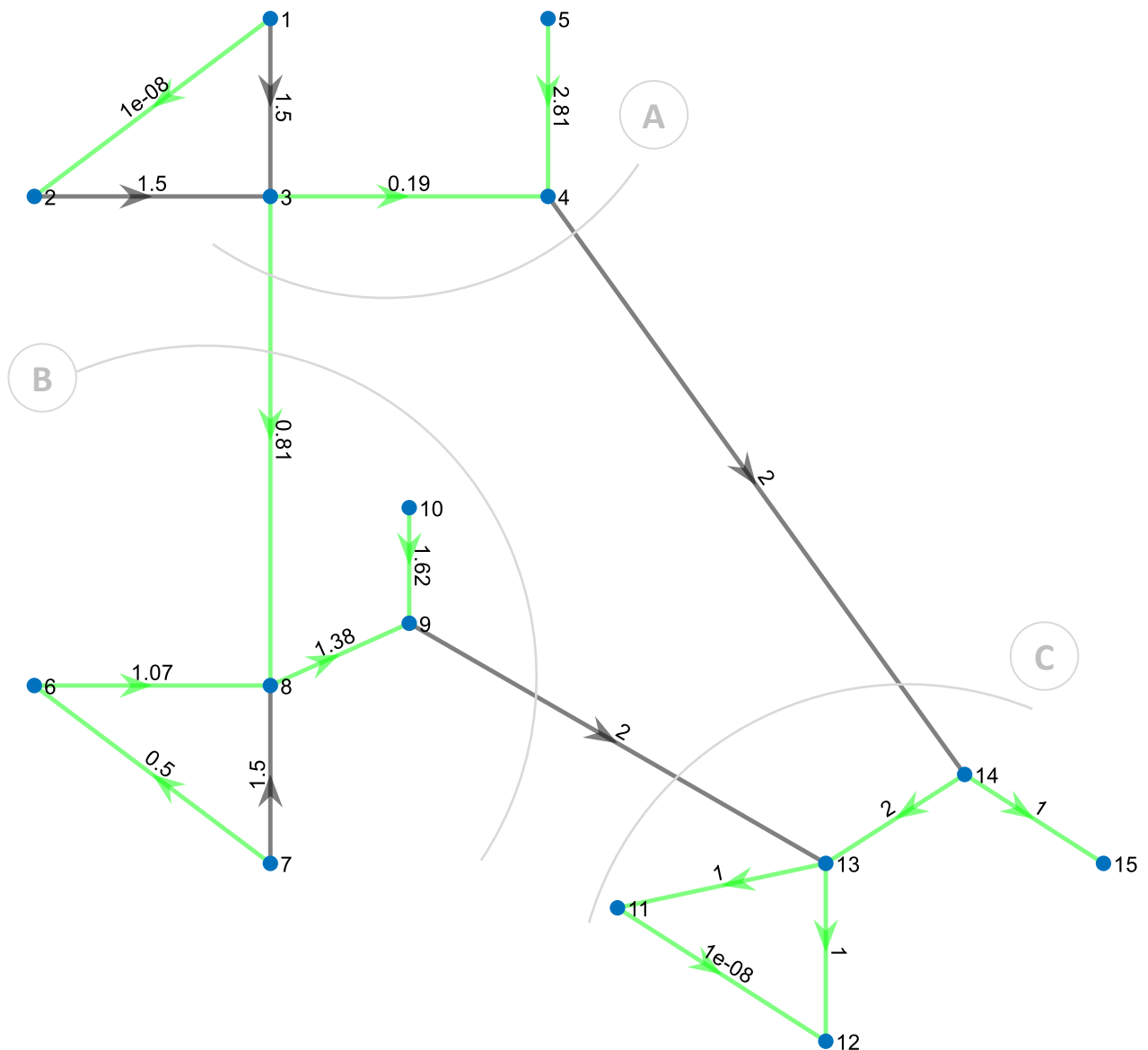


Fig. 3-3: Flow of P in lines resulted from Constrained DC OPF using MATPOWER

Information derived from Fig. 3-3:

1. Previously overloaded lines from cheap generators (1,2 and 7) are now congested (as expected to happen as we get closer to the economically best point while respecting the constraints) in addition B-C and A-C linking lines.

2. Power exchange between areas is as follows (in MW):

A->B: 81 A->C: 200 MW → A->*: 281 B->C: 200
 → B->*: -119, C->*: -400

Generators dispatch information table is shown below:

Table 3-3: Generators dispatch resulted from Constrained DC OPF using MATPOWER

Gen	P _g (MW)	MC (\$/MW-hr)	λ _p (\$/MW-hr)	Generation Income (P _g × λ _p)	Generation Cost (P _g × MC)	Generators Profit (Income – Cost)
1	250	5	5	1,250	1,250	0
2	250	6	6	1,500	1,500	0
4	0	12	8	0	0	0
5	381.0026	8	8	3,048.021	3,048.021	0
6	156.9923	13	13	2,040.9	2,040.9	0
7	300	11	11.56641	3,469.923	3,300	169.9229
9	0	21	18	0	0	0
10	262.0051	18	18	4,716.093	4,716.093	0
11	0	22	21.66667	0	0	0
12	0	23	21.66667	0	0	0
14	200	25	25	5,000	5,000	0
15	0	26	25	0	0	0
Total:	1,800	-	-	21,025 (\$)	20,855 (\$)	169.9229 (\$)

Because of congestion we have LMPs instead of SMP as shown below:

Table 3-4: LMP per bus resulted from Constrained DC OPF using MATPOWER

Bus	λ _p (\$/MW-hr)
1	5
2	6
3	11.3333
4	8
5	8
6	13
7	11.56641
8	14.66667
9	18
10	18
11	21.66667
12	21.66667
13	21.66667
14	25
15	25

3.4. Unconstrained AC OPF using MATPOWER

The resulting system profile is shown below; for each bus, "Bus number=Voltage magnitude" is displayed, and for the lines, green and red colors mean, respectively, uncongested and overloaded lines.

NOTE: Values on lines represent the real power related to (injected or received) the bus that is closest to the arrow (e.g., bus 14 injects 1.007 p.u. toward bus 15 while bus 4 receives 3.415 p.u. from bus 5's side)

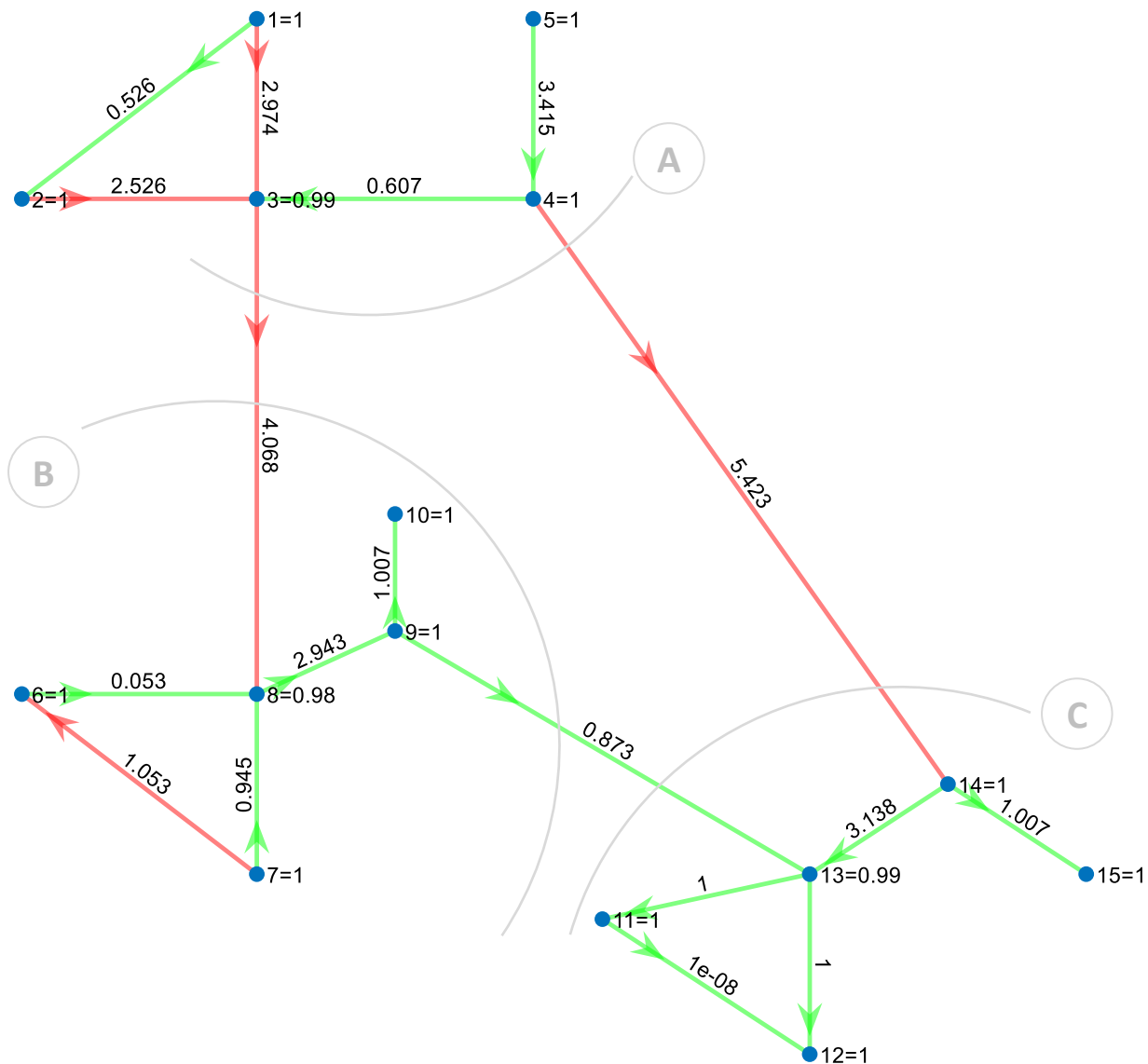


Fig. 3-4: Flow of P in lines and voltage profile from Unconstrained AC OPF using MATPOWER

Information derived from Fig. 3-4:

1. 5 lines are overloaded and the rest are uncongested similar to unconstrained DC OPF result shown in Fig. 3-2.

2. Power exchange between areas is as follows (in MW):

A->B: 406.8 A->C: 542.3 A->*: 949.1 B->C: 87.3

→ B->*: (-319.5+|Loss on 3-8|), C->*: (-629.6+|Losses of Lines|)

3. Like a DC PF, the voltage magnitude on all buses is almost one p.u.

Generators dispatch information table is shown below (values are rounded to two decimal places):

Table 3-5: Generators dispatch resulted from Unconstrained AC OPF using MATPOWER

Gen	P _g (MW)	MC (\$/MW-hr)	λ _p (\$/MW-hr)	Generation Income (P _g × λ _p)	Generation Cost (P _g × MC)	Generators Profit (Income – Cost)
1	450	5	11.92	5,365.13	2,250	3,115.13
2	300	6	11.95	3,584.76	1,800	1,784.76
4	361.76	12	12.00	4,341.10	4,341.10	0.00
5	450	8	11.41	5,136.41	3,600	1,536.41
6	0.00	13	12.89	0.01	0.01	0.00
7	300	11	12.84	3,850.57	3,300	550.57
9	0.00	21	13.47	0.00	0.00	0.00
10	0.00	18	13.66	0.00	0.00	0.00
11	0.00	22	13.70	0.00	0.00	0.00
12	0.00	23	13.70	0.00	0.00	0.00
14	0.00	25	13.01	0.00	0.00	0.00
15	0.00	26	13.20	0.00	0.00	0.00
Total:	1,861.76	-	-	22,277.98 (\$)	15,291.11 (\$)	6,986.87 (\$)

Because of losses on lines ($P_{\text{Loss}} = (1861.76 - 1800) = 61.76$ MW) we have LMPs instead of SMP as shown below (values are rounded to two decimal places):

Table 3-6: LMP per bus resulted from Unconstrained AC OPF using MATPOWER

Bus	λ _p (\$/MW-hr)
1	11.92
2	11.95
3	12.11
4	12.00
5	11.41
6	12.89
7	12.84
8	12.90
9	13.47
10	13.66
11	13.70
12	13.70
13	13.63
14	13.01
15	13.20

3.4.1. A comparison of the unrestricted OPFs of DC and AC.

Flow of real power in lines in two conditions and their difference are shown in the table below:

Table 3-7: Comparison of flow in lines resulted from Unconstrained DC and AC OPFs

Branches		Real Power Flows (MW)		
From bus	To bus	DC	AC	Difference
1	2	52.40	52.60	0.20
1	3	297.60	297.40	0.20
2	3	252.40	252.60	0.20
4	3	41.70	60.70	19.00
5	4	350.00	341.50	8.50
7	6	104.90	105.30	0.40
6	8	4.90	5.30	0.40
7	8	95.10	94.50	0.60
8	9	291.70	294.30	2.60
9	10	100.00	100.70	0.70
11	12	0.00	0.00	0.00
13	11	100.00	100.00	0.00
13	12	100.00	100.00	0.00
14	13	308.30	313.80	5.50
14	15	100.00	100.70	0.70
3	8	391.70	406.80	15.10
4	14	508.30	542.30	34.00
9	13	91.70	87.30	4.40

Due to the low values of the lines' R/X (≈ 0.1) ratio and the consequently relatively small amount of losses, Table 3-7 demonstrates that there is no discernible difference between these two approaches in this case. Because all buses except three are fixed at 1 p.u., voltage profiles are also almost identical. (As shown in Fig. 3-5)

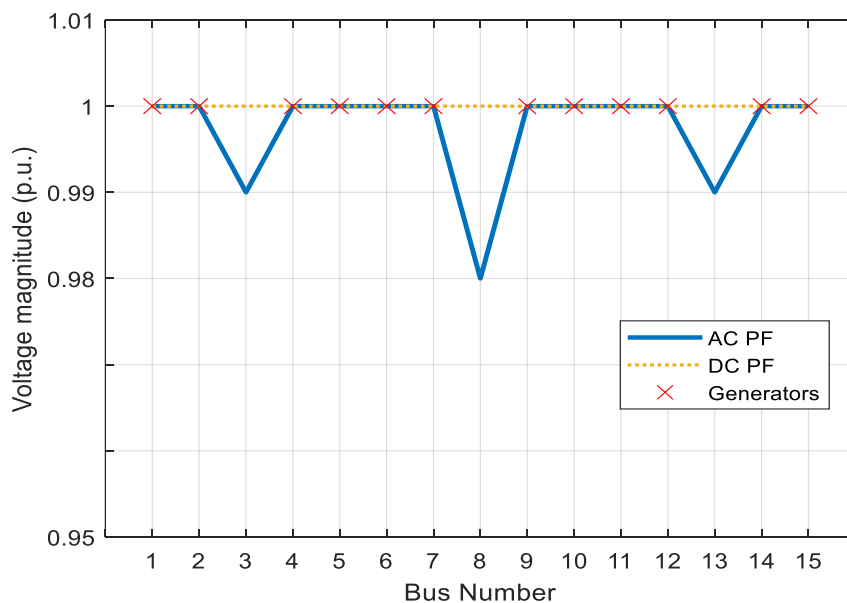


Fig. 3-5: Voltage profiles resulted from Unconstrained DC and AC OPFs

An energy price profile in two conditions is shown in the table below:

Table 3-8: Comparison of price on buses resulted from Unconstrained DC and AC OPFs

Bus	Energy Price (π) (\$/MW-hr)		
	π_{DC}	π_{AC}	$(\pi_{AC} - \pi_{DC})$
1	12	11.92	-0.08 ↓
2	12	11.95	-0.05 ↓
3	12	12.11	0.11 ↑
4	12	12.00	0.00
5	12	11.41	-0.59 ↓
6	12	12.89	0.89 ↑
7	12	12.84	0.84 ↑
8	12	12.90	0.90 ↑
9	12	13.47	1.47 ↑
10	12	13.66	1.66 ↑
11	12	13.70	1.70 ↑
12	12	13.70	1.70 ↑
13	12	13.63	1.63 ↑
14	12	13.01	1.01 ↑
15	12	13.20	1.20 ↑

In order to have a better view of changes, we can also draw energy profiles as shown below:

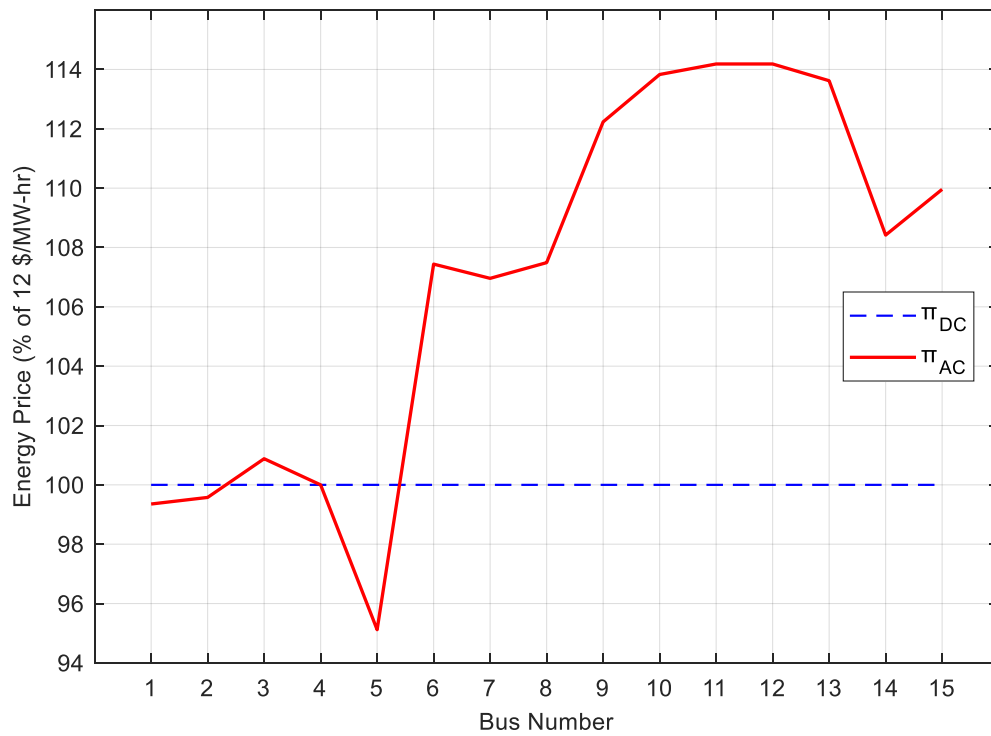


Fig. 3-6: Price profiles resulted from Unconstrained DC and AC OPFs

Fig. 3-6 shows that the most change in price is about %14 (= 1.7 \$/MW-hr) and the most affected area is 'C' area.

Figure below shows the generation income both in AC and DC power flows and the production in MW for each point.

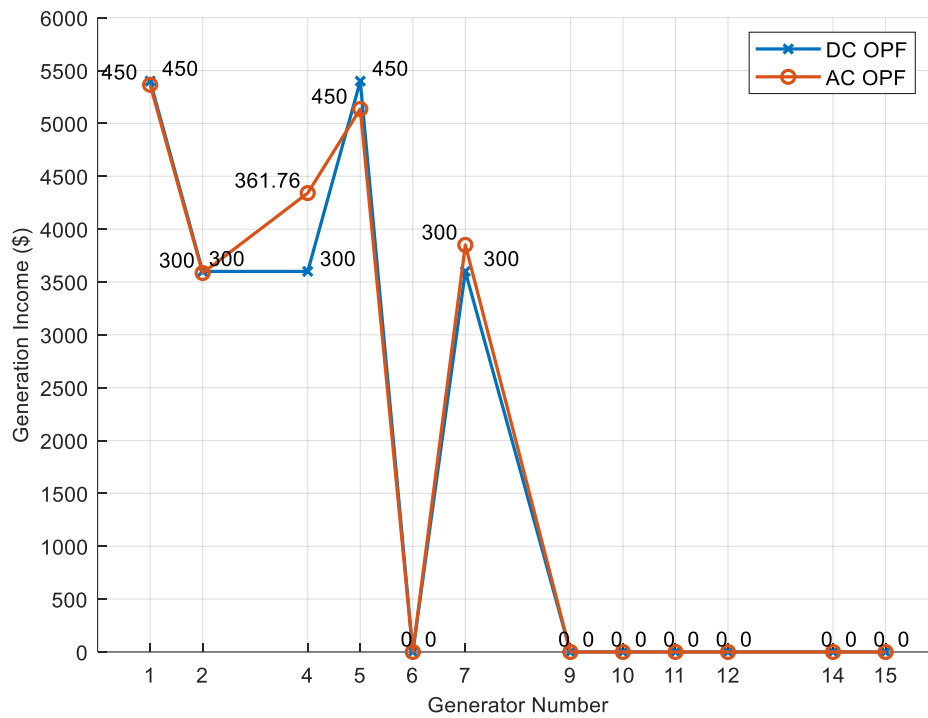


Fig. 3-7: Generation Income resulted from Unconstrained DC and AC OPFs

Changes in generation profit is shown in figure below:

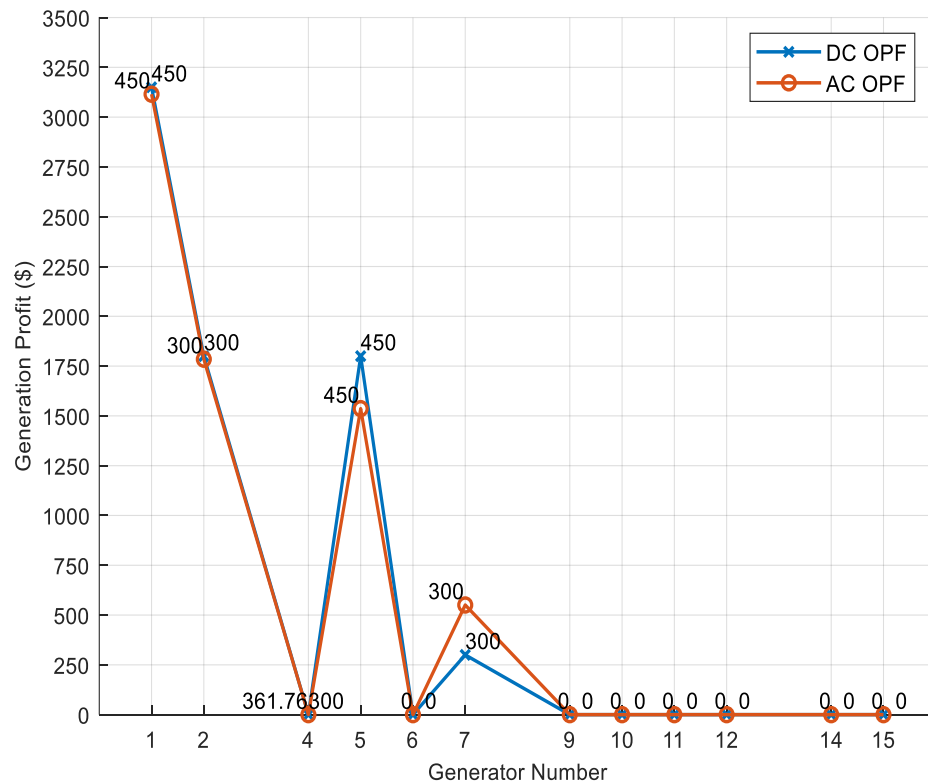


Fig. 3-8: Generation Profit resulted from Unconstrained DC and AC OPFs

Overall, generation values remain unchanged with the exception of Gen. 4, whose generation value has increased but its bus price remains constant and equal to its marginal cost under both conditions.

Gen. 7 is the only generator whose profit rises, and it does so almost 100% more.

3.5. Constrained AC OPF using MATPOWER

The resulting system profile is shown below; for each bus, "Bus number=Voltage magnitude" is displayed, and for the lines, green and black colors mean, respectively, uncongested and congested lines.

NOTE: Values on lines represent the real power related to (injected or received) the bus that is closest to the arrow.

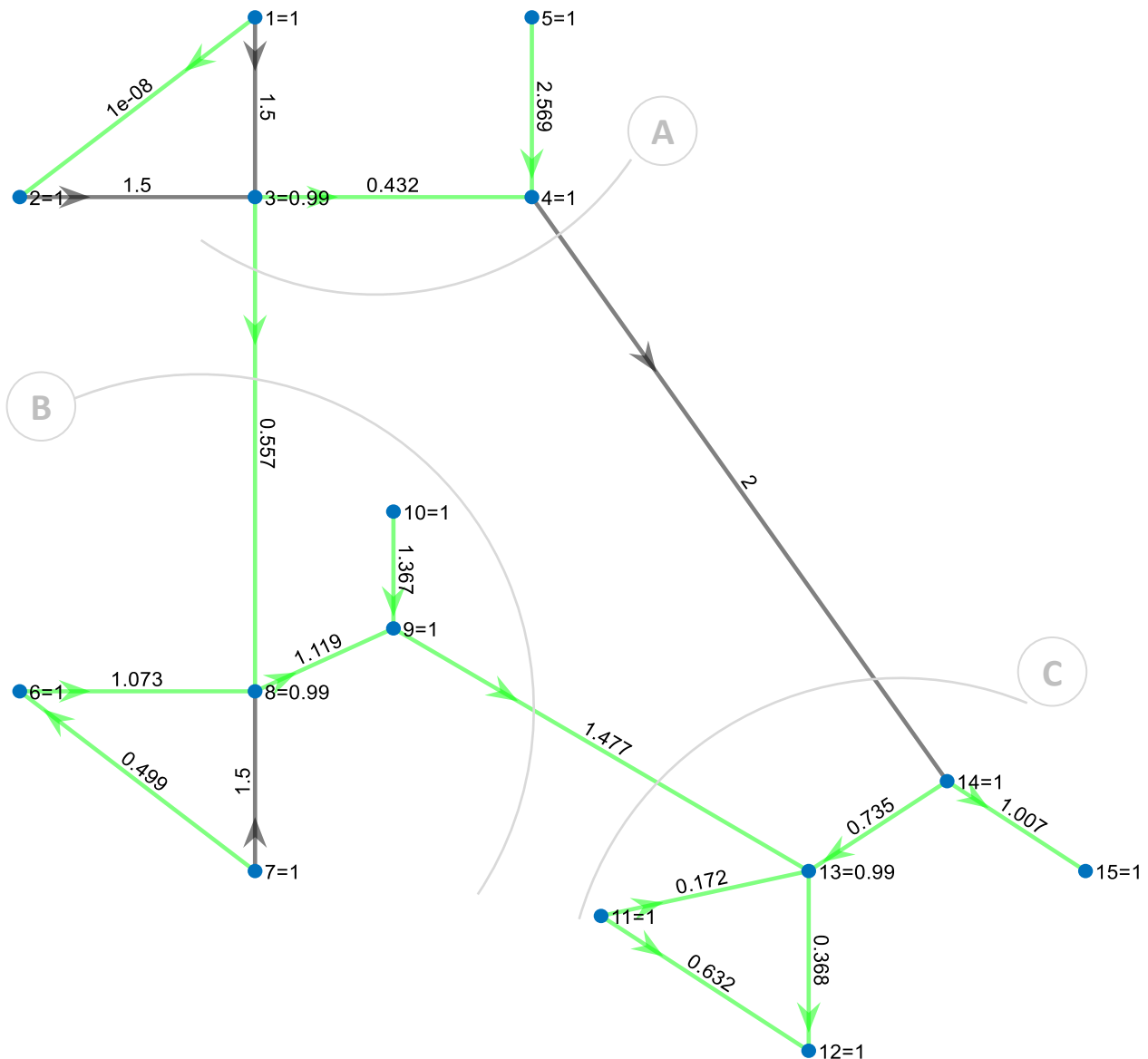


Fig. 3-9: Flow of P in lines and voltage profile from Constrained AC OPF using MATPOWER

Information derived from Fig. 3-9:

1. Three transmission lines from the cheapest generators (1,2 and 7) are congested.
2. Power exchange between areas is as follows (in MW):

A->B: 55.7	A->C: 200	→ A->*: 255.7	B->C: 147.7
→ B->*: (92+ Loss on line 3-8), C->*: (-347.7+ Losses on lines)			
3. Like a DC PF, the voltage magnitude on all buses is almost one p.u.

The following table provides information about the dispatch of generators (values are rounded to two decimal places):

Table 3-9: Generators dispatch resulted from Constrained AC OPF using MATPOWER

Gen	P _g (MW)	MC (\$/MW-hr)	λ _p (\$/MW-hr)	Generation Income (P _g × λ _p)	Generation Cost (P _g × MC)	Generators Profit (Income – Cost)
1	250.00	5	5.00	1,250.00	1,250.00	0.00
2	250.00	6	6.00	1,500.00	1,500.00	0.00
4	0.00	12	8.30	0.00	0.00	0.00
5	361.69	8	8.00	2,893.53	2,893.53	0.00
6	157.31	13	13.00	2,044.99	2,044.99	0.00
7	300.00	11	11.47	3,440.68	3,300.00	140.69
9	0.00	21	18.35	0.00	0.00	0.00
10	238.04	18	18.00	4,284.74	4,284.74	0.00
11	180.48	22	22.00	3,970.51	3,970.51	0.00
12	0.00	23	22.06	0.00	0.00	0.00
14	77.39	25	25.00	1,934.66	1,934.66	0.00
15	0.00	26	25.36	0.01	0.01	0.00
Total:	1,814.90	-	-	21,319.12 (\$)	21,178.43 (\$)	140.69 (\$)

Because of congestion and losses on lines we have LMPs instead of SMP as shown below:

Table 3-10: LMP per bus resulted from Constrained AC OPF using MATPOWER

Bus	λ _p (\$/MW-hr)
1	5.00
2	6.00
3	8.30
4	8.00
5	13.00
6	11.47
7	18.35
8	18.00
9	22.00
10	22.06
11	25.00
12	25.36
13	5.00
14	6.00
15	8.30

3.6. Constrained DC OPF using GAMS

Results of generators' dispatch and LMP on buses are shown in table below:

Table 3-11: Generators dispatch and LMPs resulted from Constrained DC OPF using GAMS

[Bus]	Gen (MW)	Load (MW)	LMP (\$/MWh)
1	250	100	5
2	250	100	6
3	–	200	11.333
4	0.00	100	8
5	381.003	100	8
6	156.992	100	13
7	300	100	11.566
8	–	200	14.667
9	0.00	100	18
10	262.005	100	18
11	0.00	100	21.667
12	0.00	100	21.667
13	–	200	21.667
14	200	100	25
15	0.00	100	25

The MATPOWER results in Table 3-3 and Table 3-4 are exactly the same as those in Table 3-11. The real power flow in the transmission lines resulted from GAMS is shown below which is also identical to that shown in Fig. 3-3.

----	167 VARIABLE Pij.L					
	1	2	3	4	5	6
1			1.500			
2			1.500			
3	-1.500	-1.500		0.190		
4			-0.190		-2.810	
5				2.810		
7						0.500
8			-0.810			-1.070
14				-2.000		
+	7	8	9	10	11	12
3		0.810				
6	-0.500	1.070				
7		1.500				
8	-1.500		1.380			
9		-1.380		-1.620		
10			1.620			
13			-2.000		1.000	1.000
+	13	14	15			
4		2.000				
9	2.000					
11	-1.000					
12	-1.000					
13		-2.000				
14	2.000		1.000			
15		-1.000				

3.7. The result of line 4-14 being absent

We know that if the line between buses 4 and 14 is taken out of service, there won't be the 200 MW of power that area A previously supplied from generators whose average MC was about $(5+6+8)/3=6.33$ \$/MWh. As a result, generators in area C would compensate for the power with an average MC of about 24 \$/MWh, and we can expect the following effects:

1. Total Cost: Using more expensive generators will increase total cost.
2. Local Marginal Prices (LMPs): According to the Table 3-3, Gen. 14 (with MC of 25) is too expensive to supply the entire 200 MW power; Gen. 11 (MC=22) and Gen. 12 (MC=23) should be used instead. On the other hand, if the LMP on buses 11 and 12 remains smaller than its generators' MCs, then they could not produce. To supply from generators on buses 11 and 12, the LMP on those buses must increase to at least 22 and 23, respectively. This will also reduce the LMP on bus 14 that is now equal to Gen.14's MC, causing Gen.14 to be turned off, and LMPs over area C will be more uniform with a minimal change in average value.

3.7.1. Solving the problem with the absence of line 4-14

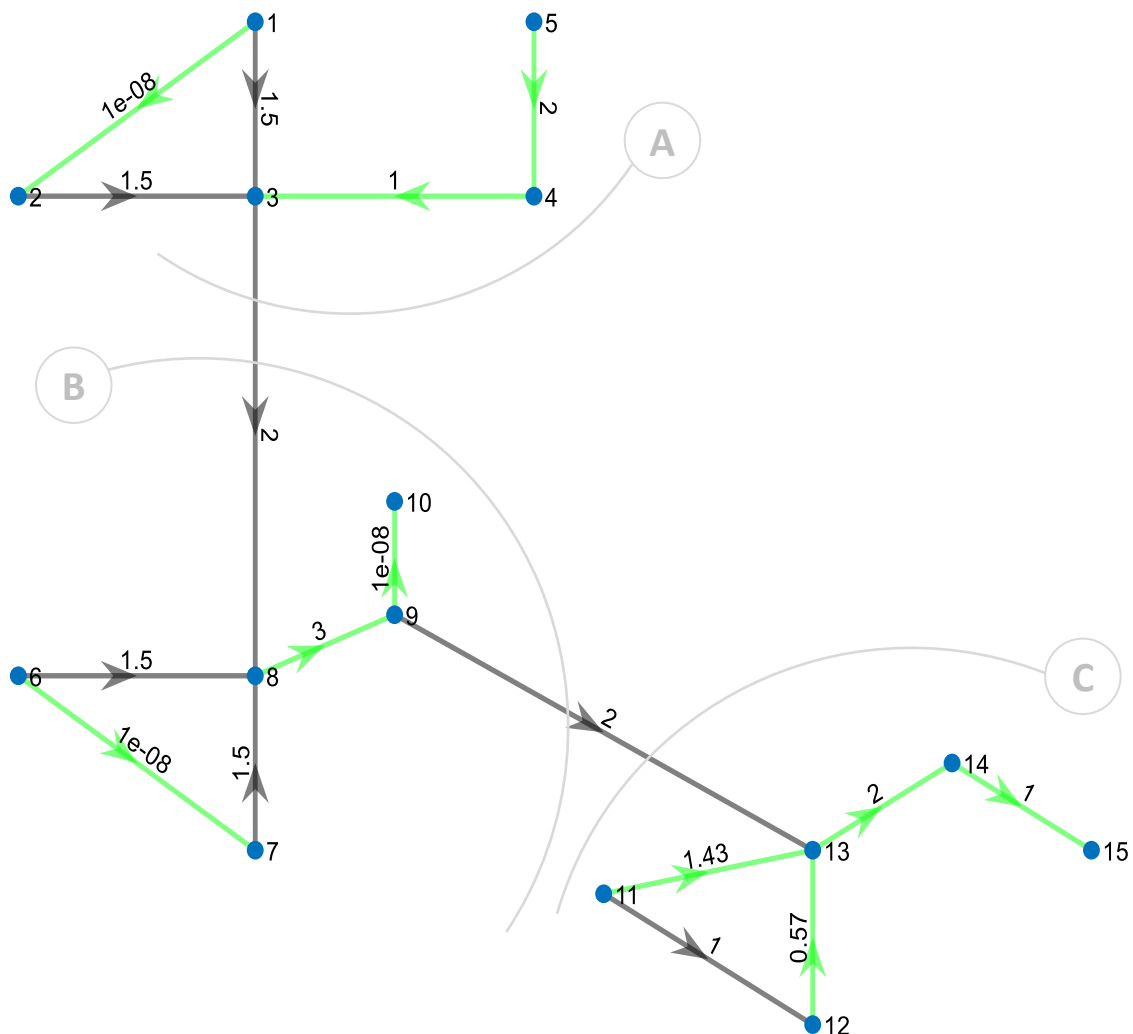


Fig. 3-10: Flow of P in lines resulted from Constrained DC OPF in absence of line 4-14

Fig. 3-10 shows flow of power in lines and the generators dispatch and LMP on buses are shown in the tables below:

Table 3-12: Generators dispatch resulted from Constrained DC OPF in absence of line 4-14

Gen	P _g (MW)	MC (\$/MW-hr)	λ _p (\$/MW-hr)	Generation Income (P _g × λ _p)	Generation Cost (P _g × MC)	Generators Profit (Income – Cost)
1	250	5	5	1,250.00	1,250.00	0.00
2	250	6	6	1,500.00	1,500.00	0.00
4	0	12	8	0.00	0.00	0.00
5	300	8	8	2,400.00	2,400.00	0.00
6	250	13	13	3,250.00	3,250.00	0.00
7	250	11	11	2,750.00	2,750.00	0.00
9	0	21	18	0.00	0.00	0.00
10	100	18	18	1,800.00	1,800.00	0.00
11	343.01	22	22	7,546.17	7,546.17	0.00
12	56.99	23	23	1,310.82	1,310.82	0.00
14	0	25	22.5	0.00	0.00	0.00
15	0	26	22.5	0.00	0.00	0.00
Total:	1,800	-	-	21,807.00 (\$)	21,807.00 (\$)	0.00 (\$)

Table 3-13: LMP per bus resulted from Constrained DC OPF in absence of line 4-14

Bus	λ _p (\$/MW-hr)
1	5
2	6
3	8
4	8
5	8
6	13
7	11
8	18
9	18
10	18
11	22
12	23
13	22.5
14	22.5
15	22.5

We can see that our predictions were correct by comparing Table 3-12 to Table 3-3 and Table 3-13 to Table 3-4.

3.8. The Most Valuable Line

The most valuable transmission line can be defined from the perspective of the power market. One way to do this is by looking at the Lagrange multipliers associated with the transmission line flow constraints in the DCOPF problem. The Lagrange multiplier (μ) represents the change in the objective function (e.g., the total generation cost) resulting from a marginal change in the transmission line flow constraint. In other words, it represents the marginal value of relaxing the transmission line flow constraint.

A higher Lagrange multiplier for a transmission line flow constraint indicates that relaxing that constraint would result in a larger reduction in the total generation cost. Therefore, from an economic perspective, transmission lines with higher Lagrange multipliers can be considered more valuable in terms of their contribution to reducing the total generation cost.

Tables below show the Lagrange multipliers in different conditions using MATLAB® and GAMS®. The branches with μ value of zero are not mentioned in tables.

Table 3-14: Lagrange multipliers resulted from DC OPF using MATPOWER

From	To	μ
1	3	7.5
2	3	4.17
7	8	4.77
4	14	20.33

Table 3-15: Lagrange multipliers resulted from DC OPF using GAMS

From/To	3	4	8	9
1	-749.592			
2	-417.075			
7			-476.692	
13				33.333
14		2033.333		

Table 3-16: Lagrange multipliers resulted from AC OPF using MATPOWER

From	To	μ
1	3	7.55
2	3	4.22
7	8	5.02
4	14	19.2

All tables above represent that Line 4-14 has the greatest μ value at both AC and DC modes.

3.9. Simultaneous Clearing of Energy and Reserves

Using GAMS® and DC load flow, we performed simultaneous clearing of the power and spinning reserve markets taking network constraints into account. Since the system needs 180 MW of spinning reserve, generators 4, 5, 9, 10, 14 and 15 are candidates to provide reserves at a cost of one third of their energy price and a maximum of 30% of their capacity. The result is shown below.

Table 3-17: Gen. and S.R. dispatch and LMP resulted from simultaneous clearing of markets

Bus	Generation (MW)	Reserve (MW)	Res.max (MW)	Res.+Gen. (MW)	Pmax (MW)	LMP (\$/MWh)	Bus
1	250	0	–	250	450	5	1
2	250	0	–	250	300	6	2
3	–	–	–	–	–	12	3
4	0	45	180	45	600	9	4
5	315	135	135	450	450	9	5
6	156.99	–	–	156.99	450	13	6
7	300	–	–	300	300	11.28	7
8	–	–	–	–	–	15	8
9	0	0	180	0	600	18	9
10	328.01	0	135	328.01	450	18	10
11	198.01	–	–	198.01	450	22	11
12	0	–	–	0	300	22	12
13	–	–	–	–	–	22	13
14	1.992	0	180	1.992	600	25	14
15	0	0	135	0	450	25	15

Only generators 4 and 5 contribute to reserve by a value of 45 MW and 135 MW, respectively, according to Table 3-17.

4. Conclusion

In this study, we explored and compared the results of Constrained and Unconstrained DC and AC Optimal Power Flow (OPF) models. We conducted these analyses using the MATPOWER and GAMS modeling language. Additionally, we implemented a Constrained DC OPF model with Reserves to account for the availability of spinning reserves in the electricity market.

By comparing the results of Constrained and Unconstrained OPF models, we observed significant differences in the optimal operating schedules of generating units.

By comparing the results on Constrained DC and AC OPF models we saw that the results were close together because of large number of generators distributed all over system that control voltage at 1 per unit so DC model can be a good and fast approximation of power market.

Furthermore, the inclusion of spinning reserves in the Constrained DC OPF model allowed us to account for the availability of reserves to maintain system reliability. By considering reserve constraints, we ensured that the total spinning reserve provided by reserve-generating units met the specified reserve requirements. Additionally, we established upper limits for the reserve capacity of individual units and maintained the combined output of reserves and generation below the maximum capacity of each unit.

Overall, our study highlights the importance of considering constraints and reserve requirements in power system optimization.

5. Appendix (Codes)

5.1. MATPOWER Case for the system

In case_market_project.m we determined the system's configuration and specifications.

```
1. function mpc = case_Market_Project
2. mpc.version = '2'; %% MATPOWER Case Format : Version 2
3. mpc.baseMVA = 100; %% system MVA base
4. %% bus data
5. % bus_i type Pd Qd Gs Bs area Vm Va baseKV zone Vmax Vmin
6. mpc.bus = [ %% (Pd and Qd are specified in MW & MVar here)
7. 1 3 100 20.30586606 0 0 1 1 0 400 1 1 1;
8. 2 2 100 20.30586606 0 0 1 1 0 400 1 1 1;
9. 3 2 200 40.61173213 0 0 1 1 0 400 1 1 1;
10. 4 2 100 20.30586606 0 0 1 1 0 400 1 1 1;
11. 5 2 100 20.30586606 0 0 1 1 0 400 1 1 1;
12. 6 2 100 25.06236244 0 0 2 1 0 400 1 1 1;
13. 7 2 100 25.06236244 0 0 2 1 0 400 1 1 1;
14. 8 2 200 50.12472487 0 0 2 1 0 400 1 1 1;
15. 9 2 100 25.06236244 0 0 2 1 0 400 1 1 1;
16. 10 2 100 25.06236244 0 0 2 1 0 400 1 1 1;
17. 11 2 100 14.24922826 0 0 3 1 0 400 1 1 1;
18. 12 2 100 14.24922826 0 0 3 1 0 400 1 1 1;
19. 13 2 200 28.49845652 0 0 3 1 0 400 1 1 1;
20. 14 2 100 14.24922826 0 0 3 1 0 400 1 1 1;
21. 15 2 100 14.24922826 0 0 3 1 0 400 1 1 1;];
22. %% Bus Vm limits
23. mpc.bus(:,end) = 0.95.*ones(1,15);
24. mpc.bus(:,end-1) = 1.05.*ones(1,15);
25. %% generator data
26. % bus Pg Qg Qmax Qmin Vg mBase status Pmax Pmin Pc1 Pc2 Qc1min Qc1max Qc2min Qc2max ramp_agc ramp_10 ramp_30 ramp_q
27. apf
28. mpc.gen = [
29. 1 0 0 inf -inf 1 100 1 450 0 0 0 0 0 0 0 0 0 0 0 0;
30. 2 0 0 inf -inf 1 100 1 300 0 0 0 0 0 0 0 0 0 0 0 0;
31. 4 0 0 inf -inf 1 100 1 600 0 0 0 0 0 0 0 0 0 0 0 0;
32. 5 0 0 inf -inf 1 100 1 450 0 0 0 0 0 0 0 0 0 0 0 0;
33. 6 0 0 inf -inf 1 100 1 450 0 0 0 0 0 0 0 0 0 0 0 0;
34. 7 0 0 inf -inf 1 100 1 300 0 0 0 0 0 0 0 0 0 0 0 0;
35. 9 0 0 inf -inf 1 100 1 600 0 0 0 0 0 0 0 0 0 0 0 0;
36. 10 0 0 inf -inf 1 100 1 450 0 0 0 0 0 0 0 0 0 0 0 0;
37. 11 0 0 inf -inf 1 100 1 450 0 0 0 0 0 0 0 0 0 0 0 0;
38. 12 0 0 inf -inf 1 100 1 300 0 0 0 0 0 0 0 0 0 0 0 0;
39. 14 0 0 inf -inf 1 100 1 600 0 0 0 0 0 0 0 0 0 0 0 0;
40. 15 0 0 inf -inf 1 100 1 450 0 0 0 0 0 0 0 0 0 0 0 0;];
41. mpc.gen(:,2)=250.*ones(1,12);
42. %% Generator bus Vm fixed at 1 pu
43. mpc.bus(mpc.gen(:,1),end-1:end)=1;
44. %% branch data
45. % fbus tbus r x b rateA rateB rateC ratio angle status angmin angmax
46. mpc.branch = [ %% (r and x specified in p.u. here, Rates are in p.u. here and converted to MW below)
47. 1 2 0.002085 0.020851 0 1 1 1 0 0 1 -360 360;
48. 1 3 0.002424 0.024241 0 1.5 1.5 1.5 0 0 1 -360 360;
49. 2 3 0.002424 0.024241 0 1.5 1.5 1.5 0 0 1 -360 360;
50. 3 4 0.00695 0.069502 0 4 4 4 0 0 1 -360 360;
51. 4 5 0.00695 0.069502 0 4 4 4 0 0 1 -360 360;
52. 6 7 0.002085 0.020851 0 1 1 1 0 0 1 -360 360;
53. 6 8 0.002424 0.024241 0 1.5 1.5 1.5 0 0 1 -360 360;
54. 7 8 0.002424 0.024241 0 1.5 1.5 1.5 0 0 1 -360 360;
55. 8 9 0.00695 0.069502 0 4 4 4 0 0 1 -360 360;
56. 9 10 0.00695 0.069502 0 4 4 4 0 0 1 -360 360;
57. 11 12 0.002085 0.020851 0 1 1 1 0 0 1 -360 360;
58. 11 13 0.002424 0.024241 0 1.5 1.5 1.5 0 0 1 -360 360;
59. 12 13 0.002424 0.024241 0 1.5 1.5 1.5 0 0 1 -360 360;
60. 13 14 0.00695 0.069502 0 4 4 4 0 0 1 -360 360;
61. 14 15 0.00695 0.069502 0 4 4 4 0 0 1 -360 360;
62. 3 8 0.00695 0.069502 0 2 2 2 0 0 1 -360 360;
63. 4 14 0.00695 0.069502 0 2 2 2 0 0 1 -360 360;
64. 9 13 0.00695 0.069502 0 2 2 2 0 0 1 -360 360;];
65. mpc.branch(:,6:8)=100.*mpc.branch(:,6:8);
66. %%----- OPF Data -----%%
67. %% generator cost data
68. % 2 startup shutdown n c(n-1) ... c0
69. mpc.gencost = [
70. 2 0 0 2 5 0;
71. 2 0 0 2 6 0;
72. 2 0 0 2 12 0;
73. 2 0 0 2 8 0;
74. 2 0 0 2 13 0;
75. 2 0 0 2 11 0;
76. 2 0 0 2 21 0;
77. 2 0 0 2 18 0;
78. 2 0 0 2 22 0;
79. 2 0 0 2 23 0;
80. 2 0 0 2 25 0;
81. 2 0 0 2 26 0;];
82. %% convert branch impedances from Ohms to p.u.
83. [PQ, PV, REF, NONE, BUS_I, BUS_TYPE, PD, QD, GS, BS, BUS_AREA, VM, ...
84. VA, BASE_KV, ZONE, VMAX, VMIN, LAM_P, LAM_Q, MU_VMAX, MU_VMIN] = idx_bus;
85. [F_BUS, T_BUS, BR_R, BR_X, BR_B, RATE_A, RATE_B, RATE_C, ...
86. TAP, SHIFT, BR_STATUS, PF, QF, PT, QT, MU_SF, MU_ST, ...
87. ANGMIN, ANGMAX, MU_ANGMIN, MU_ANGMAX] = idx_brch;
88. Vbase = mpc.bus(1, BASE_KV) * 1e3; %% in Volts
89. Sbase = mpc.baseMVA * 1e6; %% in VA
```

5.2. Script of Performing OPF using MATPOWER

In Market_Project_402.m we implement how to use MATPOWER in order to perform either DC or AC and Constrained or Unconstrained OPF.

```
1. clear
2. clc
3. %%
4. system_case = 'case_Market_Project';
5. %system_case = 'case_Market_Project_no_4_14'; `%This case only has a '%' at first of line 62 of case
   above
6. mpc=loadcase(system_case);
7. mpopt=mpoption;

8. %% Constrained/Unconstrained DC/AC OPF
9. i1=input('PowerFlow Model? AC/DC: ','s');
10. mpopt=mpoption(mpopt,'model',i1,'opf.flow_lim','P','out.all',1);
11. %% Variables
12. branch_limit=mpc.branch(:,6);
13. bus_l_limit=mpc.bus(:,end);
14. bus_u_limit=mpc.bus(:,end-1);
15. i2=input('With/WithoutNetwork Constraints? 1/0: ');
16. if i2==0
17. mpc.branch(:,6:8)=0;
18. end
19. %% Results
20. result=runopf(mpc,mpopt);
21. %% Visualizing and Printing Results
22. Visualizing_results
23. typec=["Without Constraints","With Constraints"];
24. title("PF: "+i1+" , "+typec(i2+1));
25. branches_mu=result.branch(:,[1,2,18])
```

5.3. Script of visualizing the results from MATPOWER

In Visualizing_results.m script we draw the system graph and show p values flowing in lines and generate tables.

```
1. close all
2. s=mpc.branch(:,1);
3. t=mpc.branch(:,2);
4. wp=round(result.branch(:,14),1);
5. w=wp(:,1);
6. w=w./100;
7. w(w==0)=1e-8;
8. arrow_pos=w;
9. arrow_pos(1:length(s))=0.5;
10. for i=1:length(s)
11. if i1=="AC"
12.     a. arrow_pos(i)=0.3;
13. end
14. if sign(w(i))==1
15.     a. temp=s(i);
16.     b. s(i)=t(i);
17.     c. t(i)=temp;
18.     d. if i1=="AC"
19.         i. arrow_pos(i)=0.9;
20.     end
21. end
22. end
23. w=abs(w);
24. clear i temp
25. bus_no=15;
26. eq(1:bus_no,1)="";
27. vbp=result.bus(:,8);
28. if i1=="DC"
29.     names=string((1:bus_no)');
30. else
31.     names=string((1:bus_no)')+string(char(eq))+string(round(vbp(:,1),2));
32. end
33. G = digraph(s,t,w,names);
34. c=zeros(length(s),3);
35. st=[s,t];
36. st_sorted=table2array(sortrows(table(s,t)));
37. for k=1:length(s)
38.     row=find(st(:,1)==st_sorted(k,1) & st(:,2)==st_sorted(k,2));
39.     %st(row,:)
40.     i=row;
41.     if abs(w(i))>branch_limit(i)/100
42.         a. c(i,:)= [1 0 0]; %red
43.     elseif abs(w(i))==branch_limit(i)/100
44.         a. c(i,:)= [0 0 0]; %black
45.     else
46.         a. c(i,:)= [0 1 0]; %green
47.     end
48. end
49. temp=sortrows(table(s,t,wp,w,branch_limit./100,c,(arrow_pos)));
50. c=table2array(temp(:,6));
51. arrow_pos=table2array(temp(:,7));
52. clear temp
53. G.Edges.EdgeColors=c;
54. figure
55. h3 = plot(G,'Layout','circle','EdgeLabel',G.Edges.Weight,'MarkerSize',6,'LineStyle','-','LineWidth',2,'NodeFontSize',10,'ArrowSize',15,'EdgeFontSize',10,'EdgeFontAngle','normal','EdgeColor',c,'ArrowPosition',arrow_pos);
56. if system_case == "case_Market_Project" | system_case == "case_Market_Project_no_4_14"
57. X=[1 0 1 3 3 0 1 1 2 2 3.5 5 5 6 7];
```

```

48. Y=[11 9 9 9 11 4 2 4 4.7 6 1.5 0 2 3 2];
49. X=X+0.7;
50. X([2,6])=0;
51. Y([1:5])=Y([1:5])+0.5;
52. h3.XData = X;
53. h3.YData = Y;
54. end
55. %%
56. genbus=result.gen(:,1);
57. Pg=result.gen(:,2);
58. genmc=result.gencost(:,5);
59. LAM_P=result.bus(:,end-3);
60. LAM_P_gen=LAM_P(result.gen(:,1));
61. income=Pg.*LAM_P_gen;
62. expen=Pg.*genmc;
63. prof=income-expen;
64. dis_table = table(genbus,Pg,genmc,LAM_P_gen,income,expen,prof,...
    i. 'VariableNames',{ 'Gen_Bus','Pg','MC','?p','Income','Cost','Profit'})
65. total_generation=sum(dis_table.Pg)
66. total_gen_income=sum(dis_table.Income)
67. total_gen_expen=sum(dis_table.Cost)
68. total_gen_profit=sum(dis_table.Profit)
69. LMP_table= table((1:length(LAM_P))',LAM_P,...
    i. 'VariableNames',{ 'Bus','?p'})
70. if i1=="AC"
71. figure
72. plot((1:bus_no),round(vbp(:,1),2))
73. end
74. if i1=="AC"
75. figure
76. plot((1:bus_no),100.*ones(1,15),'b--','LineWidth',1);
77. hold on
78. plot((1:bus_no),LAM_P./12.*100,'r-','LineWidth',1.5)
79. grid on
80. xlim([0 16])
81. xticks([1:15])
82. ylim(100.*[0.94 1.16])
83. %yticks(100+sort((LAM_P-12)./12.*100))
84. xlabel('Bus Number')
85. ylabel('Energy Price (% of 12 $/MW-hr)')
86. legend('?_{DC}','?_{AC}','Location','best')
87. end

```

5.4. Script of Visual Comparing Unconstrained DC and AC OPFs

By using the script `compare_acdc0.m` we can visually compare parameters resulted from solving Unconstrained DC and AC OPFs.

```
1. clear
2. clc
3. close all
4. %%
5. load dc0_ac0_compare.mat
6. %%
7. % income
8. figure
9. hold on
10. grid on
11. plot(gen,dc(:,2),'-x','LineWidth',1.5);
12. plot(gen,ac(:,2),'-o','LineWidth',1.5);
13. xlim([0 16])
14. xticks(gen)
15. ylim([0 6000])
16. yticks(0:500:6000)
17. legend('DC OPF','AC OPF','Location','best')
18. xlabel('Generator Number')
19. ylabel('Generation Income ($)')
20. text(gen,dc(:,2),strcat([' '],[num2str(dc(:,1))]),'horiz','left','vert','bottom')
21. text(gen,ac(:,2),strcat([' '],[num2str(ac(:,1))]),'horiz','right','vert','bottom')

22. % profit
23. figure
24. hold on
25. grid on
26. plot(gen,dc(:,end),'-x','LineWidth',1.5);
27. plot(gen,ac(:,end),'-o','LineWidth',1.5);
28. xlim([0 16])
29. xticks(gen)
30. ylim([0 3500])
31. yticks(0:250:3500)
32. legend('DC OPF','AC OPF','Location','best')
33. xlabel('Generator Number')
34. ylabel('Generation Profit ($)')
35. text(gen,dc(:,end),strcat([' '],[num2str(dc(:,1))]),'horiz','left','vert','bottom')
36. text(gen,ac(:,end),strcat([' '],[num2str(ac(:,1))]),'horiz','right','vert','bottom')

37. % profit per MW
38. figure
39. hold on
40. grid on
41. plot(gen,dc(:,end)./dc(:,1),'-x','LineWidth',1.5);
42. plot(gen,ac(:,end)./ac(:,1),'-o','LineWidth',1.5);
43. xlim([0 8])
44. xticks(gen)
45. ylim([0 7.5])
46. % yticks(0:250:3500)
47. legend('DC OPF','AC OPF','Location','best')
48. xlabel('Generator Number')
49. ylabel('Profit per Production ($/MW)')
50. text(gen,dc(:,end)./dc(:,1),strcat([' '],[num2str(dc(:,1))]),'horiz','left','vert','bottom')
51. text(gen,ac(:,end)./ac(:,1),strcat([' '],[num2str(ac(:,1))]),'horiz','right','vert','bottom')
```

5.5. MATLAB Script of Drawing Supply and Demand Curves

By using SD_curves.m we can draw supply and demand curve for any system defined in MATPOWER style.

```
1. gendata=sortrows(table(mpc.gencost(:,end-1),mpc.gen(:,1),mpc.gen(:,9))));
2. x=zeros(1,size(gendata,1)+1);
3. y=table2array(gendata(:,1));
4. for i=2:length(x)
5. x(i)=sum(table2array(gendata(1:i-1,end)));
6. end
7. x=x./100;
8. y=[y;y(end)];

9. figure
10. l1=stairs(x,y,'-b','LineWidth',2);
11. xlabel('Total demand (p.u.) (S_{base}=100)')
12. ylabel('Price ($/MW-hr)')
13. ylim([0 max(y)*1.1])
14. grid on
15. hold on
16. Total_D=sum(result.gen(:,2));
17. l2=plot([Total_D Total_D]./100,[0 max(y)],'-r','LineWidth',2);
18. plot([Total_D]/100,[12],'x','MarkerSize',10)
```

5.6. GAMS code for Constrained DC OPF

```

$title "DC OPF" model for Power Market Project by Hamed Najafi

$oneolcom
$eolcom //

sets
bus /1*15/           //Set of buses from 1 to 15
slack(bus) /1/       // Set of slack bus, containing only bus 1
GenNo /1,2,4,5,6,7,9,10,11,12,14,15/ //Set of generator indices

scalars
Sbase /100/
Vbase /400/
;
alias(bus,node);

table GenData(GenNo,*) Generating units characteristics
      b      pmin      pmax
1      5      0      450
2      6      0      300
4     12      0      600
5      8      0      450
6     13      0      450
7     11      0      300
9     21      0      600
10     18      0      450
11     22      0      450
12     23      0      300
14     25      0      600
15     26      0      450
;

-----
set GBconnect(bus,GenNo) connectivity index of each generating unit to each bus
/
      1      .      1
      2      .      2
      4      .      4
      5      .      5
      6      .      6
      7      .      7
      9      .      9
     10      .     10
     11      .     11
     12      .     12
     14      .     14
     15      .     15/ ;

*****
*****
Table BusData(bus,*) Demands of each bus in MW
Pd
      100
      100
      200
      100
      100
      100
      100
      200
      100
      100
      100
      100

```



```

                200
                100
                100

;
*****
set conex          Bus connectivity matrix
/
1      .      2
1      .      3
2      .      3
3      .      4
4      .      5
6      .      7
6      .      8
7      .      8
8      .      9
9      .      10
11     .      12
11     .      13
12     .      13
13     .      14
14     .      15
3      .      8
4      .      14
9      .      13

-----

/;
conex(bus,node)$(conex(node,bus))=1;

table branch(bus,node,*)      Network technical characteristics
                                x          Limit
1      .      2      0.020851      100
1      .      3      0.024241      150
2      .      3      0.024241      150
3      .      4      0.069502      400
4      .      5      0.069502      400
6      .      7      0.020851      100
6      .      8      0.024241      150
7      .      8      0.024241      150
8      .      9      0.069502      400
9      .      10     0.069502      400
11     .      12     0.020851      100
11     .      13     0.024241      150
12     .      13     0.024241      150
13     .      14     0.069502      400
14     .      15     0.069502      400
3      .      8      0.069502      200
4      .      14     0.069502      200
9      .      13     0.069502      200

-----

;

branch(bus,node,'x')$(branch(bus,node,'x')=0)=branch(node,bus,'x');
branch(bus,node,'Limit')$(branch(bus,node,'Limit')=0)=branch(node,bus,'Limit');
branch(bus,node,'bij')$conex(bus,node) =1/branch(bus,node,'x');
*****
Variables
OF
Pij(bus,node)
Pg(GenNo)
delta(bus)
;

Equations
*****

```

```

const1
const2
const3
;
*****
const1(bus,node)$( conex(bus,node)) .. Pij(bus,node)=e= branch(bus,node,'bij')*(delta(bus)-delta(node));
const2(bus)..+sum(GenNo$GBconect(bus,GenNo),Pg(GenNo))-
BusData(bus,'pd')/Sbase=e+=sum(node$conex(node,bus),Pij(bus,node));
const3 .. OF=g=sum(GenNo,Pg(GenNo)*GenData(GenNo,'b')*Sbase);

model loadflow /const1,const2,const3/;

Pg.lo(GenNo)=GenData(GenNo,'Pmin')/Sbase;
Pg.up(GenNo)=GenData(GenNo,'Pmax')/Sbase;
delta.up(bus)=pi;
delta.lo(bus)=-pi;
delta.fx(slack)=0;
Pij.up(bus,node)$(conex(bus,node))=1* branch(bus,node,'Limit')/Sbase;
Pij.lo(bus,node)$(conex(bus,node))=-1*branch(bus,node,'Limit')/Sbase;

solve loadflow minimizing OF using lp;
parameter report(bus,*);
report(bus,'Gen(MW)')= sum(GenNo$GBconect(bus,GenNo),Pg.l(GenNo))*sbase;
report(bus,'load(MW)')= BusData(bus,'pd');
report(bus,'LMP($/MWh)')=const2.m(bus)/sbase ;

display report,Pij.l,Pij.m;

```

5.7. GAMS code for DC OPF with Reserves

```

$title "DC OPF with reserve" model for Power Market Project by Hamed Najafi

$oneolcom
$eolcom //

sets
bus /1*15/          //Set of buses from 1 to 15
slack(bus) /1/      // Set of slack bus, containing only bus 1
GenNo /1,2,4,5,6,7,9,10,11,12,14,15/ //Set of generator indices

***** For Reserve market *****
ReserveGenNo(GenNo) /4,5,9,10,14,15/ // Set of generator indices for reserve-providing units
scalars
Sbase /100/ //Base power value in megawatts (MW)
Vbase /400/ //Base voltage value in kilovolts (kV)

***** For Reserve market *****
SpinningReserveRequired /180/          //Required spinning reserve in megawatts (MW)
SpinningReserveFraction /3/           //Fraction of spinning reserve capacity required (percentage)
SpinningReserveCapacityFraction /30/  //Fraction of generator's capacity available for spinning reserve
(percentages)

;
alias(bus,node); //Alias declaration for bus and node

table GenData(GenNo,*) Generating units characteristics
      b      pmin      pmax
1      5      0      450
2      6      0      300
4      12     0      600
5      8      0      450
6      13     0      450
7      11     0      300
9      21     0      600
10     18     0      450
11     22     0      450
12     23     0      300
14     25     0      600
15     26     0      450
;

* -----
set GBconnect(bus,GenNo) connectivity index of each generating unit to each bus
/
      1      .      1
      2      .      2
      4      .      4
      5      .      5
      6      .      6
      7      .      7
      9      .      9
     10      .     10
     11      .     11
     12      .     12
     14      .     14
     15      .     15/ ;

*****
*****
Table BusData(bus,*) Demands of each bus in MW
      Pd
1      100
2      100
3      200

```

```

4      100
5      100
6      100
7      100
8      200
9      100
10     100
11     100
12     100
13     200
14     100
15     100
;
*****
set conex      Bus connectivity matrix
/
1      .      2
1      .      3
2      .      3
3      .      4
4      .      5
6      .      7
6      .      8
7      .      8
8      .      9
9      .      10
11     .      12
11     .      13
12     .      13
13     .      14
14     .      15
3      .      8
4      .      14
9      .      13
* -----
/;
conex(bus,node)$(conex(node,bus))=1;

table branch(bus,node,*)      Network technical characteristics
                                x      Limit
1      .      2      0.020851      100
1      .      3      0.024241      150
2      .      3      0.024241      150
3      .      4      0.069502      400
4      .      5      0.069502      400
6      .      7      0.020851      100
6      .      8      0.024241      150
7      .      8      0.024241      150
8      .      9      0.069502      400
9      .      10     0.069502      400
11     .      12     0.020851      100
11     .      13     0.024241      150
12     .      13     0.024241      150
13     .      14     0.069502      400
14     .      15     0.069502      400
3      .      8      0.069502      200
4      .      14     0.069502      200
9      .      13     0.069502      200
* -----
;

branch(bus,node,'x')$(branch(bus,node,'x')=0)=branch(node,bus,'x');
branch(bus,node,'Limit')$(branch(bus,node,'Limit')=0)=branch(node,bus,'Limit');
branch(bus,node,'bij')$conex(bus,node) =1/branch(bus,node,'x');
*****

```

```

Variables
OF                Objective function value
Pij(bus,node)    Power flow from bus to node
Pg(GenNo)        Power generation of each generator
delta(bus)       Phase angle at each bus
Reserve(GenNo)   Reserve provided by each generator
;

Equations
*****
const1 //DC LF Equation
const2 //Load Balance Constraint at each bus
const3 //OF=sum(Pgi*bi)+sum(Rj*b_Rj) : bi:cost of Pi , b_Rj:cost of b_Rj //P(p.u.),R(MW)

***** For Reserve market *****
const4 // sum of Reserves is equal to the required spinning reserve
const5 // Ri<=Ri_max
const6 // Ri+Pi<=Pi_max
;
*****
const1(bus,node)$( conex(bus,node)) .. Pij(bus,node)=e= branch(bus,node,'bij')*(delta(bus)-delta(node));
const2(bus) .. +sum(GenNo$GBconnect(bus,GenNo),Pg(GenNo))-
BusData(bus,'pd')/Sbase=e+=sum(node$conex(node,bus),Pij(bus,node));
const3 .. OF=g=sum(GenNo,Pg(GenNo)*GenData(GenNo,'b')*Sbase)+sum(ReserveGenNo, Reserve(ReserveGenNo) *
(GenData(ReserveGenNo, 'b') / SpinningReserveFraction));


***** For Reserve market *****
const4.. sum(ReserveGenNo, Reserve(ReserveGenNo)) =e= SpinningReserveRequired;
const5(ReserveGenNo).. Reserve(ReserveGenNo)
=l=(GenData(ReserveGenNo,'pmax'))*SpinningReserveCapacityFraction/100;
const6(ReserveGenNo).. Reserve(ReserveGenNo) + Pg(ReserveGenNo) * Sbase =l= GenData(ReserveGenNo, 'pmax');
*****
model loadflow /const1, const2, const3,const4, const5, const6/;

Pg.lo(GenNo)=GenData(GenNo,'Pmin')/Sbase; //Pi>=Pi_min
Pg.up(GenNo)=GenData(GenNo,'Pmax')/Sbase; //Pi<=Pi_max
delta.up(bus)=pi; //delta_i<=pi
delta.lo(bus)=-pi; //delta_i>=-pi
delta.fx(slack)=0; //delta_slack<=0
Pij.up(bus,node)$(conex(bus,node))=1* branch(bus,node,'Limit')/Sbase; //line limit
Pij.lo(bus,node)$(conex(bus,node))=-1*branch(bus,node,'Limit')/Sbase; //line limit

***** For Reserve market *****
Reserve.lo(ReserveGenNo)=0; //Ri>=0
*****
solve loadflow minimizing OF using LP;
*****
parameter report(bus,*);
report(bus,'Gen(MW)')= sum(GenNo$GBconnect(bus,GenNo),Pg.l(GenNo))*Sbase;
report(bus,'load(MW)')= BusData(bus,'pd');
report(bus,'LMP($/MWh)')=const2.m(bus)/Sbase;

***** For Reserve market *****
report(bus,'SR(MW)') = sum(ReserveGenNo$GBconnect(bus,ReserveGenNo), Reserve.l(ReserveGenNo));
report(bus,'SRmax(MW)') = sum(ReserveGenNo$GBconnect(bus,ReserveGenNo),GenData(ReserveGenNo,
'pmax')*SpinningReserveCapacityFraction/100);
report(bus,'SR+Gen(MW)')=
sum(GenNo$GBconnect(bus,GenNo),Pg.l(GenNo))*Sbase+sum(ReserveGenNo$GBconnect(bus,ReserveGenNo),
Reserve.l(ReserveGenNo));
report(bus,'Pmax(MW)') = sum(ReserveGenNo$GBconnect(bus,ReserveGenNo),GenData(ReserveGenNo, 'pmax'));
*****
display report,Pij.l,Pij.m, Reserve.l,Pg.l;

```



بررسی جامع یک مسئله‌ی بازار برق با استفاده از نرم‌افزارهای متلب و گمز

تیرماه ۱۴۰۲

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درس: بازار برق

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