5) Sea la señal gaussiana $x(t) = e^{-at^2}$

$x(t) = e^{-at^2}$ $a \in \mathbb{R}^+$

Sistema A $y_A(t) = x^2(t)$

Sistema B un SLIT con respuesta impulso $h_B(t) = Be^{-bt^2}$

a) Salida serie

$$x(t) \xrightarrow[h_B(t)]{} y(t) \xrightarrow{\text{cuadrado}} y_A(t)$$

1) $x(t) * h_B(t) \to y(t)$

2) $y_A(t) = y^2(t)$

convolucion de $x(t) * h_B(t)$

$$y(t) = x(t) * h_B(t) = \int_{-\infty}^{\infty} x(\tau)\, h_B(t-\tau)\, d\tau$$

$$x(\tau) = e^{-a\tau^2} \quad h_B(t-\tau) = Be^{-b(t-\tau)^2}$$

$$y(t) = \int_{-\infty}^{\infty} e^{-a\tau^2} \cdot Be^{-b(t-\tau)^2}$$

$$(t-\tau)^2 = t^2 - 2t\tau + \tau^2$$

$$y(t) = \int_{-\infty}^{\infty} e^{-a\tau^2} \cdot Be^{-b(t-\tau)^2} d\tau$$

$$y(t) = B\int_{-\infty}^{\infty} e^{-a\tau^2} \cdot Be^{-b(t-\tau)^2} d\tau$$

Sustituyendo

$$y(t) = B\int_{-\infty}^{\infty} e^{-a\tau^2} e^{-b(t^2 - 2t\tau + \tau^2)} d\tau$$

$$y(t) = Be^{-bt^2} \int_{-\infty}^{\infty} e^{-(a+b)\tau^2 + 2bt\tau} d\tau$$

$$y(t) = Be^{-bt^2} \int_{-\infty}^{\infty} e^{-(a+b)\left[\left(\tau - \frac{bt}{a+b}\right)^2 - \left(\frac{bt}{a+b}\right)^2\right]} d\tau$$

$$= Be^{-bt^2} e^{\frac{b^2t^2}{a+b}} \int_{-\infty}^{\infty} e^{-(a+b)(\tau-\mu)^2} d\tau$$

$$\mu = \frac{bt}{a+b}$$

la integral gaussiana

$$\int_{-\infty}^{\infty} e^{-k(\tau-\mu)^2} d\tau = \sqrt{\frac{\pi}{k}} \quad k = a+b$$

$$y(t) = B\sqrt{\frac{\pi}{a+b}}\; e^{-bt^2 + \frac{b^2t^2}{a+b}}$$

Simplificando

$$-bt^2 + \frac{b^2t^2}{a+b} = t^2\left(\frac{-b(a+b)+b^2}{a+b}\right) = \frac{-abt^2}{a+b}$$

$$y(t) = B\sqrt{\frac{\pi}{a+b}}\; e^{-\frac{abt^2}{a+b}}$$

Aplicar $y_A(t) = y^2(t)$

$$y_A = \left(B\sqrt{\frac{\pi}{a+b}}\; e^{\frac{-abt^2}{a+b}}\right)^2$$

$$\boxed{y(t) = B^2\frac{\pi}{a+b}\; e^{-2\frac{abt^2}{a+b}}}$$

→ Salida del sistema Serile

$$x(t) \rightarrow y_A(t) = x^2(t) \xrightarrow{h_B(t)} y(t)$$

→ Aplicar A directamente

$$y_A(t) = x^2(t) = (e^{-at^2})^2$$

$$y_A(t) = e^{-2at^2}$$

convolucion con $h_B(t) = Be^{-bt}$

$$y(t) = y_A(t) * h_B(t) = \int_{-\infty}^{\infty} e^{-2a\tau^2}\cdot Be^{-b(t-\tau)^2}\,d\tau$$

$$y(t) = B\int_{-\infty}^{\infty} e^{2a\tau^2}\cdot e^{-b(b-\tau)^2}\,d\tau$$

$$\boxed{y(t) = B\sqrt{\frac{\pi}{2a+b}} * e^{-2\frac{abt^2}{2a+b}}}$$

8) Demuestre las siguientes propiedades sin utilizar tablas de propiedades.

A) $\mathcal{L}\{x(t-t_0)\} = e^{-st_0}X(s)$ $\qquad u = t-t_0 \therefore t = u+t_0$

$$\mathcal{L}\{x(t-t_0)\} = \int_0^{\infty} x(t-t_0)e^{-st}\,dt$$

$t \to \infty = u \to \infty$

$t \to \infty = u \to -\infty$

$$\mathcal{L}\{x(t-t_0)\} = e^{-st_0}\int_{-\infty}^{\infty} x(u)e^{-su}\,du = e^{-st_0}X(s)$$

B) $\mathcal{L}\{x(at)\} = \frac{1}{|a|}X(s/a)$

$$\mathcal{L}\{x(at)\} = \int_{-\infty}^{\infty} x(at)e^{-st}\,dt \quad \text{si } a>0$$

$u = at \therefore t = u/a$ $\qquad t \to +\infty \quad u \to +\infty$

$t \to -\infty \quad u \to -\infty$

$$\frac{1}{a}\int_{-\infty}^{\infty} x(u)e^{-su/a} = \frac{1}{a}X(s/a)$$

$a<0$

$$(-1)\frac{1}{a}\int_{+\infty}^{-\infty} x(u)\,e^{-s\frac{u}{a}} = -\frac{1}{a}X(s/a)$$

$$\mathcal{L}\{x(at)\} = \frac{1}{|a|}X(s/a)$$

C) $\mathcal{L}\left\{\frac{dx(t)}{dt}\right\} = sX(s) = \int_{-\infty}^{\infty} x'(t)e^{-st}\,dt$

$u = e^{-st}$ $\qquad du = -se^{-st}$

$dv = x'(t)$ $\qquad v = x(t)$

$$= e^{-st}x(t)\Big|_0^{\infty} - \int_{-\infty}^{\infty} se^{-st}x(t)\,dt = 0 - x(0) + s\mathcal{L}\{x(t)\}$$

$$\mathcal{L}\left\{\frac{dx(t)}{dt}\right\} = sX(s) - x(0)$$

$$\mathcal{L}\left\{\frac{dx(t)}{dt}\right\} = sX(s)$$

D) $\mathcal{L}\{x(t) * y(t)\} = X(s)Y(s)$

$$\mathcal{L}\{x(t)*y(t)\} = \int_{-\infty}^{\infty} (x(t)*y(t))\,e^{-st}\,dt$$

$$\mathcal{L}\{x(t)*y(t)\} = \int_{-\infty}^{\infty} x\tau\, y(\tau - t)\,dt$$

$$\mathcal{L}\{x(t)*y(t)\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(\tau)\,y(\tau-t)\,d\tau\, e^{-st}dt$$

$$= \int_{-\infty}^{\infty} x(\tau)\int_{-\infty}^{\infty} y(\tau-t)e^{-st}\,dt\,d\tau$$

$u = \tau - t \quad du = -dt \quad t = \tau + u$

$$= \int_{-\infty}^{\infty} x(\tau)(+1)\int_{-\infty}^{\infty} y(u)e^{-s(\tau+u)}\,dt\,d\tau$$

$$= \int_{-\infty}^{\infty} x(\tau)e^{-s\tau}d\tau \int_{-\infty}^{\infty} y(u)e^{-su}du$$

$$= X(s) \cdot Y(s)$$

4) Encuentre la transformada de la place. dibuje el esquema de ceros y polos y la region de convergencia (ROC) de:

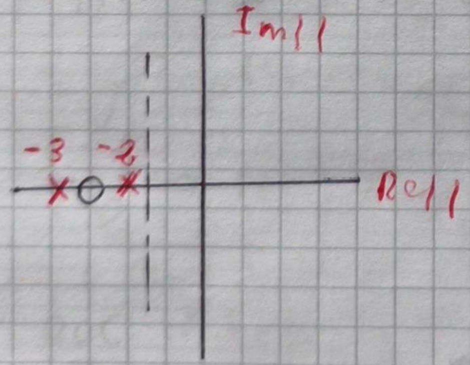
A) $e^{-2t}u(t) + e^{-3t}u(t)$

$$\mathcal{L}\{e^{-2t}u(t)\} + \mathcal{L}\{e^{-3t}u(t)\} = \int_{0}^{\infty} e^{-2t}e^{-st}dt + \int_{0}^{\infty} e^{-3t}e^{-st}dt$$

$$= \int_0^{\infty} e^{-(s+2)t}dt + \int_0^{\infty} e^{-(s+3)t}dt = -\frac{e^{-(s+2)t}}{s+2}\Big|_0^{\infty} - \frac{e^{-(s+3)t}}{s+3}\Big|_0^{\infty}$$

ROC $s > -2$ y $s > -3$ $\therefore$ $s > -3$

$$\frac{1}{s+2} + \frac{1}{s+3} = \frac{2s+5}{(s+2)(s+3)}$$ donde los polos igual a -2 y -3 cero $= -5/2$.



B) $e^{2t}u(t)+e^{-3t}u(-t)$

$\mathcal{L}\{e^{2t}u(t)\}+\mathcal{L}\{e^{-3t}u(-t)\}$

$=\int_0^\infty e^{2t}e^{-st}\,dt+\int_\infty^0 e^{3t}e^{-st}\,dt$

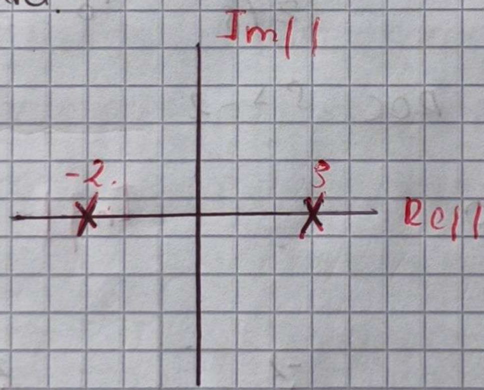
$=\int_0^\infty e^{-(s-2)t}\,dt+\int_{-\infty}^0 e^{-(3+s)t}\,dt$

$=-\dfrac{e^{-(s-2)t}}{(s-2)}\Big|_0^\infty-\dfrac{e^{(-3-s)}}{(3+s)}\Big|_{-\infty}^0$

ROC $\quad s>2 \qquad s<-3$

$=\cancel{\dfrac{1}{s+2}}\ \dfrac{1}{s-2}-\dfrac{1}{3+s}=\dfrac{5}{(s-2)(s+3)}$

2 y -3 no hay ceros si fuera posible su transformada.



C) $e^{-a|t|}$

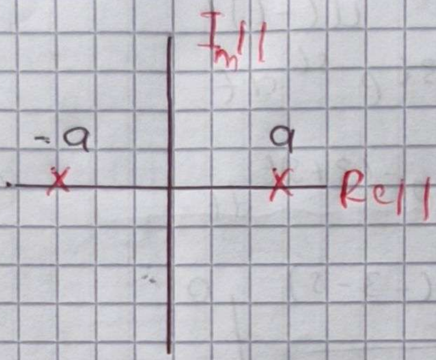
$\mathcal{L}\{e^{-a|t|}\}=\int_{-\infty}^0 e^{-a(-t)}e^{-st}\,dt+\int_0^\infty e^{-at}e^{st}\,dt$

$=\int_{-\infty}^0 e^{(a-s)t}\,dt+\int_0^\infty e^{-(a+s)t}\,dt$

$=\dfrac{e^{(a-s)t}}{a-s}\Big|_{-\infty}^0-\dfrac{e^{-(a+s)t}}{a+s}\Big|_0^\infty$

ROC: $s<a$ y $s>-a$ por lo tanto la region de convergencia. $\in(-a,a)$

$$= \frac{1}{a-s} + \frac{1}{a+s} = \frac{2a}{a^2-s^2}$$ po. lo tanto $-a$ y a ningun cero.



D) $e^{-2t}[u(t)-u(t-5)]$ $t \in [0,5]$

$$\mathcal{L}\{e^{-2t}u(t)-u(t-5)\} = \int_0^5 e^{-2t}e^{-st}\,dt$$

$$= \int_0^5 e^{-(2+s)t}\,dt = \frac{-e^{-(2+s)t}}{2+s}\Big|_0^5$$

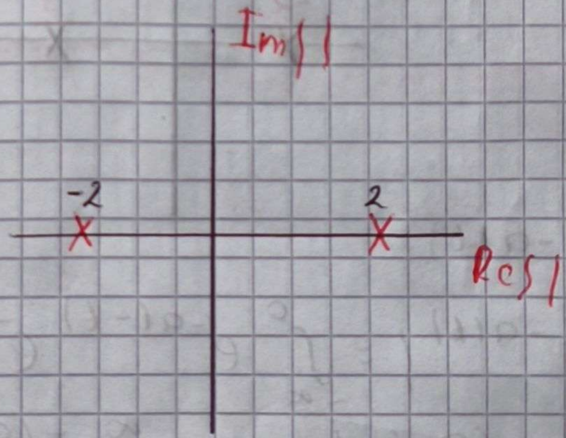
$$= \frac{1-e^{-(5s+10)}}{s+2}$$ ROC $s > -2$

$$1-e^{-(5s+10)} = 0$$

$$\ln(1) = -5s+10$$

$$0 = -5s+10$$

$$\underline{s = 2.}$$



12. Función de transferencia en lazo abierto para RLC serie y circuito RCL paralelo.

Serie.

$$i(t) = C\frac{dv_C}{dt} = C\frac{dv(t)}{dt}$$

$$i(t)R + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)\,dt = x(t)$$

$$RC\frac{dv(t)}{dt} + L\frac{d}{dt}\left(C\frac{dY(t)}{dt}\right) + \frac{1}{C}\int C\frac{dY(t)}{dt} = x(t)$$

$$x(t) = LC\frac{d^2Y(t)}{dt} + RC\frac{dY(t)}{dt} + Y(t)$$

$$X(s) = LC\left(s^2Y(s) - sY(0) - Y'(0)\right) + RC\left(sY(s) - Y(0)\right) + V(s)$$

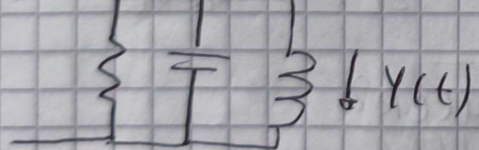
$$X(s) = LCs^2Y(s) - LsCY(0) - LCY(0) + RCsY(s) - RCY(0) + V(s)$$

$$X(s) = Y(s)\left(LCs^2 + RCs + 1\right) - Y(0)\left(LCs + RC\right) - LCY'(0)$$

$$Y(s) = \frac{X(s) + Y(0)\left(LCs^2 + RC\right) + LCY'(0)}{LCs^2 + RCs + 1}$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{X(s) + Y(0)\left(LCs + RC\right) + LCY'(0)}{X(s)\left(LCs^2 + RCs + 1\right)}$$

paralelo.



$$V_L = L\frac{di(t)}{dt} = L\frac{dY(t)}{dt}$$

$$x(t) = I_R(t) + I_C(t) + Y(t)$$

$$x(t) = \frac{V_L}{R} + C\frac{dV_L}{dt} + V(t)$$

$$R\,X(s) = L\left(sY(s) - Y(0)\right) + RCL\left(Y(s)s^2 - Y(0)s - Y'(0)\right) + Y(s) + R$$

$$R\,X(s) = Y(s)\left(RLCs^2 + Ls + R\right) - Y(0)\left(RCL + 1\right) - Y'(0)RLC$$

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$$Y(s) = \frac{R X(s) + V(0)(RLCs+1) + Y'(0) RLC}{RLCs^2 + Ls + R}$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{R X(s) + Y(0)(RLCs+L) + Y'(0) RLC}{X(s)(RLCs^2 + Ls + R)}$$

13. Encuentre la expresión de salida en tiempo para una configuración de laza cerrado del sistema en función RLC.

A) impulso

$x(t) = \delta(t) \quad \mathcal{L}\{\delta(t)\} = 1 = X(s)$

$X(0) = 1 \quad X'(0) = 1$

$$Y(s) = \frac{1}{LCs^2 + RCs + 1} + \frac{(LCs + RC)Y(0)}{LCs^2 + RCs + 1} + \frac{LCY'(0)}{LCs^2 + RCs + 1}$$

$$Y(s) = \frac{1}{LC} + \frac{1}{s^2 + \frac{R}{L}s + \frac{1}{LC}} + \frac{sV(0)}{s^2 + \frac{R}{L}s + \frac{L}{LC}} + \frac{RY(0)}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$+ \frac{V'(0)}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$V(s) = \frac{1}{LC} \times \frac{1}{\left(s + \frac{R}{2L}\right)^2 + \frac{4L - CR^2}{4L^2C}} + \frac{V_0\left(s - \frac{R}{L} + \frac{R}{2L}\right)}{\left(s + \frac{R}{2L}\right)^2 + \frac{4L - CR^2}{4L^2C}}$$

$$+ \frac{RY(0)}{\left(s + \frac{R}{2L}\right)^2 + \frac{4L - CR^2}{4LC}} + \frac{Y'(0)}{\left(s + \frac{R}{2L}\right)^2 + \frac{4LCR^2}{4L^2C}}$$

$$V(t) = \left(\frac{\frac{1}{LC} + RY(0) + Y'(0) - \frac{R}{2L}}{\sqrt{\frac{4L - CR^2}{4L^2C}}}\right) e^{-\frac{R}{2L}t} \sin\left(\sqrt{\frac{4L - CR^2}{4L^2C}}\, t\right)$$

$$+ e^{-\frac{R}{2L}t} \cos\left(\sqrt{\frac{4L-CR^2}{4L^2C}}\, t\right)$$

$$Y(s) = \frac{R + Y(0)RLCs + Y(0)L + Y'(0)RLC}{RLCs^2 + Ls + R}$$

$$Y(s) = \frac{(R + Y(0)L + Y'(0)RLC - Y(0)\, 1/RLC\, 2RC)}{RLC}$$

$$* \frac{1}{\left(s + \frac{1}{2RC}\right)^2 + \frac{4R^2C-1}{4R^2LC^2}} + Y(0) \frac{(s + 1/2RC)}{\left(s + \frac{1}{2RC}\right)^2 + \frac{4R^2C-1}{4R^2LC^2}}$$

$$Y(t) = \frac{2R + RLY(0) - LY(0)}{L\sqrt{4R^2C-L}}\, e^{-1/2RCt} * \sin\left(\sqrt{\frac{4R^2C-L}{4R^2LC^2}}\, t\right)$$

$$+ Y(0)\, e^{-1/2RCt} \cos\left(\sqrt{\frac{4R^2C-L}{4R^2LC^2}}\, t\right)$$

B Escalon unitario

$$\mathcal{L}\{u(t)\} = \frac{1}{s}$$

$$Y(s) = \frac{1/s + Y(0)(LCs + RC) + LCY'(0)}{LCs^2 + RCs + 1}$$

$$Y(s) = \frac{1 + Y(0)\, LCs^2 + RCs + LCsY'(0)}{s(LCs^2 + RCs + 1)}$$

$$Y(s) = \frac{1}{s(LCs^2 + RCs + 1)} + \frac{sY(0)LC}{(LCs^2 + RCs + 1)} + \frac{Y_0RC + LC + V(0)}{LCs^2 + RCs + 1}$$

$$\frac{1}{s(LCs^2 + RCs + 1)} = \frac{a}{s} + \frac{bs + d}{LCs^2 + RCs + 1}$$

$$a(LCs^2 + RCs + 1) + bs^2 + ds + 1$$

$$a = 1 \quad b = -L \quad d = RC$$

$$V(s)=\frac{1}{s}-\frac{LCs+RC}{LCs^2+RCs+1}+\frac{Y(0)LC}{LCs^2+RCs+1}+\frac{C\,LCY'(0)}{LCs^2+RCs+1}$$

$$Y(s)=\frac{1}{s}+(Y(0)-1)\frac{s+R/2L}{\left(s+\frac{R}{2L}\right)^2+\left(\frac{4L-CR}{4L^2C}\right)}+\Big(RC+LCY'(0)-$$

$$-\frac{RLC}{2L}+RLCY(0)\Big)$$

$$\left(\frac{1}{LCs^2+RCs+1}\right)$$

- con calculos anteriores.

$$V(t)=1+(Y(0)-1)e^{-\frac{Rt}{2L}}\cos\left(\sqrt{\frac{4L-CR^2}{4L^2C}}\,t\right)$$

$$+\left(\frac{RC+LCY'(0)-\frac{RC}{2}+\frac{RC}{2}Y(0)}{LC\sqrt{4L-CR^2/4L^2C}}\right)\left(e^{-\frac{R}{2L}t}\sin\sqrt{\frac{4L-CR^2}{4L^2C}}\right.$$

$$Y(t)=1+(Y(0)-1)e^{-\frac{RLt}{2}}\cos\left(\frac{\sqrt{4L-CR^2}}{4L^2C}\right)+\left(\frac{\sqrt{C}(R+2LY'(0)+}{}\right.$$

$$\left.\frac{R(0)\sqrt{4L-R^2C}}{4L-R^2C}\right)\left(e^{-\frac{R}{2L}t}\sin\left(\sqrt{\frac{4L-CR^2}{4L^2C}}\,t\right)\right)$$

- Ahora en el circuito en paralelo

$$V(s)=\frac{RLs+Y(0)RLCs+Y(0)L+Y'(0)LRC}{RLCs^2+Ls+R}$$

$$V(s)=\frac{Ra}{s^2(RLCs^2+Ls+R)}+\frac{Y(0)RCs}{RLCs^2+Ls+R}+\frac{LY(0)+Y(0)RLC}{RLCs^2+Ls+R}$$

- Fracciones parciales

$$\frac{Ra}{s^2(RLCs^2+Ls+R)}=\frac{bs+d}{s^2}+\frac{es+f}{RLs^2+Ls+R}$$

$$Ra=bRLCs^3+bs^2L+bRs+dRLCs^2+dLs+dR+es^3+Fs^3$$

$$Ra=dR \quad bR+dL=0 \quad d=a \quad b=-aR/R$$

$bL + dRLC + f = 0$ $\qquad$ $e + bRLC = 0$

$f = \frac{aL^2}{R} - aRLC$ $\qquad$ $e = aL^2C$

Ahora queda.

$$V(s) = \frac{-\frac{aLs}{R} + a}{s^2} + \frac{aL^2Cs + aL^2 - aRLC}{RLCs^2 + Ls + R} + \frac{Y(0)RCs}{RLCs^2 + Ls + R}$$

$$+ \frac{LY(0) + Y(0)RLC}{RLCs^2 + Ls + R}.$$

$$Y(t) = -\frac{aL}{R} + at + \left(\frac{Y(0)}{L} + \frac{aL}{R}\right) e^{-\frac{R}{2L}t} \cos\left(\sqrt{\frac{4L - CR^2}{4L^2C}}\, t\right)$$

$$+ \frac{2L\sqrt{C}\left(\frac{Y(0)}{RC} + Y(0) + \frac{aL}{R^2} - a + R\frac{Y(0)}{2L^2} + \frac{a}{2}\right)}{\sqrt{4L - CR^2}} * \ldots$$

$$\ldots * \, e^{-\frac{Rt}{2L}} \operatorname{sen}\left(\sqrt{\frac{4L - CR^2}{4L^2C}}\, t\right)$$