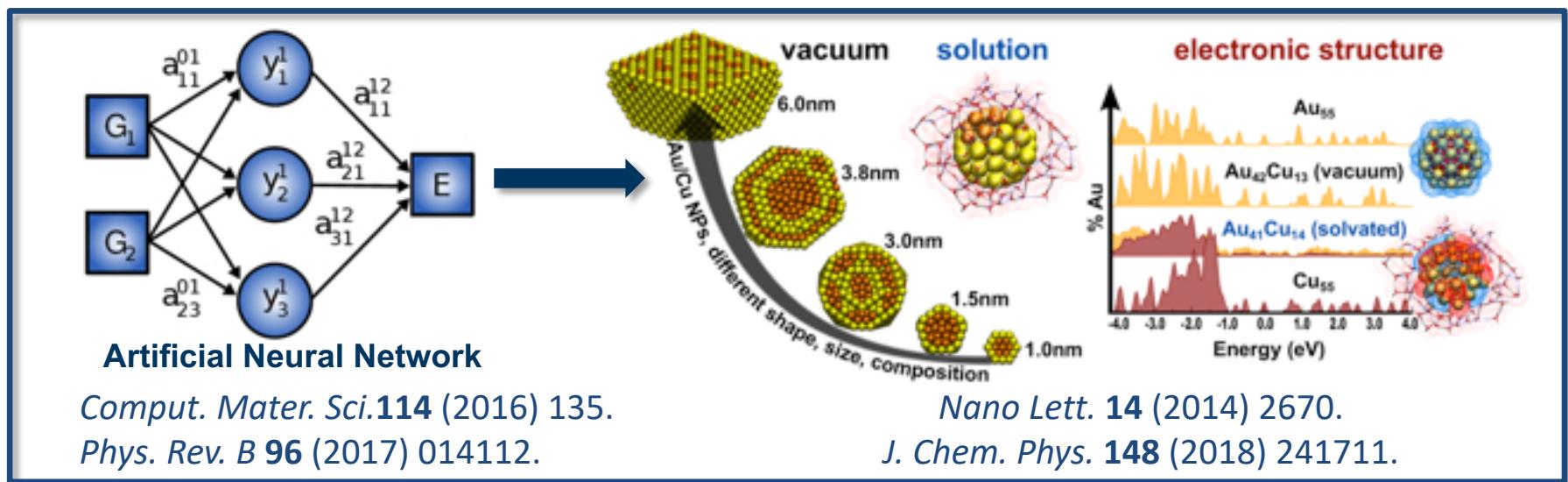


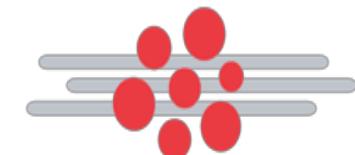
# Development of Efficient and Accurate MLPs for the Simulations of Complex Materials



## Nong Artrith

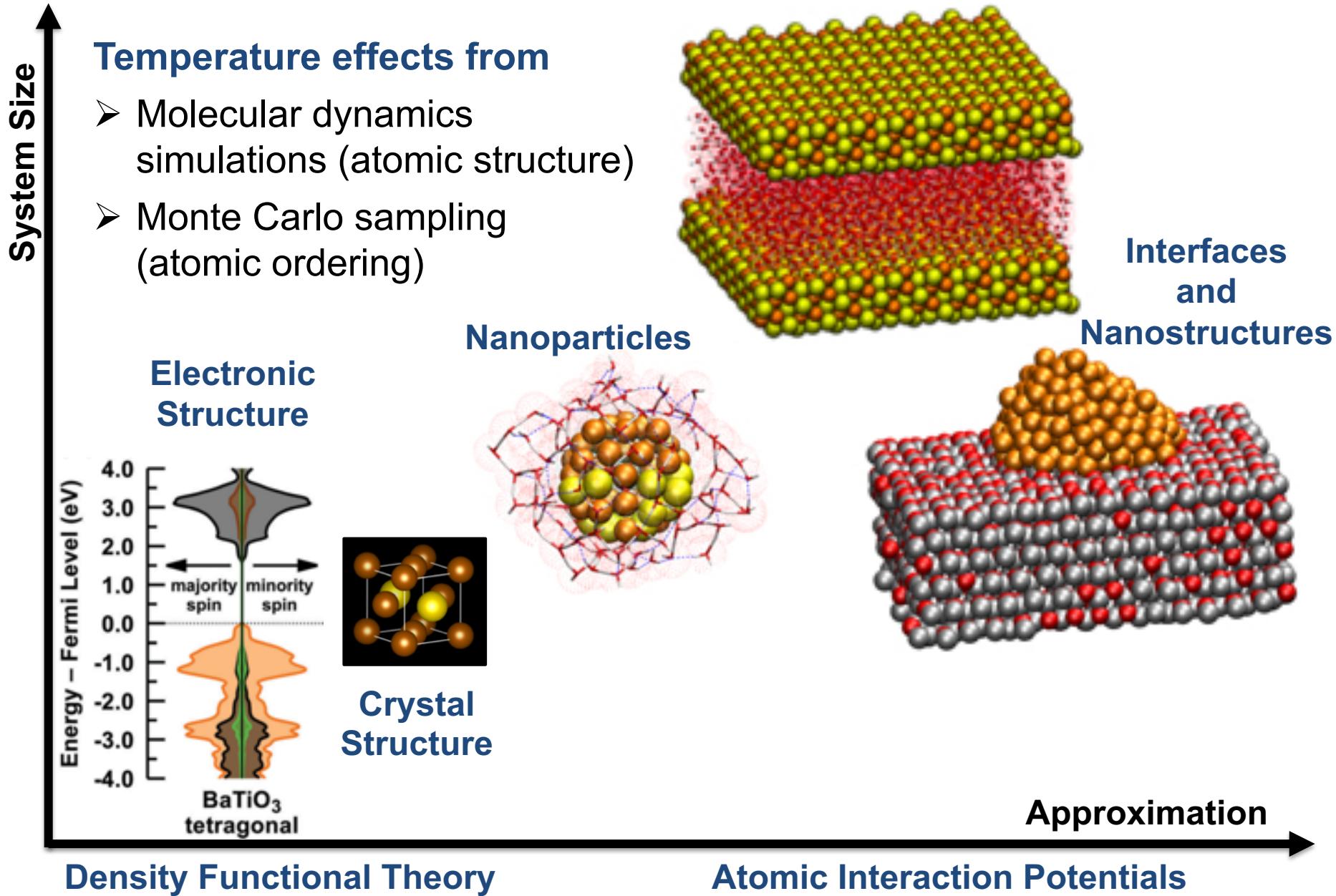
E-mail: nartrith@atomistic.net

Aalto, Finland, May 6, 2019



Center for Functional Nanomaterials  
Brookhaven National Laboratory

# Computational Methods and Length Scales



# Complex Systems: Challenging for Simulation

## Electrochemical Interfaces:

Necessary to describe

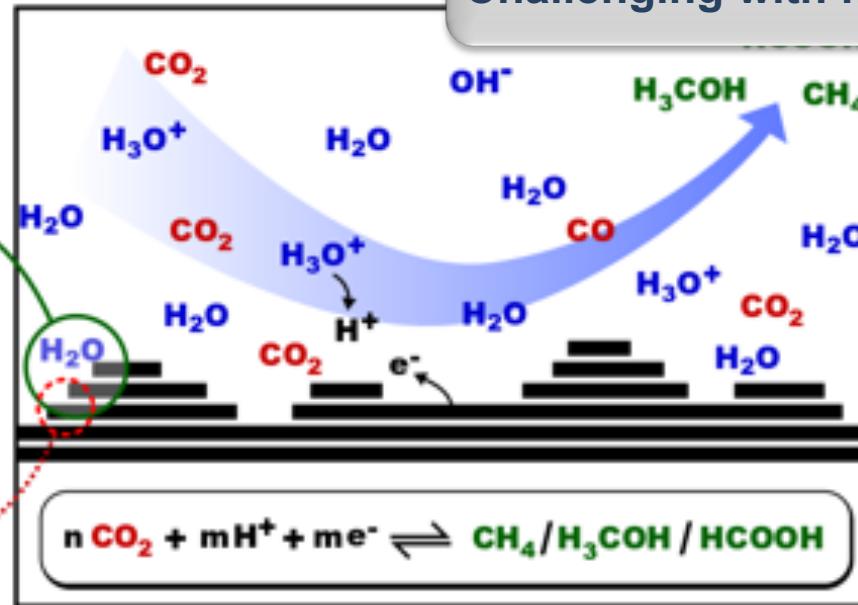
- ⇒ Breaking and making of bonds
- ⇒ Molecules in gas phase, liquids, solids, and interfaces
- ⇒ Non-ideal surfaces with various kinds of defects
- ⇒ Possibly solvent effects and effect of electric field

No Classical Molecular Potentials

Realistic length scales:  
Challenging with first principles

Solid-Liquid Interface

Complex Interfaces



# Potentials for Materials

- **Empirical Potentials:**

- + Efficient
- Simple functional form / usually not reactive
- Transferability problem

- ***Ab initio* (First Principle) MD Potentials:**

- + General + Predictive
- + Reactive  $\Rightarrow$  proton transfer (chemical reactions)
- No unique combination of, e.g., xc functional + vdW correction
- Computationally demanding

# Accelerating Simulations with Machine Learning

## State of atomistic simulations

- First-principles methods to compute accurate energies and atomic forces
  - accurate but computationally expensive
- Empirical atomic interaction potentials
  - computationally efficient but only reliable for specific applications

## Can machine learning help?

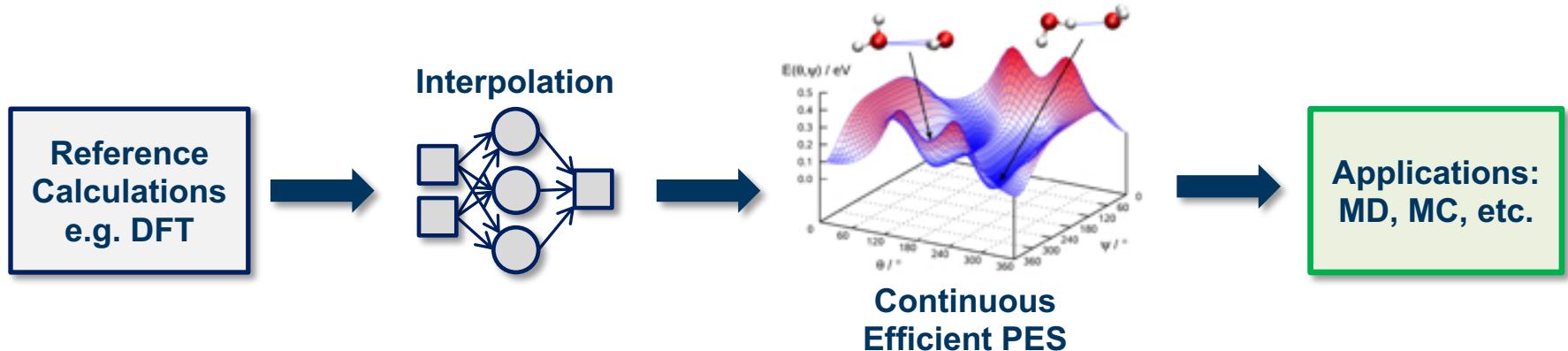
**Idea:** Train *efficient* machine-learning model to reproduce first-principles results

→ Need **descriptor** of atomic structure as input.

- Model for energy & forces: machine-learning potential
- All kind of structure-property relationships: classification, interpolation

# Potentials for Materials

- Potentials Based on *Ab Initio* Calculations:



⊕ Accurate + Efficient

Reactive → depends on approach

- Our Approach:

- Artificial neural networks for interpolation

⇒ Very accurate

- Not based on many-body expansion of the PES, no bonds/angles need to be specified

⇒ Reactive  
⇒ + Full-dimensional

# **Machine-Learning Potentials:**

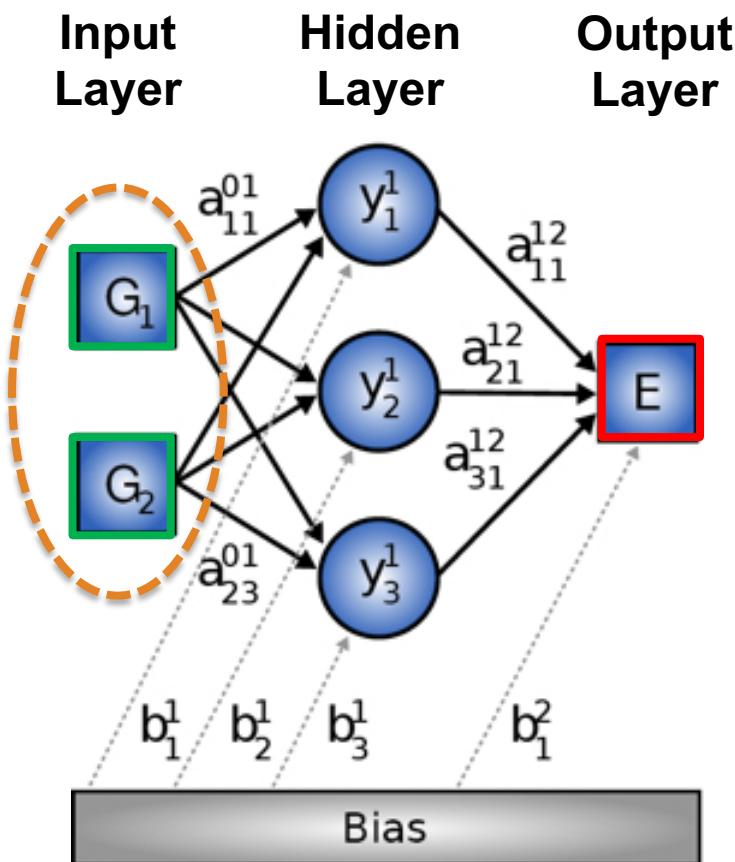
Direct application of ML to MC/MD simulations

# Artificial Neural Networks for Regression

- ANNs can approximate arbitrary continuous functions
- *Universal Approximation Theorem*  
G. Cybenko, *Math. Control Signals Syst.* **2** (1989) 303–314.  
K. Hornik, *Neural Netw.* **4** (1991) 251–257.  
B. Hanin (2017) *arXiv* 1708.02691.
- Ideal for the approximation of high-dimensional functions
- Our approach: Use ANNs to approximate the potential energy surface

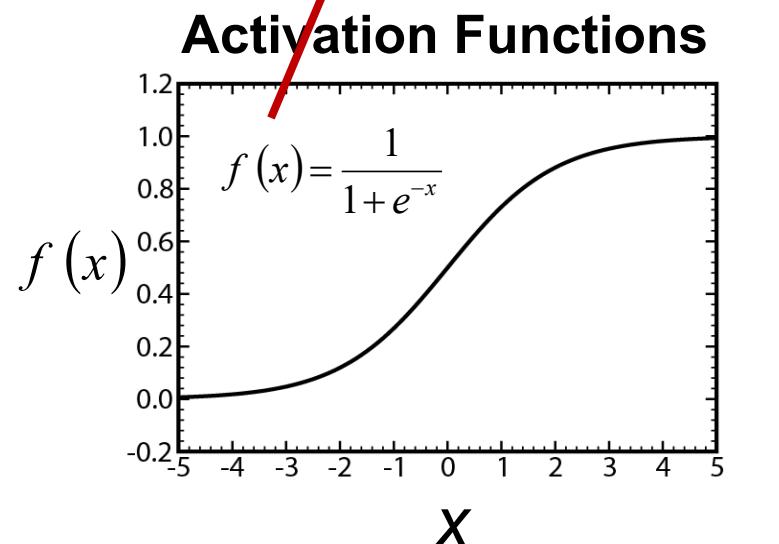
# Machine Learning for Atomic Structures

## Small Example for a Feed-Forward Neural Network



## Total Energy Expression

$$E = f_1^2 \left( b_1^2 + \sum_{j=1}^3 a_{j1}^{12} \cdot f_j^1 \left( b_j^1 + \sum_{i=1}^2 G_i \cdot a_{ij}^{01} \right) \right)$$



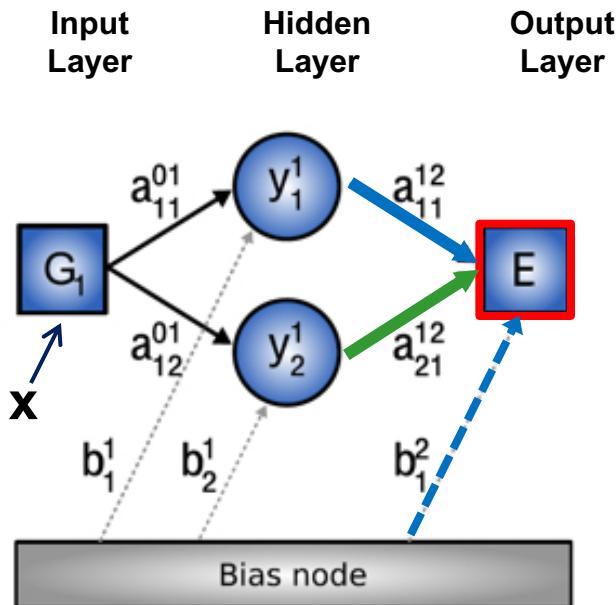
⇒ Can fit arbitrary functions

T.B. Blank, S.D. Brown, A.W. Calhoun, and D.J. Doren, *J. Chem. Phys.* 103 (1995) 4129.

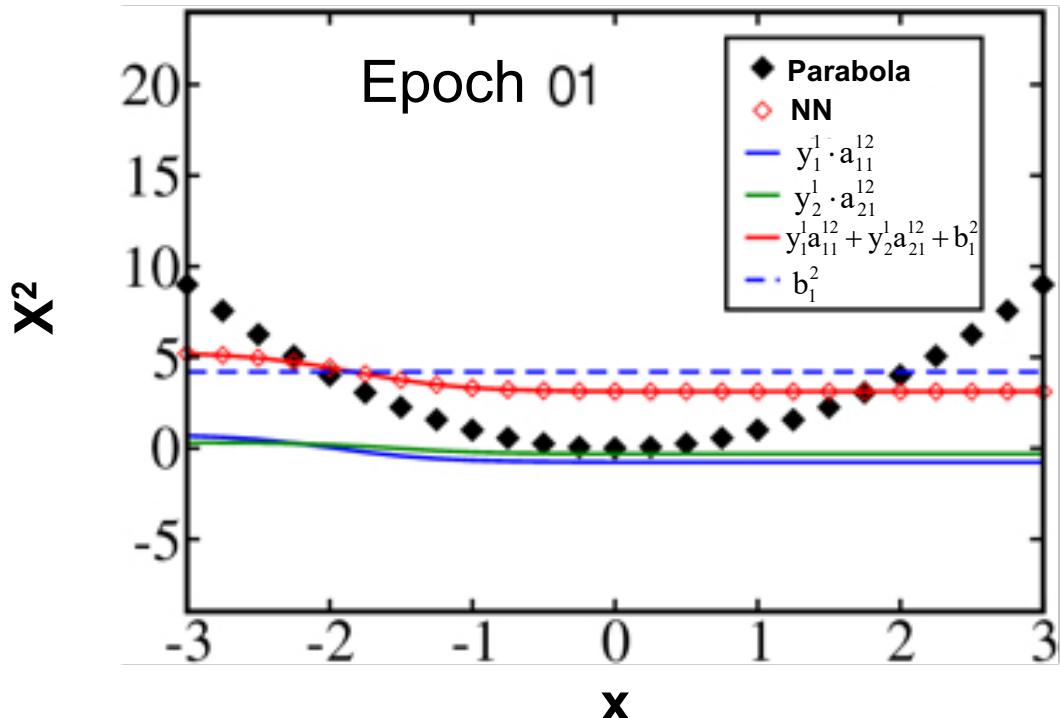
S. Lorenz, A. Groß, and M. Scheffler, *Chem. Phys. Lett.* 395 (2004) 210.

# Standard Neural Network

Training the Neural Network:



Parabola:  $x^2$



$$E_{NN} = a_{21}^{12} f^1(b_2^1 + G_1 \cdot a_{12}^{01}) + a_{11}^{12} f^1(b_1^1 + G_1 \cdot a_{11}^{01}) + b_1^2$$

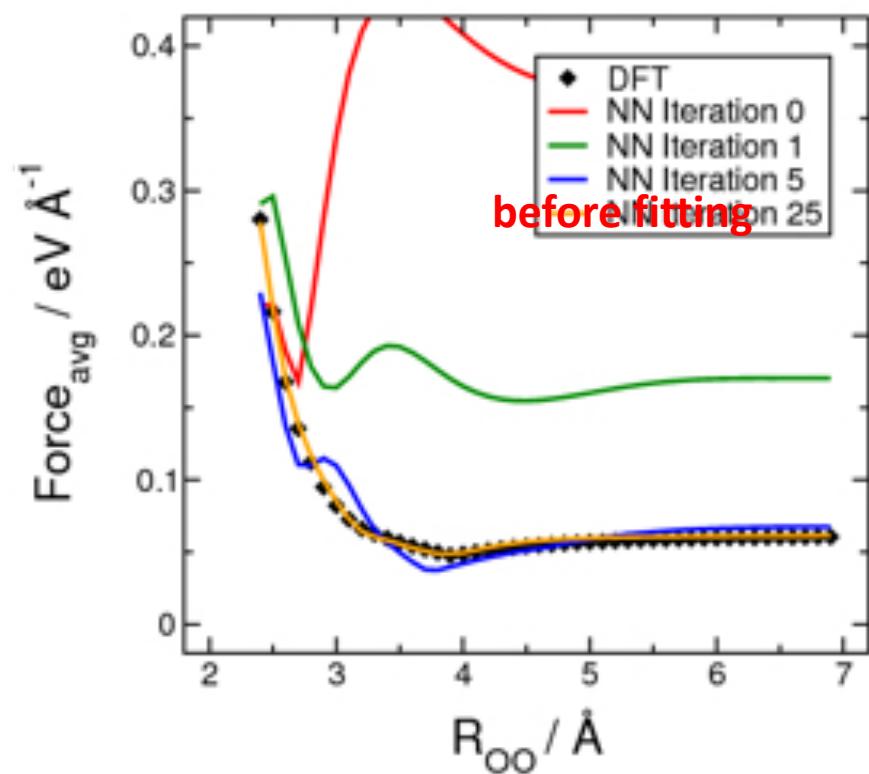
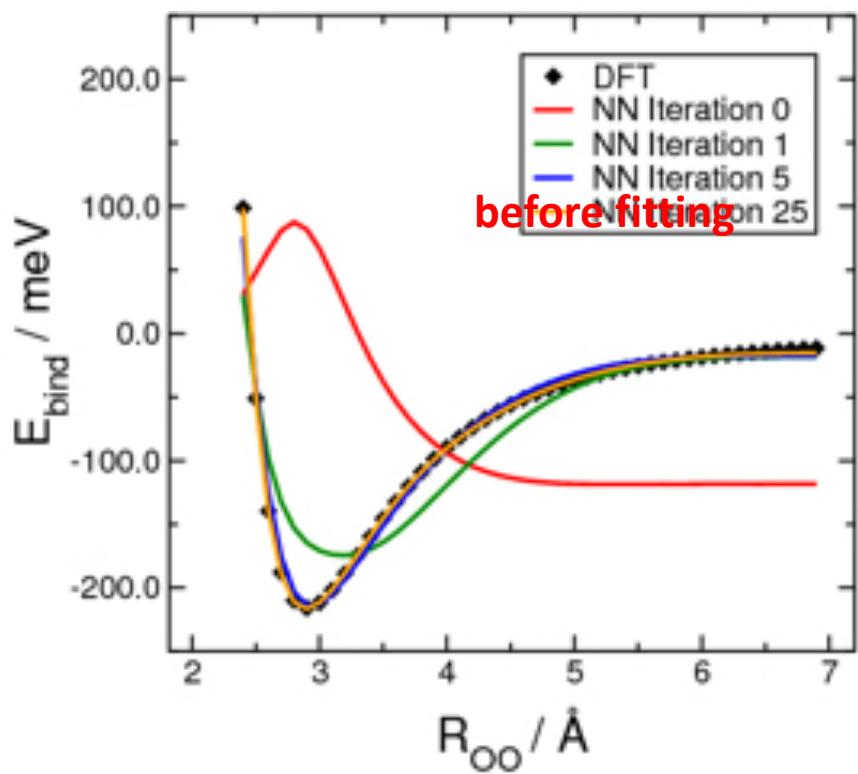
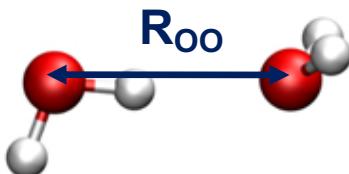
- The NN can learn the functional form of the PES

# Training the Neural Network

- Minimize error function for energy (and forces)

- Example:

1D cut of water dimer PES



⇒ The NN can learn the functional form of the PES

# Conventional ANNs are not Transferable

- ANNs have a fix input dimension
  - If the input is atomic coordinates, the ANN can only be used for one specific number of atoms
  - Hence, the ANNs are not transferable to atomic structures with different numbers of atoms
- NOT a replacement for interatomic potentials

# Conventional ANNs do not Exhibit Physical Invariants

- ANNs are not automatically invariant with respect to translation & rotation of the atomic structure
- ANNs are not invariant with respect to the exchange of two equivalent atoms

→ NOT a replacement for interatomic potentials

# The Behler-Parrinello Approach: Invariant ANN Potentials

1. The total energy is the sum of atomic energies

$$E_{\text{tot}} = \sum_i^{\text{atoms}} E_i$$

2. ANNs represent the atomic energies  $E_i$
3. The input for the ANNs are invariant representations of the local atomic environment

→ Suitable replacement for interatomic potentials

J. Behler, and M. Parrinello, *Phys. Rev. Lett.* 98, (2007) 146401.

# Universal Atomic Energy Function

Decomposition of the total structural energy into atomic contributions

$$E(\sigma) = \sum_i^{\text{atoms}} E_i(\sigma)$$

↑   ↑  
Total energy of                              Energy of the  $i$ -th atom  
structure  $\sigma$                                    in structure  $\sigma$

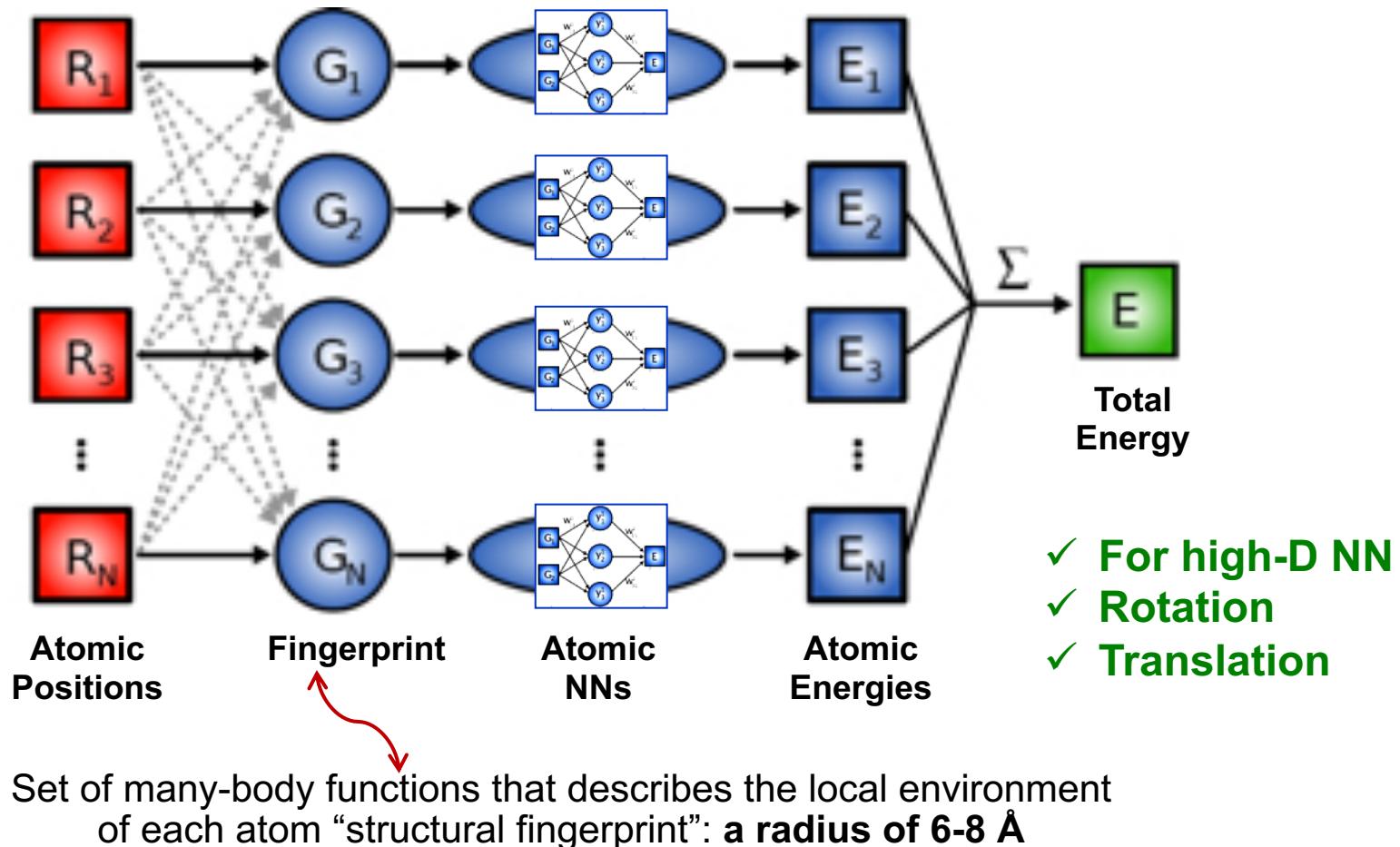
In embedded atom models (EAM) the atomic energy is

$$E_i(\sigma) = F_\alpha \left( \sum_{i \neq j} \rho_\beta(R_{ij}) \right) + \frac{1}{2} \sum_{i \neq j} \phi_{\alpha\beta}(R_{ij})$$

↑   ↑                                   ↑  
Embedding                                  Contribution to                      Pair potential  
function                                   charge density

- Physically motivated functional form, but not flexible. No dependence on bond angles. The model is **not appropriate for every structure/chemical species.**
- **Use machine learning to determine universal atomic energy function**

# High-Dimensional Neural Network



J. Behler, and M. Parrinello, *Phys. Rev. Lett.* 98, (2007) 146401.

J. Behler, R. Martoňák, D. Donadio, and M. Parrinello, *phys. stat. sol. (b)* 245, (2008) 2618.

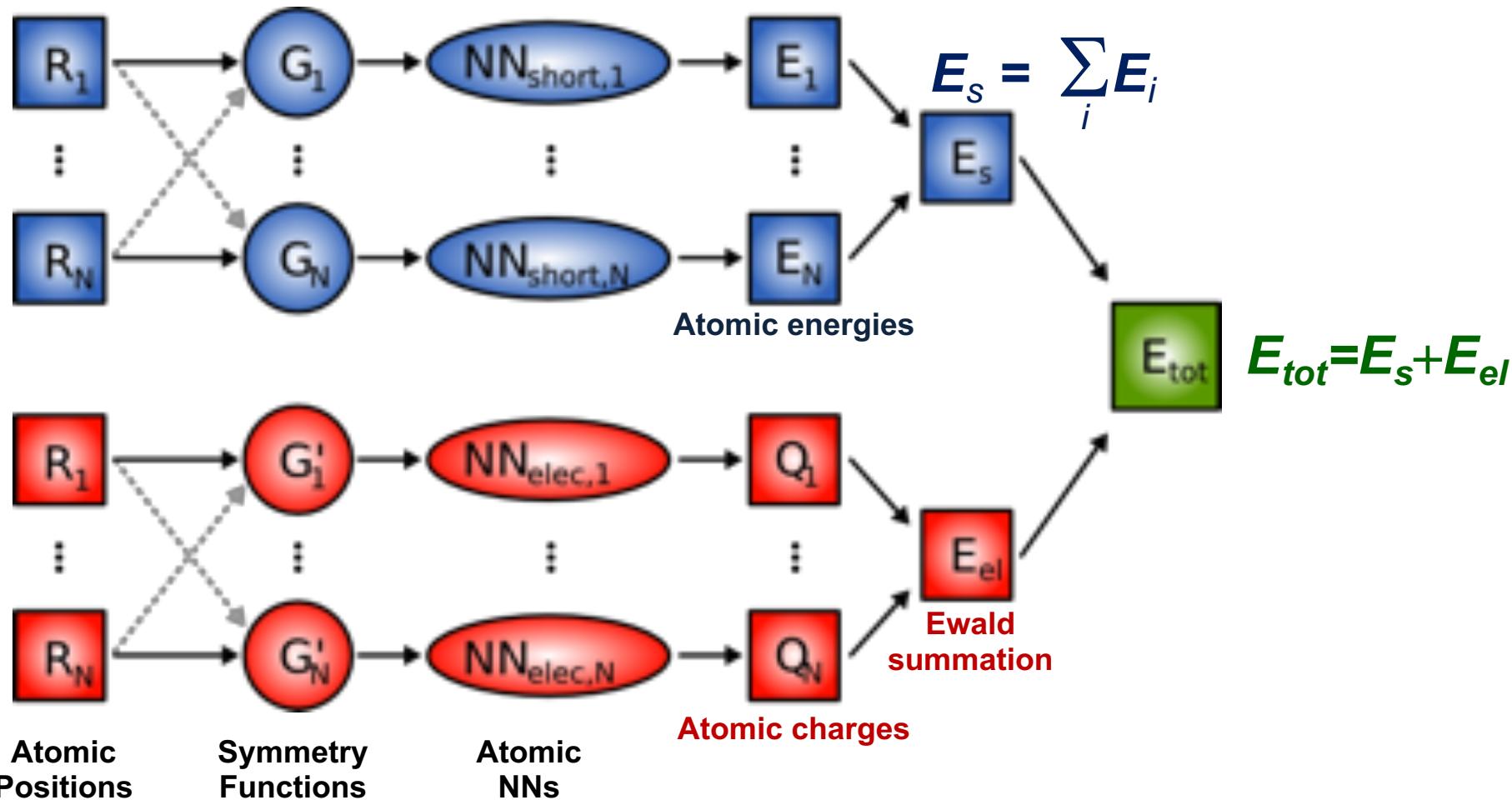
J. Behler, *J. Chem. Phys.* 134, (2011) 074106.

**N. Artrith**, T. Morawietz, and J. Behler, *Phys. Rev. B* 83, (2011) 153101.

# Short-Range and Long-Range Energy

- In systems with multiple chemical species, electrostatic interactions may become important
- Electrostatic interactions are **long-ranged**
- Cannot (strictly) decompose the electrostatic energy into atomic contributions
- However, the charge can be calculated
  - no unique way, but consistent recipes
- **Train a separate high-dimensional ANN for the atomic charges**

# Extension to Multicomponent Systems

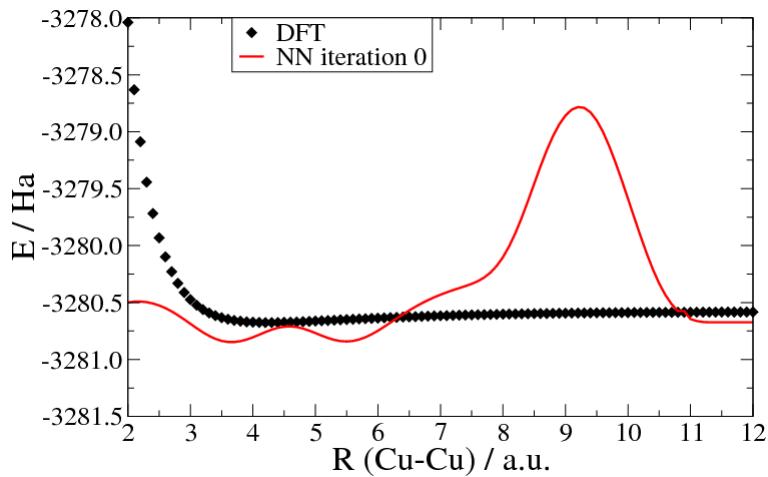


# Extension to Multicomponent Systems

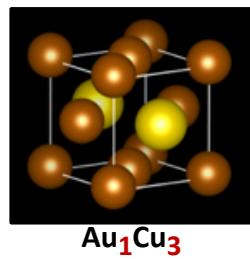
- In practice: electrostatic interactions are screened in condensed phases
- **Slightly longer-ranged potentials can often capture the effective electrostatic interactions (e.g., using 8 - 15 Å instead of 6 Å)**
- Confirmed for many different oxide materials

# Neural Network Fitting

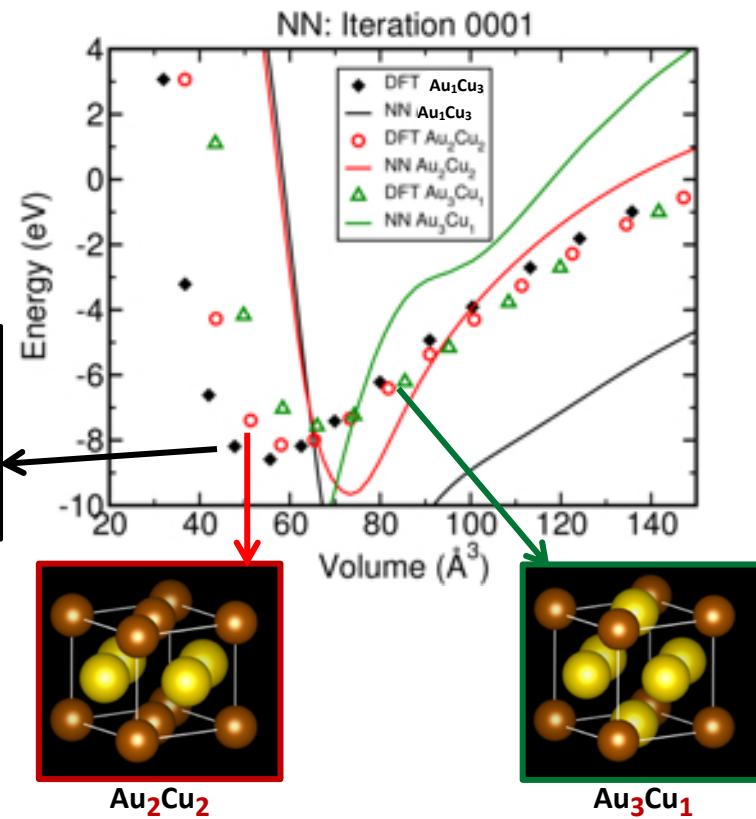
## Example: Copper Dimer



“Before fitting”  
Random parameters

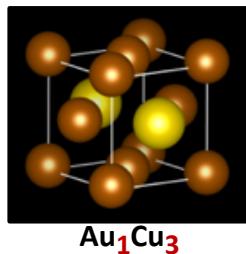
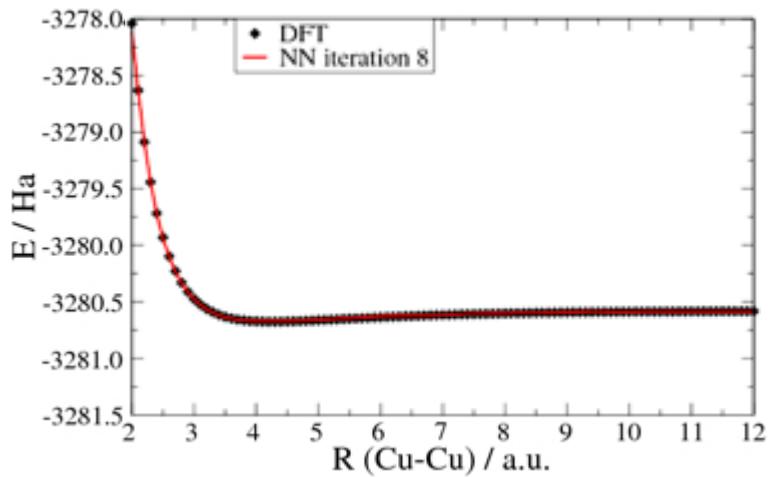


## High-Dimensional: $\text{Au}_x\text{Cu}_y$ Crystal Structures

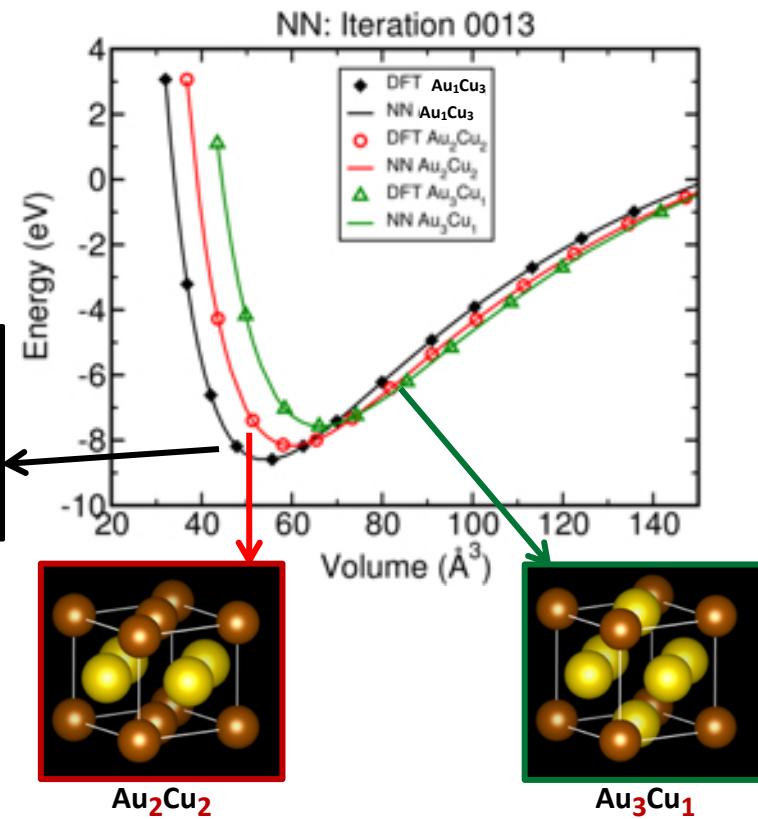


# Neural Network Fitting

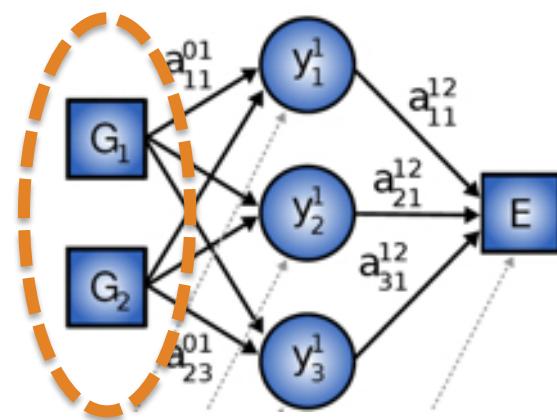
## Example: Copper Dimer



## High-Dimensional: $\text{Au}_x\text{Cu}_y$ Crystal Structures



# Descriptors of the Local Structural Environment



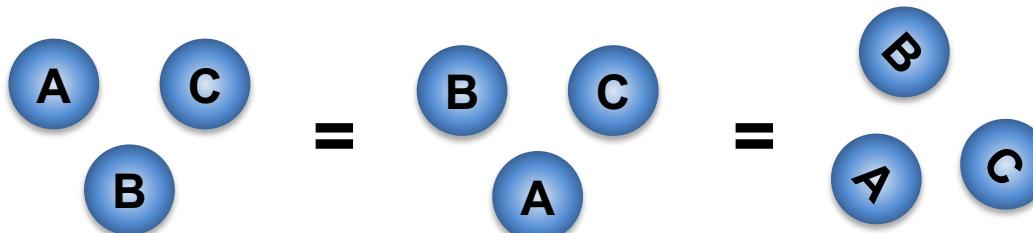
# Properties of the Atomic Energy

$$E(\sigma) = \sum_i^{\text{atoms}} E_i(\sigma)$$

Like the structural energy, the atomic energy is invariant with respect to

- Exchange of equivalent atoms (order of counting) and
- Translation/rotation of the entire structure.

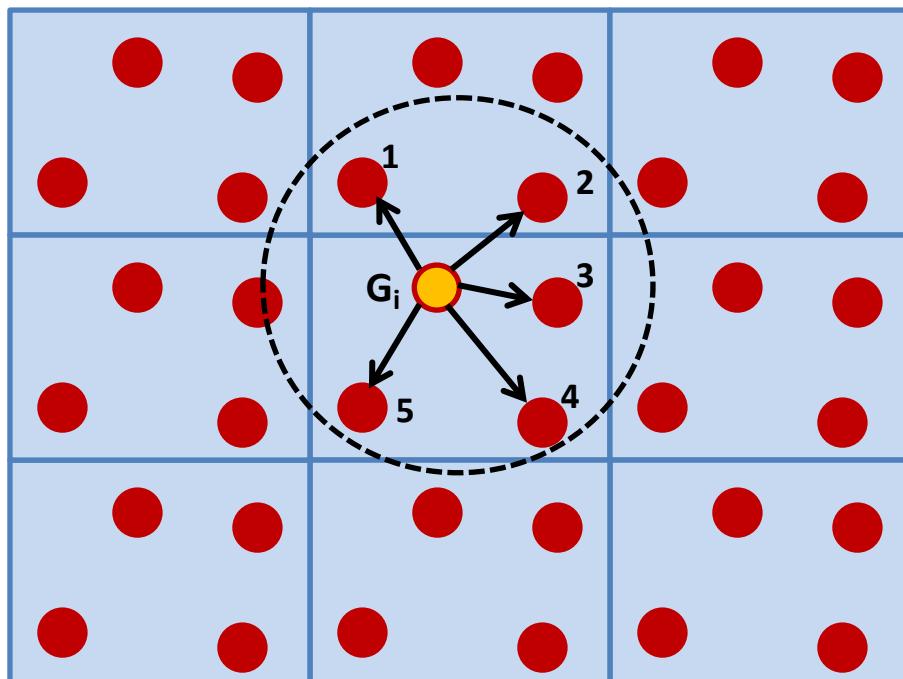
**Any (machine-learning) model must obey these invariants.**



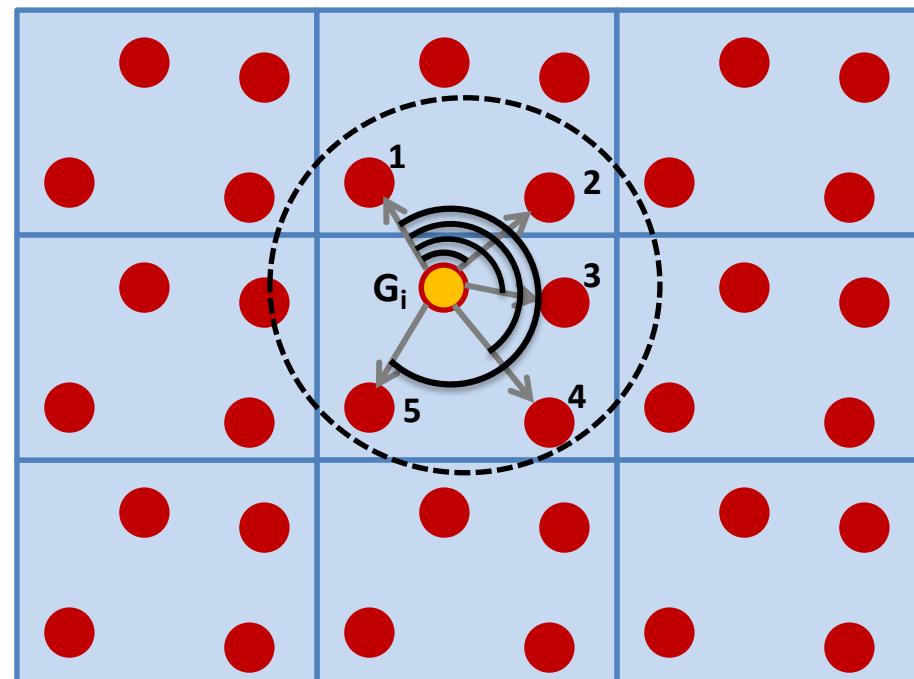
# Radial & Angular Distribution Functions

Approach: Use the radial and angular distribution of atoms and atom types as descriptor.

Atomic Radial Distribution Function



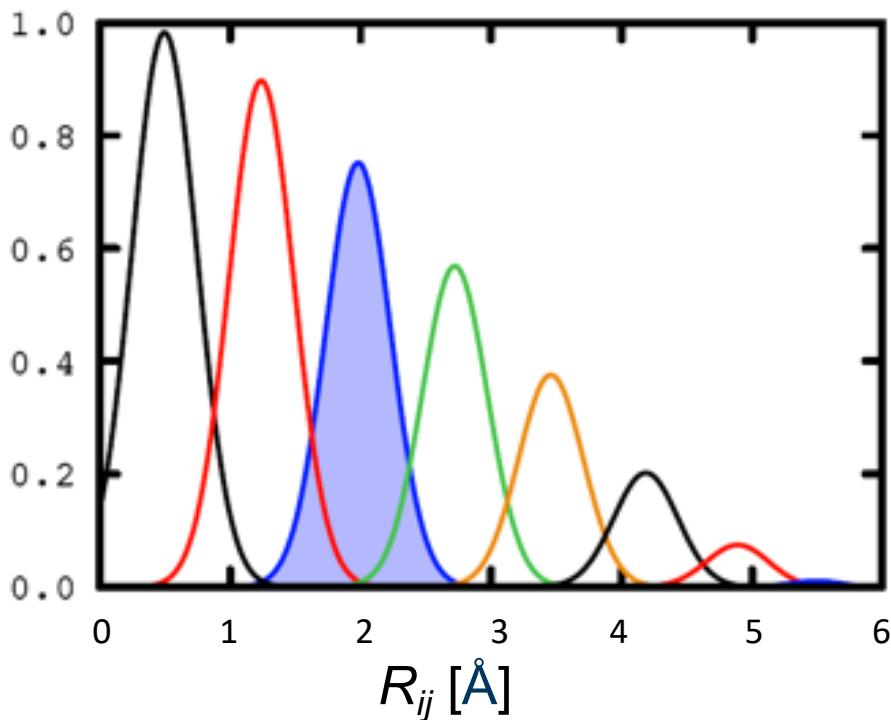
Atomic Angular Distribution Function



# Behler-Parrinello Symmetry Functions

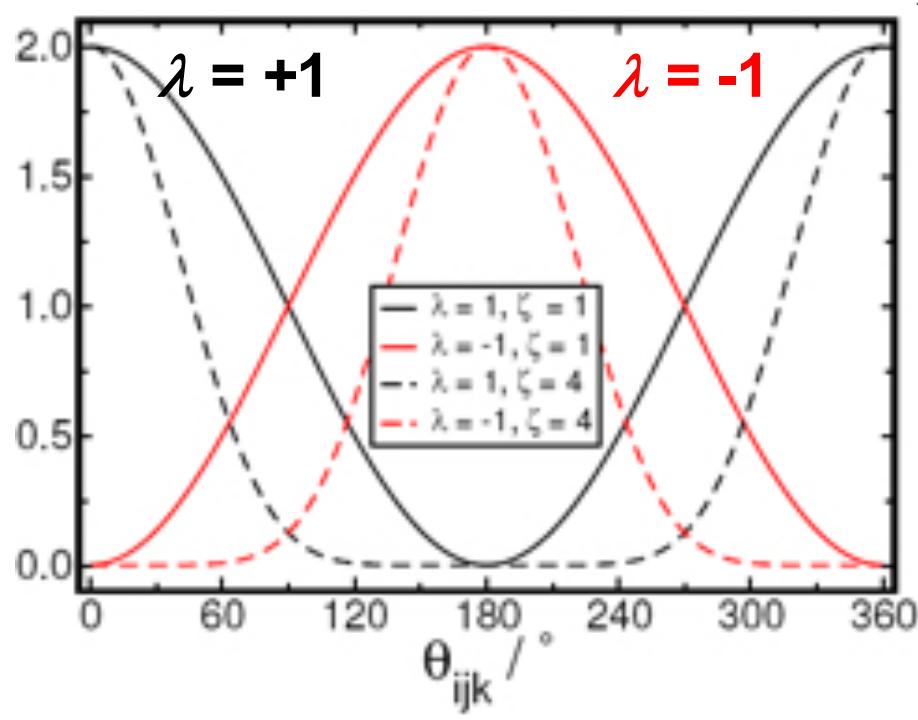
## Radial symmetry functions

$$G_i^2 = \sum_j e^{-\eta(R_{ij} - R_s)^2} \cdot f_c(R_{ij})$$



## Angular symmetry functions

$$G_i = 2^{1-\zeta} \sum_j \sum_k \left[ (1 + \lambda \cdot \cos \theta_{ijk})^\zeta \cdot e^{-\eta(R_{ij}^2 + R_{ik}^2 + R_{jk}^2)} \cdot f_c(R_{ij}) \cdot f_c(R_{ik}) \cdot f_c(R_{jk}) \right]$$



J. Behler, and M. Parrinello, *Phys. Rev. Lett.* 98, (2007) 146401. J. Behler, *J. Chem. Phys.* 134, (2011) 074106.  
N. Artrith, T. Morawietz, J. Behler, *Phys. Rev. B* 83, (2011) 153101.

# Challenge: Multicomponent Materials

**Behler- Parrinello (BP)** uses parameters for all possible combinations of species.

Example BP descriptor for 3 atomic species A, B, and C  
(potential for species A):

- Radial: A-A, A-B, A-C → factor of  $N$
  - Angular: A-A-A, A-A-B, A-A-C, A-B-B, A-B-C, A-C-C → factor of  $O(N^2)$
- **Descriptor size scales with  $N(N+1)/2$  where  $N$  is the number of species**

**Quadratic scaling!**

**Very challenging to construct MLPs with more than 4 atomic species using BP descriptor.**

# ML does not Require Complete Descriptors

BP functions are **not complete in the structural space** but are suitable for the construction of ANN potentials.

**Machine-learning techniques are useful when only incomplete descriptors are available!**

- The descriptor does not have to distinguish between all possible sets of 3-D coordinates. It is sufficient to distinguish between relevant atomic arrangements.
  - Similarly, not all chemical combinations occur in real materials. There is no need for a complete descriptor of the chemical space!
- **Construct a simple, incomplete yet refinable descriptor of the local atomic structure  $\{R\}$  and chemistry  $\{t\}$**

# Descriptor for Many Species: Structure $\{R\}$ and Chemistry $\{t\}$

N. Artrith\*, A. Urban, and G. Ceder, *Phys. Rev. B* **96** (2017) 014112.

# Descriptor to Describe Structure $\{R\}$

N. Artrith\*, A. Urban, and G. Ceder, *Phys. Rev. B* **96** (2017) 014112.

# Structure $\{R\}$ : Expansion of RDF and ADF

Expansion of radial (bond length) and angular (bond angle) distribution functions

**Pairs**       $\text{RDF}_i(r) = \sum_{j \neq i} \delta(r - R_{ij}) f_c(R_{ij})$

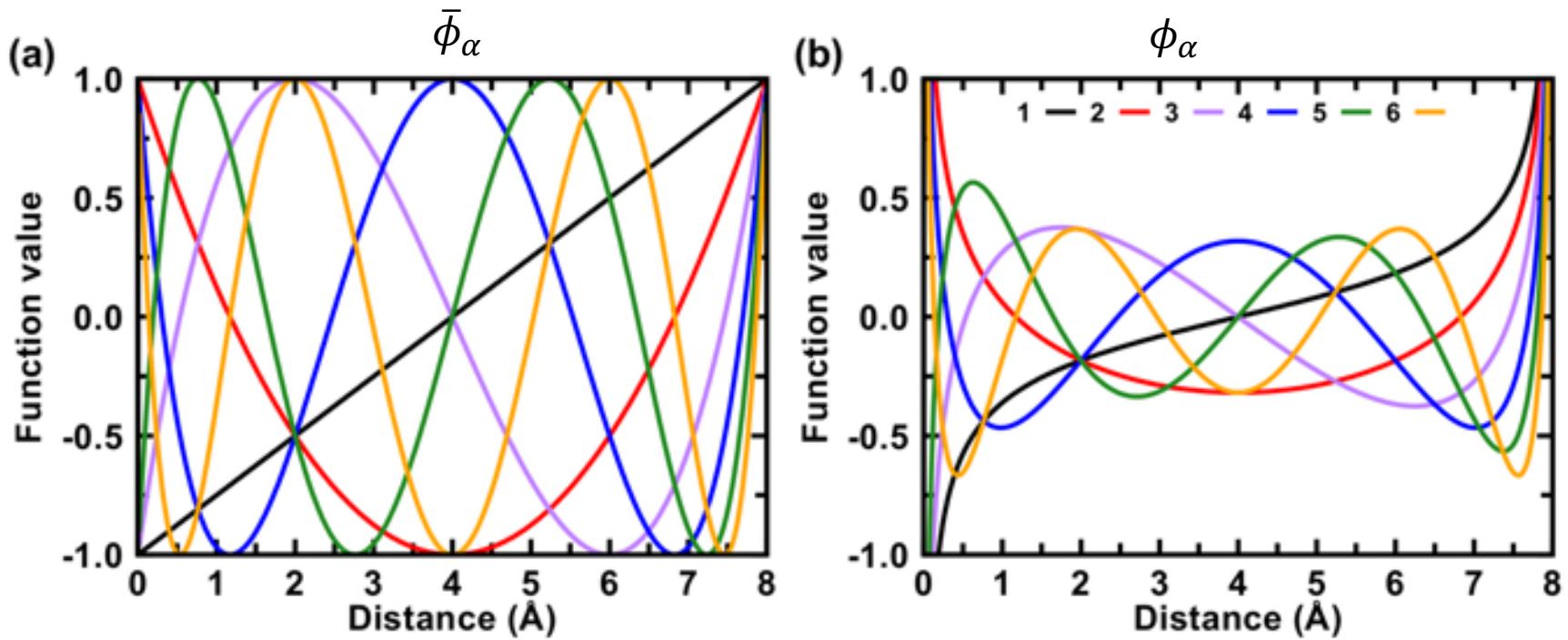
**Triplets**       $\text{ADF}_i(\theta) = \sum_{j,k \neq i} \delta(\theta - \theta_{ijk}) f_c(R_{ij}) f_c(R_{ik})$

in an orthonormal basis set  $\{\phi\}$  (**we use Chebyshev polynomials for their faster convergence properties compared to Fourier series**).

$$\text{RDF}_i(r) = \sum_{\alpha} c_{\alpha}^{(2)} \phi_{\alpha}(r) \text{ for } 0 \leq r \leq R_c \quad \text{ADF}_i(\theta) = \sum_{\alpha} c_{\alpha}^{(3)} \phi_{\alpha}(\theta) \text{ for } 0 \leq \theta \leq \pi$$

Both RDF and ADF are invariant wrt. rotation, translation, and exchange of equivalent atoms, so the coefficients  $\{c_{\alpha}\}$  can be used as descriptor.

# Visualization of the Chebyshev Polynomials

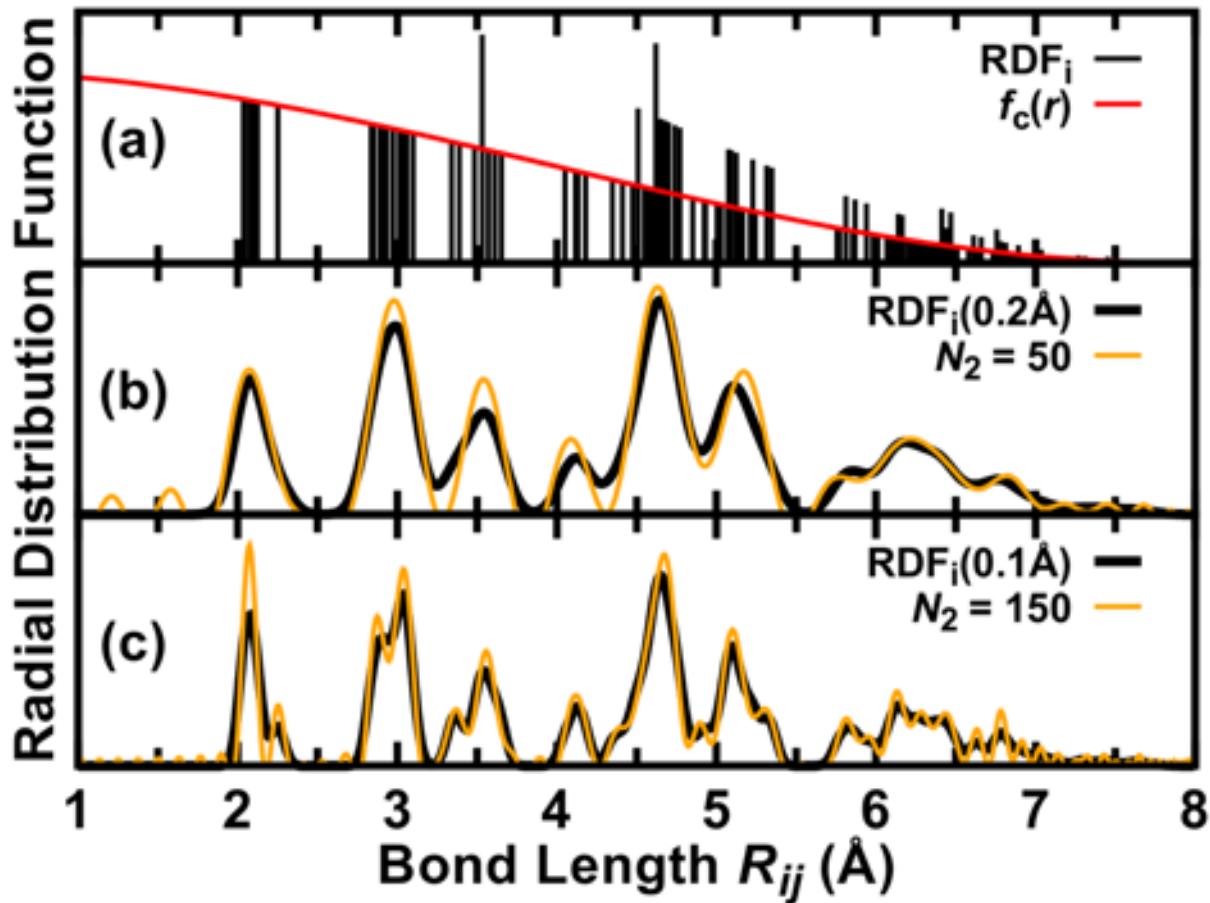


**(a)** Basis functions (i.e., rescaled Chebyshev polynomials) up to order  $\alpha = 6$  for a cutoff radius  $R_c = 8.0 \text{ \AA}$ . The polynomial of order  $\alpha = 0$  is constant 1 and not shown. **(b)** The corresponding basis functions are only needed for the reconstruction of the RDF or ADF.

# Descriptor is Systematically Refinable

The resolution of the descriptor is determined by the expansion order.

Example convergence for the radial distribution function ( $\text{Li}_2\text{MnNiO}_4$ ):



Discrete RDF multiplied with cutoff function.

RDF broadened by convolution with Gaussians, (b): 0.2 Å, (c): 0.1 Å, compared to Chebyshev expansions of orders (b):50 and (c):150.

# Descriptor for Many Species: Chemistry $\{t\}$

N. Artrith\*, A. Urban, and G. Ceder, *Phys. Rev. B* **96** (2017) 014112.

# Chemistry $\{t\}$ : Second Set of Coefficients

To describe the local chemistry, we include a species-dependent weight:

**Pairs**       $\{t\}\text{RDF}_i(r) = \sum_{j \neq i} \delta(r - R_{ij}) f_c(R_{ij}) \textcolor{red}{w}_{t_j}$

**Triplets**       $\{t\}\text{ADF}_i(\theta) = \sum_{j,k \neq i} \delta(\theta - \theta_{ijk}) f_c(R_{ij}) f_c(R_{ik}) \textcolor{red}{w}_{t_j} \textcolor{red}{w}_{t_k}$

So that the expansion coefficients become

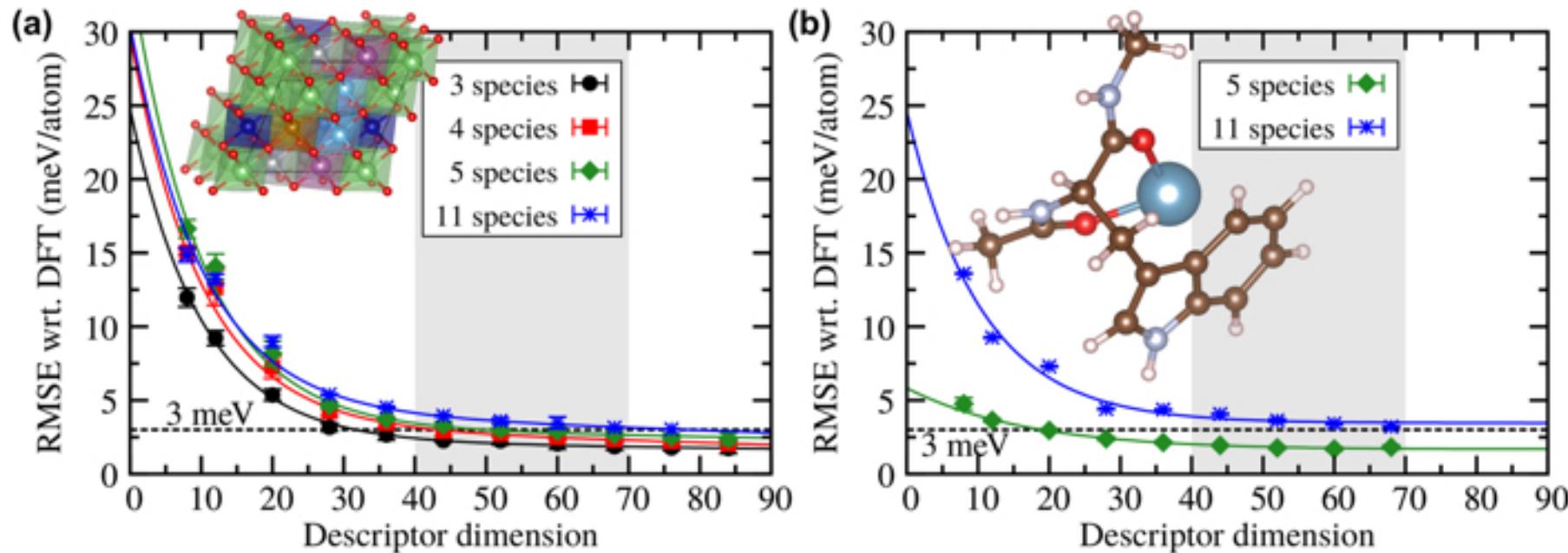
$$\{t\} c_\alpha^{(2)} = \sum_{j \neq i} \bar{\phi}_\alpha(R_{ij}) f_c(R_{ij}) \textcolor{red}{w}_{t_j} \quad \{t\} c_\alpha^{(3)} = \sum_{j,k \neq i} \bar{\phi}_\alpha(\theta_{ijk}) f_c(R_{ij}) f_c(R_{ik}) \textcolor{red}{w}_{t_j} \textcolor{red}{w}_{t_k}$$

The descriptor of the local chemistry is then

$$\{t\} \hat{\sigma}_i^{R_c} = \begin{pmatrix} \{t\} c_0^{(2)} \\ \{t\} c_1^{(2)} \\ \vdots \\ \{t\} c_0^{(3)} \\ \{t\} c_1^{(3)} \\ \vdots \end{pmatrix}$$

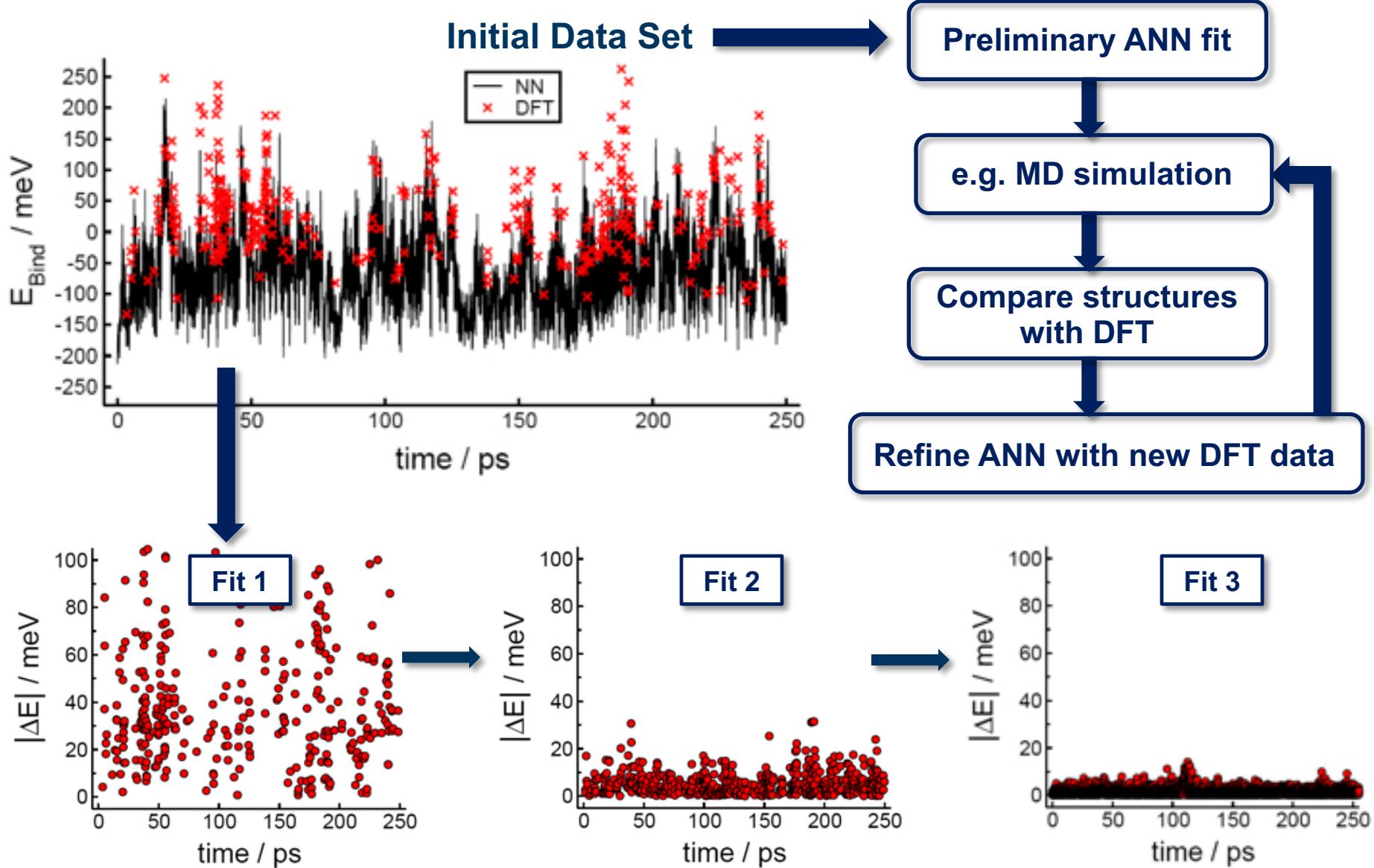
# The Same Descriptor Size is Optimal for 3-11 Species

The combined descriptor is appropriate for Li-TM oxides and amino acid complexes with **11 chemical species**. The size of the descriptor is constant.



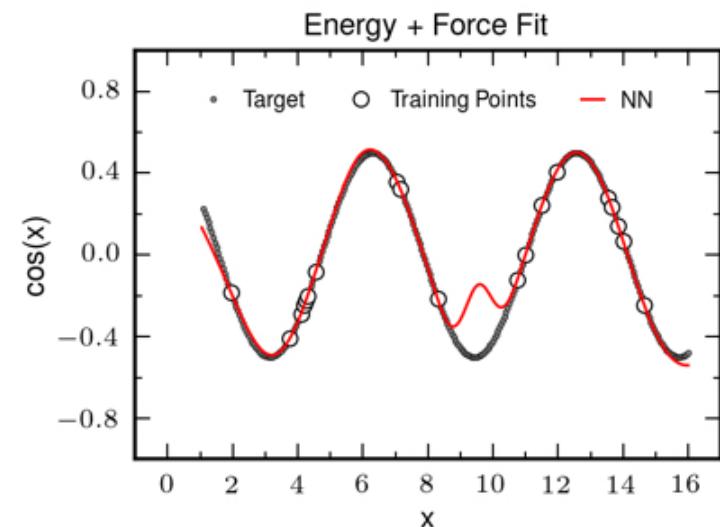
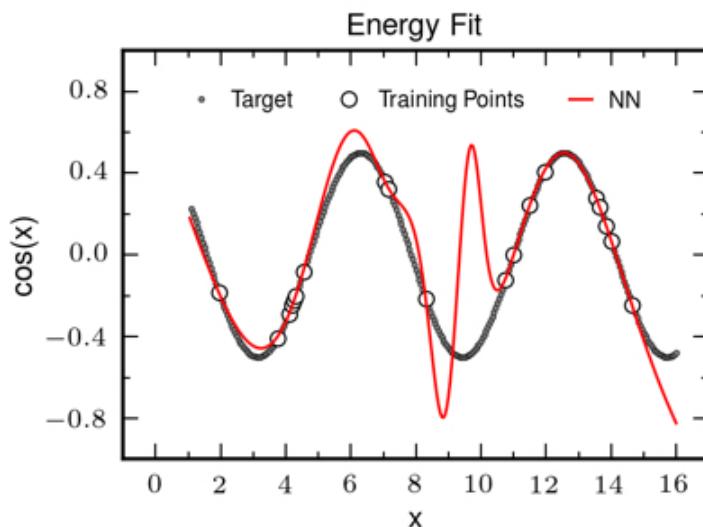
The RMSE was evaluated after 3000 training iterations for 3-5 species and after 5000 iterations for 11 species.

# Systematic Construction of the Training Set



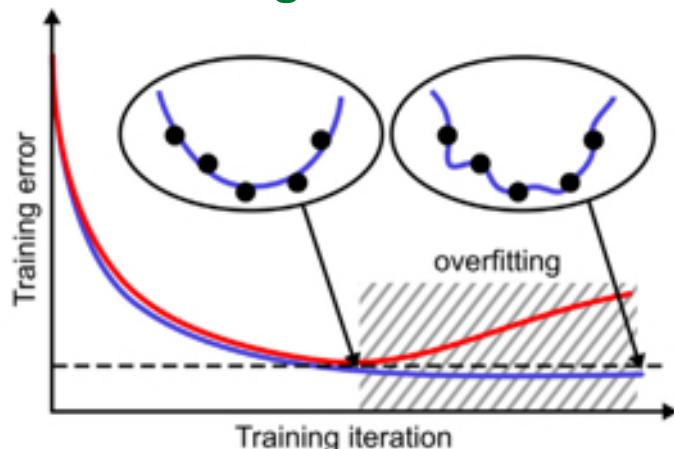
# Caution: Overfitting and Extrapolation

## Overfitting:

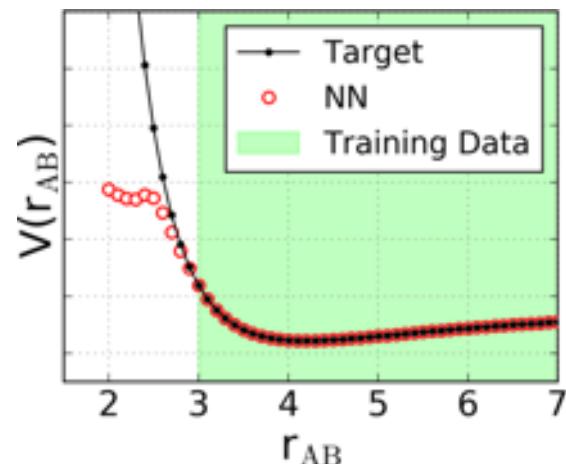


## Solutions:

- \* Early stopping
- \* Use gradient info



## Extrapolation:



## Solutions:

- \* Carefully sample repulsive regions
- \* Output “extrapolation warnings” during MD

# Summary of Part I – Theory of ANN Potentials

- ANN potentials are interatomic potentials based on artificial neural networks
- The ANNs represent atomic energies as function of the local atomic environment
- Input of the ANNs are invariant descriptors (feature vectors) of the local atomic environment
- Construction/training is done by iterative sampling of the relevant structure and composition space

# **Part II**

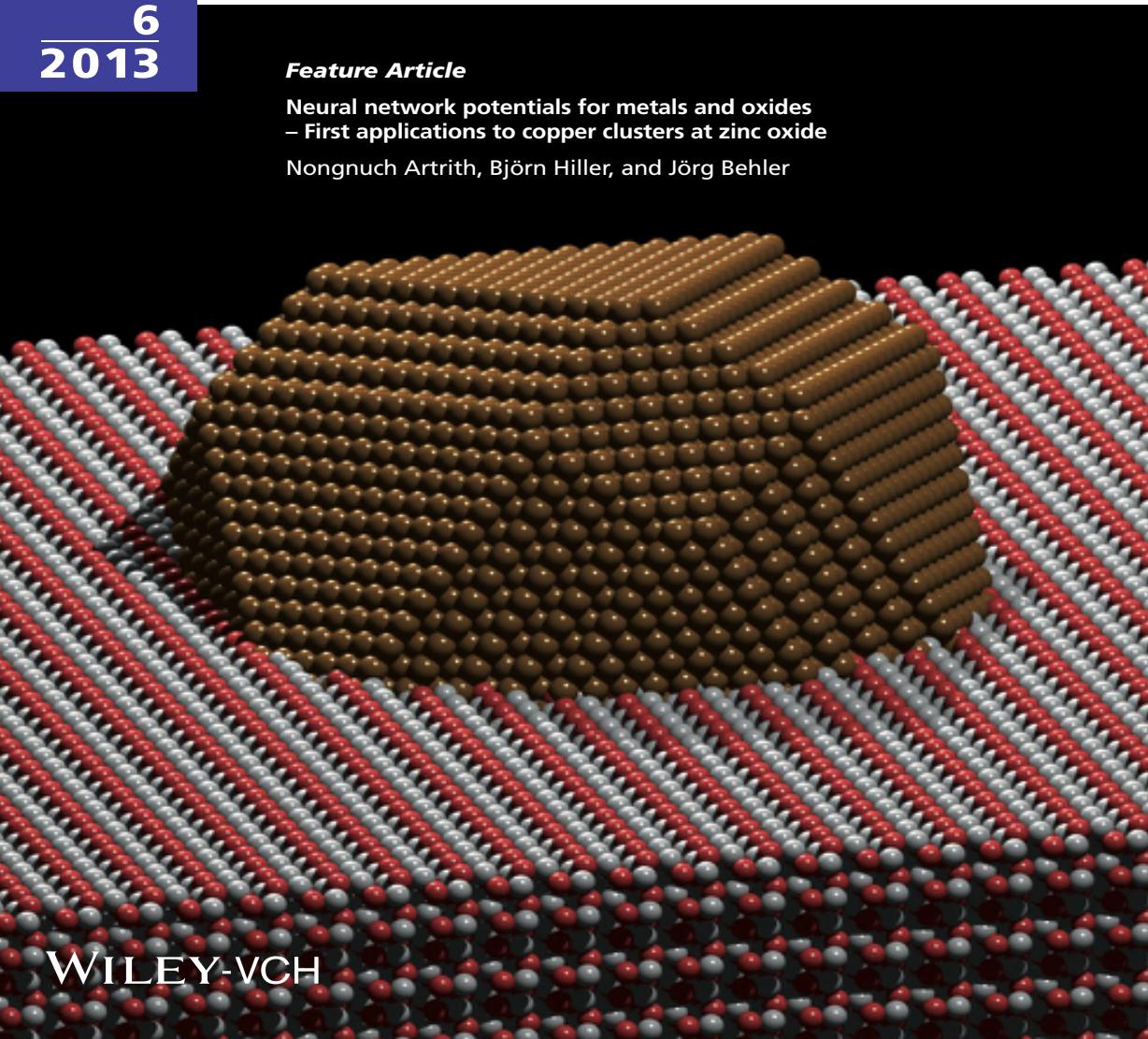
# **Complex Inorganic Materials**

# **for Energy Applications**

**Feature Article**

**Neural network potentials for metals and oxides**  
– First applications to copper clusters at zinc oxide

Nongnuch Artrith, Björn Hiller, and Jörg Behler



**N. Artrith, B. Hiller, J. Behler**  
*Phys. Stat. Sol. B* **250** (2013)  
1191 (invited feature article).

**Journal Cover**

# Neural Network Potential for Copper

## Training of the ANN potential

DFT code: FHI-aims (PBE) [a]

### Cu Structures:

- Bulk: **15,400**
- Clusters : **8,400**
- Surfaces : **13,800**

RuNNer code [b]

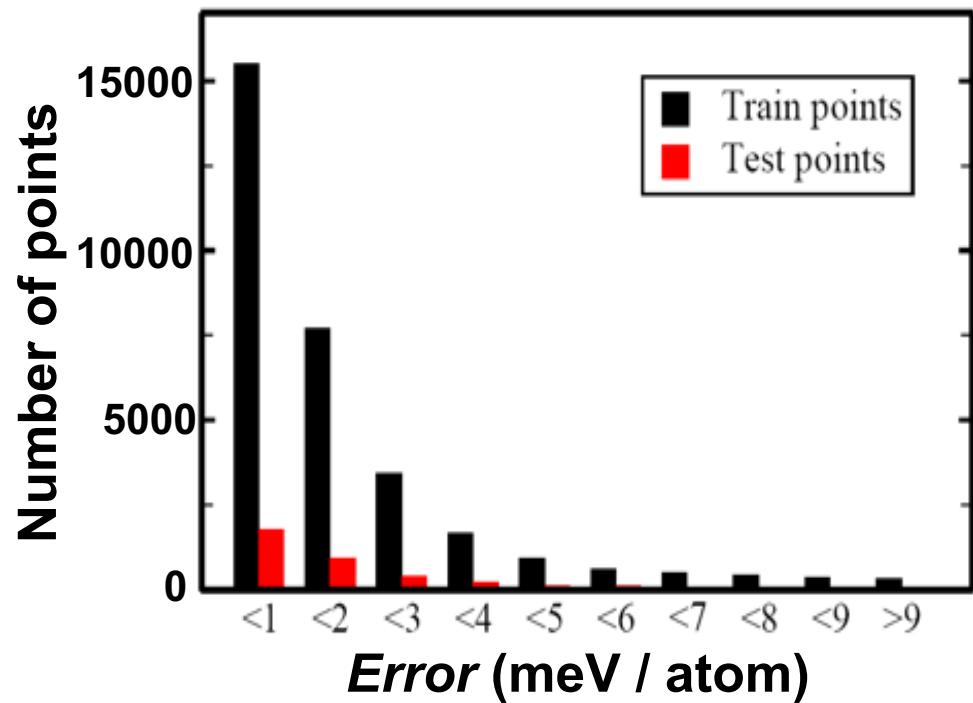
Fitting points: **32,000**

RMSEs     $E_{\text{total}}$     = **0.0036** eV/atom  
             Forces    = **0.0415** eV/Bohr

Testing points: **3,600**

RMSEs     $E_{\text{total}}$     = **0.0034** eV/atom  
             Forces    = **0.0416** eV/Bohr

Training data: 2 - 100 atoms



[a] V. Blum et al., *Comp. Phys. Comm.* 180, (2009) 2175 - 2196.

[b] J. Behler, RuNNer – A Neural Network Code for High-Dimensional PESs, Ruhr-University Bochum

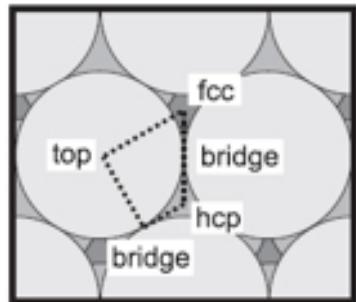
# Low-Index Copper Surfaces

## Surface Energies (fcc):

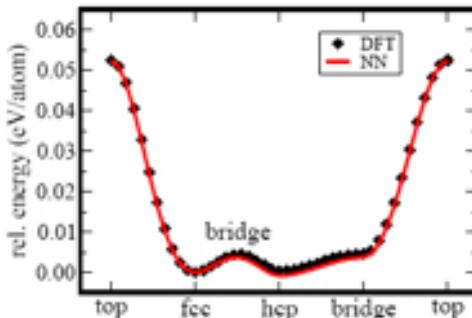
Surface	DFT (meV/Å <sup>2</sup> )	NN (meV/Å <sup>2</sup> )
Cu(111)	93.16	92.74
Cu(100)	100.53	100.99
Cu(110)	102.39	103.92
Cu(110)mr	109.93	111.69

## Energy Profile for Cu Adatom Diffusion:

**Cu(111)**

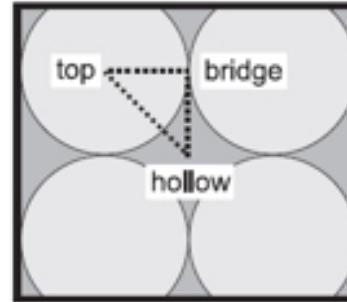


Path

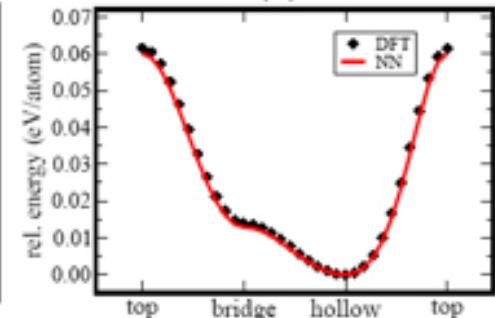


Energy profile

**Cu(100)**



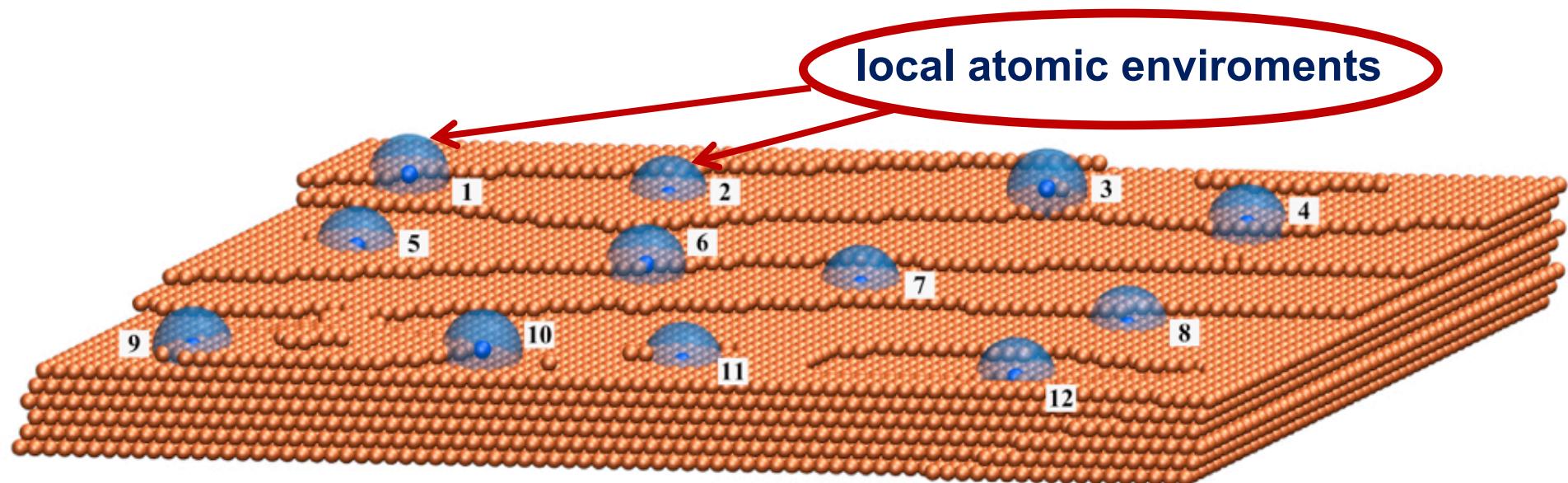
Path



Energy profile

# Cu(111): Complex Realistic Model

Model of a real surface with steps, kinks, and adatoms (29,443 atoms).

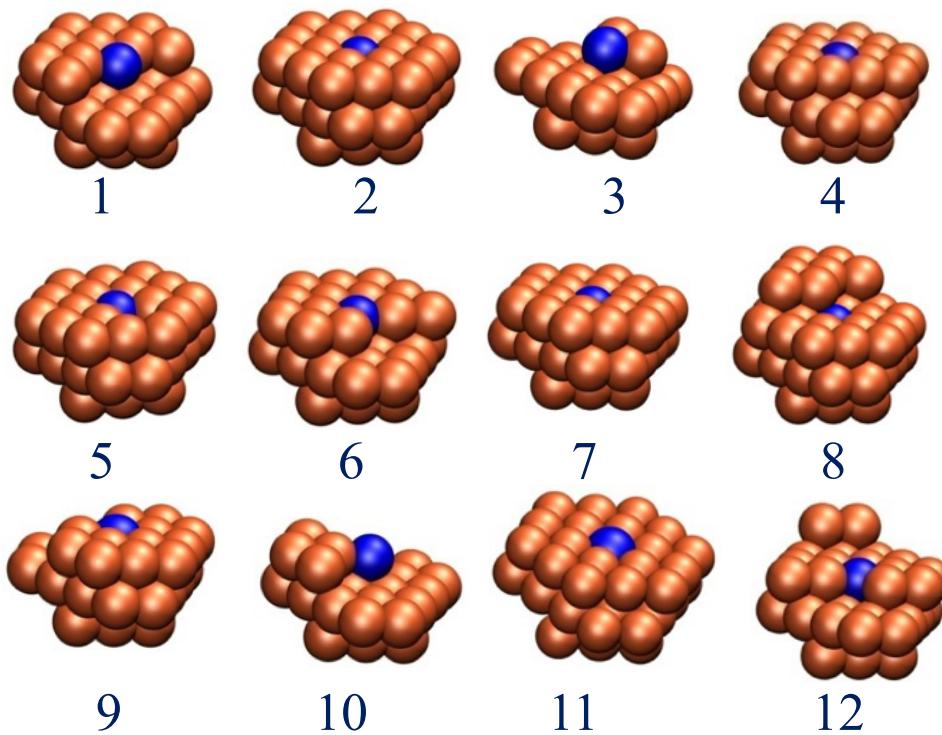


DFT  $\Rightarrow$  impossible  
NN  $\Rightarrow \approx$  few minutes (1 core)

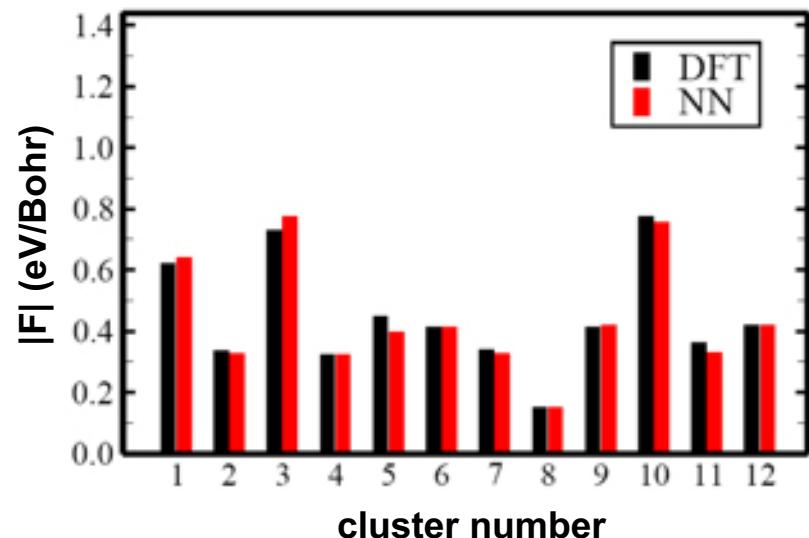
$\Rightarrow$  NN potentials can be used to study systems of this size.

# Checking the Accuracy for Large Systems

Comparison of the DFT and neural network (NN) forces acting on the central atoms of clusters cut from the slab.



Forces:



⇒ Very good agreement  
⇒ NN PES is reliable

# Cu@ZnO Catalyst for Methanol Synthesis

ANN-MD Simulation: Slab model ~8,000 atoms: NVT, MD at 1000 K

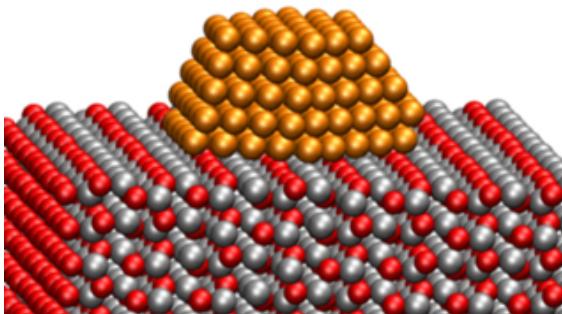
**Training and testing sets for the ANN potential:**

**Cu/Zn/O structures:** (e.g. ideal, vacancies, defects)

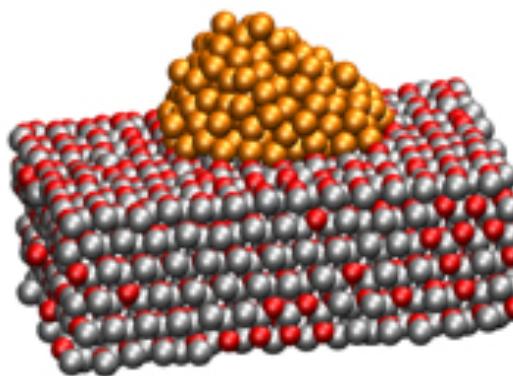
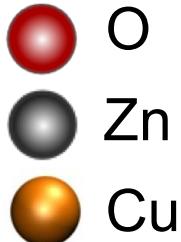
~100,000 structures (90% train, 10% test)

RMSEs  $E_{\text{total}}$ : 0.005 eV/atom

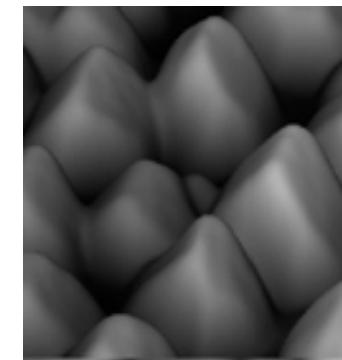
Forces: 0.090 eV/Bohr



Initial configuration/  
MD movie



Configuration at 300 ps



STM image of  
Cu@ZnO(1010), T = 290 K

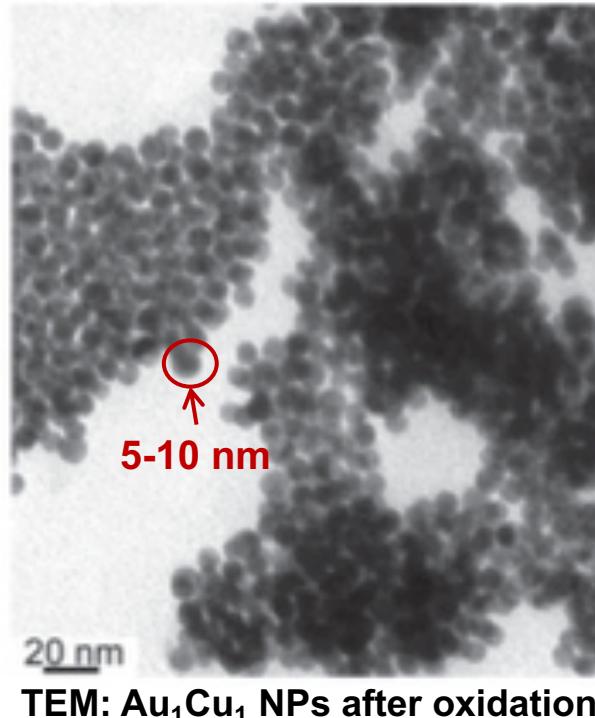
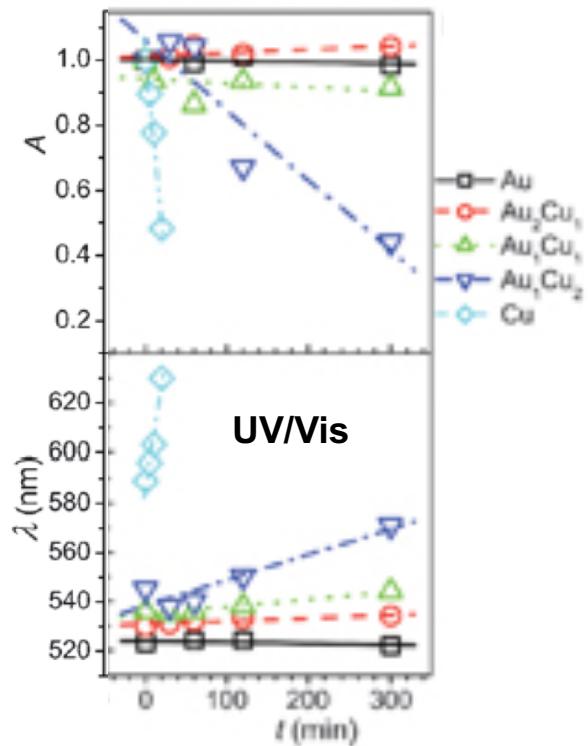
N. Artrith, B. Hiller, J. Behler  
*Phys. Stat. Sol. B* 250 (2013)  
1191 (invited feature article).

U. Köhler, et. al,  
*Phys. Status Solidi B*  
250 (2013) 1122.

- ANN potentials allow to simulate structural models with thousands of atoms while providing high accuracy close to the reference method

# ANN Potential: Cu/Au/O/H System

Au/Cu is an efficient and stable catalyst for the ORR and CO<sub>2</sub> reduction

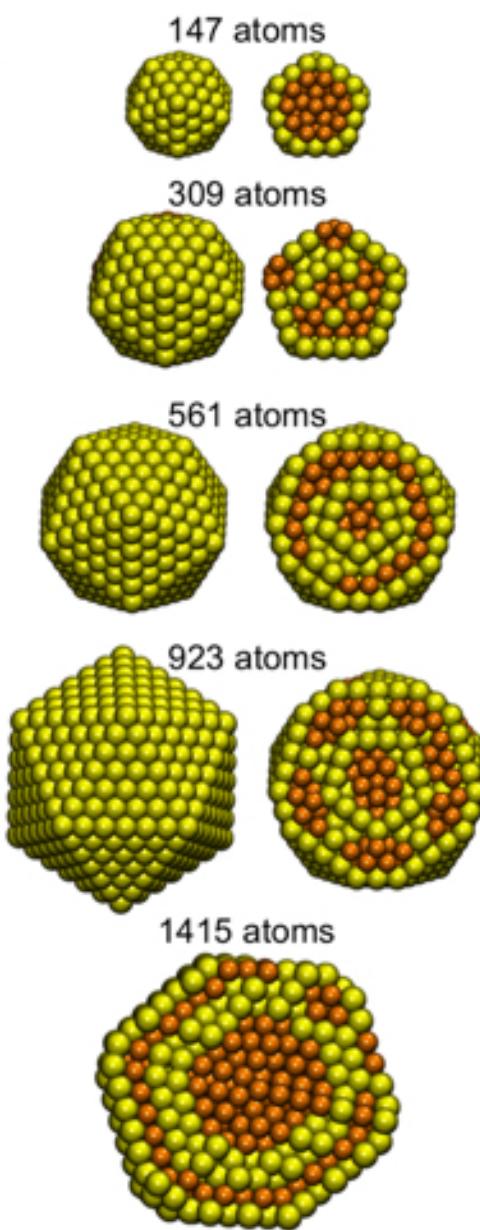
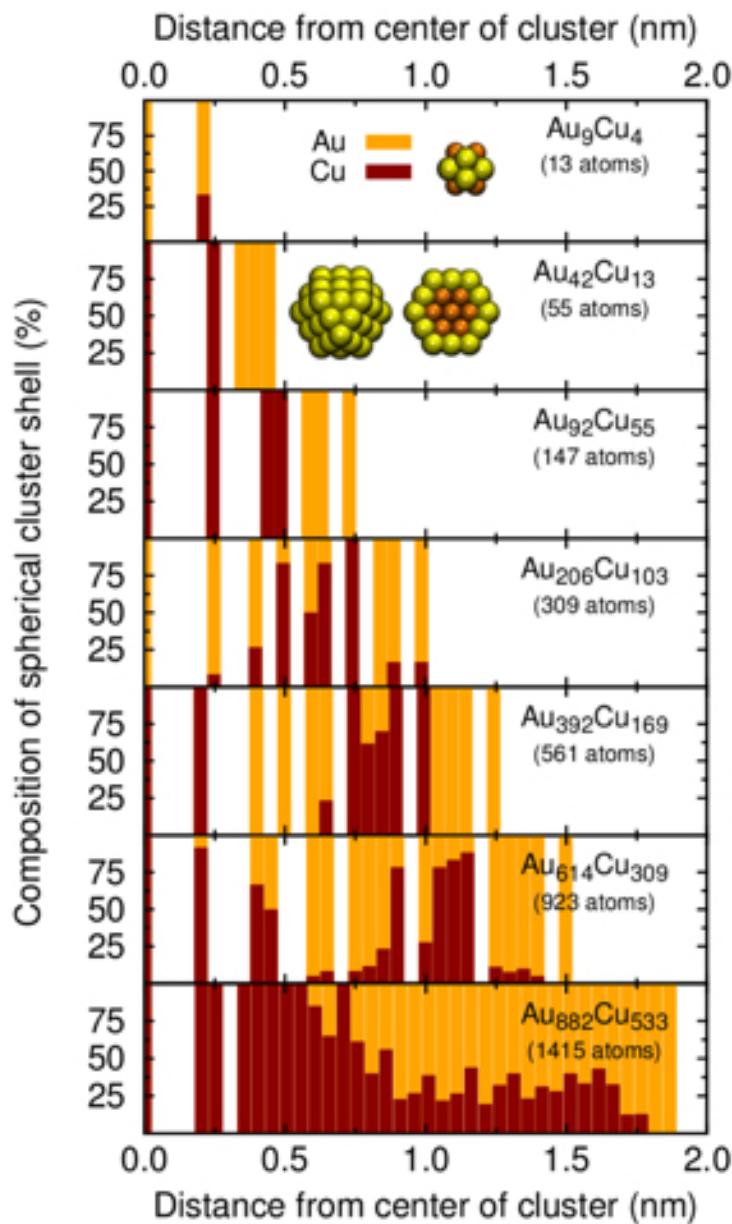


Shao-Horn,  
Hamad-Schifferli, et. al,  
*Chem. Commun.*,  
48, (2012) 5626.

**Oxidation rates of  $\text{Au}_x\text{Cu}_y$  NPs:** depend on composition,  
where  $kA$  and  $k\lambda$  exhibited a trend of  $\text{Au}_2\text{Cu}_1 < \text{Au} < \text{Au}_1\text{Cu}_1 < \text{Au}_1\text{Cu}_2 < \text{Cu}$

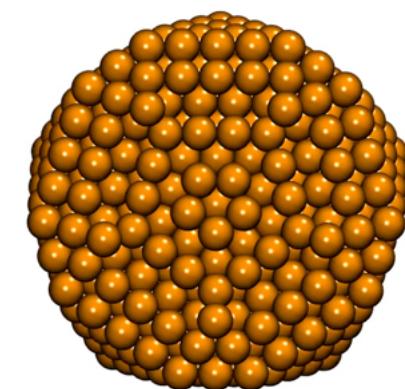
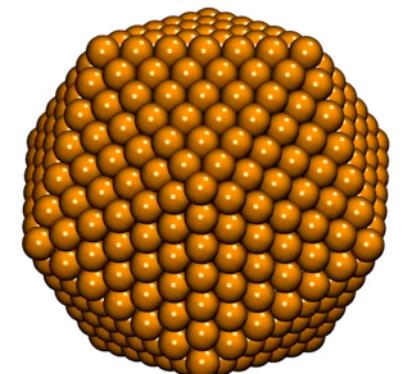
- How can we identify relevant compositions and (surface) structures?

# Optimized Compositions and Ordering of Au/Cu: MC



MC Annealing (movies):  
T = 5,000-300 K

Cluster 923 atoms

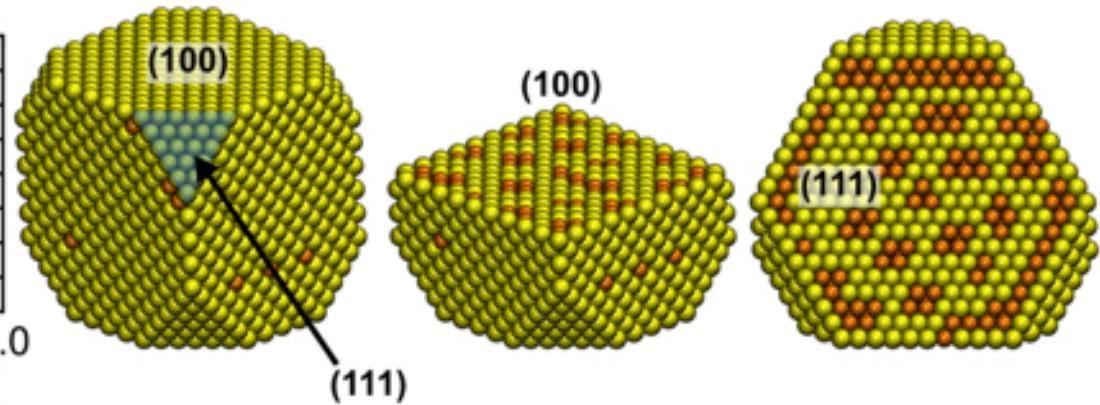
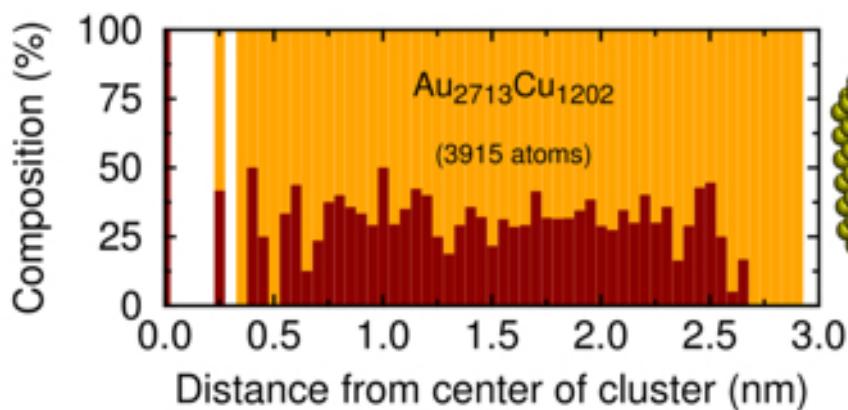


Cross-section  
Cluster 923 atoms

# Optimized Compositions and Ordering of NPs: 3,915 atoms

MC Annealing: T= 5,000-300 K

⇒ Composition of Au/Cu NP (~6 nm) and cut-through in (100) and (111) directions

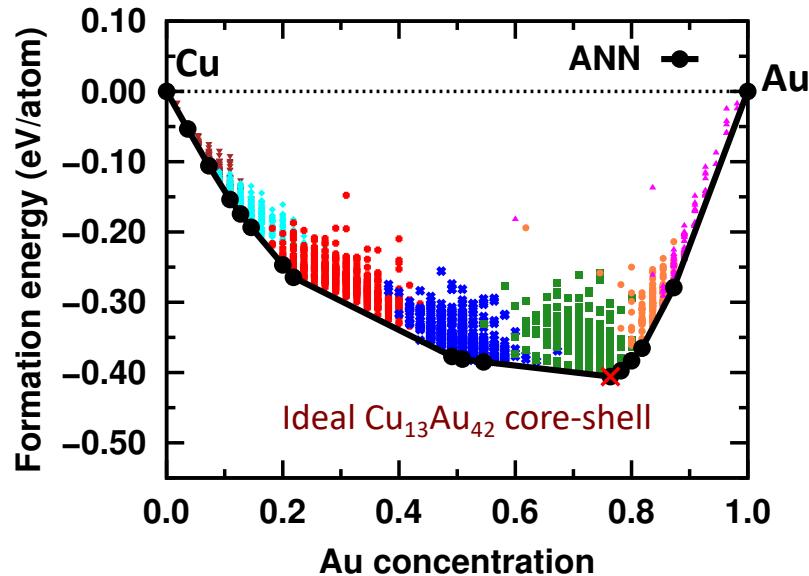


- ⇒ No longer a core-shell morphology
- ⇒ Outer layer of the NP is gold-terminated (as expected from the surface-energy)
- ⇒ Interior bulk Au and Cu atoms form a solid solution

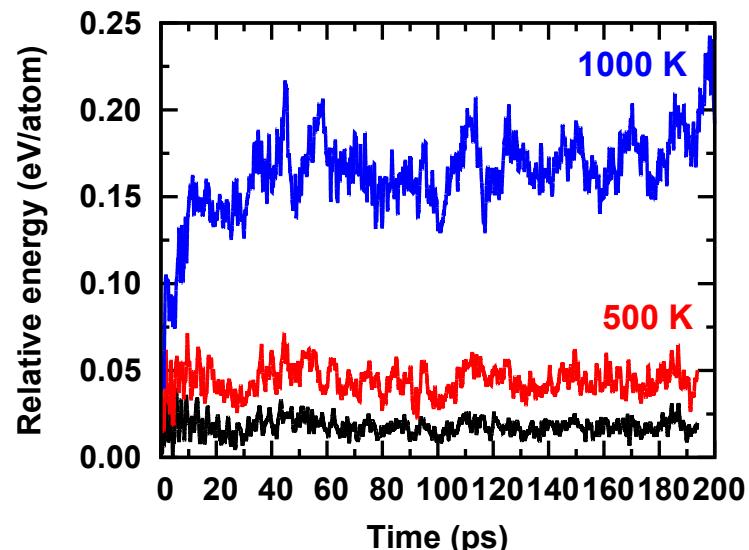
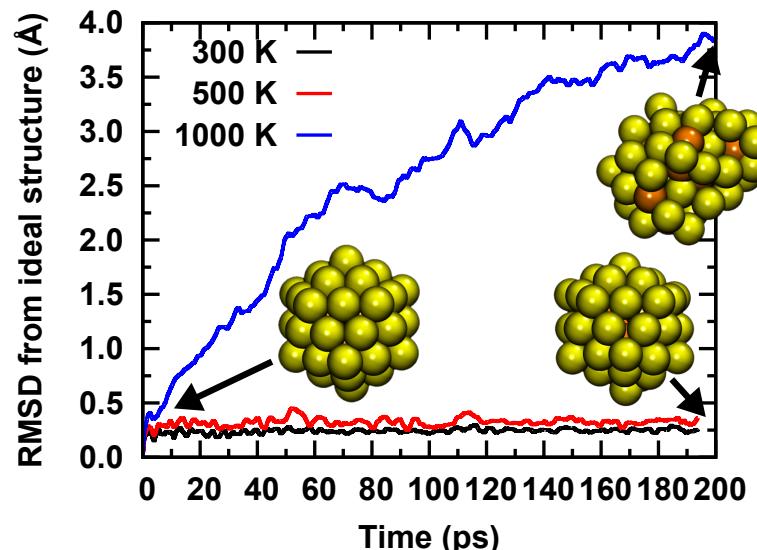
**CPU time/structure:** 147 atoms  
**NN << DFT :**  $10^4$   
**DFT :** 3.0 hours (16 cores)  
**NN :** < 1 second (1 core)

**CPU time/structure:** 3,915 atoms  
**DFT :** Very difficult  
**NN :** 59 seconds (1 core)

# Formation Energies of 55-Atom Cluster and Thermal Stability



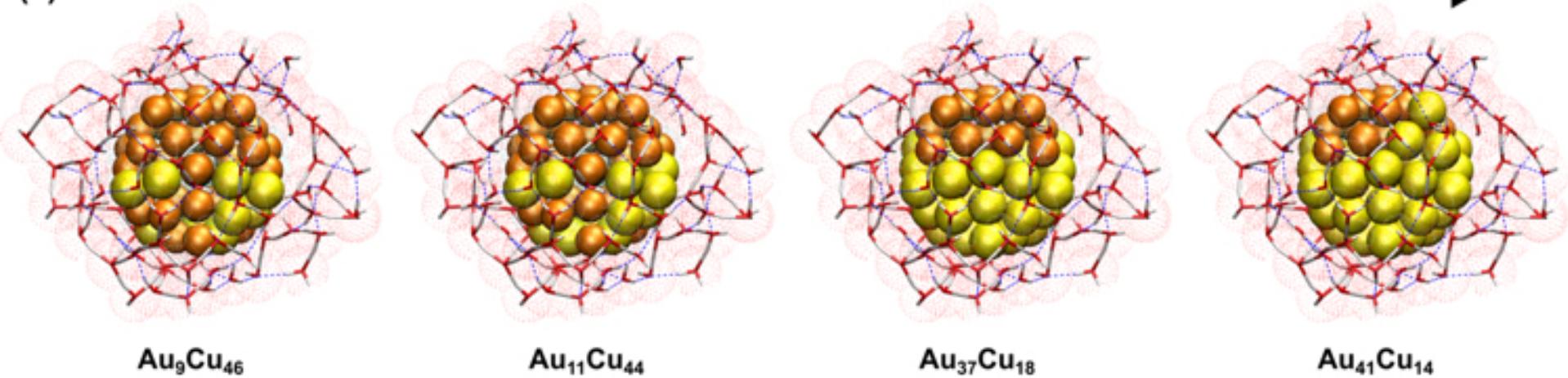
- Grand canonical MD
- Stable configurations: derived from core-shell structure
- At 1000K: cluster melts
- 300K & 500K: below melting point



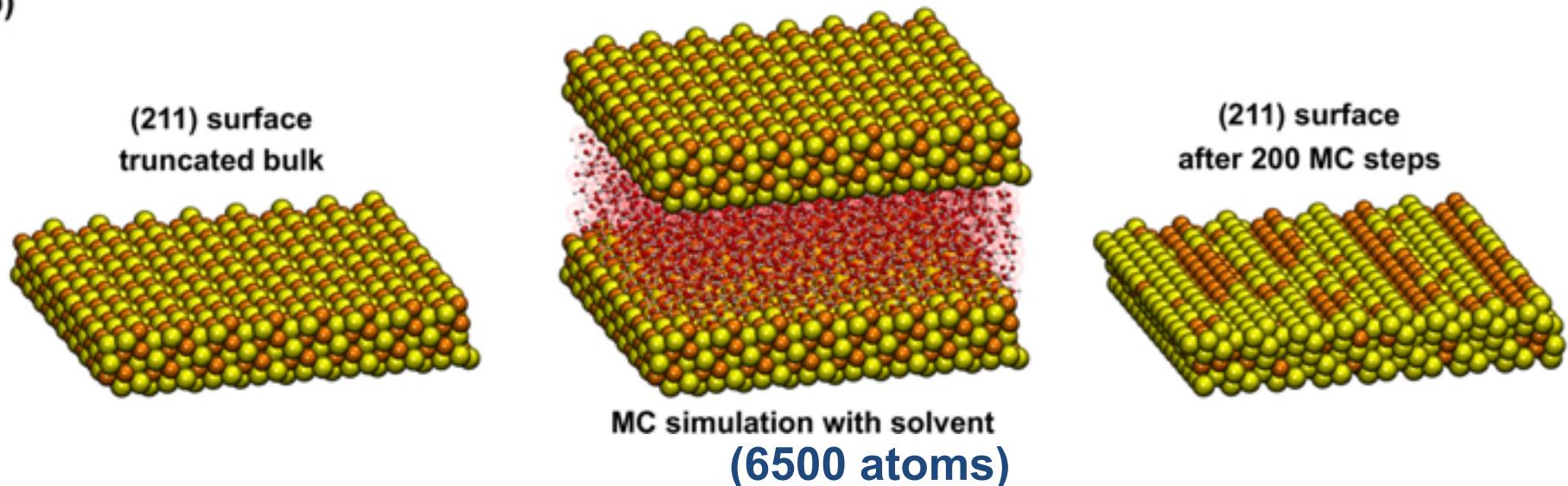
# 55-Atom Cluster in Water and (211) Surface Slab

(a)

Increasing Au chemical potential



(b)



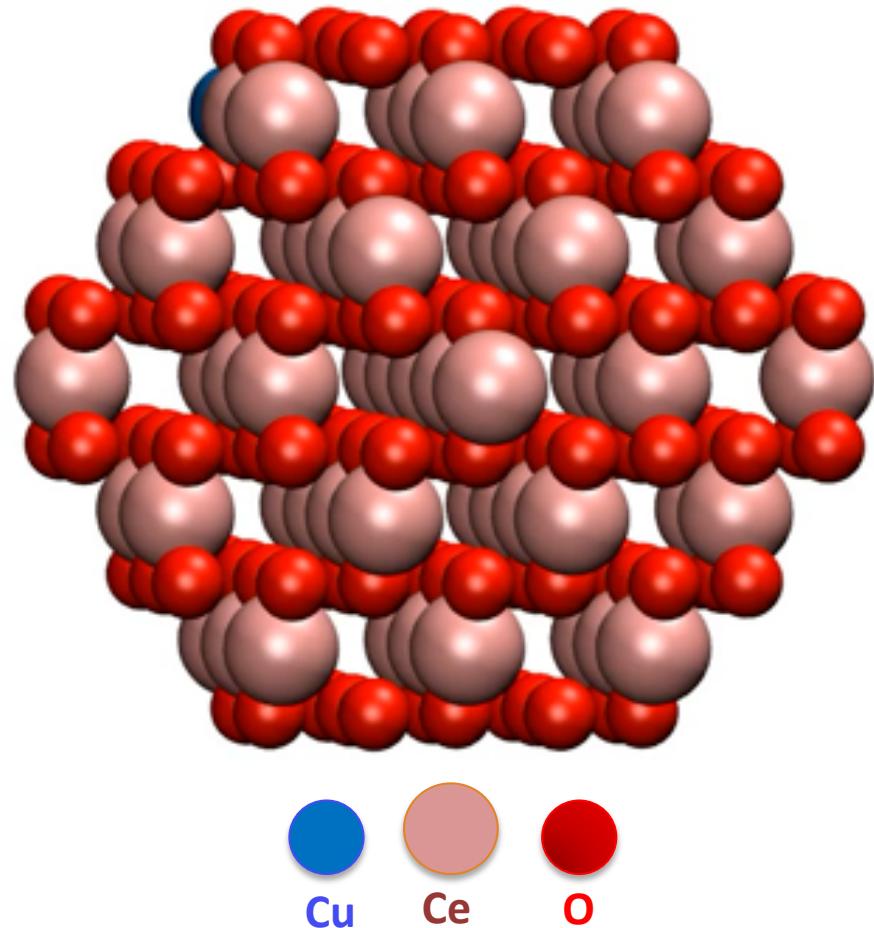
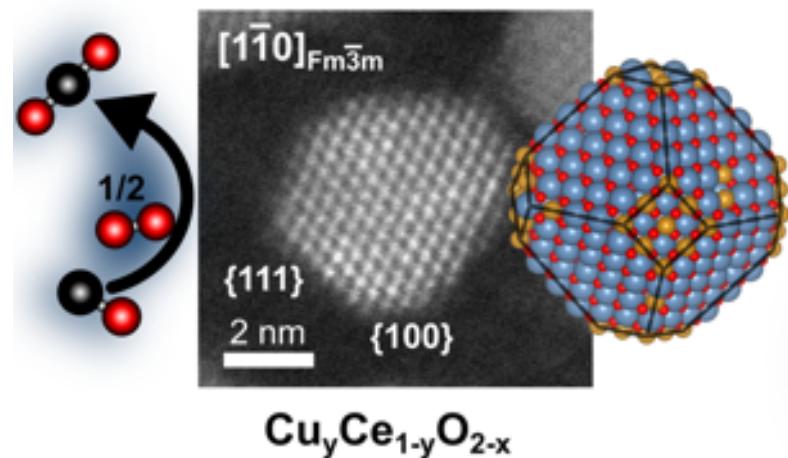
# Specialized ML Potentials for Assisting DFT Calculations

- Often, a general ML potential is not necessary
- ML potential for specific structure space sufficient for accelerated sampling

# Active Site in CuO/CeO<sub>2</sub> for CO Oxidation

## Cu Distribution in the particles

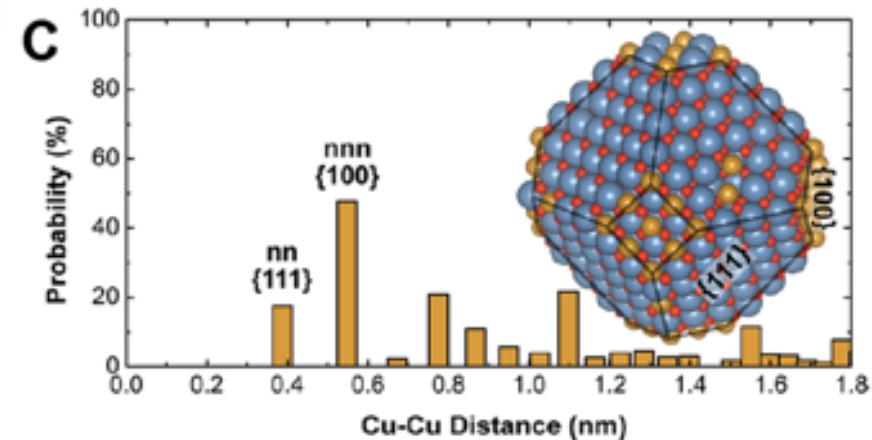
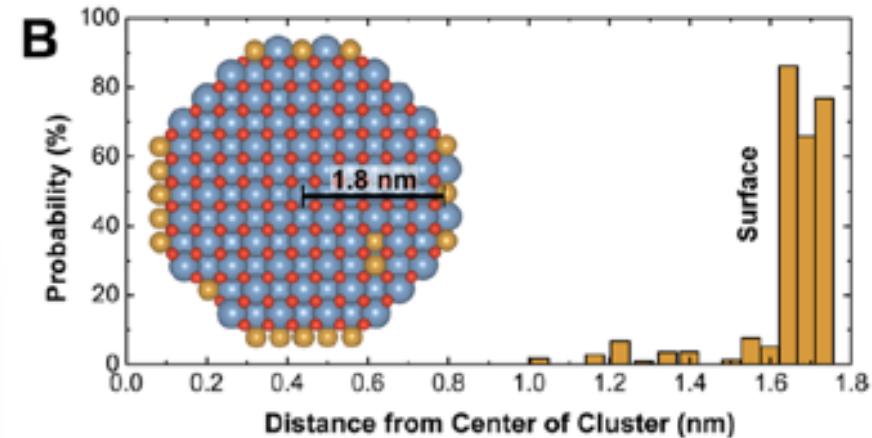
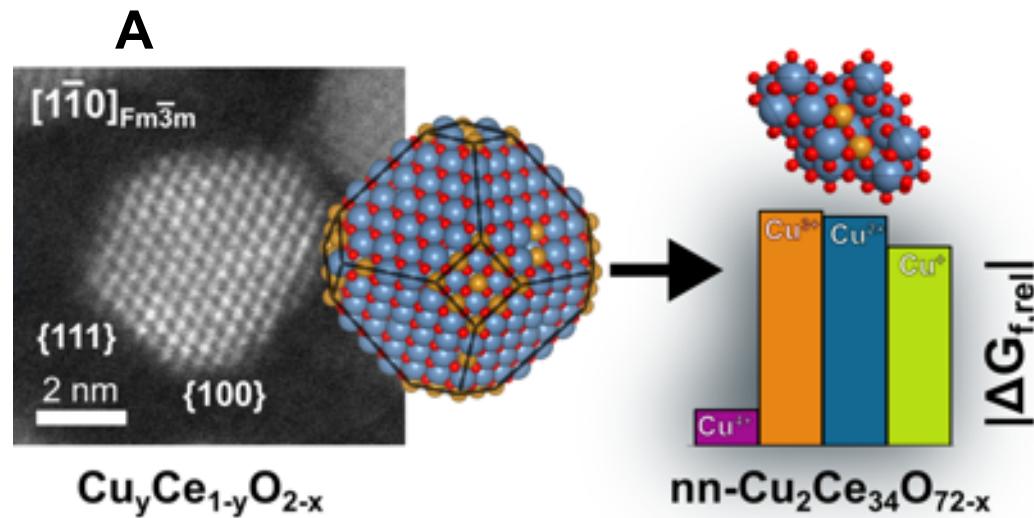
- MC simulations of a **3.5 nm (~1,300 atoms, Cu<sub>54</sub>Ce<sub>405</sub>O<sub>834</sub>)**: Cu is most stable near surface
- Cu adsorption on {100} surface and on edges favorable



# Active Site in CuO/CeO<sub>2</sub> for CO Oxidation

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- MC simulations of a **3.5 nm (~1,300 atoms, Cu<sub>54</sub>Ce<sub>405</sub>O<sub>834</sub>)**: Cu is most stable near surface
- Cu adsorption on (100) surface and on edges favorable

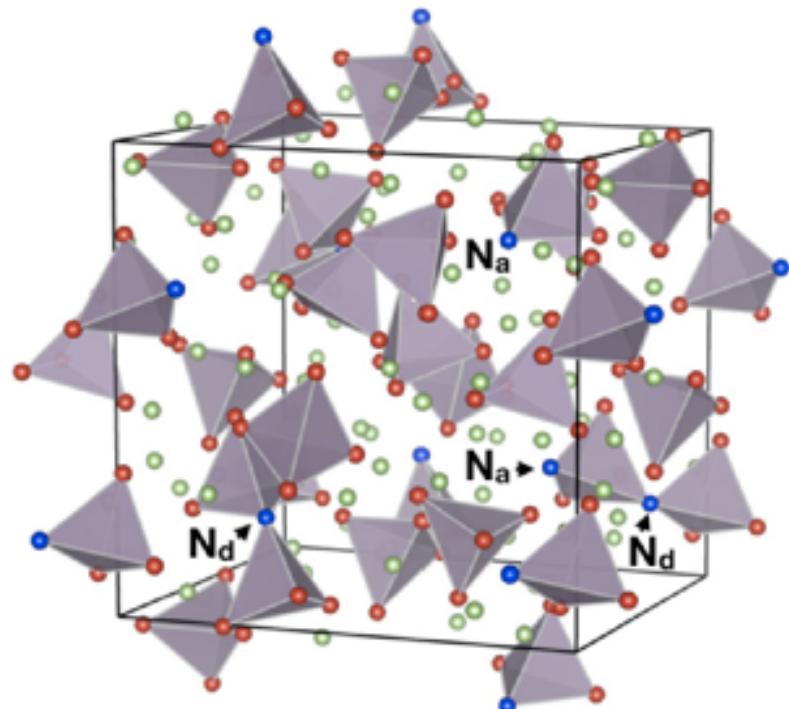


## Cu-Cu pair distribution

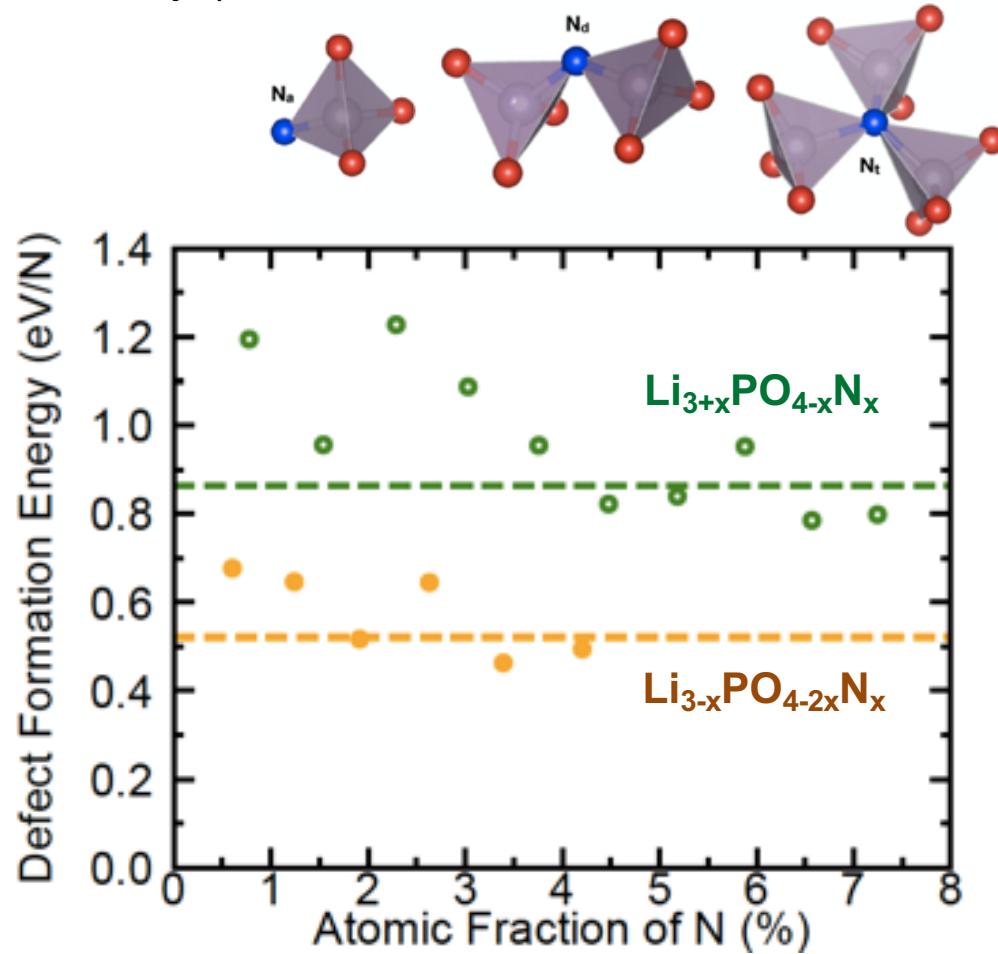
- Cu clustering: Cu-Cu pair distribution
- Combined probability that either the nearest neighbor (nn) or next-nearest neighbor (nnn) site of a Cu defect is also a Cu defect is around 70%

# MLP: Application to Amorphous LiPON

- Genetic Algorithm with MLP:
- Amorphous N doped  $\text{Li}_3\text{PO}_4$ : **Solid Electrolyte Material for Li-ion batteries**
- With N improving the conductivity and diffusivity (from MD simulations of amorphous structure models)



Diffusivity :  $\text{LiPO} = 10^{-13} \text{ cm}^2\text{s}^{-1}$   
:  $\text{LiPON} = 10^{-10} \text{ cm}^2\text{s}^{-1}$



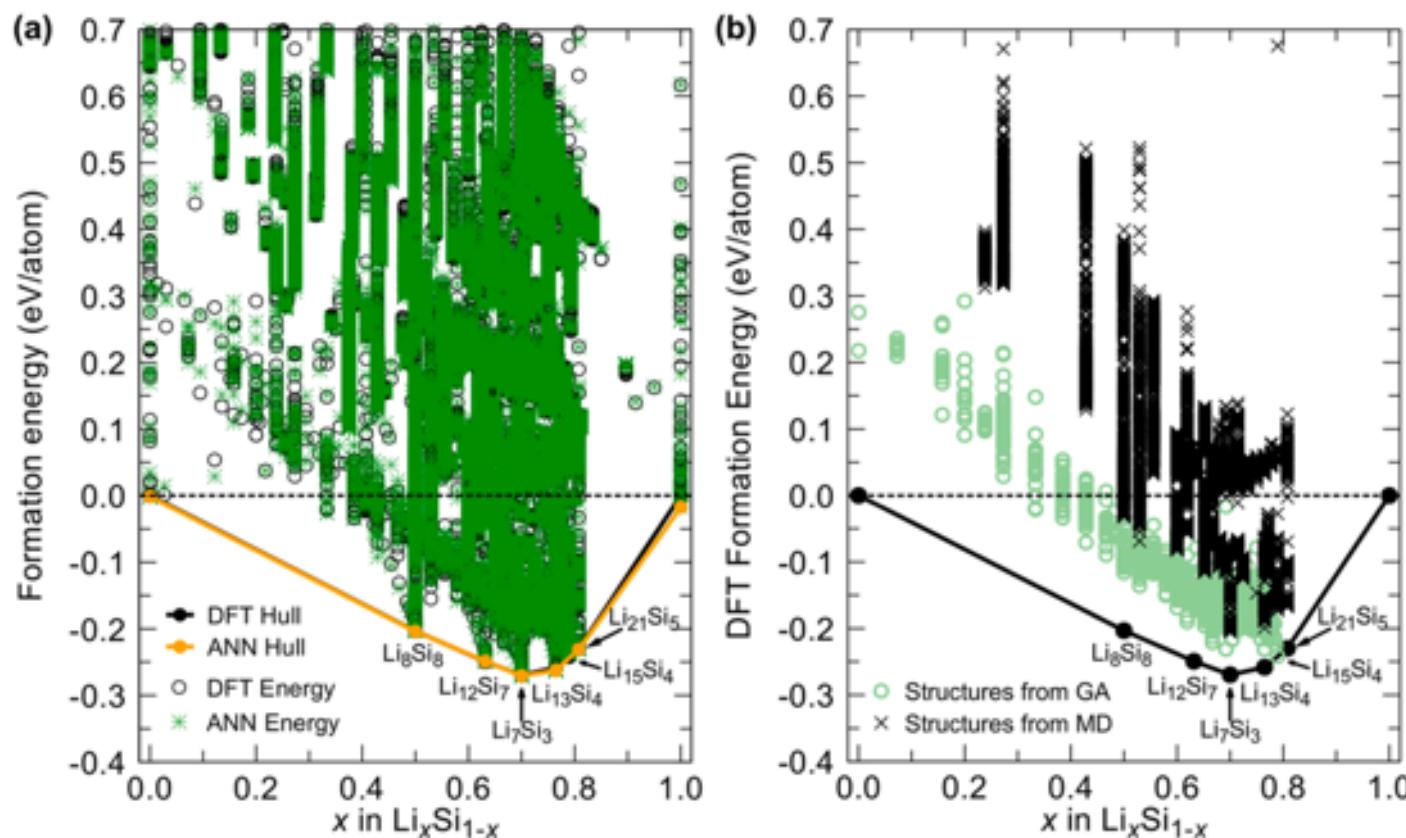
# AIMD: Application to Amorphous LiPON

- Genetic Algorithm with MLP:
- Amorphous N doped  $\text{Li}_3\text{PO}_4$ : **Solid Electrolyte Material for Li-ion batteries**
- With N improving the conductivity and diffusivity (from MD simulations of amorphous structure models)

	at. % N	# $N_a$	# $N_d$	$\rho$	D
computed					
$\text{Li}_{2.69}\text{PO}_{3.38}\text{N}_{0.31}$	4	1	4	2.04	$5 \times 10^{-10}$
$\text{Li}_{2.94}\text{PO}_{3.50}\text{N}_{0.31}$	4	2	3	2.33	$7 \times 10^{-10}$
$\text{Li}_{3.31}\text{PO}_{3.69}\text{N}_{0.31}$	4	3	2	2.30	$3 \times 10^{-12}$
$\text{Li}_{3.38}\text{PO}_{3.62}\text{N}_{0.38}$	5	6	0	2.31	$2 \times 10^{-12}$
experimental					
$\text{Li}_{2.7}\text{PO}_{3.9}^1$	0				
$\text{Li}_{2.9}\text{PO}_4^{49,50}$	0				$6 \times 10^{-13}$
$\text{Li}_{3.1}\text{PO}_{3.8}\text{N}_{0.16}^4$	2				
$\text{Li}_{3.3}\text{PO}_{3.8}\text{N}_{0.22}^4$	3				
$\text{Li}_{2.9}\text{PO}_{3.3}\text{N}_{0.46}^4$	6				

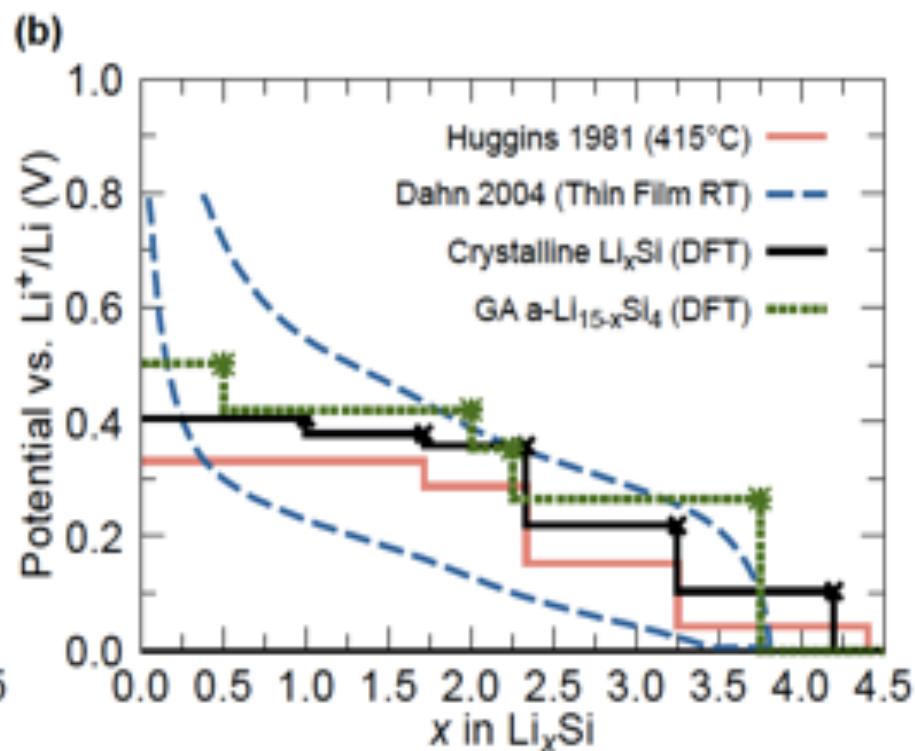
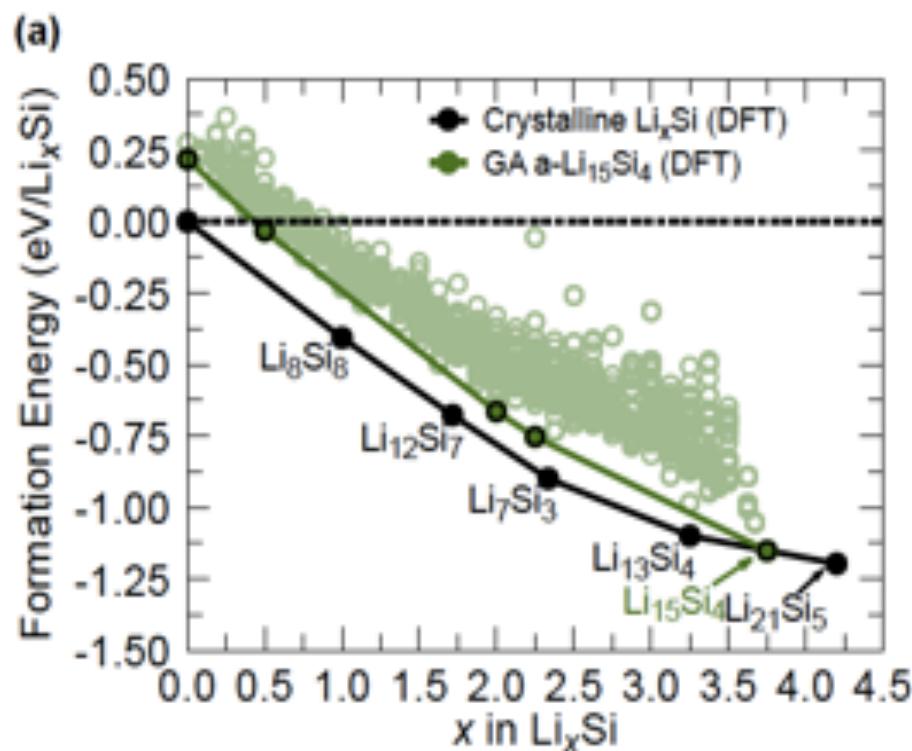
# LiSi Alloys for High-Capacity Li-Ion Battery Anodes

- A specialized ANN potential: **GA samples only a limited structure space**
- Only ~1,000 reference structures needed for the construction of a specialized ANN potential
- Identified low-energy structures are then recomputed using DFT
- The result is a first-principles phase diagram based on extensive sampling



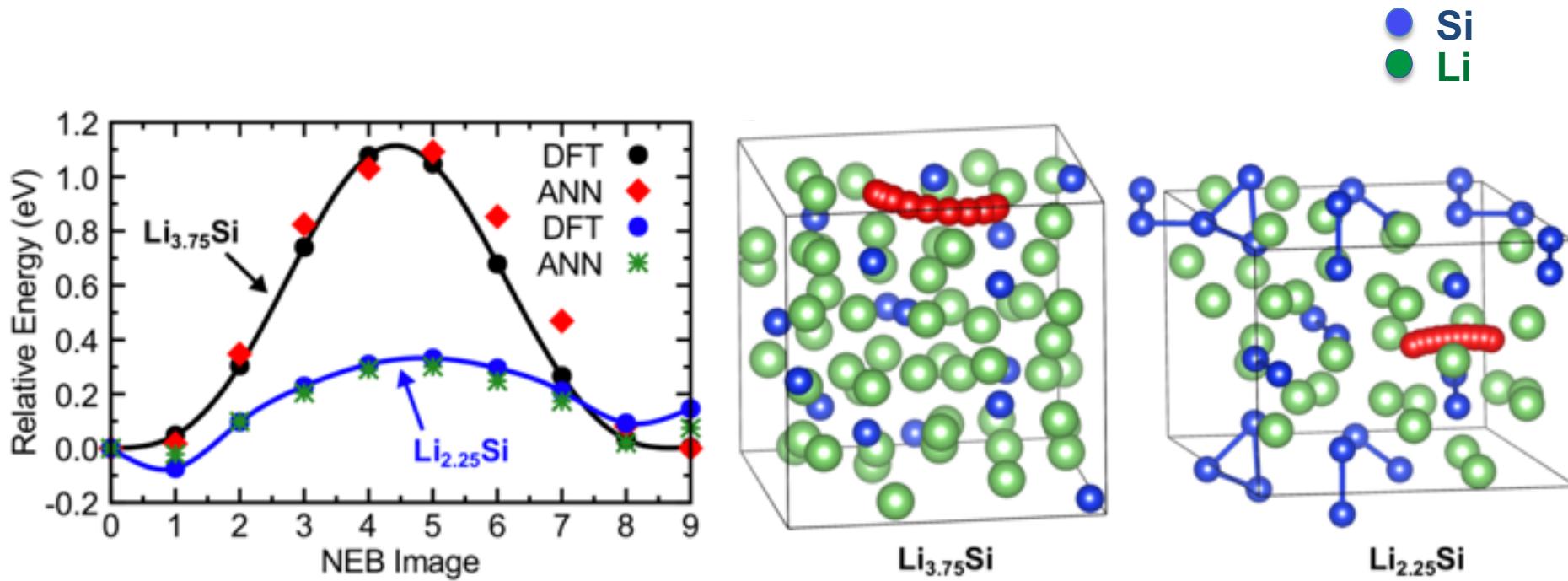
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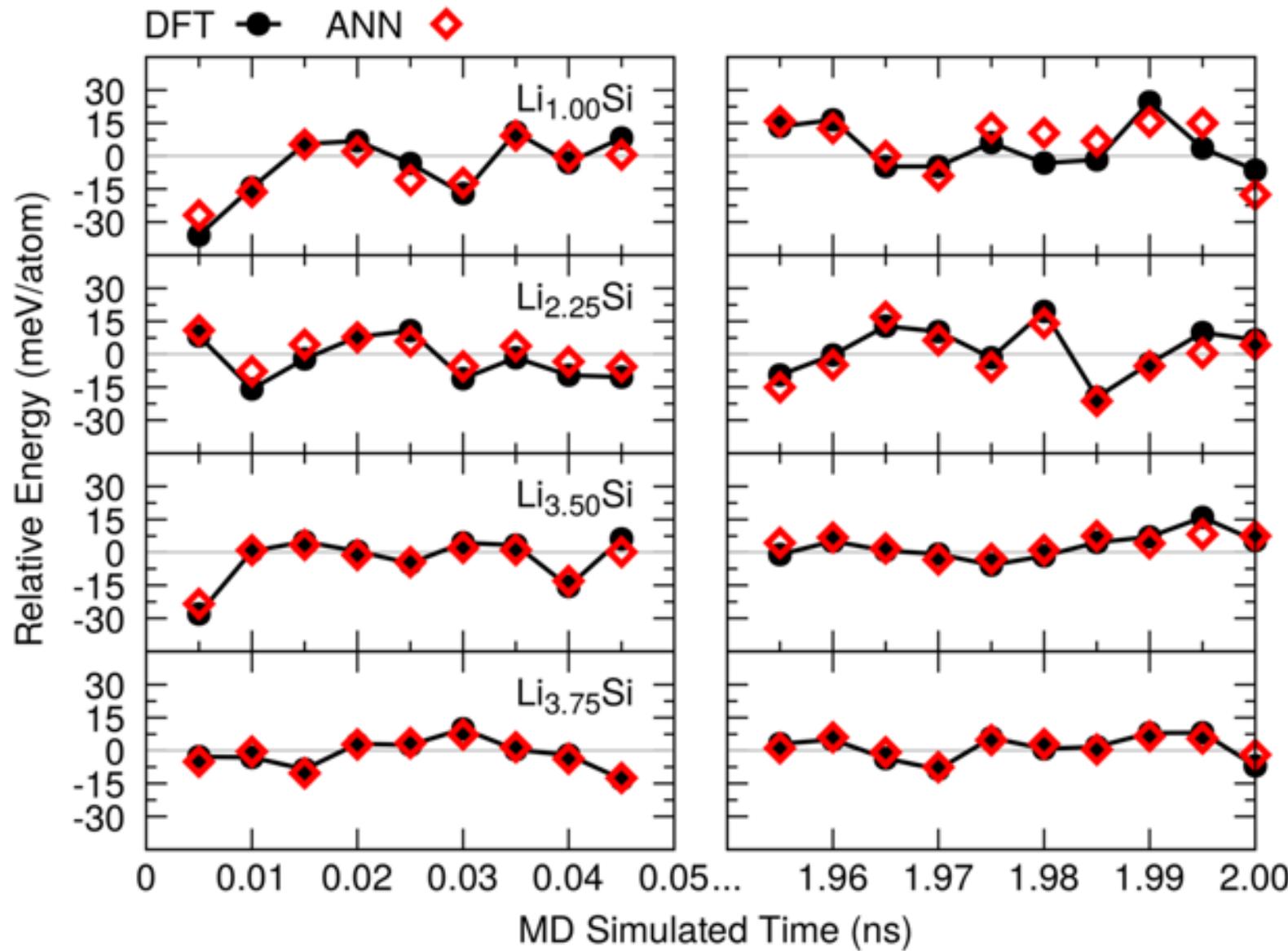
# A General Machine Learning Potential for LiSi

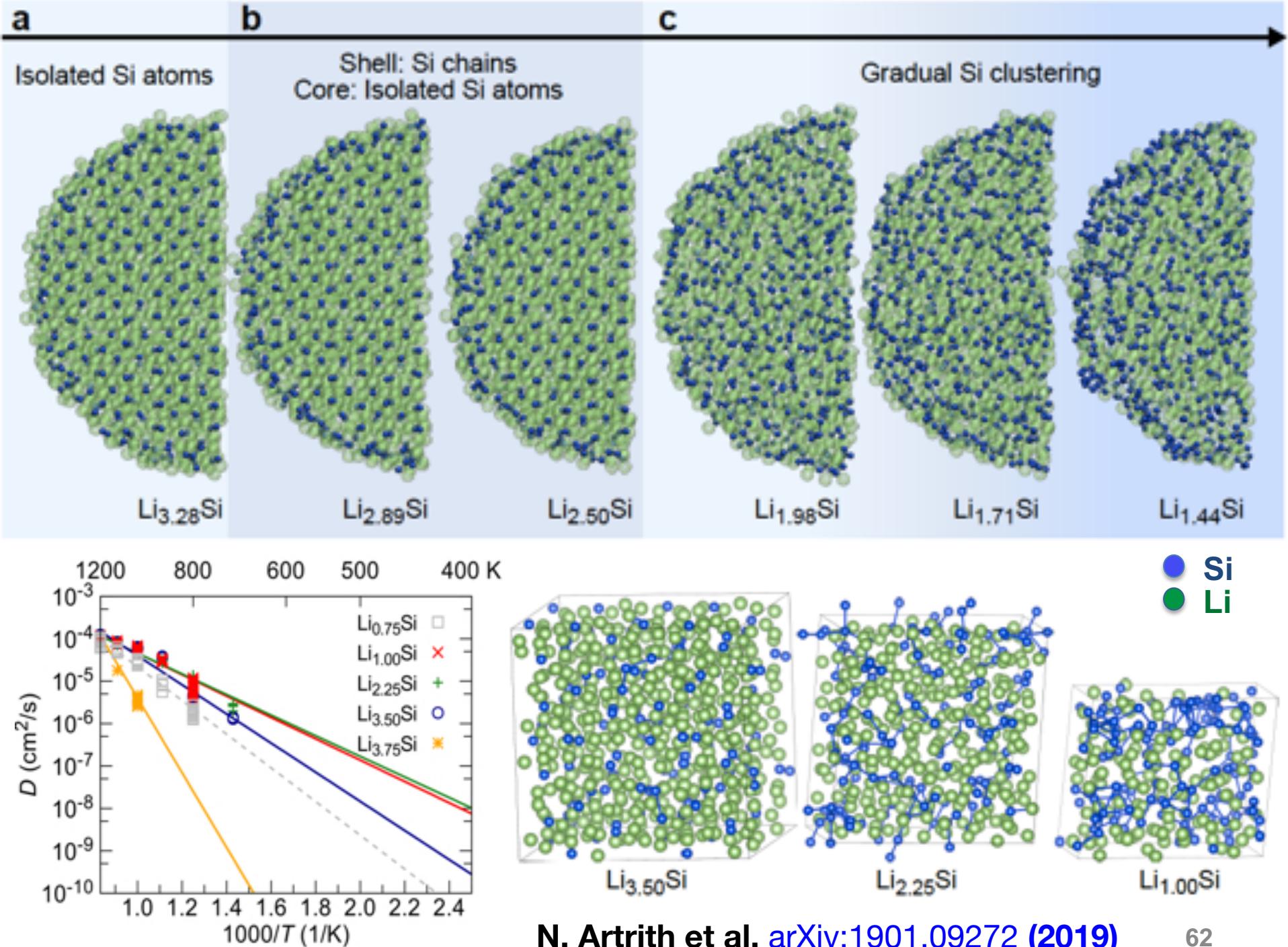
# The LiSi ANN Potential is Accurate for Diffusion



- Structures not in training set
- **Tested many different diffusion pathways in different alloy compositions**

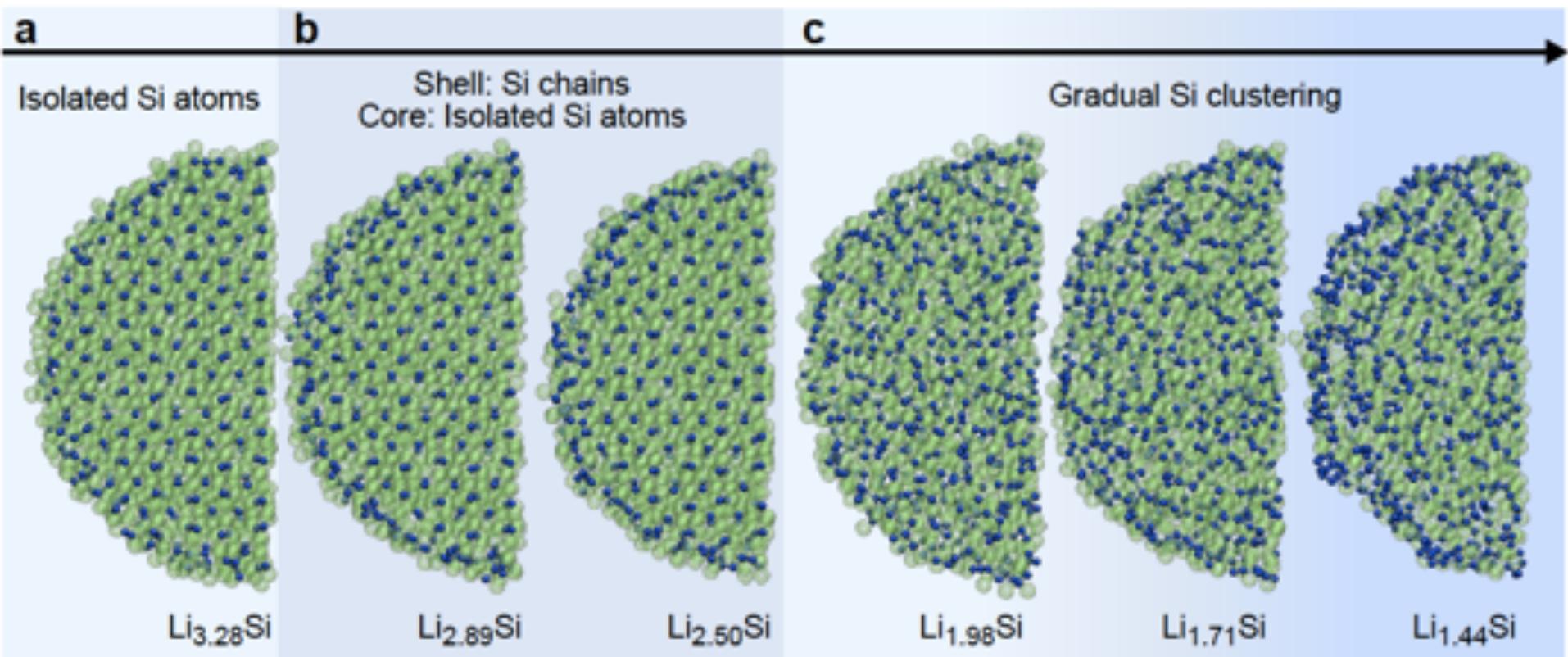
# And it is Accurate for Long MD Trajectories





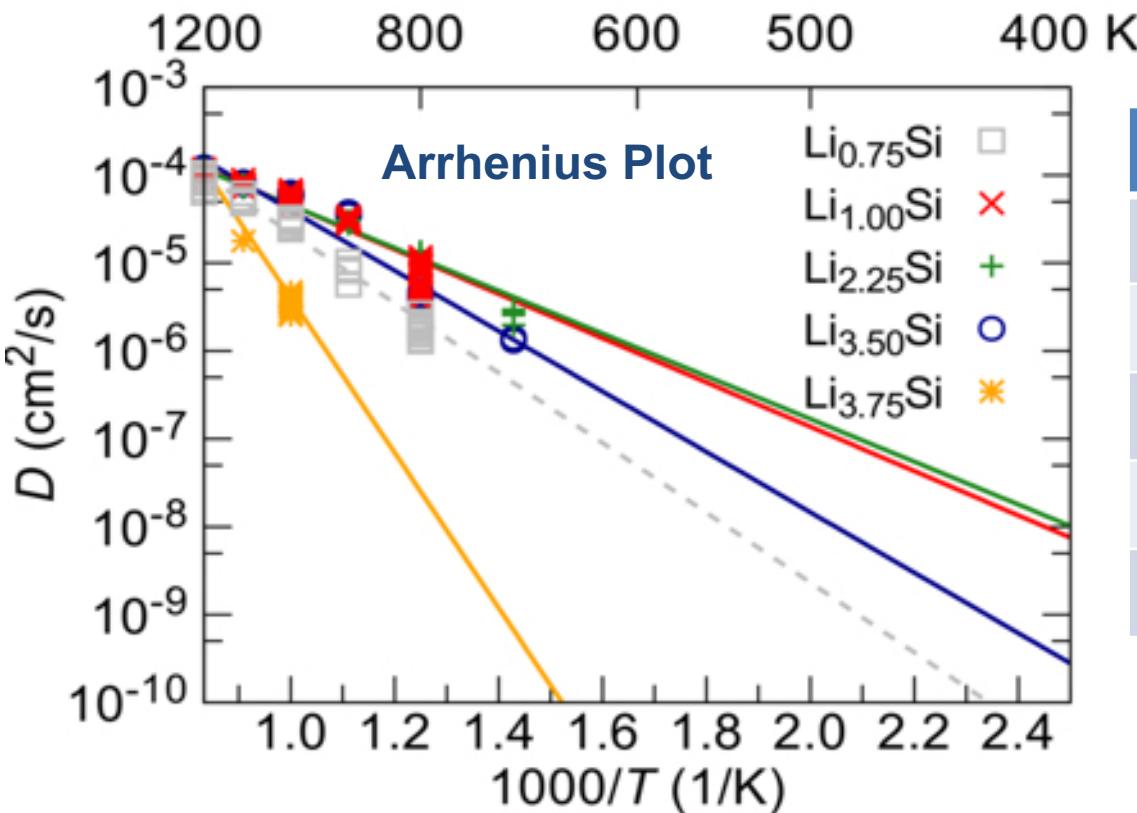
# NP Delithiation ( $d \approx 8\text{nm}$ ) Shows Si Clustering

- MD simulations, over **4 ns at 500 K** at each composition, based on nanoparticle (NP) structures with **~12,000 atoms**.
- For composition within the two-phase region a core-shell structure is most stable: in the nanoparticle bulk Si atoms are isolated and near the surface Si is clustered into short Si chains.
- Further Li extraction results in Si clustering throughout the entire NP.



# ANN MD: The Li Diffusivity Varies with Li Content

- MD simulations (5 ns) of  $\text{Li}_{480-x}\text{Si}_{128}$  structures using the ANN potential show that structures with Si clusters ( $\text{Li}_x\text{Si}$  with  $1.0 \leq x \leq 2.25$ ) exhibit the highest Li diffusivities with  $D \approx 5-10 \times 10^{-11} \text{ cm}^2\text{s}^{-1}$ .
- Isolated Si atoms ( $\text{Li}_{3.50}\text{Si}$  and  $\text{Li}_{3.75}\text{Si}$ ) and structures in which Si forms three-dimensional networks ( $\text{Li}_x\text{Si}$  with  $x < 1.0$ ) exhibit much lower diffusivities of  $D < 5 \times 10^{-13} \text{ cm}^2\text{s}^{-1}$ .



$x_{\text{Li}}$	$E_a$ (eV)	$D$ ( $\text{cm}^2\text{s}^{-1}$ )
0.75	0.789	$1.154 \times 10^{-14}$
1.00	0.500	$5.986 \times 10^{-11}$
2.25	0.483	$9.607 \times 10^{-11}$
3.50	0.682	$3.820 \times 10^{-13}$
3.75	1.750	$0.107 \times 10^{-25}$

N. Artrith et al.  
[arXiv:1901.09272 \(2019\)](https://arxiv.org/abs/1901.09272)

# ANN MD: The Li Diffusivity Varies with Li Content

## Experimental References

D (cm <sup>2</sup> /s)	Method	Reference
10 <sup>-10</sup>	EIS	[1]
10 <sup>-12</sup>	CV, EIS, GITT	[2]
10 <sup>-14</sup>	EIS, PITT	[3]
10 <sup>-14</sup> –10 <sup>-13</sup>	PITT	[4]

## ANN Results

x <sub>Li</sub>	E <sub>a</sub> (eV)	D (cm <sup>2</sup> s <sup>-1</sup> )
0.75	0.789	1.154 × 10 <sup>-14</sup>
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- [1] R. Ruffo, S. S. Hong, C. K. Chan, R. A. Huggins, and Y. Cui, *J. Phys. Chem. C* **113**, 11390 (2009).
- [2] N. Ding, J. Xu, Y. Yao, G. Wegner, X. Fang, C. Chen, and I. Lieberwirth, *Solid State Ionics* **180**, 222 (2009).
- [3] J. Xie, N. Imanishi, T. Zhang, A. Hirano, Y. Takeda, and O. Yamamoto, *Mater. Chem. Phys.* **120**, 421 (2010).
- [4] J. Li, X. Xiao, F. Yang, M. W. Verbrugge, and Y.-T. Cheng, *J. Phys. Chem. C* **116**, 1472 (2012).

# Summary Part II – ANN Potentials in Applications

- ANN potentials are a versatile tool for the modeling of complex materials such as amorphous alloys and non-ideal oxides
- With new improved structure descriptors, the method can now be used with compositions with more than 10 chemical species
- Training accurate ANN potentials for general applications may require large reference libraries (>10,000 structures), but often specialized potentials for smaller configuration spaces are sufficient if combined with DFT



# Outlook

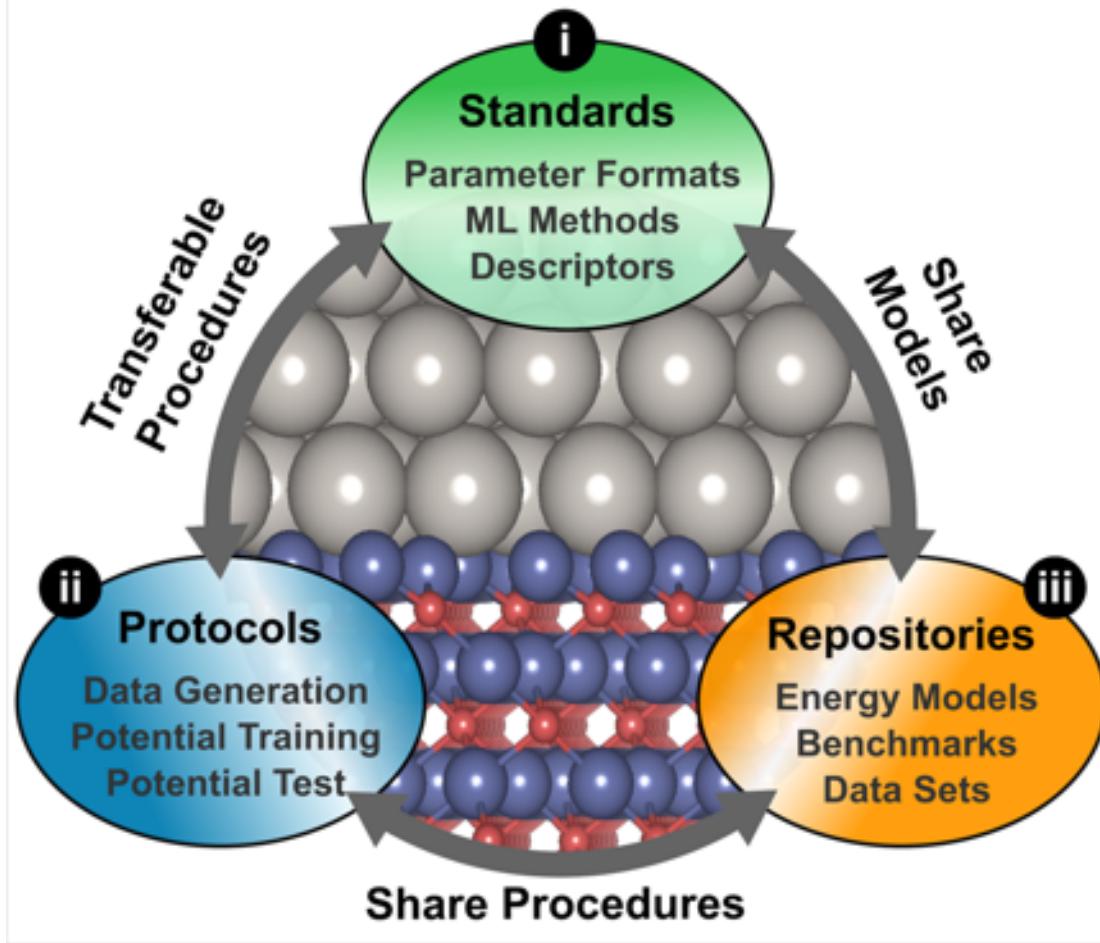
- **More systematic tests needed** to understand for which materials and applications the MLP method is successful
- Now that several implementations exist, we should have **a transferable format for ANN potentials** (collaborate on standardized format)
- How to **sample large chemical and configuration spaces** in the most efficient way?
- Including a **physically motivated baseline** could reduce the size of the reference library needed

# Outlook

Some more steps required for ANN potentials to become a standard tool (like other potentials):

- **Interfaces with standard simulation software** are needed
  - aenet interfaces in development:  
ASE, Tinker, DL\_POLY, LAMMPS, PIMD
- Implementations have to become compatible so that ANN potentials can be shared
- **Model construction (training) has to be made easier**
- **ANN potential parameter formats should be standardized**

# Outlook



**N. Artrith, J. Phys. Energy (2019) just accepted (invited review)**

# Acknowledgements

## Columbia University, USA

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## The Kolpak group, MIT, USA

: Faculty for the Future Fellowship (Schlumberger Foundation)

## The Behler group, RUB, Germany

: DFG SFB 558 Collaborative Project

## Computational resources

: Extreme Science and Engineering Discovery Environment (**XSEDE**)

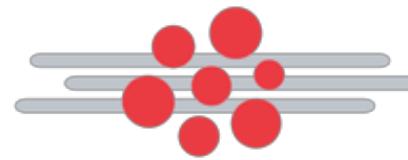
: Brookhaven National Laboratory (**CFN Computer Cluster**)



**COLUMBIA  
UNIVERSITY**



<http://ann.atomistic.net>



**Center for Functional Nanomaterials**  
Brookhaven National Laboratory



National Energy Research  
Scientific Computing Center

# Acknowledgements

## Collaborators:

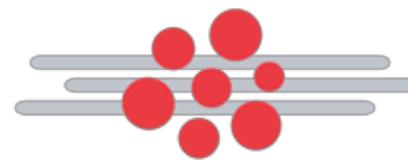
- Dr. Tobias Morawietz, Michael Chen, Prof. Tom Markland (Stanford)
- Dr. Alexander Kaiser, Prof. Michael Probst (Innsbruck University)
- April Cooper and Prof. Johannes Kästner (Stuttgart University)
- And others



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